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ELECTRICAL ENGINEERS' HANDBOOK

Electric Power

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ELECTRICAL ENGINEERS' HANDBOOK

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IV. Electric Power

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PUBLISHER'S PREFACE

In making plans for new editions of our handbooks in mechanical engineering and in electrical engineering, it soon became clear that engineering science and practice had developed to such an extent that handbooks were growing beyond all practical bounds. They had become both bulky and inconvenient and contained much duplicated material. In order to solve the problems presented by these conditions, the editors of our various handbooks were asked to serve as an advisory editorial board.

This board recommended, first, that the fundamental material underlying all engineering be published in a separate volume, and, second, that the existing handbooks as they are revised be issued in several volumes containing material closely related to the specialized branches of engineering. As a result of these recommendations, the Wiley Engineering Handbook Series has been initiated, which in the beginning will comprise the following: Eshbach's "Handbook of Engineering Fundamentals"; Kent's "Mechanical Engineers' Handbook" in two volumes, viz., "Power" and "Design and Shop Practice"; Pender's "Electrical Engineers' Handbook" in two volumes, viz., "Electric Power" and "Communication and Electronics."

This division has also made it possible to devote more space to the various topics so that the entire new series of handbooks contains more complete information on all topics than heretofore has been possible. It is our hope that this new plan will give engineers information that is more useful, more complete, and in more convenient form.

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GENERAL TABLE OF CONTENTS

Detailed tables of contents are given at the beginning of each section. An alphabetical index appears following Section 19.

SECTION 1. MATHEMATICS, UNITS, AND SYMBOLS

	PAGE
Mathematics	1-02
Mathematical Tables	1-18
Units and Conversion Factors	1-36
Symbols and Abbreviations	1-64
Constants	1-70

SECTION 2. PROPERTIES OF MATERIALS

Weights of Materials	2-02
Conductor Materials	2-05
Insulating Materials	2-12
Magnetic Materials	2-38
Thermal Properties of Materials	2-49

SECTION 3. ELECTRIC CIRCUITS AND ELECTRIC LINES

Alternating Currents	3-02
Linear Circuits	3-09
Non-linear Circuits	3-19
Polyphase Systems and Symmetrical Components	3-29
Electrical Machines	3-41
Transmission Lines	3-45
Stability of Power Systems	3-49

SECTION 4. RESISTORS, REACTORS, MAGNETS

Resistors, Rheostats	4-02
Capacitors—Condensers	4-08
Inductors	4-16
Magnets, Magnetic Devices	4-26

SECTION 5. MEASUREMENTS AND MEASURING APPARATUS

SECTION 6. PRINCIPLES OF ELECTROCHEMISTRY

SECTION 7. BATTERIES

Primary Batteries	7-02
Lead-acid Storage Batteries	7-08
Alkaline-type Storage Batteries	7-24

SECTION 8. DIRECT-CURRENT MACHINES

Direct-current Generators	8-02
Direct-current Motors	8-30

SECTION 9. ALTERNATING-CURRENT MACHINES

Alternating-current Generators	9-02
Synchronous Motors	9-28
Polyphase Induction Motors	9-36
Single-phase Induction Motors	9-58
Alternating-current Commutator Motors	9-60

SECTION 10. TRANSFORMERS

SECTION 11. CONVERTERS AND RECTIFIERS

Energy Converters	11-02
Frequency Converters	11-13
Phase Converters and Balancers	11-17
Rectifiers	11-18

SECTION 12. SWITCHING, CONTROL, AND PROTECTION

	PAGE
Circuit Elements	12-02
Circuit Breakers	12-11
Controllers and Regulators	12-29
Protection	12-42
Station Equipment	12-49
Station Structures and Layout	12-60

SECTION 13. POWER STATIONS AND SUBSTATIONS

Steam Power Stations	13-02
Hydroelectric Power Stations	13-14
Internal-combustion Power Stations	13-29
Substations	13-36
Power Station Circuits	13-46

SECTION 14. POWER TRANSMISSION AND DISTRIBUTION

Systems	14-03
Electrical Design of Transmission and Distribution Systems	14-12
Capacitance and Inductance Tables	14-37
Pole and Tower Lines	14-54
Underground Lines	14-119
Distribution	14-132
Bare Wires and Cables	14-143
Insulated Wires and Cables	14-171
Wiring of Buildings	14-226
Grounding of Electric Circuits and Accessories	14-258
Electrolysis of Underground Structures	14-267

SECTION 15. LIGHTING AND HEATING

Lighting	15-02
Household Heating Appliances and Cooking	15-44
Water Heating	15-46
Heating and Air Conditioning of Buildings	15-50

SECTION 16. INDUSTRIAL APPLICATIONS OF MOTORS

SECTION 17. TRANSPORTATION

Electric Traction	17-02
Electrical Equipment of Internal-combustion Automobiles	17-72
Electric Automobiles	17-101
Electric Propulsion of Ships	17-108

SECTION 18. ELECTROCHEMICAL AND ELECTROTHERMAL INDUSTRIES

Electrochemical Processes	18-02
Electric Furnaces and Their Products	18-12
Electric Resistance Heating	18-22
Electrical Precipitation of Suspended Particles	18-31
Electric Welding	18-34

SECTION 19. ELECTRICITY ON THE FARM

For contents of other handbooks of this series, see pages following Index of this volume.

This book is divided into sections, each section carrying its independent sequence of page numbers. For example, 3-15 indicates Section 3, page 15.

SECTION 1

MATHEMATICS, UNITS, AND SYMBOLS

MATHEMATICS

By Carl C. Chambers

1. Algebraic Formulas 02
2. Complex Quantities 06
3. Trigonometric Formulas 07
4. Exponential and Hyperbolic Formulas. 09
5. Calculus Formulas 12
6. Differential Equations 13
7. Errors of Observation 14
8. Approximations 15
9. Series 16
10. Mensuration 16

MATHEMATICAL TABLES

11. Common and Natural Logarithms of Numbers 18
12. Trigonometric Tables 20
13. Exponential and Hyperbolic Tables... 25

UNITS AND CONVERSION FACTORS

By J. G. Brainerd and Carl C. Chambers

14. Systems of Units 36
15. Conversion Tables 41

Table
1. Length (L) 41
2. Area (L^2) 42
3. Volume (L^3) 43
4. Plane Angle (No Dimensions). 45
5. Solid Angle (No Dimensions). 45
6. Time (T) 45
7. Linear Velocity (LT^{-1}) 46
8. Angular Velocity (T^{-1}) 47
9. Linear Acceleration (LT^{-2})... 47
10. Angular Acceleration (T^{-2})... 47
11. Mass (M) and Weight 48
12. Density or Mass per Unit Volume (ML^{-3}) 49
13. Force (MLT^{-2}) or (F) 49
14. Torque or Moment of Force (ML^2T^{-2}) or (FL) 50
15. Pressure or Force per Unit Area ($ML^{-1}T^{-2}$) or (FL^{-2}) 50
16. Energy, Work and Heat (ML^2T^{-2}) or (FL) 51
17. Power or Rate of Doing Work (ML^2T^{-3}) or (FLT^{-1}).... 52
18. Quantity of Electricity and Dielectric Flux (Q) 52
19. Charge per Unit Area and Dielectric Flux Density (QL^{-2}) 53
20. Electric Current (QT^{-1})..... 53
21. Current Density ($QT^{-1}L^{-2}$).. 53
22. Electric Potential and Electromotive Force ($MQ^{-1}L^2T^{-2}$) or ($FQ^{-1}L$) 54
23. Electric Field Intensity and Potential Gradient ($MQ^{-1}LT^{-2}$) or (FQ^{-1})... 54
24. Electric Resistance ($MQ^{-2}L^2T^{-1}$) or ($FQ^{-2}LT$) 55
25. Electric Resistivity ($MQ^{-2}L^3T^{-1}$) or ($FQ^{-2}L^2T$) 55
26. Electric Conductivity ($M^{-1}Q^2L^{-3}T$) or ($F^{-1}Q^2L^{-2}T^{-1}$) 56
27. Capacitance ($M^{-1}Q^2L^{-2}T^2$) or ($F^{-1}Q^2L^{-1}$) 56
28. Inductance ($MQ^{-2}L^2$) or ($FQ^{-2}LT^2$) 56
29. Magnetic Flux ($MQ^{-1}L^2T^{-1}$) or ($FQ^{-1}LT$) 57
30. Magnetic Flux Density ($MQ^{-1}T^{-1}$) or ($FQ^{-1}L^{-1}T$) 57
31. Magnetic Potential and Magnetomotive Force (QT^{-1}).. 57
32. Magnetic Field Intensity, Potential Gradient, and Magnetizing Force ($QL^{-1}T^{-1}$). 58
33. Specific Heat ($L^2T^{-2}t^{-1}$).... 58
34. Thermal Conductivity ($MLT^{-3}t^{-1}$) 58
35. Light 59

16. Gages 59

SYMBOLS AND ABBREVIATIONS

17. Abbreviations for Enginering Terms . 64
18. Letter Symbols for Electrical Quantities 65
19. Symbols for Power Apparatus 66

CONSTANTS

By Carl C. Chambers

20. Principal Physical Constants and Ratios 70

SECTION 1

MATHEMATICS, UNITS, AND SYMBOLS

MATHEMATICS

By Carl C. Chambers

1. ALGEBRAIC FORMULAS

MISCELLANEOUS FORMULAS

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$$

$$(a \pm b)^n = \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} a^k (\pm b)^{(n-k)}, \qquad n! = M(M-1)\ \ldots\ 3 \times 2 \times 1$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + b^2 = (a+jb)(a-jb), \quad j = \sqrt{-1}$$

$$a^x \times a^y = a^{(x+y)}, \quad a^\circ = 1 \text{ [for } a \neq 0], \quad (ab)^x = a^x b^x$$

$$\frac{a^x}{a^y} = a^{(x-y)}, \quad a^{-x} = \frac{1}{a^x}, \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$(a^x)^y = a^{xy}, \quad a^{1/x} = \sqrt[x]{a}, \quad \sqrt[x]{ab} = \sqrt[x]{a}\,\sqrt[x]{b}$$

$$\sqrt[x]{\sqrt[y]{a}} = \sqrt[xy]{a}, \quad a^{x/y} = \sqrt[y]{a^x}, \quad \sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

$$\log (a^x) = x \log a, \quad \log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a \pm b}{b} = \frac{c \pm d}{d} \text{ and } \frac{a-b}{a+b} = \frac{c-d}{c+d}$$

The sum of an arithmetical progression is given by

$$s = \frac{n}{2}(a + l) = \frac{n}{2}\{2a + (n-1)d\}$$

where $l = a + (n-1)d$ is the last term, a is the first term, d is the common difference, and s is the sum of the n terms.

The sum of a geometrical progression is given by

$$s = a\frac{(1-r^n)}{1-r} = \frac{lr - a}{r-1}$$

where $l = ar^{n-1}$ is the last term, a is the first term, d is the common ratio, and s is the sum of the n terms. If n approaches infinity and r^2 is less than unity

$$s = \frac{a}{1-r}$$

The multiple product represented by $n(n-1)(n-2)\ \ldots\ 3 \times 2 \times 1$ is designated by the symbol $n!$ or $\underline{|n}$ and is called "n factorial." The following list gives the value of n up to $n = 10$

1! =	1	6! =	720
2! =	2	7! =	5,040
3! =	6	8! =	40,320
4! =	24	9! =	362,880
5! =	120	10! =	3,628,800

For large values of n a good approximation for $n!$ is, from Stirling's formula,

$$n! = (2\pi n)^{1/2}\left(\frac{n}{e}\right)^n, \quad e = 2.7182818$$

This formula is accurate to about $2^1/_2$ per cent at $n = 10$ and becomes more accurate very rapidly as n is increased,

The number of permutations or arrangements of n things taken p at a time is

$$P_p^n = \frac{n!}{(n-p)!}$$

The number of combinations of n things taken p at a time is then

$$C_p^n = \frac{1}{p!} P_p^n$$

QUADRATIC EQUATION. The solution of

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If a, b, and c are real, and the discriminant, $b^2 - 4ac$, is positive, the roots are real and unequal; if it is zero, the roots are real and equal; if it is negative, the roots are conjugate complex numbers.

CUBIC EQUATIONS. The solution of

$$ax^3 + 3bx^2 + 3cx + d = 0 \tag{1}$$

is obtained as follows: Put $x = \frac{1}{a}(y - b)$; then (1) becomes

$$y^3 - 3Hy + G = 0$$

where

$$H = b^2 - ac$$
$$G = a^2 d - 3abc + 2b^3$$

For a solution let

$$A = \sqrt[3]{\frac{-G}{2} + \sqrt{\frac{G^2}{4} - H^3}}, \quad B = \sqrt[3]{\frac{-G}{2} - \sqrt{\frac{G^2}{4} - H^3}}$$

then the values of y will be given by

$$y = A + B, \quad -\frac{1}{2}(A + B) + j\frac{\sqrt{3}}{2}(A - B), \quad -\frac{1}{2}(A + B) - j\frac{\sqrt{3}}{2}(A - B)$$

If a, b, c, d are real and if $G^2 - 4H^3$, the discriminant, is positive there are one real root and two conjugate complex roots; if $G^2 - 4H^3$ is zero there are three real roots, at least two of which are equal; if $G^2 - 4H^3$ is negative there are three real and unequal roots.

The solution may be written in three other forms.

(1) Put

$$\phi = \frac{1}{3}\sin^{-1}\left[\frac{G}{2H\sqrt{H}}\right]$$

then the roots are

$$y = 2\sqrt{H}\sin\phi, \quad 2\sqrt{H}\sin(\phi + 120°), \quad 2\sqrt{H}\sin(\phi - 120°)$$

Or (2) put

$$u = \frac{1}{3}\cosh^{-1}\left[\frac{G}{2H\sqrt{H}}\right]$$

then the roots are

$$y = -2\sqrt{H}\cosh u, \quad \sqrt{H}\cosh u + \sqrt{-3H}\sinh u, \quad \sqrt{H}\cosh u - \sqrt{-3H}\sinh u$$

Or (3) put

$$u = \frac{1}{3}\sinh^{-1}\left[\frac{G}{2H\sqrt{-H}}\right]$$

Then the roots are

$$y = 2\sqrt{-H}\sinh u, \quad -\sqrt{-H}\sinh u + \sqrt{3H}\cosh u$$
$$-\sqrt{-H}\sinh u - \sqrt{3H}\cosh u$$

SIMULTANEOUS EQUATIONS. Given n independent equations in n unknowns, these n equations usually fix one or more values for each of the n unknowns. To solve

such a set of simultaneous equations in x, y, and z, say, solve each of the three equations for x in terms of y and z. Equating these three values for x gives two equations in y and z. Solving each of these two equations for y in terms of z and equating these two values of y gives a single equation in z. The solution of this last equation then gives the value of z. Then substitute this value of z in either of the equations in y and z, and solve for y. Then substitute these values of y and z in any one of the original equations and solve for x.

DETERMINANTS. In the case of linear simultaneous equations (i.e., when x, y, and z occur only to the first power), the equations may be solved by determinants. This method is a considerable time-saver when the number of unknowns is greater than three, but when the number of unknowns is three or less the straight substitution method is preferable.

The determinant of a set of simultaneous equations is formed by writing the equations one below the other with the same unknown in the same relative position in each. The block of numbers forming the coefficients of the unknowns is called the determinant. For example, the determinant of the equations

$$\begin{aligned} w + x + y + z &= 6 \\ w + \phantom{2x+{}} y + 3z &= 4 \\ w + 2x + 3y \phantom{{}+z} &= 1 \\ w + 3x + \phantom{3y+{}} z &= 3 \end{aligned}$$

is

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 0 & 1 \end{vmatrix}$$

The values of any one of the unknowns, say y, is found by writing a second determinant, D_y, exactly like the determinant D, except that the constants forming the right-hand members of these equations are substituted for the coefficients of y in the determinant, that is

$$D_y = \begin{vmatrix} 1 & 1 & 6 & 1 \\ 1 & 0 & 4 & 3 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{vmatrix}$$

Then

$$y = \frac{D_y}{D}$$

and similarly for the other unknowns.

The value of any determinant is found by making use of the following rules:

(1) If a determinant has two equal rows or columns, it is equal to zero.

(2) To any row or column one may add or substract any number of times any other row or column without altering the value of the determinant.

(3) To multiply any row or column by a number is the same as multiplying the determinant by that number.

(4) If all the terms in a row or column except one are zero, the determinant reduces to one of a lower order which may be obtained by striking out the row and column which intersect at the element of the row or column which is not zero, and multiplying the whole by that element, changing the sign of this element, however, if it is removed by an odd number of elements from the principal diagonal. The principal diagonal is the line of elements beginning at the upper left-hand corner and ending at the lower right-hand corner. Thus,

$$\begin{vmatrix} 1 & 2 & 8 & 5 \\ 3 & 4 & 6 & 9 \\ 0 & 3 & 0 & 0 \\ 6 & 7 & 4 & 3 \end{vmatrix} = -3 \begin{vmatrix} 1 & 8 & 5 \\ 3 & 6 & 9 \\ 6 & 4 & 3 \end{vmatrix}$$

the principal diagonal being that with the figures 1, 4, 0, and 3. It is immaterial whether the distance from the diagonal is counted along a row or a column.

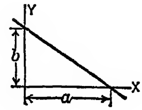
(5) The value of a determinant of the second order is

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

The reduction of determinants is effected by altering the terms according to the above rules until a row or column is obtained in which all terms but one are zero. This enables a reduction of order to be effected in accordance with rule 4. Reductions are continued until one of the second order is obtained.

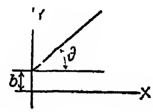
EQUATIONS OF COMMON CURVES. **Straight Line.**

$$\frac{x}{a}+\frac{y}{b}=1$$



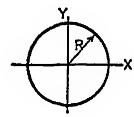
or

$$y = x \tan \theta + b.$$



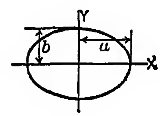
Circle.

$$x^2 + y^2 = R^2$$



Ellipse

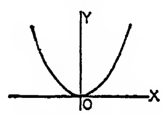
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$



Parabola (Vertical).

$$y = kx^2$$

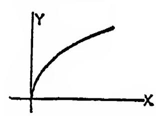
where k is a constant.



Parabola (Horizontal).

$$y = k\sqrt{x}$$

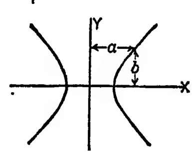
where k is a constant.



Hyperbola.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ (Horizontal)}$$

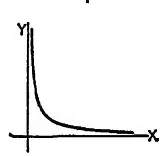
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \text{ (Vertical)}$$



Rectangular or Equilateral Hyperbola.

$$y = \frac{k}{x}$$

where k is a constant.



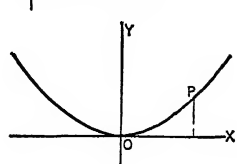
Catenary.

$$y = \frac{1}{k}\cosh kx - 1$$

where k is a constant. The length of arc from O to P is

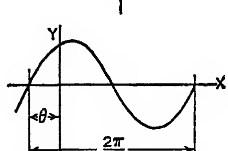
$$= \frac{1}{k}\sinh (kx)$$

See tables of hyperbolic functions.



Sinusoid.

$$y = A \sin (ax + \theta).$$



2. COMPLEX QUANTITIES

The square root of a negative quantity is called an "imaginary" quantity, or a pure imaginary. A quantity consisting of the sum or difference of a real quantity and an imaginary quantity is called a "complex" quantity. All the rules of ordinary algebra apply to pure imaginaries and complex quantities. The square root of minus one is called the imaginary unit and is usually represented by the symbol j (writers on pure mathematics use the symbol i), that is,

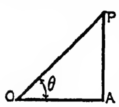
$$j = \sqrt{-1}$$

Any complex quantity may then be written

$$a + jb$$

where a and b are both real quantities.

GEOMETRICAL REPRESENTATION OF A COMPLEX QUANTITY. A positive real quantity may be represented by a line drawn on a plane in a given direction; a negative real quantity may be represented by a line drawn in the opposite direction. Multiplying a quantity by -1 then reverses its direction. Also, since multiplying a real quantity by $\sqrt{-1}$ twice is equivalent to multiplying it by -1, the operation of multiplying once by $\sqrt{-1}$ may be represented by turning the line representing the quantity through 90° in the positive direction of rotation. The positive direction of rotation is taken as the opposite direction to that in which the hands of a clock move. Hence, a complex quantity $a + jb$ may be represented by the line OP in the figure, where $OA = a$ and $AP = b$. The complex quantity $a + jb$ is then completely specified by a line of length $\sqrt{a^2 + b^2}$ making an angle θ, with the axis of reference OX where $\tan\theta = \frac{b}{a}$. The length $M = \sqrt{a^2 + b^2}$ is called the magnitude of the complex quantity, and the angle $\theta = \tan^{-1}\frac{b}{a}$ is called its angle. From the figure it is evident that the complex quantity $a + jb$ may also be written



$$a + jb = M(\cos\theta + j\sin\theta)$$

Expanding $\cos\theta$ and $\sin\theta$ into series (see Series) and adding, the resultant series obtained is the series for $e^{j\theta}$; hence

$$a + jb = Me^{j\theta} \qquad (1)$$

From the above definitions and equation (1) it is evident that complex numbers possess the following properties:

ADDITION OF TWO COMPLEX QUANTITIES.

$$(a + jb) + (a_1 + jb_1) = (a + a_1) + j(b + b_1)$$

SUBTRACTION OF TWO COMPLEX QUANTITIES.

$$(a + jb) - (a_1 + jb_1) = (a - a_1) + j(b - b_1)$$

MULTIPLICATION OF A COMPLEX QUANTITY BY A COMPLEX NUMBER.

$$(a + jb)(a_1 + jb_1) = aa_1 - bb_1 + j(ab_1 + a_1 b)$$

or, putting $\quad a + jb = Me^{j\theta}$ and $a_1 + jb_1 = M_1 e^{j\theta_1}$

where $\quad M = \sqrt{a^2 + b^2}, \quad M_1 = \sqrt{a_1^2 + b_1^2}$

$$\tan\theta = \frac{b}{a}$$

and

$$\tan\theta_1 = \frac{b_1}{a_1}$$

we have $\quad (a + jb)(a_1 + jb_1) = Me^{j\theta} M_1 e^{j\theta_1} = MM_1 e^{j(\theta + \theta_1)}$

Hence the product of two complex quantities is in general a complex quantity which has a magnitude equal to the product of the magnitudes of the two quantities and an angle equal to the sum of the angles of the two quantities.

DIVISION OF A COMPLEX QUANTITY BY A COMPLEX NUMBER.

$$\frac{a + jb}{a_1 + jb_1} = \frac{(a + jb)(a_1 - jb_1)}{(a_1 + jb_1)(a_1 - jb_1)} = \frac{aa_1 + bb_1 - j(ab_1 - a_1 b)}{a_1^2 + b_1^2}$$

or

$$\frac{a + jb}{a_1 + jb_1} = \frac{Me^{j\theta}}{M_1 e^{j\theta_1}} = \frac{M}{M_1} e^{j(\theta - \theta_1)}$$

Hence the quotient of two complex quantities is in general a complex quantity which has a magnitude equal to the quotient of the magnitudes of the two quantities and an angle equal to the difference of the angles of the two quantities.

EQUATIONS CONTAINING COMPLEX QUANTITIES. Since a real quantity cannot be equal to an imaginary quantity it follows that any equation of the form

$$A + jB = A_1 + jB_1$$

where A, B, A_1, and B_1 are all real quantities (which may, however, consist of any number of terms), is equivalent to the two equations

$$A = A_1$$

and

$$B = B_1$$

Also, if one member of an equation reduces to the form $A + jB$, then the other member of this equation must likewise contain an equal real and an equal imaginary part.

3. TRIGONOMETRIC FORMULAS

The trigonometric functions of an angle are the ratios to one another of the various sides of a right triangle having the given angle as one of its angles. Referring to Fig. 1, let B, P and H be the three sides of a triangle. Then the trigonometric functions of the angle x are

sine of x, abbreviated $\sin x = \frac{P}{H}$; cotangent of x, abbreviated $\cot x = \frac{B}{P}$

cosine of x, abbreviated $\cos x = \frac{B}{H}$; secant of x, abbreviated $\sec x = \frac{H}{B}$

tangent of x, abbreviated $\tan x = \frac{P}{B}$; cosecant of x, abbreviated $\csc x = \frac{H}{P}$

When B, P and H are limited to the three sides of a right triangle, the above definitions are directly applicable only to angles lying between 0 and 90°. The definitions, however, may be extended by considering the point A (Fig. 2) as describing a circle of radius OA

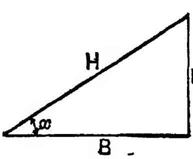


Fig. 1

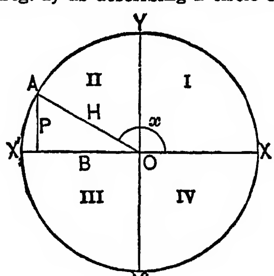


Fig. 2

with the center at O. Let XX' be the horizontal diameter and YY' the vertical diameter of this circle, and call P the perpendicular distance from A to the line XX' and B the horizontal distance from A to YY'. P is to be considered positive when A lies above XX', negative when below. B is considered positive when A is to the right of YY' and negative when to the left. The four quarters of the circle are called quadrants, and are designated as the first, second, third and fourth quadrants as indicated. The angle is said to lie in the quadrant in which the point A lies. In Fig. 2 the angle x is in the second quadrant.

Algebraic Signs of the Functions

	Sine	Cosine	Tangent
Angle in first quadrant......	+	+	+
Angle in second quadrant....	+	−	−
Angle in third quadrant.....	−	−	+
Angle in fourth quadrant....	-	+	−

Period. From the above definitions it is evident that adding 2π radians or 360° to an angle does not change the value of any of its functions, that is, these functions repeat themselves every time the angle increases by the 2π radians or 360°. They are therefore said to have a period equal to 2π radians or 360°.

Functions of Angles in Any Quadrant in Terms of Angles in First Quadrant.

$$\begin{aligned}
\sin(-x) &= -\sin x & \sin(90+x) &= \cos x\\
\cos(-x) &= \cos x & \cos(90+x) &= -\sin x\\
\tan(-x) &= -\tan x & \tan(90+x) &= -\cot x\\
\sin(180-x) &= \sin x & \sin(180+x) &= -\sin x\\
\cos(180-x) &= -\cos x & \cos(180+x) &= -\cos x\\
\tan(180-x) &= -\tan x & \tan(180+x) &= \tan x\\
\sin(270-x) &= -\cos x & \sin(270+x) &= -\cos x\\
\cos(270-x) &= -\sin x & \cos(270+x) &= \sin x\\
\tan(270-x) &= \cot x & \tan(270+x) &= -\cot x
\end{aligned}$$

Anti-functions. If $a = \sin x$, then x is the angle whose sine is a; this may be expressed symbolically $x = \sin^{-1} a$, which is read "x equals the angle whose sine is a." The angle x is also called the "anti-sine" or the "inverse sine" of a. Similar notation is used for the other functions; for example, $x = \cos^{-1} b$ is used to express the relation that x is the angle whose cosine is b. At least two "anti-functions" must be known to completely determine the quadrant in which an angle lies; for example, if $x = \sin^{-1} 0.5$ then x may be either 30° or 150°, but if we also have $x = \cos^{-1} 0.866$, then x must equal 30°, while if $x = \cos^{-1}(-0.866)$, then x must equal 150°.

Anti-functions may be taken from the Trigonometric Tables by finding the angle in the margin corresponding to the function in the table.

Example. $\sin^{-1} 0.319 = 18.6°$ or $180° - 18.6° = 161.4°$.

Versine. The expression $(1 - \cos x)$ is called the "versine" of x.

Relations Among Functions of the Same Angle.

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x} \qquad \sin^2 x + \cos^2 x = 1$$

$$\sec x = \frac{1}{\cos x} \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\csc x = \frac{1}{\sin x} \qquad 1 + \cot^2 x = \frac{1}{\sin^2 x}$$

$$\begin{aligned}
\sin(90-x) &= \cos x & \sin(-x) &= -\sin x\\
\cos(90-x) &= \sin x & \cos(-x) &= \cos x\\
\tan(90-x) &= \cot x & \tan(-x) &= -\tan x
\end{aligned}$$

Sum and Difference of Two Angles.

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Product of the Functions of Two Angles.

$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \sin y = \tfrac{1}{2}[\sin(x+y) - \sin(x-y)]$$
$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$

Functions of Twice an Angle.

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Functions of Half an Angle.

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} \qquad \tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

Functions of Three Times an Angle.

$$\sin 3x = 3\sin x - 4\sin^3 x \qquad \cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}.$$

TRIGONOMETRY. Any triangle is completely defined when, (1) two sides and the included angle are known, (2) one side and two angles are known, (3) three sides are known. Let the sides and angles of a triangle be designated as in Fig. 1.

1. Given two sides a and b, and the included angle γ. Then

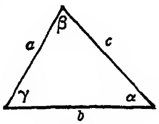


FIG. 3

$$c = \sqrt{a^2 + b^2 - 2\,ab\cos\gamma}$$

$$\sin\alpha = \frac{a}{c}\sin\gamma$$

$$\beta = 180 - \alpha - \gamma.$$

2. Given the side a and the two angles β and γ. Then

$$\alpha = 180 - \beta - \gamma$$

$$b = a\frac{\sin\beta}{\sin\alpha}$$

$$c = a\frac{\sin\gamma}{\sin\alpha}$$

3. Given the three sides a, b and c. Put

$$s = {}^1\!/_2\,(a + b + c)$$

Then

$$\sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin\beta = \frac{b}{a}\sin\alpha$$

$$\gamma = 180 - \alpha - \beta$$

Relations Between Sides and Angles. The following relations between the sides and angles of a triangle are sometimes useful:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

and similar relations for the other two angles.

4. EXPONENTIAL AND HYPERBOLIC FORMULAS

When the relation between any variable y and another variable x is such that x occurs as an exponent of one or more terms, y is said to be an exponential function of x. Of particular importance in connection with electric circuits are the exponential functions e^x and e^{-x}, where e is the base of the natural logarithms. Since x is the natural logarithm of e^x, the value of e^x can be obtained from the table of common logarithms as shown at the beginning of that table. In addition the values of e^x and e^{-x} are given in a separate table.

Hyperbolic functions are an extension of the trigonometric functions to those cases where the use of the latter gives rise to imaginary or complex angles. From the relations,

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

where $j = \sqrt{-1}$, it follows that, putting $x = jz$:

$$\cos jz = \frac{e^z + e^{-z}}{2} \tag{1}$$

$$-j\sin jz = \frac{e^z - e^{-z}}{2} \tag{2}$$

Expressions (1) and (2) are both real quantities when z is real, that is, when the angle jz is imaginary. The first expression is called the hyperbolic cosine of z, abbreviated and pronounced " cosh "; the second expression is called the hyperbolic sine of z, abbreviated sinh and pronounced " shin." Hence, using x for the variable.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The hyperbolic tangent, cotangent, secant, and cosecant are defined as follows:

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

The hyperbolic angle x is a number analogous to radians in circular measure; it is never expressed in degrees.

Adding 2π to an angle does not change the value of the trigonometric functions; they are therefore said to have a period equal to 2π radians. Hyperbolic functions, however, have no true period, but adding $2\pi j$ to the hyperbolic angle does not change the values of the functions; hence these functions have an imaginary period, $2\pi j$.

For the value of the hyperbolic functions see Table of Exponential and Hyperbolic Functions.

Approximate Formulas. Note that for x less than 0.1,

$$\text{Sinh } x = x \text{ with an error of less than 0.2 per cent}$$

$$\cosh x = 1 + \frac{x^2}{2} \text{ with an error of less than 0.09 per cent}$$

For x greater than 6,

$$\sinh x = \cosh x = \frac{e^x}{2} = \frac{1}{2}\log_{10}^{-1}(0.43429x)$$

with an error of less than 0.01 per cent.

Anti-functions. If $a = \sinh x$, then x is the angle whose hyperbolic sine is a; this may be expressed symbolically

$$x = \sinh^{-1} a$$

which is read " x equals the angle whose hyperbolic sine is a." The angle x is also called the " anti-hyperbolic sine " or the " inverse hyperbolic sine " of a. Similarly for the other hyperbolic functions. The following relations exist between the anti-hyperbolic functions and the natural logarithms:

$$\sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

Relations among Functions of the Same Angle.

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \frac{1}{\cosh^2 x}$$

$$\coth^2 x - 1 = \frac{1}{\sinh^2 x}$$

$$\sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\tanh(-x) = -\tanh x$$

See also the definitions given above.

Sum and Difference of Two Angles.

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$
$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$
$$\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$$
$$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$$
$$\tanh(x-y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$$

Product of the Functions of Two Angles.

$$\sinh x \sinh y = {}^1/_2[\cosh(x+y) - \cosh(x-y)]$$
$$\sinh x \cosh y = {}^1/_2[\sinh(x+y) + \sinh(x-y)]$$
$$\cosh x \sinh y = {}^1/_2[\sinh(x+y) - \sinh(x-y)]$$
$$\cosh x \cosh y = {}^1/_2[\cosh(x+y) + \cosh(x-y)]$$

Functions of Twice an Angle.

$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \sinh^2 x + \cosh^2 x = 2 \sinh^2 x + 1 = 2 \cosh^2 x - 1$$
$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

Functions of Half an Angle.

$$\sinh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{2}}$$
$$\cosh \frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}$$
$$\tanh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$

Functions of Three Times an Angle.

$$\sinh 3x = 3 \sinh x + 4 \sinh^3 x$$
$$\cosh 3x = 4 \cosh^3 x - 3 \cosh x$$
$$\tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$$

Relations between Hyperbolic and Trigonometric Functions.

$$\sinh(jx) = j \sin x \qquad \sin(jx) = j \sinh x$$
$$\cosh(jx) = \cos x \qquad \cos(jx) = \cosh x$$
$$\tanh(jx) = j \tan x \qquad \tan(jx) = j \tanh x$$
$$\sinh^{-1} jx = j \sin^{-1} x \qquad \sin^{-1} jx = j \sinh^{-1} x$$
$$\tanh^{-1} jx = j \tan^{-1} x \qquad \tan^{-1} jx = j \tanh^{-1} x$$
$$\cosh^{-1} jx = j \cos^{-1} jx = \log(x + \sqrt{1+x^2}) - j\frac{\pi}{2}$$

Hyperbolic Functions of a Complex Angle.

$$\sinh(x+jy) = \sinh x \cos y + j \cosh x \sin y = Me^{j\theta}$$

where $M = \sqrt{\dfrac{\cosh 2x - \cos 2y}{2}}$ and $\tan \theta = \dfrac{\tan y}{\tanh x}$

$$\cosh(x+jy) = \cosh x \cos y + j \sinh x \sin y = Ne^{j\phi}$$

where $N = \sqrt{\dfrac{\cosh 2x + \cos 2y}{2}}$ and $\tan \phi = \tanh x \cdot \tan y$

$$\tanh(x+jy) = \frac{\sinh x \cos y + j \cosh x \sin y}{\cosh x \cos y + j \sinh x \sin y} = Pe^{j\psi}$$

where $P = \sqrt{\dfrac{\cosh 2x - \cos 2y}{\cosh 2x + \cos 2y}}$ and $\psi = \tan^{-1}\left[\dfrac{\sin 2y}{\sinh 2x}\right]$

$$\tanh^{-1}(Ae^{j\alpha}) = B_1 + jB_2$$

where $B_1 = \dfrac{1}{2}\tanh^{-1}\left[\dfrac{2A \cos \alpha}{1 + A^2}\right]$ and $B_2 = \dfrac{1}{2}\tan^{-1}\left[\dfrac{2A \sin \alpha}{1 - A^2}\right]$

5. CALCULUS FORMULAS

The formula for the integration by parts is:

$$\int_a^b u\,dv = [uv]_a^b - \int_a^b v\,du$$

The following table is used in the formulas

$$\frac{df(x)}{dx} = f'(x)$$

$$\int f'(x)dx = f(x) + C$$

where C is an arbitrary constant.

$f'(x)$	$f(x)$	$f'(x)$	$f(x)$
x^m	$\frac{1}{m+1}x^{m+1}$	$\frac{1}{\cos^2 ax}$	$\frac{1}{a}\tan ax$
$\frac{1}{ax}$	$\frac{1}{a}\log_e x$	$\frac{1}{\sin^2 ax}$	$-\frac{1}{a}\cot ax$
e^{ax}	$\frac{1}{a}e^{ax}$	$\frac{1}{\sqrt{a^2+bx^2}}$	$\frac{1}{\sqrt{-b}}\sin^{-1}\sqrt{-b}\,\frac{x}{a}$
a^{bx}	$\frac{1}{b\log a}a^{bx}$	$\frac{1}{x\sqrt{x^2+a}}$	$\frac{1}{\sqrt{-a}}\cos^{-1}\sqrt{-a}\,\frac{1}{x}$
$\cos ax$	$\frac{1}{a}\sin ax$	$\frac{x}{\sqrt{a^2\pm x^2}}$	$\pm\sqrt{a^2\pm x^2}$
$\sin ax$	$-\frac{1}{a}\cos ax$	$\frac{x}{\sqrt{x^2-a^2}}$	$\sqrt{x^2-a^2}$
$\cosh ax$	$\frac{1}{a}\sinh ax$	$\frac{u\frac{dv}{dx}-v\frac{du}{dx}}{u^2}$	$\frac{v}{u}$
$\sinh ax$	$\frac{1}{a}\cosh ax$	$\log x$	$x\log x - x$
$\tan ax$	$-\frac{1}{a}\log(\cos ax)$	$\sin^2 x$	$-\frac{1}{2}(\cos x\sin x - x)$
$\tanh ax$	$\frac{1}{a}\log(\cosh ax)$	$\cos^2 x$	$\frac{1}{2}(\sin x\cos x + x)$

MAXIMA AND MINIMA. Let y be any function of a variable x, then y will be a maximum or minimum for any value of x which satisfies

$$\frac{dy}{dx} = 0 \qquad (1)$$

provided $\frac{d^2y}{dx^2}$ is not zero. If the second derivative $\frac{d^2y}{dx^2}$ is positive for this value of x, then the corresponding value of y is a minimum; if this second derivative is negative, the corresponding value of y is a maximum.

In case $\frac{d^2y}{dx^2}$ is also zero for the value of x which satisfies (1), the corresponding value of y is not a maximum or minimum unless $\frac{d^3y}{dx^3}$ is also zero and $\frac{d^4y}{dx^4}$ is not zero. When $\frac{d^3y}{dx^3} = 0$, y is a minimum if $\frac{d^4y}{dx^4}$ is positive and a maximum if $\frac{d^4y}{dx^4}$ is negative. In case $\frac{d^4y}{dx^4}$ is also zero, similar relations must hold for the fifth and sixth derivatives, etc.

6. DIFFERENTIAL EQUATIONS

Differential equations of the following forms are met with in the theory of alternating and transient currents.

The following notation is used: $e = 2.7183 \ldots =$ base of natural system of logarithms; x, y, z are variables. A, ϕ, γ, and θ are constants of integration or arbitrary constants. Other letters represent known constants.

$$\frac{dy}{dx} = ay \tag{1}$$

Solution:

$$y = Ae^{ax}$$

$$\frac{dy}{dx} + ay = 0 \tag{2}$$

Solution:

$$y = Ae^{-ax}$$

$$\frac{dy}{dx} + ay = b \tag{3}$$

Solution:

$$y = \frac{b}{a}[1 - Ae^{-ax}]$$

$$\frac{d^2 y}{dx^2} = - a^2 y \tag{4}$$

Solution:

$$y = A \sin (ax + \phi)$$

$$\frac{d^2 y}{dx^2} = a^2 y \tag{5}$$

Solution:

$$y = A \sinh (ax + \phi)$$

$$\frac{d^2 y}{dx^2} + 2u\frac{dy}{dx} + (u^2 + a^2)y = 0 \tag{6}$$

Solution:

Case I. a^2 positive: $y = Ae^{-ux} \sin (ax + \phi)$
Case II. a^2 negative: $y = Ae^{-ux} \sinh (ax + \phi)$
Case III. $a^2 = 0$: $y = A(x + \phi)e^{-ux}$

$$\frac{d^2 y}{dx^2} + 2u\frac{dy}{dx} + (u^2 + a^2)y = B \sin (\omega x + \theta) \tag{7}$$

The complete solution of this equation consists of the solution of (6) plus the term

$$\left(\frac{B \sin \delta}{2u\omega}\right) \sin (\omega x + \theta - \delta) \tag{a}$$

where

$$\delta = \tan^{-1} \frac{2u\omega}{a^2 + u^2 - \omega^2}$$

For each additional sine term added to the right-hand member of the equation, there will be a corresponding term of the same form as (*a*) in the solution.

$$\frac{d^n y}{dx^n} + a_{n-1}\frac{d^{n-1} y}{dx^{n-1}} + \ldots a_1\frac{dy}{dx} + a_0 y = B \sin (\omega x + \theta) \tag{8}$$

Solution:

$$y = A_1 e^{m_1 x} + A_2 e^{m_2 x} + \ldots A_n e^{m_n x} + KB \sin (\omega X + \theta + \delta)$$

where m_1, m_2, etc., are the n roots of the equation

$$m^n + a_{n-1} m^{n-1} + \ldots a_1 m + a_0 = 0,$$

and K and δ are found by substituting the $KB \sin (\omega x + \theta + \delta)$ by itself in the given differential equation and equating the coefficients of $\sin (\omega x + \theta)$ and $\cos (\omega x + \theta)$ respectively on the two sides of the resulting equation. When the second member of the differential equation is a constant, B, the sine term in the solution becomes simply $\frac{B}{a_0}$.

Note that all the preceding equations are merely special cases of the general equation (8).

$$\frac{d^2 y}{dx^2} + 2u\frac{dy}{dx} + (u^2 - q^2)y = \frac{1}{c^2}\frac{d^2 y}{dz^2} \tag{9}$$

The complete solution of this equation contains an infinite number of terms of the form

$$y = e^{-(u-s)x}\,[A_1\, e^{mz} \sin(\omega x + nz + \phi_1) + A_2\, e^{-mz} \sin(\omega x - nz + \phi_2)] \qquad (a)$$

where A_1, ϕ_1, A_2, ϕ_2 and two of the four constants ω, s, m, and n are integration constants (fixed by the terminal conditions). The values of m and n in terms of ω and s are

$$m = c\sqrt{ab}\cos\frac{\eta + e}{2}$$

$$n = c\sqrt{ab}\sin\frac{\eta + e}{2}$$

where $a = \sqrt{(s+q)^2 + \omega^2}$, $\qquad e = \tan^{-1}\left(\frac{\omega}{s+q}\right)$

$b = \sqrt{(s-q)^2 + \omega^2}$, $\qquad \eta = \tan^{-1}\left(\frac{\omega}{s-q}\right)$

The values of ω and s in terms of m and n are

$$\omega = \frac{\sqrt{FG}}{c}\cos\frac{\alpha + \beta}{2}$$

$$s = \frac{\sqrt{FG}}{c}\sin\frac{\alpha + \beta}{2}$$

where $F = \sqrt{(n + cq)^2 + m^2}$, $\qquad \alpha = \tan^{-1}\left(\frac{m}{n + cq}\right)$

$G = \sqrt{(n - cq)^2 + m^2}$, $\qquad \beta = \tan^{-1}\left(\frac{m}{n - cq}\right)$

The solution of equation (9) may also be written as a series of terms of the form

$$y = Me^{-(u-s)x}\sin(\omega x + \phi + \mu) \qquad (b)$$

where

$$M = \frac{A}{\sqrt{2}}\sqrt{\cosh 2(mz + \gamma) + \cos 2(nz + \theta)}$$

$$\tan\mu = \tanh(mz + \gamma)\tan(nz + \theta)$$

where A, ϕ, γ, and θ are integration constants, and the relations between the other constants ω, s, m, and n are the same as above.

In the special case when $q = 0$, the solution of equation (9) is

$$y = e^{-ux}\,[f_1(\omega x + nz) + f_2(\omega x - nz)] \qquad (c)$$

where f_1 and f_2 are any two arbitrary functions and ω and n are connected by the relation

$$\frac{\omega}{n} = \frac{1}{c}$$

$$\frac{d^2 y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = -4a^2 y \qquad (10)$$

This is known as Bessel's equation of zero order. One solution is the infinite series:

$$y = A\left[1 - (ax)^2 + \frac{(ax)^4}{(2!)^2} - \frac{(ax)^6}{(3!)^2} + \frac{(ax)^8}{(4!)^2} - \cdots\right] \qquad (a)$$

If $A = 1$, y is Bessel's function of the first kind of zero order. This series is absolutely convergent for all values of x, but for $ax > 1$ the following asymptotic series is more convenient and sufficiently accurate.

$$y = \frac{Be^{2ax}}{\sqrt{2ax}}\left[1 + \frac{1}{16ax} + \frac{3^2}{2!(16ax)^2} + \frac{3^2\cdot 5^2}{3!(16ax)^3} + \frac{3^2\cdot 5^2\cdot 7^2}{4!(16ax)^4}\right] \qquad (b)$$

---

7. ERRORS OF OBSERVATION

When a quantity is measured with all possible accuracy many times in succession, the numbers expressing the results are found to differ by amounts which, although generally small, are occasionally considerable in comparison with the quantity measured. Though these differences may be decreased by improved methods, better instruments, or greater skill, they can never be entirely removed. They are known as the errors of observation. The following formulas, which are derived from the theory of least squares, apply to such errors and not to errors which can be eliminated by correcting mistakes of the observer or defects of instruments or methods of observation. That is, they apply only to errors

which may be either positive or negative, the chance of a positive error occurring being exactly the same as the chance of a negative error occurring.

WEIGHTED OBSERVATIONS. Sometimes, in spite of the care with which observations are taken, there are reasons for believing that some observations are better than others. In this case the observations are given different "weights" or numbers expressing their relative practical worth. A weighted observation is an observation multiplied by its weight.

PROBABLE VALUE OF SEVERAL OBSERVATIONS. The most probable value of a quantity which is observed directly several times with equal care is the arithmetical mean of the measurements.

The most probable value of a quantity which is observed directly several times, but the observations of which have different weights, is equal to the sum of the weighted observations divided by the sum of the weights.

PROBABLE ERROR OF ANY ONE OF SEVERAL OBSERVATIONS. The probable error or dispersion of a number of direct observations made with equal care is given by the following formula:

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$$

where n = number of observations.
r = probable error of a single observation.
v = residual found by subtracting the arithmetical mean from each measurement.

The probable error of each of a number of direct observations, where the observations have different weight, is found by the following formula, in which p represents the weight of an observation.

$$r_1 = 0.6745\sqrt{\frac{\Sigma pv^2}{n-1}}$$

PROBABLE ERROR OF THE ARITHMETICAL MEAN. If

r = probable error of a single observation
n = number of observations,
r_0 = probable error of the arithmetical mean,

$$r_0 = \frac{r}{\sqrt{n}} \text{ for observations of equal weight}$$

or

$$r_0 = \frac{r_1}{\sqrt{\Sigma p}} \text{ for unequal weight}$$

It should be noted that the probable error of the mean decreases inversely as the square root of the number of observations.

PROBABLE ERROR IN A RESULT CALCULATED FROM THE MEANS OF SEVERAL OBSERVED QUANTITIES. Let Z = a sum or difference of several independent quantities.

Let r_1, r_2, r_3, etc., be the probable errors in these quantities. Then the probable error of Z is equal to

$$\sqrt{r_1^2 + r_2^2 + r_3^2 + \text{etc.}}$$

Let $Z = Az$, where z is an observed quantity, and A, a known number. Let r be the probable error in z. Then the probable error in Z is Ar.

Let Z be the product of two independently observed quantities z_1 and z_2 whose probable errors are r_1 and r_2 respectively. Then the error in Z is equal to

$$\sqrt{z_1^2 r_2^2 + z_2^2 r_1^2}$$

Let Z be any function of the independently observed quantities z_1, z_2, z_3, etc., whose probable errors are r_1, r_2, r_3, etc. Then the probable error in Z is equal to

$$\sqrt{\left(\frac{\partial Z}{\partial z_1}\right)^2 r_1^2 + \left(\frac{\partial Z}{\partial z_2}\right)^2 r_2^2 + \left(\frac{\partial Z}{\partial z_3}\right)^3 r_3^2 + \text{etc.}}$$

8. APPROXIMATIONS

If a is small

$$(1 \pm a)^m = 1 \pm ma$$

If m is nearly equal to n

$$\sqrt{mn} = \frac{m+n}{2}$$

If θ is small compared to a radian

$$\sin\theta = \tan\theta = \theta \text{ radians}$$

9. SERIES

Taylor's series is written

$$f(x+h) = f(x) + \frac{h}{1!}f'(x) + \frac{h^2}{2!}f''(x) + \dots$$
$$= f(h) + \frac{x}{1!}f'(h) + \frac{x^2}{2!}f''(h) + \dots$$

where the prime on the function means the derivative with respect to the argument

The following series are frequently useful.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$
$$a^x = 1 + x\log a + \frac{(x\log a)^2}{2!} + \frac{(x\log a)^3}{3!} + \dots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$
$$\cos(x\sin\theta) = J_0(x) + 2\{J_2(x)\cos 2\theta + J_4(x)\cos 4\theta + \dots$$

where $J_n(x)$ is Bessel's function of order n,

$$\sin(x\sin\theta) = 2\{J_1(x)\sin\theta + J_3(x)\sin 3\theta + \dots$$

10. MENSURATION

The term mensuration is used in this article to include the relations between the areas and volumes of geometric figures and their linear dimensions.

Triangle.

$$\text{Area} = {}^1/_2\,(\text{Base}) \times (\text{Perpendicular height})$$
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c are the lengths of the three sides respectively, and $s = {}^1/_2\,(a+b+c)$

Trapezoid.

$$\text{Area} = \left(\frac{a+b}{2}\right)d$$

where a and b are the lengths of the parallel sides respectively, and d their distance apart.

Parallelogram.

$$\text{Area} = (\text{Base}) \times (\text{Perpendicular height})$$

Parabola.

$$\text{Area} = {}^2/_3\,(\text{Area of circumscribing triangle})$$

Cycloid.

$$\text{Area} = {}^3/_4\,\pi x\,(\text{Altitude})^2$$

the altitude being the diameter of the rolling circle.

Circle.

$$\text{Circumference} = 2\pi r = \pi d$$
$$\text{Area} = \pi r^2 = \frac{\pi}{4}d^2$$

where r is the radius and d the diameter.

$$\text{Area of segment} = \frac{r^2}{2}(\theta - \sin\theta)$$

where θ is the angle in radians (see Angles) subtended by the arc of the segment. If n is the height of the segment, measured along the radius perpendicular to the chord,

$$\text{Area of segment} = \pi r^2 M - A(r-n)$$

where $$A = \sqrt{n(2r-n)} \quad \text{and} \quad M = \frac{1}{180}\sin^{-1}\left(\frac{A}{r}\right)$$

Ellipse.

$$\text{Area} = \pi ab$$

where a and b are the principal semi-axes.

Prism with Parallel Sides and Parallel Ends.

$$\text{Volume} = (\text{Area of end}) \times (\text{Perpendicular distance between ends})$$

Right Circular Cylinder.

$$\text{Volume} = \frac{\pi}{4} d^2 l$$

where d is the diameter and l the length.

$$\text{Total surface of right cylinder} = \pi d(l + {}^1\!/_2 d)$$

Right Circular Cone.

$$\text{Volume} = {}^1\!/_3 \text{ (Area of base)} \times \text{(Height)}$$
$$= {}^1\!/_3 \text{ (Volume of circumscribing cylinder)}$$

where r is the radius of base and h the height of the cone.

$$\text{Area of curved surface of a right circular cone} = \pi r \sqrt{h^2 + r^2}$$

Right Pyramid.

$$\text{Volume} = {}^1\!/_3 \text{ (Area of base)} \times \text{(Height)},$$
$$\text{Volume of frustum of pyramid} = {}^1\!/_3 \text{ (Height)} (A + \sqrt{aA})$$

where A and a are the areas of the ends respectively.

Sphere.

$$r = \text{radius}$$
$$\text{Area of surface} = 4\pi r^2 = {}^2\!/_3 \text{ (total area of circumscribing cylinder)}$$

Area of the surface of a zone of a sphere = area of zone of the same height as this zone projected on to a cylinder.

$$\text{Volume} = {}^4\!/_3 \pi r^3 = {}^2\!/_3 \text{ (volume of circumscribing cylinder)}$$

Volume of a frustum of a sphere $= \pi r^2 (k \pm h) - \frac{\pi}{3} (k^3 \pm h^3)$, where k is the distance of its outer face from center and h the distance of its inner face from the center, the negative signs in the brackets to be used if both faces are on the same side of the center and the positive signs if on opposite sides of the center.

Ellipsoid.

$$\text{Volume} = {}^4\!/_3 \pi abc$$

where a, b, and c are the three principal semi-axes, respectively.

Paraboloid. Volume of a paraboloid of revolution equals one-half that of the circumscribing cylinder.

MATHEMATICAL TABLES

11. COMMON AND NATURAL LOGARITHMS OF NUMBERS

The common logarithm of a number is the index of the power to which the base 10 must be raised in order to equal the number.

The common logarithm of every positive number not an integral power of 10 consists of an *integral* and a *decimal part*. The integral part or whole number is called the *characteristic* and may be either *positive or negative*. The decimal or fractional part is a *positive* number called the *mantissa* and is the same for all numbers which have the same sequential digits.

The characteristic of the logarithm of any positive number greater than one is positive and is one less than the number of digits before the decimal point.

The characteristic of the logarithm of any positive number less than one is negative and is one more than the number of ciphers immediately after the decimal point.

A negative number or number less than zero has no real logarithm.

EXAMPLES: $\text{Log}_{10}\ 25400. = 4.404834$ $\text{Log}_{10}\ 0.0254 = \bar{2}.404834$ or $8.404834 - 10$

The two systems of logarithms in general use are the common or Briggsian logarithms, introduced in 1615 by Henry Briggs, a contemporary of John Napier, the inventor of logarithms, and the natural or less appropriately termed Napierian or hyperbolic logarithms, which developed somewhat accidentally from Napier's original work. The latter have a base denoted by e, an irrational number, which is:

$$e = \text{Lim}_{u=\infty}\left(1 + \frac{1}{u}\right)^u = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots = 2.7182818$$

To obtain the natural logarithm, the common logarithm given below is multiplied by $\log_e 10$ which is 2.302585, or $\log_e N = 2.302585 \log_{10} N$.

N	0	1	2	3	4	5	6	7	8	9
0		**000000**	**301030**	**477121**	**602060**	**698970**	**778151**	**845098**	**903090**	**954243**
1	000000	041393	079181	113943	146128	176091	204120	230449	255273	278754
2	301030	322219	342423	361728	380211	397940	414973	431364	447158	462398
3	477121	491362	505150	518514	531479	544068	556303	568202	579784	591065
4	602060	612784	623249	633468	643453	653213	662758	672098	681241	690196
5	698970	707570	716003	724276	732394	740363	748188	755875	763428	770852
6	778151	785330	792392	799341	806180	812913	819544	826075	832509	838849
7	845098	851258	857332	863323	869232	875061	880814	886491	892095	897627
8	903090	908485	913814	919078	924279	929419	934498	939519	944483	949390
9	954243	959041	963788	968483	973128	977724	982271	986772	991226	995635
10	**000000**	**004321**	**008600**	**012837**	**017033**	**021189**	**025306**	**029384**	**033424**	**037426**
1	041393	045323	049218	053078	056905	060698	064458	068186	071882	075547
2	079181	082785	086360	089905	093422	096910	100371	103804	107210	110590
3	113943	117271	120574	123852	127105	130334	133539	136721	139879	143015
4	146128	149219	152288	155336	158362	161368	164353	167317	170262	173186
5	176091	178977	181844	184691	187521	190332	193125	195900	198657	201397
6	204120	206826	209515	212188	214844	217484	220108	222716	225309	227887
7	230449	232996	235528	238046	240549	243038	245513	247973	250420	252853
8	255273	257679	260071	262451	264818	267172	269513	271842	274158	276462
9	278754	281033	283301	285557	287802	290035	292256	294466	296665	298853
20	**301030**	**303196**	**305351**	**307496**	**309630**	**311754**	**313867**	**315970**	**318063**	**320146**
1	322219	324282	326336	328380	330414	332438	334454	336460	338456	340444
2	342423	344392	346353	348305	350248	352183	354108	356026	357935	359835
3	361728	363612	365488	367356	369216	371068	372912	374748	376577	378398
4	380211	382017	383815	385606	387390	389166	390935	392697	394452	396199
5	397940	399674	401401	403121	404834	406540	408240	409933	411620	413300
6	414973	416641	418301	419956	421604	423246	424882	426511	428135	429752
7	431364	432969	434569	436163	437751	439333	440909	442480	444045	445604
8	447158	448706	450249	451786	453318	454845	456366	457882	459392	460898
9	462398	863893	465383	466868	468347	469822	471292	472756	474216	475671
30	**477121**	**478566**	**480007**	**481443**	**482874**	**484300**	**485721**	**487138**	**488551**	**489958**
1	491362	492760	494155	495544	496930	498311	499687	501059	502427	503791
2	505150	506505	507856	509203	510545	511883	513218	514548	515874	517196
3	518514	519828	521138	522444	523746	525045	526339	527630	528917	530200
4	531479	532754	534026	535294	536558	537819	539076	540329	541579	542825
5	544068	545307	546543	547775	549003	550228	551450	552668	553883	555094

N	0	1	2	3	4	5	6	7	8	9
5	544068	545307	546543	547775	549003	550228	551450	552668	553883	555094
6	556303	557507	558709	559907	561101	562293	563481	564666	565848	567026
7	568202	569374	570543	571709	572872	574031	575188	576341	577492	578639
8	579784	580925	582063	583199	584331	585461	586587	587711	588832	589950
9	591065	592177	593286	594393	595496	596597	597695	598791	599883	600973
40	**602060**	**603144**	**604226**	**605305**	**606381**	**607455**	**608526**	**609594**	**610660**	**611723**
1	612784	613842	614897	615950	617000	618048	619093	620136	621176	622214
2	623249	624232	625312	626340	627366	628389	629410	630428	631444	632457
3	633468	634477	635484	636488	637490	638489	639486	640481	641474	642465
4	643453	644439	645422	646404	647383	648360	649335	650308	651278	652246
5	653213	654177	655138	656098	657056	658011	658965	659916	660865	661713
6	662758	663701	664642	665581	666518	667453	668386	669317	670246	671173
7	672098	673021	673942	674861	675778	676694	677607	678518	679428	680336
8	681241	682145	683047	683947	684845	685742	686636	687529	688420	689309
9	690196	691081	691965	692847	693727	694605	695482	696356	697229	698100
50	**698970**	**699838**	**700704**	**701568**	**702431**	**703291**	**704151**	**705008**	**705864**	**706718**
1	707570	708421	709270	710117	710963	711807	712650	713491	714330	715167
2	716003	716838	717671	718502	719331	720159	720986	721811	722634	723456
3	724276	725095	725912	726727	727541	728354	729165	729974	730782	731589
4	732394	733197	733999	734800	735599	736397	737193	737987	738781	739572
5	740363	741152	741939	742725	743510	744293	745075	745855	746634	747412
6	748188	748963	749736	750508	751279	752048	752816	753583	754348	755112
7	755875	756636	757396	758155	758912	759668	760422	761176	761928	762679
8	763428	764176	764923	765669	766413	767156	767898	768638	769377	770115
9	770852	771587	772322	773055	773786	774517	775246	775974	776701	777427
60	**778151**	**778874**	**779596**	**780317**	**781037**	**781755**	**782473**	**783189**	**783904**	**784617**
1	785330	786041	786751	787460	788168	788875	789581	790285	790988	791691
2	792392	793092	793790	794488	795185	795880	796574	797268	797960	798651
3	799341	800029	800717	801404	802089	802774	803457	804139	804821	805501
4	806180	806858	807535	808211	808886	809560	810233	810904	811575	812245
5	812913	813581	814248	814913	815578	816241	816904	817565	818226	818885
6	819544	820201	820858	821514	822168	822822	823474	824126	824776	825426
7	826075	826723	827369	828015	828660	829304	829947	830589	831230	831870
8	832509	833147	833784	834421	835056	835691	836324	836957	837588	838219
9	838849	839478	840106	840733	841359	841985	842609	843233	843855	844477
70	**845098**	**845718**	**846337**	**846955**	**847573**	**848189**	**848805**	**849419**	**850033**	**850646**
1	851258	851870	852480	853090	853698	854306	854913	855519	856124	856729
2	857332	857935	858537	859138	859739	860338	860937	861534	862131	862728
3	863323	863917	864511	865104	865696	866287	866878	867467	868056	868644
4	869232	869818	870404	870989	871573	872156	872739	873321	873902	874482
5	875061	875640	876218	876795	877371	877947	878522	879096	879669	880242
6	880814	881385	881955	882525	883093	883661	884229	884795	885361	885926
7	886491	887054	887617	888179	888741	889302	889862	890421	890980	891537
8	892095	892651	893207	893762	894316	894870	895423	895975	896526	897077
9	897627	898176	898725	899273	899821	900367	900913	901458	902003	902547
80	**903090**	**903633**	**904174**	**904716**	**905256**	**905796**	**906335**	**906874**	**907411**	**907949**
1	908485	909021	909556	910091	910624	911158	911690	912222	912753	913284
2	913814	914343	914872	915400	915927	916454	916980	917506	918030	918555
3	919078	919601	920123	920645	921166	921686	922206	922725	923244	923762
4	924279	924796	925312	925828	926342	926857	927370	927883	928396	928908
5	929419	929930	930440	930949	931458	931966	932474	932981	933487	933993
6	934498	935003	935507	936011	936514	937016	937518	938019	938520	939020
7	939519	940018	940516	941014	941511	942008	942504	943000	943495	943989
8	944483	944976	945469	945961	946452	946943	947434	947924	948413	948902
9	949390	949878	950365	950851	951338	951823	952308	952792	953276	953760
90	**954243**	**954725**	**955207**	**955688**	**956168**	**956649**	**957128**	**957607**	**958086**	**958564**
1	959041	959518	959995	960471	960946	961421	961895	962369	962843	963316
2	963788	964260	964731	965202	965672	966142	966611	967080	967548	968016
3	968483	968950	969416	969882	970347	970812	971276	971740	972203	972666
4	973128	973590	974051	974512	974972	975432	975891	976350	976808	977266
5	977724	978181	978637	979093	979548	980003	980458	980912	981366	981819
6	982271	982723	983175	983626	984077	984527	984977	985426	985875	986324
7	986772	987219	987666	988113	988559	989005	989450	989895	990339	990783
8	991226	991669	992111	992554	992995	993436	993877	994317	994757	995196
9	995635	996074	996512	996949	997386	997823	998259	998695	999131	999565
100	**000000**	**000434**	**000868**	**001301**	**001734**	**002166**	**002598**	**003029**	**003461**	**003891**

12. TRIGONOMETRIC TABLES

The following tables give the values of sin x, cos x, and tan x for values of x from 0 to 90° in intervals of 0.1 degree. By making use of the periodic character of these functions, the values can be determined from these tables for all values of x to an accuracy of 0.1 degree. (See Trigonometric Formulas.)

If the angle is given in radians multiply the number of radians by $\frac{180}{\pi}$ (57.295) to obtain the number of degrees.

Trigonometric Functions **0.0°–15.9°**

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0 0157
0	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0 0262	0.0279	0.0297	0.0314	0.0332
1	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0 0471	0.0488	0.0506
2	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
3	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
4	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
5	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0 9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
6	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
7	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	9.1299	0.1317	0.1334	0.1352	0.1370	0.1388
	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
8	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1663	0.1685	0.1702	0.1719
9	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
10	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
11	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
12	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
	sin	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402
13	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
14	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
15	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849

Trigonometric Functions 16.0°–35.9°

Angle in Degrees	Name of Function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
		Value of Function for Each Tenth of a Degree									
16	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
	cos	0.9563	0.9558	0.9553	0.9548	0 9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
	cos	0.9387	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
30	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239

Trigonometric Functions **36.0°–55.9°**

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
36	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
	sin	0 6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
37	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
38	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
39	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
40	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6653	0.6665	0.6678
41	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0 7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
42	cos	0.7431	0.7420	0.7408	0.7396	0 7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
43	cos	0.7314	0.7302	0.7290	0.7278	0 7266	0.7254	0.7242	0 7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
44	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
45	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
46	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
47	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
48	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
49	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
50	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
51	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
52	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
53	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
54	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
55	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770

Trigonometric Functions

56.0°–75.9°

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
56	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
57	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
58	cos	0.5299	0 5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0 5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
59	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
60	cos	0.5000	0 4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
61	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
62	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
63	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
64	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0 4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
65	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
66	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
67	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
68	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
69	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0 9444	0.9449
70	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
71	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
72	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
73	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
74	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
75	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812

Trigonometric Functions

76.0°–89.9°

Angle in Degrees	Name of Function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
76	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
77	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
78	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
79	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
80	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
81	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
82	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
83	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
84	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
85	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
86	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.89	15.46	16.35	16.83	17.34	17.89	18.46
	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
87	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
88	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
89	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0

13. EXPONENTIAL AND HYPERBOLIC TABLES

The following tables give values of e^x, e^{-x}, $\sinh x$, $\cosh x$ and $\tanh x$ for values of x from 0.00 to 6.00 in intervals of 0.01.

To facilitate computations involving multiplication, the common logarithms of e^x, $\sinh x$, $\cosh x$, and $\tanh x$ are also given.

For values of x greater than 6, e^x may be computed from the relationship $e^x = \log^{-1} (x \log_{10} e) = \log^{-1} 0.43429x$; e^{-x} approaches zero; $\sinh x$ and $\cosh x$ are approximately equal and become $0.5\, e^x$; and $\tanh x$ and $\coth x$ have values approximately equal to unity.

Where more accurate values of the exponentials and functions are required they may be computed from the following relationships.

$$e = 2.71828\ 18285 \qquad \frac{1}{e} = 0.36787\ 94412$$

$$M = \log_{10} e = 0.43429\ 44819 \qquad \frac{1}{M} = \log_e 10 = 2.30258\ 50930$$

$$e^x = \log^{-1} Mx \qquad e^{-x} = \log^{-1} - Mx$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

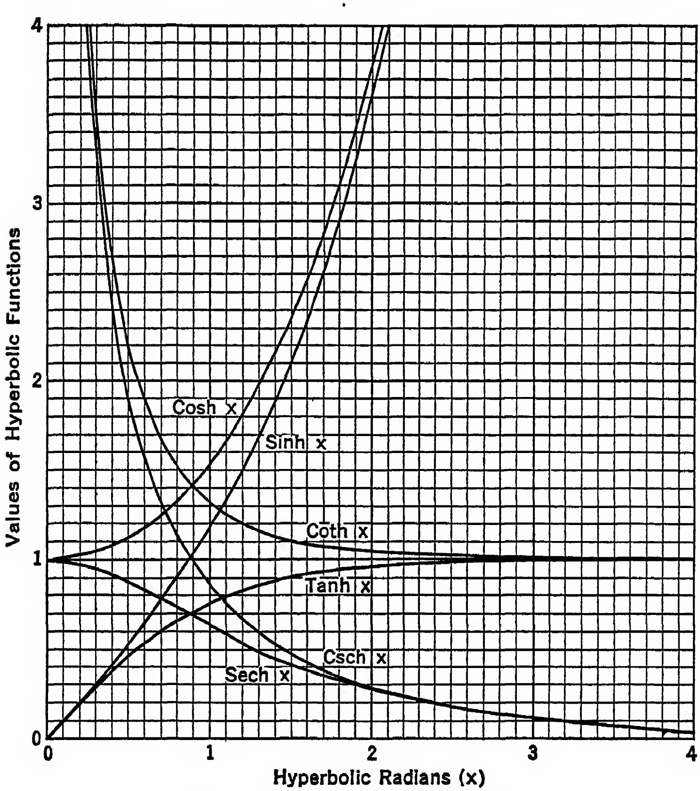


Chart of the Hyperbolic Functions.

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
0.00	**1.0000**	**1.0000**	**0.0000**	**1.0000**	**.00000**	**0.00000**	**$-\infty$**	**0.00000**	**$-\infty$**
0.01	1.0101	.99005	0.0100	1.0001	.01000	.00434	$\bar{2}$.00001	.00002	$\bar{3}$.99999
0.02	1.0202	.98020	0.0200	1.0002	.02000	.00869	.30106	.00009	$\bar{2}$.30097
0.03	1.0305	.97045	0.0300	1.0005	.02999	.01303	.47719	.00020	.47699
0.04	1.0408	.96079	0.0400	1.0008	.03998	.01737	.60218	.00035	.60183
0.05	1.0513	.95123	0.0500	1.0013	.04996	.02171	.69915	.00054	.69861
0.06	1.0618	.94176	0.0600	1.0018	.05993	.02606	.77841	.00078	.77763
0.07	1.0725	.93239	0.0701	1.0025	.06989	.03040	.84545	.00106	.84439
0.08	1.0833	.92312	0.0801	1.0032	.07983	.03474	.90355	.00139	.90216
0.09	1.0942	.91393	0.0901	1.0041	.08976	.03909	.95483	.00176	.95307
0.10	**1.1052**	**.90484**	**0.1002**	**1.0050**	**.09967**	**0.04343**	**$\bar{1}$.00072**	**0.00217**	**$\bar{2}$.99856**
0.11	1.1163	.89583	0.1102	1.0061	.10956	.04777	.04227	.00262	$\bar{1}$.03965
0.12	1.1275	.88692	0.1203	1.0072	.11943	.05212	.08022	.00312	.07710
0.13	1.1388	.87810	0.1304	1.0085	.12927	.05646	.11517	.00366	.11151
0.14	1.1503	.86936	0.1405	1.0098	.13909	.06080	.14755	.00424	.14330
0.15	1.1618	.86071	0.1506	1.0113	.14889	.06514	.17772	.00487	.17285
0.16	1.1735	.85214	0.1607	1.0128	.15865	.06949	.20597	.00554	.20044
0.17	1.1853	.84366	0.1708	1.0145	.16838	.07383	.23254	.00625	.22629
0.18	1.1972	.83527	0.1810	1.0162	.17808	.07817	.25762	.00700	.25062
0.19	1.2092	.82696	0.1911	1.0181	.18775	.08252	.28136	.00779	.27357
0.20	**1.2214**	**.81873**	**0.2013**	**1.0201**	**.19738**	**0.08686**	**$\bar{1}$.30392**	**0.00863**	**1.29529**
0.21	1.2337	.81058	0.2115	1.0221	.20697	.09120	.32541	.00951	.31590
0.22	1.2461	.80252	0.2218	1.0243	.21652	.09554	.34592	.01043	.33549
0.23	1.2586	.79453	0.2320	1.0266	.22603	.09989	.36555	.01139	.35416
0.24	1.2712	.78663	0.2423	1.0289	.23550	.10423	.38437	.01239	.37198
0.25	1.2840	.77880	0.2526	1.0314	.24492	.10857	.40245	.01343	.38902
0.26	1.2969	.77105	0.2629	1.0340	.25430	.11292	.41986	.01452	.40534
0.27	1.3100	.76338	0.2733	1.0367	.26362	.11726	.43663	.01564	.42099
0.28	1.3231	.75578	0.2837	1.0395	.27291	.12160	.45282	.01681	.43601
0.29	1.3364	.74826	0.2941	1.0423	.28213	.12595	.46847	.01801	.45046
0.30	**1.3499**	**.74082**	**0.3045**	**1.0453**	**.29131**	**0.13029**	**1.48362**	**0.01926**	**$\bar{1}$.46436**
0.31	1.3634	.73345	0.3150	1.0484	.30044	.13463	.49830	.02054	.47775
0.32	1.3771	.72615	0.3255	1.0516	.30951	.13897	.51254	.02107	.49067
0.33	1.3910	.71892	0.3360	1.0549	.31852	.14332	.52637	.02323	.50314
0.34	1.4049	.71177	0.3466	1.0584	.32748	.14766	.53981	.02463	.51518
0.35	1.4191	.70469	0.3572	1.0619	.33638	.15200	.55290	.02607	.52682
0.36	1.4333	.69768	0.3678	1.0655	.34521	.15635	.56564	.02755	.53809
0.37	1.4477	.69073	0.3785	1.0692	.35399	.16069	.57807	.02907	.54899
0.38	1.4623	.68386	0.3892	1.0731	.36271	.16503	.59019	.03063	.55956
0.39	1.4770	.67706	0.4000	1.0770	.37136	.16937	.60202	.03222	.56980
0.40	**1.4918**	**.67032**	**0.4108**	**1.0811**	**.37995**	**0.17372**	**$\bar{1}$.61358**	**0.03385**	**$\bar{1}$.57973**
0.41	1.5068	.66365	0.4216	1.0852	.38847	.17806	.62488	.03552	.58936
0.42	1.5220	.65705	0.4325	1.0895	.39693	.18240	.63594	.03723	.59871
0.43	1.5373	.65051	0.4434	1.0939	.40532	.18675	.64677	.03897	.60780
0.44	1.5527	.64404	0.4543	1.0984	.41364	.19109	.65738	.04075	.61663
0.45	1.5683	.63763	0.4653	1.1030	.42190	.19543	.66777	.04256	.62521
0.46	1.5841	.63128	0.4764	1.1077	.43008	.19978	.67797	.04441	.63355
0.47	1.6000	.62500	0.4875	1.1125	.43820	.20412	.68797	.04630	.64167
0.48	1.6161	.61878	0.4986	1.1174	.44624	.20846	.69779	.04822	.64957
0.49	1.6323	.61263	0.5098	1.1225	.45422	.21280	.70744	.05018	.65726
0.50	**1.6487**	**.60653**	**0.5211**	**1.1276**	**.46212**	**0.21715**	**1.71692**	**0.05217**	**1.66475**
0.51	1.6653	.60050	0.5324	1.1329	.46995	.22149	.72624	.05419	.67205
0.52	1.6820	.59452	0.5438	1.1383	.47770	.22583	.73540	.05625	.67916
0.53	1.6989	.58860	0.5552	1.1438	.48538	.23018	.74442	.05834	.68608
0.54	1.7160	.58275	0.5666	1.1494	.49299	.23452	.75330	.06046	.69284
0.55	1.7333	.57695	0.5782	1.1551	.50052	.23886	.76204	.06262	.69942
0.56	1.7507	.57121	0.5897	1.1609	.50798	.24320	.77065	.06481	.70584
0.57	1.7683	.56553	0.6014	1.1669	.51536	.24755	.77914	.06703	.71211
0.58	1.7860	.55990	0.6131	1.1730	.52267	.25189	.78751	.06929	.71822
0.59	1.8040	.55433	0.6248	1.1792	.52990	.25623	.79576	.07157	.72419
0.60	**1.8221**	**.54881**	**0.6367**	**1.1855**	**.53705**	**0.26058**	**1.80390**	**0.07389**	**$\bar{1}$.73001**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
0.60	**1.8221**	**.54881**	**0.6367**	**1.1855**	**.53705**	**0.26058**	**$\bar{1}$.80390**	**0.07389**	**$\bar{1}$.73001**
0.61	1.8404	.54335	0.6485	1.1919	.54413	.26492	.81194	.07624	.73570
0.62	1.8589	.53794	0.6605	1.1984	.55113	.26926	.81987	.07861	.74125
0.63	1.8776	.53259	0.6725	1.2051	.55805	.27361	.82770	.08102	.74667
0.64	1.8965	.52729	0.6846	1.2119	.56490	.27795	.83543	.08346	.75197
0.65	1.9155	.52205	0.6967	1.2188	.57167	.28229	.84308	.08593	.75715
0.66	1.9348	.51685	0.7090	1.2258	.57836	.28663	.85063	.08843	.76220
0.67	1.9542	.51171	0.7213	1.2330	.58498	.29098	.85809	.09095	.76714
0.68	1.9739	.50662	0.7336	1.2402	.59152	.29532	.86548	.09351	.77197
0.69	1.9937	.50158	0.7461	1.2476	.59798	.29966	.87278	.09609	.77669
0.70	**2.0138**	**.49659**	**0.7586**	**1.2552**	**.60437**	**0.30401**	**$\bar{1}$.88000**	**0.09870**	**$\bar{1}$.78130**
0.71	2.0340	.49164	0.7712	1.2628	.61068	.30835	.88715	.10134	.78581
0.72	2.0544	.48675	0.7838	1.2706	.61691	.31269	.89423	.10401	.79022
0.73	2.0751	.48191	0.7966	1.2785	.62307	.31704	.90123	.10670	.79453
0.74	2.0959	.47711	0.8094	1.2865	.62915	.32138	.90817	.10942	.79875
0.75	2.1170	.47237	0.8223	1.2947	.63515	.32572	.91504	.11216	.80288
0.76	2.1383	.46767	0.8353	1.3030	.64108	.33006	.92185	.11493	.80691
0.77	2.1598	.46301	0.8484	1.3114	.64693	.33441	.92859	.11773	.81086
0.78	2.1815	.45841	0.8615	1.3199	.65271	.33875	.93527	.12055	.81472
0.79	2.2034	.45384	0.8748	1.3286	.65841	.34309	.94190	.12340	.81850
0.80	**2.2255**	**.44933**	**0.8881**	**1.3374**	**.66404**	**0.34744**	**$\bar{1}$.94846**	**0.12627**	**$\bar{1}$.82219**
0.81	2.2479	.44486	0.9015	1.3464	.66959	.35178	.95498	.12917	.82581
0.82	2.2705	.44043	0.9150	1.3555	.67507	.35612	.96144	.13209	.82935
0.83	2.2933	.43605	0.9286	1.3647	.68048	.36046	.96784	.13503	.83281
0.84	2.3164	.43171	0.9423	1.3740	.68581	.36481	.97420	.13800	.83620
0.85	2.3396	.42741	0.9561	1.3835	.69107	.36915	.98051	.14099	.83952
0.86	2.3632	.42316	0.9700	1.3932	.69626	.37349	.98677	.14400	.84277
0.87	2.3869	.41895	0.9840	1.4029	.70137	.37784	.99299	.14704	.84595
0.88	2.4109	.41478	0.9981	1.4128	.70642	.38218	.99916	.15009	.84906
0.89	2.4351	.41066	1.0122	1.4229	.71139	.38652	0.00528	.15317	.85211
0.90	**2.4596**	**.40657**	**1.0265**	**1.4331**	**.71630**	**0.39087**	**0.01137**	**0.15627**	**$\bar{1}$.85509**
0.91	2.4843	.40252	1.0409	1.4434	.72113	.39521	.01741	.15939	.85801
0.92	2.5093	.39852	1.0554	1.4539	.72590	.39955	.02341	.16254	.86088
0.93	2.5345	.39455	1.0700	1.4645	.73059	.40389	.02937	.16570	.86368
0.94	2.5600	.39063	1.0847	1.4753	.73522	.40824	.03530	.16888	.86642
0.95	2.5857	.38674	1.0995	1.4862	.73978	.41258	.04119	.17208	.86910
0.96	2.6117	.38289	1.1144	1.4973	.74428	.41692	.04704	.17531	.87173
0.97	2.6379	.37908	1.1294	1.5085	.74870	.42127	.05286	.17855	.87431
0.98	2.6645	.37531	1.1446	1.5199	.75307	.42561	.05864	.18181	.87683
0.99	2.6912	.37158	1.1598	1.5314	.75736	.42995	.06439	.18509	.87930
1.00	**2.7183**	**.36788**	**1.1752**	**1.5431**	**.76159**	**0.43429**	**0.07011**	**0.18839**	**$\bar{1}$.88172**
1.01	2.7456	.36422	1.1907	1.5549	.76576	.43864	.07580	.19171	.88409
1.02	2.7732	.36059	1.2063	1.5669	.76987	.44298	.08146	.19504	.88642
1.03	2.8011	.35701	1.2220	1.5790	.77391	.44732	.08708	.19839	.88869
1.04	2.8292	.35345	1.2379	1.5913	.77789	.45167	.09268	.20176	.89092
1.05	2.8577	.34994	1.2539	1.6038	.78181	.45601	.09825	.20515	.89310
1.06	2.8864	.34646	1.2700	1.6164	.78566	.46035	.10379	.20855	.89524
1.07	2.9154	.34301	1.2862	1.6292	.78946	.46470	.10930	.21197	.89733
1.08	2.9447	.33960	1.3025	1.6421	.79320	.46904	.11479	.21541	.89938
1.09	2.9743	.33622	1.3190	1.6552	.79688	.47338	.12025	.21886	.90139
1.10	**3.0042**	**.33287**	**1.3356**	**1.6685**	**.80050**	**0.47772**	**0.12569**	**0.22233**	**$\bar{1}$.90336**
1.11	3.0344	.32956	1.3524	1.6820	.80406	.48207	.13111	.22582	.90529
1.12	3.0649	.32628	1.3693	1.6956	.80757	.48641	.13649	.22931	.90718
1.13	3.0957	.32303	1.3863	1.7093	.81102	.49075	.14186	.23283	.90903
1.14	3.1268	.31982	1.4035	1.7233	.81441	.49510	.14720	.23636	.91085
1.15	3.1582	.31664	1.4208	1.7374	.81775	.49944	.15253	.23990	.91262
1.16	3.1899	.31349	1.4382	1.7517	.82104	.50378	.15783	.24346	.91436
1.17	3.2220	.31037	1.4558	1.7662	.82427	.50812	.16311	.24703	.91607
1.18	3.2544	.30728	1.4735	1.7808	.82745	.51247	.16836	.25062	.91774
1.19	3.2871	.30422	1.4914	1.7957	.83058	.51681	.17360	.25422	.91938
1.20	**3.3201**	**.30119**	**1.5095**	**1.8107**	**.83365**	**0.52115**	**0.17882**	**0.25784**	**$\bar{1}$.92099**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
1.20	**3.3201**	**.30119**	**1.5095**	**1.8107**	**.83365**	**0.52115**	**0.17882**	**0.25784**	**1.92099**
1.21	3.3535	.29820	1.5276	1.8258	.83668	.52550	.18402	.26146	.92256
1.22	3.3872	.29523	1.5460	1.8412	.83965	.52984	.18920	.26510	.92410
1.23	3.4212	.29229	1.5645	1.8568	.84258	.53418	.19437	.26876	.92561
1.24	3.4556	.28938	1.5831	1.8725	.84546	.53853	.19951	.27242	.92709
1.25	3.4903	.28650	1.6019	1.8884	.84828	.54287	.20464	.27610	.92854
1.26	3.5254	.28365	1.6209	1.9045	.85106	.54721	.20975	.27979	.92996
1.27	3.5609	.28083	1.6400	1.9208	.85380	.55155	.21485	.28349	.93135
1.28	3.5966	.27804	1.6593	1.9373	.85648	.55590	.21993	.28721	.93272
1.29	3.6328	.27527	1.6788	1.9540	.85913	.56024	.22499	.29093	.93406
1.30	**3.6693**	**.27253**	**1.6984**	**1.9709**	**.86172**	**0.56458**	**0.23004**	**0.29467**	**$\bar{1}$.93537**
1.31	3.7062	.26982	1.7182	1.9880	.86428	.56893	.23507	.29842	.93665
1.32	3.7434	.26714	1.7381	2.0053	.86678	.57327	.24009	.30217	.93791
1.33	3.7810	.26448	1.7583	2.0228	.86925	.57761	.24509	.30594	.93914
1.34	3.8190	.26185	1.7786	2.0404	.87167	.58195	.25008	.30972	.94035
1.35	3.8574	.25924	1.7991	2.0583	.87405	.58630	.25505	.31352	.94154
1.36	3.8962	.25666	1.8198	2.0764	.87639	.59064	.26002	.31732	.94270
1.37	3.9354	.25411	1.8406	2.0947	.87869	.59498	.26496	.32113	.94384
1.38	3.9749	.25158	1.8617	2.1132	.88095	.59933	.26990	.32495	.94495
1.39	4.0149	.24908	1.8829	2.1320	.88317	.60367	.27482	.32878	.94604
1.40	**4.0552**	**.24660**	**1.9043**	**2.1509**	**.88535**	**0.60801**	**0.27974**	**0.33262**	**$\bar{1}$.94712**
1.41	4.0960	.24414	1.9259	2.1700	.88749	.61236	.28464	.33647	.94817
1.42	4.1371	.24171	1.9477	2.1894	.88960	.61670	.28952	.34033	.94919
1.43	4.1787	.23931	1.9697	2.2090	.89167	.62104	.29440	.34420	.95020
1.44	4.2207	.23693	1.9919	2.2288	.89370	.62538	.29926	.34807	.95119
1.45	4.2631	.23457	2.0143	2.2488	.89569	.62973	.30412	.35196	.95216
1.46	4.3060	.23224	2.0369	2.2691	.89765	.63407	.30896	.35585	.95311
1.47	4.3492	.22993	2.0597	2.2896	.89958	.63841	.31379	.35976	.95404
1.48	4.3929	.22764	2.0827	2.3103	.90147	.64276	.31862	.36367	.95495
1.49	4.4371	.22537	2.1059	2.3312	.90332	.64710	.32343	.36759	.95584
1.50	**4.4817**	**.22313**	**2.1293**	**2.3524**	**.90515**	**0.65144**	**0.32823**	**0.37151**	**$\bar{1}$.95672**
1.51	4.5267	.22091	2.1529	2.3738	.90694	.65578	.33303	.37545	.95758
1.52	4.5722	.21871	2.1768	2.3955	.90870	.66013	.33781	.37939	.95842
1.53	4.6182	.21654	2.2008	2.4174	.91042	.66447	.34258	.38334	.95924
1.54	4.6646	.21438	2.2251	2.4395	.91212	.66881	.34735	.38730	.96005
1.55	4.7115	.21225	2.2496	2.4619	.91379	.67316	.35211	.39126	.96084
1.56	4.7588	.21014	2.2743	2.4845	.91542	.67750	.35686	.39524	.96162
1.57	4.8066	.20805	2.2993	2.5073	.91703	.68184	.36160	.39921	.96238
1.58	4.8550	.20598	2.3245	2.5305	.91860	.68619	.36633	.40320	.96313
1.59	4.9037	.20393	2.3499	2.5538	.92015	.69053	.37105	.40719	.96386
1.60	**4.9530**	**.20190**	**2.3756**	**2.5775**	**.92167**	**0.69487**	**0.37577**	**0.41119**	**$\bar{1}$.96457**
1.61	5.0028	.19989	2.4015	2.6013	.92316	.69921	.38048	.41520	.96528
1.62	5.0531	.19790	2.4276	2.6255	.92462	.70356	.38518	.41921	.96597
1.63	5.1039	.19593	2.4540	2.6499	.92606	.70790	.38987	.42323	.96664
1.64	5.1552	.19398	2.4806	2.6746	.92747	.71224	.39456	.42725	.96730
1.65	5.2070	.19205	2.5075	2.6995	.92886	.71659	.39923	.43129	.96795
1.66	5.2593	.19014	2.5346	2.7247	.93022	.72093	.40391	.43532	.96858
1.67	5.3122	.18825	2.5620	2.7502	.93155	.72527	.40857	.43937	.96921
1.68	5.3656	.18637	2.5896	2.7760	.93286	.72961	.41323	.44341	.96982
1.69	5.4195	.18452	2.6175	2.8020	.93415	.73396	.41788	.44747	.97042
1.70	**5.4739**	**.18268**	**2.6456**	**2.8283**	**.93541**	**0.73830**	**0.42253**	**0.45153**	**$\bar{1}$.97100**
1.71	5.5290	.18087	2.6740	2.8549	.93665	.74264	.42717	.45559	.97158
1.72	5.5845	.17907	2.7027	2.8818	.93786	.74699	.43180	.45966	.97214
1.73	5.6407	.17728	2.7317	2.9090	.93906	.75133	.43643	.46374	.97269
1.74	5.6973	.17552	2.7609	2.9364	.94023	.75567	.44105	.46782	.97323
1.75	5.7546	.17377	2.7904	2.9642	.94138	.76002	.44567	.47191	.97376
1.76	5.8124	.17204	2.8202	2.9922	.94250	.76436	.45028	.47600	.97428
1.77	5.8709	.17033	2.8503	3.0206	.94361	.76870	.45488	.48009	.97479
1.78	5.9299	.16864	2.8806	3.0492	.94470	.77304	.45948	.48419	.97529
1.79	5.9895	.16696	2.9112	3.0782	.94576	.77739	.46408	.48830	.97578
1.80	**6.0496**	**.16530**	**2.9422**	**3.1075**	**.94681**	**0.78173**	**0.46867**	**0.49241**	**$\bar{1}$.97626**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
1.80	**6.0496**	**.16530**	**2.9422**	**3.1075**	**.94681**	**0.78173**	**0.46867**	**0.49241**	**$\bar{1}$.97626**
1.81	6.1104	.16365	2.9734	3.1371	.94783	.78607	.47325	.49652	.97673
1.82	6.1719	.16203	3.0049	3.1669	.94884	.79042	.47783	.50064	.97719
1.83	6.2339	.16041	3.0367	3.1972	.94983	.79476	.48241	.50476	.97764
1.84	6.2965	.15882	3.0689	3.2277	.95080	.79910	.48698	.50889	.97809
1.85	6.3598	.15724	3.1013	3.2585	.95175	.80344	.49154	.51302	.97852
1.86	6.4237	.15567	3.1340	3.2897	.95268	.80779	.49610	.51716	.97895
1.87	6.4883	.15412	3.1671	3.3212	.95359	.81213	.50066	.52130	.97936
1.88	6.5535	.15259	3.2005	3.3530	.95449	.81647	.50521	.52544	.97977
1.89	6.6194	.15107	3.2341	3.3852	.95537	.82082	.50976	.52959	.98017
1.90	**6.6859**	**.14957**	**3.2682**	**3.4177**	**.95624**	**0.82516**	**0.51430**	**0.53374**	**$\bar{1}$.98057**
1.91	6.7531	.14808	3.3025	3.4506	.95709	.82950	.51884	.53789	.98095
1.92	6.8210	.14661	3.3372	3.4838	.95792	.83385	.52338	.54205	.98133
1.93	6.8895	.14515	3.3722	3.5173	.95873	.83819	.52791	.54621	.98170
1.94	6.9588	.14370	3.4075	3.5512	.95953	.84253	.53244	.55038	.98206
1.95	7.0287	.14227	3.4432	3.5855	.96032	.84687	.53696	.55455	.98242
1.96	7.0993	.14086	3.4792	3.6201	.96109	.85122	.54148	.55872	.98272
1.97	7.1707	.13946	3.5156	3.6551	.96185	.85556	.54600	.56290	.98311
1.98	7.2427	.13807	3.5523	3.6904	.96259	.85990	.55051	.56707	.98344
1.99	7.3155	.13670	3.5894	3.7261	.96331	.86425	.55502	.57126	.98377
2.00	**7.3891**	**.13534**	**3.6269**	**3.7622**	**.96403**	**0.86859**	**0.55953**	**0.57544**	**$\bar{1}$.98409**
2.01	7.4633	.13399	3.6647	3.7987	.96473	.87293	.56403	.57963	.98440
2.02	7.5383	.13266	3.7028	3.8355	.96541	.87727	.56853	.58382	.98471
2.03	7.6141	.13134	3.7414	3.8727	.96609	.88162	.57303	.58802	.98502
2.04	7.6906	.13003	3.7803	3.9103	.96675	.88596	.57753	.59221	.98531
2.05	7.7679	.12873	3.8196	3.9483	.96740	.89030	.58202	.59641	.98560
2.06	7.8460	.12745	3.8593	3.9867	.96803	.89465	.58650	.60061	.98589
2.07	7.9248	.12619	3.8993	4.0255	.96865	.89899	.59099	.60482	.98617
2.08	8.0045	.12493	3.9398	4.0647	.96926	.90333	.59547	.60903	.98644
2.09	8.0849	.12369	3.9806	4.1043	.96986	.90768	.59995	.61324	.98671
2.10	**8.1662**	**.12246**	**4.0219**	**4.1443**	**.97045**	**0.91202**	**0.60443**	**0.61745**	**$\bar{1}$.98697**
2.11	8.2482	.12124	4.0635	4.1847	.97103	.91636	.60890	.62167	.98723
2.12	8.3311	.12003	4.1056	4.2256	.97159	.92070	.61337	.62589	.98748
2.13	8.4149	.11884	4.1480	4.2669	.97215	.92505	.61784	.63011	.98773
2.14	8.4994	.11765	4.1909	4.3085	.97269	.92939	.62231	.63433	.98798
2.15	8.5849	.11648	4.2342	4.3507	.97323	.93373	.62677	.63856	.98821
2.16	8.6711	.11533	4.2779	4.3932	.97375	.93808	.63123	.64278	.98845
2.17	8.7583	.11418	4.3221	4.4362	.97426	.94242	.63569	.64701	.98868
2.18	8.8463	.11304	4.3666	4.4797	.97477	.94676	.64015	.65125	.98890
2.19	8.9352	.11192	4.4116	4.5236	.97526	.95110	.64460	.65548	.98912
2.20	**9.0250**	**.11080**	**4.4571**	**4.5679**	**.97574**	**0.95545**	**0.64905**	**0.65972**	**$\bar{1}$.98934**
2.21	9.1157	.10970	4.5030	4.6127	.97622	.95979	.65350	.66396	.98955
2.22	9.2073	.10861	4.5494	4.6580	.97668	.96413	.65795	.66820	.98975
2.23	9.2999	.10753	4.5962	4.7037	.97714	.96848	.66240	.67244	.98996
2.24	9.3933	.10646	4.6434	4.7499	.97759	.97282	.66684	.67668	.99016
2.25	9.4877	.10540	4.6912	4.7966	.97803	.97716	.67128	.68093	.99035
2.26	9.5831	.10435	4.7394	4.8437	.97846	.98151	.67572	.68518	.99054
2.27	9.6794	.10331	4.7880	4.8914	.97888	.98585	.68016	.68943	.99073
2.28	9.7767	.10228	4.8372	4.9395	.97929	.99019	.68459	.69368	.99091
2.29	9.8749	.10127	4.8868	4.9881	.97970	.99453	.68903	.69794	.99109
2.30	**9.9742**	**.10026**	**4.9370**	**5.0372**	**.98010**	**0.99888**	**0.69346**	**0.70219**	**$\bar{1}$.99127**
2.31	10.074	.09926	4.9876	5.0868	.98049	1.00322	.69789	.70645	.99144
2.32	10.176	.09827	5.0387	5.1370	.98087	.00756	.70232	.71071	.99161
2.33	10.278	.09730	5.0903	5.1876	.98124	.01191	.70675	.71497	.99178
2.34	10.381	.09633	5.1425	5.2388	.98161	.01625	.71117	.71923	.99194
2.35	10.486	.09537	5.1951	5.2905	.98197	.02059	.71559	.72349	.99210
2.36	10.591	.09442	5.2483	5.3427	.98233	.02493	.72002	.72776	.99226
2.37	10.697	.09348	5.3020	5.3954	.98267	.02928	.72444	.73203	.99241
2.38	10.805	.09255	5.3562	5.4487	.98301	.03362	.72885	.73630	.99256
2.39	10.913	.09163	5.4109	5.5026	.98335	.03796	.73327	.74056	.99271
2.40	**11.023**	**.09072**	**5.4662**	**5.5569**	**.98367**	**1.04231**	**0.73769**	**0.74484**	**$\bar{1}$.99285**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
2.40	**11.023**	**.09072**	**5.4662**	**5.5569**	**.98367**	**1.04231**	**0.73769**	**0.74484**	**$\bar{1}$.99285**
2.41	11.134	.08982	5.5221	5.6119	.98400	.04665	.74210	.74911	.99299
2.42	11.246	.08892	5.5785	5.6674	.98431	.05099	.74652	.75338	.99313
2.43	11.359	.08804	5.6354	5.7235	.98462	.05534	.75093	.75766	.99327
2.44	11.473	.08716	5.6929	5.7801	.98492	.05968	.75534	.76194	.99340
2.45	11.588	.08629	5.7510	5.8373	.98522	.06402	.75975	.76621	.99353
2.46	11.705	.08543	5.8097	5.8951	.98551	.06836	.76415	.77049	.99366
2.47	11.822	.08458	5.8689	5.9535	.98579	.07271	.76856	.77477	.99379
2.48	11.941	.08374	5.9288	6.0125	.98607	.07705	.77296	.77906	.99391
2.49	12.061	.08291	5.9892	6.0721	.98635	.08139	.77737	.78334	.99403
2.50	**12.182**	**.08208**	**6.0502**	**6.1323**	**.98661**	**1.08574**	**0.78177**	**0.78762**	**$\bar{1}$.99415**
2.51	12.305	.08127	6.1118	6.1931	.98688	.09008	.78617	.79191	.99426
2.52	12.429	.08046	6.1741	6.2545	.98714	.09442	.79057	.79619	.99438
2.53	12.554	.07966	6.2369	6.3166	.98739	.09877	.79497	.80048	.99449
2.54	12.680	.07887	6.3004	6.3793	.98764	.10311	.79937	.80477	.99460
2.55	12.807	.07808	6.3645	6.4426	.98788	.10745	.80377	.80906	.99470
2.56	12.936	.07730	6.4293	6.5066	.98812	.11179	.80816	.81335	.99481
2.57	13.066	.07654	6.4946	6.5712	.98835	.11614	.81256	.81764	.99491
2.58	13.197	.07577	6.5607	6.6365	.98858	.12048	.81695	.82194	.99501
2.59	13.330	.07502	6.6274	6.7024	.98881	.12482	.82134	.82623	.99511
2.60	**13.464**	**.07427**	**6.6947**	**6.7690**	**.98903**	**1.12917**	**0.82573**	**0.83052**	**$\bar{1}$.99521**
2.61	13.599	.07353	6.7628	6.8363	.98924	.13351	.83012	.83482	.99530
2.62	13.736	.07280	6.8315	6.9043	.98946	.13785	.83451	.83912	.99540
2.63	13.874	.07208	6.9008	6.9729	.98966	.14219	.83890	.84341	.99549
2.64	14.013	.07136	6.9709	7.0423	.98987	.14654	.84329	.84771	.99558
2.65	14.154	.07065	7.0417	7.1123	.99007	.15088	.84768	.85201	.99566
2.66	14.296	.06995	7.1132	7.1831	.99026	.15522	.85206	.85631	.99575
2.67	14.440	.06925	7.1854	7.2546	.99045	.15957	.85645	.86061	.99583
2.68	14.585	.06856	7.2583	7.3268	.99064	.16391	.86083	.86492	.99592
2.69	14.732	.06788	7.3319	7.3998	.99083	.16825	.86522	.86922	.99600
2.70	**14.880**	**.06721**	**7.4063**	**7.4735**	**.99101**	**1.17260**	**0.86960**	**0.87352**	**$\bar{1}$.99608**
2.71	15.029	.06654	7.4814	7.5479	.99118	.17694	.87398	.87783	.99615
2.72	15.180	.06587	7.5572	7.6231	.99136	.18128	.87836	.88213	.99623
2.73	15.333	.06522	7.6338	7.6991	.99153	.18562	.88274	.88644	.99631
2.74	15.487	.06457	7.7112	7.7758	.99170	.18997	.88712	.89074	.99638
2.75	15.643	.06393	7.7894	7.8533	.99186	.19431	.89150	.89505	.99645
2.76	15.800	.06329	7.8683	7.9316	.99202	.19865	.89588	.89936	.99652
2.77	15.959	.06266	7.9480	8.0106	.99218	.20300	.90026	.90367	.99659
2.78	16.119	.06204	8.0285	8.0905	.99233	.20734	.90463	.90798	.99666
2.79	16.281	.06142	8.1098	8.1712	.99248	.21168	.90901	.91229	.99672
2.80	**16.445**	**.06081**	**8.1919**	**8.2527**	**.99263**	**1.21602**	**0.91339**	**0.91660**	**$\bar{1}$.99679**
2.81	16.610	.06020	8.2749	8.3351	.99278	.22037	.91776	.92091	.99685
2.82	16.777	.05961	8.3586	8.4182	.99292	.22471	.92213	.92522	.99691
2.83	16.945	.05901	8.4432	8.5022	.99306	.22905	.92651	.92953	.99698
2.84	17.116	.05843	8.5287	8.5871	.99320	.23340	.93088	.93385	.99704
2.85	17.288	.05784	8.6150	8.6728	.99333	.23774	.93525	.93816	.99709
2.86	17.462	.05727	8.7021	8.7594	.99346	.24208	.93963	.94247	.99715
2.87	17.637	.05670	8.7902	8.8469	.99359	.24643	.94400	.94679	.99721
2.88	17.814	.05613	8.8791	8.9352	.99372	.25077	.94837	.95110	.99726
2.89	17.993	.05558	8.9689	9.0244	.99384	.25511	.95274	.95542	.99732
2.90	**18.174**	**.05502**	**9.0596**	**9.1146**	**.99396**	**1.25945**	**0.95711**	**0.95974**	**$\bar{1}$.99737**
2.91	18.357	.05448	9.1512	9.2056	.99408	.26380	.96148	.96405	.99742
2.92	18.541	.05393	9.2437	9.2976	.99420	.26814	.96584	.96837	.99747
2.93	18.728	.05340	9.3371	9.3905	.99431	.27248	.97021	.97269	.99752
2.94	18.916	.05287	9.4315	9.4844	.99443	.27683	.97458	.97701	.99757
2.95	19.106	.05234	9.5268	9.5791	.99454	.28117	.97895	.98133	.99762
2.96	19.298	.05182	9.6231	9.6749	.99464	.28551	.98331	.98565	.99767
2.97	19.492	.05130	9.7203	9.7716	.99475	.28985	.98768	.98997	.99771
2.98	19.6 8	.05079	9.8185	9.8693	.99485	.29420	.99205	.99429	.99776
2.99	19.886	.05029	9.9177	9.9680	.99496	.29854	.99641	.99861	.99780
3.00	**20.086**	**.04979**	**10.018**	**10.068**	**.99505**	**1.30288**	**1.00078**	**1.00293**	**$\bar{1}$.99785**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
3.00	**20.086**	**.04979**	**10.018**	**10.068**	**.99505**	**1.30288**	**1.00078**	**1.00293**	**$\bar{1}$.99785**
3.01	20.287	.04929	10.119	10.168	.99515	.30723	.00514	.00725	.99789
3.02	20.491	.04880	10.221	10.270	.99525	.31157	.00950	.01157	.99793
3.03	20.697	.04832	10.325	10.373	.99534	.31591	.01387	.01589	.99797
3.04	20.905	.04783	10.429	10.477	.99543	.32026	.01823	.02022	.99801
3.05	21.115	.04736	10.534	10.581	.99552	.32460	.02259	.02454	.99805
3.06	21.328	.04689	10.640	10.687	.99561	.32894	.02696	.02886	.99809
3.07	21.542	.04642	10.748	10.794	.99570	.33328	.03132	.03319	.99813
3.08	21.758	.04596	10.856	10.902	.99578	.33763	.03568	.03751	.99817
3.09	21.977	.04550	10.966	11.011	.99587	.34197	.04004	.04184	.99820
3.10	**22.198**	**.04505**	**11.077**	**11.122**	**.99595**	**1.34631**	**1.04440**	**1.04616**	**$\bar{1}$.99824**
3.11	22.421	.04460	11.188	11.233	.99603	.35066	.04876	.05049	.99827
3.12	22.646	.04416	11.301	11.345	.99611	.35500	.05312	.05481	.99831
3.13	22.874	.04372	11.415	11.459	.99618	.35934	.05748	.05914	.99834
3.14	23.104	.04328	11.530	11.574	.99626	.36368	.06184	.06347	.99837
3.15	23.336	.04285	11.647	11.689	.99633	.36803	.06620	.06779	.99841
3.16	23.571	.04243	11.764	11.807	.99641	.37237	.07056	.07212	.99844
3.17	23.807	.04200	11.883	11.925	.99648	.37671	.07492	.07645	.99847
3.18	24.047	.04159	12.003	12.044	.99655	.38106	.07927	.08078	.99850
3.19	24.288	.04117	12.124	12.165	.99662	.38540	.08363	.08510	.99853
3.20	**24.533**	**.04076**	**12.246**	**12.287**	**.99668**	**1.38974**	**1.08799**	**1.08943**	**$\bar{1}$.99856**
3.21	24.779	.04036	12.369	12.410	.99675	.39409	.09235	.09376	.99859
3.22	25.028	.03996	12.494	12.534	.99681	.39843	.09670	.09809	.99861
3.23	25.280	.03956	12.620	12.660	.99688	.40277	.10106	.10242	.99864
3.24	25.534	.03916	12.747	12.786	.99694	.40711	.10542	.10675	.99867
3.25	25.790	.03877	12.876	12.915	.99700	.41146	.10977	.11108	.99869
3.26	26.050	.03839	13.006	13.044	.99706	.41580	.11413	.11541	.99872
3.27	26.311	.03801	13.137	13.175	.99712	.42014	.11849	.11974	.99875
3.28	26.576	.03763	13.269	13.307	.99717	.42449	.12284	.12407	.99877
3.29	26.843	.03725	13.403	13.440	.99723	.42883	.12720	.12840	.99879
3.30	**27.113**	**.03688**	**13.538**	**13.575**	**.99728**	**1.43317**	**1.13155**	**1.13273**	**$\bar{1}$.99882**
3.31	27.385	.03652	13.674	13.711	.99734	43751	.13591	.13706	.99884
3.32	27.660	.03615	13.812	13.848	.99739	.44186	.14026	.14139	.99886
3.33	27.938	.03579	13.951	13.987	.99744	.44620	.14461	.14573	.99889
3.34	28.219	.03544	14.092	14.127	.99749	.45054	.14897	.15006	.99891
3.35	28.503	.03508	14.234	14.269	.99754	.45489	.15332	.15439	.99893
3.36	28.789	.03474	14.377	14.412	.99759	.45923	.15768	.15872	.99895
3.37	29.079	.03439	14.522	14.556	.99764	.46357	.16203	.16306	.99897
3.38	29.371	.03405	14.668	14.702	.99768	.46792	.16638	.16739	.99899
3.39	29.666	.03371	14.816	14.850	.99773	.47226	.17073	.17172	.99901
3.40	**29.964**	**.03337**	**14.965**	**14.999**	**.99777**	**1.47660**	**1.17509**	**1.17605**	**$\bar{1}$.99903**
3.41	30.265	.03304	15.116	15.149	.99782	.48094	.17944	.18039	.99905
3.42	30.569	.03271	15.268	15.301	.99786	.48529	.18379	.18472	.99907
3.43	30.877	.03239	15.422	15.455	.99790	.48963	.18814	.18906	.99909
3.44	31.187	.03206	15.577	15.610	.99795	.49397	.19250	.19339	.99911
3.45	31.500	.03175	15.734	15.766	.99799	.49832	.19685	.19772	.99912
3.46	31.817	.03143	15.893	15.924	.99803	.50266	.20120	.20206	.99914
3.47	32.137	.03112	16.053	16.084	.99807	.50700	.20555	.20639	.99916
3.48	32.460	.03081	16.215	16.245	.99810	.51134	.20990	.21073	.99918
3.49	32.786	.03050	16.378	16.408	.99814	.51569	.21425	.21506	.99919
3.50	**33.115**	**.03020**	**16.543**	**16.573**	**.99818**	**1.52003**	**1.21860**	**1.21940**	**$\bar{1}$.99921**
3.51	33.448	.02990	16.709	16.739	.99821	.52437	.22296	.22373	.99922
3.52	33.784	.02960	16.877	16.907	.99825	.52872	.22731	.22807	.99924
3.53	34.124	.02930	17.047	17.077	.99828	.53306	.23166	.23240	.99925
3.54	34.467	.02901	17.219	17.248	.99832	.53740	.23601	.23674	.99927
3.55	34.813	.02872	17.392	17.421	.99835	.54175	.24036	.24107	.99928
3.56	35.163	.02844	17.567	17.596	.99838	.54609	.24471	.24541	.99930
3.57	35.517	.02816	17.744	17.772	.99842	.55043	.24906	.24975	.99931
3.58	35.874	.02788	17.923	17.951	.99845	.55477	.25341	.25408	.99933
3.59	36.234	.02760	18.103	18.131	.99848	.55912	.25776	.25842	.99934
3.60	**36.598**	**.02732**	**18.285**	**18.313**	**.99851**	**1.56346**	**1.26211**	**1.26275**	**$\bar{1}$.99935**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
3.60	**36.598**	**.02732**	**18.285**	**18.313**	**.99851**	**1.56346**	**1.26211**	**1.26275**	**$\bar{1}$.99935**
3.61	36.966	.02705	18.470	18.497	.99854	.56780	.26646	.26709	.99936
3.62	37.338	.02678	18.655	18.682	.99857	.57215	.27080	.27143	.99938
3.63	37.713	.02652	18.843	18.870	.99859	.57649	.27515	.27576	.99939
3.64	38.092	.02625	19.033	19.059	.99862	.58083	.27950	.28010	.99940
3.65	38.475	.02599	19.224	19.250	.99865	.58517	.28385	.28444	.99941
3.66	38.861	.02573	19.418	19.444	.99868	.58952	.28820	.28878	.99942
3.67	39.252	.02548	19.613	19.639	.99870	.59386	.29255	.29311	.99944
3.68	39.646	.02522	19.811	19.836	.99873	.59820	.29690	.29745	.99945
3.69	40.045	.02497	20.010	20.035	.99875	.60255	.30125	.30179	.99946
3.70	**40.447**	**.02472**	**20.211**	**20.236**	**.99878**	**1.60689**	**1.30559**	**1.30612**	**$\bar{1}$.99947**
3.71	40.854	.02448	20.415	20.439	.99880	.61123	.30994	.31046	.99948
3.72	41.264	.02423	20.620	20.644	.99883	.61558	.31429	.31480	.99949
3.73	41.679	.02399	20.828	20.852	.99885	.61992	.31864	.31914	.99950
3.74	42.098	.02375	21.037	21.061	.99887	.62426	.32299	.32348	.99951
3.75	42.521	.02352	21.249	21.272	.99889	.62860	.32733	.32781	.99952
3.76	42.948	.02328	21.463	21.486	.99892	.63295	.33168	.33215	.99953
3.77	43.380	.02305	21.679	21.702	.99894	.63729	.33603	.33649	.99954
3.78	43.816	.02282	21.897	21.919	.99896	.64163	.34038	.34083	.99955
3.79	44.256	.02260	22.117	22.140	.99898	.64598	.34472	.34517	.99956
3.80	**44.701**	**.02237**	**22.339**	**22.362**	**.99900**	**1.65032**	**1.34907**	**1.34951**	**$\bar{1}$.99957**
3.81	45.150	.02215	22.564	22.586	.99902	.65466	.35342	.35384	.99957
3.82	45.604	.02193	22.791	22.813	.99904	.65900	.35777	.35818	.99958
3.83	46.063	.02171	23.020	23.042	.99906	.66335	.36211	.36252	.99959
3.84	46.525	.02149	23.252	23.274	.99908	.66769	.36646	.36686	.99960
3.85	46.993	.02128	23.486	23.507	.99909	.67203	.37081	.37120	.99961
3.86	47.465	.02107	23.722	23.743	.99911	.67638	.37515	.37554	.99961
3.87	47.942	.02086	23.961	23.982	.99913	.68072	.37950	.37988	.99962
3.88	48.424	.02065	24.202	24.222	.99915	.68506	.38385	.38422	.99963
3.89	48.911	.02045	24.445	24.466	.99916	.68941	.38819	.38856	.99964
3.90	**49.402**	**.02024**	**24.691**	**24.711**	**.99918**	**1.69375**	**1.39254**	**1.39290**	**$\bar{1}$.99964**
3.91	49.899	.02004	24.939	24.960	.99920	.69809	.39689	.39724	.99965
3.92	50.400	.01984	25.190	25.210	.99921	.70243	.40123	.40158	.99966
3.93	50.907	.01964	25.444	25.463	.99923	.70678	.40558	.40591	.99966
3.94	51.419	.01945	25.700	25.719	.99924	.71112	.40993	.41025	.99967
3.95	51.935	.01925	25.958	25.977	.99926	.71546	.41427	.41459	.99968
3.96	52.457	.01906	26.219	26.238	.99927	.71981	.41862	.41893	.99968
3.97	52.985	.01887	26.483	26.502	.99929	.72415	.42296	.42327	.99969
3.98	53.517	.01869	26.749	26.768	.99930	.72849	.42731	.42761	.99970
3.99	54.055	.01850	27.018	27.037	.99932	.73284	.43166	.43195	.99970
4.00	**54.598**	**.01832**	**27.290**	**27.308**	**.99933**	**1.73718**	**1.43600**	**1.43629**	**$\bar{1}$.99971**
4.01	55.147	.01813	27.564	27.583	.99934	.74152	.44035	.44063	.99971
4.02	55.701	.01795	27.842	27.860	.99936	.74586	.44469	.44497	.99972
4.03	56.261	.01777	28.122	28.139	.99937	.75021	.44904	.44931	.99973
4.04	56.826	.01760	28.404	28.422	.99938	.75455	.45339	.45365	.99973
4.05	57.397	.01742	28.690	28.707	.99939	.75889	.45773	.45799	.99974
4.06	57.974	.01725	28.979	28.996	.99941	.76324	.46208	.46233	.99974
4.07	58.557	.01708	29.270	29.287	.99942	.76758	.46642	.46668	.99975
4.08	59.145	.01691	29.564	29.581	.99943	.77192	.47077	.47102	.99975
4.09	59.740	.01674	29.862	29.878	.99944	.77626	.47511	.47536	.99976
4.10	**60.340**	**.01657**	**30.162**	**30.178**	**.99945**	**1.78061**	**1.47946**	**1.47970**	**$\bar{1}$.99976**
4.11	60.947	.01641	30.465	30.482	.99946	.78495	.48380	.48404	.99977
4.12	61.559	.01624	30.772	30.788	.99947	.78929	.48815	.48838	.99977
4.13	62.178	.01608	31.081	31.097	.99948	.79364	.49249	.49272	.99978
4.14	62.803	.01592	31.393	31.409	.99949	.79798	.49684	.49706	.99978
4.15	63.434	.01576	31.709	31.725	.99950	.80232	.50118	.50140	.99978
4.16	64.072	.01561	32.028	32.044	.99951	.80667	.50553	.50574	.99979
4.17	64.715	.01545	32.350	32.365	.99952	.81101	.50987	.51008	.99979
4.18	65.366	.01530	32.675	32.691	.99953	.81535	.51422	.51442	.99980
4.19	66.023	.01515	33.004	33.019	.99954	.81969	.51856	.51876	.99980
4.20	**66.686**	**.01500**	**33.336**	**33.351**	**.99955**	**1.82404**	**1.52291**	**1.52310**	**$\bar{1}$.99980**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
4.20	**66.686**	**.01500**	**33.336**	**33.351**	**.99955**	**1.82404**	**1.52291**	**1.52310**	**$\bar{1}$.99980**
4.21	67.357	.01485	33.671	33.686	.99956	.82838	.52725	.52745	.99981
4.22	68.033	.01470	34.009	34.024	.99957	.83272	.53160	.53179	.99981
4.23	68.717	.01455	34.351	34.366	.99958	.83707	.53594	.53613	.99982
4.24	69.408	.01441	34.697	34.711	.99958	.84141	.54029	.54047	.99982
4.25	70.105	.01426	35.046	35.060	.99959	.84575	.54463	.54481	.99982
4.26	70.810	.01412	35.398	35.412	.99960	.85009	.54898	.54915	.99983
4.27	71.522	.01398	35.754	35.768	.99961	.85444	.55332	.55349	.99983
4.28	72.240	.01384	36.113	36.127	.99962	.85878	.55767	.55783	.99983
4.29	72.966	.01370	36.476	36.490	.99962	.86312	.56201	.56217	.99984
4.30	**73.700**	**.01357**	**36.843**	**36.857**	**.99963**	**1.86747**	**1.56636**	**1.56652**	**$\bar{1}$.99984**
4.31	74.440	.01343	37.214	37.227	.99964	.87181	.57070	.57086	.99984
4.32	75.189	.01330	37.588	37.601	.99965	.87615	.57505	.57520	.99985
4.33	75.944	.01317	37.966	37.979	.99965	.88050	.57939	.57954	.99985
4.34	76.708	.01304	38.347	38.360	.99966	.88484	.58373	.58388	.99985
4.35	77.478	.01291	38.733	38.746	.99967	.88918	.58808	.58822	.99986
4.36	78.257	.01278	39.122	39.135	.99967	.89352	.59242	.59256	.99986
4.37	79.044	.01265	39.515	39.528	.99968	.89787	.59677	.59691	.99986
4.38	79.838	.01253	39.913	39.925	.99969	.90221	.60111	.60125	.99986
4.39	80.640	.01240	40.314	40.326	.99969	.90655	.60546	.60559	.99987
4.40	**81.451**	**.01228**	**40.719**	**40.732**	**.99970**	**1.91090**	**1.60980**	**1.60993**	**$\bar{1}$.99987**
4.41	82.269	.01216	41.129	41.141	.99970	.91524	.61414	.61427	.99987
4.42	83.096	.01203	41.542	41.554	.99971	.91958	.61849	.61861	.99987
4.43	83.931	.01191	41.960	41.972	.99972	.92392	.62283	.62296	.99988
4.44	84.775	.01180	42.382	42.393	.99972	.92827	.62718	.62730	.99988
4.45	85.627	.01168	42.808	42.819	.99973	.93261	.63152	.63164	.99988
4.46	86.488	.01156	43.238	43.250	.99973	.93695	.63587	.63598	.99988
4.47	87.357	.01145	43.673	43.684	.99974	.94130	.64021	.64032	.99989
4.48	88.235	.01133	44.112	44.123	.99974	.94564	.64455	.64467	.99989
4.49	89.121	.01122	44.555	44.566	.99975	.94998	.64890	.64901	.99989
4.50	**90.017**	**.01111**	**45.003**	**45.014**	**.99975**	**1.95433**	**1.65324**	**1.65335**	**$\bar{1}$.99989**
4.51	90.922	.01100	45.455	45.466	.99976	.95867	.65759	.65769	.99989
4.52	91.836	.01089	45.912	45.923	.99976	.96301	.66193	.66203	.99990
4.53	92.759	.01078	46.374	46.385	.99977	.96735	.66627	.66637	.99990
4.54	93.691	.01067	46.840	46.851	.99977	.97170	.67062	.67072	.99990
4.55	94.632	.01057	47.311	47.321	.99978	.97604	.67496	.67506	.99990
4.56	95.583	.01046	47.787	47.797	.99978	.98038	.67931	.67940	.99990
4.57	96.544	.01036	48.267	48.277	.99979	.98473	.68365	.68374	.99991
4.58	97.514	.01025	48.752	48.762	.99979	.98907	.68799	.68808	.99991
4.59	98.494	.01015	49.242	49.252	.99979	.99341	.69234	.69243	.99991
4.60	**99.484**	**.01005**	**49.737**	**49.747**	**.99980**	**1.99775**	**1.69668**	**1.69677**	**$\bar{1}$.99991**
4.61	100.48	.00995	50.237	50.247	.99980	2.00210	.70102	.70111	.99991
4.62	101.49	.00985	50.742	50.752	.99981	.00644	.70537	.70545	.99992
4.63	102.51	.00975	51.252	51.262	.99981	.01078	.70971	.70979	.99992
4.64	103.54	.00966	51.767	51.777	.99981	.01513	.71406	.71414	.99992
4.65	104.58	.00956	52.288	52.297	.99982	.01947	.71840	.71848	.99992
4.66	105.64	.00947	52.813	52.823	.99982	.02381	.72274	.72282	.99992
4.67	106.70	.00937	53.344	53.354	.99982	.02816	.72709	.72716	.99992
4.68	107.77	.00928	53.880	53.890	.99983	.03250	.73143	.73151	.99993
4.69	108.85	.00919	54.422	54.431	.99983	.03684	.73577	.73585	.99993
4.70	**109.95**	**.00910**	**54.969**	**54.978**	**.99983**	**2.04118**	**1.74012**	**1.74019**	**$\bar{1}$.99993**
4.71	111.05	.00900	55.522	55.531	.99984	.04553	.74446	.74453	.99993
4.72	112.17	.00892	56.080	56.089	.99984	.04987	.74881	.74887	.99993
4.73	113.30	.00883	56.643	56.652	.99984	.05421	.75315	.75322	.99993
4.74	114.43	.00874	57.213	57.222	.99985	.05856	.75749	.75756	.99993
4.75	115.58	.00865	57.788	57.796	.99985	.06290	.76184	.76190	.99993
4.76	116.75	.00857	58.369	58.377	.99985	.06724	.76618	.76624	.99994
4.77	117.92	.00848	58.955	58.964	.99986	.07158	.77052	.77059	.99994
4.78	119.10	.00840	59.548	59.556	.99986	.07593	.77487	.77493	.99994
4.79	120.30	.00831	60.147	60.155	.99986	.08027	.77921	.77927	.99994
4.80	**121.51**	**.00823**	**60.751**	**60.759**	**.99986**	**2.08461**	**1.78355**	**1.78361**	**$\bar{1}$.99994**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
4.80	**121.51**	**.00823**	**60.751**	**60.760**	**.99986**	**2.08461**	**1.78355**	**1.78361**	**$\bar{1}$.99994**
4.81	122.73	.00815	61.362	61.370	.99987	.08896	.78790	.78796	.99994
4.82	123.97	.00807	61.979	61.987	.99987	.09330	.79224	.79230	.99994
4.83	125.21	.00799	62.601	62.609	.99987	.09764	.79658	.79664	.99994
4.84	126.47	.00791	63.231	63.239	.99987	.10199	.80093	.80098	.99995
4.85	127.74	.00783	63.866	63.874	.99988	.10633	.80527	.80532	.99995
4.86	129.02	.00775	64.508	64.516	.99988	.11067	.80962	.80967	.99995
4.87	130.32	.00767	65.157	65.164	.99988	.11501	.81396	.81401	.99995
4.88	131.63	.00760	65.812	65.819	.99988	.11936	.81830	.81835	.99995
4.89	132.95	.00752	66.473	66.481	.99989	.12370	.82265	.82269	.99995
4.90	**134.29**	**.00745**	**67.141**	**67.149**	**.99989**	**2.12804**	**1.82699**	**1.82704**	**$\bar{1}$.99995**
4.91	135.64	.00737	67.816	67.823	.99989	.13239	.83133	.83138	.99995
4.92	137.00	.00730	68.498	68.505	.99989	.13673	.83568	.83572	.99995
4.93	138.38	.00723	69.186	69.193	.99990	.14107	.84002	.84006	.99995
4.94	139.77	.00715	69.882	69.889	.99990	.14541	.84436	.84441	.99996
4.95	141.17	.00708	70.584	70.591	.99990	.14976	.84871	.84875	.99996
4.96	142.59	.00701	71.293	71.300	.99990	.15410	.85305	.85309	.99996
4.97	144.03	.00694	72.010	72.017	.99990	.15844	.85739	.85743	.99996
4.98	145.47	.00687	72.734	72.741	.99991	.16279	.86174	.86178	.99996
4.99	146.94	.00681	73.465	73.472	.99991	.16713	.86608	.86612	.99996
5.00	**148.41**	**.00674**	**74.203**	**74.210**	**.99991**	**2.17147**	**1.87042**	**1.87046**	**$\bar{1}$.99996**
5.01	149.90	.00667	74.949	74.956	.99991	.17582	.87477	.87480	.99996
5.02	151.41	.00660	75.702	75.710	.99991	.18016	.87911	.87915	.99996
5.03	152.93	.00654	76.463	76.470	.99991	.18450	.88345	.88349	.99996
5.04	154.47	.00647	77.232	77.238	.99992	.18884	.88780	.88783	.99996
5.05	156.02	.00641	78.008	78.014	.99992	.19319	.89214	.89217	.99996
5.06	157.59	.00635	78.792	78.798	.99992	.19753	.89648	.89652	.99997
5.07	159.17	.00628	79.584	79.590	.99992	.20187	.90083	.90086	.99997
5.08	160.77	.00622	80.384	80.390	.99992	.20622	.90517	.90520	.99997
5.09	162.39	.00616	81.192	81.198	.99992	.21056	.90951	.90955	.99997
5.10	**164.02**	**.00610**	**82.008**	**82.014**	**.99993**	**2.21490**	**1.91386**	**1.91389**	**$\bar{1}$.99997**
5.11	165.67	.00604	82.832	82.838	.99993	.21924	.91820	.91823	.99997
5.12	167.34	.00598	83.665	83.671	.99993	.22359	.92254	.92257	.99997
5.13	169.02	.00592	84.506	84.512	.99993	.22793	.92689	.92692	.99997
5.14	170.72	.00586	85.355	85.361	.99993	.23227	.93123	.93126	.99997
5.15	172.43	.00580	86.213	86.219	.99993	.23662	.93557	.93560	.99997
5.16	174.16	.00574	87.079	87.085	.99993	.24096	.93992	.93994	.99997
5.17	175.91	.00568	87.955	87.960	.99994	.24530	.94426	.94429	.99997
5.18	177.68	.00563	88.839	88.844	.99994	.24965	.94860	.94863	.99997
5.19	179.47	.00557	89.732	89.737	.99994	.25399	.95294	.95297	.99997
5.20	**181.27**	**.00552**	**90.633**	**90.639**	**.99994**	**2.25833**	**1.95729**	**1.95731**	**$\bar{1}$.99997**
5.21	183.09	.00546	91.544	91.550	.99994	.26267	.96163	.96166	.99997
5.22	184.93	.00541	92.464	92.470	.99994	.26702	.96597	.96600	.99997
5.23	186.79	.00535	93.394	93.399	.99994	.27136	.97032	.97034	.99998
5.24	188.67	.00530	94.332	94.338	.99994	.27570	.97466	.97469	.99998
5.25	190.57	.00525	95.281	95.286	.99994	.28005	.97900	.97903	.99998
5.26	192.48	.00520	96.238	96.243	.99995	.28439	.98335	.98337	.99998
5.27	194.42	.00514	97.205	97.211	.99995	.28873	.98769	.98771	.99998
5.28	196.37	.00509	98.182	98.188	.99995	.29307	.99203	.99206	.99998
5.29	198.34	.00504	99.169	99.174	.99995	.29742	.99638	.99640	.99998
5.30	**200.34**	**.00499**	**100.17**	**100.17**	**.99995**	**2.30176**	**2.00072**	**2.00074**	**$\bar{1}$.99998**
5.31	202.35	.00494	101.17	101.18	.99995	.30610	.00506	.00508	.99998
5.32	204.38	.00489	102.19	102.19	.99995	.31045	.00941	.00943	.99998
5.33	206.44	.00484	103.22	103.22	.99995	.31479	.01375	.01377	.99998
5.34	208.51	.00480	104.25	104.26	.99995	.31913	.01809	.01811	.99998
5.35	210.61	.00475	105.30	105.31	.99995	.32348	.02244	.02246	.99998
5.36	212.72	.00470	106.36	106.36	.99996	.32782	.02678	.02680	.99998
5.37	214.86	.00465	107.43	107.43	.99996	.33216	.03112	.03114	.99998
5.38	217.02	.00461	108.51	108.51	.99996	.33650	.03547	.03548	.99998
5.39	219.20	.00456	109.60	109.60	.99996	.34085	.03981	.03983	.99998
5.40	**221.41**	**.00452**	**110.70**	**110.71**	**.99996**	**2.34519**	**2.04415**	**2.04417**	**$\bar{1}$.99998**

x	Natural Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
5.40	**221.41**	**.00452**	**110.70**	**110.71**	**.99996**	**2.34519**	**2.04415**	**2.04417**	**$\bar{1}$.99998**
5.41	223.63	.00447	111.81	111.82	.99996	.34953	.04849	.04851	.99998
5.42	225.88	.00443	112.94	112.94	.99996	.35388	.05284	.05285	.99998
5.43	228.15	.00438	114.07	114.08	.99996	.35822	.05718	.05720	.99998
5.44	230.44	.00434	115.22	115.22	.99996	.36256	.06152	.06154	.99998
5.45	232.76	.00430	116.38	116.38	.99996	.36690	.06587	.06588	.99998
5.46	235.10	.00425	117.55	117.55	.99996	.37125	.07021	.07023	.99998
5.47	237.46	.00421	118.73	118.73	.99996	.37559	.07455	.07457	.99998
5.48	239.85	.00417	119.92	119.93	.99997	.37993	.07890	.07891	.99998
5.49	242.26	.00413	121.13	121.13	.99997	.38428	.08324	.08325	.99999
5.50	**244.69**	**.00409**	**122.34**	**122.35**	**.99997**	**2.38862**	**2.08758**	**2.08760**	**$\bar{1}$.99999**
5.51	247.15	.00405	123.57	123.58	.99997	.39296	.09193	.09194	.99999
5.52	249.64	.00401	124.82	124.82	.99997	.39731	.09627	.09628	.99999
5.53	252.14	.00397	126.07	126.07	.99997	.40165	.10061	.10063	.99999
5.54	254.68	.00393	127.34	127.34	.99997	.40599	.10495	.10497	.99999
5.55	257.24	.00389	128.62	128.62	.99997	.41033	.10930	.10931	.99999
5.56	259.82	.00385	129.91	129.91	.99997	.41468	.11364	.11365	.99999
5.57	262.43	.00381	131.22	131.22	.99997	.41902	.11798	.11800	.99999
5.58	265.07	.00377	132.53	132.54	.99997	.42336	.12233	.12234	.99999
5.59	267.74	.00374	133.87	133.87	.99997	.42771	.12667	.12668	.99999
5.60	**270.43**	**.00370**	**135.21**	**135.22**	**.99997**	**2.43205**	**2.13101**	**2.13103**	**$\bar{1}$.99999**
5.61	273.14	.00366	136.57	136.57	.99997	.43639	.13536	.13537	.99999
5.62	275.89	.00362	137.94	137.95	.99997	.44074	.13970	.13971	.99999
5.63	278.66	.00359	139.33	139.33	.99997	.44508	.14404	.14405	.99999
5.64	281.46	.00355	140.73	140.73	.99997	.44942	.14839	.14840	.99999
5.65	284.29	.00352	142.14	142.15	.99998	.45376	.15273	.15274	.99999
5.66	287.15	.00348	143.57	143.58	.99998	.45811	.15707	.15708	.99999
5.67	290.03	.00345	145.02	145.02	.99998	.46245	.16141	.16142	.99999
5.68	292.95	.00341	146.47	146.48	.99998	.46679	.16576	.16577	.99999
5.69	295.89	.00338	147.95	147.95	.99998	.47114	.17010	.17011	.99999
5.70	**298.87**	**.00335**	**149.43**	**149.44**	**.99998**	**2.47548**	**2.17444**	**2.17445**	**$\bar{1}$.99999**
5.71	301.87	.00331	150.93	150.94	.99998	.47982	.17879	.17880	.99999
5.72	304.90	.00328	152.45	152.45	.99998	.48416	.18313	.18314	.99999
5.73	307.97	.00325	153.98	153.99	.99998	.48851	.18747	.18748	.99999
5.74	311.06	.00321	155.53	155.53	.99998	.49285	.19182	.19182	.99999
5.75	314.19	.00318	157.09	157.10	.99998	.49719	.19616	.19617	.99999
5.76	317.35	.00315	158.67	158.68	.99998	.50154	.20050	.20051	.99999
5.77	320.54	.00312	160.27	160.27	.99998	.50588	.20484	.20485	.99999
5.78	323.76	.00309	161.88	161.88	.99998	.51022	.20919	.20920	.99999
5.79	327.01	.00306	163.51	163.51	.99998	.51457	.21353	.21354	.99999
5.80	**330.30**	**.00303**	**165.15**	**165.15**	**.99998**	**2.51891**	**2.21787**	**2.21788**	**$\bar{1}$.99999**
5.81	333.62	.00300	166.81	166.81	.99998	.52325	.22222	.22222	.99999
5.82	336.97	.00297	168.48	168.49	.99998	.52759	.22656	.22657	.99999
5.83	340.36	.00294	170.18	170.18	.99998	.53194	.23090	.23091	.99999
5.84	343.78	.00291	171.89	171.89	.99998	.53628	.23525	.23525	.99999
5.85	347.23	.00288	173.62	173.62	.99998	.54062	.23959	.23960	.99999
5.86	350.72	.00285	175.36	175.36	.99998	.54497	.24393	.24394	.99999
5.87	354.25	.00282	177.12	177.13	.99998	.54931	.24828	.24828	.99999
5.88	357.81	.00279	178.90	178.91	.99998	.55365	.25262	.25262	.99999
5.89	361.41	.00277	180.70	180.70	.99998	.55799	.25696	.25697	.99999
5.90	**365.04**	**.00274**	**182.52**	**182.52**	**.99998**	**2.56234**	**2.26130**	**2.26131**	**$\bar{1}$.99999**
5.91	368.71	.00271	184.35	184.35	.99999	.56668	.26565	.26565	.99999
5.92	372.41	.00269	186.20	186.21	.99999	.57102	.26999	.27000	.99999
5.93	376.15	.00266	188.08	188.08	.99999	.57537	.27433	.27434	.99999
5.94	379.93	.00263	189.97	189.97	.99999	.57971	.27868	.27868	.99999
5.95	383.75	.00261	191.88	191.88	.99999	.58405	.28302	.28303	.99999
5.96	387.61	.00258	193.80	193.81	.99999	.58840	.28736	.28737	.99999
5.97	391.51	.00255	195.75	195.75	.99999	.59274	.29171	.29171	.99999
5.98	395.44	.00253	197.72	197.72	.99999	.59708	.29605	.29605	.99999
5.99	399.41	.00250	199.71	199.71	.99999	.60142	.30039	.30040	.99999
6.00	**403.43**	**.00248**	**201.71**	**201.72**	**.99999**	**2.60577**	**2.30473**	**2.30474**	**.99999**

UNITS AND CONVERSION FACTORS

By J. G. Brainerd and Carl C. Chambers

14. SYSTEMS OF UNITS

The magnitude of a physical quantity has no tangible meaning except as the relative magnitude of that quantity as compared with some other quantity of the same nature. Thus, 50 ohms is a resistance having a magnitude 50 times the resistance of 1 ohm. Therefore, whenever it is necessary or desirable to talk about the magnitude of a physical quantity, it is necessary to have a basis for comparison. This basis for a quantity is called the *unit* of magnitude of that quantity. In order to communicate the idea of magnitude between different people, it is necessary that they at least know the relative magnitudes of their units. It is the purpose of this section to act as tool for the specification of the relative magnitudes of the more commonly used systems of units for physical quantities.

Because of the relations defining physical laws, there are relations between the magnitudes of physical quantities. It is desirable that these physical relations be expressed alike in the different systems of units. For instance, the relation mass $\times$ acceleration = force should be independent of the system of units. Therefore, unit mass times unit acceleration should equal unit force. This gives a relation among these three units.

Because of such physical relations, all the mechanical units can be derived from the units for three fundamental quantities. The three quantities ordinarily taken as fundamental are mass, length, and time. Thermal quantities are conveniently derived from these three quantities together with another fundamental quantity, temperature. Photometric quantities are derived from the three fundamental mechanical quantities together with luminous intensity as a fourth fundamental quantity.

Similarly, electrical and magnetic quantities are derived from the three fundamental mechanical quantities and one fundamental electrical or magnetic quantity.

Two systems of mechanical units are in use in English-speaking countries, the English and the metric systems. The metric system is used universally by physicists and to a great extent by engineers, although the English system is still very common in engineering. The English system uses the foot, the pound, and the second as the units for length, mass, and time, respectively. The metric system (as used in the current literature—see MKS system below) employs the meter, the kilogram, and the second as the units for length, mass, and time, respectively.

STANDARDS OF THE FUNDAMENTAL UNITS. The physical units upon which these fundamental units are based and the legalized standards of the United States and Great Britain are described below.

Standard of Length. The standard meter (100 cm) is the distance between two lines on a platinum-iridium bar carefully preserved at the Bureau of Weights and Measures, at Sèvres, France, when the bar is kept at a uniform temperature of 0 deg cent throughout. In the United States the yard (3 ft) was defined by Act of Congress, July 28, 1866, as

$$1 \text{ U. S. yard} = \frac{3600}{3937} \text{ meter}$$

and similarly the British imperial yard is defined by law as

$$1 \text{ British imperial yard} = \frac{3600}{3937.079} \text{ meter}$$

For engineering purposes the U. S. and British yards may be considered as identical.

Standard of Mass and Force. The standard kilogram (1000 grams) as a unit of mass is a cylinder of platinum preserved at the Bureau of Weights and Measures, at Sèvres, France. The U. S. pound avoirdupois is defined by law (Act of Congress, 1866) as $\frac{1}{2.2046}$ kg, but in 1893, the Superintendent of Weights and Measures, with the approval of the Secretary of the Treasury, declared the pound to be

$$1 \text{ U. S. lb} = \frac{1}{2.204622} \text{ kg}$$

The British imperial pound has the same value.

The same relations between the pound and kilogram hold whether these units be taken as units of mass or as units of force, the unit of force being defined in both cases as the pull of the earth on unit mass at 45 deg latitude and sea level.

Standard of Time. The standard second universally adopted is the $\frac{1}{86,400}$ part of a mean solar day. The solar day is the interval of time between two successive transits of the sun across a meridian of the earth at the point of observation: this interval varies in length at different times during the year, but the average length of the interval for one year is constant as far as can be determined by any known methods of observation.

Standard of Temperature. Two units of temperature, or temperature scales, are commonly employed, viz., the centigrade and the fahrenheit units. The relation between these two units results solely from the manner in which they are defined. One degree centigrade = $^9/_5$ fahr. Owing to the difference in the zeros of the two scales, a temperature of t_f degrees fahrenheit corresponds to a temperature of $t_c = {}^5/_9\,(t_f - 32)$ degrees centigrade, and vice versa, $t_f = {}^9/_5\, t_c + 32$ degrees fahrenheit.

Standard of Luminous Intensity. The standard of luminous intensity is the mean intensity in the horizontal plane from a group of incandescent lamps maintained by the U. S. Bureau of Standards, in cooperation with similar custodians in France, Great Britain, and Germany. The International candle is a point source of light having an intensity of a definite fraction of this standard intensity.

ELECTRIC UNITS. Three systems of electric and magnetic units are in general use, viz., (1) the cgs electrostatic system, (2) the cgs electromagnetic system, and (3) the practical system. In the cgs electrostatic system the dielectric coefficient, k, of air* at 0 deg cent and 760 mm mercury pressure is arbitrarily chosen as unity. In the cgs electromagnetic system the magnetic permeability of air under the same standard conditions is arbitrarily chosen as unity. In the practical system a concrete standard of the unit of resistance (called the ohm) and of the unit of current (ampere) is arbitrarily chosen (see below); the unit of resistance is closely equal to 10^9 times the unit of resistance in the cgs electromagnetic system and the unit current is approximately 0.1 that in the latter system. Occasionally other (special) systems are used, most of which are designed to get rid of a factor 4π which frequently appears in the usual systems. The most popular of these others is the Heaviside–Lorentz system in which the unit of electric charge is $1/\sqrt{4\pi}$ of the unit in the electrostatic system. (See MKS system.)

Use of the Prefixes "Stat" and "Ab." To designate the electric and magnetic units in the electrostatic and electromagnetic systems of units respectively, the prefixes "stat" and "ab" may be used with the name of the corresponding practical unit. For example, the cgs electrostatic unit of quantity may be called the statcoulomb and the cgs electromagnetic unit of quantity may be called the abcoulomb, etc.†

Relations among the Three Systems of Electrical Units. The fundamental relations, experimentally determined, between the cgs electrostatic and the cgs electromagnetic system is that 1 abfarad = 8.9878×10^{20} statfarads, which may be approximated for engineering purposes to

$$1 \text{ abfarad} = 9 \times 10^{20} \text{ statfarads}$$

which, as a consequence of the definition of the various terms, is equivalent to

$$1 \text{ abcoulomb} = 3 \times 10^{10} \text{ statcoulombs}$$

the erg being the unit of energy in both systems. Rigorously,

$$1 \text{ abcoulomb} = 2.9979 \times 10^{10} \text{ statcoulombs}$$

(See the article by Birge, *Rev. of Mod. Phys.*, Vol. 1, p. 1, July 1929.)

The fundamental relations between the cgs electromagnetic system and the practical system are

$$1 \text{ abcoulomb} = 10 \text{ coulombs}$$
$$1 \text{ erg} = 10^{-7} \text{ watt-second or joule}$$

the erg being the unit of energy in the cgs electromagnetic system and the joule (or watt-second) that in the practical system.

Practical Electrical Units. The legal units of electrical measure in the United States are given in an Act of Congress, July 12, 1894. Unfortunately, the units there defined are not consistent with one another; for example, the unit of power (watt) there given is not equal to the unit of power derived from the units of current (ampere) and voltage (volt)

* Rigorously, k of free or empty space is chosen unity; for air at 0 deg cent and 76 cm mercury pressure $k = 1.000585$; see International Critical Tables, Vol. 6, p. 77, for the value of k for air under various conditions.

† This abcoulomb, the unit of quantity of electricity in the electromagnetic system, should not be confused with an "absolute coulomb," which is a unit closely equal to the coulomb and is what the latter would be if 1 international or practical ohm equaled *exactly* 10^9 abohms and 1 ampere equaled *exactly* 0.1 abamp. For engineering purposes, the difference between an absolute coulomb and a coulomb is negligible.

as defined in the Act. Recently an amendment to the Treaty on Weights and Measures has given control of the electrical units to the International Committee on Weights and Measures, and within a few years the practical units in use in the United States will be defined in terms of the cgs electromagnetic units:

$$1 \text{ ohm} = 10^9 \text{ abohms and } 1 \text{ ampere} = 0.1 \text{ abampere}$$

At present the practical units (the so-called international units) are based on the following two definitions:

The unit of resistance is the (international) ohm and is equal to the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and 106.300 cm in length.

The unit of current is the (international) ampere and is equal to the unvarying electric current which, when passed through a solution of nitrate of silver in accordance with certain specifications, deposits silver at the rate of 0.00111800 gram per second.

The unit of electromotive force, the (international) volt, is derived from the above by Ohm's law. Other international units are derived from these.

The reason for adopting these units is that it has been considered easier and more accurate to make electrical comparisons with these standards than to make absolute measurements in terms of fundamental (cgs electromagnetic) units. However, though it is comparatively easy to set up secondary standards of resistance from the primary standard defined above, it is not practical to set up secondary standards of current. Instead, the Weston normal cell is used as a standard of voltage, its voltage being determined from the primary standards of resistance and current above, and secondary standards of voltages (standard cells) are used. It will be seen from the data below (from Birge, cited above) that for practical purposes

$$\begin{aligned}
&1 \text{ ohm} &&= 10^9 \text{ abohms and } 1 \text{ ampere} = 0.1 \text{ abampere}\\
&1 \text{ international ohm} &&= 1.00051 \times 10^9 \text{ abohms}\\
&1 \text{ international ampere} &&= 0.99995 \times 0.1 \text{ abampere}\\
&1 \text{ international volt} &&= 1.00046 \times 10^8 \text{ abvolts}\\
&1 \text{ international joule} &&= 1.0004 \text{ absolute joules}\\
&\text{Weston normal cell voltage} &&= 1.01830 \text{ international volts}
\end{aligned}$$

The Act of 1894 defined the international ohm as above, but defined the ampere as 0.1 abampere. These units give rise to the so-called "semi-absolute" system, which is seldom used. For further details, see Circular 60 of the Bureau of Standards and references in the article by Birge cited above.

THE MKS SYSTEM OF UNITS. In 1904 Giorgi proposed a system of units in which the fundamental units were the meter, the kilogram, the second, and the ohm. If, using this system of fundamental units, the permeability of free space is taken as $\mu_0 = 4\pi \times 10^{-7}$ henry per meter, the equations of electricity and magnetism, using the practical units, become equations without factors such as 10^8, etc. Such a system is similar to the so-called absolute systems such as the cgs electromagnetic and the cgs electrostatic systems. It follows from the theory of radiation of electromagnetic waves that the dielectric coefficient $\epsilon_0 = \dfrac{1}{\mu_0 c^2}$, where c is the ratio of electrostatic to electromagnetic units, which can be taken as the velocity of light in free space.

The International Committee of Weights and Measures at its meeting in October 1935 at Sèvres, France, decided that the actual substitution of this absolute system of electrical units for the international system shall take place on January 1, 1940.

The units are then defined by a set of definitions such as follows:

(a) **Ampere.** The ampere is the constant current which, maintained in two parallel rectilinear conductors of infinite length separated by a distance of 1 meter, produces between these conductors a force equal to 2×10^{-7} mks (meter-kilogram-second) units of force per meter of length.

(b) **Volt.** The volt is the difference of electrical potential between two points of a conductor carrying a constant current of 1 ampere when the power dissipated between these points is equal to 1 mks unit of power (watt).

(c) **Coulomb.** The coulomb is the quantity of electricity transported each second by a current of 1 ampere.

(d) **Ohm.** The ohm is the electrical resistance between two points of a conductor when a constant difference of potential of 1 volt, applied between these points, produces in the conductor a current of 1 ampere, the conductor not being the seat of an electromotive force.

(e) **Weber.** The weber is the magnetic flux which, traversing a circuit of a single turn, would produce an electromotive force of 1 volt, if brought to zero in 1 second with uniform diminution.

(f) **Henry.** The henry is the inductance of a closed circuit in which an electromotive force of 1 volt is produced when the electric current traversing the circuit varies uniformly at the rate of 1 ampere per second.

(g) **Farad.** The farad is the electrical capacitance between the armatures of which appears an electrical difference of potential of 1 volt, when charged with 1 coulomb of electric quantity.

The original Giorgi mks system chose the ohm as the fourth fundamental unit. This choice has not been confirmed. The electrical fundamental unit could be almost any of the electrical units. No particular unit has as yet been chosen as fundamental. The preferences seem to be divided between the ampere, the ohm, the permeability, and the coulomb.

The original Giorgi mks system chose $\mu_0 = 4\pi 10^{-7}$, the 4π factor causing the electromagnetic formulas expressing rectilinear symmetry, such as the Maxwell equations, to be free of the factor 4π, and the electromagnetic formulas expressing circular symmetry, such as Coulomb's law, to contain the factor 4π. Such a system is called a rationalized system as contrasted with a non-rationalized system, examples of which are the electromagnetic and the electrostatic cgs systems. The non-rationalized mks system corresponding to the original Giorgi system is defined by the choice of $\mu_0 = 10^{-7}$. This changes the values of some of the units as shown in the table below.

These two questions, namely, the choice of the fundamental unit and the question of rationalization, will probably be decided before the official substitution date, January 1, 1940. The choice of the fundamental unit will be made in such a way as to conform with the definitions given above so that the units will not be changed. The question of rationalization will change the value of a few of the units by a factor 4π or $1/4\pi$ as shown below.

Rationalized MKS Units and Corresponding CGS Electromagnetic Units

Multiply mks units by F to obtain cgs units

Quantity	*Symbol*	MKS *Unit*	CGS *Unit*	F
Mechanical				
Length	L	m	cm	10^2
Mass	M	kg	g	10^3
Time	T	sec	sec	1
Area	S	sq m	sq cm	10^4
Volume	V	cu m (stere)	cu cm	10^6
Frequency	f	cycle per sec (hertz)	cycle per sec	1
Density	d	kg per cu m	g per cu cm	10^{-3}
Velocity	v	m per sec	cm per sec	10^2
Acceleration	a	m per sec per sec	cm per sec per sec	10^2
Force	F	j per m	dyne	10^5
Pressure	p	j per cu m	dyne per sq cm	10
Angle	α, β	radian	radian	1
Angular velocity	ω	radian per sec	radian per sec	1
Torque	τ	j per radian	dyne cm	10^7
Moment of inertia	J	kg-sq m	g-sq cm	10^7
Energetics				
Work or energy	W	j	erg	10^7
Volume energy or energy density	w	j per cu m	erg per cu cm	10
Active power	P	w	erg per sec	10^7
Reactive power	jQ	var	erg per sec	10^7
Thermal				
Quantity of heat	Q	k cal	g cal	10^3
Temperature	θ	C or K	C or K	1
Luminous				
Intensity	I	candle	candle	1
Luminous flux	ψ	1	1	1
Illumination	E	lux	phot	10^{-4}
Brightness	b	candle per sq m	stilb	10^{-4}
Electrical				
Electromotive force	E	volt	abvolt	10^8
Potential gradient or electric field intensity	ε	volt per m	abvolt per cm	10^6
Resistance	R	Ohm	abohm	10^9
Resistivity	ρ	ohm-m	abohm-cm	10^{11}
Conductance	G	siemens, mho	abmho	10^9
Conductivity	γ	mho per m	abmho per cm	10^{-1}
Quantity or displacement	Q	coulomb	abcoulomb	10^{-1}
Current	I	amp	abamp	10^{-1}
Electric flux	ψ	coulomb	abcoulomb	10^{-1}
Flux density	D	coulomb per sq m	abcoulomb per sq cm	10^{-5}
Current density	i	coulomb per sq m	abcoulomb per sq cm	10^{-5}
Capacitance	C	farad	abfarad	10^{-9}
Specific inductive capacity	ϵ/ϵ_0	numeric	numeric	1
Dielectric coefficient for space or space permittivity	ϵ_0	$10^7/4\pi c^2 = 8.854 \times 10^{-12}$	$\frac{1}{c^2} = 1.113 \times 10^{-21}$	
Magnetic				
Magnetomotive force	M	amp-turn	gilbert	$4\pi 10^{-1}$
Magnetizing force or magnetic field intensity	H	amp-turn per m	gilbert per cm	$4\pi 10^{-3}$
Space permeability	μ_0	$4\pi 10^{-7} = 1.257 \times 10^{-6}$	1	
Relative permeability	μ/μ_0	numeric	numeric	1
Magnetic flux	ϕ	weber	maxwell	10^8
Flux density	B	weber per sq m	gauss	10^4
Reluctance	$\mathcal{R}$	amp-turn per weber	oersted	$4\pi 10^{-9}$
Permanence	$\mathcal{P}$	weber per amp-turn	(oersted)$^{-1}$	$10^9/4\pi$
Inductance	L	henry	abhenry	10^9
Pole strength	m	weber	maxwell/4π	$10^8/4\pi$
Magnetization	$\mathfrak{I}$	weber per sq m		$10^4/4\pi$
Magnetic moment	$\mathfrak{M}$	weber-m	maxwell-cm/4π	$10^{10}/4\pi$

Rationalized MKS Units and Corresponding Non-rationalized Units

Multiply non-rationalized mks units by F to obtain rationalized mks units

Quantity	*Symbol*	*Name of Rationalized* MKS *Units*	F
Electrical			
Electric flux	ψ	coulomb	4π
Flux density	D	coulomb per sq m	4π
Space permittivity	ϵ_0	farad per m	4π
Magnetic			
Magnetomotive force	M or $\mathfrak{F}$	amp-turn	$1/4\pi$
Magnetizing force	H	amp-turn per m	$1/4\pi$
Space permeability	μ_0	henry per m	4π
Permanence	P	weber per amp-turn	4π
Reluctance	R	amp-turn per weber	$1/4\pi$
Pole strength	m	weber	4π
Magnetic moment	$\mathfrak{M}$	weber-m	4π
Magnetization	$\mathfrak{I}$	weber per sq m	4π

15. CONVERSION TABLES

Table 1. Length [L]

Multiply Number of → by to Obtain ↓	Centimeters	Feet	Inches	Kilometers	Knots (Nautical miles)	Meters	Mils	Miles	Millimeters	Yards
Centimeters	1	30.48	2.540	10^5	1.853×10^5	100	2.540×10^{-3}	1.609×10^5	0.1	91.44
Feet	3.281×10^{-2}	1	8.333×10^{-2}	3281	6080.27	3.281	8.333×10^{-5}	5280	3.281×10^{-3}	3
Inches	0.3937	12	1	3.937×10^4	7.296×10^4	39.37	0.001	6.336×10^4	3.937×10^{-2}	36
Kilometers	10^{-5}	3.048×10^{-4}	2.540×10^{-5}	1	1.853	0.001	2.540×10^{-8}	1.609	10^{-6}	9.144×10^{-4}
Knots (Nautical miles)		1.645×10^{-4}		0.5396	1	5.396×10^{-4}		0.8684		4.934×10^{-4}
Meters	0.01	0.3048	2.540×10^{-2}	1000	1853	1		1609	0.001	0.9144
Mils	393.7	1.2×10^4	1000	3.937×10^7		3.937×10^4	1		39.37	3.6×10^4
Miles	6.214×10^{-6}	1.894×10^{-4}	1.578×10^{-5}	0.6214	1.1516	6.214×10^{-4}		1	6.214×10^{-7}	5.682×10^{-4}
Millimeters	10	304.8	25.40	10^6		1000	2.540×10^{-2}		1	914.4
Yards	1.094×10^{-2}	0.3333	2.778×10^{-2}	1094	2027	1.094	2.778×10^{-5}	1760	1.094×10^{-3}	1

Metric Multiples

10^6 microns = 10^3 millimeters = 10^2 centimeters = 10 decimeters = 1 meter = 10^{-1} dekameter = 10^{-2} hectometer = 10^{-3} kilometer = 10^{-4} myriameter = 10^{-6} megameter = 10^{10} Angstrom Units.

Land Measure

7.92 inches = 1 link
25 links = 1 rod = 16.5 feet = 5.5 yards (1 rod = 1 pole = 1 perch)
4 rods = 1 chain (Gunther's) = 66 feet = 22 yards = 100 links
10 chains = 1 furlong = 660 feet = 220 yards = 1000 links = 40 rods
8 furlongs = 1 mile = 5280 feet = 1760 yards = 8000 links = 320 rods = 80 chains

Ropes and Cables

2 yards = 1 fathom   120 fathoms = 1 cable's length

Nautical Measure

6080.27 feet = 1 knot = 1 nautical mile = 1.15156 statute miles
3 nautical miles = 1 league (U. S.)   3 statute miles = 1 league (Gr. Britain)

(NOTE.—A knot, or nautical mile, is the length of a minute of longitude of the earth at the equator at sea level. The British Admiralty uses the round figure of 6080 feet. The word "knot" is frequently used also to denote "nautical miles per hour.")

Miscellaneous

3 inches = 1 palm   9 inches = 1 span
4 inches = 1 hand   2 1/2 feet = 1 military pace

Table 2. Area [L^2]

Multiply Number of → by to Obtain ↓	Acres	Circular mils	Square centimeters	Square feet	Square inches	Square kilometers	Square meters	Square miles	Square millimeters	Square yards
Acres	1			2.296 $\times 10^{-5}$		247.1	2.471 $\times 10^{-4}$	640		2.066 $\times 10^{-4}$
Circular mils		1	1.973 $\times 10^{5}$	1.833 $\times 10^{8}$	1.273 $\times 10^{6}$		1.973 $\times 10^{9}$		1973	
Square centimeters		5.067 $\times 10^{-6}$	1	929.0	6.452	10^{10}	10^{4}	2 590 $\times 10^{10}$	0.01	8361
Square feet	4.356 $\times 10^{4}$		1.076 $\times 10^{-3}$	1	6.944 $\times 10^{-3}$	1.076 $\times 10^{7}$	10.76	2.788 $\times 10^{7}$	1.076 $\times 10^{-5}$	9
Square inches	6,272,640	7.854 $\times 10^{-7}$	0.1550	144	1	1.550 $\times 10^{9}$	1550	4.015 $\times 10^{9}$	1.550 $\times 10^{-3}$	1296
Square kilometers	4 047 $\times 10^{-3}$		10^{-10}	9.290 $\times 10^{-8}$	6.452 $\times 10^{-10}$	1	10^{-6}	2.590	10^{-12}	8.361 $\times 10^{-7}$
Square meters	4047		0 0001	9 290 $\times 10^{-2}$	6.452 $\times 10^{-4}$	10^{6}	1	2 590 $\times 10^{6}$	10^{-6}	0.8361
Square miles	1.562 $\times 10^{-3}$		3 861 $\times 10^{-11}$	3 587 $\times 10^{-8}$		0 3861	3 861 $\times 10^{-7}$	1	3.861 $\times 10^{-13}$	3.228 $\times 10^{-7}$
Square millimeters		5.067 $\times 10^{-4}$	100	9.290 $\times 10^{4}$	645.2	10^{12}	10^{6}		1	8.361 $\times 10^{5}$
Square yards	4840		1.196 $\times 10^{-4}$	0.1111	7 716 $\times 10^{-4}$	1.196 $\times 10^{6}$	1.196	3.098 $\times 10^{6}$	1.196 $\times 10^{-6}$	1

Land Measure

30 1/4 square yards = 1 square rod = 272 1/4 square feet
16 square rods = 1 square chain = 484 square yards = 4356 square feet
2 1/2 square chains = 1 rood = 40 square rods = 1210 square yards
4 roods = 1 acre = 10 square chains = 160 square rods
640 acres = 1 square mile = 2560 roods = 102,400 square rods
1 section of land = 1 square mile; 1 quarter section = 160 acres

Architect's Measure

100 square feet = 1 square

Circular Inch and Circular Mil

A circular inch is the area of a circle 1 inch in diameter = 0.7854 square inch
1 square inch = 1.2732 circular inches
A circular mil is the area of a circle 1 mil (or 0.001 inch) in diameter = 0.7854 square mil
1 square mil = 1.2732 circular mils
1 circular inch = 10^6 circular mils = 0.7854 $\times$ 10^6 square mils
1 square inch = 1.2732 $\times$ 10^6 circular mils = 10^6 square mils

Metric Multiples

1 square meter = 1 centiare = 10^{-2} are = 10^{-4} hectare
= 10^{-6} square kilometer = 10^{-8} square myriameter

Table 3. Volume $[L^3]$

Multiply Number of → by to Obtain ↓	Bushels (dry)	Cubic centimeters	Cubic feet	Cubic inches	Cubic meters	Cubic yards	Gallons (liquid)	Liters	Pints (liquid)	Quarts (liquid)
Bushels (dry)	1		0.8036	4.651×10^{-4}	28.38			$2\ 838 \times 10^{-2}$		
Cubic centimeters	3.524×10^{4}	1	2.832×10^{4}	16.39	10^{6}	7.646×10^{5}	3785	1000	473.2	946.4
Cubic feet	1.2445	3.531×10^{-5}	1	5.787×10^{-4}	35.31	27	0.1337	3.531×10^{-2}	1.671×10^{-2}	3.342×10^{-2}
Cubic inches	2150.4	6.102×10^{-2}	1728	1	6.102×10^{4}	46,656	231	61.02	28.87	57.75
Cubic meters	3.524×10^{-2}	10^{-6}	2.832×10^{-2}	1.639×10^{-5}	1	0.7646	3.785×10^{-3}	0.001	4.732×10^{-4}	9.464×10^{-4}
Cubic yards		1.308×10^{-6}	$3\ 704 \times 10^{-2}$	2.143×10^{-5}	1.308	1	4.951×10^{-3}	1.308×10^{-3}	6.189×10^{-4}	1.238×10^{-3}
Gallons (liquid)		$2\ 642 \times 10^{-4}$	7.481	4.329×10^{-3}	264.2	202.0	1	0.2642	0.125	0.25
Liters	35.24	0.001	28.32	1.639×10^{-2}	1000	764.6	3.785	1	0.4732	0.9464
Pints (liquid)		2.113×10^{-3}	59.84	3.463×10^{-2}	2113	1616	8	2.113	1	2
Quarts (liquid)......		$1\ 057 \times 10^{-3}$	29.92	1.732×10^{-2}	1057	807.9	4	1.057	0.5	1

Metric Multiples

10 milliliters	= 1 centiliter	= 0.338 fluid ounce
10 centiliters	= 1 deciliter	= 0.845 liquid gill
10 deciliters	= 1 liter	= 1.0567 liquid quarts
10 liters	= 1 dekaliter	= 2.6417 liquid gallons
10 dekaliters	= 1 hectoliter	= 2.8375 U. S. bushels
10 hectoliters	= 1 kiloliter (or stere)	= 28.375 U. S. bushels

Cubic Measure

1 cord of wood = a pile cut 4 feet long, piled 4 feet high and 8 feet on the ground = 128 cubic feet

1 perch of stone = a quantity 1 $^1/_2$ feet thick, 1 foot high and 16 $^1/_2$ feet long = 24 $^3/_4$ cubic feet

(NOTE.—A perch of stone is, however, often computed differently in different localities; thus, in most if not all of the States and Territories west of the Mississippi, stonemasons figure rubble by the perch of 16 $^1/_2$ cubic feet. In Philadelphia, 22 cubic feet are called a perch. In Chicago, stone is measured by the cord of 100 cubic feet. Check should be made against local practice.)

Board Measure

In board measure, boards are assumed to be one inch in thickness. Therefore, feet board measure of a stick of square timber = length in feet × breadth in feet × thickness in inches.

Shipping Measure

For register tonnage or measurement of the entire internal capacity of a vessel, it is arbitrarily assumed, to facilitate computation, that:

100 cubic feet = 1 register ton

For the measurement of cargo:

40 cubic feet = 1 U. S. shipping ton = 32.143 U. S. bushels
42 cubic feet = 1 British shipping ton = 32.703 Imperial bushels

Dry Measure

One U. S. Winchester bushel contains 1.2445 cubic feet or 2150.42 cubic inches. It holds 77.601 pounds distilled water at 62° F.

(NOTE.—The above is a *struck* bushel. A *heaped* bushel in general equals 1 $^1/_4$ struck bushels, although for apples and pears it contains 1.2731 struck bushels = 2737.72 cubic inches.)

One U. S. gallon (dry measure) = $^1/_8$ bushel and contains 268.8 cubic inches.

(NOTE.—This is not a legal U. S. *dry measure* and therefore is given for comparison only.)

One British Imperial bushel contains 1.2843 cubic feet or 2219.36 cubic inches. It holds 80 pounds distilled water at 62° F.

One British Imperial gallon = $^1/_8$ Imperial bushel and contains 277.42 cubic inches.

1 Winchester bushel = 0.9694 Imperial bushel
1 Imperial bushel = 1.032 Winchester bushels

Same relations as above maintain for gallons (dry measure)

(NOTE.—1 U. S. gallon (dry) = 1.164 U. S. gallons (liquid)).

U. S. Units

2 pints	= 1 quart	=	67.2 cubic inches
4 quarts	= 1 gallon * = 8 pints	=	268.8 cubic inches
2 gallons*	= 1 peck = 16 pints = 8 quarts	=	537.6 cubic inches
4 pecks	= 1 bushel = 64 pints = 32 quarts = 8 gallons*	=	2150.42 cubic inches

1 cubic foot contains 6.428 gallons (dry measure)*

Liquid Measure

One U. S. gallon (liquid measure) contains 231 cubic inches. It holds 8.336 pounds distilled water at 62° F.

One British Imperial gallon contains 277.42 cubic inches. It holds 10 pounds distilled water at 62° F.

1 U. S. gallon (liquid) = 0.8327 Imperial gallon
1 Imperial gallon = 1.201 U. S. gallons (liquid)

(NOTE.—1 U. S. gallon (liquid) = 0.8594 U. S. gallon (dry)).

U. S. Units

4 gills	= 1 pint	=	16 fluid ounces
2 pints	= 1 quart = 8 gills	=	32 fluid ounces
4 quarts	= 1 gallon = 32 gills = 8 pints	=	128 fluid ounces

1 cubic foot contains 7.4805 gallons (liquid measure)

Apothecaries' Fluid Measure

60 minims = 1 fluid drachm. 8 drachms = 1 fluid ounce

In the U. S. a fluid ounce is the 128th part of a U. S. gallon, or 1.805 cu in. or 29.58 cu cm. It contains 455.8 grains of water at 62° F. In Great Britain the fluid ounce is 1.732 cu in. and contains 1 ounce avoirdupois (or 437.5 grains) of water at 62° F.

* The *gallon* is not a U. S. legal *dry measure*.

Table 4. Plane Angle [*No Dimensions*]

Multiply Number of → by to Obtain ↓	Degrees	Minutes	Quadrants	Radians *	Revolutions * (Circumferences)	Seconds
Degrees	1	1.667×10^{-2}	90	57.30	360	2.778×10^{-4}
Minutes	60	1	5400	3438	2.16×10^{4}	1.667×10^{-2}
Quadrants	1.111×10^{-2}	1.852×10^{-4}	1	0.6366	4	3.087×10^{-6}
Radians *	1.745×10^{-2}	2.909×10^{-4}	1.571	1	6.283	4.848×10^{-6}
Revolutions * (Circumferences)	2.778×10^{-3}	4.630×10^{-5}	0.25	0.1591	1	7.716×10^{-7}
Seconds	3600	60	3.24×10^{5}	2.063×10^{5}	1.296×10^{6}	1

* 2π radians = 1 circumference = 360 degrees by definition.

Table 5. Solid Angle [*No Dimensions*]

Multiply Number of → by to Obtain ↓	Hemispheres	Spheres *	Spherical right angles	Steradians †
Hemispheres	1	2	0.25	0.1592
Spheres *	0.5	1	0.125	7.958×10^{-2}
Spherical right angles	4	8	1	0.6366
Steradians †	6.283	12.57	1.571	1

* A sphere is the total solid angle about a point. † 4π steradians = 1 sphere by definition.

Table 6. Time [*T*]

Multiply Number of → by to Obtain ↓	Days	Hours	Minutes	Months (average)*	Seconds	Weeks
Days	1	4.167×10^{-2}	6.944×10^{-4}	30.42	1.157×10^{-5}	7
Hours	24	1	1.667×10^{-2}	730.0	2.778×10^{-4}	168
Minutes	1440	60	1	4.380×10^{4}	1.667×10^{-2}	1.008×10^{4}
Months (average) *	3.288×10^{-2}	1.370×10^{-3}	2.283×10^{-5}	1	3.806×10^{-7}	0.2302
Seconds	8.64×10^{4}	3600	60	2.628×10^{6}	1	6.048×10^{5}
Weeks	0.1429	5.952×10^{-3}	9.921×10^{-5}	4.344	1.654×10^{-6}	1

* One common year = 365 days; one leap year = 366 days; one average month = 1/12 of a common year.

Table 7. Linear Velocity [LT^{-1}]

Multiply Number of → by to Obtain ↓	Centimeters per second	Feet per minute	Feet per second	Kilometers per hour	Kilometers per minute	Knots per hour *	Meters per minute	Meters per second	Miles per hour	Miles per minute
Centimeters per second	1	0.5080	30.48	27.78	1667	51.48	1.667	100	44.70	2682
Feet per minute	1.969	1	60	54.68	3281	101.3	3.281	196.8	88	5280
Feet per second	3.281×10^{-2}	1.667×10^{-2}	1	0.9113	54.68	1.689	5.468×10^{-2}	3.281	1.467	88
Kilometers per hour	0.036	1.829×10^{-2}	1.097	1	60	1.853	0.06	3.6	1.609	96.54
Kilometers per minute	0.0006	3.048×10^{-4}	1.829×10^{-2}	1.667×10^{-2}	1	3.088×10^{-2}	0.001	0.06	2.682×10^{-2}	1.609
Knots per hour,*	1.943×10^{-2}	9.868×10^{-3}	0.5921	0.5396	32.38	1	3.238×10^{-2}	1.943	0.8684	52.10
Meters per minute	0.6	0.3048	18.29	16.67	1000	30 88	1	60	26.82	1609
Meters per second	0.01	5.080×10^{-3}	0.3048	0.2778	16.67	0.5148	1.667×10^{-2}	1	0.4770	26.82
Miles per hour	2.237×10^{-2}	1.136×10^{-2}	0.6818	0.6214	37.28	1.152	3.728×10^{-2}	2.237	1	60
Miles per minute	3.728×10^{-4}	1.892×10^{-4}	1.136×10^{-2}	1.036×10^{-2}	0.6214	$1\ 919 \times 10^{-2}$	6.214×10^{-4}	3.728×10^{-2}	1.667×10^{-2}	1

* Usually called "knots."

The Miner's Inch

(Used in Measuring Flow of Water)

An Act of the California legislature, May 23, 1901, makes the standard miner's inch 1.5 cu ft per minute, measured through any aperture or orifice.

The term Miner's Inch is more or less indefinite, for the reason that California water companies do not all use the same head above the center of the aperture, and the inch varies from 1.36 to 1.73 cu ft per minute, but the most common measurement is through an aperture 2 in. high and whatever length is required, and through a plank 1 1/4 in. thick. The lower edge of the aperture should be 2 in. above the bottom of the measuring-box, and the plank 5 in. high above the aperture, thus making a 6-in. head above the center of the stream. Each square inch of this opening represents a miner's inch, which is equal to a flow of 1.5 cu ft per minute.

Table 8. Angular Velocity [T^{-1}]

to Obtain ↓ / Multiply Number of → by	Degrees per second	Radians per second	Revolutions per minute	Revolutions per second
Degrees per second	1	57.30	6	360
Radians per second	1.745×10^{-2}	1	0.1047	6.283
Revolutions per minute	0.1667	9.549	1	60
Revolutions per second	2.778×10^{-3}	0.1592	1.667×10^{-2}	1

Table 9. Linear Acceleration * [LT^{-2}]

to Obtain ↓ / Multiply Number of → by	Centimeters per second per second	Feet per second per second	Kilometers per hour per second	Meters per second per second	Miles per hour per second
Centimeters per second per second	1	30.48	27.78	100	44.70
Feet per second per second	3.281×10^{-2}	1	0.9113	3.281	1.467
Kilometers per hour per second	0.036	1.097	1	3.6	1.609
Meters per second per second	0.01	0.3048	0.2778	1	0.4770
Miles per hour per second	2.237×10^{-2}	0.6818	0.6214	2.237	1

* The (standard) acceleration due to gravity (g_0) = 980.7 cm per sec per sec, = 32.17 feet per sec per sec = 35.30 km per hour per sec = 9.807 meters per sec per sec = 21.94 miles per hour per sec.

Table 10. Angular Acceleration [T^{-2}]

to Obtain ↓ / Multiply Number of → by	Radians per second per second	Revolutions per minute per minute	Revolutions per minute per second	Revolutions per second per second
Radians per second per second	1	1.745×10^{-3}	0.1047	6.283
Revolutions per minute per minute	573.0	1	60	3600
Revolutions per minute per second	9.549	1.667×10^{-2}	1	60
Revolutions per second per second	0.1592	2.778×10^{-4}	1.667×10^{-2}	1

Table 11. Mass [M] and Weight *

Multiply Number of → by to Obtain ↓	Grains	Grams	Kilograms	Milligrams	Ounces †	Pounds †	Tons (long)	Tons (metric)	Tons (short)
Grains	1	15.43	1.543 $\times 10^{4}$	1 543 $\times 10^{-2}$	437.5	7000			
Grams	6.481 $\times 10^{-2}$	1	1000	0.001	28.35	453.6	1.016 $\times 10^{6}$	10^{6}	9 072 10^{5}
Kilograms	6.481 $\times 10^{-5}$	0.001	1	10^{-6}	2.835 $\times 10^{-2}$	0.4536	1016	1000	907.2
Milligrams	64.81	1000	10^{6}	1	2.835 $\times 10^{4}$	4.536 $\times 10^{5}$	1.016 $\times 10^{9}$	10^{9}	9.072 $\times 10^{8}$
Ounces †	2.286 $\times 10^{-3}$	3.527 $\times 10^{-2}$	35.27	3.527 $\times 10^{-5}$	1	16	3 584 $\times 10^{4}$	3 527 $\times 10^{4}$	3 2 $\times 10^{4}$
Pounds †	1.429 $\times 10^{-4}$	2 205 $\times 10^{-3}$	2.205	2.205 $\times 10^{-6}$	6.250 $\times 10^{-2}$	1	2240	2205	2000
Tons (long)		9.842 $\times 10^{-7}$	9.842 $\times 10^{-4}$	9.842 $\times 10^{-10}$	2.790 $\times 10^{-5}$	4.464 $\times 10^{-4}$	1	0.9842	0.8929
Tons (metric)		10^{-6}	0.001	10^{-9}	2.835 $\times 10^{-5}$	4.536 $\times 10^{-4}$	1.016	1	0.9072
Tons (short)		1 102 $\times 10^{-6}$	1.102 $\times 10^{-3}$	1.102 $\times 10^{-9}$	3.125 $\times 10^{-5}$	0.0005	1.120	1.102	1

* These same conversion factors apply to the *gravitational* units of force having the corresponding names. The dimensions of these units when used as gravitational units of force are MLT^{-2}; see table for *Force*.

† Avoirdupois pounds and ounces.

Metric Multiples

10^6 micrograms = 10^3 milligrams = 10^2 centigrams = 10 decigrams = 1 gram = 10^{-1} dekagram = 10^{-2} hectogram = 10^{-3} kilogram = 10^{-4} myriagram = 10^{-6} megagram

Avoirdupois Weight

(Used Commercially)

27.343 grains	= 1 drachm
16 drachms	= 1 ounce (oz) = 437.5 grains
16 ounces	= 1 pound (lb) = 7000 grains
28 pounds	= 1 quarter (qr)
4 quarters	= 1 hundredweight (cwt) = 112 pounds
20 hundredweight	= 1 gross or long ton *
2000 pounds	= 1 net or short ton

(* NOTE.—The long ton is used by the U. S. custom-houses in collecting duties upon foreign goods. It is also used in freighting coal and selling it wholesale.)

14 pounds	= 1 stone; 100 pounds = 1 quintal

Troy Weight

(Used in weighing gold or silver)

24 grains	= 1 pennyweight (dwt)
20 pennyweights	= 1 ounce (oz) = 480 grains
12 ounces	= 1 pound (lb) = 5760 grains

The grain is the same in Avoirdupois, Troy and Apothecaries' weights. A carat, for weighing diamonds = 3.086 grains = 0.200 gram. (International Standard, 1913.)

1 pound troy	= .8229 pound avoirdupois
1 pound avoirdupois	= 1.2153 pounds troy

Apothecaries' Weight

(Used in compounding medicines)

20 grains = 1 scruple (℈)
3 scruples = 1 drachm (ʒ) = 60 grains
8 drachms = 1 ounce (℥) = 480 grains
12 ounces = 1 pound (lb) = 5760 grains

The grain is the same in Avoirdupois, Troy and Apothecaries' weights.

1 pound apothecaries = 0.82286 pound avoirdupois
1 pound avoirdupois = 1.2153 pounds apothecaries

Table 12. Density or Mass per Unit Volume $[ML^{-3}]$

Multiply Number of → by to Obtain ↓	Grams per cubic centimeter	Kilograms per cubic meter	Pounds per cubic foot	Pounds per cubic inch
Grams per cubic centimeter	1	0.001	1.602×10^{-2}	27.68
Kilograms per cubic meter	1000	1	16.02	2.768×10^{4}
Pounds per cubic foot	62.43	6.243×10^{-2}	1	1728
Pounds per cubic inch	3.613×10^{-2}	3.613×10^{-5}	5.787×10^{-4}	1
Pounds per mil foot *	3.405×10^{-7}	3.405×10^{-10}	5.456×10^{-9}	9.425×10^{-6}

* Unit of volume is a volume one foot long and one circular mil in cross-section area.

Table 13. Force * $[MLT^{-2}]$ or $[F]$

Multiply Number of → by to Obtain ↓	Dynes	Grams	Joules per cm	Joules per meter	Kilo-grams	Pounds	Poundals
Dynes	1	980.7	10^{7}	10^{5}	9.807×10^{5}	4.448×10^{5}	1.383×10^{4}
Grams	1.020×10^{-3}	1	1.020×10^{4}	102.0	1000	453.6	14.10
Joules per cm	10^{-7}	9.807×10^{-5}	1	.01	9.807×10^{-2}	4.448×10^{-2}	1.383×10^{-3}
Joules per meter	10^{-5}	9.807×10^{-3}	100	1	9.807	4.448	0.1383
Kilograms	1.020×10^{-6}	0.001	10.20	0.1020	1	0.4536	1.410×10^{-2}
Pounds	2.248×10^{-6}	2.205×10^{-3}	22.48	0.2248	2.205	1	3.108×10^{-2}
Poundals	7.233×10^{-5}	7.093×10^{-2}	723.3	7.233	70.93	32.17	1

* Conversion factors between absolute and gravitational units apply only under standard acceleration due to gravity conditions. (See Sec. 3.)

Table 14. Torque or Moment of Force $[ML^2T^{-2}]$ or $[FL]$ *

to Obtain ↓ / by / Multiply Number of →	Dyne-centimeters	Gram-centimeters	Kilogram-meters	Pound-feet
Dyne-centimeters	1	980.7	9.807×10^7	1.356×10^7
Gram-centimeters	1.020×10^{-3}	1	10^5	1.383×10^4
Kilogram-meters	1.020×10^{-8}	10^{-5}	1	0.1383
Pound-feet	7.376×10^{-8}	7.233×10^{-5}	7.233	1

* Same dimensions as energy.

Table 15. Pressure or Force per Unit Area $[ML^{-1}T^{-2}]$ or $[FL^{-2}]$

to Obtain ↓ / by / Multiply Number of →	Atmospheres *	Baryes or dynes per square centimeter †	Centimeters of mercury at 0° C ‡	Inches of mercury at 0° C ‡	Inches of water at 4° C	Feet of water at 4° C	Kilograms per square meter §	Pounds per square foot	Pounds per square inch	Tons (short) per square foot
Atmospheres *	1	9.869×10^{-7}	1.316×10^{-2}	3.342×10^{-2}	$2\ 458 \times 10^{-3}$	2.950×10^{-2}	9.678×10^{-5}	4.725×10^{-4}	6.804×10^{-2}	0.9450
Baryes or dynes per square centimeter †		1					98.07	478 8	6.895×10^4	9.576×10^5
Centimeters of mercury at 0° C ‡	76.00	7.501×10^{-5}	1	2 540	0.1868	2.232	7.356×10^{-3}	3.591×10^{-2}	5.171	71.83
Inches of mercury at 0° C ‡	29.92	$2\ 953 \times 10^{-5}$	0.3937	1	7.355×10^{-2}	0.8826	$2\ 896 \times 10^{-3}$	1.414×10^{-2}	2.036	28.28
Inches of water at 4° C	406.8	4.015×10^{-4}	5.354	13.60	1	12	$3\ 937 \times 10^{-2}$	0.1922	27.68	384.5
Feet of water at 4° C	33.90	3.346×10^{-5}	0.4460	1.133	$8\ 333 \times 10^{-2}$	1	3.281×10^{-3}	1.602×10^{-2}	2.307	32.04
Kilograms per square meter §	1.033×10^4	1.020×10^{-2}	136.0	345.3	25.40	304.8	1	4.882	703.1	9765
Pounds per square foot	2117	2.089×10^{-3}	27.85	70.73	5.204	62.43	0.2048	1	144	2000
Pounds per square inch	14.70	1.450×10^{-5}	0.1934	0.4912	3.613×10^{-2}	0.4335	1.422×10^{-3}	6.944×10^{-3}	1	13.89
Tons (short) per square foot	1.058							0.0005	0.072	1

* Definition: One atmosphere (standard) = 76 cm of mercury at 0° C.
† Sometimes called a bar.
‡ To convert height h of a column of mercury at t degrees Centigrade to the equivalent height h_0 at 0° C. use $h_0 = h\left\{1 - \frac{(m - l)t}{1 + mt}\right\}$ where $m = 0.0001818$ and $l = 18.4 \times 10^{-6}$ if the scale is engraved on brass; $l = 8.5 \times 10^{-6}$ if on glass. This assumes the scale is correct at 0° C; for other cases (any liquid) see International Critical Tables, vol. 1, p. 68.
§ 1 gram per sq cm = 10 kilograms per sq m.

Table 16. Energy, Work and Heat * $[ML^2T^{-2}]$ or $[FL]$

Multiply Number of → by to Obtain ↓	British thermal units †	Centimeter-grams	Ergs or centimeter-dynes	Foot-pounds	Horsepower-hours	Joules ‡ or watt-seconds	Kilogram-calories †	Kilowatt-hours	Meter-kilograms	Watt-hours
British thermal units †	1	9.297 $\times 10^{-8}$	9.480 $\times 10^{-11}$	1.285 $\times 10^{-3}$	2545	9.480 $\times 10^{-4}$	3.969	3413	9 297 $\times 10^{-3}$	3.413
Centimeter-grams	1.076 $\times 10^{7}$	1	1.020 $\times 10^{-3}$	1.383 $\times 10^{4}$	2.737 $\times 10^{10}$	1.020 $\times 10^{4}$	4 269 $\times 10^{7}$	3.671 $\times 10^{10}$	10^{5}	3.671 $\times 10^{7}$
Ergs or centimeter-dynes	1.055 $\times 10^{10}$	980.7	1	1.356 $\times 10^{7}$	2 684 $\times 10^{13}$	10^{7}	4 186 $\times 10^{10}$	3 6 $\times 10^{13}$	9.807 $\times 10^{7}$	3.6 $\times 10^{10}$
Foot-pounds	778.0	7.233 $\times 10^{-5}$	7.367 $\times 10^{-8}$	1	1.98 $\times 10^{6}$	0.7376	3087	2.655 $\times 10^{6}$	7.233	2655
Horsepower-hours	3 929 $\times 10^{-4}$	3 654 $\times 10^{-11}$	3.722 $\times 10^{-14}$	5.050 $\times 10^{-7}$	1	3.722 $\times 10^{-7}$	1.559 $\times 10^{-3}$	1.341	3 653 $\times 10^{-6}$	1.341 $\times 10^{-3}$
Joules ‡ or watt-seconds	1054.8	9 807 $\times 10^{-5}$	10^{-7}	1.356	2 684 $\times 10^{6}$	1	4186	3.6 $\times 10^{6}$	9.807	3600
Kilogram-calories †	0.2520	2.343 $\times 10^{-8}$	2.389 $\times 10^{-11}$	3.239 $\times 10^{-4}$	641.3	2.389 $\times 10^{-4}$	1	860.0	2.343 $\times 10^{-3}$	0.8600
Kilowatt-hours	2 930 $\times 10^{-4}$	2 724 $\times 10^{-11}$	2.778 $\times 10^{-14}$	3.766 $\times 10^{-7}$	0 7457	2.778 $\times 10^{-7}$	1.163 $\times 10^{-3}$	1	2 724 $\times 10^{-6}$	0.001
Meter-kilograms	107.6	10^{-5}	1.020 $\times 10^{-8}$	0.1383	2.737 $\times 10^{5}$	0.1020	426.9	3.671 $\times 10^{5}$	1	367.1
Watt-hours	0.2930	2.724 $\times 10^{-8}$	2 778 $\times 10^{-11}$	3.766 $\times 10^{-4}$	745.7	2.778 $\times 10^{-4}$	1.163	1000	2.724 $\times 10^{-3}$	1

* See note at the bottom of Table 43.

† Mean calorie and Btu used throughout. One gram-calorie = 0.001 kilogram-calorie; one Ostwald calorie = 0.01 kilogram-calorie.

The IT cal, 1000 international steam-table calories, has been defined as the 1/860th part of the international kilowatthour (see *Mechanical Engineering*, Nov., 1935, p. 710). Its value is very nearly equal to the mean kilogram-calorie, 1 IT cal = 1.00037 kilogram-calories (mean). 1 Btu = 251.996 IT cal.

‡ Absolute joule, defined as 10^7 ergs. The international joule, based on the international ohm and ampere, equals 1.0003 absolute joules.

Table 17. Power or Rate of Doing Work * $[ML^2T^{-3}]$ or $[FLT^{-1}]$

Multiply Number of → by to Obtain ↓	British thermal units per minute	Ergs per second	Foot-pounds per minute	Foot-pounds per second	Horsepower *	Kilogram-calories per minute	Kilowatts	Metric horsepower	Watts
British thermal units per minute	1	5.689 $\times 10^{-9}$	1 285 $\times 10^{-3}$	7.712 $\times 10^{-2}$	42.41	3.969	56.89	41.83	5.689 $\times 10^{-2}$
Ergs per second	1 758 $\times 10^{8}$	1	2.259 $\times 10^{5}$	1.356 $\times 10^{7}$	7.457 $\times 10^{9}$	6.977 $\times 10^{8}$	10^{10}	7.355 $\times 10^{9}$	10^{7}
Foot-pounds per minute	778 0	4.426 $\times 10^{-6}$	1	60	3.3 $\times 10^{4}$	3087	4.426 $\times 10^{4}$	3.255 $\times 10^{4}$	44.26
Foot-pounds per second	12.97	7.376 $\times 10^{-8}$	1.667 $\times 10^{-2}$	1	550	51.44	737.6	542.5	0.7376
Horsepower *	2.357 $\times 10^{-2}$	1.341 $\times 10^{-10}$	3.030 $\times 10^{-5}$	1.818 $\times 10^{-3}$	1	9.355 $\times 10^{-2}$	1.341	0.9863	1.341 $\times 10^{-3}$
Kilogram-calories per minute	0.2520	1 433 $\times 10^{-9}$	3.239 $\times 10^{-4}$	1.943 $\times 10^{-2}$	10.69	1	14.33	10.54	1.433 $\times 10^{-2}$
Kilowatts	1.758 $\times 10^{-2}$	10^{-10}	2.260 $\times 10^{-5}$	1.356 $\times 10^{-3}$	0.7457	6 977 $\times 10^{-2}$	1	0.7355	10^{-3}
Metric horsepower	2.390 $\times 10^{-2}$	1.360 $\times 10^{-10}$	3.072 $\times 10^{-5}$	1.843 $\times 10^{-3}$	1.014	9.485 $\times 10^{-2}$	1.360	1	1.360 $\times 10^{-3}$
Watts	17.58	10^{-7}	2.260 $\times 10^{-2}$	1.356	745.7	69.77	1000	735.5	1

1 Cheval-vapeur = 75 kilogram-meters per second
1 Poncelet = 100 kilogram-meters per second

* The "horsepower" used in these tables is equal to 550 foot-pounds per second by definition. Other definitions are one horsepower equals 746 watts (U. S. and Great Britain) and one horsepower equals 736 watts (continental Europe). Neither of these latter definitions is equivalent to the first; the "horsepowers" defined in these latter definitions are widely used in the rating of electrical machinery.

Table 18. Quantity of Electricity and Dielectric Flux $[Q]$

Multiply Number of → by to Obtain ↓	Abcoulombs	Ampere-hours	Coulombs	Faradays	Stat-coulombs
Abcoulombs	1	360	0.1	9649	3.335 $\times 10^{-11}$
Ampere-hours	2.778 $\times 10^{-3}$	1	2.778 $\times 10^{-4}$	26.80	9.259 $\times 10^{-14}$
Coulombs	10	3600	1	9.649 $\times 10^{4}$	3.335 $\times 10^{-10}$
Faradays	1.036 $\times 10^{-4}$	3.731 $\times 10^{-2}$	1.036 $\times 10^{-5}$	1	3.457 $\times 10^{-15}$
Statcoulombs	2.998 $\times 10^{10}$	1.080 $\times 10^{13}$	2.998 $\times 10^{9}$	2.893 $\times 10^{14}$	1

Table 19. Charge per Unit Area and Dielectric Flux Density $[QL^{-2}]$

to Obtain ↓ by Multiply Number of →	Abcoulombs per square centimeter	Coulombs per square centimeter	Coulombs per square inch	Statcoulombs per square centimeter
Abcoulombs per square centimeter	1	0.1	1.550×10^{-2}	3.335×10^{-11}
Coulombs per square centimeter	10	1	0.1550	3.335×10^{-10}
Coulombs per square inch	64.52	6.452	1	2.151×10^{-9}
Statcoulombs per square centimeter	2.998×10^{10}	2.998×10^{9}	4.647×10^{8}	1

Table 20. Electric Current $[QT^{-1}]$

to Obtain ↓ by Multiply Number of →	Abamperes	Amperes	Statamperes
Abamperes	1	0.1	3.335×10^{-11}
Amperes	10	1	3.335×10^{-10}
Statamperes	2.998×10^{10}	2.998×10^{9}	1

Table 21. Current Density $[QT^{-1}L^{-2}]$

to Obtain ↓ by Multiply Number of →	Abamperes per square centimeter	Amperes per square centimeter	Amperes per square inch	Statamperes per square centimeter
Abamperes per square centimeter	1	0.1	1.550×10^{-2}	3.335×10^{-11}
Amperes per square centimeter	10	1	0.1550	3.335×10^{-10}
Amperes per square inch	64.52	6.452	1	2.151×10^{-9}
Statamperes per square centimeter	2.998×10^{10}	2.998×10^{9}	4.647×10^{8}	1

Table 22. Electric Potential and Electromotive Force $[MQ^{-1}L^2T^{-2}]$ or $[FQ^{-1}L]$

Multiply Number of → by, to Obtain ↓	Abvolts	Microvolts	Millivolts	Statvolts	Volts
Abvolts	1	100	10^5	2.998×10^{10}	10^8
Microvolts	0.01	1	1000	2.998×10^8	10^6
Millivolts	10^{-5}	0.001	1	2.998×10^5	1000
Statvolts	3.335×10^{-11}	3.335×10^{-9}	3.335×10^{-6}	1	3.335×10^{-3}
Volts	10^{-8}	10^{-6}	0.001	299.8	1

Table 23. Electric Field Intensity and Potential Gradient $[MQ^{-1}LT^{-2}]$ or $[FQ^{-1}]$

Multiply Number of → by, to Obtain ↓	Abvolts per centimeter	Microvolts per meter	Millivolts per meter	Statvolts per centimeter	Volts per centimeter	Kilovolts per centimeter	Volts per millimeter	Volts per inch	Volts per mil
Abvolts per centimeter	1	1	1000	2.998×10^{10}	10^8	10^{11}	10^9	3.937×10^7	3.937×10^{10}
Microvolts per meter	1	1	1000	2.998×10^{10}	10^8	10^{11}	10^9	3.937×10^7	3.937×10^{10}
Millivolts per meter	0.001	0.001	1	2.998×10^7	10^5	10^8	10^6	3.937×10^4	3.937×10^7
Statvolts per centimeter	3.335×10^{-11}	3.335×10^{-11}	3.335×10^{-8}	1	3.335×10^{-3}	3.335	3.335×10^{-2}	1.313×10^{-3}	1.313
Volts per centimeter	10^{-8}	10^{-8}	10^{-5}	299.8	1	1000	10	0.3937	393.7
Kilovolts per centimeter	10^{-11}	10^{-11}	10^{-8}	0.2998	0.001	1	0.01	3.937×10^{-4}	0.3937
Volts per millimeter	10^{-9}	10^{-9}	10^{-6}	29.98	0.1	100	1	3.937×10^{-2}	39.37
Volts per inch	2.540×10^{-8}	2.540×10^{-8}	2.540×10^{-5}	761.6	2.540	2540	25.40	1	1000
Volts per mil	2.540×10^{-11}	2.540×10^{-11}	2.540×10^{-8}	0.7616	2.540×10^{-3}	2.540	2.540×10^{-2}	0.001	1

Table 24. Electric Resistance $[MQ^{-2}L^2T^{-1}]$ or $[FQ^{-2}LT]$

Multiply Number of → by to Obtain ↓	Abohms	Megohms	Microhms	Ohms	Statohms
Abohms	1	10^{15}	1000	10^9	8.988×10^{20}
Megohms	10^{-15}	1	10^{-12}	10^{-6}	8.988×10^5
Microhms	0.001	10^{12}	1	10^6	8.988×10^{17}
Ohms	10^{-9}	10^6	10^{-6}	1	8.988×10^{11}
Statohms	1.112×10^{-21}	1.112×10^{-6}	1.112×10^{-18}	1.112×10^{-12}	1

Electrical Conductance $[F^{-1}QL^{-1}T^{-1}]$

1 mho = 1 ohm^{-1} = 10^{-6} megmho = 10^6 micromho

Table 25. Electric Resistivity * $[MQ^{-2}L^3T^{-1}]$ or $[FQ^{-2}L^2T]$

Multiply Number of → by to Obtain ↓	Abohm-centimeters	Microhm-centimeters	Microhm-inches	Ohms (mil, foot)	Ohms (meter, gram) †
Abohm-centimeters	1	1000	2540	166.2	$\frac{10^5}{\delta}$
Microhm-centimeters	0.001	1	2.540	0.1662	$\frac{100}{\delta}$
Microhm-inches	3.937×10^{-4}	0.3937	1	6.545×10^{-2}	$\frac{39.37}{\delta}$
Ohms (mil, foot)	6.015×10^{-3}	6.015	15.28	1	$\frac{601.5}{\delta}$
Ohms (meter, gram) †	$10^{-5}\delta$	0.01δ	$2.540 \times 10^{-2}\delta$	$1.662 \times 10^{-3}\delta$	1

* In this table δ is density in grams per cm.³ The following names, corresponding respectively to those at the tops of columns, are sometimes used: abohms per cm cube; microhms per cm cube; microhms per inch cube; ohms per mil-foot; ohms per meter-gram. The first four columns are headed by units of *volume* resistivity, the last by a unit of *mass* resistivity. The dimensions of the latter are $Q^{-2}L^6T^{-1}$; not those given in the heading of the table.

† One ohm (meter, gram) = 5710 ohms (mile, pound).

Table 26. Electric Conductivity * $[M^{-1}Q^2L^{-3}T]$ or $[F^{-1}Q^2L^{-2}T^{-1}]$

Multiply Number of → by, to Obtain ↓	Abmhos per cm	Mhos (mil, foot)	Mhos (meter, gram)	Micromhos per cm	Micromhos per inch
Abmhos per cm	1	6.015×10^3	$10^{-5}\delta$	0.001	3.937×10^{-4}
Mhos (mil, foot)	166.2	1	$1.662 \times 10^{-3}\delta$	0.1662	6.524×10^{-2}
Mhos (meter, gram)	$10^5/\delta$	$601.5/\delta$	1	$100/\delta$	$39.37/\delta$
Micromhos per cm	1000	6.015	0.01δ	1	0.3937
Micromhos per inch	2540	15.28	$2.540 \times 10^{-2}\delta$	2.540	1

* See footnote of Table 51, Electric Resistivity. Names sometimes used are abmho per cm cube, mho per mil-foot, etc. Dimensions of mass conductivity are $Q^2L^{-6}T$.

Table 27. Capacitance $[M^{-1}Q^2L^{-2}T^2]$ or $[F^{-1}Q^2L^{-1}]$

Multiply Number of → by, to Obtain ↓	Abfarads	Farads	Microfarads	Statfarads
Abfarads	1	10^{-9}	10^{-15}	1.112×10^{-21}
Farads	10^9	1	10^{-6}	1.112×10^{-12}
Microfarads	10^{15}	10^6	1	1.112×10^{-6}
Statfarads	8.988×10^{20}	8.988×10^{11}	8.988×10^5	1

Table 28. Inductance $[MQ^{-2}L^2]$ or $[FQ^{-2}LT^2]$

Multiply Number of → by, to Obtain ↓	Abhenries *	Henries	Microhenries	Millihenries	Stathenries
Abhenries *	1	10^9	1000	10^6	8.988×10^{20}
Henries	10^{-9}	1	10^{-6}	0.001	8.988×10^{11}
Microhenries	0.001	10^6	1	1000	8.988×10^{17}
Millihenries	10^{-6}	1000	0.001	1	8.988×10^{14}
Stathenries	1.112×10^{-21}	1.112×10^{-12}	1.112×10^{-18}	1.112×10^{-15}	1

* An abhenry is sometimes called a "centimeter." See footnote to Table 56 on "Magnetic Flux Density."

Table 29. Magnetic Flux $[MQ^{-1}L^2T^{-1}]$ or $[FQ^{-1}LT]$

Multiply Number of → by to Obtain ↓	Kilolines	Maxwells (or lines)	Webers
Kilolines	1	0.001	10^5
Maxwells (or lines)	1000	1	10^8
Webers	10^{-5}	10^{-8}	1

Table 30. Magnetic Flux Density $[MQ^{-1}T^{-1}]$ or $[FQ^{-1}L^{-1}T]$

Multiply Number of → by to Obtain ↓	Gausses (or lines per square centimeter)	Lines per square inch	Webers per square centimeter	Webers per square inch
Gausses * (or lines per square centimeter)	1	0.1550	10^8	1.550×10^7
Lines per square inch	6.452	1	6.452×10^8	10^8
Webers per square centimeter	10^{-8}	$1.550 \times {}^{-9}$	1	0.1550
Webers per square inch	6.452×10^{-8}	10^{-8}	6.452	1

* The name "gauss" is sometimes used for the unit of magnetic field intensity (1 gauss = 1 gilbert per cm). Since flux density = permeability × field intensity ($B = \mu H$) these two quantities have the same units if μ is considered dimensionless, just as 1 abhenry = μ × 1 cm hence the occasional name centimeter for an abhenry. The A.I.E.E. sanctions "gauss" for both B and H; physicists usually do not. In 1930 the International Electrotechnical Commission, of which the U. S. National Committee is the electrical standards committee of the American Standards Association, adopted the following names for units in the cgs electromagnetic system: Magnetomotive force, gilbert; magnetizing force, oersted; magnetic flux, maxwell; magnetic flux density, gauss. The name oersted has been used for a unit of reluctance in the U. S.

Table 31. Magnetic Potential and Magnetomotive Force $[QT^{-1}]$

Multiply Number of → by to Obtain ↓	Abampere-turns	Ampere-turns	Gilberts
Abampere-turns	1	0.1	7.958×10^{-2}
Ampere-turns	10	1	0.7958
Gilberts	12.57	1.257	1

Table 32. Magnetic Field Intensity, Potential Gradient and Magnetizing Force $[QL^{-1}T^{-1}]$

Multiply Number of → by to Obtain ↓	Abampere-turns per centimeter	Ampere-turns per centimeter	Ampere-turns per inch	Gilberts per centimeter *
Abampere-turns per centimeter	1	0.1	3.937×10^{-2}	7.958×10^{-2}
Ampere-turns per centimeter	10	1	0.3937	0.7958
Ampere-turns per inch	25.40	2.540	1	2.021
Gilberts per centimeter	12.57	1.257	0.4950	1

* Called "oersteds" by the I.E.C. (1930). See footnote of Table 56 on "Magnetic Flux Density."

Table 33. Specific Heat $[L^2T^{-2}t^{-1}]$

(t = temperature)

To change specific heat in gram-calories per gram per degree Centigrade to the units given in any line of the following table, multiply by the factor in the last column.

Unit of Heat or Energy	Unit of Mass	Temperature Scale*	Factor
Gram-calories	Gram	Centigrade	1
Kilogram-calories	Kilogram	Centigrade	1
British thermal units	Pound	Centigrade	1.800
British thermal units	Pound	Fahrenheit	1.000
Joules	Gram	Centigrade	4.186
Joules	Pound	Fahrenheit	1055.
Kilowatt-hours	Kilogram	Centigrade	1.163×10^{-3}
Kilowatt-hours	Pound	Fahrenheit	2.930×10^{-4}

* Temperature conversion formulas:

t_c = temperature in Centigrade degrees
t_f = temperature in Fahrenheit degrees
1 deg fahr = (5/9) deg cent.
$t_c = \frac{5}{9}(t_f - 32)$
$t_f = \frac{9}{5}t_c + 32$

Table 34. Thermal Conductivity $[MLT^{-3}t^{-1}]$

(t = temperature)

To convert thermal conductivity, in gram-calories transmitted per second from one face of a cube 1 cm on edge to the opposite face per degree Centigrade temperature difference between these faces, to the units given in any line of the following table, multiply by the factor in the last column.

Units of				Temperature Scale	Factor
Heat	Area	Thickness	Time		
Gram-calories	cm^2	cm	second	Centigrade	1
Kilogram-calories	m^2	cm	hour	Centigrade	3.6×10^4
British thermal units	ft^2	inch	hour	Fahrenheit	2903.
Joules	cm^2	cm	second	Centigrade	4.186
Joules	ft^2	inch	second	Fahrenheit	850.6
Kilowatt-hours	m^2	cm	hour	Centigrade	41.86
Kilowatt-hours	ft^2	inch	hour	Fahrenheit	0.8506

Table 35. Light

Multiply Number of → by to Obtain ↓	International candles	Hefners	10-cp pentanes	Carcels	Bougie decimales	English candles	German candles
International candles	1.00	0.90	10.0	9.61	1.00	1.04	1.055
Hefners	1.11	1.00	11.1	10.66	1.11	1.154	1.17
10-cp pentanes	0.10	0.09	1.00	0.96	0.10	0.104	0.105
Carcels	0.104	0.094	1.04	1.00	0.104	0.1	0.109
Bougie decimales	1.00	0.90	10.0	9.61	1.00	1.04	1.055
English candles	0.96	0.864	9.6	9.24	0.96	1.00	1.02
German candles	0.95	0.855	9.5	9.19	0.95	0.98	1.00

16. GAGES

Sheet Metal Gages

The important sheet metal gages in use in the United States are: the United States Standard Gage for sheet and plate iron and steel, the American Wire Gage (also called the Brown and Sharpe W.G.) for copper, aluminum, and brass and other non-ferrous alloys, the Tin Plate Gage, the Galvanized Sheet Gage, the American Zinc Gage, and the Birmingham Wire (or Stubs' Iron Wire) Gage. In Canada and England the Birmingham Gage (different from the Birmingham Wire Gage) and the Imperial Standard Wire Gage (S.W.G.) are used. Still other gages are used elsewhere. In Japan standard thickness of sheet metal is denoted by the thickness in millimeters. A standard Decimal Gage, in which the standard thicknesses are denoted by decimal parts of an inch and not by gage numbers, has been used in the United States. Copper sheets may be obtained with thicknesses any integral multiple of $^1/_{16}$ of an inch up to 2 inches. Heavy copper sheets may be obtained in definite weights per square foot. Each ounce of weight is equivalent to approximately 0.001352 inch thickness. Lead is usually ordered in this manner, each pound being equivalent to approximately 0.017 inch thickness.

The United States Standard Gage for sheet iron and steel (Act of Congress, March 3, 1893; formerly the legal standard for duties) is a *weight* gage based on a density for wrought iron of 480 pounds per cubic foot. Since 1893, steel (density of 489.6 lb per cu ft) has come into general use. A given gage number of this gage represents a fixed weight per unit area, hence a steel sheet will have a smaller thickness than a wrought iron sheet of the same gage number (but monel metal sheets are rolled to the thickness given for wrought iron to without regard to weight, which is about 552.2 lb per cu ft. Practice among steel manufacturers is irregular, some keeping the *thickness* constant for a given gage number irrespective of weight. If this practice is followed, the weight per square foot and per square meter given in the second and third columns of Table 36 will vary, whereas thickness will remain that given for wrought iron.

The American Wire Gage specifies thicknesses without regard to weight. For the basis of this gage see the next section (Wire Gages), where are also given the Birmingham W.G. and the S.W.G.

Tables of Thickness and Weight corresponding to United States Standard gage and American Wire gage numbers are shown in Tables 36 and 37. These tables are taken from Circular No. 391 of the Bureau of Standards, in which are given all the gages mentioned above and the tolerances customary in commerce. A committee of the American Standards Association is now working (1936) on the standardization of sheet metal gages.

Table 36. United States Standard Gage * for Sheet and Plate Iron and Steel, and Its Extension †

Gage No.	Weight per square foot		Weight per square meter	Approximate thickness			
				Wrought iron 480 lb/ft³		Steel and open-hearth iron 489.6 lb/ft³	
	Ounces	Pounds	kg	Inch	mm	Inch	mm
0000000	320	20.00	97.65	0.500	12.70	0.490	12.45
000000	300	18.75	91.55	.469	11.91	.460	11.67
00000	280	17.50	85.44	.438	11.11	.429	10.90
0000	260	16.25	79.34	.406	10.32	.398	10.12
000	240	15.00	73.24	.375	9.52	.368	9.34
00	220	13.75	67.13	.344	8.73	.337	8.56
0	200	12.50	61.03	.312	7.94	.306	7.78
1	180	11.25	54.93	.2812	7.14	.2757	7.00
2	170	10.62	51.88	.2656	6.75	.2604	6.62
3	160	10.00	48.82	.2500	6.35	.2451	6.23
4	150	9.375	45.77	.2344	5.95	.2298	5.84
5	140	8.750	42.72	.2188	5.56	.2145	5.45
6	130	8.125	39.67	.2031	5.16	.1991	5.06
7	120	7.500	36.62	.1875	4.76	.1838	4.67
8	110	6.875	33.57	.1719	4.37	.1685	4.28
9	100	6.250	30.52	.1562	3.97	.1532	3.89
10	90	5.625	27.46	.1406	3.57	.1379	3.50
11	80	5.000	24.41	.1250	3.18	.1225	3.11
12	70	4.375	21.36	.1094	2.778	.1072	2.724
13	60	3.750	18.31	.0938	2.381	.0919	2.335
14	50	3.125	15.26	.0781	1.984	.0766	1.946
15	45	2.812	13.73	.0703	1.786	.0689	1.751
16	40	2.500	12.21	.0625	1.588	.0613	1.557
17	36	2.250	10.99	.0562	1.429	.0551	1.400
18	32	2.000	9.765	.0500	1.270	.0490	1.245
19	28	1.750	8.544	.0438	1.111	.0429	1.090
20	24	1.500	7.324	.0375	.952	.0368	.934
21	22	1.375	6.713	.0344	.873	.0337	.856
22	20	1.250	6.103	.0312	.794	.0306	.778
23	18	1.125	5.493	.0281	.714	.0276	.700
24	16	1.000	4.882	.0250	.635	.0245	.623
25	14	.8750	4.272	.0219	.556	.0214	.545
26	12	.7500	3.662	.0188	.476	.0184	.467
27	11	.6875	3.357	.0172	.437	.0169	.428
28	10	.6250	3.052	.0156	.397	.0153	.389
29	9	.5625	2.746	.0141	.357	.0138	.350
30	8	.5000	2.441	.0125	.318	.0123	.311
31	7	.4375	2.136	.0109	.278	.0107	.272
32	6 1/2	.4062	1.983	.0102	.258	.0100	.253
33	6	.3750	1.831	.0094	.238	.0092	.233
34	5 1/2	.3438	1.678	.0086	.218	.0084	.214
35	5	.3125	1.526	.0078	.198	.0077	.195
36	4 1/2	.2812	1.373	.0070	.179	.0069	.175
37	4 1/4	.2656	1.297	.0066	.169	.0065	.165
38	4	.2500	1.221	.0062	.159	.0061	.156
39	3 3/4	.2344	1.144	.0059	.149	.0057	.146
40	3 1/2	.2188	1.068	.0055	.139	.0054	.136
41	3 3/8	.2109	1.030	.0053	.134	.0052	.131
42	3 1/4	.2031	.9917	.0051	.129	.0050	.126
43	3 1/8	.1953	.9536	.0049	.124	.0048	.122
44	3	.1875	.9155	.0047	.119	.0046	.117

* For the Galvanized Sheet Gage, add 2.5 ounces to the weight per square foot as given in the table. Gage numbers below 8 and above 34 are not used in the Galvanized Sheet Gage.

† Gage numbers greater than 38 were not in the standard as set up by law, but are in general use.

Table 37. American Wire Gage—Weights of Copper, Aluminum and Brass Sheets and Plates

Gage No.	Thickness		Approximate weight * per sq ft in lb		
	Inch	mm	Copper	Aluminum	Commercial (high) brass
0000	0.4600	11.68	21.27	6.49	20.27
000	.4096	10.40	18.94	5.78	18.05
00	.3648	9.266	16.87	5.14	16.07
0	.3249	8.252	15.03	4.58	14.32
1	.2893	7.348	13.38	4.08	12.75
2	.2576	6.544	11.91	3.632	11.35
3	.2294	5.827	10.61	3.234	10.11
4	.2043	5.189	9.45	2.880	9.00
5	.1819	4.621	8.41	2.565	8.01
6	.1620	4.115	7.49	2.284	7.14
7	.1443	3.665	6.67	2.034	6.36
8	.1285	3.264	5.94	1.812	5.66
9	.1144	2.906	5.29	1.613	5.04
10	.1019	2.588	4.713	1.437	4.490
11	.0907	2.305	4.195	1.279	3.996
12	.0808	2.053	3.737	1.139	3.560
13	.0720	1.828	3.330	1.015	3.172
14	.0641	1.628	2.965	0.904	2.824
15	.0571	1.450	2.641	.805	2.516
16	.0508	1.291	2.349	.716	2.238
17	.0453	1.150	2.095	.639	1.996
18	.0403	1.024	1.864	.568	1.776
19	.0359	0.9116	1.660	.506	1.582
20	.0320	.8118	1.480	.451	1.410
21	.0285	.7230	1.318	.402	1.256
22	.0253	.6438	1.170	.3567	1.115
23	.0226	.5733	1.045	.3186	0.996
24	.0201	.5106	0.930	.2834	.886
25	.0179	.4547	.828	.2524	.789
26	.0159	.4049	.735	.2242	.701
27	.0142	.3606	.657	.2002	.626
28	.0126	.3211	.583	.1776	.555
29	.0113	.2859	.523	.1593	.498
30	.0100	.2546	.4625	.1410	.4406
31	.00893	.2268	.4130	.1259	.3935
32	.00795	.2019	.3677	.1121	.3503
33	.00708	.1798	.3274	.0998	.3119
34	.00630	.1601	.2914	.0888	.2776
35	.00561	.1426	.2595	.0791	.2472
36	.00500	.1270	.2312	.0705	.2203
37	.00445	.1131	.2058	.0627	.1961
38	.00397	.1007	.1836	.0560	.1749
39	.00353	.0897	.1633	.0498	.1555
40	.00314	.0799	.1452	.0443	.1383

* Assumed specific gravities or densities in grams per cubic centimeter; Copper, 8.89; Aluminum, 2.71; brass, 8.47.

Wire Gages

The sizes of wires having a diameter less than 1/2 inch are usually stated in terms of certain arbitrary scales called "gages." The size or gage number of a solid wire refers to the cross-section of the wire perpendicular to its length; the size or gage number of a stranded wire refers to the total cross-section of the constituent wires, irrespective of the pitch of the spiraling. Larger wires are usually described in terms of their area expressed in circular mils. A circular mil is the area of a circle 1 mil in diameter, and the area of any circle in circular mils is equal to the square of its diameter in mils.

There are a number of wire gages in use, the principal ones being the following:

American or Brown and Sharpe Wire Gage.—This gage is the one commonly used in the United States for copper, aluminum and resistance wires. The gage is designated by either of the abbreviations A.W.G. or B. & S.

Basis of the A.W.G. or B. & S. Gage.—The diameters of wires having successive numbers on this gage are in the ratio of $\sqrt[39]{92}$ (= 1.1229 approx.) to 1, and the No. 36 wire has a diameter of 5 mils. No. 35 A.W.G., therefore, has a diameter of $5 \times 1.1229 = 5.61$ mils and so on until No. 0000 is reached, having a diameter of 460 mils.

The ratio $\sqrt[39]{92}$ is approximately equal to $\sqrt[6]{2}$, which is 1.1225. This circumstance makes it possible to have a group of wires of regular gage size with an aggregate area approximately equal to that of another regular gage size. For example, a reduction of three gage numbers (as from gage No. 36 to No. 33) results in a new gage number representing a diameter approximately $\sqrt{2}$ times that represented by the original gage number—or an area approximately two times as great.

The following approximate relations are also useful:

An increase of 1 in the number increases the resistance 25 per cent.
An increase of 2 in the number increases the resistance 60 per cent.
An increase of 3 in the number increases the resistance 100 per cent.
An increase of 10 in the number increases the resistance 10 times.

A No. 10 A.W.G. copper wire has the following approximate characteristics:

Ohms per 1000 feet	1
Circular mils area	10,000
Weight, pounds per 1000 feet	32

A No. 10 A.W.G. aluminum wire has the following approximate characteristics:

Ohms per 1000 feet	1.6
Circular mils area	10,000
Weight, pounds per 1000 feet	9.5

Remembering these rules it is easy to find the approximate size, resistance, area, or weight of any size wire. For example, a No. 12 A.W.G. copper wire has a resistance of 1 plus 60 per cent = 1.6 ohms per 1000 ft approximately. Its area, being inversely as its resistance, is $10{,}000/1.6 = 6250$ circular mils; its diameter is therefore $\sqrt{6250} = 79$ mils and its weight, $32/1.6 = 20$ pounds per 1000 feet.

U. S. Steel Wire Gage.—This gage, known also as the "Washburn and Moen," "Roebling," "American Steel and Wire Co.'s gage," is the one usually employed in the United States for steel and iron wire. It is frequently abbreviated "S.W.G.," but to avoid confusion with the British Standard Wire Gage (*see below*) it should be abbreviated "Stl. W.G." or "A. (steel) W.G."

Birmingham (or Stubs' Iron) Wire Gage.—This gage is still used in the United States for some purposes, e.g., to designate the size of brass wire, and is also employed to a limited extent in Great Britain. It is usually abbreviated "B.W.G." It is sometimes referred to as the "Stubs' Iron Wire Gage," but it should not be confused with the Stubs' Steel Wire Gage.

British Standard Wire Gage.—This gage, usually called simply the "Standard Wire Gage," and abbreviated "S.W.G." is also known as the "New British Standard" (abbreviated "N.B.S."), the English Legal Standard, or the Imperial Wire Gage, and is the legal standard of Great Britain for all wires, as fixed by order in Council, August 23, 1883. It was constructed by modifying the Birmingham Wire Gage, so that the differences between successive diameters were the same for short ranges, i.e., so that a graph representing the diameters consists of a series of a few straight lines.

Edison Wire Gage.—The size of a wire on this gage is equal to its cross-sectional area in circular mils divided by 1000. For example, a solid wire 0.2 inch in diameter has the number $(200)^2/1000 = 40$. This gage is now rarely used.

Metric Wire Gage.—The gage number is ten times the diameter in millimeters.

Other Gages.—In addition wire sizes are sometimes specified in terms of the "Old English Wire Gage," known also as the "London Gage," and the "Stubs' Steel Wire Gage." The Old English Wire Gage is the same as B.W.G. for all gage numbers under 20.

Comparison of Wire Gages.—A comparison of the different gages, in terms of the diameters (in mils or thousandths of an inch) of solid wires corresponding to the various numbers, is given in Table 38. The cross-section in circular mils is the square of the diameter in mils.

Table 38. Comparison of Wire Gage Diameters in Mils
(Bureau of Standards, Circulars No. 31 and No. 67)

Gage No.	American wire gage (B. & S.)	Steel wire gage	Birmingham wire gage (Stubs')	Old English wire gage (London)	Stubs' steel wire gage	(British) Standard wire gage	Metric gage *	Gage No.
7-0	....	490.0	....	....	....	500	....	7-0
6-0	....	461.5	....	....	....	464	....	6-0
5-0	....	430.5	....	....	....	432	....	5-0
4-0	460	393.8	454	454	....	400	....	4-0
3-0	410	362.5	425	425	....	372	....	3-0
2-0	365	331.0	380	380	....	348	....	2-0
0	325	306.5	340	340	....	324	....	0
1	289	283.0	300	300	227	300	3.94	1
2	258	262.5	284	284	219	276	7.87	2
3	229	243.7	259	259	212	252	11.8	3
4	204	225.3	238	238	207	232	15.7	4
5	182	207.0	220	220	204	212	19.7	5
6	162	192.0	203	203	201	192	23.6	6
7	144	177.0	180	180	199	176	27.6	7
8	128	162.0	165	165	197	160	31.5	8
9	114	148.3	148	148	194	144	35.4	9
10	102	135.0	134	134	191	128	39.4	10
11	91	120.5	120	120	188	116	....	11
12	81	105.5	109	109	185	104	47.2	12
13	72	91.5	95	95	182	92	....	13
14	64	80.0	83	83	180	80	55.1	14
15	57	72.0	72	72	178	72	....	15
16	51	62.5	65	65	175	64	63.0	16
17	45	54.0	58	58	172	56	....	17
18	40	47.5	49	49	168	48	70.9	18
19	36	41.0	42	42	164	40	....	19
20	32	34.8	35	35	161	36	78.7	20
21	28.5	31.7	32	31.5	157	32	....	21
22	25.3	28.6	28	29.5	155	28	....	22
23	22.6	25.8	25	27.0	153	24	....	23
24	20.1	23.0	22	25.0	151	22	....	24
25	17.9	20.4	20	23.0	148	20	98.4	25
26	15.9	18.1	18	20.5	146	18	....	26
27	14.2	17.3	16	18.75	143	16.4	....	27
28	12.6	16.2	14	16.50	139	14.8	....	28
29	11.3	15.0	13	15.50	134	13.6	....	29
30	10.0	14.0	12	13.75	127	12.4	118	30
31	8.9	13.2	10	12.25	120	11.6	....	31
32	8.0	12.8	9	11.25	115	10.8	....	32
33	7.1	11.8	8	10.25	112	10.0	....	33
34	6.3	10.4	7	9.50	110	9.2	....	34
35	5.6	9.5	5	9.00	108	8.4	138	35
36	5.0	9.0	4	7.50	106	7.6	....	36
37	4.5	8.5	....	6.50	103	6.8	....	37
38	4.0	8.0	....	5.75	101	6.0	....	38
39	3.5	7.5	....	5.00	99	5.2	....	39
40	3.1	7.0	....	4.50	97	4.8	157	40
41	....	6.6	....	....	95	4.4	....	41
42	....	6.2	....	....	92	4.0	....	42
43	....	6.0	....	....	88	3.6	....	43
44	....	5.8	....	....	85	3.2	....	44
45	....	5.5	....	....	81	2.8	177	45
46	....	5.2	....	....	79	2.4	....	46
47	....	5.0	....	....	77	2.0	....	47
48	....	4.8	....	....	75	1.6	....	48
49	....	4.6	....	....	72	1.2	....	49
50	....	4.4	....	....	69	1.0	197	50

* For diameters corresponding to metric gage numbers, 1.2, 1.4, 1.6, 1.8, 2.5, 3.5, and 4.5, divide those of 12, 14, etc., by ten.

SYMBOLS AND ABBREVIATIONS

17. ABBREVIATIONS FOR ENGINEERING TERMS

Note: This list is a selection of American Tentative Standard abbreviations, for scientific and engineering terms, recommended by the American Standards Association. (See ASA, Z10i—1932.)

Absolute	abs
Acre	acre
Alternating-current (as adjective)	a-c
Ampere	amp
Ampere-hour	amp-hr
Angstrom unit	A
Atomic weight	at. wt
Atmosphere	atm
Average	avg
Avoirdupois	avdp
Barometer	bar.
Barrel	bbl
Baumé	Bé
Boiler pressure	bp
Boiling point	bp
Brake horsepower	bhp
Brake horsepower-hour	bhp-hr
Brinell hardness number	Bhn
British thermal unit	Btu or B
Calory	cal
Candlepower	cp
Centigram	cg
Centiliter	cl
Centimeter	cm
Centimeter-gram-second (system)	cgs
Chemically pure	cp
Circular	cir
Circular mils	cir mils
Coefficient	coef
Cologarithm	colog
Concentrate	conc
Conductivity	cond
Constant	const
Cord	cd
Cosecant	csc
Cosine	cos
Cotangent	ctn
Coulomb	spell out
Counter electromotive force	counter emf
Cubic	cu
Cubic centimeter	cu cm, cm^3, cc
Cubic foot	cu ft
Cubic inch	cu in.
Cubic meter	cu m or m^3
Cubic yard	cu yd
Decibel	db
Degree	deg or °
Degree Centigrade	C
Degree Fahrenheit	F
Degree Kelvin	K
Degree Réaumur	R
Diameter	diam
Direct-current (as adjective)	d-c
Dozen	doz
Dram	dr
Efficiency	eff
Electric	elec
Electromotive force	emf
Equation	eq
External	ext
Farad	spell out
Foot	ft
Foot-candle	ft-c
Foot-Lambert	ft-L
Foot-pound	ft-lb
Foot-pound-second (system)	fps
Freezing point	fp
Frequency	spell out
Fusion point	fnp
Gallon	gal
Grain	spell out
Gram	g
Gram-calory	g-cal
Henry	h
Horsepower	hp
Horsepower-hour	hp-hr
Hour	hr
Hundred	C
Hyperbolic sine	sinh
Hyperbolic cosine	cosh
Hyperbolic tangent	tanh
Inch	in.
Inch-pound	in-lb
Internal	int
Joule	j
Kilocycle	kc
Kilogram	kg
Kilogram-meter	kg-m
Kiloliter	kl
Kilometer	km
Kilovolt	kv
Kilovolt-ampere	kva
Kilowatt	kw
Kilowatthour	kwhr
Lambert	L
Latitude	lat
Linear foot	lin ft
Liter	l
Liquid	liq
Logarithm (common)	log
Logarithm (natural)	$\log_e$ or ln
Longitude	long.
Lumen	l
Lumen-hour	l-hr
Magnetomotive force	mmf
Mass	spell out
Mathematics (ical)	math
Maximum	max
Melting point	mp
Meter	m
Meter-kilogram	m-kg
Mho	spell out
Microampere	μa or mu a
Microfarad	μf or mu f
Micromicron	$\mu\mu$ or mu mu
Micron	μ mu
Microwatt	μw or mu w
Mile	spell out
Milliampere	ma
Millifarad	mf
Milligram	mg
Millihenry	mh
Milliliter	ml
Millimeter	mm
Millimicron	mμ or m mu
Million	spell out
Millivolt	mv

Abbreviations for Engineering Terms—*Continued*

Minimum	min
Minute	min
Minute (angular measure)	′
Ohm	spell out
Ounce	oz
Ounce-foot	oz-ft
Once-inch	oz-in.
Peck	pk
Pint	pt
Potential	spell out
Pound	lb
Pound-foot	lb-ft
Pound-inch	lb-in.
Pounds per square foot	lb per sq ft
Pounds per square inch	lb per sq in.
Power factor	spell out
Quart	qt
Radian	spell out
Reactive kilovolt-ampere	rkva
Reactive volt-ampere	rva
Revolutions per minute	rpm
Revolutions per second	rps
Rod	spell out
Root mean square	rms
Secant	sec
Second	sec
Second (angular measure)	″
Sine	sin
Specific gravity	sp gr
Specific heat	sp ht
Spherical candle power	scp
Square	sq
Square centimeter	sq cm or cm^2
Square foot	sq ft
Square inch	sq in.
Square kilometer	sq km or km^2
Square meter	sq m or m^2
Square root of mean square	rms
Standard	std
Tangent	tan
Temperature	temp
Thousand	M
Ton	spell out
Versed sine	vers
Volt	v
Volt-ampere	va
Watt	w
Watthour	whr
Weight	wt
Yard	yd
Year	yr

18. LETTER SYMBOLS FOR ELECTRICAL QUANTITIES

(Adopted by the Board of Directors, A.I.E.E., October 18, 1928)

Name of Quantity	Symbol
1. Admittance	Y, y
2. Angular frequency	ω
3. Angular velocity	ω
4. Capacitance	C
5. Capacity, electrostatic (*see* capacitance)	
6. Conductance	G, g
7. Conductivity	γ
8. Current	I, i
9. Difference of potential, electric (*see* note 4)	E, e
10. Dielectric constant	K or ϵ
11. Dielectric flux	Ψ
12. Dielectric flux density	D
13. Efficiency	η
14. Electric potential (*see* note 4)	E, e
15. Electrical tension (*see* voltage)	
16. Electromotive force	E, e
17. Electrostatic capacity (*see* capacitance)	
18. Electrostatic flux (*see* dielectric flux)	
19. Electrostatic flux density (*see* dielectric flux density)	
20. Energy	W
21. Flux density, electrostatic (*see* dielectric flux density)	
22. Flux density, magnetic (*see* magnetic flux density)	
23. Frequency	f
24. Impedance	Z, z
25. Inductance	L
26. Magnetic intensity	H
27. Magnetic flux	Φ
28. Magnetic flux density	B
29. Magnetomotive force	$\mathcal{F}$
30. Mutual inductance	M
31. Number of conductors or turns	N
32. Permeability	μ
33. Permeance	$\mathcal{P}$
34. Permittance (*see* capacitance)	
35. Period	T
36. Permittivity (*see* dielectric constant)	
37. Phase displacement	θ or φ
38. Power	P, p
39. Quantity, electric	Q, q
40. Quantity of electricity	Q, q
41. Reactance	X, x
42. Resistance	R, r
43. Resistivity	ρ
44. Reluctance	$\mathcal{R}$
45. Reluctivity	ν
46. Self-inductance	L
47. Susceptance	b
48. Speed of rotation	n
49. Voltage	E, e
50. Work	W

NOTES: 1. Where distinctions between maximum, instantaneous, effective (root-mean-square), and average values are necessary, E_m, I_m, P_m are recommended for maximum values; e, i, p for instantaneous values, E, I for effective (rms) values, and P for average value.

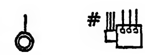
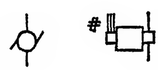
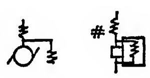
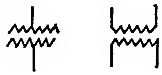
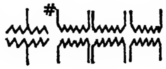
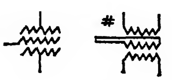
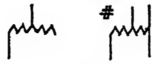
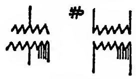
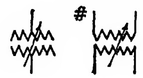
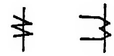
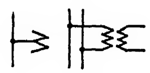
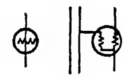
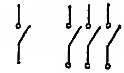
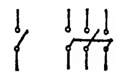
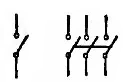
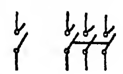
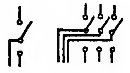
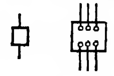
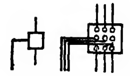
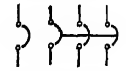
2. In accordance with the practice in other branches of engineering, it is recommended that quantities per unit volume, area, length, etc., be represented as far as practicable by lower-case letters corresponding to the capitals which represent the total quantities.

3. In print, vector or complex quantities should be represented by bold-face letters. In typing, overscoring may be used to indicate bold-face letters (vectors).

4. Where a distinction between electromotive force and difference of electric potential is desirable, the symbols E e, and V v, respectively, may be used.

19. SYMBOLS FOR POWER APPARATUS

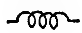
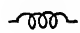
(Approved by the Board of Directors, A.I.E.E., May 20, 1932)
(Approved by American Standards Association, September 1, 1933)

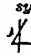
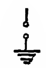
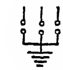
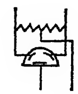
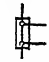
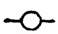
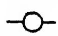
Name	One Line	Complete*
1. A-c Generator or Motor—Basic Symbol For motor, use "M" within symbol if necessary to distinguish.		
2. Induction Motor		
3. Induction Rotor with Slip Ring Rotor		
4. Synchronous Generator or Motor with Separately Excited Field		
5. Synchronous Converter (See also "D-c Generator or Motor")		
6. D-c Generator or Motor—Basic Symbol For motor, use "M" within symbol if necessary to distinguish.		
7. D-c Generator or Motor with Shunt and Series Field		
8. Direct Connected Units—Basic Symbol Use particular symbols and join as here shown.		
*9. Single-phase Two-winding Transformer — Basic Symbol***		
*10. Polyphase Two-winding Transformer—Basic Symbol***		
*11. Three-winding Transformer***		
*12. Auto-transformer***		
*13. Transformer with Taps***		
*14. Constant Current Transformer***		
*15. Current Transformer***		
*16. Potential Transformer***		
17. Induction Voltage Regulator		
18. Disconnecting Switch—Basic Symbol		
19. Knife Switch, Single-throw		
20. Disconnecting Switch, Group Operated		
21. Air Break Switch, Horn Gap, Group Operated		
22. Double-throw Switch		
23. Oil Circuit Breaker, Single-throw		
24. Oil Circuit Breaker, Double-throw		
25. Air Circuit Breaker		

Note: Use symbol () for windings of apparatus as required, and connect to suit particular case.

* The "complete" symbol is intended to illustrate the method of treatment for any desired polyphase combination rather than to show the exact symbol required.

*** This symbol has not been approved as American Standard because in radio symbols it is preferred to use looped lines to indicate the winding and in the case of iron core transformers to use parallel lines between the windings to indicate the iron core. At present these are irreconcilable.*

Name	One Line	Complete*
26. Fuse		
27. Resistor *This symbol has not been approved as American Standard because in radio symbols the symbol is*  *. At present theseareirreconcilable.*		
28. Rheostat		
29. Reactor		
30. Capacitor *This symbol has not been approved as American Standard because in radio symbols the symbol is*  *. At present these are irreconcilable.*		

Name	One Line	Complete*
31. Lightning Arrester—Basic Symbol *This symbol has not been approved as American Standard because in railway symbols the symbol is*  *. At present these are irreconcilable.*		
32. Pothead Cable Terminal		
33. Mercury Arc Rectifier		
34. Battery Polarity should be indicated.		
35. Instrument Shunt		
36. Lamp		

* The "complete" symbol is intended to illustrate the method of treatment for any desired polyphase combination rather than to show the exact symbol required.

Symbols for Instruments and Relays

Instruments

100. Indicating Instrument—Basic Symbol	# I	†
101. Graphic Instrument—Basic Symbol	Graph	†
102. Ampere-hour Meter	AH	†
103. Ammeter	A	†
104. Frequency Meter	F	†
105. Generator Voltage Regulator	Reg	†
106. Ground Detector	GD	†
107. Power-factor Meter	PF	†
108. Reactive Volt-ampere Meter	RVA	†
109. Maximum Demand Meter	MD	†
110. Watthour Meter	WH	†
111. Wattmeter	W	†
112. Synchroscope	S	†
113. Voltmeter	V	†
114. Contact Making Voltmeter	CMV	†
115. Ammeter Switch	‡ A.Sw.	†
116. Voltmeter Switch	‡ V.Sw.	†
117. Voltmeter Receptacle	‡ V.Rec.	†

Relays

200 Relay—Basic Symbol	* R	**

Relay Function Designation

(For use with relay and oil circuit breaker symbols.)

201. Over-current		**
202. Over-voltage	V	**
203. Over-power	W	**
204. Under-current		**
205. Under-voltage	V	**
206. Under-power	W	**
207. Power Directional		**
208. Distance or Impedance, Non-directional	Z	**
209. Distance or Impedance, Directional	Z	**
210. Differential Current	X	**
211. Differential Power	X	**
212. Pilot Wire		**
213. Temperature		**
214. Phase Balance Current		**
215. For Ground Relay Application Add Ground Symbol to Relay Symbol, Thus		**

Relay Time Designations

(When necessary to establish time functions us the following in addition to function designations.)

250. Instantaneous—No Time Delay	Inst.	**
251. Inverse Time Delay	I.T.	**
252. Definite Time Delay	D.T.	**

Letter within circle indicates type of instrument if but one is used. If more than one instrument is used "I" appears within the circle with abbreviation alongside.

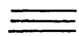
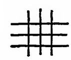
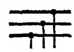
† For complete symbol show outline approximating that of rear view of actual device and indicate terminals in actual relative location, current terminals by open circles and potential terminals by solid circles. Scale range and type number may be marked adjacent to symbol, if desired.

‡ For use adjacent to instrument symbol.

* Relay designations appear beside the "R" circle. For system diagrams, current transformer symbols may be omitted and relay designations placed beside the oil circuit breaker symbol.

** For complete line symbol show outline approximating that of rear view of actual device and indicate terminals in actual relative location, current terminals by open circles and potential terminals by solid circles. Tripping circuit may be identified by representation of contacts within outline.

General Symbols for Maps and Connection Diagrams

Name	One Line	Complete
Wiring Diagrams †		
300. Conductors		
301. Conductors, Crossing but not connected		
302. Conductors, Crossing and Electrically Connected		
303. Bus (with Branches)		
304. Conductors with Branches		
305. Ground Connection		

Name	Symbol
Winding Connections	
400. Two-phase, Three-wire	
401. Two-phase, Four-wire	
402. Three-phase, Delta (or Mesh)	
403 Three-phase Y (or Star)	
404. Three-phase Y (or Star) with Neutral Brought Out and Grounded	
405. Three-phase, zigzag	
406. Three-phase T	
407. Six-phase, Double Delta	
408. Six-phase, Hexagonal (or Chordal)	
409. Six-phase, Diametrical (or Star)	
410. Six-phase Diametrical (or Star) with Neutral Brought Out and Grounded	
Maps	
500. Generating Station	
501. Substation	
502. One or More Circuits Overhead	
503. One or More Circuits Underground	
504. Overhead Line on Pole	
505. Overhead Line on Tower	
506. Street Lamp	

† The thickness of the line varies according to the importance of the circuits; compare "Conductors" with "Bus," for example.

CONSTANTS

By Carl C. Chambers

20. PRINCIPAL PHYSICAL CONSTANTS AND RATIOS*

Velocity of light	$(2.99796 \pm 0.00004) \times 10^{10}$ cm sec^{-1}
Ratio of electrostatic to electromagnetic units	$(2.9971 \pm 0.0001) \times 10^{10}$ cm$^{1/2}$ sec$^{-1/2}$ (int ohms)$^{-1/2}$ $(2.9979 \pm 0.0001) \times 10^{10}$ cm sec^{-1} (in absolute units)
Volume of a perfect gas (0° C and normal atmospheric pressure)	$(22.4141 \pm 0.0008) \times 10^{3}$ cm^3 mole^{-1}
Normal atmospheric pressure	$(1.013249 \pm 0.000003) \times 10^{6}$ dynes cm^{-2}
45° atmospheric pressure	$(1.013199 \pm 0.000003) \times 10^{6}$ dynes cm^{-2}
Ice point (absolute scale)	273.18 ± 0.03° K
Mechanical equivalent of heat (20° C)	4.1813 ± 0.0006 abs joule cal^{-1}
Electrical equivalent of heat (20° C)	4.1796 ± 0.0007 int joule cal^{-1}
Faraday constant	96494 ± 5 int coulombs g-equiv^{-1}
Electronic charge	$(4.770 \pm 0.005) \times 10^{-10}$ abs-es unit $(1.5910 \pm 0.0016) \times 10^{-20}$ abs-em unit
Planck constant	$(6.547 \pm 0.008) \times 10^{-27}$ erg sec^{-1}
Acceleration of gravity	980.665 cm sec^{-2}
Electrochemical equivalent of silver	$(1.11800 \pm 0.00005) \times 10^{-3}$ g. (int coulombs)$^{-1}$
Wave-length of red cadmium line (15° C, normal atmospheric pressure)	6438.4696 I.A.†
Effective grating space of calcite (20° C)	$(3.0279 \pm 0.0010) \times 10^{-8}$ cm
Avogadro's number	$(6.064 \pm 0.006) \times 10^{23}$ mole^{-1}
Boltzmann constant	$(1.3708 \pm 0.0014) \times 10^{-16}$ erg deg^{-1}
Stefan-Boltzmann constant	$(5.735 \pm 0.011) \times 10^{-5}$ erg cm^{-1} deg^{-4} sec^{-1}
Mass of the electron	$(9.01 \pm 0.03) \times 10^{-28}$ g
Ratio of mass of H to mass of electron (measured by deflection)	1847 ± 2

* Values taken from Birge, *Rev. of Mod. Phys.*, Vol. 1, No. 1, p. 1 (July, 1929).

† This defines the international angstrom unit (I.A.). The unit is of the order of 1 part in several million different from 10^{-8} cm.

SECTION 2

PROPERTIES OF MATERIALS

BY

D. F. MINER

WEIGHTS OF MATERIALS

ART. PAGE

1. International Atomic Weights........ 02
2. Weights (Masses) of Materials....... 03

CONDUCTOR MATERIALS

3. Classification...................... 05
4. Copper.............................. 05
5. Aluminum............................ 07
6. Metallic Resistor Materials.......... 09
7. Carbon Contact Materials........... 10

INSULATING MATERIALS

8. Classification...................... 12
9. Discussion of Properties............ 12
10. Solid Insulating Materials.......... 18
11. Plastic Insulating Materials......... 26
12. Transformer and Switch Oil......... 29

ART. PAGE

13. Other Liquid Insulating Materials.... 32
14. Gases.............................. 36

MAGNETIC MATERIALS

15. Classification...................... 38
16. Non-retentive Materials............. 41
17. Retentive Materials................ 47
18. Special Magnetic Materials.......... 48

THERMAL PROPERTIES OF MATERIALS

19. Thermal Capacity and Specific Heat.. 49
20. Melting or Freezing Point and Heat of Fusion.......................... 50
21. Transfer of Heat................... 52
22. Low-temperature Heat-insulating Materials........................... 54
23. Expansion Due to Heating.......... 54
24. Flash Point of Oils................ 59

PROPERTIES OF MATERIALS

By D. F. Miner

WEIGHTS OF MATERIALS

1. INTERNATIONAL ATOMIC WEIGHTS

1936

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	Symbol	Atomic Number	Atomic Weight		Symbol	Atomic Number	Atomic Weight
Aluminum	Al	13	26.97	Molybdenum	Mo	42	96.0
Antimony	Sb	51	121.76	Neodymium	Nd	60	144.27
Argon	A	18	39.944	Neon	Ne	10	20.183
Arsenic	As	33	74.91	Nickel	Ni	28	58.69
Barium	Ba	56	137.36	Nitrogen	N	7	14.008
Beryllium	Be	4	9.02	Osmium	Os	76	191.5
Bismuth	Bi	83	209.00	Oxygen	O	8	16.0000
Boron	B	5	10.82	Palladium	Pd	46	106.7
Bromine	Br	35	79.916	Phosphorus	P	15	31.02
Cadmium	Cd	48	112.41	Platinum	Pt	78	195.23
Calcium	Ca	20	40.08	Potassium	K	19	39.096
Carbon	C	6	12.00	Praseodymium	Pr	59	140.92
Cerium	Ce	58	140.13	Radium	Ra	88	225.97
Cesium	Cs	55	132.91	Radon	Rn	86	222.
Chlorine	Cl	17	35.457	Rhenium	Re	75	186.31
Chromium	Cr	24	52.01	Rhodium	Rh	45	102.91
Cobalt	Co	27	58.94	Rubidium	Rb	37	85.44
Columbium	Cb	41	92.91	Ruthenium	Ru	44	101.7
Copper	Cu	29	63.57	Samarium	Sm	62	150.43
Dysprosium	Dy	66	162.46	Scandium	Sc	21	45.10
Erbium	Er	68	167.64	Selenium	Se	34	78.96
Europium	Eu	63	152.0	Silicon	Si	14	28.06
Fluorine	F	9	19.00	Silver	Ag	47	107.880
Gadolinium	Gd	64	157.3	Sodium	Na	11	22.997
Gallium	Ga	31	69.72	Strontium	Sr	38	87.63
Germanium	Ge	32	72.60	Sulfur	S	16	32.06
Gold	Au	79	197.2	Tantalum	Ta	73	181.4
Hafnium	Hf	72	178.6	Tellurium	Te	52	127.61
Helium	He	2	4.002	Terbium	Tb	65	159.2
Holmium	Ho	67	163.5	Thallium	Tl	81	204.39
Hydrogen	H	1	1.0078	Thorium	Th	90	232.12
Indium	In	49	114.76	Thulium	Tm	69	169.4
Iodine	I	53	126.92	Tin	Sn	50	118.70
Iridium	Ir	77	193.1	Titanium	Ti	22	47.90
Iron	Fe	26	55.84	Tungsten	W	74	184.0
Krypton	Kr	36	83.7	Uranium	U	92	238.14
Lanthanum	La	57	138.92	Vanadium	V	23	50.95
Lead	Pb	82	207.22	Xenon	Xe	54	131.3
Lithium	Li	3	6.940	Ytterbium	Yb	70	173.04
Lutecium	Lu	71	175.0	Yttrium	Y	39	88.92
Magnesium	Mg	12	24.32	Zinc	Zn	30	65.38
Manganese	Mn	25	54.93	Zirconium	Zr	40	91.22
Mercury	Hg	80	200.61				

2. WEIGHTS (MASSES) OF MATERIALS

The density of any substance is the mass of that substance per unit volume. Or, using weight in the ordinary sense as equivalent to mass, the density may also be defined as the *weight* per unit volume. The numerical value of the density of any substance depends upon the unit in which the mass or weight is expressed and also upon the unit of volume used; see Units and Conversion Factors. However, it is quite common to state the density of a substance in grams per cubic centimeter, without naming the units, since when so expressed the density is numerically equal (practically) to the specific gravity.

The "specific gravity" of a substance is defined as the ratio of the weight (mass) per unit volume of that substance to the weight (mass), expressed in the same unit of an equal volume of water. To make such a statement exact the temperature of the water should be specified. There is no general agreement as to the temperature of reference, though water at 0 deg cent is commonly taken as the reference temperature. For gases, air at 0 deg cent and 760 mm mercury pressure is frequently taken as the reference substance instead of water.

VARIATION OF DENSITY OF WATER WITH TEMPERATURE. The following table gives the results of measurements by Thiesen, Scheel, and Diesselhorst (Landolt, Börnstein and Roth, Physikalisch-chemische Tabellin, 1913).

Table 1. Density of Water; Grams per CC

Degrees Cent	0	1	2	3	4	5	6	7	8	9
0	0.99987	0.99993	0.99997	0.99999	1.00000	0.99999	0.99997	0.99993	0.99988	0.99981
10	0.99973	0.99963	0.99953	0.99940	0.99927	0.99913	0.99897	0.99880	0.99862	0.99843
20	0.99823	0.99802	0.99780	0.99756	0.99732	0.99707	0.99681	0.99654	0.99626	0.99597
30	0.99567	0.99537	0.99505	0.99473	0.99440	0.99406	0.99371	0.99336	0.99299	0.99262
40	0.99224	0.99186	0.99147	0.99107	0.99066	0.99025	0.98982	0.98940	0.98896	0.98852
50	0.98807	0.98762	0.98715	0.98669	0.98621	0.98573	0.98525	0.98475	0.98425	0.98375
60	0.98324	0.98272	0.98220	0.98167	0.98113	0.98059	0.98005	0.97950	0.97894	0.97838
70	0.97781	0.97723	0.97666	0.97607	0.97548	0.97489	0.97429	0.97368	0.97307	0.97245
80	0.97183	0.97121	0.97057	0.96994	0.96930	0.96865	0.96800	0.96734	0.96668	0.96601
90	0.96534	0.96467	0.96399	0.96330	0.96261	0.96192	0.96122	0.96051	0.95981	0.95909
100	0.95838	0.95765	0.95693	......	......	......	......	......	......	......

Example: The density of water at 33 deg cent is 0.99473.

WEIGHTS PER CUBIC FOOT AND SPECIFIC GRAVITY. In the following table are given the values of the density in pounds per cubic foot of the more commonly used substances. The specific gravity, or density in grams per cubic centimeter, corresponding to any weight per cubic foot w is equal to $w/62.43$; for the conversion factors necessary to convert these figures into densities for other units of mass and volume, see Units and Conversion Factors.

Table 2. Specific Gravity and Pounds per Cubic Foot of Various Materials at Room Temperatures

Specific Gravities all referred to water at 0 deg cent.
(See References at end of table)

Material	Lb per cu ft		Average Spec. Grav.	Material	Lb per cu ft		Average Spec. Grav.
	From	To			From	To	
Air *	0.0807	......	0.00129	Bronze	545	555	8.80
Acetylene gas *	0.0733	......	0.00117	Carbon	125	144	2.25
Aluminum, cast	160	161	2.57	" dioxide *	0.124	......	0.00199
" wire	168	......	2.70	" monoxide *	0.0781	......	0.00125
Ammonia *	0.0482	......	0.000771	Caoutchouc	57	62	0.955
Antimony	414	......	6.64	Cement, loose	72	105	1.42
Asbestos	125	175	2.40	" set	170	190	2.85
Asphaltum	69	94	1.30	Charcoal	17	35	0.421
Basalt	150	190	2.7	Clay, hard	129	133	2.10
Bismuth	604	618	9.78	" soft	118	......	1.89
Brass	511	542	8.45	Coal, anthracite	81	106	1.50
Brick, red	111	128	1.92	" anthracite piled loose	47	58	0.84
" fire	110	......	1.76				

* At a temperature of 0 deg cent a pressure of 760 mm mercury.

Table 2 (*Continued*)

Material	Lb per cu ft From	Lb per cu ft To	Average Spec. Grav.
Coal, bituminous	78	88	1.33
" bituminous, piled loose	44	54	0.79
" lignite	52	......	0.83
Cobalt	530	563	8.71
Coke	62	105	1.35
" piled loose	23	32	0.45
Concrete, 1 : 2 : 4	146	......	2.34
" 1 : 1 1/2 : 3	139	.....	2.23
" 1 : 3 : 6	156	......	2.50
Copper, cast	549	558	8.87
" wrought	552	558	8.90
" wire	555	558	8.89†
Cork	15.6	......	0.25
Ebonite	72	.....	1.15
Flint	164	......	2.63
German silver (52 Cu + 26 Zn + 22 Ni)	527	......	8.45
Glass, common	150	175	2.6
" flint	180	370	4.4
Gold, cast	1200	......	19.3
Granite	165	172	2.7
Gravel	90	147	1.9
Gutta percha	61.1	......	0.980
Gypsum or plaster of Paris	142	145	2.26
Hydrogen *	0.00561	......	0.0000900
Ice	57.2	......	0.917
Iridium	1400	.....	22.42
Iron, pure	490	492	7.86
" gray cast	439	445	7.08
" white cast	473	482	7.65
" wrought	487	492	7.85
" steel	474	494	7.76
Lead	708	......	11.34
Leather, dry	54	......	0.86
" greased	64	......	1.02
Lime	81	87	1.35
Limestone	167	171	2.72
Loam	65	88	1.23
Marble	160	177	2.72
Masonry	100	165	2.12
Mercury at 0° C	849	......	13.6
Mercury at 20° C	846	......	13.55
Mica	165	200	2.9
Molybdenum	636	......	10.2
Mortar, hard	103	111	1.75
Muck	40	74	0.915
Mud	80	130	1.68
Nickel	540	550	8.8
Nitrogen *	0.0782	......	0.00125
Nitrous oxide *	0.1234	......	0.00198
Oil, cotton-seed	57.8	......	0.926
" gasoline	41	43	0.675
" lard	57.4	......	0.920
" linseed	58.8	......	0.942
" mineral, lubricating	56.2	57.7	0.912
" petroleum	54.8	......	0.878
Oil, turpentine	54.2	......	0.873
" whale	57.3	57.7	0.921
Osmium	1400	......	22.5
Oxygen *	0.0892	......	0.00142
Palladium	686	749	11.5
Paper	44	72	0.92
Paraffin	54	57	0.89
Pitch	67	......	1.07
Platinum	1320	1350	21.37
Porcelain	143	156	2.4
Pumice stone	23	56	0.63
Quartz	165	......	2.65
Rhodium	686	775	12.44
Salt	50	70	0.965
Sand	90	120	1.68
Sandstone	134	147	2.25
Selenium	300	......	4.82
Silver	650	657	10.5
Slate	162	205	2.85
Snow, fresh fallen	5	12	0.136
" wet compact	15	50	0.520
Soapstone	162	175	2.7
Steel (see Iron)	......	......	....
Sulfur	120	130	2.05
Tantalum	1035	......	16.6
Tar	62.4	......	1.00
Tile, hollow terra cotta, building block	26	38	0.51
Tile, flat and segmental arches	31	45	0.608
Tile partitions‡	12	26	....
Tin	455	......	7.29
Trap-rock	187	190	3.02
Tungsten	1160	1190	18.8
Turf	20	30	0.400
Water, max. density	62.4	......	1.00
" sea	64.0	64.3	1.03
Wax, bees	60.5	......	0.965
Wood, ash	40	53	0.75
" butternut	24	......	0.38
" cedar	30	35	0.53
" chestnut	38	41	0.63
" cypress	32	37	0.55
" elm	34	37	0.57
" fir	34	35	0.55
" hemlock	25	29	0.43
" hickory	37	58	0.75
" lignum vitae	73	83	1.25
Wood, mahogany	41	53	0.75
" maple	39	47	0.68
" oak	37	56	0.75
" pine, white	22	31	0.42
" " yellow	23	37	0.50
" poplar	22	31	0.42
" red wood	30	......	0.481
" spruce	25	32	0.457
" walnut	40	43	0.67
Zinc	428	448	7.10

* At a temperature of 0 deg cent and a pressure of 760 mm mercury.
† This value has been adopted internationally as representing the average density at 20 deg cent; see reference.
‡ Including air spaces.

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CONDUCTOR MATERIALS

3. CLASSIFICATION

Theoretically the terms conductor and insulator are only relative. There is no sharp dividing line between them. For practical purposes, however, conducting materials are those selected to carry current in an electrical circuit and insulating materials are those chosen to restrict the current in the conductor to desired paths.

Conductors are mostly all metals, but there is a wide difference in conductivity (as much as 70 to 1 ratio in metals). They are usually separated into:

a. Good conductors such as copper, aluminum, silver.

b. Poor conductors (resistors) such as iron; alloys of nickel, iron, copper, chromium; and carbon products.

4. COPPER

(See also Electrochemical Processes, Industrial; Wires and Cables, Bare.) The following discussion applies primarily to copper for electrical conductors.

ROLLING-MILL PROCESSES. The refined copper comes to the rod mill in bars weighing about 200 lb each. These bars frequently have ridges along the sides, due to faults in castings, and the surface is often covered with a layer of oxide. They are heated in a furnace until sufficiently soft for rolling and are passed through a series of rolls diminishing in size until a rod of the proper diameter is obtained. The rod is then coiled up and immersed in a pickling liquid (10 per cent H_2SO_4) in order to dissolve the oxide formed during rolling. It is then washed.

WIRE-MILL PROCESSES. The rods having cooled are connected together by brazing and are drawn through a series of dies of decreasing diameter. The rod is lubricated by passing through a ball of tallow just before entering the die. Drawing the wire hardens the outside of the wire and increases its strength almost in proportion to the reduction in cross-sectional area. The smaller the wire, the larger the proportion of hardened metal, and hence the higher the tensile strength.

DEFECTS IN WIRE. One of the most serious defects in wire results from ridged bars. As the bar enters the rolls, these ridges are sometimes folded over, enclosing the oxidized surface or scale. Subsequent passes hide this fold, sometimes completely, but it remains a serious flaw even through the final drawing operations. Careless handling, uneven welding of the rods, and unequal temperature of the wire while passing through the dies will all produce noticeable effects in the quality of the finished wire, so that care throughout the mill is absolutely necessary for the best results.

It, therefore, appears that the most efficient wire must possess not only high conductivity but also the maximum torsion and tensile strength possible in commercial copper, and that to obtain these qualities it is necessary to use high-grade copper and to prevent an excess of cuprous oxide entering it at any stage of the manufacture.

ANNEALING. All wire when first drawn is more or less hard. It may be softened by annealing, i.e., by heating to a temperature between 450 and 600 deg cent. The hardness is not affected by the rate of cooling. Annealed copper has a crystalline structure, whereas hard-drawn copper consists of grains elongated in the direction of drawing.

OXYGEN-FREE HIGH-CONDUCTIVITY COPPER. A special grade of copper is available which is made under carefully controlled conditions. The atmosphere present in the furnace and the method of casting prevent entrance of oxygen and formation of copper oxide. Wire or other shapes made from this material exhibit much superior characteristics in two respects. The ductility, as evidenced in elongation, torsion and bending, is made greater. OFHC wire may be twisted two to three times as many turns as regular copper. This copper also may be subjected to the action of reducing gases at elevated temperatures (as in brazing) without the usual danger of embrittlement. Conductivity and tensile strength are the same as for tough pitch electrolytic copper. The fatigue resistance is apparently unchanged.

Because of the superior ductility of oxygen free copper, it is useful in difficult operations of deep drawing, spinning or edge bending. Where the embrittlement hazard must be avoided in welding, brazing, or hot working operations, this copper may be used to advantage, in spite of the higher cost.

Mechanical Properties

(See Wires and Cables, Bare.) The more important mechanical properties of copper are discussed in some detail below.

TENSILE STRENGTH AND ELONGATION OF SOFT ANNEALED COPPER. The tensile strength of soft annealed copper is about 30,000 to 33,000 lb per sq in. with an elongation of about 25 per cent (in 10 in.) at the fracture. It has no true elastic limit, permanent elongation being produced by very small loads.

TENSILE STRENGTH AND ELONGATION OF HARD-DRAWN COPPER. Modern hard-drawn copper is equally affected by drawing throughout the section, no hard skin being produced. According to D. R. Pye, hard-drawn copper in wires up to $^1/_2$-in. diameter varies in tensile strength with the diameter according to a linear law of the form

$$T = 70{,}000 - 45{,}000\,D$$

where T = tensile strength, pounds per square inch; and D = diameter of wire, inches.

The above constants agree approximately with the tables of the American Society for Testing Materials and represent values somewhat under those usually obtained.

The elongation at fracture is approximately represented by

$$E = 4\sqrt{D}$$

where E = per cent elongation at fracture.

Tests by G. C. Batson on 50-ft lengths of hard-drawn copper showed a tensile strength only $^1/_2$ per cent less than that of 10-ft lengths, a fact which indicates the material is very uniform.

MODULUS OF ELASTICITY OF HARD-DRAWN COPPER. The modulus of elasticity of hard-drawn copper varies somewhat with diameter, higher values being shown by small wires. The usual figure taken is 16×10^6 as an average for all copper forms.

COMPRESSION TEST. Copper of good quality does not fracture under compression; it yields and flattens. According to Thurston the resistance to compression may be calculated, within the limits $e < {}^1/_2$, from the formula

$$C = 145{,}000\sqrt[3]{e}$$

where C = resistance in pounds per square inch of original area, and e = fractional compression.

MISCELLANEOUS MECHANICAL PROPERTIES. The torsional strength, shearing strength, hardness, resistance to impact, and fatigue of copper are all discussed in Bureau of Standards Circular No. 73 on Copper.

Table 1. Typical Properties of Copper

	Brinell Hardness	Tensile Strength lb per sq in.	Yield Strength lb per sq in.	Elongation	Endurance Limit lb per sq in.
As Cast	36	30,000	17,000	45	6,000
Bars					
Soft	40	32,000	12,000	38	10,000
Hard	70	50,000	46,000	18	15,000
Sheet					
Soft	40	32,000	12,000	37	10,000
Hard	100	51,000	48,000	4	15,000
Wire					
Soft	..	38,000	12,000	36	10,000
Hard	..	60,000	39,000	3	15,000

DENSITY. The density of copper or, for all practical purposes, its specific gravity referred to water, is 8.89 at 20 deg cent. This is the value which has been adopted as standard by the American Institute of Electrical Engineers, and most other authorities in the past. Measurements by the Bureau of Standards, the Calumet & Hecla Smelting Works, and the Reichsanstalt have indicated this as a mean.

Conductivity and Resistivity

F. A. Wolff and J. H. Dellinger give the resistivities of 89 samples of commercial copper from 14 important refiners and wire manufacturers in this and other countries. The mean for annealed wire is: Resistivity in ohms per meter-gram at 20 deg cent = 0.15292; per cent conductivity = 100.25. (Per cent conductivity is computed on the basis of 100 per cent conductivity corresponding to the standard resistivity of 0.15328 ohm per meter-gram at 20 deg cent.)

CONDUCTIVITY OF HARD-DRAWN COPPER. The conductivity of hard-drawn No. 12 B. & S. wires was found to be less than the conductivity of annealed wires by a mean value of 2.7 per cent. The difference between the conductivity of annealed and hard-drawn wires increases as the diameter of the wire decreases. The minimum conductivity usually specified for annealed wire is 98 per cent. Resistivity of annealed wire is taken as 10.371 ohms per circular mil foot at 20 deg cent.

TEMPERATURE COEFFICIENT OF RESISTIVITY. It has been found by J. H. Dellinger (Bulletin 147, Bur. Standards, 1910, Vol. 7, No. 1) that the temperature coefficient of copper is proportional to the conductivity instead of being virtually a constant, as hitherto assumed. This fact may be expressed by saying that the change of resistivity per degree centigrade of a sample of copper is 0.000597 ohm per meter-gram or 0.00681 microhm per centimeter cube. The 20 deg cent temperature coefficient of a sample of copper is found by multiplying the per cent conductivity by 0.00393 and dividing by 100. These rules apply only to copper furnished for electrical use and to the temperature range of 10 to 100 deg cent over which the temperature coefficient was found to be linear. Table 2 gives the temperature coefficients α_T in the formula:

$$R_t = R_T[1 + \alpha_T(t - T)]$$

Table 2

Ohms per meter-gram at 20 deg cent	Per cent conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}
0.16134	95	0.00403	0.00380	0.00373	0.00367	0.00360
0.15966	96	0.00408	0.00385	0.00377	0.00370	0.00364
0.15802	97	0.00413	0.00389	0.00381	0.00374	0.00367
0.15753	97.3	0.00414	0.00390	0.00382	0.00375	0.00368
0.15640	98	0.00417	0.00393	0.00385	0.00378	0.00371
0.15482	99	0.00422	0.00397	0.00389	0.00382	0.00374
0.15328	**100**	0.00427	0.00401	**0.00393**	0.00385	0.00378
0.15176	101	0.00431	0.00405	0.00397	0.00389	0.00382

The bold-face values in the table have been adopted as standard by the American Institute of Electrical Engineers.

5. ALUMINUM

The method of manufacturing aluminum wire is similar to that employed in making copper wire. Commercial conductors are generally made from aluminum having a purity of about 99.5 per cent.

PHYSICAL CHARACTERISTICS (COPPER, ALUMINUM, AND STEEL COMPARED). The weights of the different materials are as follows. For aluminum, copper, and steel stranded conductors 2 per cent is added to the wire weights.

Material	Weight in Pounds of One Million Circular Mils, 1000 Ft Long		Weight in Kilograms, of One Square Millimeter, One Kilometer Long	
	Wire	Cable	Wire	Cable
Aluminum	920	938	2.702	2.756
Copper	3029	3090	8.896	9.074
Steel	2640	2690	7.753	7.908

Modulus of elasticity of aluminum, 9,000,000 lb per sq in.
Modulus of elasticity of copper, 16,000,000 lb per sq in.
Modulus of elasticity of steel, 30,000,000 lb per sq in.

The following values are used for hard-drawn material:

Elastic limit of aluminum, 21,000 lb per sq in.
Elastic limit of copper, 30,000 lb per sq in.
Elastic limit of steel, 130,000 lb per sq in.
Ultimate stress of aluminum, 24,000 lb per sq in.
Ultimate stress of copper, 60,000 lb per sq in.
Ultimate stress of steel, 190,000 lb per sq in.

Coefficient of expansion of aluminum (per degree cent) 0.0000231
Coefficient of expansion of copper (per degree cent) 0.0000167
Coefficient of expansion of steel (per degree cent) 0.0000115

CONDUCTIVITY AND RESISTIVITY. The Aluminum Company of America gives, as the average of many thousands of separate determinations, the figure of 2.828 microhms per centimeter cube, or 17.002 ohms per circular mil foot, at 20 deg cent, as the resistivity of commercial aluminum wire. This corresponds to a conductivity of 61 per cent of the annealed copper standard.

EFFECT OF HARDNESS ON CONDUCTIVITY. The resistance of aluminum depends upon its hardness; for example, the resistance of aluminum wire of hardness corresponding to a tensile strength of about 20 kg per sq mm (28,500 lb per sq in.) is approximately 2 per cent greater than that of the same wire when thoroughly annealed, and consequently soft (H. M. Hobart).

In the tables of stranded conductors, the resistances are 2 per cent greater than the equivalent solid conductor. This allows for increase of length due to an average length of lay as recommended by the American Institute of Electrical Engineers.

In computing the resistances of Aluminum Cable Steel Reinforced no deduction is made for the conductance of the steel core.

TEMPERATURE COEFFICIENT OF RESISTIVITY. The 20 deg cent temperature coefficient of 61 per cent conductivity annealed aluminum wire is 0.00403 per degree centigrade, and the 0 deg cent coefficient is 0.00444. Grassi also found that the temperature coefficient of aluminum is proportional to its per cent conductivity (as is true also of copper).

Table 3. Comparison of Copper and Aluminum Wires for Equal Resistances per Unit Length

Item	Copper	Aluminum
Cost	1	$0.488 \times \frac{p}{P}$
Cross-section	1	1.61
Diameter	1	1.27
Weight	1	0.488
Breaking strength	1	0 64

p = unit price of aluminum wire.
P = unit price of copper wire.

DISADVANTAGE OF LOW TENSILE STRENGTH. The lower tensile strength of aluminum for equal length and conductance as compared with copper affects the cost of an aerial line in two ways: first, by making it necessary to erect the spans with a greater sag or less length in order to reduce the stresses, thereby either increasing the height or the number of poles; and second, by making it necessary to increase the distance between wires on account of the increased sag. The increase in the height of poles for the same spacing amounts to about 10 per cent (C. L. Johnson). Aluminum cables with steel core are now used and overcome these difficulties.

EFFECT OF LARGE ELONGATION OF ALUMINUM. The extraordinarily great elongation of aluminum enables it to withstand severe mechanical overloads by stretching and thus increasing the sag. However, in dealing with a single solid wire this cannot be relied on, as a scratch or a single imperfection will often cause the wire to break without any appreciable elongation. This is one reason why cables are to be preferred to single-wire conductors.

CORONA FORMATION. At very high potentials, such as 100,000 volts, aluminum conductors possess a marked advantage over copper in the lower corona loss due to their greater diameter for the same conductance.

MELTING OF ALUMINUM CABLES. Because of the lower melting point of aluminum (660 deg cent compared to 1085 deg cent for copper), it was once the opinion that aluminum transmission cables would be subject to more damage from flashover arcs on transmission lines. Recent tests have shown that aluminum cable and copper are equally good in resisting arcs. In some test cases it was noted that the arc spreads more on aluminum than on copper, and the effect was therefore less concentrated. Furthermore, the steel core provides ample strength even if the aluminum is badly burned.

ALUMINUM CONDUCTORS IN ELECTRICAL MACHINES. The use of aluminum in motors and generators, particularly for field coils, has been advantageous where weight is a primary consideration, as in railway motors. The coils must, however, either occupy more space or be operated at a higher temperature. The difficulty of making a satisfactory and permanent joint to other portions of the circuit, which may be of copper, has retarded the use of aluminum conductors in machines.

6. METALLIC RESISTOR MATERIALS

There are on the market a large number of alloy wires used for resistance purposes. They cover a wide range of resistivities, operating temperature limits, physical properties. temperature coefficient of resistance, and thermal emf. For detailed description of these products the catalogs of manufacturers should be consulted. In general, the trade-name products can be grouped in approximately ten classes of alloys. Table 4, naming a few

Table 4. Properties of Metals and Alloys Used in Resistors

Material	Ni	Cr	Fe	Mn	Cu	Zn	Al	Resistivity at 20 deg cent, ohms per circular mil foot	Temperature Coefficient of Resistance		Coefficient of Linear Expansion		Maximum Working Temperature, deg cent
									α Temp. Coeff.	t deg cent Diff. in Temp.	α Coeff. of Exp.	t deg cent Diff. in Temp.	
Ohmax *	...	...	...	...	...	...	...	1000	0.00035	20–500	...	...	500
Radiohm *	...	...	...	...	...	...	...	800	0.0007	20–500	...	...	500
Al–Cr–Fe (Ohmalloy) *	...	12.5	83	...	...	...	4.5	750	0.0003	...	...	...	850
Nickel chromium (Chromel C * Nichrome * Tophet C) *	59	16	22	3	...	...	...	675	0.00013	...	0.0000137	...	1000
Nickel chromium (Chromel A * Nichrome IV * Tophet A) *	80	20	...	...	...	...	...	650	0.0001	...	0.0000132	...	1100
525 Alloy	...	...	...	...	...	...	...	600	0.00034	20–500	0.0000151	20–500	500
Ni–Cr–Fe	38	16	45	1	...	...	...	575	0.00044	...	0.0000171	...	800
Comet*	...	...	...	...	...	...	...	570	0.00088	20–500	0.000015	20–500	600
Nilvar *	...	...	...		...	...	...	450	...	...	0.000001	20–100	200
Copper-nickel (Advance * Cupel * Cupron * Constantan * Ideal,* etc.)	45	...	...	.	55	...	...	295	±0.00002	...	0.0000144	...	500
Manganin*	4	...	...	12	84	...	...	290	0.00002	...	...	...	100
Lucero *	...	...	...	...	...	...	...	270	0.0010	20–250	0.0000125	20–100	500
Monel *	2/3	...	...	...	1/3	...	...	256	0 00145	...	0.000014	...	500
Nickel silver (Nickel silver German silver)	18	...	...	...	65	17	...	190	0.00019	...	0.0000173	...	250
Midohm *	...	...	...	...	...	...	...	180	0.00018	20–100	...	...	200
Hytemco *	...	...	...	...	...	...	...	120	0.0061	20–500	0.000015	20–1000	500
Iron, pure	...	...	100	...	...	...	...	61.1	0.0062	...	0.0000114	...	...
Nickel, pure	100	...	...	...	...	...	...	60	0.0048	...	0 000126	...	500

* Trademark names.

products under the class name, gives the approximate composition and electrical characteristics of resistance wires commonly used. Tables of current-carrying capacities at various chosen operating temperatures are available in the manufacturers' catalogs.

COPPER ALLOYS AS RESISTORS IN ROTATING MACHINES. Damper bars in the fields of a–c machines and rotor bars in some designs of a–c motors make use of resistance material composed chiefly of copper. Some of the alloys commonly used for these purposes are given in the table below.

Name	Chemical Composition (parts per 100)								Resistivity, ohms per cir mil ft
	Cu	Zn	Pb	Sn	Si	Mn	Ni	Fe	
Copper	100	...	...	...	...	...	...	...	10.371
Brass	62	35	3	...	...	...	...	...	41.46
Gun bronze	92	...	...	8	...	...	...	...	79.8
Silicon bronze	96	...	...	...	3	1	...	...	155.0
Telephone bronze	98.25	...	...	1.75	...	...	...	...	29.62
Monel metal	24	...	...	...	...	3	60	.3	268.

7. CARBON CONTACT MATERIALS

Carbon and combinations of carbon and metal are commonly used as contact materials where current is frequently interrupted. The object is usually to provide terminals which will not melt and weld together, as metal contacts sometimes do. Relays, contactors, and various control devices use buttons or studs of this type, making contact with moving parts of metal, usually brass or copper. Where high conductivity is required, copper-graphite or silver-graphite material is used. These are similar to metal-graphite brush material, giving non-sticking contacts with low contact drop and some degree of lubrication. Silver-graphite, although more expensive, gives lower contact drop after long service than copper-graphite. It is therefore used in low-energy circuits such as relays, where reliable operation, even after long periods of idleness, is important.

CARBON BRUSHES. The chief use of carbon in electrical manufacturing is for current collection. Carbon brushes form an important part of the electrical circuit in commutator machines, and many types have been developed to afford most satisfactory operation of motors and generators. Carbon brushes are used, rather than metal, in order to provide a resistance path for the short-circuit current which flows when a brush spans two or more commutator bars. The resistance offered must be high enough to prevent excessive coil currents, yet low enough so that the main load current led in or out of the brush will not cause excessive drop in total machine potential.

The essential features of brush manufacture and composition are as follows:

The three purest forms of carbon are diamond, petroleum coke, and lamp black. The last two ingredients are extensively used in the manufacture of carbon brushes and other carbon products.

Petroleum coke is the residue left on the bottom and sides of the still after the refining of crude oil. After all other products have been collected, this petroleum coke forms a shell of varying thickness and is dug out in chunks. This by-product, or green petroleum coke, is purchased in large quantities by all manufacturers of carbon.

Lamp black is made by the incomplete combustion of oil or gas, depending upon the type of black desired.

The first step in the process of manufacture is the driving out of all impurities or volatile matter from the green coke. This is done by subjecting the green product to an intense heat in calcining furnaces. The resulting product is pure carbon, or commonly termed, calcined coke. This calcined coke is then put through a grinding and sieving operation, similar to the operation of a wheat flour mill. All types of mills, including roller mills and pulverizers, are used. The resulting product is a fine, pure carbon flour, mainly 200 mesh and finer.

The carbon flour is then mixed with graphite, lamp black, and other ingredients, according to the formulas necessary to attain certain electrical and physical characteristics desired in standard grades of brushes. Graphite is added for lubricating purposes and to increase the current-carrying capacity of carbon brush material.

The highest-grade graphite brushes are made by transforming pure carbon brushes to graphite by subjecting the material to an intense heat in an electric furnace.

The method of making carbon graphite and graphite brushes is as follows:

1. Calcined petroleum coke, lamp black, graphite, or other flours are mixed with a pitch or other binder in steam-heated mixers.
2. After thorough mixing, material for molded brushes is permitted to cool and reground and pulverized, after which the fine powder mix is then molded under hydraulic pressure into plates.
3. The molded carbon graphite plates are then baked at various temperatures.
4. This completes the manufacturing process for carbon graphite and natural graphite plates from which brushes of these two classes are cut.
5. The carbon graphite plates from which electro-graphitic brushes are to be cut are given an additional high-temperature treatment in an electric furnace which removes all impurities and in addition changes some of the carbon into graphite.

Metal graphite grades are made by mixing fine mesh copper, lead, zinc, tin, or other materials with graphite. This material is mixed, molded, and baked in a manner similar to carbon graphite material.

Resin bonded brushes contain a certain amount of resin of high electrical resistance such as a condensation product resulting from the reaction of formaldehyde upon phenol. The brush structure is of a laminated texture which results in a cross electrical resistance approximately five times greater than the lengthwise electrical resistance. This has the effect of limiting circulating currents in the brush and providing better commutation.

The various formulas necessary to make grades of different electrical and physical

properties are governed by the characteristics of the electrical equipment for which the brushes are intended. The higher the metal content of brush material, the higher the current-carrying capacity of that material. Plain carbon graphite grades carry between 35 and 50 amp per sq in., whereas the highest content graphite grade carries about 70 amp per sq in. continuously. High metal content grades are rated at 125–150 amp per sq in. but will carry 450–500 amp per sq in. momentarily without undue heating.

The many grades of brushes on the market may be classified generally into five grades known as (1) carbon graphite, (2) electro-graphite, (3) natural graphite, (4) resin bonded, and (5) metal graphite.

Carbon Graphite. These are survivals of the original types and are used mostly on older-design machines where a cleaning action on the commutator is required. They are not suitable for high speeds or heavy currents.

Electro-graphite brushes have largely superseded the carbon graphite type. They have lower friction, higher current capacity, less ash, lower abrasive effect, and greater mechanical strength.

Natural Graphite brushes are an improvement over the carbon graphite type where abrasive action is beneficial but where low friction is required for higher-speed operation.

Resin Bonded brushes have a laminated structure whose distinctive characteristic is that the resistance across these laminations is from five to eight times the resistance parallel to the laminations. This is effective in reducing short-circuit current in the face of the brush. They are particularly suited to machines with high commutating voltage.

Metal Graphite brushes have replaced the early leaf copper types. They are used where high current capacity and low contact drop are important, as in electroplating generators or slip rings on a–c machines.

Navy Specifications. The following specification values of the U. S. Navy may be taken as illustrative of the range of properties existing in brushes in common use:

	Grade A*	Grade B*	Grade D*	Grade E*
Specific resistance at 30 deg cent, ohms per in.^{3}	0.00176–0.0021	0.0015–0.0019	0.0006–0.0012	0.000005–0.000025
Specific resistance at 250 deg cent, ohms per in.^{3}	0.0013–0.0017	0.0011–0.0015	0.0004–0.0010	....
Contact drop, volts at rated current	1.5 min 3.2 max	1.8 min 3.0 max	1.6 min 3.0 max	low
Coefficient of friction (no current)	0.50 max	0.55 max	0.60 max	0.40 max
Coefficient of friction (carrying current)	0.35 max	0.40 max	0.45 max	....
Transverse strength, lb per in.^{2}	2100 min	3000 min	1000 min	2500 min
Hardness, scleroscope	43 to 58	50 to 65	8 to 16	8 to 20
Per cent ash	0.25 max	0.25 max	2.3 max	....
Density, lb per in.^{2}	0.054–0.058	0.058–0.065	0.042–0.050	....
Percentage of graphite	......	......	......	20–30

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INSULATING MATERIALS

8. CLASSIFICATION

Insulation materials may be classified according to:

a. Thermal limits (A.I.E.E. operating temperature classes).
b. Physical state (solid, liquid, etc.).
c. Quality [good dielectric (e.g., mica) or poor dielectric (e.g., asbestos)].

The Thermal Classification as defined in A.I.E.E. Standards is as follows:

Class	Description of Material
O	Class O insulation consists of cotton, silk, paper, and similar organic materials when neither impregnated * nor immersed in oil.
A	Class A insulation consists of cotton, silk, paper, and similar organic materials when impregnated * or immersed in oil; also enamel as applied to conductors.
B	Class B insulation consists of inorganic materials such as mica and asbestos in built-up form combined with binding substances. If Class A material is used in small quantities in conjunction, for structural purposes only, the combined material may be considered as Class B, provided that the electrical and mechanical properties of the insulated winding are not impaired by the application of the temperature permitted for Class B material. (The word "impair" is here used in the sense of causing any change which could disqualify the insulating material for continuous service.)
C	Class C insulation consists of inorganic materials such as pure mica, porcelain, and quartz.

Physical Classification. To give an idea of the variety of types of materials used as insulators, a list of physical classifications is herewith shown.

Solids
- Natural (marble, slate, asbestos, quartz, mica, lava, soapstone).
- Vitreous (glass, porcelain).
- Fibrous, organic (cotton, silk, linen, threads, and fabrics, paper, wood).
- Rubber materials and substitutes (rubber, gutta percha, Duprene, Thiokol).

Plastics
- Synthetic resins (phenolic, urea, resorcinol, vinyl, casein, etc., used as impregnation of fibrous insulation or as molded composition and laminated sheet).
- Waxes (natural and synthetic, beeswax, halowax, paraffin).
- Gums (natural gums, asphalt, shellac, copal).

Liquids
- Natural oils (linseed, tung, castor).
- Mineral oil (transformer and switch).
- Varnishes.
- Solvents (benzine, alcohol, naphtha, toluol, etc.).

Gases
- Air
- CO_2
- N_2
- H_2

9. DISCUSSION OF PROPERTIES

DIELECTRIC STRENGTH. The electric strength of a dielectric material is the maximum potential gradient that unit thickness of the material can withstand without rupture. With solids, breakdown is frequently termed puncture. With thin sheet

* Impregnated cotton, paper, or silk: An insulation is considered to be "impregnated" when a suitable substance replaces the air between its fibers, even if this substance does not completely fill the spaces between the insulated conductors. The impregnating substance, in order to be considered suitable, must have good insulating properties, must entirely cover the fibers and render them adherent to each other and to the conductor, must not produce interstices within itself as a consequence of evaporation of the solvent or through any other cause, must not flow during the operation of the machine at full working load or at the temperature limit specified, and must not unduly deteriorate under prolonged action of heat.

insulation the dielectric strength is commonly expressed in volts per unit thickness, as volts per mil.

The breakdown or puncture voltage of any sample is influenced by a number of factors—size and shape of electrodes, the frequency and wave shape of the applied voltage, time of voltage application, temperature, moisture content. The methods of test published by Am. Soc. for Testing Materials are generally recognized as standard. It is essential that the conditions of test should be known in comparing different materials.

Table 1 illustrates the effect of the thickness of the test specimen upon the dielectric strength of porcelain. The effect may be greater or less for other insulating materials. It is obvious, however, that a standard thickness of the test specimen should be adopted in order to obtain comparable values of dielectric strength of the various insulations.

Table 1. Dielectric Strength of Porcelain for Different Thicknesses of Test Specimen

(Peek: Dielectric Phenomena in High Voltage Engineering)

Total Thickness, mm	Kv per mm	Total Thickness, mm	Kv per mm
0.5	16.0	5.0	11.0
1.0	14.5	10.0	9.6
2.0	12.2	15.0	9.2

Effect of Time of Voltage Application. Time is an important factor in dielectric breakdown. This is due to effect of heating, cumulative products of ionization, or lining up of particles of low dielectric strength. The following data, taken with rod electrodes (spherical ends) at large spacings in transformer oil, show that a voltage much lower than the short-time breakdown value of a given dielectric will cause breakdown if applied for a long time. Instantaneous values can therefore not safely be used in design unless a generous factor of safety is allowed.

Table 2. Variation of Dielectric Strength of Oil with Time of Applied Voltage

1/4-in. Diameter Spherical Electrodes

Spacing, inches	Instantaneous Breakdown, kv	Voltage for Breakdown at 1 Min, kv	Voltage for Breakdown at 3 min, kv	Voltage for Breakdown at 10 Min, kv
2	295	180	125	90
4	320	230	155	145
8	345	280	225	225
16	420	375	365	365

Effect of Temperature. Table 3 illustrates the effect of time of applied voltage on a solid dielectric. It demonstrates also the reduction of dielectric strength with increase in temperature. Care must be taken in applying data on commercial insulating materials. The breakdown values, resistivity, and other properties are quite likely to be greatly reduced at operating temperatures of electrical apparatus.

Table 3. Variation of Dielectric Strength with the Time of Applied Voltage

(Peek: Dielectric Phenomena in High Voltage Engineering)

Material	Thickness, mm	Time to Puncture, min	25 deg cent kv per mm	100 deg cent kv per mm
Oil-impregnated paper, 30 layers, 60-cycle, 10-cm diameter disks, round edges.	1.90	"Inst."	39.4	32.0
		1	33.1	27.3
		2	31.0	25.7
		4	29.2	24.5
		6	28.2	23.6
		10	26.8	22.7
		20	25.5	21.6
		40	23.6	20.5
		60	22.7	19.7
		80	22.1	19.3
		100	21.6	19.0

Table 3a. Micarta Tubing—Effect of Time on Puncture Strength, Tested Under Oil

Wall Thickness, inches	KV Rapid Test	KV 1 Min. to Breakdown	KV 1 Hr. to Breakdown
0.0625	55	34	24
.125	90	53	38
.250	154	78	48
.50	...	116	55

SURFACE LEAKAGE is due to a film of moisture on the surface of the insulation. This moisture may contain soluble salts and condensed gases, such as ammonia, and carbon dioxide from the air. This film may indeed be very thin; for example, on cleaned quartz glass, it has been found to be from 3 to 6 $\times$ 10^{-6} mm. Waxy materials have no surface film because the moisture condensed on the surface tends to draw into drops instead of spreading over the surface. Surface resistivity according to Curtis is independent of voltage and also temperature, the relative humidity being constant. The temperature range which he investigated was 25 to 30 deg cent with relative humidity at 25 per cent.

Surface resistivity of most insulating materials decreases upon exposure to sunlight or ultra-violet light. The results obtained by Curtis indicate that a few hours of exposure to ultra-violet light will produce the same or sometimes a greater effect than would be produced in years by sunlight.

Surface resistivity is defined as the resistance between two opposite edges of a surface film, 1 cm square. The values in Table 4 are those obtained by Curtis at temperatures from 25 to 29 deg cent and are given for 30 and 90 per cent relative humidity. The test voltage was 200 volts.

On high voltages, apparent surface leakage should be distinguished from actual surface leakage. The surface moisture film under high voltage is soon dispelled by the heat generated with the leakage current. Then a new leakage film is produced by the oxidation of the insulation with ozone generated at the surface due to corona formed by local ionization of the surrounding air. If the surface of the insulation is covered with an oil such as blown linseed oil prior to the application of the voltage the oxidized film can be greatly reduced, and the arcover or flashover voltage will be increased. Even though all the actual surface leakage were removed, the flashover voltage would still be limited because of the apparent surface leakage, which, for a given leakage distance, depends upon the position and shape of the electrodes, relative specific inductive capacities of air, and the insulation and the shape of the insulation (contour of the surface). If no surface leakage at all existed the flashover voltage would then be the breakdown voltage of the air between the electrodes. (Adapted from Peek, Dielectric Phenomena in High Voltage Engineering).

In high-voltage designs surface breakdown must be prevented by providing sufficient creepage distance. The characteristics of sheet insulating materials such as mica, laminated Bakelite, fullerboard, etc., have been investigated. As an example, Fig. 1 shows the surface breakdown voltage for mica plate in air. The high voltage electrode was a 2-in. square block of brass, $^1/_2$ in. thick, placed on the mica sheet at given distances from the edge. The mica was placed on a large sheet of metal, which formed the grounded electrode. Failure was thus over the surface and across the thickness of mica sheet to the metal plate.

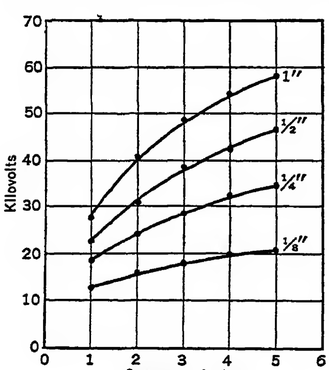

FIG. 1. Surface Creepage. Mica plate 1/8″ to 1″ thick

VOLUME RESISTIVITY of insulating materials in general follows Ohm's law. The only exceptions of any consequence are a few vegetable oils, as for example castor oil, in which the resistance decreases with increase in the applied voltage. The volume resistivity varies with the amount of moisture present and with the temperature. In Shrader's work on the effect of moisture, he found, for example, an insulating material, which is generally considered non-hygroscopic, had a volume resistivity of 26 $\times$ 10^{10} ohm-cm ordinarily, but after being heated in vacuum, had a resistivity of 8 $\times$ 10^{14} ohm-cm, or 3000 times as great. As to the effect

of temperature, Curtis found that in 53 of the insulations which he investigated, the ratio of the resistivity at 20 deg cent to that at 30 deg cent varied from 1 to 5.3.

The values given in Table 4 are for room temperature at ordinary atmospheric humidity. The unit used is the resistance to the current flowing through the material between two opposite faces of a centimeter cube, no surface leakage being present.

Table 4. Electrical Properties of Insulating Materials

Material	Dielectric Strength		Resistivity			SIC
	Specimen thickness, mm	Kv per mm (a)	Volume ohm-cm	Surface (b) Ohms, 30%	Surface (b) Ohms, 90%	Air Unity
Asbestos paper	1.2	4.2	1.6×10^{11}	...	...	...
Asphalt (Byerlyte)	3.6	14.0	...	...	...	2.7
Bakelite, wood molding mixture	...	17.7 to 21.6	1×10^{12}	...	...	4.5 to 5.5
Bakelite, asbestos molding mixture	...	up to 9.8	4×10^{11}	...	...	...
Bakelite, laminated X	...	up to 31.4	5×10^{11}	...	...	5
Cellophane	0.022	51 to 66	...	...	...	8
Celluloid (clear)	.25	12 to 28	2×10^{10}	8×10^{10}	2×10^{9}	...
Ceresin	...	...	over 5×10^{18}	8×10^{16}	8×10^{16}	...
Cellulose acetate	.019	48	...	...	...	5
Empire cloth, muslin	.38	48.0	...	...	...	...
Fiber, vulcanized, including hard fiber, all colors	3.2 6.4	4.9 to 10.8 3.9 to 8.9	5 to 20×10^{9} 5 to 20×10^{9}	3×10^{10} 3×10^{10}	1×10^{7} 1×10^{7}	5 5
Glass (ordinary)	...	8 to 9	9×10^{13}	...	...	5.5 to 9.1
Glass (plate)	...	...	2×10^{13}	3×10^{13}	2×10^{7}	5.5 to 9.1
Jute (impregnated)	6	1.2	...	...	...	3 to 4
Lava	...	3 to 10	2×10^{10}	6×10^{11}	1×10^{8}	...
Marble	...	6.5	1 to 100×10^{9}	8×10^{10}	2×10^{7}	8.3
Mica	0.6	21 to 28	0.04 to 200×10^{15}	2×10^{13}	3×10^{9}	5 to 7
Micabond, plate	1.6	37.5	...	...	...	...
Micabond, flexible	1.6	23.1	...	...	...	...
Oil, insulating	2.54	10 to 16	...	...	...	2.5
Paper	0.13	8.7	5×10^{4}	...	...	2.6
Paraffin (parowax)	...	11.5	1×10^{16}	1.5×10^{16}	5×10^{15}	2.1
Porcelain	20	8.0	3×10^{14}	4×10^{13}	5×10^{8}	4.4
Pressboard (oiled)	1.58	29.2	...	...	...	5.0
Pressboard (varnished)	1.58	15.5	...	...	...	3
Rosin	...	...	5×10^{16}	8×10^{14}	2×10^{14}	2.5
Rubber (hard)	0.5	70	1×10^{18}	6×10^{15}	1×10^{9}	2.0 to 3.5
Shellac	...	...	1×10^{16}	2×10^{14}	6×10^{9}	3.0 to 3.7
Slate	10.3	1.3	1×10^{18}	2×10^{8}	1×10^{7}	6.6 to 7.4
Sulfur	...	...	1×10^{17}	1×10^{16}	1×10^{14}	2.9 to 3.2
Wood (maple), paraffined	15.2	4.6	3×10^{10}	1×10^{12}	2×10^{9}	4.1

(a) To obtain volts per mil multiply kilovolts per millimeter by 25.4.
(b) At 30 per cent and 90 per cent relative humidity.

DIELECTRIC LOSS is due to the following causes:

With either direct or alternating current.

1. Ionization of occluded gases.
2. I^2R losses due to leakage current.

With alternating current only.

3. Oscillatory movement of particles of higher or lower specific capacity than the surrounding medium, due to the charges induced on them being alternately attracted and repelled by the electrodes.
4. I^2R losses due to displacement current.

Generally, dielectric loss varies as the square of the voltage up to a certain dielectric stress at which the loss begins to increase faster than the square of the voltage. The increase occurs when the occluded gases or other particles of low specific inductive capacity begin to ionize. This has the effect of greatly increasing the dielectric stress on the particles of high specific capacity which soon rupture. Dielectric loss therefore has a

bearing upon the dielectric strength of insulations especially when the voltage is applied indefinitely.

Table 5 gives dielectric loss of varnished cloth and oil-treated paper.

Table 5. Dielectric Losses

(Peek: Dielectric Phenomena in High Voltage Engineering)

(Effective Sine Wave 60 Cycles)

Total Thickness, mm	Insulation	No. of Layers	Temp., deg cent	Volts per mm	Watts per cc
4.0	Varnished cloth	15	25	4,000	0.005
				6,000	0.015
				8,000	0.035
				10,000	0.060
				12,000	0.090
4.0	Varnished cloth	15	90	4,000	0.025
				6,000	0.075
				8,000	0.150
				10,000	0.240
				12,000	0.350
2.5	Oil-treated paper	30	25	10,000	0.040
				14,000	0.070
2.5	Oil-treated paper	30	60	10,000	0.043
				14,000	0.080
2.5	Oil-treated paper	30	90	10,000	0.050
				14,000	0.100
2.5	Oil-treated paper	30	120	10,000	0.067
				14,000	0.138

These losses may be lower or very much higher, depending upon the condition of the insulation.

The power factor of insulation is the ratio of the power dissipated in it (i.e., the dielectric loss) to the product of volts across it and amperes in the circuit.

Dielectric loss is of great importance in some high-voltage devices, such as cables and condenser-type terminals. It is also a determining factor in the choice and use of materials for capacitors. High losses lead to heating of the dielectric and low efficiency. If too high, losses cause heating resulting in still higher losses, and so on cumulatively to failure of the dielectric.

The variation of power factor and specific inductive capacity with temperature on cable and cloth tape are shown in Table 6. It is apparent that operating temperatures, whether from ambient conditions or internal losses, should be kept low.

Table 6. Power Factor and Specific Inductive Capacity (SIC)

Temp. °C	Tan Tape		Black Tape		Cable	
	Per Cent Power Factor	SIC	Per Cent Power Factor	SIC	Per Cent Power Factor: Single Cond.	Per Cent Power Factor: Multiple Cond.
20	3.4	4.5	3.0	3.9	7	10
40	4.3	5.0	3.0	4.2	7	10
60	5.4	5.5	3.2	4.5	10	15
80	9.2	6.0	4.8	4.6	16	25
90	14.5	6.4	6.5	4.7	—	—

POWER FACTOR TEST OF TREATED MATERIAL (PAPER AND COTTON). Measurements of power factor are usually made at two temperatures, 25° C. and 80° C. and at two voltage gradients, 30 and 100 volts per mil. The Schering bridge, or a more recent modification, the Atkinson bridge, is used to obtain the values. Great care must be taken in preparation of the sample to insure reliable results. If entrapped air or moisture is present, a proper test is impossible. One procedure which is tedious but reliable is to prepare a sample of tape wrapped on a metal tube about 2″ in diameter and about 12″ in effective length. To eliminate air, hot insulating oil is applied to the tube and to each carefully wrapped layer. Over the outside a foil electrode and guard rings are placed. A successful method of testing flat samples of tape (approx. 1″ × 6″) has

been developed and standardization of procedure is being completed by A.S.T.M. To insure consistent results, the sample is dried at least three hours at 105° C. and many authorities follow this with sixteen hours of vacuum drying. A considerable pressure of electrodes is desirable, such as 10 lbs. per sq. in.

IMPULSE CHARACTERISTICS. The behavior of insulating materials when subjected to steep wave front surges is of great importance in all apparatus used in the transmission and distribution of electric power. Voltage surges may be due to lightning or to switching disturbances and may vary greatly in steepness, maximum voltage, decay of wave, and may be of either polarity. The various insulating parts of a system should be

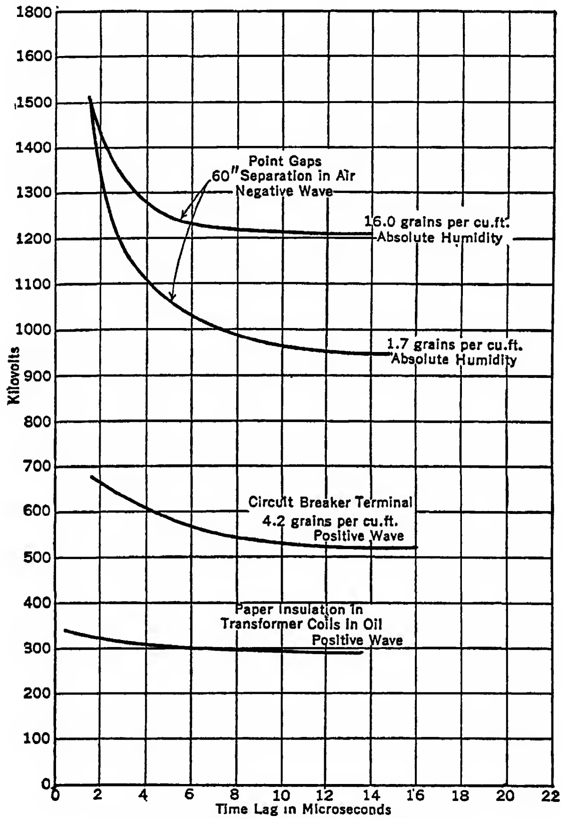

FIG. 2. Time Lag Curves (1.5 × 40 wave)

coordinated so that selective protection is obtained. For example, the line insulators should flash over and give relief if necessary before the circuit breaker terminals, and the terminals should break down before the transformer insulation. Only in recent years have laboratory tests on materials and apparatus been coordinated sufficiently to permit comparison of different products and to evaluate the protection installed on any system. Two types of waves are in general use, the 1 × 5 and the 1.5 × 40, tentatively agreed upon by A.I.E.E. subcommittees. The first figure is the time in microseconds to the crest of the wave and the second figure is the time in microseconds from zero to half crest value on the tail of the wave.

In the literature various investigators have given results on impulse tests of various materials but they are not of great use because of the former lack of standardization of test waves and procedure. In surge breakdown of insulation, time is a major factor. For example, if a wave of which the crest voltage is slightly above the 60 cycle crest

breakdown is applied, breakdown may occur perhaps 15 microseconds after the start. If a wave with a crest voltage 50% higher is applied, breakdown will occur more quickly, perhaps in 2 microseconds. Such tests over a range of voltages give data for time-lag curves which are of great use in coordinating the insulation of various structures. Time lag curves are of hyperbolic shape in general, approaching the 60 cycle crest value as an asymptote (see Fig. 2). The degree of curvature varies with the material. In the case of air the curve is very steep. The crest value of 1.5 × 40 waves which will cause breakdown of air in 2 microseconds may be 1 1/2 to 2 times the crest value of the 1.5 × 40 wave which will produce breakdown in 15 microseconds. With solids such as terminals and transformer coil insulation the shape is much flatter, there being less difference in voltage for breakdown for differences in time chosen.

10. SOLID INSULATING MATERIALS

Insulating materials are often described in general terms without specific data. A material is said to be "heat proof," which may mean that it will not deteriorate when used continuously at some temperature within the range of 30 to 250 deg cent. Dielectric strength is sometimes quoted without reference to the thickness of the test specimen. If the test has been made on a very thin specimen, the dielectric strength obtained will be misleading to those who are not aware of the law of the increase of dielectric strength with the decrease of the thickness of the test specimen. Claims may be made that an insulating material is insoluble in certain solvents and is weatherproof. Such claims are not absolute. They are only general and refer to ordinary conditions found in the use of insulations. If the working conditions are unusual and severe, specific information should be obtained on the appropriateness of the insulation under consideration.

The more important types of insulation are described in the following pages.

Natural

ASBESTOS is a mineral consisting chiefly of silica, magnesia, lime, alumina, water, and oxide of iron. The structure is of innumerable fibers the ultimate fiber of which is thought to be a single row of the molecular structure of the crystal. The fibers are exceedingly smooth and glossy and have very little friction to hold them together when they are spun into a yarn, this resulting in a low tensile strength. This difficulty, however, has been partly overcome so that now threads are made which have fair tensile strength. The more important varieties of asbestos are amianthus and amphibole and are used in the form of asbestos paper, cardboard, yarn, cloth, tape, and as a filler in molding mixtures.

Asbestos contains small particles of iron oxide or grit which cannot be entirely removed and affect to a slight degree its insulating qualities. It is hygroscopic and should therefore not be used on high voltage, in general not over 3300 volts. It is unaffected by oils, acids, and alkalies and withstands very high temperatures. Above 500 deg cent it loses its mechanical strength, and it melts at about 1300 deg cent. It has an extensive use as a heat insulator.

LAVA is a mineral talc, machined in its natural condition and then baked at a temperature of 1100 deg cent to a condition of extreme hardness. It is then unaffected by any subsequent temperature short of its baking temperature. It is slowly attacked by hydrochloric acid but is not affected by other acids or alkalies.

MARBLE is the name given to any limestone which is sufficiently compact to admit of a polish. Pure marble is white, but the presence of iron oxide or other impurities gives it different colors. It is used principally for switchboard work and should not contain metallic veins, which reduce its insulating qualities. If used on circuits of 1000 volts or more, it should be saturated with an insulating varnish and baked. It shows oil spots, and for that reason it is sometimes stained black and given a so-called marine finish. Very little marble is now used except to match old installations.

SLATE is any rock having a fissile structure, the common variety being composed principally of silica, alumina, and oxides of iron. It is hygroscopic. It is often permeated by metallic veins, making it unfit for use unless the electrical connections are insulated by bushings. It is useful for switchboard and switch-base work owing to its desirable mechanical and fireproof qualities. Its dielectric strength decreases rapidly as the temperature increases, and at a high temperature slate becomes a conductor. When a high voltage is impressed upon a piece of slate for some time, the slate usually is not punctured but, owing to the consequent rise in temperature, the slate acts as a short circuit to the impressed voltage. The breakdown is thus only apparent as the specimens regain their dielectric properties after cooling.

Slate from some localities is a better insulator than that from others. Purple slate from Vermont and vicinity is considered superior to black slate from Pennsylvania. The use of slate for electrical purposes is rapidly declining.

QUARTZ a form of silica, naturally formed into hexagonal crystals, has a density of about 2.7. It may be fused and cast into usable shapes and is sometimes used as a ceramic type of insulation for high temperatures.

SOAPSTONE, a form of talc or steatite, is gray in color and feels smooth and soapy. It is very easily machined and is sometimes used as an insulator and structural part where resistance to heat and chemicals is desired. It is weak mechanically and hygroscopic.

MICA is an anhydrous silicate of aluminum and potash or sodium. It crystallizes in a laminated mass, some grades of which may be subdivided down to a thickness of 0.0008 mm. The ultimate thickness of cleavage layers is unknown and may be finally but one layer of the molecular structure. It is useful as an insulator because of its high insulating qualities and its ability to withstand high temperatures. Owing to its impurity, lack of flexibility, and excessive surface leakage in the natural state, the laminae are separated and sorted into various grades of purity and are then cemented together to form plate or flexible reconstructed mica of any thickness or purity.

Two kinds of mica are in common use—muscovite (white mica), usually obtained from India; and phlogopite (amber mica), from Madagascar.

Mica can be built into various useful forms, including plate, flexible mica, tape, coil wrappers, and tubing. Mica is combined with thin paper or cloth as a backing when made into tape or wrappers. The bond is frequently shellac although asphalts, synthetic resins, and other materials are preferred for special purposes. White mica is hard and is used in building segment mica for undercut commutators. The maximum safe temperature for white mica is 500 deg cent.

Amber mica is softer and is used in mica plate for flush commutators. It is also used in making plate for heating-appliance insulation. It will withstand temperatures up to 800 deg cent.

Flexible mica or molding mica may be made for hot or cold molding. White mica is generally used with a bond which is plastic at the molding temperature. Shellac bond may be flexibilized with castor oil.

Sheet insulation is made with mica and some other insulation such as fishpaper, rope paper, or Kraft paper for special purposes.

Solid-vitreous

GLASS. Glass is essentially a mixture of silicates of sodium, potassium, calcium, or barium. A given specimen may contain one or more of these silicates, the composition being chosen so as to give the desired properties. Other metallic oxides are often added, and if a low-expansion, heat-resisting glass is desired, boron trioxide is used, to make the borosilicate glass.

At normal temperatures glass is a good insulator but at red heat is a fair conductor. It resists most chemicals except hydrofluoric acid. Most glass is brittle and weak on heat shock. Newer glasses of the borosilicate type (Pyrex) have lower coefficients of expansion and do not shatter with wide temperature changes. Successful line insulators have been placed on the market, competing with porcelain.

PORCELAIN. Porcelain is distinguished from other forms of earthenware by a vitrified and non-porous structure. It is composed of china clay, ball clay, flint, and feldspar. China clay, sometimes called kaolin, is slightly plastic whereas ball clay is very tough and plastic. By a proper combination of the two the desired plasticity is obtained. Flint is in the form of pure sand or quartz ground so finely as to be entirely free from any gritty feeling. Feldspar is a natural rock and is ground as fine as the flint. These four ingredients are mixed in proper proportions and passed through the manufacturing processes, which require very skilful care and experience upon which depends the quality of the product.

Insulators are made by a wet process and are non-porous, which is necessary for high-voltage use. The glaze is provided to protect the porcelain from dust and deterioration in the weather and should have the same temperature coefficient of expansion as the porcelain.

In the wet process the ingredients are thoroughly ground in a ball mill and then mixed with water to a consistency about the same as that of paint. After several hours of stirring (blunging) this liquid (slip) is pumped through a filter press where the water is extracted and press cakes left. These are stacked in bins for use, being kept moist. Thorough mixing of the press cake material is accomplished by a screw type of mixer called a pug mill. The wet clay is extruded in cylindrical form and sections cut off for forming into

insulators or other parts. Forming is usually done either by hot pressing the plastic clay soon after it leaves the pug mill, or by turning operations performed after the clay has partially dried. Wet-process material may also be cast into plaster of paris molds, a special liquid body being prepared for this purpose. Porcelain is fired at temperatures ranging from 1250 to 1300 deg cent for several days.

Such articles as knobs, cleats, and lamp sockets are made by a dry process. Dry-process body is a moist powder when molded. Such porcelain is porous and not usually suitable for high-voltage outdoor applications. It is of value because of the accuracy with which intricate pieces can be molded and is suitable for indoor or low-voltage purposes.

The mechanical strength depends upon the flint content, the heat resistance upon the clay, and the dielectric strength upon the feldspar which unfortunately also adds brittleness. In a special type of porcelain for spark plugs, magnesia is used which adds mechanical strength at high temperatures. Porcelain, because of its low tensile strength, should be used under a compressive strain. It is comparatively inexpensive, chemically inert, and not sensitive to changes of temperature. The fracture of good porcelain is conchoidal, fine grained, white and bright.

Fibrous Organic

COTTON. Cotton occurs in many forms as an insulator, such as cloth, tape, and yarn. These products serve as mechanical supports or separators of conductors and are usually impregnated with an insulating liquid or compound. The insulating value of cotton is considered only equivalent to the same thickness of air, reliance being placed usually on the impregnation for dielectric strength and moisture proofing. Cloth is used in making varnished "cambric" of various types. Tape is used largely in insulating coils of all types, and yarn in covering wires or insulating layers of enameled wire in "universal" wound magnet coils.

SILK is not frequently used as an insulator, except where space is an important factor. It is sometimes used as a varnished fabric or as wire covering for appearance and because it can easily be obtained thinner than cotton materials. The cost is higher, however. Its insulating properties are approximately the same as those of cotton.

LINEN thread or twine is used for fastening the coils of rotating machines because of its high unit strength.

PAPER. A great variety of papers are used for insulation, usually impregnated the same as cotton. Paper insulation occurs as sheets, treated either before or after application for layer insulation; as tape for insulating conductors; or in combination with mica, asbestos, or cloth for various applications. Paper is principally of rag, wood fiber, or vegetable fiber origin.

INSULATING PAPERS. The principal papers used in electrical apparatus are described below.

Absorbent Papers of 100 per cent purified sulfite wood pulp are used in making laminated phenolic products. The paper has a soft fiber, thoroughly cleaned of natural resin, and it takes the impregnating phenolic varnishes readily. A plate with a high ratio of impregnation weight to paper weight can be made with absorbent papers. A common thickness is 0.010 in.

Another type of absorbent paper is made from 100 per cent cotton rag stock to be used in filter presses employed in purifying insulating oils. It is very porous, to absorb water readily, and yet let oil through. The thickness of sheet is approximately 0.025 in.

Asbestos Paper is not all asbestos fiber, the usual grades containing not more than 80 per cent asbestos and the remainder sulfite wood pulp fibers. Thin papers from 0.005 to 0.010 in. are used in insulating heavy copper used in field coils and in tape form as a covering for wire. Certain grades of heat-resisting laminated phenolic plate are made of asbestos paper impregnated with materials like Bakelite. Asbestos papers are not high-grade insulators owing to conducting particles and moisture absorption.

Rag Base Papers. (*a*) Tough insulating papers used where resistance to mechanical damage, as in motor and generator slots, is important, are manufactured from 100 per cent cotton rags. Most of them are identified by trade names, such as Duro, Capacio, and Armco papers.

(*b*) Fish paper is a special type of rag paper made entirely from old cotton rags. The paper during manufacture is treated with zinc chloride which partially dissolves the cellulose. A paper is obtained without a definite fiber structure. It is hard, tough in both directions, and does not dislaminate. Fish paper may be obtained in thicknesses from 0.007 to 0.056 in. and is used for slot insulation, for layer insulation of coils, and as switch barriers. Paraffin impregnation or varnish treatments are frequently used to overcome the inherent hygroscopic property of fish paper.

Fullerboard (or Pressboard). (*a*) Rag fullerboard is composed of 75 per cent rag stock and 25 per cent sulfate wood pulp. It is made on a cylinder machine, cut off, dried, and pressed in sheets. It may be obtained in thickness from 0.007 to 1/2 in. and in sheets as large as 84 by 120 in. Fullerboard is particularly adapted for oil impregnation and is extensively used in transformer construction for barriers, coil sides, and washers. It may be formed into angles and channels. Shrinkage and distortion are low.

(*b*) Kraft fullerboard made of 100 per cent sulfate wood pulp in the same general manner as rag fullerboard is a cheaper type of material and is valuable for many applications for which rag fullerboard was once employed. It is used for barriers, coil spacers, washers, etc., for electrical machinery. It has much greater shrinkage and warping tendencies than rag material. Forming is not as readily done. These limitations result in the continued use of both rag and Kraft types. Varnish treatment is frequently applied when fullerboard is to be used in air.

(*c*) A similar material in thin papers with a highly calendered surface is called express paper. It is used making varnish-treated papers where only a surface coating and not impregnation is desired.

(*d*) A compromise material containing 60 per cent rag and 40 per cent woodpulp has been developed which combines practically all the stability and toughness of rag fullerboard with the cheapness of Kraft fullerboard.

Kraft Papers contain 100 per cent sulfate wood pulp and may be obtained in a range of thickness from 0.0005 in. to fullerboard thicknesses. They are brown in color, are readily impregnated, and are among the most useful electrical papers. Many grades of laminated resin (phenolic or other) plate and tubing are made with Kraft paper. It is also used, shellac coated, in making rolled paper bushings and terminals. Very thin Kraft tissue (0.0005 in.) is now used in capacitors largely replacing linen and cotton tissue formerly employed.

Linen. So called linen papers are part cotton, sometimes as much as 60 per cent. They are strong and tough and have been used in very thin grades for condenser (capacitor) insulation. The use of linen in all forms (cloth, paper, and twine) is not nearly so extensive as it once was. Cheaper satisfactory substitutes have been found.

Japanese Paper is imported from Japan and is made of mulberry pulp fiber. The particular feature of this fiber is the great length, giving good strength, although the paper looks frail and not dense. Tissue (0.001 in.) finds use as a backing for mica tape and as wire insulation. It is not suitable for capacitors because of small holes and lack of uniform structure.

Rope Paper may be manufactured either from manila hemp fiber or from old manila hemp rope. It is very strong, resisting tear better than cotton paper. It is frequently impregnated with varnish and used in combination insulations such as mica and rope paper. When made from old rope, the paper frequently contains metal particles. For this reason, other papers are usually preferred for high-insulation duty. Rope paper can be folded and crimped. It is thus used in holding wires in some types of coils where flanges are omitted.

CELLOPHANE is a pure cellulose product made from wood pulp by chemical treatment. It is transparent. It is obtainable in sheet, tubing, and tape forms. The dielectric strength and specific inductive capacity are high (see table), and inflammability is no greater than that of paper. Because of its purity it has much promise as an insulator and has been used as wire insulation and coil taping. Some varieties named "flameproof" and "moisture proof" are now available. Moisture proofing is accomplished by a lacquer coating. Some grades of cellophane are flexibilized with the addition of glycerin.

CELLULOSE ACETATE is obtained by acetic anhydride treatment of cotton fiber or purified wood pulp. It has high dielectric properties and uniformity of characteristics. It can be obtained in the same forms as cellophane and is used for similar applications. It is comparable in inflammability to paper. It is not as hygroscopic as regular cellophane.

VARNISH-TREATED CLOTH. Varnished cloth, usually muslin, is widely used as an insulating material. It is sometimes known as varnished cambric or empire cloth. It enters the manufacture of cloth-insulated cables, is used as coil wrappings, especially on end turns of rotating machines, and in combination with paper as a lining for coil slots. Coil terminals on large transformers are taped with varnished cloth.

The fabric is most commonly an unbleached cotton cloth of 60 to 64 threads per inch and in various thickness from 0.005 to 0.015 in. Treatment consists of impregnating and coating with varnish. This is done in a tower through which the cloth passes continuously. It passes through successive varnish baths and is dried by heat between dips. When it emerges it has had from two to four dips and bakes. The varnishes generally used are two of the natural oil types, tan cloth varnish or black. The tan is somewhat more oilproof and less moisture proof than the black. The black is usually asphaltic but

can be made satisfactorily oilproof. The dielectric properties of black cloth are generally superior to those of tan. Black cloth finds a large use in cable manufacture. For this purpose it must possess low power factor and losses. The surface coat of varnish may be made relatively hard and smooth; tacky so that, in taping, successive layers will stick together; or greasy so that layers of tape on irregular forms will slip into place easily.

Some properties of insulating fabrics and papers are shown in the following table:

Table 7. Insulating Fabrics and Papers

Material	Thickness	Breakdown Volts	
		1 Layer	Per Mil
Treated Cloths			
Asbestos cloth, varnished	0.047	3,780	80
Asbestos tape, varnished	.037	3,145	85
Cambric tape, varnished black, bias	.010	13,640	1364
Cambric tape, varnished tan, bias	.010	11,515	1151
Cambric, rolls, varnished black, straight	.010	13,320	1332
Cambric, rolls, varnished tan, straight	.007	7,850	1121
Drilling, rolls, varnished black, flex	.020	9,250	462
Duck, rolls, varnished black, flex	.030	8,947	298
Friction, cloth taper, commer	.015	3,290	219
Friction, cloth taper, bias	.015	1,480	99
Friction cloth	.024	1,815	76
Silk, oiled	.004	4,450	1112
Surgical tape, varnished	.023	1,240	54
Treated Papers			
Asbestos paper, shellac 1 side	0.010	2,120	212
Asbestos paper, shellac 1 side	.029	2,610	90
Express paper, paraffin 2 sides	.008	5,370	671
Fishpaper, shellac 1 side	.003	1,710	570
Fishpaper, paraffin 1 side	.010	8,840	884
Fullerboard, shellac 2 sides	.013	3,390	260
Fullerboard, varnished 2 sides	.035	13,130	375
Jap. paper, shellac 2 sides	.003	1,136	379
Kraft paper, tan, shellac 1 side	.0045	1,270	282
Rope cement paper, varnished 2 sides	.008	9,746	1218
Rope cement paper, shellac 2 sides	.016	8,850	553

Cloths and papers tested flat between 2-in. circular electrodes. Average of 10 breaks. (60 cycles.) Tapes tested by wrapping on 1-in. diameter rod in half lapped layers. Average of 10 breaks. (60 cycles.)

The effect of pull on the dielectric strength and stretch of tape is of importance. Tape must be capable of conforming to irregular forms and must therefore stretch to an extent, but must be neither too stiff nor too weak. The curve of Fig. 3 shows satisfactory performance of a black tape.

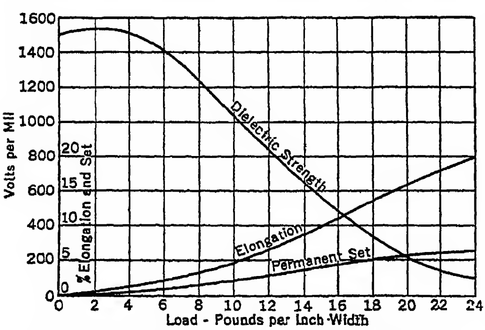

Fig. 3. Effect of Stretch on Black Varnished Tape

WOOD. Dry wood is a good dielectric and possesses the advantages of low cost, light weight, and strength with toughness. Probably the most frequently used woods are maple and hickory. White pine is a good insulator, especially when oil treated, but is not strong. Wood is generally treated with oil (linseed, tung, or transformer) or some wax or gum to fill the pores and keep out moisture. For use in air it is frequently given a finish of baking varnish.

FIBER is pure cotton cellulose chemically treated to form a hard, bone-like material. It is known by various trade names such as Fiber, Vulcanized Fiber, Hard Fiber, Horn Fiber. Fiber of best quality has its origin in old rags, because the natural oil of the virgin cotton fiber is deleterious and cannot be easily removed by chemical processes.

This oil is removed in the wearing and washing of the garment. All dirt and grit must be removed from the rags. The quality also depends on the softness of the rags.

Vulcanized Fiber sheets are made by passing cotton rag paper through a strong acid or zinc chloride bath and rolling it up on a large drum where each layer of paper sticks to the layer beneath it. When the proper thickness is obtained the acid soaked material is cut from the drum and cut in half forming two sheets of raw fiber. These sheets are put through a soaking process in large wooden tubs, each subsequent tub containing a weaker solution, the last tub containing pure water. (Catalog of the Continental Fiber Co.)

The soaking process requires from one week to one year, depending on the thickness of the sheet. A very small amount of acid or chloride will remain in spite of all the soaking. The sheets are air dried and seasoned at a constant temperature. During the drying, they warp and shrink to one-half the original thickness. They are flattened in a steam-heated hydraulic press and then calendered to exact thickness.

According to Almy the mechanical and physical properties of fiber vary between wide limits according to the manipulation of the chemical treatment and the varying quality of the original paper or rags. A fiber for a particular use should therefore be selected with great care.

Fiber is not waterproof. Dilute acids and alkalies cause no other effects than water, but concentrated acids cause disintegration. Organic solvents and oils have no effect whatever. At a sustained temperature of 80 to 100 deg cent, it loses its water of condition and becomes brittle, and it chars in a short period of time at 200 deg cent.

Fiber is widely used because of its strength, toughness, and comparatively high insulating values when dry. It is easily formed and machined. It resists the action of arcs on the surface to a great degree. The following specification properties of fiber have been taken from the N.E.M.A. Standards, 1931.

Table 8. N.E.M.A. Specifications for Hard Fiber

Minimum Tensile Strength at 20 Deg Cent

(Pounds per Square Inch)

Thickness, in.	Bone		Commercial Grades	
	Crosswise	Lengthwise	Crosswise	Lengthwise
Up to 1/8 incl.	6500	8500	6000	8000
1/8 to 1/2	6000	8000	5500	7500
Over 1/2	....	....	5500	7000

Minimum Transverse Strength

(Pounds per Square Inch)

Commercial Grade	Crosswise	Lengthwise
Load applied on	Face or edge	Face or edge
1/2 in. thick and over	11,000	13,000

Adopted Standard 9/24/1926.

Minimum Dielectric Strength

All Grades, in.	Volts per Mil
Up to 1/16 incl.	175
Over 1/16 to 1/8 incl.	150
Over 1/8 to 3/8 incl.	100
Over 3/8 to 1/2 incl.	50
Over 1/2 at least (total volts)	25,000

Minimum Dielectric Strength for Thin-weight Fiber and Fish Paper

Thickness, in.	Volts per Mil
0.004 to 0.005 incl.	200
Over 0.005 to 0.015 incl.	300
Over 0.015 to 0.040 incl.	250

Adopted Standard 10/22/1930.

MAXIMUM WATER ABSORPTION

Per Cent Change in Weight After 1 Hour Immersion

Thickness, in.	Bone	Commercial
Up to 1/16 incl	45.0	65.0
Over 1/16 to 1/8	20.0	35.0
Over 1/8 to 3/8	10.0	15.0
Over 3/8	5.0	10.0

MINIMUM BRINELL HARDNESS

Thickness, in.	Bone	Commercial
1/4 and over	15.0	10.0

MINIMUM SPECIFIC GRAVITY

Thickness, in.	Bone	Commercial
Up to 1/16	1.30	1.05
1/16 to 1/2 incl	1.30	1.10
1/2 to 1	....	1.05
1 and over	....	1.01

Rubber Materials

RUBBER. (See also Gutta-percha; Wires and Cables.) Rubber is derived from the milky secretion or latex of certain tropical trees, creepers, and shrubs found chiefly in America, Africa, Ceylon, and Malacca. When these plants are tapped, a thick milky looking fluid or latex exudes from them. This latex is composed of very minute oil-like refractive globules, varying in size, which are in a state of rapid Brownian movement in a clear transparent liquid, called the serum. Besides these *caoutchouc* globules, or rubber-gum proper, the serum contains resins, protein, enzymes, and various organic and inorganic compounds. Rubber or India rubber is the dried-up or coagulated latex. The best rubber is from a tree known as the *Hevea brasiliensis*, and is known as *Hevea* rubber. It grows wild in Brazil and is cultivated in Ceylon and the Malay Peninsula. In Brazil coagulation is effected principally by dry heat or smoking. A wooden paddle is dipped in the latex and held over a smoky fire until the latex has coagulated. This process is repeated until the caoutchouc layers have become sufficiently thick, when the lump of raw rubber is cut off, dried for several days, and dispatched usually as "fine Para biscuits" to a trading center. The plantation rubber is coagulated in sheets by means of acetic acid. If subsequently smoked it is known as "smoked sheets." Para entrefine, Negro Heads, and Sernamby are usually prepared from fine Para rubber which adheres to the tree during tapping or to the vessels containing the latex.

A relatively recent development is the shipping of the preserved latex in the liquid form. This may then be treated to precipitate the rubber particles, or used directly for impregnation of cords or fabrics.

Acetone Extract. In addition to the pure rubber gum, washed rubber contains resins and proteins which are soluble in acetone and are therefore often known as "acetone extract."

Impurities in Wild Rubber. Wild rubber contains, in addition to the caoutchouc, a number of foreign substances, such as sand and bark, which can be removed by mechanical washing, followed by drying.

Manufacture of Rubber Insulation. The rubber is milled passed between heavy rollers and broken down to a plastic mass. It is then mixed with a large proportion of fine powder consisting usually of inert mineral substances, waxy hydrocarbons and sulfur. The mixture is thus masticated until all its constituents are thoroughly mixed and a smooth homogeneous paste obtained. This process is known as compounding.

Fillers Used in Compounding. Experience has shown that 60 or 70 per cent of mineral filler, or even a greater proportion of rubber substitute, may be added to rubber gum, before the essential qualities of the rubber cease to predominate. The majority of commercial 30 per cent insulating compounds have compositions which fall within the following limits.

Ingredient	Per cent
Rubber	30–32
Whiting	0–30
Zinc oxide	28–67
Litharge	1–12
Ozokerite or paraffin	2– 4

In addition to the above fillers, from 2 per cent to 4 per cent of sulfur is added to the compound, the greater part of which combines with the rubber in the vulcanizing process. Sulfur bearing organic substances are also used for vulcanizing.

Barium sulfate, sublimed white lead, lead carbonate, carbon black, talc, magnesium carbonate, red lead, barium carbonate, black hydro-carbons, and other substances are also used in small quantities.

Applying the Compound to Wires. The rubber compound is applied to the wire by "tubing" machines, or is applied in strips, and the wire thus covered with the compound is coiled up ready for vulcanizing.

Vulcanizing. If exposed for a long time to air or sunlight, rubber loses its elasticity and finally oxidizes completely into resinous matter soluble in acetone. By vulcanization, however, rubber is rendered more or less immune from deterioration by weathering. Vulcanization is the chemical union of rubber gum with sulfur. It takes place at a temperature between 248 and 302 deg fahr.

The coils of wire, covered with the compound as above described, are placed in a suitable chamber to which steam at the proper temperature is admitted. The time required for vulcanization depends upon the thickness of the insulation, the nature of the compound, the temperature and pressure of the steam, etc., ranging from 2 to 8 hours.

Accelerators and Anti-oxidants. Certain classes of organic compounds are used in small amounts to accelerate the cure, thus largely decreasing the investment in vulcanizing equipment for a given production. Anti-oxidants, such as aniline and related compounds, are often added to the mix to increase the useful life of the articles made from it.

Specific Resistance. The specific resistance is not an indication of the quality of rubber insulation as it depends more upon the dryness, the proportion of mineral wax, and the degree of vulcanization, than upon the quality of the ingredients. When the megohms are low they can almost invariably be raised by drying the insulation in a desiccator. The specific resistance of very poor-quality rubber compound, however, sometimes is so low that, when a cable lies in damp earth, sufficient leakage current may flow to permit the passage of water by endosmose, when the conductor is negative to the ground.

Dielectric Strength. The disruptive strength of rubber insulation is generally given as between 350 and 450 kv per in. or about 140 to 180 kv per cm effective a-c values. The pressures which should be used in commercial testing do not depend entirely upon the dielectric strength of the rubber, as air in the rubber or between the rubber and conductors becomes ionized at a pressure of about 30 kv per cm with consequent generation of ozone, which rapidly oxidizes and destroys the rubber. Hence test pressures for rubber insulation, if sufficiently high to permit ionization of the air, should not last long enough to permit the formation of an appreciable amount of ozone.

Specific Inductive Capacity. The specific inductive capacity of pure rubber is about 2.3 (Floy), but the vulcanized compounds used for insulation have specific capacities ranging between 3 and 5, the latter value being nearer the average.

Water Absorption. Rubber contains proteins which, when the rubber is immersed in water, act as membranes to carry the water into the rubber. There are natural electrolytes in the rubber, such as quebrachitol, which go into solution in the absorbed water, the rate of absorption depending on the difference between the external and internal osmotic pressures. Water absorption may be detected by increase of capacitance and conductance while the rubber is immersed in water.

HARD RUBBER OR EBONITE. If, instead of using the small proportions of rubber indicated above for vulcanization, the proportion be increased to between 20 and 35 per cent, a hard material will be obtained which can be used for instrument panels, etc., as it can be machined, drilled, etc. The principal properties will be as follows:

Tensile strength	1000 lb/sq in. or more
Dielectric strength, between flat electrodes	1000 to 2000 volts/mil
Dielectric constant (S.I.C.)	2.0 to 3.5
Resistivity	10^{15} to 10^{18}

Hard rubber is not hygroscopic but is attacked to some extent by oil and ozone. Sunlight reduces its surface resistivity. (See Bureau of Standards Scientific Paper No. 234.) Data on dielectric strength are given by F. M. Farmer, *Trans. A.I.E.E.*, 1913, Vol. 32, p. 2101.

FACTICE. Vegetable oil, such as cotton seed oil, rape seed (colza) oil, corn oil, etc., may be converted into a rubber-like substance by heating to about 470 deg fahr and pouring into it from 30 to 40 per cent by volume of molten sulfur. The mass is stirred until a thorough combination has been obtained, and evolution of gas has ceased. When cooled, the resultant mass of *vulcanized oil* is a soft rubber-like material of low tensile strength but water resistant and unaffected by ozone. It mills readily with rubber and

adds its characteristics in considerable measure. Rubber insulation for high voltages almost invariably contains factice in large proportions, so that it may be ozone resistant.

KOROSEAL is a synthetic rubber developed by the Goodrich Rubber Co. which is highly oil resistant and quite immune to the action of ozone. It is a thermoplastic material, and makes an excellent filling for oil proofing fabrics.

DUPRENE is a synthetic rubber developed by the DuPont Company. It is a chlorinated derivative of acetylene, and is similar in molecular structure to rubber. The advantages of Duprene are its resistance to age brittleness and to the action of oil. The material is permanently flexible and in accelerated aging at high temperature shows superiority. Light mineral oils soften Duprene somewhat but not so seriously as they do rubber. It may be obtained in sheets and is used chiefly for gaskets.

THIOKOL, chemically olein polysulfide, derived from natural gas, is an oilproof rubber substitute made by the Thiokol Corporation. It is unaffected by oil, gasoline, or similar materials. In accelerated aging tests, its life compares with high-grade long-life rubber. The "cold flow" under pressure (as in a gasket) is greater than that of either rubber or Duprene. Because of the composition, Thiokol produces corrosion in contact with copper; and with transformer oil, sludging and acid formation result. It cannot be used as transformer gaskets with usual designs. The chief application is for oil and gasoline hose. It may be used as a surface layer over rubber for gaskets on lubricating oil and gasoline containers. Also, an outside coating of thiokol on rubber-insulated cables is useful for cables subjected to oily conditions.

GUTTA-PERCHA. Gutta-percha is derived from the milky secretion or latex of the bark of certain trees of the order of Sapotaceæ, especially the Dichopsia Gutta, found chiefly in the Straits Settlements and Malaccan Archipelago. The trees are felled just after the rainy season, and the gum collected as it exudes from incisions in the bark. Latex is also extracted from the leaves by digesting them in toluol. The latex is boiled in water and it is then ready for export. Gutta-percha is becoming quite scarce, and practically the whole available supply is used by British cable makers.

The chemical composition of gutta-percha is represented by the formula $C_{10}H_{16}$. It resembles dark brown leather at temperatures between 0 and 27 deg cent. At higher temperatures it softens, and at 65 deg cent it is plastic and capable of being molded or rolled. On cooling it returns to the non-plastic condition. Gutta-percha oxidizes when exposed to the air, changing from dark brown or black to yellowish gray and becoming brittle.

Preparation of Gutta-percha Insulation. For insulating purposes gutta-percha is shredded and squeezed in warm water. It is then kneaded and strained through fine wire gauze and rolled into sheets. Its further refinement is carried on differently by various manufacturers, the processes being more or less trade secrets. Like rubber it is applied to the wire either by a tubing machine or by strips. Unlike rubber it is used in the pure state without mixture with minerals. Gutta-percha is less porous than rubber and therefore more waterproof, a quality which makes it the best material for submarine cables. Its specific gravity is just above unity.

Specific Resistance. The constant K in the formula

$$M = K \log \frac{D}{d}$$

has the value 900 approximately, at 75 deg fahr after one minute electrification. See also article on Rubber.

Temperature Coefficient of Resistance. The temperature coefficient of resistance of gutta-percha is of the same nature as that of rubber (see article on Rubber), i.e.,

$$R_T = R_{75}\,\epsilon^{(75-T)C}$$

where R_{75} is the resistance at 75 deg fahr, R_T the resistance at T deg fahr, and C a constant which varies from 0.065 to 0.085. For values of ϵ^x see Exponential Functions.

Effect of Pressure upon Resistance. Gutta-percha being used principally for submarine cables, the effect of pressure upon its resistance is important. Let R = its resistance at atmospheric pressure, R_p = resistance under pressure of p pound per square inch. Then

$$R_p = R\,(1 - 0.00023\,p)$$

11. PLASTIC INSULATING MATERIALS

Synthetic Resin Products

In recent years great advances have been made in the manufacture of countless varieties of insulating products using some organic filler (cloth, paper, or other fiber) bonded with some type of synthetic resin. The first important material of this type was Bakelite,

developed by Dr. Bakeland. This is a condensation product resulting from the combination of phenol and formaldehyde under certain conditions. The resin in its initial state is thermoplastic and soluble in alcohol-benzol solvent or acetone. Further heat polymerizes the resin and renders it insoluble and infusible. In treating materials with Bakelite, the first stage is used in varnish form and the impregnated filler is then pressed with accompanying heat to cause polymerization.

Many other resins have been introduced since Bakelite was developed. Some are modifications of the phenolic class, but many are of distinct types. A useful series of resins is known as Glyptal. These are made of glycerin and pthalic anhydride. Other types are resorcinal-formaldehyde, furfural-formaldehyde, vinyl resins, and urea resins.

Table 9. Properties of Laminated Phenolic Sheet Materials (N.E.M.A. Standards, 1931)

Grade	Description	Average Tensile Strength, lb per sq in.	Average Flexural Strength, lb per sq in.	Average Dielectric Strength, volts per mil		1,000,000 Cycles			Moisture Absorption Percentage in 24 hr
				Short Time	Step by Step	Power Factor Percent	SIC	Dielectric Loss Factor	
X	Paper base	12,000	21,000	700	500	....	....	....	4.0
P	Paper base	8,000	15,000	600	400	....	....	....	4.0
XX	Paper base	8,000	16,000	700	500	4.5	5.5	0.25	1.3
XXX	Paper base	7,000	15,000	650	450	3.5	5.0	0.18	1.0
C	Heavy fabric base for mechanical purposes	10,000	20,000	150	100	10.0	7.0	0.70	1.7
CE	Heavy fabric base electrical and mechanical	9,500	19,000	425	275	5.5	5.5	0.30	1.5
L	Fine weave fabric mechanical	10,000	20,000	150	100	10.0	7.0	0.70	2.0
LE	Fine weave fabric electrical and mechanical	9,000	19,000	500	300	4 5	5.0	0.22	1.2

Table 10. Properties of Typical Laminated Tubing

Description	Color	Specific Gravity	Water Absorption Per cent Increase in Weight after 24 Hours	Ultimate Strength, lb per sq in. Axial Stresses		Dielectric Strength 1/16 Wall, Volts per mil	
				Tensile	Compression	In Oil	In Oil after 24 Hours in Water
Kraft paper and synthetic resin. Applications where machining qualities are desirable. Superior mechanical and electrical properties. Special tools for shapes other than round	Dark tan	1.29	3	11,000	20,000	1000	800
Kraft paper and shellac. For general use (except brush holder tubes) under 75 deg cent. Will collapse at 100 deg cent unless mechanically supported. Resists arcing slightly more than phenolic materials. Made in round and other shapes. Not baked	Brown	1.12	55	8,000	7,000	800	
Fine weave fabric and synthetic resin. Where extra strength and high impact are desired. Has lowest moisture absorption of any grade, and is preferred for most chemical applications	Tan	1.25	2	5,800	22,000	350	300
Kraft paper and synthetic resin	Black	1.29	3	11,000	20,000	650	500

indicative of satisfactory oil as far as laboratory tests can determine. The following values may be taken as representative of acceptable unused transformer oil which is also suitable for most circuit breakers:

Specific gravity	0.898 at 15.5 deg cent
Flash point	132 deg cent
Fire point	149 deg cent
Viscosity (Saybolt)	57 sec at 40 deg cent
	280 sec at 0 deg cent
Pour test	−45.6 deg cent
Demulsibility (resistance to emulsion value)	25 sec
Neutralization value	0.03 mg of KOH per gram

(See A.S.T.M. Test Methods for Oil.)

Maintenance

CAUSES OF DETERIORATION OF OIL IN TRANSFORMERS. The principal causes of deterioration of insulating oil in service are water and oxidation. The oil may be exposed to moisture through condensation from moist air due to breathing of the transformer especially when the transformer is not continuously in service. The moist air drawn into the transformer condenses moisture on the surface of the oil and inside of the tank. The oil may also be contaminated with water through leakage such as from leaky cooling coils or covers. Sludge is an oxidation product, the amount formed in a given oil being dependent upon the temperature and the time of exposure of the oil to the air. By careful refining, the components of oil which are most readily oxidized to form sludge can be removed, so as to provide an insulating oil which will not sludge under normal operating conditions. Excessive temperatures may cause sludging of any transformer oil, regardless of how well it is refined.

Transformer oil which has begun to sludge will continue to do so after it has been purified by means of the centrifuge or filter press, as these methods of purification do not remove the deterioration products which are in process of formation but have not yet been thrown down as sludge. No method is yet available in the field which will remove these products and bring sludged oil back to its original condition when new. Such oil can be refined so as to be equivalent to new oil, but this would require equipment which is available only in an oil refinery. It is not economical to send used oil to the refinery as it will only allow fuel oil price, which would probably be less than the cost of transportation.

Another effect of oxygen is gradually to produce organic or "fatty" acids in oil in service. These should not be confused with the mineral acid such as sulfuric acid used in refining, as in small amounts they do not have a deteriorating effect upon insulation. No method is available in the field of purifying oil of high organic acidity.

In an effort to produce oil which has less tendency to sludge, it is possible to "over-refine" it. Such an oil develops organic acidity, which, when once started, increases rapidly to a point where it becomes a menace to insulation. This high acid development is characteristic of some of the so-called water-white oils.

CAUSES OF DETERIORATION OF OIL IN CIRCUIT BREAKERS. The principal causes of deterioration of insulating oil in circuit breakers in service are:

1. Water.
2. Carbonization of the oil caused by operation of the circuit breaker.

Insulating oils do not absorb water, but they may receive water through condensation on the surface of the oil or on the inside of the tank owing to the entrance of moist air.

All oil in circuit breakers is subject to carbonization due to arcing between the contacts. Part of the carbon formed is deposited on the mechanism and at the bottom of the tank while the remainder continues in suspension in the oil.

Carbonization takes place not only when the circuit breaker opens heavy short circuits, but also whenever an arc is formed, even during such light service as the opening of the charging current of the line, and this latter service repeated may eventually produce enough carbon to be a source of trouble.

The carbon reduces the dielectric strength of the oil, lowers the surface resistance of the insulation if water is present, and also lowers the resistance to emulsification. The carbon may not be detected by the dielectric test, particularly if the oil is free from moisture.

In cold weather, a larger amount of carbon is formed than in warm weather on account of the increased viscosity of the oil at low temperatures. Also the carbon is not as readily dispersed through the oil.

TEST METHOD. (Instructions for testing correspond in general to the recommendations of A.S.T.M.) Dielectric strength.

Apparatus. The transformer and the source of supply of energy shall not be less than $1/2$ kva, and the frequency shall not exceed 100 cycles per second. Regulation shall be so controlled that the high-tension testing voltage taken from the secondary of the testing transformer can be raised gradually without opening either primary or secondary circuit. The rate of rise shall approximate 3000 volts per second. The voltage may be measured by any approved method which gives root-mean-square values.

The test cup for holding the sample of oil shall be made of a material having a suitable dielectric strength. It must be insoluble in and unattacked by mineral oil and gasoline and non-absorbent as far as moisture, mineral oil, and gasoline are concerned.

The electrodes in the test cup between which the sample is tested shall be circular disks of polished brass or copper, 1 in. in diameter and having square edges. The electrodes shall be mounted in the test cup having their axes horizontal and coincident, with a gap of 0.100 in. between their adjacent faces, and with top of electrodes about $1^1/_4$ in. below the top of the cup.

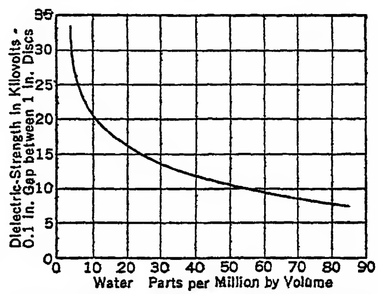

FIG. 4. Relation between Dielectric Strength and Amount of Water in Insulating Oil

Procedure. (*a*) The electrodes and the test cup shall be wiped clean with dry calendered tissue paper or with a clean dry chamois skin and thoroughly rinsed with oil-free dry gasoline or benzine until they are entirely free from fibers.

(*b*) The spacing of electrodes shall be checked with a standard round gage having a diameter of 0.100 in. and the electrodes then locked in position. Care shall be taken not to touch the electrodes with the gage or in any other manner after cleaning the electrodes and cup, so as to avoid any possible contamination.

(*c*) The test cup shall be filled with dry gasoline or benzine and voltage applied with uniform increase at the rate of approximately 3000 volts (rms) per second until breakdown occurs. If the dielectric strength is not less than 25 kv, the cup shall be considered in suitable condition for testing the oil. If a lower test value is obtained the cup shall be cleaned with gasoline and the test repeated.

(*d*) The temperature of the test cup and of the oil when tested shall be the same as that of the room which should be between 20 and 30 deg cent (68 and 86 deg fahr). Testing at lower temperatures is likely to give variable results which may be misleading.

(*e*) The sample in the container shall be agitated with a swirling motion to avoid introducing air, so as to mix the oil thoroughly before filling the test cup. This is even more important with used oil than with new oil, as the impurities may settle to the bottom and the test may be misleading.

(*f*) The cup shall be filled with oil to a height of no less than 0.79 in. (20 mm) above the top of the electrodes.

(*g*) The oil shall be gently agitated by rocking the cup and allowing it to stand in the cup for 3 min before the first and 1 min before each succeeding puncture. This will allow air bubbles to escape.

(*h*) Voltage shall be applied and increased uniformly at a rate of approximately 3000 volts (rms) per second until breakdown occurs as indicated by a continuous discharge across the gap. (Occasional momentary discharges which do not result in a permanent arc may occur; these should be disregarded.)

(*i*) Provision shall be made for opening the circuit as promptly as possible after breakdown has occurred in order to prevent unnecessary carbonization of the oil. After each puncture, the testing vessel shall be jarred to loosen particles of carbon adhering to the electrodes and the oil gently agitated but not with sufficient violence to introduce air bubbles.

(*j*) Five breakdowns shall be made on each filling after which the vessel shall be emptied and refilled with fresh oil from the original sample. The test shall be continued until the averaged values of at least three fillings do not differ from their mean by more than 10 per cent.

PURIFICATION. The purification of oil used in circuit breakers and transformers consists principally in the removal of water, carbon, and sludge and the restoration of its

resistance to emulsification, putting the oil in the best condition to separate out any water which may later be introduced.

Three types of equipment for simple purification of oil in circuit breakers and transformer service are in general use, the centrifuge, the blotter filter press, and the combination centrifuge and filter press. The combination of centrifuge with chemical treatment is particularly well adapted for the purification of carbonized circuit breaker oil. The application of chemical treatment for purifying sludged oil or oil having high acidity for transformer service is still in the experimental stage.

Blotter Filter Press. The filter press is essentially a number of sets of filter papers in parallel, each set containing several thicknesses. The oil is pumped through the filter paper which absorbs the water and strains out the sediment.

The filter press is not intended to remove large amounts of free water from the oil. Obviously the changing of filter papers necessary for obtaining dry oil would so reduce the capacity as to make this method of purification impractical. In such cases the water may be removed by a centrifuge, or should be allowed to settle out and be drawn off from the bottom of the container before passing the oil through a filter press.

Filtering through blotters does not materially reduce organic acidity or improve resistance to emulsification except as the latter is affected by the presence of carbon, although the dielectric strength may be restored to a satisfactory value.

The Centrifuge is the most convenient equipment known for removing water from oil. It also removes solid material other than finely divided carbon. The temperature of the oil should be maintained at 48.9 to 51.7 deg cent (120 to 125 deg fahr) in order to insure removal of all the water at full capacity of the machine. A higher temperature gives no advantage and, if excessive, is harmful to the oil. (A 6-kw heater will raise the oil about 15.6 deg cent (60 deg fahr) per 100 gallons per hour.) The centrifuge equipment may be arranged to act as a separator, discharging the oil and water by different outlets, or as a clarifier, discharging the oil but retaining in the bowl, the water and other impurities. When there is considerable water in the oil it should be operated as a separator.

The centrifuge will remove the coarser particles of carbon from the oil. For removal of fine particles of carbon, the blotter filter press or centrifuge with suitable chemical treatment should be used.

Soluble impurities developed by constant use of the oil in circuit breakers or transformers *can be removed only by chemical treatment.*

The Combination Centrifuge and Filter Press. The combination centrifuge and filter press meets the need for a compact unit which may be used advantageously in the purification of insulating oil. It consists briefly of a motor-driven centrifuge, electric heaters, regulating float valve, pumps and filter press or presses, mounted upon a truck.

The centrifuge and chemical treatment of circuit-breaker oil includes chiefly the agitation of warm oil with a solution of silicate of soda, the passing of this mixture through a centrifuge operating as a separator which removes the sodium silicate solution containing nearly all the impurities, then to a mixing tank where it receives a definite amount of inert earth, and finally passing the oil through another centrifuge operating as a clarifier, which removes the inert earth and any impurities or moisture present. The process is continuous.

13. OTHER LIQUID INSULATING MATERIALS

INSULATING VARNISHES. A varnish is generally considered a solution of gums (fossil or synthetic) in (1) volatile solvents or (2) drying oils. The evaporation of the solvent in the first type (spirit varnishes), and the oxidation (or polymerization) of the oils in the second type (oil varnishes), leave a hard smooth film which is a protection against moisture and weak chemical solutions. Varnishes may be differentiated from paints in that varnishes usually contain no pigment. Varnishes may, however, be used as vehicles for pigments, forming enamel paints.

Insulating varnishes have good electrical properties and also serve to protect and preserve the insulating properties of other materials used as insulation. Most of the widely used materials, such as cotton, paper, wood, and asbestos, are excellent insulators when dry, but absorb moisture readily, and when moist become more or less conducting. Varnishes fill air spaces in coils and offer better heat conductivity than air, as well as cementing coils together.

Spirit Varnishes are usually those composed of shellac (various grades) or synthetic resins dissolved in alcohol or other volatile solvents. They are used for rapid air-drying applications and find use more as finishes or surface coatings than impregnating purposes. They usually possess low flexibility and good cementing properties.

Oil Varnishes consist of varnish gums or resins, such as copal, succin, dammar, mastic, combined with a vehicle compounded from linseed oil and tung oil. In black varnishes, clear gums are replaced by asphaltic materials or pitches. The drying time is governed by the proportion of gums and oils and by the quantity of oxidizing agents or driers in the varnish. The greater the proportion of oils, the longer the drying time.

Some of the quick air-drying black varnishes may contain no drying oil. Such varnishes are neither oil type nor spirit varnishes and may be termed asphaltic varnishes. They may consist of an asphaltic base and a solvent derived from petroleum or coal tar.

In the manufacture of spirit varnishes, the gums are cut cold in the solvent; the oil type and asphaltic varnishes are made by fluxing the various ingredients together in a kettle at a high temperature.

Varnishes to be satisfactory must possess the desirable properties in all three states—as a liquid, during drying, and when solid. The most important characteristics to consider are:

Liquid	While Drying	Solid
Color	Spreading	Hardness
Viscosity	Draining	Toughness
Specific gravity	Drying time	Cementing ability
Per cent solids	Relation of surface to subsurface drying	Dielectric strength
Nature of solvent	Chemical action on metals or insulating materials	Softening temperature
Penetration		Resistance to water, oil, chemicals
Flow		
Tendency to skin in tanks		

It is not possible to combine in one varnish the maximum degree of all desirable attributes. There are, therefore, a number of useful types of insulating varnish, adapted for particular needs. For example, it is not yet possible to make a quick-drying varnish that has the ultimate degree of flexibility and heat life. Further, a plastic varnish is not considered the best for a hard surface finishing material.

For most electrical work the following varnishes cover the requirements satisfactorily: a clear and a black varnish for both air drying and baking, and a plastic coil impregnation varnish, making five varieties (see Table 11). These varnishes are useful as well for finishing of apparatus (metal and insulation) to provide good appearance and a surface easily kept clean.

Air-drying varnishes do not produce as hard a moisture-resistant coating as baking varnishes under usual conditions but are necessary for apparatus that cannot be baked or for repairs away from the factory. The black varnishes are generally more moisture resistant than the clear but are less oil resistant. Black varnishes are somewhat superior to clear varnishes in dielectric properties.

Varnishes may be applied by brushing, dipping, or spraying. Coils are usually dipped for sufficient time to drive out air, then drained, and then baked. The temperature of baking depends on many factors such as time allowable and type of material. Short-time baking (15 min) may be done up to 200 deg cent. A more usual temperature for baking time of 1 to 3 hours is 150 deg cent. Good oven circulation to remove solvent, and thus hasten drying, is important. Vacuum drying of coils is frequently employed to remove moisture and air. The impregnating material may be admitted to the tank while under vacuum to prevent entrapping air.

Testing of Insulating Varnishes. It is very difficult to describe a varnish comprehensively by any list of physical properties. The true test of a varnish is its performance in the application chosen. However, a number of standardized tests have been worked out by A.S.T.M. for evaluating insulating varnishes. They are of importance in checking uniformity and as a general basis for specifications. They cover: specific gravity; viscosity; flash point; evaporation; draining; drying time; dielectric strength; water absorption; heat endurance (life); acid, alkali, and oil resistance; and non-volatile content.

WIRE ENAMELS. Insulating varnishes with special characteristics are used in coating wires with an insulating film. Wire enamels are composed of fossil gums (in most cases) combined with drying oils. They differ from usual insulating varnishes in containing less oil (linseed and tung) and are thinned with coal-tar solvents quite similar to kerosene instead of the naphthas.

In the coating operation, the wire is passed successively through the tank of liquid enamel ("dope pan") and vertically through a baking chamber, over sheaves, and the cycle is repeated. From three to six layers are applied, depending on the size of wire and the service required. Baking is best done by controlled electric heat in the range 400–500 deg cent. The speed of wire travel depends on height of baking oven, tempera-

Table 11. Insulating Varnishes

	Drying Time, hours	Moisture Resistance Rating	Oil Resistance Rating	Acid Resistance, seconds till failure		Dielectric strength, volts per mil		Life at 110 deg cent, hours	Chemical Analysis										Applications
				H_2SO_4	HNO_3	Dry	Wet		Rosin	Asphaltum	Lime	Linseed Oil	Chinawood Oil	Lead and Manganese	Iron Oxide Dryer	Benzine 50 to 60 deg Baumé	HeavyBenzine 40 to 45 deg Baumé	Turpentine	
Black plastic baking	4	90	65	72	150	1923	947	770											Plastic insulator for coils and stators
Black coil baking	5	90	80	128	65	1890	610	528	6.1%	14.9%		23%	11.5%	0.77%		44%			For coils, armatures, and wound apparatus, also finish
Clear coil baking	1	55	100	61	40	1722	403	600	9.4%		0.51%		43.3%	1.02%		46%			For wound armatures and cloth finish
Black air dry	4	65	0	107	141	1642	527	24		29%		13%			0.43%	59%	54%		General use where baking cannot be done
Clear air dry	4	75	100	27	22	1128	246	675	8.2%			3%	27%	1.4%		55%		6.1%	General use where baking cannot be done

Specific gravity of all varnishes, 0.835 or 0.850. Thinner for all varnishes, benzine.

ture, and size of wire. Heavy wires (No. 10 A.W.G.) may travel 12 ft per min, whereas fine wire (No. 40–44 A.W.G.) may pass through at 80 ft per min.

Common American enamels are mahogany red in color when properly baked. Color is a fair but not infallible indicator of degree of bake or cure, since the color changes with time. Underbaked wire (copper red) is tougher and will elongate without rupture of enamel, but may be less resistant to abrasion and not oilproof. On the other hand, a black wire (overbaked) will probably be brittle but harder and more oil resistant. A good wire enamel properly baked should have the following characteristics:

a. Should produce a uniform coating—no beading and no thin spots.

b. Should be flexible to withstand elongation in coil winding. Should show no breaks when wire is stretched to breaking point.

c. Should resist abrasion.

d. Should have reasonably high dielectric strength (1500 volts per mil).

e. Should withstand hot oil and varnish. Withstand varnish at 80 deg cent for 16 hours without softening.

f. Should have long life. Elongation should not decrease appreciably when wire is stored for 6 months.

SHELLAC. Shellac is a resinous substance produced by insects of the plant lice species. These insects suck the sap of certain trees in India, and convert this sap into a resin which is exuded through the pores and finally completely surrounds the insects. Being a product formed by natural processes, it is subject to a variety of ills caused by climatic changes during its formation, and by natural enemies such as parasitic and predatory insects which prey on the minute shellac insects. The branches on which the insects lived are removed from the trees and the shellac incrustations scraped off. In this form, with some twigs, bark, and insect bodies included, the shellac is known as sticklac. This is ground into smaller pieces and washed thoroughly in water to remove dirt and all traces of the red coloring matter known as lac dye. After washing, the material, pale orange in color, is known as seedlac. The large granules are used in the highest grades of orange shellac; the smaller dark granules are marketed as lower grades of orange shellac and garnet shellacs. After purification, the lumps of shellac resin are packed in bags for shipment. When used, the lumps are ground and cut in alcohol, forming a spirit varnish. Some grades of shellac are dissolved and then dried to flake form for easy solution. The various grades of shellac differ in color, flow properties, and freedom from impurities such as dirt, dye, and wax. Shellac which has poor flow characteristics is sometimes improved by addition of rosin. For some purposes, this does no harm, but in general the adhesive and mechanical strength is impaired by rosin additions.

Shellac is most frequently used in varnish form, applied to insulating materials (paper, cloth, yarn, or asbestos). The treated material is subsequently heated to melt the dried shellac and cement the parts together. To obtain satisfactory uniformity of performance certain tests have been developed for acceptance of shellac.

Flow Test is an important indication of shellac fluidity. It is made with a device in which 2 grams of powdered shellac is melted and at 125 deg cent allowed to flow down a trough inclined 60 deg. The time to flow 5 in. is measured. Typical results of usual grades are shown below.

VSO orange shellac	60– 100 sec at 125 deg cent
CVTN orange shellac	100– 200 " " " " "
Pale orange shellac	80– 200 " " " " "
Garnet orange shellacs	200–1500 " " " " "
Soda lac	Too sluggish to flow

Polymerizing Test. The length of time a shellac retains its fluidity at high temperatures (125–250 deg cent) before becoming infusible is important in selecting shellacs for uses where repeated melting is necessary.

Representative figures of "life" (time until "rubbery" consistency) at various temperatures are:

Shellac	Life at 125 deg cent, min	Life at 150 deg cent, min	Life at 200 deg cent, min
VSO orange shellac	60	36	10
CVTN orange shellac	35	14	6
Pale orange shellac	50	30	9
Garnet shellac	20	10	4
Soda lac	40	20	7

Viscosity Test. Used in determining solubility.

Volatile Test. A measure of amount of gas evolved.

SOLVENTS. It is important to know the insulating characteristics of ordinary solvents, since some quantity of solvent is usually still present after coils are baked or insulating structures finished.

Because of its high affinity for water, alcohol must be thoroughly removed from spirit varnish impregnation. In deep coils this is difficult, and drying operations must be carefully done.

Turpentine, benzine, toluol, benzol, and other related solvents do not mix with water and are good insulators. It is not therefore required for insulating reasons that all solvent be completely removed in impregnation processes. Evaporation will continue for some time. For mechanical reasons (to prevent throwing of "wet" varnish or distortion of coils) it is important to carry evaporation far enough to produce a hardening of the varnish or compound. Under some conditions, organic acids develop in "wet" varnish.

LINSEED OIL, used as an impregnating material and as a major constituent of paints and varnishes, comes from flaxseed. The density at 15 deg cent is 0.934. Linseed oil dries principally by atmospheric oxidation, finally to a hard gum. With "raw" linseed this process in air is quite slow. "Boiled" oil will oxidize fairly rapidly in air. The action is accelerated by heat or chemical driers. (See Insulating Varnishes.) Linseed oil is commonly used to impregnate wood. It produces a moisture-proof product, capable of withstanding weather. Impregnation is usually done hot (60 deg cent) to aid in penetration. Baking oxidizes the oil to give a satisfactory surface coat.

NON-DRYING OILS, such as castor oil, rosin oil, or olive oil, are good insulators, but almost always are used in combination with other materials.

TUNG OIL (china wood oil) comes from the nuts of an oriental tree and is used mainly in the manufacture of varnishes. It hardens chiefly by polymerization rather than oxidization and thus differs from linseed oil. It may be used for impregnating coils or wood.

14. GASES

Gases, when stressed below rupturing gradients are excellent insulators. Their specific inductive capacity under normal atmospheric conditions is nearly the same as a vacuum, being less than 1 per cent greater. Dielectric strength is rather low, being 31 kv (crest) per cm for air in a uniform electrostatic field. Where the field is non-uniform, local breakdown (corona) induces complete rupture at much lower values. The dielectric strength increases directly with pressure.

AIR. Air occurs as an insulator on every piece of electrical apparatus. The distance externally between live parts or between terminals and a grounded part must be great enough to prevent breakdown or excessive leakage in an air path. It is true that frequently the effect of a solid insulation creepage surface is important but in many cases air is the effective external dielectric. In solid insulation, air is a detriment. Its low specific inductive capacity causes concentration of voltage gradient on the air layers or pockets (inversely proportional to specific inductive capacity) and may lead to corona and progressive breakdown.

The dielectric strength of air gaps depends largely on the type of electrodes with their local electrostatic fields. Needle point gaps have a fairly uniform strength of approximately 4 kv per cm. Sphere gaps or gaps between large curved surfaces have much higher unit breakdown strength, especially at spacings less than the diameter of the spheres. (See Sphere Gaps.) With most types of air gaps, humidity, air density, and frequency greatly influence the dielectric strength.

CARBON DIOXIDE. Compressed carbon dioxide has been used as an insulator in high-voltage condensers for apparatus used in measurement of power factor and capacitance. It is an inert gas, obtainable in a relatively pure state and sometimes used as a gaseous insulator in containers.

NITROGEN. There is little difference in the dielectric properties of the common gases. The choice usually depends on cost or chemical properties. Nitrogen is inert and finds application as an atmosphere in sealed transformers. It prevents absorption of oxygen by the oil and thus eliminates formation of organic acids and sludge ("Inertaire" type of transformers).

HYDROGEN. Because of its high specific heat (3.41 compared to 0.237 for air), hydrogen is useful as a dielectric to absorb heat from electrical machinery. It is used in a closed system for cooling large generators. Care must be taken to keep a positive gas pressure in the system to prevent influx of sufficient air to produce an explosive mixture.

Although the dielectric strength of gases varies roughly as the pressure, there are small differences in behavior of the various common gases. Considering the strength of

air as 1.00 at the various pressures, the following table shows the relative dielectric strengths as a factor of the air values.

Relative Dielectric Strengths of Gases (Air = 1.00)
(Wolf)

Atmospheres Pressure	Carbon Dioxide	Nitrogen	Hydrogen
1	1.20	1.16	0.87
2	1.10	1.15	0.76
3	1.05	1.15	0.72
4	1.03	.1.14	0.69
5	1.02	1.14	0.68

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MAGNETIC MATERIALS

15. CLASSIFICATION

Materials may be assigned to one of three classes on the basis of magnetic properties, namely:

(*a*) **Diamagnetic,** having a permeability less than that of a vacuum. Bismuth is a material in this class.

(*b*) **Paramagnetic,** having a permeability slightly greater than that of a vacuum, and approximately independent of the magnetizing force.

(*c*) **Ferromagnetic,** having a permeability considerably greater than that of a vacuum, which will in general be a function of the previous magnetic history of the material.

Only the ferromagnetic materials find general commercial application on the basis of their magnetic properties.

Definitions

The following general concepts and definitions, which have been adopted by the A.S.T.M., are useful in considering the magnetic properties of materials.

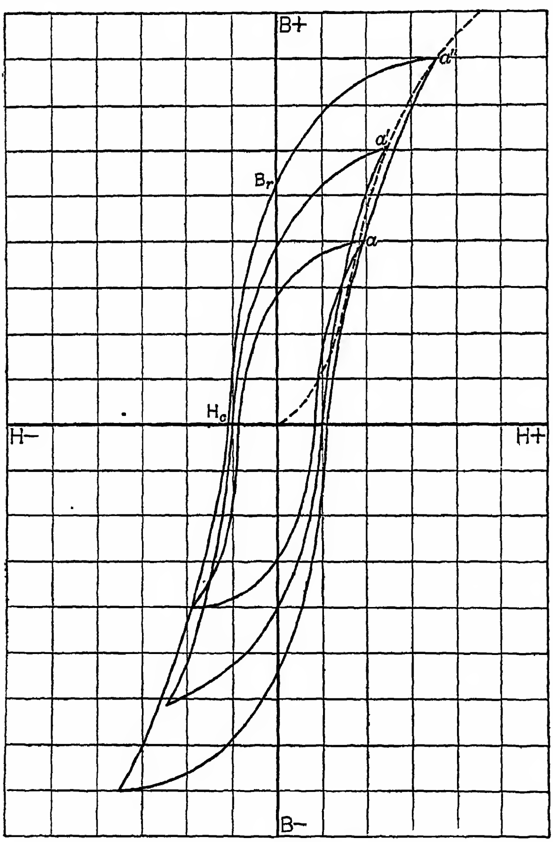

FIG. 1. Typical Hysteresis Loops

Aging of Magnetic Materials: The normal or accelerated change in magnetic properties of a magnetic material under continued normal or specified artificial aging conditions. When used in reference to core loss, this term, unless otherwise modified, implies an increase in loss. When used in reference to permeability or remanence, the term, in a positive sense, indicates a decrease in these quantities.

Aging Coefficient: The percentage change in the standard core loss after continued heating at 100 deg cent for 600 hours.

Coercive Force, H_c: The reversed magnetizing force (distance to H_c in Fig. 1) that is just sufficient to reduce the residual induction in a material to zero. Coercivity is the property of a material, measured by the coercive force required to reduce to zero the induction remaining after the removal of an applied magnetizing force corresponding to the saturation induction for the material.

Core Loss (Iron Loss), W: The power expended in a magnetic material subjected to a varying induction. The standard core loss, $W_{B/f}$ is the total power in watts per pound expended in the magnetic material which is subjected to a harmonically varying induction of a specified maximum value B

and of a specified frequency f. The usual units are watts per pound at 60 cycles and 10,000 gausses expressed as $W_{10/60}$.

Eddy Current Loss, W_e: That part of the core loss due to currents circulating in the magnetic material as a result of electromotive forces induced by the varying induction.

Hysteresis Loss, W_h: That part of the core loss represented by the energy converted into heat as a result of magnetic hysteresis.

Intrinsic Induction in a Ferromagnetic Material (Ferric Induction), B_i: For a given value of the magnetizing force, the excess of the normal induction in the material over that in vacuum.

Normal Induction in a Ferromagnetic Material (Normal Induction), B: The induction for a given magnetizing force when the material is in a symmetrical cyclic condition with respect to the magnetizing force, that is, when the material as a result of previous magnetic experience has no magnetic bias. The magnitude of the normal induction for a given magnetizing force is the same for both positive and negative values of the magnetizing force. Unit: gauss.

Permeability, μ: That property of an isotropic medium which determines, under specified conditions, the magnitude relation between magnetic induction and magnetizing force in the medium. Permeability is measured as the ratio of the magnetic induction to the magnetizing force.

Permeability Differential, μ_d: The ratio of the positive increase of normal induction to the positive increase of magnetizing force when these increases are vanishingly small.

Permeability, Incremental, μ_Δ: The ratio of the cyclic change in induction to the corresponding cyclic change in magnetizing force when the mean induction differs from zero.

Permeability, Initial, μ_o: The normal permeability when both the magnetizing force and the induction are vanishingly small.

Permeability Intrinsic, μ_i: The ratio of the intrinsic induction to the corresponding magnetizing force.

Permeability, Normal, μ: The ratio of the normal induction to the corresponding magnetizing force.

Permeability, Reversible, μ_r: The incremental permeability when the change in induction is vanishingly small.

Permeability, Space: That factor which expresses the ratio of magnetic induction to magnetizing force in vacuum. In the cgs electromagnetic system of units the permeability of a vacuum is arbitrarily taken as unity.

Permeance, P: In a portion of a magnetic circuit extending between two equipotential surfaces, the ratio of the flux through any cross-section, to the magnetic potential difference between the surfaces, when taken within the portion under consideration.

Permeance, $$P = \frac{\phi}{F} \text{ is equivalent to } \frac{\mu A}{L} \text{ for uniform } \mu$$

where μ = permeability; A = area in square centimeters, and L = length in centimeters.

Reluctance, R: The reciprocal of permeance.

Reluctance, $$R = \frac{F}{\phi} \text{ is equivalent to } \frac{L}{\mu A} \text{ for uniform } \mu$$

Reluctivity: The reciprocal of the permeability of a medium.

Remanence: The magnetic induction remaining in a magnetic circuit after the removal of an applied magnetizing force. If there is an air gap in the magnetic circuit, the remanence will be less than the residual induction. Unit: gauss.

Residual Induction, B_r: The magnetic induction (distance (OB_r) in Fig. 1) remaining in a symmetrically cyclicly magnetized magnetic material when the effective magnetizing force has been reduced to zero at every point. Unit: gauss.

Retentivity: That property of a material measured by the residual induction corresponding to an applied magnetizing force sufficient to produce saturation induction in the material. Unit: gauss.

Saturation Induction, B_s: The maximum intrinsic (ferric) induction possible in a material. Unit: gauss.

Criteria of Magnetic Quality

In judging the suitability of magnetic materials for various applications it is customary to make use of various standard forms of data.

Normal Induction Curves or tables constitute the locus of the extremes of a series of hysteresis loops (O–a–a'–a'', etc., in Fig. 1). A typical normal induction curve for 1 per cent silicon steel is shown in Fig. 2. In using such data the effect of air gaps or joints in the magnetic circuit should be considered, since these joints may have appreciable reluctance.

Normal Permeability Curves or tables are obtained from the normal induction curves by dividing the flux density in gausses by the magnetizing force in oersteds.

$$\mu = B/H$$

A typical normal permeability curve is shown in Fig. 2.

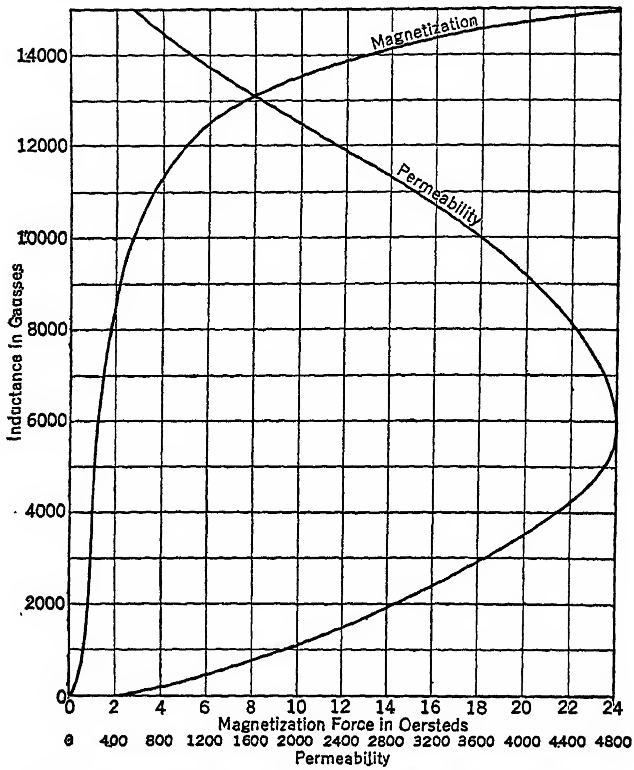

FIG. 2. Magnetization and Permeability Curves for Low Silicon Steel

Hysteresis Loops represent the relation, in an initially demagnetized material, between induction and magnetizing force, as the induction is varied from a positive maximum to a negative maximum and back to the initial point. Fig. 1 shows a family of such hysteresis loops taken at various values of peak induction. The areas of such loops represent the loss due to magnetic hysteresis and are sometimes used in comparing magnetic materials. The values of the residual induction and coercive force, criteria particularly useful in selecting permanent magnet materials, may also be found from such loops.

Iron Loss Curves or tables are usually prepared to show the relation between the summation of the hysteresis and eddy current components of the core loss, and maximum induction, at a definite frequency, and when the induction is varying harmonically. In using iron loss data obtained from tests on small samples, it is important to remember that extra losses, not present in the test sample, will be found in most apparatus due to eddy currents in various parts of the magnetic path or in the structural members, stray flux in structural parts, and possibly to harmonics in the flux wave.

Incremental Permeability Data are frequently useful in the design of equipment where an alternating flux is superposed on a steady flux. The length and number of the joints or air gaps in the magnetic circuit must be carefully considered in using such data, which are frequently of much value in the design of radio equipment, such as audio transformers and filter reactors.

The value of permanent magnet materials is frequently judged on the basis of the value of the product of the residual induction and the coercive force available at that

induction, at that point in the demagnetization quadrant of the hysteresis loop where this product is a maximum. This figure represents the maximum amount of useful energy that a magnet material can deliver; it is sometimes known as the *maximum energy product* and written (B_dH_d) max.

Classes of Commercial Magnetic Material

Magnetic materials may be divided into three groups for the sake of convenience of description, namely: (*a*) the *non-retentive* or magnetically "soft" materials, (*b*) the *retentive* or magnetically "hard" materials, and (*c*) those *special magnetic materials* having properties suited only to certain special applications.

(*a*) The non-retentive or magnetically "soft" materials are by far the most important, on the basis of the amount used. Materials falling in this class range from cast iron, which is one of the poorest, to Hipernik, which is one of the best, and include also cast steel, ordinary low-carbon sheet or plate, ingot iron, and the electrical (silicon) sheet materials. These materials are in general characterized by low coercive force, comparatively low hysteresis loss, relatively high permeability, and, usually, a fairly high saturation induction; they are used in the electromagnetic circuits of electrical apparatus.

(*b*) The retentive or magnetically "hard" materials find application in the permanent magnets of meters and relays, sound-reproducing units, magnetos, and other equipment where a steady magnetic field is required and it is not convenient to obtain this field electromagnetically. These materials are characterized by high coercive force, comparatively high hysteresis loss, and, usually, low permeability.

(*c*) Special alloys include those materials having special temperature-permeability relations, used in compensating for changes in flux in a magnetic circuit due to temperature changes, materials having unusually good properties only at very low inductions, materials especially suited to operation at very high flux densities, and those materials which can be used at very high frequencies.

16. NON-RETENTIVE MATERIALS

General Considerations

EFFECT OF IMPURITIES. Generally speaking, the magnetic properties of iron and its alloys are adversely affected by the presence of impurities; carbon, sulfur, and oxygen tend to increase the hysteresis loss and lower the permeability at all inductions. Manganese, phosphorus, and copper have little effect if present in small quantities, but, with the exception of cobalt, all additions to pure iron tend to lower the saturation induction and the permeability at high flux densities. Silicon, aluminum, and arsenic tend to decrease the hysteresis loss and to improve the permeability at moderate and low flux densities, largely because of the elimination of oxygen and other impurities; but even these elements impair the permeability at high flux densities. Silicon is also of considerable value because it increases the resistivity of the alloy and, therefore, reduces the eddy current losses. (Aluminum and arsenic act in the same way, but are seldom used.)

EFFECT OF MECHANICAL STRAIN. A stress beyond the elastic limit tends to increase the hysteresis loss and reduce the permeability of magnetic materials. This effect is likely to be roughly proportional to the amount of distortion or strain, and usually the best magnetic materials are the most sensitive to mechanical abuse. Shearing or punching strains will increase the 10-kilogauss, 60-cycle loss of a 1 $^3/_{16}$-in. wide strip of silicon steel 10 or 15 per cent and will decrease the permeability at this induction more than 30 per cent. This effect is greater the thicker the material, and is likely to be more marked on the softer grades. The percentage effect of shearing or punching is, of course, an inverse function of the width of the piece sheared. It may be very large in narrow sections such as are found in the teeth of small motors.

Stresses within the elastic limit will change the magnetic characteristics of materials. These changes may improve or impair the magnetic properties, depending on the kind and the amount of the stress, the induction at which the test is made, and the kind of material. Changes in magnetic condition are likewise reflected in changes in the length, or, if confined, in the stress on such materials. These effects are usually small, but may become significant at low inductions or for very high-quality materials.

EFFECT OF DIRECTION OF GRAIN. Electrical sheet or strip steels are likely to show an appreciable change in properties depending on the direction of the flux in the material relative to the final rolling direction. Usual sheet or strip material will show 10 or 15 per cent higher loss when test strips are cut perpendicular to the direction of final rolling than when they are cut parallel to this direction, and the cross-grain samples will

have 20 to 30 per cent lower permeability. Samples sheared at 45 degrees to the direction of rolling usually show loss and permeability characteristics not far different from the straight-grain specimens.

EFFECT OF TEMPERATURE. The magnetic properties of iron and iron silicon alloys are only slightly affected by ordinary temperature changes. In general, the permeability at low and moderate flux densities is improved, and the hysteresis loss is reduced, as the temperature is raised. These changes are greater at low flux densities, but may usually be neglected in the commercial application of materials over ordinary temperature ranges. Iron and the silicon iron alloys become non-magnetic at about 750 deg cent and the magnetic properties are in general more seriously impaired as temperatures of about 500 deg cent are exceeded. The magnetic properties of materials are sometimes impaired after being held for some time at temperatures of the order of 100 deg cent. This "aging" effect used to be of considerable importance, but electrical steels produced today are not seriously affected.

EFFECT OF HEAT TREATMENT. It is usually possible to effect a marked improvement in the magnetic properties of materials by a proper heat treatment or annealing cycle. This heat treatment usually involves heating the material to a suitable temperature, holding at this temperature for a short time, and cooling slowly. The best annealing temperature, if only the release of punching strains is desired, will usually be about 725 deg cent. The punchings can be held at this temperature for several hours, and cooled at a rate of about 30 deg cent per hour. It is important that every precaution be taken to exclude oxidizing or carburizing gases from the material while it is above 600 deg cent. This is effected by either placing the material to be annealed under a tightly sealed metal box, or, if an electric furnace, by supplying an inert or reducing atmosphere to the annealing chamber.

If the material to be annealed has been given no other annealing treatment, the maximum temperature will range between 750 and 900 deg cent for such materials as cast iron, cast steel, ordinary structural or low-carbon steel, "pure" iron, and low-silicon steel. High-silicon steel is annealed at temperatures ranging over 1000 deg cent, and Hipernik at temperatures above 1200 deg cent. The importance of proper atmospheric control increases as the annealing temperature is increased.

IRON LOSSES. When a changing magnetic flux is established in a magnetic material, heat is developed through (*a*) magnetic hysteresis and (*b*) eddy currents.

(*a*) The hysteresis loss (W_h) of a homogeneous magnetic material, which is proportional to the area of the hysteresis loop, has been shown by Steinmetz to be given by the empirical relation

$$W_h = pVfB^x_{\max}$$

where p is the Steinmetz hysteresis coefficient; V is the volume of iron, in cubic centimeters; f is the frequency of the alternating flux, in cycles per second; B is the maximum flux density, in gausses; x is the hysteresis exponent; and W_h is the hysteresis loss in ergs per second.

Values of p are given elsewhere in this section. The values of the hysteresis exponent may be taken as 1.6 over a range of 1000 to 12,000 gausses. At lower inductions the exponent approximates 2, and at higher induction it may considerably exceed this figure. The hysteresis loss is not appreciably affected by the shape of the wave form, according to M. G. Lloyd, provided the voltage wave does not pass through zero more than twice per cycle.

If an alternating flux is superposed on a steady flux or if an alternating flux oscillates about some induction other than zero the hysteresis loss may be considerably increased, depending on the amount of the flux displacement. When a high-frequency flux is superposed on a low-frequency flux, the total hysteresis loss is likewise increased.

(*b*) The eddy current loss is the result of circulating currents set up in a magnetic material due to the changing flux in the material. The eddy current loss, W_e, is given by the expression

$$W_e = Kf^2B^2t^2$$

and where "skin" effect is slight it has been shown that theoretically

$$W_e = \frac{f^2B^2t^2}{6e \times 10^{16}}$$

where f is the frequency, in cycles per second; B is the induction, in gausses; t is the thickness of the sheets, in centimeters; and e is the resistivity, in ohm-centimeters.

However, a theoretical figure for the eddy current loss as computed by this method is usually far less than that determined by actual test, and no completely satisfactory explanation for the discrepancy has been advanced. Four per cent silicon steel may

Table 1. General Data on Magnetic Properties of Commercial Materials

Figures represent approximate average properties; individual samples may differ somewhat

Property	Perm-alloy Western Electric	Hipernik Westing-house Electric	Si Steel 0.60 Loss	Si Steel 0.66 Loss	Si Steel 0.72 Loss	Si Steel 0.82 Loss	Si Steel 1.01 Loss	Si Steel 1.17 Loss	Si Steel 1.30 Loss	Pure Iron Armco or Norway	Low-carbon Sheet or Shapes	Cast Steel An-nealed	Cast Iron An-nealed
Specific gravity, grams per cc	8.6	8.3	7.5*	7.5*	7.5*	7.5*	7.5*	7.7*	7.7*	7.85	7.85	7.8	7.0
Sheet weight, lb per sq ft in 29 gage†	0.63	0.60	0.54	0.54	0.54	0.54	0.54	0.55	0.55	0.56	0.56	....	....
Ultimate strength, lb per sq in	....	65,000	75,000	75,000	72,000	70,000	60,000	45,000	40,000	40,000	45,000	60,000	25,000
Yield point, lb per sq in	....	20,000	55,000	55,000	52,000	50,000	35,000	22,000	20,000	20,000	25,000	30,000	....
Per cent elongation (2 in.)	....	50	3	3	3.5	5	14	18	20	25	18	20	....
Erichsen draw, mm (ductility index)	....	8	2.5	2.5	2.5	4.5	5	7	7	10	9	....	....
Resistivity, microhms per cc	22	45	60	58	56	48	41	26	18	10.7	13	15	100
Steinmetz, hysteresis coefficient	0.0001	0.00015	0.00046	0.00051	0.00056	0.00065	0.00081	0.00088	0.001	0.002	0.003	0.005	0.012
Typical coercive force, oersteds B (max) = 10,000 gausses	0.04	0.06	0.32	0.36	0.39	0.46	0.56	0.70	0.85	1.0	2.0	5	11
W 10/60–29 gage iron loss, watts per lb‖	....	0.25	0.60	0.66	0.72	0.82	1.01	1.17	1.30	....	....	....	....
Maximum aging, core loss	....	Nil	Nil	Nil	Nil	2%	3%	5%	....	....	....	....	....
Typical maximum permeability	100,000	90,000	8,000	8,000	6,500	6,000	5,500	5,200	5,000	4,500	2,500	1,500	500
Saturation (ferric induction)	11,000	15,600	19,500	19,500	19,800	20,200	20,500	21,000	21,200	21,600	21,200	21,000	14,000
Typical initial permeability	9,000	6,000	750	700	600	500	400	350	325	275	250	175	125
Approximate relative cost, in percentage	....	2,500	100	93	80	73	64	50	43	36	32	20	17
Approximate percentage silicon	‡	§	4.5	4.5	4.4	3.5	2.5	1.0	0.5	low	low	0 4	2 0
Typical application	Telephone equipment	High-quality audio and instrument transformers, relays	Distribution and power transformers		High-efficiency rotating machines and small transformers		Small motors, a-c magnets, starting transformers			Pole pieces, relays	Fields and frames of d-c and synchronous machines	Frames and solid poles	Frames

* A.S.T.M. Standard, not quite actual.

† For magnetic sheet of low silicon content (2% and less) the table of sheet iron gages and weights should be used. For material of medium and high silicon grades (over 2%) the gage dimensions in the sheet iron table are satisfactory but the weights should be multiplied by 0.975.

‡ Seventy-eight per cent nickel.

§ Fifty per cent nickel.

‖ The guaranteed maximum loss at 10,000 gausses, 60 cycles, in 29 gage, is used to designate the grade of electrical sheet steel.

show more than double the calculated eddy current loss, and 50 per cent excess of test over calculated values is not uncommon for ordinary open-hearth sheet and low-silicon steel. Eddy currents tend, by virtue of their counter magnetomotive force, to damp out flux in the center of the cross-section of a magnetic material. Rosenberg has shown that, when this "skin effect" is large, the flux penetration a is

$$a = 5000\sqrt{\frac{e}{\mu f}}$$

where a is the flux penetration, in centimeters; e is the resistivity in ohm-centimeters; f is the frequency; and μ is the permeability. Although probably not very accurate, the formula is very useful in that it will indicate at least the order of magnitude of the flux penetration.

Curves and Data on Commercial Magnetic Materials

In using data on commercial magnetic materials it is important to remember that the magnetic properties of all such materials are likely to vary considerably, depending upon chemistry, kind and amount of mechanical work done on the material, heat treatment, and factors at present beyond control. It is impossible to give exact figures representing the various magnetic characteristics, and the values presented (Tables 1–6) will be those corresponding to the average of good practice. In general, the range over which the values will fluctuate will be smaller for the materials of higher magnetic quality. Silicon steel sheets, for example, will usually show variations in standard iron loss of less than 10 per cent, although permeability figures for this class of material will vary over a considerably wider range. In general, both hysteresis losses, and low and moderate flux

Table 2. Normal Induction Data (Average Figures)

Value of Magnetizing Force (Oersteds) Corresponding to Induction

Induction in Gausses	Permalloy (Western Electric Company)	Hipernik (Westinghouse Electric Company)	High (About 4%) Silicon Steel	Medium (About 2.5%) Silicon Steel	Low (About 1%) Silicon Steel	Commercially Pure Iron Annealed	Ordinary Low-Carbon Steel Annealed	Cast Steel Annealed	Cast Iron Annealed	Induction Lines per sq in.
10	0.0010	0.0015	0.010	0.017	0.021	0.033	0.038	0.050	0.08	64
16	0 0017	0.0021	0.016	0.025	0.030	0.05	0.041	0.075	0.13	103
25	0.0026	0.0034	0.025	0.038	0.044	0.07	0.093	0.11	0.19	161
40	0.0039	0 0058	0.04	0.05	0.063	0.11	0.14	0.15	0.29	258
64	0.0057	0.0089	0.06	0.08	0.095	0.16	0.21	0.22	0.44	423
100	0.008	0.0107	0.09	0.12	0.14	0.21	0.29	0.30	0.65	645
160	0.011	0.0130	0.12	0.17	0.20	0.29	0.41	0.42	0.91	1,030
250	0.016	0.017	0.16	0.22	0.27	0.40	0.52	0.57	1.2	1,610
400	0.021	0.023	0.19	0.30	0.35	0.55	0.67	0.79	1.6	2,580
640	0.026	0.029	0.23	0.38	0.45	0.68	0.82	1.03	2.0	4,230
1,000	0.030	0.036	0.28	0.45	0.53	0.75	1.02	1.23	2.5	6,450
1,250	0.033	0.041	0.31	0.49	0.58	0.78	1.13	1.38	2.8	8,100
1,600	0.036	0.047	0.35	0.54	0.64	0.82	1.24	1.57	3.3	10,300
2,000	0.038	0.053	0.38	0.58	0.69	0.87	1.4	1.8	4.0	12,900
3,000	0.044	0.069	0.46	0.66	0.82	0.94	1.6	2.3	7.0	19,400
4,000	0.049	0.100	0.56	0.74	0.93	1.0	1.8	2.8	14.	25,800
5,000	0.060	0.137	0.64	0.85	1.06	1.15	2.1	3.3	25.	32,300
6,000	0.080	0.185	0.75	1.00	1.25	1.4	2.4	3.9	45.	38,700
7,000	0.12	0.25	0.90	1.19	1.50	1.6	2.8	4.5	70.	45,200
8,000	0.20	0.35	1.05	1.45	1.8	1.8	3.2	5.2	81.	51,600
9,000	0.50	0.50	1.22	1.8	2.2	2.0	3.8	6.2	160.	58,100
10,000	2.0	0.76	1.40	2.3	2.7	2.3	4.5	7.5	220.	64,500
11,000	20.0	1 40	1.80	3.2	3.6	2.8	5.3	9.5	310.	71,000
12,000	500.	2.50	2.6	4.5	5.0	3.3	6.6	12.5	410.	77,400
13,000	....	4.8	4.1	6.8	7.6	4.1	8.6	16.	620.	83,900
14,000	....	10.0	9.0	11.0	13.0	5.5	12.8	22.	1000.	90,300
15,000	....	40.	23.00	24.0	26.0	10.	17.	32.	....	96,800
16,000	....	400.	52.	47.0	41.0	20.	28.	50.	....	103,200
17,000	....	....	110.	93.	80.0	40.	52.	82.	....	109,700
18,000	....	....	200.	170.	140.0	80.	95.	130.	....	116,100
19,000	....	....	380.	300.	220.0	145.	165.	200.	....	122,600
20,000	....	....	900.	590.	360.0	230.	280.	380.	....	129,000
21,000	....	....	1800.	1250.	700.	360.	500.	800.	....	135,500
22,000	....	....	2800.	2100.	1400.	940.	1100.	1800.	....	141,900

Permeability at any induction may be found from the relation $\mu = B \div H$.

Table 3 (Part 1). Approximate Frequency and Induction Core Loss Factors for 29 Gage (0.014 in.) Material

Induction, Gausses	15 Cycles	25 Cycles	40 Cycles	50 Cycles	60 Cycles	80 Cycles	100 Cycles	160 Cycles	250 Cycles	Induction Lines per Sq In.
10	0.35×10^{-6}	0.60×10^{-6}	0.10×10^{-5}	0.13×10^{-5}	0.16×10^{-5}	0.22×10^{-5}	0.28×10^{-5}	0.47×10^{-5}	0.80×10^{-5}	64
16	$.10\times10^{-5}$	$.17\times10^{-5}$	$.30\times10^{-5}$	$.36\times10^{-5}$	$.46\times10^{-5}$	$.64\times10^{-5}$	$.80\times10^{-5}$	$.14\times10^{-4}$	$.24\times10^{-4}$	103
25	$.29\times10^{-5}$	$.50\times10^{-5}$	$.80\times10^{-5}$	$.10\times10^{-4}$	$.12\times10^{-4}$	$.18\times10^{-4}$	$.23\times10^{-4}$	$.40\times10^{-4}$	$.72\times10^{-4}$	161
40	$.80\times10^{-5}$	$.14\times10^{-4}$	$.23\times10^{-4}$	$.28\times10^{-4}$	$.35\times10^{-4}$	$.51\times10^{-4}$	$.64\times10^{-4}$	$.12\times10^{-3}$	$.21\times10^{-3}$	258
64	$.22\times10^{-4}$	$.41\times10^{-4}$	$.65\times10^{-4}$	$.82\times10^{-4}$	$.10\times10^{-3}$	$.15\times10^{-3}$	$.18\times10^{-3}$	$.32\times10^{-3}$	$.58\times10^{-3}$	423
100	$.58\times10^{-4}$	$.10\times10^{-3}$	$.18\times10^{-3}$	$.22\times10^{-3}$	$.26\times10^{-3}$	$.38\times10^{-3}$	$.47\times10^{-3}$	$.80\times10^{-3}$	$.15\times10^{-2}$	645
160	$.15\times10^{-3}$	$.28\times10^{-3}$	$.44\times10^{-3}$	$.56\times10^{-3}$	$.66\times10^{-3}$	$.92\times10^{-3}$	$.12\times10^{-2}$	$.21\times10^{-2}$	$.38\times10^{-2}$	1,030
250	$.36\times10^{-3}$	$.64\times10^{-3}$	$.10\times10^{-2}$	$.13\times10^{-2}$	$.16\times10^{-2}$	$.22\times10^{-2}$	$.30\times10^{-2}$	$.50\times10^{-2}$	$.90\times10^{-2}$	1,610
400	$.90\times10^{-3}$	$.15\times10^{-2}$	$.26\times10^{-2}$	$.30\times10^{-2}$	$.38\times10^{-2}$	$.56\times10^{-2}$	$.72\times10^{-2}$	$.12\times10^{-1}$	$.22\times10^{-1}$	2,580
640	$.20\times10^{-2}$	$.35\times10^{-2}$	$.62\times10^{-2}$	$.75\times10^{-2}$	$93.\times10^{-2}$	$.13\times10^{-1}$	$.17\times10^{-1}$	$.30\times10^{-1}$	$.53\times10^{-1}$	4,230
1,000	$.43\times10^{-2}$	$.74\times10^{-2}$	$.13\times10^{-1}$	$.16\times10^{-1}$	$.22\times10^{-1}$	$.28\times10^{-1}$	$.36\times10^{-1}$	$.61\times10^{-1}$	.11	6,450
1,600	$.10\times10^{-1}$	$.16\times10^{-1}$	$.28\times10^{-1}$	$.36\times10^{-1}$	$.43\times10^{-1}$	$.60\times10^{-1}$	$.78\times10^{-1}$	.14	.26	10,300
2,000	$.14\times10^{-1}$	$.24\times10^{-1}$	$.40\times10^{-1}$	$.52\times10^{-1}$	$.62\times10^{-1}$	$.88\times10^{-1}$	.11	20	.38	12,900
2,500	$.20\times10^{-1}$	$.34\times10^{-1}$	$.60\times10^{-1}$	$.75\times10^{-1}$	$.92\times10^{-1}$	.13	.16	.29	.55	16,100
3,000	$.29\times10^{-1}$	$.46\times10^{-1}$	$.81\times10^{-1}$	.10	.12	.18	.22	.37	.74	19 400
4,000	$.45\times10^{-1}$	$.73\times10^{-1}$	.13	.17	.20	.28	.36	.64	1 25	25,800
5,000	$.66\times10^{-1}$	.11	.19	.25	.29	.41	.54	96	1 80	32,300
6,000	$.90\times10^{-1}$	.15	.26	.34	.40	.56	.72	1 30	2 50	38,700
7,000	.12	.20	.35	.44	.52	.72	.96	1 70	3 30	45,200
8,000	.15	.25	.44	.55	.66	.92	1.20	2.10	4 1	51,600
9,000	.19	.31	.54	.68	.82	1.14	1.48	2 60	5.1	58,100
10,000	.23	38	.64	.82	1 00	1.40	1.80	3 3	6.2	64,500
11,000	.28	.46	.76	.98	1.20	1.70	2.25	4 0	7 7	71,000
12,000	.33	.55	91	1.15	1.45	2.05	2.8	4 9	9 5	77,400
13,000	.39	.64	1.07	1.37	1.70	2.5	3.3	5.9	11 3	83,900
14,000	.45	.74	1.25	1 60	2 00	2 90	3.9	7.1	13.0	90,300
15,000	52	.86	1.47	1 90	2 35	3 40	4 6	8 5	15 0	96,800

NOTE: To find the approximate iron loss at any induction or frequency, multiply the loss, at 60 cycles, 10,000 gausses, by the factor shown in the table. Values are most accurate for about a 4 per cent silicon steel, but are reasonably exact for lower silicon steels and may be used to give reasonably approximate values for Hipernik.

Table 3 (Part 2). Approximate Frequency and Induction Core Loss Factors for 29 Gage (0.014 in.) Material

Induction, Gausses	400 Cycles	640 Cycles	1000 Cycles	1600 Cycles	2500 Cycles	4000 Cycles	6400 Cycles	10,000 Cycles	15,000 Cycles	Induction Lines per Sq In.
10	0.14×10^{-4}	0.30×10^{-4}	0.56×10^{-4}	$0\ 13\times10^{-3}$	0.39×10^{-3}	0.79×10^{-3}	0.16×10^{-2}	0.38×10^{-2}	0.80×10^{-2}	64
16	$.47\times10^{-4}$	$.95\times10^{-4}$	$.20\times10^{-3}$	$.43\times10^{-3}$	$.10\times10^{-2}$	$.24\times10^{-2}$	$.56\times10^{-2}$	$.11\times10^{-1}$	$.30\times10^{-1}$	103
25	$.14\times10^{-3}$	$.28\times10^{-3}$	$.58\times10^{-3}$	$.14\times10^{-2}$	$.32\times10^{-2}$	$.80\times10^{-2}$	$.19\times10^{-1}$	$.44\times10^{-1}$	.10	161
40	$.40\times10^{-3}$	$.85\times10^{-3}$	$.18\times10^{-2}$	$.44\times10^{-2}$	$.95\times10^{-2}$	$.24\times10^{-1}$	$.58\times10^{-1}$	.14	.31	258
64	$.11\times10^{-2}$	$.24\times10^{-2}$	$.51\times10^{-2}$	$.11\times10^{-1}$	$.27\times10^{-1}$	$.64\times10^{-1}$	.15	.38	.84	423
100	$.28\times10^{-2}$	$.59\times10^{-2}$	$.13\times10^{-1}$	$.29\times10^{-1}$	$.67\times10^{-1}$	.15	.38	.96	2.0	645
160	$.70\times10^{-2}$	$.15\times10^{-1}$	$.32\times10^{-1}$	$.70\times10^{-1}$	.16	.38	.95	2.3	5.0	1,030
250	$.17\times10^{-1}$	$.36\times10^{-1}$	$.74\times10^{-1}$	.17	.38	.93	2.2	5.3	11.0	1,610
400	$.41\times10^{-1}$	$.86\times10^{-1}$	.18	.39	.88	2.2	5 2	13.0	27.0	2,580
640	$.95\times10^{-1}$	.20	.42	.90	2.2	5.1	13 0	30.0	62.0	4,230
1,000	.22	.45	.94	2.2	5.0	12.0	29.0	72.0	....	6,450
1,600	.48	1.0	2.15	4.9	11.5	28.0	65.0	....	....	10,300
2,000	.71	1.5	3.2	7.4	17.0	42.0	....	....	....	12,900
2,500	1.10	2.2	4.6	11.0	25.0	63.0	....	....	....	16,100
3,000	1.40	3.1	6.4	15.0	35.0	....	....	....	....	19,400
4,000	2.30	5.0	11.0	26.0	58.0	....	....	....	....	25,800
5,000	3.50	7.7	16.0	38.0	82.0	....	....	....	....	32,300
6,000	5.0	10.0	22.0	50.0	....	....	....	....	....	38,700
7,000	6.6	14.0	29.0	65.0	....	....	....	....	....	45,200
8,000	8.5	18.0	37.0	....	....	....	....	....	....	51,600
9,000	10.9	22.0	46.0	....	....	....	....	....	....	58,100
10,000	13.4	27.0	58.0	....	....	....	....	....	....	64,500
11,000	16.0	32.0	....	....	....	....	....	....	....	71,000
12,000	19.0	38.0	....	....	....	....	....	....	....	77,400
13,000	22.0	....	....	....	....	....	....	....	....	83,900
14,000	25.	....	....	....	....	....	....	....	....	90,300
15,000	29.	....	....	....	....	....	....	....	....	96,800

NOTE: To find the approximate iron loss at any induction or frequency, multiply the loss, at 60 cycles, 10,000 gausses, by the factor shown in the table. Values are most accurate for about a 4 per cent silicon steel, but are reasonably exact for lower silicon steels and may be used to give reasonably approximate values for Hipernik.

density permeabilities, will vary over a considerably greater range than the permeabilities at high values of magnetizing force.

Although electrical sheets are usually marketed under the trade names of the various manufacturers, the number of grades and the loss characteristics of these grades have been fairly well standardized (the "radio" grades are essentially the same as the standard). Accordingly, the figure corresponding to the 29 gage (0.014″) iron loss, expressed in watts per pound when tested at 10,000 gausses and 60 cycles, has been used to designate the grade of electrical sheet material.

Table 4. Loss in Watts per Pound of Electrical Sheets as a Function of Gage Commercial Grades in Common Use

The grade is designated by the guaranteed maximum iron loss when tested at 10,000 gausses, 60 cycles, A.S.T.M. method.

Induction in Gausses

Grade	Gage	3000	4000	5000	6000	7000	8000	9000	10,000	11,000	12,000	13,000	14,000	15,000
0.72	30	0.096	0.160	0.237	0.317	0.402	0.495	0.60	0.71	0.84	1.00	1.18	1.38	1.63
.72	29	.096	.160	.237	.317	.402	.495	.60	.71	.84	1.00	1.18	1.38	1.63
.72	28	.104	.172	.250	.329	.417	.52	.63	.75	.89	1.05	1.24	1.45	1.71
.72	27	.112	.184	.255	.344	.438	.54	.66	.79	.94	1.11	1.31	1.53	1.80
.72	26	124	.196	.271	.360	.464	.57	.70	.83	.99	1.17	1.38	1.62	1.91
.72	25	.137	.214	.302	392	.492	.61	.74	.89	1.06	1.25	1.48	1.74	2.05
.72	24	.152	231	.320	.410	.53	.66	.80	.97	1.17	1.36	1.59	1.86	2.19
.82	30	098	158	.237	.318	.415	.53	.64	.78	.94	1.10	1.27	1.47	1.68
.82	29	098	.162	.242	.330	.430	.55	.66	.81	.97	1.13	1.32	1.53	1.76
.82	28	.104	.173	.253	.347	.450	.57	.69	.85	1.01	1.18	1.38	1.61	1.85
.82	27	.112	.183	.264	.368	.477	.60	.73	.90	1.06	1.24	1.45	1.69	1.95
.82	26	123	.198	.285	.390	.51	.64	.77	.96	1.11	1.30	1.52	1.78	2.07
.82	25	.137	.218	.313	.423	.55	.68	.83	1.03	1.17	1.38	1.61	1.89	2.21
.82	24	.154	240	.350	.468	.60	.74	.89	1.10	1.26	1.48	1.72	2 02	2.38
1.01	30	.129	.209	.313	.423	.54	.66	.81	.98	1.16	1.36	1.58	1.86	2.15
1.01	29	.132	.214	.319	.435	.56	.69	.84	1.01	1.19	1.40	1.62	1.91	2.20
1.01	28	.139	.224	.331	.452	.58	.72	.87	1.06	1.24	1.47	1.70	1.99	2.30
1.01	27	.150	.237	.346	.475	.61	.75	.91	1.12	1.30	1.54	1.79	2.08	2.42
1.01	26	.164	.253	.363	.498	.64	.79	.96	1.18	1.37	1.62	1.89	2.20	2.56
1.01	25	.180	.271	.384	.53	.68	.84	1.01	1.24	1.45	1.71	2.02	2.34	2.72
1.01	24	.198	.293	.408	.56	.73	.90	1.09	1.30	1.55	1.82	2.16	2.51	2.91
1.17	30	.144	.247	.350	.450	.58	.74	.91	1.10	1.31	1.55	1.82	2.11	2.50
1.17	29	.155	.258	.360	.460	.61	.76	.93	1.15	1.34	1.58	1.86	2.16	2.55
1.17	28	.165	268	.370	.480	.63	.78	.96	1.17	1.37	1.62	1.90	2.22	2.61
1.17	27	.178	.285	.388	.50	.65	.80	1.00	1.21	1.43	1.69	2.00	2.35	2.74
1.17	26	.197	.308	.425	.53	.70	.87	1.07	1.28	1.52	1.82	2.18	2.55	2.98
1.17	25	.224	.342	.47	.60	.78	.97	1.19	1.43	1.70	2.04	2.43	2.84	3.31
1.17	24	.260	.385	.53	.72	.91	1.15	1.39	1.65	1.96	2.33	2.78	3.32	3.96
1.30	30	.172	.280	.395	.53	.68	.85	1.03	1.24	1.49	1.77	2.10	2.47	2.91
1.30	29	.178	.290	.410	.55	.71	.88	1.07	1.28	1.53	1.81	2.15	2.55	3.00
1.30	28	.186	.305	.435	.58	.74	.92	1.12	1.33	1.58	1.88	2.22	2.67	3.14
1.30	27	.198	.325	.465	.62	.79	.98	1.19	1.41	1.67	2.00	2.37	2.86	3.38
1.30	26	.216	.350	.50	.67	.85	1.06	1.28	1.53	1.82	2.18	2.60	3.11	3.67
1.30	25	.235	.375	.55	.75	.95	1.17	1.41	1.69	2.02	2.42	2.90	3.47	4.10
1.30	24	.260	.420	.61	.85	1.09	1.37	1 68	2.03	2.44	2.94	3.54	4.25	5.10

Table 5. Apparent Incremental Permeability, Standard E and I Laminations

Length of magnetic circuit = 5.6 in.
Two lap joints
29 gage material, 0.66 grade

A-c Induction	D-c Magnetizing Force, in oersteds							
	0	.5	1.0	1.5	2	3	4	5
10 gausses....	650	600	480	390	330	260	220	210
100 gausses....	1230	1000	750	580	475	360	300	280
1000 gausses....	2400	1450	1070	850	700	560	500	480

Data by American Rolling Mill Company, Middletown, Ohio.
Data shown are for 60 cycles; 1000 cycles shows almost identical result.

Table 6. Incremental Permeability D-c Tests

29 gage material, 0.60 grade

	B = 10	B = 30	B = 100	B = 300	B = 1000	B = 3000
Steady magnetizing force = 0.0 oersted..	1000	1440	1970	2770	4460	7320
Steady magnetizing force = 0.1 oersted..	1000	1350	1910	2550	4030	6650
Steady magnetizing force = 0.3 oersted..	840	1090	1470	1985	3120	5200
Steady magnetizing force = 1.0 oersted..	578	740	934	1130	1570	2750
Steady magnetizing force = 3.0 oersteds.	200	204	214	250	450	1000
Steady magnetizing force = 10.0 oersteds.	62	63	65	70	100	310

Data from Westinghouse Electric tests.
Ring samples, no air gaps.
60 cycle a-c tests with no air gaps have checked these figures closely.

The thickness of electrical sheets is ordinarily given in terms of the U. S. Standard sheet gage (see Sect. 1). For magnetic sheet of low silicon content (2 per cent and less) the table of sheet-iron gages and weights should be used. For material of medium- and high-silicon grades (above 2 per cent) the gage dimensions in the sheet-iron table are satisfactory, but the weights will be somewhat less. The approximate weight can be obtained by using the sheet-iron figures multiplied by a correction factor of 0.975.

17. RETENTIVE MATERIALS

General Considerations

The magnetic properties and the application of the various retentive or permanent magnet materials depend, in the main, on four factors, namely: (1) the physical dimen-

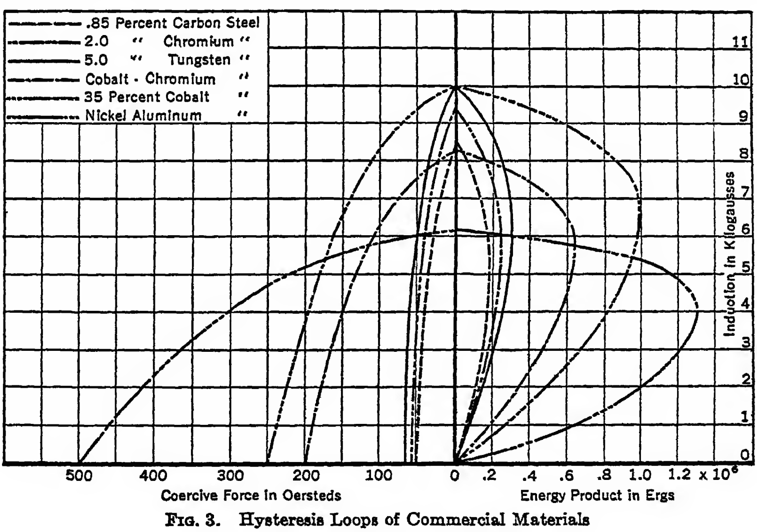

Fig. 3. Hysteresis Loops of Commercial Materials

sions of the magnetic circuit, (2) the heat treatment of the material, (3) the subsequent "stabilization," and (4) small variations in composition.

1. The relative lengths of the retentive material, of the air gap, and, if present, of the non-retentive material, together with cost and weight or size considerations, will determine, in general, the choice of magnetic material. When the air gap must be relatively large, or when weight or size are important considerations, materials having high coercive force will be used.

2. The desired magnetic properties of these materials are not developed, as a rule, until a final heat treatment has been given. Usually, precautions must be taken to avoid cracking, excessive oxidation, decarburization, or overheating; the temperatures must be rather closely controlled; and it is ordinarily desirable to keep these materials at high

temperatures as short a time as is practicable. Permanent magnet materials are ordinarily quite difficult to forge or machine.

3. After the material has been heat-treated and magnetized it is frequently necessary to put it in such condition that very little or no further change in magnetic properties will occur. This may be accomplished, but only by sacrificing a part of the initially available magnetic energy, by one or all of three methods: (*a*) prolonged "aging" at a slightly elevated temperature, usually about 100 deg cent for about 10 hours; (*b*) subjecting the material to mechanical shock or vibration, as by dropping it several feet to a hard surface; (*c*) applying a small negative, or alternating, demagnetizing force, as by introducing a slightly larger air gap than will later be used in the magnetic circuit.

4. Small amounts of various impurities will sometimes produce changes in the magnetic properties of retentive materials. Small percentages of sulfur, phosphorus, or oxygen are believed harmful, and loss of carbon in heat treatment may damage some grades considerably.

Data on Commercial Materials

1. Hysteresis loops are shown in Fig. 3.
2. General data on permanent magnet materials are shown in Table 7.

Table 7. Properties of Permanent Magnet Materials, Approximate Values

Characteristic	0.85 Per Cent Carbon Steel	2.0 Per Cent Chromium Steel	5.0 Per Cent Tungsten Steel	Cobalt Chromium Steel	Cobalt Steel	Nickel Aluminum Steel
Per cent carbon	0.85	0.90	0.65	0.9	0.85[e]	....
Per cent chromium	....	2.0	0.50	10.0	4.2	....
Per cent cobalt	....	....	....	15.0	37.0	....
Per cent tungsten	....	....	5.50	....	2.3	....
Per cent manganese	0.25	0.30	0.30	0.40	0.4	5.0[f]
Per cent aluminum	....	....	....	....	....	10
Per cent nickel	....	....	....	....	....	25
Forging temperature	850° C	1000° C	1000° C	1000° C	1000° C	[b]
Annealing temperature	875° C[a]	900° C[a]	850° C[a]	750° C[a]	[d]	[c]
Hardening temperature	760° C	800° C	840° C	[g]	950° C	[c]
Quenching medium	Oil	Oil	Oil	Oil	Oil	[c]
Magnetizing force, oersteds	300	400	400	1000	1000	2000
Residual induction (B_r)	8500	9500	10,000	8300	10,000	6000
Coercive force (H_c)	50	55	65	200	250	500
Maximum energy product ($B_d \times H_d$) max	180,000	230,000	300,000	650,000	1,000,000	1,300,000
Approximate relative cost in percentage	100	100	200	1000	1900	300
Maximum magnetic energy per unit cost	1.8	2.3	1.5	0.65	0.52	4.2

[a] Cooled in air.
[b] Cannot be forged or machined, may be ground, and casts fairly well.
[c] Heat to 1200 deg cent, quench, and age 4 hr at 700 deg cent.
[d] Annealing impairs the magnetic properties of this material and should be avoided. If necessary for machining, hold at 830 deg cent for 1 hr, cool 30 deg cent per hr to 600 deg cent, cool in air.
[e] Carbon fairly critical.
[f] The manganese is not essential, but increases the coercive force from 350 to 500, and lowers the residual induction from 7500 to 6000.
[g] Heat rapidly to 1160 deg cent, cool in air, heat slowly to 725 deg cent, cool in air, heat to 1000 deg cent, cool in moving air to 300 deg cent, and quench in oil, according to Darwins, Ltd., Sheffield, England.

18. SPECIAL MAGNETIC MATERIALS

LOW-INDUCTION MATERIALS. There are several magnetic materials which have remarkable magnetic characteristics at very low flux densities. These materials include those known as (1) "Conpernik" and (2) "Perminvar."

1. Conpernik is characterized by a constant permeability of a little over 1000 at flux densities up to several hundred gausses, and by very low hysteresis loss at these low inductions. The composition of Conpernik is the same as that of Hipernik, and the unusual properties are secured by a low temperature heat treatment. The hysteresis loss of Conpernik is about 1.6×10^{-2} erg per cc per cycle at an induction of 100 gausses. The material may be used to advantage in several ways, for example, in producing reactors with a reactance to resistance ratio well over 150, at frequencies of about 500 cycles. In order to keep eddy current losses low, it is usually used in 0.005-in. thickness.

2. Perminvar is characterized by somewhat similar properties, and is an alloy of iron, nickel, and cobalt, in one of several proportions. The magnetic properties are developed by special heat treatment.

HIGH-INDUCTION MATERIALS. An alloy of iron and cobalt, preferably in the ratio of 65 per cent iron to 35 per cent cobalt, has remarkable properties at very high inductions. The saturation induction is about 12 per cent higher than that of pure iron, or a little over 24,000 gausses; and at magnetizing forces between 50 and 200 oersteds the inductions are about 25 per cent higher than for iron. The material is used only for special applications because of its relatively high cost.

TEMPERATURE EFFECTS. 1. Alloys of copper and nickel, in the proportions of 70 per cent copper to 30 per cent nickel, with the addition of small amounts of manganese and silicon, and alloys of iron and nickel in the ratio 3 to 1 respectively, will show substantial and approximately linear changes in permeability with temperature, and are used as shunts for compensating the magnetic circuits of electrical instruments for changes in temperature.

2. An alloy of iron and nickel, with about 35 per cent nickel (Invar composition), will become non-magnetic at a slightly elevated temperature and is used in magnetic-thermal relays.

HIGH-FREQUENCY MATERIALS. For data on magnetic materials at high frequency see bibliography and Volume V, Section 2.

BIBLIOGRAPHY

The Properties and Testing of Magnetic Materials, by T. Spooner, published by McGraw-Hill, contains a great deal of practical information on commercial magnetic materials, and their application, and a very complete bibliography. Other works on the subject are Pender, H., Electricity and Magnetism for Engineers; Karapetoff, V., The Magnetic Circuit; Ewing, J. A., Magnetic Induction in Iron and Other Metals; Williams, S. R., Magnetic Phenomena; and The International Critical Tables, Vol. VI.

THERMAL PROPERTIES OF MATERIALS

19. THERMAL CAPACITY AND SPECIFIC HEAT

The thermal capacity of a body is defined as the heat absorbed by the body per unit increase in its temperature, there being during this change in temperature no change of state (e.g., no change from solid to liquid or from liquid to gaseous form or no chemical change) and no transfer of heat energy from the body in question to other bodies. The thermal capacity *per unit mass* of a substance is approximately constant, but increases slightly with increase in temperature; in the case of iron the increase with temperature is quite marked. Calling C the thermal capacity per unit mass of a substance, the heat absorbed by a homogeneous mass M when its temperature increases from t_1 to t_2 is

$$H = CM\,(t_2 - t_1) \tag{1}$$

provided C is constant.

The mean thermal capacity per unit mass of water (between 0 and 100 deg cent), when expressed in mean gram-calories per gram per degree centigrade, is numerically equal to unity. The ratio of the thermal capacity per unit mass of any substance to the mean thermal capacity of water is called the specific heat of the substance. The specific heat of a substance does not depend upon the units in which the various quantities are measured; its thermal capacity per unit mass does. When heat is expressed in mean gram-calories, mass in grams and temperature in degrees centigrade, the thermal capacity per unit mass is equal to its specific heat; compare with density and specific gravity.

CALCULATION OF HEAT ABSORBED OR GIVEN OUT.

C = specific heat (gram-calories per gram per degree centigrade),
M = mass heated,
$t_2 - t_1$ = rise of temperature.

Then for any set of units the heat absorbed is

$$H = kCM\,(t_2 - t_1) \tag{2}$$

where k has the following values:

Values of k

Unit of Heat or Energy	Unit of Mass	Temperature Scale	Value of k
Gram-calorie	Gram	Centigrade	1.000
Kilogram-calorie	Kilogram	Centigrade	1.000
Btu	Pound	Centigrade	1.800
Btu	Pound	Fahrenheit	1.000
Watt-second (joule)	Gram	Centigrade	4.183
Watt-second (joule)	Pound	Fahrenheit	1054
Kilowatt-hour	Kilogram	Centigrade	1.162×10^{-3}
Kilowatt-hour	Pound	Fahrenheit	2.928×10^{-4}

VALUES OF SPECIFIC HEAT. In the table below are given the values of the specific heat for the more common substances used in engineering work. These numbers, are also equal to the thermal capacity per unit mass, when mass is expressed in grams, temperature in degrees centigrade, and heat in gram-calories.

Table 1. Specific Heat of Some Common Substances

(From Landolt-Börnstein Tables; see also article on Pyrometers.)

Substance	Temperature, °C	Mean Specific Heat, C	Substance	Temperature, °C	Mean Specific Heat, C
Air *	−102 to 440	0.237	Iridium	0 to 100	0.032
Aluminum	15 to 435	.236	Iron, cast	18 to 100	.113
Ammonia	23 to 216	.520	Lead	17 to 100	.031
Antimony	22 to 600	.052	Manganin ¶	18	.097
Asbestos	20 to 98	.195	Manganin ¶	100	.100
Bismuth	−79 to 100	.029	Marble	0 to 100	.206
Brass ¶	20 to 100	.092	Mercury	0	.0335
Bronze ‡	20 to 100	.104	Mercury	100	.0326
Carbon (gas carbon)	20 to 1040	.315	Mica	20 to 98	.208
Carbon (graphite)	0 to 3000	.535	Molybdenum	20 to 550	.072
Carbon dioxide *	−78 to 7	.184	Nickel	0 to 105	.108
Carbon dioxide *	0 to 200	.215	Nitrogen *	0 to 200	.244
Carbon monoxide *	23 to 198	.243	Nitrous oxide *	13 to 172	.231
Cement (Portland)	28 to 30	.271	Oil, transformer	20	.44
Chlorine	13 to 202	.124	Oxygen *	20 to 440	.224
Cobalt	15 to 350	.109	Osmium	19 to 98	.031
Concrete, tamped	20 to 100	.156	Palladium	0 to 100	.059
Concrete, tamped	800	.219	Palladium	0 to 1265	.071
Constantan *	18	.098	Paraffin	25 to 30	.589
Constantan *	100	.102	Petroleum	21 to 58	.511
Copper	−188 to 20	.080	Petroleum	18 to 99	.498
Copper	0 to 100	.094	Platinum	0 to 100	.032
Copper	300	.098	Rhodium	10 to 97	.058
Copper	900	.126	Silver	0 to 260	.057
Cork	....	.485	Steam **	....	....
Cotton	0 to 100	.362	Steel	20 to 100	.118
Ebonite	....	.339	Tantalum	−185 to 20	.033
German silver	0 to 100	.095	Tin	17 to 100	.056
Glass	0 to 19	.171	Tungsten	20 to 100	.034
Glass	56 to 78	.192	Wax (yellow)	26 to 42	.820
Gold	0 to 100	.032	Wood's metal ‖	5 to 50	.035
Hydrogen *	−28 to 198	3.41	Wool	....	.393
Ice	−78 to −18	0.463	Zinc	20 to 100	.093

* At constant pressure of 1 atmosphere.
† 60 Cu + 40 Zn.
‡ 88.7 Cu + 11.3 Al.
§ 60 Cu + 40 Ni.
¶ 84 Cu + 4 Ni + 12 Mn.
‖ 25.85 Pb + 6.99 Cd + 52.43 Bi + 14.73 Sn.
** See article on Steam.

20. MELTING OR FREEZING POINT AND HEAT OF FUSION

Certain chemically simple substances when heated to a definite temperature pass from the solid to the liquid state with no increase in temperature during this change in state, provided the solid and liquid are kept thoroughly mixed, but the change is accompanied

Table 2. Fusion and Vaporization

(At atmospheric pressure, i.e., 760 mm mercury)

From International Critical Tables

Substance	Melting Point, ° C *	Heat of Fusion, g-cal per g †	Boiling Point, ° C *	Heat of Vaporization g-cal per g †
Aluminum	657	87	1800	2000
Ammonia	−75	108	−33.04	327
Antimony	630	38.9	1380	373
Bismuth	270	10.2	1450	220
Brass	900±	....	....	....
Bronze	900±	....	....	....
Cadmium	321	11.1	767	206
Carbon	over 3600	....	over 4200	....
Carbon dioxide	−56.2	45.3	−79	....
Carbon monoxide	−206	8.0	−192	51.6
Cesium	28.5	3.6	670	132
Chlorine	−103.5	23	−34.6	69
Chromium	1610	31.6	2200	1470
Cobalt	1480	58.5	2900	1540
Copper	1084	41.6	2300	1750
German silver	1100±	....	....	....
Glass, flint	1300	....	....	....
Gold	1064	15.9	2600	446
Gutta-percha	100	....	....	....
Hydrogen	−260.6	14.1	−252.7	107
Iridium	2350(?)	....	4800	....
Iron	1535	23.0 to 50	3000	1625
Lead	327	5.6	1620	221
Manganese	1260	36.6	1900	1045
Marble	2500±	....	....	....
Mercury	−38.7	2.7	356.9	68.0
Molybdenum	2620	....	3700	1770
Nickel	1450	72.8	2900	1545
Nitrogen	−210	6.1	−195.8	47.65
Nitric oxide	−160.6	....	−153	....
Oxygen	−219	3.3	−183.0	50.97
Osmium	2700	....	5300	....
Palladium	1545	36.3	2200	....
Paraffin	52.4	35.1	....	....
Platinum	1755	26.9	4300	635
Rhodium	1955	....	2500	....
Rubber	100	....	....	....
Selenium	220	....	688	94
Silicon	1420	....	2600	....
Silver	961	25.9	1950	552
Steel	1300 to 1475	....	....	....
Sulfur	115 to 119	8.9 to 13.2	444.6	69.1
Tantalum	2850	....	4100	....
Tin	232	13.4	2260	655
Tungsten	3370	....	5900	1180
Vanadium	1710	....	3000	....
Water	0	80	100	539
Wax, bees	62	42.3	....	....
Wood's metal	75.5	8.40	....	....
Zinc	419	25.5	907	363

* Let t_c be the value in degrees centigrade; then the value in degrees fahrenheit is $t_f = 32 + 1.8\,t_c$.

† Let H be the value in gram-calories per gram; then the corresponding heat of fusion or of vaporization

In kg-cal per kg is	$1.000\,H$,
In watt-seconds per gram is	$4.183\,H$,
In kwhr per kg is	$1.162 \times 10^{-3}\,H$,
In kwhr per lb is	$5.271 \times 10^{-4}\,H$,
In kwhr per ton (2000 lb) is	$1.054\,H$.

References

Bureau of Standards, Circ. 35.
Smithsonian Physical Tables, 1920.
Landolt-Börnstein-Roth, Physikalisch-Chemische Tabellen, 1912.
International Critical Tables.

by a considerable absorption of heat. The temperature at which the change takes place is called the melting point or freezing point (the reverse change takes place at the same

temperature), and the heat absorbed per unit mass is called the heat of fusion or heat of liquefaction; this same amount of heat is given out when the body solidifies. Many substances, however, have no definite melting point, the change from one state to the other being gradual; such substances begin to melt at a lower temperature than that at which solidification begins during cooling. The melting points and heats of fusion for some common substances are given in Table 2.

21. TRANSFER OF HEAT

When a body is at a higher temperature than the surrounding bodies, energy is transferred from the hotter to the colder bodies, as is manifested by the changes in temperature or state which tend to, or actually do, take place, even though the intervening space is entirely void of matter. The energy thus transferred from one body to another through empty space is called radiant energy or radiant heat, and is similar in nature to the energy radiated in the form of light waves and electromagnetic waves. The waves of radiant heat have a length greater than that of light waves and less than that of the ordinary electromagnetic waves used in wireless telegraphy. Radiant heat is absorbed by, transmitted through and reflected by, ordinary matter in much the same way that light waves are absorbed, transmitted and reflected. Matter which is transparent to light waves, however, may be practically opaque to heat waves; e.g., water absorbs practically all the heat waves which fall upon it.

When a hot and a cold body are separated by a fluid which is free to circulate, heat is transferred from the hot to the cold body by currents of the fluid itself flowing from one to the other; similarly, all parts of a fluid which is being heated quickly come to approximately the same temperature. This transfer of heat by currents of the fluid itself is called convection.

In the case of a hot and a cold body separated by a solid the transfer of heat, which may be very rapid, particularly when the separating medium is a metal, is probably due to an extremely rapid to-and-fro motion of the molecules which constitute the medium. In any event, the process is essentially different from the transfer of heat either by radiation or by convection; it is described by the term *conduction* of heat. In a fluid heat is transferred both by conduction and convection.

RADIATION, ABSORPTION, AND REFLECTION OF HEAT. The rate at which a hotter body radiates heat, and a colder body absorbs heat, depends upon the state of the surfaces of the bodies as well as on their temperatures. The rate of radiation and of absorption are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish. For this reason the covering of steam pipes and boilers should be smooth and of a light color; uncovered pipes and steam-cylinder covers should be polished.

The heat radiated by a body at a given temperature T to surrounding bodies at a lower temperature is equal to the heat which this body would absorb at this same temperature T from surrounding bodies at a higher temperature. When a given quantity of radiant heat strikes a body, only part of the heat is, as a rule, absorbed, the rest being reflected.

Let H_i be the incident heat, H_r the reflected heat, and H_a the absorbed heat, at temperature t, and let H_e be the heat which the body would emit at this same temperature to bodies at a lower temperature; then

$$H_i = H_r + H_a$$
$$H_a = H_e$$

DEFINITION OF "BLACK BODY"; STEFAN-BOLTZMANN LAW. A "black body" is defined as one that absorbs all radiations falling upon it, neither reflecting nor transmitting any. The radiation of such a body is a function of the temperature alone, and is identical with the radiation inside an inclosure all parts of which have the same temperature. By heating the walls of an inclosure as uniformly as possible and observing the radiation through a very small opening, a practical realization of a black body is obtained.

RADIATING AND REFLECTING POWERS. The ratio of the heat radiated per unit area by any surface at a given temperature T to the heat radiated per unit area by an absolutely black surface at this same temperature T is called the radiating power, or relative emissivity, of the surface at that temperature. The difference between this ratio and unity is a measure of the heat which would be reflected by this surface at the same temperature, and is defined as the reflecting power of the surface. The radiating power of a surface depends upon the temperature of the surface; at very high temperatures the radiating power of every surface approaches the value of unity, i.e., at high tempera-

tures the total energy radiated by any surface approaches in value the total energy radiated by an absolutely black body. Table 3, taken in part from Kent's Mechanical Engineer's Pocketbook and in part from Langmuir's paper (*Trans. A.I.E.E.*, 1913), gives the approximate value of the radiating power of some common surfaces at ordinary temperatures.

Oiling a polished surface may increase its radiating power from 2 to 3 times, but oiling does not seriously affect the radiating power of a rough surface.

CONDUCTION OF HEAT. Whenever a difference of temperature is maintained between any two parts of the same body there is a transfer of heat from the hotter to the colder part by the process described by the term conduction, as distinguished from radiation and convection.

Table 3. Approximate Radiating and Absorbing Powers

Surface	Radiating or Absorbing Power	Surface	Radiating or Absorbing Power
Lamp black	1.00	Zinc, polished	0.19
Water	1.00	Steel, polished	0.17
Carbonate of lead	1.00	Platinum, polished	0.24
Writing-paper	0.98	Platinum in sheet	0.17
Ivory, jet, marble	0.93–0.98	Tin	0.15
Ordinary glass	0.90	Brass, cast, dead polished	0.11
Ice	0.85	Brass, bright polished	0.07
Gum lac	0.72	Copper, varnished	0.14
Silver leaf on glass	0.27	Copper, hammered	0.07
Cast iron, bright polished	0.25	Copper, oxidized	0.568
Cast iron, oxidized	0.71	Copper, calorized	0.27
Aluminum paint on cast iron	0.47	Monel metal, polished	0.40
Gold enamel on cast iron	0.39	Monel metal, oxidized	0.44
Mercury, about	0.23	Gold, plated	0.05
Wrought iron, polished	0.23	Gold on polished steel	0.03
		Silver, polished bright	0.03

Consider a flat layer *within* a substance, the two sides of the layer being parallel and its thickness small compared with its area. Let one side of this be maintained at a constant temperature T, the same at all points of this surface, and the other side of the layer be maintained at a constant temperature T_1; let A be the area of the layer (i.e., of one of its flat surfaces) and x the thickness of the layer. Then the amount of heat transferred through the layer in time t is

$$H = \frac{K A (T - T_1)t}{x} \tag{3}$$

where K, called the thermal conductivity, is approximately a constant for a given material, and is independent of the temperature difference $T - T_1$, when this difference is small; K is not constant, however, for wide temperature variations, particularly in the cases of gases (see paragraph following Table 5). Values of K are given in Tables 4 and 5. The reciprocal of the thermal conductivity, viz., $\rho = 1/K$, is called the thermal resistivity.

The values of K given in the tables below are the values of this factor when H is expressed in gram-calories, A in square centimeters, x in centimeters, $(T - T_1)$ in degree centigrade, and t in seconds; i.e., K is the number of gram-calories transmitted per second through a cube 1 cm on each edge when a difference of temperature of 1 deg cent is

Unit of Heat	Unit of Area	Unit of Thickness	Unit of Time	Unit of Temp. Diff.	Multiply K by
Gram-calorie	sq cm	cm	second	°C	1.000
Kg-calorie	sq m	cm	hour	°C	3.600×10^4
Btu	sq ft	inch	hour	°F	2902
Watt-seconds	sq cm	cm	second	°C	4.183
Watt-seconds	sq ft	inch	second	°F	850.8
Kwhr	sq m	cm	hour	°C	41.83
Kwhr	sq ft	inch	hour	°F	0.8508

maintained between opposite faces of this cube. For other units the value of K as given should be multiplied by the factor noted in the table above.

TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY. For metals the variation of the internal thermal conductivity with temperature may be expressed with a fair degree of approximation by the relation

$$K = K_0 (1 + at) \tag{4}$$

where K_0 is the conductivity at 0 deg cent, say, and K is the conductivity at any other temperature of t deg cent, and a is a constant. The coefficient a may be either positive or negative; its value for some of the common metals is given in Table IV.

VALUES OF THERMAL CONDUCTIVITY (K) OF MATERIALS. In the following tables, 4 and 5, are given the thermal conductivities of certain common non-metallic and metallic substances respectively. The data are from the following sources: Landolt-Börnstein, Physikalish-Chemische Tabellen, 1923; Randolph, C. P., *Trans. Am. Electrochem. Soc.*, 1912; Smithsonian Physical Tables, 1920; Langmuir, *A.I.E.E.*, *Trans.*, 32, p. 301, February, 1913; International Critical Tables, Vol. V, 1929.

THERMAL CONDUCTIVITY OF LAMINATED STEEL. The thermal conductivity of a laminated iron core perpendicular to the laminations depends upon the conductivity of the iron and the oxide scale or varnish on the laminations and also upon the pressure on the laminations. G. E. Luke (*Elec. World*, 70, p. 562, Sept. 22, 1917) gives the following values for K in gram-calories per cm³ per deg cent.

Material	Pressure, pounds per square inch			
	25	50	75	100
Silicon steel, 15.5 mils thick, 3 per cent oxide layer..........................	0.00096	0.00115	0.00125	0.00132
Ordinary varnished steel, 18.35 mils thick, 5 per cent varnish layer.......	0.00094	0.00098	0.00109	0.00120

THERMAL CONDUCTIVITY OF GASES. The thermal conductivity of a gas depends not only upon the nature of the gas but also upon its specific heat at constant volume, C_v, and its absolute temperature T. Langmuir (*Phys. Rev.*, 34, p. 408, 1912) gives the following values for the *watts* conducted from a plane surface through an adhering film A square centimeters in area and B centimeters thick.

$$W = \frac{A}{B} (\phi_2 - \phi_1) \tag{5}$$

where ϕ_1 and ϕ_2 are functions of the absolute temperatures at the two surfaces of the film, as given in the following table:

Values of ϕ

Abs. Temp. °C	Hydrogen	Air	Mercury Vapor	Abs. Temp. °C.	Hydrogen	Air	Mercury Vapor
0	0.0000	0.0000	....	1700	5.945	0.931	0.228
100	0.0329	0.0041	....	1900	7.255	1.138	0.284
200	0.1294	0.0168	....	2100	8.655	1.363	0.345
300	0.278	0.0387	....	2300	10.18	1.608	0.411
400	0.470	0.0669	....	2500	11.82	1.871	0.481
500	0.700	0.1017	0.0165	2700	13.56	......	0.556
700	1.261	0.189	0.0356	2900	15.54	......	0.636
900	1.961	0.297	0.0621	3100	17.42	......	0.719
1100	2.787	0.426	0.0941	3300	19.50	......	0.807
1300	3.726	0.576	0.1333	3500	21.79	......	0.898
1500	4.787	0.744	0.1783				

22. LOW-TEMPERATURE HEAT-INSULATING MATERIALS

Table 4, based on tests made by the Bureau of Standards, gives a comparison of thermal conductivities of several heat-insulating materials on the market intended for use in low-temperature applications, such as refrigerators, rooms, and cars.

23. EXPANSION DUE TO HEATING

Most substances expand when heated, but water between 0 and 4 deg cent, quartz glass below −84 deg cent, and a few other substances contract with increase of temperature. When the temperature of a solid is changed by rapid cooling, slow changes in its

Table 4. Thermal Conductivity of Non-metallic Substances

Substance	Temp. Range, ° C		Therm. Conduct., K ‡		Temperature Coefficient per ° C a
	From *	To	From †	To †	
Asbestos	100	500	0.00016	0.00050	....
Asbestos paper	....	....	0.00043	0.00060	....
Brick building	15	1100	0.00149	....	....
Brick, dust	15	30	0.000461	....	....
Brick, fire	0	1300	0.00140	0.0054	....
Carborundum, brick	150	1200	0.0032	0.027	....
Cardboard	Below 0	....	0.000394	....	....
Cement, Portland	35	90	0.000712	0.00217	....
Chalk	....	....	0.00219	....	....
Cloth, empire	....	....	0.00060	....	....
Concrete	0	....	0.002	....	....
Cork	20	20,000	0.000076	0.000717	....
Cotton batting, loose	....	....	0.000096	....	....
Cotton batting, tightly packed	−150	150	0.000091	0.00018	....
Ebonite	6	90	0.00038	....	−0.0019
Eiderdown, loose	150	....	0.00015	....	....
Eiderdown, tightly packed	150	....	0.000045	....	....
Feathers	20	155	0.000163	....	....
Felt	21	175	0.000087	0.000225	....
Flannel	50	....	0.00012	....	....
Fullerboard	....	....	0.00034	....	....
Glass	....	....	0.0014	0.002	....
Glycerin	....	....	0.00068	....	....
Granite	100	200	0.0045	0.0097	....
Hair	20	155	0.000148	....	....
Ice	−160	0	0.0053	0.0057	....
Infusorial earth	20	350	0.00032	0.00040	....
Lamp black	100	500	0.000074	0.000107	....
Leather	....	....	0.00015	0.00042	....
Limestone	40	350	0.0046	0.0035	....
Linen	....	....	0.00021	....	....
Liquids, hydrocarbons, oils, etc.	....	....	about 0.0003	....	....
Magnesia, carb.	20	188	0.00023	....	....
Magnesia, calcined	20	155	0.000165	0.000173	....
Magnesia, asbestos	100	400	0.000162	0.000178	....
Marble	15	30	0.00770	0.00910	−0.0005
Mica, pure	....	....	0.00086	0.0012	....
Mica, paper	....	....	0.00038	....	....
Micanite	....	30	0.000050	0.00010	....
Oil, paraffin	....	....	0.00033	....	....
Oil, castor	....	....	0.000425	....	....
Oil, petroleum	0	34	0.000355	0.000382	+0.0110
Oil, turpentine	13	....	0.000325	....	+0.0067
Paper	....	....	0.00011	0.00031	....
Paper, treated	....	....	0.00014	0.00041	....
Paraffin	....	30	0.000473	0.00062	....
Pasteboard	....	....	0.000450	....	....
Plaster	....	....	0.00130	....	....
Plaster of paris	20	155	0.000425	....	....
Plumbago	20	155	0.00100	....	....
Poplox, made from Na_2SiO_3	200	500	0.0000920	0.000162	....
Porcelain	95	....	0.00249	....	....
Pumice stone	20	155	0.000428	....	....
Quartz glass	....	....	0.0036	....	....
Rubber, Para	....	....	0.00038	....	....
Rubber, vulcanized	....	....	0.00034	0.00054	....
Sand	20	155	0.000855	0.000867	....
Sawdust	....	30	0.000143	....	....
Shellac	....	....	0.00060	....	....
Silk	50	100	0.000095	0.000141	....
Slag	50	....	0.00264	....	....
Slate	94	....	0.0048	....	....
Snow	....	0	0.00038	....	....
Strawboard	....	....	0.000330	....	....
Tape, treated cloth	....	....	0.00036	0.00065	....
Tape, rubber	....	....	0.00103	....	....
Water	0	25	0.00150	0.00136	....
Wood	....	....	0.000087	0.00086	....
Wool, sheep's, loose	....	30	0.00010	....	....
Wool, sheep's, tightly packed	....	....	0.000055	....	....
Wool, mineral	0	175	0.0000930	0.000128	....
Wool, steel	100	....	0.000192	0.000216	....
Woolen	100	....	0.0000553	0.000119	....

* When only one temperature is given the measurement was made at that temperature.

† Range of determination by different experimenters except where only one value of K is given, in which case the range is that to which the given value of K applies.

‡ In gram-calories per centimeter-cube per degree centigrade per second; see Section 1 for multiplying factors when other units are employed.

Table 5. Thermal Conductivity of Metals and Various Forms of Carbon

Substance	Temp. Range, °C		Therm. Conduct. K *		Temp. Coeff. per °C a
	From †	To	From ‡	To ‡	
Aluminum	18	100	0.480	0.492	+0.0030
Aluminum	200	400	0.545	0.760	+0.0020
Aluminum	500	600	0.885	1.01	+0.0014
Brass, yellow	0	100	0.204	0.254	+0.0024
Brass, red	0	100	0.246	0.283	+0.0015
Calorite	20	250	0.038	....	....
Carbon, amorphous	100	360	0.089	....	....
Carbon, amorphous	100	842	0.129	....	....
Carbon, Ach. graph	100	390	0.338	....	....
Carbon, Ach. graph	100	914	0.291	....	....
Carbon, graphite brick	300	700	0.024	....	....
Charcoal	20	155	0.00019	....	....
Coal	....	....	0.00030	....	....
Constantan	18	100	0.054	0.064	+0.00227
Copper	−54	−14	0.921	1.059	+0.0053 to
Copper	74	167	0.914	1.024	+0.00047
Copper	100	197	0.908	....	....
Copper	100	837	0.858	....	....
Copper, commercial	18	....	0.835	....	....
German silver	0	100	0.070	0.089	+0.0027
Gold	100	....	0.703	....	−0.00007
Iron	0	....	0.167	0.207	−0.0008 to
Iron	100	....	0.142	0.163	−0.0001
Iron	200	....	0.136	....	....
Iron	100	727	0.202	....	....
Iron	100	1245	0.191	....	....
Iron, cast	100	....	0.096	....	....
Lead	0	100	0.0836	0.0764	{ −0.00086 to −0.00016
Manganin	18	100	0.52	0.63	+0.0026
Mercury	0	50	0.0148	0.0189	+0.0055
Nickel	18	....	0.142	....	−0.00031
Nickel	300	....	0.126	....	−0.00095
Nickel	1200	....	0.058	....	−0.00047
Platinum	18	100	0.166	0.173	+0.00051
Platinoid	18	....	0.060	....	....
Silver	18	100	1.006	0.992	−0.00017
Steel	....	....	0.108	0.111	−0.0006
Steel, transformer §	20	250	0.077	....	....
Tin	0	100	0.155	0.145	−0.0007
Zinc	18	100	0.265	0.262	−0.00016

* In gram-calories per centimeter-cube per degree centigrade per second; see Section 1 for multiplying factors when other units are employed.

† When only one temperature is given the measurement was made at that temperature.

‡ Range of determinations by different experimenters except where only one value of K is given, in which case the range is that to which the given value of K appues.

§ See also paragraph Thermal Conductivity of Laminated Steel on page 2–54.

dimensions continue long after it has attained the same uniform temperature throughout. This effect, which is particularly marked in glass, is known as thermal hysteresis. It can be largely eliminated by prolonged heating at a high temperature followed by a very gradual cooling, i.e., by annealing.

COEFFICIENT OF EXPANSION OF GASES. Gases present a remarkable uniformity, all the so-called permanent gases expanding about $^1/_{273}$ part of their *initial volume at 0* deg cent per degree centigrade increase of temperature, irrespective of the pressure, provided this remains constant throughout. That is, for any of the ordinary gases,

$$V = V_0\left(1 + \frac{t_c}{273}\right)$$

where V_0 is the volume at 0 deg cent, and V the volume at any other temperature t_c deg cent, the pressure being the same at both temperatures. If the temperature is expressed in fahrenheit degrees and V_0 is the volume at 0 deg fahr, then

$$V = V_0\left(1 + \frac{t_f}{460}\right)$$

Note that −273 and −460 are the absolute zeros on the centigrade and fahrenheit scales respectively.

Table 6. Conductances of Various Thicknesses of Insulations Computed from Conductivities per Inch of Thickness

Material	Description	Density (lb per cu ft)	Mean Temp (deg fahr)	Thickness of Insulation, In.														
				1/3	3/8	7/16	1/2	2/3	3/4	7/8	1	1 1/2	1 5/8	1 3/4	2	3	3 5/8	5 5/8
Balsam wood	Chemically treated wood fiber	2.2	90				0.54				0.27							
Cabots quilt	Eel grass between Kraft paper	4.6	90	0.78			0.52	0.39			0.26							
Cabots quilt	Eel grass between Kraft paper	3.4	90	0.75			0.50	0.375			0.25							
Celotex	Board form insulation made from sugar-cane fiber	13.2	90			0.777	0.68			0.389	0.34	0.227		0.194	0.17	0.113		
Corkboard	Pure; no added binder	14.0	90								0.34	0.227			0.17	0.113		
Corkboard	Pure; no added binder	10.6	90								0.30	0.20			0.15	0.10		
Corkboard	Pure; no added binder	7.0	90								0.27	0.18			0.135	0.09		
Corkboard	Pure; no added binder	5.4	90								0.25	0.167			0.125	0.083		
Corkboard (Eureka)	Asphaltic binder	14.5	90								0.32	0.213			0.16	0.107		
Dry zero	Kapok between burlap or paper	2.0	90								0.25							
Dry zero	Kapok between burlap or paper	1.0	90								0.24							
Fibrofelt	Flax and rye fiber	13.6	90	0.96	0.853		0.64		0.426		0.32							
Flaxlinum	Flax fiber	13.0	90				0.62				0.31							
Hairinsul	(75 per cent hair, 25 per cent jute)	6.3	90				0.54		0.36		0.27							
Hairinsul	(50 per cent hair, 50 per cent jute)	6.1	90				0.52		0.347		0.26							
Hair felt	Felted cattle hair	13.0	90				0.52				0.26							
Hair felt	Felted cattle hair	11.0	90				0.52				0.26							
Insulex or Pyrocell	Cellular gypsum	30.0	90								1.00		0.615				0.276	0.178
Insulex or Pyrocell	Cellular gypsum	24.0	90								0.77		0.473				0.212	0.137
Insulex or Pyrocell	Cellular gypsum	18.0	90								0.59		0.363				0.163	0.105
Insulex or Pyrocell	Cellular gypsum	12.0	90								0.44		0.271				0.121	0.078
Insulite	Board form insulation made from wood	16.9	90				0.68				0.34							
Linofelt	Flax fibers between paper	4.9	90		0.747		0.56				0.28							
Lith	Rock wool, flax, and straw pulp with binder	14.3	90								0.40	0.267			0.20	0.133		
Regranulated cork	About 3/16-in. particles	8.1	90								0.31		0.191				0.086	0.055
Rock wool	Fibrous material, made from rock	10.0	90								0.27		0.166				0.075	0.048
Sprayo-flake	Shredded paper with silica binder	4.2	94				0.56		0.373	0.32	0.28	0.186			0.14	0.093		
Thermofelt	Jute and asbestos fibers, felted	10.0	90				0.74				0.37							
Thermofelt	Hair and asbestos fibers, felted	7.8	90				0.56				0.28							
Thermofill	Dry, fluffy, flaked gypsum	34.0	90								0.60		0.369				0.165	0.107
Thermofill	Dry, fluffy, flaked gypsum	26.0	90								0.52		0.32				0.143	0.092
Torfoleum	Peat moss compressed into sheet form	10.2	91.5								0.29	0.193		0.166	0.145	0.097		

Table 7. Coefficients of Linear Expansion

$l = l_0 (1 + at + bt^2)$, temperature in deg cent *
a_{20} = "true" coefficient at 20 deg cent

(From Landolt-Börnstein's Tables, 1923 Edition)

Substance	Temp., ° C		a	b	a_{20}
	From	To			
Aluminum	0	610	0.235×10^{-4}	0.707×10^{-8}	0.238×10^{-4}
Brass (73.7 Cu + 24.2 Zn + 1.5 Sn + 0.6 Pb)	0	80	0.179×10^{-4}	0.456×10^{-8}	0.181×10^{-4}
Bronze (81.2 Cu + 8.6 Zn + 9.9 Sn + 0.2 Pb)	0	80	0.176×10^{-4}	0.469×10^{-8}	0.177×10^{-4}
Carbon, gas-carbon	40	....	0.054×10^{-4}	....	....
Carbon, graphite	40	....	0.079×10^{-4}	....	....
Constantan (60 Cu + 40 Ni)	0	500	0.148×10^{-4}	0.402×10^{-8}	0.150×10^{-4}
Copper	0	625	0.167×10^{-4}	0.403×10^{-8}	0.169×10^{-4}
Glass, Jena	0	100	0.077×10^{-4}	0.350×10^{-8}	0.079×10^{-4}
Glass, French	2	100	0.072×10^{-4}	0.544×10^{-8}	0.075×10^{-4}
Gold	9	95	0.136×10^{-4}	1.12×10^{-8}	0.140×10^{-4}
German silver	0	100	0.184×10^{-4}	....	....
Ice	−27	−2	0.514×10^{-4}	....	....
Iron, cast	0	625	0.098×10^{-4}	0.566×10^{-8}	0.102×10^{-4}
Iron, wrought	0	500	0.117×10^{-4}	0.525×10^{-8}	0.119×10^{-4}
Lead	14	94	0.273×10^{-4}	0.74×10^{-8}	0.276×10^{-4}
Marble, white	15	100	0.117×10^{-4}	....	....
Mica, parallel to cleavage	5	80	0.077×10^{-4}	1.200×10^{-8}	0.082×10^{-4}
Mica, perpendicular to cleavage	4	82	0.076×10^{-4}	0.490×10^{-8}	0.079×10^{-4}
Nickel	0	1000	0.135×10^{-4}	0.332×10^{-8}	0.136×10^{-4}
Nickel steel (24% Ni)	0	38	0.175×10^{-4}	0.711×10^{-8}	0.178×10^{-4}
Phosphor bronze (97.6 Cu + 2.2 Sn + 0.2 P)	0	80	0.167×10^{-4}	0.462×10^{-8}	0.168×10^{-4}
Platinum	0	1000	0.0887×10^{-4}	0.1324×10^{-8}	0.0892×10^{-4}
Porcelain, Berlin	20	100	0.027×10^{-4}	0.306×10^{-8}	0.028×10^{-4}
Porcelain, Bayeux	0	600	0.034×10^{-4}	0.107×10^{-8}	0.035×10^{-4}
Rubber, hard	17	25	0.77×10^{-4}	....	....
Rubber, hard	25	35	0.84×10^{-4}	....	....
Silver	0	750	0.1827×10^{-4}	0.4793×10^{-8}	0.1846×10^{-4}
Steel	0	300	0.092×10^{-4}	0.336×10^{-8}	0.093×10^{-4}
Tin	8	95	0.203×10^{-4}	2.63×10^{-8}	0.214×10^{-4}
Vulcanite	0	18	0.636×10^{-4}	....	....
Zinc	9	96	0.274×10^{-4}	2.34×10^{-8}	0.284×10^{-4}

* When the temperature is expressed in fahrenheit degrees, the formulas become

$$l = l_{32}\left[1 + \frac{a\,(t_f - 32)}{1.8} + \frac{b\,(t_f - 32)^2}{3.24}\right]$$

$$a_{68} = \frac{a_{20}}{1.8}$$

where a, b, and a_{20} have the values given in the above table.

Table 8. Coefficients of Cubical Expansion

$V = V_0 (1 + \alpha t + \beta t^2)$, temperatures in degrees centigrade
α_{20} = "true" coefficient at 20 deg cent

(From Landolt-Börnstein's Tables, 1923 edition)

Substance	Temp., ° C		α	β	α_{20}
	From	To			
Caoutchouc, crude gray	0	75	6.62×10^{-4}	24.2×10^{-8}	6.80×10^{-4}
Gutta-percha, pure rolled	0	40	4.96×10^{-4}	496×10^{-8}	6.94×10^{-4}
Paraffin	0	33	5.84×10^{-4}	99.2×10^{-8}	5.88×10^{-4}
Petroleum, sp. gr. 0.8467	24	120	8.99×10^{-4}	140×10^{-8}	9.55×10^{-4}
Wax, white solid	10	57	10.7×10^{-4}	-5580×10^{-8}	3.06×10^{-4}

Mercury (− 10 to 300 deg cent.):

$$V = V_0 (1 + 1.805553 \times 10^{-4}\, t + 1.2444 \times 10^{-8}\, t^2 + 2.539 \times 10^{-11}\, t^3)$$

24. FLASH POINT OF OILS

The flash point of an oil is the temperature to which the oil must be raised before the vapor immediately above it will take fire upon the application of a flame. This temperature depends to an appreciable extent upon the size of the flame, the method of applying it, and the shape and dimensions of the containing vessel. The following values of the flash point for various oils are taken from J. Lewkowitisch, Chemical Technology and Analysis of Oils, Fats and Waxes.

Table 9. Flash Point of Oils

Oils	Spec. Grav. at 60° F	Flash Point, ° F
Mineral oils:		
Refined American	0.875 to 0.920	325 to 425
Refined Russian	0.895 to 0.915	300 to 425
Scotch	0.875 to 0.895	300 to 350
Natural (dark) American	0.880 to 0.895	325 to 425
Natural (dark) Russian	0.910 to 0.915	250 to 300
Natural filtered American	0.885 to 0.905	450 to 575
Animal oils:		
Sperm	0.8804 to 0.8807	446 to 457
Lard	0.9172	494
Tallow	0.951	265
Neat's foot	0.9178	470
White whale	0.9207	476
Vegetable oils:		
Castor	0.963	275
Linseed	0.930	285
Olive	0.914	305
Rape, crude	0.920	265
Rape, refined	0.911	305

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SECTION 3

ELECTRIC CIRCUITS AND ELECTRIC LINES

BY

IRVEN TRAVIS

ART. PAGE

ALTERNATING CURRENTS

1. General Definitions, Periodic Quantities 02
2. Maximum, Average, and RMS or Effective Values 02
3. Form Factor, Crest or Peak Factor, Deformation Factor 03
4. Power, Power Factor, Volt-amperes, Reactive Power 03
5. Sinusoidal Currents and Voltages, Phase Angle 04
6. Power and Power Factor for Sinusoidal Current and Voltage 05
7. Vector Representation of Sinusoids 05
8. Fourier Series, Harmonics 06
9. RMS Value and Power of Non-sinusoidal Currents 09

LINEAR CIRCUITS

10. Circuit Equations and Vector Impedances 09
11. Impedance, Admittance, Reactance, Susceptance 10
12. Impedances in Series and Parallel 11
13. Vector Power 11
14. Equivalent Impedances of a Receiver 12
15. Complicated Circuits 12
16. Reciprocal Theorem 13
17. Thévenin's Theorem 13
18. Equivalent Transducers (T and π) 14
19. General Circuit Constants 15
20. General N-terminal Network 16
21. General Four-terminal Network 17

NON-LINEAR CIRCUITS

22. Ferromagnetism 19
23. Eddy Currents 21
24. Skin Effect 21
25. Electric Arcs 24
26. Corona 25
27. Variable Inductance 25
28. Special Mathematical Methods 26
29. Iron-cored Transformers 27

POLYPHASE SYSTEMS AND SYMMETRICAL COMPONENTS

30. Polyphase Currents and Voltages 29
31. Mesh and Star Connections 29
32. Phase and Line Currents and Voltages 30
33. Power in Polyphase Systems 31
34. Two-phase (or Quarter-phase) System 31
35. Three-phase System, Delta and Wye Connections 31
36. Balanced Three-phase Circuits 32
37. Symmetrical Components of Currents and Voltages 32
38. Graphical Calculation of Symmetrical Components 34
39. Symmetrical Components of Impedances 34
40. Matrix Formulation of the Three-phase Problem 35
41. Symmetrical Components for a Symmetrical Network 37
42. Partially Symmetrical Network 37
43. Receiver Currents in Terms of Line to Ground Voltages 38
44. Symmetrical Component Equations for a Three-phase Transducer 39
45. Symmetrical Components of Instantaneous Values 40

ELECTRICAL MACHINES

46. Magnetomotive Forces of Windings 41
47. Rotating Fields 42
48. Flux Linkages in Synchronous Machines 43
49. Nominal EMF and Armature Reactances 44

TRANSMISSION LINES

50. Transmission Line Equations 46
51. Steady-state Current and Voltage Distribution 46
52. General Circuit Constants 47
53. Propagation Constant, Characteristic Impedance 47
54. Short- and Open-circuit Impedances 48
55. Electrical Length 48
56. Transients in Transmission Lines 48

STABILITY OF POWER SYSTEMS

57. Definitions of Stability Terms 49
58. General Stability Problem 50
59. Line between Infinite Buses 51
60. Torque-angle Characteristics 51
61. Rotor Acceleration 53
62. One Machine on an Infinite Bus 53
63. Two-machine Problem 54
64. Methods of Improving Stability 55

ELECTRIC CIRCUITS AND ELECTRIC LINES

By Irven Travis

ALTERNATING CURRENTS

The definitions given below are applicable to currents, electromotive forces, potential differences, or any other function of time. These definitions are those recommended by the Sectional Committee on Electrical Definitions of the A.I.E.E.

1. GENERAL DEFINITIONS, PERIODIC QUANTITIES

(*a*) An *oscillating quantity* is a quantity which as a function of some independent variable (such as time) alternately increases and decreases in value, always remaining within finite limits.

(*b*) A *periodic quantity* is an oscillating quantity the values of which recur for equal increments of the independent variable.

(*c*) The *period* of a periodic quantity is the smallest value of the increment of the independent variable which separates recurring values of the quantity.

(*d*) A *cycle* is the complete series of values of a periodic quantity which occur during a period.

(*e*) The *frequency* of a periodic quantity in which time is the independent variable is the reciprocal of the period.

(*f*) The *angular velocity* of a periodic quantity is the frequency multiplied by 2π.

(*g*) An *alternating quantity* is a periodic quantity which has alternately positive and negative values.

In order to avoid repetition the following statements will be given in terms of electric currents; they apply equally well to electromotive forces, drops of potential, etc.

If a given current is represented by the equation

$$i = f(t) \tag{1}$$

and if the function $f(t)$ has the property that

$$f(t) = f(t + T) \tag{2}$$

in which T is a constant, then the current is said to be periodic in time and T is the period. If time is measured in seconds, then $1/T$ represents the number of periods per second and is usually denoted by f. The frequency f may also be said to be the number of cycles per second. If, as often happens, the current is expressible more simply as

$$i = f(\omega t) \tag{3}$$

in which ω is defined by

$$f(\omega t) = f(\omega t + 2\pi) \tag{4}$$

then the constant ω, being equal to 2π divided by the period or 2π times the frequency, is the angular velocity as defined in (*f*). That is

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{5}$$

2. MAXIMUM, AVERAGE, AND RMS OR EFFECTIVE VALUES

The *instantaneous value* of an alternating current is the value of the current at any instant. Instantaneous values of current, potential difference, and electromotive force will be designated by lower-case letters throughout this section, viz., i, v, and e, respectively.

The *maximum value* of an alternating current is the numerical value of its maximum instantaneous value. Maximum values will be designated by capital letters with the subscript m.

The *half-period average value* of a symmetrical alternating current * is the absolute

*A symmetrical alternating quantity is one of which all values separated by a half-period have the same magnitude but opposite sign. The term half-period average has no meaning for alternating currents which are not symmetrical.

value of the algebraic average of the values of the current taken throughout a half period, beginning with a zero value. If the current has more than two zeros during a cycle, that zero shall be taken which gives the largest half-period average value. The expression for the half-period average of a symmetrical alternating current $i(t)$ having a period T is

$$I_{av} = \frac{2}{T}\int_{t_0}^{t_0+\frac{1}{2}T} i(t)\,dt \tag{6}$$

where t_0 is chosen such that $i(t) = 0$ at t_0, and I_{av} is the largest value obtainable.

The square root of the means of the squares of the instantaneous values of an alternating current over a complete period is called the rms, or the *effective* value of the alternating current. In specifying the value of an alternating current as so many amperes this rms value is always meant unless specifically stated otherwise. In the same manner the square root of the mean of the squares of the instantaneous values of an alternating potential difference over a complete period is called the rms value of the alternating potential difference. When the value of an alternating potential difference is specified as so many volts, this rms value is always meant unless specifically stated otherwise.

The reason for selecting this particular function of the instantaneous values of an alternating current or a potential difference as a measure of the current or the potential difference is that the deflection of all instruments used in alternating-current measurements is a function of this rms value. (See Measurements.) Moreover, the average power dissipated as heat in a resistance, r, when an alternating current of rms value I flows through it, is rI^2.

Rms values will be designated throughout this section by capital letters without subscripts.

The general expression for the rms value of an alternating current is

$$I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \tag{7}$$

and similarly for an alternating potential difference.

3. FORM FACTOR, CREST OR PEAK FACTOR, DEFORMATION FACTOR

The *form factor* of a symmetrical alternating current is the ratio of the effective value of the current to its half-period average value.

The *peak* or *crest factor* of an alternating current is the ratio of the maximum value of the current to its effective value; this is also called the *amplitude factor*.

The *equivalent sinusoidal current* of a given alternating current is a sinusoid having the same period and the same effective value of the given alternating current.

The *deformation factor* of an alternating current is the ratio to the maximum value of the equivalent sinusoidal current *of* the maximum difference between the corresponding values of the current considered and the equivalent sinusoid, when the two are superimposed in such a way as to make this difference a minimum.

4. POWER, POWER FACTOR, VOLT AMPERES, REACTIVE POWER

Let v be the value at any instant of the potential drop from any point 1 to any other point 2, and let i be the instantaneous value of the current from 1 to 2 at this same instant; then the *power input* at this instant is

$$p = vi \tag{8}$$

When v and i are both positive (i.e., in the direction from 1 to 2, say) or when they are both negative, the power input is positive, but when v is positive and i negative, or vice versa, the power input is negative, i.e., there is an actual power output.

The average value of the product vi over a complete period for both v and i (or over any whole number of periods) is the *average power* input or output, usually called simply the power input or output (input when the average of vi is positive, output when the average of vi is negative), the word average being understood. That is, the average power input is

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T vi\,dt \tag{9}$$

T being a complete period. For the actual measurement of alternating-current power see Section 5, Measurements.

Only in certain special cases (see below) is the average power input P equal to the product of the rms value V of the potential difference by the rms value I of the current; it can never be greater and as a rule is less. The ratio of the average power P to the product of the rms value V of the potential difference by the rms value I of the current is called the *power factor* of the circuit between the terminals considered, i.e.,

$$\text{Power factor} = \frac{P}{VI} \tag{10}$$

When V is expressed in volts and I in amperes, then P must be in watts; when V is expressed in kilovolts and I in amperes, P must be in kilowatts.

The product of the rms volts across the terminals of a circuit and the rms amperes through it is called the *volt-amperes* or *apparent power* taken by the circuit; this product divided by 1000 is called the kilovolt-ampere input. Or, when V is in volts and I in amperes,

$$\text{Volt-amperes} = VI \tag{11}$$

$$\text{Kilovolt-amperes} = \frac{VI}{1000} \tag{11a}$$

Kilovolt-amperes is usually abbreviated kva, kv-a, or K.V.A., the first form being that recommended by the American Institute of Electrical Engineers and used in this book.

Reactive power has no accepted definition when either the current or emf is non-sinusoidal. (See below.)

5. SINUSOIDAL CURRENTS AND VOLTAGES, PHASE ANGLE

A *simple sinusoidal current* (simple harmonic current) is an alternating current the instantaneous values of which are equal to the product of a constant and the sine of an angle having values varying linearly with time. Thus

$$i = I_m \sin(\omega t + \theta) \tag{12}$$

where t represents time in seconds, measured from any arbitrarily chosen instant; I_m the maximum value of the current; $\omega = 2\pi f = 2\pi/T$, where f is the frequency in cycles per second and T the period as a fraction of a second; and θ a constant which depends upon the instant chosen as the zero of time.

The quantity I_m is often called the *amplitude* of the sinusoidal current.

The ratio of I_m to the rms value I (the amplitude factor) for a sinusoidal current is $\sqrt{2}$.

The *phase* of a periodic current, for a particular value of the independent variable, is the fractional part of a period through which the independent variable has advanced measured from an arbitrary origin. In the case of a simple sinusoidal current, the origin is usually taken as the last previous passage through zero from the negative to positive direction. The *phase angle* is the angle obtained by multiplying the phase by 2π if the angle is to be expressed in radians, or by 360 deg if the angle is to be expressed in degrees.

The phase angle is thus equal to $(\omega t + \theta)$. In electric circuit theory the phase differences between various quantities having the same frequency are of importance. Since these differences in phase are independent of the instant at which they are evaluated, it is customary to refer to the angle θ (which is the phase angle when $t = 0$) as the phase angle of the current given by eq. (12).

In general, when a sine-wave electromotive force is impressed on a circuit the resulting current is likewise a sine function of time after a very brief interval, (see Handbook of Engineering Fundamentals, Eshbach, John Wiley & Sons, " Transients ") having the same frequency, but the emf and current do not reach their maximum values simultaneously. Let the current be represented by eq. (12) and let the voltage be given by

$$v = V_m \sin \omega t \tag{12a}$$

where t is the time measured from the instant when $v = 0$ and is increasing in the positive direction. The voltage reaches its maximum value when $t = \dfrac{\pi}{2\omega}$, while the current reaches its maximum value when $t = \dfrac{\pi}{2\omega} - \dfrac{\theta}{\omega}$. Hence when θ is positive the voltage reaches its maximum value θ/ω seconds after the current reaches its maximum, or the current reaches its maximum value θ/ω seconds before the voltage reaches its maximum; when θ is negative the current reaches its maximum value θ/ω seconds after the voltage reaches its maximum. In the first case the current is said to " lead " the voltage and in the second case the current is said to " lag " the voltage. The angle θ is called the angular phase difference between the current and voltage.

When the phase difference is zero the current and voltage are said to be "in phase;" when the phase difference is $\pi/2$ radians or 90 deg the current and voltage are said to be "in quadrature;" when the phase difference is π radians or 180 deg the current and voltage are said to be "in opposition."

6. POWER AND POWER FACTOR FOR SINUSOIDAL CURRENT AND VOLTAGE

Let the voltage drop from terminal 1 to terminal 2 through any piece of apparatus be $v = \sqrt{2}V \sin(\omega t + \theta_v)$, and the current from terminal 1 to terminal 2 be $i = \sqrt{2}I \sin(\omega t + \theta_i)$, where V and I are the rms values and therefore $\sqrt{2}V$ and $\sqrt{2}I$ are the maximum values. Then the instantaneous power input is

$$p = vi = VI[\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)] \tag{13}$$

A study of Fig. 1 will show the physical meaning of this expression. The average power input is

$$P = VI \cos(\theta_v - \theta_i) \tag{13a}$$

where $(\theta_v - \theta_i)$ is the angular difference in phase between the current and voltage. Putting θ for this difference in phase, viz., $\theta = \theta_v - \theta_i$, eq. (13a) may be written

$$P = VI \cos\theta \tag{13b}$$

Whence the power factor of the load supplied to the apparatus is, from eq. (10),

$$\cos\theta = \frac{P}{VI} \tag{14}$$

Since in the case of sine-wave currents and voltages the power factor is equal to the cosine of the angle which expresses the difference in phase between them, this difference in phase is frequently called the "power-factor angle." When the wave shape is not a pure sine curve, the power factor cannot be interpreted as the cosine of the phase difference, for phase difference has no definite meaning except in reference to sine waves; see definitions above. A non-sinusoidal voltage and current may both reach their zero values at the same instant, and in a sense may be said to be "in phase," but the power factor as defined by eq. (10) may be far from unity.

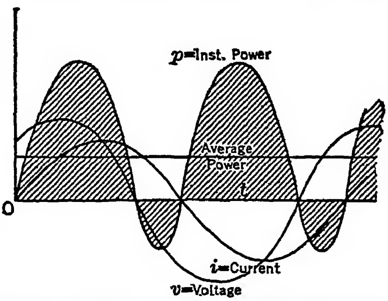


FIG. 1. Power *versus* Time in the Case of a Sinusoidal Current and Voltage

The *reactive power* in a circuit in which a sinusoidal current is flowing is equal to the effective emf times the effective current times the sine of the phase difference between them. When the emf and current are in volts and amperes respectively the reactive power is in *vars*.

7. VECTOR REPRESENTATION OF SINUSOIDS

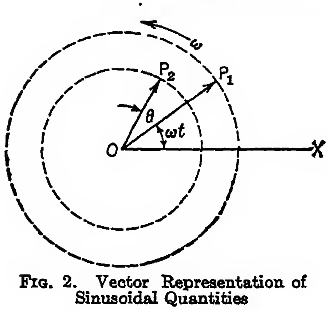


FIG. 2. Vector Representation of Sinusoidal Quantities

Consider any sine function

$$i = I_m \sin \omega t$$

The value of i at any instant may be represented graphically, see Fig. 2, by the vertical projection (i.e., the vertical distance from P_1 to OX) of a point P_1 at the end of a radius $OP_1 = I_m$ which revolves* at a constant angular velocity ω about a fixed point O, the angle ωt being measured from the horizontal fixed line OX. Similarly, any other sine function

$$v = V_m \sin(\omega t + \theta)$$

may be represented by the vertical projection of the point P_2 at the end of a radius $OP_2 = V_m$ also revolving about O with a constant angular velocity ω, the angle between OP_1 and OP_2, when the fre-

* Counter-clockwise rotation has been adopted (1911) as standard by the International Electrotechnical Commission.

quency of both i and v is the same, remaining fixed in value and equal to the difference in phase θ between v and i. That is, v and i may be represented by rotating vectors. When v and i are of the same frequency the relative position of the two vectors remains fixed. Similarly any number of currents and voltages of the same frequency may be represented by rotating vectors which remain fixed with respect to one another.

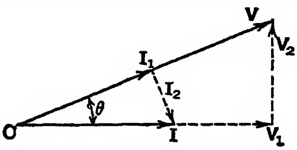


FIG. 3. Components of Vectors

Instead of referring the various rotating vectors to a fixed line OX, this line of reference may also be considered as rotating with the same speed as the various vectors, or any one of the vectors may be chosen as the line of reference, for example, the vector OP_1 in Fig. 2. The rotating vectors referred to this rotating line of reference are then fixed with respect to this line of reference, and the entire diagram may be considered as fixed, as in Fig. 3.

Instead of making the vectors equal in length to the maximum values of the sine functions they may be chosen equal in length to their rms values. This merely introduces a factor $\sqrt{2}$ so that the instantaneous value of the quantity which any rotating vector is considered to represent is equal to $\sqrt{2}$ times the perpendicular distance from the end of the vector to the fixed line of reference.

Since the rms values and phase relations of sine-wave currents and voltages may be represented by vectors, sine-wave currents may be added in exactly the same manner as vectors, or forces, are added, and similarly for sine-wave voltages. To add any two sine-wave currents or voltages not only must their effective values be known but also their phase relation; the resultant of two alternating voltages of rms values V_1 and V_2 is never the arithmetical sum of V_1 and V_2, except when the two voltages are exactly in phase, and similarly for alternating currents.

In Fig. 3, considering OI as equal to the rms value I of the current and OV as representing the rms value V of the voltage, the voltage V may be considered as made up of two components, viz.: $V_1 = V \cos\theta$ in phase with I, $V_2 = V \sin\theta$ in quadrature with I. The average power corresponding to the component $V_1 = V\cos\theta$ is, from eq. (13b), $V_1 I = VI\cos\theta$, and is equal to the total power corresponding to V and I. The average power corresponding to the component $V_2 = V\sin\theta$, since the angle between the current and this component of the voltage is 90 deg, is equal to zero. The voltage component $V_1 = V\cos\theta$ is therefore frequently called the "power" component of the voltage, and the component $V_2 = V\sin\theta$ is frequently called the "wattless" component of the voltage. These terms, however, are not recommended. It is preferable to refer to these two components as the in-phase and quadrature components, respectively. The terms active and reactive components are also used.

Similarly, the current I may be considered as made up of two components, viz.:

$$I_1 = I\cos\theta \text{ in phase with } V$$

$$I_2 = I\sin\theta \text{ in quadrature with } V$$

The first component is called the in-phase component of the current, and the second the quadrature component of the current.

8. FOURIER SERIES, HARMONICS

It can be shown that any single-valued, periodic function (which fulfils certain mathematical conditions always fulfilled by electric currents) may be represented by a sum of sinusoids of which the frequencies form an arithmetical progression. Hence any current $i(t)$ can be written

$$\left.\begin{aligned} i(t) &= A_1 \sin 2\pi\frac{t}{T} + A_2 \sin 2\pi\frac{2t}{T} + \ldots + A_n \sin 2\pi\frac{nt}{T} + \ldots \\ &B_0 + B_1\cos 2\pi\frac{t}{T} + B_2\cos 2\pi\frac{2t}{T} + \ldots + B_n\cos 2\pi\frac{nt}{T} + \ldots \end{aligned}\right\} \tag{15}$$

where the values of the coefficients A and B are given by the integrals:

$$A_n = \frac{2}{T}\int_0^T i(t)\sin 2\pi\frac{nt}{T}\,dt \tag{15a}$$

$$B_n = \frac{2}{T}\int_0^T i(t)\cos 2\pi\frac{nt}{T}\,dt \tag{15b}$$

The series of eq. (15) is called a *Fourier series*.

If the terms involving the same frequencies are combined, and if $\frac{2\pi}{T} = \omega$, eq. (15) may be written

$$i(t) = I_0 + I_1 \sin(\omega t - \theta_1) + I_2 \sin(2\omega t - \theta_2) + \ldots + I_n \sin(n\omega t - \theta_n) + \ldots \quad (16)$$

The term $I_1 \sin(\omega t - \theta_1)$ is called the *fundamental*, and the terms $I_n \sin(n\omega t - \theta_n)$ are called *harmonics*. The *order* of a harmonic is the ratio of the harmonic frequency to the frequency of the fundamental. Each harmonic is characterized by two constants, its amplitude I_n and the angle θ_n. It should be noted that this angle may be given in either of two ways:

$$I_n \sin(n\omega t - \theta_n) \quad (16a)$$

$$I_n \sin n(\omega t - \theta'_n) \quad (16b)$$

In the first notation the angle θ_n is taken relative to the period of the harmonic, whereas in the second notation the angle θ'_n is taken relative to the period of the fundamental. It follows that

$$\theta_n = n\theta'_n \quad (16c)$$

The mathematical statement of the definition given above for a symmetrical alternating current is

$$i(\omega t + \pi) = -i(\omega t) \quad (17)$$

from which it follows by substitution of eq. (16) that

$$a_2 = a_4 = a_6 = \ldots = a_{2k} = \ldots = 0 \quad (18)$$

i.e., all the even harmonics of a symmetrical alternating current are zero.

If the two half-periods are alike and if, further, each is symmetrical about an axis erected at a quarter of a period from a null point then not only are the even harmonics zero but the θ's in eq. (16) are all either zero or π. In this case all the odd harmonics are in phase or in direct opposition. This type of symmetry often exists in commercial alternating currents. If this more restricted type of symmetry holds, the current may be written

$$i(t) = I_1 \sin \omega t + I_3 \sin 3\omega t + I_5 \sin 5\omega t + \ldots \quad (19)$$

or in terms of cosine functions,

$$i(t) = I_1 \cos \omega t + I_3 \cos 3\omega t + I_5 \cos 5\omega t + \ldots \quad (19a)$$

The odd harmonics may be divided into two groups, those of order $4k - 1$ and those of order $4k + 1$. Harmonics of the first class have the same sign in (19*a*) as in (19); those of the second class have opposite signs in (19) and (19*a*). Thus harmonics of order $4k - 1$ are in phase at their maximum values if they are in phase at their zero values; harmonics of order $4k + 1$ are in opposition at their maximum values if they are in phase at their zero values, and conversely. These facts are of importance in the consideration of harmonics in synchronous machines and induction motors.

If an analytic expression for $i(t)$ is available and if the integrations (15*a*) and (15*b*) can be performed, the terms of the Fourier series (15) can be written out. In practice the function $i(t)$ will usually be obtainable only in the form of a plotted curve and the evaluation of the A's and B's must be carried out by graphical means. This process is called *harmonic analysis*. Various means of harmonic analysis have been devised, the Fisher-Hinnen method being perhaps the most convenient in ordinary cases.

This method is quite simple when the wave contains only the fundamental and the third, fifth, and seventh harmonics. When even harmonics are present, or when higher odd harmonics than the seventh exist, certain corrections must be applied. Since voltage and current waves usually contain only odd harmonics, and seldom contain higher harmonics than the seventh, the simple method without corrections is usually sufficiently accurate.

WAVES CONTAINING ONLY THE THIRD, FIFTH, AND SEVENTH HARMONICS. To determine the nth harmonic (n equals 3, 5, or 7), divide the base of a half-wave into $2n$ equal parts and measure the ordinates of the wave at the beginning of each of these sections of the base. Call these ordinates $y_1, y_2, y_3 \ldots y_{2n}$, taking the y's positive if above the base line, negative if below. y_1 will be zero, since the first section begins where the resultant wave crosses the base line.

Then the ordinates of this harmonic at the points 1 and 2 are, respectively,

$$A_n = \frac{1}{n}[(y_5 + y_9 + \ldots y_{2n-1}) - (y_3 + y_7 + \ldots y_{2n-3})]$$

$$B_n = \frac{1}{n}[(y_2 + y_6 + \ldots y_{2n}) - (y_4 + y_8 + \ldots y_{2n-2})]$$

The maximum value of this harmonic is

$$Y_n = \sqrt{A_n^2 + B_n^2}$$

and the phase angle, calling the wavelength of the fundamental 360 deg, is

$$\phi_n = \frac{1}{n}\tan^{-1}\frac{A_n}{B_n}$$

These formulas give the third, fifth, and seventh harmonics (n = 3, 5, and 7, respectively). The fundamental is found by calculating

$$A_1 = -(A_3 + A_5 + A_7)$$
$$B_1 = y_0 + B_3 - B_5 + B_7$$

where y_0 is the mid-ordinate of the half-wave. Then

$$Y_1 = \sqrt{A_1^2 + B_1^2}$$
$$\phi_1 = \tan^{-1}\frac{A_1}{B_1}$$

The equation of the given wave is then

$$y = Y_1 \sin(x + \phi_1) + Y_3 \sin 3(x + \phi_3) + Y_5 \sin 5(x + \phi_5) + Y_7 \sin 7(x + \phi_7).$$

The *effective* value of the given wave is

$$Y = \sqrt{\frac{Y_1^2 + Y_3^2 + Y_5^2 + Y_7^2}{2}}$$

and the average value is

$$Y_{av} = \frac{2}{\pi}[Y_1 \cos\phi_1 + {}^1/_3\, Y_3 \cos 3\phi_3 + {}^1/_5\, Y_5 \cos 5\phi_5 + {}^1/_7\, Y_7 \cos 7\phi_7]$$

In using the above formulas strict attention must be paid to algebraic signs.

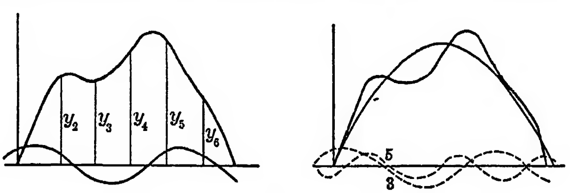


FIG. 4. Fisher-Hinnen Method of Wave Analysis

Example. Find the third harmonic in the wave shown in Fig. 4. The values of the six ordinates are found by measurement to be 0, 676, 660, 940, 1004, and 554, respectively. Then

$$A_3 = {}^1/_3(1004 - 660) = 114.7$$
$$B_3 = {}^1/_3(676 + 554 - 940) = 96.7$$
$$Y_3 = \sqrt{(114.7)^2 + (96.7)^2} = 150$$
$$\phi_3 = {}^1/_3 \tan^{-1}\frac{114.7}{96.7} = 16.6°$$

Similarly, for the fifth harmonic,

$$A_5 = -92.8 \text{ and } B_5 = 37.4$$
$$Y_5 = 100 \quad\text{and}\quad \phi_5 = -13.6°$$

For the fundamental

$$A_1 = -114.7 + 92.8 = -21.9$$
$$B_1 = 940 + 96.7 - 37.4 = 999.3$$
$$Y_1 = \sqrt{(21.9)^2 + (999.3)^2} = 1000$$
$$\phi_1 = \tan^{-1}\frac{-21.9}{999.3} = -1.25°$$

Hence the complete expression for the given wave, taking as the origin the point at which the resultant wave crosses the base line in the rising direction, is

$$y = 1000 \sin(x - 1.25°) + 150 \sin 3(x + 16.6°) + 100 \sin 5(x - 13.6°)$$

The *effective* value is then 718, and *average* value 673.

WAVES CONTAINING ANY NUMBER OF HARMONICS, ODD OR EVEN. See *Elec. J.*, 1908, Vol. 5, p. 386; *Electrotechm. Zeit.*, 1913, Vol. 34, p. 936.

9. RMS VALUE AND POWER OF NON-SINUSOIDAL CURRENTS

The rms value of a non-sinusoidal current can be obtained directly from eq. (3), or if the rms values I_1, I_2, I_3, etc., of the fundamental and harmonics are known the rms value of the resultant current is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2} + \ldots \tag{20}$$

A like relation holds for the rms value of a non-sinusoidal voltage. Similarly, if for example a 25-cycle emf, say E_{25}, and a direct emf, say E_d, are acting in series in the same circuit, the resultant emf of the combination is

$$E = \sqrt{E_{25}^2 + E_d^2} \tag{21}$$

The power corresponding to non-sinusoidal currents and voltages may be computed as follows:

Let I_1 be the rms value of the fundamental of the current, V_1 the rms value of the fundamental of the voltage, and θ_1 the difference in phase between these two fundamentals, both being of the same frequency; let I_2, V_2, and θ_2 be the corresponding quantities for the second harmonic; let I_3, V_3, and θ_3 be the corresponding values for the third harmonic, etc. Then the average power is

$$P = V_1 I_1 \cos\theta_1 + V_2 I_2 \cos\theta_2 + V_3 I_3 \cos\theta_3 + \ldots \tag{22}$$

That is, each harmonic contributes an amount to the total power equal to the power it would develop were the other harmonics not present. If, for example, the third harmonic is not present in the current wave, then this harmonic contributes nothing to the average power even though there may be a large third harmonic in the voltage wave. Again, when a 25-cycle alternating emf E_{25} and a direct emf E_d are acting in series on the same circuit the power developed is the sum of the powers which each would develop if they acted separately, but the resultant emf of the combination is not $E_{25} + E_d$, but, as noted above,

$$\sqrt{E_{25}^2 + E_d^2}$$

LINEAR CIRCUITS

10. CIRCUIT EQUATIONS AND VECTOR IMPEDANCES

Practically all steady-state calculations of electric circuit problems are carried out by means of the *complex number* or *vector* method based on the differential equations for the electric charge in terms of the physical parameters of the electric circuit and the impressed voltages. This method is given in detail in Handbook of Engineering Fundamentals, Eshbach, John Wiley & Sons, "Electricity and Magnetism," and is abstracted here. This method is valid only when the resistances, inductances, and capacitances in the network under consideration are constants. Various means are available for extending the method in order to obtain good approximations when these parameters are not constants. Some of these are given in the chapters entitled "Non-Linear Circuits" and "Electrical Machines."

The following are the rules for finding the steady-state currents produced in a network by a group of sinusoidal electromotive forces of a given frequency:

(*a*) Write the differential equations for the instantaneous values of the currents in the network. These will be of the form

$$r_{11} i_1 + L_{11}\frac{di_1}{dt} + \frac{1}{C_{11}}\int i_1\,dt - r_{12} i_2 - L_{12}\frac{di_2}{dt} - \frac{1}{C_{12}}\int i_2\,dt \pm M_{12}\frac{di_2}{dt}$$

$$+ \text{ other terms involving the other currents in the network}$$
$$= \text{impressed voltage in mesh 1.}$$

(*b*) Replace each electromotive force in the network by a complex number having a modulus equal to the rms value of the given electromotive force, and a phase angle equal to the phase angle of the given electromotive force (when expressed in cosine form).

(*c*) Replace $\frac{d}{dt}$ by $j\omega$ and $\int dt$ by $\frac{1}{j\omega}$, where ω is the angular velocity of the given sinusoidal electromotive forces and j is the square root of minus one.

(*d*) Replace each current in the network by an unknown complex number, and solve for these complex numbers.

(*e*) The rms value of the various currents are the moduli of the corresponding complex numbers and the phase angles of the currents (when expressed in cosine form) are the phase angles of these complex numbers.

The coefficients of each of the complex number currents in the equations constructed as indicated above will also be complex numbers; these are called *vector impedances* or often simply *impedances*.

Example. The differential equation for the current in a single isolated mesh having resistance, inductance, and capacitance when a 60-cycle voltage having a modulus value E is impressed is

$$ri + L\frac{di}{dt} + \frac{1}{C}\int i dt = \sqrt{2}\,E \cos 120\pi t \tag{1}$$

The complex number * equation is

$$r\mathbf{I} + j\,120\pi L\mathbf{I} + \frac{1}{j\,120\pi C}\mathbf{I} = \mathbf{E} \tag{1a}$$

hence the vector impedance is

$$\mathbf{Z} = r + j\left(120\pi L - \frac{1}{120\pi C}\right) \tag{2}$$

The current in the circuit is

$$i = \sqrt{2}I \cos (120\pi t - \theta) \tag{3}$$

in which

$$I = \frac{E}{\sqrt{r^2 + \left(120\pi L - \dfrac{1}{120\pi C}\right)^2}} \tag{3a}$$

and

$$\theta = \tan^{-1}\frac{120\pi L - \dfrac{1}{120\pi C}}{r} \tag{3b}$$

11. IMPEDANCE, ADMITTANCE, REACTANCE, SUSCEPTANCE

The *impedance* of a portion of an electric circuit to a completely specified periodic current and potential difference is the ratio of the effective value of the potential difference between the terminals to the effective value of the current, there being no source of emf in the portion under consideration. Impedance defined in this way is a real number which in the case of a sinusoidal current and voltage reduces to the modulus value of the vector impedance defined above.

Admittance is defined as the reciprocal of the impedance. The *vector admittance* is the reciprocal of the complex number representing the vector impedance.

Impedance and admittance, as thus defined, both depend upon the frequency and wave shape of the current. Impedance is expressed in the same units as resistance (e.g., ohms), and admittance in the same units as conductance (e.g., mhos).

The *reactance* of a portion of a circuit for a sinusoidal current, and hence for any one of the frequencies of a periodic current, is the ratio of the quadrature component of the potential difference for a particular frequency to the value of the current for that frequency, there being no source of emf in the portion of the circuit under consideration.

The *susceptance* of a portion of a circuit for a sinusoidal potential difference, and hence for any one of the frequencies of a periodic potential difference, is the ratio of the quadrature component of the current for a particular frequency to the value of the potential difference for that frequency, there being no source of emf in the portion of the circuit under consideration.

From the definitions given above it may be shown that for sine-wave currents and voltages of a given frequency the following relations hold for any portion of a circuit:

$$\left.\begin{array}{ll}
\mathbf{Z} = r + jx & \mathbf{Y} = g - jb \\
Z = \sqrt{r^2 + x^2} & Y = \sqrt{g^2 + b^2} \\
\mathbf{Z} = \dfrac{1}{\mathbf{Y}} & \mathbf{Y} = \dfrac{1}{\mathbf{Z}} \\
r = \dfrac{g}{y^2} & g = \dfrac{r}{Z^2} \\
x = \dfrac{b}{y^2} & b = \dfrac{x}{Z^2}
\end{array}\right\} \tag{4}$$

* Vectors are printed in bold face, the modulus value of a vector $\mathbf{A}$ being printed in light face, A.

where r = effective resistance, x = reactance (taken positive when inductive and negative when capacitive), g = effective conductance, b = susceptance (taken positive when inductive and negative when capacitive), Z = impedance, Y = admittance, $\boldsymbol{Z}$ = vector impedance, and $\boldsymbol{Y}$ = vector admittance, all for the given portion of circuit.

12. IMPEDANCES IN SERIES AND PARALLEL

In general any vector impedance can be written in the form

$$\boldsymbol{Z} = r + jx \tag{5}$$

where r is the sum of the resistances and x is the algebraic sum of the reactances of the several portions of the network which make up this impedance.

The vector impedance may also be expressed in the form

$$\boldsymbol{Z} = Z\angle\theta \tag{6}$$

where

$$Z = \sqrt{r^2 + x^2} \tag{7}$$

is the modulus of the complex number $(r + jx)$ and

$$\theta = \tan^{-1}\left(\frac{x}{r}\right) \tag{8}$$

is the phase angle of this complex number.

In eq. (6) the symbol $\angle$ is read "at an angle" (or "phase") and has the same mathematical significance as e^j. Thus $Z\angle\theta$ is *identical* with $Ze^{j\theta}$.

When two or more vector impedances $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, etc., are connected in *series*, so that the *same current I flows through each*, the total impedance drop through the group is

$$\begin{aligned} \boldsymbol{V} &= \boldsymbol{Z}_1\boldsymbol{I} + \boldsymbol{Z}_2\boldsymbol{I} + \dots \\ &= (\boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 + \dots)\boldsymbol{I} \end{aligned} \tag{9}$$

hence two or more vector impedances in series are equivalent to a single vector impedance equal to the sum of several impedances.

When two or more vector impedances $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, etc., are connected in *parallel*, so that the *voltage drop is the same for each* the total current through the group is

$$\begin{aligned} \boldsymbol{I} &= \frac{\boldsymbol{V}}{\boldsymbol{Z}_1} + \frac{\boldsymbol{V}}{\boldsymbol{Z}_2} + \dots \\ &= \left(\frac{1}{\boldsymbol{Z}_1} + \frac{1}{\boldsymbol{Z}_2} + \dots\right)\boldsymbol{V} \end{aligned} \tag{10}$$

It is customary to express this law in terms of the vector admittances, $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, ... Two or more vector admittances in parallel are equivalent to a single vector admittance equal to the sum of the several admittances.

13. VECTOR POWER

In Art. 4 average power is defined, and in Art. 6 the average power due to a sinusoidal current and voltage of the same frequency was shown to be

$$P = VI\cos(\theta_v - \theta_i) \tag{11}$$

The vector voltage and vector current in this case are

$$\boldsymbol{V} = Ve^{j\theta_v}$$

$$\boldsymbol{I} = Ie^{j\theta_i}$$

of which the product *is not* the average power. The average power is given by the *real part* of the product of the vector voltage by the *conjugate* of the vector current. The *complex power* is given by

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{V} \times (\text{conjugate of } \boldsymbol{I}) \\ &= VIe^{j(\theta_v - \theta_i)} \\ &= VI\cos(\theta_v - \theta_i) + jVI\sin(\theta_v - \theta_i) \end{aligned} \tag{12}$$

As stated above, the real part of the complex power is the average power. The imaginary part of the complex power is the reactive power. The average power is the actual average rate of transformation by the receiver of electrical energy into some other form of energy. The reactive power measures the rate of interchange of unused electrical energy between the source and receiver.

This power surges back and forth between source and receiver but is not transformed

into any other type of energy; it does not therefore represent a rate of dissipation of energy in the receiver.

When the current and voltage are written in the form

$$V = V_1 + jV_2$$
$$I = I_1 + jI_2$$

the average power is given by

$$P = V_1 I_1 + V_2 I_2 \tag{13}$$

14. EQUIVALENT IMPEDANCES OF A RECEIVER

The receiver at the end of a transmission line is often a motor or other device which contains a back electromotive force. The vector impedance equations for a network containing such a device therefore involve this back electromotive force as one of the electromotive forces in the network. It is often convenient, however, to express the emf in terms of an equivalent impedance drop.

Let the back emf be denoted by E and let the current through the device be I, then the equivalent impedance of the back emf is simply $E/I = Z_e$. If the true impedance of the remainder of the circuit is Z then the impedance equation becomes

$$(Z_e + Z)I = V \tag{14}$$

where V is the voltage impressed across the device.

If the emf leads the current by an angle θ then an amount of electric power

$$P = EI \cos \theta \tag{15}$$

is converted into some other form of power. A resistance which will absorb this same amount of power when the current I flows through it is

$$r_e = \frac{P}{I^2} = \frac{E}{I} \cos \theta \tag{16}$$

This resistance is called the equivalent resistance of the back emf; it is the real part of the vector impedance Z_e. The equivalent reactance of the back emf is

$$x_e = r_e \tan \theta \tag{17}$$

It should be noted that the equivalent impedance of a back emf as given above is a function of the current through the receiver. The quantity $(Z_e + Z)$ is the total equivalent impedance of the receiver.

15. COMPLICATED CIRCUITS

When an electrical network is extremely complicated the process of combining series and parallel impedances in order to solve for the currents in the various branches is quite difficult. In such cases it is usually easier to set up the problem in terms of *mesh currents* instead of branch currents. If currents are assumed to flow around each of the meshes of a network so that the current in any branch is the difference of the mesh currents in the two meshes on either side of that branch, then the impedance equations resulting from the application of Kirchhoff's second law will contain the Kirchhoff point equations. See Handbook of Engineering Fundamentals, Eshbach, John Wiley & Sons, "Direct Current Circuits" and "Alternating Current Circuits."

The mesh equations for a network having n meshes, each mesh containing impressed voltages, are

$$\left.\begin{array}{l} Z_{11} I_1 + Z_{12} I_2 + \dots + Z_{1n} I_n = V_1 \\ Z_{21} I_1 + Z_{22} I_2 + \dots + Z_{2n} I_n = V_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ Z_{n1} I_1 + Z_{n2} I_2 + \dots + Z_{nn} I_n = V_n \end{array}\right\} \tag{18}$$

in which Z_{jj} is the self-impedance of the jth mesh and Z_{jk} is the mutual impedance of the jth mesh with respect to the kth mesh. Z_{jk} is always equal to Z_{kj} in passive networks and in active networks in which there is no valve action.

A representative network of this type is shown in Fig. 1; in this network the number of meshes is easy to count. In general, as for example when the network cannot be drawn on the surface of a sphere, the number of mesh currents necessary to formulate the problem completely is

$$B - V + S$$

where B = the number of branches, V = the number of intersections or vertices, and S = the number of separate parts. A circuit which closes on itself (single mesh) is assumed to have one intersection.

These impedance equations can be solved by the substitution method or by the method of determinants. For numerical calculations it is usually simpler to solve by substitution and elimination of currents. In theoretical work, however, particularly in the development of the general properties of electrical networks, the method of determinants is much more powerful.

The values of the currents in eq. (18) can be calculated by means of Cramer's rule which is developed in the theory of linear algebraic equations and determinants.

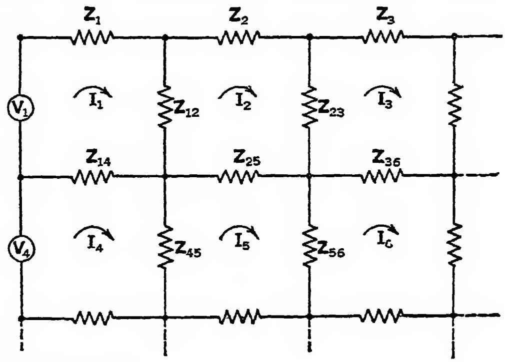
Let D denote the determinant of order n whose elements are the Z's in eqs. (18), i.e.,

$$D = \begin{vmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{vmatrix} \tag{19}$$

and let M_{jk} denote the determinant of order $n - 1$ obtained from D by deleting the jth row and the kth column of that determinant. M_{jk} is called a minor of D. Further let

$$A_{jk} = (-1)^{j+k} M_{jk} \tag{20}$$

A_{jk} is called a cofactor of D.



$Z_{11}=Z_1+Z_{12}+Z_{14}$

$Z_{22}=Z_2+Z_{12}+Z_{23}+Z_{25}$

Etc.

FIG. 1. Complicated Network. The network shown is *not* perfectly general but is given here for illustration; the network corresponding to eqs. (18) cannot be drawn on a plane

Then the current I_k is given by

$$I_k = \sum_{k=1}^{n} \frac{A_{jk} V_j}{D} \tag{21}$$

16. RECIPROCAL THEOREM

Let a single impressed voltage V be applied to, say, the jth mesh of a network, all other impressed voltages being zero. The current in the kth mesh is

$$I_k = \frac{A_{jk} V}{D}$$

Now let the voltage be removed from the jth mesh and inserted in the kth mesh. The current in the jth mesh in this case is

$$I_j = \frac{A_{kj} V}{D}$$

Since $Z_{jk} = Z_{kj}$ it follows also that $A_{jk} = A_{kj}$, hence the theorem commonly called the *reciprocal theorem*:

The current in a given branch of a network due to an electromotive force in another branch of the network is the same as the current which would be produced in the second branch of the network if the electromotive force were transferred to the first branch.

The quantity A_{kj}/D is called the *transfer admittance* from the kth to the jth mesh. The reciprocal of this quantity is the *transfer impedance*.

17. THÉVENIN'S THEOREM

A theorem due to Thévenin, which can be deduced from eqs. (18), is extremely valuable in network problems inasmuch as it allows the calculation of the performance of a device from its terminal properties only, without inquiring into its internal make-up.

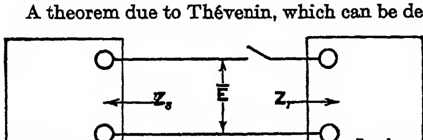


FIG. 2. Junction of a Source and Receiver

Consider any complicated network having internal electromotive forces and having two available points of entry, such as that shown in Fig. 2. Such a network is capable of acting as a source of electrical

energy. Let E be the open-circuit terminal voltage of this source, and let the impedance *looking into* the source terminals be Z_s. (By impedance "looking into" a pair of terminals is meant the input impedance measured at these terminals with the rest of the network disconnected, and all emf's made equal to zero.) Let this source be connected to a receiver having two points of entry and containing no internal emf's. The impedance looking into the receiver terminals will be taken as Z_r. Then Thévenin's theorem states that the current which flows from source to receiver is given by

$$(Z_r + Z_s)I = E \tag{22}$$

A generalization of this statement may be stated thus:

If two networks are connected together by means of a two-conductor line the line current is equal to the algebraic sum of the open-circuit voltages which would appear if the line were cut, divided by the sum of the impedances looking toward the source and toward the receiver from the cut.

A simple use of this theorem is the calculation of the current delivered to a motor by a generator. In this case the generator emf minus the motor back emf divided by the sum of the two internal impedances plus the line impedance yields the line current.

18. EQUIVALENT TRANSDUCERS (T AND π)

It is often desirable to study the performance of a network at two pairs of its terminals when various voltages or various impedances are connected to them. In such cases it is convenient to call one pair of terminals the input terminals and the other pair the output terminals. The input terminals are those usually connected to a source of energy, and the output terminals are those usually connected to a receiver. Both terminals may, of course, be connected to sources. A device of this type having one pair of input terminals and one pair of output terminals is called a *transducer*. A transformer, an underground cable, a transmission line together with its terminal transformers, etc., are transducers.

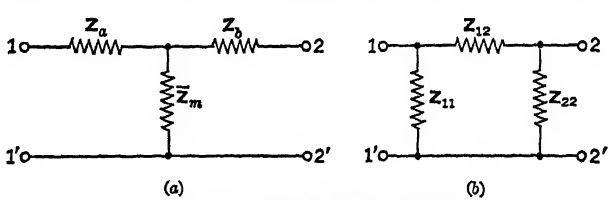


FIG. 3. Equivalent T and π Sections

Any *passive* transducer, i.e., one which does not contain a source of electromotive force, may, for a given frequency, be replaced by an equivalent circuit containing only three independent impedances. These impedances may be arranged as in Fig. 3*a* or as in Fig. 3*b*. The former arrangement is called an equivalent T; the latter arrangement is called an equivalent π of the given transducer.

The constants of the equivalent T may be computed from measurements made at the terminals of the given network. Let:

Z_{10} = impedance measured at the input terminals when the output terminals are open.
Z_{20} = impedance measured at the output terminals when the input terminals are open.
Z_{1s} = impedance measured at the input terminals when the output terminals are short-circuited.
Z_{2s} = impedance measured at the output terminals when the input terminals are short-circuited.

These quantities are not all independent, the relation

$$Z_{10}\, Z_{2s} = Z_{20}\, Z_{1s} \tag{23}$$

providing a means for calculating any one when the other three are known.

In terms of these open- and short-circuit impedances the parameters of the equivalent T are:

$$Z_m = \sqrt{Z_{20}(Z_{10} - Z_{1s})} \tag{24}$$

$$Z_a = Z_{10} - Z_m \tag{25}$$

$$Z_b = Z_{20} - Z_m \tag{26}$$

Other forms for these equations may be obtained by the use of eq. (23). It is sometimes convenient to make use of the transfer impedance of the transducer in these calcula-

tions. The transfer impedance Z_{ts} can of course be measured directly; it is related to the open- and short-circuit impedances by:

$$Z_{ts} = Z_{1s}\sqrt{\frac{Z_{20}}{Z_{10} - Z_{1s}}} \tag{27}$$

The shunt arm Z_m of the equivalent T may also be expressed as:

$$Z_m = \frac{Z_{10}\,Z_{2s}}{Z_{ts}} \tag{28}$$

The impedances in the equivalent π may be calculated from those of the equivalent T by the formulas:

$$Z_{11} = Z_a\,Z_m\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_m}\right) \tag{29}$$

$$Z_{22} = Z_b\,Z_m\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_m}\right) \tag{30}$$

$$Z_{12} = Z_a\,Z_b\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_m}\right) \tag{31}$$

19. GENERAL CIRCUIT CONSTANTS

Another method of specifying a transducer is by means of the four constants A, B, C, and D defined by the equations:

$$E_s = AE_r + BI_r \tag{32}$$

$$I_s = CE_r + DI_r \tag{33}$$

in which the subscript s stands for source and the subscript r stands for receiver.* These constants are no more "general" than those described in the preceding article; they have the advantage, however, of expressing the source conditions, directly in terms of the receiver conditions. The constants A, B, C, and D are not independent. Any one can be calculated, when the other three are known, by the equation:

$$AD - BC = 1 \tag{34}$$

This relation may be used to deduce from eqs. (32) and (33) the alternative forms:

$$E_r = DE_s - BI_s \tag{35}$$

$$I_r = -CE_s + AI_s \tag{36}$$

These equations have the advantage of expressing the receiver conditions directly in terms of the conditions which obtain at the source.

General circuit constants are of great value in the calculation of the performance of power systems and in studies of stability (q.v.).

The general circuit constants may be calculated from the constants of the equivalent T by the formulas:

$$\left.\begin{aligned} A &= 1 + \frac{Z_a}{Z_m} \\ B &= Z_a + Z_b + \frac{Z_a\,Z_b}{Z_m} \\ C &= \frac{1}{Z_m} \\ D &= 1 + \frac{Z_b}{Z_m} \end{aligned}\right\} \tag{37}$$

The general circuit constants may also be calculated from the constants of the equivalent π, thus

$$\left.\begin{aligned} A &= 1 + \frac{Z_{12}}{Z_{22}} \\ B &= Z_{12} \\ C &= \frac{1}{Z_{11}} + \frac{1}{Z_{22}} + \frac{Z_{12}}{Z_{11}\,Z_{22}} \\ D &= 1 + \frac{Z_{12}}{Z_{11}} \end{aligned}\right\} \tag{38}$$

* E_s and E_r are voltage *rises* in source and receiver, respectively; I_s is the current flowing *into* the network from the source and I_r is the current flowing *out* of the network to the receiver.

It is interesting to note that A and D are pure numerics; from eqs. (32) and (33) it is apparent that A is the open-circuit voltage ratio and that D is the short-circuit current ratio of the transducer.

The constant B has the dimensions of impedance whereas the constant C has the dimensions of admittance. From eq. (32) the constant B may be recognized as the transfer impedance of the transducer. The constant C, from eq. (33), may be seen to be the ratio of source current to receiver voltage when the receiver terminals are open. These physical meanings which attach themselves to the general circuit constants provide means for their direct measurement.

If two transducers are connected in series, the general circuit constants of the resulting transducer are easily expressible in terms of the general circuit constants of the separate transducers. Let A_1, B_1, C_1, and D_1 be the general circuit constants of the first and A_2, B_2, C_2, and D_2 the general circuit constants of the second transducer, then if A_s, B_s, C_s, D_s be the general circuit constants of the transducer formed by the two in series, we have:

$$\left.\begin{aligned} A_s &= A_1 A_2 + B_1 C_2 \\ B_s &= A_1 B_2 + B_1 D_2 \\ C_s &= C_1 A_2 + D_1 C_2 \\ D_s &= C_1 B_2 + D_1 D_2 \end{aligned}\right\} \qquad (39)$$

If A_p, B_p, C_p, D_p be the general circuit constants of the transducer formed by the two transducers in parallel we have:

$$\left.\begin{aligned} A_p &= \frac{A_1 B_2 + B_1 A_2}{B_1 + B_2} \\ B_p &= \frac{B_1 B_2}{B_1 + B_2} \\ C_p &= C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \\ D_p &= \frac{B_1 D_2 + D_1 B_2}{B_1 + B_2} \end{aligned}\right\} \qquad (40)$$

Since A, B, C, and D for a given transducer are not independent, slightly more work is done in calculation of these constants than if three independent constants were used; this, however, affords an extremely valuable check of the numerical computation. Eq. (34) may be checked at various stages throughout a lengthy calculation, it being extremely unlikely that this equation will be satisfied if a numerical error has been made.

20. GENERAL N-TERMINAL NETWORK

It is possible, given an electrical network with any number of points of entry, to obtain a circuit having the same number of points of entry which cannot be distinguished from the given circuit by any single-frequency electrical tests made at the terminals. This equivalence is independent of any arbitrary pairing of terminals and hence is more general than the transducer equivalence discussed in Arts. 18 and 19.

The performance of a network having $(n + 1)$ points of entry is determined by $n(n + 1)/2$ independent impedance links. This becomes apparent if the mesh equations are written down. Referring to the network of Fig. 4, assume that equations have been written down for all the currents in the network and that the resulting system of equations has been reduced by elimination of currents until there remain n equations in n of the $(n + 1)$ branch currents indicated. The remaining branch current is then determined since the sum of all $(n + 1)$ currents must be zero. The impedance equations are of the form given by eq. (18). Since $Z_{jk} = Z_{kj}$ the number of independent impedances in eq. (18) is $n(n + 1)/2$.

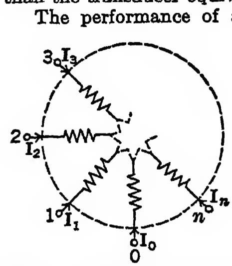


FIG. 4. General Network Having n Points of Entry

If a new network is set up, having the same number of points of entry as the given network, in which each point of entry is connected with every other point of entry through an impedance, the total number of impedances in the new network is the number of combinations of $(n + 1)$ things taken 2 at a time, which is $n(n + 1)/2$.

It is possible to solve for the $n(n + 1)/2$ new impedances in terms of the $n(n + 1)/2$ impedances of the given network so as to preserve equality of branch currents in any set

of single-frequency impressed voltages. The method of doing this is illustrated in Art. 21 by means of a four-terminal network. This may readily be generalized to any number of terminals.

An equivalent network of this type is sufficiently general for any steady-state calculations and is valuable in the representation of complex networks on calculating boards.

21. GENERAL FOUR-TERMINAL NETWORK

Let the general four-terminal network be represented by Fig. 5*a*, and let the network which we propose to make equivalent to it be represented by Fig. 5*b*. Terminal 0 will be considered as a reference or "ground" terminal in both cases. In other words the impressed voltages will be thought of as being V_{10}, V_{20}, and V_{30}, all other voltages being differences of these. All quantities in (*a*) will be differentiated from corresponding quantities in (*b*) by a prime. It is required that the currents I'_1, I'_2, and I'_3 be equal to I_1, I_2, and I_3, respectively, when V'_{10}, V'_{20}, and V'_{30} are equal to V_{10}, V_{20}, and V_{30}, respectively.

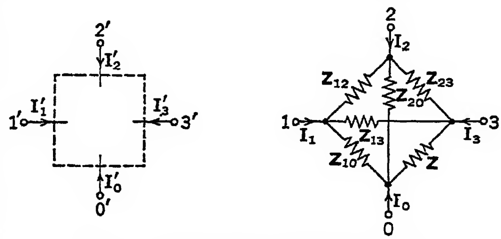


FIG. 5. General Four-terminal Networks

The equations for the currents in the given network are of the form

$$\left.\begin{aligned} A_{11} I'_1 + A_{12} I'_2 + A_{13} I'_3 &= V'_{10} \\ A_{12} I'_1 + A_{22} I'_2 + A_{23} I'_3 &= V'_{20} \\ A_{13} I'_1 + A_{23} I'_2 + A_{33} I'_3 &= V'_{30} \end{aligned}\right\} \tag{41}$$

The solution of these equations may be found by a simple transformation to be

$$\left.\begin{aligned} I'_1 &= B_{11} V'_{10} + B_{12} V'_{20} + B_{13} V'_{30} \\ I'_2 &= B_{12} V'_{10} + B_{22} V'_{20} + B_{23} V'_{30} \\ I'_3 &= B_{13} V'_{10} + B_{23} V'_{20} + B_{33} V'_{30} \end{aligned}\right\} \tag{42}$$

in which the B's are equivalent admittances calculable from the actual network impedances by ordinary algebra of complex numbers.

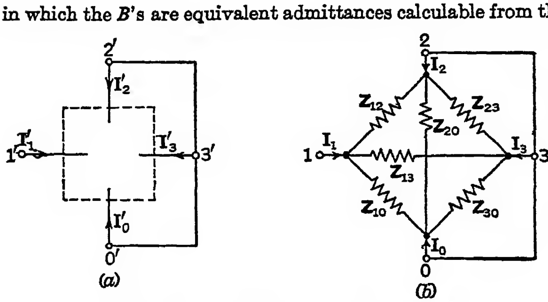


FIG. 6. General Four-terminal Networks

Short all terminals except the first to the reference terminal as shown in Fig. 6 (*b*). It follows that in the (*a*) network the currents are given by

$$\left.\begin{aligned} I'_1 &= B_{11} V'_{10} \\ I'_2 &= B_{12} V'_{20} \\ I'_3 &= B_{13} V'_{30} \end{aligned}\right\} \tag{43}$$

In the (*b*) network a current entering the common bus from any terminal is simply the current in that branch of the network which joins the given terminal with terminal 1. This is due to the fact that no current flows in any of the branches which are short-circuited by the common bus. The currents are

$$\left.\begin{aligned} I_2 &= -\frac{V_{10}}{Z_{12}} \\ I_3 &= -\frac{V_{10}}{Z_{13}} \\ I_0 &= -\frac{V_{10}}{Z_{10}} \end{aligned}\right\} \tag{44}$$

Comparing eqs. (43) and (44) the two simple relations

$$\left.\begin{aligned} Z_{12} &= -\frac{1}{B_{12}} \\ \text{and} \qquad Z_{13} &= -\frac{1}{B_{13}} \end{aligned}\right\} \tag{45}$$

are observed. By shorting the terminals in other groupings the following general equations may be verified:

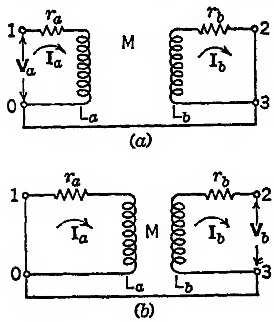


FIG. 7. Air-core Transformer

$$Z_{jk} = -\frac{1}{B_{jk}} \tag{46}$$

It may be shown that the equivalent circuit of a network having *any* number of points of entry follows the same scheme and that eq. (46) also holds.*

Example.—The general equivalent circuit of a two-winding transformer is easily found by the method outlined. Let the primary terminals be 0 and 1 and let the secondary terminals be 2 and 3. The mesh equations when all terminals except terminal 1 are strapped together are (refer to Fig. 7*a*)

$$\begin{aligned} (r_a + j\omega L_a)I_a - j\omega M I_b &= V_a \\ -j\omega M I_a + (r_b + j\omega L_b)I_b &= 0 \end{aligned} \tag{47}$$

in which I_a and I_b are the primary and secondary mesh currents. The following relations are apparent, comparing the transformer with Fig. 6:

$$\left.\begin{aligned} I_a &= I'_1 = -I'_0 \\ I_b &= I'_3 = I'_2 \\ V_a &= V'_{10} \end{aligned}\right\} \tag{48}$$

Hence we have

$$B_{12} = -\frac{j\omega M}{D} \tag{49}$$

$$B_{13} = \frac{j\omega M}{D} \tag{50}$$

in which

$$D = (r_a + j\omega L_a)(r_b + j\omega L_b) + \omega^2 M^2 \tag{51}$$

Similarly, when all terminals except terminal 2 are strapped together the mesh equations are (see Fig. 7*b*):

$$\begin{aligned} (r_a + j\omega L_a)I_a - j\omega M I_b &= 0 \\ -j\omega M I_a + (r_b + j\omega L_b)I_b &= V_b \end{aligned} \tag{52}$$

In this case the relations between I_a and I_b and the currents of Fig. 6 remain unchanged but V_b now corresponds to V_{02}. Hence we have

$$B_{12} = -\frac{j\omega M}{D} \tag{53}$$

$$B_{23} = -\frac{r_a + j\omega L_a}{D} \tag{54}$$

From the relation $B_{jk} = -\frac{1}{Z_{jk}}$ and from symmetry the Z's of the equivalent network are

$$Z_{10} = \frac{(r_a + j\omega L_a)(r_b + j\omega L_b) + \omega^2 M^2}{r_b + j\omega L_b}$$

$$Z_{23} = \frac{(r_a + j\omega L_a)(r_b + j\omega L_b) + \omega^2 M^2}{r_a + j\omega L_a}$$

$$Z_{12} = Z_{30} = \frac{(r_a + j\omega L_a)(r_b + j\omega L_b) + \omega^2 M^2}{j\omega M}$$

$$Z_{13} = Z_{20} = \frac{(r_a + j\omega L_a)(r_b + j\omega L_b) + \omega^2 M^2}{-j\omega M}$$

The circuit is shown in Fig. 8. This circuit cannot be realized in practice owing to the fact that negative resistances would be required. In order to eliminate this difficulty

* Starr, F. M., Equivalent circuits—I. *Trans. A.I.E.E*, Feb., 1932.

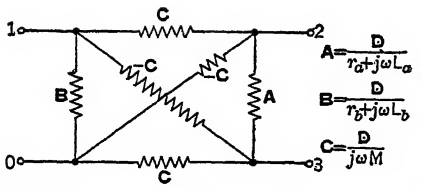


FIG. 8. Equivalent Circuit of a Two-winding Transformer

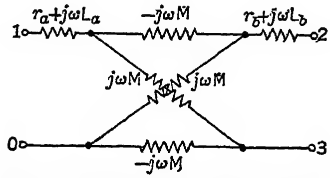


FIG. 9. Equivalent Circuit of a Two-winding Transformer (having positive resistance only)

and to obtain a circuit which could be used on a calculating board it is desirable to transform this network, if possible, into one having positive resistances only. This has been done by Starr (*loc cit.*); the equivalent circuit is shown in Fig. 9.

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NON-LINEAR CIRCUITS

A non-linear circuit is one in which one or more of the circuit elements, resistance, inductance, or capacitance, depends upon one or more of the currents flowing in the circuit. Variable electric circuits in which the resistances, inductances, or capacitances are functions of time or position in space, but not dependent upon the currents, are not non-linear but require special methods for their solutions. Some of these are given in the chapter on Electrical Machines.

The outstanding feature of a non-linear circuit is the production of component frequencies of current other than those corresponding to impressed voltages or natural periods.

In the present chapter the most important types of non-linearity arising in electric power circuits are described; the various methods for solving networks involving them are outlined.

22. FERROMAGNETISM

A ferromagnetic material is one having a permeability greater than that of a vacuum. The permeability of such materials is ordinarily a function of the magnetizing force and dependent upon the previous history of the sample under consideration.

When any completely demagnetized ferromagnetic substance is subjected to an increasing magnetizing force H, a curve such as the dotted curve in Fig. 1*a* results. Such a curve is variously termed a *rising characteristic*, a *virgin curve*, or a *neutral curve*; all these names are reserved for the special case of a magnetization curve of a *demagnetized* body.

If, from any positive value of H, the magnetizing force is decreased, the flux density

will also decrease, but not along the same curve as the original rising characteristic. For the decreasing values of H, B is greater than for the corresponding values on the rising characteristic. The magnetized body retains a part of its magnetization. It will be noticed that, when the magnetizing force is reduced to zero, there is still a positive value of B, which is dependent upon the maximum positive value of H on the rising characteristic, before H is decreased. The flux density which remains when H has been reduced to zero is called the *residual magnetism* and is in the nature of permanent or semi-permanent magnetism. If now the magnetizing force is reversed (e.g., by reversing the direction of the current through the magnetizing coil), a value of H will be found which will reduce B to zero. This value of H is called the *coercive force.* By proceeding through a cycle of values for the magnetizing force—from zero to a positive maximum, to zero, to a negative maximum, and back to zero—a loop of flux densities will be obtained. Such a loop is called a *hysteresis loop*; each substance is capable of having an infinity of different hysteresis loops depending on the conditions of magnetization. During part of the magnetizing cycle the electric circuit is transferring energy to the magnetic circuit,

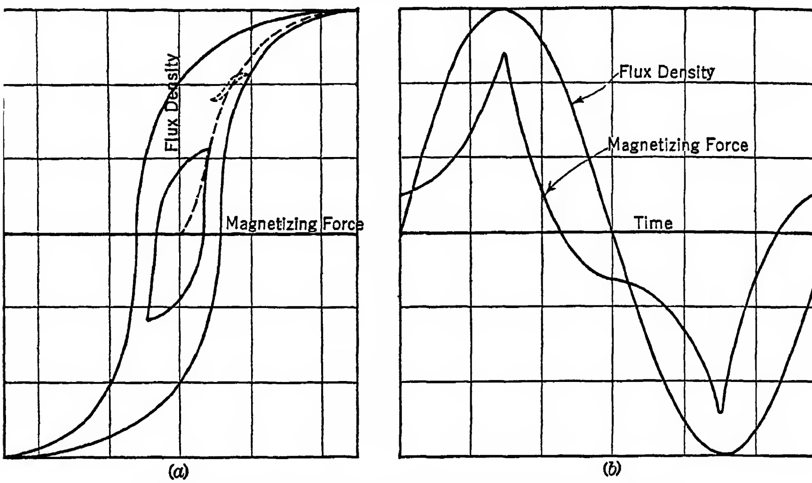

FIG. 1. Magnetization Curves and Wave Form of Magnetizing Force

and during the remainder of the cycle energy is retransferred from the magnetic to the electric circuit. When any ferromagnetic substance is present in the magnetic circuit more energy is transferred to the magnetic circuit than is retransferred to the electric circuit. The difference between these two amounts of energy is a loss and is dissipated by heat. Part of this loss is called the *hysteresis loss* and is presumably due to molecular friction accompanying the change in orientation of the elementary molecular magnets within the ferromagnetic substance. It may be shown that the hysteresis loss per cycle per cubic centimeter in any magnetic substance due to a complete cycle of changes of the flux density and the magnetizing force is equal to $\frac{1}{4\pi}$ times the area of the corresponding hysteresis loop, provided this loop is determined under such conditions that there is no mechanical motion and no change in the relative distribution of the lines of force. This relation is based on the further assumption that unit distance on one scale represents one gauss and that unit distance on the other scale represents one oersted. The following empirical formula is useful in determining hysteresis loss.

$$w = \eta B_m^{1.6} \text{ ergs per cubic centimeter per cycle} \qquad (1)$$

where η is a coefficient depending upon the substance and B_m is the maximum flux density reached during the cycle.

The power loss per unit weight of magnetic material is given by

$$P_h = kfB_m^{1.6} \text{ watts per pound} \qquad (2)$$

where f is the frequency in cycles per second and k is constant.

If the value of B varies sinusoidally with time, as is approximately true in many practical cases, the value of H at any instant of time can be found graphically from the

hysteresis loop as shown in Fig. 1(*b*). This is of value in calculating exciting currents of transformers and machines.

If, as sometimes happens, the variation of magnetizing force with time is such that there is more than one maximum and one minimum value per cycle, the above formulas for power loss and the above graphical construction are not valid. In such cases the hysteresis loop has interior loops as indicated by the dotted loop in Fig. 1(*a*). The shape of such interior loops and the power loss corresponding to them will depend upon the value of H at which the loop occurs and the width of the loop. No simple method has been devised to calculate circuits in which this occurs even when all the data are known since ordinarily the character of the B versus H curve cannot be found except by direct measurement under the specified conditions.

For detailed information, including tables and curves, refer to Sect. 2, Magnetic Materials.

23. EDDY CURRENTS

Since a varying magnetic field induces an emf in any closed circuit which it links, it will in general cause a flow of current in the magnetic materials composing the magnetic circuit. Such currents are called *eddy currents* or *Foucault currents* and cause rI^2 losses. Eddy currents can be distinguished from ordinary induced currents by the fact that they are due solely to the lines of force which pass *through* the space occupied by the conducting mass. Ordinary induced currents are produced by lines of force which *link* the conductor but do not pass through the space occupied by the conducting mass. It is customary wherever possible to laminate magnetic cores in order to increase the resistance of the path of induced eddy currents. The eddy-current loss in metal sheets is given by

$$P_e = \frac{\pi^2}{6 \times 10^{16}} \rho (afB_m)^2 \text{ watts per cubic centimeter} \tag{3}$$

where ρ is the resistivity of the material, a is the thickness of each sheet, f is the frequency of the eddy currents, and B_m is the maximum value of the flux density during a cycle. Eq. (3) holds provided (*a*) that the lines of force are parallel to the planes of the laminations, (*b*) that the laminations are thoroughly insulated from each other, and (*c*) that the thickness of each lamination is small in comparison with its other dimensions. The combination of hysteresis loss and eddy current is called the *total core loss*. A method of calculating circuits in which eddy currents are of importance is given in Art. 29.

24. SKIN EFFECT

A conductor of finite cross-section may be looked upon as made up of separate filaments each carrying its portion of the total current. When the same potential gradient is established through all the filaments of the conductor the exterior filaments are linked by fewer flux lines than the interior filaments. If the emf producing the potential gradient through the wire is an alternating one, the induced back emf in the interior filaments will be greater than in those nearer the surface. Since the potential drop is the same across all the filaments the resistance drops in the internal filaments are less than in the external filaments. This can be brought about only by the current distributing itself over the cross-section of the conductor in such a manner that the current density in the interior of the wire will be less than at the surface, i.e., the current is forced toward the surface filaments or " skin " of the wire; hence the term skin effect is applied to this phenomenon.

The self-induced emf depends not only upon the amount of flux set up but also upon the rapidity of its variations; hence the skin effect becomes more pronounced the greater the frequency of the impressed emf. It is also greater the larger the cross-section of the conductor, the greater the conductivity of the conductor, and the greater its magnetic permeability. It also depends slightly upon the temperature since the conductivity changes with temperature.

As a consequence of the skin effect the effective resistance of a conductor to alternating currents is greater than to direct currents, but the *internal* inductance *decreases* with the frequency; the external inductance is not altered. Whereas, however, the internal inductance with increasing frequency approaches a limiting value, the resistance increases indefinitely as the frequency approaches an infinite value. The change of resistance is always relatively much larger than the change in the total inductance.

The effects just described are, for the most part, negligible at low frequencies, except in heavy conductors and in coils wound with stout wire in several layers. In the latter case, however, the diminution of the inductance, due to the irregular distribution of the current, is masked, to a greater or less degree, by the effect of the capacitance between the

windings of the coil, which gives rise to an *increase* of the inductance with the frequency. For the same reason the resistance is increased more than it would be by the eddy currents alone.

Unfortunately, the rigorous or approximate solution of the problem at high frequencies for the various cases which arise in practice is in many instances very difficult, if not impossible.

The above has considered the redistribution of the current stream lines due to the current in the conductor itself. If there are other current carrying conductors in the vicinity a further redistribution of the current stream lines will occur, producing an additional change in resistance and inductance. This is called the *proximity effect.*

For a mathematical analysis of skin effect refer to A. Russel, *Phil. Mag.* 17, p. 524, 1909; J. R. Carson, *Phil. Mag.* 41, p. 607, April, 1921; Sallie Pero Mead, *B.S.T.J* 4, No. 2, April, 1925.

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SKIN EFFECT IN STRAIGHT ROUND WIRES. An accurate solution for the case of straight *solid* * wires of circular cross-section has been given in a number of different forms by various scientists. A summary of the formulas is given in a paper by Rosa and Grover, Bull. Bur. Standards, Vol. 8, p. 172, 1912. The calculations are most conveniently made by the use of the tables given in Rosa and Grover's paper, which are given in a condensed form in Table 1. Let

f = frequency in cycles per second,
μ = permeability of the wire, *assumed constant,*
R = direct-current resistance, in ohms, of 1000 ft of the wire; see tables in the article on Wires and Cables, Bare, Sect. 14, §47.
L = direct-current inductance, in millihenrys per 1000 ft. of a *non-magnetic* wire of the same *cross-section* as that of the given wire, and at the given spacing between wires, see tables in the article on Inductance and Inductive Reactance, Sect. 14, §22.

Calculate the quantity
$$x = 0.02768\sqrt{\frac{\mu f}{R}} \tag{4}$$

and take from Table 1 the corresponding values of K_1 and K_2. Then the alternating-current resistance at the frequency f is

$$R' = K_1 R \qquad \text{ohms per 1000 ft} \tag{5}$$

and the alternating-current inductance of the given wire at the frequency f is

$$L' = L + 0.01524\,(\mu K_2 - 1) \qquad \text{millihenrys per 1000 ft} \tag{6}$$

For x greater than 7 the following relations hold to within less than 1 per cent, the error being less the greater the value of x:

$$R' = \left(\frac{x}{2.828} + 0.25\right)R \tag{7}$$

$$L' = L + 0.01524\left(\frac{2.828\,\mu}{x} - 1\right) \tag{8}$$

SKIN EFFECT IN THIN STRIPS AND TUBES. The following formulas are exact for a flat strip of infinite width and at an infinite distance from any other conductor carrying a current; they also apply with a close degree of approximation to a tube which has a circumference 10 or more times its thickness, provided no other conductor carrying a current is closer than a distance of 10 times the thickness of the strip or tube.

Let t = thickness of strip, or *twice* the thickness of the wall of a tube, in centimeters,
w = width of strip or *half* the mean circumference of tube, in centimeters,
l = length of strip or tube in centimeters,

* Tests at the Massachusetts Institute of Technology (*Proc. A.I.E.E.*, Sept., 1915), indicate that the formulas for solid wires also apply to round *stranded* wires of the *same cross-section of metal,* not the same over-all diameter, for frequencies up to 1200, but above this frequency there is an increase in the skin effect, due to the spiraling of the component wires of the cable.

Table 1. Skin Effect Factors for Solid Round Wires

x	K_1	K_2	x	K_1	K_2	x	K_1	K_2	x	K_1	K_2
0.0	1.000	1.000	2.9	1.286	0.860	6.6	2.603	0.424	17.0	6.268	0.166
0.1	1.000	1.000	3.0	1.318	0.845	6.8	2.673	0.412	18.0	6.621	0.157
0.2	1.000	1.000	3.1	1.351	0.830	7.0	2.743	0.400	19.0	6.974	0.149
0.3	1.000	1.000	3.2	1.385	0.814	7.2	2.813	0.389	20.0	7.328	0.141
0.4	1.000	1.000	3.3	1.420	0.798	7.4	2.884	0.379	21.0	7.681	0.135
0.5	1.000	1.000	3.4	1.456	0.782	7.6	2.954	0.369	22.0	8.034	0.128
0.6	1.001	1.000	3.5	1.492	0.766	7.8	3.024	0.360	23.0	8.387	0.123
0.7	1.001	0.999	3.6	1.529	0.749	8.0	3.094	0.351	24.0	8.741	0.118
0.8	1.002	0.999	3.7	1.566	0.733	8.2	3.165	0.343	25.0	9.094	0.113
0.9	1.003	0.998	3.8	1.603	0.717	8.4	3.235	0.335	26.0	9.447	0.109
1.0	1.005	0.997	3.9	1.641	0.702	8.6	3.306	0.327	28.0	10.154	0.101
1.1	1.008	0.996	4.0	1.678	0.686	8.8	3.376	0.320	30.0	10.861	0.094
1.2	1.011	0.995	4.1	1.715	0.671	9.0	3.446	0.313	32.0	11.568	0.088
1.3	1.015	0.993	4.2	1.752	0.657	9.2	3.517	0.306	34.0	12.275	0.083
1.4	1.020	0.990	4.3	1.789	0.643	9.4	3.587	0.299	36.0	12.982	0.079
1.5	1.026	0.987	4.4	1.826	0.629	9.6	3.658	0.293	38.0	13.689	0.074
1.6	1.033	0.983	4.5	1.863	0.616	9.8	3.728	0.287	40.0	14.395	0.071
1.7	1.042	0.979	4.6	1.899	0.603	10.0	3.799	0.282	42.0	15.102	0.067
1.8	1.052	0.974	4.7	1.935	0.590	10.5	3.975	0.268	44.0	15.809	0.064
1.9	1.064	0.968	4.8	1.971	0.579	11.0	4.151	0.256	46.0	16.516	0.061
2.0	1.078	0.961	4.9	2.007	0.567	11.5	4.327	0.245	48.0	17.223	0.059
2.1	1.094	0.953	5.0	2.043	0.556	12.0	4.504	0.235	50.0	17.930	0.057
2.2	1.111	0.945	5.2	2.114	0.535	12.5	4.680	0.226	60.0	21.465	0.047
2.3	1.133	0.935	5.4	2.184	0.516	13.0	4.856	0.217	70.0	25.001	0.040
2.4	1.152	0.925	5.6	2.254	0.498	13.5	5.033	0.209	80.0	28.536	0.035
2.5	1.175	0.913	5.8	2.324	0.481	14.0	5.209	0.202	90.0	32.071	0.031
2.6	1.201	0.901	6.0	2.394	0.465	14.5	5.386	0.195	100.0	35.607	0.028
2.7	1.228	0.888	6.2	2.463	0.451	15.0	5.562	0.188			
2.8	1.256	0.875	6.4	2.533	0.437	16.0	5.915	0.176			

ρ = specific resistance of conductor, in microhms per centimeter cube, at the given temperature,
μ = magnetic permeability of conductor in absolute units,
f = frequency in cycles per second,
$x = 0.1987\, t\sqrt{\frac{\mu f}{\rho}}$.

Then d-c resistance is $$R = \frac{10^{-6}\,\rho l}{wt} \text{ ohms}$$

The d-c internal inductance is

$$L_i = 1.047 \times 10^{-6}\, l\frac{\mu t}{w} \text{ millihenrys}$$

The ratio of the a-c to the d-c resistance is

$$\frac{R'}{R} = \frac{x}{2}\left(\frac{\sinh x + \sin x}{\cosh x - \cos x}\right)$$

and the ratio of the a-c to the d-c internal inductance is

$$\frac{L'_i}{L_i} = \frac{3}{x}\left(\frac{\sinh x - \sin x}{\cosh x - \cos x}\right)$$

For x less than unity these ratios are unity to within 0.6 and 0.2 per cent, respectively, i.e., the a-c resistance is practically equal to the d-c resistance and the a-c internal inductance is practically equal to the d-c internal inductance. For x greater than 6 the following formulas are accurate to within 0.5 per cent:

$$\frac{R'}{R} = \frac{x}{2} \quad \text{and} \quad \frac{L'_i}{L_i} = \frac{3}{x}$$

To a very rough degree of approximation the skin effect in a conductor of any shaped cross-section may be approximated by using the above formulas for a strip, taking for the effective width w one-half the perimeter of the section (in centimeters) and for its effective thickness twice the area of the section (in square centimeters) divided by its perimeter. In general, then, for the same area the skin effect will be less the greater the perimeter of the section. If the section approaches more nearly that of a solid circle than that of an elongated rectangle, the formulas for a solid round wire will give more accurate results.

SKIN EFFECT IN BUS-BARS (STRIPS OF FINITE WIDTH). The formulas given above are for strips of infinite width; the skin effect in narrow strips, such as used for bus-bars, is much greater than the theoretical value for infinite strips, owing to the crowding of the current to the edge of the strip. This latter effect has been called the "edge effect." Experimental results obtained by Kennelly, Laws and Pierce (*Proc. A.I.E.E.*, September, 1915), for strip conductors, outgoing and return conductor 60 cm apart, are given in Fig. 2. For other spacings the original paper should be consulted.

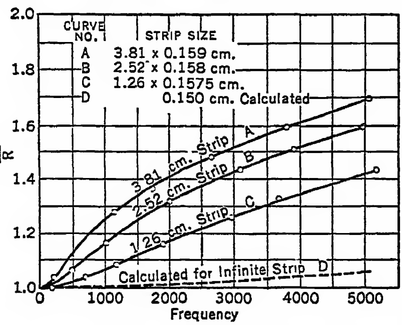

FIG. 2. Skin Effect in Finite Strips. Return conductor 60 cm away

In the *Journal of the Franklin Institute* for March, 1918, are reported tests on copper bus-bars 4 in. by 1/4 in., showing a ratio of a-c to d-c resistance of 1.3 at 60 cycles and 1.1 at 24 cycles. It is stated that this increase in resistance is due chiefly to the crowding of the current toward the edge of the strip, and not to the sides. Three such bars in parallel with an air space of 0.25 in. between them gave a ratio of a-c to d-c resistance of 2.2 at 60 cycles.

SKIN EFFECT IN TRANSMISSION LINES. For a comprehensive treatment of effective resistance and inductance of bimetallic (copper-clad steel) and also of iron wires, see Miller, J. M., Bull. Bur. Standards, 12, p. 207, 1915. For tables of skin-effect ratios for concentric-lay and annular conductors see Sect. 14, Art. 50.

25. ELECTRIC ARCS

An electric arc is an incandescent vapor bridge consisting of material electrically impelled from a negative to a positive electrode. A spark is also an incandescent vapor bridge, but differs from an arc in not depending upon the electrodes for its material medium. The establishment and maintenance of an arc require the expenditure of energy for the latent heat of evaporation of the electrode and for the motion of the vapor stream. As no energy can be expended for these purposes until a current flows, an arc cannot start spontaneously. The following expedients are, therefore, adopted to start arcs:

1. Bringing the electrodes into contact with each other and separating them after the current has commenced to flow.
2. Stressing the dielectric between the conductors by overvoltage until it breaks down electrically and becomes conducting.
3. Using a subsidiary arc to furnish the initial vapor bridge.

The first of these methods is used in carbon-arc lamps and the last in mercury-arc rectifiers.

A peculiarity of the carbon arc is that, with any particular length of arc, if the current be increased the difference of potential across the carbons will decrease. This occurs continuously up to a certain point when in the open arc the voltage drops quite suddenly. If the current is still increased the voltage will again become steady at a much lower value. Between the values before and after the drop the arc is unstable and emits a hissing sound. In inclosed arcs the hissing point is absent and the curve is continuous.

The relation between the drop of potential across the arc and the current flowing through it depends upon whether the arc is in the steady state corresponding to the current flowing or whether the current has been changed without giving time to the vapor column and electrodes to accommodate themselves to the new strength of current. Let v be the drop of potential across an arc, and i be the current flowing. If the current be increased at a rate rapid enough that a steady state is not set up, then $\frac{dv}{di}$ is the resistance of the arc to the rapidly changing current. Throughout ordinary ranges of frequency (less than 100 kc) the ratio $\frac{dv}{di}$ is negative. Thus the alternating-current resistance of an arc is negative and is a function of the current.

26. CORONA

When the voltage between two conductors becomes sufficiently great, the electric field, which is greatest at the surface, ionizes the air at the surfaces of the conductor. The increase in the conductivity of the air or other gas surrounding the conductor is equivalent to an increase in the effective diameter of the conductor to the value at which the decreasing electric field is balanced by the dielectric strength of the air. The phenomenon is known as *corona*. When the corona is formed, there is visible a faint violet light near the conductor. If the voltage is raised, a *brush discharge* occurs in which bluish streaks like the bristles of a brush are visible near the surface of the conductor. If the voltage is still further increased, a disruptive spark discharge takes place between conductors.

In transmission lines (q.v.), corona is accompanied by power losses frequently of serious proportions. It can be controlled by the use of larger-diameter conductors and by operating with voltages low enough to prevent its formation.

The current leaving the line as corona is a function of the instantaneous voltage on the line and therefore introduces non-linearity into the network.

27. VARIABLE INDUCTANCE

The electromotive force produced by electromagnetic induction in a closed circuit having no sliding contacts is

$$e_1 = \frac{d\lambda_1}{dt} \tag{9}$$

in which λ_1 is the total flux linkages in the circuit. The flux linkages λ_1 will in general be produced by the current in the given circuit, by the currents in neighboring circuits, and by permanent magnets in the vicinity of the given circuit. The flux linkages may change, owing to changes in the currents or to motion. If there is motion the space coordinates giving the relative positions of the circuits and magnets in the group are functions of time and can be eliminated in the expression for λ. Hence λ_1 may be written

$$\lambda_1 = \lambda_1(i_1, i_2, \ldots, i_n, t) \tag{10}$$

where i_1 is the current in the circuit under consideration, $i_2, \ldots, i_n$ are the currents in neighboring circuits, and t is the time.

It follows that

$$e = \frac{\partial \lambda_1}{\partial i_1}\frac{di_1}{dt} + \frac{\partial \lambda_1}{\partial i_2}\frac{di_2}{dt} + \ldots + \frac{\partial \lambda_1}{\partial t} \tag{11}$$

The coefficient of $\frac{di_1}{dt}$ in this expression for the emf is called the *coefficient* of *self-induction* or simply the *inductance* of the circuit.

The coefficient of $\frac{di_n}{dt}$ in this expression for the emf is called the *coefficient* of *mutual induction* or simply the *mutual inductance* of the nth mesh with respect to the first.

There result the following definitions:

The *self-inductance* of a given circuit is the factor L by which the rate of increase of current in that circuit must be multiplied to give the induced electromotive force in that circuit when at rest, the currents in all neighboring circuits being zero.

The *mutual inductance* of a given circuit with respect to a second circuit is the factor M by which the rate of increase of current in the first circuit must be multiplied to give the induced electromotive force in the second circuit when these circuits are at rest, the currents in the second and all neighboring circuits being zero.

In terms of these definitions the general expression for the induced emf is

$$e_1 = L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt} + \ldots + M_{1n}\frac{di_n}{dt} + \frac{\partial \lambda_1}{\partial t} \tag{12}$$

There are several special cases which arise in practical problems, each of which will be considered separately.

(*a*) *If λ_1 is a function of i_1 only* (as in an isolated iron-cored choke coil at rest),

$$\frac{\partial \lambda_1}{\partial t} = \frac{di_2}{dt} = \frac{di_3}{dt} = \ldots = \frac{di_n}{dt} = 0$$

from which it follows that

$$e_1 = L_1\frac{di_1}{dt} \tag{13}$$

If the permeance of the core is constant L is a constant; in general, however, L will be a function of the current i_1.

(*b*) *If* λ_1 *is a function of* i_n *only* (as in an open winding of a multi-winding transformer in which another winding carries current),

$$\frac{\partial \lambda_1}{\partial t} = \frac{di_1}{dt} = \frac{di_2}{dt} = \ldots = \frac{di_{n-1}}{dt} = 0$$

from which it follows that
$$e_1 = M_{1n} \frac{di_n}{dt} \tag{14}$$

If the permeance of the core is constant M_{1n} is a constant; in general, however, M will be a function of the current i_n.

(*c*) *If* λ_1 *is composed of a sum of terms each of which is directly proportional to one of the currents*, and is also a function of t (as when several circuits move with respect to each other but have constant permeances, for example, an air-core alternator)

$$\lambda_1 = i_1 A_1 (t) + i_2 A_{12} (t) + \ldots + i_n A_{1n} (t) \tag{15}$$

The electromotive force is, from eq. (11),

$$e_1 = A_1 (t) \frac{di_1}{dt} + A_{12} (t) \frac{di_2}{dt} + \ldots + A_{1n} (t) \frac{di_n}{dt} \tag{16}$$
$$+ \frac{\partial}{\partial t} [i_1 A_1 (t) + i_2 A_{12} (t) + \ldots + i_n A_{1n} (t)]$$

and comparing coefficients with eq. (12) it is seen that

$$\begin{aligned} A_1 (t) &= L_1 \\ A_{12} (t) &= M_{12} \\ A_{1n} (t) &= M_{1n} \end{aligned} \tag{17}$$

and

hence

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + \ldots + M_{1n} \frac{di_n}{dt} + i_1 \frac{dL_1}{dt} + i_2 \frac{dM_{12}}{dt} + \ldots + i_n \frac{dM_{1n}}{dt} \tag{18}$$

Eq. (18) may be written

$$e_1 = \frac{d}{dt} (L_1 i_1 + M_{12} i_2 + M_{13} i_3 + \ldots + M_{1n} i_n) \tag{19}$$

Eq. (19) holds when the self and mutual inductances are functions of t only, but does not hold when they are functions of the current. In other words, eq. (19) may be used for generator and motor calculations if the magnetization curve can be assumed to be linear; whereas, for calculations when the magnetization curve cannot be assumed to be straight (even in the case of a transformer), eq. (12) must be used.

28. SPECIAL MATHEMATICAL METHODS

There is no general method of solution applicable to differential equations which are not linear. Therefore the electric circuit equations arising from non-linear circuits are not capable of solution by any standard scheme. Various special methods, satisfactory for limited classes of problems, have been devised, some of which are outlined in this article. The methods may be classified as follows:

REDUCTION OF THE NETWORK EQUATIONS TO APPROXIMATE LINEAR EQUATIONS. In many circuits the non-linearity is in the nature of a second order effect. In such circuits it is desirable to solve for those currents which would flow if the network were linear, and to superimpose upon them correction currents to take account of the non-linearity. An excellent example of this is the iron-core transformer which is given in some detail in Art. 29. The method of solution outlined in that article may be applied to any circuit falling in this classification.

SUBSTITUTION OF AN EMF FOR THE NON-LINEAR ELEMENT. If the non-linearity consists in the sudden opening or closing of a circuit the currents may be found by making use of the two principles:

(*a*) The effect at any succeeding instant of time of a short circuit at time $t = 0$ is precisely the same as the insertion at $t = 0$ of a voltage $-v(t)$, equal and opposite to the voltage $v(t)$ which exists at that instant across the terminals to be short-circuited.

The resultant currents are therefore composed of two components: the currents which would exist in the invariable network in the absence of the short circuit and the currents due to the emf $v(t)$ inserted at $t = 0$. Both these components are calculable by usual methods.

(*b*) The effects at any succeeding instant of time of opening a branch of a network at

time $t = 0$ is the same as the insertion at $t = 0$ of a voltage $v(t)$ which produces in the branch to be opened a current $-i(t)$ equal and opposite to the current $i(t)$ which would exist in the branch in the absence of the open circuit.

The resultant currents are therefore composed of two components: the currents which would flow in the network in the absence of the open circuit and the currents which flow due to the emf $v(t)$ inserted at $t = 0$. Both these components are calculable by usual methods.

In a network having a non-linear circuit element, the mesh equation for the mesh involving this element is

$$Z(p)i + \phi(i) = e(t) \tag{20}$$

in which $Z(p)$ is the operational impedance of the linear part of the mesh, $\phi(i)$ is the voltage drop across the non-linear circuit element, and $e(t)$ is the impressed emf in the mesh.

If this equation is rewritten as

$$Z(p)i = e(t) - \phi[i(t)] \tag{20a}$$

it is possible to solve formally for the current by treating the right-hand member as the impressed voltage. Hence

$$i(t) = \frac{d}{dt}\int_0^t h(t-\lambda)\{e(\lambda) - \phi[i(\lambda)]\}d\lambda \tag{21}$$

which may be written

$$i(t) = i_0(t) - \frac{d}{dt}\int_0^t h(t-\lambda)\phi[i(\lambda)]d\lambda \tag{21a}$$

in which $i_0(t) =$ the current which would flow in the invariable part of the circuit due to $e(t)$; $h(t) =$ the indicial admittance of the invariable part.

This is an integral equation which can be solved in general only in the form of an infinite series. For methods of solution see Lovitt, Linear Integral Equations, McGraw-Hill Book Co.

SUCCESSIVE APPROXIMATIONS, NUMERICAL INTEGRATION. When the non-linear differential equations for the currents in a non-linear network are written down their exact solution becomes a purely mathematical problem. The process of obtaining an infinite series solution by successive approximations is given in any standard text on differential equations. For the numerical integration of differential equations refer to Runge and König, Numerisches Rechnen, Julius Springer, 1924. These methods are usually very laborious and should be avoided if possible.

A method of solution of non-linear differential equations of the particular type found in vacuum-tube problems was developed by Carson. (See Bibliography and Vol. 5, Electric Circuits, Lines, and Fields.) This method is based on a Taylor's series expansion and is of great practical value to the communication engineer. It is not easily extended to other problems.

Another special scheme which has been applied to non-linear communication problems is the *isoclyne* method. This method involves the plotting of families of curves using the derivative as a parameter; it has been used by Van der Pol in the analysis of relaxation oscillators.

THE DIFFERENTIAL ANALYZER. Perhaps the most powerful tool available for the numerical solution of non-linear circuit problems is a machine called the differential analyzer. This device is capable of solving mechanically differential equations in which the coefficients are functions of the dependent variables. The solution may be obtained in the form of a plotted curve or in tabular form, the latter giving the greater accuracy. The precision is adequate for all engineering purposes. Two such machines are at present available in this country, one at the Massachusetts Institute of Technology and one at the Moore School of Electrical Engineering, University of Pennsylvania. For a complete description of its principle of operation see V. Bush, The differential analyzer, *Journal Franklin Institute,* Vol. 212, No. 4, October, 1931.

29. IRON-CORED TRANSFORMERS

The mesh equations of a two-winding transformer upon the primary of which is impressed a voltage v_1, and which supplies a voltage v_2 to a receiver connected to its secondary, are:

$$r_1 i_1 + L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} = v_1 \tag{22}$$

$$r_2 i_2 + L_2 \frac{di_2}{dt} - M\frac{di_1}{dt} = -v_2 \tag{22a}$$

in which i_1 and i_2 are the primary and secondary mesh currents. Owing to the variable permeability of the iron core the values of the inductances depend upon the currents. The equations are therefore non-linear and cannot be solved by ordinary means. In the design of power transformers for usual purposes, an attempt is made to keep the leakage as small as possible, whereas the mutual inductance is made relatively large. This fact makes possible an approximate method of solution which yields results sufficiently accurate for most practical purposes.

The self-inductances of the windings may be expressed in terms of the leakage and mutual inductances:

$$L_1 = L_1' + a\,M \tag{23}$$

$$L_2 = L_2' + \frac{M}{a} \tag{23a}$$

in which L_1' and L_2' are the primary and secondary leakage inductances and a is the turn ratio. L_1' and L_2' are constants and are quite small in comparison to M. Substitution of eqs. (23) and (23a) in eqs. (22) and (22a) gives the following mesh equations.

$$aM\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right) = v_1 - r_1 i_1 - L_1'\frac{di_1}{dt} \tag{24}$$

$$M\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right) = v_2 + r_2 i_2 + L_2'\frac{di_2}{dt} \tag{24a}$$

It is apparent that if r_1 and L_1' are small the right-hand side of eq. (24) does not differ greatly in wave form from v_1. Hence the quantity $M\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right)$ does not differ greatly in wave form from v_1. It should be noted that, since M is variable, the quantity $\left(i_1 - \frac{i_2}{a}\right)$ may be considerably different in wave form from v_1. The secondary current i_2 will be essentially the same in wave form as v_1 since the term $M\frac{d}{dt}\left(i - \frac{i_2}{a}\right)$ may be thought of as an electromotive force impressed upon a linear mesh in which the impedance drop is

$$v_2 + r_2 i_2 + L_2'\frac{di_2}{dt}$$

The primary current can thus be treated as the sum of two components $\left(\frac{i_2}{a}\right)$ and $\left(i_1 - \frac{i_2}{a}\right)$, the first component having essentially the same wave shape as the impressed voltage, and therefore representable as a current flowing in an equivalent linear network, and the second component being distorted in wave shape and representing the non-linear effect of the transformer. The first component $\left(\frac{i_2}{a}\right)$ is often called the load component of the primary current; the second component $\left(i_1 - \frac{i_2}{a}\right)$ is called the magnetizing or exciting current of the transformer.

The scheme used in eqs. (24) and (24a) of splitting mesh equations into two parts, one containing all the non-linearity and the other being entirely linear, is extremely useful in problems involving iron cores, especially electrical machines.

If the core of a transformer is conducting, as it always is in practical cases, there are an infinite number of conducting paths, each having mutual inductances with respect to the primary and secondary. The currents flowing in these paths are called eddy currents.

Let M_{1i} be the mutual inductance between the ith eddy-current path and the primary winding, and let M_{2i} be the mutual inductance between the ith eddy-current path and the secondary winding. If, as in eqs. (22) and (22a) we take M_{12} with a negative sign, then M_{1i} and M_{2i} must have opposite signs.

Adopt the convention that all the M_{1i}'s have negative signs, then all the M_{2i}'s have positive signs. The mesh equations for the primary and secondary currents (neglecting non-linearity of the core) are

$$(r_1 + j\omega L_1)I_1 - j\omega M_{12} I_2 - j\omega \sum_{i=3}^{\infty} M_{1i} I_i = V_1 \tag{25}$$

$$(r_2 + j\omega L_2)I_2 - j\omega M_{12} I_1 + j\omega \sum_{i=3}^{\infty} M_{2i} I_i = -V_2 \tag{26}$$

Corresponding to the eddy-current paths there will exist an infinite number of equa-

tions. If in these the terms involving I_1 and I_2 are transposed to the right-hand side it is possible, at least formally, to solve for each of the currents I_i in terms of I_1 and I_2. If these values are then substituted back in eqs. (25) and (26), two equations will result which involve only parameters of the network and the currents I_1 and I_2. These are the equations of an equivalent T network, in which the quantity multiplying I_2 in the first equation will be the same as the quantity multiplying I_1 in the second equation. This quantity will be a complex number, both real and imaginary parts of which are functions of the frequency ω.

Making the assumptions (*a*) that the resistance of a given eddy-current path is large in comparison with its inductance, (*b*) that the mutual inductance between an eddy-current path and either the primary or secondary is large in comparison with the mutual inductance between two eddy-current paths, and (*c*) that the leakage is small, the mesh equations for the primary and secondary currents become

$$(r_1 + j\omega L_1')\, I_1 + a(r_e + j\omega M_{12})\left(I_1 - \frac{I_2}{a}\right) = V_1 \tag{27}$$

$$(r_2 + j\omega L_2')\, I_2 - (r_e + j\omega M_{12})\left(I_1 - \frac{I_2}{a}\right) = -V_2 \tag{28}$$

in which L_1' and L_2' are the primary and secondary leakage inductances, a is the turn ratio, and r_e is a resistance representing the effect of the eddy currents. The resistance r_e is proportional to the square of the frequency of the impressed voltage.

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POLYPHASE SYSTEMS AND SYMMETRICAL COMPONENTS

30. POLYPHASE CURRENTS AND VOLTAGES

The term polyphase alternating voltages is applied to the voltages of a group in which the modulus values are approximately the same and in which the difference in phase between successive voltages in the group is approximately equal to 2π divided by the number of voltages in the group. If these conditions are exactly instead of only approximately true, the voltages are said to be *balanced* polyphase voltages. If these conditions are not even approximately true the voltages are still termed polyphase voltages (but badly unbalanced) if they arise in a system in which the normal or ideal voltages are polyphase.

Polyphase currents are defined in the same way as polyphase voltages.

A polyphase circuit is a circuit in which the normal or ideal currents are polyphase currents. Thus a circuit having three meshes may be a three-phase circuit or not, depending upon its use.

31. MESH AND STAR CONNECTIONS

Consider n separate coils or windings, which may be mounted on a common armature or be entirely distinct, as for example n groups of lamps. When these n windings are connected end to end so that they are all in series, forming a closed chain, as in Fig. 1, and terminals are brought out from the n junctions, they are said to be connected in *mesh*. When one terminal of each of these windings is connected to a common junction point,

as in Fig. 2, and terminals are brought out from the free ends, the windings are said to be connected in *star*, and the common point is called the *neutral point*.

When such a group of n windings, as shown in Fig. 1 or 2, is connected to a generator or other source of emf having n separate windings and therefore developing n different emf's which differ in phase from one another, the system is called an n-phase system, each winding being called a *phase*. For example, when there are three separate windings on

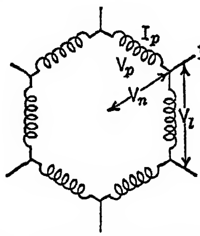

FIG. 1. Mesh Connected Network

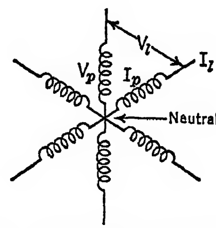

Fig. 2. Star Connected Network

the generator, three line wires and three windings constituting the load, the system is a three-phase system.

A system is said to be balanced if the phase impedances are equal and the impressed voltages are balanced.

32. PHASE AND LINE CURRENTS AND VOLTAGES

The current I_p in any winding or phase of an n-phase system, see Figs. 1 and 2, is called the *phase current*, and the drop (or rise) of potential V_p through this winding is called the *phase voltage*. The current I_l in the line leaving any terminal of an n-phase system is called the *line current*, and the voltage V_l between adjacent line wires or terminals is called the *line voltage*, except in the special case of a two-phase connection (see below) when the voltage between diametrically opposite terminals is called the line voltage. In the case of a star connection the voltage between any terminal and the neutral point is called the *voltage to neutral*; in the case of a balanced mesh connection, by voltage to neutral is meant the voltage which would exist between any terminal and the neutral of a star connection connected to the terminals of the actual device, the impedance of all the legs of the star connection forming this *artificial neutral* being equal and sufficiently large not to take an appreciable current.

The relations between these various currents and voltages for a balanced n-phase system are as follows:

	Mesh	Star
Number of phases	n	n
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \dfrac{1}{2 \sin \frac{\pi}{n}} I_l$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \dfrac{1}{2 \sin \frac{\pi}{n}} V_l$
Voltage to neutral	$V_n = \dfrac{1}{2 \sin \frac{\pi}{n}} V_p$	$V_n = V_p$
Total volt-amperes	$nV_p I_p = \dfrac{n}{2 \sin \frac{\pi}{n}} V_l I_l$	$nV_p I_p = \dfrac{n}{2 \sin \frac{\pi}{n}} V_l I_l$

33. POWER IN POLYPHASE SYSTEMS

The total power in any polyphase system is the sum of the powers in the separate phases of the system. Thus for an unbalanced n-phase system the total average power is

$$P = \sum_{k=1}^{n} V_k I_k \cos\theta_k \tag{1}$$

in which θ_k is the phase power factor of the kth phase and V_k and I_k are the phase voltage and current respectively of that phase. This is true for either a star or a mesh connection.

In the special case of a balanced n-phase system the power may be expressed as

$$P = nV_p I_p \cos\theta_p = \frac{nV_l I_l \cos\theta_p}{2\sin\frac{\pi}{n}} \tag{2}$$

Note that even when the power is expressed in terms of line voltages and currents the power factor is the phase power factor.

34. TWO-PHASE (OR QUARTER-PHASE) SYSTEM

Strictly, the so-called single-phase system is a star-connected two-phase system, since the currents from the two terminals are in opposite directions at any instant, the current leaving by one and entering by the other. However, in practice the name two-phase system is used for a system supplied from a generator or other source of emf having two windings in which are developed two emf's differing in phase by 90 deg; i.e., a two-phase system is in reality two distinct single-phase systems each with two terminals. Two of the four terminals may be connected to each other, in which case but three line wires are required. Or the two single-phase systems may be connected at their middle points; in this case the two-phase system may be considered as a four-phase, or, as it is usually called, a "quarter-phase" system.

35. THREE-PHASE SYSTEM, DELTA AND WYE CONNECTIONS

For a three-phase system the generators and motors are designed with three windings or phases which may be connected either in mesh, usually called a *delta connection* since the diagram of the three windings forms a Greek delta, or in star, usually called a *wye connection* since the diagram of the three windings forms a Y. The relations between line and phase currents and voltages for a balanced three-phase system are as follows:

	Delta	Y
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \frac{I_l}{\sqrt{3}}$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \frac{V_l}{\sqrt{3}}$
Voltage to neutral	$V_n = \frac{V_l}{\sqrt{3}}$	$V_n = V_p$
Total volt-amperes	$3V_p I_p = \sqrt{3}V_l I_l$	$3V_p I_p = \sqrt{3}V_l I_l$
Phase power factor	$\cos\theta_p$	$\cos\theta_p$
Total power	$\sqrt{3}V_l I_l \cos\theta_p$	$\sqrt{3}V_l I_l \cos\theta_p$

As was pointed out in Art. 20 any network having n points of entry can be completely represented by a network having n points of entry in which each point is connected by an impedance to every other point. If $n = 3$ the equivalent network is a delta. Hence any wye, balanced or unbalanced, may be transformed into an equivalent delta. The inverse transformation also exists. The transformation formulas are given by

$$Z_1 = \frac{Z_{31}Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{12}Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_3 = \frac{Z_{23} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$Z_{23} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_{31} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

In which Z_1, Z_2, and Z_3 are the wye impedances from lines 1, 2, and 3 to neutral and Z_{12}, Z_{23}, Z_{31} are the delta impedances between lines 1 and 2, 2 and 3, and 3 and 1.

36. BALANCED THREE-PHASE CIRCUITS

Any problem in regard to a *balanced* three-phase circuit may be solved by reducing all parts of the circuit to an equivalent Y connection, provided the currents and emf's are sine waves. The transformations are made as follows:

Any Δ-connected motor or generator is considered as equivalent to a Y-connected generator or motor in which

$$E_y = \frac{E_\Delta}{\sqrt{3}}, \quad r_y = \frac{r_\Delta}{3}, \quad x_y = \frac{x_\Delta}{3}$$

where the quantities E_y, r_y, and x_y are the emf, resistance, and reactance per phase of the Y-connected machine equivalent to the emf, resistance, and reactance per phase of the actual Δ-connected machine.

Each of the line wires is in series with a corresponding phase of the equivalent Y-connected machine.

When all parts of the circuit have thus been reduced to equivalent Y's, each of the three phases may be treated as a single-phase circuit, each circuit considered completed by a wire having zero impedance connecting all the neutrals together, since all the neutrals are at the same potential.

The voltages thus calculated are the voltages to neutral, and the currents are line currents. To find the line voltage multiply the calculated voltage by $\sqrt{3}$; similarly, to find the actual phase current in the Δ-connected generator or load divide the calculated current by $\sqrt{3}$.

37. SYMMETRICAL COMPONENTS OF CURRENTS AND VOLTAGES

The general solution of unbalanced polyphase networks presents a very formidable problem when attacked by the method of ordinary mesh equations. It was discovered independently by Stokvis and Fortescue that the equations for such problems often exhibited a striking symmetry, and that a general method could be developed which had peculiar advantages when applied to certain types of unbalanced polyphase networks, in particular those involving rotating machines. The outstanding feature of this method consists in a choice of suitable symmetrical systems of vectors in terms of which to formulate the problem. This procedure is analogous to the selection of normal coordinates in certain physical problems, a process which has proved extremely fruitful. The method is capable of considerable generalization, but for the sake of simplicity the three-phase system only will be given.

It should be pointed out here that, for ordinary networks which have no moving parts, and which therefore give rise to differential equations having constant coefficients, the method of symmetrical components has no advantages unless the network is partly symmetrical and partly unsymmetrical. If, however, the network has one portion which is symmetrical and one portion which is unsymmetrical, as, for example, an unbalanced load connected to a symmetrical transmission line fed by a symmetrical generator, a considerable simplification is obtained by the use of symmetrical components. In rotating machines, as is demonstrated in the chapter on Electrical Machines, the advantages of the symmetrical component method are even greater.

The complex number representing one of the cube roots of unity will be of importance in this analysis. Let the complex number $e^{j(2\pi/3)}$ be designated by

$$a = e^{j(2\pi/3)} = {}^1/_2(-1 + j\sqrt{3}) \qquad (3)$$

then it follows that

$$a^2 = e^{j(4\pi/3)} = 1/2\,(-1 - j\sqrt{3}) \tag{4}$$

and that

$$a^3 = 1 \tag{5}$$

Note also that

$$1 + a + a^2 = 0 \tag{6}$$

The following are identities, in which A, B, and C are three arbitrary (complex or real) quantities, and may be readily verified:

$$A = \frac{A + B + C}{3} + \frac{A + aB + a^2 C}{3} + \frac{A + a^2 B + a^4 C}{3} \tag{7}$$

$$B = \frac{A + B + C}{3} + \frac{1}{a}\frac{A + aB + a^2 C}{3} + \frac{1}{a^2}\frac{A + a^2 B + a^4 C}{3} \tag{8}$$

$$C = \frac{A + B + C}{3} + \frac{1}{a^2}\frac{A + aB + a^2 C}{3} + \frac{1}{a^4}\frac{A + a^2 B + a^4 C}{3} \tag{9}$$

By writing eq. (7) in the form

$$A = 1/3(1 + 1 + 1)A + 1/3(1 + a + a^2)B + 1/3(1 + a^2 + a)C$$

these relations follow immediately from eq. (6).

If the quantities A, B, and C are complex numbers they may be represented by vectors. Eqs. (7), (8), and (9) may therefore be regarded as the vector equations for A, B, and C in terms of certain components chosen in a particular manner. Note that the three first components of A, B, and C are the same, that the three second components differ from each other only by factors which are powers of a, and that the three last components also differ from each other only by factors which are powers of a.

Let these components be denoted by the letters with subscripts 0, 1, and 2.

$$\left.\begin{aligned} A &= A_0 + A_1 + A_2 \\ B &= B_0 + B_1 + B_2 \\ C &= C_0 + C_1 + C_2 \end{aligned}\right\} \tag{10}$$

where the nine complex quantities on the right-hand side are defined by

$$A_0 = B_0 = C_0 = \frac{A + B + C}{3} \tag{11}$$

$$A_1 = aB_1 = a^2 C_1 = \frac{A + aB + a^2 C}{3} \tag{11a}$$

$$A_2 = a^2 B_2 = aC_2 = \frac{A + a^2 B + aC}{3} \tag{11b}$$

The group (A_0, B_0, C_0) consists of three coincident vectors, while each of the groups (A_1, B_1, C_1) and (A_2, B_2, C_2) consists of a symmetrical three-phase set. The nine vectors on the right-hand sides of eqs. (11) are therefore called the *symmetrical components* of the original vectors. The vectors A_0, B_0, C_0 are called the *zero sequence* components; the vectors A_1, B_1, and C_1 are called the *positive sequence* components; and the vectors A_2, B_2, and C_2, the *negative sequence* components.

If the three complex quantities A, B, and C denote sine-wave electrical quantities by the usual complex number notation for linear electric circuits, then all the symmetrical components of these complex numbers will represent component electrical quantities. Since superposition holds in all problems in which the ordinary complex number notation can be used, the symmetrical component transformation yields actual electrical quantities which can be used in the solution of network problems. That is, any problem may be solved for the various components separately and the result obtained by addition. For example, it may be that the complex numbers represent currents having an angular velocity ω, in which case the positive sequence components will have the form

$$\left.\begin{aligned} &A_1 = Ie^{j\phi} \text{ which corresponds to } \sqrt{2}I \cos(\omega t + \phi) \\ &B_1 = a^2 A_1 = Ie^{j\phi} e^{j(4\pi/3)} \text{ which corresponds to } \sqrt{2}I \cos(\omega t + \phi - 120°) \\ &C_1 = aA_1 = Ie^{j\phi} e^{j(2\pi/3)} \text{ which corresponds to } \sqrt{2}I \cos(\omega t + \phi - 240°) \end{aligned}\right\} \tag{12}$$

This illustration gives the justification for calling the vectors having the subscript 1 positive sequence vectors, i.e., these vectors when used in electric circuits will represent time functions such that A_1 precedes B_1 which precedes C_1 in time.

38. GRAPHICAL CALCULATION OF SYMMETRICAL COMPONENTS

Eq. (7) suggests a simple geometrical construction for the determination of the symmetrical components of three vectors A, B, and C. The sum of A, B, and C, is $3A_0$. If B be rotated through a positive angle of 120 deg and C be rotated through a positive angle of 240 deg and the resulting vectors added to A the resultant is $3A_1$. If B be rotated through a negative angle of 120 deg and C be rotated through a negative angle of 240 deg and the resulting vectors added to A the resultant is $3A_2$. The graphical representation is shown in Fig. 3.

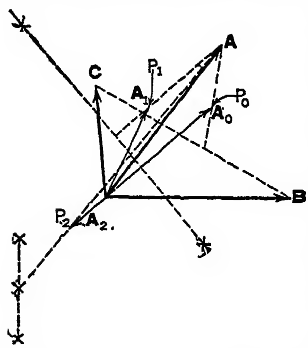

FIG. 3. Construction of Symmetrical Components of Three Arbitrary Complex Quantities

In the construction of Fig. 3, use is made of the theorem of geometry which states that one-third the resultant of three vectors is a vector from the origin to the centroid of the triangle whose vertices lie at the ends of the three vectors. Thus the vector A_0 is OP_0 where P_0 is the centroid of the triangle ABC. The point P_0 is located two-thirds the distance from A along a line from A to the midpoint of BC. The points of P_1 and P_2 are located in a similar manner after rotating the vectors B and C through the appropriate angles.

Having gotten the symmetrical components of the vector A with respect to B and C, it is a simple matter to obtain the symmetrical components of B and C. Eqs. (11) give the necessary information for this construction, viz.: B_0 and C_0 are coincident with A_0, B_1 is equal in modulus to A_1 and displaced from it in a counter-clockwise direction by an angle of 240 deg, B_2 is equal in modulus to A_2 and displaced from it in a counter-clockwise direction by an angle of 120 deg, etc.

39. SYMMETRICAL COMPONENTS OF IMPEDANCES

A network consisting of three impedances Z_1, Z_2, and Z_3 connected in *wye* gives the following equations in terms of phase currents and phase voltages

$$\left.\begin{aligned} Z_1 I_1 &= V_1 \\ Z_2 I_2 &= V_2 \\ Z_3 I_3 &= V_3 \end{aligned}\right\} \tag{13}$$

which in terms of symmetrical components are

$$\left.\begin{aligned} Z_1 (I_{10} + I_{11} + I_{12}) &= V_{10} + V_{11} + V_{12} \\ Z_2 (I_{20} + I_{21} + I_{22}) &= V_{20} + V_{21} + V_{22} \\ Z_3 (I_{30} + I_{31} + I_{32}) &= V_{30} + V_{31} + V_{32} \end{aligned}\right\} \tag{14}$$

From eqs. (11) the following relations hold

$$\begin{aligned} V_{10} &= V_{20} = V_{30} & I_{10} &= I_{20} = I_{30} \\ V_{21} &= a^2 V_{11} & I_{21} &= a^2 I_{11} \\ V_{31} &= aV_{11} & I_{31} &= aI_{11} \\ V_{22} &= aV_{12} & I_{22} &= aI_{12} \\ V_{32} &= a^2 V_{12} & I_{32} &= a^2 I_{12} \end{aligned}$$

hence eqs. (14) may be written

$$\left.\begin{aligned} Z_1 (I_{10} + I_{11} + I_{12}) &= V_{10} + V_{11} + V_{12} \\ Z_2 (I_{10} + a^2 I_{11} + aI_{12}) &= V_{10} + a^2 V_{11} + aV_{12} \\ Z_3 (I_{10} + aI_{11} + a^2 I_{12}) &= V_{10} + aV_{11} + a^2 V_{12} \end{aligned}\right\} \tag{15}$$

Adding eqs. (15)

$$\frac{Z_1 + Z_2 + Z_3}{3} I_{10} + \frac{Z_1 + a^2 Z_2 + aZ_3}{3} I_{11} + \frac{Z_1 + aZ_2 + a^2 Z_3}{3} I_{12} = V_{10}$$

Multiplying the three equations of (15) by 1, a, and a^2 respectively and adding

$$\frac{Z_1 + aZ_2 + a^2 Z_3}{3} I_{10} + \frac{Z_1 + Z_2 + Z_3}{3} I_{11} + \frac{Z_1 + a^2 Z_2 + aZ_3}{3} I_{12} = V_{11}$$

Multiplying the three equations of (15) by 1, a^2, and a respectively and adding there results:

$$\frac{Z_1 + a^2 Z_2 + aZ_3}{3} I_{10} + \frac{Z_1 + aZ_2 + a^2 Z_3}{3} I_{11} + \frac{Z_1 + Z_2 + Z_3}{3} I_{12} = V_{12}$$

Hence the following equations may be written

$$\left.\begin{aligned} Z_{10} I_{10} + Z_{12} I_{11} + Z_{11} I_{12} &= V_{10} \\ Z_{11} I_{10} + Z_{10} I_{11} + Z_{12} I_{12} &= V_{11} \\ Z_{12} I_{10} + Z_{11} I_{11} + Z_{10} I_{12} &= V_{12} \end{aligned}\right\} \qquad (16)$$

in which the Z's are symmetrical components of the impedances Z_1, Z_2, and Z_3, and are defined by equations similar to eqs. (10).

It should be pointed out that the Z's of eqs. (16) are symmetrical components of impedances in a purely formal sense, and that no particular physical significance can be attached to them. In certain electric circuits in which the parameters are functions of time, in particular in rotating machines, it is possible to obtain equivalent impedances which have different values depending upon whether the currents flowing through them have positive or negative phase sequence. The *positive* (negative, zero) *sequence impedance* in this sense has an important physical significance and is not related to the above defined *positive* (negative, zero) *sequence impedance*. Care should be taken to keep the two concepts distinct; the context will usually leave no doubt as to which is meant. See the chapter on Electrical Machines.

40. MATRIX* FORMULATION OF THE THREE-PHASE PROBLEM

It is convenient, in the development of circuit equations in terms of the symmetrical components of currents, voltages, and impedances, to make use of the algebra of matrices.

The properties of the particular matrix S defined by eq. (17), and designated by the term *sequence matrix*, are used.

$$S = {}^1/_3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \qquad (17)$$

Consider the product of any arbitrary square matrix having three rows and columns by the matrix S.

Let the arbitrary matrix (in which the elements are complex numbers) be

$$R = \begin{vmatrix} A & D & G \\ B & E & H \\ C & F & K \end{vmatrix} \qquad (18)$$

Then the product matrix is

$$SR = {}^1/_3 \begin{vmatrix} (A + B + C) & (D + E + F) & (G + H + K) \\ (A + aB + a^2 C) & (D + aE + a^2 F) & (G + aH + a^2K) \\ (A + a^2 B + aC) & (D + a^2 E + aF) & (G + a^2 H + aK) \end{vmatrix} \qquad (19)$$

and in terms of the symmetrical components of the elements of the matrix R taken in the grouping (A, B, C); (D, E, F); (G, H, K) the matrix SR becomes

$$SR = \begin{vmatrix} A_0 & D_0 & G_0 \\ A_1 & D_1 & G_1 \\ A_2 & D_2 & G_2 \end{vmatrix} \qquad (20)$$

in which A_0 is the zero sequence component of A with respect to B and C, A_1 is the positive sequence component of A with respect to B and C, etc. Thus multiplication of a given matrix by S has the effect of replacing the elements in any column of the given matrix by the symmetrical components of the leading element of the column taken with respect to the other two elements of that column.

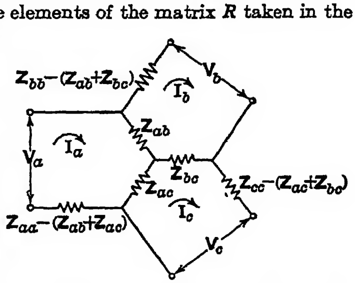

FIG. 4. General Three-phase Network

Suppose that it is desired to solve the network shown in Fig. 4 in terms of symmetrical components. The mesh equations are:

$$\left.\begin{aligned} Z_{aa} I_a - Z_{ab} I_b - Z_{ac} I_c &= V_a \\ -Z_{ab} I_a + Z_{bb} I_b - Z_{bc} I_c &= V_b \\ -Z_{ac} I_a - Z_{bc} I_b + Z_{cc} I_c &= V_c \end{aligned}\right\} \qquad (21)$$

* For the algebra of matrices refer to Bôcher, Introduction to Higher Algebra.

This set of equations may be written down as a group by means of the matrix equation

$$ZI = V \tag{21a}$$

where the matrices Z, I, and V are defined by

$$Z = \begin{vmatrix} Z_{aa} & -Z_{ab} & -Z_{ac} \\ -Z_{ab} & Z_{bb} & -Z_{bc} \\ -Z_{ac} & -Z_{bc} & Z_{cc} \end{vmatrix}$$

$$I = \begin{vmatrix} I_a & 0 & 0 \\ I_b & 0 & 0 \\ I_c & 0 & 0 \end{vmatrix}$$

and

$$V = \begin{vmatrix} V_a & 0 & 0 \\ V_b & 0 & 0 \\ V_c & 0 & 0 \end{vmatrix}$$

Let both sides of eq. (21a) be multiplied by the matrix S. Since multiplication of matrices is associative

$$(SZ)I = (SV) \tag{22}$$

which contains on the right-hand side the symmetrical components of voltages, which is desired. Since $S^{-1}S$ is the unit matrix it follows that for any matrices P and Q

$$(P)(Q) = P(S^{-1}S)Q = (PS^{-1})(SQ) \tag{23}$$

Applying this operation to the left-hand side of eq. (22) there results

$$(SZS^{-1})(SI) = (SV) \tag{24}$$

which is a matrix equation containing symmetrical components of currents and voltages.

The inverse of the matrix S is

$$S^{-1} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \tag{25}$$

The expansion of the matrix (SZS^{-1}) may, after some rearrangement of terms, be written

$$(SZS^{-1}) = \begin{vmatrix} (Z_{aa0} - 2Z_{bc0}) & (Z_{aa2} + Z_{bc2}) & (Z_{aa1} + Z_{bc1}) \\ (Z_{aa1} + Z_{bc1}) & (Z_{aa0} + Z_{bc0}) & (Z_{aa2} - 2Z_{bc2}) \\ (Z_{aa2} + Z_{bc2}) & (Z_{aa1} - 2Z_{bc1}) & (Z_{aa0} + Z_{bc0}) \end{vmatrix} \tag{26}$$

in which the Z's are symmetrical components of impedances taken in the groupings (Z_{aa}, Z_{bb}, Z_{cc}) and (Z_{ab}, Z_{bc}, Z_{ac}). That is

$$\begin{aligned} Z_{aa0} &\equiv {}^1/_3(Z_{aa} + Z_{bb} + Z_{cc}) \\ Z_{aa1} &\equiv {}^1/_3(Z_{aa} + aZ_{bb} + a^2 Z_{cc}) \\ Z_{aa2} &\equiv {}^1/_3(Z_{aa} + a^2 Z_{bb} + aZ_{cc}) \\ Z_{bc0} &\equiv {}^1/_3(Z_{bc} + Z_{ac} + Z_{ab}) \\ Z_{bc1} &\equiv {}^1/_3(Z_{bc} + aZ_{ac} + a^2 Z_{ab}) \\ Z_{bc2} &\equiv {}^1/_3(Z_{bc} + a^2 Z_{ac} + aZ_{ab}) \end{aligned}$$

Upon expanding eq. (24) and equating corresponding elements, the following equations relating the symmetrical components of currents, voltages, and impedances result:

$$\left.\begin{aligned} (Z_{aa0} - 2Z_{bc0})\, I_{a0} + (Z_{aa2} + Z_{bc2})\, I_{a1} + (Z_{aa1} + Z_{bc1})\, I_{a2} &= V_{a0} \\ (Z_{aa1} + Z_{bc1})\, I_{a0} + (Z_{aa0} + Z_{bc0})\, I_{a1} + (Z_{aa2} - 2Z_{bc2})\, I_{a2} &= V_{a1} \\ (Z_{aa2} + Z_{bc2})\, I_{a0} + (Z_{aa1} - 2Z_{bc1})\, I_{a1} + (Z_{aa0} + Z_{bc0})\, I_{a2} &= V_{a2} \end{aligned}\right\} \tag{27}$$

The principle of superposition, which holds for any network in which the parameters are constants or functions of the time only, states that the current in any branch of a network is the sum of the currents which would be produced in this branch by the electromotive forces singly or in any arbitrary grouping. It is permissible, therefore, to solve sets of equations of the form (27) for the symmetrical components of the currents when the symmetrical components of one voltage only are applied, and to add such solutions. Hence the current I_a of eq. (21) is given by

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

in which I_{a0}, I_{a1}, and I_{a2} are the solutions of (27). Similarly

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$$

and

$$I_c = I_{a0} + aI_{a1} + a^2 I_{a2}$$

41. SYMMETRICAL COMPONENTS FOR A SYMMETRICAL NETWORK

Consider the special case in which the network in Fig. 4 is perfectly symmetrical, i.e., when

$$\left.\begin{aligned} Z_{ab} = Z_{bc} = Z_{ac} \\ Z_{aa} = Z_{bb} = Z_{cc} \end{aligned}\right\} \tag{28}$$

In this case the symmetrical components of the impedances have the values

$$\left.\begin{aligned} Z_{aa0} &= Z_{aa} \\ Z_{aa1} &= 0 \\ Z_{aa2} &= 0 \\ Z_{bc0} &= Z_{bc} \\ Z_{bc1} &= 0 \\ Z_{bc2} &= 0 \end{aligned}\right\} \tag{29}$$

and therefore eqs. (27) reduce to the simple form

$$\left.\begin{aligned} (Z_{aa} - 2Z_{bc})\, I_{a0} &= V_{a0} \\ (Z_{aa} + \ \ Z_{bc})\, I_{a1} &= V_{a1} \\ (Z_{aa} + \ \ Z_{bc})\, I_{a2} &= V_{a2} \end{aligned}\right\} \tag{30}$$

Note that in this case positive sequence currents are due to positive sequence voltages only, negative sequence currents are due to negative sequence voltages only, and zero sequence currents are due to zero sequence voltages only. Hence independent networks can be set up for the three sequences; these are called *sequence networks*. The positive and negative sequence networks are identical,* whereas the zero sequence network is different from these.

Eqs. (30) are equivalent to the single-phase method of calculation which is given in Art. 36 and which applies to all three-phase networks which are perfectly balanced. Therefore no real advantage is gained by the use of symmetrical components in the case of a completely symmetrical network. Furthermore, eqs. (27) are no easier to solve than eqs. (21) from which they were deduced, and therefore the work done in transforming to symmetrical components in a completely unbalanced network is a waste of time.

If the network to be solved has a portion which is unsymmetrical and a portion which is symmetrical, as usually happens in power systems, the advantages of the method of symmetrical components are great; this is illustrated in Art. 42.

42. PARTIALLY SYMMETRICAL NETWORK

As an illustration of the application of symmetrical components to a network which is partially balanced and partially unbalanced, consider the circuit of Fig. 5. This network, consisting of a wye-connected generator, transmission line, and delta-connected

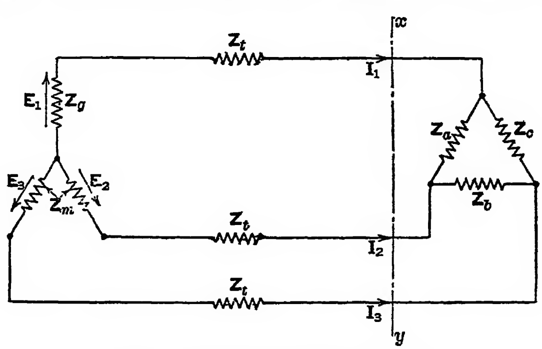

FIG. 5. Partially Symmetrical System

* This is not true of rotating machines.

receiver is symmetrical to the left of section x–y and unsymmetrical to the right. The generator phase voltages will be assumed to have positive phase sequence:

$$\left.\begin{aligned} E_1 &= E_g \angle 0 \\ E_2 &= E_g \angle -120° \\ E_3 &= E_g \angle -240° \end{aligned}\right\} \tag{31}$$

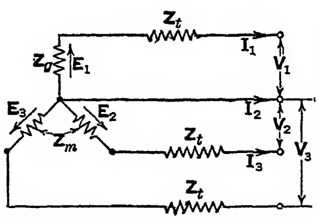

FIG. 6. Symmetrical Generator and Transmission Line

Let the unbalanced load be replaced by three voltages measured from the neutral of the generator. The network to the left of section x–y then appears as shown in Fig. 6 in which V_1, V_2, and V_3 are the voltages replacing the load. Since this network is symmetrical the equations for it may be written down directly by the principle stated in the last article.

$$\left.\begin{aligned} E_{10} - (Z_g + Z_t + 2Z_m)\, I_{10} &= V_{10} \\ E_{11} - (Z_g + Z_t + Z_m)\, I_{11} &= V_{11} \\ E_{12} - (Z_g + Z_t + Z_m)\, I_{12} &= V_{12} \end{aligned}\right\} \tag{32}$$

But V_{10}, I_{10}, E_{10} and E_{12} are all zero so that eq. (32) is simply

$$\left.\begin{aligned} E_{11} - (Z_g + Z_t + Z_m)\, I_{11} &= V_{11} \\ - (Z_g + Z_t + Z_m)\, I_{12} &= V_{12} \end{aligned}\right\} \tag{32a}$$

There remains now the problem of expressing the voltages V_{11} and V_{12} in terms of the symmetrical components of the line currents delivered to the receiver. This is somewhat involved but once solved affords a general formula which can be used in all cases of unbalanced loads. This is done in Art. 43.

43. RECEIVER CURRENTS IN TERMS OF LINE TO GROUND VOLTAGES

In Fig. 7 let P be any arbitrarily chosen point (later to be taken as the neutral of the generator) and let the voltages V_1, V_2, and V_3 be the voltage drops from lines 1, 2, and 3, respectively, to point P. The receiver has been chosen as a delta for convenience; if the actual receiver in a given problem is connected in wye the impedances given here become the impedances of the *equivalent* delta. Note that the point P is perfectly arbitrary and therefore has no relation to the neutral of the wye in a wye-connected receiver. Point P can, for example, be taken at ground potential, whether or not the system is grounded. The currents and voltages are related by the following equations:

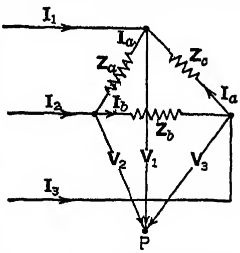

FIG. 7. Unsymmetrical Load

$$\left.\begin{aligned} V_1 - V_2 &= V_a = I_a Z_a \\ V_2 - V_3 &= V_b = I_b Z_b \\ V_3 - V_1 &= V_c = I_c Z_c \end{aligned}\right\} \tag{33}$$

$$\left.\begin{aligned} I_a - I_c &= I_1 \\ I_b - I_a &= I_2 \\ I_c - I_b &= I_3 \end{aligned}\right\} \tag{34}$$

Substitution of (33) in (34) gives

$$\left.\begin{aligned} I_1 &= \frac{V_1 - V_2}{Z_a} - \frac{V_3 - V_1}{Z_c} \\ I_2 &= \frac{V_2 - V_3}{Z_b} - \frac{V_1 - V_2}{Z_a} \\ I_3 &= \frac{V_3 - V_1}{Z_c} - \frac{V_2 - V_3}{Z_b} \end{aligned}\right\} \tag{35}$$

It is more convenient to manipulate the equations if admittances are substituted in place of impedances. Put

$$\left.\begin{aligned} \frac{1}{Z_a} &= A \\ \frac{1}{Z_b} &= B \\ \frac{1}{Z_c} &= C \end{aligned}\right\} \tag{36}$$

in which case eqs. (35) become

$$\left.\begin{aligned} I_1 &= (A + C)V_1 - AV_2 - CV_3 \\ I_2 &= - AV_1 + (A + B)V_2 - BV_3 \\ I_3 &= - CV_1 - BV_2 + (A + C)V_3 \end{aligned}\right\} \tag{37}$$

It is interesting to note that the solutions of eqs. (37) do not exist; i.e., it is impossible to express the V's in terms of the I's. Mathematically this can be proved by observing that the determinant of the coefficients of the V's is zero. Physically this should be expected since the V's are measured to an arbitrary point.

Eqs. (37) are of the same form as eqs. (21) and therefore can be transformed into symmetrical component form by the formulas (27). These symmetrical component equations are

$$\left.\begin{aligned} I_{11} &= (A + B + C)V_{11} + (B + a^2C + aA)V_{12} \\ I_{12} &= (B + aC + a^2A)V_{11} + (A + B + C)V_{12} \end{aligned}\right\} \tag{38}$$

The zero sequence equation and all zero sequence terms drop out. This means physically that there is no relation between the zero sequence components of line current and voltage to ground. Furthermore, the determinant of the coefficients in eqs. (38) is *not* zero; hence a solution exists. This means that although the V's are not determined when the I's are known the *positive* and *negative sequence components* of the V's are determined when the *positive* and *negative* sequence components of the I's are known. In other words, as the point P is moved from place to place the *zero sequence* components of the V's change but the other components remain unchanged.

Solving eqs. (38) for V_{11} and V_{12} and rewriting these in terms of impedances there results

$$\left.\begin{aligned} \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{3(Z_a + Z_b + Z_c)} I_{11} - \frac{a^2 Z_a Z_b + aZ_b Z_c + Z_a Z_c}{3(Z_a + Z_b + Z_c)} I_{12} &= V_{11} \\ - \frac{aZ_a Z_b + a^2 Z_b Z_c + Z_a Z_c}{3(Z_a + Z_b + Z_c)} I_{11} + \frac{Z_a Z_b + Z_b Z_c + Z_a Z_c}{3(Z_a + Z_b + Z_c)} I_{12} &= V_{12} \end{aligned}\right\} \tag{39}$$

Hence the general theorem: in any problem involving a three-terminal network the three line currents may be uniquely expressed in terms of the symmetrical components of a group of three-phase voltages having positive and negative sequence components which are unique but of which the zero sequence is arbitrary. This expression is eq. (39).

To complete the problem of the partially symmetrical network of Art. 42 it is necessary to equate the values of V_{11} and V_{12} as given in eqs. (32*a*) and (39). This general scheme yields a convenient means for numerical calculation of unbalanced loading problems and short-circuit currents in three-phase systems.

44. SYMMETRICAL COMPONENT EQUATIONS FOR A THREE-PHASE TRANSDUCER

The general three-phase transformer bank has three pairs of input terminals and three pairs of output terminals. The mesh equations for such a network are

$$\left.\begin{aligned} Z_{aa} I_a + Z_{ab} I_b + Z_{ac} I_c - Z_{au} I_u - Z_{av} I_v - Z_{aw} I_w &= V_a \\ Z_{ab} I_a + Z_{bb} I_b + Z_{bc} I_c - Z_{bu} I_u - Z_{bv} I_v - Z_{bw} I_w &= V_b \\ Z_{ac} I_a + Z_{bc} I_b + Z_{cc} I_c - Z_{cu} I_u - Z_{cv} I_v - Z_{cw} I_w &= V_c \\ - Z_{au} I_a - Z_{bu} I_b - Z_{cu} I_c + Z_{uu} I_u + Z_{uv} I_v + Z_{uw} I_w &= V_u \\ - Z_{av} I_a - Z_{bv} I_b - Z_{cv} I_c + Z_{uv} I_u + Z_{vv} I_v + Z_{vw} I_w &= V_v \\ - Z_{aw} I_a - Z_{bw} I_b - Z_{cw} I_c + Z_{uw} I_u + Z_{vw} I_v + Z_{ww} I_w &= V_w \end{aligned}\right\} \tag{40}$$

This set of equations, when proper values are assigned to the Z's, may be used to express the performance of single-phase transformers in Y–Δ, Δ–Y, etc., or of three-phase transformers. It may also express the performance of synchronous or induction machines. In this case an additional equation (the torque equation) is required. The impedances in eq. (40) are actual self and mutual impedances, not leakage impedances. For numerical accuracy the values of these impedances should be calculated from open- and short-circuit tests in order to eliminate the error which would otherwise arise owing to differences of large quantities. (See Art. 29 of this section.)

Transformation to symmetrical components yields the equations in Table 1.

Table 1

Coefficient of						In equation for
I_{a0}	I_{a1}	I_{a2}	I_{u0}	I_{u1}	I_{u2}	
$(Z_{aa0}+2Z_{bc0})$	$(Z_{aa2}-Z_{bc2})$	$(Z_{aa1}-Z_{bc1})$	$-(Z_{au0}+Z_{av0}+Z_{aw0})$	$-(Z_{au2}+a^2Z_{av2}+aZ_{aw2})$	$-(Z_{au1}+aZ_{av1}+a^2Z_{aw1})$	V_{a0}
$(Z_{aa1}-Z_{bc1})$	$(Z_{aa0}-Z_{bc0})$	$(Z_{aa2}+2Z_{bc2})$	$-(Z_{au1}+Z_{av1}+Z_{aw1})$	$-(Z_{au0}+a^2Z_{av0}+aZ_{aw0})$	$-(Z_{au2}+aZ_{av2}+a^2Z_{aw2})$	V_{a1}
$(Z_{aa2}-Z_{bc2})$	$(Z_{aa1}+2Z_{bc1})$	$(Z_{aa0}-Z_{bc0})$	$-(Z_{au2}+Z_{av2}+Z_{aw2})$	$-(Z_{au1}+a^2Z_{av1}\;aZ_{aw1})$	$-(Z_{au0}+aZ_{av0}+a^2Z_{aw0})$	V_{a2}
$-(Z_{au0}+Z_{av0}+Z_{aw0})$	$-(Z_{au2}+Z_{av2}+Z_{aw2})$	$-(Z_{au1}+Z_{av1}+Z_{aw1})$	$(Z_{uu0}+2Z_{vw0})$	$(Z_{uu2}-Z_{vw2})$	$(Z_{uu1}-Z_{vw1})$	V_{u0}
$-(Z_{au1}+aZ_{av1}+a^2Z_{aw1})$	$-(Z_{au0}+aZ_{av0}+a^2Z_{aw0})$	$-(Z_{au2}+aZ_{av2}+a^2Z_{aw2})$	$(Z_{uu1}-Z_{vw1})$	$(Z_{uu0}-Z_{vw0})$	$(Z_{uu2}+2Z_{vw2})$	V_{u1}
$-(Z_{au2}+a^2Z_{av2}+aZ_{aw2})$	$-(Z_{au1}+a^2Z_{av1}+aZ_{aw1})$	$-(Z_{au0}+a^2Z_{av0}+aZ_{aw0})$	$(Z_{uu2}-Z_{vw2})$	$(Z_{uu1}+2Z_{vw1})$	$(Z_{uu0}-Z_{vw0})$	V_{u2}

in which the symmetrical components of currents and voltage are defined in the usual manner and in which the symmetrical components of

Z_{aa} are taken with respect to Z_{bb} and Z_{cc}
Z_{ab} " " " " " Z_{bc} and Z_{ca}
Z_{au} " " " " " Z_{bv} and Z_{cw}
Z_{av} " " " " " Z_{bw} and Z_{cu}
Z_{aw} " " " " " Z_{bu} and Z_{cv}
Z_{uu} " " " " " Z_{vv} and Z_{ww}
Z_{uv} " " " " " Z_{vw} and Z_{wu}

45. SYMMETRICAL COMPONENTS OF INSTANTANEOUS VALUES

The application of the method of symmetrical components is by no means restricted to steady-state calculations in which all the quantities are complex numbers. Symmetrical components of instantaneous values are of great advantage in the solution of transient problems, especially in synchronous machines.

What has been said of vectors is, in general, true of instantaneous values of currents and electromotive forces. The following identities hold:

$$i_a = \frac{i_a + i_b + i_c}{3} + \frac{i_a + ai_b + a^2 i_c}{3} + \frac{i_a + a^2 i_b + ai_c}{3}$$

$$i_b = \frac{i_a + i_b + i_c}{3} + \frac{1}{a}\,\frac{i_a + ai_b + a^2 i_c}{3} + \frac{1}{a^2}\,\frac{i_a + a^2 i_b + ai_c}{3}$$

$$i_c = \frac{i_a + i_b + i_c}{3} + \frac{1}{a^2}\,\frac{i_a + ai_b + a^2 i_c}{3} + \frac{1}{a}\,\frac{i_a + a^2 i_b + ai_c}{3}$$

in which i_a, i_b, and i_c are perfectly arbitrary. Hence the symmetrical components of instantaneous values are defined in the same way as were the symmetrical components of the vector values.

If i_a, i_b, and i_c are sinusoids of the same frequency, the symmetrical components become:

$$i_{a0} = \frac{(I_a e^{j\theta_a} + I_b e^{j\theta_b} + I_c e^{j\theta_c})}{6} e^{j\omega t} + \frac{(I_a e^{-j\theta_a} + I_b e^{-j\theta_b} + I_c e^{-j\theta_c})}{6} e^{-j\omega t}$$

$$i_{a1} = \frac{(I_a e^{j\theta_a} + I_b e^{j\theta_b} e^{j(2\pi/3)} + I_c e^{j\theta_c} e^{j(4\pi/3)})}{6} e^{j\omega t} + \frac{(I_a e^{-j\theta_a} + I_b e^{-j\theta_b} e^{-j(2\pi/3)} + I_c e^{-j\theta_c} e^{-j(4\pi/3)})}{6} e^{-j\omega t}$$

$$i_{a2} = \frac{(I_a e^{j\theta_a} + I_b e^{j\theta_b} e^{j(4\pi/3)} + I_c e^{j\theta_c} e^{j(2\pi/3)})}{6} e^{j\omega t} + \frac{(I_a e^{-j\theta_a} + I_b e^{-j\theta_b} e^{-j(4\pi/3)} + I_c e^{-j\theta_c} e^{-j(2\pi/3)})}{6} e^{-j\omega t}$$

Note that, in the special case in which the currents i_a, i_b, and i_c form a balanced positive sequence group,

$$i_{a0} = 0$$

$$i_{a1} = \frac{I_a}{2} e^{j\omega t}$$

$$i_{a2} = \frac{I_a}{2} e^{-j\omega t}$$

If i_a, i_b, and i_c are not sinusoids but are harmonic quantities having the same fundamental frequency the result is quite complicated. In such cases, however, a type of symmetry such that i_b is obtained from i_a by replacing ωt by $\left(\omega t - \frac{2\pi}{3}\right)$ and i_c from i_a by replacing ωt by $\left(\omega t - \frac{4\pi}{3}\right)$ usually obtains. Further, in power systems, only the odd harmonics appear.

With this type of symmetry the symmetrical components are

$$i_{a0} = \frac{I_a}{2} (e^{j\theta_3} e^{j3\omega t} + e^{j\theta_9} e^{j9\omega t} + \ldots) + \frac{I_a}{2} (e^{-j\theta_3} e^{-j3\omega t} + e^{-j\theta_9} e^{-j9\omega t} + \ldots)$$

$$i_{a1} = \frac{I_a}{2} (e^{j\omega t} + e^{j\theta_7} e^{j7\omega t} + e^{j\theta_{13}} e^{j13\omega t} + \ldots) + \frac{I_a}{2} (e^{-j\omega t} + e^{-j\theta_7} e^{-j7\omega t} + \ldots)$$

$$i_{a2} = \frac{I_a}{2} (e^{j\theta_5} e^{j5\omega t} + e^{j\theta_{11}} e^{j11\omega t} + \ldots) + \frac{I_a}{2} (e^{-j\theta_5} e^{-j5\omega t} + e^{-j\theta_{11}} e^{-j11\omega t} + \ldots)$$

It is also possible to generalize the concept of symmetrical components of impedances so that instead of being restricted to complex numbers they may be differential operators and may if required contain explicit functions of time. The application of this method to eq. (40) affords a method of calculating transients in machines.

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ELECTRICAL MACHINES

The calculation of the performance of any electrical machine involves the solution of the electric circuit equations for a group of circuits, some of which move with respect to the others. There are in general two groups of circuits, one group fixed in space (usually called the stator winding) and one group in motion (usually called the rotor winding). Since the self and mutual inductances involved are in general functions of the various currents and of time, many simplifying assumptions are ordinarily made and certain special methods of calculation must be used. Only fundamental considerations are treated in this chapter; for design and performance of machines refer to the particular type of machine of interest.

46. MAGNETOMOTIVE FORCES OF WINDINGS

When a current flows through a constant self- or mutual inductance the voltage produced can easily be calculated by multiplying $\frac{di}{dt}$ by the inductance in question. When the inductance is not constant it is often more convenient to make this calculation in two parts, namely, to calculate first the magnetomotive forces produced by the currents in the various windings and then to find the flux linkages which these magnetomotive forces produce. Thus instead of using the expression

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} + \ldots \tag{1}$$

magnetomotive forces of the type

$$\text{mmf} = 0.4\pi(N_1 i_1 + N_2 i_2 + \ldots) \tag{2}$$

are calculated. The saturation in any particular flux path can then be determined and the flux in that path evaluated by the relation

$$\phi = (\text{mmf})\ (\text{permeance}) \tag{3}$$

The voltage in the winding designated as No. 1, due to this flux, is then

$$e = 10^{-8} \frac{d}{dt} (N_1 \phi) \tag{4}$$

This method of calculating emf, coupled with the methods used normally in electric circuit theory, offers a valuable tool for determining the performance of electrical machines.

In most machines the windings are distributed along the air gap in such a way that the mmf may be assumed to be proportional to the cosine of the angle from the center of a given coil. For example, the magnetomotive force produced at the air gap of a three-phase induction motor or synchronous machine by a direct current I flowing in the stator (armature) windings may be assumed to be given by

$$\left.\begin{aligned} A_a &= NI \cos x \\ A_b &= NI \cos (x - 120°) \\ A_c &= NI \cos (x - 240°) \end{aligned}\right\} \tag{5}$$

in which x is the distance in electrical degrees measured from the center of one of the coil groups of phase a, and N is a factor depending upon the number and arrangement of turns. The resultant magnetomotive force at any point on the stator is the sum of these three magnetomotive forces. Thus with direct current in the windings the resultant mmf A is

$$A = NI[\cos x + \cos (x - 120°) + \cos (x - 240°)] = 0 \tag{6}$$

If the armature windings are excited by means of three alternating currents which form a balanced positive sequence group, the magnetomotive force will be a function of the time as well as of the angle x. Write this mmf as $A_1(x, t)$. The three currents may be written as

$$\left.\begin{aligned} i_{a1} &= \sqrt{2}\, I_{a1} \sin (\omega t - \beta_1) \\ i_{b1} &= \sqrt{2}\, I_{a1} \sin (\omega t - \beta_1 - 120°) \\ i_{c1} &= \sqrt{2}\, I_{a1} \sin (\omega t - \beta_1 - 240°) \end{aligned}\right\} \tag{7}$$

The armature mmf becomes

$$\left.\begin{aligned} A_1 (x, t) &= \sqrt{2}\, NI_{a1} [\sin (\omega t - \beta_1) \cos x + \sin (\omega t - \beta_1 - 120°) \cos (x - 120°) \\ &\quad + \sin (\omega t - \beta_1 - 240°) \cos (x - 240°)] \\ A_1 (x, t) &= \frac{3NI_{a1}}{\sqrt{2}} \sin (\omega t - x - \beta_1) \end{aligned}\right\} \tag{8}$$

47. ROTATING FIELDS

In eq. (8), $A_1 (x, t)$ is the resultant magnetomotive force at a point on the stator at a distance x from the center of one of the coil groups of phase a, expressed as a function of time. If it is desired to find the resultant magnetomotive force at some point on the rotor in terms of the distance from a datum on the rotor structure, say the center of a pole, x may be put equal to the distance from the center of a pole to the point in question plus the distance between the centers of the pole and the coil group. Let the origin of time be taken as the instant at which the center of a coil group of phase a coincides with the center of a pole, so that the distance between the center of the coil group and the center of the pole is simply $\omega_r t$. Let the distance between the center of the pole and the point on the rotor at which the magnetomotive force is desired be y. That is, put

$$x = \omega_r t + y \tag{9}$$

then we have

$$A_1 (y,t) = -\frac{3NI_{a1}}{\sqrt{2}} \sin [y + \beta_1 - (\omega - \omega_r) t] \tag{10}$$

in which $A_1 (y,t)$ is the mmf at a distance $(\omega_r t + y)$ from the center of coil a, or at a distance y from the center of a pole. This quantity is not a function of time when ω and ω_r are equal. Physically this means that the armature currents produce a magnetomotive force which is sinusoidal in space and which rotates at the angular velocity ω. When ω_r is equal to ω there is no relative motion between the armature flux and the field conductors. The result is not limited to the case of three phases; it is readily shown that *any* polyphase winding through which balanced positive sequence currents flow produces

a similar rotating magnetomotive force. The more general theorem is known as the "Theorem of Ferraris."

This principle makes possible certain graphical solutions of synchronous machine problems which are of use when non-linearity of the saturation curve is not neglected. The resultant mmf of the field and armature can be found; and since this resultant is independent of time so far as the field iron is concerned the flux which this mmf produces can be found from a direct-current saturation curve. In this way the problem of *armature reaction* (q.v.) in the synchronous machine is rendered relatively simple. It is important to stress the fact that *armature reaction* is simply a special name applied to a component magnetomotive force involved in the simultaneous solution of two or more mesh equations. Armature reaction is not used in transformer theory because the mmf of the secondary is not constant with time and is not a convenient quantity to use in transformer calculations. Ordinary circuit theory methods are more suitable for that problem.

48. FLUX LINKAGES IN SYNCHRONOUS MACHINES

The space wave form of flux around the periphery of the armature of a synchronous machine is the same as that of the mmf only if the air gap is uniform and if saturation is negligible. Since flux linkages and not mmf determine the electromotive forces in the machines, flux linkage is the quantity which must be evaluated. An approximate method applicable to machines having salient poles was developed by Blondel, on the empirical theory of alternators, *L'Industrie Électrique*, Nov. 10 and 25, 1899. He resolved the resultant mmf acting across the air gap into two sinusoidal components, one having its maximum value at the center of a pole and one having its maximum value at a point midway between the poles.

It is well known that in d-c machines the armature reaction is transverse when the brushes are in the neutral position, and direct when they are shifted 90 electrical degrees from the neutral. It is customary to treat the two reactions separately and to split any armature reaction occurring with the brushes in some intermediate position into two parts. The reaction in a salient pole alternator is fundamentally the same phenomenon. Blondel assumed that the resultant flux could be found by multiplying the magnetomotive force due to the first component by the permeance p_d of the path along the polar axis, multiplying the second component by the permeance p_q of the path along an axis midway between the poles, and adding the results. There is no necessity to restrict this process to balanced positive sequence armature currents, or indeed to restrict it to the armature at all. All magnetomotive forces acting in the air gap can be split into direct and quadrature components, and the flux linkages which these magnetomotive forces produce can then be found. The subsequent analysis follows closely, except for his use of per-unit quantities, the work of Park.*

The components of mmf in the direct and quadrature axes due to the field will be denoted by F_d and F_q. Then the field flux in maxwells per electrical degree is given by

$$F = p_d F_d \cos y + p_q F_q \sin y \tag{11}$$

The mmf in ampere-turns due to the armature was shown in the preceding article to be

$$A(x, t) = N[i_a \cos x + i_b \cos (x - 120°) + i_c \cos (x - 240°)] \tag{12}$$

Putting $x = \omega t + y$ and expanding, there results

$$\begin{aligned} A(x, t) = {} & N \cos y\,[i_a \cos \omega t + i_b \cos (\omega t - 120°) + i_c \cos (\omega t - 240°)] \\ & - N \sin y[i_a \sin \omega t + i_b \sin (\omega t - 120°) + i_c \sin (\omega t - 240°)] \end{aligned}$$

which is seen to be separated into components, one in the direct polar axis and one in the quadrature polar axis. The resultant flux density in maxwells per electrical degree in the air gap is

$$\begin{aligned} b_r = {} & p_d F_d \cos y + p_q F_q \sin y \\ & + 0.4\pi p_d N \cos y[i_a \cos \omega t + i_b \cos (\omega t - 120°) + i_c \cos (\omega t - 240°)] \\ & - 0.4\pi p_q N \sin y[i_a \sin \omega t + i_b \sin (\omega t - 120°) + i_c \sin (\omega t - 240°)] \end{aligned} \tag{13}$$

which may be rewritten in the form

$$\begin{aligned} b_r = {} & p_d F_d \cos (\omega t - x) - p_q F_q \sin (\omega t - x) \\ & + 0.4\pi N \frac{p_q + p_d}{2} [i_a \cos x + i_b \cos (x - 120°) + i_c \cos (x - 240°)] \\ & + 0.4\pi N \frac{p_d - p_q}{2} [i_a \cos (2\omega t - x) + i_b \cos (2\omega t - x - 120°) \\ & + i_c \cos (2\omega t - x - 240°)] \end{aligned} \tag{14}$$

* Park, R. H., Definition of an ideal synchronous machine, *G. E. Rev.*, June, 1928.

The flux linkages in phase a due to the flux b_r are proportionate to the flux density at the axis of coil a. This is the value of eq. (14) when $x = 0$. If N' is the constant of proportionality between flux density and flux linkages, we have

$$\psi_a = N'p_d F_d \cos \omega t - N'p_q F_q \sin \omega t + 0.4\pi NN' \frac{p_d + p_q}{2}\left[i_a - \frac{i_b + i_c}{2}\right]$$
$$+ 0.4\pi NN' \frac{p_d - p_q}{2}[i_a \cos 2\omega t + i_b \cos (2\omega t - 120°) + i_c \cos (2\omega t - 240°)] \quad (15)$$

The value of ψ_a given by eq. (14) represents the total flux linkages in coil a due to flux which crosses the air gap. There will be additional flux linkages due to flux which links coil a only, and due to flux which links coils b and c but which does not cross the air gap. This flux is mostly slot leakage and flux encircling the end connections, and hence has its paths mostly in the air.

The total flux linkages in coil a may be written

$$\psi_a = N'p_d F_d \cos \omega t - N'p_q F_q \sin \omega t + \frac{x_{l\sigma}}{10^{-8}\omega} i_a - \frac{x_m}{10^{-8}\omega}(i_b + i_c)$$
$$+ 0.4\pi NN' \frac{p_d + p_q}{4}\left[i_a - \frac{i_b + i_c}{2}\right]$$
$$+ 0.4\pi NN' \frac{p_d - p_q}{2}[i_a \cos 2\omega t + i_b \cos (2\omega t - 120°) + i_c \cos (2\omega t - 240°)] \quad (16)$$

where $x_{l\sigma}$ = self-leakage reactance of one phase of the armature, and x_m = mutual leakage reactance of any two phases.

It is convenient to write the armature currents in terms of their symmetrical components and to split the armature flux linkages into four parts, the flux linkages due to the field flux and those due to each of the symmetrical components of the armature currents, thus

$$\psi_a = \psi_{af} + \psi_{a0} + \psi_{a1} + \psi_{a2} \quad (17)$$

The flux linkages due to the field are given by

$$\psi_{af} = N'p_d F_d \cos \omega t - N'p_q F_q \sin \omega t \quad (18)$$

The flux linkages due to positive sequence armature currents are

$$\psi_{a1} = \left(\frac{x_{l\sigma}}{\omega 10^{-8}} + \frac{x_m}{\omega 10^{-8}} + 1.2\pi NN' \frac{p_d + p_q}{4}\right)\sqrt{2} I_{a1} \sin (\omega t - \beta_1)$$
$$-1.2\pi NN' \frac{p_d - p_q}{4}\sqrt{2} I_{a1} \sin (\omega t + \beta_1) \quad (19)$$

The flux linkages due to negative sequence armature current are

$$\psi_{a2} = \left(\frac{x_{l\sigma}}{\omega 10^{-8}} + \frac{x_m}{\omega 10^{-8}} + 1.2\pi NN' \frac{p_d + p_q}{4}\right)\sqrt{2} I_{a2} \sin (\omega t - \beta_2)$$
$$+ 1.2\pi NN' \frac{p_d - p_q}{4}\sqrt{2} I_{a1} \sin (3\omega t - \beta_2) \quad (20)$$

The flux linkages due to zero sequence armature currents are

$$\psi_{a0} = \left(\frac{x_{l\sigma}}{\omega 10^{-8}} - \frac{2x_m}{\omega 10^{-8}}\right)\sqrt{2} I_{a0} \sin (\omega t - \beta_0) \quad (21)$$

49. NOMINAL EMF AND ARMATURE REACTANCES

The electromotive forces due to the various flux linkages are easily found. The open-circuit or nominal electromotive force is

$$e_a = - 10^{-8}\frac{d}{dt}\psi_{af} \quad (22)$$

The net induced emf in phase a of the armature is equal to the derivative with respect to time of ψ_a hence the total armature inductive drop is equal to $\frac{d}{dt}(\psi_a - \psi_{af})$. The total inductive drop per unit armature current is the armature reactance. It is apparent that the character of the armature reactance will depend upon whether or not there are salient poles and whether or not the armature currents are balanced. It is customary to ascribe several different " reactances " to a synchronous machine armature, each reactance suitable for a particular type of computation.

Consider a machine having non-salient poles and having equal receiver impedances

connected to the three phases. If there are no salient poles the permeance is the same along any radius and hence

$$p_d = p_q$$

If the receiver impedances are equal the currents form a positive sequence group, hence

$$I_{a0} = I_{a2} = 0$$

It follows, therefore, that the total armature inductive drop is

$$-\left[x_{l\sigma} + x_m + {}^3/_2\,(4\pi\omega NN'p_d\,10^{-9})\right]\sqrt{2}I_{a1}\cos(\omega t - \beta_1) \tag{23}$$

The quantity in the square bracket in eq. (23) is called the *synchronous reactance* of the machine. The synchronous reactance is the effective leakage reactance of the armature plus the combined reactive effect, on one phase of the armature, of all the armature flux which traverses the polar path. The first part is usually called *armature* reactance, and the second part is usually called *armature reaction.* Denote the synchronous reaction by X_s, then

$$X_s = x_{l\sigma} + x_m + {}^3/_2\,\omega(4\pi NN'\,p_d\,10^{-9}) \tag{24}$$

The *synchronous impedance* of the machine is an impedance having a reactive part X_s and a resistive part equal to the armature effective resistance,

$$Z_s = r_a + jX_s \tag{25}$$

The vector equation for the terminal voltage is

$$E_a - Z_s I_a = V_a \tag{26}$$

The synchronous reactance given by eq. (24) is the value of reactance for a salient pole machine when the power factor is such that the armature current is in quadrature with the nominal emf. It does not depend on the permeance of the quadrature path and hence is called the *direct axis synchronous reactance*, and is usually denoted by X_d.

The *quadrature axis synchronous reactance*, denoted by X_q, is defined by the equation

$$X_q = x_{l\sigma} + x_m + {}^3/_2\,\omega(4\pi NN'\,p_q\,10^{-9}) \tag{27}$$

and may readily be seen to be the value of the synchronous reactance for a salient pole machine when the armature current is in phase with the nominal electromotive force.

The reactance to armature currents having zero sequence is often called the *zero sequence reactance* and is denoted by x_0. From eq. (21) this reactance is

$$X_0 = x_{l\sigma} - 2x_m \tag{28}$$

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TRANSMISSION LINES

Transmission lines and cables, in contrast to electric circuits such as transformers and machines, have parameters which are *distributed.* The *lumped-parameter* electric circuit theory outlined above cannot be used for transmission line calculations except in special cases in which the lines are electrically short. (The *electrical length* of a line is defined in Art. 55.) A *smooth* line has series resistance, series inductance, shunt conductance or leakance, and shunt capacitance, uniformly distributed along the line. The series constants are usually termed the conductor impedance; the shunt constants are usually called the dielectric admittance.

50. TRANSMISSION LINE EQUATIONS

Consider a transmission line consisting of two parallel wires and having constants *per unit length* given by

Resistance $= r$ ohms
Inductance $= L$ ohms
Leakance $= g$ mhos
Capacitance $= C$ farads

Let x be the distance measured from the receiving end of the line. Consider the change

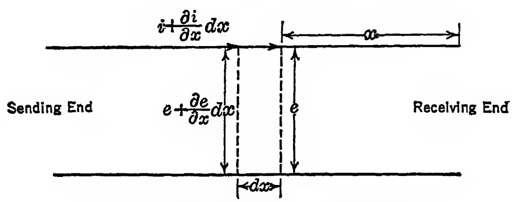

Fig. 1. Section of a Transmission Line

in voltage and current in a differential element dx, Fig. 1. The impedance drop across dx is $\left(rdxi + Ldx\frac{\partial i}{\partial t}\right)$, and the current through the admittance is $\left(gdxe + Cdx\frac{\partial e}{\partial t}\right)$; hence the two equations

$$ri + L\frac{\partial i}{\partial t} = \frac{\partial e}{\partial x} \tag{1}$$

$$ge + C\frac{\partial e}{\partial t} = \frac{\partial i}{\partial x} \tag{2}$$

give the current and voltage at any point on the line at any instant. These partial differential equations are perfectly general and therefore are applicable to both steady-state and transient calculations. Although these equations were established for two parallel wires they may be used for polyphase as well as single-phase calculations. In the polyphase case the series constants are taken per unit length of *wire* instead of per unit length of *line*, and the shunt constants are taken per unit length of wire to neutral. The current is then *per wire* and the voltage is *wire to neutral.*

51. STEADY-STATE CURRENT AND VOLTAGE DISTRIBUTION

If the voltages impressed at the ends of the line have the angular velocity ω, then in the steady state eqs. (1) and (2) become

$$(r + j\omega L)I = \frac{dE}{dx} \tag{3}$$

$$(g + j\omega C)E = \frac{dI}{dx} \tag{4}$$

in which E and I are the vector voltage and vector current at a distance x from the receiver. A more symmetrical form may be obtained by differentiating eqs. (3) and (4) with respect to x and solving for E and for I separately:

$$\frac{d^2E}{dx^2} = (r + j\omega L)(g + j\omega C)E \tag{5}$$

$$\frac{d^2I}{dx^2} = (r + j\omega L)(g + j\omega C)I \tag{6}$$

The solutions of these equations may be written in either of two alternative forms:*

$$E = Me^{\gamma x} + Ne^{-\gamma x} \tag{7}$$

$$I = M'e^{\gamma x} + N'e^{-\gamma x} \tag{7a}$$

* In the following γ is a complex number; due to printing limitations it appears in light face type when in an exponent.

or

$$E = M_1 \cosh \gamma x + N_1 \sinh \gamma x \tag{8}$$

$$I = M'_1 \cosh \gamma x + N'_1 \sinh \gamma x \tag{8a}$$

where

$$\gamma = \sqrt{(r + j\omega L)(g + j\omega C)}$$

The constants M and N in eqs. (7) and (8) are determined by the terminal currents and voltages. Let E_r and I_r be the voltage and current at the receiver and let E_s and I_s be the voltage and current at the source. The E's are both voltage rises; the I's are currents *into* the line from the source and *out* of the line to the receiver. Only the form eqs. (8) and (8*a*) will be given; these are usually most convenient for calculation.

$$E = E_r \cosh \gamma x + Z_c I_r \sinh \gamma x \tag{9}$$

$$I = I_r \cosh \gamma x + \frac{E_r}{Z_c} \sinh \gamma x \tag{9a}$$

$$E = E_s \cosh \gamma y - Z_c I_s \sinh \gamma y \tag{10}$$

$$I = I_s \cosh \gamma y - \frac{E_s}{Z_c} \sinh \gamma y \tag{10a}$$

in which Z_c is defined by

$$Z_c = \sqrt{\frac{r + j\omega L}{g + j\omega C}} \tag{11}$$

and in which y is defined by

$$y = (\text{length of line}) - x \tag{12}$$

Thus the current and voltage at any point in the line may be expressed in terms of the conditions at and distance from the sending end or receiving end, whichever is the more convenient.

52. GENERAL CIRCUIT CONSTANTS

If in eqs. (9) and (9*a*) x is put equal to the length of the line l, the current I and voltage E become those of the source, hence

$$E_s = E_r \cosh \gamma l + I_r Z_c \sinh \gamma l \tag{13}$$

$$I_s = E_r \frac{1}{Z_c} \sinh \gamma l + I \;\cosh \gamma l \tag{14}$$

Comparing these with eqs. (32) and (33) of Linear Circuits, p. 3–15, the general circuit constants are

$$\left.\begin{aligned} A &= \cosh \gamma l \\ B &= Z_c \sinh \gamma l \\ C &= \frac{1}{Z_c} \sinh \gamma l \\ D &= \cosh \gamma l \end{aligned}\right\} \tag{15}$$

By means of these constants the transmission line can be treated in the same way as any other impedance link in a given transmission system. The usual laws for combination in series and in parallel can, of course, be used. See Art. 19. When lines of different characteristics are connected together, as for example an aerial line and an underground cable, the general circuit constants may be calculated separately and then combined.

53. PROPAGATION CONSTANT, CHARACTERISTIC IMPEDANCE

Although less useful in practical calculations the exponential form of solution of the transmission line equations is more valuable in the development of theory. Consider eq. (7).

$$E = Me^{\gamma x} + Ne^{-\gamma x} \tag{7}$$

Both M and N must have the dimensions of voltage; M is often called the *incident* voltage and N the *reflected* voltage. The incident voltage increases in magnitude as we go farther and farther away from the receiver, and the reflected voltage decreases as we go farther and farther away from the receiver. Viewed from the source the incident voltage is *attenuated* as it progresses along the line toward the receiver and the reflected wave builds up, i.e., is attenuated from the receiver. Let γ be written in the form

$$\gamma = \alpha + j\beta \tag{16}$$

then the factor $e^{\gamma x}$ may be written

$$e^{\gamma x} = e^{\alpha x} \angle \beta x \tag{17}$$

Hence α is the quantity which causes a change in magnitude of the incident or reflected voltage, and β is the quantity which causes a phase shift in the voltage.

For this reason α is called the *attenuation* constant, β the *phase shift* constant, and γ the *propagation* constant of the line.

In the special case in which the receiver impedance is equal to the impedance Z_c defined by eq. (11),

$$I_r Z_c = E_r$$

and eqs. (9) and (9a) become

$$E = E_r(\cosh \gamma x + \sinh \gamma x) = E_r e^{\gamma x} \tag{18}$$

$$I = I_r(\cosh \gamma x + \sinh \gamma x) = I_r e^{\gamma x} \tag{19}$$

If such is the case the reflected voltage and current disappear and the line is said to be *smoothly terminated.*

The impedance at any point in the line, from eqs. (9) and (9a), is

$$Z = \frac{E}{I} = \frac{(E_r + I_r Z_c)e^{\gamma x} + (E_r - I_r Z_c)e^{-\gamma x}}{(E_r + I_r Z_c)e^{\gamma x} - (E_r - I_r Z_c)e^{-\gamma x}} Z_c \tag{20}$$

For an infinite line the impedance is

$$Z_\infty = \lim_{x \to \infty} Z = Z_c \tag{21}$$

The impedance Z_c is therefore the impedance at any point of an infinite line; it is also the impedance at any point of a smoothly terminated line. This impedance is called the *characteristic* or *surge impedance* of the line.

54. SHORT- AND OPEN-CIRCUIT IMPEDANCES

If the receiving end of the line is short-circuited E_r is zero and the input impedance becomes

$$Z_{ss} = Z_c \tanh \gamma l \tag{22}$$

If the receiving end is open, I_r is zero and the input impedance becomes

$$Z_{s0} = Z_c \coth \gamma l \tag{23}$$

From these relations the following simple expression for the characteristic impedance is derived

$$Z_c = \sqrt{Z_{ss} Z_{s0}} \tag{24}$$

We have also

$$\gamma = \tanh^{-1} \frac{Z_{ss}}{Z_c} = \coth^{-1} \frac{Z_{s0}}{Z_c} \tag{24a}$$

55. ELECTRICAL LENGTH

One *wavelength* is a length of line such that when smoothly terminated the phase shift in current or voltage is 360 deg. The same phase shift will, of course, occur in the same length of an infinite line having the same constants. Thus in general the wavelength λ of a line is

$$\lambda = \frac{2\pi}{\beta} \tag{25}$$

A line whose physical length is short in comparison with one-quarter wavelength is said to be electrically short. If the length of line is an appreciable fraction of one-quarter wave it is said to be electrically long. Most power transmission lines are electrically short.

56. TRANSIENTS IN TRANSMISSION LINES

The calculation of transmission line transients is extremely specialized and will be considered in outline only.

The steady-state transfer impedance from the sending end of the line to any other point at a distance y from the sending end may be shown to be

$$Z_y = Z_c \left[\frac{(Z_r + Z_c)e^{\gamma l} + (Z_r - Z_c)e^{-\gamma l}}{(Z_r + Z_c)e^{\gamma(l-y)} - (Z_r - Z_c)e^{-\gamma(l-y)}} \right] \tag{26}$$

In "Electric Circuits, Transient State," (Handbook of Engineering Fundamentals Eshbach, John Wiley & Sons), the expression

$$\frac{1}{\beta Z(\beta)} = \int_0^{\infty} e^{-\beta t}\, h(t) dt \tag{27}$$

is given for the *indicial admittance* $h(t)$. In this expression the quantity $Z(\beta)$ is the transfer impedance as a function of the parameter β.

Hence for a transmission line the indicial admittance is given by the Laplace integral

$$\int_0^{\infty} e^{-\beta t}\, h_y(t) dt = \frac{1}{\beta Z_c(\beta)}\left[\frac{1 + U(\beta)}{e^{-y\gamma(\beta)} - U(\beta) e^{y\gamma(\beta)}}\right] \tag{28}$$

in which

$h_y(t)$ = transfer indicial admittance from the sending end to a point y from the sending end.

$Z_c(\beta) = \sqrt{\frac{r + \beta L}{g + \beta C}}$.

$\gamma(\beta) = \sqrt{(r + \beta L)(g + \beta C)}$.

$U(\beta) = \left[\frac{Z_r(\beta) - Z_c(\beta)}{Z_r(\beta) + Z_c(\beta)}\right] e^{-2l\gamma(\beta)}$.

$Z_r(\beta)$ = impedance of the receiver impedance when the angular frequency $j\omega$ is replaced by β.

For methods of solving eq. (28) refer to Carson, J. R., Electric Circuit Theory and Operational Calculus, McGraw-Hill Book Co.; and Campbell and Foster, Fourier integrals for practical applications, *B.S.T.J.*, September, 1931.

Having determined the indicial admittance the current due to any applied emf is

$$i_y(t) = \frac{d}{dt}\int_0^{t} h_y(t - \lambda) E(\lambda) d\lambda \tag{29}$$

in which

$i_y(t)$ = current at a distance y from the sending end.
$E(t)$ = sending-end voltage.
λ = a variable of integration.

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STABILITY OF POWER SYSTEMS

The term stability as applied to a power system has a comparatively loose meaning. In general it may be defined as the ability of the system to operate intact without loss of synchronism of the various component generating units. This concept of stability is essentially different from that of dynamic stability defined in the chapter entitled "Transients," Handbook of Engineering Fundamentals, Eshbach, John Wiley & Sons.

A brief non-mathematical treatment of the subject is given in Sect. 14, Art. 18.

57. DEFINITIONS OF STABILITY TERMS

The following definitions were reported by the Committee on Definitions of the A.I.E.E. in January, 1932, and will be used in this chapter.

Stability, when used with reference to a power system, is that quality of the system or part of the system which enables it to develop restoring forces between the elements equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.

Steady-state Stability * exists in a power system if it operates with stability when there is no aperiodic disturbance on the system.

Transient Stability * exists in a power system if after an aperiodic disturbance has taken place the system regains steady-state stability.

A Stability Limit is the maximum power transfer through some particular point in the system when the entire system or the part of the system to which the stability limit refers is operating with stability.

The Steady-state Stability Limit is the maximum power transfer through some particular point in the system when the entire system or the part of the system to which the stability limit refers is operating with steady-state stability.

The Transient Stability Limit is the maximum power transfer through some particular point in the system when the entire system or the part of the system to which the stability limit refers is operating with transient stability.

A Stability Factor is the ratio of a stability limit to the nominal power transfer at the point of the system to which the stability limit is referred.

The Steady-state Stability Factor of a system or part of a system is the ratio of the steady-state stability limit to the nominal power transfer at the point of the system to which the stability limit is referred.

The Transient Stability Factor of a system or part of a system is the ratio of the transient stability limit to the nominal power transfer at the point of the system to which the stability limit is referred.

Infinite Bus is frequently used in discussions of power system stability. An infinite bus is a point at which the voltage is constant in modulus, phase, and frequency, independent of the active and reactive power which flows in or out at the point. This condition is approached when the generating capacity connected to the point is extremely large in comparison to the power flow under consideration. The limiting condition of zero synchronous machine impedance and infinite capacity may be thought of as an infinite bus.

58. GENERAL STABILITY PROBLEM

Because of the almost unlimited complexity of electrical power systems and their interconnections, complete stability studies, taking into consideration separately each of the synchronous machines connected to the system and each impedance link involved in their interconnection, would entail so much calculation as to be impracticable. On the other hand, many simplifying assumptions can be made which serve to reduce the labor greatly but still yield results sufficiently accurate for all practical purposes. The simplifications of networks, solution by calculating boards (either of the alternating- or direct-current type), and the construction of mechanical models have all played important parts in the study of system stability.

For the purpose of analysis it is customary to break the general problem down into component problems each simple enough to allow calculation and each representing an important practical case. These component problems are:

1. *Line between infinite buses.* The steady-state power limit of a given transmission link is one limiting factor in system stability. It is apparent that a transmission link operating at its power limit offers no possibility of additional power transfer between the synchronous apparata at its terminals; hence the system is inherently unstable. This represents the practical case of two metropolitan areas, each having a large installed capacity, connected by a tie line. This case is given in Art. 59.

2. *One machine on an infinite bus.* The steady-state and transient stability limits of one machine connected by means of a transducer to an infinite bus present an extremely important practical problem. This arises when a distant hydroelectric station of relatively small capacity is connected by a transmission line to a metropolitan system of large capacity. It may be assumed in this case that all the water-wheel generators act as a unit at the hydro-station bus and that all the turbo-generators combine to produce an effective infinite bus at the receiving end of the line. This case is given in Art. 62.

3. *Two-machine problem.* If the turbo-generator capacity in the above case is not large in comparison with the hydro station, or if two machines or groups of machines of nearly equal capacity are connected together, it is not permissible to assume one to act as a fixed reference; hence the infinite bus concept cannot be used. This case is given in Art. 63.

4. *Many-machine problem.* If a more rigorous analysis is desired than can be obtained by reduction of the network to one of the above simple cases, it is usually necessary to

* If automatic devices are used to aid stability their use should be indicated by the following phrases: steady-state stability with automatic devices; transient stability with automatic devices.

resort to some mechanical or quasi-experimental method of calculation. References applying to such means of calculation are given in the bibliography.

59. LINE BETWEEN INFINITE BUSES

Let the transmission link (transmission line, terminating transformers, etc.) between two points which may be considered as infinite buses be represented by its general circuit constants.

$$E_s = AE_r + BI_r \tag{1}$$

$$I_s = CE_r + DI_r \tag{2}$$

in which E_s and E_r are the voltages maintained by the infinite buses, I_s is the current flowing into the line from the source, and I_r is the current flowing out of the line at the receiver (Fig. 1).

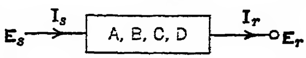

FIG. 1. Single Line Diagram of a General Circuit

If the constants A, B, C, and D are written in the form

$$A = A\ \angle\alpha$$
$$B = B\ \angle\beta$$
$$C = C\ \angle\gamma$$
$$D = D\ \angle\delta$$

and if the angle by which E_r lags E_s is denoted by θ then the input to the line at the sending end is

$$P_s = E_s^2 \frac{D}{B} \cos(\beta - \delta) - \frac{E_r E_s}{B} \cos(\beta + \theta) \tag{3}$$

$$P_r = -E_r^2 \frac{A}{B} \cos(\beta - \alpha) + \frac{E_r E_s}{B} \cos(\beta - \theta) \tag{4}$$

The maximum value which the power input P_s can have, when the modulus values E_r and E_s are constant, occurs when the phase angle between E_r and E_s is given by

$$\frac{dP_s}{d\theta} = 0 \tag{5}$$

or when

$$\sin(\beta + \theta) = 0 \tag{5a}$$

The value $\theta = -\beta$ corresponds to minimum power, and the value

$$\theta = 180° - \beta \tag{5b}$$

gives the condition for the power to be a maximum.

This maximum value is

$$P_{s(\max)} = E_s^2 \frac{D}{B} \cos(\beta - \delta) + \frac{E_r E_s}{B} \tag{5c}$$

The maximum value which the power output P_r can have when the modulus values E_r and E_s are constant occurs when θ satisfies the equation

$$\frac{dP_r}{d\theta} = 0 \tag{6}$$

or when

$$\sin(\beta - \theta) = 0 \tag{6a}$$

The value $\theta = \beta$ corresponds to the smallest absolute value of the negative power, and the value

$$\theta = 180° + \beta \tag{6b}$$

gives the condition for the negative power to have its greatest absolute value. This maximum value is

$$P_{r(\max)} = -\left[E_r^2 \frac{A}{B} \cos(\beta - \alpha) + \frac{E_r E_s}{B}\right] \tag{6c}$$

When the transmission link is operating at the condition given by either eq. (5c) or (6c) no synchronizing power can be transferred as a result of a phase shift of either of the terminal voltages; hence the system is unstable.

60. TORQUE-ANGLE CHARACTERISTICS

The following idealizing assumptions are made:

(a) That all synchronous machines can be represented by equivalent cylindrical rotor machines.

(b) That all loads can be represented by equivalent constant impedance shunts or equivalent synchronous machines.

(c) That damping can be neglected.

(d) That the generator emf's remain constant in modulus value.

Eqs. (3) and (4) represent the power input and output of a given transmission link as a function of the angle between its terminal electromotive forces. Suppose that a synchronous machine is connected through a transducer to an infinite bus. Take E_r as the voltage at the infinite bus. Then the general circuit constants may be made to include the synchronous machine impedance, and E_s may be taken as the machine emf. If the calculations are for steady state the synchronous impedance and nominal emf of the machine should be used. If the calculations are for transients the transient reactance and emf due to air-gap flux should be used. (This neglects the effect of subtransient reactance, which is permissible in most stability calculations.) When these substitutions have been made, eq. (3) gives the electrical power output of the generator as a function of its rotor position with respect to the reference established by the infinite bus. Such a curve is called a *power-angle characteristic*. Since the speed of the machine is constant under steady-state conditions and may be assumed to be constant with good accuracy so far as torque calculations are concerned, this characteristic when expressed in per-unit quantities may also be regarded as a *torque-angle characteristic*.

If, instead of a machine connected to an infinite bus, two machines of finite capacity are connected together through a transducer, the powers are given by eqs. (3) and (4) providing the E's are regarded as the generator electromotive forces and the general circuit constants include the impedance of both synchronous machines.

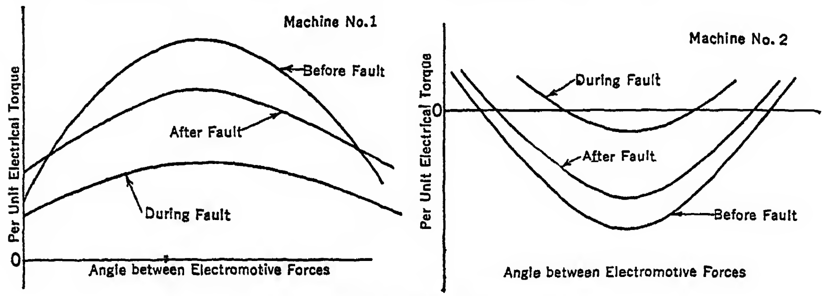

FIG. 2. Torque-angle Characteristics

Since in these applications of eqs. (3) and (4) the terms "source" and "receiver" lose their significance (both will frequently be sources), it will be convenient to rewrite the equations in a slightly different and more symmetrical form:

$$P_1 = \frac{E_1^2}{Z_{11}} \sin \alpha_{11} + \frac{E_1 E_2}{Z_{12}} \sin (\theta - \alpha_{12}) \tag{7}$$

$$P_2 = \frac{E_2^2}{Z_{22}} \sin \alpha_{22} + \frac{E_1 E_2}{Z_{12}} \sin (\theta + \alpha_{12}) \tag{8}$$

In these equations E_1 = modulus value of electromotive force of machine 1 (nominal emf or voltage-behind-transient-reactance as required).

E_2 = modulus value of electromotive force of machine 2 (nominal emf or voltage-behind-transient-reactance as required).

θ = angle between E_1 and E_2.

$Z_{11} = Z_{11}\angle\theta_{11}$ = input impedance to the transducer at machine 1.

$Z_{22} = Z_{22}\angle\theta_{22}$ = input impedance to the transducer at machine 2.

$Z_{12} = Z_{12}\angle\theta_{12}$ = transfer impedance of the transducer.

$\alpha_{11} = 90° - \theta_{11}$.

$\alpha_{22} = 90° - \theta_{22}$.

$\alpha_{12} = 90° - \theta_{12}$.

P_1 = power *output* of machine 1.

P_2 = power *output* of machine 2.

Eqs. (7) and (8) apply to a particular transducer. If a fault occurs, or if a line within the transducer is switched in or out, the input and transfer impedances will change. Representative torque angle characteristics before, during, and after the clearing of a fault are shown in Fig. 2.

61. ROTOR ACCELERATION

Let θ be the angular position of the rotor of a given machine with respect to an arbitrary reference which runs at synchronous speed. Then the acceleration is given by

$$\frac{d^2\theta}{dt^2} = \frac{\pi f \Delta P}{H'} \tag{9}$$

in which f is the frequency in cycles per second, ΔP is the power available to produce acceleration, and H' is the kinetic energy of rotation of the machine and its prime mover. H' in kilowatt-seconds is given by

$$H' = 2.31 \text{ (moment of inertia) (rpm)}^2 \times 10^{-7} \tag{10}$$

Let H be the value of H' per unit *kva*.

$$H = \frac{H'}{\text{base } kva} \text{ seconds} \tag{11}$$

Then in per-unit quantities

$$\frac{d^2\theta}{dt^2} = \pi f \frac{\Delta T}{H} \tag{12}$$

in which ΔT is the net torque available to accelerate the rotor.

If T = per-unit mechanical torque (positive for input to a machine) and T_e = per-unit electrical torque (positive for output from the machine) then the accelerating torque is

$$\Delta T = T - T_e \tag{13}$$

The torque ΔT is in general a function of the angle θ. If eq. (12) is multiplied by $\frac{d\theta}{2dt}$ and integrated with respect to t, the following integral equation results

$$\left(\frac{d\theta}{dt}\right)^2 = 2\pi f \int_{\theta_0}^{\theta} \frac{\Delta T}{H}\, d\theta \tag{14}$$

This gives the rotor velocity as an area under the accelerating-torque versus angle characteristic.

62. ONE MACHINE ON AN INFINITE BUS

Consider a synchronous machine connected to an infinite bus by means of a transducer. Eq. (7) gives the power output of the machine which, if expressed in per-unit quantities, is also the electrical torque; hence

$$T_{e1} = \frac{E_1^2}{Z_{11}} \sin\alpha_{11} + \frac{E_1 E_2}{Z_{12}} \sin(\theta - \alpha_{22}) \tag{15}$$

In this equation, θ is the angle between the machine rotor and the reference established by the infinite bus.

The torque T_{e1} is taken with the positive sign if it acts *against* the direction of rotation, i.e., positive for an output from the machine. For static stability the electrical torque should increase when θ increases and decrease when θ decreases; this gives rise to a counterbalancing torque tending to pull the machine in synchronism when, for any reason, there is an incremental swing of the rotor away from synchronism. It is assumed that all changes occur slowly so that inertial effects are negligible. The condition for stability is

$$\frac{dT_{e1}}{d\theta} > 0$$

which results in the equation

$$\cos(\theta - \alpha_{12}) > 0 \tag{16}$$

or θ must satisfy the inequality

$$\alpha_{12} - \frac{\pi}{2} < \theta < \alpha_{12} + \frac{\pi}{2} \tag{16a}$$

In terms of the angle θ_{12} of the transfer impedance Z_{12} this becomes

$$-\theta_{12} < \theta < \pi - \theta_{12} \tag{17}$$

In order to determine whether or not the machine is operating with transient stability it is necessary to investigate the acceleration equation. Suppose the machine to be operating in equilibrium so that the mechanical torque and electrical torque are equal. This is represented by the operating point O in Fig. 3, at which point θ has the value θ_0. If a fault occurs, changing the torque-angle characteristic from the curve A to the curve B, an excess of mechanical over electrical torque arises which tends to speed up the machine. During the interval of time after the occurrence of a fault and before the clearing of the

fault the machine continues to be accelerated. At the instant of clearing, say at an angle θ', the accelerating torque changes from the value by cb on Fig. 3 to the value given by dc. Note that the sign has changed. During the interval after clearing and before reaching some point such as g, at which the accelerating torque again changes sign, the rotor slows up. If there exists a value of θ such as θ'' at which the velocity of swing is zero the machine will not lose synchronism on the first swing. It will be assumed that, because of damping, if synchronism is not lost on the first swing it will not be lost at all.

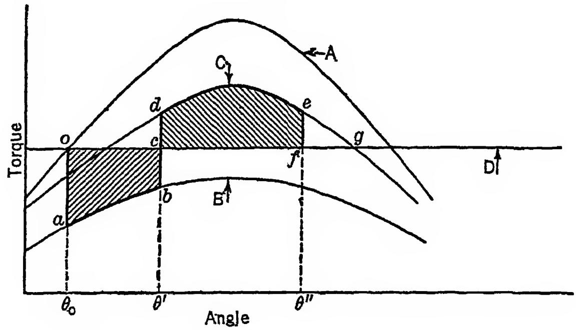

FIG. 3. Torque-angle Characteristics Showing Effect of Switching.

From eq. (14) it is apparent that the condition for the existence of a point of zero velocity is that

$$\int_{\theta_0}^{\theta} \frac{\Delta T}{H} d\theta = 0 \tag{18}$$

or in other words that the area $oabc$ in Fig. 3 be equal to the area $cdef$. This is known as the "equal area" criterion for stability.

After the angle θ', at which clearing of the fault must occur for the system to remain stable, has been determined from a diagram such as Fig. 3, it is necessary to find the time interval within which the relays must operate. The solution of the differential equation (14) can be accomplished by step-by-step processes, or by mechanical means such as a differential analyzer.

63. TWO-MACHINE PROBLEM

If, instead of one machine connected to an infinite bus, two machines of comparable capacities must be considered, a slight modification of the analysis in Art. 62 may be used.

In the study of steady-state stability both ends of the transducer must be considered. Hence the condition for steady-state stability is that both

$$\frac{dT_{e1}}{d\theta} > 0$$

and

$$\frac{dT_{e2}}{d\theta} > 0$$

simultaneously, if the machines are not to lose synchronism with each other. These conditions result in the equations:

$$\cos(\theta - \alpha_{12}) > 0 \tag{19}$$

$$\cos(\theta + \alpha_{12}) > 0 \tag{20}$$

or θ must satisfy the two inequalities

$$\alpha_{12} - \frac{\pi}{2} < \theta < \alpha_{12} + \frac{\pi}{2}$$

$$-\alpha_{12} - \frac{\pi}{2} < \theta < -\alpha_{12} + \frac{\pi}{2}$$

In terms of the angle θ_{12} of the transfer impedance Z_{12} these conditions become

$$-\theta_{12} < \theta < \pi - \theta_{12}$$

$$\theta_{12} - \pi < \theta < \theta_{12}$$

In other words, the angle θ between the two generator electromotive forces must be greater than the greater of the two quantities $-\theta_{12}$ and $(\theta_{12}-\pi)$ and less than the smaller of the two quantities

$$\theta_{12} \quad \text{and} \quad (\pi - \theta_{12})$$

In calculating the conditions for transient stability in the two-machine problem the *relative* motion of the two rotors is of importance. If the system accelerates or decelerates as a whole but does not lose synchronism it is still stable according to the definition given in Art. 57. The two equations.

$$\frac{d^2\,\theta_1}{dt^2} = \pi f \frac{\Delta T_1}{H_1} \tag{22}$$

and

$$\frac{d^2\,\theta_2}{dt^2} = \pi f \frac{\Delta T_2}{H_2} \tag{23}$$

give the accelerations of the two rotors with respect to some arbitrary datum. Put $\delta = \theta_1 - \theta_2$; then δ is the angle of relative swing of the rotors with respect to each other.

$$\frac{d^2\,\delta}{dt^2} = \frac{d^2\,\theta_1}{dt^2} - \frac{d^2\,\theta_2}{dt^2}$$

hence

$$\left(\frac{d\,\delta}{dt}\right)^2 = 2\pi f \int_{\delta_0}^{\delta} \left(\frac{\Delta T_1}{H_1} - \frac{\Delta T_2}{H_2}\right) d\,\delta \tag{24}$$

This expression can be used to calculate the condition for relative synchronous operation between two machines in exactly the same way as eq. (14) was used to calculate the condition for absolute synchronous operation of one machine with respect to a fixed system of reference. The angle δ is the angle between the two generator electromotive forces; therefore the electrical-torque versus angle characteristics are as shown in Fig. 2. The condition for stability is that there must exist a value of δ for which the equation

$$\int_{\delta_0}^{\delta} \left(\frac{\Delta T_1}{H_1} - \frac{\Delta T_2}{H_2}\right) d\,\delta = 0 \tag{25}$$

is satisfied. If a switching angle can be found so as to make eq. (25) true then the differential equation (24) determines the interval of time during which the clearing of a fault must be accomplished.

64. METHODS OF IMPROVING STABILITY *

The following means are available for generally improving the stability of power systems:

1. Reduction of series reactance.
2. Suitable arrangement of buses.
3. Grounding of the neutral.
4. Improvement in generator design.
5. Rapid response excitation.
6. Ultra rapid relays and circuit breakers.
7. Regulation of prime movers.
8. Artificial loading of generators during fault period.
9. Intermediate synchronous condenser stations.
10. Protection against lightning and suppression of arcs.
11. Transmission by means of low frequency or direct current.

REDUCTION OF SERIES REACTANCE. The most evident way of increasing the limits of stability of a system consists in decreasing the value of the reactance of the synchronous machines, for thus the synchronising power which can be exchanged is increased.

The reactance of a transmission line can be reduced by increasing the diameter of the conductor, by using a metal having low conductivity or by using a hollow cable; however, the characteristics of conductors are habitually fixed for economical reasons totally unrelated to stability. Another means of lowering the reactance of a line is to reduce the spacing of the conductors. This again is normally determined by other considerations, such as protection against lightning and against the spread of arcs on one phase to another.

The placing of capacitances in series furnishes another means of diminishing the reactance of transmission systems. However, at the instant at which a disturbance occurs, the current which goes through the condenser greatly increases the voltage drop

* Most of the material in this article was abstracted from R. D. Evans and C. F. Wagner, La stabilité des systèmes de transmission d'énergie électrique, *C.I.E.*, Vol. VI, No. 5.

across it so that it is necessary to short-circuit it to avoid the breakdown. The condenser is therefore suppressed at the moment when there is the greatest need for it.

The reactance of transformers is largely determined by economic considerations; therefore there is little opportunity for improvement in stability in this way.

The most practical procedure for lowering the reactance of transmission lines is to have several lines in parallel or to use circuits with higher voltage. There are the two alternatives; a great number of lines at low voltage in parallel, or a smaller number of circuits at high voltage. An important consideration is that, with a small number of circuits, the break of a line reduces the limit of stability by a relatively larger amount.

ARRANGEMENT OF BUSES. The placing in parallel of the high-tension sides of transmission lines at their termination, or at intermediate points, produces a smaller increase of reactance at the moment of clearing of a fault than when the connection is made on the low-tension side, for in the latter case, not only the transmission line itself but also the corresponding transformers are removed from the circuit. On the other hand, for the duration of the fault, the shock sustained by the system is more violent if the connection is made on the high-tension side than if it is made on the low-tension side. It is impossible to draw any general conclusions as regards the advantage of one or the other of these arrangements, for in each particular case the effect depends on the relations between the different reactances of the system, the type of fault, its duration, and the type of ground used.

The scheme known by the term "synchronization at the load," such as is applied to metropolitan networks, employs another method of paralleling. In this scheme there are no direct connections between the synchronous machines, but only an indirect coupling afforded by the multiple connections in the low-voltage network. With this arrangement, the faults on the secondary network have no serious repercussions on the system and can be eliminated by fusion. A short circuit on one of the generator buses causes the disconnection of that unit, but the other machines, although submitted to a violent shock, become accelerated or slowed down as a group.

Although the "synchronized at the load" system was developed for supplying metropolitan areas with power, the general principle can be applied to long-distance transmission. The modification of the scheme in this case consists in connecting the different lines only on the low-voltage sides at the receiving end.

GROUNDING OF THE NEUTRAL. In America, the current practice is to ground the neutral directly on the high-tension systems and, either indirectly or by means of a resistance, on the intermediate-voltage networks. Petersen earth coils have not been adopted, although they have been tried in two places. The introduction of impedances in the neutral, by limiting the seriousness of the faults, raises the limits of stability. This may come about in either of two ways. If the impedance is a pure reactance the fault current is reduced, and it follows that the synchronizing power is increased. If the impedance is a resistance, it absorbs some energy, loads the generator, and thus decreases the acceleration of the latter. In general, however, the type of ground used is usually determined by considerations other than stability.

IMPROVEMENT IN GENERATOR DESIGN. The best criterion, from the viewpoint of stability, of the functioning of a generator is its transient reactance. This must be as small as possible. This result can usually be obtained by a slight additional cost with respect to that of the normal machine. The short-circuit ratio which is approximately the inverse of the synchronous reactance does not matter from the viewpoint of stability, except in cases of static operation without voltage regulators.

The increase in inertia of the rotor of the generator or of that of the prime mover decreases the acceleration at the instant of a disturbance, and in this way is favorable to stability. The improvement which can be expected from this source will depend on the arrangement and the constants of the system, but it is small in general.

The seriousness of disymmetrical faults on a network depends upon the negative sequence impedance of the machine. Machines without amortisseurs have large negative sequence reactances and from this point of view are more desirable; on the other hand, machines with amortisseurs have large negative sequence resistances. The improvement gained by the use of high-resistance amortisseurs is quite appreciable for prolonged faults, but it is much less for faults whose duration is of the order that can be attained now with ultra-rapid interrupters. Low-resistance amortisseurs are much more effective than those of high resistance in damping out rotor oscillations but are worthless during faults. A double squirrel-cage arrangement in which the exterior cage is highly resistive and the inner cage is of low resistance and high reactance unites these principles.

The increase in size of the generating units, causing a greater concentration of power on a single bus, has increased the work of the circuit breakers and the extent of the zone affected by a given fault. This effect has been greatly reduced with the use of alternators

with two armature windings in which the two circuits of the stator are linked only by their mutual inductance. Each armature is connected to a separate bus; if a fault occurs on one of them, the disturbance limits itself to this bus. The tendency of the rotor to accelerate will thus be very much decreased. Generators of this type lend themselves well to be incorporated as units in " synchronized at the load " systems.

RAPID RESPONSE EXCITATION. The regulating of the excitation of synchronous machines furnished a practical means of raising the limits of both dynamic and static stability. Short circuits increase the demagnetizing effect of the armature, and this effect can be counterbalanced by an increase of excitation at a suitable rate. A system of excitation with sufficiently rapid response for this purpose requires that the regulator have rapid response, that the exciter itself be capable of responding rapidly, and that the exciter have a large ratio of maximum voltage to normal voltage.

The field circuits of exciters for this purpose must have low time constants; this is accomplished by placing them in series with exterior resistors, laminating the stator, etc. Often a pilot exciter, which in turn supplies the main exciter, is used.

The means of rapid excitation are effective not only when there is a question of counterbalancing the demagnetizing action of short-circuit currents, but also in supplying an important lack of excitation arising somewhere in the network; for example, following the cutting of the excitation of a unit, or the sudden disconnection of a machine which is very much overexcited. This peculiarity of rapid response excitation permits anticipation of serious disturbances against which ultra-rapid circuit breakers and similar devices are ineffective.

ULTRA-RAPID RELAYS AND CIRCUIT BREAKERS. As shown in Arts. 62 and 63 the duration of a fault up to its clearing is of paramount importance in determining whether or not the system remains stable. This is illustrated in Fig. 4. These curves show the limits of dynamic stability of a standard system whose constants are indicated in the figure as a function of the duration of the fault. The dotted curves give the transmissible load for permanent short circuits of the indicated types. Rapid response excitation is assumed to exist.

The importance of ultra-rapid relays and circuit breakers cannot be overstressed since their use is essential to the satisfactory operation of practically all large transmission systems. They constitute the most practical means yet developed to improve stability.

It has been suggested that the cutting of the defective phase only may be advantageous in cases of line to ground faults. In this way the remaining phases serve to maintain load on the generator so that the acceleration during the fault period is reduced.

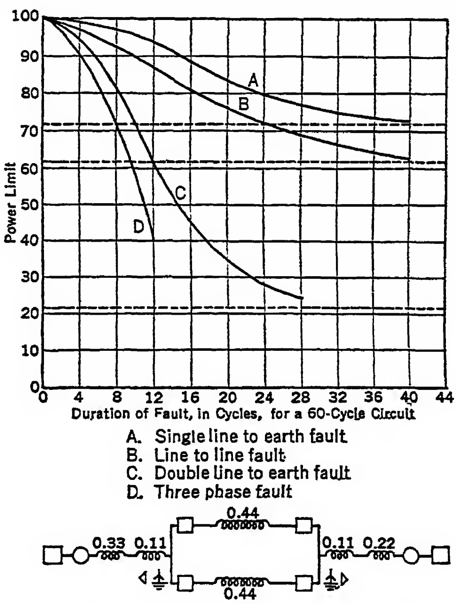

FIG. 4. Effect of the Duration of Fault on the Power Limit for Various Types of Fault. (This figure is reprinted from Evans, R. D., and Wagner, C. F., La Stabilité des Systèmes de Transmission d' Énergie Électrique, Vol. VI, Comptes Rendué du Congrès International d' Electricité.

The other means suggested for the improvement of stability are for the most part in the development stage and will not be discussed in detail.

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SECTION 4

RESISTORS, REACTORS, MAGNETS

BY

A. DEXTER HINCKLEY

ART.	RESISTORS, RHEOSTATS	PAGE
1.	General Considerations	02
2.	Metal Alloy Resistors, not mounted on ceramic forms	03
3.	Metal Alloy Resistors, mounted on ceramic forms	04
4.	Non-metallic Resistors	08
	CAPACITORS—CONDENSERS	
5.	Properties of Condensers	09
6.	Construction and Fields of Application	12
7.	Capacitors for Special Uses	15

ART.	INDUCTORS	PAGE
8.	Air-core Inductor	16
9.	Inductor with Ferrous Core	18
10.	Adjustable Inductors, Variometers	23
11.	Current Limiting Reactors for Power Systems	24
	MAGNETS, MAGNETIC DEVICES	
12.	Permanent Magnets	26
13.	Electromagnet Windings	29
14.	Electromagnets, Lifting and Plunger	35
15.	Induction Coils	41

RESISTORS, REACTORS, MAGNETS

By A. Dexter Hinckley

RESISTORS, RHEOSTATS

A resistor is a device, the primary purpose of which is to introduce resistance into an electric circuit. It consists essentially of an electrical conductor, usually of relatively high resistivity, formed in such a manner that a relatively high resistance is enclosed in a relatively small volume. A rheostat is an adjustable resistor so constructed that its resistance may be adjusted without opening the circuit in which it is connected.

Generally, resistors and rheostats find their applications in circuits for current and voltage adjustment. For applications in the field of power engineering such duties are performed on direct or low-frequency currents. Since the effects of residual inductance and capacitance in a resistor are generally negligible for continuous currents or currents of low frequency, the resistor will be considered, in this chapter, purely as a device for energy dissipation.

1. GENERAL CONSIDERATIONS

ENERGY DISSIPATION. As a circuit element, the resistor generally has current flowing through it, either continuously or intermittently. The electrical energy dissipated in the device may be measured from the power,

$$P = I^2r$$

where P is the power in watts, I the effective current in amperes, and r the resistance in ohms for the conditions under which the resistor is operating. If the current remains constant for t seconds the energy dissipated in the resistor is

$$\text{Energy} = I^2 rt \text{ joules (or watt-seconds)}$$

This mathematical statement of Joule's law indicates that the total energy dissipated in the resistor goes into the form of heat.

TEMPERATURE OF RESISTOR. The various materials making up resistors are variously affected by the resultant temperature change. Conductor resistivities vary, conductor and supporting insulators change their dimensions, and qualities of physical and electrical strength may be seriously affected. Temperatures may be reached which constitute a definite fire hazard to nearby materials. All these factors must be considered in the design and construction of the resistor, and each bears different weight in the different resistor forms. In order to safeguard against the various possibilities noted above, certain standards of temperature rise have been designated. The American Standard states that " The temperature rise of resistors above the temperature of the cooling air when the test is made in accordance with the rating, shall not exceed 350 degrees Centigrade when the thermometer is placed in contact with the resistive conductor,* or 250 degrees Centigrade when the thermometer is placed in contact with the embedding material. The temperature rise of the issuing air, when measured one inch from the inclosure, shall not exceed 175 degrees Centigrade." Accordance with standard practice results in the rating of resistors to meet the above specifications. In addition, however, care must be exercised against the possibility of mechanical failure or excessive resistance variations, and these matters are considered in this section under the individual resistors.

Many instances arise where the natural convection currents of air, and heat conduction to other parts of the structure, do not remove the energy rapidly enough and the resistor temperature rises to excessive values. To aid in the dissipation of the excessive amounts of energy, forced cooling may be employed. This involves moving more cooling fluid past the resistor than would flow by natural convection, and for this purpose air, gas, water, oil, and other fluids may be circulated in an appropriate fashion over the resistor.

* The N.E.M.A. Standard states: " For bare resistive conductors the temperature rise shall not exceed 375 deg. cent. . . ."

CLASSIFICATION. The resistive conductors which have a general use in the construction of resistors are nickel, chromium, and iron usually combined to form an alloy, also molybdenum and platinum; non-metallic materials such as silicon-carbide compounds and carbon in various forms; and various liquids. The metallic resistive elements are prepared as wire, ribbon, or cast grids and are supported on various structures to give mechanical strength and electrical insulation and possibly to aid in energy dissipation by conduction (see Section 2, Materials). The materials which serve for supports in many forms are mica, micarta, sand, porcelain, pottery, lava, asbestos, vitreous enamel, and other compositions and refractories. The non-metallic resistor elements may be self-supporting, but carbon disks are usually supported in a tube from which they are electrically insulated. Liquids of various constitutions can be used for the dissipation of large amounts of energy, and various resistivities are secured by the use of different acids or salts. The solution is contained in an appropriate vessel, and the quantity used depends upon the energy to be dissipated.

The various elements noted above, assembled in the several forms of fixed and adjustable resistors, serve numerous applications. The commercial forms which these resistor elements take will be described in this section, for the applications in the power field. Precision resistors as used in laboratory precision measurements are considered in Section 5.

2. METAL ALLOY RESISTORS, NOT MOUNTED ON CERAMIC FORMS

CLASSIFICATION, DESCRIPTION, APPLICATION. This type of resistor consists of resistive elements in the form of grids, ribbon, or wire, supported on racks or panels and electrically insulated therefrom by mica, refractory, or composition. These elements may be built into groups to form a fixed resistance unit, or by means of connections, contact points, and a sliding contactor the resistance may be varied so that the device may serve for current adjustment in a connected circuit. For a given material, the resistance of a group of resistive elements may be changed by changing the physical dimensions of the individual elements, or the connection of the individual elements may be altered. Thus, placing elements in parallel will decrease the resistance across the terminals, and incidentally increase the current-carrying capacity of the device. The metal grid or ribbon resistors very readily supply the need for large-sized, relatively low-resistance units for large current-carrying capacities. Both the grid and ribbon may be built mechanically rigid, a characteristic not easily attained for coils or other forms of wire. Ribbon and wire are frequently used for heating and loading units, and in this application the elements are permanently assembled on a supporting frame.

The assembly of grid, ribbon, or wire resistors into a compact unit must follow certain definite considerations. After satisfaction of the resistance and current-carrying requirements there are the mechanical considerations. The elements must be spaced in such a manner that they do not touch each other. Long coils or spans of wire may have this defect, not possessed by rigid grid or ribbon. Spacing is also essential for the free movement of air past the elements to carry away the generated heat energy. It is essential that the element survive cycles of heating and cooling and physical shock, without cracking or breaking. Ribbon-type resistors have the advantage here, for they are generally unbreakable, can be assembled in a continuous strip, suffer no stress when expanded by heating, and weigh less in units of equivalent dissipation ratings.

Grids are built with a wide range of current ratings running from 15 to about 350 amp. Ribbon is constructed with a current rating up to about 25 amp. By paralleling units any desired current rating can be obtained.

The grid and ribbon resistor elements find their greatest use in applications requiring moderate and large current-carrying capacity such as for a-c and d-c speed regulation and starting of large motors. For these uses resistors are designed to comply with the American Engineering Standards. When correctly applied the resistors will operate with an average temperature rise not exceeding 350 deg cent. The A.E.S. Classification Table, Table 1, shows the service for which each class of resistors is designed together with the approximate starting torque and starting current obtained in the motors of one manufacturer. This table is the A.E.S. Table modified to include plugging resistors for steel mill service and resistors for dynamic braking in the off position, and also for dynamic braking while lowering for use on certain types of hoists.

RIBBON RESISTOR ELEMENTS. The size of the grid or ribbon type resistor when assembled in a fixed or adjustable resistance unit varies widely depending upon the amount of auxiliary equipment and the complexity of the assembly. Typical of the simple ribbon resistor are the weights given in Table 2 for various groups of units pro-

Table 1. A.E.S. Resistor Service Classification (Courtesy Westinghouse E. & M. Co.)

Percentage Full-load Current on 1st Point of Controller	Starting Torque in Percentage of Full-load Torque					Resistor Classification Number				
	D-C Motors			Wound Rotor Motors		Starting Duty		Intermittent Duty		Continuous Speed-Regulating Duty
						Light	Heavy	Light	Heavy	
	Series Motors	Compound Motors	Shunt Motors	Single-phase Starting	Three-phase Starting	15 Seconds on out of 4 Minutes	30 Seconds on out of 4 Minutes	1 Minute on out of 4 Minutes	2 Minutes on out of 4 Minutes	
25	8	12	25	15	25	11	31	51	71	91
50	30	40	50	30	50	12	32	52†	72†	92*
70	50	60	70	40	70	13	33	53	73	93
100	100	100	100	55	100	14	34	54‡	74‡	94
150	170	160	150	85	150	15	35	55§	75	95
200	250	230	200	..	200	16	36	56§	76§	96

* The letter V indicates that the resistor is designed for varying torque applications where the horsepower varies approximately as the cube of the speed.
† The letter D indicates additional capacity for dynamic braking while lowering.
‡ The letter P indicates additional capacity for plugging service.
§ The letter B indicates additional capacity for dynamic braking in off position.

duced by one manufacturer. The ribbon is best in zigzag shape and held between porcelain blocks which have recesses for each bend of the ribbon. The blocks are securely interlocked in steel channels which are fastened together at the end to form unit resistors of different lengths. The tabulation is shown for assemblies made up of different unit resistors No. 3, 4, 5, and 6. Each No. 3 unit when frame assembled with other, similar units occupies a volume of about the dimensions 16 by 3 by 6 1/2 in. The other units are longer (20, 24, and 28 in., respectively) but have similar dimensions otherwise.

Table 2. Approximate Ratings of Ribbon-Type Resistors
(Courtesy Square D. Co.)

Unit No.	No. of Units per Section	Continuous Watts per Section to 375° C	All Units Connected in Multiple		All Units Connected in Series		Shipping Weight, Lb
			Minimum Ohms	Amperes	Maximum Ohms	Amperes	
	3	3330	0.06	235	7.80	20.6	28
3	6	4680	.03	395	15.60	17.3	42
	9	5760	.02	535	23.40	15.7	56
	12	6960	.015	680	31.20	14.9	70
	3	4440	.08	235	10.42	20.6	38
4	6	6240	.04	395	20.84	17.3	56
	9	7650	.027	535	31.26	15.7	75
	12	9240	.02	680	41.68	14.9	95
	3	5550	.10	235	13.05	20.0	48
5	6	7800	.05	395	26.10	17.3	70
	9	9540	.033	535	39.15	15.7	95
	12	11520	.025	680	52.20	14.9	120
	3	6660	.12	235	15.60	20.6	57
6	6	9360	.06	395	31.20	17.3	84
	9	11520	.04	535	46.80	15.7	112
	12	13920	.03	680	62.40	14.9	145

3. METAL ALLOY RESISTORS, MOUNTED ON CERAMIC FORMS

This type of resistor is produced in two general forms. In the first, the resistor elements are imbedded in a refractory material and supported by it, sometimes in conjunction with a metal plate. In the second form the resistor ribbon or wire is wound around an insulating form, which generally has a cylindrical shape.

FIXED RESISTANCE UNIT. The first of these forms (resistance wire imbedded in refractory) is generally produced as a heater unit. Encased in sheaths of rust-resisting

iron or heat-resisting chrome steel, these units can be carried to high temperatures for air heating or for use in all types of appliances such as ranges, incubators, and melting pots. Such forms likewise are applied to the heating of liquids in the immersion-type unit. One manufacturer gives the relation shown in the curve of Fig. 1 below, for the watts rating of the strip type heater for different heater temperatures.

Since there are approximately 3 1/2 sq in. of surface per inch of heated length of this type of unit a calculation will show the relation between the watts per square inch and the temperature of the radiator surface. If it is assumed that the resistor unit rises in temperature above an ambient value of 70 deg fahr, a calculation may be made of the temperature rise per watt of power for each square inch of surface. The result of this

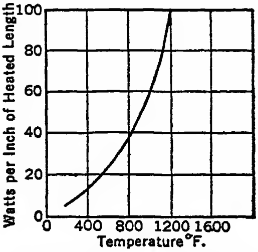

FIG. 1. Operating Temperatures of a 1 1/2-inch Strip Heater Element, Operating in Horizontal Position in Still Air. (Chromalox—Courtesy E. L. Wiegand and Co.

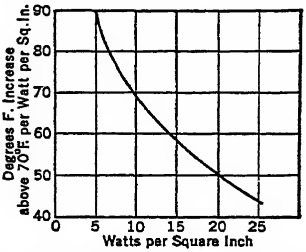

FIG. 2. Temperature Rise above 70° F of a 1 1/2-inch Strip Heater Element, per watt per sq. in. for Various Rates of Energy Dissipation

calculation is shown in the curve of Fig. 2, which indicates that the temperature increment per watt per square inch decreases with increasing heat dissipation. Approximately the same values under similar temperature conditions will be found in the various other types of imbedded resistor including the plate type and vitreous tube type.

PLATE-TYPE RHEOSTAT, CONSTRUCTION. The "plate-type" rheostat has a very general use for small and moderately sized current regulation requirements such as in motor starting, field current regulation, lamp circuit dimming, battery charging, and other uses. Probably the most widespread use occurs in the control of field current in d-c machines. The adjustable resistance type of unit is the general form, and is built up in the following steps on a "plate" of metal or some insulating material. Using a pressed steel plate as a base this plate is sandblasted on one side to remove foreign particles and to provide a softly roughened surface. A ground coat of vitreous enamel is then baked on this surface of the plate, protecting this surface and forming an electrical insulating but heat-conducting layer. The resistance wire used in the plate type rheostat is first crimped into a "wave" form. Various lengths of this wave wire are attached between the metal buttons which subsequently serve to contact the moving contact arm. As the wire is very springy it must be held in a form during its assembly and spot-weld connection to the buttons. While still on the form, the wire is heated and annealed to remove its springiness. The resistance wire and button assembly are then carefully spread on the prepared plate and baked in enough vitreous enamel to securely cover and hold the wire in place. The coating likewise serves to prevent corrosion and mechanical injury, and it aids in conducting away the generated heat. The last step in the manufacture is the cleaning and leveling of the contact buttons and the assembly of contact arm, mounting, and other auxiliary parts. An important test which must be passed by this type of unit is that the resistance wire circuit be continuous, not grounded to the metal base plate, and with the proper resistance value between adjacent buttons.

RHEOSTATS, GENERAL. This last point is an important one for all types of adjustable resistance devices. With the motion of the contact arm from one contact point to the next it is desired to change the resistance in the circuit by a definite number of ohms, or perhaps a definite percentage. Depending upon the type of circuit, the resistance between points may be uniform around the rheostat or may taper according to some particular design. Generally, the motion of the contact arm is accompanied by a change of current through the rheostat. It is seen from this that the current-carrying capacity of the various intercontact points need not be the same, and for this reason, rheostats are generally constructed with a current rating which varies from point to point on the rheostat. This must be given consideration in both the plate and box-type rheostat with the careful design of grid, ribbon, wire in vitreous enamel, or on open form units, to suit the particular circuit need. The combination of different resistance values

and different current values will result in a variable wattage rating at different points about the rheostat. In the usual construction the resistor elements are mounted on a supporting insulated plate on which are also mounted the contact buttons, contact arm, and other auxiliary equipment. For certain applications the resistor elements are surrounded by a metal enclosure filled with sand. The sand aids in supporting the elements and in the conduction of the heat energy from the resistor. The thermal conditions are different from those surrounding resistors in free air, and this must be considered in the design of the rheostat.

The assembly of the adjustable resistance rheostat, in many of its uses, requires considerable auxiliary equipment including contact arms, no-voltage release, overload circuit breaker, equipment for manual or motor operation, mounting, and case, and these variations are numerous and cause wide ranges in design. Specific information on these matters is given in Section 12 on Control and Protective Equipment.

WOUND VITREOUS ENAMELED UNITS. This type of unit finds its use where a resistor of relatively small capacity and of fixed value is needed. Certain variations on the main type are in use for special purposes, but generally the unit consists of a single winding running from one end to the other on a cylindrical ceramic tube. For power applications this type of unit is used primarily to dissipate small amounts of energy. The major application is found in the communication field where the vitreous enamel unit serves as a voltage divider in radio receivers and other equipment. Some application of this type of unit has been made in the construction of small rheostats.

WINDING. Windings on refractory porcelain tubes vary in wire size from strap down to diameters of about 0.002 in. This winding when imbedded in its protective coat of vitreous enamel reaches fairly high temperatures. For the maintenance of a small tolerance in resistance variation the wire should have a low temperature coefficient (see Section 2, Art. 6). The standard resistance tolerance on standard vitreous enamel resistors is $\pm$ 5 per cent. The temperature coefficient of expansion is likewise important for this type of resistor. Expansion and contraction in the winding and porcelain tube with temperature change should be similar so that the winding will not tear away from the tubing or be unduly stressed. The winding is run on the tube with the use of a lathe and automatic pitch setting. It is customary in resistors of this type for the units to be wound substantially full with wire of such size that the winding pitch will not be less than twice the wire diameter. Terminals are then applied to the ends of the winding, and to any tap points along the winding. Terminals of copper, monel metal, or some other alloy are secured to the ends of the winding usually by mechanical interlocking and then by brazing with special alloys or silver. The terminals are produced in many forms including lug type, flexible leads, ferrule type, cartridge type, A.R.A. unit type, and standard Edison base units.

VITREOUS ENAMEL COATING, ON WOUND UNITS. Certain requirements are of importance for the vitreous enamel used to coat the wound resistors. The primary requirements are that the coat should protect the wire and provide a conducting path for the heat generated in the wire to the outside air. In addition the vitreous enamel must maintain its insulation strength and must be moisture and corrosion proof. To maintain these latter characteristics the thermal expansion and contraction of the coating should be as nearly as possible equal to those of the winding and of the refractory base. This will aid in preventing chipping and cracking of the enamel. The unit is subjected to a red heat during the process of firing the enamel, and after cooling, the enamel should completely cover the winding and hold it in place without any mechanical strain, and should be free from bubbles or other imperfections.

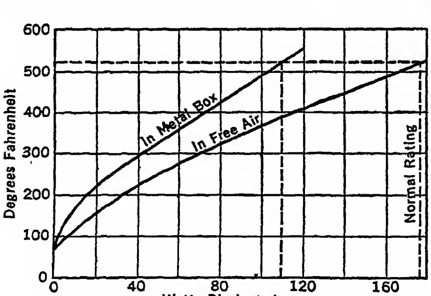

FIG. 3. Effect of Enclosing Vitreous Enameled Resistor in Metal Box. Unit when mounted in metal box reaches maximum temperature when dissipating only about 62 per cent of normal rating. (Courtesy Ohmite Mfg. Co.)

VITREOUS ENAMEL UNIT RATING. According to the construction of each unit, it is given a "normal rating" which is based on the assumption that the resistor, when mounted, will be surrounded with at least 1 ft of free air space, and that the circulation of air will be unrestricted. When so mounted the vitreous enameled resistor will dissipate energy at approximately 7 watts per square inch of surface area with a tem-

perature rise of not more than 250 deg cent when the temperature of the cooling medium does not exceed 40 deg cent. This is in accordance with the Standards of the National Electric Manufacturers Association. Whenever resistor units are mounted in a small enclosed space where air circulation is limited, the units should not be submitted to "normal rating." Where such conditions exist (say if the resistor operates in an enclosure), or if the resistor is submitted to over-rating surges, or one or more taps are built in, the unit should be worked at half or even quarter normal rating. The effect of enclosing, on the temperature of the hottest point (at the middle of the resistor) of a vitreous enameled resistor, is shown in the curves of Fig. 3. For operation in a metal box it is seen that a unit dissipates only 62 per cent of normal rating before reaching the maximum temperature.

VITREOUS ENAMEL UNIT, DIMENSIONS. Standard vitreous enameled units are made in a very wide range of resistance values and in normal ratings up to about 250 watts. A representative line of resistors is given in Table 3 along with approximate dimensions. Units may be had with various terminals, various numbers of taps, or adjustable taps, at slightly higher cost.

Table 3. Approximate Dimensions of Vitreous Enameled Resistors

Normal Rating, watts	Made in Units of Resistance, ohms	Approximate Size, inches Diameter × Length
5	1 to 10,000	1/4 × 1
10	1 to 30,000	3/8 × 1 3/4
20	5 to 100,000	1/2 × 2
30	5 to 100,000	5/8 × 3
50	10 to 100,000	5/8 × 4
75	100 to 100,000	5/8 × 5
100	25 to 100,000	7/8 × 6 1/2
200	25 to 100,000	1 1/4 × 8 1/2

SPECIAL FORMS. Low-resistance vitreous enamel resistors intended for heavy duty generally employ a strip of ribbon for the resistor element instead of wire. In this type the metal alloy resistance ribbon is wound on edge on the refractory tube. The support of the ribbon may be derived from grooves in the core, or the ribbon may be crimped into a wave form thus supporting itself on edge. In the latter case it is the practice to cover the element partially with vitreous enamel, holding the element in place and aiding in the heat dissipation. The fin-like form of the ribbon brings considerable area into contact with the cooling air. Both the enamel covered and uncovered units with edgewound resistors find their primary application in moderate rating power uses. Individual units are rated up to about 500 watts for continuous duty, and in proper combinations they serve for starting and control on motors up to about 50 hp.

Many applications of the wire-wound resistor require that the resistance from one unit be semi-variable. In the vitreous enamel unit a narrow contact strip along one side is not covered with enamel, and free contact with the resistance wire is possible. An adjustable lug is pressed into contact with the winding at the desired point, and together with the end terminals constitutes a three-terminal resistance. These units may be had in total resistance values of 1 to 100,000 ohms and in ratings up to 200 watts.

A refinement of the above resistor form appears in the sliding contact tubular rheostats which are designed for accurate control of currents up to about 25 amp. The winding is usually mounted on a glazed porcelain tube although an iron tube surfaced with vitreous enamel is sometimes used. Oxidized resistance strip or wire is used on the tube, the oxide providing insulation between the turns. A slider bar, mounted above the tube, supports laminated phosphor-bronze contact brushes, which make contact on the winding surface. Fine resistance adjustment may be effected by use of a micrometer drive, and for rough adjustment the slider is moved by hand. Such units may serve as rheostats or potentiometers according to connections and have wide application in laboratory and precision measurements. The various units obtainable range from about 0.4 to 40,000 ohms, in current-carrying capacities from 25 amp down to 0.1 amp.

Among the many other special forms of resistors for power use there is the current-limiting resistor for potential transformer fuses. This type of resistor is intended for use in series with potential transformer fuses to limit the short-circuit current, in case of short circuit, to such values that the fuses will interrupt the circuit safely. The resistors are of the wire type and consist of coiled nickel-chromium wire, wound in grooves of a heavy wet-process porcelain tube. These units are built in resistance values up to 225 ohms for voltages up to 25,000 volts.

4. NON-METALLIC RESISTORS

COMPOSITION TYPE. For industrial applications in the power field composition resistors are usually made of some silicon carbide composition, and used in rod or tube form as a resistance heating element or for special applications. The chief advantage of this type of resistor is that it can be operated continuously at temperatures up to about 2400 deg fahr which is above the permissible safe range of nickel-chromium resistors. The " Globar " type of resistor can, therefore, dissipate heat at a greater rate than a metallic resistor, and dissipation rates of 5 to 10 kw per sq ft are possible with the elements mounted adjacent to or in contact with refractory surfaces. This is about twice the safe rate for metallic resistors under similar conditions. The composition rod resistor manufactured under the name of Zircon may be used in all types of circuits where a non-inductive resistance is required. Its applications are numerous and include uses as resistor in series with potential transformer fuses; as relay shunts, for all purposes in surge testing circuits; for all d-c and a-c testing outfits; as charging resistors for electrolytic lightning arrestors; as high-resistance shunts for power-limiting reactors and many others. Zircon rods are manufactured in resistance values from 20 to 1,000,000 ohms, and although they may be run at temperatures as high as 150 deg cent they are not intended for use as heater units, and energy will be dissipated at only about 100 watts in the 1 by 12-in. rod.

CARBON RESISTORS. Carbon resistors with fixed resistance values find wide application in the field of radio communication and are therefore considered in the volume on Electrical Communication under "Resistors." Carbon rheostats, however, have considerable use for current regulation in many types of power circuits including the use as manual or automatic motor starting equipment. The carbon rheostat is generally made up of plates of carbon enclosed in a refractory-lined steel tube or frame. The variation of resistance is effected by applying varying degrees of pressure on the column of graphite disks. This varies the numerous points of contact between the individual disks and produces a contact resistance variation inversely as the pressure applied. The resistance of the column consists almost entirely of the contact resistance between the disks, and the graphite disks serve merely as contactors and heat dissipators. The advantages claimed for this type of compression rheostat include stepless control, durability, high overload capacity, wide resistance range amounting approximately to 75 to 1, freedom from sliding contacts, and full wattage capacity at any resistance setting. Applications of these characteristics range from battery-charging installations to the starting and control of 500-hp motors. Such control applications are discussed in Section 12.

LIQUID RHEOSTATS. Water as from the tap supply or containing small quantities of a salt, acid, or base may be used as an electrical resistance. Some of the substances which have been used are $NaCl$, NH_4Cl, KOH, and H_2SO_4. The device may serve as a rheostat by the movement of one electrode with respect to the other. A single phase load of 100 amp at 100 volts may be carried continuously in a 40-gallon barrel without causing the water to boil. The plates should have a surface (one side only) of at least 1 sq in. per amp.

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CAPACITORS—CONDENSERS

An electric condenser is a device the primary purpose of which is to introduce capacitance into an electric circuit. It consists essentially of two conductors with their surfaces relatively close together, separated by an insulating medium called the dielectric. The dielectric is that part of the condenser which is mainly responsible for the properties of the condenser itself. The characteristics of materials generally used as dielectrics are presented in Section 2, Materials. The action of the condenser as a circuit element is presented in Section 3.

5. PROPERTIES OF CONDENSERS

CLASSIFICATION. Condensers are usually classified in terms of the material used for dielectric. This classification serves readily to distinguish most condensers, but two special forms must be considered separately. These are the synchronous condenser and the electrolytic condenser. The synchronous condenser is a synchronous motor operated with its excitation so adjusted that it has the properties of a condenser. It is operated in this manner for power-factor correction on transmission lines. A discussion of the synchronous motor in this and other applications is given in Section 9, A-c Generators and Motors. To distinguish condensers used for power factor correction, not of the synchronous machine form, the classification " static " is given. This classification may well include all condensers without moving parts. The other special form of condenser is the electrolytic. The capacitance action of this cell arises from the formation of an extremely thin insulating layer at the anode on an electrolytic cell. The insulating layer serves as the dielectric between the metal anode and electrolyte which are the "plates." This form of condenser has its major application in power supplies for radio receivers. It will therefore be discussed in the volume on Electrical Communication.

Classification of condensers according to the dielectric used between the conducting plates will form a division into the following groups: (1) impregnated paper condensers, (2) mica condensers, (3) glass condensers, (4) condensers with a fluid dielectric (oil, air, etc.). This classification might be extended to include the listing of practically every insulating material known. Those noted above are, however, the most important and the most widely used. In order to understand the factors in the design and use of these condensers, the development of some of the important condenser formulas will be given. Condensers may be divided into two other broad groups—those with fixed capacitance (generally solid dielectric such as impregnated paper, or mica) and those in which the capacitance can be varied. The latter generally have fluid dielectrics (air, oil) and have wide use in radio circuits. The characteristics of the variable condenser will therefore be treated in the volume on Electrical Communication under " Condensers." The following development applies particularly to solid-dielectric fixed-capacitance condensers.

CAPACITANCE. The capacitance, which is a measure of the electrical charge that a condenser will retain under a given potential, depends upon three factors: the area of the dielectric separating the plates, the thickness of the dielectric, and its specific inductive capacity (dielectric constant). When the two conducting plates are parallel, close together, and of large area, as in most paper, mica, and glass condensers, the capacitance is given by the expression,

$$C = 0.0885 \times 10^{-6} KS/\tau$$

where C is in microfarads, S = area of one side of one conducting plate in square centimeters, τ = thickness of dielectric between the plates in centimeters, and K the dielectric constant (= 1 for air and between 1 and 10 for most ordinary substances). This formula assumes that the charge is uniformly distributed over the surfaces of the conductors, no corrections being made for edge or end effects. It is seldom worth while, however, to apply a correction on this account, because the capacitance to the condenser case or other conductors is ordinarily not calculable, so that the actual capacitance of a condenser can be calculated only approximately. The actual value is likely to be in excess of that calculated. When very accurate values are required they must be obtained by measurement.

POWER LOSS. In an ideal condenser, the conductors or plates would have zero resistance and the dielectric infinite resistivity in all its parts. If an alternating emf is applied to such a condenser, a-c current will flow into the condenser. At the moment when the emf is a maximum no current will be flowing, and when the emf is zero the current will be a maximum. Hence, in a perfect condenser the current and voltage are 90 deg out of phase. In actual condensers the conditions as to resistance in the plates and dielectric are not fulfilled, and in consequence an alternating current flowing in a condenser is not exactly 90 deg out of phase with the impressed voltage. The difference between 90 deg and the actual phase angle is called the " phase difference " or phase defect angle. In an ideal condenser there would be no consumption of power; the existence of a phase difference means a power loss, which appears as a production of heat in the condenser. The amount of power loss is given, as for any part of a circuit, by $P = EI \cos \theta$, where θ is the phase angle between current and voltage and $\cos \theta$ is the power factor. This is equivalent to

$$P = EI \sin \psi$$

where ψ is the phase difference and $\sin \psi$ is the power factor of the condenser. In all except extremely poor condensers, ψ is small, $\sin \psi = \psi$, and thus the phase difference and power factor are synonymous. The power loss is given by $P = \omega CE^2 \sin \psi$. This shows that, for constant voltage, the power loss is proportional to the frequency, to the capacitance, and to the power factor. The power loss in a condenser may be due to either imperfection of the dielectric or to resistance in the metal plates or leads. The dielectric may cause a power loss either by current leakage, by brush discharge, or more commonly by the phenomenon described later under the head of "Dielectric Absorption."

LEAKAGE. The leakage of electricity by ordinary conduction through the dielectric or along its surface contributes to the phase difference at low frequencies. The effect of leakage on the power factor may be seen as follows: A condenser having leakage may be represented by a pure capacitance with a resistance in parallel. The current divides between the two branches, the current I_r through the resistance being in phase with the applied E, and the current I_c through the capacitance leading E by 90 deg. The resultant I leads E by an angle which is less than 90 deg by the phase difference ψ. From Fig. 1, $\tan \psi = \dfrac{1}{r\omega C}$.

The effect of r may be shown by an example. A condenser of 0.01–μf capacitance with an insulation resistance as low as 10 megohms has, at a frequency of 60 cycles per second, a power factor $= \dfrac{1}{(10)^7\, 377 (10)^{-8}} = 0.027 = 2.7$ per cent. This is a very appreciable quantity; 2.7 per cent of the current flows by conduction instead of by dielectric displacement. This effect, however, decreases as the frequency increases, for the dielectric current increases in proportion to the frequency while the leakage current does not. For instance, at 10,000 cycles, the power factor = 0.00016.

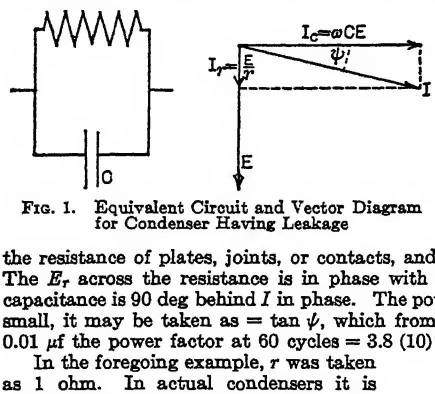

Fig. 1. Equivalent Circuit and Vector Diagram for Condenser Having Leakage

SERIES RESISTANCE. A resistance within a condenser in series with the capacitance affects the power factor very differently from a resistance in parallel. The series resistance includes the resistance of plates, joints, or contacts, and the leads from binding posts to plates. The E_r across the resistance is in phase with the current I, and the emf E_c across the capacitance is 90 deg behind I in phase. The power factor $= \sin \psi$, and since ψ is usually small, it may be taken as $= \tan \psi$, which from Fig. 2 is $r\omega C$. If $r = 1$ ohm and $C = 0.01\ \mu$f the power factor at 60 cycles $= 3.8\,(10)^{-6}$. This is a negligible quantity.

In the foregoing example, r was taken as 1 ohm. In actual condensers it is sometimes greater than this. A plate resistance of several ohms is common in the ordinary paper condenser. In most other condensers, a high series resistance indicates a defect.

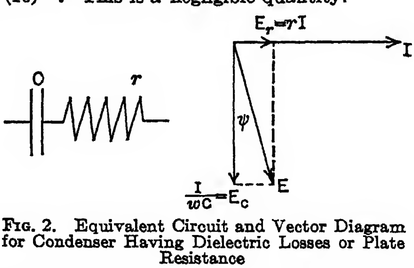

Fig. 2. Equivalent Circuit and Vector Diagram for Condenser Having Dielectric Losses or Plate Resistance

DIELECTRIC ABSORPTION. When a condenser is connected to a source of emf such as a battery, the instantaneous charge is followed by the flow of a small and steadily decreasing current into the condenser. The additional charge seems to be absorbed by the dielectric. Similarly, the instantaneous discharge of a condenser is followed by a continuously decreasing current. It follows that the maximum charge in a condenser cyclically charged and discharged varies with the frequency of charge. The phenomenon is similar to viscosity in a liquid, and is sometimes called dielectric viscosity.

Dielectric absorption is always accompanied by a power loss, which appears as a production of heat in the condenser. Since energy is expended in the periodic stressing of the dielectric material, the phenomenon is similar to magnetic hysteresis in the ferrous metals. It therefore is frequently termed dielectric hysteresis. The existence of a power loss signifies that there is a component of emf in phase with the current. The effect of absorption is thus equivalent to that of a resistance either in series or in parallel with the condenser. It is found most convenient to represent absorption in terms of a series resistance, which is spoken of as an equivalent resistance of the condenser. An absorbing

condenser is, therefore, considered from the standpoint of Fig. 2 and the power factor $= r\omega C$. The equivalent resistance r is constant for a given frequency but is different for different frequencies.

EQUIVALENT SERIES RESISTANCE. In the foregoing paragraphs it was seen that losses in a condenser could be classified under the following heads:

1. Leakage of current through or around the dielectric, appreciable when the insulation resistance of the material is low.
2. Actual conductor series resistance of leads, connections, and the plates themselves.
3. Dielectric absorption or hysteresis.
4. Corona from plate edges, important at high voltages.

In the discussion, the effect of each of these, with the exception of corona, has been individually shown on the power loss and power factor of the condenser. In practice, all the losses exist to some degree and each contributes to the power loss of the condenser. Generally, it is difficult to determine the individual losses separately, and therefore for any given set of conditions they are determined together. Grouping these effects, we attribute the losses in the condenser entirely to an "equivalent series resistance." This is indicated in Fig. 3. Except when insulation resistance is intended, a discussion of the "resistance" of a condenser always means the equivalent series resistance. The value of this resistance when multiplied by the square of the alternating current flowing in the condenser represents the rate of energy consumption in the condenser. Repeating the expression for power in a condenser

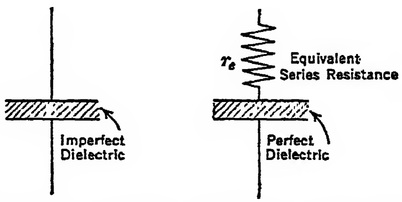

FIG. 3. Equivalent Circuit of Actual Solid Dielectric Condenser Showing all Loss Effects Lumped into One Equivalent Series Resistance

$$P = \omega CE^2 \sin\psi = I^2 r_{\text{equivalent series}}$$

DIELECTRIC STRENGTH. When the leakage of electricity by conduction through the dielectric becomes excessive, the dielectric ceases to be an insulator and breakdown occurs. The leakage, in any given material, depends upon the voltage on the plates, the shape of the dielectric, its size, homogeneity, and temperature. For any given dielectric under a set of physical conditions there is a limiting voltage beyond which the dielectric strength of the material will be exceeded. See Sect. 2, Art. 9.

COMPARATIVE MERITS OF DIELECTRICS. In a comparison of dielectrics the following characteristics are of importance and must be considered as under the conditions of intended use: (1) the physical constitution of the material (fluid, solid); (2) the dielectric constant; (3) the power factor; (4) dielectric strength. The product of the dielectric constant and the power factor is sometimes used as another means for comparing dielectrics. From the expression previously stated for the power loss in a condenser, $P = E^2\omega C \sin\psi$. When the phase difference angle of the condenser is small, $\sin\psi = \psi$. If V is the volume of dielectric used, E the voltage across it, f the frequency, K the dielectric constant, ψ the phase difference, P the power used in the dielectric due

Table 1. Dielectric Data*

Approximate Low-Frequency Characteristics

Material	K	Power Factor $10^4 \times \tan\psi$	KV eff. per centimeter
Paraffin	2	0.1–1	200
Halawax	3.7†	50†	...
Mica	6	10	400
Mineral oil (transformers)	2	1–10‡	100
Bakelite	5–6	1–10	100
Quartz	4–5	1	1100–1500§
Porcelain	5–6	400	200
Flint glass	7	40	400
Crown glass	6–7	40	400

* From Theory of Dielectrics, A. Schwaiger, John Wiley & Sons, 1932.
† From J. A. Lee and H. H. Lowry, *Indust. & Eng. Chem.*, February, 1927, p. 302.
‡ From Liquid Dielectrics, A. Gemant, John Wiley & Sons, 1933.
§ From The Electrical Properties of Glass, J. T. Littleton and G. W. Morey, John Wiley & Sons, 1933.

to dielectric hysteresis, and m a constant depending upon the various units employed, the above expression becomes

$$\frac{P}{V} = mE^2f(K\psi)^*$$

Thus, the product $K\psi$ gives a measure of the relative merits of two dielectrics so far as power loss is concerned, and it is called the loss factor.

MEASUREMENT OF CONDENSER CHARACTERISTICS. Measurement of any condenser characteristic should always be made, if possible, under the same physical and electrical conditions as will later surround the actual use of the condenser. Temperature, humidity, atmospheric pressure, the surroundings of the unit under test; the circuit voltage, wave form, and frequency all have some effect on the various characteristics (see Section 5). It should be carefully noted that none of the " constants " of a condenser are constant. As the above conditions vary, condenser capacitance, power factor, equivalent series resistance, and the dielectric insulation resistance, dielectric constant, and breakdown strength all vary. These variations have major importance in many applications of condensers both in the power and communications fields of development.

6. CONSTRUCTION AND FIELDS OF APPLICATION

The choice of a condenser for a particular application is governed by several considerations of an economic and electrical nature. Usually the first consideration is the amount of capacitance required in the unit to be constructed. Then, the voltage across the condenser, and the circuit frequency must be determined. With these known the current flow through the condenser is calculated. For a given set of electrical conditions the questions next answered are: In how much volume can this condenser be built? Are there any restrictions as to size? The variation of any one of these factors affects practically all the others, and dominating all, is the factor of condenser cost. To build a condenser for high voltage requires the use of a pure, uncontaminated dielectric of the requisite high dielectric strength. To build a small condenser requires the use of a thinner dielectric, of high dielectric constant and the requisite dielectric strength. To build a condenser in which the capacitance will be unaffected by humidity, and which will remain constant, requires special care in impregnating and encasing. To build a condenser in which the capacitance of all the units is practically the same with set limits of tolerance requires uniform material and uniform processing. All these factors, affecting as they do the choice of dielectric, the impregnation of the dielectric, and the assembly of the components essentially determine the cost of a condenser. The various factors are weighed against each other and a final selection made.

CONDENSERS WITH PAPER DIELECTRIC. More units of this type of condenser are produced and put in use each year than of any other type. The materials of construction and the variety of forms permit a wide range of quality, and therefore, cost. The field of telephone communication has the most extensive use for this type of condenser. The paper condenser is practically always produced as a fixed capacitance winding of some metal foil interleaved with one or more sheets of impregnated paper. Such rolls of multiple layers of foil and insulation build up a comparatively large capacitance in relatively small volume. The unit is usually encased in a metal or cardboard container to protect it from mechanical damage. The paper used as the insulating dielectric between the foil plates must be thin, uniform, and continuous, must be neutral chemically and free of conducting particles. The paper usually is 0.5 mil thick, but to secure further decreases in size a 0.4-mil paper is extensively used. The paper is usually impregnated with some compound to give it a higher effective dielectric constant in the finished state. The most common impregnating material has been paraffin. Many other oils and preparations are in use, and a chlorinated naphthalene, commercially known as halawax, has shown excellent characteristics for this service.

Materials most generally used as conductors include tin foil, aluminum foil, and metallized paper. Aluminum and tin foil are prepared in thicknesses down to 0.25 mil. Aluminum foil is more desirable generally, being lighter, but it is more difficult to solder. Metallized paper or Mansbridge condensers use an extremely thin sheet of foil deposited on the paper to form a conducting plate. In this type of condenser, should a dielectric failure occur, the thin metal surface around the fault vaporizes and the fault is usually cleared. This self-healing is a most valuable characteristic and usually possible only in fluid-dielectric condensers. The very thin conductor plates, however, lead to high conductor resistance and do not readily permit the construction of a non-inductive winding.

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* After Hoch, *Bell Sys. Tech. J.*, November, 1922.

The assembly process consists of rolling the two sheets of metal foil, and in one particular case, four sheets of paper so that the completed unit will have two layers of paper between adjacent sheets of tin foil. The winding may be either inductive or non-inductive. Inductive windings use foil which is about 0.5 in. narrower than the paper. The section is wound up with equal paper margins on each side. Tinned copper strips are laid in at the end of the winding to enable a connection to be made. The non-inductive winding uses foil the same width as the paper, foils of opposite polarity protruding about 0.25 in. on the opposite edges of the winding. High-frequency applications usually require the non-inductive winding, and this type of winding facilitates heat dissipation by thermal conduction through the electrical connections. In such a condenser the maximum distance the charging current has to flow is the width of the foil, whereas in the inductive winding the charging current must run the whole length of the winding.

After winding, the unit is pressed into compact shape and then while still in a high vacuum (approximately 0.25 in. mercury) is impregnated with the oil or wax. If the air were not completely removed the effective dielectric constant of the insulating medium would be small owing to the effect of the low dielectric constant of the air. Moisture in a condenser very seriously increases its losses, and therefore both air and water vapor are carefully removed. After impregnation the unit is pressed to the required size, forcing out the excess wax or oil, and the soldering lugs are fastened to the metal contact strips which are in contact with or soldered to the sheets of tin foil. If, during the cooling of a wax impregnated condenser, the surface coating cracks, moisture and air will re-enter and spoil the condenser. The ends of the cooling condensers are sometimes covered with a layer of soft non-cracking wax to prevent this. The unit is then ready for potting in a metal container. These are partially filled with a sealing compound, which, when the condenser units are inserted, completely fills the container and seals the condenser against all entrance of moisture. This sealing compound likewise performs the valuable function of conducting the condenser energy dissipation to the outside cooling can and air. The condenser is now ready to withstand severe conditions of continuously applied voltage, high humidity, high temperature, and mechanical abuse.

Since 1926 the standard condensers produced by the Western Electric Company have gone through considerable development. With the increased use of hydraulic press, power winding, prepressing, and automatic sorting, and with the development of aluminum foil, thinner papers, and the new wax, the cost of manufacture of a unit has been about halved. At the same time the volume of the standard 1-μf unit in container has been decreased from 5.85 cu in. to 1.84 cu in. The winding length which was 238 in. in 1926 is reduced to only 95 in.

A paraffin-impregnated condenser using a good grade of 0.5-mil paper will stand from 100 to 200 volts for each layer of paper used. For high voltages it is necessary to use as many layers of paper as are necessary to withstand the dielectric stress. With dielectric loss occurring in a condenser when it is used in an a-c circuit, it is necessary to rate the condenser for a lower alternating voltage than would be permissible in a d-c circuit. Since at high frequencies the voltage across a condenser (for any current) becomes less, it is necessary to rate the condenser in terms of safe heating of the dielectric. This requires that a current limitation be placed on the use of the condenser, and obviously the safe volt-ampere rating will vary with frequency.

In a well-made impregnated-paper condenser the insulation resistance will be from 100 to 1000 megohms per microfarad. Since insulation resistance varies inversely as the capacitance of a condenser, the smaller condensers will have greater insulation resistance. Manufacturing specifications have set definite limits on the insulation resistance for condensers in certain uses. For instance, the N.E.M.A. Standard 334–216 states "... the insulation resistance shall not be less than 500 megohm-microfarads, at a capacitor temperature of 68° F (20° C)." This particular standard applies to fixed paper condensers used in power supply circuits for radio equipment.

Table 2 represents the construction practice of one manufacturer of condensers:

CONDENSERS WITH MICA DIELECTRIC. Mica as a condenser dielectric has established itself as one of the most efficient materials for use in fixed low-capacitance but high-voltage condensers. When pure, it has a very low dielectric loss, but it acts as a fair adsorbent of water. Even a thin layer of water only one molecule thick on the faces of the mica will enormously increase its dielectric loss. Mechanical injuries such as surface scratches or any disturbance of the crystalline structure will cause very large dielectric losses. The assembly of mica dielectric and metal foil sheets as a condenser follows the main steps in the similar process for paper-condenser manufacture. The mica condenser, however, is made of alternate flat sheets of insulation and foil and is not rolled. Built for high voltage, a mica condenser consists of a combination of individual condensers called sections. These are connected in series forming stacks, which may be

Table 2. Insulation and Voltage Specifications
For Paper Condensers

Maximum D-c Working Voltage	Maximum A-c Working Voltage	* D-c Retest Voltage	Total No. of Papers	No. of Papers between Plates	Thickness of Each Paper, mils
200	125	400	4	2	0.5
300	175	600	6	3	0.4
400	250	800	6	3	0.5
500	300	1000	8	4	0.4
600	350	1200	8	4	0.5
800	440	1600	10	5	0.5
1000	600	2000	12	6	0.5
1000	600	2000	6	3	1.0

* N.E.M.A. Standard 334–111: "The standard voltage test for paper capacitors shall be a single application of two times the working voltage—and the immediate discharge through a resistor of sufficient ohmic resistance to limit the discharge current to not more than 1 ampere, the entire cycle of operation not to exceed 15 seconds."

connected in parallel with other stacks. A section is made of a definite number of sheets of mica and a number of metal foils. The usual form and size of the mica is a rectangle about 1.5 in. square, and generally not over 2 by 3 in. Thicknesses of 1.0 mil to 4.0 mils are used. The foil area is correspondingly smaller to allow for at least $^1/_8$-in. margin. Odd and even foils protrude on opposite sides of a section, and the foils of like polarity are soldered together. After moisture and air are driven off, the stacks are adjusted for capacitance, encased, and clamped. The many small mica units which have extensive communication circuit use are encased in molded Bakelite. Metal containers, however, are used for the larger units and for high-voltage work usually require insulator bushing terminals of high dielectric strength. For a mica condenser to hold its accuracy, have no drift with time, and not vary with atmospheric conditions, the case of the condenser must be moisture proof and maintain a constant pressure on the condenser unit. For precision work this requires some constant-pressure device, pressing on the sections at all times. Mica condensers serve extensively in high-voltage circuits for both the power and communication fields. Among these uses there are the radio transmitter, high-frequency furnace circuits, and carrier current communication systems. Their use in small sizes, either fiber or molded Bakelite encased, in radio receivers is very extensive.

CONDENSERS WITH GLASS, RUBBER AND OTHER SOLID DIELECTRICS. As in the mica and paper condensers, numerous other solid dielectrics are used in the construction of fixed condensers. Perhaps the most familiar of these is the historic Leyden jar in units of 0.002 to 0.004 μf. This consists of an open-mouthed cylindrical jar usually with an inner and outer surface coating of foil or copper. These have had extensive use and exist now in numerous special forms. Moscicki's modification of the Leyden jar diminished the loss in this type of condenser due to brush discharge. This was accomplished by increasing the thickness of the dielectric near the edges of the metal coatings. Another modification submerges the whole jar in oil to reduce brush discharge loss. The materials suggested in the above heading along with many others are generally used as insulators separating the moving from the fixed plates in a variable-capacitance condenser.

CONDENSERS WITH FLUID DIELECTRICS. Air and various inert gases and certain mineral and vegetable oils are used as dielectrics in condensers. It has already been seen that wax and oil improve the dielectric constant of paper in a condenser. Between the supported plates of an air condenser a good oil will permit a high voltage, will have a low brush and dielectric loss, and usually will have a relatively large dielectric constant. The last feature permits a smaller condenser for a given capacitance. Precisely the same effects result when air or an inert gas is compressed and serves between open plates. The breakdown voltage becomes very high; as much as 35,000 volts may be used between plates spaced 3 mm apart. Fluid-dielectric condensers have their major use where a continuously variable capacitance is desired. Inert gas under pressure is occasionally used to permeate the paper or linen dielectric of high-voltage condensers used for power-factor correction.

ELECTROLYTIC CONDENSERS. The electrolytic condenser has a most extensive use in the field of communications, and a complete discussion of its characteristics is given in the volume on Electrical Communication. However, numerous applications have been developed in the industrial field and include use as a filter condenser, in spark suppression and noise filters on d-c contactors, circuit breakers, commutators, motors, and other d-c apparatus. A brief description of the major characteristics follows:

Certain metals, notably aluminum and tantalum, when placed as anodes in a suitable electrolyte, become coated with a film having unusual electrical properties. Films formed in this manner are characterized by the influence of impressed potential on their electrical resistance. This resistance imparts to the film the capability of conducting current more freely in one direction than in the other; of breaking down as an insulation between the metallic electrode and the solution when voltages above a critical value are applied; and, in combination with the thinness of the film, of holding a substantial charge at potentials below the breakdown voltage. This last characteristic brings the device into the category of a condenser. As a typical case; if an aluminum plate in an ammonium borate solution were formed to 30 volts the film is probably about 0.00001 mm thick and shows the very large capacitance of 0.2 μf per sq cm. Forming to a higher voltage of 350 volts increases the thickness of the film and reduces the capacitance to about 0.018 μf per sq cm of area. The use of this type of condenser is complicated with several special requirements. A polarizing direct voltage must be maintained on the condenser in the proper polarity at all times. There is a resulting leakage current flowing at all times. To this loss must be added the high loss in the electrolyte which results in a power factor ranging from 10 per cent up. The performance of these condensers is very sensitive to temperature, and the characteristics vary with temperature, frequency, forming voltage, polarizing voltage, operating voltage, and time. These facts coupled with the high loss make this condenser useful only in circuits where a large capacitance is needed, and this explains its appearance generally in filter circuits. A more convenient form of this condenser is the " dry " type in which the oxide film has been formed on one of two aluminum sheets and these sheets are separated by an electrolyte-saturated gauze.

7. CAPACITORS FOR SPECIAL USES

POWER-FACTOR CORRECTION. To be of use in power-factor correction a capacitor must possess two important qualifications. First, in order to compete with the synchronous condenser, it must have very low losses, for the losses of a synchronous condenser are only about 1 1/2 to 5 per cent of its kva rating. The static capacitor meets this requirement quite well by normally having losses amounting to about 0.25 per cent of its rating. When connected into a high-tension circuit by use of an auto-transformer the over-all power factor is between 1 and 1 1/2 per cent. Second, the static condenser must have a dielectric of very sure and quite high dielectric strength. There are several means of developing high dielectric strength, through the use generally of solid dielectric materials permeated with oil, special preparations, or inert gases. Oil replaces wax in the construction of condensers for high-voltage service for the combination of oil and paper is a much better conductor of heat than that of wax and paper. Mineral oil of about the same grade used in transformers is commonly used as an impregnating material. The several types of papers which have been used include Kraft, a combination of wood pulp and cotton and pure linen tissue.

There has been described * a new impregnating fluid for use in capacitors which is called Pyranol. This synthetic, non-inflammable, insulating liquid is used as a treating and filling material to replace waxes and mineral oils formerly used. It has a high dielectric constant and has chemical stability under high temperatures and electrical stress. Condensers impregnated with Pyranol are used in high-voltage surge generators and in general where great reliability at high voltage is required.

Illustrative of the capacitor using an inert gas as part of its dielectric is the " Permittor," † a commercial static capacitor used generally for power-factor correction. The Permittor makes use of nitrogen under pressure which serves as a cooling medium, and, in combination with a high-grade insulating paper, as the dielectric. The penetration of the gas into the paper results in a dielectric of high dielectric strength, low losses, and perfect chemical stability. The heat-transfer properties at the 225-lb-per-sq-in. pressure used are similar to those obtained with insulating oils. The condenser units are built up into rolls consisting of alternate layers of aluminum foil and layers of the high-grade tissue. The foils of opposite polarity protrude at opposite ends of the roll, and flanged disks of aluminum cover the ends, making good electrical and thermal contact. The rolls are assembled in a tank of pressed steel which serves as the chamber for the high-pressure nitrogen. The losses of this type of capacitor range between 0.2 and 0.4 per cent of the kva rating through the operating range of temperatures. This type of unit is built for voltages from 440 volts up to 33,000 volts.

COUPLING CONDENSERS TO HIGH-TENSION TRANSMISSION LINES. A very definite need exists for communication between the various parts of a power

* General Electric Co. † Products Protection Corp.

system. A means, immediately available, are the transmission lines themselves which link the various generating and distributing units together. Means must be had for this purpose, to connect to the power line without loss of power and without danger to the users of the communication system. The ideal linking unit is the capacitor with its high impedance to the power-frequency current and relatively low impedance to carrier-frequency currents. Another use that may be served through the power lines is through tapping the lines directly to secure a small amount of power for relay or signal operation. The capacitor coupling has also been used with synchronizing equipment, voltage indication, and metering.

The general form taken by the coupling capacitor is the condenser type bushing, originally developed as a means of surge protection for high-tension equipment. This type of bushing has inherent capacitance potentiometer characteristics and readily provides a source of low voltage. The insulator potentiometer has relatively high capacitance, and as it is constructed of concentric metal-foil cylinders about the high-voltage conductor, it forms an ideal capacitance potentiometer. The metal foil cylinders are the plates and the micarta is the dielectric of a series of condensers which can be considered as connected in series from the high-voltage conductor to ground. By adding a tap to the last step to ground a potentiometer is easily obtained.

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INDUCTORS

An inductor is a device the primary purpose of which is to introduce inductance into an electric circuit. The term reactor is given to devices which introduce reactance into a circuit. There are numerous applications for such devices in electrical circuits, and the characteristics of these as used in power circuits are presented in this chapter. Current control is probably the most important function of an inductor, and this function is performed both continuously and intermittently. Among the applications are the following: short-circuit current limitation in power systems; surge and lightning protection in transmission lines; synchronous converter compounding; shunting of field windings, lamps, and other devices; ripple suppression in filters, welding equipment; current control through lamps, relays, motors, circuits; high-frequency oscillation suppression; parallel operation of unlike transformers; control of potential as in the auto-transformer; and numerous other circuit uses. The actual applications in the particular uses are presented in the appropriate sections of this volume.

8. AIR-CORE INDUCTOR

INDUCTANCE FORMULAS. The inductance of a conductor is a direct function of its dimensions and its arrangement, and for any given conductor the inductance may be increased by forming the conductor into a coil. With a given length of wire, wound with a given pitch, that single-layer coil has the greatest inductance, which has such a shape that the ratio $\frac{\text{diameter}}{\text{length}} = 2.46$ approximately. A measure of this increase of inductance for a given length of conductor may be had in the computation of the inductance of a conductor when straight and when formed to the optimum coil. An expression giving the approximate value of the inductance of a single-layer solenoid

(see Fig. 2) is $L = 0.03948a^2n^2K/b$ microhenrys, where n = number of turns of the winding, a = radius of the coil, measured from the axis to the center of any wire, b = length of the coil, and K is a function of $2a/b$, values of which have been calculated by Nagaoka. Dimensions a and b are expressed in centimeters. For a 10-turn winding of

Table 1. Value of the Constant K (Nagaoka's Constant) for Various Ratios of Diameter to Length. Single-Layer Solenoid

Diameter/Length	K	Diameter/Length	K	Diameter/Length	K	Diameter/Length	K
0.00	1.000	0.50	0.818	1.00	0.688	3.50	0.394
.05	.979	.55	.803	1.10	.667	4.00	.365
.10	.959	.60	.789	1.20	.648	4.50	.341
.15	.939	.65	.775	1.40	.611	5.00	.320
.20	.920	.70	.761	1.60	.580	6.00	.285
.25	.902	.75	.748	1.80	.551	7.00	.258
.30	.884	.80	.735	2.00	.526	8.00	.237
.35	.867	.85	.723	2.50	.472	9.00	.219
.40	.850	.90	.711	3.00	.429	10.00	.203
.45	.834	.95	.700				

No. 10 A. W. G. round copper wire formed into a compact coil of 2.46-in. diameter, and length of about 1 in., a calculation from the formula above gives an inductance value of 7.2 microhenrys. The inductor of this problem contains 77 in. of wire, and its inductance, when straight and distant from other conductors, is given by the expression $L = 0.002l\left(\log_e \frac{2l}{r} - \frac{3}{4}\right)$ microhenrys, where l = length of wire, r = radius of wire, measured in centimeters. A calculation from this expression for the straight wire inductance gives a value of about 2.85 microhenrys. The comparison of the two calculations shows an increase of over 2 1/2 times in the inductance value by the change from the straight wire to coil form.

Brooks has determined (*Bur. Standards J. Research*, Vol. 7, p. 289, August, 1931) that there is a most efficient multilayer coil form to produce the maximum inductance with a given length of conductor. This most efficient inductor was produced as a compact multilayer cylindrical coil with a mean diameter 2.95 times the side of the square cross-section. Other proportions varying somewhat from the optimum affect the inductance

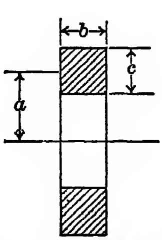

FIG. 1. Multiple Layer Inductor with Winding of Rectangular Cross-section

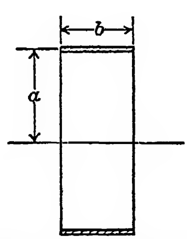

FIG. 2. Inductor with Single Layer Winding

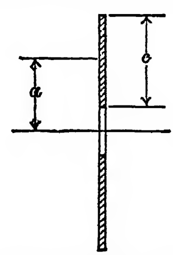

FIG. 3. Inductor with Spiral Winding

only slightly. When the ratio of mean diameter to side of square cross-section is 2.80, the resulting inductance is only 0.04 per cent less than the maximum value. It is generally convenient and within limits of accuracy to consider the optimum form as having the dimensions $2a = 3b$ with $b = c$ (see Fig. 1). For the optimum form of multilayer coil the accurate expression of inductance has been modified, giving the expression $L = 0.04274an^2$ microhenrys where a is measured in inches and n = the number of turns. This formula presupposes a uniform current distribution over the square cross-section of the coil.

Some general simplified formulas for the inductance calculation of air-core inductors have been given by Wheeler (*Proc. I.R.E.*, Vol. 16, p. 1398, October, 1928) and apply to the multilayer (Fig. 1) single-layer helical (Fig. 2), and single-layer spiral coil (Fig. 3).

The expression for the inductance of the multilayer winding

$$L = \frac{0.8a^2n^2}{6a + 9b + 10c} \text{ microhenrys} \tag{1}$$

is accurate to ± 1 per cent when the terms in the denominator are about equal. All dimensions are inches. The single-layer helical winding (Fig. 2) has an inductance expressed as

$$L = \frac{a^2 n^2}{9a + 10b} \text{ microhenrys} \tag{2}$$

which is accurate within ± 1 per cent when $b > 0.8a$. The inductance of the single-layer spiral coil (Fig. 3) is given in the expression

$$L = \frac{a^2 n^2}{8a + 11c} \text{ microhenrys} \tag{3}$$

or in terms of Fig. 2

$$L = \frac{a^2 n^2}{8a + 11b} \tag{4}$$

This expression is accurate to ± 5 per cent when $c > 0.2a$ (Fig. 3), or $2a > b > 0.2a$ (Fig. 2). In general form eq. 2 should be used for helical coils, whenever $b > a$. For short helical coils eq. 4 is more accurate, particularly when $a > b > 0.2a$. The accuracy of these expressions may be less than stated for coils of very few turns or when skin effect or distributed capacitance is appreciable.

If the coil is mounted on a toroidal form instead of a tubular cylinder, its inductance in microhenrys is

$$L = 0.01257n^2 (R - \sqrt{R^2 - a^2}) \tag{5}$$

in which R = distance from the axis to the center of the winding cross-section, a = radius of each turn of the winding, and n = number of such turns (all dimensions in cms). Using the 77 in. of conductor formed into a toroid an inductance of several microhenrys could be designed.

CONSTRUCTION. The turns of an inductor winding generally must be supported, and where no magnetic core is used, the coil is formed on a frame of wood, composition, concrete, or some other good insulator. If the winding has sufficient stiffness to hold its form it may have no support but rest entirely on the terminal connections. Provision must be made in the construction of an inductor for the electrical insulation of closely adjacent turns, the liberation of the energy generated in the winding, and the strengthening against interwinding stresses. This last factor is important to the construction of series reactors built for short-circuit protection in electric generating systems.

ENERGY DISSIPATION. The losses in the inductor are due to the heat energy liberated in the wire following the flow of current. A measure of this loss is had from the expression for the power which is $P = I^2 r$, in which P the power in watts, I the effective current in amperes, and r the resistance of the winding. The liberated heat energy may be carried away by fluid convection through the winding or by the supporting frame. Construction according to accepted practice will limit the temperature rise of the windings to values specified in the standards of the A.I.E.E. Normal operation will require that the insulation between turns withstand certain potentials. Generally, insulation will be of such a character that it will withstand potentials in excess of normal operation, but occasional abnormal conditions place very high stress on the insulation. Such conditions arise when an attempt is made to open an inductive circuit. As the circuit switch is opened the current must decrease from its steady value to zero. As the current diminishes rapidly, the voltage of self-induction is very high, and maintains an arc at the switch contacts until the energy of the magnetic field is dissipated. The energy stored in the field is $LI^2/2$ joules, with L in henrys and I effective amperes of current. In circuits of high inductance value, provision is generally made to dissipate this energy safely in a discharge resistance which is connected temporarily into the circuit as the switch is opened.

DISTRIBUTED CAPACITANCE. Since the adjacent turns of the winding on an inductor are at a difference of potential, and electrically insulated, they constitute the elements of a capacitor. The capacitance between any turn to the others of a winding affects the inductance action of the device, particularly at high frequencies. The effect is not of great importance in power applications, except when an inductor is being used to suppress a high-frequency oscillation. In this use the distributed capacitance would by-pass the high-frequency currents and nullify the choking action of the inductor.

9. INDUCTOR WITH FERROUS CORE

INDUCTANCE FORMULA. The use of a ferrous core to form a complete magnetic path for the flux increases the amount of magnetic flux set up and so increases the induc-

tance of the winding many times. The inductance of an iron torus with a closely spaced winding is

$$L = 0.4\pi n^2 \frac{\mu A}{l} \text{ microhenrys}$$

where n is the number of turns of the winding, A the cross-section of the winding, l the mean length of the winding (dimensions in centimeters), and μ the average permeability for the conditions of use (see Section 2, Magnetic Materials, for a discussion of the proper value of permeability to use). This is a fundamental equation for any magnetic circuit and has a very general use in magnetic calculations. The expression may be written

$$L = 0.4\pi n^2 P$$

where P = the permeance of the magnetic circuit computed from the dimensions of μ, A, and l. The permeance of a magnetic circuit may be represented as $1/\mathcal{R}$, where $\mathcal{R}$ = the total reluctance of the circuit. This reluctance is composed of the reluctances of all the parts of the magnetic circuit and may include different kinds of iron, as well as air or other non-magnetic material.

LOSSES. The use of an iron or ferrous alloy core for an inductor usually increases the losses of the inductor. These losses occur from two causes, hysteresis and eddy currents. A complete discussion of them and the method of calculating their magnitude is contained in Section 2, Magnetic Materials.

EQUIVALENT SERIES RESISTANCE. Since the core of an inductor is an integral part of the device, energy losses which occur in this material are considered as energy losses of the inductor. The losses in the ferrous core must therefore be added to the loss due to the current flow in the inductor winding. This gives a total loss

$$W_{\text{total}} = W_e + W_h + I^2 r$$

where I is the effective current flowing through the winding with an ohmic resistance r under a given set of conditions. The total loss may be set equal to a quantity ($I^2 r_{\text{equiv}}$), where r_{equiv} is the equivalent series resistance of the inductor. From its origin, it can be seen that r_{equiv} does not have a single constant value for a given inductor but will vary with the magnitude and frequency of the current through the winding as well as with the temperature of the device. Methods for the measurement of the losses, effective inductance, and other characteristics of reactors are presented in Section 5, Measurements and Measuring Apparatus.

CURRENT WAVE FORM. In an iron-cored inductor upon which is impressed a sinusoidal emf the flow of current is very complex and usually not of sine-wave form because of the variation of permeability during the current cycle. Neglecting the small resistance reaction, the emf of self-induction must be a sinusoidal voltage equal and contrary to the impressed voltage. This requires a sinusoidal time variation of flux, and this flux must be produced in the iron core by the current in the winding. For a core material being worked through a certain hysteresis loop the resulting current flow can be predicted as shown in Fig. 4. If the hysteresis loop carries into the saturation region the current rises to very high peaks to produce the necessary flux changes.

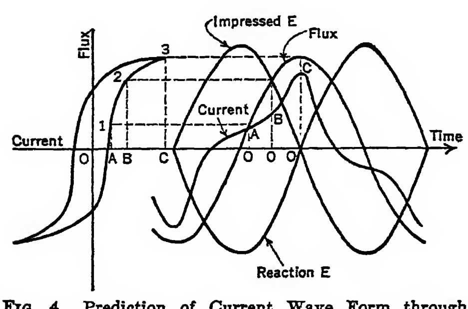

FIG. 4. Prediction of Current Wave Form through Hypothetical Inductor on which Sinusoidal emf is Applied

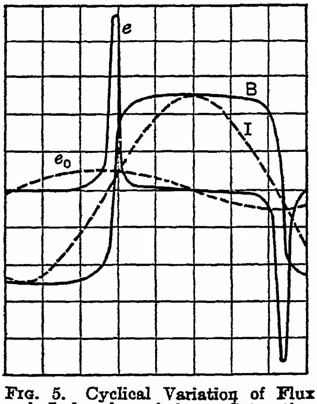

FIG. 5. Cyclical Variation of Flux and Induced emf in a Saturating Reactor through which a Sinusoidal Current is Flowing

If the iron-core inductor is only one of a series of circuit elements, it may be that a sinusoidal alternating current will be flowing through the circuit. It has been shown that under these circumstances the voltage across the inductor may reach very high peak values. The curves of Fig. 5 show the cyclical variations of a saturating reactor, through which a sinusoidal current is flowing. In the curves of Fig. 5 are shown the wave of magnetic flux, of the induced voltage e, the sine wave of current, and the voltage e_0 which would be induced if the flux density B were a sine wave of the same maximum value. The variation of the induced voltage e approximately represents the cyclical variation of the instantaneous inductance. It is apparent from the curves of Figs. 4 and 5 and from preceding paragraphs that the voltages, currents, inductance, and resistance which may describe the actions of an inductor are far from sinusoidal or constant quantities. Therefore measurements on devices of this type are generally interpreted in terms of equivalent or effective values.

TIME CONSTANT. In many circuits involving the use of an inductor there are periodic changes in the flow of direct current. It is sometimes required that the inductor control the rate of change of current in such a circuit. The growth of current in an inductor of constant inductance L is expressed as

$$i = \frac{E}{r}(1 - \epsilon^{-rt/L})$$

where i = current at any time t seconds after the circuit is closed; E = constant impressed voltage; r = resistance of the winding; ϵ = Napierian logarithmic base 2.7183. The curve of Fig. 6 shows the current rise in an air-core inductor and follows very closely the equation above. However, with the iron-core inductor the inductance is not constant

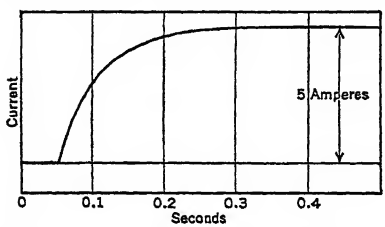

FIG. 6. Current Rise in an Air Core Inductor, L = 0.62 henry, r = 13.2 ohms (After Morecroft)

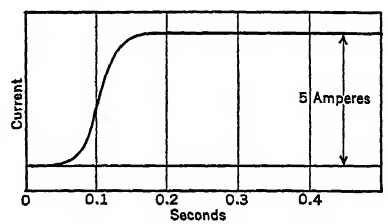

FIG. 7. Current Rise in Saturating Reactor, L variable, r = 18.7 ohms (After Morecroft)

and varies with increase of current. This effect is seen in the curve of Fig. 7, the form being very unlike that of Fig. 6. These effects must be considered when it is required that the inductor control the current growth. The magnitude of the exponent in the expression above determines the rate of current rise in the inductor. For this reason the quantity L/r is a characteristic constant (when L and r are constant) of the inductor and is given the name "time constant." If the numerical value of L/r, in seconds, is substituted in the above equation it is seen that it is the time taken for the current to reach 63.2 per cent of its final value. The time constant is generally used in the comparison of inductors and, from its dimensions, indicates the effectiveness of a given winding in producing inductance for a given resistance.

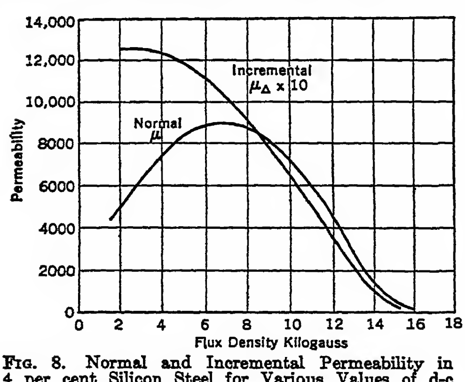

FIG. 8. Normal and Incremental Permeability in 4 per cent Silicon Steel for Various Values of d-c Saturation (C. R. Hanna, *Trans. A.I.E.E.*)

ACTION OF D-C MMF WITH A-C MMF. Many applications of inductors require their use in circuits carrying both direct and alternating currents of more than one frequency. Both these situations result in unusual effects on the inductance of the device. Since some of the effects avail themselves for application in certain control circuits, it is common for

an inductor, in this work, to have two windings, one carrying the circuit alternating current and the other carrying a direct current. The effective inductance offered to the alternating current depends upon the magnitude of both the direct current and the alternating current. The state of magnetic saturation due to d-c mmf depends upon the magnitude and direction of the direct current. For any particular direct current

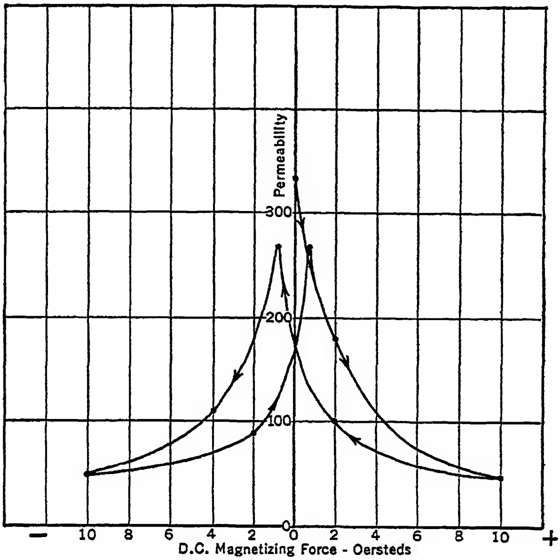

FIG. 9. Butterfly Curve. Variation of incremental permeability to a small a-c mmf for various d-c mmfs acting on a medium silicon steel core

the iron core is brought to some operating point on the hysteresis loop. When an alternating current superimposes its mmf upon this as an origin, a small hysteresis loop is executed. It is the average slope and the area of this small displaced hysteresis loop which give a measure of the permeability and losses of the iron under these conditions; and these determine the effective inductance and resistance of the inductor for that particular alternating current. The permeability determined from the displaced hysteresis loop is termed the "incremental" permeability (see Section 2, Magnetic Materials). Generally the mmf due to the direct current decreases the permeability for the alternating current with the exception of the case of low d-c magnetizing forces acting in conjunction with residual flux from some previous magnetization. The curve of Fig. 8 shows the variation of incremental permeability for various values of d-c saturation in a 4 per cent silicon steel.

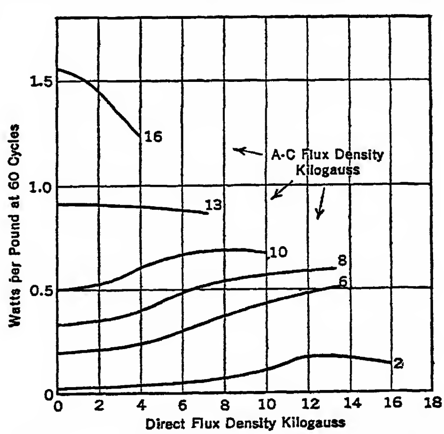

FIG. 10. Variation of Hysteresis Loss in 4 per cent Silicon Steel for Various Conditions of a-c and d-c Flux Density (Edgar, *Elec. Eng.*)

When the d-c magnetization in a core is varied with increasing values first in one direction and then the opposite, the effect on the a-c permeability is as shown in Fig. 9. In this "butterfly" curve it is seen that, under certain conditions, the impressed direct mmf increases the a-c permeability.

Both of the energy losses in iron due to hysteresis and the flow of eddy currents are

affected by the combined a-c and d-c saturation. Edgar (*Elec. Eng.*, Vol. 53, No. 9, p. 318, February, 1934) has determined that, in the range of alternating flux density below a critical value, hysteresis loss tends first to rise, reaches a maximum, and then decreases as the d-c magnetization is superimposed upon a fixed a-c flux component. It has been found that the critical value of the alternating flux density for silicon steel lies in the range of 13,200 to 13,800 gausses. The curves of Fig. 10 show the variation of hysteresis loss with direct flux density for various values of a-c flux density. Sidhu (*Elec. Eng.*, Vol. 52, No. 9, p. 625, September, 1933) has developed expressions for the hysteresis loss in materials carried through displaced or unsymmetrical hysteresis loops. These expressions are of the general type of the Steinmetz formula, but for high magnetizations an expression of another form more accurately expresses the loss, and each case requires particular value of coefficients and exponents. Spooner has shown that, for a given B increment, the hysteresis loss increases with the displacement from the normal position. The ratio of the loss of the displaced loop to that of the normal loop is termed the displacement factor. Under certain conditions this displacement factor may amount to 10 or more.

When alternating currents of more than one frequency impress their mmf on the core of an inductor a very complicated effect results. This may be summarized in two parts: (1) the effective inductance to each frequency is altered by the presence of the other current; (2) the inductance offered to each frequency varies cyclically at the frequency of the other component.

EFFECT OF AIR GAP. When it is essential that an iron-core inductor have a more constant inductance, minimizing the effects of d-c saturation and high a-c mmf's,

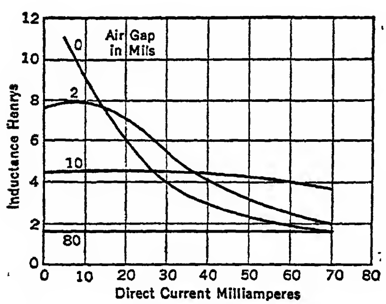

FIG. 11. Variation of Effective a-c Inductance with d-c Magnetization for Various Gaps. Alternating current of 1.0 milliampere (Turner, *Proc. I.R.E.*)

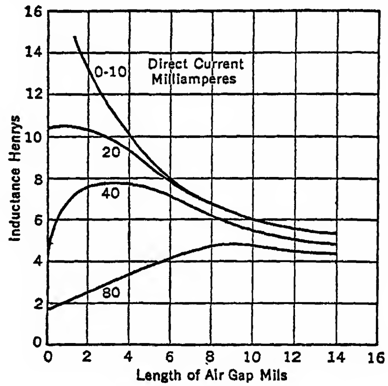

FIG. 12. Variation of Effective a-c Inductance with Air Gap for Various d-c Magnetizations. Alternating current of 10.0 milliamperes (Turner, *Proc. I.R.E.*)

an air gap is left in the magnetic circuit. This causes a reduction of the amount of self-induction possible with a given winding. The amount of decrease depends upon the comparative lengths of iron and air in the magnetic circuit and upon the degree of saturation of the iron. Hanna (*Trans. A.I.E.E.*, Vol. 46, p. 155, 1927), has shown a method for the design of reactances and transformers which carry direct current, including a determination of the best value of air gap. H. M. Turner (*Proc. I.R.E.*, Vol. 17, p. 1822, October, 1929) has experimentally shown the effect of initial magnetic state, air gap, and superposed direct currents on the inductance to alternating current of an inductor. The curves of Figs. 11 and 12 show the results of measurements made on an inductor with a winding consisting of 2500 turns of No. 36 A. W. G. wire. This winding served as a path for both the d-c and a-c currents. The core consisted of two L-sections each having 44 laminations, 14 mils in thickness, of 3.5 per cent silicon steel. The dimensions of the magnetic circuit were: length 16.3 cm, cross-section 2.82 sq cm, and volume 46.2 cc.

The curves of Fig. 11 show the characteristic decrease of effective inductance with d-c magnetization, with the slight initial increase due to remnant magnetization. For large air gaps and with no iron core, the inductor has very little change in inductance indicating the action of an air-core inductor.

The curves of Fig. 12 show that under certain conditions the inductance of the inductor is increased by an increase of air gap. This is due to the action of the air gap in lowering

the degree of saturation brought about by the direct current. Increasing the air gap corresponds to diminishing the ampere-turns per centimeter of iron of the d-c magnetizing force, and it is this which causes the rise in effective inductance for the curves of Fig. 12.

A very large number of magnetic circuits for inductors are made up in the manner of the inductor for which curves are given above. To complete the magnetic circuit, which is usually in two sections, these are brought together forming butt or lap joints. At these butt joints the air gap is introduced, using spacers of paper or fiber. Owing to the irregularities of the ends of the laminations it is difficult to secure an accurately measured, uniform air gap, and with no spacers in the joint there is still a considerable effective gap. The effective value of air gap may be more or less than the measured gap not only because of irregularity of the laminations, but also on account of fringing of the flux at the gap. Since the effective inductance is inversely proportional to the reluctance of the magnetic path, the results of Fig. 11 may be used to determine an incremental reluctance of the magnetic circuit. For low saturations this reluctance is practically all due to the air gap.

PRACTICAL DESIGN CONSIDERATIONS. The usual commercial reactor is built up with a laminated silicon steel core, an air gap, usually adjustable, and a winding of turns sufficient to give the desired inductance. In such reactors the air gap is commonly of such length as to constitute the major reluctance of the magnetic circuit. The reluctance of the iron portion of the magnetic circuit may then be neglected, provided the flux density in the iron is not exceeding about 80,000 lines per square inch. Under such conditions, the expression for inductance given earlier in this article becomes

$$L = \frac{3.2n^2 A}{10^8 l} \text{ henrys,}$$

where n = number of turns in coil; A = cross section of path in gap, square inches; l = length of gap, inches. To check that the flux density is not excessive it should be calculated from

$$B_m = \frac{3.2\sqrt{2}\, nI}{l}$$

where I = effective value of current, amperes; B_m = maximum value of flux density.

The iron-core reactor is generally built up using a magnetic circuit of either the toroidal or the E form. These are analogous to the core type and shell type of cores which are used for the magnetic circuits of transformers. Economy is effected by separate assembling of winding and core, and this is possible when the core is made up of laminations. These are punched in such form that the winding may slip over part of the core (middle leg of the E), after which the magnetic circuit is completed by an additional stack of laminations. The windings are constructed with insulated copper wire, the insulation being enamel, cotton covering, silk covering, or a combination of these. Adjacent layers are separated by one or more layers of paper, cloth, or cambric, and the whole winding is supported on a paper form or spool.

The design of reactors follows very closely those principles laid down in the design of the modern commercial power transformer, and reference may be made to any good source on transformer design (see Section 10). In a power transformer the core losses are essentially constant, and copper loss varies according to the conditions of loading. Reactors, however, are submitted to widely varying conditions, and the energy dissipation due to core and winding loss will vary in a wide range of values. Thus the reactor surface which must be allowed for radiating the dissipated energy will vary between wide limits. An allowance of anywhere from 1 to 6 sq in. of radiating surface per watt loss will usually not cause a temperature rise exceeding 55 deg cent, but this of course depends upon the cooling conditions. Experience has shown that, for a reactor of ordinary dimensions, continuous operation will be possible if there is a radiating surface of 2 sq in. per watt of power. Safe current ratings for a reactor winding vary widely, and any rating will depend upon the total reactor losses, not only on the copper loss. Considering this we find that current ratings may vary from 500 to 2000 amp per sq in.

Costs of reactors will vary widely depending upon the design, number of windings, materials, and conditions of use. An average figure for the cost of small reactors would be $5.00 per kva of rating.

10. ADJUSTABLE INDUCTORS, VARIOMETERS

From the characteristics of inductors operating under the various conditions detailed in this chapter, it is seen that inductance may be adjusted by one of several means. For a given inductor it is possible to vary the number of effective turns in the winding, and this is frequently done by arranging a sliding contact on the winding or on tap points to the

winding. When the full potential is impressed across the whole winding the sliding contact permits of a potentiometer use for the adjustable supply of voltage. This is the essential feature of an auto-transformer, or a starting compensator. The influence of an air gap on the inductance of an inductor was seen, and this effect is used for adjustment. Adjustment is effected by the control of a butt-joint air gap or through the motion of a wedge which can be moved in and out of a wedge-shaped cut in the magnetic circuit. The adjustment of a-c inductance by variation of the d-c saturation is probably the most widespread method of control, and accomplishes the desired end without moving parts and with small energy expenditure.

A very convenient continuously variable inductor is formed of two windings, generally in series and mounted so that one can rotate within the other. Although the self-inductance of the two coils is constant, the total inductance of the combination changes with the change of the mutual induction between the two coils. The inductance of this device is

$$L_{\text{total}} = L_1 + L_2 \pm 2M$$

where L_1 and L_2 are the individual self-inductances and M = the mutual induction. Another variometer effects the inductance adjustment by means of a variation of the overlap between adjacent flat coils. In variometers of these two types it is possible to adjust the inductance in a range of values of about 10 to 1. Adjustable inductors of the several forms have wide use in laboratories, and the principle of this device is applied in the development of the induction-type feeder voltage regulator.

11. CURRENT LIMITING REACTORS FOR POWER SYSTEMS

Reactors for short-circuit current limitation in power systems are generally installed at one or more of the following locations in a system: (1) generator reactors on the leads of generators feeding bus-bars; (2) bus tie reactors on the bus-bars between different bus sections; (3) feeder reactors on feeder circuits fed from bus-bars to protect switching equipment. The prevention of heavy currents on short circuits avoids severe mechanical stresses in connected apparatus. The placing of reactors in feeder circuits prevents a serious voltage drop in feeders other than the one in which the short circuit occurs, and reduces the probability of apparatus falling out of step.

SPECIAL DESIGN REQUIREMENTS. Inductors designed for use as short-circuit reactors must meet certain special requirements, and in consideration of these, the turns of the inductor cannot be wound in the close arrangement which would produce the largest inductance value. H. B. Brooks has noted (*Bur. Standards J. Research*, Vol. 7, p. 289, August, 1931) the important considerations to be: (*a*) ability to withstand heating caused by heavy overloads; (*b*) ability to withstand destructive forces during short circuits; (*c*) high flashover voltage; (*d*) low skin-effect resistance ratio. To minimize the effects of the heating and interwinding forces (both of which are proportional to the square of the current), layers and adjacent turns are well spaced. To give the winding mechanical strength the turns and layers are spaced apart and held by strong cleats of heat-resisting insulator material, or they may have concrete supports cast around them. During short-circuit conditions, the normally low turn-to-turn voltage is increased many fold and the spacing of turns must be such as to prevent flashover. To aid flashover prevention and to provide protection against short-circuited turns, the cable winding is insulated with a material having a high dielectric strength and impregnated with compounds which render it heat and fire resisting. For higher line voltage, fewer turns are placed in the end layers to reduce the stress on the insulation. It is important that copper loss in the winding be minimized during normal operation. This is effected by stranding the conductor, the individual wires of which are enameled and insulated one from another.

CONSTRUCTION AND PERFORMANCE. Short-circuit reactors are built for outdoor or indoor service and are designed for either air cooling (dry-type) or for cooling by oil immersion (wet-type). The application and development of large current-limiting reactors have occurred mainly in the forms using no core material chiefly from reasons of economy. Reactor windings are generally formed into parallel multilayer spiral disks. These windings are symmetrically spaced and wound so that the total current will be equally divided and circulating current will be prevented.

For a given operating frequency the reactor will possess a reactance $X = 2\pi fL$. From this value the "per cent reactance" of the reactor may be calculated as $\frac{(100\,IX)}{V}$, where I is the normal full-load current through the reactor and V the normal voltage across the circuit in which the reactor is placed. As the normal voltage drop through the reactor is equal to IX, the per cent reactance is actually the per cent ratio of this voltage drop to

the circuit voltage in which it is used. When the reactor winding is inserted in one branch of a Y, the voltage, V, is volts to neutral.

The standards of the A.I.E.E. specify normal performance and tests for current-limiting reactors. According to the standard, the rating of current-limiting reactors shall be expressed in kilovolt-amperes, absorbed at the rated current and frequency on a circuit of specified voltage rating. Since the resistance of the reactor is usually negligible compared with the reactance, the kva rating may be determined as I^2X, and the value of reactance may be used as the impedance of the reactor. The standard further requires that " current limiting reactors having an impedance of 3 per cent or more shall be capable of withstanding without injury for five seconds the maximum current that would result from any short circuit on the system with normal line voltage maintained at the supply terminals and with only the inherent impedance of the reactors in the circuit. Reactors having an impedance of less than 3 per cent shall be capable of withstanding without injury for five seconds a current equal to 33 1/3 times the rated current."

TEMPERATURE RISE, LOSSES. For the purposes of standardization the temperature of the copper under short-circuit conditions given above shall not exceed 250 deg cent for class A insulation, or 350 deg cent for class B insulation where calculated by the formula given below, assuming: (*a*) that all the heat is stored in the copper, and (*b*) an initial temperature of 105 deg cent for class A insulation and 125 deg cent for class B insulation.

The formula for the increase in temperature during short-circuit conditions is:

$$\theta = At\left[\frac{B}{2\theta_1} + \frac{618.4E}{B}\right] + \theta_0$$

where θ = final temperature, degrees centigrade; θ_0 = initial temperature, degree centigrade; θ_1 = absolute initial temperature = $(\theta_0 + 234.5)$; t = time, seconds; E = ratio of eddy current to I^2R loss at 75 deg cent; $A = \dfrac{\text{watts per pound (at } \theta_0)}{180}$; $B = 2\theta_1 + At$.

" The losses to be considered in current-limiting reactors shall be the load losses. These losses consist of I^2R losses in the winding due to load current, and stray losses due to stray fluxes in the windings and other metallic parts. The stray load losses shall not include the losses produced by the field of the reactor in adjacent apparatus or materials." Actually, when reactors are installed at a distance less than half their diameter from any metallic structure, appreciable eddy current and hysteresis losses will be produced in such structure. Thus in the oil-immersed reactor there is considerable energy loss in the laminated iron strips, used to shield the oil tank walls, and this increases the effective resistance of the reactor.

RATINGS. Current-limiting reactors are constructed in a wide range of sizes running up to normal current ratings of 7500 amp. Reactors are generally limited to use in circuits of 13 kv or less, but reactors for grounding neutrals of 220,000-volt transformer banks have been constructed. For generator, bus, and feeder installation, reactors are constructed with a per cent reactance up to 5 per cent. Typical of the service for large generator and bus tie circuits one manufacturer * supplies a reactor rated at 11 kv, 3750 amp, 2850 kva, 25 cycles. The guaranteed efficiency of reactors of this type is given as 99.56 per cent. The costs of current-limiting reactors vary over a wide range. As a general statement, based on a copper cost of 10 cents, the cost per kva may run as low as 65 cents on very large reactors, and it may run as high as $16 per kva for small reactors.

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MAGNETS, MAGNETIC DEVICES

Magnets and magnetic devices function through the use of magnetic flux. This flux may be intermittently generated by means of a coil carrying electric current, or it may be residual in a body of iron. This permits the grouping of magnetic devices broadly into two classes, those employing electromagnets and those employing permanent magnets.

Magnetic devices are very widely used in all electrical circuits and serve many utilitarian purposes. In the electric power service, circuit breakers, switches, and overload relays find extensive application. Among the many industrial and commercial uses there are ore separation, lifting and holding magnets, valve operation. Magnets are an indispensable component of electrical motors and generators as well as of electrical meters and relays. The control and interconnection of communication circuits is effected by magnetic relays.

Magnetic devices generally are made up with some type of ferrous alloy core. The chemical constitution and the physical and thermal treatment vary according to the requirements producing a magnetically "hard" or "soft" material. The iron used in the structure not only increases the flux above the value which could be set up in air, but directs it to the point of application. The relative value of materials as to the setting up of flux is represented by the permeability of the material, and generally it is desired that the material have a reasonably high permeability and, in the case of permanent magnets, a high retentivity (see Section 2, Magnetic Materials).

12. PERMANENT MAGNETS

A "permanent" magnet to be considered such must retain the major portion of its magnetism after being subjected to moderate demagnetizing forces. It usually is not of much value as a magnet unless it can produce a magnetic field of fairly high intensity.

When a piece of steel has been magnetized it will contain magnetic flux to a certain extent throughout its mass. The term "remanence" is used to designate the magnetic induction at the magnetic equator of a permanent magnet with no external magnetizing or demagnetizing force. Most important in the design of permanent magnets is the deter-

mination of the relation between the remanence of a magnet and the various factors which determine its value. These factors include the magnetic properties of the steel and the shape and dimensions of the magnet. Of particular interest to the manufacturer are the problems involved in chemical composition, melting and rolling practice, hot forming, and annealing and hardening treatments. The last group bears an important relation to the physical characteristics of the product as well.

GENERAL DESIGN FACTORS, EFFECT OF AIR GAP. The type of steel to be used in a permanent magnet is determined by the permissible size of the magnet, the magnetic properties of the steel, and the cost of the steel. Cheaper steels of comparatively low coercive force may be used for some magnets in which size and weight are not important. It is often possible to compensate for a lower coercive force by increasing the length of the magnet, since the ability of a magnet of given retentivity to hold its magnetism is proportional to its length. The magnetic flux density in a bar magnet is usually lower than the remanence of the steel because the ends of the magnet exert a demagnetizing force on the rest of the bar. This effect is greater in bars whose ratio of length to cross-section is small, and is negligible in very long thin magnets.

A knowledge of the coercive force of the material is of value when estimating the relative magnetic strengths of short magnets of the same size but of different materials. If the coercive force is very high, as it is in cobalt magnet steel, short magnets may be made which will have as high magnetic strength as considerably longer magnets made of material whose coercive force is lower. However, the relative strength of long thin magnets is dependent primarily on the residual induction of the material used. Long, slender magnets are capable of greater magnetic strength if made of tungsten steel than if made of any other common steel, because the residual induction of tungsten steel is relatively high.

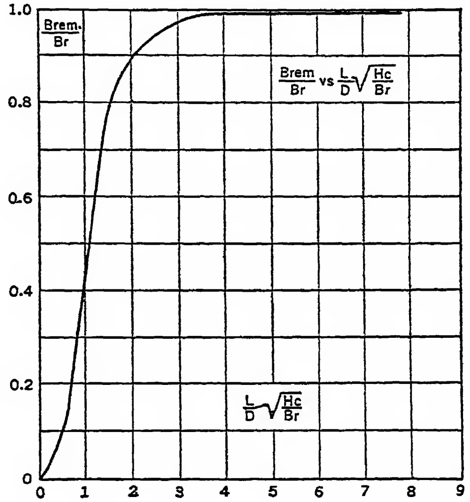

FIG. 1. Curve of Generalized Expression for Permanent Magnets, Relating Remanence and Physical and Magnetic Dimensions (Scott, *Trans. A.I.E.E.*)

THE DESIGN OF PERMANENT MAGNETS. Extensive investigations have determined a relation between the remanence of any permanent magnet and the parameters of physical dimensions and the axial intercepts of the demagnetization curve. This generalized relation is shown in the curve of Fig. 1 below and holds for bars of widely different compositions, magnetic properties, dimensions, and heat treatment.

In this curve the quantities related are:

B_{rem} = remanence, the magnetic induction at the magnetic equator of a permanent magnet with no external magnetizing or demagnetizing force. Values of B_{rem} given are without pole pieces on the magnets, in gausses.

B_r = residual induction, the magnetic induction in a ring or infinitely long straight bar after the value of H has been reduced from H_{max} to 0. The value of B at the intersection of the hysteresis loop with the B axis, in gausses.

L = the actual developed length of a magnet.

D = the equivalent diameter of a magnet $= 2\sqrt{\frac{\text{section area}}{\pi}}$

H_c = coercive force, the value of H required to reduce B from B_r to 0 in a ring or infinitely long bar. The value of H at the intersection of the hysteresis loop with the H axis, in oersteds.

L/D = the dimension ratio of a permanent magnet.

The form of this relation indicates that, with other factors constant, the remanence of a magnet is roughly proportional to $\sqrt{B_r}$, and for large values of dimension ratio or coercive force is practically equal to B_r.

Although the results given in Fig. 1 were derived from data on bar magnets, the general conclusions drawn are applicable to magnets of any shape if L is assumed to stand for the effective length of the magnet. By effective length is meant the length of a straight bar magnet of equal cross-section and magnetic properties, having the same remanence as the given magnet.

CRITERIA OF MAGNET QUALITY. A number of quantities have been used as criteria of the magnetic quality of magnet steel. The quantity $(BH)_{max}$ has been demonstrated as a measure of the maximum amount of external energy which can be supported per unit volume of a given magnet steel, and that this in turn defines the term "magnetic quality" of a magnet steel.

An experimental verification has been made of this criterion, and a relation between $(BH)_{max}$ and (B_rH_c) has been given to facilitate the determination of "magnetic quality." The curves of Fig. 2 above give a graphical comparison of the demagnetization curves and (BH) curves for various kinds of magnet steel.

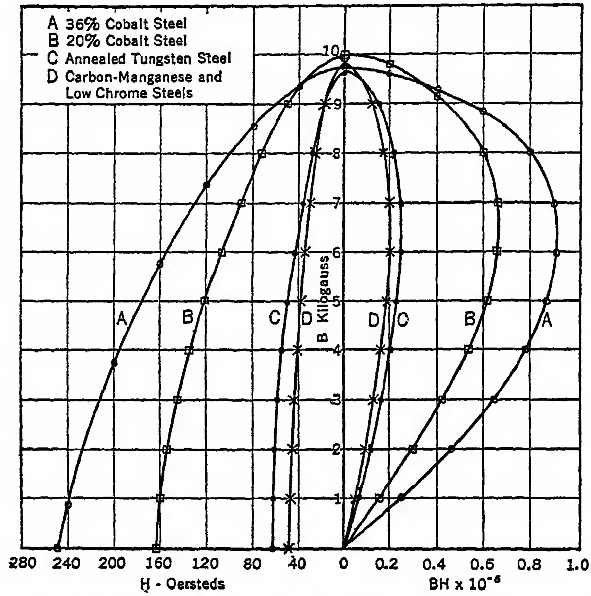

FIG. 2. Typical Demagnetization Curves and Curves of BH vs. B for Various Kinds of Magnet Steels

MAGNETIZATION, CONSERVATION. To obtain the theoretically maximum strength, magnets should be magnetized with a force sufficient to magnetize the steel completely. Practically, it has been found that satisfactory results are obtained with common magnet steels when they are magnetized with a force approximately five times the value of the coercive force of the steel. Well-designed commercial magnets have values of flux density in the middle region at about two-thirds the residual induction of the steel. The magnetizing force may be applied to a material by placing it in the magnetic field inside a solenoidal winding which is carrying a large direct current. A very large current acting for a short duration will be effective, this current being gradually reduced to a small value. If more convenient the magnet may be applied to the poles of a strong electromagnet. If possible the magnet should be magnetized in its final condition, with pole pieces attached. If magnetization is effected on the poles of an electromagnet, and the magnet is then withdrawn and pole pieces applied, the residual magnetism will be less than if the magnet has the pole piece attached when the magnetizing force is applied. It is therefore of importance that soft-iron pole pieces be constantly across the poles when they are magnetized and until the magnet is installed in the apparatus.

13. ELECTROMAGNET WINDINGS

The electromagnet is usually composed of a winding and an iron-air magnetic circuit more or less permanently related to the winding. The winding, when carrying an electric current, sets up a magnetic field which is a direct function of the number of turns and the current flow. To localize the magnetic field and to intensify it, a core or shell, or both, of magnetic material are added to the winding. When the core is fixed within a solenoidal winding, a bar electromagnet results. If the core is movable the electromagnet is of the plunger variety.

The function of the electromagnet is to provide a magnetic field in space. This magnetic field may be used in the operation of electrical rotating machinery, or may serve to magnetize materials under chemical or physical research. In addition to these fundamental uses there is the action used in almost an infinite number of devices—that of the attraction of magnetic objects in a magnetic field.

In switches, relays, and kindred magnetic devices the iron attracted in the magnetic field is usually in the form of an armature or clapper linked or hinged to the shell or core of the structure. The attracted magnetic material may also be in the form of a free body such as the material lifted by an ore separator or lifting magnet.

CORE MATERIAL IN ELECTROMAGNETS—MAGNETIC CHARACTERISTICS. The magnetic characteristics of the iron used in the core and shells of electromagnets will be considerably different from those desired in permanent magnets. An electromagnet is generally intended for intermittent use, that is, magnetizing current is applied when it is desired that the magnet act. When it is desired that the electromagnet cease to act, the magnetizing current is disconnected. Ideally, then, when the current is reduced to zero, it is desired that the attractive force of the electromagnet immediately cease and become zero. This would require the material being acted on by the magnetizing force to have zero residual induction and zero coercive force. An attempt to secure this ideal condition is made by the manufacture and use of magnetically soft materials such as carbon-free iron, low-carbon silicon steel, and the soft alloys of iron.

Beyond the magnetic requirements of low coercive force and residual induction it is generally desired that the core and shell material have the possibility of a high maximum induction. This requires the use of material with reasonably high magnetic permeability. In certain relays used in communication circuits special iron alloys with nickel and cobalt are useful in setting up high flux densities. Such use is of particular importance in telephone exchanges where space is at a premium and devices must be small. To set up the necessary action forces in small relays requires high flux densities, and to do this with material of small section requires material capable of high flux densities.

FUNDAMENTAL ACTION IN ELECTROMAGNETS. The fundamental law underlying the motion of a magnetic body in a magnetic field is that the free body always is forced to move in a direction such as to decrease the total reluctance of the magnetic circuit. For a given change in reluctance the force causing this motion is proportional to the square of the number of lines of magnetic flux threading through the body. It is obvious from these facts that a large amount of flux is desirable to secure sizable forces, and this flux should be concentrated in small localities so that it may act on small magnetic bodies. This desirable result is most difficult of achievement. When an iron torus is wound with a uniform winding through which current is passed, practically all the magnetic flux set up by the winding is contained in the iron. To expose some object to the flux requires the introduction of an air gap in such a toroid. With even very small air gaps the magnetic flux in the iron ring does not pass directly from one face of the gap to the other. There is fringing of the flux around the gap which, in large gaps, appreciably diminishes the amount and concentration of flux available in the working space.

The stray and fringing flux is generally not useful when it is the intention to have some magnetic material acted on. The quantity and distribution of leakage flux around any magnetic structure will vary widely depending upon the design of the magnetic device. If a magnetic circuit is almost entirely completed with unsaturated magnetic material, and air gaps are relatively short, the leakage flux is small. In many magnetic devices, however, the air gap must be sizable to get the necessary action on the attracted armature. With this the case, leakage flux will be relatively large, and as indicated later in this article, relations dependent upon flux values will be relatively less precise.

CONTROL OF FORCE AND MOTION IN ELECTROMAGNETS. The motion resulting directly from the action of the forces in an electromagnet is generally a linear one. Whether the motion is through a short or long distance, energy up to some limited quantity can be transferred through the magnetic field. Large forces may act through relatively short distances, or if motion through considerable distances is required, only a small force can be exerted. In view of these conditions various designs are created for the

securing of the required force and distance characteristics. By the use of proper links, hinges, and levers, as well as core and frame forms, forces, distances, and directions may be altered in the particular application of an electromagnet.

In the motion of the plunger, clapper, or other moving part of the electromagnet certain features in the design are important. The *initial force* acting must be sufficient to overcome frictional and gravitational forces on the moving part. At the end of the travel of the moving part the *final pull* exerted on it must be sufficient to make the action definite and non-chattering. This is effected by the creation of a good *sealing pull* which holds the moving part firmly at the end of its travel. The travel of the moving part is usually arrested by part of the electromagnet frame, by a magnetic cushion, or mechanically by means of a spring, stop, or dash pot. Certain magnetic devices are designed to utilize the energy resident in the moving part to cause a *hammer blow* as in a relay trip or in the electric hammer.

To make the action of electromagnets quick, positive, and non-sticking, it is common to use a spring or some other energy-storing means against which the electromagnet force must work. By adjusting this stored spring force to exceed any residual forces after current is disconnected, the action of the electromagnet is made positive and definite. When the back force and electromagnet force are about equal, a condition of fluttering or chattering may arise until one of the forces becomes large enough to hold contrary to the other.

The speed of action in an electromagnet is primarily controlled by the mechanical and electrical characteristics of the device. For quick action the self-inductance should be small, the mass to be accelerated small, with little initial friction. These characteristics will permit a quickly rising, relatively large current with a subsequent quick but usually short action. Heavy-duty, long-stroke electromagnets are generally slow-acting, being of relatively large size and large self-inductance.

When the moving part of an electromagnet makes a good physical contact with the poles of the electromagnet a good sealing pull results from the high magnetization set up in the closed magnetic circuit. This condition may give rise to *sticking* when the electromagnet current is cut off. This sticking depends primarily on the remnant magnetism and may be overcome by reversing the polarity for a moment, or by originally limiting the current with an external resistor. A non-magnetic gap or air gap in the magnetic circuit reduces the remnant magnetism and therefore the sticking.

CLASSIFICATION OF ELECTROMAGNETS. The choice of an electromagnet for any particular application depends upon the following factors: (1) voltage available (a-c or d-c), permitted current drain; (2) distance over which electromagnet force must act; (3) speed of action required.

When these factors are known and a suitable type of electromagnet is chosen, the actual design as to dimensions of the various parts is made. This design will consider the size of load, the action distance, speed of action, the excitation, and any limitations placed on the shape and size of the device.

Electromagnets may be grouped in several ways, but a most general classification may be made on the following basis: (1) no moving parts; (2) part moved for holding (portative); (3) part moved through a distance against forces, thus doing work (tractive).

These general forms may be further subdivided in classes as to current used, physical shape, etc. The following brief classification indicates the major forms in each category.

1. *No Moving Parts* (operate on direct current).
   Electrical machine field structures, electromagnets for physical and chemical research.
2. *Portative* (for attracting and holding magnetic materials; operate on direct current, or with alternating current and shading coil.)
   Lifting magnets, magnetic chucks, clutches, couplings and brakes, ore and trash separators, low-voltage release on starting rheostats.
3. *Tractive* (with associated moving parts; operate on direct current and alternating current; various combinations of size of armature, length of stroke, speed of action) and including:
   bells and buzzers; telegraph and telephone relays; current, time, and voltage limit switches and relays; mechanical switch and valve control; and many forms of indicators and magnetic mechanical control.

DESIGN AND CONSTRUCTION. The problem is to construct a coil which, for a given voltage across its terminals, will produce a given number of ampere-turns, and which will not overheat. Usually one or more of the dimensions of the coil are also fixed by the conditions under which the coil is to be used. The following factors must be taken into account in designing a winding:

Insulation from Core; Design of Bobbin. When the core is fixed, two washers of hard rubber or vulcanized fiber are forced on at either end of the core. The core is then insulated with a wrapping of paper, mica, or oiled linen, and is then ready to be wound. When the core is movable, the two end washers are forced on to the ends of a brass tube or a tube made of the same material as the washers. When a metallic tube is used the washers are sometimes made of the same metal. In the case of a quick-acting plunger magnet a metallic bobbin, if used, should be slotted, in order to avoid eddy currents; this also applies to all forms of a-c electromagnets.

Insulation of Wires; Insulation Between Layers; Baking. The wire may be insulated with a cotton, silk, or asbestos wind or by a coating of enamel. In high-voltage solenoids the various layers of wire are insulated from each other by paper, mica, or oiled linen wrappings, or the entire winding is divided into several sections separated by vertical washers of insulating material. The wound coil may be further insulated by dipping it into an insulating varnish in a vacuum or at atmospheric pressure, after which it is either air-dried or baked.

Aluminum wires and ribbons are used extensively by some manufacturers, especially abroad. In this case the layer of oxide on the wire is the only insulation which is used, but inasmuch as this insulation cannot be destroyed by heat, the coils can be run at much higher temperatures, which is of special value for lifting magnets and similar devices. The oxide layer will stand a potential stress of about 0.5 volt.

Temperature Rise of Winding; Watts per Square Inch. The rise of temperature of the winding will depend primarily upon the average rate at which heat is developed by the electric current and the amount of exposed surface from which this heat can be radiated; the temperature rise will also depend upon the depth of winding, the circulation of the air, etc. The hottest spot in the winding should never reach a higher temperature than 105 deg cent. When the interior of the winding is at 105 deg cent the temperature of the external surface, as measured by a thermometer, will usually be much less (15 deg cent less), as will also the average temperature measured by the change of resistance method.

As a rough approximation a solenoid winding should be so designed that the average power developed will not exceed 0.5 watt per square inch of radiating surface for an open winding, and will not exceed 0.7 watt per square inch of radiating surface for an iron-clad solenoid. In figuring the radiating surface of an open winding, the surface of the hole through the solenoid is not included, and the end surfaces are included only when the solenoid is short. By the radiating surface in the case of an iron-clad solenoid is meant the surface of the winding which is in contact with the iron. A radiation of 0.5 watt per square inch and 0.7 watt per square inch for an open and an iron-clad winding respectively corresponds roughly to an average temperature rise of approximately 60 deg cent; for other rates of radiation the temperature rise will be approximately proportional to the watts per square inch radiated.

For short-time service, i.e., when the solenoid is energized only for short intervals with long intervals between the applications of power, the thermal capacity of the solenoid will permit of a greater dissipation of energy in the winding without overheating it.

Space Factors; Round Versus Square Wire; Layer Versus Haphazard Windings. By the space factor of a winding is meant the ratio of the space occupied by the conductors to the total space occupied by the conductors, the insulation on the conductors, and the voids between conductors. The space factor for strips and square wires is greater than for round wires, but strip and square wires are not extensively used in small sizes because of the increased amount of insulation required for a given section of conductor, and because of the tendency of such wires to twist in winding so that they lie upon their corners instead of upon their faces. However, for conductors of larger section than No. 10 A. W. G., square wire is often used.

In winding wires larger than No. 18 A. W. G. it always pays to wind them carefully in smooth layers ("layer" wound), but for smaller sizes used for open solenoids (as distinguished from iron-clad solenoids) the gain in space factor does not as a rule warrant this care and the wires are wound in a more or less haphazard fashion ("haphazard" wound). For iron-clad solenoids, however, a layer winding is always used, for economy of material requires that the winding space be kept as small as possible. The dotted curves A and B in Fig. 3 for haphazard windings are taken from an article by F. A. Willard (*Elec. World*, 1906, Vol. 47, p. 823).

Round wires are sometimes so wound that the wires of one layer lie in the hollows between the wires of the layer underneath; the wires in this case are said to be imbedded. Objections to this procedure, however, are that each layer must be started from the same end and the insulation on the wires becomes tightly compressed and therefore is less

effective; in most instances the extra labor and the diminution of insulating quality offset the small gain in space factor, which seldom exceeds 3 per cent.

The space factor s for a layer winding of round wire without imbedding, and making no allowance for extra insulation between layers, is

$$s = \frac{\pi}{4}\left(\frac{d}{d + 2t}\right)^2 \tag{1}$$

where d is the diameter of the conductor and t the thickness of insulation. Values of s for various sizes of wire and various thickness of insulation are given by the curves in

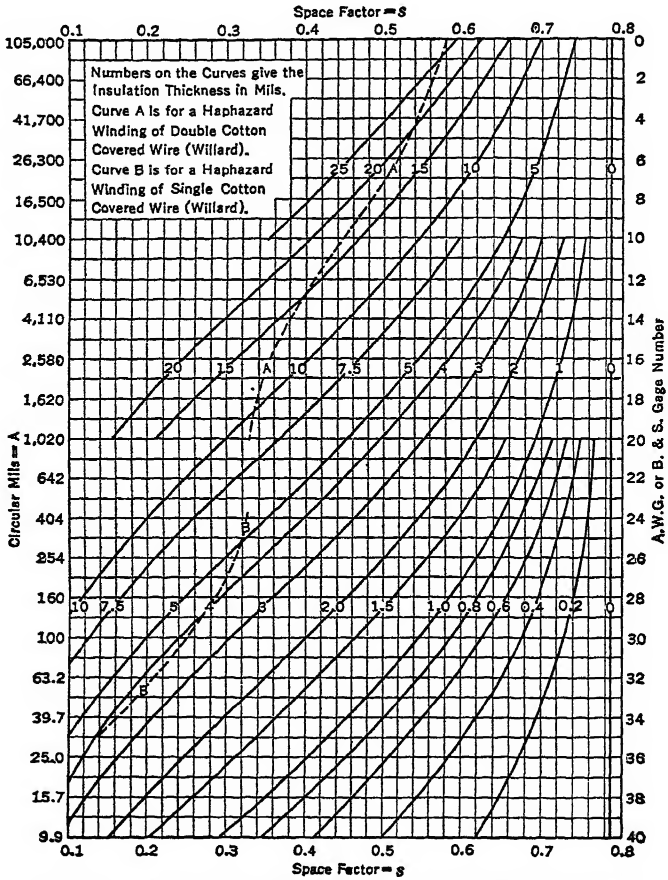

Fig. 3. Space Factor Curves for Close Layer and Haphazard Windings

Fig. 3. The "over-all" space factor, including the allowance for the space occupied by the extra insulation, if any, between layers, is equal to the value of s from these curves multiplied by $(1 - e)$, where e is the ratio of the space occupied by this extra insulation to the total winding space.

Thickness of Insulation. Magnet wire is usually referred to as "single covered," "double covered," and "triple covered," depending upon the number of layers of insulating threads wrapped around it. Different manufacturers use different thicknesses for these layers, with the result that the thickness of the insulation on a "double cotton covered" wire of a given size or gage number depends upon the manufacturer of the wire. Table 1 gives the standard specifications on thickness of insulation for round copper wire.

Table 1. A.I.E.E. Standards Magnet Wire Insulation

Permissible additions to the diameter of bare round
Bare copper wire, with one layer of insulation

Size of Wire A.W.G.	Maximum Addition Single Cotton Covered	* Maximum Addition Single Silk Covered	Size of Wire A.W.G.	Minimum Addition Enamel Covered	Maximum Addition Enamel Covered
0000–8	0.008 in.	.....	8–15	0.0015 in.	0.0025 in.
9	0.007	.....	16–19	0.0012	0.0020
10	0.006	.....	20–22	0.0010	0.0018
11–14	0.005	.....	23–25	0.0009	0.0015
15–21	0.005	0.002 in.	26–30	0.0007	0.0012
22–35	0.0045	0.002	31–32	0.0006	0.0010
36–40	0.004	0.002	33–34	0.0005	0.0008
			35–36	0.0004	0.0007
			37–38	0.0003	0.0006
			39–40	0.0002	0.0005

* The minimum permissible addition to the diameter of the bare wire shall be 75 per cent of the maximum.

Winding Calculations for Direct-current Solenoids. Round solid wires are assumed throughout. Let

s = space factor, from Fig. 3.

k = ratio of specific resistance of conductor used to that of standard annealed copper at 20 deg cent. For copper of 100 per cent conductivity at 20 deg cent, $k = 1$; for copper of any other per cent conductivity, say C per cent, at any other temperature, say t deg cent, $k = \frac{100}{C} + 0.004\,(t - 20)$. See also Section 14, Art. 50.

A = cross-section of wire in circular mils (= square of diameter in thousandths of an inch).

$n = \frac{1{,}270{,}000\,s}{A}$ = number of conductors per square inch, the square inch being taken perpendicular to the direction in which the wire is wound.

$\rho = \frac{1{,}100{,}000\,sk}{A^2}$ = resistance of the winding per cubic inch of the winding space, excluding the space, if any, occupied by extra insulation between layers; ρ is in ohms.

$w = 0.271\,s + 0.040$ = weight of the winding (copper and cotton insulation) per cubic inch of the winding space, exclusive of the space, if any, occupied by extra insulation between layers; w is in pounds.*

E = impressed volts.

(NI) = ampere-turns, where N is the total number of turns and I the current in amperes.

p = allowable watts per square inch of radiating surface (may be taken approximately as 0.5 for open and 0.7 for iron-clad solenoids, see above).

S = radiating surface, in square inches, calculated as described above under Temperature Rise of Winding.

l = mean length of turn in inches (see Fig. 4 for a simple solenoid).

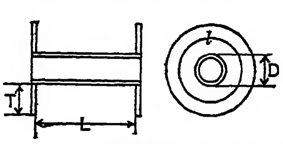

FIG. 4. Winding Form for Simple d-c Solenoid

T = depth of winding space in inches (see Fig. 4), excluding the space occupied by the extra insulation, if any, between layers.

L = length of winding space in inches (see Fig. 4).

$V = LlT$ = volume of winding space in cubic inches, excluding space occupied by the extra insulation, if any, between layers.

* This formula also holds approximately for other kinds of insulation and for most alloy resistance wires. Calling g_c the specific gravity of the conductor and g_i the specific gravity of the insulation, the exact formula is

$$w = 0.0362\,(g_c - g_i)s + 0.0284 g_i$$

Underhill gives g_i as 1.6 for asbestos, 1.4 for cotton, and 1.0 for silk.

The problem is usually to find the size of wire and necessary winding space for a coil which will give, without overheating, a required number of ampere-turns (NI) at a given impressed voltage E. The diameter of the core or spool is also usually known, or at least must not exceed certain fairly well-defined limits, depending upon the service for which it is to be used. The procedure is then to assume a reasonable value for the mean length of turn l. The size of wire is then immediately fixed by the relation

$$A = \frac{kl(NI)}{1.16E} \tag{2}$$

From Fig. 3 the corresponding size of wire (A. W. G.) and the corresponding space factor s may then be found.

The volume V and radiating surface S of the coil must then satisfy the relation

$$\frac{LTS}{l} = \frac{k(NI)^2}{1{,}470{,}000\ ps} \tag{3}$$

The length L and depth T of the winding space must be so chosen that this relation will be satisfied; as a rule this can be done only by cut and try. Note that changing the depth T of the winding space will also change the mean length of turn l, unless the diameter of the core or spool is so altered as to keep l constant. The cross-section of the wire varies directly as l, as shown by eq. 2, and therefore the value of the space factor s to be used in eq. 3 will depend upon l, but only to a slight extent except in very small wires.

Having determined the cross-section of the wire (A), the mean length of turn (l) and the dimensions (L and T) of the winding space so that both (2) and (3) are satisfied, the total number of turns in the winding will be

$$N = nLT = \frac{1{,}270{,}000\ sLT}{A} \tag{4}$$

and the current is then equal to the given number of ampere-turns divided by N and the total length of wire is equal to Nl.

The total resistance r and total weight W (including insulation) of the wire may then be found directly from a wire table, or may be calculated from the formulas

$$r = \rho V = \frac{klN}{1.16A} = \frac{1{,}100{,}000 skLTl}{A^2} \tag{5}$$

$$W = wV = (0.271s + 0.040)LTl. \tag{6}$$

See note at bottom of preceding page regarding w.

Calculation of Open Solenoid of Circular Cross-section. In the case of a coil wound on a spool of diameter D, see Fig. 4, the mean length of turn is $l = \pi(D + T)$. Assuming that the outside cylindrical surface of the winding is the only radiating surface, which is only approximately true, as pointed out above, $S = \pi(D + 2T)L$. Whence, putting

$$Q = \frac{k(NI)^2}{1{,}470{,}000ps} \tag{7}$$

the required length of coil for a given diameter of core D and depth of winding T is

$$L = \sqrt{\frac{(D + T)Q}{(D + 2T)T}} \tag{8}$$

When L is given instead of T, then the required depth of winding is

$$T = \frac{1}{4}\left[\left(\frac{Q}{L^2} - D\right) + \sqrt{\left(\frac{Q}{L^2} - D\right)^2 + \frac{8QD}{L^2}}\right] \tag{9}$$

In either case, the number of turns, total resistance, weight, etc., are found as described above for the general case.

Example. Required to design an open solenoid 10 in. long and having an internal diameter of 1.5 in., to give 12,000 amp-turns at 110 volts, the heat developed not to exceed 0.5 watt per square inch of radiating surface (taken as the outside cylindrical surface of the coil). Assume a mean length of turn equal to 10 in., then from eq. 2 the required cross-section of the wire, assuming copper at 70 deg cent, is $A = 1130$ cir mils. From Fig. 3 the space factor is then $s = 0.62$, assuming single cotton-covered wire with 2-mil insulation wound in layers. From eq. 7 the value of Q is then $Q = 380$, whence from eq. 9 the required depth of winding is $T = 2.36$ in. The actual mean length of turn is then $l = 12.1$ in. which substituted in eq. 2 gives, for the cross-section of the wire required, $A = 1368$. This wire section gives a space factor, see Fig. 3, of $s = 0.63$, which agrees practically with the value $s = 0.62$ used above. The nearest commercial size of wire is No. 19 A. W. G., having a cross-section of 1288 cir mils. If this size of wire is used the actual ampere-turns will be $NI = 11{,}300$. From eq. 4 the total number of turns will

then be $N = 14{,}700$, and from eq. 5 the total resistance will be $R = 143$ ohms. The current is then $I = 0.77$ amp. The total weight of the winding is then, from eq. 6,

$$W = 60 \text{ lb}$$

14. ELECTROMAGNETS, LIFTING AND PLUNGER

(See also Section 3, Electricity and Magnetism, Principles of.) A solenoid provided with a movable magnetic core, or with a fixed core and movable "armature," serves as a very convenient means of causing an electric current to produce a direct mechanical pull. This principle is utilized in various forms of lifting magnets, relays, contactors, electric brakes, clutches, etc. In the paragraphs following are given the formulas required in calculating the pull of various kinds of electromagnets in terms of their dimensions and ampere-turns, and also a brief statement of the applications of the various types.

Approximation of Formulas; Leakage Factor. In applying the formulas given below, it should be noted that in general the effect of magnetic leakage is neglected. The leakage factor varies so greatly with the different forms of electromagnets that it is impossible to go into this matter in detail in the limited space available for this article. The designer, in making an allowance for leakage, has to rely chiefly upon his previous experience with other magnets of the same general form and dimensions. Merely as a rough guide to the designer who has not had this experience, it may be stated that for magnets of reasonable shape and dimensions, the formulas for pull given below may be relied upon to give the actual pull with an error of less than ± 10 per cent, the actual pull usually being less than the calculated pull. Under certain conditions the agreement between the actual pull and calculated pull may be much closer than the difference just stated.

SIMPLE SOLENOID AND PLUNGER. The simplest type of electromagnet is a simple solenoid, as shown in Fig. 5, consisting of a cylindrical coil of circular or rectangular

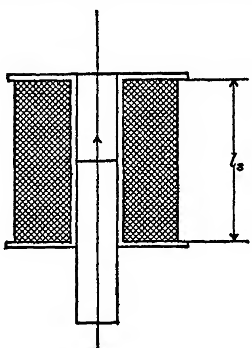

FIG. 5. Simple Electromagnet

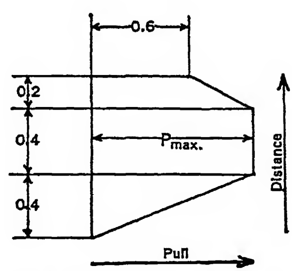

FIG. 6. Approximation Variation of Pull During Stroke for Simple Electromagnet

section and an iron plunger which fits into the inside of this coil. When current passes through the coil the plunger is attracted.

VARIATION OF PULL OF SIMPLE SOLENOID DURING THE STROKE. The pull of the simple solenoid varies between approximately zero when the plunger is at the lower end of the coil and a maximum which is nearly constant over approximately 40 per cent of the length of the coil. The maximum is reached when the plunger has entered the coil a distance of approximately 40 per cent. When the plunger has entered the coil 80 per cent of its length, the pull decreases again, reaching a value of about 0.6 of the maximum pull when the plunger is even with the top of the coil. The approximate pull variation is shown in the diagram, Fig. 6, of course the actual variation is a smooth curve, such as shown by curve A in Fig. 10.

Calculation of Pull of Simple Solenoid. The calculation of the exact pull of such a magnet is very complicated, being dependent not only upon the ampere-turns, but also upon the shape of the coil, the size and length of the plunger, and the induction in same. For practical purposes the maximum pull, when the plunger is at least as long as the coil, can be represented by the formula

$$P_1 = \frac{cANI}{l_s} \text{ pounds} \tag{10}$$

where c = the pull in pounds per square inch per ampere-turn per inch of coil length;
A = the area of the cross-section of plunger, in square inches;
I = the current in the coil, in amperes;
N = the number of turns in the coil;
l_s = the length of the coil, in inches.

It has been found that c varies between 9×10^{-3} and 10.5×10^{-3} when the plunger is *magnetically saturated*. For practical purposes sufficiently close results for a *saturated* plunger are obtained if c is taken equal to 10^{-2}. The formula shows that the maximum pull for a *saturated* plunger is directly proportional to the current, which makes this type of magnet especially suitable for relays and instruments which should be very sensitive. When the flux density in the plunger is well below the knee of the B–H curve, the factor c varies almost directly as the ampere-turns, and the maximum pull, therefore, varies approximately as the square of the ampere-turns.

The pull at any point in the stroke for a *saturated* plunger may then be expressed by the formula

$$P_1 = \frac{10^{-2}ANI}{l_s} \cdot k \text{ pounds} \tag{11}$$

where k is a factor which gives the ratio of the pull at any point of the stroke to the maximum pull; Fig. 6 gives the approximate value of k at various points of the stroke.

IRON-CLAD ELECTROMAGNET WITH FLAT-END PLUNGER. The simple solenoid is a very inefficient type of magnet, because a large percentage of the reluctance of the magnetic path is found in the long air path outside the coil. Therefore, the plunger magnet is modified by putting an iron return circuit around the outside of the coil, thus reducing considerably the reluctance of the path and increasing the work which can be obtained with a certain amount of power, and with a certain expenditure of energy in the coil. Fig. 7 shows this type of "iron-clad" magnet. In its highest position the plunger

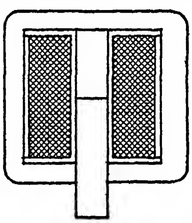

FIG. 7. Iron-clad Electromagnet, Flat-end Plunger

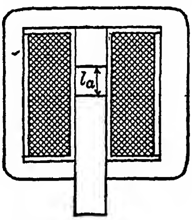

FIG. 8. Iron-clad Electromagnet, with Stop

strikes against the frame, so that in reality the magnet represented is a special form of magnet with stop, which is described below. If it is desired to have the pull decrease towards the end of the stroke, a hole similar to the one on the lower end is drilled in the upper end of the frame, and the plunger permitted to protrude through it.

Use of Stop to Increase Pull. The total pull on a plunger depends on the total number of flux lines which pass through it. For a given number of ampere-turns the total number of flux lines may be increased by decreasing the reluctance of the magnetic circuit. This reluctance may be still further reduced by using a plug or stop in the upper part of the solenoid of such a length that the air gap is central to the coil for the maximum travel which is required. This form is shown in Fig. 8.

Pull of Stop on Plunger; "Air-gap Pull." The pull P_2 between the stop and plunger may be expressed by the formula

$$P_2 = \frac{B^2A}{72 \times 10^6} \text{ pounds} \tag{12}$$

where B is the flux density in the air gap * perpendicular to the end surface of the plunger, in lines (maxwells) per square inch, and A the area of the end of the plunger, in square inches. The value of B may be calculated from the ampere-turns of the coil and the dimensions of the magnetic circuit in the same manner as the flux due to the field coils of a generator is calculated. Or, neglecting leakage and the reluctance of the iron part of the path, B may be calculated approximately from the formula

$$B = \frac{3.19NI}{l_a} \text{ lines per square inch} \tag{13}$$

* Strictly, B in this formula is the actual air-gap flux density less the flux density which would be produced by the solenoid were there no iron whatever in its magnetic circuit. This correction however, is smaller than the probable error in calculating B and may therefore be neglected.

where NI is the total ampere-turns of the coil and l_a is the length of the air gap in inches. Combining eqs. 12 and 13 gives*

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l_a}\right)^2 \text{ pounds} \tag{14}$$

This pull of the stop on the plunger is usually referred to as the "air-gap pull." The actual variation of this air-gap pull with the length of the air gap, for a particular magnet, is shown in curve B, Fig. 10.

Total Pull of Iron-clad Solenoid with Stop. The total pull of the iron-clad solenoid with stop may be looked upon as due to two components, the solenoid effect P_1 given by eq. 11 and the pull P_2 between stop and plunger given by eq. 14; hence the total pull is

$$P = 10^{-2} ANI\left(\frac{k}{l_s} + \frac{1.4 \times 10^{-5} NI}{l_a^2}\right) \text{ pounds} \tag{15}$$

When the air gap is short and at the center of the solenoid $k = 1$. The variation of this total pull with length of air gap for a particular electromagnet is shown in curve C, Fig. 10.

IRON-CLAD SOLENOID WITH CONED PLUNGER. It will be noted that for a given air-gap pull P_2, the ampere-turns required are directly proportional to the length of the air gap. If, therefore, for the same stroke the length of the path for the magnetic lines through the air gap can be reduced, the pull for the same ampere-turns will be increased, or if the pull remains constant the ampere-turns to obtain it can be decreased. This is accomplished by coning the end of the plunger, as shown in Fig. 9. For such a magnet the stroke should not be much in excess of the plunger diameter, as the leakage increases considerably with longer strokes, and this defeats the object of the coning. As the induction in the iron plunger is greater than the induction in the air gap, there is a limit to the practicable value which may be given to the cone angle; this limit is about reached for cast iron for a cone angle of 28 deg and for cast steel for a cone angle of 19 deg.

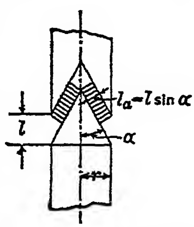

FIG. 9. Iron-clad Electromagnet, with Coned Plunger

"Air-gap Pull" on Coned Plunger. In the following it is assumed for simplicity's sake that a uniform flux at right angles to and distributed over the entire surfaces of the cones passes between plunger and plug. This is not strictly correct, especially for long strokes and the pull calculation is therefore only approximately correct.

Let l = length of stroke, in inches;
A = total cross-section of plunger, in square inches, $= \pi r^2$ where r is the radius of the plunger, see Fig. 9;
α = angle of the cone, in degrees, see Fig. 9;
NI = total ampere-turns of coil.

Then the flux density in the plunger is

$$B_i = \frac{3.19 NI}{l \sin^2 \alpha} \text{ lines per square inch} \tag{16}$$

and the air-gap pull in the direction of the stroke is

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l \sin \alpha}\right)^2 \text{ pounds} \tag{17}$$

The solenoid pull, as found by experiment, is practically the same as for a flat-end plunger, eq. 10. Whence the total pull on the coned plunger is

$$P = 10^{-2} ANI \left(\frac{1}{l_s} + \frac{1.4 \times 10^{-5} NI}{l^2 \sin^2 \alpha}\right) \text{ pounds} \tag{18}$$

where l_s is the length of the solenoid winding assuming the gap at the center of the solenoid ($k = 1$).

Note that $l \sin \alpha = l_a$ is the "effective" length of the air gap, i.e., the length perpendicular to the surface of the cone. Hence, comparing with eq. 15, it is seen that the pull on a coned plunger for the same effective air gap is the same as on a flat-end plunger, but the length of the stroke is increased in the ratio of $\dfrac{1}{\sin \alpha}$. Comparing eq. 16 with eq. 13, it is seen that this advantage is gained by increasing the flux density in the plunger by the square of $\dfrac{1}{\sin \alpha}$. For small air gaps, therefore, the coned plunger becomes satu-

* This formula applies only when the flux density in the magnetic circuit is below the point of saturation.

rated much more quickly for the same current than does the flat-end plunger, and the leakage and the reluctance of the iron part of the path produce an appreciable effect, causing the pull to become almost constant for small air gaps instead of increasing as shown by the approximate equation 18. This effect is clearly shown by the curves D and E in Fig. 10.

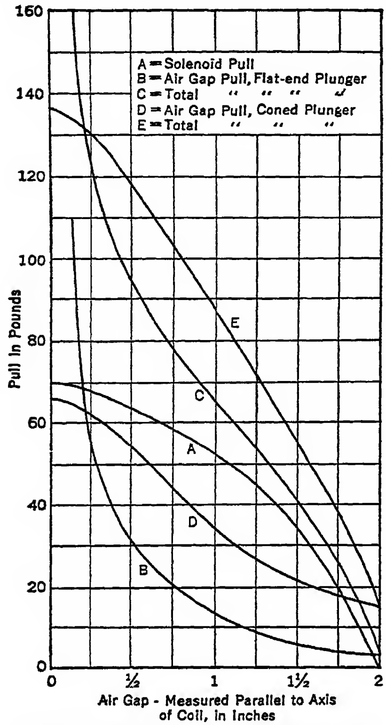

Fig. 10. Relative Pulls of Electromagnet Plungers

HORSESHOE-TYPE ELECTROMAGNETS. Another type of electromagnet which is used quite extensively is the horseshoe type, shown in its simplest form in Fig. 11. This magnet is not suitable for very long air gaps, because the leakage increases very rapidly with increasing distance between armature and poles. The total pull on the "armature" K of such a magnet, neglecting the leakage, may be expressed approximately by the relation

$$P = 2.8 \times 10^{-7} A \left(\frac{NI}{2l_a + l'}\right)^2 \text{ pounds} \quad (19)$$

where A = cross-section of each pole, in square inches; NI = total ampere-turns; l_a = length of each air gap in inches; and l' = length of air gap equivalent to the reluctance of the iron. This last depends on the permeability and dimensions of the iron part of the circuit; calling l_i the mean length of this iron circuit and μ its permeability, and assuming the mean cross-section of this path to be the same as the cross-section A of each pole, then $l' = l_i/\mu$. A more exact calculation of the pull may be made by calculating, as for a generator field, the ampere-turns required to establish a given flux density of B lines per square inch in the gap, and then applying eq. 12 to determine the pull.

SPEED OF MOVEMENT OF PLUNGER. In order to obtain quick action, the flux, upon closing the circuit, should reach its full value in as short a time as possible. The flux being a function of the current, the speed depends upon the rapidity with which the current reaches its full value. The time required for the current to reach its full value depends upon the quotient of the inductance L divided by the "effective" resistance r of the circuit. The larger this ratio L/r, which is called the "time constant" of the circuit, the longer the time required for the current to reach its full value. The inductance L is proportional to the square of the number of turns in the solenoid winding and inversely proportional to the total reluctance of the circuit. The effective resistance r depends not only upon the d-c or ohmic resistance of the winding, but also upon the eddy currents and hysteresis loss set up when the current is changing.

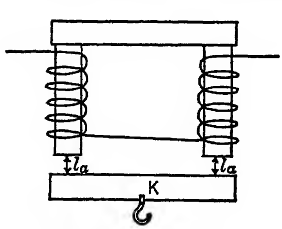

Fig. 11. Horseshoe Type Electromagnet

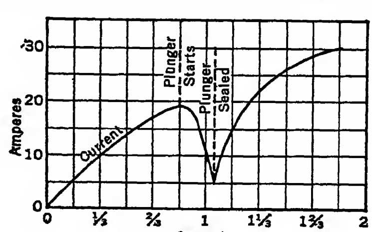

Fig. 12. Current-time Characteristic of D-c Electromagnet

In Fig. 12 is given current-time curve showing the change of the current during the

switching-in period of a d-c magnet. It will be seen that, after the closure of the circuit, the current first rises to a certain value which corresponds to a flux just sufficient to cause a movement of the armature and lift the plunger. As the plunger moves the flux increases, thereby causing a counter electromotive force, which tends to reduce the current. This counter emf depends upon the speed of the plunger. In the case shown the current drops off continuously until the plunger strikes against the stop, at which moment it has a value of approximately one-third of the value which started the motion of the plunger. After the plunger has come to rest, the current again increases and gradually reaches the value which is dependent upon the terminal voltage and the resistance of the coil.

Methods of Obtaining Quick Action. To reduce the eddy-current effect, the cross-section of the magnet frame should be as small as consistent with other considerations. Where very quick action is required it is sometimes advisable to slot this frame at right angles to the direction of the eddy currents or laminate it similar to transformers. In such cases it is also advisable to eliminate the brass tube on which the coil is very frequently wound and which acts as a guide for the plunger, or to slot this brass tube parallel to its axis. It is also advisable to slot or laminate the plunger.

Another method of obtaining quick action is to impress at the start a high voltage on the coil and insert resistance into the circuit of the coil as the plunger rises, in order to protect the magnet from overheating. This reduces the ampere-turns at the end of the stroke, which is permissible in most cases, because usually the pull of the magnet increases very rapidly toward the end of the stroke, as is indicated by the formulas given above.

ALTERNATING-CURRENT ELECTROMAGNETS. Electromagnets for producing a mechanical pull may also be designed to operate on alternating current. The flux in an a-c magnet passes through zero twice per cycle. The pull, which varies with the square of the current, therefore becomes zero twice every cycle, and it can be shown that it also varies according to a sine curve when the current is sinusoidal. The average effective pull is one-half of the maximum pull. Whenever the pull is less than the load, there is a tendency for the plunger to move away from the stop, and this causes rattling or humming of the magnet. This humming may be overcome in different ways, one method being to use a "shading coil," described below.

Polyphase Electromagnets. It can be shown that if three magnets are used and each is supplied with current from one phase of a three-phase source, or if two magnets are used and each is supplied with current from one phase of a two-phase source, then the resultant pull will be constant at any moment, and if the plungers are rigidly connected there will then be no chattering. The most common form of three-phase magnet is shown in Fig. 13. This consists of a core having three poles and a plunger of similar construction. Over each pole there is wound a coil which is supplied from one of the three phases of the circuit. There are various modifications of the polyphase magnet, but their general principle is the same. In calculating the total pull, the pull of each pole is figured separately and the several pulls, which are equal to one another, are combined vectorially, since they differ in time phase.

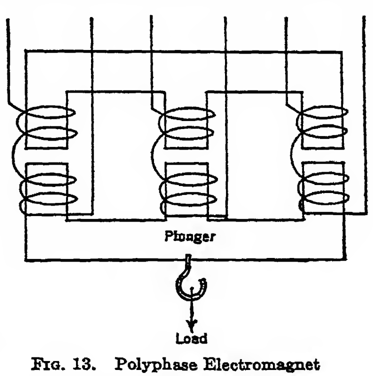

FIG. 13. Polyphase Electromagnet

Calculation of Pull of Single-phase Electromagnets, or of One Phase of Polyphase Electromagnets. The formulas for pull given above for d-c electromagnets also hold for a-c magnets, provided I is taken as the effective value of the current, B as the effective value of the flux density, and P as the average or effective value of the pull.

In contrast to the d-c electromagnet, the flux in an a-c electromagnet for a given impressed emf is approximately constant irrespective of the length of the air gap. This is due to the fact that the opposition to the flow of current through the winding is due almost entirely to the back emf, due to the alternation of the flux, and only to a very small extent to the resistance of the winding, the action in this respect being similar to that of a transformer (q.v.) or induction motor. The back emf being practically equal to the impressed emf, the flux producing this back emf is also proportional to the impressed emf, and is therefore practically constant when the impressed emf is constant.

Since the flux remains practically constant, the current I, as may be seen from eq. 13, must vary approximately proportionally to the length of the air gap l_a; this propor-

tionality holds only approximately, since eq. 13 neglects the magnetic leakage and the reluctance of the iron part of the magnetic circuit. The actual variation of the current with the length of the air gap for a particular a-c electromagnet is shown in Fig. 14.

Using the same notation as used above for d-c electromagnets and in addition putting E = effective value of impressed voltage per phase, and f = frequency of impressed voltage in cycles per second, the current taken by each phase of the magnet, neglecting the iron losses, leakage, and reluctance of the iron part of the magnetic circuit, will be

$$I = \frac{10^7 E l_a}{2fNA^2} \quad \text{amperes} \tag{20}$$

A more accurate calculation of the current, taking into account the losses and the magnetic leakage, may be effected by the method used for calculating the current in a transformer.

Substitution of the value of the current given by eq. 20 in eq. 15 for the total pull on a plunger with a flat end gives the approximate formula

$$P = \frac{10^5 E}{2fN}\left(\frac{kl_a}{l_s} + \frac{72E}{fNA}\right) \quad \text{pounds} \tag{21}$$

This equation also applies to a coned plunger, when l_a is taken equal to the length of the air gap perpendicular to the face of the cone.

Fig. 14 shows the pull curve of an a-c magnet with flat-end stop and plunger. It will be noted that the current is roughly proportional to the air gap. The solenoid pull reaches a maximum at an air gap of about 2 1/4 in. and then falls again approximately proportional to the air gap; the air-gap pull is constant over a wide range of the travel. The result is a total pull curve which has a maximum at approximately 2 1/4 in. and which drops until an air gap of approximately 3/4 in. is reached, and from there on it increases again, due to the increase of the air-gap pull, which is caused by the diminution of the leakage flux as the air gap decreases.

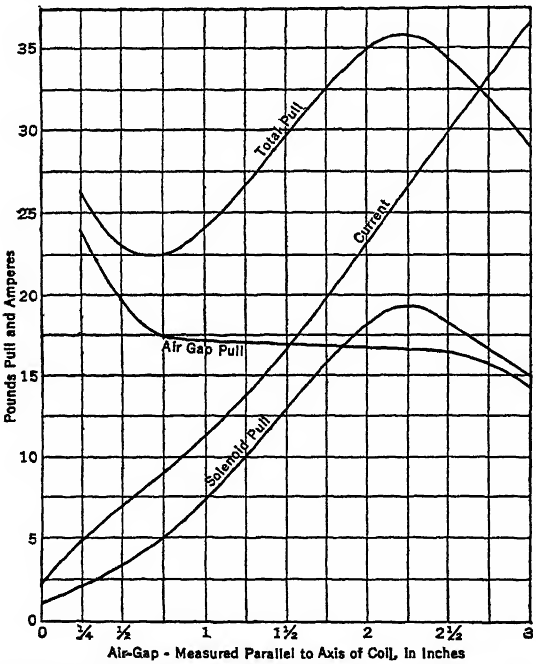

FIG. 14. Pull and Current Curves of a-c Magnet. Length of solenoid 5 inches; internal diameter of solenoid 3 inches; external diameter 4 1/2 inches; diameter of plunger 2 3/8 inches; stop projects 1 1/2 inches inside of coil; turns 144; voltage 220; frequency 60 cycles per second

Limiting Flux Density and Losses in A-c Electromagnets. Attention must be paid to the fact that the iron losses, like those of the transformer, increase with the flux density, and therefore the design must be such that the flux density is not too high. The flux density in the iron in the case of a flat-end plunger, from eq. 13, which neglects the magnetic leakage and reluctance of the iron part of the magnetic path, is

$$B = \frac{1.6 \times 10^7 E}{fNA} \quad \text{maxwells per square inch} \tag{22}$$

and in the case of a coned plunger, under the same assumptions, from eq. 16 is

$$B = \frac{1.6 \times 10^7 E}{fNA \sin \alpha} \quad \text{maxwells per square inch} \tag{23}$$

where α is the cone angle, see Fig. 9.

These relations are approximate only. The magnetizing component of the exciting current, and from it the flux density can be more accurately calculated by the methods employed in the design of transformers or induction motors.

The iron and copper losses can also be calculated by similar methods, and from these losses the energy component of the exciting current can be determined. The total current and power factor of the electromagnet can then be deduced.

Shading Coil for Single-phase Electromagnets. The humming of single-phase magnets may be greatly reduced by introducing a so-called "shading coil" in the pole face. This shading coil is nothing more than a short-circuited secondary winding, consisting of one or more turns, which encloses only part of the total flux passing through the plunger. Owing to the leakage reactance of this turn, the current induced in it is out of phase with the inducing flux, so that at the moment when the inducing flux, due to the main winding, is zero, there still remains a flux due to the current in the shading coil, which flux produces a pull. The result is that the combined pull from the main flux and the shading coil flux never becomes zero. Fig. 15 shows the arrangement of the shading coil. This coil is mounted in the plunger or in the plug close to the pole face, in order to reduce the length of the path for the magnetic lines which are interlinked with the shading coil. Naturally the shading coil has no effect with long air gaps, and it is therefore imperative that a good magnetic contact be obtained when the plunger is in the sealed position to get the greatest possible effect of the shading coil.

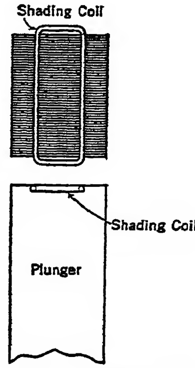

Fig. 15. Shading Coil

Incidentally the shading coil also increases considerably the maximum pull for a given impressed emf as the maximum pull is also due to the combined main and local flux. As an example, a plunger magnet without shading coil, which gave a minimum pull of zero and a maximum pull of 28 lb, had, after the introduction of the shading coil, a minimum pull of 18 lb and a maximum pull of 143 lb.

15. INDUCTION COILS

(See also Section 3.) An induction coil is a device which transforms a low direct emf to a high alternating emf of unsymmetrical form. The single-winding coil, called a primary coil, is used extensively in gas-engine ignition and in automatic gas lighting, and the double-winding coil, called a secondary coil, is used for the excitation of x-ray tubes, gas-engine ignition, automatic gas lighting, electrotherapeutics, and wireless telegraphy.

PRIMARY INDUCTION COIL. The primary induction coil consists of a single coil wound upon an iron core made up of a compact bundle of soft-iron wires. To obtain a spark from such a coil it is connected in series with a battery and some kind of "make-and-break" contact. When the circuit is closed, the current increases gradually according to the expression,

$$i_t = \frac{E}{r}(1 - \epsilon^{-rt/L})$$

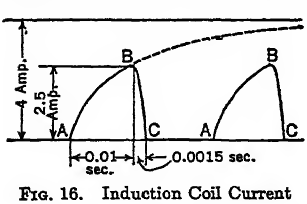

Fig. 16. Induction Coil Current

where i_t equals the current at any time t seconds after the circuit is closed, E equals the emf of the battery, r equals the resistance in ohms and L equals the inductance in henrys of the entire circuit, and ϵ the base of the natural system of logarithms. The shape of the current curve at "make" is shown in the curve AB (Fig. 16),* which is an oscillograph record taken when an emf of 4 volts is impressed upon a primary coil of 1-ohm resistance and 0.01-henry inductance.

If the circuit is opened when the current reaches some value, such as B (Fig. 16), the current decreases rapidly to zero as shown in the curve BC. Since the emf induced in such a coil at any instant equals $L\frac{di}{dt}$ when the current falls from B to C, the emf, induced in the coil will be many times that of the battery emf and a spark will be established between the open contacts at "break." For use in connection with gas engines the "make-and-break" contacts are located within the cylinder. Reliable ignition is

* This and the following oscillograph records are due to Bailey, B. F., *Elec. World*, 1910, Vol. 55, p. 948.

obtained when about 0.02 joule is dissipated in the spark at "break." Since the energy dissipated as heat in the spark is converted from the electromagnetic energy stored up in the coil, it follows that the coil must have an inductance and carry a current at "break" such that the energy stored up, i.e., $1/2\, LI^2$, will exceed the required value of 0.02 joule. In practice it is customary to design the coil and time of contact so that $1/2\, LI^2$ equals about 0.04 joule, such coils having an efficiency of about 50 per cent. One-half of the energy put into the coil is then used up in heating the conductors and metallic parts by I^2R and hysteresis losses.

SECONDARY INDUCTION COIL. The secondary coil has two separate windings wound about an iron core, one of few turns called the primary winding and the other of many turns called the secondary winding. The primary winding is connected in series with a battery and an interrupter (described below). The spark is produced between the terminals of the secondary winding. In most cases a condenser is shunted across the interrupter to prevent sparking at that point. Since the emf induced in each turn linked by the magnetic flux is the same, the emf induced in the secondary winding, neglecting leakage, will equal $\frac{N_s}{N_p}$ times that induced in the primary winding, where N_s and N_p equal the respective number of turns on the secondary and primary windings. Secondary coils which give sparks as long as 5 ft have been constructed in this manner.

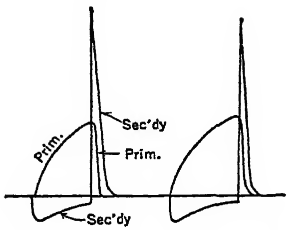

FIG. 17. Interrupter Current; No Condenser

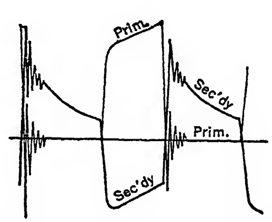

FIG. 18. Interrupter Current at Break; Condenser Shunt

Effect of Shunting Interrupter with Capacitance. When the interrupter is shunted by a capacitance the emf induced in the secondary winding becomes oscillatory. This effect of the capacitance is well shown in the oscillograph records given in Figs. 17 and 18. In each case a high non-inductive resistance is connected across the secondary terminals, and the curves indicate the variation of the primary current and the secondary emf. In Fig. 17 the interrupter is unshunted by a condenser, and as a result the secondary emf increases at "break" and decreases to zero without oscillation. In Fig. 18 the interrupter is shunted by a condenser and the secondary emf becomes oscillatory. The form of the emf curve in any actual case is not shown accurately by Fig. 18 except at the time of "break," since the resistance between the terminals of a spark gap decreases rapidly after the spark is established. In Fig. 19 a low resistance is connected across the secondary terminals and shows the general form of the emf curve after the spark is established.

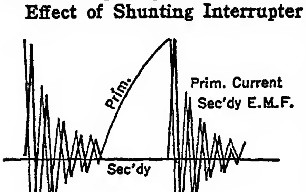

FIG. 19. Interrupter Current after Establishment of Arc; Condenser Shunt

Although not shown by the curves owing to difference in scale, the capacitance shunted around the interrupter also increases the secondary emf above the value obtained without the capacitance. Assuming that no spark is formed at the interrupter, it may be shown that the emf of the secondary equals $\frac{N_s}{N_p}\sqrt{\frac{L}{C}}\,I_b$, where L equals the inductance of the primary in henrys, C equals the capacitance of the condenser in farads, and I_b equals the primary current at "break" in amperes. If C is made so low that a spark appears at the interrupter, the above formula no longer holds and the secondary emf will be greatly reduced. C should then be made as small as possible but of sufficient size to suppress the spark at the interrupter.

Insulation. The conductors in the secondary winding must be heavily insulated to

withstand the very high emf induced in that winding. In most cases a double-covered silk insulation impregnated with some insulating compound is used. In large induction coils the secondary winding is built up of several flat coils insulated from each other by ebonite or fiber disks. The coils are so wound that the electrical connections are made alternately at the top and at the bottom of the respective coils.

Dimensions of 3-inch Coil. The general dimensions of parts of a 3-in. induction coil, as reported by S. R. Bottone, *Radiography* (London, 1898). are given following· Iron, bundle of No. 20 (A. W. G.) annealed iron wire, 1 1/4 in. diameter. 13 in. long. Primary winding, four layers of No. 12 double silk-covered copper wire, about 4 1/4 lb. Ebonite tube over primary, 12 in. long, 2 in. inside diameter, 2 1/2 in. outside. Two ebonite heads, 5 in. square, 1/2 in. thick. Seven vulcanized fiber circlets (for sections), 4 1/2 in. diameter, 1/8 in. thick, 2 1/2 in. central aperture. Secondary winding, 4 lb No. 36 double-silk-covered wire. Platinum-tip contact breaker, height from base to center of iron hammer, 2 1/2 in., size of iron head of hammer 3/4 in. diameter, 3/4 in. long. Base (fitted with false bottom to contain condenser), 18 in. long by 9 in wide by 2 3/4 in. deep. Condenser, 144 sheets of tin foil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper 8 by 13 in.

Dimensions of 6-inch Coil. Similar dimensions for a 6-in. induction coil, as reported by Bottone, are given following: Iron bundle of No. 16 (A. W. G.) annealed iron wire, 1 1/2 in. diameter, 15 in. long. Primary winding, four layers of No. 12 double silk-covered copper wire, about 5 lb. Ebonite tube over primary 14 in. long, 2 1/4 in. inside diameter, 2 3/4 in. outside. Ebonite heads 6 in. square, 3/4 in. thick. Seven vulcanized fiber circlets 5 1/4 in. diameter, 1/8 in. thick, with 2 3/4 in. central hole. Secondary winding, 7 lb No. 38 double-silk-covered copper wire. Platinum-tip contact breaker, height from base to center of hammer, 3 in. size of hammer head, 1 in. diameter, 1 in. long. Base (fitted with false bottom to contain condenser), 20 in. long by 10 in. wide by 3 1/2 in. deep. Condenser, 144 sheets of tin foil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper, 8 by 13 in.

Interrupters. An interrupter should be so designed that it will close the circuit for a definite time by easy adjustment and will open the circuit at the end of that time as quickly as possible. Two distinct types, the mechanical and the electrolytic interrupter, are in common use. Mechanical interrupters may be divided into the following forms. hammer, atonic, commutator and mercury interrupters. Of the electrolytic interrupters the Wehnelt type is the most popular.

Hammer Interrupter. In the hammer interrupter, shown in Fig. 20, the circuit is opened at A when the core B attracts the iron mass C mounted at the free end of the

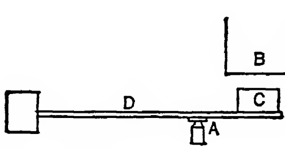

Fig. 20. Hammer Interrupter

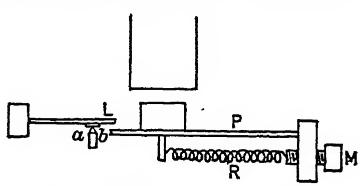

Fig. 21. Atonic Interrupter

spring blade D. Since the core attracts the iron mass only when the circuit is closed and loses its attraction when the circuit is opened, the spring blade is set in vibration and opens and closes the circuit in rapid succession. This type is used extensively in connection with small coils. The contacts at A are usually tipped with platinum to withstand the intense heat developed by sparking. The rapidity of the " break " is not great enough for large coils, and when a large current must be broken, trouble is experienced in keeping the contact points in good condition.

Atonic Interrupter. In the atonic interrupter, shown in Fig. 21, the free end of the iron strip P is attracted as in the hammer interrupter and is returned to its original position by the spring R. The circuit is opened at the contacts ab when the free end of P strikes the blade L. Interrupters constructed in this manner open the circuit quicker than do the hammer type since the attracted member is moving faster at the instant of " break." The period of the atonic interrupter may be varied within wide limits by regulating the tension on the spring R by means of the thumb-screw M.

Commutator Interrupters are often used when the primary current is too large to be broken by the hammer or atonic interrupters. A brush bearing upon a revolving disk, built up of conducting and insulating segments, makes and breaks the circuit at any desired rate, depending upon the speed of the motor which drives the disk.

Mercury Interrupters are specially adapted to circuits of high emf. In the *plunger*

type a pointed electrode is alternately immersed and withdrawn from a cup of mercury, the moving electrode being set in vibration magnetically as in the hammer interrupter, or by the positive action of a cam driven by a motor. The mercury cup is covered with a layer of oil so that the spark is quickly extinguished when the moving electrode is withdrawn into the oil. In the *turbine* type a small stream of mercury is directed upon the periphery of a revolving toothed wheel. The circuit is made when the mercury stream strikes a tooth and is broken when the mercury stream passes through a slot between the teeth. The toothed wheel is rotated at high speed by a motor which also drives a small mercury pump.

Wehnelt Interrupter. The Wehnelt interrupter has two fixed electrodes which are immersed in dilute sulphuric acid. The anode consists of a small platinum wire insulated by glass except at its tip end. The cathode usually consists of a sheet of lead. If the electrolyte is well circulated, an interrupter of this kind will give about 450 interruptions per second with 24 volts impressed upon the primary circuit. When used with large coils the electrolyte heats up quickly, and a cooling coil is sometimes immersed in the electrolyte to control the temperature.

Tesla Coil. In some cases, as in wireless telegraphy, electrotherapeutics, etc., where a unidirectional emf is not required, but an emf of high frequency is desired, a Tesla coil may be used in place of an induction coil. The construction of a simple Tesla coil is shown in Fig. 22. In the primary circuit a primary winding of few turns and a small spark gap are shunted by a condenser. Secondary and primary windings are wound together upon an air core. If an alternating emf of from 5000 to 10,000 volts is impressed upon the primary the condenser is charged until the voltage across the primary gap breaks it down. The condenser then discharges through the gap, producing an oscillating current in the primary winding. The oscillating current in the primary induces a high-frequency emf in the secondary, which is sufficient to break down the long secondary spark gap.

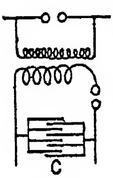

FIG. 22. Tesla Coil

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SECTION 5

MEASUREMENTS AND MEASURING APPARATUS

BY

H. B. BROOKS

ART.		PAGE
1.	Standards	2
2.	Galvanometers	2
3.	Standard Cells	6
4.	Time Standards	9
5.	Resistance Standards	9
6.	Resistance Boxes	11
7.	Bridges	12
8.	Potentiometers	19
9.	Laboratory Measurements of D-C Power and Energy	26
10.	Laboratory Measurements of Alternating Current, Voltage, and Power	26
11.	Electrical Indicating Instruments	27
12.	Electrical Integrating Meters	45
13.	Instrument Transformers	53
14.	Electrometers	61
15.	Rotary Voltmeter for High D-C Voltages	63
16.	Spark Gaps for Measuring High Voltages	63
17.	Corona Voltmeter	67
18.	Oscillographs	67
19.	Testing of Magnetic Materials	72
20.	Testing of Insulating Materials	81
21.	Photometry	87
22.	Electrical Recording Instruments	95
23.	Telemetering	96
24.	Measurement of Non-electrical Quantities by Electrical and Magnetic Means	96
25.	Measurement of Temperature	96
26.	Bibliography	104

The following members of the scientific staff of the National Bureau of Standards collaborated with Dr. Brooks in the preparation of this section.

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MEASUREMENTS AND MEASURING APPARATUS

By H. B. Brooks

1. STANDARDS

In the measurement of electrical, magnetic, thermal, and optical quantities, reference must ultimately be made to established units and verified standards. Experience has indicated the most suitable units and standards and also the most practicable procedures for carrying out the precise laboratory measurements upon which rest the less accurate but indispensable practical measurements.

DISTINCTION BETWEEN UNITS AND STANDARDS. The distinction between unit and standard should be carefully noted. A **unit** is a quantity in terms of which similar quantities are to be measured; a **standard** is a physical structure by which the unit is realized and made available. For example, the unit of electromotive force, the volt, is an invisible, intangible quantity, the very nature of which is unknown; but the **standard** of electromotive force is a particular kind of physical structure known as a standard cell.

The so-called "international" electrical units now (1936) in use will probably be replaced within a few years by those decimal multiples of the cgs electromagnetic units which the international units were originally intended to represent.

BASIC STANDARDS FOR ELECTRICAL MEASUREMENTS. All measurements of electric current, voltage, power, and energy, in either d-c or a-c circuits, are ultimately referred to resistance standards, standard cells, and time standards. Standards of mass (standard "weights") require some form of balance for their comparison. Similarly, the Wheatstone bridge and other bridges are used in the comparison of resistance standards, and potentiometers serve to measure electromotive forces in terms of the known electromotive force of a standard cell. In the use of bridges and potentiometers, adjustments are made in one or more arms of a network of conductors until a particular condition of balance is established, as shown by a suitable detector, usually by a galvanometer.

2. GALVANOMETERS

A galvanometer is an instrument for measuring a small electric current. A d-c galvanometer consists essentially of a permanent magnet and a coil or wire through which the current may flow. In the **moving-magnet galvanometer** the coil is fixed and relatively large; the magnet is very small and light and is suspended within the magnetic field of the coil so that it may readily turn as a result of the interaction of the field of the magnet and that of the coil. In the **moving-coil galvanometer** (often called d'Arsonval galvanometer) the coil is very light and is arranged to rotate within the field of the relatively large fixed permanent magnet. The **string galvanometer** is a variant of the moving-coil galvanometer, and has a conducting "string" (wire or metal-coated fiber) stretched within the field of a strong magnet. The string is deflected laterally when a current flows through it. (See article on Oscillographs.) In all forms of galvanometer the motion is opposed and brought to rest by a counter torque (or counter force, in the string galvanometer), by electromagnetic damping, and by friction. The last is made up of air friction, molecular friction due to imperfect elasticity of the suspensions or control springs, and (in pivoted galvanometers) of bearing friction.

MOVING-MAGNET GALVANOMETERS have certain advantages over moving-coil galvanometers which cause them to be used for some purposes in spite of their inherent disadvantages. Their advantages are (*a*) lightness of the moving element; (*b*) absence of current in the moving element, permitting the use of very delicate suspensions which (like silk and quartz fibers) may be non-conducting; (*c*) very high sensitivity as a result of (*a*) and (*b*); (*d*) the sensitivity may be changed by regrouping sections of the fixed coil, as well as by varying the degree of astaticism of astatic galvanometers; (*e*) the damping is virtually unaffected by changes in resistance of the external circuit. Their disadvantages are (*a*) great susceptibility to disturbance by variations of the local magnetic

field; (b) the damping is not as readily controlled as in the moving-coil galvanometer. Disturbances resulting from variation of the local magnetic field are reduced either by using an astatic construction or by surrounding the galvanometer with an iron shield or a number of concentric shields, or by both precautions. In the **astatic galvanometer** the moving system is a double one with magnetic polarities arranged in such a way that the moving system would be unaffected by a change in the strength of the local magnetic field, provided this change were such as to affect both elements of the astatic system equally. In practice such an ideal condition is seldom realized, and the user of astatic galvanometers (or other nominally astatic instruments) should take all reasonable care to avoid exposing them to stray magnetic fields.

MOVING-COIL GALVANOMETERS. Originally much inferior to moving-magnet galvanometers in sensitivity, moving-coil galvanometers have been refined and improved until they now serve for all but extreme requirements. They have the great advantages of being almost completely unaffected by even large variations in the local magnetic field, and of being readily damped by proper choice and adjustment of external circuit conditions. In some forms the damping may be regulated by a magnetic shunt.

MOTION OF GALVANOMETER MOVING ELEMENT. If the moving coil (or moving magnet) of a galvanometer be deflected from its zero position and then released, it may come to rest at the zero position only after one or more swings through this position. For this case the motion is said to be **periodic.** Under other conditions, the coil or magnet will return to its equilibrium position without passing through it. This constitutes **aperiodic** motion. When the damping is just sufficient to make the motion aperiodic, the galvanometer is said to be **critically damped.** In this condition the time required to come to apparent rest is a minimum. When critically damped, the moving system will reach its final deflected position, within 0.1 per cent of the deflection, in about 1.5 times the free period. Theoretically this is the ideal condition of operation, but a slight amount of underdamping is usually preferable because it allows the coil to overshoot the rest position slightly and then return to it. This gives assurance that sticking is not present.

The **damping ratio** of a galvanometer is the ratio, expressed as a positive number, of a given deflection to the next deflection in the opposite direction. The greater this ratio, the greater the degree of damping. The natural logarithm of this ratio is called the **logarithmic decrement.**

Much time will be lost, in using moving-coil galvanometers, if the total resistance in the galvanometer circuit is widely different from the value which gives aperiodic motion. If one is constrained to choose between using a galvanometer in a circuit having (for example) only one-half the resistance for critical damping, or in a circuit of twice the critical value, the latter condition, giving underdamping, is preferable to the former because with it the moving system comes to apparent rest in a shorter time.

SENSITIVITY OF GALVANOMETERS. The sensitivity of a galvanometer may be defined either as its response (in specified deflection units) to unit stimulus, or as the magnitude of the stimulus required to produce a specified unit deflection. The second form of definition gives a numerical value of sensitivity (for a given galvanometer) which is the reciprocal of the value given by the first definition. For example, the sensitivity of a given galvanometer may be stated either as 10 mm per microvolt, or as 0.1 microvolt per mm. Both systems of definition are in successful use, and the ease with which the sensitivity may be translated from one form to the other evidently explains the lack of confusion which would ordinarily attend the use of radically different definitions for the same thing.

The second form of definition has been found more convenient in American manufacturing practice. In galvanometers having attached scales the specified unit deflection is assumed to be one scale division. In reflecting galvanometers for use with separate reading devices the standard unit deflection is assumed to be 1 mm, with a scale distance of 1 meter. The **current sensitivity** is usually expressed as the number of microamperes to produce the specified unit deflection. The **voltage sensitivity** is expressed as the number of microvolts that must be impressed on the circuit consisting of the galvanometer coil and the external resistance for critical damping to produce the specified unit deflection.

The **ballistic sensitivity** is expressed as the number of microcoulombs which must be discharged through the galvanometer to produce the specified unit deflection. The **megohm sensitivity** is expressed as the number of megohms in the galvanometer circuit for which an impressed emf of 1 volt gives the specified unit deflection.

Particular care is necessary in making and in interpreting statements concerning the microvolt sensitivity of galvanometers, because some makers state the microvolt sensitivity as the microvolts per millimeter (or reciprocally, as the millimeters per microvolt) at the galvanometer terminals. Most modern moving-coil galvanometers are so

designed that they cannot be effectively used with the zero value of external resistance implied in this definition, because this condition causes excessive overdamping. All statements of microvolt sensitivity should contain definite information as to the total resistance to which they refer. When such a statement is not given in the maker's catalog, his basis for the definition of microvolt sensitivity can often be discovered from other data given, namely, the microampere sensitivity and the resistance of the galvanometer. A statement of the external resistance for critical damping should always be included in makers' statements concerning their moving-coil galvanometers.

The use of the above-defined second method of stating sensitivity gives numerical values (for galvanometers of relatively high sensitivity) in the form of very small decimal fractions. To avoid this, some foreign makers who follow this principle in defining sensitivity use units of current and voltage smaller than the microampere and microvolt respectively. For example, in a single circular of one maker the current sensitivity of various galvanometers is given in terms of 10^{-7}, 10^{-8}, 10^{-9} and 10^{-10} amp, and the voltage sensitivity is given in millimeters for 10^{-8} volt. This lack of uniformity in the units used in stating galvanometer sensitivity would be a source of inconvenience to the user if it were not so easy to convert a given sensitivity from one form to another. The need for a named unit of current smaller than the microampere (and continuing the sequence, ampere, milliampere, microampere) has induced some foreign authors to use the word **nanoampere** to signify 10^{-9} amp.

The attempt is sometimes made to increase the effective sensitivity of a galvanometer by placing the scale at a comparatively great distance from it. In doing this, some users overlook the fact that the instability of the zero is magnified in the same ratio as the deflections. The use of an increased scale distance is justifiable only when the stability, including the effect of vibrations, is in excess of the precision of reading.

GALVANOMETER SHUNTS. For many purposes it is essential to reduce the sensitivity of a galvanometer to a definite fraction of its value. This is conveniently done by connecting a resistance across the galvanometer terminals to bypass the greater part of the current. If the resistance of this bypass ("galvanometer shunt") is $^1/_9$, $^1/_{99}$, or $^1/_{999}$ of the galvanometer resistance, the current sensitivity will thereby be reduced to $^1/_{10}$, $^1/_{100}$, or $^1/_{1000}$ of its original value, respectively. A simple shunt of this sort has two drawbacks, namely, it must be adjusted for a particular galvanometer, and unless it is wound with copper wire it will not have the correct shunting effect except at some one temperature. To overcome these defects, the Ayrton-Mather shunt was designed. See Fig. 1. It may be shown that if the combination of galvanometer and shunt has a given value of current sensitivity with the slider on the stud b, the current sensitivity with the slider on any other stud x will be lower in the ratio of r to R, regardless of the resistance of the galvanometer. In choosing an Ayrton-Mather shunt for a moving-coil galvanometer the value of R should be selected with reference to the external critical damping resistance and the probable value of resistance of the circuit to be connected to the terminals A and B so that the damping will be satisfactory.

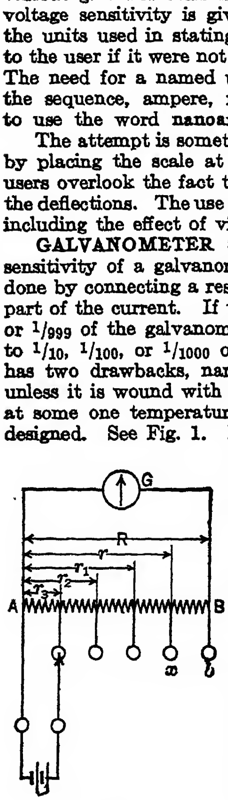

FIG. 1

The Ayrton-Mather shunt was originally devised for use in circuits of high resistance, such as were encountered in measurements of insulation resistance. When this shunt is used in such a circuit the variations of resistance produced by the operation of its range-changing switch have an inappreciable effect on the magnitude of the current in the external circuit. However, when the Ayrton-Mather shunt is used with the low-resistance external circuits so common at this time, large variations in the current in the external circuit may be caused by changing the range of the shunt. While a compensating resistance could be used, it is often preferable and less expensive to use an ordinary galvanometer shunt.

CHOICE OF A TYPE OF GALVANOMETER. A moving-coil galvanometer should always be preferred to a moving-magnet galvanometer except for some very special purposes. For any application the most robust, simple, and inexpensive galvanometer which has sufficient sensitivity with a suitable period should be chosen. In general, superfluous sensitivity increases the first cost of the galvanometer and tends toward greater delicacy and hence fragility, as well as toward a longer period and consequently greater time required for observations. For the most precise work of the standardizing laboratory it is necessary to use delicate, expensive galvanometers of relatively long period and to employ a reading arrangement consisting of a telescope and scale or a lamp

and scale. Such moving-coil galvanometers are made in this country in two grades of sensitivity and at corresponding prices. For work requiring only moderate sensitivity, with shorter periods, portable lamp-and-scale galvanometers are supplied as compact, self-contained units. For work requiring still lower sensitivity, robust portable suspended-coil pointer galvanometers are available having the advantages of low price and ease of exchange or replacement of the coil system. For some purposes for which moderate sensitivity is sufficient and the deflection should be proportional to the current over a relatively large angle, pivoted pointer galvanometers are made which embody the standard features of construction of d-c indicating instruments.

Certain old theorems state that for maximum sensitivity the coil of the galvanometer should be wound to have a resistance equal to the "internal resistance" of the source of the emf which is to produce the deflection. Although this can be proved to be true as a matter of mathematics, the statement is largely of academic interest, as one can depart very widely from the theoretically correct value of coil resistance with but little loss of sensitivity. Furthermore, most moving-coil galvanometers as now made are designed to be used with an external resistance of, say, 3 to 15 times the coil resistance in order to get suitably damped motion. (This statement does not apply to galvanometers having damping frames or damping windings to increase still further the value of external resistance for critical damping.) A more practical rule for choosing a moving-coil galvanometer is as follows: If a given small emf in an external circuit of resistance R is to be detected or measured, select the galvanometer which has a value of external critical damping resistance most nearly equal to R; if no available galvanometer has just this value, select the galvanometer for which the external resistance for critical damping falls short of R by the smallest amount. The resulting motion will be underdamped, which is much to be preferred to overdamping; if desired, the damping may be improved by suitably shunting the galvanometer.

PARASITIC THERMAL EMF IN GALVANOMETERS. Moving-coil galvanometers as made without special precautions against parasitic emf contain a number of junctions of dissimilar metals. The resultant thermal emf will depend upon thermal conditions, and may amount to 10 or 20 microvolts under very bad conditions, for example, in winter weather when a descending current of cold air strikes the top of a galvanometer but does not sensibly affect the temperature of the bottom. Older forms of galvanometers with a long tube at the top containing the suspension are particularly liable to this defect, which is of consequence chiefly when relatively low voltages are to be detected or measured by the galvanometer. The thermal emf may be minimized by enclosing the galvanometer in a case made (*a*) of metal, such as copper, aluminum, or brass, of high thermal conductivity; or (*b*) of material of high thermal resistivity (cork, balsa wood, etc.). In (*a*) the metal enclosure will have nearly the same temperature in all its parts; in (*b*) the galvanometer itself, protected from radiation and convection, will have a nearly uniform temperature in all its parts. Some makers supply galvanometers in which not only the coil but also the suspensions, binding posts, etc., are made of copper to minimize thermal emf. It is important that ballistic galvanometers should be so constructed.

BALLISTIC GALVANOMETERS. Ballistic galvanometers are used to measure a quantity of electricity, such as is given up by a condenser during discharge. They are usually constructed to have a relatively long period for one or both of the following reasons: first, because a longer period makes it easier to read the momentary maximum deflection; second, because in some applications the time required for the passage of the electrical quantity is relatively great enough to introduce an error if a galvanometer of short period should be used.

Ballistic galvanometers may be used either undamped or damped. When critically damped they have only about one-third the sensitivity when undamped, but for most applications the sensitivity is so ample that the reduction which accompanies critical damping can well be tolerated for the sake of the much greater convenience in operation.

The **ballistic constant** of a galvanometer is conveniently expressed as the number of microcoulombs per millimeter deflection with a scale distance of 1 meter. It is usually determined by discharging through the moving coil a known quantity of electricity from a charged condenser or from the secondary winding of a mutual inductor. In the former case the quantity in microcoulombs is the product of the capacitance of the condenser in microfarads and the difference of potential in volts to which it is charged. Ballistic galvanometers have important applications in magnetic testing. For a discussion of their use in this field, and the method of calibrating them by means of a mutual inductor, see Testing of Magnetic Materials.

A-C GALVANOMETERS. The two forms of a-c galvanometer most used in this country are the vibration galvanometer and the moving-coil separately excited galva-

nometer. In its usual form, the moving-coil vibration galvanometer differs from the d-c moving-coil galvanometer chiefly in having a very narrow coil and stiff suspensions. The stiffness of the suspensions is adjustable to bring the natural frequency of the moving system into resonance with the frequency of the a-c supply circuit. For any other frequency the sensitivity is relatively very small, and harmonics present in the wave form are substantially without effect on the deflection. This valuable property of the vibration galvanometer often makes possible the use of the simple mathematical relations based on the assumption of a sine-wave form, even though the available a-c supply voltage contains harmonics.

The moving-coil separately excited a-c galvanometer differs from the ordinary d-c moving-coil galvanometer mainly in having a laminated electromagnet and a laminated core in place of a permanent magnet and solid core. The theory of this class of galvanometer has been thoroughly treated by Weibel; see Bibliography. Unlike the vibration galvanometer, the separately excited a-c galvanometer is not restricted to a particular frequency (or frequency range). It must be excited by a current of the same frequency as that used in the measurement circuit (bridge or potentiometer). It responds only to that component of the current in its moving coil which is in time phase with the magnetic flux in which the coil is located, and consequently for maximum sensitivity the current in the coil must be in phase with the flux. It is also possible to overload and damage the coil without producing a large deflection, if the current is nearly in quadrature with the flux. In some cases the in-phase condition of current and flux is readily and simply attained; for example, in using the galvanometer with a resistance bridge, the exciting coil of the galvanometer is connected to the supply circuit through a suitable non-inductive resistance, and the current for the bridge is taken from potential taps on this resistance. To obtain proper damping in reflecting galvanometers of this type, the circuit external to the galvanometer must have suitable values of resistance, inductance, and capacitance. When these conditions are fulfilled, the separately excited a-c galvanometer has a possible sensitivity but little inferior to that of d-c moving-coil galvanometers of corresponding construction. In the use of the robust pointer-type a-c galvanometers no particular attention has to be paid to the external inductance and capacitance in order to obtain good damping in the usual circuits.

SUPPORTS FOR GALVANOMETERS. Sensitive reflecting galvanometers should preferably be mounted on stable masonry walls or piers free from vibration. Vibration galvanometers are particularly susceptible to mechanical vibrations set up in buildings by the operation of motors or alternators at the frequency for which the galvanometers are tuned. When trouble is experienced with either d-c or a-c galvanometers mounted on the best available firm supports, recourse must be had to expedients, of which the Julius suspension is among the oldest. See bibliography, paper by W. P. White in *Phys. Rev.*, 1904, vol. 19, p. 323, and book by Werner. At the Bureau of Standards, vibration galvanometers are protected as follows: A slab of concrete weighing about 400 lb is supported by four helical steel springs. On top of the slab is a layer of hair felt 1.5 in. thick on which is a board upon which the galvanometers are placed. This spring-supported slab is a vibrating system with a natural period so great in proportion to the period of the galvanometer system that it has no tendency to pick up vibrations of the frequency of the galvanometer system. The felt pad is effective in damping out vibrations of higher frequency which may be transmitted along the length of the steel wires composing the helical springs. Hartsough (see Bibliography) has stated that the shielding of a sensitive apparatus from vibration is very nearly perfect when it is supported on thin inflated rubber bags. In his work three interconnected bags were used, containing air at a pressure of about 50 cm (20 in.) of water, and a mass of about 4 lb was placed on each bag.

3. STANDARD CELLS

These are primary cells made of pure materials in accordance with exact specifications. They have the special characteristic of maintaining a very constant emf when suitably cared for and used. They are of great and increasing importance as standards of emf.

The only kinds of standard cell of technical importance at present are the **saturated cadmium cell** called by international agreement the "Weston Normal Cell," which serves as the primary standard of emf in national and other important standardizing laboratories, and the **unsaturated cadmium cell** which is used generally (except in England) in engineering laboratories and industrial plants as a secondary or working standard of emf.

WESTON NORMAL CELL. Fig. 2 shows the construction of this cell. The H-form glass vessel, with a platinum lead-in wire at the bottom of each limb, contains mercury covered with mercurous sulfate paste as the depolarizer in the positive limb, cadmium

amalgam in the negative limb, and cadmium sulfate solution with an excess of cadmium sulfate crystals above the paste and amalgam as shown. For the exceedingly exact work for which these cells are used, they must be kept at a very constant temperature. In the standard-cell laboratory of the National Bureau of Standards the normal cells are kept in baths of oil maintained very accurately at 28 deg cent by a sensitive thermostat. The choice of this temperature was made because of climatic conditions in Washington, in order to avoid the condensation of moisture from the air upon parts of the oil bath in humid summer weather. Aside from such considerations, other temperatures would be equally suitable. The normal cells of England, France, and Germany are kept at about 20 deg cent.

For very exact work the saturated cells should be maintained at as near a constant temperature as practicable for at least several days before electrical comparisons are made, because, in addition to having an appreciable temperature coefficient, these cells do not immediately assume their true emf after a change in temperature.

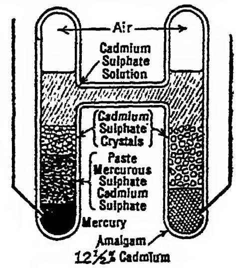

FIG. 2

UNSATURATED CADMIUM CELL. This cell differs from the normal cell in having no excess crystals of cadmium sulfate. The concentration of the cadmium-sulfate solution is intended to be such that it will reach saturation when the temperature is reduced to approximately 4 deg cent, a feature which insures a minimum effective temperature coefficient of the cell within the temperature range permissible in normal use. The unsaturated cell also differs from the normal cell shown in Fig. 2 in being made portable by the use of some form of porous plug or septum above the active material in each limb to keep these materials in place even when the cell is inverted.

CHARACTERISTICS OF THE TWO KINDS OF CELL. The Weston Normal (saturated) Cell, made to specifications from carefully purified materials, has a highly reproducible value of emf which usually remains very constant for years if the cell receives proper care. It has, however, an appreciable temperature coefficient of emf. The unsaturated cell is not as reproducible in emf, and there is usually a slow decrease of emf with time. Only very general statements may be made in regard to the rate of decrease of emf. Some very exceptional unsaturated cells have decreased over a long period of years by only 10 to 15 microvolts per year. A decrease of 25 to 50 microvolts per year may be taken as average performance. Some cells have shown a decrease of 100 microvolts per year. Any rate of decrease much in excess of this figure may be taken as an indication of either abuse of the cell or defect in materials or workmanship.

The advantages of the unsaturated cadmium cell are its portability and the very low temperature coefficient of cells which are within certain limits of emf. To be acceptable, new cells should have values of emf within the range 1.0185 to 1.0195 international volts; such cells have extremely small temperature coefficients provided all parts of the cell are at the same temperature and no abrupt changes of temperature have recently occurred. However, as the emf of the cell decreases with age and use, an appreciable temperature coefficient of emf develops. The National Bureau of Standards does not issue certificates for cells having an emf lower than 1.0180 international volts. For very accurate work it is advisable to use cells for which the emf is not lower than 1.0183 international volts. (At the time this is being written, it seems evident that the arbitrary international electrical units will be replaced, possibly by the year 1940, by the absolute electrical units. The present international volt is approximately 0.0004 volt larger than the "absolute volt" equal to 10^8 cgs electromagnetic units of emf. After the absolute units are adopted, the numerical values of emf given in this paragraph should be increased by 4 units in the fourth decimal place.)

It is important to realize that the low temperature coefficient of the unsaturated cadmium cell is actually the small difference between two relatively large temperature coefficients of opposite signs (those of the two limbs), and consequently depends upon the existence of a high degree of temperature equality between the two limbs.

MEASUREMENT OF EMF OF STANDARD CELLS. The emf of a standard cell is measured by comparing it with that of another cell, the "reference cell," which is taken as a basis. In national standardizing laboratories the reference cell is compared at suitable intervals with the cells composing the primary standards of the institution. In industrial and engineering laboratories which do not maintain a group of Weston normal (saturated) cells, the reference cell is one for which the emf has been certified by a standardizing laboratory.

Standard cells must be measured by the null (potentiometer) method to avoid taking

any current from them. Although it is possible to measure the entire emf of the cell under test with a potentiometer, the opposition method is much more accurate and can be carried out with a very much simpler and inherently less accurate potentiometer. In the opposition method the emf of the cell under test is opposed to that of the reference cell and their small difference is measured with a potentiometer. Attention must be paid to the algebraic sign of this difference, so that its numerical value may be added to, or subtracted from, the value of the emf of the reference cell to obtain that of the cell under test.

INTERNAL RESISTANCE OF STANDARD CELLS. The internal resistance of the normal (saturated) cell, of the usual construction and in good condition, may be taken as from 600 to 800 ohms. The internal resistance of the portable (unsaturated) cell, when made up in glass vessels of the size used by the principal national standardizing laboratories for normal cells, is about 500 ohms. For some purposes, particularly for use with a deflection potentiometer, this high value of resistance is a drawback. One American manufacturer regularly supplies portable cells with an internal resistance of about 100 ohms, while another maker supplies two forms having nominal internal resistances of 500 ohms and 100 ohms, respectively. Cells having the latter value are particularly desirable for use with the deflection potentiometer.

To measure the internal resistance of a standard cell, first measure its emf in terms of that of any other cell used as a reference cell in the manner above described, then shunt the cell under test with a resistance of 1 megohm and note the resulting apparent decrease in emf. The numerical value of this decrease, expressed in microvolts, is (with sufficient accuracy) the numerical value of the internal resistance in ohms. The current through the 1-megohm resistor should be allowed to flow only long enough to obtain the reading. This resistor should be of the wire-wound type.

HIGH RESISTANCE RESULTING FROM GAS BUBBLE. In time, a gas bubble may form between the amalgam and the septum of a portable cell. Though harmless at first, the bubble may grow until it has forced out the solution between the amalgam and the septum and thereby open-circuited the cell. Before this extreme condition is reached, the internal resistance increases sufficiently to impair the usefulness of the cell by reducing the working sensitivity of the galvanometer through which the emf of the cell is opposed to some other emf or potential difference.

If periodic measurements of the internal resistance of a cell show increasing values which are variable through rather wide limits, depending upon whether the cell is in the normal upright position or slightly inclined, the existence of a gas bubble at the amalgam limb is indicated. This test may be applied without the necessity of breaking the maker's seals.

PRECAUTIONS IN USING STANDARD CELLS. No appreciable current can be taken from a standard cell without some alteration of its emf. If the current is small and flows for only a short time, the change is not permanent, and the cell gradually recovers its original emf; of course the cell is unreliable until the recovery is complete. Consequently, standard cells should be used only in methods in which their emf is opposed to an equal emf, as in connection with potentiometers. They should always be protected by a key or set of keys, which are closed only momentarily, and for preliminary adjustments a resistance of at least several thousand ohms should be in series with the cell. When an approximate balance has been obtained this resistance may be cut out and the final balance then made.

In using portable standard cells, care should be taken to protect them from drafts of air, from sunlight, and from heat radiation from lamps, rheostats, radiators, steam pipes, etc., to prevent unequal heating of the two limbs of the cell. Even the heat of the hand, applied for several minutes, may so unbalance the temperature as to produce a noticeable change in the emf. The magnitude of this change, for various kinds of temperature disturbance which might occur in practice, has been determined by J. H. Park (see Bibliography). Cells may be protected by a thick wrapping of felt or other heat-insulating material. In the electrical instrument laboratory of the National Bureau of Standards each cell is kept in a thick-walled copper pot jacketed with heat-insulating material. Such precautions are justified when one wishes to avoid errors of the order of 0.01 per cent, as is necessary when acceptance tests are made on instruments with a guaranteed accuracy of 0.1 per cent.

At least three portable standard cells should be provided for a laboratory having a single potentiometer. The cells should be intercompared at least once a month; oftener if they are frequently used. A change of emf of one of the cells, from improper handling or other cause, will be revealed by its failure to check with the other two.

It is preferable not to ship standard cells in extremely cold weather, although when the cells are properly packed instances of damage from exposure to low temperatures

during shipment seem to be infrequent. It is much more objectionable to *carry* the cells from place to place in cold weather, either without packing or so insufficiently packed as to afford little protection from the cold. After exposure to temperatures outside the permissible range (4 to 40 deg cent; that is, 39 to 104 deg fahr) the cell may recover its normal emf after a rest of, say, several weeks at a reasonably constant temperature within the permissible range.

A standard cell should never be connected to a voltmeter in the attempt to measure its emf or to check the reading of the voltmeter. The voltmeter reading will be lower than the emf of the cell by an indeterminate amount, and unless the voltmeter be of relatively high resistance the emf of the cell will be altered, at least for a time.

4. TIME STANDARDS

Time enters into electrical measurements in passing from the measurement of power (rate of flow of energy) to the measurement of energy itself. Time also enters in a-c measurements; for example, the frequency of an alternating voltage or current is the reciprocal of the time required for a complete cycle. Time measurements in the laboratory cover an extremely wide range as to the magnitude of the time, the accuracy required, and the time-measuring devices available for various purposes. Among these may be mentioned clocks, stop-watches, synchronous timers, tuning forks, and piezoelectric oscillators, the last-named being capable of an accuracy which threatens to expose some of the failings of the most accurate astronomical clocks.

For the measurement of time in the testing of portable standard watthour meters stop-watches are often used. In the effort to avoid some of the errors of the stop-watch, special devices have been developed by meter engineers which make use of pendulum clocks which are in turn checked against radio time signals from Arlington. These have been described in the Reports of the N.E.L.A. Meter Committee.

5. RESISTANCE STANDARDS

MAINTENANCE OF THE OHM.—Although a specified column of pure mercury (the "mercury ohm") has been involved for many years in the laws of many civilized nations as the ultimate material standard of electrical resistance, the national standards laboratories have found it impracticable to maintain the unit of resistance by frequent reference to it, and instead, have maintained the international ohm largely by reference to the mean value of selected manganin coils. The German Reichsanstalt has used the same four 1-ohm coils since 1892 for this purpose. The National Bureau of Standards has maintained its unit of resistance since about 1910 by the use of ten 1-ohm manganin coils, selected for constancy from a larger group. From time to time, however, any of the ten coils which have not maintained their values with a satisfactory degree of constancy have been replaced by others chosen from the larger group.

At its meeting in Paris in 1930, the (international) Advisory Committee on Electricity recommended to the International Committee of Weights and Measures that no further mercury-ohm determinations should be made. The General Conference of Weights and Measures of 1933 approved in principle the abandonment of the present "international" ohm, volt, and ampere, with their arbitrary definitions in terms of mercury and silver, in favor of the absolute ohm, volt, and ampere, equal to 10^9, 10^8, and 10^{-1} of the corresponding cgs electromagnetic units. It appears evident that this step will be taken within the decade 1930-1940. The "international" ohm is larger than the absolute ohm by about 0.05 per cent; the international ampere is about 0.01 per cent smaller than the absolute ampere; and the international volt about 0.04 per cent larger than the absolute volt.

Absolute measurements of resistance will soon form the ultimate basis of reference by the national laboratories, but for years to come the maintenance standards of resistance will continue to be coils of wire of low temperature coefficient. Although better alloys are hoped for and sought after, manganin has for many years been virtually the only resistance alloy used for the "wire standards," not only for the reference coils just mentioned, but also for working standards of all degrees of importance and accuracy.

FORMS OF CONSTRUCTION OF RESISTANCE STANDARDS. These standards, for resistances of 0.1 ohm and above, are usually wound with manganin wire, and for the most accurate work are designed for immersion in an oil bath or contain oil, surrounding the coil, in a sealed case. They are generally provided with copper terminals for immersion in mercury cups formed in copper connection blocks. In the older Reichs-

anstalt type (Fig. 3) the distance between centers of the copper terminals is 160 mm (6.3 in.), and the case is perforated to permit the oil of the bath to circulate around the coil. In the type developed at the National Bureau of Standards by E. B. Rosa (Fig. 4), the case contains purified oil and is hermetically sealed to prevent the entrance of moisture. The distance between the centers of the terminals of the Rosa-type coil is 75 mm (3.0 in.). In the more recent resistance standard devised by J. L. Thomas at the National Bureau of Standards (see Fig. 5), the general dimensions and the distance between centers of terminals are the same as in the Reichsanstalt type, but the coil is hermetically sealed in the space between two coaxial brass tubes; it is wound on (and hence in good thermal contact with) the inner tube, which is in contact with the oil of the bath. The manganin wire is thus protected from chemical attack which might occur if the oil should deteriorate. The wire of the Thomas standard is annealed at a red heat after winding, and is not

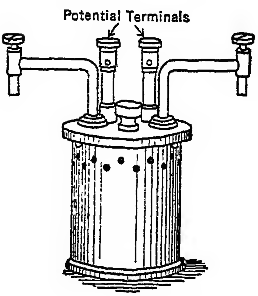

Fig. 3

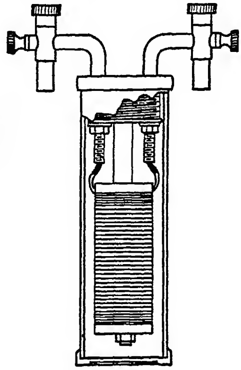

Fig. 4

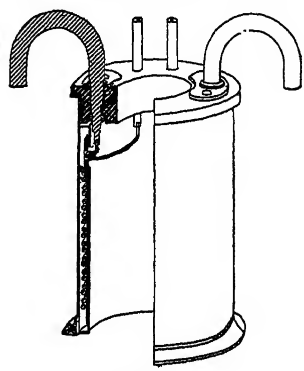

Fig. 5

bent or otherwise deformed after this annealing. Coils of the Rosa type and the Thomas type are exempt from change of resistance which occurs in unsealed coils as a result of variation of atmospheric humidity.

Resistance standards of values lower than 0.1 ohm are usually made of manganin sheets hard-soldered to copper terminals, and are of the four-terminal type to avoid contact-resistance errors at the terminals; that is, the resistance is adjusted to a stated value between two potential terminals which are connected to points on the resistor intermediate between those at which the current enters and leaves the standard. These standards, when used with a potentiometer for the measurement of current, are often colloquially called "shunts" because of their similarity to ammeter shunts.

AIR-COOLED VS. OIL-COOLED RESISTANCE STANDARDS. For a given difference of potential at rated current the power loss (and hence the rate of liberation of heat) increases with the current. Some forms of resistance standard for large currents are made for oil immersion, the oil being cooled by circulating water. Other forms are designed primarily for use in still air. To avoid excessive weight and cost, standards of the latter type are made with progressively lower values of potential difference as the rating in amperes increases. Oil-immersed heavy-current standards may have a larger value of this potential difference, with which certain sources of error (such as thermal emf) have correspondingly less effect on the accuracy of the measurements. The use of oil makes it possible to determine the temperature of the manganin more accurately. These advantages are accompanied by the usual disadvantages of oil immersion, among which are the necessity for providing for cooling water; the possibility of leakage or creeping of oil; the tendency for oil to become rancid in time and to attack the manganin. Both oil-cooled and air-cooled standards for large currents are used, according to the properties upon which the user lays the most emphasis. Air-cooled standards may on occasion be used in oil, or in a forced current of air. The Bureau of Standards uses non-inductive resistance standards, cooled by a forced air current, in testing current transformers up to 500 amp. Beyond this value it is necessary to resort to oil cooling to obtain an adequate voltage drop with a sufficiently low temperature rise and suitable dimensions.

LIMIT OF ERROR AND RATED CURRENT OF RESISTANCE STANDARDS. Single-value standards of 1 ohm or decimal multiples of this value, not exceeding 10,000

ohms, are usually adjusted to a stated limit of error of 0.01 per cent when used with a current corresponding to 0.1 watt loss. For 0.1 ohm and lower decimal submultiples of the ohm, the stated limit is generally 0.02 per cent, but this figure refers to a current corresponding to 1 watt loss. Standards of 1 ohm and above may be used with a load of 1 watt, and those of 0.1 ohm and below with 10 watts, with reduced accuracy during use but without any permanent change in resistance as a result of the load. The statements in this paragraph apply to standards having terminals for mercury-cup connections, and used in an oil bath.

Resistance standards ("shunts") of 0.1 to 0.00001 ohm, for measuring large currents, have various values of permissible power loss. The user should obtain information on this point from the maker, or from experiments to show the temperature rise of the manganin and the consequent change in the resistance of the standard.

6. RESISTANCE BOXES

CONSTRUCTION OF RESISTORS. A resistance box or precision rheostat is a group of resistors assembled in a case and provided with means for varying the resistance in circuit between two terminals. The resistors are usually of manganin wire and differ from those used in the best grade of resistance standards chiefly in being of smaller dimensions and lower accuracy of adjustment. It is standard practice to wind the resistors noninductively. For use with direct current, and with alternating current of usual power frequencies, the ordinary bifilar winding is usually satisfactory. Its use results in a coil having a time-constant nearly equal to zero, for coils of 100 ohms: for lower values the inductance predominates, for higher values the coil behaves like a pure resistance shunted by a capacitance. The bifilar winding is objectionable for coils of high resistance (1000 ohms and above) when used with frequencies above those for power and lighting, and various special types of winding have been devised to minimize both inductance and capacitance. The greatly increased use of frequencies in the audio range and higher has stimulated the development of resistors and boxes of very low time-constant; for example, a recent line of resistance boxes by Leeds & Northrup Co. is stated to have a time-constant of less than 0.1 microsecond for all settings above 5 ohms.

The coils of resistance boxes are usually wound on spools of wood, brass, porcelain, lava, or molded insulating material. In some cases the wire is wound on cards of mica or other sheet insulating material, or woven, with threads of silk or other insulating materials, to form a fabric. These constructions give a very low time-constant and good facility for the escape of heat from the winding.

ARRANGEMENT OF RESISTORS. The resistance between the terminals of a resistance box is varied by changing the manner of connection of its resistors, either by shifting plugs or by turning rotary switches. The 1, 2, 3, 4 and the 1, 2, 2, 5 plug arrangements are the most usual. Using the former, a resistance box of 1110 ohms total would contain resistors of the following resistances: 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400 ohms. The resistors are in series between the terminals, and the junctions of adjacent coils are connected to brass blocks which are reamed to receive taper plugs between them. See Fig. 6. The insertion of a plug short-circuits a resistor. The disadvantages of these arrangements are the time and mental effort required to add up the values of the resistors unplugged, the likelihood of errors, and the risk of loss of plugs. A further disadvantage of this construction is that there are a maximum number of plug contacts in circuit when the resistance setting is a minimum. Thus the lower the resistance setting the greater the likelihood of error or variability of resistance arising from imperfect contacts at the plugs. The decade form of plug resistance box avoids all these disadvantages except the risk of loss of plugs; the amount of resistance in circuit can be read off directly from the position of a single plug in each decade; the decade plan is usually preferred. While the original decade arrangements had 9 (or 10) resistors per decade, special arrangements have been devised to obtain 9 (or 10) steps with a smaller number of resistors. The object is to reduce the cost of the box for a given total resistance. Among these arrangements may be mentioned Northrup's, using coils of 1, 2, 3, and 3 units; Smith's, using coils of 1, 2, 2, 2, and 2 units; these and other arrangements are discussed by Northrup; see book section of Bibliography. When some of these plans are used with the rotary-switch construction the resistance of the decade goes through some undesired value as the brush passes from one stud to the adjacent one. Behr's new arrangement for a 10-step decade, free from this disadvantage, con-

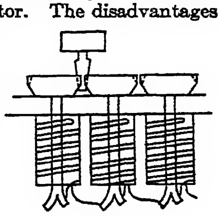

Fig. 6

sists of 6 resistors of 2 units each, two being connected in parallel when the resistance of the decade is to have an odd value.

PLUGS VS. SWITCH CONTACTS. Plug contacts, when carefully made, cared for and used, give (in general) a lower and more definite contact resistance than switch contacts. However, well-made switch contacts, when properly cared for, have a satisfactory definiteness of contact resistance and have the advantages of more rapid manipulation and reading, with no loose parts to be mislaid or lost. They require more care than plug contacts. Switch contacts are now generally preferred for most purposes. They permit placing all the live parts inside the box, and inside the metal shield in the case of shielded resistance boxes for a-c use. The location of the live parts where the operator cannot come in contact with them is an advantage when the box is used in a high-voltage circuit. For resistance boxes for use on high frequencies the switch construction makes it possible to keep the geometry of the internal circuit fixed. In the most recent boxes for this purpose the number of resistors in circuit is constant for all settings, even the connectors in the zero position being in effect resistors which are balanced as to resistance and inductance.

The angle of taper of the plug is an important detail. A taper of 1 in 12 is very satisfactory. Taper reamers having this angle ("1 in. per ft.") are supplied as standard articles by U. S. tool manufacturers. A taper of 1 in 10 is much used. If the taper of the plug is too pronounced, the plug is likely to loosen spontaneously; if the taper is much less pronounced than the above values, the lateral forces tending to spread the contact blocks apart will be excessive and the plug may seat so tightly as to require undue force for its removal.

MAINTENANCE OF CONTACTS. In both the plug and the switch constructions the contact surfaces must be kept clean and properly lubricated if the contact resistance is to be kept low and definite. It has been shown that the use of a suitable petroleum oil on these surfaces not only avoids cutting and scoring but also reduces the contact resistance and makes it more definite. The purified petroleum oils used internally in medicine (such as "Nujol") are very suitable. Keinath states that to give a satisfactory contact the force between the brush and the contact segment should be of the order of magnitude of 1 to 2 kg (2 to 4 lb), and that, to avoid the resulting danger of scoring, the contact surfaces should be kept greased with vaseline. The use of vaseline is satisfactory when the contacts are effectively protected from dust, but for open switch contacts vaseline becomes thickened by the dust and eventually makes the contact resistance high and irregular. When exposed contacts are lubricated with a suitable oil this difficulty does not arise.

It is an advantage to have the switches inclosed to hinder the access of dirt, but the construction should be such as to allow ready access for cleaning and oiling.

Further information applying to plug and switch contacts as used in resistance boxes is given in the section on Bridges under the heading Accuracy of Measurements with Wheatstone Bridge.

7. BRIDGES

GENERAL. Instruments embodying forms of networks known as **bridge circuits** are used for the measurement of resistance, inductance, capacitance, frequency, etc. The operation of a bridge consists in varying the constants of its component circuits or "arms" until the response of a suitable detector connected between two points on the network is reduced to zero or to an amount measurable by the detector for the purpose of interpolation.

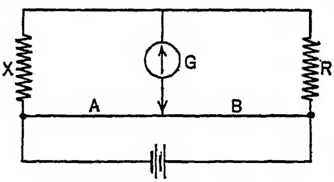

FIG. 7

SLIDE-WIRE BRIDGE (Fig. 7). This very simple form of bridge, of low cost and moderate accuracy, is identical in principle with the Wheatstone bridge (see below), the difference being that a simple wire AB forms the two ratio arms. It may be a wire of uniform resistance per unit length, stretched along a meter scale divided into millimeters. All connections in the X and R circuits are of low resistance. A slider is arranged to make contact at any desired point along the wire. When the bridge is balanced,

$$X = AR/(1000-A) \tag{1}$$

where A is the distance in millimeters between the slider and the left-hand end of the slide wire. The accuracy will be greater when the slider is nearer to the center of the wire. For this case the error may be minimized by taking the mean of two values of X,

between which the relative positions of X and R are interchanged. The accuracy may be increased by adding end coils to the slide wire, but at the expense of reduction of the range of measurement.

WHEATSTONE BRIDGE. Fig. 8 shows diagrammatically the arrangement of resistances in an ordinary Wheatstone bridge. A and B represent two coils or sets of

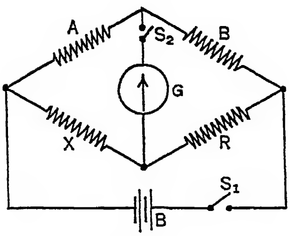

FIG. 8

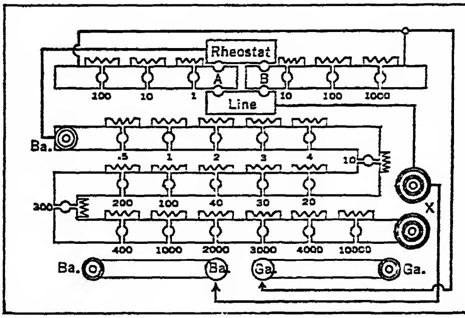

FIG. 9

coils, usually called the "ratio coils" or "ratio arms." Their relative resistances must be known but their absolute accuracy of adjustment is immaterial. R is usually a set of coils which can be connected to give any value of resistance, by small equal steps, from zero to the maximum; X is the unknown resistance, G a galvanometer connected in one diagonal of the bridge, and B the battery in the other diagonal. The operation of the bridge consists in successively adjusting R until the galvanometer shows no deflection, or until two values of R are found which differ by the smallest step and between which lies the value of R for no deflection, obtained by interpolation of the galvanometer deflections. The value of the unknown resistance is then

$$X = \frac{AR}{B} \tag{2}$$

where the letters represent the resistances of the four arms.

Forms of Wheatstone Bridge. Wheatstone bridges are made in a variety of forms, sizes, and degrees of accuracy to meet various requirements. For miscellaneous testing where the highest accuracy is not required, low cost is essential, and the bridge may be or must be taken to the object of the measurement, small portable bridges ("testing sets") are usually provided with self-contained battery and galvanometer. Their usual error limits are 0.1 per cent and 0.05 per cent for rheostat coils and ratio arms respectively. Larger bridges for laboratory purposes usually require an outside battery and galvanometer, although all the coils and the keys are usually mounted in one box. The coils are usually adjusted to a smaller limit of error than those in the portable bridges. Characteristic differences in laboratory bridges relate to (*a*) whether plug or dial contacts are used; (*b*) the arrangement of the ratio coils; (*c*) the number and arrangement of the coils in the rheostat arms.

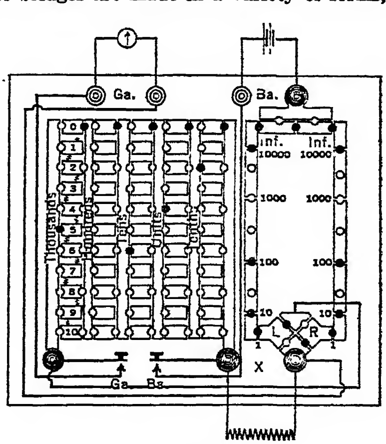

FIG. 10

The differences of arrangement of ratio coils and rheostat coils may best be brought out by brief descriptions of several classical types of bridge. Fig. 9 shows the so-called Post Office type, of English origin, with the rheostat coils arranged on the 1, 2, 3, 4 plan. The advantage of this design is the small number of coils; the disadvantage is the necessity for adding the values of the coils unplugged in the rheostat arm. Fig. 10 shows a decade bridge of the type originally devised by Professor Anthony. The resistance in the rheo-

stat arm may be read off by inspection—an advantage which is obtained at the expense of a relatively large number of coils. An important advantage of the Anthony form of bridge is that the relative values of the coils may be checked by the user. To obtain the direct-reading feature with a relatively small number of coils, special methods of connection are used in other forms of bridge by which a 9-coil "decade" is obtained with as few as 4 coils.

Special Arrangements of Ratio Coils. In the Wheatstone bridge shown diagrammatically in Fig. 11 the ratio is changed by a single sliding contact in the battery circuit where variations of its resistance have no effect on the accuracy of measurement. A comparison of the improved arrangement of ratio coils indicated in Fig. 12 with the old Post Office

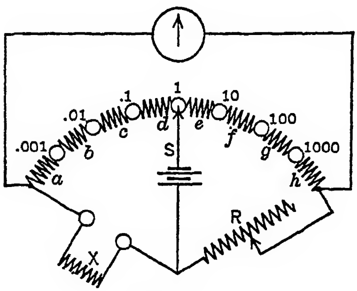

FIG. 11

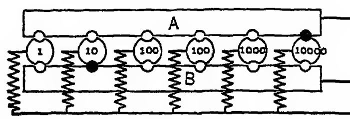

FIG. 12

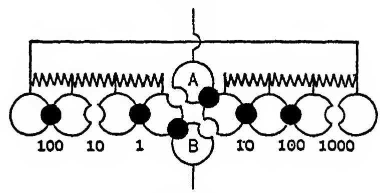

FIG. 13

arrangement of Fig. 13 shows that the former has only 2 plugs to manipulate while the latter has 6.

Accuracy of Measurements with Wheatstone Bridge. The degree of accuracy with which a resistance can be measured with a Wheatstone bridge is determined by the following conditions:

1. The limit of error to which the various coils are adjusted to their stated resistances. Limits stated by the maker usually range from 0.2 to 0.02 per cent.

2. The relative values of the coil resistances and the contact resistances at plugs, contact brushes, and binding posts. A well-fitting clean plug has a contact resistance which is definite to the order of 0.0001 ohm or better; a poorly fitting or dirty plug may fail to repeat by 0.01 ohm or more. The contact resistance between a binding post and a well-clamped wire may range from 0.0001 to 0.001 ohm. The magnitude and definiteness of the contact resistance of rotary switches depend greatly on their design. Each leaf of the laminated brush should press independently against the contact block. Rotary switches in which the thrust of the brushes is balanced about the axis of rotation are preferable, other things being equal, to those in which a considerable unbalanced force exists.

3. The relative values of the coil resistances and the insulation resistances between the contact blocks. Dust and moisture on the hard-rubber top may cause an appreciable error in high-resistance coils. Hard rubber (ebonite), which is largely used for the tops of bridges, resistance boxes, potentiometers, and volt boxes, undergoes surface deterioration when exposed to light. Such apparatus should never be exposed to direct sunlight, and should be covered when not actually in use. The deterioration goes on, though more slowly, even in diffused daylight. Dust and other impurities present in the air of cities and manufacturing centers aggravate the deterioration, which may reach the stage of a surface film of dilute acid formed from excess sulfur present in the hard rubber.

4. The effects of changes of temperature in changing the resistances of the coils and in setting up parasitic thermal emf's. Small errors from the latter source may be eliminated by taking the mean of the results of two measurements, between which the direction of flow of the battery current through the bridge is reversed.

5. Changes of resistance resulting from heating by the battery current. For any given type and size of resistor and given environment the temperature rise depends on the watts loss. Only very general figures can be given as a guide. Northrup states that the safe watt rating of the coils of a Wheatstone bridge ranges from 0.25 to 4 watts per coil, according to its construction, and that, as a rule, 1 watt should be the limit. If many coils are in use, lower values of watts per coil must be employed. It is better

to obtain sensitivity by using a sensitive galvanometer rather than by using large currents through the coils.

6. The sensitivity of the galvanometer and its relative suitability (as regards its value of external resistance for critical damping) for use with given values of resistance in the bridge arms.

Best Value of Galvanometer Resistance and Preferable Location of Galvanometer. When a choice may be made between two or more moving-coil galvanometers which differ only in the resistance of their coils, the galvanometer should be selected for which the external resistance for critical damping most nearly equals the resultant resistance R_r of the arms $A + X$ in parallel with $B + R$ (Fig. 8); namely,

$$R_r = \frac{(A + X)(B + R)}{A + B + X + R} \tag{3}$$

This is the resistance of the bridge network between the points to which the galvanometer is connected, because for the condition of balance the resistance in the battery diagonal has no effect on the value of R_r. Tradition states that the galvanometer should be selected for which the coil resistance most nearly equals the value of R as given by the above formula; that is, for example, if A, B, R, and X are each 100 ohms, that a galvanometer with a 100-ohm coil is most suitable. Such a rule could be followed when moving-magnet galvanometers were used, but to apply it to moving-coil galvanometers as now made would either result in excessive overdamping or require the insertion of additional resistance with consequent loss of sensitivity. Present-day moving-coil galvanometers which are damped only through the external circuit usually require that the external resistance for critical damping must be from, say, 3 to 15 times the coil resistance. If no galvanometer is available which complies sufficiently well with this requirement, the one which most nearly meets it may be used, resistance being placed in series with it to correct any overdamping, or in parallel with it to correct underdamping. Whenever one must choose between a moderate degree of overdamping or an equal degree of underdamping, the latter saves time and has other advantages.

In practice, the user is limited to a relatively small number of values of galvanometer resistance, and galvanometer sensitivities are usually more than ample for all ordinary purposes.

The resistance of modern batteries is so low that the old question of "best location of the battery" no longer arises; in fact, it is often advisable to insert some resistance in series with the battery to prevent damage to low-resistance bridge arms. For example, particular care in this respect is necessary when the resistance of a millivoltmeter is being measured, lest the pointer be bent and the moving coil and springs be damaged by too great a current.

Precautions in Using a Wheatstone Bridge. The following rules will conduce to accuracy of measurement and maintenance of the bridge in good condition.

1. See that all binding posts are screwed down tight and that the plugs and switch contacts are clean and properly lubricated. Plugs should be inserted and removed with a twisting motion. Only moderate force should be used in inserting them. After withdrawing a plug, retighten adjacent plugs, if necessary. The contact surfaces of the battery key and galvanometer key may need cleaning at infrequent intervals. This is conveniently done with a fine flat file.

2. For measuring all but very high resistances, use a battery of only a few volts, for example, one to three dry cells in series. The minimum voltage required to obtain the necessary sensitivity should be used. To get sufficient sensitivity when measuring high resistances (say, 100,000 ohms up to several megohms) it is generally necessary to use a greater number of cells, together with very unequal ratio arms; for example, 1 ohm and 1000 ohms. In such a case care should be taken not to use a battery voltage which will pass too great a current through any coil of the bridge, and the battery should be connected in that diagonal of the bridge which gives minimum current in the bridge coils.

3. For a preliminary balance use a 1 to 1 ratio and protect the galvanometer by shunting it or preferably by inserting a resistance of several hundred ohms (or more, according to the circumstances) in series with the battery. This resistance may later be reduced or cut out entirely.

4. Avoid touching the metal parts of keys, plugs, blocks, etc., forming part of the bridge circuit. When the circuit under measurement contains appreciable capacitance or inductance, always close the battery key first, then tap the galvanometer key momentarily to note the direction of the swing; when balance is more closely approached the galvanometer key may be held down. Always open the galvanometer key before opening the battery key when measuring coils of high time-constant, such as motor, transformer, and electromagnet windings. For these latter, allow time for the current to build up

before closing the galvanometer key. When circuits which are substantially non-inductive are under measurement, particularly in the case of low resistances, it is sometimes advantageous to close the galvanometer key first, to allow the galvanometer coil to assume a "false zero" position under the action of thermal emf, and then to close the battery key and adjust the rheostat arm to produce the condition of balance with respect to this zero position. This procedure gives the same general result as the conventional one of reversing the direction of the battery current through the bridge, but has the advantages of saving time and of eliminating the effect of all thermal emf, including that in the galvanometer.

5. If it is necessary to use connecting leads between the X posts of the bridge and the object of measurement, the outer ends of the leads should be clamped together and the lead resistance measured before making measurements on the object. This precaution is especially necessary when the leads are of flexible stranded conductor with attached terminal lugs. Defective soldered joints, or the breaking of strands of wire, may make the lead resistance variable, in which case the fact should be discovered at the beginning. If the leads are moved while the keys are depressed, small deflections of the galvanometer may occur because the leads cut the earth's magnetic field. This effect should be carefully distinguished from deflections caused by an actual variation of the lead resistance.

Care of Bridge. The hard-rubber top should be kept free from moisture and dirt, especially between the blocks; it should **never be exposed to direct sunlight,** and should be **covered** when not in use. The plugs should be free from tarnish and dirt. When necessary, they should be cleaned with whiting (never with sandpaper or emery paper) and then lightly oiled; see Resistance Boxes, Plug and Switch Contacts.

THOMSON (KELVIN) DOUBLE BRIDGE. When the resistance of a conductor is 0.1 ohm or less, an accurate measurement of its resistance cannot be made with a Wheatstone bridge, used in the ordinary way, because of the relatively large uncertainty introduced by contact resistances at the binding posts. Furthermore, in applications where the resistance of a conductor between certain points must be accurately measurable and remain definite in use, it is necessary to attach so-called potential leads to these points, thus converting the simple two-terminal conductor (or resistor) into a four-terminal resistor. Its "four-terminal resistance" is defined as the ratio of e to i, where e is the difference of potential set up between the two potential leads by a current i which enters and leaves the conductor at its ends (the "current terminals").

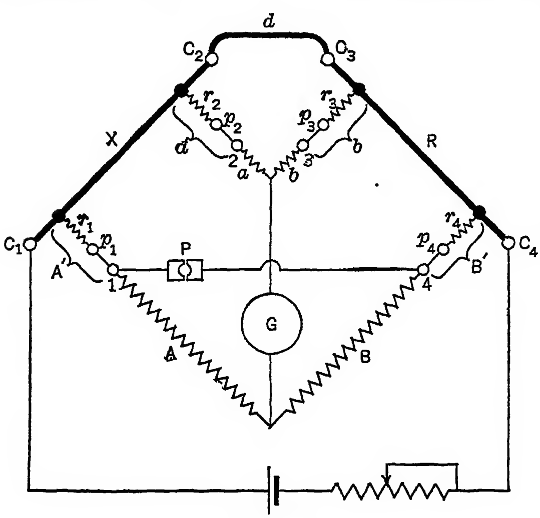

Fig. 14

A bridge network for the accurate comparison of two four-terminal resistances was devised by Wm. Thomson (Lord Kelvin) and is known as the Thomson (or Kelvin) **double bridge.** It is shown diagrammatically in Fig. 14, in which the four-terminal resistors X and R to be compared are connected in series by the link d which joins their current terminals c_2 and c_3. Current from a battery enters and leaves at the current terminals c_1 and c_4, respectively. The Kelvin bridge consists of the main ratio arms A and B and the auxiliary ratio arms a and b. In several forms of Kelvin bridge made by Leeds & Northrup Co. these ratio arms may be selected from a comparatively few values, and the four-terminal resistance of the standard R is continuously adjustable to balance the bridge by moving the potential taps. In other forms the ratio arms B and b may be selected from a few fixed values (for example, 50, 100, and 200 ohms and values made up of two or more of these), and A and a are adjustable from 0 to 1000 ohms by steps of 0.1 ohm. The arms A and a, in some constructions, are mechanically connected so that both must

have the same nominal value at all times. The binding posts 1, 2, 3, 4 in Fig. 14 are the terminals of the double bridge itself, and laboratory checks of its coils give values of resistance of A, B, a, and b up to these binding posts.

The four-terminal resistance X is actually the resistance of that part of the conductor $c_1 c_2$ lying between two points within its structure where the leads to the potential terminals $p_1 p_2$ effectively branch off. The resistances of the conductors from these branch points to p_1 and p_2 are denoted in Fig. 14 by r_1 and r_2. These resistances are usually very small, but in some special cases may be relatively large; for example, in some external shunts for d-c ammeters in which the four-terminal resistance is intentionally made too high and calibrating resistance (sometimes as much as several ohms) is added at r_1 or r_2. In any event, it is necessary to determine the effect on the measurement, not only of r_1, r_2, r_3, and r_4, but also of the resistances of the leads joining p_1 to 1, p_2 to 2, and so on. Let the resistances of r_1 plus the lead to terminal 1, r_2 plus the lead to terminal 2, and so on, be denoted by A', a', b', and B', respectively.

It may be shown that the necessary and sufficient condition for zero current through the galvanometer is

$$X = \frac{A + A'}{B + B'}\cdot R + \frac{(b + b')d}{a + a' + b + b' + d}\left(\frac{A + A'}{B + B'} - \frac{a + a'}{b + b'}\right) \tag{4}$$

This expression for X contains five unknown quantities A', B', a', b', and d, and in its present form requires a knowledge of a and b also. Most double bridges of the type under discussion are made with the intention that a/b shall equal A/B, but the lack of perfect adjustment of the coils makes these ratios slightly unequal. Certain auxiliary procedures have been devised which make it unnecessary to determine the values of A', B', a', b', d, and the exact relation between the ratios A/B and a/b. Each of these procedures makes it possible to adjust the lead resistances to give the condition that

$$\frac{A'}{B'} = \frac{A}{B} = \frac{a + a'}{b + b'} \tag{5}$$

whereupon the above expression for X assumes the convenient form

$$X = \frac{A}{B}\cdot R \tag{6}$$

One of these auxiliary procedures requires the provision of a low-resistance connection (shown in Fig. 14) which will join the terminals 1 and 4 when a plug is inserted at P. The procedure is as follows:

(a) With link d in place, joining c_2 and c_3, and plug out at P, balance the bridge by varying A and a, or B and b, so as to keep the ratios A/B and a/b nominally equal. If the extra resistances A' and B' are small compared with A and B respectively, this balance will give nearly the correct value of A/B.

(b) With link d and plug P in place, and A and B left unchanged, adjust the lead resistance forming part of A' (or of B') until balance again exists; this makes A'/B' equal to A/B, to a very close approximation.

(c) With link d out and plug out, and previous adjustments unchanged, adjust the lead resistance forming part of a' (or of b') until balance exists; this makes $(a + a')/(b + b')$ very closely equal to $(A + A')/(B + B')$ and therefore to A/B.

(d) With link replaced and plug removed, readjust the settings of A and a to get a new and more accurate balance. If the amount of readjustment required is relatively small, this balance may be regarded as final and the value of X may be found directly from the simple relation (6). If a considerable amount of readjustment is required, as may occur when a resistance of several ohms is present in one of the potential leads of X, the steps (b), (c), and (d) may be repeated until the balance obtained in any step (d) agrees closely with that obtained in the previous step (d).

Although the procedure above outlined eliminates the effect of the resistance of the link, it is important to keep this resistance as low and as definite as possible. It is customary to use a heavy copper link with its ends arranged for mercury contacts.

Other methods of making the auxiliary adjustments in measurements with the Kelvin bridge are given in papers by Wenner and Weibel (see Bibliography).

As may be seen from a consideration of step (c) in the above procedure, the coils composing the arms a and b need to be adjusted to only a very moderate degree of accuracy, and there is no object in knowing their corrections. It is also obvious that the unit in terms of which A and B are adjusted is immaterial because only the ratio of A to B is significant. Nothing would be gained, for example, by readjusting the coils of such a double bridge in terms of the "absolute" ohm to be adopted in the near future.

Besides its use for the very accurate measurements of low resistances in the laboratory,

the Kelvin bridge, in simplified form, has been adapted by several American makers for shop and field measurements of generator and transformer windings, samples of wire and cables, etc. Such instruments usually have a range of measurement of 0.0001 to about 25 ohms. The Kelvin bridge has been used by Leeds & Northrup Co. in its apparatus for continually measuring the resistance of the copper field windings of generators while in operation. The indicating (or indicating and recording) part of the apparatus is calibrated to show the mean temperature of the field winding.

HOOPES CONDUCTIVITY BRIDGE. This bridge is a modification of the Kelvin bridge, designed for the rapid determination of the relative conductivity of samples of wire. It is extensively used in wire factories. A diagram of the connections is shown in Fig. 15. The standard A–B and the unknown C–D are of the same metal; consequently if care be taken that they are at the same temperature, all corrections for temperature are avoided. The arms p, r, p_1, r_1 are in the same case and are made of material of low temperature coefficient so that their relative values will not change. They are

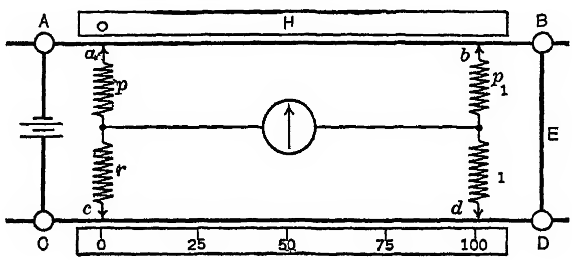

Fig. 15

adjusted so that $p = r$ and $p_1 = r_1$; consequently at balance the resistance of c–d equals the resistance of a–b.

The sample C–D is placed alongside of scale I divided into 100 parts, so that the graduations represent percentages of the total length of the scale. Accompanying the standard wire A–B is a scale H, on which are laid off a number of points corresponding to the weights of the standard length (38 in.) of a range of sizes of sample wires.

To make a conductivity reading, the weight of the standard length of the sample C–D is found to an accuracy of $^1/_{20}$ per cent. The contact b is set at the point on scale H corresponding to this weight, the contacts a and c being at the zero points of their respective scales. After the case has been closed a sufficient length of time to allow the standard and sample to assume the same temperature, the contact d is moved until the galvanometer shows no deflection; this will occur when the resistance between c and d is equal to that between a and b. The scale reading corresponding to the position of d for a balance is equal to the per cent conductivity. The bridge is designed for use with standard wire samples, each of which covers a range of sizes equal to 3 numbers of the American Wire Gage. Any number of standards can be supplied with a bridge, so that it can cover an extensive range of sizes and can also be used for wires of different materials. In order to keep the standard wire and the test wire at the same temperature the bridge is mounted in a metal-lined case and the scale is read through a glass window in the case, the window being closed by a metal screen when readings are not being taken.

BRIDGES FOR MEASURING THE RESISTIVITY OF ELECTROLYTES. The resistance of an electrolyte contained in a suitable cell can be measured by using a Wheatstone bridge with an a-c detector and an a-c source substituted for the battery. Alternating current must be used on account of the polarizing action of a direct current. Suitable **a-c sources** are: ordinary 25-cycle or 60-cycle circuits; a small 1000-cycle generator; a "microphone hummer" which is a combination of electrically operated 1000-cycle fork, a microphone, and a transformer, the secondary of which supplies 1000-cycle current; a buzzer operated by dry cells, with a transformer having its primary winding in series with the buzzer and its secondary winding connected to the bridge.

A telephone receiver is a suitable detector when a 1000-cycle source is used, but is not suitable for low frequencies such as 60 cycles. For the latter, an a-c galvanometer of the separately excited moving-coil type, or of the vibration type, is necessary. With the moving-coil galvanometer separately excited by a current in phase with the bridge currents it is sufficient if the three arms of the bridge (other than the one formed by the cell containing the electrolyte) are non-reactive. With the vibration galvanometer or

the telephone receiver there must be a condition not only of resistance balance but also of phase balance, and the coils must have good a-c and good d-c characteristics. One way to obtain the phase balance is to use an adjustable air condenser connected across one of the arms of the bridge.

Cells for containing the electrolyte are usually made of glass and have fixed electrodes of platinum or gold. Each cell is calibrated by one or more measurements with the cell filled with liquid of known resistivity, in order that the measured *resistance* of any sample of liquid may be reduced to its *resistivity*. A cell of the immersion type for approximate measurements may be extemporized from a glass tube of known internal diameter, in the ends of which circular electrodes are secured at a measured distance apart. There should be several small holes in the wall of the tube. In use, the cell is immersed in a vessel containing the electrolyte to be tested. The resistivity is the product of the cross-sectional area of the tube and the observed resistance, divided by the distance between the electrodes.

A-C BRIDGES. Bridges for the measurement of inductance, capacitance, and other quantities of importance in a-c circuits have been developed in a great variety of forms and for use over widely varying ranges of frequency. With few exceptions, they have hitherto usually been assembled in the laboratory from component pieces of apparatus. Recently, however, a-c bridges as self-contained units have been developed and are being manufactured. Space limitations prevent an extended treatment of this subject, for which the reader is referred to the book by Hague (see Bibliography). See also Schering Bridge, in the section on Testing of Insulating Materials.

8. POTENTIOMETERS

DEFINITIONS. A potentiometer is an instrument for measuring an unknown emf or potential difference by balancing it, wholly or in part, by a known potential difference produced by the flow of known currents in a network of circuits of known electrical constants. In d-c potentiometers the only network constants involved are the resistances, but in a-c potentiometers the capacitances and the self and mutual inductances of the network also enter into the result, as well as the frequency.

In potentiometers of the **null type** the unknown emf or potential difference is completely balanced, and the galvanometer (or other detector) merely indicates the absence of a current. In the **deflection potentiometer** a part of the unknown voltage produces a current through the galvanometer, which is calibrated to read directly the corresponding part of the measured value.

USES OF THE D-C POTENTIOMETER. For the checking of d-c ammeters, voltmeters, and wattmeters the d-c potentiometer is recognized as the most accurate instrument available. In most forms it has the unique advantage that only the *relative* values of its coils are of importance, and these values, in other than complicated designs, can often be adequately checked by the user even though the absolute values of his standards are not known. With suitable resistance standards of appropriate current ratings, the potentiometer may be used to measure direct currents over a wide range; and with suitable "volt boxes," it will measure voltages up to several thousand volts.

Auxiliary devices for the potentiometer proper for d-c measurements are a cell or battery to supply the auxiliary current, a rheostat to adjust this current, a standard cell, and a galvanometer. In some forms of potentiometer, some or all of these auxiliary devices are built into the instrument.

PRINCIPLE OF THE D-C NULL-TYPE POTENTIOMETER. The d-c potentiometer in its simplest form (Fig. 16) consists of a uniform wire stretched over a scale divided into a number of equal divisions; for example, 1500. A storage cell B, having an emf of about 2 volts, and a rheostat R are connected in series with the wire, and two contact points are provided, A being fixed at the zero point and S being movable. G is a galvanometer and K is a key for the galvanometer circuit. Between the terminals at P there may be connected at will a standard cell or any unknown voltage. At P is first connected a standard cell, for example, an unsaturated cadmium cell having an emf of 1.0190 volts. The terminal of B and that of the standard cell which are connected to the point A must have like polarities. The contact S is set at 1019 divisions on the

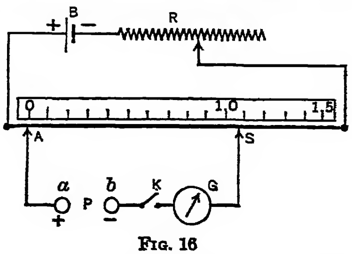

FIG. 16

scale, and the rheostat R is adjusted by trial until the closing of the key K produces no deflection of the galvanometer.

To measure any other potential difference the standard cell is disconnected from the terminals a and b and the unknown potential difference, for example, that of an ammeter shunt, is connected to a and b. The contact S is then moved until a position is found such that the closing of K produces no deflection. The corresponding position of S on the wire, as read from the scale, gives the value of the unknown potential difference.

Although the simple slide-wire potentiometer is sufficiently accurate for some technical purposes, its range and accuracy are inadequate for precision laboratory measurements, such as the checking of high-grade indicating instruments. Many forms of precision potentiometer have been devised, of which a few typical ones only can be described.

POTENTIOMETERS WITH ONE DIAL AND A SLIDE WIRE. An obvious modification of the above simple potentiometer is the insertion, in series with the slide wire, of a set of coils, each having a resistance equal to that of the slide wire between the zero and the point 1.0. Fig. 17 shows such a potentiometer (Leeds and Northrup type K). The slide wire DB is 11 turns of manganin wound in an external screw thread cut on a Bakelite cylinder. It is shunted to a resistance of 5.5 ohms and is in series with fifteen

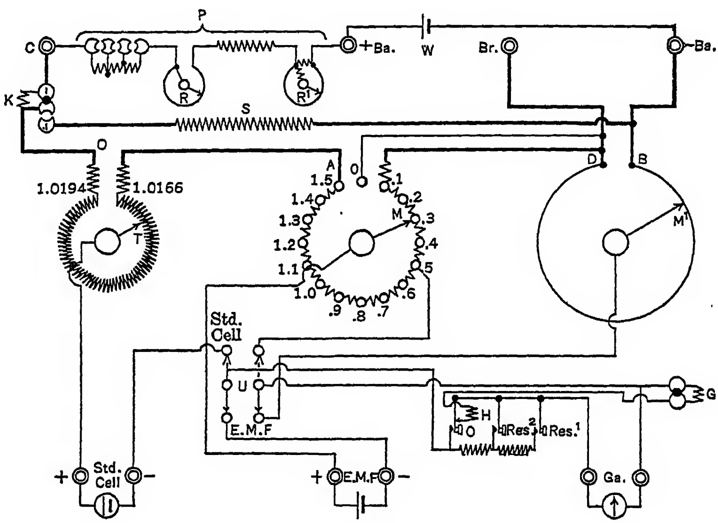

FIG. 17

5-ohm coils connected to studs, over which runs the brush M. The rheostats P are for adjustment of the auxiliary current. The other slide wire, over which runs the slider T, adapts the potentiometer to be direct reading with any standard cell having an emf within the slide-wire range. Although this range permits the use of standard cells of emf as low as 1.0166 volts, cells having an emf below 1.0180 should not be used for precise measurements because unsaturated cadmium cells of lower value are apt to be unreliable under ordinary conditions of varying room temperature.

A double-throw switch U inserts the galvanometer, with its keys and protective resistors, either in the standard-cell circuit or in the unknown-emf circuit. Keys Res. 1, Res. 2, and 0 are closed in the order stated to protect the galvanometer from violent deflections and the standard cell from currents which might (at least temporarily) alter its emf.

The function of coil K, the plug switch near it, and the coil S is to permit, after adjusting the battery current properly, with the plug set at 1, the shunting of nine-tenths of the current through coil S while keeping the total battery current unchanged. The potentiometer current is thus reduced to one-tenth of its normal value, and the normal range of 0 to 1.61 volts is thereby reduced to 0 to 0.161 volt.

The procedure of operation just outlined is typical for d-c potentiometers. For any particular potentiometer, the maker's instructions should be consulted for details.

The **Type K-2** potentiometer (Leeds & Northrup Co.) retains the slide wire and

set of coils of the Type K potentiometer, but includes a new low range of 0 to 0.016 volt. The constancy of the auxiliary current can be checked, by reference to a standard cell, regardless of which range factor (1, 0.1, or 0.01) is in use. This is accomplished as shown in Fig. 18. The coils of the main dial and the slide wire are shunted by four coils in series having resistances of 424.5, 300, 72.45, and 8.05 ohms, respectively. Taps from

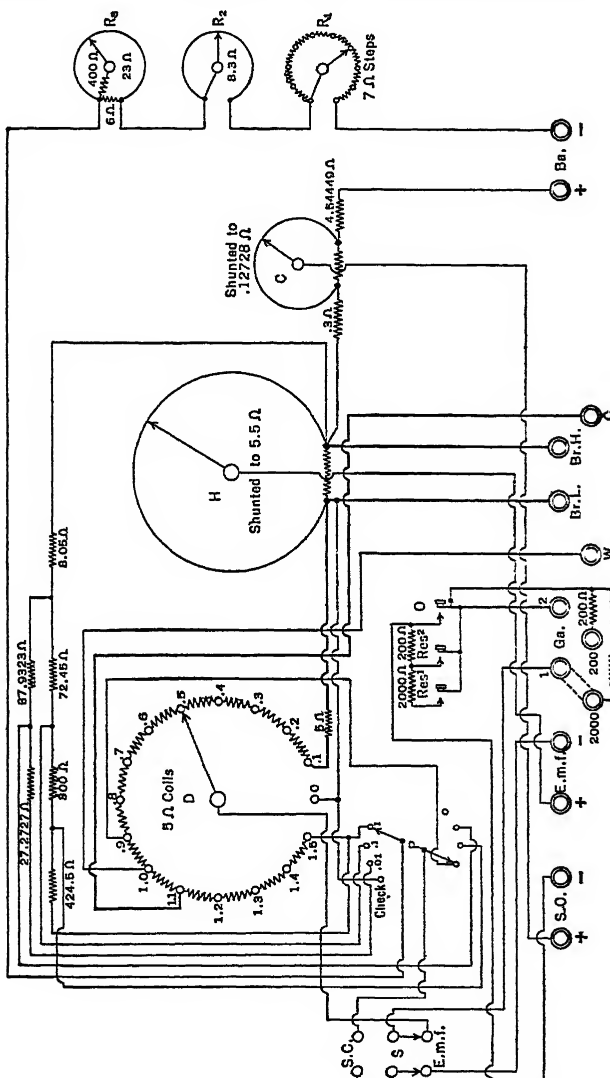

FIG. 18

junction points in this shunt circuit run to the points marked 1, 0.1, 0.01 on the factor switch P, the tap to the point 0.01 containing two coils in series having resistances of 27.2727 and 37.9323 ohms, respectively. These resistances are such that if the total current from the auxiliary battery Ba be adjusted in the usual manner to produce a current of 0.02 amp through the main dial and the slide wire with the factor switch set at 1,

the shifting of the factor switch to the position marked 0.1 will redistribute the auxiliary current, without altering its total value, so that only 0.002 amp flows through the main dial and the slide wire. With the factor switch at 0.01, the current is again redistributed, without change of total value, so that the current through the main dial and the slide wire is reduced to 0.0002 amp. For each distribution of current the lower part of the factor switch P connects the − pole of the standard cell to a junction point on the network such that, if the condition of balance (zero current in the galvanometer) existed with the factor switch set at 1, it will exist for each of the other two positions of this switch.

The Type K-2 potentiometer has three extra taps from the main-dial coils to external binding posts. Two of these taps include exactly 1 volt and are used with a volt box in wattmeter testing where it is required to hold the test voltage constant at 100 volts. This feature makes it possible to check the voltage without regard to the positions of the dial switch D and the slider H, and in effect makes the potentiometer read directly in watts. One of the preceding taps and a third tap serve in the checking of the 11 turns of the slide wire H against the first 11 coils of the main dial.

The Type K-2 potentiometer differs from the Type K in the following structural details: All contacts are inclosed; the battery rheostats are all dial-controlled; the change from emf to standard-cell check, and the change of range, are effected by dial switches instead of the rocker switch and plug switch, respectively, of the Type K.

WHITE COMBINATION POTENTIOMETER. This is a low-resistance instrument designed to minimize parasitic thermal emf, and is intended particularly for precise measurements of temperature with thermocouples. A simplified diagram of the White "single potentiometer" is shown in Fig. 19. In the White "double potentiometer," an additional set of dials is provided, to permit measurements with two separate thermocouples at widely different temperatures without extensive and repeated changes of the dial settings.

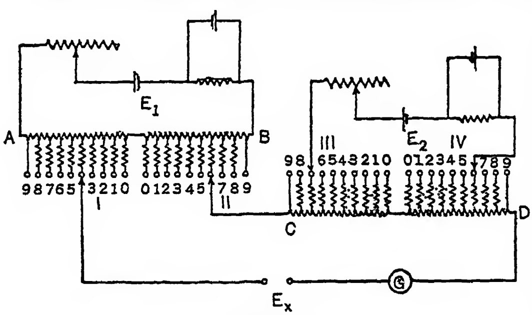

FIG. 19

WENNER POTENTIOMETER. This is a recent low-resistance five-dial potentiometer, in which the galvanometer circuit resistance is maintained substantially constant at either 40 or 174 ohms. The instrument has two ranges of 0 to 1.9111 and 0 to 0.19111 volt. It is described by Behr in *Rev. Sci. Insts.*, 1932, vol. 3, p. 109.

WOLFF FIVE-DIAL POTENTIOMETER, FEUSSNER TYPE. In this potentiometer, shown diagrammatically in Fig. 20, the auxiliary current entering at $B+$ flows through the nine 100-ohm coils of the upper right-hand dial, then through the coils of the lower halves of the 0.1-, 1-, and 10-ohm double dials, through the 1000-ohm coils, then through the upper halves of the 10-, 1-, and 0.1-ohm double dials. These three

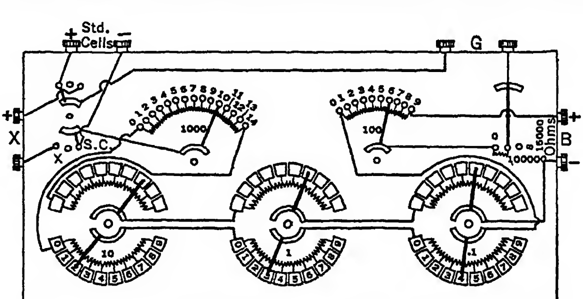

FIG. 20

double dials vary the resistance between the zero studs of the two upper dials from 0 to 99.9 ohms by steps of 0.1 ohm while keeping the total resistance between the terminals B constant at 14,999.9 ohms. With the auxiliary current commonly used (0.0001 amp) the range of measurement is 0 to 1.5 volts, approximately. An external rheostat is required for adjusting the current, which may be supplied by a single storage cell or by two dry cells in series. In either case it is not necessary to open the battery circuit when the potentiometer is out of use. In one form of Wolff potentiometer, in order to check the value of the auxiliary current it is necessary to set the dials to the value of the emf of the standard cell and to set the double-pole double-throw switch in the upper left-hand corner to S.C. In a later form a separate standard-cell dial is provided, having 11 steps for 1.0180 to 1.0190 volts. This feature makes it possible to check the auxiliary current quickly without the necessity of setting the regular five dials to the value of the standard cell.

EPPLEY FIVE-DIAL POTENTIOMETER, FEUSSNER TYPE. This potentiometer is similar to the Wolff potentiometer in plan of circuits and values of coils, but has a number of refinements, including: the elimination of six of the twelve contact resistances in the measuring circuit; an additional standard-cell dial by which the standard-cell setting can be made to one more decimal place (namely, to 0.00001 volt); copper binding posts to minimize parasitic thermal emf; built-in rheostats for the adjustment of the auxiliary current; and provision for oil immersion and control of oil temperature. The additional standard-cell dial is obtained by bringing out taps from one of the 1000-ohm coils.

INCREASING THE RANGE OF VOLTAGE MEASUREMENT. To measure voltages greater than the maximum directly measurable with the potentiometer, an accessory variously known as a "multiplier," "potential divider," or "volt box" is used. (The first two names imply respectively that the accessory *multiplies* the range of the potentiometer or *divides* the line voltage to get a fraction which can be measured.) It consists essentially of a high resistance to be connected to the points between which the voltage is to be measured, with the potentiometer connected across a fractional part of the resistance. The reading of the potentiometer, multiplied by the reciprocal of this fraction, gives the value of the voltage to be measured. Volt boxes are usually multirange. Two general types of volt-box circuit are shown in Fig. 21. In (*a*) the total resistance is varied to change the range, and in (*b*) the resistance across which the potentiometer is connected is changed. In (*a*) there is the danger of burning out the coils by applying too high a voltage to a part of the total resistance meant for measurements of relatively low voltage. In (*b*) this danger does not exist, but when the lower ranges are in use the volt box may introduce an excessive amount of resistance into the measurement circuit, thereby reducing the sensitivity. There is the further disadvantage in arrangement (*b*) that if a high voltage is applied to the line posts while the switch is set for a low ratio an abnormally high voltage will be impressed on the potentiometer and the galvanometer is very easily damaged. This trouble is possible even with arrangement (*a*), but experience shows that it is very rare. Experience shows also that separate "line" binding posts for the various ranges are safer than a rotary switch.

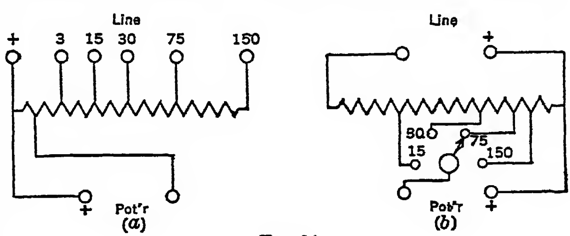

FIG. 21

The combination of a potentiometer with a volt box is not a null instrument, and for accurate work a correction must be made for the resistance of the leads connecting the line terminals of the volt box to the points between which the voltage is to be measured. For a given resistance in the leads, this correction is smaller as the resistance of the volt box is greater. Furthermore, a high value of "ohms per volt" is desirable because it reduces the self-heating of the volt box. The considerations which act to set an upper limit to the ohms-per-volt are the greater cost and lower stability of high-resistance coils, the greater effect of leakage over the insulating surfaces, and the reduction in sensitivity caused by the large resistance which the volt box contributes to the measurement circuit.

MEASUREMENT OF DIRECT CURRENTS. The current to be measured is passed through a four-terminal resistance standard having its potential terminals joined to the emf terminals of the potentiometer. The measured potential difference, divided by

the four-terminal resistance of the standard, gives the value of the current in amperes. For convenience, the values of the four-terminal resistance are often decimal submultiples of an ohm (0.1, 0.01, etc.), in which case the numerical result of the measurement in volts, multiplied by 10, 100, etc., gives the value of the current in amperes.

DEFLECTION POTENTIOMETERS. In the potentiometers described above, the galvanometer is commonly used merely as a detector, although it is feasible to interpolate between adjacent steps on the lowest dial by noting the corresponding galvanometer deflections. In the deflection potentiometer the greater part of the measured result is read from the dial and the galvanometer is calibrated to indicate directly the last two figures of the result. The only difference between the circuits of an ordinary potentiometer and those of a deflection potentiometer is that the latter has supplementary coils which maintain a constant resistance in the galvanometer circuit for all positions of the switch of the potentiometer dial and those of the battery rheostats, as well as for all values of resistance in the volt box and in the shunts used for current measurements.

The accuracy and precision of the deflection potentiometer are intermediate between those of an ordinary ammeter or voltmeter and those of a high-precision null-type potentiometer. It is used for the checking of ammeters, voltmeters, and wattmeters, and for current and voltage measurements in photometry.

PRECAUTIONS IN THE USE OF D-C POTENTIOMETERS. The emf of a storage cell is unsteady immediately after charging and decreases rapidly for a time. The cell is unfit for use with a potentiometer during this period. The emf of storage cells and of dry cells changes with temperature, and for the most accurate results the cells should be protected from abrupt changes of temperature. The temperature coefficient of emf of dry cells is about twice that of storage cells, the values being about 0.02 per cent and 0.01 per cent per degree centigrade, respectively.

Potentiometers should not be unnecessarily exposed to dust, moisture, or chemical fumes, and should be *kept covered* when not in use. When the top is of hard rubber it should *not be exposed to direct sunlight*, which will cause deterioration of its surface and greatly impair its surface resistivity.

The galvanometer and all the other accessories of the potentiometer should be carefully insulated to avoid leakage currents from other d-c circuits. It is often necessary or desirable to apply the Price "guard-wire" method of protection, in which the potentiometer and each of its auxiliaries, including the galvanometer, are supported by, but insulated from, metal plates which in turn are preferably insulated from the table or other support. These guard plates are connected together and to that pole of the circuit, supplying the current or the voltage under measurement, which is more nearly at earth potential. The line terminal of the volt box (the + terminal in Fig. 21) which in use is connected directly to the potentiometer is to be connected to this pole of the supply circuit, and the shunt used for current measurement should be connected in this side of the line. If the potential of this side of the line unavoidably differs appreciably from earth potential, it is advisable to apply the guard plates to the observer's chair also.

The wires connecting the guard plates together should be protected against breakage. W. P. White has proposed a series connection of the guard plates, so that a single test of the resulting loop checks all the wires for continuity.

When the atmosphere is very dry, as in heated buildings during the winter, disturbances of the galvanometer may result from static charges on the clothing of the observer. This difficulty has been found very troublesome with fur-trimmed garments.

The following practice should be observed in order to avoid accidental short-circuiting of the standard cell. In connecting a standard cell to a potentiometer, first attach *both* wires to the potentiometer, then connect the outer ends of the wires to the cell. In disconnecting the cell, first remove the wires from it. It is well to disconnect one wire from the cell at the end of the day even though the apparatus will be used the next morning. The cell should never be removed with the two wires still attached to it.

A-C POTENTIOMETERS. Referring to the simple potentiometer of Fig. 16, let a milliammeter be inserted in circuit to measure the current in the slide wire. It should be of such type as to give equally accurate readings on direct current and on alternating current of the frequency to be used. After the direct current through the slide wire has been adjusted to the standard value, the reading of the milliammeter should be noted; if this reading depends somewhat on the direction of current flow through the milliammeter, the mean of two readings, between which the direction of flow through the milliammeter is reversed, should be taken. The battery B is then replaced by an a-c source and the rheostat R is adjusted until the reading of the milliammeter is the same as on direct current. The unknown alternating voltage is then connected at P and the d-c galvanometer replaced by a suitable a-c galvanometer or other detector. The unknown voltage must have the same frequency as the current through the potentiometer.

This condition is ordinarily fulfilled by taking both from the same source, or from generators which are mechanically coupled. To obtain a balance, the potential difference taken from the slide wire not only must have the same magnitude as the unknown potential difference but also must be in phase with it. Means must therefore be provided for shifting the phase of the current through the slide wire. In the **Drysdale potentiometer** this is done by means of a phase-shifting transformer. This potentiometer is of the polar-coordinate type; that is, it gives the magnitude of the unknown emf and its phase with respect to a reference vector.

The first a-c potentiometer of the cartesian-coordinate type was devised by **Larsen,** and its principle of operation is shown in Fig. 22. An alternating current i of a standard value flows through the slide wire AB and the primary winding of the adjustable mutual inductor M. The potential difference which balances the unknown potential difference E_x is made up of a component taken from the slide wire, in phase with i, and a quadrature component $2\pi fiM$ induced in the secondary winding of M. Since the potentiometer must usually be available for a range of frequencies, the scale of M must be marked in terms of mutual inductance, and its readings must be multiplied by $2\pi fi$ to reduce them to volts.

Erlang and Pedersen proposed other forms of a-c potentiometer, especially suitable for investigations of telephone apparatus, but not so well adapted for power frequencies.

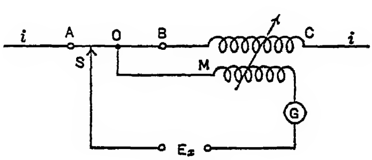

FIG. 22

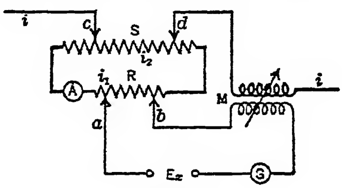

FIG. 23

See Bibliography. The Gall a-c potentiometer, made by H. Tinsley & Co., is an assembly of two potentiometers to form a single piece of apparatus. In use, the two potentiometers are traversed by currents mutually in quadrature. The **Geyger a-c potentiometer,** made by Hartmann & Braun, consists of two center-zero slide wires, one fed from the secondary of an insulating transformer through a non-inductive resistance, the other from the secondary of an air-core transformer having its primary in series with the first slide wire. The **Campbell-Larsen a-c potentiometer,** made by Cambridge Instrument Co., is shown diagrammatically in Fig. 23. The operation is like that of the Larsen potentiometer, but the troublesome multiplication by $2\pi fi$ is avoided. To balance the unknown voltage E_x, two components are required; one is taken from the non-inductive resistance R by adjusting the sliders a and b, and the other by adjusting the value of the mutual inductance M. The current i through the primary of M is composed of the current i_1 through R plus the current i_2 through S. The current i_1 is set at a standard value by means of the device A, which consists of a heating device, a thermocouple, a resistor, and a galvanometer to indicate when the effective value of i_1 is equal to an auxiliary direct durrent of standard value. By adjusting the position of the contacts c and d the total current i corresponding to the standard value of i_1 may be made such that the scale of M indicates directly the emf induced in the secondary of M, in millivolts, at the given frequency. The range of frequency for which this can be done is 25 to 2000 cycles per second. The scales for the sliders a and b are also graduated in millivolts.

Uses of the A-C Potentiometer. Although the a-c potentiometer was originally advocated as a standard instrument for checking ammeters, voltmeters, and wattmeters, the inherent limitations on its accuracy and the other improved apparatus now available for a-c instrument checking make it inadvisable to purchase an a-c potentiometer for this purpose. The accuracy of measurement of the effective value of a given alternating emf or current with the average a-c potentiometer may be assumed to be from 0.5 to 1 per cent for the usual power and lighting frequencies, when the wave form of the quantity to be measured and that of the current through the potentiometer are closely alike and sinusoidal. The real usefulness of the a-c potentiometer lies in fields of engineering research, testing, and investigation where an accuracy of about 1 per cent is adequate and where the measurements either could not be made at all by other means or would be very difficult to carry out. The a-c potentiometer has been used with good results for studying the actions which take place in induction meters and other a-c measuring

apparatus; in magnetic analysis; for measuring the magnetic properties of laminated steel, the ratio and phase angle of current transformers, and the amplification factors of amplifiers. The success of work of this kind depends very much on the manner of setting up and manipulating the potentiometer and the related apparatus, and those who wish to apply the a-c potentiometer method to their problems may consult with advantage papers by Spooner, Geyger, and von Krukowski (see Bibliography).

9. LABORATORY MEASUREMENTS OF D-C. POWER AND ENERGY

For the precise measurement of d-c power in the laboratory, where the current and voltage are supplied by batteries and can be kept very constant, the accepted technique is to measure the current and the voltage by means of a potentiometer, volt box, resistance standard, and standard cell, as indicated in the preceding section on Potentiometers. Measurements of d-c power with such equipment are made in checking the correctness of wattmeters. The same procedure, with the addition of the measurement of time intervals, is the accepted method for the checking of portable standard d-c energy meters (watthour meters).

10. LABORATORY MEASUREMENTS OF ALTERNATING CURRENT, VOLTAGE, AND POWER

GENERAL. The measurement of the instantaneous values of alternating current, voltage, and power requires the use of instruments such as are described in the section on Oscillographs. The majority of a-c measurements, however, refer to conventionally defined and universally recognized effective (or root-mean-square) values. An alternating current of 1 amp, effective value, flowing through a circuit having a resistance of 1 ohm, will liberate heat at the same rate as if 1 amp of direct current were flowing through this circuit. It follows that electrothermic instruments are suitable in principle for the measurement of effective values of alternating currents. Although such instruments are necessary for measurements at high frequencies, they have limitations which have caused two other types to be preferred for precision laboratory measurements at the frequencies used for power and lighting. These are the electrostatic and the electrodynamic types.

METHODS USED BY NATIONAL LABORATORIES. Methods employing electrostatic instruments are preferred for precision a-c measurements by the British National Physical Laboratory. The National Bureau of Standards at Washington prefers instruments of the electrodynamic type. These instruments are of the reflecting type, with the moving-coil system carried by upper and lower suspension strips. The deflection is observed by means of a lamp-and-scale arrangement. The moving-coil system has two coils of unlike instantaneous polarity in order to make the instrument astatic as to uniform stray fields. The arrangement and connection of the windings in order to serve for current, voltage, and power measurements are the same (in the older instruments long used at the Bureau) as the corresponding features in the portable and laboratory-standard electrodynamic instruments so widely used, and the reader is referred to the subsequent treatment of such instruments. In Bureau practice, these instruments are checked on direct current, using a potentiometer, standard cells, and resistance standards, immediately before and after a pair of a-c observations; that is, they serve to carry over or transfer the a-c measurements to the units embodied in these basic d-c standards. They are accordingly spoken of as "transfer instruments." Although this method avoids some of the sources of error which attend the use of standard instruments which are checked at only occasional intervals (of weeks or months), some sources of error remain. These have to do chiefly with the limited accuracy of reading the deflection and the not wholly satisfactory elastic performance of the bronze suspension strips which carry the moving-coil system.

In recent years newer transfer instruments have been devised at the Bureau to overcome these limitations. One is the Harris suppressed-zero electrodynamic voltmeter, the other the Silsbee composite-coil electrodynamic ammeter. In the former the entire deflecting torque is balanced by the torque of the suspensions, as in the older transfer instruments, but the coil system is restrained by two stops so that it cannot return to zero when the current ceases to flow. The continual, nearly constant stress in the suspensions eventually results in a steady state such that "zero drift" is eliminated to a very high degree. In the Silsbee composite-coil ammeter each of the fixed coils and each of the moving coils is wound with a pair of wires, and in operation the deflecting torque resulting from the alternating current is nearly balanced by the counter torque set up

by a precisely adjusted direct current in the other circuit through the coil system. Small variations in the magnitude of the alternating current are measured by observing the deflection of the coil system to the right or to the left of its central zero position.

TRANSFER INSTRUMENTS FOR GENERAL USE. Reflecting electrodynamometers are used for the measurement of very small amounts of power at low power factor, as in the testing of samples of cable for dielectric loss. For many purposes, however, in commercial laboratories electrodynamic voltmeters, wattmeters, and ammeters of the pivoted type, with pointer and scale, are of satisfactory accuracy and are much more convenient. The inductance of their circuits can be kept small by suitable design, and corrections are usually small and not difficult to apply. However, for the measurement of very small amounts of power at high voltage, or for measurements at frequencies much above the usual power and lighting frequencies, the effects of the unavoidable inductances of the electrodynamic instrument become sufficiently pronounced to warrant the use of an electrometer in spite of its more delicate construction and much longer period.

11. ELECTRICAL INDICATING INSTRUMENTS

GENERAL. An **electrical indicating instrument** is an instrument in which the present value of the quantity under observation is indicated by the position of a pointer relative to a scale. The term "instrument" may include the instrument proper, as defined below, or it may include the instrument proper together with any necessary auxiliary devices, such as shunts, shunt leads, resistors, reactors, capacitors, or instrument transformers. The **instrument proper** consists of the actuating mechanism together with those auxiliary devices (scale, resistors, shunts, etc.) which are built into the instrument case or otherwise made an integral part of its structure.

The large output of electrical instruments and the wide diversity of their applications have made an American code of standard definitions and terminology necessary in order to facilitate a clear understanding between the specifying engineer, the purchasing agent, and the manufacturer. The beginning of such a code has been provided by the A.I.E.E. in its Standards No. 33 for Electrical Indicating Instruments.

DISTINCTION BETWEEN ERROR AND CORRECTION; ACCURACY OF AN INSTRUMENT. The error of indication of an instrument is found by subtracting from the value which it indicates the true value of the quantity measured by it, as determined by reference to some standard of a higher order than the instrument itself. It is the quantity which must be **algebraically subtracted** from the indication to obtain the true value of the quantity. A positive error thus denotes that the instrument "reads high," that is, that its indication is greater than the true value. The **correction to the indication** is found by subtracting the indicated value from the true value, that is, the correction has the same numerical value as the error of indication, but the opposite sign. **The correction, algebraically added** to the indication, gives the true value.

The **accuracy of an instrument** may be expressed in various ways, some of which are ambiguous and likely to cause disagreement between maker and purchaser. It is generally defined as the ratio of the error of indication to some like quantity taken as a basis. If the indication were taken as the basis it would be necessary to have a sliding scale of accuracy with the permissible percentage error increasing in some manner as the deflection decreased. The majority of American manufacturers consequently favor the simple definition of accuracy as the percentage ratio of the limit of error at any point on the scale to the full-scale value. In instruments having uniform scales this is equivalent to saying that no division mark is out of its proper position by more than a specified distance. This definition of accuracy applies to the usual case in which the zero point is at the lower end of the scale. For instruments in which the zero point is in an intermediate position on the scale, the sum of the full-scale readings to the right and to the left of the zero point should be used as the basis for reckoning the accuracy.

The accuracy of an electrical instrument depends on the operating principle; the quality of the design, materials, and workmanship; and the conditions of use. In general, the accuracies stated by American instrument makers are about as follows: switchboard instruments, 4 in. diameter and larger, 1 per cent; 3 in. diameter and smaller, 2 per cent; laboratory-standard instruments, 0.1 per cent; portable instruments, highest-grade, 0.2 to 0.25 per cent, intermediate-grade, 0.5 to 0.75 per cent; miniature instruments, 2 per cent. Exceptions: high-grade portable polyphase wattmeters, 0.5 per cent; rectifier instruments, for which relatively large errors must be tolerated; frequency meters, power-factor meters, phase-angle meters, and capacitance meters, the accuracy of which must be stated in special ways for which the makers' catalogs should be consulted.

Although a high degree of accuracy is important in much laboratory work and in such

Electrothermic instruments depend for their operation on the heating effect of a current. Two markedly distinct types are (1) the **expansion type,** commonly known as the hot-wire type, and (2) the **thermocouple type.** In the hot-wire instrument the indication is effected by mechanically multiplying the expansion of a wire or strip which is heated by the current. Hot-wire instruments have been virtually superseded by instruments of the thermocouple type, in which one or more thermojunctions are heated (directly or indirectly) by the current and supply a direct current which actuates a suitable d-c instrument mechanism. Both the hot-wire and the thermocouple types may be made as ammeters, voltmeters, and wattmeters. Among the **advantages of the thermocouple instrument** over the hot-wire instrument are: much lower power loss, greater sensitivity, and ability to measure currents down to very much smaller values. Like the hot-wire instruments, the thermocouple instruments are **easily damaged by overload.** For example, Hartmann and Braun state that the heating element of their vacuum thermocouple will burn out when the current reaches 1.6 times its rated value; for a 1-amp and a 10-amp heating element the fusing currents are given as twice rated current and three times rated current, respectively. The "thermal inertia" which brings most instruments through momentary inadvertent overloads without thermal damage is so small in the thermocouple instrument that even careful users must count on burning out a heating element occasionally.

Ammeters of the thermocouple type are suitable for use on alternating current of frequencies ranging from the lowest up to those employed in radio circuits. They are not ordinarily used at power frequencies, but have advantages in special cases where minimum resistance and negligible inductance are necessary. They are indispensable for measurements at audio and higher frequencies. Self-contained thermocouple ammeters are made in ranges from 2 ma up to 50 amp. They are supplied with external heating elements up to 1000 amp. Multiple-range milliammeters, because of the necessity for passing part of the current through a shunt around the heating element, are limited to a maximum frequency of 3000 to 5000 cycles. This limitation does not apply to multiple-range instruments in which a separate heating element is used for each range.

Voltmeters of the thermocouple type may be used with frequencies up to 3000 to 5000 cycles.

Electrostatic instruments depend for their operation on the forces of attraction or repulsion between electrically charged fixed metal parts and moving metal parts. The relatively weak forces developed in this way tend toward an undesirably long period and other operating drawbacks, and limit the application of electrostatic instruments to particular purposes for which their special advantages make them preferable or even indispensable. They are useful when voltage measurements are to be made without drawing any appreciable current from the source, or when the cost of series resistors or of voltage transformers for high voltages would be prohibitive. They are practically the only means available for measuring direct voltages of 100 kv or more as used in cable testing. They are designed either for direct connection to the points between which the voltage is to be measured, or through an electrostatic voltage divider analogous to the "volt box" used with a potentiometer. The use of such voltage dividers is feasible for a-c voltage measurements only.

Electronic (thermionic) instruments utilize for their operation the properties of the electronic tube. These instruments are in the early stages of development, and their construction and application have not yet been standardized. Results may be obtained with them that are not possible with instruments of any other type. For example, the thermionic voltmeter described by S. C. Hoare (see Bibliography), having a 5-volt range, can be made to function as if its internal resistance were 50,000 ohms when a 6-volt auxiliary battery is used. With a 24-volt auxiliary battery the voltmeter behaves as if its resistance were infinite. The same type of instrument can be made as a microammeter giving full-scale deflection for 0.1 ma and having an equivalent resistance of 3.3 megohms. These instruments are subject to errors when the wave form is other than that used in their calibration, and they have some other special limitations concerning which Mr. Hoare's article and others (see Bibliography) should be consulted.

RECTIFIER INSTRUMENTS. A rectifier instrument is the combination of an instrument responding only to direct current and a device for rectifying the alternating current to be measured. Instruments of this type are usually of the permanent-magnet moving-coil type in combination with **copper-oxide rectifiers** in the Wheatstone-bridge full-wave arrangement. They have been developed to meet the need for low-range a-c ammeters and voltmeters of very small power consumption. They are used for those a-c measurements for which the relatively large power consumption of electrodynamic, thermocouple, and moving-iron instruments makes them ill-adapted or even inappli-

cable, also for a-c measurements where only moderate accuracy (say 5 per cent) is sufficient and ruggedness and ability to sustain overloads are important. The deflection of a rectifier instrument is proportional to the **average value** of the rectified wave, but since the user is generally concerned with the effective value, it is customary to mark the scales in terms of the effective (rms) values for an assumed (usually sine) wave form. It is possible to make a-c voltmeters of the rectifier type with as high as 2000 to 4000 ohms per volt, and ammeters giving full-scale deflection for as little as 500 ma. Rectifier instruments not only are subject to relatively large **wave-form errors** but also are noticeably affected by changes of room temperature. For details as to the magnitude of these effects, methods of compensation for temperature, etc., reference should be made to the maker's instructions and to papers by Sahagen and others (see Bibliography). **Rectifier ammeters** are available in ranges of from 100 to 500 ma. **Rectifier voltmeters** are made in ranges of from 2 to 150 volts. Rectifier instruments are of relatively recent origin and have not reached the stage of standardization which characterizes most other types. The makers should be consulted for the latest information about rectifier instruments for specific requirements, particularly since the accuracy of these instruments is so dependent on temperature and wave form.

WATTMETERS. General. A wattmeter measures the average value of the product ei of the instantaneous voltage by the instantaneous current; that is, the average power. Types of instrument which may be adapted for use as wattmeters include the electrodynamic, induction, electrostatic, hot-wire, and thermocouple. Even the moving-iron instrument may be so used, but though patents for such wattmeters have been issued, no practical use appears to have been made of them. For the use of electrostatic wattmeters, see **Electrometers.** For most purposes the electrodynamic wattmeter is preferred, and the discussion in this section is limited to this type.

The **electrodynamic wattmeter** consists essentially of two coils, one of relatively coarse wire carrying the load current, and the other, of fine wire, in series with a noninductive resistor, connected in parallel with the load. The current in the fine-wire coil being very nearly in phase with the voltage e across the load, the instantaneous torque will be closely proportional to the instantaneous power. Because the natural period of vibration of the moving coil is so much greater than the period of an alternating current, the coil does not follow the rapid cyclic changes of the instantaneous power but assumes a deflected position which indicates the average power. The torque produced by the currents in the coils is opposed and balanced by a counter torque, usually that of a spring. In nearly all wattmeters the current coil is made the fixed coil because the current in it is ordinarily too great to be carried by the springs. In most cases the fixed coils surround the moving coil.

Current and Voltage Ranges of Wattmeters. It was formerly customary to construct portable wattmeters for currents up to 200 to 400 amp, but the present tendency is to avoid such high current ranges. It is very difficult to construct a 200-amp current coil so that the distribution of an alternating current over the cross-section of the coil will be accurately identical (as to production of torque) with the distribution of the direct current with which the wattmeter must be checked. Also, as the current range is made greater, it becomes increasingly difficult to guard against errors occasioned by stray field from the current leads to the wattmeter. Recent great improvements in the ratio and phase-angle accuracy of current transformers make it possible to obtain better accuracy from a 5-amp wattmeter and a current transformer than from a wattmeter of large current rating connected directly in the line. However, when a wattmeter is to be used to measure the total power in an a-c circuit in which flows a d-c component, the current transformer cannot be used. Although single-phase switchboard wattmeters are made for currents up to 400 amp, there is a recent tendency to restrict the current range to 20 amp or less.

Wattmeters are relatively expensive instruments, and when they are used to check portable standard watthour meters the accuracy of their indications is of great commercial importance. It is therefore usual to build portable wattmeters with two current ranges and with two (sometimes three) voltage ranges, thus facilitating the obtaining of the largest possible deflection for the amount of power under measurement. Abroad, wattmeters are frequently made with three current ranges in the ratio 1 : 2 : 4, and as an extreme case, with six current ranges, which, however, are obtained at a sacrifice of 50 per cent in torque.

A **polyphase wattmeter** contains two single-phase wattmeter mechanisms with the moving coils attached to a common shaft. Its use makes it possible to measure power in two-phase or three-phase three-wire circuits with a single instrument.

Methods of Connecting Wattmeter in Circuit; Correction for Power Loss in Wattmeter. A wattmeter may be connected to the load which it is to measure in either of

the two ways illustrated in Figs. 24 and 25. When connected as in Fig. 24 the current through the current circuit of the wattmeter is equal to the vector sum of the current in the load and that in the voltage circuit of the wattmeter; hence the wattmeter will read the sum of the watts in the load and the watts loss in the voltage circuit, which is equal to E^2/R_p, where R_p is the resistance of the voltage circuit and E the voltage across the load. For precise work this correction should always be made, unless the wattmeter is of relatively high current range or is "compensated"; see below.

In the second scheme of connection, Fig. 25, the current through the current circuit is the same as that taken by the load, but the voltage across the voltage circuit of the wattmeter is higher than the voltage across the load by the drop through the wattmeter current coil, and the wattmeter reads too high by an amount equal to the loss in this coil. This loss in equal to $R_c I^2$, where R_c is the resistance of the current circuit and I the load current; it is usually less than the loss in the voltage circuit, hence if no correction is made for the wattmeter loss the scheme of connections shown in Fig. 25 should be used. If the highest accuracy is required, especially with a wattmeter of low current rating, the connections shown in Fig. 24 should be used, and a correction should be applied.

When a voltmeter is connected to the load across AA' in Fig. 24, while the wattmeter reading is being taken, a correction may need to be made for the power taken by the voltmeter. Calling R_v the resistance of the voltmeter and multiplier, if any, and E the voltage across the load, the loss in the voltmeter is E^2/R_v. In most cases the voltage-circuit losses in the wattmeter and voltmeter are best determined by a direct measurement with the wattmeter, using the test voltage and leaving the load circuit open. If the voltmeter is connected across BB' in Fig. 25, the power loss in it is not read by the

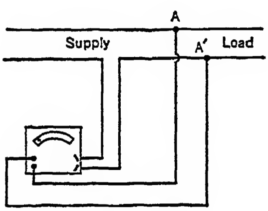

Fig. 24

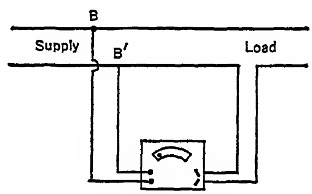

Fig. 25

wattmeter, and the voltmeter reading equals the load voltage plus the impedance drop in the wattmeter current coil.

Compensation for Loss in Voltage Circuit of Wattmeter. In the so-called compensated wattmeters a stationary compensating coil, in series with the moving coil, is placed so that the current through it produces on the moving coil a torque equal and opposite to that produced by this same current flowing through the current coil. To make this compensation accurate for all positions of the moving coil, the compensating coil must be intermingled with the current coil. A compensated wattmeter should always be connected to the circuit, as shown in Fig. 24; when so connected no correction for the loss in the wattmeter is necessary. As a check on the correctness of connections, the load circuit may be opened, whereupon the wattmeter reading should be zero.

Phase Angle of Voltage Circuit of a Wattmeter. The inductance of the moving coil of a simple (uncompensated) wattmeter tends to cause the alternating current in the voltage circuit to lag slightly behind the impressed voltage. Distributed capacitance in the series resistor tends to counteract this tendency, but only slightly in the flat-card resistors now generally used. The resultant lag of the current is known as the phase angle α of the wattmeter. It is usually about 2 to 4 minutes (at 60 cycles) in high-grade wattmeters of the deflection type having a voltage circuit rated at 125 to 150 volts; for the Westinghouse precision wattmeter, 150-volt range, it is about 12 minutes; it varies directly as the frequency and inversely as the voltage rating. The error which it introduces into the measurement of a-c power becomes relatively greater as the power factor is lowered. The angle α is taken as positive when the positive reactance of the wattmeter moving coil predominates over any negative (capacitive) reactance in the voltage circuit. Such a wattmeter will give a larger reading on an inductive load than on a non-inductive load, for the same actual power.

Effect of Mutual Inductance on Wattmeter Reading. In most wattmeters of the deflection type the mutual inductance between the fixed coils and the moving coil changes sign at approximately half-scale deflection. For any other relative position of the coils there is an error caused by mutual inductance which is negligible in good wattmeters

of current ratings of 5 amp or more and voltage ratings of 75 volts or over, used with ordinary power frequencies. For higher frequencies, lower current ranges, or lower voltage ranges it may be advisable to check the magnitude of the mutual-inductance error; formulas for this purpose are given in *Trans. A.I.E.E.*, vol. 39, pp. 563-567, 1920.

Effect of Eddy Currents on Wattmeter Reading. Eddy currents in fixed conducting masses near the fixed coils will set up magnetic fluxes which combine with the flux of the current coil to form a resultant flux which differs from that of the current coil in both magnitude and phase. There is no simple way to correct for the resulting error, and it is up to the maker to use as little metal as possible, and that of high resistivity, and to arrange it in such a way as to minimize the presence of closed paths for eddy currents. Appreciable eddy-current errors resulted in one case from the inadvertent omission of insulating washers in a metal assembly holding the field coils. In another, short-circuited turns in the compensating winding caused a similar result. One or more short-circuited turns in a current coil would produce the same kind of result but to a greater extent, but such a condition should be revealed by its effect on the d-c performance of the wattmeter.

Compensation for Inductance of Voltage Circuit of Wattmeter. The phase angle of a wattmeter may be reduced to zero (that is, the inductance of its voltage circuit may be compensated) by shunting a capacitor of capacitance C around a portion R/n of the series resistance of the voltage circuit. If the inductance of the voltage circuit is L, the relation which gives compensation of phase angle is

$$C = \frac{Ln^2}{R^2} \tag{7}$$

The practical application of this formula requires precautions which are summarized in *Trans. A.I.E.E.*, vol. 39, pp. 562-563, 1920.

Correction Factor for Self-inductance of the Voltage Circuit of a Wattmeter. If a wattmeter is used to measure the power of a sinusoidal current I which lags by an angle θ behind the emf E, and if the inductance and resistance of the wattmeter voltage circuit are L and R, so that $\tan \alpha = \omega L/R$ is the tangent of the angle of lag in the voltage circuit, the correction factor C by which the reading of the wattmeter must be multiplied to correct it for the effect of inductance in the voltage circuit is

$$C = \frac{1 + \tan^2 \alpha}{1 + \tan \alpha \tan \theta} \tag{8}$$

In a wattmeter which is fit to use, $\tan \alpha$ will be small and its square negligible. Consequently the term $\tan^2 \alpha$ may be omitted from the numerator. This formula becomes useless for power factors near zero, at which point $\tan \theta$ becomes infinite. To avoid this difficulty, **Drysdale** proposed an additive correction for the self-inductance of the voltage circuit. It is outlined, with numerical examples, in *Trans. A.I.E.E.*, vol. 39, pp. 560-562, 1920.

The tangent formula above given (eq. 8) has been much used in America in a form which is trigonometrically identical with it but in which the wattmeter phase angle α is extended to include also the phase displacements introduced by a current transformer and a voltage transformer.

Correction of Wattmeter Reading for Phase Angles of Wattmeter and of Instrument Transformers. Let α denote the phase angle of the wattmeter as above defined, β and γ the phase angles of the current transformer and the voltage transformer, respectively. Let P_2 denote the wattmeter reading corrected for scale error and multiplied by the product of the corrected ratios of the current and voltage transformers, E the voltmeter reading corrected for scale error and multiplied by the ratio of the voltage transformer, and I the ammeter reading corrected for scale error and multiplied by the corrected ratio of the current transformer. Then the apparent power factor is

$$\cos \theta_2 = \frac{P_2}{EI} \tag{9}$$

and the true power is

$$P = P_2 \frac{\cos (\theta_2 + \alpha + \beta - \gamma)}{\cos \theta_2} \tag{10}$$

The angle θ_2 is to be taken as positive when the current lags behind the voltage, and as negative when the current leads. In the case of β and γ a positive angle denotes that the reversed secondary quantity leads the primary quantity. Using this definition, formula (10) shows that a leading phase angle γ tends to offset the effect of a leading phase angle β. Values of the correction factor $\frac{\cos (\theta_2 + \alpha + \beta - \gamma)}{\cos \theta_2}$ are given in Tables I and II. Note carefully the conditions, stated at the head of each table, to which each

table applies. It should be noted that formula (10), like the tangent formula (8), becomes useless as the power factor of the load approaches zero.

Table I. Phase-Angle Correction Factors: for Lagging Current When ($\alpha + \beta - \gamma$) is Positive; for Leading Current When ($\alpha + \beta - \gamma$) is Negative.

Phase Angle ($\alpha+\beta-\gamma$)	Apparent Power Factor ($\cos\theta_2$)													
	0.10	0 15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0 90	0.95	0.99	1.00
5′	0.9855	0.9904	0.9929	0.9944	0.9954	0.9967	0.9975	0 9981	0.9985	0 9989	0.9993	0.9995	0.9998	1.0000
10′	0.9711	0.9808	0.9857	0.9887	0.9907	0.9933	0.9950	0.9961	0 9970	0.9978	0.9986	0.9990	0.9996	1.0000
15′	0.9566	0 9712	0.9786	0.9831	0 9861	0.9900	0.9924	0 9942	0 9955	0.9967	0 9979	0 9986	0.9994	1 0000
20′	0.9421	0.9616	0.9715	0.9775	0.9815	0.9867	0.9899	0.9922	0.9940	0.9956	0.9972	0.9981	0.9992	1.0000
25′	0 9276	0.9520	0.9643	0.9718	0.9768	0.9833	0.9874	0 9903	0.9926	0.9945	0.9965	0.9976	0.9989	1 0000
30′	0.9131	0.9424	0.9572	0.9662	C.9722	0 9800	0 9848	0.9883	0.9911	0.9934	0.9957	0.9971	0.9987	1 0000
40′	0 8842	0.9232	0.9429	0 9549	0.9629	0.9733	0.9798	0 9844	0.9881	0.9912	0.9943	0.9961	0.9983	0.9999
50′	0 8552	0.9040	0.9286	0.9436	0.9536	0.9666	0.9747	0.9805	0.9851	0.9890	0.9929	0.9951	0.9978	0.9999
1° 0′	0.8262	0.8848	0.9143	0.9323	0.9444	0 9599	0.9696	0 9766	0.9820	0.9868	0.9914	0 9941	0.9974	0.9998
10′	0.7972	0 8656	0.9000	0.9209	0.9350	0.9531	0.9645	0.9726	0.9790	0.9845	0.9899	0.9931	0.9969	0 9993
20′	0.7682	0.8464	0.8857	0.9096	0.9257	0.9464	0.9594	0 9687	0.9760	0.9823	0.9885	0.9921	0.9964	0.9997
30′	0.7392	0.8271	0.8714	0.8983	0.9164	0.9397	0.9543	0.9648	0 9730	0.9800	0.9870	0.9911	0.9959	0.9997
40′	0 7102	0.8079	0.8571	0.8869	0.9071	0 9329	0.9492	0 9608	0.9699	0.9778	0.9855	0 9900	0.9954	0.9996
50′	0.6812	0 7886	0.8428	0.8756	0.8978	0.9262	0.9441	0.9568	0.9668	0.9755	0.9840	0.9890	0.9949	0.9995
2° 0′	0.6521	0 7694	0.8284	0.8642	0.8884	0 9194	0.9389	0.9529	0.9638	0.9732	0.9825	0.9879	0.9944	0.9994
10′	0.6231	0.7501	0.8141	0.8529	0.8791	0.9127	0.9338	0.9489	0.9607	0.9709	0 9810	0 9869	0.9939	0.9993
20′	0.5941	0.7308	0.7997	0.8415	0.8697	0.9059	0.9287	0.9449	0.9576	0.9686	0.9795	0.9858	0.9934	0.9992
30′	0.5650	0 7115	0.7854	0.8301	0.8603	0.8991	0.9235	0.9409	0.9545	0.9663	0.9779	0.9847	0.9928	0 9990
40′	0.5360	0 6923	0.7710	0.8187	0 8510	0.8923	0.9183	0.9369	0.9515	0.9640	0.9764	0.9836	0.9923	0 9989
50′	0.5069	0.6730	0.7566	0.8073	0.8416	0.8855	0.9132	0.9329	0.9483	0.9617	0.9748	0.9825	0.9917	0.9988
3° 0′	0.4779	0.6537	0.7422	0.7959	0.8322	0.8787	0.9080	0.9288	0.9452	0.9594	0.9733	0.9814	0.9912	0.9936
10′	0.4488	0.6344	0.7279	0.7845	0.8228	0.8719	0.9028	0.9248	0.9421	0.9570	0.9717	0.9803	0.9905	0.9985
20′	0.4198	0.6151	0.7135	0.7731	0.8134	0.8651	0.8976	0.9208	0.9390	0.9547	0.9701	0.9792	0.9900	0.9983
30′	0.3907	0.5957	0.6991	0.7617	0.8040	0.8583	0.8924	0.9167	0.9359	0.9523	0.9686	0.9781	0.9894	0.9981
40′	0.3616	0.5764	0.6847	0.7503	0.7946	0.8514	0.8872	0.9127	0.9327	0.9500	0.9670	0.9769	0.9888	0.9980
50′	0.3326	0.5571	0.6702	0.7388	0 7852	0.8446	0.8820	0.9086	0.9296	0.9476	0.9654	0.9758	0.9882	0.9978
4° 0′	0.3035	0.5378	0.6558	0.7274	0.7758	0.8377	0.8767	0.9046	0.9264	0.9452	0.9638	0.9746	0.9876	0.9976
10′	0.2744	0.5185	0.6414	0.7160	0.7663	0.8309	0.8715	0.9005	0.9232	0.9429	0.9622	0.9735	0.9870	0.9974
20′	0.2453	0.4991	0.6270	0.7045	0.7569	0.8240	0.8663	0.8964	0.9201	0.9405	0.9605	0.9723	0.9864	0.9971
30′	0.2163	0.4793	0.6125	0.6930	0.7474	0.8171	0.8610	0.8923	0.9169	0.9381	0.9589	0.9711	0.9857	0.9969
40′	0.1872	0.4604	0.5981	0.6816	0.7380	0.8103	0.8558	0.8882	0.9137	0.9357	0.9573	0.9699	0.9851	0.9967
50′	0.1581	0.4411	0.5837	0.6701	0.7285	0.8034	0.8505	0.8841	0.9105	0.9333	0.9556	0.9687	0.9844	0.9964
5° 0′	0.1290	0.4217	0.5692	0.6586	0.7191	0.7965	0.8452	0.8800	0.9073	0.9308	0.9540	0.9675	0.9838	0 9962
10′	0 0999	0.4024	0.5548	0.6472	0.7096	0.7896	0.8400	0.8759	0.9041	0.9284	0.9523	0.9663	0.9831	0.9959
20′	0.0708	0.3830	0.5403	0.6357	0.7001	0.7827	0.8347	0.8717	0.9008	0.9260	0.9507	0.9651	0.9824	0.9957

Interpolation for correction factors corresponding to values of ($\alpha + \beta - \gamma$) lying between those given in the table may be made without error. Interpolation for correction factors corresponding to values of $\cos\theta_2$ lying between those given in the table may be made without exceeding an error of 0.0010 in the sections of the table lying between the heavy black lines; outside of these sections, and in all cases where the adjacent values of $\cos\theta_2$ are separated by the heavy black lines, the maximum error in interpolation will exceed 0.0010.

The above convention in regard to the algebraic sign of the phase angle γ of a voltage transformer (positive sign for secondary voltage *leading* the reversed primary voltage) is consistent with the universally adopted convention as to the sign of the phase angle of a current transformer; it has been used for many years by the National Bureau of Standards and by the principal foreign national laboratories, and was adopted in 1930 by the International Electrotechnical Commission. The opposite convention, namely, γ positive for secondary voltage *lagging* the reversed primary voltage, is still in use. When it is used, the correction factor becomes $\dfrac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos\theta_2}$. In both of these forms of the correction factor the + and − signs in the numerator are addition (or subtraction) *operators*, and the angles α, β, and γ may be intrinsically either + or −. The angles α and β, however, are nearly always intrinsically positive. The following examples illustrate the use of the correction tables.

Example 1. Given a single-phase circuit with lagging current in which the wattmeter reading corrected for scale error and multiplied by the corrected ratios of current and

Table II. Phase-Angle Correction Factors: for Lagging Current When $(\alpha + \beta - \gamma)$ is Negative; for Leading Current When $(\alpha + \beta - \gamma)$ is Positive

Phase Angle $(\alpha+\beta-\gamma)$	Apparent Power Factor ($\cos \theta_2$)													
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99	1.00
5′	1.0145	1.0096	1.0071	1.0056	1.0046	1.0033	1.0025	1.0019	1.0015	1.0011	1.0007	1.0005	1.0002	1.0000
10′	1.0289	1.0192	1.0142	1.0113	1.0092	1.0067	1.0050	1.0039	1.0030	1.0022	1.0014	1.0010	1.0004	1.0000
15′	1.0434	1.0288	1.0214	1.0169	1.0139	1.0100	1.0075	1.0058	1.0044	1.0033	1.0021	1.0014	1.0006	1.0000
20′	1.0579	1.0383	1.0285	1.0225	1.0185	1.0133	1.0101	1.0077	1.0059	1.0043	1.0028	1.0019	1.0008	1.0000
25′	1.0723	1.0479	1.0356	1.0281	1.0231	1.0166	1.0126	1.0097	1.0074	1.0054	1.0035	1.0024	1.0010	1.0000
30′	1.0868	1.0575	1.0427	1.0338	1.0277	1.0200	1.0151	1.0116	1.0089	1.0065	1.0042	1.0028	1.0012	1.0000
40′	1.1157	1.0766	1.0569	1.0450	1.0369	1.0266	1.0201	1.0154	1.0118	1.0087	1.0056	1.0038	1.0016	0.9999
50′	1.1446	1.0958	1.0711	1.0562	1.0461	1.0332	1.0251	1.0193	1.0147	1.0108	1.0069	1.0047	1.0020	0.9999
1° 0′	1.1735	1.1149	1.0853	1.0674	1.0553	1.0398	1.0301	1.0231	1.0177	1.0129	1.0083	1.0056	1.0023	0.9998
10′	1.2024	1.1340	1.0995	1.0787	1.0645	1.0464	1.0351	1.0269	1.0206	1.0151	1.0097	1.0065	1.0027	0.9998
20′	1.2313	1.1531	1.1137	1.0898	1.0737	1.0530	1.0400	1.0308	1.0235	1.0172	1.0110	1.0074	1.0030	0.9997
30′	1.2601	1.1722	1.1279	1.1010	1.0829	1.0596	1.0450	1.0346	1.0264	1.0193	1.0123	1.0083	1.0034	0.9997
40′	1.2890	1.1913	1.1421	1.1122	1.0921	1.0662	1.0500	1.0384	1.0292	1.0214	1.0137	1.0091	1.0037	0.9996
50′	1.3178	1.2104	1.1562	1.1234	1.1012	1.0728	1.0549	1.0421	1.0321	1.0235	1.0150	1.0100	1.0040	0.9995
2° 0′	1.3466	1.2294	1.1704	1.1346	1.1104	1.0794	1.0598	1.0459	1.0350	1.0256	1.0163	1.0109	1.0044	0.9994
10′	1.3755	1.2485	1.1845	1.1457	1.1195	1.0859	1.0648	1.0497	1.0379	1.0276	1.0176	1.0117	1.0047	0.9993
20′	1.4043	1.2675	1.1986	1.1569	1.1286	1.0925	1.0697	1.0535	1.0407	1.0297	1.0189	1.0126	1.0050	0.9992
30′	1.4331	1.2866	1.2127	1.1680	1.1377	1.0990	1.0746	1.0572	1.0435	1.0318	1.0202	1.0134	1.0053	0.9990
40′	1.4618	1.3056	1.2268	1.1791	1.1469	1.1055	1.0795	1.0610	1.0464	1.0338	1.0215	1.0142	1.0055	0.9989
50′	1.4906	1.3246	1.2409	1.1902	1.1560	1.1120	1.0844	1.0647	1.0492	1.0359	1.0227	1.0150	1.0058	0.9988
3° 0′	1.5194	1.3436	1.2550	1.2013	1.1650	1.1185	1.0893	1.0684	1.0520	1.0379	1.0240	1.0158	1.0061	0.9986
10′	1.5481	1.3626	1.2691	1.2124	1.1741	1.1250	1.0942	1.0721	1.0548	1.0399	1.0252	1.0166	1.0063	0.9985
20′	1.5768	1.3816	1.2832	1.2235	1.1832	1.1315	1.0990	1.0758	1.0576	1.0419	1.0265	1.0174	1.0066	0.9983
30′	1.6056	1.4005	1.2972	1.2346	1.1923	1.1380	1.1039	1.0795	1.0604	1.0439	1.0277	1.0182	1.0068	0.9981
40′	1.6343	1.4195	1.3113	1.2456	1.2013	1.1445	1.1087	1.0832	1.0632	1.0459	1.0289	1.0190	1.0071	0.9980
50′	1.6630	1.4384	1.3253	1.2567	1.2103	1.1509	1.1136	1.0869	1.0660	1.0479	1.0301	1.0197	1.0073	0.9978
4° 0′	1.6916	1.4573	1.3393	1.2677	1.2194	1.1574	1.1184	1.0906	1.0687	1.0499	1.0313	1.0205	1.0075	0.9976
10′	1.7203	1.4763	1.3533	1.2788	1.2284	1.1638	1.1232	1.0942	1.0715	1.0519	1.0325	1.0212	1.0077	0.9974
20′	1.7489	1.4952	1.3673	1.2898	1.2374	1.1703	1.1280	1.0979	1.0742	1.0538	1.0337	1.0220	1.0079	0.9971
30′	1.7776	1.5141	1.3813	1.3008	1.2464	1.1767	1.1328	1.1015	1.0770	1.0558	1.0349	1.0227	1.0081	0.9969
40′	1.8062	1.5329	1.3953	1.3118	1.2554	1.1831	1.1376	1.1052	1.0797	1.0577	1.0361	1.0234	1.0083	0.9967
50′	1.8348	1.5518	1.4092	1.3228	1.2644	1.1895	1.1424	1.1088	1.0824	1.0596	1.0373	1.0241	1.0085	0.9964
5° 0′	1.8634	1.5707	1.4232	1.3337	1.2733	1.1959	1.1472	1.1124	1.0851	1.0616	1.0384	1.0248	1.0086	0.9962
10′	1.8920	1.5895	1.4371	1.3447	1.2823	1.2023	1.1519	1.1160	1.0878	1.0635	1.0396	1.0255	1.0088	0.9959
20′	1.9205	1.6083	1.4510	1.3557	1.2912	1.2086	1.1567	1.1196	1.0905	1.0654	1.0407	1.0262	1.0089	0.9957

Interpolation for correction factors corresponding to values of $(\alpha + \beta - \gamma)$ lying between those given in the table may be made without error. Interpolation for correction factors corresponding to values of $\cos \theta_2$ lying between those given in the table may be made without exceeding an error of 0.0010 in the sections of the table lying between the heavy black lines; outside of these sections, and in all cases where the adjacent values of $\cos \theta_2$ are separated by the heavy black lines, the maximum error in interpolation will exceed 0.0010.

voltage transformers equals 24,520 watts, and the product of the voltmeter and ammeter readings, similarly corrected, equals 35,600 volt-amperes. Then $\cos \theta_2 = 24{,}520/35{,}600 = 0.689$. If the equivalent phase angle α of the wattmeter is $+ 4'$ and if from examination of characteristic curves the current-transformer phase angle is found to be $+ 48'$ and the voltage-transformer phase angle is $+ 10'$, then $\alpha + \beta - \gamma = + 42'$, and from Table I, the correction factor is 0.9871. Whence the true power equals $24{,}520 \times 0.9871 = 24{,}204$ watts.

Example 2. Given a single-phase circuit with leading current, in which the wattmeter reading, corrected as in example No. 1, equals 12,266 watts and the product of the voltmeter and ammeter readings, similarly corrected, equals 24,532 volt-amperes. Then $\cos \theta_2 = 12{,}266/24{,}532 = 0.5$. If the equivalent phase angle α of the wattmeter is $+ 5'$, the phase angle β of the current transformer $+ 2° 33'$ and the phase angle γ of the voltage transformer is $- 38'$, then $\alpha + \beta - \gamma = + 3° 16'$ and therefore from Table II the correction factor to be used is 1.0971. Whence the true power equals $12{,}266 \times 1.0971 = 13{,}457$ watts.

In measuring badly unbalanced three-phase loads with two wattmeters, corrections should be applied separately, as above, to each wattmeter. When the circuit is balanced a single correction based on $\cos \theta_2$ for the whole circuit may be used.

Accuracy of Wattmeters. High-grade portable wattmeters have a stated accuracy of 0.2 to 0.25 per cent of full-scale value for single-phase or 0.5 per cent for polyphase wattmeters. It is not practicable to obtain as high accuracy in the latter, on account of the impossibility of making both elements of the polyphase wattmeter follow exactly the same scale law. This theoretical disadvantage of the polyphase wattmeter is offset by its greater convenience and the reduction in number of observations necessary. High-grade switchboard wattmeters have a stated accuracy of 1 per cent of full-scale value. Laboratory standard wattmetters have a stated accuracy of 0.1 per cent of full-scale value.

Wattmeters are usually tested for accuracy on direct current, using a potentiometer and suitable accessories. For convenience in computing the values of watts, a voltage of 100 volts may be used for wattmeters for use on 110-volt circuits. The current required to produce a given deflection having been recorded, a second observation is made after reversing the currents in both of the windings of the wattmeter. The mean of two such readings which do not differ more than several per cent is free from error due to stray magnetic field in unshielded wattmeters, or from accidental magnetic polarity in the iron shield of shielded wattmeters, such as may arise from an accidental large overload in the current coil. This test on "reversed direct current" having been made at a sufficient number of points on the scale, it is necessary, for a new wattmeter, and advisable for one that has been in service for some time, to determine the difference between the a-c power and the d-c power required to produce a given deflection. For this test another wattmeter must be used, of known or calculable a-c performance. This test may be made only at power factor 0.5, for wattmeters of proved construction, but for a new design the test should be made at zero power factor also.

Measurement of Three-Phase Power. The measurement of the power supplied to a three-phase load may be effected in one or more of the following ways. The connections may be made either directly or through proper instrument transformers.

Single-element Wattmeter on Three-wire Three-phase Load. The current circuit of the wattmeter is connected in series with one of the mains supplying the load, and the voltage circuit of the wattmeter is connected between the corresponding terminal of the load and the neutral. If the neutral point of the load, or of the transformers supplying the load, is not available, a "Y-box" (see below) can be used to establish an artificial neutral. If the load is perfectly balanced the power input is then three times the wattmeter reading. However, a three-phase load is seldom sufficiently well balanced, even in the case of a three-phase motor load, to render this method of measurement an accurate one.

Y-box. The simplest form of Y-box consists of two equal non-inductive resistors connected in series, each of the free ends and the junction point being connected to a binding post. Each resistor has a resistance equal to that of the voltage circuit of the wattmeter. One terminal of the voltage circuit of the wattmeter is connected to the junction terminal of the Y-box and the other terminal of the voltage circuit to the line wire in which the current circuit of the wattmeter is connected. The other two terminals of the Y-box are connected to the other two line wires, respectively.

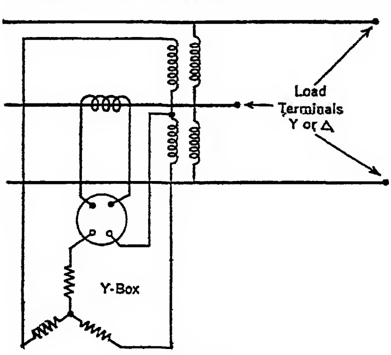

FIG. 26

In the case of wattmeters designed especially for use with a Y-box, part of the resistance of the potential circuit of the wattmeter is placed in the Y-box, being connected permanently to the junction point between the other two resistors. A similar arrangement may be used as a multiplier. The connections of such a Y-box, wattmeter, and instrument transformers are shown diagrammatically in Fig. 26.

Two-wattmeter Method for a Three-wire Three-phase Load. The simple arrangement of two wattmeters shown in Fig. 27 will give exactly the total power in any three-wire circuit, provided each wattmeter by itself gives accurate indications. Aside from the sources of error, as noted above, which may affect the accuracy of a wattmeter on a single-phase circuit, the arrangement shown in Fig. 27 will give the true power for any condition of unbalancing, wave form, frequency, etc. It is also immaterial whether the load be Y or Δ connected. The connections may be made directly as shown or through two current transformers and two voltage transformers, the connections in the latter

case being the same as in Fig. 28 except that two separate wattmeters instead of a polyphase wattmeter are used.

Rule for Adding or Subtracting Readings. With the arrangement shown in Fig. 27 the total power is always the *algebraic* sum of the readings of the two wattmeters. Since a wattmeter reads in only one direction, usually to the right, the two instruments must be so connected to the line that the needle of each instrument is deflected over the scale. For a balanced three-phase load having a power factor greater than 50 per cent, the sum of the two wattmeter readings, when the connections are thus made, gives the total power; for a power factor less than 50 per cent the difference of the two readings must be taken. When the power factor is not known, one can determine whether the sum or the difference of the readings should be taken by interchanging the two wattmeters, leaving unaltered the potential connections to the third wire (C in Fig. 27); if the pointers of the two wattmeters deflect in the same direction as before, add the two readings; if one of the pointers deflects in the opposite direction (i.e., against the stop), take the difference. In making this test, care must be exercised to connect the source side of each of the two lines (in which the current coil of each wattmeter is connected in succession) to the same binding post of the given wattmeter in each of the two positions.

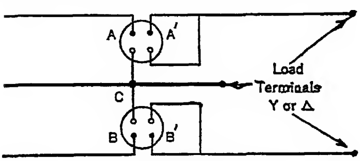

Fig. 27

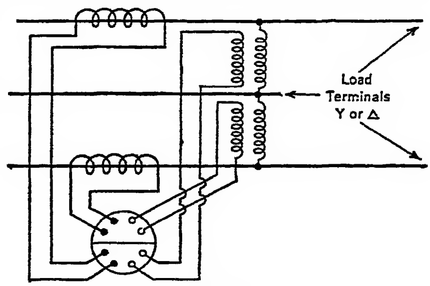

Fig. 28

When the load is balanced, and the two wattmeters are connected, as in Fig. 27, so that each gives a positive reading, the question as to whether the power factor is above or below 50 per cent can be determined by changing the potential lead of the lower-reading wattmeter from the common connection C to the line in which the current coil of the other wattmeter is connected. If the reading thus obtained is positive, the power factor is more than 50 per cent and the readings should be added; if the reading is negative the power factor is less than 50 per cent and the readings should be subtracted.

When three wattmeters are used for the measurement of a three-phase four-wire load they should be connected in a manner similar to that used for two wattmeters on a three-wire system, i.e., each of the three voltage circuits should be connected between a main wire and the neutral wire, and each of the three current circuits should be connected in series with the corresponding main wire.

Special Precautions in Use of Wattmeters. In addition to the general precautions given under the heading "Precautions in Placing and Using Instruments," the following special points must be noted. A wattmeter has three distinct limits, namely, current, voltage, and power, no one of which should be exceeded. The potential of the moving coil must not be greatly different from that of the fixed coil, and the connections must always be such as to ensure this condition, both in the use and in the checking of a wattmeter. This precaution is especially necessary when an external multiplier is used for high voltages. Unshielded wattmeters, not of astatic construction, are very susceptible to the effect of stray magnetic field because of the relatively weak operating field of the current in the current coil. In using such wattmeters, special care should be taken to locate them where this effect will be negligible. To check this point, allow current to flow in the voltage circuit only. Any deflection will then be due to stray field. In applying this test to a wattmeter with more than one current range which is compensated for the loss in its voltage circuit, the switch for changing the compensating-coil connections (as the current range is varied) must be correctly set, or an error will result. To minimize stray-field error, in wattmeters of the deflection type, it is the practice of American makers to surround the wattmeter coils with a laminated magnetic shield.

FREQUENCY METERS. Indicating frequency meters have been made in a great variety of types, of which relatively few are now used in American practice. The following may be mentioned.

In the vibrating-reed frequency meter a number of thin steel reeds, tuned to respond to a closely graded series of frequencies, are mounted so that their free ends are aligned

with a scale marked in cycles per second. An electromagnet is arranged to act upon the reeds, either directly or by vibrating a common frame supporting all the reeds. The electromagnet is so designed that the normal line voltage will produce such moderate forces on the reeds that only those reeds with frequencies very close to the line frequency will respond. The ends of the reeds are provided with white flags which appear to widen as the amplitude of vibration increases. Frequency meters of this type cannot be read as closely as those of the pointer type, but they have some advantages which make them valuable, especially for use in the laboratory. These are as follows: The indications being dependent on the fundamental frequency, the effect of distortion of the voltage wave is very small. The indications are not influenced by moderate changes of voltage; nevertheless, extreme changes must be avoided. The indications are only very slightly dependent on the temperature of the reeds, the change in frequency being 0.01 per cent per degree centigrade. An increase of temperature reduces the normal frequency of the reeds, that is, causes the indicated frequency to be too high. Vibrating-reed frequency meters must be protected from intense mechanical vibrations.

In American practice the vibrating-reed frequency meter has been restricted to laboratory uses. For use on switchboards some form of indicating frequency meter has always been preferred. Frequency meters of the induction type (Westinghouse), the resistance-reactance type (General Electric), and the bridge type (Weston) have been superseded in recent years by modern types capable of the greater sensitivity and accuracy demanded by present-day practices, including system interconnection and the operation of synchronous-motor clocks.

The **tuned-circuit frequency meter** resembles the earlier pointer-type frequency meters but may be made much more sensitive and much less susceptible to error from variation of wave form because the tuned circuits act as filters to pass only the fundamental. The extreme sensitiveness obtainable is shown by the fact that switchboard frequency meters are made by the General Electric Co. in which the entire length of the scale corresponds to the range 59.5-60.5 cycles. Even narrower ranges would be possible under this principle of construction, the practical limitations arising from the temperature coefficient and other minor changes in the available capacitors. The **General Electric frequency meter** has a pair of coaxial fixed coils which are composed of two equal interwound windings of opposite polarity. There is a single moving winding which carries the sum of the currents in the two fixed windings. The torque depends upon the product of the vector sum of the two currents by their vector difference, and changes greatly for a small change in the relative value of the two currents. To make the instrument indicate frequency, a control is necessary; and to make the indications independent of the line voltage this control must vary as the square of the line voltage. Such a control is realized by the addition to the moving element of a nickel-iron-alloy vane which tends to align itself with the field of the fixed coils.

The **Westinghouse resonance-type frequency meter** (Fig. 29) consists of a pair of stationary field coils within which rotate two moving coils fixed on a common shaft at right angles with each other. A reactor 8 is connected in series with the fixed coils, and a capacitor 10 is shunted across the fixed coils and the moving coil 3. The relation between capacitance and inductance is such that parallel-circuit resonance occurs at the normal frequency. The moving coil 2 is connected in series with the parallel-resonance circuit. There is no mechanical control of the moving system, which assumes a position depending on the relation between the forces set up by the currents in the coils. The sensitivity to frequency changes is such that a range of 58 to 62 cycles is obtainable for use on 60-cycle systems. The resonant circuit employed makes the instrument very insensitive to temperature changes and keeps the burden down to a low figure, namely, 3.2 volt-amperes at 110 volts, 60 cycles.

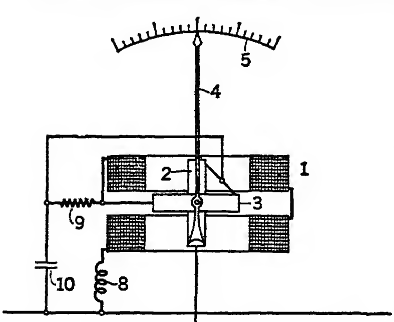

FIG. 29

The **Weston frequency meter** is of the moving-iron type. Two fixed coils mounted at right angles to each other act upon a single iron needle carried by a shaft which also carries the pointer and a damping vane. There is no spring or other mechanical control. The connections are shown in Fig. 30. Current from the upper line wire passes through the reactor X_1, resistor R, and field coils 1; at the junction point P_1 the current divides

into two components mutually displaced in phase, namely, a lagging component through field coils 2 and reactor X, and a leading component through capacitor C. At the junction point P_2 these components unite and flow through reactor X_2 to the lower line wire.

The undivided current in coils 1 produces a magnetic field tending to displace the needle from the central position in which it is shown and which corresponds to the normal operating frequency. The lagging component of current flowing through coils 2 tends to hold the needle in its central position. At normal frequency the effect of the fundamental of this current is very great because the circuit 2, X, C becomes resonant. For lower frequencies the lagging component of the current in coils 1 preponderates because of the decrease in the reactance of X, and the increase in the reactance of C, and the needle is deflected to the left. The converse effect takes place when the frequency exceeds the normal value. The reactors X_1, X_2, are used only when the wave form of the line voltage is very much distorted. For ordinary commercial wave forms the action of the resonant circuit in responding selectively to the fundamental makes the accuracy of indication independent of the wave form.

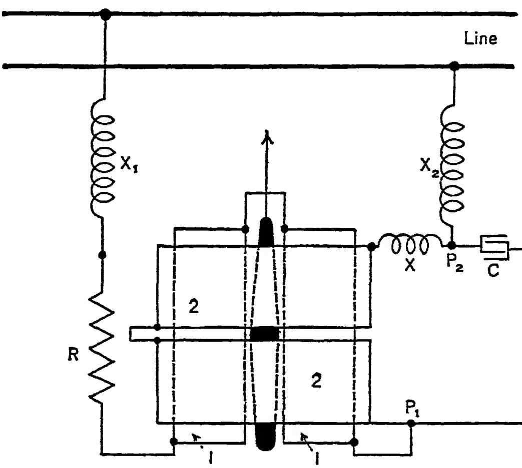

Fig. 30

The **Leeds & Northrup frequency recorder** records and indicates the frequency. It contains a bridge network connected to the line voltage. Two arms are resistors, a third is a capacitor shunted by a resistor, and the fourth is a capacitor in series with a resistor. Sliding contacts, actuated by the auxiliary motor of the recorder in accordance with the position of the galvanometer pointer, move with a change of frequency until a balance of the bridge is re-established. This instrument is very sensitive to changes of frequency and very insensitive to changes of temperature and line voltage. In one 60-cycle recorder the pen traverses the 10-in. chart for the change from 58 to 62 cycles, and in another for the change from 59 to 61 cycles.

POWER-FACTOR METERS. The phase relation between the current and the voltage in a circuit or a polyphase system can be measured by indicating instruments of a type in which the mechanical angle between the instrument pointer and a reference mark is equal to the electrical phase angle between current and voltage. If the scale is graduated in equal angular intervals the instrument is called a **phase meter.** If the scale is graduated according to the cosine of the phase angle, the instrument is called a **power-factor meter.** If the graduations are marked according to the sine of the phase angle it is called a **reactive-factor meter.**

Strictly speaking, the power factor as defined for single-phase systems is the quotient obtained by dividing the power by the product of the effective values of the voltage and the current. The power factor may therefore be computed directly from the readings of a wattmeter, a voltmeter, and an ammeter, but if the wave forms are not sinusoidal the power factor will not be measured exactly by an instrument whose indication depends only on phase displacement.

For a polyphase system the power factor has been defined by arbitrary convention as the ratio of the total power to the total vector volt-amperes, the total vector volt-amperes, in turn, being defined as the square root of the sum of the squares of the total active power and the total reactive power. For sine waves this definition is consistent with that for the single-phase case, but for non-sinusoidal wave forms the polyphase definition is theoretically inconsistent with any logical generalization of the single-phase definition.

Power-factor meters of the indicating type depend for their operation on the interaction of a pulsating single-phase magnetic field, produced by a single coil, and a rotating magnetic field produced by two or more other coils, which are so placed that the mechan-

ical (space) angle between their axes is equal to the electrical (time-phase) angle between the currents in them. The actual moving element may be either the single coil or the two or more other coils mentioned, or an iron vane magnetized by the single coil. In any event, the moving element will take up a position such that at the instant the pulsating field has its maximum value the rotating field has the same direction as that of the pulsating field.

In all types of these instruments the moving element is intended to be free from all mechanical constraint and hence on open circuit it will point indifferently in any direction. Because of the relatively weak electrical restoring torque, the indication becomes definite only when the product of the applied current and voltage is at least half the normal value.

In instruments for use on single-phase circuits the rotating magnetic field is obtained (usually from the voltage circuit) by supplying one coil through a non-inductive resistance while the other is in series with a highly inductive iron-cored reactance coil. When used on polyphase circuits the rotating field can be obtained more easily (usually from the voltage circuits) by using a non-inductive resistor in series with each of the two or three coils spaced 120 deg apart which produce the rotating field. If a single current coil connected in series with one line is used, the instrument reading will depend only on the phase displacement between that current and the corresponding star voltage, and will correctly represent the entire polyphase load only if this is balanced. An instrument having three such elements will correctly indicate the polyphase power factor even on an unbalanced load if the wave form is sinusoidal.

Some instruments, especially those of the magnetic-vane type, are provided with a scale covering the complete 360 deg and serve to indicate the direction of flow of power as well as the phase relation of current to voltage.

REACTIVE-POWER METERS. These instruments, also called reactive-volt-ampere meters or reactive-component meters, are essentially wattmeters so connected in an a-c line as to make the voltage across the voltage circuit (or circuits) of the instrument 90 deg out of phase with the voltage which would be applied in the case of power measurement. If a polyphase reactive-power meter is connected in circuit so that the shift of phase is obtained by suitable connections to the secondary terminals of ordinary voltage transformers, the indication in watts which the instrument would give if connected as wattmeter must be multiplied by a suitable factor (which depends on the connection used) to obtain the reactive power in "vars." (At the meeting of the International Electrotechnical Commission in Oslo in 1930 it was voted to adopt "the name or term var for designating the unit of reactive power"; i.e., 1 var results from 1 volt acting in quadrature with 1 amp). If special voltage transformers, or if auto-transformers connected to the secondary of the voltage transformers, are used, the correction factor can be automatically allowed for and the meter will read directly in vars.

Any reactive-power measuring device which depends on cross-connection of the voltage circuits will read incorrectly if both the voltages and the currents are unbalanced. Those in which the number of current coils is less by two than the number of wires in the circuit will be incorrect on unbalanced currents, even if the voltages are symmetrical.

SYNCHROSCOPES. A synchroscope is essentially a robust form of phase meter in which the pointer is free to rotate. It is provided with a single scale mark which denotes synchronism, that is, equality of frequency and coincidence of phase of two a-c sources. In use, one of the two windings is connected to the bus, the other to a machine which is to be connected to the bus. Synchroscopes have superseded, in great measure, the various forms of synchronizing devices employing lamps.

In the General Electric synchroscope two split-phase windings are located on an iron core arranged to rotate. The windings are connected through slip rings to the incoming machine, and the phase displacement is obtained by a reactor in one winding and a resistor in the other. A stator surrounding the rotor has its windings connected to the bus. When exact synchronism is reached, a pointer attached to the rotor shaft remains at rest in a vertical position. The synchroscope is usually connected through voltage transformers; on polyphase circuits only one phase is used.

In the Westinghouse synchroscope an iron vane, free to rotate, is magnetized by a fixed coil connected to the incoming machine. The vane is located within a rotating field produced by current from the bus passing through two coils which are fixed at approximately right angles with each other. A reactor in series with one of the coils and a resistor in series with the other produce a phase displacement between the two currents. The pointer attached to the rotating vane indicates at any moment the phase displacement between the voltages of the bus and the incoming machine, and the speed and direction of its rotation depend on the difference between the two frequencies.

The Weston synchroscope consists of an instrument mechanism similar to that of an electrodynamic voltmeter. The fixed winding is connected to the bus through a

resistor, and the moving coil is connected to the incoming machine through a capacitor. When the voltage of the incoming machine is either in phase with or in opposition to the bus voltage no torque is exerted on the moving coil and the pointer stands in its central (vertical) position. For any other phase relation the pointer will be deflected to the right or to the left, when the incoming machine is ahead or behind, respectively. The pointer is behind a translucent screen, and only its shadow, cast by a synchronizing lamp, can be seen. This lamp is operated by a double-primary transformer connected for synchronizing "light." When the shadow of the pointer is in the vertical position, the incoming machine is in phase. The pointer will be vertical for phase opposition also, but cannot be seen then because for this condition the synchronizing lamp is dark. When the incoming machine is too fast or too slow a succession of shadows of the pointer will be seen, and the shadow will appear to rotate in the clockwise or counter-clockwise direction, respectively.

INSTRUMENTS FOR MEASURING RECTIFIED CURRENTS. A rectified current will not give identical indications when measured with ammeters of various types. A rectified current may be regarded as equivalent to the superposition of a direct current upon an alternating current. The proper type of ammeter to use depends entirely upon the function to be performed by the rectified current. For charging storage batteries or for electroplating, the desired result is proportional to the d-c component of the rectified current, and a permanent-magnet moving-coil instrument should be used because it responds only to this component. For rough measurements the less expensive polarized-vane instrument may be used if it has been calibrated for use on the particular wave form. If the rectified current is used for incandescent lamps or for electric heating devices, its effective (rms) value is significant, and ammeters of either the electrodynamic type or the electrothermic type will theoretically give the correct value for the given purpose. Electrodynamic and electrothermic ammeters, however, are not generally available, which makes it desirable to measure the effective value of rectified currents with a moving-iron ammeter of good quality if an accuracy of, say, 1 or 2 per cent is sufficient. If the wave form of the rectified current is very much distorted in comparison with a half-cycle of a sinusoidal current, or if doubt exists as to the quality of the design of the moving-iron ammeter and the magnetic material used in it, the ammeter should be checked on the given current against an electrodynamic or electrothermic ammeter.

When the rectified current is used to operate an arc lamp and is to be adjusted to produce the same light output as would be given by a stated direct current, the permanent-magnet moving-coil ammeter gives approximately correct results and is the only kind to be used. However, its indications will usually be very erroneous as regards the heating effect of the current in the carbons, the rheostats, cables, etc., and if overheating is to be avoided, a moving-iron ammeter should also be in circuit. This precaution is needed also in using rectified currents for battery charging and electroplating.

For a full description of the measurement of rectified currents for various purposes, see paper by W. N. Goodwin, Jr. (Bibliography).

The choice of a voltmeter for voltage measurements with rectified currents is based on the preceding considerations for the ammeter. For battery charging or electroplating, a permanent-magnet moving-coil voltmeter should be used; for regulating the voltage applied to incandescent lamps, an electrodynamic, electrothermic, or (with reservations as for the ammeter) a moving-iron voltmeter; the product of the effective voltage, so measured, by the effective current will give the watts only if the circuit is non-reactive. If reactance is present the power must be measured with a wattmeter.

The a-c component of a rectified current may be measured by a "selective ammeter" if the d-c component is 5 times the a-c component, or less. This instrument is of the iron-core electrodynamic type and has a self-contained current transformer with an air gap in its magnetic circuit.

OHMMETERS. The name "ohmmeter" is applied to two radically different types of portable instrument for the rapid and simple measurement of resistance with moderate accuracy. One of these is a simplified Wheatstone bridge and the other is essentially a voltmeter with an ohm scale. The Roller-Smith ohmmeters are slide-wire bridges. They are built in three forms. Self-contained batteries are the source of current in each case. In two forms a built-in galvanometer is the only detector provided. In the third form either a galvanometer or a telephone receiver may be used as the detector, and the battery may be used either as a d-c source or to operate an induction coil, the primary of which may be used as an a-c source. This latter ohmmeter serves for the usual resistance measurements; for the measurement of inductance, capacitance, and resistivity of electrolytes; and also for fault location.

In the **Leeds & Northrup d-c ohmmeter** the slide wire is mounted within the instrument on the edge of a circular disk which may be rotated by means of a handle above the

panel. A d-c galvanometer and a battery are mounted within the case. The **Leeds & Northrup a-c ohmmeter** closely resembles the d-c ohmmeter but has an a-c galvanometer and an insulating transformer. In the ordinary applications of these ohmmeters any one of five built-in standard resistors may be used, according to the magnitude of the resistance under measurement. Provision is also made for an external standard, such as a copper coil, against which nominally like coils are to be checked. In this case the scale reads the ratio of the resistance of the unknown coil to that of the known coil.

Permanent-magnet moving-coil ohmmeters are made in various types which differ somewhat radically in principle of operation, including: volt-ohmmeters, ohmmeters with differentially wound moving coil, and quotient-type ohmmeters.

Volt-ohmmeters differ from ordinary voltmeters mainly in having an extra scale figured in ohms. It is usual to have the full-scale value of voltage coincide with the zero of the ohm scale. In use, a battery of such voltage as to give full-scale deflection is connected to the instrument; if the unknown resistance be now introduced into the circuit, the pointer will drop back to a position corresponding to the reduced value of the current, when the resistance may be read off directly from the ohm scale. As it is obviously difficult to ensure that an available battery will produce exactly full-scale deflection, the more accurate instruments of this type, intended only for use as ohmmeters, have an adjustment, usually a magnetic shunt, by which the pointer may be brought to the full-scale mark in spite of normal variation of the battery voltage.

Any **d-c voltmeter may be used as an ohmmeter** in connection with a constant-voltage source. If the deflection is d_1 with the voltmeter connected directly to the source, and d_2 with the unknown resistance x in series with the voltmeter, the unknown resistance is given by the relation

$$x = \left(\frac{d_1}{d_2} - 1\right) r \tag{11}$$

where r is the resistance of the voltmeter.

Ohmmeters with differentially wound moving coil are exemplified by the Weston Model 1 direct-reading ohmmeter. This permanent-magnet moving-coil instrument has two windings in the moving coil, the effect of the current in one winding being to deflect the pointer up scale and that of the other to deflect it down scale. Suitable resistances in the instrument, controlled by a plug switch, provide for three ranges, for example, 0-200, 0-1000, and 1000-2000 ohms. To take care of variations in battery voltage, a magnetic shunt is adjusted to bring the pointer to the full-scale mark with no connection between the X posts and with the plug in the checking position.

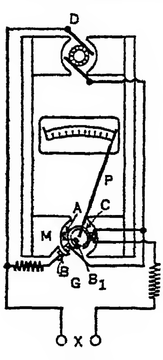

Fig. 31

The **quotient-type ohmmeter**, in principle, is independent of the value of the battery voltage because its indication depends upon the ratio of two oppositely directed turning moments produced by currents flowing in two coils. One of these coils is in a circuit of fixed resistance; the other contains the resistance under measurement. A change of the voltage changes both the currents but does not alter their ratio. The **Evershed megger** is an ohmmeter of the quotient type, intended principally for measuring relatively high resistances in megohms, whence the name. Its construction is indicated diagrammatically in Fig. 31 in which D is a small hand-driven d-c magneto and G a special form of moving-coil instrument. One pair of bar magnets supplies the flux for both. The galvanometer contains three coils rigidly attached at a fixed angular distance apart to the shaft which carries the pointer P. The unknown resistance is connected at X.

The coils BB_1, called the pressure coils, are permanently connected through a resistance across the terminals of D. The current coil A is made to move through an annular gap in such a manner that the field in which it moves is uniform, whereas the pressure coils BB_1 move from a position midway between the poles, where the field is at a minimum, into a stronger field, the connections being such that the torque due to the current in the current coil is opposed by the torque due to the current in the pressure coil. With no current in the current coil, that is to say, when the resistance to be measured is infinite, current through the pressure coils will cause them to come to rest with their plane at right angles to the magnetic field. When the current through the current coil is increased by putting in lower resistances, the current coil drags the moving system round in a clockwise direction; since the pressure coils come into a stronger and stronger field, the resistance to this motion becomes greater and greater. Hence a definite position is assumed by the system for the particular resistance at X. An increase in voltage would

increase the current in both current and pressure coils in the same proportion; consequently the instrument is independent of the voltage of the generator.

In the less-expensive meggers the generator gives a variable voltage depending upon the speed of rotation of the handle. In higher-priced meggers the generator is driven through a slipping clutch to give a constant voltage for a considerable range in the speed of driving. The voltage at which an insulation test is made is often of some importance. Constant-voltage meggers are especially useful for testing insulation resistance associated with a large capacitance, where a varying test voltage would cause charging and discharging currents to flow with resulting fluctuations in the readings.

Meggers are made with generators giving a maximum of 2500 volts and with a maximum measurement range of 4 to 10,000 megohms. The successful operation of these instruments to measure such high values of resistance is possible because the Price guard-wire scheme is used to prevent leakage of current across and through insulating supports. Meggers may be checked for accuracy, using known resistances having only a small fraction of the full-scale value; see Bibliography, Ohmmeters, paper by H. B. Brooks.

The most recent form of megger is of particular interest to instrument designers and users because its small size and light weight (3 lb) are obtained by the use of modern magnetic alloy for the permanent magnets of the 500-volt generator and of the ohmmeter.

MICROFARADMETERS. Direct-reading pointer instruments for capacitance measurements are made by a number of manufacturers under the names of condenser meters, capacitance meters, and microfaradmeters. They are usually made to be operated by alternating current from ordinary lighting circuits. For details as to their operating principles and internal circuits, see reference under Microfaradmeters in Bibliography.

FLUXMETERS. The Grassot fluxmeter is essentially a moving-coil galvanometer, the suspension fiber of which exerts a negligible torque on the moving system. The motion of the coil is very heavily overdamped. Fig. 32 is a diagram showing its construction. The coil *C* swings in a uniform field in the air gap between the pole pieces *NS* of a permanent magnet and the soft-iron core *A*. The system is supported at the top by a single cocoon fiber, the upper end of this fiber being attached to a flat spiral spring *R*, to minimize the effect of shocks. The torsional force exerted by such a fiber is extremely small. S and S_1 are very thin silver strips, serving to lead the current to and from the coil *C*. These strips are in the form of springs, which, however, are extremely weak and exert the minimum practicable torque on the coil. An index attached to the coil swings over a calibrated scale. The fluxmeter is also provided with a mirror in addition to the pointer so that it may be used in conjunction with a lamp and scale or with a telescope and scale.

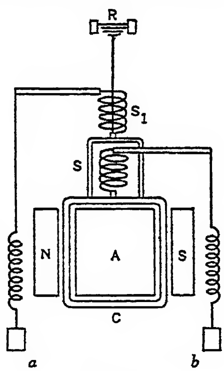

Fig. 32

When a given quantity of electricity is discharged through the moving coil, for example, by changing the flux threading an exploring coil connected to the terminals *a* and *b*, a force is exerted upon the coil tending to deflect it. The only opposing forces (neglecting the small torsional forces) are the mechanical friction to motion and the back emf induced in the coil by its motion through the field of the permanent magnet. The back emf is proportional at any instant to the velocity of the coil, and the frictional force is also approximately proportional to this velocity. If both forces are directly proportional to the velocity, when a given quantity of electricity is discharged through the circuit, the coil comes to rest at a definite point, depending only upon its initial or "zero" position and the total quantity discharged through it. As the quantity of electricity discharged through an exploring coil, when the magnetic flux threading it is changed, is proportional to the change in flux, the instrument can be calibrated to read directly the change in the flux produced in the region occupied by the coil. In practice the friction is not exactly proportional to the velocity, and the fiber and leading-in strips usually exert an appreciable force on the coil. The instrument therefore has not proved altogether satisfactory.

In motor or in dynamo work an exploring coil, consisting of one or more turns of wire, may be fixed or wound in position and the change in flux of induction observed on exciting the field magnet. Even in the largest work, where some minutes may be taken to reach the limit of magnetization, no serious error is thus introduced. The fluxmeter can also be used for the measurement of magnetic field strength, pole strength, and the distribution of magnetism in bar magnets. It is also adapted to the measurement of permeability and hysteresis.

PROTECTION OF INSTRUMENT MECHANISM FROM STRAY MAGNETIC FIELD. Because of the low reluctance of iron as compared with that of air, a soft-iron shield placed around the instrument mechanism will greatly reduce the disturbing effect of an external magnetic field on the reading of an instrument. Many types of switchboard and portable instruments are thus shielded. Permanent-magnet moving-coil instruments are not seriously affected except by relatively strong external fields, and ordinarily do not need shielding other than that afforded by the iron case used with switchboard instruments.

The modern high-permeability nickel-iron alloys, such as permalloy C and mumetal, are very suitable for use as magnetic shields. Using them, a given degree of shielding ratio can be obtained with very much thinner and lighter shields than when silicon steel is used.

PRECAUTIONS IN PLACING AND USING INSTRUMENTS. The weight of the moving element of an electrical instrument or meter is carried by a steel pivot resting on a jewel bearing, and the area of the surfaces in contact is so minute that the pressure is very great. The operating forces are very small. Accidental shocks which would not damage a steam gage or a gas meter may blunt the pivot and destroy the usefulness of an electrical instrument. If portable instruments are to be used where vibration is unavoidably present, they should be placed on pads of folded cloth, felt, or similar material. **Do not hammer on a table or bench on which instruments are placed.** Switchboard instruments should preferably not be put in place until all work on or near the boards has been finished.

Instruments to be transported by automobile should preferably be packed in excelsior in suitable boxes. If this is not done, they should be placed on the seat cushions, upside down or on edge, in order to minimize the liability of damage to the lower pivot and its jewel. Instruments to be shipped by rail should be upside down when the packing case is right side up.

Tapping an instrument while it is being read is usually unnecessary if it is properly constructed and in good order, but when slight friction is present its effect can be eliminated by gently tapping the instrument. Hard tapping defeats its object, and may cause damage to pivots and jewels.

Portable instruments are nearly always calibrated with the scale in a horizontal plane. For the most accurate results they should be used in this position. The moving system should be mechanically well balanced so that no appreciable error will be introduced if the instrument is not exactly horizontal. New instruments should be examined for balance by tipping them about 10 deg from the horizontal, in various directions. If the zero reading varies more than a few tenths of a division during this test, the moving system is not well balanced. Tipping an instrument by too great an angle (say 45 deg or more) may give misleading results by causing the point of contact of pivot and jewel to shift, thereby changing the axis of rotation.

Avoid wiping the glass cover over the pointer of an instrument before taking readings because the resulting electrostatic charge on the glass will attract the pointer. Very large errors from this cause are possible with some types of instrument. If for any reason the cover glass has recently been rubbed, breathe on it to dispel the charge.

Wires attached to instruments should preferably extend back over the table and away from the observers. When such wires are draped over the front of the table they are sometimes accidentally caught by the observer and the instrument is thrown to the floor. If it is impossible to avoid having the wires placed in this way, they should be clamped to the table near the instrument.

Instruments should be kept at a distance from motors, generators, transformers, or cables carrying large currents, in order to avoid error caused by **stray magnetic field.** Unshielded instruments should not be placed close together; for example, a particular type of unshielded d-c instrument, widely used, will be affected by about 1 per cent by a similar instrument placed alongside it, and about 2 per cent if the instruments are in contact with their magnetic poles facing each other. This disturbance occurs regardless of whether the second instrument is in circuit.

Instruments should not be used in **extreme temperatures,** if this can be avoided. Even though no permanent injury may result, the indications of all instruments change to some extent with temperature. This effect is very small in d-c voltmeters and in moving-iron ammeters, but may be large in d-c ammeters and millivoltmeters.

In any current-carrying part of an instrument the heating increases at least as the square of the current. Great care should be used to **avoid passing excessive currents** through instruments. All set-ups should be checked over carefully before the current is turned on, to see that no instrument will be overloaded. In using **multiple-range instruments** with unknown or doubtful values of current or voltage, always begin with

the highest range. Special care is necessary in the use of **millivoltmeters**, which will usually be damaged by connection to a single dry cell. Wattmeters have three operating limits, namely, current, voltage, and power, of which only the last named is shown by the position of the pointer. The fact that the current or the voltage is excessive, in cases where the power is relatively small, can reveal itself only by smoke or other sign of thermal damage. In using a **millivoltmeter with shunts, always attach both of its leads to the shunt** before connecting the other ends to the millivoltmeter, and in disconnecting **take the leads off the millivoltmeter first.** When the reverse procedure is followed, and only one end of one of the leads remains to be connected to the shunt, an accidental contact between this end and a switch blade or other live part, or a grounded metal object, may burn out the millivoltmeter instantly.

In connecting instruments into circuit, see that all **contact surfaces** of wires, lugs, binding posts, etc., are clean and that connections are firmly made. If the test is to last for a number of hours, it is advisable to provide **switches for bypassing** the current around ammeters and the current coils of wattmeters. (Modern watthour meters are made to carry three times rated current.) The switches are to be opened when readings are to be taken. Switches may be provided also to disconnect a-c voltmeters and the voltage circuits of wattmeters, if these instruments do not have keys. The power required by d-c voltmeters is so small that this precaution is unnecessary for them.

If an ammeter must be connected directly in a line, it is advisable to put it in the **grounded wire**, if available, to protect the operator and the instrument.

If an instrument has been dropped, overloaded, or exposed to abnormal temperatures, it should be checked, say by comparing it with a similar instrument not so exposed, before continuing to rely on its indications.

A heavy short circuit in nearby conductors, even though of extremely short duration, is likely to partially demagnetize the magnet of a d-c instrument. After such an exposure the instrument should be rechecked, even though it has sustained no visible injury.

Repairs to instruments, if at all extensive, are usually best made at the factory. Electrical instruments are so delicate that repairs elsewhere should not be attempted unless equipment and some experience are available. Instruments should be opened only in a clean dry place free from lint, dust, metal filings, etc. Direct-current instruments have very strong permanent magnets and very small clearances for the moving coil, and should never be opened on a workbench where iron filings may be present.

Small displacements (say a few tenths of a division) of the pointer from the zero mark, when no current is flowing through the instrument, may be corrected by turning the zero adjuster. Large displacements (several divisions) may indicate bending of the pointer or other derangement, and the consequent need for repairs. The use of the zero corrector to bring a bent pointer back to zero usually introduces appreciable errors.

The heads of binding posts should be screwed down before instruments are put away. Instruments are sometimes picked up by the binding posts, and this always unsafe procedure reaches a maximum of insecurity when the binding-post heads happen to be loose.

DETAILS OF DESIGN AND STRUCTURE OF ELECTRICAL INSTRUMENTS. Space limitations permit only references to sources of information on the numerical design of the windings, springs, magnets, damping mechanisms, etc., of electrical instruments. The more important ones are Drysdale and Jolley's books on Electrical Measuring Instruments, Edgcumbe and Ockenden's book on Industrial Electrical Measuring Instruments, Keinath's book, Die Technik der elektrischen Messgeräte, and the technical journal *Archiv für technisches Messen*. The *Journal of Scientific Instruments* (London) publishes, from time to time, numerical data on the construction and functioning of electrical measuring instruments.

12. ELECTRICAL INTEGRATING METERS

Amperehour Meters

Amperehour or quantity (coulomb) meters may be classified as electrolytic meters and motor meters. They are used in this country chiefly for storage-battery work. Amperehour meters of small current rating are used abroad as substitutes for watthour meters, and are calibrated to read directly in kilowatthours at some assumed voltage.

ELECTROLYTIC AMPEREHOUR METERS. This type of meter is used extensively in Europe, but is rarely (if ever) used in this country. The meter generally consists of a glass vessel or U-tube containing an electrolyte, such as water, or an aqueous solution of a salt of mercury or of caustic soda. The solution is decomposed by the current, and since the rate of decomposition is proportional to the current, the deposit of metal or the quantity of gas evolved can be used to measure the electrical quantity

in amperehours. In some types the entire current to be measured is passed through the electrolyte, and in others the bulk of the main current passes through a shunt and only a very small part through the electrolyte.

MOTOR AMPEREHOUR METERS. The essential parts of the motor amperehour meter are the motor element, the brake or speed-regulating device, and the register. The motor consists of an armature rotating in the field of a permanent magnet. Usually only a shunted portion of the current flows through the armature. Motor amperehour meters are intended to be cheaper and if possible more rugged than the ordinary watthour meter. The absence of a constantly excited voltage circuit in the meter avoids the constant loss of energy in such a circuit.

COMMUTATOR AMPEREHOUR METER. The commutator amperehour meter in its simplest form usually consists of one or two permanent magnets which provide a driving field within which rotates an armature with three coils and a three-part commutator. In some designs the armature coils are mounted flat on an aluminum disk which in addition to supporting the coils produces the retarding action in connection with the field magnet. In another design there is a stationary iron core between the poles of the field magnet and a drum armature rotating in the gap between this iron core and the pole faces of the field magnet. No braking device is used in this latter type, and the motor runs at a speed proportional to the impressed emf, taking only sufficient power to overcome the friction in the bearings and in the gear train.

Amperehour meters of the commutator type are gradually passing out of use because of their inferior accuracy at light loads and the difficulty of adequately insulating the windings from the framework of the meter.

MERCURY-MOTOR AMPEREHOUR METER. In the mercury-motor amperehour meter, current is carried to the armature by mercury, which takes the place of the brushes and commutator of the ordinary d-c motor. One form consists of a copper disk armature mounted in jeweled bearings and immersed in mercury. The current is led into and out of the mercury chamber through suitable terminals, and since mercury has about 55 times the resistivity of copper, the greater part of the current goes diametrically across the disk. A device used to compensate for the increase of fluid friction with increase of speed takes some such form as an auxiliary coil, in series with the motor element, which produces a flux which augments the flux cutting the driving disk.

SPECIAL FEATURES OF AMPEREHOUR METERS. Amperehour meters used for showing the state of charge of storage batteries are sometimes provided with special features. For example, a "compensation for overcharge" causes the meter to run more slowly on a given value of charging current than on the same value of current during discharge. The percentage difference in the two cases is chosen to provide for the excess that must be put into the battery because it is not 100 per cent efficient. This is accomplished by a "variable resistor element" in parallel with the main mercury chamber. A pivoted copper bar in this element is immersed in mercury, and is cut by some of the flux from the driving magnets. It will thus change its position with the reversal of current direction, and will vary the resistance of the element. The "compensation for light-load accuracy and internal battery losses" consists of a thermocouple heated by a potential winding and delivering a small current continually to the motor element in the direction corresponding to battery discharge. This action may be adjusted to cause the meter to record the loss of charge when the battery is not in use. The "thermal shunt method" of compensation for high discharge rates consists in making the shunt to the motor element of iron or other material of high temperature coefficient, and so designing it that it will heat up at the heavier loads. This causes the amperehour meter to have a higher percentage registration as the load increases.

Amperehour meters are sometimes provided with contacts which actuate auxiliary devices, such as circuit breakers, bells, or other signaling devices. This feature is used in connection with battery charging and electroplating.

Watthour Meters

GENERAL. A watthour meter consists essentially of (1) a small electric motor, which may be either of the commutator type, mercury and disk type, or induction type; (2) a brake system composed of a disk of non-magnetic conducting material (usually aluminum) mounted on a spindle and arranged to rotate between the poles of one or more permanent magnets; and (3) a register indicating a number proportional to the number of revolutions of the disk. One winding, called the voltage coil, is connected in shunt with the load; and the other winding, called the current coil, is connected in series with the load, the connections being the same as for an indicating wattmeter. These connections may be made either directly or through suitable instrument transformers.

PRINCIPLE OF OPERATION OF WATTHOUR METERS. Watthour meters are so constructed that the average torque exerted by the motor is proportional to the average power in the load. The brake system is so designed that the opposing torque, due to the eddy currents induced in the disk, is proportional to the speed. When the speed is steady the driving torque must be equal to the opposing torque, and therefore proportional to the speed. Hence the speed of the disk is proportional to the average power in the load, and the total number of revolutions which the disk makes during any interval must be proportional to the total energy which has passed through the meter during this interval, whether the power remains constant or varies. To determine the energy input to the load in watthours or kilowatthours during a given interval it is therefore only necessary to take the difference between the dial readings at the beginning and end of the interval, and to multiply by the proper constant if the meter is not direct reading.

SOURCES OF ERROR IN WATTHOUR METERS. In a d-c watthour meter the chief sources of error are the somewhat variable friction of the brushes, friction of the bearings and register, and air friction. In a-c watthour meters, the lack of exact 90-deg phase relation between the impressed voltage and the magnetic flux due to the current in the voltage coil may cause additional errors which vary with the frequency and power factor and also with the distortion of the wave form; in well-designed modern meters these errors are practically negligible. Instrument transformers introduce additional sources of error which may be undesirably large if the transformers are not properly designed or if the burden is excessive.

COMMUTATOR TYPE OF WATTHOUR METER. The commutator type of meter, used only on d-c circuits, consists of a set of stationary field coils which carry the load current, and an armature wound with fine wire which is connected in shunt with the supply circuit. A resistor is usually connected in series with the armature, and the ir drop in it and in the armature itself is almost exactly equal to the line voltage at any practicable speed of revolution. In other words, the counter emf of the armature is too small to have any appreciable effect on the speed, which explains why an increase in the current in the fixed (field) windings increases the speed instead of reducing it as in ordinary shunt motors. The connection to the armature is by a commutator and brushes, as in an ordinary d-c commutator motor. Special designs are sometimes adopted for very large capacities, such as double armatures astatically arranged, and damping magnets inclosed in a laminated iron shield in order to reduce stray-field errors where heavy currents are used and heavy short circuits may occur. A four-pole construction with a single armature is also used for the same purpose.

Compensation of Commutator-type Watthour Meters for Friction. Friction in a well-designed d-c watthour meter should have a noticeable effect only on light loads. To compensate for it, a "light-load adjusting coil" is added, which is an auxiliary or compounding coil connected in series with the armature. It is placed adjacent to the field coils so that its field strengthens the main field and produces a slight torque independent of the load and just sufficient to compensate for friction.

Details of Construction, Bearings, etc. The design of the modern d-c commutator watthour meter includes close electromagnetic coupling between the armature and field coils, a light-weight armature, a commutator of very small diameter, and brushes having very small friction but able to stand the wear and carry the current without undue sparking and pitting of the contact parts. The lower bearing, which carries all the weight, usually consists of a steel pivot supported by a cupped and polished jewel, usually of sapphire. In some meters, particularly in high-capacity ones with astatic armatures and a correspondingly heavy weight, diamond jewels are used.

Current Ratings of Commutator Watthour Meters. Commutator-type d-c meters are built with current ratings ranging from 5 to 10,000 amp, 100 to 600 volts. Meters of this type are furnished with double current circuits for three-wire service, the maximum ampere capacity for three-wire meters being about 6000 amp. Special meters have been supplied for 1200- and 2400-volt railway circuits.

Accuracy of Commutator-type D-c Watthour Meters. Tests on d-c commutator meters show that a meter of good design should start on 2 per cent of full load and should give accuracies about as follows: to within $3^1/_2$ per cent from 5 per cent to $^1/_4$ load, 2 per cent from $^1/_4$ to full load. The Code for Electricity Meters (see Bibliography) requires d-c watthour meters to run continuously on normal voltage and 2 per cent of rated current, and for higher loads to be accurate within the following limits:

Per cent rated current.........	5	10	20	50	100	150
Maximum deviation, per cent...	6.0	3.0	2.0	2.0	2.0	2.0

For a summary of the meter accuracies required by commission regulations, city

ordinances, and state statutes, see Bureau of Standards Circular 56, Standards for Electric Service.

MERCURY-MOTOR WATTHOUR METER. The mercury-motor d-c watthour meter has a copper disk armature submerged in mercury. The line current enters and leaves the mercury through two electrodes diametrically apart. Since mercury has about 55 times the resistivity of copper, the major part of the current traverses the disk. A laminated iron electromagnet having many turns of fine wire is connected across the line, and sends through the copper disk a magnetic flux proportional to the line voltage. A shaft attached to the disk passes through tubes constructed to prevent the mercury from escaping when the meter is inverted. Externally, this shaft carries a worm which engages with the register train, and an aluminum damping disk moving in the field of a permanent magnet. The submerged copper disk carries a float so proportioned that there is a slight upward thrust. In consequence, the lower bearing is a ring-stone jewel, while the upper end of the shaft is carried in a ring-stone end-stone bearing. The driving torque is proportional to the power supplied to the load. The damping at small and moderate loads is nearly all caused by the drag disk, but at full load and overloads the fluid friction of the mercury increases at a rate which would cause a drop in the accuracy curve unless compensated. Compensation is effected by passing the line current through a few turns of heavy wire around the voltage electromagnet. The copper disk is slotted radially to reduce the damping effect which would be caused by the flux from the voltage electromagnet. This expedient reduces the influence of variation of voltage on the accuracy of the meter.

Compensation of Mercury-motor Watthour Meter for Friction. The compensation for friction is effected by a thermocouple shunted around the mercury chamber and heated from a resistance coil connected in series with the voltage circuit. The small emf generated in the thermocouple sends a current through the armature which is sufficient to overcome the friction of the moving parts.

Application of Mercury-motor Watthour Meters. On account of the low resistance of the current circuit, mercury-motor watthour meters are particularly well adapted for use with external shunts, and are used for switchboard work where large currents must be metered. They withstand vibration and shocks better than commutator meters.

Current Ratings of Mercury-motor Watthour Meters. Direct-current mercury-motor watthour meters have a full-load current rating of 10 amp. For all higher ranges external shunts are used. Shunts with ratings up to 60,000 amp have been furnished. Meters designed for voltages up to 700 volts are regularly furnished and special meters have been supplied for 1200- to 3000-volt d-c railway circuits.

Accuracy of Mercury-motor Watthour Meters. The load-accuracy curve at normal voltage is said to deviate from a straight line by less than 0.5 per cent from 5 per cent to 200 per cent of rated load.

INDUCTION WATTHOUR METER. The essential elements of a typical induction watthour meter are shown in Fig. 33. A laminated electromagnet has three cores carrying a voltage coil and two current coils. The voltage coil, of many turns of fine wire, is connected across the line; the current coils, of a few turns of heavy wire, carry the load current. The magnetic flux set up by the voltage coil would be in quadrature with the applied voltage but for the iron loss and copper loss in the voltage electromagnet. Exact quadrature is obtained by expedients of which that shown in Fig. 33 is typical, namely, the use of a so-called lag coil closed through an external resistance (not shown). This resistance is adjusted by trial to give the quadrature relation at rated frequency. In some designs a metal plate forming a closed loop is used instead of a wound lag coil. The flux of the current electromagnet is practically in phase with the load current.

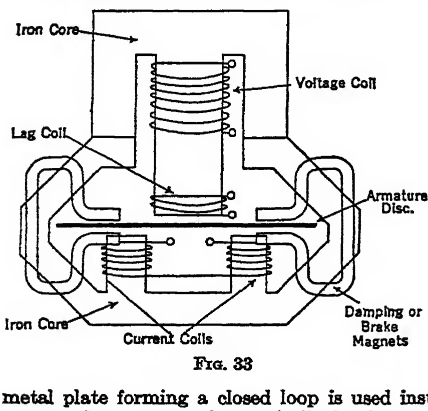

FIG. 33

Principle of Operation of Induction Watthour Meter. Currents induced in the disk flow about each pole in the general form of concentric circles. The driving torque results from the interaction of each induced current with the flux which does not induce it. When the load power factor is unity, each induced current will be in phase with the flux

with which it reacts to produce torque, and as the power factor of the load becomes less than unity, this torque will decrease in proportion to the cosine of the angle of lag (or lead) in the load. Hence the driving torque is proportional to the power in watts taken by the load. The retarding torque of the damping magnets is proportional to the speed of rotation, and the speed will therefore be proportional to the power taken by the load. The total number of revolutions over any given period will be proportional to the energy in watthours.

Compensation of Induction Watthour Meter for Friction. To maintain accuracy down to very light loads, the friction of the bearings and of the register is usually compensated by placing a metal plate, forming a closed loop, near the pole of the voltage electromagnet with means for adjusting it in a plane at right angles to the axis of the voltage coil. The current induced in this loop produces a flux out of phase with the flux of the voltage coil, and the interaction between each of these two fluxes and the current induced in the disk by the other produces a slight turning moment, independent of the load current, which is adjusted by shifting the light-load plate until the frictional moment is just balanced.

Recent Improvements in Induction Watthour Meters. Cooperating with the meter committees of the electric power industry American manufacturers have steadily improved their meters. Notable improvements made in recent years include (1) compensation for temperature changes, (2) extension of the range of accurate registration up to 300 per cent of rated load. Compensation for temperature, for loads of unity power factor, is obtained by locating pieces of magnetically temperature-sensitive alloy near the retarding magnets so as to bypass some of their flux. The permeability of these alloys changes appreciably with room temperature, and by proper design the effective braking flux may be made to change with temperature by the right amount to keep the accuracy of the meter unchanged with loads of unity power factor. The change of resistance of the voltage coil with changing temperature causes a small shift in the phase of its flux with respect to the line voltage, and in consequence, a change in the accuracy of the meter on loads of low power factor. Compensation for this effect of temperature change is obtained, to a greater or less extent, by various expedients which affect the flux of the voltage electromagnet.

Multi-element Induction Watthour Meters. Polyphase induction meters are usually made by combining two (or three) single-phase meter elements with their disks mounted on a common shaft. Two-element meters are used for 3-wire 3-phase circuits, and also for metering 3-wire single-phase loads with an accuracy which is independent of current and voltage unbalance. Three-element meters are used to meter 3-phase 4-wire loads. Although it is possible to meter 3-phase 4-wire loads with a special 2-element meter equipped with a third current circuit divided between the two current electromagnets, such meters are subject to errors from unbalance of the voltages.

Accuracy of Induction Watthour Meters. For the induction meters now in American production the makers' stated accuracy is 0.5 per cent or better from "light load" of, say, 5 per cent of rated watts up to 300 per cent of rating. Such a performance is needed for residence meters because electrically operated household appliances are widely used. Meters of the older designs of larger current rating, say 15 amp, would not correctly register on very light loads.

Current transformers do not have a large continuous overload capacity; for example, a secondary current of 7.5 amp would probably be considered an upper limit for a secondary rating of 5 amp. The use of a modern 5-amp watthour meter with a current transformer is inefficient, and meters rated at 2.5 amp are coming into use for this purpose.

Performance Data of Induction Watthour Meters. For the single-phase watthour meters now in production (1936) the following representative data are given by the manufacturers: weight of moving element, 13 to 16 grams; torque at rated load in watts, 40 to 53 millimeter-grams; rpm at rated load in watts, 28.75 in 3 makes, 16 in the fourth. Loss in voltage coil at rated voltage, 0.9 to 1.4 watts; loss in current coil at rated current, for 5-amp meters, 0.15 to 0.30 watt.

Standardization of Induction Watthour Meters. Within recent years, two important steps toward uniformity have been taken by the four American makers. The direction of rotation of the disk has been made the same in all induction meters now in production, namely, counter-clockwise as seen from above the disk. Three of the four makers have adopted a uniform value of disk constant, namely, $1/3$ watthour per disk revolution for a 5-amp 115-volt 2-wire meter. The fourth maker has adopted a disk constant of 0.6. Studless glass covers are being used by all makers. The greater overload capacity, with sustained accuracy, permits a reduction in the number of ampere ratings used. The so-called "new sequence" (meter, switch, fuse) now permissible in customers' installations is stimulating the development of meters and weatherproof cases for outdoor use. Outdoor

meter installations save the time of meter readers and testers, and tend to discourage the theft of current.

MAINTENANCE AND TESTING OF WATTHOUR METERS. The use of a portable standard watthour meter ("portable test meter" or "rotating standard") is the accepted method not only for checking customers' meters in position but also for testing repaired or exchanged meters in the meter laboratory. These standard meters should be checked against an indicating wattmeter and a time standard, at intervals depending upon the conditions of use. To facilitate the testing of customers' meters, a large number of artificial-load and other devices are available. For details, consult the Handbook for Electrical Metermen (see Bibliography). Additional valuable information on devices and methods for watthour meter testing and on demand meters, indicating instruments, potentiometers, etc., is contained in the subsequent Reports of the N.E.L.A. Meter Committee under the heading of New Developments.

Demand Meters

The charge to any customer should logically be made up of a part fixed by the maximum power in kilowatts taken at any time during a certain definite period, and another part fixed by the total energy used during the same period. The demand system of charging tends to improve the load factor of the station and to distribute its fixed charges more equitably among its customers.

A large number of variables are involved in the problem of demand metering, including the rating of generators, lines, and transformers, and the diversity factor, load factor, and power factor of the system. No practical meter has yet been devised to take into account all these variables; consequently, charges based on the use of demand meters in addition to the usual energy meters must necessarily be somewhat arbitrary.

REQUIREMENTS OF A DEMAND METER. The energy supplied to the customer is measured by a watthour meter; the maximum power (in kilowatts) taken by him is measured by some form of demand meter. For d-c supply at practically constant voltage a device which measures the maximum current is as satisfactory as one which measures maximum power, but when the voltage or power factor (in case of an a-c system) varies, the maximum-current indicator is not suitable. In any case, the demand meter should not measure the instantaneous peak but rather the average of the power demanded by the customer over a specified time interval, for the maximum demand recorded should not be unduly influenced by short circuits, excessive starting current of motors, or any other abnormal demand over a time too short to have any real effect on the generator and line capacity which must be provided. This time interval should theoretically differ with the character of the installation and the relation of its maximum power demand to the maximum station capacity. In relatively large customers' installations the time should be carefully chosen with reference to the time that the central station can endure an overload successfully. Hydroelectric plants have a very definite power limit which is reached with maximum gate opening, while steam-power plants have considerable thermal inertia and their boilers and generating apparatus can usually carry relatively large overloads for a considerable time. These considerations require much shorter time intervals for large users of hydroelectric power than would be necessary in steam plants of corresponding magnitude.

Demand intervals of 15 minutes, 30 minutes, and 1 hour are those in general use at the present time. For very special installations, intervals as short as 1 minute and as long as 3 hours have been used. The general tendency is toward the use of a single interval for a given system, and toward the use of intervals larger than those formerly employed.

To improve the load factor, the off-peak system of rate schedules has been developed, which gives a preferential rate to customers utilizing current during the times of light load. Demand meters giving time of day as well as demand are used for this class of metering.

CLASSIFICATION OF DEMAND METERS. Demand meters may be classified into (1) **curve-drawing instruments,** giving the load-time curve of the installation; (2) **integrated-demand meters** consisting of an integrating meter combined with a device which registers the energy consumption from time to time in such a way as to indicate or record the maximum demand; (3) **lagged-demand meters,** which require a certain time interval for the indication to reach the value corresponding to the load. Under each of these types there are a number of further distinctions, such as length of demand interval, whether the demand intervals begin at specified times of the day or may be so chosen as to include the maximum average load during any time interval of the given length, and whether the maximum demand is simply indicated without reference to time or is recorded so that the time of its occurrence may be determined.

CURVE-DRAWING INSTRUMENTS AS DEMAND METERS. The most complete knowledge of the conditions attending the customer's use of energy may theoretically be obtained by using a curve-drawing wattmeter, with which the average demand for any time interval may be found and also the time at which it occurred. In practice, however, unless the paper speed is fairly high, the lines will run together so as to make it difficult or impossible to determine the demand with any approach to accuracy. Curve-drawing wattmeters are expensive, and their indications are difficult of comprehension for the ordinary customer. They are used largely by central stations to accumulate demand data. Curve-drawing ammeters may be used on the assumption of constant voltage and power factor, but they do not give as accurate information as wattmeters.

INTEGRATED-DEMAND METERS INDICATING MAXIMUM DEMAND ONLY. The Types M-16 and M-19 demand meters of the General Electric Co. and the Type G demand meter of the Sangamo Electric Co. consist essentially of a demand-registering and a timing mechanism mechanically connected and mounted within the same case. The demand-registering element is electrically operated by means of a contact actuated by the watthour meter in conjunction with which the demand meter is used. The registering element consists of a train of gears which drive a demand pointer forward over a dial. This train is actuated by a ratchet-and-pawl mechanism operated by an electromagnet which receives an impulse every time a certain amount of energy is registered by the watthour meter. The speed of advance of this pointer is therefore proportional to the power. The pointer-driving mechanism is reset to zero position at the end of each time interval, leaving the pointer itself in the highest position reached. This resetting is accomplished by a mechanism controlled either by a clock or by a constant-speed motor. These demand meters are for use on a-c circuits, and are arranged for intervals of 15, 30, or 60 minutes. The Type M-16 demand meter, for a-c service, is timed by a Warren synchronous motor; the G.E. Type M-19, and the Sangamo Type G, for d-c service, have a motor-wound spring-driven clock. The Type M-14 demand register, when used in appropriate General Electric a-c watthour meters instead of the regular register, serves the same purpose as the M-16 and M-19 demand meters and also has the usual four dials showing the energy registration in kilowatthours. The Westinghouse demand attachment, the Sangamo Type HB maximum demand register, and the Duncan T-1 demand register, when installed in the corresponding watthour meters in place of the usual registers, operate in the same general way and serve the same purposes as the General Electric Type M-14 demand register.

The General Electric Type I-16 MW, Type D-14 MW, and Type D-15 MW watthour demand meters perform the same functions as the I-16, D-14, and D-15 watthour meters equipped with a demand register as outlined in the preceding paragraph, but have a regular kilowatthour register and a separate demand registering mechanism. The timing element is a Warren synchronous motor.

INTEGRATED-DEMAND METERS WITH ARBITRARY TIME INTERVAL, RECORDING ALL DEMANDS AND TIME OF OCCURRENCE. The earliest instrument of this class was the printometer, later made by the General Electric Co. as the Type P demand meter. This instrument printed on a paper tape at regular time intervals the time of record and the energy consumption registered up to that time by a watthour meter. Cyclometer type-wheels were moved forward, at a rate proportional to the power, by means of a solenoid, the energizing circuit of which was closed through a contact wheel fixed to one of the spindles of the gear train of the watthour meter. The reading of the cyclometer was printed on a paper tape by a copying ribbon and a rubber platen actuated by a solenoid which was energized at regular time intervals by means of a contact-making clock. The time was also printed opposite each cyclometer reading. Thus, the difference between consecutive recorded values on the tape was proportional to the energy consumption in kilowatthours in the circuit during a definite time interval. The average demand in kilowatts over a period of time, corresponding to the interval between successive operations of the printing solenoid, was obtained by dividing the kilowatthours of energy consumption during a time interval by the length of the interval in hours.

The Type PD demand meter is based on the same general principles as the Type P demand meter just described, but contains new features which reduce the labor of discovering the maximum demand. The printing wheels automatically return to zero at the end of each time interval, hence each printed record shows the demand during the interval just elapsed. This eliminates the necessity of subtracting readings in order to obtain the demand. The PD demand meter also has three dials giving, respectively: an indication of the maximum demand since the meter was last reset; a reading of the demand at any instant; and the total number of contact impulses delivered to the demand meter, which may be used in checking the total registration of the demand meter with that of the watthour meter.

The Type RA recording-demand watthour meter of the Westinghouse Co. consists of a polyphase watthour meter combined with a mechanism for obtaining a permanent graphic record of the integrated demand in kilowatts over successive predetermined time intervals. The total energy consumption is shown in the usual way by the register of the watthour meter. The time interval is controlled either by a hand-wound 35-day clock or by a synchronous-motor clock. A separate spring mechanism advances the record paper. The pen is moved across the paper by the gear train of the watthour meter, and at the end of each time interval a release mechanism frees it from the gear train and allows a weight to move it back to zero where it is again meshed with the gear train to repeat its advance during the next time interval. Just before the release occurs, the record paper is advanced slightly so that the pen makes a distinct record of the maximum travel, which is a measure of the integrated demand. The time of occurrence is shown by the time figures printed on the paper. This meter is made with time intervals of 5, 15, 30, or 60 minutes. It is a self-contained structure, with the metering and demand-recording mechanisms mechanically connected.

The Westinghouse Type RB recording-demand watthour meter is a development from the Type RA and functions in the same general manner. It employs OB watthour-meter elements and has a synchronous motor which drives the timing device and the chart-driving mechanism.

The General Electric Type G-9 demand meter is used in connection with a watthour meter, and gives a graphic record on a circular chart of the demands integrated over definite time intervals, with the time of day and day of week of their occurrence. It consists essentially of a registering element and a timing element. The registering element is electrically connected to the watthour meter, which has a contact device either on one of the spindles of the register or in a separate mechanism operated by the rotating element. This contact device alternately makes and breaks the circuit of a solenoid in the registering element each time a definite amount of energy is registered by the watthour meter. An armature operates a ratchet-and-pawl mechanism to advance the stylus by a small step for each operation of the contacts. At the end of each time interval a cam mechanism operated by the timing device allows the stylus to return to the zero position, where it is re-engaged ready to integrate the energy used in the following interval. The timing mechanism rotates the chart continuously, giving a record in the form of a saw-tooth polar curve. The chart gives a record for one week or one month. The time interval is either 15, 30, or 60 minutes.

The General Electric Type GM-10 demand meter is a development from the G-9 demand meter, and is for the purpose of assisting the plant operator to keep the maximum demand at the lowest practicable level. Alarm contacts used for this purpose in previous demand meters had the serious limitation that they closed after the predetermined maximum demand had been reached. The GM-10 demand meter avoids this limitation by having the alarm contacts so arranged that they will close if for any part of the time interval the integrated load (number of impulses transmitted by the watthour meter to the demand meter) has exceeded a predetermined and adjustable value for the elapsed part of the time interval. To accomplish this, a so-called "ideal-demand" mechanism has been added to the G-9 demand meter. Type GM-10 demand meters are made for use on a-c circuits only, and employ the Warren clock motor as the timing element.

The Type DG-1 watthour demand meter of the General Electric Co. is a combination of a two-element polyphase watthour meter and a block-interval demand meter which records the demand for each definite time interval on a strip chart. The watthour-meter disk shaft has the usual worm at the top which drives the register and a second worm at the bottom which drives the recording pen mechanism through a gear train and clutch. The timing is accomplished by a Warren motor which also drives and rerolls the chart, winds a resetting spring, and actuates the resetting mechanism. Standard time intervals are 15, 30, and 60 minutes.

THERMAL-STORAGE LAGGED-DEMAND METERS. These meters all show maximum demand only, and the indication depends on the duration of the load as related to the demand interval of the meter, as well as on the previous history of the load. The demand interval of this type of meter is usually taken as the time required for the indication to reach 90 per cent of the full value of a steady load which is thrown suddenly on it.

The Type H-2 demand meter of the General Electric Co. is an ampere-demand instrument based on the differential thermometer principle, using a pair of strips of thermostatic metal. One of these is in proximity to a heating element traversed by the load current, while the other nominally follows the room temperature. For a given change in temperature the two strips tend to move in opposite directions, and with no current flowing through the heating device the pointer remains at zero. Current flowing through the

heating device heats the adjoining strip and moves the pointer over the scale. The pointer remains at the highest point reached until manually reset.

The Lincoln Type A.D. demand meter indicates maximum ampere demand on either d-c or a-c circuits. In the two-wire Type A.D. meter a shaft carries the demand pointer and two bimetallic spiral coils which tend to rotate the shaft in opposite directions when their temperature is increased. A heater carrying the line current (or a definite part of it, when shunts or current transformers are used for the larger capacities) is arranged so as to heat only one of the spirals. The angular deflection of the pointer, when a steady state is reached, is proportional to the square of the current, but the scale is marked either in terms of the first power of the current or in kilowatts for an assumed value of line voltage. The three-wire Type A.D. demand meter operates in a similar manner but has three heating coils, two of equal resistance being connected in the outside wires and acting on one bimetallic spiral while the third heater, having one-half the resistance of each of the others, is connected in the neutral wire and acts on the second spiral. This arrangement ensures that the maximum ampere demand of the three-wire circuit will be correctly measured, regardless of the degree of unbalance of the load.

The Lincoln Type W. D. demand meter is constructed on the principle of the hot-wire wattmeter and indicates demand in kilowatts. In construction it resembles the two-wire Lincoln Type A.D. meter just described, but differs in that two heaters are used, one acting on each of the two bimetallic spirals. The meter contains a small transformer supplying a low-voltage current to the two heating coils connected in series. The line current is brought in at the central point on the secondary winding of the transformer and flows through the two heating coils in parallel and on to the load. One heating coil therefore carries one-half the line current plus a current proportional to the line voltage, and the other heating coil carries one-half the line current minus the current which is proportional to the line voltage. It may be shown that the resulting difference in rate of heat liberation in the heaters, and therefore the angular deflection of the pointer, will be proportional to the load in kilowatts, for any power factor. The Type W.D. meter is internally shunted in the larger capacities.

DEMAND METERS TAKING ACCOUNT OF POWER FACTOR. The increasing appreciation of the effect of low power factor on the cost of energy supply has resulted in the development of demand devices taking power factor into account. The Westinghouse Type RI kva recording demand meter resembles the Type RA recording demand meter described above but has also a second polyphase meter which is connected by means of phasing transformers to register the kilovarhours (reactive kilovolt-ampere hours). This meter and the energy meter drive two small aluminum disks on which rests an aluminum ball. A third disk, in contact with the ball, is caused to rotate at a speed which is equal to the vector sum of the speeds of the other two disks. The total kilowatthours and total kilovarhours are shown by the usual registers on the energy meter and the reactive-component meter. The graphic record shows the kilowatt demand and the kilovolt-ampere demand during the predetermined time interval, from which the average power factor during any time interval may be found.

The Sangamo Type S kva demand meter is controlled by two watthour meters which measure the energy in kilowatthours and the reactive component (kilovarhours), respectively. It gives a record on a single chart from which the following information about the load can be obtained: (1) the kw demand; (2) the kva demand; (3) the maximum kw demand; (4) the power factor at time of demand; (5) the reactive-component demand; (6) the time of occurrence of the demand; (7) the energy component in kilowatthours; (8) the reactive component in kilovarhours (reactive kilovolt-ampere hours). The power triangle for any given demand interval is drawn on a strip chart. The energy is measured by a watthour meter, and the registration is transferred by electrical means to move a stylus horizontally across a vertical paper chart which is driven (by electrical means) by a second watthour meter connected so that it integrates the reactive component. Both the kilowatthours and the kilovarhours are shown by registers in the kva meter. A pusher arm mechanically connected to the energy stylus advances a friction pointer which shows the maximums kw demand. A small synchronous motor drives an auxiliary stylus across 13 equally spaced time (hour) divisions at the right of the chart, and also trips the energy stylus and returns it to zero at the end of each predetermined time interval.

13. INSTRUMENT TRANSFORMERS

APPLICATIONS. An instrument transformer is a transformer in which the conditions of current or voltage and of phase position in the primary circuit are represented with acceptable accuracy in the secondary circuit. An instrument transformer may be

either an instrument current transformer or an instrument potential (voltage) transformer. These terms are usually abbreviated to current transformer and potential (voltage) transformer, respectively. Both kinds of transformer are applicable for measurement and control purposes. The current transformer is intended to have its primary winding connected in series with the circuit carrying the current to be measured or controlled, and the voltage transformer is intended to be connected in parallel with the circuit.

The use of instrument transformers separates the secondary measurement and control devices from the dangerous line voltage and also permits all these devices to have a single rated current and a single rated voltage. For measurement purposes high accuracy is important; for the operation of control and protective devices a very moderate accuracy is sufficient but reliability and ruggedness are of the highest importance. It is therefore frequently advisable to use separate transformers for the two functions, even at an increased cost.

DEFINITIONS. The **true ratio** of a current transformer or a voltage transformer is the ratio of the rms primary current or voltage, as the case may be, to the rms secondary current or voltage, under specified conditions. The quotient obtained by dividing the true ratio by the nominal ratio (as marked on the name plate) is often called the "ratio factor" or "ratio correction factor."

The **phase angle** of a current transformer or a voltage transformer is the angle between the primary current or voltage vector and the secondary current or voltage vector reversed. This angle, by international agreement, is considered as positive when the reversed secondary current or voltage vector leads the primary current or voltage vector. The phase angle of a current transformer is often denoted by the letter β, that of a voltage transformer by γ. The phase angles β and γ may be inherently either positive or negative, depending on the characteristics of the particular transformer and its burden.

If a wattmeter is connected to the secondary circuits of a voltage transformer of ratio R_v and phase angle γ, and of a current transformer of ratio R_c and phase angle β, the true power P in the primary circuit is given by

$$P = P_s R_v R_c \frac{\cos(\theta_2 + \alpha + \beta - \gamma)}{\cos \theta_2} \tag{12}$$

where P_s is the power indicated by the wattmeter; θ_2 is the apparent phase displacement by which the voltage leads the current as deduced from the indications of the instruments in the secondary circuits; and α is the phase angle of the voltage circuit of the wattmeter. Tables of values of the correction factor $\frac{\cos(\theta_2 + \alpha + \beta - \gamma)}{\cos \theta_2}$ are given in the section on Wattmeters. The correction factor for energy measurements, when a watthour meter is used with instrument transformers, is the same as that given above for power measurements.

The **burden** of an instrument transformer is that property of the circuit connected to its secondary winding which determines the flow of active power and reactive power from the transformer. It is expressed either as total ohms impedance, together with the effective resistance and reactance components of the impedance, or as the total volt-amperes and power factor of the secondary devices and connecting leads. The values expressing the burden apply conventionally to the condition of rated secondary current or voltage of the instrument transformer and a stated frequency, both of which must be included with the burden expression. The impedance expression is the more convenient for current transformers, the volt-ampere power-factor expression for voltage transformers.

POLARITY MARKINGS. The primary terminals and secondary terminals of instrument transformers should be so marked that, when a lead is connected to a marked secondary terminal, the polarity will be the same as if the primary conductor running to the marked primary terminal were detached from the transformer and connected directly to the lead; that is, the relation of the marked terminals is such that if the instantaneous direction of the current in the marked primary lead is toward the transformer, the instantaneous direction of current in the marked secondary lead is away from the transformer, or vice versa.

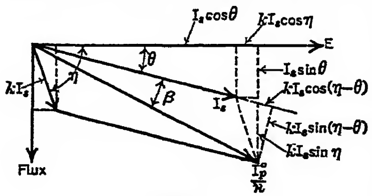

FIG. 34

RATIO AND PHASE ANGLE OF CURRENT TRANSFORMER. Refer to Fig. 34 and let

n = ratio of the number of secondary to the number of primary turns.

I_p = primary current (reversed).

I_s = secondary current.
k = ratio of exciting current (referred to secondary) to the secondary current.
η = phase angle between the exciting current and primary induced emf.
θ = phase angle between the secondary current and secondary induced emf (positive for lagging current).
β = the "phase angle" of the transformer, i.e., the angle between the primary current and the secondary current reversed.

Then the ratio of transformation is

$$\frac{I_p}{I_s} = n\left[(\cos\theta + k\cos\eta)^2 + (\sin\theta + k\sin\eta)^2\right]^{1/2}$$
$$= n\,[1 + k\cos(\eta - \theta)\ldots]\ \text{approx.} \tag{13}$$

and the phase angle is

$$\beta = \tan^{-1}\frac{k\sin(\eta-\theta)}{1 + k\cos(\eta-\theta)} \tag{14}$$

The value of the ratio k for a given primary current depends upon the impedance of the burden connected to the secondary; the higher this impedance the greater the value of k, and therefore increasing the secondary burden tends to increase the difference between the ratio of turns and the true ratio of the primary and secondary currents; increasing the secondary burden also tends to increase the phase angle β of the transformer. The true ratio I_p/I_s and the phase angle β also depend upon the power factor of the burden. The core flux required for a given induced voltage is inversely proportional to the frequency; consequently the exciting current and hence the phase angle and the departure of the true ratio from the ratio of turns will also vary approximately in inverse proportion to the frequency.

In order to use these formulas for numerical results it is necessary to know the amount and power factor of the exciting current under the particular conditions of burden. Since this is difficult to predetermine, it is customary to use curves similar to those given in Fig. 37 below, obtained by plotting actual test results.

SPECIAL DESIGNS OF CURRENT TRANSFORMERS. Although the design of current transformers is necessarily based on the same fundamental principles as that of power transformers, the conditions under which they operate (nearly short-circuited secondary and varying terminal voltage) are so special that the resulting design is very different. The flux density under normal conditions is very low (a few hundred gausses). The ampere-turns at rated primary current should be at least 1000 for good accuracy under normal conditions if silicon steel is used. In high-range current transformers this condition is met by the use of a single primary conductor running through the center of the core opening. A transformer of this type has the great advantage that when subjected to severe overloads there is no mechanical force tending to distort the primary. Moreover, in high-voltage circuits a single-turn transformer can be made by slipping a core and secondary winding over the bushing of a power transformer or oil switch, thus saving the cost of additional insulation. In ratings below 600 amp the ratio error and phase angle of single-turn transformers become large unless some special means is provided for reducing them.

The ratio error at any chosen burden may be made small for one value of current by choosing a number of secondary turns slightly less than that corresponding to the nominal ratio of the transformer. The phase angle for a given burden can be reduced somewhat by providing a closed tertiary circuit of suitable impedance.

Much greater improvement applicable over a considerable range of current, burden, and frequency can be obtained by certain other procedures. One of these is the use of a **two-stage current transformer** (see paper by Brooks and Holtz, Trans. A.I.E.E. 41, p. 382, 1922), which is provided with an auxiliary magnetic circuit encircled by primary and secondary windings having a turn ratio equal to the nominal transformer ratio. Any departure from perfect performance of the main (or first-stage) unit induces a corrective current in a third winding on the auxiliary core. This corrective current, passed through an auxiliary current coil in a secondary instrument, will add its effect vectorially to that of the main current in the regular instrument coil, with the result that the effective ratio of primary current to the vector sum of the main and auxiliary secondary currents is very close to the nominal value. By the use of a suitably adjusted impedance network, two-stage transformers can be used with ordinary instruments and meters having single current windings.

Another method for improving the accuracy of current transformers is the use of **special nickel-iron alloys** (such as hipernik; see paper by Spooner, Trans. A.I.E.E. 45, p. 701, 1926) which have very low losses and high permeability. The errors are reduced

roughly in proportion to the losses, and high accuracy with low rated ampere-turns or moderate accuracy with a very small and light transformer can be obtained.

A third scheme is to provide a hole through the core splitting the magnetic circuit into two parallel paths over part of its length (see paper by Wilson, Trans. A.I.E.E. 48, p. 783, 1929). A few turns of the secondary circuit are wound through this hole and cause one path to operate at a flux density different from that in the other. A change in the current changes the relation between the reluctances of the paths, and the effective secondary linkages can be made to vary in such a way as to maintain a nearly constant ratio. A short-circuited turn through the hole tends to compensate the phase angle in a similar fashion. A saturated-core reactor of suitable design in parallel with the secondary burden can be used with similar results.

When a short circuit occurs on the line in which a current transformer is connected the primary current may momentarily attain many times the rated value. The design of the primary winding should be such that the transformer will not be damaged by the mechanical or thermal effects of specified overloads. If the design is such that the core is carried well up to saturation at these large currents, the secondary current will not increase in proportion to the primary current. Such a characteristic is often desirable since the secondary instruments and relays are thus spared much of the shock of the short circuit, though the increase in secondary current may be ample to cause the relays to function. In other applications, however, it is important that the current transformer maintain its ratio reasonably constant even on severe overloads. Transformers can be designed to have either type of characteristic.

It is sometimes desired to measure the current in a cable without cutting it to insert the primary winding of a transformer. This can be done by the use of a current transformer, the core of which is made in two parts hinged together, and provided with a secondary winding. Such a transformer can be clamped around the cable which thus forms a single-turn primary winding. Although not capable of very high accuracy, such transformers are very useful, in approximate measurements; for example, of the current taken by motors, the distribution of load among transformers banked in parallel, etc.

Most portable current transformers and certain types of switchboard transformers are provided with two or more sections of primary winding which can be connected in series or in parallel, to give various primary ranges. The use of such switchboard transformers makes it possible to take care of growth or redistribution of the station loads without replacing the transformers.

Certain types of current transformers, especially oil-filled high-voltage transformers, are made with duplicate cores and duplicate secondary windings, placed in a common oil tank and linked with the same primary winding. Such "double-secondary" transformers constitute in effect two transformers, one of which can be used with a light burden for accurate metering, while the other supplies relays or other control equipment of higher burden. This construction avoids the duplication of the expensive high-voltage bushing and the oil tank.

TESTING CURRENT TRANSFORMERS FOR RATIO AND PHASE ANGLE.

General. For accurate measurements the ratio error and the phase angle may not be negligible. It is also possible for damage to the transformer windings to be caused by surges, overloads, or other disturbance, and it has therefore become the practice of many power companies and the rule of a number of public utility commissions to require the periodic testing, at installation and at intervals of 5 or 10 years thereafter, of all instrument transformers used in important metering installations.

If the core of a current transformer becomes permanently magnetized, as by the passage of direct current through its winding or by the opening of its secondary circuit while primary current is flowing, the ratio and phase angle may be seriously changed. The original values may be restored by **demagnetizing the core.** This is conveniently done by sending an alternating current of 5 amp through the secondary winding, while the primary is open, and gradually reducing this current to zero. An alternative method is to send rated current through the primary winding while the secondary is connected to a resistance of several hundred ohms and gradually to reduce this resistance to zero.

Methods of test fall into two classes, direct and relative. In the former method the ratio and phase angle of a single transformer are determined directly; in the latter the transformer under test is compared with a standard transformer of the same nominal range, the ratio and phase angle of which are already known as the result of some previous test. In general, relative methods are simpler, quicker, and more convenient. Direct methods can be made more accurate but are usually less convenient and require the use of more sensitive and hence more delicate equipment. They are, however, a necessary link in the chain of measurement. Equipments for making direct measurements on instrument transformers are maintained in the laboratories of the National Bureau of

Standards, of certain state public utility commissions, of the larger manufacturing companies, and of a few of the larger power companies. Relative testing methods are regularly used by other state commissions and by a large number of power companies, their standard transformers being checked at regular intervals by reference to one of the direct equipments.

Direct Methods for Ratio and Phase Angle. In these methods the IR drops in two known non-inductive resistors, carrying the primary current and the secondary current respectively, are adjusted to be equal and opposite. The ratio of currents is then the reciprocal of the ratio of resistances. The value of the phase angle is obtained from the value of mutual inductance or of capacitance required to bring the two IR drops into exact opposition.

These direct null methods can be forced to a precision of setting of 0.01 per cent, which is enough to reveal unavoidable changes in the performance of most current transformers caused by self-heating, magnetization, etc. The principal sources of error are uncertainties as to the residual inductance and the resistance at rated current of the primary resistor and the effects of the stray magnetic field produced by the large primary current of the transformer under test. The design of suitable non-inductive primary resistors becomes very difficult for currents above 1000 amp. The testing of current transformers of high ranges is probably best done by a relative method, using as a standard a special transformer provided with a number of equal sections of primary windings. Such a transformer can be standardized by a direct method on a low range with the sections of primary winding in series and then used as a standard of higher range with its sections in parallel. The ratio can be considered as strictly inversely proportional to the number of series primary turns, and the phase angle as strictly constant, provided only that each section of the primary winding carries its equal share of the total current when the sections are in parallel, or that all sections have equally close magnetic coupling to the secondary winding. This latter condition can be very closely approximated by the use of a ring core with a uniformly distributed secondary winding. This same principle is made use of in the Baker test-ring method for the direct testing of current transformers.

Relative Methods for Ratio and Phase Angle. Of the relative methods the simplest is that in which the primary windings of the test and standard transformers are connected in series while each secondary winding is connected to one of a pair of similar measuring instruments, of which the difference in the readings gives directly the difference in the two transformers. If only the ratio is desired the two instruments may be 5-amp ammeters. If a phase-angle comparison also is desired, two wattmeters or two watthour meters are used. In any case the effect of errors in calibration of the instruments can be almost entirely eliminated by interchanging them and using the mean of the two sets of readings thus obtained. The wattmeter or watthour-meter readings made with the voltage circuits excited in phase with the primary current give directly a comparison of the ratios of the two current transformers. The readings made with the voltage circuits excited in quadrature with the current give a measure of the difference in phase angle of the two transformers.

The accuracy of methods of this type is limited by the precision of reading the ammeters or wattmeters and becomes very poor at low currents. By using two watthour meters and allowing the registration to accumulate over a longer time at small currents the precision can be considerably increased.

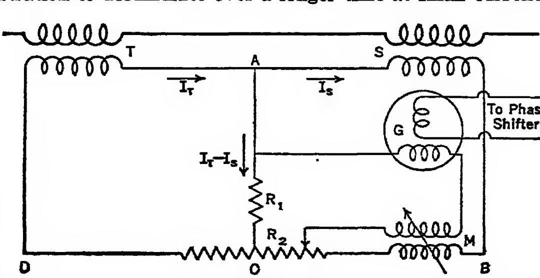

FIG. 35

The advantages of the relative method of test are much greater when balance methods are used. As indicated in Fig. 35, if the primary windings of two transformers T and S are connected in series, the secondary windings may also be connected in series so that their induced emf's aid in sending current through the main circuit $ABCD$. If for instance the transformer under test T has a smaller ratio (and hence a larger secondary current I_T) than does the standard transformer S, the excess secondary current $I_T - I_S$ will flow through the bridge circuit R_1 between points A and C. This vector difference in current may be measured directly by inserting in R_1 one coil of a separately excited

electrodynamic instrument (a 1-amp wattmeter fixed coil or a 30-volt wattmeter moving coil is convenient; a 10-amp ammeter should be first tried as a precaution against damage to the wattmeter if the transformers are connected with the wrong polarity). The difference in ratio and the phase angle (in radians) of the two transformers are given directly with ample accuracy by the ratio of the in-phase and quadrature components respectively of the difference in current, to the total secondary current of either transformer.

A more convenient modification of this method is to measure the vector difference $(I_T - I_S)$ in current by a null method. This is done by inserting a fixed resistance R_1 (of, say, 0.2 ohm) in the bridge circuit AC and balancing the IR drop of the difference current in it against the sum of the drop of the secondary current flowing in a slide wire of resistance R_2 and the emf induced in the secondary of the mutual inductor M. The detector at G is thus brought to a zero deflection and its calibration need not be known. The difference in ratio of the two transformers is then R_2/R_1, and the phase angle (in radians) is $\omega M/R_1$. Portable test sets (known as the Silsbee Current Transformer Test Set) embodying the principles of Fig. 35 are widely used for routine current transformer testing.

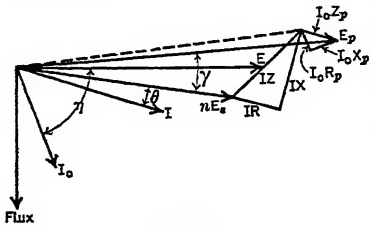

Fig. 36

RATIO AND PHASE ANGLE OF POTENTIAL (VOLTAGE) TRANSFORMERS. The relations of the quantities involved are shown in the vector diagram, Fig. 36.

$$\text{Ratio} = \frac{E_p}{E_s} = n\left[1 + \frac{IR\cos\theta + IX\sin\theta}{nE_s} + \frac{(IR\sin\theta - IX\cos\theta)^2}{2n^2E_s^2} + \frac{I_0R_p\cos\eta + I_0X_p\sin\eta}{nE_s}\right] \quad (15)$$

$$\text{Phase angle } \gamma \text{ in minutes} = \frac{3438}{E_p}[IR\sin\theta - IX\cos\theta + I_0R_p\sin\eta - I_0X_p\cos\eta] \quad (16)$$

where n = ratio of the primary turns to the secondary turns.
R and X = equivalent resistance and reactance of the transformer, referred to the primary side.
R_p and X_p = resistance and reactance of the primary winding.
E_p = primary impressed voltage reversed.
E_s = secondary terminal voltage.
E = primary induced voltage.
$I = I_s/n$ = load current in the primary winding, I_s denoting the secondary current.
I_0 = exciting current.
η = angle by which secondary induced voltage leads exciting current.

It is assumed that
$$\frac{R_p}{R} = \frac{X_p}{X} \quad (17)$$

because the reactance of the primary and secondary windings cannot be directly determined. The third term within the brackets in the expression for the ratio is very small and can be omitted with very little error. Hence it is evident that at constant power factor and constant secondary voltage the ratio and phase angle will vary linearly with the secondary current. Also, if the impedance of the burden is kept constant as the voltage is changed the ratio and phase angle of high-grade voltage transformers are found to change only to the very slight extent which results from lack of exact proportionality between I_0 and E_s.

The phase angle γ of a voltage transformer, i.e., the angle between the primary terminal voltage and the reversed secondary terminal voltage, may range from a negative angle (reversed secondary voltage lagging behind primary voltage) to a considerable positive angle (reversed secondary voltage leading primary voltage), depending on the exciting current, power factor, and impedance of the secondary burden and the impedance of the transformer windings. Under the no-load condition this angle is nearly always positive, and at high core densities where the exciting current is large and of very low power factor, it may be very large. The general tendency of non-inductive secondary burdens is to cause γ to vary in the negative direction (secondary voltage to lag) and of inductive (lagging current) burdens to cause it to vary in the positive direction (secondary voltage to lead). It is frequently possible to bring γ practically to zero for a single voltage and burden by adding a non-inductive burden in suitable amount.

SPECIAL VOLTAGE-TRANSFORMING DEVICES. For high-voltage circuits the size and hence the cost of voltage transformers of the conventional type increases very rapidly (roughly in proportion to the square of the rated primary voltage) because of the large amount of insulation required and the need for using in the primary winding a size of wire of reasonable mechanical strength, much larger than would be required to carry the load current. To avoid this expense European manufacturers have developed several alternative types of construction. See Bibliography, papers by Imhof, Keinath, and Goldstein; for a summary of these developments in English see chapter 24 of book, Industrial Electrical Measuring Instruments, by Edgcumbe and Ockenden.

TESTING VOLTAGE TRANSFORMERS FOR RATIO AND PHASE ANGLE. The phase angle and the departure from nominal ratio are usually less in voltage transformers than in current transformers, but are not always negligible. Methods for testing voltage transformers also can be classified as direct or relative, the former being more accurate but slower and requiring more expensive and delicate equipment. The relative methods have the further advantage that the circuits which the operator has to handle are completely insulated from the high voltage, which is not true of direct methods.

Resistance Method. This method requires the use of a high-resistance non-inductive resistor which can be connected across the line supplying the full rated primary voltage of the transformer under test. The current flowing in the resistor is usually 0.1 or 0.05 amp. The secondary voltage of the transformer under test is balanced against the IR drop in a known portion of the high resistance at its grounded end. The ratio of primary voltage to secondary voltage is then given by the ratio of the total resistance to the resistance of the portion across which the secondary is balanced. The effect of the transformer phase angle can be balanced by inserting an adjustable self inductor in one portion of the high-resistance circuit, by shunting part of the other portion with a condenser, or by connecting a mutual inductor with its primary in series with the resistor and its secondary in series with the detector. An alternative procedure for determining phase angle is first to make the ratio balance with an electrodynamic instrument excited in phase with the applied voltage and then to measure the phase angle by noting the deflection of a second calibrated electrodynamic instrument excited in quadrature.

Because of the high applied voltage this method is capable of very great precision even when used with insensitive and rugged detectors. It also has the advantage that the transformer can be tested with zero burden. The principal sources of error arise from the self-heating of the main resistor and from the effects of stray capacitance between sections of the main resistor and ground. For voltages above 5000 volts it is essential to minimize these effects by subdividing the resistor into a number of sections, each enclosed in a metal shield. An auxiliary guard circuit (usually having the same resistance as the main resistor) is connected in parallel across the line. Each shield of the main resistor is connected to a tap on the guard circuit so located that the potential of the shield is midway between the potentials of the ends of that section of the main resistor which it encloses. Resistors of this type can be made for use up to 50 kv, and by using a set of autotransformers in place of the guard resistance an equipment has been built for 132 kv.

Capacitance Method. In this method, which is applicable above 10 kv, the secondary of the transformer under test is connected in series with the primary, at its grounded end, with such polarity that the difference of potential between the extreme ends of the circuit thus formed is greater than the potential difference applied to the primary alone. A circuit consisting of an air capacitor of small capacitance but of high voltage rating, in series with an adjustable mica or air capacitor of much larger capacitance but of low voltage rating, is connected across the extreme terminals. The detector is connected between the junction of the two capacitors and the (grounded) junction of the two transformer windings. When a balance has been obtained by adjustment of the low-voltage capacitor the ratio of the transformer voltages is equal to the ratio of the capacitance of the low-voltage capacitor to that of the high-voltage capacitor. The effect of phase angle can be balanced by a mutual inductor which has its secondary in series with the detector while its primary, in series with a resistance of several thousand ohms, is connected across the secondary of the transformer under test.

The capacitance of the mica capacitor can be measured by the usual methods, and the effect of its phase angle on the apparent phase angle of the transformer can readily be allowed for. The capacitance of the high-voltage air capacitor may be computed from its dimensions or measured at audio frequency in terms of other capacitances or inductances. An alternative procedure is to determine at a moderate voltage the ratio of the two capacitances by comparison with a voltage-transformer testing equipment of the resistance type, and then to use this ratio in later testing work at higher voltages, beyond the range of the resistance equipment.

Another method of extending the range of testing equipment is to use a standard voltage transformer the primary winding of which consists of a number of sections of nearly equal impedance which also have nearly equal magnetic effects on the core and secondary. The ratio and phase angle of such a transformer may be determined by resistance equipment when the sections of the primary winding are in parallel, and the transformer may then be used as a standard in some relative testing method, with the primary sections in series. Under ideal conditions the phase angle would be the same on both connections and the ratio would be proportional to the number of primary turns. The principal departure from these ideal relationships would be expected to result from the effect of the stray capacitance between the sections of the primary winding. Careful measurements on several transformers at the National Bureau of Standards have shown these capacitance effects to be less than the error of measurement in all the cases examined.

Relative Methods. The use of two voltmeters, two wattmeters, or two watthour meters connected respectively to the standard and the test voltage transformers offers, of course, a possible method of test analogous to those used with current transformers. These methods are little used, however, because it is simpler to compare the two transformers by an opposition method. In this method (see paper by Brooks, Bull. Bur. Standards, 10, p. 419, 1914), the primary windings of the two transformers are supplied in parallel at rated voltage. The secondary windings are connected in series opposition, and the small difference in voltage is measured by a suitable separately excited electrodynamic instrument. This instrument may be a low-range wattmeter or one having fine-wire windings on both fixed and moving coils as in the Weston "comparator voltmeter." The difference in voltage as read with the instrument excited in phase with the supply voltage, when divided by the secondary voltage of either transformer, gives the difference in ratio factor of the two transformers; the corresponding quotient obtained when the excitation is in quadrature gives (in radians) the difference in the two phase angles.

This procedure can be made into a null method of high precision. A resistance of about 2000 ohms is connected across the secondary of the transformer which has the higher secondary voltage. The secondary voltage of the other transformer is then balanced against the IR drop in this resistance between one end and an adjustable sliding tap near the other end. The phase angle can be balanced by using a mutual inductor, the primary of which, in series with about 4000 ohms, is connected across the secondary of the standard transformer while the secondary of the inductor is in series with the detector. If the standard transformer is chosen to have a ratio, say, 2 per cent lower than nominal, the two measuring circuits will always be connected to it and will not constitute a burden on the transformer under test. This method avoids the error in reading any indicating instruments, and if the transformers have nearly the same ratio even a large error in the 2000-ohm resistance has a negligible effect on the result. The main limitation on the accuracy, therefore, lies in the calibration of the standard transformer. Another and somewhat similar relative method is used in the Leeds & Northrup voltage transformer testing set.

SECONDARY VOLT-AMPERE RATING OF INSTRUMENT TRANSFORMERS. Current transformers intended for use with a single measuring instrument usually have a secondary rating of 25 volt-amperes, and for use with more than one measuring instrument a rating of 50 volt-amperes, at rated secondary current.

Voltage transformers usually have a rating of 50 or 200 volt-amperes at rated secondary voltage.

COMPENSATION FOR RATIO ERROR. By properly proportioning the number of turns in the windings of a current transformer, it is possible to raise the secondary current to overcome the ratio error, with a given condition of burden. It is usual to compensate a current transformer for one-half its rated secondary burden, with secondary power factor of 80 per cent and frequency of 60 cycles.

The actual ratio of turns in a voltage transformer differs from the marked ratio by an amount sufficient to make up the voltage drop in the transformer at a specified burden. Voltage transformers are usually compensated for one-fifth their volt-ampere rating, with a secondary power factor of 80 per cent, and at rated frequency. The effect of phase displacement, however, cannot be compensated for, as it depends not only on the constants of the transformer itself, but on the power factor of the burden on the voltage transformer, and on the power factor of the load which is being measured with the help of the transformers.

ACCURACY OF INSTRUMENT TRANSFORMERS. Fig. 37 gives curves typical of the performance of modern current transformers under operating conditions as indicated in the following schedule:

Curve	Grade of Transformer	Frequency	Burden	
			Ohms	Microhenrys
A	Small and cheap	60	0.68	920
B	Average	60	0.68	920
C	"	60	0.97	4020
D	"	25	0.68	920
E	Excellent	60	0 68	920

The burden for curve *C* is typical of a relay circuit; that for the other curves includes a Silsbee current transformer testing set and indicating instruments. For a watthour meter only, or for indicating instruments only, the burden would be materially less.

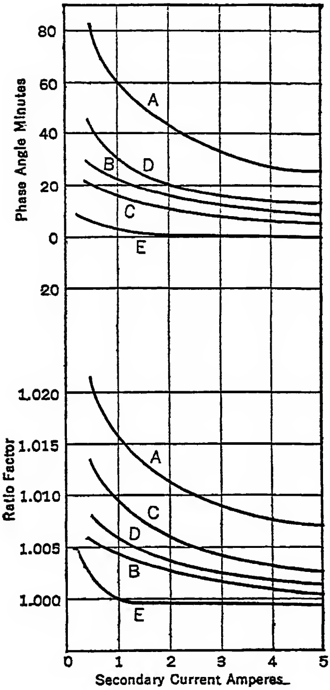

Fig. 37

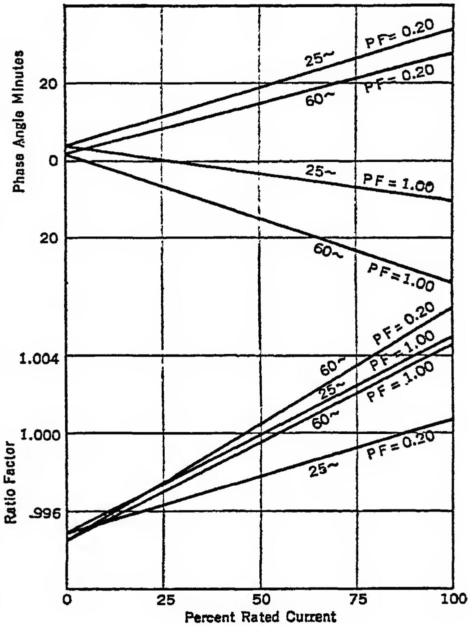

Fig. 38

Fig. 38 gives performance curves of a typical voltage transformer, ratio and phase angle being plotted as ordinate against percent of rated current as abscissa, for the operating conditions indicated on the curves.

14. ELECTROMETERS

GENERAL. An electrometer is primarily an instrument for measuring potential differences, but under certain conditions may also be used as a wattmeter; in the latter case it is usually called an electrostatic wattmeter. The deflection of the instrument is due to the attraction or repulsion of electrostatic charges. An electrometer may be used for either d-c or a-c measurements.

There are many forms of electrometers. On account of the care required in its use, the ordinary quadrant electrometer is seldom employed except for laboratory purposes, but various modifications of the electrometer provided with pointer and scale so as to be direct reading are used commercially for high-voltage measurements. Such instruments are known as electrostatic voltmeters.

The advantage of the electrometer over the galvanometer or electrodynamometer is that it takes no current when used for d-c voltage measurements. Owing to its capacitance, however, it does take a certain amount of charge which should always be allowed for if the capacitance of the electrometer is appreciable compared with any other capacitance which affects its reading. Also, when used for a-c measurements, the charging current taken by the electrometer should be allowed for, if appreciable.

APPLICATIONS. Electrometers are used in the measurement of the very small currents, 10^{-8} amp or less, dealt with in measuring insulation resistance and gaseous ionization, especially in studying x-rays and atmospheric electricity; in chemical work when electrode potentials are used to indicate the end points of certain titrations; and in investigations of static electricity. Electrostatic wattmeters are used in measuring small amounts of power at low power factor and high voltage, as in cable testing, and by the (British) National Physical Laboratory as a basic transfer instrument for the measurement of a-c power.

QUADRANT ELECTROMETER FOR VOLTAGE MEASUREMENTS. In this common type (Fig. 39) the two opposite quadrants a and a' are permanently connected together as are b and b'. Each pair is insulated from the other pair and from the case. The "needle" n (a light aluminum vane shaped like the figure 8) is suspended symmetrically between the upper and lower faces of the quadrants, by a fiber of silvered quartz, and is insulated from the case and quadrants. The complete expression for the electrostatic torque τ acting on the moving system is

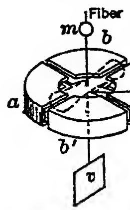

FIG. 39

$$\tau = \alpha V_n^2 + \beta_a V_a^2 + \beta_b V_b^2 + \gamma_a V_n V_a + \gamma_b V_n V_b + \delta V_a V_b + \eta_n V_n + \eta_a V_a + \eta_b V_b + \eta_0 \quad (18)$$

where V_n, V_a and V_b are the potentials, relative to the case, of the needle and of the quadrant pairs a and b respectively. The coefficients η result from the contact potentials of the metals used and are generally small. The effect of η_0 is merely to stress the suspension slightly when the zero reading is made. The other η terms, if appreciable, can be eliminated by suitable reversals of polarity. The coefficient δ is usually extremely small. If the instrument is perfectly symmetrical, α is also zero and

$$\beta_b = -\beta_a = \frac{\gamma_a}{2} = -\frac{\gamma_b}{2} = \kappa \quad (19)$$

and for these conditions the torque becomes

$$\tau_s = \kappa(V_a - V_b)\left(V_n - \frac{V_a + V_b}{2}\right) \quad (20)$$

In general the coefficients α, β, etc., will depend on the angular position of the needle as well as upon its size, shape, etc. If, for instance, the coefficient $\alpha = b\theta$ and the stiffness of the suspending fiber is U, the other coefficients having the values given in (19), the deflection is given by the relation

$$\theta = \frac{\kappa}{U - bV_n^2}(V_a - V_b)\left(V_n - \frac{V_a + V_b}{2}\right) \quad (21)$$

When a small difference of potential is to be measured, it is usually applied between the quadrants, while a suitable larger potential V_n from an auxiliary battery is applied to the needle. This is known as the "heterostatic connection." The term $\frac{V_a + V_b}{2}$ in eq. 21 is then negligible, and it will be seen that the deflection will be proportional to the first power of the measured difference $V_a - V_b$ and will increase rapidly with V_n, both because V_n is a factor in the numerator and because (if b is positive) an increase in V_n decreases the denominator. For too large values of V_n the instrument becomes unstable. In the Compton electrometer one quadrant is made adjustable so that b can be given any desired value.

When relatively large voltages are to be measured, V_a and V_n are made zero and the voltage to be measured is applied between the b quadrants and the case. This is known as the "idiostatic connection". The deflection is then proportional to the square of the applied voltage.

EXTENSION OF RANGE OF ELECTROMETER BY USE OF AUXILIARY CAPACITORS. By connecting the potential difference to be measured across two condensers in series and measuring the voltage across only one of them, the instrument

may be used for the measurement of high voltages of practically any magnitude. The connections are as shown in Fig. 40. If the condensers have no leakage, then

$$V = V_1 \frac{C_1 + C_2 + c}{C_2}, \tag{22}$$

where V_1 is the voltage read by the electrometer, C_1 and C_2 the capacitances of the two condensers, and c the capacitance of the condenser formed by the electrometer quadrants and needle. If C_1 and C_2 are large compared with c, then c may be neglected, but the larger C_1 and C_2 the greater will be the charge (and therefore the charging current, in case of an a-c measurement) taken by the measuring circuit.

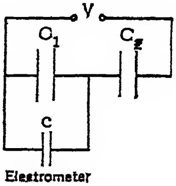

FIG. 40

QUADRANT ELECTROMETER AS ELECTROSTATIC WATTMETER (Fig. 41). The quadrant electrometer may be used to measure, with a fair degree of precision, very small amounts of power, of the order of 1 watt, when the voltage giving this power is 5000 volts or more. It therefore serves as a very convenient means of measuring the power loss in small samples of insulating materials at high voltages. See Bibliography.

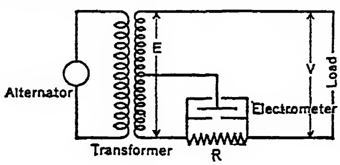

FIG. 41

STRING (FIBER) ELECTROMETERS. In this class of instruments the moving element consists of one or two fine fibers (usually of silvered quartz) held taut by a thicker quartz filament. Each fiber moves sideways under the influence of the electrostatic forces, and is viewed through a microscope provided with a scale in the eyepiece. Instruments of this class have the advantage of being quickly responsive and well damped and of having relatively low capacitance. The scale length and voltage sensitivity, however, are rather less than can be obtained with the quadrant type.

15. ROTARY VOLTMETER FOR HIGH D-C VOLTAGES

The **rotary voltmeter** ("generating voltmeter") for measuring high d-c voltages consists of an armature formed of two semicylindrical metal plates rotating in the electrostatic field between two electrodes which are connected to the points between which the voltage is to be measured. The two plates are connected through a two-part commutator to a galvanometer. The armature is rotated by a motor at a known constant speed. The current through the galvanometer varies directly as the voltage between the two inducing electrodes. The results of a calibration made at low and moderate voltages may therefore be extrapolated to high voltages, provided the electric field in the vicinity of the rotor is not distorted by space charges such as might result from corona at high voltages. The range is limited only by the clearances between live parts. One terminal of the galvanometer is grounded, and the galvanometer may be located at any desired distance from the high-voltage circuit. The sensitivity may be varied by altering a shunt around the galvanometer.

16. SPARK GAPS FOR MEASURING HIGH VOLTAGES

GENERAL. The voltage (crest value) required to break down a given gap depends principally upon: (1) the shape, size, and arrangement of the electrodes; (2) the presence of other conductors or insulators in the vicinity of the gap; (3) the time of application of the voltage; and (4) the temperature, pressure, and humidity of the air. Other factors of lesser significance are: (1) the composition and surface condition of the electrodes; (2) the circuit arrangements and constants; (3) the wave form and frequency of the voltage; (4) the extent to which the air in the gap is ionized; (5) the degree of irradiation of the spheres. Used under specified conditions, the spark gap is a commercially accurate device for measuring crest voltage in the testing of insulating materials and insulating structures.

In American practice the **needle gap** and the **sphere gap** have been used. The needle gap has the advantages of being readily extemporized and of being several times as long (for a given voltage) as the sphere gap, but its sparking distance is appreciably affected by variation of atmospheric humidity; that of the sphere gap, within the useful range of gap length, is only very slightly affected. The needle gap is now regarded as inferior to the sphere gap. The former A.I.E.E. sphere-gap tables have been extended down-

ward to include spheres 20 mm in diameter to cover the range 10 to 30 kv, rms, for which A.I.E.E. standards formerly recommended the needle gap.

For higher voltages the sphere gap has the advantages of occupying less space, of freedom from time lag, and of breaking down with a sharply defined spark discharge without previous formation of corona, provided the gap length does not greatly exceed the sphere diameter. The sphere gap has the further advantage that the electrodes do not have to be renewed after each discharge, as they do with the needle gap.

CONSTRUCTION OF SPHERE SPARK GAP. Fig. 42 shows the arrangement recommended by Farnsworth and Fortescue. The top sphere is stationary but slightly adjustable in height so as just to make contact with the lower sphere when it is set for zero separation. The lower sphere is mounted on a piece of brass tubing which carries a threaded bushing on its lower end. This bushing works on a carefully threaded rod having a pitch of two threads per centimeter. The bushing being graduated to fiftieths on its circumference, separation may be measured to the nearest 0.01 cm directly.

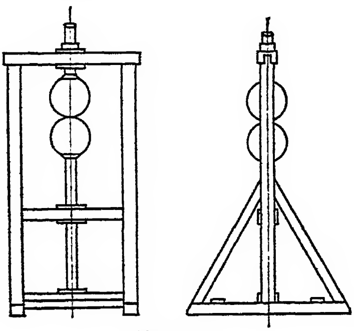

Fig. 42

The following specifications for the two forms of gap are taken from A.I.E.E. Standards No. 4 (May, 1928), Measurement of Test Voltage in Dielectric Tests.

NEEDLE GAP. The needle spark gap shall be between new sewing needles, supported axially at the ends of linear conductors at least twice as long as the gap. There must be a clear space around the gap for a radius at least twice the gap length.

The sparking distances in air between No. 00 double-long sewing-needle points for various rms sinusoidal voltages shall be assumed to be as shown in Table III.

Table III. Needle-gap Spark-over Voltages

At 25 deg cent and 760 mm barometric pressure.

Sinusoidal rms kilovolts	Millimeters	Sinusoidal rms kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33	50	90
30	41		

The values in this table refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

A.I.E.E. STANDARD SPHERE SPARK GAP. The standard sphere spark gap shall be between two suitably mounted spheres. No extraneous conducting body, other than the supporting shanks, should be nearer the gap than twice the sphere diameter. Insulating bodies, such as supporting structures, should not be nearer than one sphere diameter to the gap. The shanks should not be greater in diameter than $^1/_5$ the sphere diameter. Metal collars, etc., through which the shanks extend, shall be as small as practicable and shall not, during any measurement, come closer to the spheres than the maximum gap length used in the measurement. The sphere curvature measurement by a spherometer should not vary more than 1 per cent from that of a true sphere of the required diameter. In using the spherometer to measure curvature, the distance between the points of contact of the spherometer feet shall be within the limits indicated in Table IV.

Table IV. Spherometer Specifications

Diameter of Sphere in mm	Distance between contact points, in mm	
	Maximum	Minimum
20	12.5	7.5
62.5	35	25
125	45	35
250	65	45
500	100	65
750	135	80

Table V. Sphere Gap Sparkover Voltages. International Electrochemical Commission, 1935

Crest Values at 25° C and 760 mm Pressure

Crest Kilovolts	Sparking Distances in Millimeters, One Sphere Grounded					
	Diameter of Sphere, Millimeters					
	62.5	125	250	500	750	1000
	For crest voltages below 50 kw see insert below					
50	16.9	16.8				
60	21.0	20.7	20.5			
70	25.7	24.4	.....			
80	31.1	28.8	28.0			
90	37.9	33.0	.....			
100	46.5	37.4	35.5			
120		47.0	43.5			
140		58.0	51.5	51.5		
160		71.3	60.0	59.5		
180		89.5	69.0	67.5		
200			78.5	76.0	75.0	75.0
220			88.5	84.0	83.0	.....
240			99.0	92.0	91.0	.....
250			.....	.....	.....	95.5
260			111.5	100	99.5	.....
280			126	109	108	.....
300			142	118	116	115
320			159	127	124	.....
340			177	137	132	.....
350				.....	.....	135
360				147	141	.....
380				157	150	.....
400				168	159	155
420				179	168	.....
440				192	179	.....
450				.....	.....	177
460				206	189	.....
480				221	199	.....
500				236	209	199
520				252	220	.....
540				268	231	.....
550				.....	.....	222
560				285	242	.....
580				306	254	.....
600				330	267	247
620				358	280	.....
640					293	.....
650					.....	273
660					305	.....
680					319	.....
700					334	300
720					350	.....
740					368	.....
750					.....	328
760					385	.....
780					402	.....
800					422	360
820					442	.....
840					465	.....
850					.....	393
860					490	.....
880					512	.....
900					537	430
950						469
1000						514
1050						561
1100						622
1150						683
1200						751

Crest Kilovolts	Sparking Distances, Millimeters Sphere Diameter, mm	
	20	62 5
14	3.8	4.0
16	4.4	....
18	5.1	
20	5.8	6.0
22	6.5	....
24	7.2	....
26	7 9	
28	8.75	....
30	9.6	9 4
32	10.6	....
34	11.8	....
36	13.0	....
38	14.14	....
40	15.9	13.1

NOTE.—This table has been given instead of the A.I.E.E. table in Standard No. 4, because the latter is now (1936) in process of revision, and the I.E.C. figures are known to be more accurate. For sine waves, the rms voltage is 0.707 times the crest voltage. For example, 500 kv crest voltage corresponds to 353.5 rms voltage, the sparking distance being 236 mm with 500 mm spheres.

The standard sphere gap should be set up for use in a space comparatively free from external dielectric fields. Care should be taken that conducting bodies forming part of the circuit, or at circuit potential, are not so located with reference to the gap that their dielectric fields are superimposed on that of the gap; for example, the protective resistor should not be arranged so as to present large surfaces near the gap, even at a distance of twice the sphere diameter. If one sphere is grounded the spark point of this sphere should be not less than 5 and not greater than 10 times its diameter from any grounded surface. When sphere gaps are used as specified, with corrections properly applied, the accuracy of voltage measurement should be approximately 2 per cent.

SPHERE-GAP SPARKING DISTANCES. The sparking distance between spheres for various crest voltages shall be assumed to be as shown in Table V, when the air density is that corresponding to a barometric pressure of 760 mm mercury and a temperature of 25 deg cent. For any other air density the voltage at which the gap breaks down, for any given spacing, is equal to the breakdown voltage for this spacing at standard pressure and standard temperature multiplied by the correction factor given in Table VI. The relative air density is found from the formula:

$$\text{Relative air density} = \frac{0.392b}{273 + t} \tag{23}$$

where b = barometric pressure in millimeters and t = temperature in degrees centigrade. To determine the gap spacing for a required spark-over voltage, divide the required voltage by the correction factor obtained from Table VI and use the new voltage thus

Table VI. Air-density Correction Factors for Sphere Gaps

Relative Air Density	Diameter of Standard Spheres in Millimeters					
	20	62.5	125	250	500	750
0.50	0.573	0.547	0.535	0.527	0.519	0.517
0.55	0.617	0.594	0.583	0.575	0.567	0.565
0.60	0.661	0.640	0.630	0.623	0.615	0.613
0.65	0.705	0.686	0.677	0.670	0.663	0.661
0.70	0.748	0.732	0.724	0.718	0.711	0.709
0.75	0.791	0.777	0.771	0.766	0.759	0.757
0.80	0.833	0.821	0.816	0.812	0.807	0.805
0.85	0.875	0.866	0.862	0.859	0.855	0.854
0.90	0.917	0.910	0.908	0.906	0.904	0.903
0.95	0.959	0.956	0.955	0.954	0.952	0.951
1.00	1.000	1.000	1.000	1.000	1.000	1.000
1.05	1.041	1.044	1.045	1.046	1.048	1.049
1.10	1.082	1.090	1.092	1.094	1.096	1.097

obtained to find the corresponding spacing from Table V, using a graph of the latter, if more convenient.

PRECAUTIONS IN SPHERE-GAP MEASUREMENTS. With the exception of its own support or parts, all objects at ground potential should be separated from all live parts of the apparatus under test by a distance in inches not less than the quotient obtained from dividing by 5000 the maximum test voltage which will be applied.

As a precaution against overvoltage oscillations, and to limit the resulting current at the time of breakdown, a non-inductive resistance of about 1 ohm per volt of test voltage to be used should be inserted in series with the sphere gap. If the test is made with one terminal grounded, the entire resistance should be in series with the non-grounded electrode; if neither terminal is grounded, one-half of the resistance should be in series with each electrode. In either case this resistance shall be electrically as near the spark gap as possible and not in series with the apparatus under test. Water-tube resistors are used for this purpose. Carbon resistors of high resistivity should not be used because their resistance may become very low at high voltages.

The sphere gap should be set up for use in a space comparatively free from external dielectric fields. Conducting bodies forming part of the circuit, or at circuit potential, should not be so located with reference to the gap that their dielectric fields are superimposed on that of the gap. For example, the protective resistor should not be arranged to present large surfaces toward the gap, even at a distance of twice the sphere diameter.

If one sphere is grounded the spark point of this sphere should be not less than 5 and not more than 10 times its diameter from any grounded surface.

It is important that the surfaces of the spheres be kept free from dust and moisture, and that the spheres do not become heated appreciably by repeated discharges.

Irradiation of the gap with ultraviolet light considerably reduces the scattering of the values of breakdown voltage, and is particularly useful with high frequencies and essential with impulse voltages. For this purpose an open arc or a quartz mercury-vapor lamp may be used.

The sphere gap is more sensitive than the needle gap to momentary rises of voltage, and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap.

For additional detailed information on the characteristics of the testing equipment, its arrangement, and methods for voltage control, consult the latest edition of A.I.E.E. Standards for the Measurement of Test Voltage in Dielectric Tests.

17. CORONA VOLTMETER

The corona voltmeter, originated by Whitehead, has certain advantages over the sphere gap for the measurement of crest voltages. For a discussion of this instrument and a bibliography, see Brooks and Defandorf, *Bur. Standards J. Research*, vol. 1, p. 589, 1928; reprinted as Bur. Standards Research Paper 21.

18. OSCILLOGRAPHS

GENERAL. An oscillograph is essentially an instrument for observing rapid variations in voltage or current. These variations are automatically plotted (usually against time) on a suitable screen or (if a permanent record is desired) on a photographic film.

The moving system of the oscillograph may be a conducting loop or an iron vane (the galvanometer oscillograph); a single conducting filament (Einthoven's string galvanometer); two insulated conducting filaments (the electrostatic oscillograph); or an electron beam (the cathode-ray oscillograph). The moving system is situated in a magnetic or electric field and is actuated either by changes in its own current or potential or by changes in the field in which it lies. The inertia of the moving system is kept small in order that it may follow very rapid variations in the phenomenon to be observed. An oscillograph will record without appreciable distortion only those events which have frequencies much less than the natural frequency of its moving element. Distortion of high-frequency harmonics in a wave varies with the type of moving system and with its damping but is generally negligible for frequencies up to $^1/_5$ of the natural frequency of the vibrator.

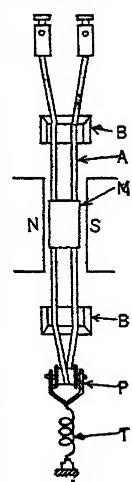

Fig. 43

The **moving-coil oscillograph** is ordinarily used for frequencies up to 2000 cycles per second and, where high sensitivity is required, may be used with a suitable amplifier. The **cathode-ray oscillograph** may be used at any frequency, but its chief application is at frequencies beyond the range of the moving-coil oscillograph. The string galvanometer is useful as an oscillograph at low frequencies where extremely high sensitivity is required.

MOVING-COIL OSCILLOGRAPH. The moving-coil oscillograph is essentially a galvanometer of very short period. It is used for voltage observations with a suitable multiplier, or for current observations with a shunt. Single-phase and three-phase wattmeter oscillograph elements give directly a record of instantaneous power. Current and wattmeter elements are interchangeable in the same mountings.

The vibrator element (see Fig. 43) is usually a loop *A* of very light ribbon of silver alloy or aluminum stretched across ivory bridges *BB* and passed around a pulley *P*. Tension is applied to the loop and may be varied by a spring *T* and screw arrangement attached to the pulley. A small mirror *M* is cemented to the loop between the bridges. The vibrator element with its mounting is immersed in a damping liquid and may be removed from the oscillograph as a unit for repair or replacement. It is placed in the narrow gap between the pole pieces *NS* of a magnet. Older designs use an electromagnet, but recently these have been replaced by small permanent magnets which provide an intense field with a large reduction in weight and with the advantage of requiring no d-c source.

Instead of the carbon arc formerly used, most recent designs use a low-voltage fila-

ment lamp operated at a considerable overvoltage only at the instant of making a photographic record. Light from this source is reflected by the vibrator mirror to the slit of the recording camera or to an oscillating mirror, driven by a synchronous motor, which projects the oscillogram as a standing wave, with linear time scale, on a screen or tracing table. Some models are equipped for simultaneously viewing and photographing a wave. A zero line is obtained on the oscillogram by interposing a small stationary mirror in the light beam just above and in front of the vibrator mirror. A photographic record may be made either with a film wound around a rotating drum or with film or sensitized paper fed at constant speed from a magazine past the camera slit.

Oscillographs are made with from 1 to 9 vibrator elements. Completely self-contained portable instruments are available with from 1 to 6 elements and weighing up to 60 lb. Oscillographs which are completely automatic in operation are made for use in recording switching and other disturbances in power systems. The disturbance may be made to start the record in less than a single cycle. The duration of the record may be predetermined. Such oscillographs are used with magazines taking 200 ft of sensitized paper and will record without attention as many as 100 disturbances.

SENSITIVITY OF MOVING-COIL OSCILLOGRAPH. The sensitivity of the oscillograph vibrator varies greatly with its free period and damping, increasing with greater free period and with diminished damping. The following table gives the sensitivity of some available elements together with their natural (undamped) frequencies.

Table VII. Sensitivity of Oscillograph Elements

Natural Frequency, cycles per second	Sensitivity, radians per ampere
8000	0.2
5000	0.4
3000	1.2
1200	12.0

The resistance of a vibrator element is ordinarily about 1 ohm. The distance from the vibrator mirror to the screen varies in different models from about 25 to 65 cm.

Distortion may occur in the higher-frequency components of an oscillogram since both the sensitivity and the phase displacement of the vibrator with respect to the applied voltage change with frequency.

Assuming unit deflection for a given steady direct voltage, the maximum deflection θ for a sine-wave voltage of any frequency and having the same crest value is given by the formula

$$\theta = \frac{1}{[p^4 + p^2(4N^2 - 2) + 1]^{1/2}} \tag{24}$$

where $p = f/f_r$ is the ratio of the applied frequency to the resonance frequency of the vibrator and $N = n/n_c$ is the relative damping.

The phase displacement of the vibrator with respect to the applied voltage is given by the formula

$$\tan \Phi = \frac{2Np}{1 - p^2} \tag{25}$$

The phase displacement of a harmonic of frequency f_h with respect to its fundamental of frequency f_0 is a measure of phase distortion in an oscillogram. This distortion δ, measured in degrees of the fundamental, is given by the formula

$$\delta = \frac{f_0}{f_h}\Phi_h - \Phi_0 \tag{26}$$

The resonance frequency to be used in these formulas is not that which would be found in air (without appreciable damping) but about half that value. The damping liquid is to some extent moved with the vibrator, adding to its effective inertia and thus increasing its natural period. This increase in period depends on the material of the vibrator element, being about 40 per cent for the usual silver-alloy loop and nearly 60 per cent for aluminum, and should be added to the air-damped period which is usually stated by the manufacturers.

Table VIII gives values computed from the above formulas for a damping (70 per cent of critical damping) which gives nearly the minimum distortion in oscillograms. The symbols are explained above. The last column gives typical values for the nth harmonic ($n = 50p$) of a 60-cycle oscillogram for the element commonly used, having a resonance frequency in air of about 5000 cycles per second.

Table VIII. Performance of Oscillograph Vibrator for Damping Equal to 70 Per Cent of Critical Damping

Ratio of Applied Frequency to Resonant Frequency p	Ratio of Deflection at Applied Frequency to Deflection with Direct Current θ	Phase Displacement of Vibrator in Degrees Φ	Phase Distortion $\frac{0.02}{p}\Phi - 1.60$ in Degrees δ
0.01	1.000	0.80	0.00
0.02	1.000	1.60	0.00
0.1	1.000	8.05	0.01
0.2	1.000	16.2	0.02
0.3	0.998	24.8	0.05
0.4	0.991	33.7	0.08
0.5	0.975	43	0.12
0.6	0.947	53	0.16
0.7	0.905	63	0.19
0.8	0.850	72	0.20
0.9	0.785	81	0.21
1.0	0.714	90	0.20
1.5	0.394	121	0.01
2.0	0.243	137	−0.23

The actual damping of the vibrator element depends on the viscosity of the damping liquid and therefore on its temperature. It can be determined for any particular operating condition with a single observation, as follows: A constant direct voltage is impressed on the element and then suddenly removed. If the element is underdamped it will deflect past its equilibrium zero position and will oscillate about zero. The amount of the first overshoot past zero is measured and its ratio to the initial d-c deflection is computed. If this ratio is x the relative damping is given by the formula

$$N = \frac{-\log_e x}{[\pi^2 + (\log_e x)^2]^{1/2}} = \frac{-\log_{10} x}{[1.86 + (\log_{10} x)^2]^{1/2}} \tag{27}$$

CATHODE-RAY OSCILLOGRAPH. The moving system of the cathode-ray oscillograph is a stream of electrons. It is therefore practically without inertia and will respond equally to any frequency.

In an evacuated vessel (see Fig. 44), electrons emitted from a cathode A are accelerated by a suitable voltage toward an anode B which has a small hole in its center. Those electrons passing through the hole form an "electron beam" of sensibly uniform velocity coinciding with the axis of the tube. The beam passes between parallel metal deflection plates C and strikes a fluorescent screen D where it gives rise to a luminous spot. If a photographic film is substituted for the screen, a latent image (subject to development by ordinary means) appears where the beam strikes the active emulsion of the film. In the presence of an electric or magnetic field at C, at right angles to the axis of the tube, the electrons are accelerated perpendicularly to their direction of motion in the beam, resulting in the deflection of the spot at D. Phenomena to be examined are made to produce such a deflecting field at C. A time axis is supplied to the oscillogram by a second field, perpendicular to the first, and varying in a known manner with time.

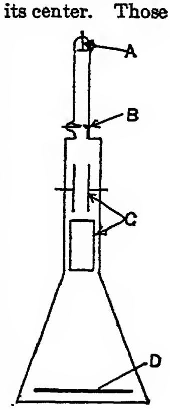

FIG. 44

There are two general types of cathode-ray oscillograph. The **hot-cathode oscillograph** utilizes the electron emission of an incandescent filament. The beam is formed by a relatively low voltage impressed between cathode and anode, usually between 300 and 3000 volts. The **cold-cathode oscillograph** utilizes the electrons freed from the surface of the cathode under positive ion bombardment in a gas discharge tube. The voltage required to generate the beam is high, 50,000 volts or more. In any case the diameter of the beam, and consequently that of the recording spot, are primarily determined by the diameter of the hole in the anode through which the beam passes. This is generally a few tenths of a millimeter.

Sensitivity of Cathode-Ray Oscillograph. The electrostatic sensitivity of the oscillograph is inversely proportional to the cathode-anode voltage and is given approximately by the formula

$$S = \frac{LA}{2VD} \tag{28}$$

in which S = deflection per unit difference of potential between the deflection plates.
L = length of parallel plates (C in figure).
A = distance from plates to screen.
V = potential difference between cathode and anode of the tube.
D = separation of parallel plates.

The sensitivity of available low-voltage (hot-cathode) tubes ranges from 0.2 mm/volt to 1 mm/volt; and for high-voltage (cold-cathode) tubes is about 0.04 mm/volt.

The electromagnetic sensitivity is inversely proportional to the square root of the cathode voltage. It depends on the shape, size, and position of the deflection coils at C and may amount to 1 mm/ampere-turn for a low-voltage tube and 0.1 mm/ampere-turn for a 50,000-volt tube.

Recording Speed. For the low-voltage tube operating at less than 1000 volts, photographic records may be made from the fluorescent screen with a camera only if the trace on the screen be repeated many times. Exposures of some seconds are necessary. The exposure time may be greatly reduced by placing the film in direct contact with the rear surface of the glass window on which the fluorescent material is deposited, but only with a very considerable loss in clearness and definition of the resulting record. For a tube operating at 2000 volts, records of a single passage of the recording spot across the fluorescent screen may be made with a camera when the speed of tracing is less than 10 meters per second.

High-voltage cold-cathode tubes, in which the photographic film is placed in the evacuated chamber and comes in direct contact with the electron beam, are conveniently divided into two classes.

In one type the voltage is applied to the cathode continuously. The electrons are focused on the hole in the anode by a coil situated between the cathode and anode, providing a magnetic field parallel to the axis of the tube. The vacuum in the discharge tube may be controlled by an adjustable gas leak in the side of the tube. Its value is indicated by reading the discharge current with a milliameter. An intense beam is formed in this way and is kept off the photographic plate by an electrostatic field or some suitable beam-blocking device (e.g., the Norinder relay) until the instant the record is to be made. For this type of oscillograph, operating under the best conditions, no practical limit to the recording speed has been found. Records have been made in which the recording spot moved across the film at a velocity of 10^{10} cm per sec ($^1/_3$ of the velocity of light).

In a second type of cold-cathode oscillograph, voltage is applied to the cathode only at the instant the record is to be made. The intensity of the beam is controlled by the gas pressure in the discharge tube. This pressure is read by means of a hot-wire vacuum gage. For a given vacuum, the reading of such a gage depends on the nature of the gas and may be greatly affected by the gases and water vapor given off by the photographic film in the vacuum chamber. Hence the beam intensity is not as closely under control as in the other type of tube and cannot in general be made as great. The maximum recording velocity, depending on the beam intensity, may be made to vary over wide limits as the pressure in the discharge tube is changed. The beam intensity may be made great enough for any except very high recording velocities by ensuring that the cathode is clean and well polished and by adjusting the vacuum in the discharge tube until an intense beam is secured.

A theoretical limiting condition for recording a cathode-ray oscillogram without distortion is that no appreciable change shall take place in the field deflecting the electron beam during the time taken by an electron to pass through the field. This time varies with the voltage used to generate the beam and with the length of the deflection plates. If deflection plates 1 cm long are used in an oscillograph in which the beam is generated at 50,000 volts, this time of passage through the deflecting field is about 10^{-10} sec. Hence this theoretical limit need not be considered in any ordinary engineering applications.

Life of Cathode-ray Oscillograph. The life of the low-voltage hot-cathode oscillograph is determined by the life of the filament and is 200 hours or more. In one type the cathode filament is replaceable. This type requires the continuous use of a pump to maintain the proper vacuum. Other types are adjusted and permanently sealed off when made and must be discarded when the filament burns out.

The electron-beam intensity of the high-voltage cold-cathode oscillograph falls off rapidly as the cathode becomes pitted by the discharge. The discharge tube of the oscillograph must then be replaced, or in some models the cathode may be removed from the tube and polished. When the cathode surface is restored to its initial condition, the oscillograph will again operate satisfactorily. In a cold-cathode oscillograph which is excited only at the instant of recording, a very heavy discharge is necessary to insure an intense beam, and the integrated life of the cathode is a few seconds. This, however,

may amount to a thousand exposures or more when working with high-speed phenomena. The continuously excited discharge tube utilizes a much lower current and secures beam intensity by the focusing action of a magnetic field. The useful life of the cathode is roughly 50 hours.

Circuits Used with Cathode-ray Oscillograph. When using low-voltage, high-sensitivity oscillographs the maximum voltage which may be impressed on the deflection plates is about 50 to 100 volts. The maximum deflection-plate voltage for the high-voltage oscillograph is roughly 1000 volts. Above these limits, imposed by the size of the screen or film, a potential divider, made of resistors or capacitors or some combination of the two, must be used. The choice of a divider requires detailed examination to ensure that two conditions are satisfied; namely, (1) the divider must make no appreciable change in the character of the circuit, that is, the impedance of the divider should be very great compared with that of the circuit; (2) the form of the wave impressed on the deflection plates must be the same as that impressed on the potential divider. Account must also be taken of distributed capacitance, capacitance to ground, and capacitance of the deflection plates; or the parts of the divider must be so arranged that the effect of these capacitances is negligible. The divider must be arranged so that the deflection plates do not differ greatly in potential from their surroundings in order to avoid spurious induced effects in the oscillogram.

In general, capacitance dividers are used for high-frequency and impulse phenomena; resistance dividers are used for low and intermediate frequencies, being limited in their application by two considerations, namely, (1) at high frequencies, unavoidable stray capacitance will change the ratio of voltage division; (2) for impulse work, especially with steep wave fronts, the rate at which potential can change on the deflection plates is limited by the resistance in series with them. The oscillogram recorded may show only the exponential charge and discharge, through a series resistor, of the capacitor formed by the deflection plates, regardless of the form of the voltage wave impressed on the divider. This effect can be reduced by decreasing the resistance of the divider; but, as the resistance decreases, the effect of the divider on the circuit becomes greater.

A number of auxiliary circuits are needed in obtaining cathode-ray oscillograms. The time axis of the oscillogram may be furnished by an oscillator which sweeps the beam repeatedly across the recording film in simple harmonic motion of known frequency. The resulting oscillograms are often difficult to interpret, especially if the phenomenon observed continues over a number of cycles of the timing wave. There is, however, an advantage in simplicity of the circuit, especially if the observed phenomenon is one whose occurrence cannot be predetermined, since the timing motion is continuous and need not be synchronized with the phenomenon. Another type of timing motion much used is a single sweep across the film obtained by impressing the voltage of a capacitor across the timing plates while it is being charged or discharged through a resistor. Such a time scale is exponential. A linear time scale may be obtained by charging the capacitor at a constant rate through a vacuum tube. Such timing circuits must be synchronized with the phenomenon to be observed. A number of relay circuits have been developed for this purpose. If the cathode is continuously excited, another relay circuit may be needed to actuate the beam-blocking device at the proper instant. If the cathode is excited only at the time of recording, the cathode excitation must also be properly timed.

For low-frequency work the switching may be mechanical in the timing and relay circuits. When used with high frequencies or impulse voltages, the timing and relay circuits are usually actuated by the breakdown of spark gaps. This timing may be made to operate in 10^{-8} second.

For details of timing and relay circuits, their calibration and uses, see Bibliography.

STRING GALVANOMETER USED AS OSCILLOGRAPH. The string galvanometer can be used for low-frequency phenomena where extremely high sensitivity is required. It consists essentially of a fine conducting filament, stretched between the pole pieces of a powerful electromagnet. The filament is illuminated through a microscope mounted in a hole bored in one pole piece. Its motion is observed through a second microscope mounted in the opposite pole piece, and it appears as a shadow against a luminous background. Magnifications up to 1000 diameters are used in observing the motion.

For a photographic record, the image of the string is projected on a slit placed parallel to the direction of motion of the image. A sensitized film behind the slit is moved at constant speed to provide a time axis. The film may be mounted on a rotating drum for short records, or fed across the slit from a reel for continuous records. An incandescent filament lamp may be used for film speeds up to 400 mm per sec at a magnification of 500 diameters. At higher speeds an arc lamp must be used.

As a rule the string is a gold-coated glass or quartz fiber about 0.001 mm in diameter, with a resistance greater than 1000 ohms. It is stretched over bridges, and its tension

is varied by a suitable slow-motion screw and lever arrangement. The string and mounting may be removed from the galvanometer as a unit. Models are available in which two or more strings can be used simultaneously with the same optical system, and in one model six strings are mounted in one optical system for simultaneous use.

Sensitivity of String Galvanometer. By changing the tension of the string the sensitivity and damping may be varied over wide limits, the sensitivity being increased by decreasing the tension with a corresponding increase in the free period. The sensitivity of a typical instrument with a 14-cm string, using a magnification of 600 diameters, is 3000 mm per μa when the string is adjusted to a period of 0.5 sec and 100 mm per μa at a period of 0.008 sec. Instruments are available with strings as short as 0.5 cm with a free period of less than 5×10^{-5} sec. Some instruments are so constructed that a number of strings of varying length can be used interchangeably, covering a wide range of sensitivity and free period.

19. TESTING OF MAGNETIC MATERIALS

MAGNETIC CHARACTERISTICS. The usual routine tests of magnetic materials consist of (1) determination of normal-induction curves and hysteresis loops, (2) determination of core loss, (3) determination of a-c permeability and core loss at low inductions, and (4) determination of the permeability of slightly magnetic materials. In commercial practice the specifications of the American Society for Testing Materials are generally followed. As these specifications are revised periodically the latest edition should be consulted.

Normal Induction. The induction produced by a given magnetizing force depends upon the previous magnetic condition of the specimen and upon the mode of approach to the given magnetizing force. In order to obtain consistent and reproducible results it is therefore necessary to follow a definite test procedure. The effect of previous magnetic history can be removed by demagnetization, that is, by subjecting the specimen to a succession of reversals of magnetizing force gradually decreasing from a certain maximum value to one somewhat lower than the lowest at which a determination is to be made. Demagnetization should be started from an initial value of magnetizing force well above that corresponding to maximum permeability, but not necessarily higher than any previously experienced by the specimen. After demagnetization, points on the induction curve are obtained by observing the induction caused by a given magnetizing force after a sufficient number of reversals to bring the material to a cyclic condition, thus closing up the hysteresis loop. After a determination has been made, further measurements can be made at higher values of magnetizing force without demagnetizing again, but if a lower point is desired, the demagnetization process must be repeated. Values of induction so obtained are called "normal induction" and are reproducible for a given specimen within the limits of experimental error. The normal-induction curve is the locus of the tips of a succession of hysteresis loops as illustrated in Fig. 45.

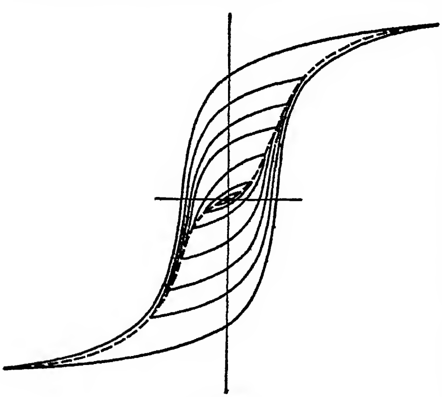

FIG. 45

Normal Hysteresis. When points on a normal hysteresis loop are to be determined, cyclic condition is first obtained by a number of reversals of the magnetizing force corresponding to the desired tip, and the induction is noted when the magnetizing force is reduced to some lower value either in the same or opposite direction. Before each determination, the material is brought back to a cyclic condition by reversals of the maximum magnetizing force. This procedure differs somewhat from the "step-by-step" method, but gives more consistent and reliable results.

MEASUREMENT OF NORMAL INDUCTION AND HYSTERESIS. CLASSIFICATION OF METHODS AND APPARATUS. Determinations of normal induction and hysteresis are ordinarily made by methods which may be classified according to the way the magnetic induction is determined, as (*a*) magnetometric, (*b*) traction, (*c*) air-gap, and (*d*) ballistic.

In the magnetometric method the induction is measured in terms of the deflection of a suspended magnetic needle placed in a definite position with respect to the test sample. The method is especially suitable for observing slow changes in induction, for instance, such as occur when a specimen is heated or cooled. The magnetometer, being very delicate and sensitive to variations in the earth's field, is not recommended for general magnetic testing.

In the traction method the induction is estimated in terms of the mechanical force of attraction between the surfaces of two magnetized bodies, one of which is the test specimen. Careful machining of the specimens is required, and the results are not of high accuracy. Traction methods are not in general use.

In the air-gap method the magnetic circuit is closed, except for a transverse air gap in which is located some measuring device, such as a deflecting coil, a rotating armature, or a bismuth spiral. The method is subject to rather large and uncertain errors.

The ballistic method employs a ballistic galvanometer (or fluxmeter) for the measurement of magnetic induction. Ballistic methods are the ones most generally used for both laboratory and shop testing. The A.S.T.M. Standards (edition of 1933) recommend for the determination of normal induction and hysteresis one of the following methods according to the type of data required and the nature of the material to be tested: (1) the ring method; (2) the Burrows permeameter; (3) the Fahy Simplex permeameter; (4) the isthmus method; (5) the Babbitt (or J) permeameter; and (6) the Simplex high-magnetization adapter.

Ring Method. The ring method is simple in principle and the easiest to apply, but it is not often used for routine testing because of the cost of preparing and winding the specimens. A diagram of connections is shown in Fig. 46. The magnetizing winding is

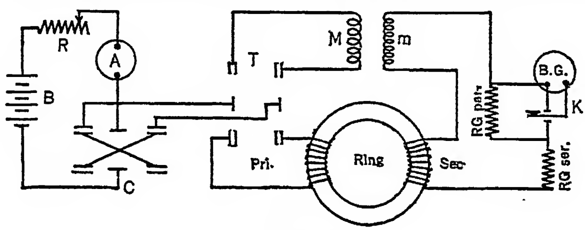

FIG. 46

uniformly distributed around the ring, and the magnetizing force in oersteds (gilberts per centimeter) is calculated from the formula

$$H = \frac{0.4\,\pi NI}{L} \tag{29}$$

in which N = total number of turns.
I = current in amperes.
L = mean circumference of ring in centimeters.

The difference in concentration of the winding over the inner and the outer part of the ring causes the magnetizing force to vary over the cross-section, with an error in the result which depends upon the ratio of the mean diameter to the radial width of the ring. If this ratio is greater than 10, however, the error can usually be neglected.

The magnetic induction is measured by the deflection of a ballistic galvanometer connected to a suitable test coil wound on the specimen, preferably under the magnetizing winding. When the flux linked with this test coil is changed, an electromotive force is induced, and a certain quantity of electricity, proportional to the change in flux, flows through the galvanometer. If the flux is reversed in direction by reversing the magnetizing current, the galvanometer deflection will be proportional to twice the flux.

The galvanometer is calibrated by means of a standard mutual inductor, the secondary of which is permanently connected in series with the test coil. It is usually convenient to adjust the sensitivity of the galvanometer by means of suitable series and parallel resistances so that the scale is direct reading in terms of magnetic induction. It is customary to make 1 cm deflection correspond to the reversal of an induction of 1000 gausses. The calibrating current to be reversed in the primary of the mutual inductor depends upon the value of the mutual inductance, the number of turns in the test coil, and the cross-section of the specimen, and is calculated from the following formula:

$$I_c = \frac{BAN}{M \times 10^8} \tag{30}$$

in which I_c = calibrating current in amperes.
B = induction in gausses.
A = cross-sectional area in square centimeters.
N = number of turns in the test coil.
M = mutual inductance in henrys.

It is often convenient to use 100 turns in the test coil. When points on the hysteresis loop are to be determined, an auxiliary series resistance is suddenly inserted in the magnetizing circuit. In this case, since the galvanometer is calibrated for reversal of the induction, the readings must be multiplied by 2.

Burrows Permeameter. It is generally most convenient to prepare samples in the form of straight bars of moderate length and uniform cross-section. Of the various testing methods adapted to such samples the most accurate one, for samples of uniform permeability along their length and having a maximum permeability not exceeding 5000, is the Burrows compensated double-yoke method. In this method auxiliary compensating coils provide the extra magnetomotive force required for the yoke and joints in the magnetic circuit, the current in them being adjusted so as to produce a uniform induction in the test bar. Fig. 47 shows the magnetic circuit of the Burrows permeameter and the relative positions of the magnetizing and the test coils. The test rod and its auxiliary, which should be of the same size and material, are joined at the ends by soft-iron yokes which make good magnetic joints. The magnetizing coils T and A are located over the test rod and auxiliary, respectively. Coil J is in four sections, connected in series, and located over the ends of the rods as near to the joints as possible. In operation, the currents in these three windings are so adjusted before each reading that there is equal flux in the two rods, and no leakage from the greater part of the test rod; for this

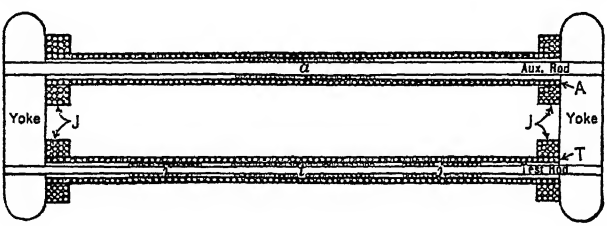

FIG. 47

condition, the value of the applied magnetizing force can be calculated from the current and number of turns per centimeter in the solenoid surrounding the test rod. For testing the compensation and determining the value of the induction when the compensation is properly adjusted there are three test coils designated as t, a, and j, respectively. These coils are each of the same number of turns and are distributed as shown in the figure; t is wound over the middle of the test bar, a over the middle of the auxiliary bar, and j half over one end and half over the other end of the test bar far enough away from the yokes and joints to avoid disturbances from these causes. When the reversal of the magnetizing currents causes no deflection of the galvanometer whether connected to t and a or to t and j in series-opposition, the magnetizing currents are properly adjusted, the induction may be measured in terms of the deflection of the galvanometer connected to t alone, and the magnetizing force is proportional to the current in the winding T. As usually constructed, the magnetizing coil is so wound that the magnetizing force is 100 times the current in amperes. A diagram of connections is given in Fig. 48. The switches ST, SA, and SJ serve to insert resistances in the corresponding circuits during the determination of points on the hysteresis loop. ST should make contact on the forward point before breaking at the other. The calibration of the ballistic galvanometer is carried out as indicated for the ring method.

In all ballistic methods a correction must be made to the observed value of induction because of the flux in the space between the specimen and the test coil. The amount to be subtracted from the observed value of induction is proportional to the magnetizing force H and depends upon the relative areas of the test coil and specimen. It is equal to kH where

$$k = \frac{a - A}{A}$$

a = area of test coil
A = area of specimen

This "air correction" may be eliminated by means of an adjustable mutual inductor having its primary in series with the main magnetizing winding and its secondary in series with the main test coil. If desired, the two main magnetizing coils may be connected in series and the average values for the two samples determined by connecting the two middle test coils in series and using them as one. In this arrangement, four auxiliary test coils near the ends of the specimens are connected in series and used as one coil for adjusting the current in the compensating magnetizing coils.

The Burrows permeameter is an absolute instrument in that its constants can be derived from measurements of its dimensions, and it is therefore suitable for standardization work under proper conditions. It is extremely sensitive to variations in permeability along the length of the specimen, and the results obtained on inhomogeneous material should be interpreted with this fact in mind.

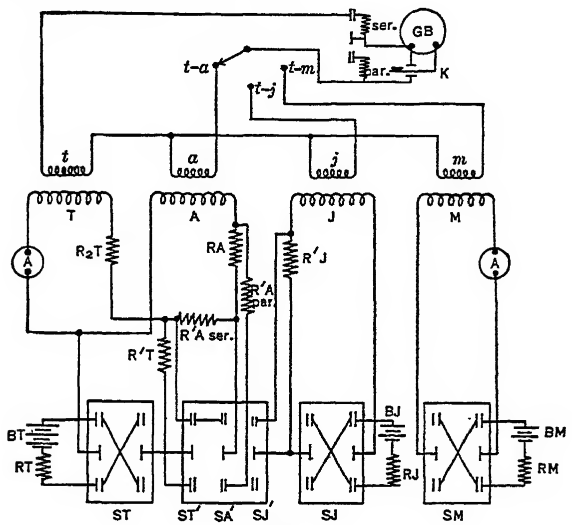

FIG. 48

Fahy Simplex Permeameter. This permeameter requires but a single specimen. The magnetizing force is applied by means of an electromagnet, across the poles of which the specimen is clamped. A uniformly wound test coil extends over the whole length of the specimen. A ballistic galvanometer connected to this test coil indicates the induction in the specimen when the magnetizing current is reversed. The magnetizing force is measured by means of a test coil uniformly wound on a non-magnetic form and extending between two iron blocks which are clamped to the ends of the specimen.

When the magnetizing force is changed in value, the deflection of a ballistic galvanometer connected to this coil is proportional to the change in magnetic potential between the ends of the coil. The function of the iron blocks is in effect to transfer the ends of the test coil to the ends of the specimen. When the magnetic circuit is properly constructed of suitable materials, this method is capable of giving very satisfactory results. It does not require an auxiliary specimen, and no compensation is necessary. It is also less sensitive than the Burrows permeameter to the effect of magnetic inhomogeneity along the length of the specimen. The galvanometer is calibrated by means of a standard mutual inductor as in the methods described above, and the correction for air-flux linked with the test coil but not in the specimen is applied.

Isthmus Method. The isthmus method was developed originally for the measurement of saturation values of intrinsic induction. It is based on the theory that the magnetizing force just outside the surface of a specimen is equal to that within. Two modifications of this method have been developed at the Bureau of Standards. The first modification is suitable for tests within the range from 100 to 3000 oersteds. It employs an electromagnet with its pole pieces pierced coaxially to receive a specimen 6 mm in diameter. The du Bois form of electromagnet has been found most convenient. The gap between the pole pieces is 2 cm long. Surrounding the specimen are two con-

centric and coextensive test coils each having the same number of turns. The inner coil is of as small diameter as possible (about 7 mm); the outer coil has a diameter of approximately 12.5 mm. It is essential that the proper dimensions of specimen, gap, and test coils be used in order to ensure uniformity of the magnetizing force throughout the space occupied by the test coils. Normal-induction measurements are made upon reversal of the magnetizing current. When the galvanometer is connected to the two coils in series-opposition, the deflection upon reversal is proportional to the magnetizing force. When the galvanometer is connected to the inner coil alone, the deflection upon reversal is proportional to the induction in the specimen. Special switching arrangements are required for the determination of points on the hysteresis loop.

Since it is often inconvenient to prepare specimens 6 mm in diameter, a second modification of the isthmus method has been developed to accommodate test specimens of rectangular cross-section. The range of this apparatus is from 100 to 1000 oersteds. The test specimens are clamped in pole pieces between two similar U-shaped yokes of laminated silicon steel upon which the magnetizing coils are wound. The length of the gap and the dimensions of the test coils are so chosen that the magnetizing force throughout the space occupied by the test coils is substantially uniform. The coil surrounding the specimen for the measurement of induction is sufficiently large to accommodate the largest specimen for which the apparatus is adapted. In order to obviate the necessity for making a large air-flux correction a compensating coil having the same value of area-turns is connected in series-opposition. The compensating coil and the test coil for the measurement of magnetizing force are wound on a common form located within the uniform part of the field between the pole pieces.

Babbitt (or J) Permeameter. This permeameter is designed especially for testing in the range 40 to 1000 oersteds. The specimen is clamped between the poles of a laminated yoke made up of different kinds of material such as permalloy, silicon steel, and ordinary iron in such proportions that the resultant permeability is practically constant throughout the range of flux density carried by the yoke. Compensation for the yoke reluctance is effected by a winding surrounding the yoke and connected in series with the main magnetizing winding around the test specimen. The value of induction is determined ballistically by means of a test coil surrounding the specimen. The air correction is eliminated by means of an auxiliary test coil placed within the magnetizing coil but not surrounding the specimen. The area-turns of this auxiliary coil are adjusted to equal the area-turns of the main test coil to which it is connected in series-opposition. The readings thus give values of intrinsic induction ($B - H$). The value of magnetizing force is determined either in terms of the magnetizing current (using an experimentally determined constant) or by means of an air coil mounted within the magnetizing solenoid. Care must be exercised in the use of this instrument to avoid excessive heating of the specimen.

Simplex High-Magnetization Adapter. This is an attachment to be used with the Fahy Simplex permeameter. By its use the span of the instrument is reduced to 1.25 in. Test coils similar to those used with the regular instrument, but shorter, serve for the determination of induction and magnetizing force. According to the A.S.T.M. specifications, this apparatus can be used for the determination of maximum induction, B_m, residual induction, B_r, and coercive force H_c, of materials having high coercive force with the reservation that the observed values of coercive force may be low by from 0 to 10 or 12 per cent, the error generally increasing with the thickness of the specimen.

NOTE. At the 1935 meeting of the A.S.T.M. two additional instruments were tentatively approved for tests at magnetizing forces in the range of 100 to 2500 oersteds, namely the Simplex Super-H Adapter and the Saturation Permeameter.

GENERAL PRECAUTIONS. In magnetic testing the following precautions should be observed: (1) in order to obtain consistent and reproducible results, the specimen (and yoke) must be thoroughly demagnetized before the measurements are made; (2) stray fields, especially from parts of the electrical circuit in which current changes during an observation, should be avoided (this can generally be accomplished by twisting together the current-carrying conductors and also the secondary circuits); (3) specimens should be clamped so as to be free from mechanical strain; (4) heating of the specimen should be avoided; (5) specimens should be protected from mechanical vibration.

ACCURACY OF MAGNETIC TESTS. The most common sources of error in magnetic testing are (1) lack of uniformity in permeability along the length of the specimen; (2) mechanical strain; and (3) temperature effects. Errors due to these causes may be large and cannot be estimated with any degree of certainty. The methods mentioned above may be expected to give the following accuracies for specimens having a satisfactory degree of uniformity, clamped so as to be free from strain and kept at a constant temperature within 5 deg cent: For normal induction up to the point of maximum permeability,

values of magnetizing force corresponding to given values of induction should be accurate within 3 per cent provided that the permeability does not exceed 10,000. This limitation as to permeability does not apply to the ring method. Values of induction corresponding to given values of magnetizing forces above the point of maximum permeability should be accurate within 1 per cent. For residual induction, values should be accurate within 2 per cent and for coercive force within 5 per cent, except in the case of the Simplex Adapter as noted above. For magnetically inhomogeneous material the accuracies specified above cannot be obtained with the Burrows permeameter. The Fahy Simplex permeameter and the Babbitt permeameter, each used within its proper range, give values more nearly representative of the average properties of non-uniform material. Unless a specimen has been tested for uniformity, it is usually safe to assume that it is non-uniform.

TESTING MATERIALS OF LOW PERMEABILITY. GENERAL LABORATORY METHOD. The methods described above are not adapted to the testing of materials of very low permeability. The commercial importance of such tests lies in the relationship that has been found between the magnetic permeability and the resistance to corrosion of certain types of corrosion-resistant steel. The permeability of such materials ranges from about 1.01 to about 5.0. A ballistic method is generally employed. A straight solenoid is used which has a test coil of several hundred turns mounted within it, and in which the specimen can be inserted. Because of the very low value of magnetization in the specimen, the self-demagnetizing effect is so small that it can be neglected. A variable mutual inductor is used, with its primary connected in series with the magnetizing solenoid and its secondary in series-opposition with the test coil. This mutual inductor is so adjusted as to balance the mutual inductance between the solenoid and the test coil. This adjustment is made with no specimen in the test coil, so that there is no deflection of the ballistic galvanometer when the primary current is reversed. The galvanometer can then be used at its maximum sensitivity. When a specimen is inserted within the test coil and the magnetizing current is reversed, the deflection of the galvanometer is proportional to the intrinsic induction of the specimen. The magnetizing force is calculated from the current and the constants of the solenoid in the usual way.

The Fahy Low-Mu Permeameter. This instrument is a modification of the one described above. The variable mutual inductor is replaced by an air coil mounted within the magnetizing solenoid and adjacent to the test coil. The value of area-turns of this air coil is slightly greater than that of the test coil. The value of effective area-turns of the air coil is adjusted by means of a variable resistance connected in parallel with the coil. The adjustment of the shunt resistance is correct when the galvanometer connected to the two secondary coils is not deflected upon reversal of the magnetizing current. The deflection obtained upon reversal with a specimen inserted in the test coil is proportional to the intrinsic induction ($B - H$). The magnetizing force is measured by means of the auxiliary coil with the shunt resistance disconnected. Calibration is carried out with a standard mutual inductor in the usual way as described under the ring method.

Measurement of Core Loss

The use of the wattmeter method for the measurement of core loss is now almost universal. Large quantities of electrical sheet are purchased annually on specifications as to their core-loss values. In the interest of uniformity, producers and consumers of this type of material have standardized on the Epstein method as specified by the American Society for Testing Materials.

A.S.T.M. 1933 STANDARD CORE-LOSS TESTS. Test Specimens. Test specimens shall consist of 10 kg (22 lb) of strips 50 cm (19 $^{11}/_{16}$ in.) in length and 3 cm (1 $^{3}/_{16}$ in.) in width cut with sharp shears from two or more sheets taken at random from shipment. One-half of the strips shall be cut parallel to the direction of rolling, and the other half at right angles to this direction. (See Note 3.) It is recommended that a test specimen shall represent not more than 5000 kg (11,000 lb). The strips shall be cut symmetrically from the sheet as nearly as may be practicable.

Place on the balance a pile of test strips weighing 2.5 kg. Add a second pile of the same kind, bringing the weight up to 5 kg. In each case the weight shall be taken to the nearest strip. Add in succession two piles of 2.5 kg each, of the other kind of strips, bringing the weight up to 7.5 kg and 10 kg, respectively. Care should be taken to keep separate the strips which are cut at different directions of the grain.

NOTE. If desired, for convenience, the bundles may be loosely secured by tape or string (not wire).

NOTE 2. If desired, a 5-kg specimen may be used for routine acceptance tests, provided it is cut from at least two sheets, except in case of a dispute between the manufacturer and the purchaser, when the 10-kg specimen shall be used.

NOTE 3 (Adopted by A.S.T.M., 1935). When the material is to be used with the flux in any one direction with respect to the direction of rolling or has preferred properties in any one direction with respect to the direction of rolling, strips may be cut in a specified direction if there is mutual agreement between manufacturer and purchaser. It must always be specifically stated when tests are so made. Only one type of strip will than be necessary for such tests.

Apparatus (Epstein). The magnetizing winding shall consist of four solenoids surrounding the four sides of the magnetic circuit and joined in series. A secondary coil shall be used for energizing the voltmeter and the voltage coil of the wattmeter.

These solenoids shall be wound on a form of any non-magnetic, non-conducting material of the following dimensions: inside cross-section, 4 by 4 cm; thickness of wall, not over 0.3 cm; winding length, 42 cm. The primary winding on each solenoid shall consist of 150 turns of copper wire uniformly wound over the 42-cm length. The total resistance of the magnetizing winding shall be between 0.3 and 0.5 ohm. The secondary winding of 150 turns of copper wire on each solenoid shall be similarly wound beneath the primary winding. Its resistance shall not exceed 1 ohm.

A voltmeter and the voltage coil of a wattmeter shall be connected in parallel to the terminals of the secondary winding of the apparatus. The current coil of the wattmeter shall be connected in series with the primary winding. A low-power-factor wattmeter shall be used.

Test Procedure. The standard core loss shall be measured under the following conditions: The magnetic circuit shall consist of 10 kg (22 lb) of the test material, one-half parallel and one-half at right angles to the direction of rolling, made up into four equal bundles, two containing material parallel and two containing material at right angles to the direction of rolling, and finally built into the four sides of a square with butt joints and opposite sides consisting of material cut in the same manner. No insulation other than the natural scale of the material (except in the case of scale-free material) shall be used between laminations, but the corner joints may be separated by tough paper approximately 0.1 mm (0.004 in.) in thickness.

An electromotive force having a wave form approximating a sine curve shall be applied to the primary winding and adjusted until the voltage of the secondary circuit is that determined by the equation.

$$E = \frac{4ff\,NfBM}{4lD10^5} \tag{31}$$

where ff = form factor of primary emf = 1.11 for sine wave.
N = number of secondary turns = 600.
f = number of cycles per second = 60.
B = maximum induction in kilogausses = 10 kilogausses.
M = total mass in grams = 10,000 grams.
l = length of strips in centimeters = 50 cm.
D = density in grams per cubic centimeter, = 7.5 grams per cm^3 for high-resistivity steel, 7.7 grams per cm^3 for low-resistivity steel.
E = 106.6 volts for high-resistivity steel for sine voltage, 103.8 volts for low-resistivity steel for sine voltage.

NOTE. For the 14-kilogauss test the voltages will be as follows: E = 149.2 volts for high-resistivity steel for sine voltage, 145.2 volts for low-resistivity steel for sine voltage.

The form factor of the applied electromotive force shall not vary more than 1 per cent from the value of 1.11. If desired, an average-value voltmeter can be substituted for or used in conjunction with the rms voltmeter.

NOTE. See G. Camilli, A flux voltmeter for magnetic tests, *J. A. I. E. E.*, 1926, vol. 45, p. 989; also B. M. Smith and C. Concordia, Measuring core loss at high densities, *Elec. Eng.*, 1932, vol. 51, p. 36.

As an alternative to the use of a sine wave, the correct value of a non-sinusoidal voltage may be determined by applying it to an Epstein apparatus containing a sample of similar material which has been standardized by means of a sine-wave electromotive force. The wattmeter is brought to the correct reading for the standard sample. The corresponding voltage is the correct one to use. This alternative method may be modified by comparing standard and test samples by means of a differential wattmeter. For this test, the secondary voltage needs to be only approximately correct. When using these comparison methods, the form factor of the induced voltage wave should not differ from 1.11 by more than 5 per cent.

For silicon steels, a density of 7.7 grams per cm^3 shall be assumed if the silicon content is 2 per cent or less and a density of 7.5 grams per cm^3 shall be assumed if the silicon content is above 2 per cent.

For nickel-iron alloys the density shall be assumed from the nickel content as given by the straight lines joining the points defined as follows:*

Nickel, per cent	Density, grams per cm^3
0	7.85
30	8.00
100	8.90

For other alloys a density corresponding to the actual measured or estimated value shall be used.

Apply the alternating voltage to the primary coil and tap the joints together until the current has a minimum value, as shown by an ammeter in series. Then clamp the corners firmly by some suitable device.

Shunt the ammeter and adjust the primary current until the voltmeter indicates the proper value. This adjustment may be made by an auto-transformer, by varying the field of the alternator, or by both, but not by the insertion of resistance or inductance in the primary circuit. Simultaneously, the frequency shall be adjusted to 60 cycles.

Read the wattmeter when the voltage and frequency are correct. The wattmeter gives the power consumed in the iron and the secondary circuit. The loss in the secondary circuit is given in terms of the total resistance and voltage. This loss $= E^2/R$ where E is the secondary rms voltage and R is the parallel resistance of the voltmeter and the shunt circuit of the wattmeter. Subtract from the wattmeter reading the loss in the secondary circuit, which will be constant for any set of instruments and voltage, and divide by the weight of the sample in kilograms. The result is the standard core loss.

NECESSITY FOR A-C PERMEABILITY AND CORE-LOSS DATA AT LOW INDUCTIONS. In current transformers, and in many forms of apparatus used in communication, magnetic core material is run at very low values of flux density ranging from 10 gausses (or less) to 1000 gausses. The ordinary testing methods are not suitable for testing at such low inductions. The A.S.T.M. specifications describing two methods suitable for testing at low inductions are as follows:

A.S.T.M. 1933 STANDARD TESTS AT LOW INDUCTIONS. Scope. These methods provide for the determination of the a-c permeability and core loss of magnetic materials at low inductions. These properties are indicative of the effect of the material on the impedance and loss characteristics of apparatus in which the core materials are worked at low inductions.

Standard Values. Tests shall be made either at a frequency of 60 cycles per second or at 1000 cycles per second. Standard values at 60 cycles per second are those corresponding to a maximum induction of 10 gausses or 1000 gausses, according to the use for which the material is intended. At 1000 cycles per second, the standard values are those corresponding to a maximum induction of 10 gausses.

NOTE. Apparatus often operates under conditions in which the alternating magnetization is superposed upon a steady unidirectional magnetization, but it is not necessary to introduce this complication into the standard tests, as the relative quality of materials is adequately indicated by the simple a-c tests.

Test Specimens. Test specimens shall consist of not less than 1 kg (2.2 lb) of the test material cut with sharp shears into strips 25 cm (9 27/32 in.) in length and 3 cm (1 3/16 in.) in width, one-half parallel and one-half at right angles to the direction of rolling, made up into four equal groups, two containing material parallel and two containing material at right angles to the direction of rolling. The four groups shall then be assembled in the test coils to form a square with the corners so arranged that the layers alternately butt and lap. The opposite sides of the square shall consist of material cut in the same manner. When testing for core loss, the separate laminations shall be effectively insulated from one another.

The tests may be made either by the bridge method or by the a-c potentiometer method, both of which are specified below.

Power Supply. The supply voltage shall be of approximately sine-wave form, having not more than 10 per cent of total harmonics present.

Test Winding. The test winding shall consist of four solenoids surrounding the four sides of the magnetic circuit and connected in series. These solenoids shall be wound on forms of any non-magnetic, non-conducting material of the following dimensions: inside cross-section, 3.2 by 1 cm; thickness of wall, not over 0.3 cm; winding length,

* For 50 per cent nickel content the density is 8.26 grams per cm^3. For 80 per cent nickel content the density is 8.64 grams per cm^3

17 cm. There shall be two windings on each solenoid. The inside winding shall consist of 25 turns of insulated copper wire uniformly wound over the 17-cm length, and the total series resistance of the four coils shall not exceed 0.05 ohm. The second winding shall consist of 250 turns per solenoid uniformly wound over the first, and the total series resistance shall not exceed 2 ohms.

Apparatus for Bridge Method. The apparatus shall consist of the following (see Fig. 49): Any voltmeter capable of indicating the voltage supplied to the bridge at the test frequency with an accuracy of at least ±2 per cent may be used. A voltmeter of the multi-range vacuum-thermocouple type is recommended. The **fixed condenser,** C_3, shall be a high-grade mica condenser having a capacitance of 1.0 μf ±0.25 per cent, and a power factor of not more than 0.0015 at 1000 cycles per second. The **adjustable condenser,** C_4, shall be a three-decade condenser having steps of $9 \times (0.1 + 0.01 + 0.001)$ μf adjusted to an accuracy of ±(0.25 per cent + 50 $\mu\mu f$), and having a power factor of not more than 0.0015 at 1000 cycles per second. In addition, C_4 shall include a single 1-μf mica condenser of the same accuracy, with suitable means for inserting it in the circuit. The **fixed resistor,** R_1, shall be a non-reactive resistor of 10 ohms ±0.1 per cent, with a time constant not to exceed 1 microsecond, or a 100-ohm resistor of the same percentage accuracy. (Time constant is defined as the inductance in henrys divided by the resistance in ohms, or the shunt capacitance in farads multiplied by the resistance in ohms.) The **adjustable resistor,** R_4, shall be a four-decade non-reactive resistor having steps of $10 \times (1{,}000 + 100 + 10 + 1)$ ohms adjusted to an accuracy of ±(0.1 per cent +0.05 ohm) with a time constant not to exceed 1 microsecond. Any **zero-current detector** having the necessary sensitivity may be used. For the 60-cycle test, a vibration galvanometer is recommended; for the 1000-cycle test a telephone receiver is recommended. A single-stage vacuum-tube amplifier preceding the galvanometer or receiver is recommended when it is necessary to increase the sensitivity. The **supply transformer,** T, shall be a suitable transformer providing efficient and distortionless inductive coupling and substantial electrostatic isolation between the source of supply and the bridge network.

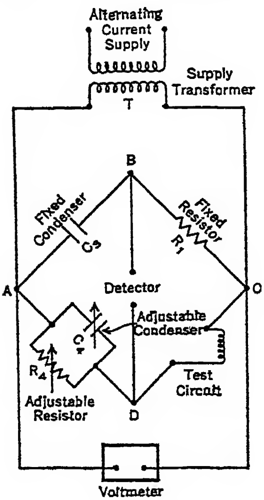

FIG. 49

Procedure for Bridge Method. For the 60-cycle test on exceptionally high-permeability low-loss material, such as hipernik or permalloy, use the 25-turn windings and $R_1 = 10$ ohms. For silicon steel, and materials of similar magnetic properties, use the 250-turn windings and $R_1 = 100$ ohms. For the 1000-cycle test use the 25-turn windings and $R_1 = 10$ ohms for both classes of material. The test specimen shall be thoroughly demagnetized before testing.

Set the voltage, E, to give the desired induction in accordance with the relation

$$E = 0.0707AB \tag{32}$$

where B = the maximum induction in gausses.

A = the cross-sectional area of the specimen in square centimeters, calculated from values of length, mass, and density. Values of density either may be assumed as indicated on p. 78, or may be experimentally determined.

Adjust R_4 and C_4 to balance the bridge. Then

$$\mu = \frac{0.795R_4}{A}\cdot\frac{R_1}{N^2}\cdot 10^4 \tag{33}$$

For $N = 100$ and $R_1 = 10$

$$\mu = \frac{7.95R_4}{A} \tag{34}$$

For $N = 1000$ and $R_1 = 100$

$$\mu = \frac{0.795R_4}{A} \tag{35}$$

and the loss

$$P = E^2\left[C_4\omega^2(C_3R_1) - R_{dc}\left(\frac{1}{R_4^2} + C_4^2\omega^2\right)\right] \tag{36}$$

where P = the total loss in watts.
A = the cross-section in square centimeters.
C_4 = the value of adjustable condenser in farads.
ω = 2π times the frequency in cycles per second.
N = the number of turns.
R_{dc} = the resistance of winding in ohms.
R_4 = the value of adjustable resistance in ohms.

NOTE. P divided by the mass in kilograms gives the loss in watts per kilogram.

Apparatus for Potentiometer Method. The apparatus shall consist of the following (see Fig. 50): The a-c potentiometer shall be of the coordinate type, indicating voltage in terms of two components having a quadrature phase relation. A suitable phase-shifting device shall be provided to adjust the phase of either the potentiometer supply or the magnetizing current. The resistor, R, shall be a non-inductive resistor of such a resistance as to give a potential drop of suitable magnitude. The resistances should be adjusted to an accuracy of ±0.1 per cent, with a time constant not to exceed 1 microsecond.

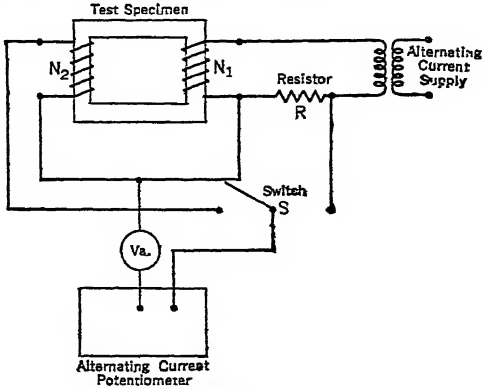

FIG. 50

Procedure for Potentiometer Method. Use the 100-turn winding for the primary and the 1000-turn winding for the secondary, unless the secondary voltage exceeds the range of the potentiometer. Under these conditions, which may occur for the 1000-gauss test, the two windings may be interchanged. The test specimen shall be thoroughly demagnetized before testing.

Throw the switch, S (Fig. 50), so as to connect the potentiometer to the secondary winding and adjust the phase so that the total voltage is read on the in-phase dial, the quadrature dial being set to zero. Set the voltage E to give the desired induction in accordance with the relation:

$$B = \sqrt{2}\pi f N_2 AB \times 10^{-8} \tag{37}$$

where the symbols have the same significance as in the bridge method; f = frequency in cycles per second; and N_2 = number of secondary turns.

Throw switch S to connect the potentiometer to the resistor, R, and read the voltage. The power component is then indicated by the in-phase voltage, and the magnetizing component is indicated by the quadrature voltage. The corresponding values of current are obtained by dividing the voltage by the resistance, R. Denoting the in-phase and quadrature currents by I_P and I_Q, respectively:

$$\mu = 56.3 \frac{B}{N_1 I_Q} \tag{38}$$

and

$$P = \frac{N_1}{N_2} E I_P \tag{39}$$

where the symbols have the same significance as above, and N_1 = number of primary turns. P divided by the mass in kilograms gives the loss in watts per kilogram.

20. TESTING OF INSULATING MATERIALS

The most important electrical properties of an insulating material are its dielectric strength, resistivity, dielectric constant, and dielectric loss. Some of the common methods of measuring these quantities are described below.

Measurement of Dielectric Strength

The dielectric strength of an insulating material is defined as the puncturing voltage per unit thickness, the thickness being measured either in centimeters or in mils (1 mil = 0.001 in.). The voltage required is in general higher than can be conveniently obtained from d-c sources of emf, and alternating emf's are therefore used in such tests.

The materials to be tested are placed between electrodes connected to the high-voltage terminals of a transformer which receives power from a low-voltage alternator.

TRANSFORMERS AND ALTERNATORS FOR DIELECTRIC TESTING. For making routine tests of dielectric strength, any well-designed high-voltage transformer connected to a suitable a-c supply may be used. For the purpose of comparison and computation of the maximum emf from the effective value, it is essential that the high-voltage transformer deliver as nearly a pure sine-wave voltage as possible. Both transformer and alternator should be large enough to operate with good voltage regulation at all testing loads, so that no serious distortion of the wave form will be produced by the charging current. The transformer and source of supply should have a rating of at least 2 kva for voltages under 50,000 volts, and at least 5 kva for voltages above 50,000 volts. For ordinary tests the frequency should not exceed 100 cycles per second.

Special tests, or tests calling for extremely high voltages or frequencies, will require special equipment.

CONTROL OF VOLTAGE. The method of voltage control should be such that the high voltage from the secondary of the testing transformer can be raised gradually from any point. In no case should the control of voltage be by discontinuous steps of more than 500 volts each. The control may be effected by means of generator field regulation, an induction regulator, a variable-ratio auto-transformer, or a combination of these methods. By using a variable-ratio auto-transformer and varying the field excitation of the alternator, the variation of field saturation in the alternator may be reduced to a minimum and the shape of the voltage wave kept almost constant over a wide range of secondary voltage. Any method of regulating the voltage may be considered satisfactory for ordinary work which does not distort the wave more than 10 per cent from a sinusoidal shape.

MEASUREMENT OF VOLTAGE. The voltage impressed upon the material under test may be determined by an adjustable spark gap shunted across the electrodes, a voltmeter and multiplier connected across the electrodes, an electrostatic voltmeter connected across the electrodes, a voltmeter connected across the low-voltage terminals of the transformer, the readings of which are to be multiplied by the step-up ratio of transformation, a voltage transformer with a voltmeter connected across its low-voltage terminals, or a special voltmeter winding placed in the middle of the high-voltage windings of the step-up transformer. The needle-spark-gap method, though convenient because of its indication of the maximum rather than the effective value of the voltage, is tedious in use, its readings are dependent upon the circulation of air, the condition of the needles, the proximity of neighboring objects, etc. The sphere spark gap is preferable; see Spark Gaps for Measuring High Voltages. The use of the voltmeter and multiplier, though flexible and convenient, is not recommended because of the load which is placed upon the transformer and the possibility of leakage in the multiplier. The use of an electrostatic voltmeter is desirable, except that at high voltages the moving element must be immersed in oil, making the instrument sluggish in action. The instrument is also liable to breakdown. In any method involving the ratio of transformation of the transformer, results which are based on the assumption that this ratio is constant are questionable.

The most satisfactory and accurate measurements of voltage are obtained, as a rule, with the separate step-down voltage transformer and voltmeter, or with a voltmeter connected to a special voltmeter coil in the testing transformer.

PROTECTION OF APPARATUS. Some protection of the measuring apparatus is necessary when using voltages such as are commonly employed in making breakdown tests. A circuit breaker should be provided in the low-voltage side of the test transformer to prevent damage to the transformer or alternator when the specimen breaks down. In addition, some protection is needed in the high-voltage circuit of the test transformer, when the voltage is 25,000 volts or over, to prevent dangerous surges of current in this circuit when the specimen punctures. It is desirable, however, to have as much energy available as possible when the puncture occurs. If impedance in the form of choke coils is used in series with the high-voltage terminals, it should not be greater than that which will limit the high-voltage current to double the normal rated current of the transformer. See Precautions in Use of Sphere Gap.

FORMS OF ELECTRODES. The breakdown voltage of a specimen depends to a marked extent upon the size and shape of the electrodes used and the nature of the surrounding medium. Inge and Walther (see Bibliography) have made extensive experimental investigations of the effects of these and other variables upon the breakdown voltage of various substances. If the edges of the electrodes used are not rounded, the increased flux density at the edges produces an excessive stress on the dielectric, and failure of the specimen almost invariably occurs at these points. At higher voltages, say over a few thousand volts, some means must always be provided to prevent corona and flash-

over at the edges of the electrodes; otherwise the resultant heating of the specimen would entirely vitiate the results.

The American Society for Testing Materials recommends that the electrodes for testing sheet materials be 2-in. cylinders, 1 in. in length, with edges rounded to a radius of $^1/_4$ in. For oils, the electrodes recommended are circular disks of polished copper or brass 1 in. in diameter and having square edges. The oil electrodes are mounted in a test cup with their axes horizontal and coincident, with a gap of 0.1 in. between their adjacent faces. By far the most complete single summary of current practice with respect to testing electrical insulating materials will be found in the specifications of this Society (see Bibliography).

METHOD OF APPLYING VOLTAGE. Materials may be tested by impressing the puncturing voltage, (1) instantaneously; (2) as a continuously increasing voltage, at a specified rate, beginning with an initial low voltage; (3) in steps of a specified value held for a given duration of time. The method of application of voltage is a significant factor in the comparison of breakdown voltages. It is customary to use method (2), with a rate of increase of voltage of approximately 1000 volts per second, or method (3), with steps of 500 volts applied for 1 minute each. Procedure with respect to these last two points is not as yet completely standardized.

CAUSES OF VARIATIONS IN RESULTS. The puncturing voltage of an insulating material is affected by its previous history, its precise condition when tested, its size and thickness, its temperature, the nature of the electric field to which it is subjected, the frequency of the applied voltage, and the method by which the puncturing voltage is applied. For crystalline materials such as mica the puncturing voltage (expressed as volts per centimeter or volts per mil) is independent of temperature and increases with decreasing thickness of the specimen. For fibrous and synthetic materials the puncturing voltage is probably affected more by temperature than by any other factor. In any case it is necessary to prevent corona discharge and flashover at the edges of the electrodes. This is commonly accomplished by immersing the specimen in oil. The nature of the electric field to which the specimen is subjected is largely determined by the size of the specimen, the size and shape of the electrodes used, and the nature of the surrounding medium. Results of dielectric-strength tests are not immediately comparable unless obtained under strictly comparable experimental conditions.

Measurement of Insulation Resistance

The method commonly used is one of substitution, that is, the deflection of a galvanometer is noted when a standard resistance R_s (0.1 or 1 megohm as the case may be) is inserted in series with the galvanometer and a source of emf and again when the unknown resistance R_x is inserted in the same circuit. A galvanometer should be used the sensitivity of which is such as to give deflections of the order of 1 mm at a scale distance of 1 meter, for a current of 10^{-9} amp. The deflections of the galvanometer should be proportional to the current. The galvanometer should be provided with an Ayrton-Mather universal shunt (see Galvanometer Shunts) such that the current under measurement will be a known multiple, K, of the galvanometer current. The shunt factor K may be 10, 100, etc. Voltages from 100 to 500 volts may be used in determining the unknown insulation resistance. Because of the difficulty of producing and maintaining resistance standards of extremely high values it will generally be necessary not only to decrease the sensitivity of the galvanometer, but also to decrease the voltage used when calibrating the galvanometer with the standard resistance, in order to keep the galvanometer deflection within the scale limits.

The test is made by observing the shunt factor, the voltage, and the galvanometer deflection when the standard resistance is inserted in the circuit and again when the unknown is inserted. The unknown resistance is then calculated from the equation,

$$R_x = R_s \frac{K_s E_x D_s}{K_x E_s D_x} \tag{40}$$

where, when measuring the specimen, K_x is the shunt factor, E_x the applied voltage, and D_x the galvanometer deflection; K_s, E_s, and D_s represent the corresponding quantities when employing the standard resistance, and R_s represents the value of the standard resistance. With such an arrangement resistances of 10^4 to 10^{11} ohms can be measured, the accuracy being of the order of 10 per cent.

Where the resistance is too high to make the deflection method feasible, the leakage current through the specimen is allowed to charge a condenser for a known interval. The condenser is then discharged through the galvanometer. Then

$$R_x = \frac{Et}{KD} \tag{41}$$

where E is the applied voltage, t the time of charge, K the ballistic constant, and D the deflection. By this method resistances from 10^{11} to 10^{15} ohms can be measured.

Resistances from 10^{15} to 10^{17} ohms can be measured if a quadrant electrometer is used to measure the current through the unknown resistance. The equation for R_x is identical in form with that given above for the ballistic-galvanometer method; E is the applied voltage, t is the time required to produce a deflection V, and K is the quantity of electricity necessary to produce unit deflection. By this method R_x can be determined at any instant.

The **volume resistivity** of a material is defined as the resistance to the current flowing through the material between two opposite faces of a centimeter cube. The **surface resistivity** of a material is defined as the resistance to the current flowing between opposite edges of a surface film which is 1 cm square. When results are to be expressed in terms of volume resistivity or of surface resistivity it is necessary to use electrodes of known dimensions, in good contact with the specimen. Metallic electrodes, wrapped in tin foil and carefully pressed against the specimen, may be used. Where the nature of the specimen permits, the specimen may be floated on a pool of mercury with an upper electrode of mercury contained within a suitable dam. Great care must be taken to provide suitable guard rings in making such measurements, for a discussion of which see paper by H. L. Curtis, Insulating properties of solid dielectrics, *Bull. Bur. Standards*, 1915, vol. 11, p. 359; reprinted as Bur. Standards Sci. Paper 234. In any case it is essential to note the temperature of the material, the relative humidity, the voltage used, and the time of application of the voltage. Since these resistance values in general vary with time after the application of voltage, it has become customary to take the resistance 1 minute after the application. Since surface resistivity is largely a surface-film phenomen, the past history of the sample, such as exposure to sunlight, moisture, corrosive vapors, etc., will materially affect its value.

Self-contained instruments are available for measuring insulation resistance (see Ohmmeters).

Measurement of Dielectric Constant, Dielectric Loss, and Power Factor

The dielectric constant of any substance is defined as the ratio of the capacitance of a condenser in which the substance in question fills the space between the plates to the capacitance of the same condenser in vacuum, that is, to the geometric capacitance of the condenser. This ratio, designated by K, may vary with the temperature of the substance, the time after the application of voltage to the dielectric, and therefore with the frequency when tested with alternating voltages. The dielectric constant of most substances decreases as the temperature increases. The dielectric constant in general increases with time after the application of voltage and hence decreases as the frequency increases.

The dielectric losses in a substance are caused by direct conduction through the substance, by true dielectric absorption in the substance, and in some cases by electrolytic conduction through the substance. For most engineering purposes, however, it is unnecessary to differentiate between these various sources of dielectric loss. The total dielectric loss of a substance is expressed in terms of the power factor of that substance. The power factor of a condenser made up with a good dielectric between its plates will be small, that is, the current and voltage will be almost in quadrature. For this reason this property of a dielectric is often defined by the phase-defect angle or dielectric-loss angle which is the small angle by which the current through and the voltage across the test condenser depart from exact quadrature.

The determination of dielectric constant and power factor requires the measurement of the capacitance and power factor of a test condenser in which the space between the plates is first filled with air and then with the substance under investigation. The dielectric constant will be the ratio of the latter capacitance to the former, and the power factor will be the power factor of the condenser when the dielectric is the substance under investigation. The power factor of a properly constructed air condenser will be zero to a high degree of precision. In the case of solids it is preferable to use a test condenser whose capacitance can be computed. It should be remembered that the computed capacitance will be in electrostatic units and the measured capacitance will be in electromagnetic units. For a parallel-plate condenser the dielectric constant is given by the formula

$$K = \frac{11.3Ct}{S} \tag{42}$$

where K is the dielectric constant, C is the measured capacitance (in micromicrofarads)

of the test condenser made from the substance under test, t is the thickness of the specimen in centimeters, and S is the area of the condenser plates in square centimeters.

MEASUREMENT OF DIELECTRIC CONSTANT AND POWER FACTOR AT RADIO FREQUENCIES. The measurement of dielectric constant and power factor at radio frequencies calls for special technique. In Fig. 51, which shows a circuit for this purpose, L represents a suitable radio-frequency inductor, C_s a standard adjustable condenser of known or negligible losses, C_x a condenser made from the material under test, T and G a shielded thermo-element galvanometer. The galvanometer G should be a sensitive low-resistance instrument whose deflections within reasonable limits are proportional to the square of the current in the heating element. The points 1, 2, 3, 4, 5, 6 are points of a special mercury-cup switch into which are inserted short-circuiting links or the terminals of special non-inductive resistors.

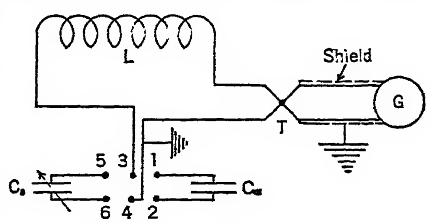

FIG. 51

Short-circuiting links are inserted between the points 1–3 and 2–4, a suitable radio-frequency generator of known frequency is coupled inductively to the coil L, and the circuits are tuned to resonance. The deflection of the galvanometer is then noted when the short-circuiting links are in place and again when a series of suitable non-inductive resistors are inserted between the points 1–3 and 2–4. The equivalent resistance of the circuit is then computed from the equation

$$R = \frac{R_1}{[d_0/d_1 - 1]^{1/2}} \tag{43}$$

where R is the equivalent resistance of the circuit, R_1 is the resistance inserted, d_0 is the galvanometer deflection with the short-circuiting links inserted, and d_1 the galvanometer deflection with R_1 inserted.

Short-circuiting links are now inserted between 3–5 and 4–6 and the circuit tuned to resonance with C_s, the generating circuit being left unchanged. The capacitance of C_s now gives the capacitance of C_x. The equivalent resistance of the circuit containing C_s is now determined as before. The difference between the two equivalent resistances just determined will give the equivalent resistance of the condenser C_x, since C_s is assumed to have negligible resistance.

The power factor in percentage is then computed from the equation

$$\text{Power factor (in percentage)} = 6.283 \times RCf \times 10^{-7} \tag{44}$$

where R is the equivalent resistance of the sample condenser in ohms, C is the capacitance of the sample condenser in micromicrofarads, and f is the test frequency in kilocycles.

The dielectric constant K is computed from the equation

$$K = \frac{11.3Ct}{S} \tag{45}$$

where C is the capacitance of the sample condenser in micromicrofarads, t is the thickness of the specimen in centimeters, and S is the area of the sample condenser (parallel-plate condenser) in square centimeters. For further details of the measurements see the Bibliography.

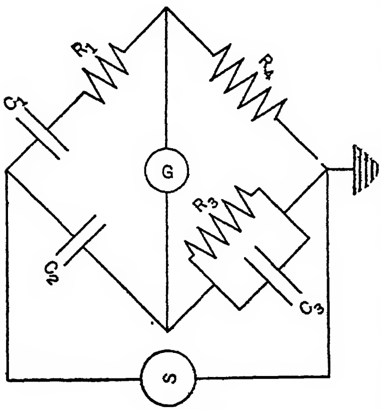

FIG. 52

MEASUREMENT OF DIELECTRIC LOSS AT HIGH VOLTAGES: THE SCHERING BRIDGE. One of the most important measurements in modern high-voltage technology is the determination of the dielectric losses in samples of insulating materials under high dielectric stress. For such measurements the Schering-bridge method has great technical advantages and may now be regarded as the standard method. Referring to Fig. 52, C_1 is the effective capacitance and R_1 the equivalent series resistance of the imperfect condenser to be tested, C_2 is a standard no-loss

condenser which must be capable of withstanding the full test voltage, C_3 is an adjustable condenser, and R_3 and R_4 are non-reactive resistors. When the bridge is balanced so that no current flows through the detector circuit G, it can be shown that

$$R_1 = \frac{C_3}{C_2} R_4 \tag{46}$$

and

$$\tan \theta_1 = \omega R_3 C_3 \tag{47}$$

Here θ_1 is the loss angle of the tested condenser, by which its phase angle differs from 90 deg, and ω is 2π times the frequency. A knowledge of the capacitance of C_2 is not necessary for a determination of the power factor of C_1, but it is essential that the losses of C_2 be negligible or known. In actual practice, C_1 and C_2 are small and consequently these arms are of high impedance in comparison with the two remaining arms. If E is the voltage applied to the bridge, the power loss in the branch containing the unknown condenser (C_1, R_1) is given to a high degree of approximation by the expression

$$\text{Power in watts} = E^2 \omega^2 R_3^2 \frac{C_2 C_3}{R_4} \tag{48}$$

MEASUREMENT OF DIELECTRIC LOSS WITH COMPENSATED ELECTRODYNAMIC WATTMETER. It is sometimes convenient to measure the dielectric loss of a specimen with a compensated electrodynamic wattmeter. The circuit is shown in Fig. 53. The test sample is subjected to a high voltage from a test transformer, and the power absorbed by it is measured by means of a sensitive reflecting wattmeter. The phase angles of the voltage transformer and the wattmeter voltage coil $V.C.$ are compensated by means of a condenser C. The effective capacitance of this condenser may be varied by changing the value of the shunting resistance r. Before making a test a zero-loss air condenser is substituted for the test sample and the resistance r varied until the wattmeter shows zero deflection. The test sample is then substituted for the air condenser and the test repeated, the deflection indicating the power lost in the sample. To measure the current, the voltage coil of the wattmeter is connected across the resistance S, through the variable resistance R_c. This puts the current in the two coils in phase and the instrument will read as an ammeter. The wattmeter is calibrated by direct current, using a potentiometer and standard cell.

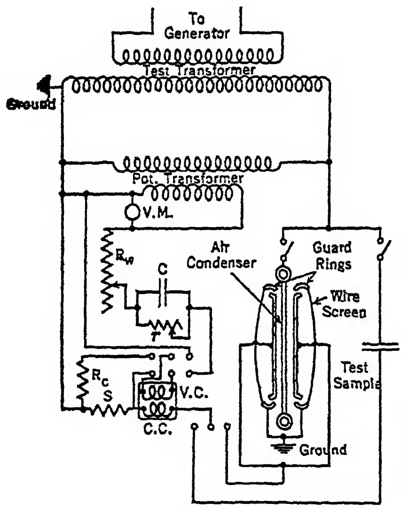

FIG. 53

SHIELDING OF BRIDGE CIRCUITS. In using such a-c bridges as have been described, both at radio frequencies and at power frequencies, the proper shielding of the bridge circuit is of as great importance as the choice of a suitable bridge circuit. See references on Shielding and Guarding Instruments and Circuits in the Bibliography.

USE OF CATHODE-RAY OSCILLOGRAPH IN DIELECTRIC RESEARCH. Although not capable of as great precision as the Schering bridge, the cathode-ray oscillograph is a powerful instrument for the investigation of short-time phenomena in dielectric researches. In using it, a voltage in phase with the voltage across the specimen is applied to one of the pairs of deflection plates, and a voltage in phase with the current through the specimen is applied to the other pair of plates. The electron beam is deflected by the fields resulting from these voltages and, upon striking a fluorescent screen or photographic plate, traces a figure, generally an ellipse, whose shape is determined by the phase relation existing between the deflecting voltages. When a pure sine-wave voltage is applied to an imperfect condenser the beam traces an ellipse from whose area and orientation the power factor can be deduced. See the article on Cathode-Ray Oscillograph.

21. PHOTOMETRY

Considering light as radiant energy evaluated according to its capacity to produce visual sensation, the chief aspects of photometry are the determination of (*a*) luminous intensity (candlepower) of light sources in a single direction, (*b*) candlepower distribution of light sources, usually in the horizontal and vertical planes, (*c*) luminous flux from light sources, (*d*) illumination of surfaces, (*e*) brightness of surfaces, (*f*) reflecting properties of surfaces, and (*g*) transmission properties of substances. Of these (*f*) and (*g*) are determinations of the properties of materials, and together with (*b*) can be made without reference to any standard since they can be expressed as ratios and not in terms of any units. All the other determinations, however, must be made directly or indirectly against primary or fundamental reference standards of light.

PRIMARY STANDARDS OF LIGHT. A primary standard of light is one by which the unit of light is established and from which the values of other standards are derived. A satisfactory primary standard must be reproducible from specifications.

Electric Lamps. Precise photometry dates from the adoption of electric incandescent lamps as standards in 1909. An agreement, entered into in 1909 by the National Bureau of Standards, the National Physical Laboratory, and the Laboratoire Central d'Électricité, brought the photometric units of the United States, England, and France to a single value, within the limits of experimental error. These national standardizing laboratories agreed to maintain the new unit constant and to call it the international candle. This name was formally adopted by the International Commission on Illumination in 1921. It was not specified in the agreement of 1909 how the new unit was to be maintained, but the agreement had been reached by the consideration of results of international comparisons of carbon-filament lamps, and the unit has since been maintained by means of carbon lamps. It is assumed that the average luminous intensity of the lamps maintained by each laboratory has not changed since individual values were assigned to the lamps in 1909. Numerous intercomparisons have shown that this assumption is not ill-founded.

Waidner-Burgess Standard of Light. Electric incandescent lamps maintain their values over long periods of time and are convenient to use, but their use as primary or fundamental standards is not justifiable in theory. The National Bureau of Standards has recently proposed as a suitable primary standard, the Waidner-Burgess standard of light (See *Bur. Standards J. Research*, vol. 6, pp. 1103-1117, June, 1931, reprinted as Bur. Standards Research Paper 325.) It consists of a black-body radiator immersed in platinum heated to a temperature slightly above its melting point and allowed to cool. When the freezing point is reached, the temperature remains constant thereafter until all the platinum is frozen; the temperature of the black body and its brightness also remain constant. The brightness of a black body at the freezing point of platinum (1773.5 deg cent) is reported by the Bureau of Standards to be 58.84 candles per sq cm.

Flame Standards. Standard candles are no longer of practical importance. The name of the unit of luminous intensity was derived from these earliest standards. The only flame standards of practical importance now are the Hefner lamp and the Harcourt 10-cp pentane lamp. These are used, respectively, in Germany and other continental European countries, and in England.

The **Hefner standard lamp** is a wick lamp burning amyl acetate of very high purity. It is equipped with an optical device so that its flame can be adjusted very accurately to the correct height of 40 mm. A variation of 1 mm in the height of the flame causes 2.7 per cent change in candlepower. In its use, allowance must be made for atmospheric pressure and humidity according to the formula

$$I = 1 + 0.0055(8.8 - e) - 0.00011(760 - b) \tag{49}$$

where I is the luminous intensity of the lamp in Hefner Kerze when burning in an atmosphere at a pressure of b millimeters of mercury, with a humidity of e liters of water vapor per cubic meter of air. A correction is also necessary for the amount of CO_2 in the atmosphere, but it is best to obviate the necessity for this correction by arranging efficient ventilation of the room in which the lamp burns.

The **Harcourt 10-cp pentane standard lamp** is essentially an argand burner supplied with pentane-air gas and preheated air. The fuel is formed by passing air over pentane in a saturator box subdivided by baffles. Under standard atmospheric conditions, viz.: a barometric pressure of 760 mm and a humidity of 8 liters of water vapor per cubic meter of air at a temperature of 15 deg cent, the pentane standard lamp is considered to have a luminous intensity of 10 candles. Corrections for differences from the atmospheric conditions specified are made in accordance with the formula

$$I = 10[1 + 0.0052(8 - e) - 0.00085(760 - b) + 0.001(15 - t)] \tag{50}$$

where e and b have the same meaning as above in the case of the Hefner lamp, I is the luminous intensity of the lamp in international candles, and t is the temperature in degrees centigrade.

SECONDARY STANDARDS OF LIGHT. A secondary standard is one calibrated by comparison with a primary standard. For want of a satisfactory reproducible primary standard the unit of light, the international candle, is maintained at the National Bureau of Standards by means of 45 carbon-filament lamps, which are loosely designated as the "primary" standards. Photometric measurements as generally made are all in terms of secondary standards which derive their assigned values from the "primary" lamps whose values were agreed upon in 1909. Certified lamps can be obtained from the National Bureau of Standards and from testing laboratories.

WORKING STANDARDS OF LIGHT. A working standard of light is any standardized luminous source for daily use in photometry. Electric incandescent lamps as working standards are superior to all other types of lamps. After being properly aged, lamps are standardized for candlepower and current at a definite voltage. The standardization can then usually be relied upon as long as the current taken by the lamps has not changed when the lamps are operated at the originally assigned voltages.

VISUAL PHOTOMETRY. In visual photometry, the eye is used to compare the brightnesses of two surfaces. Because of its power of adaptation to different brightnesses, the eye cannot *measure* light with any degree of accuracy. Measurements must therefore depend on a *judgment of equality* of brightness of two surfaces. These surfaces are presented, usually side by side, to the eye for comparison by means of a photometer. Visual photometers now in general use are of three general types, according to the method of comparison employed, viz.: (1) by equality of brightness of two surfaces visible simultaneously, (2) by equality of contrast between two pairs of surfaces visible simultaneously, and (3) by disappearance of flicker when two surfaces are viewed in rapid succession. Besides serving the purpose of presenting two surfaces to the eye for comparison, photometers must supply a means for varying the brightness of one or both of these surfaces according to some known law, so that a photometric balance may be obtained. The most common methods of doing this are: (1) varying the distance of one or both light sources from the surface compared, according to the inverse-square law of illumination; (2) interposing a rotating sector disk or other absorbing media between one of the light sources and the surface it illuminates; and (3) polarizing the light from the two surfaces in planes perpendicular to each other and using a Nicol prism, capable of rotation, to obtain a photometric balance. Combinations of these methods are commonly used.

PHOTOMETER HEADS OR SIGHT BOXES. An optical device, variously called a photometer head, sight box, or simply "the photometer," is used to make a precise judgment of the equality of brightness or contrast of two surfaces illuminated by different sources. It consists of two diffusing surfaces, each illuminated by one of the sources of light, with accessories to facilitate the comparison and to enable the eye to view the two surfaces simultaneously. The surfaces are often called photometer screens. Many photometer heads or sight boxes have been devised.

INVERSE-SQUARE METHOD OF PHOTOMETRY. If a source of candlepower I_1 is at a distance d_1 from a surface S_1, the illumination on S_1 is I_1/d_1^2 (if the light is incident perpendicularly). If the reflection factor of S_1 is ρ_1, the brightness of S_1 is $I_1\rho_1/d_1^2$. Similarly, if a source of candlepower I_2 is at a distance d_2 from a surface S_2 of reflection factor ρ_2, the brightness of S_2 is $I_2\rho_2/d_2^2$. If these two surfaces are of equal brightness,

$$\frac{I_1\rho_1}{d_1^2} = \frac{I_2\rho_2}{d_2^2} \tag{51}$$

Hence, if ρ_1 and ρ_2 are equal

$$\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2} \tag{52}$$

If I_2 is known, and d_1 and d_2 are measured, I_1 can be calculated. This method of comparing directly I_1 against I_2 assumes that the sources are point sources, that the light is incident normally on the surfaces, and that the reflection factors of the surfaces are equal. These assumptions sometimes would lead to appreciable errors. This difficulty may be obviated by the use of the substitution method. In this a third source whose candlepower must be constant, but need not be known, is used as a comparison lamp on one side of the photometer head, while the two sources to be compared are placed in turn on the other side of the photometer. A photometric balance with the comparison lamp is made in each case. If the candlepower of the comparison lamp is assumed to be I_c, then the required ratio

$$I_1/I_2 = (I_1/I_c)/(I_2/I_c). \tag{53}$$

I_c cancels out, and the required ratio is obtained independent of any lack of symmetry

in the photometer. Usually, also, the distance between the photometer head and comparison lamp is fixed, so that the brightness of the comparison surface is constant. If, then, photometric balance be obtained with the photometer head at distances d_1, d_2, and d_3, from sources of candlepower I_1, I_2, and I_3, respectively, $I_1/d_1^2 = I_2/d_2^2 = I_3/d_3^2$. Much calculation is thereby avoided. Such photometric measurements are made on a photometer bench.

ROTATING SECTOR DISK: TALBOT'S LAW. By Talbot's law, if a disk with an angular aperture in it be rotated between a lamp and a photometer head so that the light from the lamp reaches the photometer screen for only a certain fraction of the time, and if the rotation be so fast that the eye perceives no flicker, the effective candlepower of the lamp is reduced in the ratio of the time of exposure to the total time. If the aperture of the disk is a sector of angle θ degrees, the effective candlepower is reduced by the factor $\theta/360$. The sector disk has advantages over other absorbing media in that it is not affected by time and is independent of color.

SQUARED-TANGENT LAW OF POLARIZATION. The use of this law in photometry involves the production of images of the two surfaces to be compared, by means of an optical train which includes a device for plane polarizing the light from the two surfaces in planes perpendicular to each other. If then an analyzing Nicol prism be interposed in the path of these polarized planes the intensity of one image will be reduced by the factor $\cos^2\theta$ and that of the other image by the factor $\sin^2\theta$, where θ is the angle between the optic axis of the Nicol prism and the plane of polarization of the light forming the first image. If, therefore, the analyzing Nicol be rotated until a photometric balance is obtained at the angle θ, it follows that the ratio of the brightnesses of the two images, with no Nicol interposed, would be $\tan^2\theta$.

BUNSEN PHOTOMETER. In the Bunsen photometer the screen is a disk of white diffusing paper, a well-defined region of which is made translucent by impregnation with oil or paraffin. The disk is set transversely in a sight box, as shown on the plan in Fig. 54. The interior of the sight box is blackened. Light from the lamps to be compared enters the apertures A–A, and falls normally on the disk surfaces. Dihedral mirrors M_1 and M_2 enable both sides of the disk to be viewed at the sight tube T. The opaque portion of the disk reflects diffusely, while the translucent region partially reflects and partially transmits the light received. A photometric balance exists when the two sides of the disk appear alike. If both lamps are alike in color and both regions of the disk have the same absorption, the boundary disappears and both sides appear uniformly bright. With unequal absorption, balance exists when equal contrast exists between the opaque and translucent regions on both sides of the disk. The contrast principle is of distinct advantage with slight color differences. The sensitiveness of the screen depends largely on the definition of the boundary of the impregnated portion.

The Bunsen photometer was the first really accurate photometer devised. It is not as sensitive, however, as later types of photometers, chiefly because each of the surfaces receives light from both sources being compared. This difficulty is avoided in the photometers described below. The Leeson disk is a modification of the Bunsen screen.

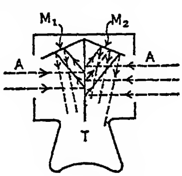

Fig. 54

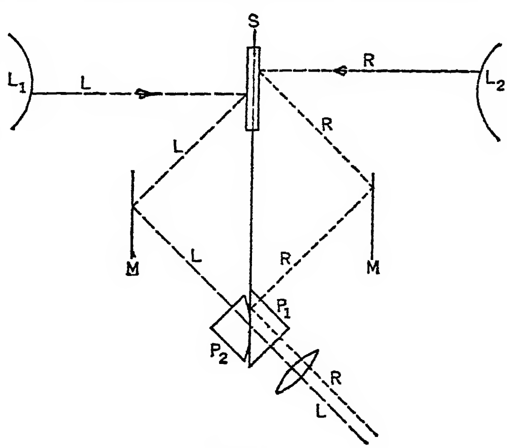

Fig. 55

LUMMER-BRODHUN HEAD OR SIGHT BOX. The plan of this box is shown in Fig. 55. An opaque diffusely reflecting screen S receives light from the sources to be compared and reflects it along the paths indicated by aid of the mirrors M–M and the prisms P_1–P_2, often referred to as a Lummer-Brodhun cube. The prisms present to the eye a

composite field in which the brightnesses of the two sides of S can be conveniently compared. Fig. 55 also shows the arrangement for equality-of-brightness working. The prisms are in optical contact over an elliptical portion of their hypotenusal faces, and the remainder of one is cut away. The central portion of the field is illuminated by direct transmission through the contact area, the outer portion by total reflection from the face of the uncut prism.

For equality-of-contrast working, the arrangement of the prisms is as shown in Fig. 56. The hypotenusal face of P_2 is recessed over an area that provides a pattern as shown in (B). The face of P_1 is plane. Two thin glass absorbing strips C_1 and C_2 are set before the faces of P_1 and P_2 as shown in the plan. The two trapezoids L and R are darkened by the absorption of light in C_1 and C_2. In a state of balance L' and R' appear equally bright and L and R equally dark in contrast. The degree of contrast created by C_1 and C_2 is ordinarily about 8 per cent. Accuracy of adjustment and cleanliness of all parts are essential in photometers of the Lummer-Brodhun type.

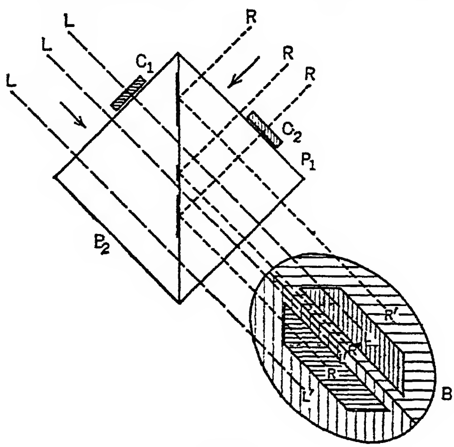

Fig. 56

PHOTOMETER BAR FOR MEASUREMENT OF LUMINOUS INTENSITY (CANDLEPOWER). The devices above described are best suited for use in connection with a fixed photometer bar in a laboratory. Using the substitution method of photometry the most convenient arrangement of the equipment is as shown in Fig. 57. The photometer head P and the comparison lamp C are clamped to a movable carriage so that the distance between them is constant. The carriage can be moved with respect to the lamp socket T which is fixed on the bar. Lamp C is maintained at constant voltage throughout any series of measurements. Working standard lamps and test lamps whose candlepower is to be determined are in turn placed in socket T and maintained at any desired voltage while photometric balance is obtained by setting the movable carriage. In precision photometry, more than one standard is used. A common procedure is to use a set of

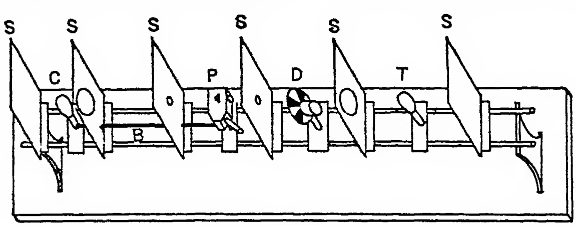

Fig. 57

six working standards, reading three at the start of a run and the other three after the test lamps have been read.

For measurements of candlepower in a fixed direction the lamps must be mounted so that they may be oriented on the bar in that direction. For measurements of mean horizontal candlepower, socket T is rotated at about 120 rpm. The bar should be provided also with a series of screens S of dead black material (preferably velvet) having graded apertures along the photometric axis, and with solid screens at the ends. These screens should completely exclude all extraneous light from the photometer head and should protect the eye of the observer from the direct light of the lamps. If these conditions are met the photometric room need not be blackened, although it is advisable to keep it as dark as is convenient.

PORTABLE PHOTOMETERS. Many forms of portable photometers and illuminometers are available for use both in the laboratory and for measurement of candlepower and illumination in the field. Such instruments are the Sharp-Millar photometer, Macbeth

illuminometer, Holophane light meter, and the foot-candle meter. The Weston illuminometer, a photoelectric-cell device, is the most recent development. The Weber portable photometer, Martens polarization photometer, Bechstein universal photometer, and Bechstein illuminometer are also well-known types

PHOTOMETERS AND ILLUMINOMETERS BASED ON INVERSE-SQUARE LAW. The Sharp-Millar photometer, the Macbeth illuminometer, and the Weber photometer are portable inverse-square-law photometers, consisting essentially of a tube a foot or more in length, a comparison lamp of low voltage, designed to operate on a storage battery or dry cells, a Lummer-Brodhun cube of special design, and a diffusing glass photometer screen. In the Sharp-Millar and Macbeth photometers the comparison lamp is movable; in the Weber photometer the screen is movable. The instruments are provided with scales (graduated according to the inverse-square law) the range of which may be extended by means of neutral filters that can be placed on the test side or comparison side of the cube. An external test plate or standard surface is placed at the point at which illumination is to be measured. The illuminated test plate then serves as the second comparison screen. For details of these photometers see the papers listed in the bibliography.

MARTENS POLARIZATION PHOTOMETER. In this type of photometer, a Fresnel biprism is used to get contiguous images of the surfaces to be compared. In Fig. 58 such a biprism together with a Wollaston prism is shown diagrammatically. Here S_1 and S_2 are the surfaces to be compared. The light from S_1 passes through a plano-convex lens C into a Wollaston prism P. Here it is split up into two beams: one, shown by the full line, is polarized in the plane of the paper; the other, shown by the broken line, is polarized in a plane perpendicular to the paper. The light from S_2 is treated similarly, and the angle of the biprism is designed so that images of S_1 and S_2 oppositely polarized coincide at the eye, while the remaining six images are stopped by a diaphragm. It follows that one-half of the biprism B appears bright because of the light from S_1 which is polarized in the plane of the paper, while the other half appears bright because of light from S_2, which is polarized in a perpendicular plane. A Nicol prism and graduated circle capable of rotation about the axis of the instrument complete the photometer.

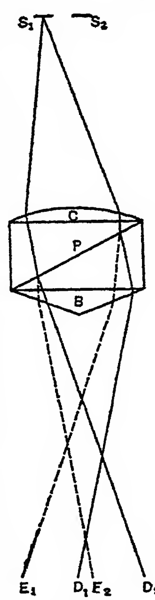

FIG. 58

FOOT-CANDLE METER. A foot-candle meter is a convenient, inexpensive, and self-contained device for making approximate measurements of illumination. It consists of a small metal box, part of whose top consists of a paper screen, opaque except for a row of Bunsen spots on it. The under surface of this screen is illuminated by a small lamp fed by three tubular dry cells. This lamp is at one end of the box, so that the Bunsen spots decrease in brightness from the end where the lamp is placed. Measurement is made by observing which spot disappears when the scale is illuminated from above.

HETEROCHROMATIC PHOTOMETRY. The illuminants in use today, i.e., vacuum and gas-filled tungsten-filament lamps, arc lamps, mantle gas lamps, and gaseous-discharge lamps, give light different in color from that of the fundamental reference-standard lamps. The measurement of the light emitted by modern lamps therefore involves, somewhere in the process, heterochromatic photometry, i.e., photometry with a color difference.

If the eye is required to decide when equality of brightness or equality of contrast exists between the comparison surfaces, with a color difference existing, it encounters a difficulty which increases as the color difference becomes greater.

Cascade Method. One method of bridging the color difference is the cascade method. It involves dividing up the color difference by making the photometric settings in a number of steps. When lamps of efficiencies different from the efficiency of carbon lamps are to be measured against carbon-filament standards it is possible to divide the color difference into a number of steps, by using lamps operating at intermediate efficiencies. Tungsten lamps are blue in color in comparison with the carbon-filament standards, but by reducing the voltage on tungsten lamps a series of color steps can be interposed between the carbon standard and the tungsten test lamp.

Compensation Method. In the compensation or mixture method the color difference is reduced by illuminating one or both of the comparison sources with light from both the sources being compared.

Flicker-Photometer Method. A reliable means of comparing two lights of different colors is the flicker photometer, in which the comparison field is alternately illuminated

by the two sources of light to be compared. This alternate illumination causes a flicker which is due to difference of color and difference in brightness. Above a certain moderate rate of alternation the color sensations blend and the disappearance of flicker is a true indication of equal brightness. The following conditions are required for high precision: (*a*) a field illumination of about 25 meter-candles; (*b*) a photometric field of 2 deg diameter; and (*c*) a background field about 25 deg in diameter surrounding the photometric field and about equal to it in brightness.

The Ives-Kingsbury flicker photometer consists of an ingenious attachment to a Lummer-Brodhun contrast-type photometer, described above, with the contrast glasses removed. It consists essentially of an optical train including (*a*) a lens for magnifying the field, (*b*) a rotating prism, (*c*) a disk with a hole limiting the field to 2 deg, (*d*) an eye lens, and (*e*) a lamp for illuminating the background field. For a complete description of this photometer see *J. Franklin Inst.*, 1915, vol. 180, p. 215.

The Ives-Kingsbury photometer, and most of the other flicker photometers, have so many optical parts that the brightnesses of the comparison surfaces are very materially reduced. Where only low illuminations of these surfaces are available, this is a distinct disadvantage. The Guild flicker photometer obviates this difficulty. It has no optical parts between the eye and the comparison surfaces. These surfaces are (1) a stationary surface of magnesium oxide illuminated by one source and (2) a rotating sector disk of magnesium oxide illuminated by the second source. The stationary surface is viewed through the rotating disk, whose speed of rotation can be adjusted. Thus the eye sees alternately the stationary surface and the sector disk, the two comparison surfaces. The sight tube is arranged so as to supply the background illumination. For a complete description of the Guild flicker photometer, see *J. Sci. Insts.*, 1924, vol. 1, p. 182.

Color-Equalizing Filters. Where a large color difference exists between two sources to be compared directly, flicker photometry gives results which are more consistent and dependable than those obtained by the use of the equality-of-contrast photometers, which are, however, much more convenient and easy to use. If two sources of light to be compared are of different color, the color difference can be removed by placing a suitably colored filter between the photometer and one of the sources and so causing the light from this source to match that from the other source. The problem of heterochromatic photometry is thus reduced to one of homochromatic photometry and can be handled very conveniently with a Lummer-Brodhun photometer. It then becomes necessary to determine the transmission factor of the filter for light of the color given by the source with which it is used. The transmission factor of the filter can be determined by means of a flicker photometer, or by a spectrophotometer.

SPECTROPHOTOMETRY. In ordinary photometry, the light emitted by a source is measured as a whole, regardless of its spectral distribution. In spectrophotometry, the light emitted by the source under examination is compared, wavelength for wavelength, with that given by a standard source. If the spectral distribution of the light given by the standard source is known, the spectral distribution of the light from the test source can be obtained. The acetylene flame has frequently been used as a standard but a tungsten-filament lamp is a more convenient standard of reference. A tungsten lamp is slightly selective as a radiator, and when vacuum lamps are used as reference standards they are operated at a definite "color temperature," as for example at a color temperature of 2360 deg K, and gas-filled lamps at 2680 or 2848 deg K. The "color temperature" of a lamp is the temperature of a black body that gives light of the same color as given by the lamp.

To make a measurement, it is necessary to disperse the light by some device, such as a prism, and then measure the intensity of each wavelength, or in practice, a small group of wavelengths, as compared with the reference standard. A spectrophotometer consists essentially of (1) a device for analyzing the light of each source and making a photometric comparison of the two sources for each small group of wavelengths, and (2) auxiliary equipment for changing the brightness of the components of the field to obtain photometric balance.

Spectrophotometric measurements are particularly applicable to the determination of the spectral transmission of colored filters. From such measurements, the overall transmission of a filter for light of known distribution of energy can be calculated if the spectral characteristics of the average eye are known, for the eye responds very differently for different wavelengths, its limits of sensitivity being about 400 and 750 millimicrons, with the maximum sensitivity at 550 millimicrons. The most generally accepted values of the relativity sensitivity of the "average eye" are those proposed by Gibson and Tyndall, which have been provisionally accepted for international use by the International Commission on Illumination. (See Illuminating Engineering Nomenclature and Photometric Standards, A.S.A., Z7-1932, p. 8, for Table of Values of Relative Visibility (V_λ).

MEASUREMENT OF CANDLEPOWER DISTRIBUTION AND LIGHT FLUX. No actual light source is a point source, so no light source used in practice has the same luminous intensity (candlepower) in all directions. Therefore the measurement of the candlepower of a source of light in any one direction gives no information as to the light output of the source, or the distribution of the light. The determination of the luminous intensity of an incandescent lamp in, say, 18 directions (10-deg intervals) in the horizontal plane and in the vertical plane is sufficient for plotting polar curves of light distribution. To make such determinations, various arrangements of mirrors and means for rotating the lamp and mirrors have been devised. A typical three-mirror device is shown diagrammatically in Fig. 59. The lamp may remain stationary or be rotated about its vertical axis. The mirrors may be turned about the horizontal photometric axis in steps.

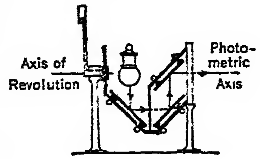

FIG. 59

The mean spherical candlepower or lumen output of an incandescent lamp may be determined: (1) by measuring the candlepower of the lamp in many directions in space and computing the mean; (2) by measurement of mean horizontal candlepower, and computation by using the reduction factor; and (3) by measurement of total flux or mean spherical candlepower in an integrating-sphere photometer.

Total flux of luminaires may be measured with sufficient accuracy for general purposes by making candlepower measurements on a distribution photometer at the middle of each 10-deg zone, from 0 to 180 deg, angles being measured counter-clockwise from the "six o'clock" position. If then the mean candlepower for each 10-deg zone be multiplied by zone factors, which are the zonal areas on the unit sphere, total lumens for any zone or the complete sphere can be computed.

INTEGRATING-SPHERE PHOTOMETER. An integrating-sphere photometer (Ulbricht sphere) provides the means of making measurements of mean spherical candlepower or total flux of light in lumens from a lamp or luminaire by a single observation, if a standard of luminous flux is available for comparison. Sphere photometers range in size from 15 in. diameter to 100 in. or even more. The inside of the sphere is painted with a specially prepared white paint, as non-selective as possible and of highest diffusing power obtainable.

When a lighted lamp is put into the sphere, the illumination on any element of area A of the sphere wall is made up of two parts: (1) the direct light from the lamp E_d; and (2) the light diffusely reflected, E_r, from all parts of the sphere wall. The component E_r of the total illumination on A is proportional to the total flux of the source and independent of the location of the source within the sphere. It is sufficient therefore to measure the component E_r. This is done by inserting a small translucent glass window flush with the inner surface of the sphere wall, and screening the window from the direct light of the source by the smallest screen that will completely cut off all direct light from the window. The sphere window then serves as a transmitting test plate, and its brightness may be measured by a suitable arrangement of a bar photometer, with Lummer-Brodhun cube, or by means of a portable photometer.

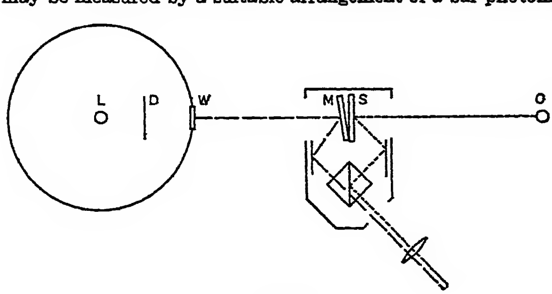

FIG. 60

Fig. 60 is a diagrammatic sketch showing a commonly used method for visual readings. The mirror M in the Lummer-Brodhun sight box makes the external sphere-window surface W one of the comparison-screen surfaces. The photometric balance is obtained by moving the comparison lamp, C keeping the photometer head fixed. Other arrangements are also used.

The substitution method is used in making measurements, standards of luminous flux and the test lamps or luminaires to be measured being placed in the sphere in turn, and photometer settings made, either visually or by physical photometry (photoelectric-cell methods). Auxiliaries such as sector disks and colored glass filters are easily applied.

Attention to details is essential. The sphere window should be non-selective in transmission, have good diffusing qualities, and be no larger than about one-tenth the

sphere diameter and preferably smaller. The shield in the sphere between the source and window must be so located and of such size as to reduce to a minimum the area of the sphere wall from which first reflections cannot reach the area, but fully screen all direct light of the source. If the source is at the center of the sphere, the best position of the screen is at a point 0.387 times the radius of the sphere from the window. All objects in the sphere introduce errors, including the lamps to be measured, particularly if such lamps are blackened from use, or if the test lamps differ much in size from the standards. The paint should be non-selective and renewed periodically.

Sphere-photometry errors are discussed at length by Weigel and Knoll in a paper in *Licht und Lampe*, 1928, vol. 17, p. 753.

MEASUREMENT OF BRIGHTNESS, REFLECTION FACTOR AND TRANSMISSION FACTOR. Measurement of Brightness. Brightness is defined as the quotient of the luminous intensity of a surface, measured in a given direction, by the area of this surface projected on the plane perpendicular to the direction considered. The brightness of an illuminated surface or transmitting medium may therefore be obtained by measuring the candlepower of the surface in a given direction and dividing this value by the projected area. However, this direct method is applicable practically only to very bright sources. For illuminating engineering purposes the brightness of surfaces may be measured satisfactorily with a Macbeth or Sharp-Millar portable photometer. The scale of the photometer is ordinarily calibrated in foot-candles to read illumination on a test plate. If the reflection factor of the test plate is known the photometer may be calibrated to read brightness directly, if scale readings are multiplied by the quantity ρ/π, where ρ is the reflection factor of the test plate. Test plates of magnesium oxide or magnesium carbonate (reflection factor 98 per cent), freshly prepared, are useful in such calibrations.

Measurement of Reflection Factor. The measurement of reflection factors is most readily made by means of a Taylor reflectometer. This instrument consists of a small integrating sphere from which a segment has been cut away, and a lighting tube, containing a lamp and lens in a movable mounting so fixed into the sphere that the spot of light produced by the lamp and lens in the tube may be projected either on the wall of the sphere, or on the test surface to be measured, the test surface forming that part of the wall of the sphere that has been cut away. Into a third opening in the sphere the sight tube of a Macbeth illuminometer is inserted. A small screen prevents light from the directly illuminated surface from falling on the observed point on the sphere wall. Readings are taken when the beam illuminates the test surface and when it illuminates the sphere wall directly. The ratio of the readings is the reflection factor.

The Taylor reflectometer may be used for measuring reflection factors of polished surfaces (regular reflection) and reflection factors of matt, diffusing surfaces. The reflection factor of a material depends upon the color and the direction of the incident light.

Measurement of Transmission Factor. The measurement of transmission of transparent media has been discussed under spectrophotometry. For illuminating engineering purposes, the transmission factor of enclosing diffusing globes and other types of glassware is of importance. It is commonly spoken of as the "efficiency" of the glassware. Measurements are most conveniently made in a large integrating sphere, by measuring, first, the lumen output of the bare lamp, and then the lumen output of the lamp in its inclosing equipment. The inverse ratio of the measured values is the percent transmission, or "efficiency." In making such measurement, the inclosing unit should be placed somewhere in the sphere while the working standard lamps and the test lamp to be used in the inclosing globe are measured.

PHOTOMETRY OF PROJECTION APPARATUS. The photometric measurement of light projectors, such as automobile headlamps, airway beacons, airport and hangar floodlights, locomotive headlamps, searchlights, and general floodlighting equipment, constitutes a special problem. The approval by state authorities of automobile headlamp devices has brought about the development by testing laboratories of special equipment for the measurement of candlepower distribution in headlamp beams. A photometric range of 100 ft is desirable. The Sharp-Millar, Macbeth, or Bechstein portable photometers with accessory equipment are suitable for making candlepower-distribution measurements to determine compliance of types of headlamps with special state requirements or with the standard specification of the American Standards Association covering the laboratory testing of automobile headlamp devices. (See Specifications of Laboratory Tests for Approval of Electric Headlighting Devices for Motor Vehicles. Illum. Engng. Soc., New York, A.S.A. Tentative Standard D2-1922. See also Joint IES-SAE Standard, *Trans. Illum. Engng. Soc.*, 1930, vol. 25, p. 835).

In making candlepower measurements on projectors, it is usually the case that the whole aperture of the projection device is filled with light and appears equally bright

all over. The area of the projector opening is therefore the source, and the values determined are in terms of apparent candlepower, measured at a specified distance. The apparent candlepower of an extended source of light, measured at a specified distance, is the candlepower of a point source of light which would produce the same illumination at that distance.

For floodlighting equipment and airway beacons, maximum beam candlepower, lumen output in the beam, and horizontal and vertical angular beam spread are required. Reference may be made to the General Procedure for Photometry of Incandescent Floodlights, *Railway Electrical Engineer*, 1927, vol. 18, p. 315, where full and detailed description of methods of measurement and computation tables will be found.

For locomotive headlighting see the Report of the Committee on Locomotive Electric Lighting, *Railway Electrical Engineer*, 1924, vol. 15, p. 335.

PHOTOELECTRIC-CELL PHOTOMETRY. The very rapid development of photoelectric cells within recent years has made photoelectric-cell photometry practicable for use in testing laboratories and in incandescent lamp factories, and, by the application of certain types of cells to portable instruments (illuminometers), for use in making illumination measurements in the field.

A photoelectric-cell photometer, in its simplest form, consists of a light-sensitive cell, one or more electric batteries, a suitable protective resistor (10 to 100 megohms), and a galvanometer. Both gas-filled and vacuum photoelectric cells are used in photometry. The electric current produced is very small, and for photometers for routine use amplification is desirable. An ideal photoelectric cell for photometric use should have, among others, the following characteristics: (*a*) The electric current produced by the cell when illuminated should be directly proportional to the illumination of the cell. This property is commonly called "linearity of response." (*b*) The response of the cell from day to day should be constant. (*c*) The color sensitivity should be the same as the color sensitivity of the average eye. By careful selection of cells and by the use of filters all these conditions can be met satisfactorily for practical use. Manufacturers of photometric apparatus now supply photometers equipped with photoelectric cells and auxiliary devices complete for laboratory installation as well as several types of portable photometers.

Reproducibility of results by photoelectric-cell photometers is much better than by visual photometers. Great care is necessary, however, for the elimination of disturbing influences. The cell and electron tubes must be carefully and completely shielded, and leakage caused by moisture must be guarded against. Dark rooms are not necessary for photoelectric-cell photometry. Candlepower distribution, transmission, and color temperature methods are being developed.

22. ELECTRICAL RECORDING INSTRUMENTS

A recording instrument is defined as an instrument which makes a graphic record of the value of an electrical quantity as a function of time. The term "recording instrument" may be replaced by the shorter term "recorder." For other definitions, and some of the more general requirements to be met, see A.I.E.E. Report on Standards for Electrical Recording Instruments, No. 40, June, 1933.

An electrical recording instrument produces and controls the motion of a marking device which leaves a record on a chart advanced at a controlled speed by clockwork, a synchronous motor, or equivalent. The operating forces set up by nearly all indicating-instrument mechanisms are so small that the attempt to make them move a pen over a chart, without serious errors from pen friction, is beset with difficulties. The design must be modified to secure a relatively high operating torque (or force), even at the cost of increased power required for operation. This difficulty is obviated, though at the expense of complication and increased cost, in relay-type recording instruments, in which the marking device is moved by a secondary driving force controlled by the moving element. By this means it is possible to measure small electrical quantities which would not be able to operate a direct-acting recording instrument, as in relay-type recording potentiometers.

Recording instruments are made in many forms and for an ever-increasing list of applications in industrial processes for which accurate control of conditions is essential to the maintenance of quality, minimum cost of production, or both. For detailed descriptions and operating instructions the makers' publications should be consulted. Detailed treatments of the construction of the various types of recording instruments are given in the books and articles listed in the Bibliography.

23. TELEMETERING

The A.I.E.E. Committee on Instruments and Measurements defines "telemeter" as "Measuring, transmitting and receiving apparatus for indicating, recording or integrating at a distance by electrical means the value of a quantity." The art of telemetering ("remote metering" or, in British usage, "distant indication") came into existence as a matter of necessity as the interconnection of systems became more widespread and the control of scattered power plants became more centralized. In 1926 the importance of the subject prompted the setting up by the A.I.E.E. Committee on Instruments and Measurements of a subcommittee to survey the field and collect information. The reports of this subcommittee should be consulted (see Bibliography) for recommended definitions of terms and the classification of remote-metering systems. A subcommittee of the A.I.E.E. Automatic Stations Committee cooperated with the subcommittee on telemetering in the preparation of a report on telemetering and supervisory control systems and related communication systems. At this writing (1936) this report has been published only in mimeographed form.

24. MEASUREMENT OF NON-ELECTRICAL QUANTITIES BY ELECTRICAL AND MAGNETIC MEANS

Electrical and magnetic means and measuring instruments have been applied for a vast number of kinds of measurement of non-electrical quantities. These have been classified by Borden (see Bibliography), as follows:

1. Measurement of temperature.
2. Measurement of stress, strain, or small changes in physical dimensions.
3. Measurement of flow of liquids and gases.
4. Measurement of angular and of linear velocity.
5. Measurement of work.
6. Measurement of radiant energy.
7. Determination of chemical and molecular condition.
8. Navigational measurement, and detection of hidden conditions.
9. Physiological and allied measurements.

The number of measuring devices coming under these nine headings is so great that reference must be made to the paper by Borden, which contains a valuable bibliography of 330 entries. A paper by H. L. Curtis, immediately following Borden's paper, gives a detailed account of the manner in which moving-coil oscillographs were used to measure the important quantities relating to the firing of heavy guns on shipboard, for example, the velocity of the projectile and the time-displacement curve of the gun during recoil.

At the Winter Convention of the A.I.E.E. in January-February, 1929, a session was devoted to electrical measurement of non-electrical quantities, with papers by Sanford, Pratt, Miner and Batten, Spooner and Folta, and Marriott (see Bibliography). For subsequent A.I.E.E. papers on the measurement of noise of machinery, see Bibliography.

25. MEASUREMENT OF TEMPERATURE

THE PRIMARY STANDARD SCALE OF TEMPERATURE. The International Temperature Scale, adopted by 31 nations in 1927, is based upon a number of fixed and reproducible equilibrium temperatures to which numerical values are assigned, and upon the indications of specified interpolation instruments calibrated according to a specified procedure at the fixed temperatures. This scale conforms, as closely as is possible with present knowledge, with the Thermodynamic Centigrade Scale, on which the temperature of melting ice, and the temperature of condensing water vapor, both under a pressure of 1 atmosphere, are numbered 0 and 100 deg, respectively. Temperatures on the international scale are ordinarily designated by the abbreviations deg cent, °C, or C. By a recently approved American Tentative Standard (ASA Z10i-1932) the abbreviation C denotes a temperature scale or points thereon, but numbers expressing *intervals* on this scale are followed by the abbreviation °C.

THE FAHRENHEIT SCALE. This scale is now derived from the international scale by means of the relations

$$t_f = \frac{9}{5} t_c + 32 \tag{54}$$

$$t_c = \frac{5}{9} (t_f - 32) \tag{55}$$

where t_f = temperature in degrees fahrenheit.
t_c = temperature in degrees centigrade.

A degree fahrenheit is abbreviated deg fahr, °F, or F.

OTHER TEMPERATURE SCALES. A number of other temperature scales have been used or proposed, of which only the Réaumur survives to any extent in the United States. In this scale, the interval between the ice and steam points is taken as 80 deg, the ice point being 0 deg R.

ABSOLUTE TEMPERATURE SCALES. Thermodynamic reasoning introduces the concept of absolute temperature, on which basis a temperature of − 273.1 deg on the International Temperature Scale would be called zero temperature absolute. The two absolute temperature scales are: the centigrade absolute or Kelvin scale, abbreviated °K or K, and the fahrenheit absolute or Rankine scale, abbreviated °R′. A temperature in degrees centigrade absolute is numerically equal to the same temperature in degrees centigrade plus 273.1, and a temperature in degrees fahrenheit absolute is numerically equal to the same temperature in degrees fahrenheit plus 459.6.

Thermometers

CHOICE OF THERMOMETERS. The selection of a thermometer for a particular purpose is governed by the temperature range to be covered, the precision required, the nature of the substance whose temperature is to be measured, the space available, and the type of reading (indication or record) desired.

LIQUID-IN-GLASS THERMOMETERS. For the measurement of temperatures below about 500 deg cent, in applications not requiring the high accuracy obtainable with precision thermocouples and resistance thermometers, liquid-in-glass thermometers are extensively used. The fluids generally used are alcohol, pentane, toluene, and mercury, the first three being used for temperatures below the freezing point of mercury. For all temperatures above its freezing point, mercury is much superior to the other liquids.

Mercury-in-glass thermometers are made in various ranges, and are of all grades as to quality of the glass and of workmanship. For engineering and laboratory measurements, thermometers of the laboratory type (sometimes called chemical thermometers), having the scale marked directly on the tube, are preferable to those in which the scale is not integral with the tube. The latter type, if of good workmanship, is suitable for industrial use where only moderate accuracy is required.

Mercury-in-glass thermometers are made to cover the range − 35 to 550 deg cent. By using fused silica instead of hard glass, the upper limit is extended to 800 deg cent, but such thermometers are very little used. They are fragile and expensive, their emergent-stem correction is large and uncertain, and some other form of pyrometer is much to be preferred.

When using thermometers calibrated for total immersion (i.e., calibrated with the entire liquid column of the thermometer at one temperature) a correction must be applied to the reading if part of the column is in the stem of the thermometer at a temperature materially different from that of the bulb. For mercury-in-glass centigrade thermometers the approximate "emergent-stem correction" is given by the formula

$$\text{Correction} = +0.00016n(t_b - t_s) \tag{56}$$

where the temperatures are in degrees centigrade and n = number of degrees of the column of mercury exposed to the emergent-stem temperature, t_s, and t_b = temperature of the bulb. For more precise corrections see Smithsonian Physical Tables.

RESISTANCE THERMOMETERS. In its usual laboratory form the resistance thermometer consists of a coil of platinum wire wound on a mica frame and enclosed in a protecting tube through which leads are brought out for connection to a bridge for measuring the resistance of the wire. Such thermometers are particularly useful in calorimetry and other laboratory work where high accuracy is required. For technical purposes some form of thermocouple is usually preferable. In the form of embedded temperature detectors resistance thermometers are built into the slots of generators to measure the temperature of the windings during operation.

PRESSURE-GAGE THERMOMETERS. Two types of thermometers are similar in that each has a metal bulb coupled to a pressure gage by a capillary tube. In the vapor-pressure type the bulb contains a liquid which has a measurable vapor pressure in the temperature range for which the thermometer is designed. In the liquid-expansion type the bulb, tube, and gage are completely filled with a liquid. In each case the gage is graduated to read the temperature of the bulb. The vapor-pressure thermometer has the advantage of indicating the temperature of the bulb regardless of the relative volume of the capillary tube and its temperature. It is theorectically unaffected by

leakage of the liquid, so long as some liquid remains in the bulb, but its readings will be affected by leakage of air or other fluid into the bulb. The reading of the liquid-expansion thermometer is dependent on the relative volume and temperature of the emergent stem, and is affected by leakage outward. The liquid-expansion type has the advantage of uniform length of scale divisions.

BIMETALLIC THERMOMETERS. The bimetallic thermometer contains a strip of so-called "thermostatic metal." One end of the strip is fixed, and the motion of the free end with change of temperature actuates the pointer through a magnifying mechanism.

The pressure-gage and the bimetallic thermometers, though especially suitable for recording, are neither as accurate nor as reliable as mercury-in-glass thermometers.

CHECKING THERMOMETERS. The graduation errors of a thermometer can best be determined by comparing it with a secondary standard mercury-in-glass thermometer which has been tested and certified by a competent laboratory, such as that of the National Bureau of Standards in Washington. Only the zero point need be checked subsequently on mercury-in-glass thermometers. This can be done by noting the reading of the thermometer after total immersion for several minutes in a mixture of crushed ice and distilled water. Pressure-gage thermometers, however, may need to be recalibrated frequently over the working range.

PRECISION AND RANGE OF MEASUREMENT OF TYPES OF THERMOMETERS. Typical figures, intended merely to guide in the selection of suitable types, are given in Table IX.

Table IX. Precision and Range of Measurement of Types of Thermometers

Type of Thermometer	Range °C	Precision	
		Best Grade °C	Average Grade °C
Mercury—ordinary	−35 to 300	0.01	0.5
hard glass	0 to 550	0.1	1
Alcohol	−100 to 40	..	1
Electrical resistance	0 to 100	0.002	..
	−200 to 500	0.01	..
	500 to 1200	..	1 to 5
Thermoelectric	−200 to 1000	0.05 to 0.1	1 to 10
Bimetallic	0 to 500	..	1 to 20
Pressure-gage	0 to 750	..	1 to 10

LOCATION OF THERMOMETERS. In locating a thermometer, it must be realized that it indicates its own temperature, which may or may not be that of its surroundings. To indicate the temperature of its surroundings, a thermometer must be in good thermal contact with them; it must not be exposed to radiation from hotter bodies, nor able to radiate to cooler ones; in short, it must not "see" anything which is at a different temperature. Equal precaution should be taken to prevent loss of heat by conduction or convection.

Pyrometers

GENERAL. A pyrometer is a device intended primarily for measuring temperatures beyond the range of a mercury-in-glass thermometer, say above 500 deg cent. Certain types of pyrometers are frequently found useful in measuring lower temperatures also.

Numerous types of pyrometers, now of historical interest only, are treated in detail in Burgess and Le Chatelier's High Temperature Measurements (see Bibliography). This section is restricted to current pyrometric data and devices.

THERMOELECTRIC PYROMETERS. The most widely used pyrometric device consists of a thermocouple and an instrument capable of indicating or recording electromotive force. Although the standard thermocouple, one wire platinum and the other wire an alloy consisting of 10 per cent rhodium and 90 per cent platinum, is used in defining the temperature scale, the majority of couples in actual use are of less expensive materials ("base metals"). The commonly used thermocouples and their ranges of usefulness are given in Table X. Constantan is also sold under a number of trade names, namely, Ia Ia, Ideal, Advance, Copel, Eureka, etc.

TEMPERATURE-EMF RELATIONS FOR VARIOUS THERMOCOUPLES. In Table XI are given representative calibration data for couples of the six types commonly used. These data may be used to obtain approximate temperatures, good to the equivalent of about 0.5 per cent in emf, except for couples having constantan as one element. For more accurate work each individual couple must be calibrated. Values of thermal

Table X. Range of Usefulness of Thermocouples

Type of Thermocouple	Useful Range	
	For Long-time Service	* For Short-time Service
Platinum vs. 90% platinum—10% rhodium...	0 to 1450° C	1700° C
" " 87% " —13% " ...	0 to 1450° C	1700° C
Chromel vs. Alumel..........................	0 to 1100° C	1350° C
Iron vs. constantan..........................	0 to 800° C	1000° C
Chromel vs. constantan.....................	0 to 800° C	1000° C
Copper vs. constantan......................	− 190 to 350° C	600° C

* Thermocouples which have been subjected to these temperatures cannot be depended upon to retain their calibrations and should not be used for accurate work subsequently without being recalibrated.

emf for couples having constantan as one element may differ by as much as 2 per cent from those in the table. However, most thermocouple manufacturers take pains to supply a uniform quality of constantan, and the calibrations of individual couples by a reputable maker can be depended to agree with the curve which he issues within about 0.5 per cent in emf.

COLD-JUNCTION CORRECTIONS. The data in Table XI give the relation between the emf of the couple and the temperature of the measuring junction, when the

Table XI. Representative Data on Thermocouples

With reference junction at 0 deg cent.

Emf, millivolts	Platinum to Platinum—10% Rhodium	Platinum to Platinum—13% Rhodium	Copper to Constantan	Emf, millivolts	Iron to Constantan	Chromel P to Alumel	Chromel P to Constantan
0	0° C	0° C	0° C	0	0° C	0° C	0° C
1	147	145	25	5	93	122	80
2	265	259	49	10	183	246	153
3	374	361	72	15	272	367	221
4	478	457	94	20	363	485	287
5	578	550	115	25	454	602	351
6	675	638	136	30	543	720	413
7	769	723	156	35	629	841	475
8	861	806	176	40	711	966	537
9	950	886	195	45	789	1096	599
10	1037	964	213	50	865	1232	661
11	1122	1040	232	55	946	1376	723
12	1206	1114	250				
13	1290	1187	268				
14	1374	1259	285				
15	1458	1332	302				
16	1543	1404	319				
17	1628	1477	336				
18	1714	1550	353				

reference junction is at 0 deg cent. In many measurements the reference junction is not at 0 deg cent; for example, when the thermocouple wires are connected directly to the measuring instrument, the temperature of the reference junction is that of the instrument. Since the table shows the change in emf of the couple as the temperature of either junction is changed, it may be used to correct an observed emf to that which would have been observed if the reference junction had been at 0 deg cent. For example, if the observed emf of a copper-constantan couple is 10 mv with the reference junction at 25 deg cent, the table shows that the emf would have been 1 mv greater, that is, 11 mv, if the reference junction had been at 0 deg cent. The temperature of the measuring junction was accordingly that corresponding to 11 mv in the table, that is, 232 deg cent. It can be seen that the correct value is not obtained when one adds the cold-junction temperature, 25 deg cent, to the value in the table, 213 deg cent, corresponding to the measured emf, 10 mv. If the indicator is graduated in temperature, so that the emf is not observed, correction for the cold-junction temperature t'_0 may be made by adding to the observed temperature the quantity $(t'_0 - t_0)K$, where t_0 is the cold-junction temperature for which the thermocouple was calibrated. Values of K for typical couples are as follows:

Table XII. Values of Cold-junction Correction Factor K*

Platinum to Platinum—10% rhodium		Platinum to Platinum—13% rhodium		Copper to Constantan		Chromel to Constantan	
Deg C	K	Deg C	K	Deg C	K	Deg C	K
250–400	0.65	250–400	0.60	0–50	1.00	0–50	1.00
400–650	0.60	400–600	0.55	50–80	0.95	50–100	0.95
650–950	0.55	600–900	0.50	80–110	0.90	100–150	0.90
950–1550	0.50	900–1550	0.45	110–150	0.85	150–200	0.85
				150–200	0.80	200–300	0.80
				200–270	0.75	300–750	0.75
				270–350	0.70		

Chromel to Alumel		Iron to Constantan	
Deg C	K	Deg C	K
0–300	1.00	0–50	1.00
300–800	0.95	50–550	0.95
800–1000	1.00	550–650	0.90
1000–1100	1.05	650–750	0.85
1100–1250	1.10	750–900	0.80

* Values of K in this table are based on calibrations with a cold-junction temperature of 0 deg cent.

COLD-JUNCTION COMPENSATORS. Various devices are used to avoid the necessity for making the calculations described in the preceding paragraphs. Simple indicators may be set, on open circuit, to read the emf corresponding to the cold-junction temperature, or the cold-junction temperature itself, depending upon the way the scale is graduated. In many deflection-type instruments this setting is done automatically by means of a bimetallic spiral which moves the outer end of one of the control springs of the moving coil as the temperature of the instrument changes. Potentiometric instruments may be equipped with either manually or automatically operated cold-junction compensators. Evidently, the automatic compensators are effective only if the temperature of the cold junction is the same as that of the instrument. The cold junction may be brought to the instrument by the use of extension leads, which for base-metal couples are of the same materials as the couple. For rare-metal couples the extension leads consist of one wire of copper and one of a suitable alloy of copper and nickel. The extension leads may be carried directly to the indicator, or they may be brought to some point at constant temperature, for example, the bottom of a pipe driven about 10 ft into the ground. When the cold junction is not at the instrument, the compensator setting must correspond to the temperature of the cold junction.

MEASUREMENT OF EMF. The instruments used to indicate or record the emf of thermocouples may be classed as potentiometers (null instruments) and millivoltmeters (galvanometers). Instruments using a combination of the galvanometric and potentiometric principles have been devised but are seldom used.

Galvanometers. Since the emf's are small, the galvanometers must be very sensitive to voltage. This could be accomplished by making the galvanometer coil of low resistance, but on account of the variable line resistance of a thermoelectric circuit it is necessary to make the resistance of the galvanometer as high as possible, consistent with substantial construction. A large part of the resistance should be that of a manganin series resistor in order that the indicated emf may be nearly independent of the temperature of the instrument. If the indicator is graduated to read emf at its terminals the relation between scale reading e_0 and true emf of the couple e becomes

$$e_0 = R_g e/(R_g + R_c + R_l), \tag{57}$$

where R_g, R_c, and R_l denote the resistances of the galvanometer, couple, and lead wires, respectively. If R_g is large compared with $R_c + R_l$ the above equation reduces to $e_0 = e$ approximately and the indicator reads correctly. When R_g is low it is possible to graduate the scale to read true emf for a definite line resistance, but if this changes on account of deterioration of the couple, etc., serious error may result. Thus a 300-ohm indicator designed for use with a 2-ohm line resistance will be in error by only 7 deg at 1000 deg cent if the line resistance changes to 4 ohms, while a 10-ohm indicator under similar conditions would read 140 deg cent too low. This emphasizes the importance of using galvanometers of high resistance. Most pyrometer manufacturers can supply galvanometers (millivoltmeters) with a resistance of several hundred ohms.

The advantages of the galvanometer method are quick reading, simplicity, and moderate initial cost. The disadvantages lie largely in the effects due to thermocouple and lead resistance mentioned above.

Potentiometers. The most accurate instrument for measuring thermoelectric emf's is the potentiometer. Portable potentiometers are available for either rare-metal or base-metal couples, or for both with a double scale. There are several important advan-

tages in the potentiometer method. The emf or temperature scale is easily made very open, thus permitting accurate readings. The calibration of the scale is in no way dependent upon the constancy of magnets, springs, or jewel bearings, nor upon the level of the instrument. From the pyrometric standpoint, however, the greatest advantage is the complete elimination of any error due to ordinary changes in the resistance of the couple or of the lead wires. Thus if a potentiometer is correctly balanced and the resistance of the thermocouple is then greatly increased, the balance is unchanged. However, excessive resistance in the couple circuit reduces the sensitivity of a setting. The objections to the potentiometer are the greater initial cost and the fact that usually a manual adjustment must be made to obtain a setting.

At least four manufacturers are now making recording potentiometers designed for use with thermocouples. In these recorders the balancing is done automatically.

THERMOCOUPLES FOR SPECIAL USES. Many special types of thermocouple adapted for special uses are available. One of these is the pipe-type couple in which one element is a pipe or tube of some metal such as iron or chromel, and the other element is a wire passing through the pipe and insulated from it, except at the closed end where it is welded to the pipe to form the hot junction. There are a number of special types of couple designed for measuring the temperature of molten metals and alloys.

PYROMETER PROTECTION TUBES. In the measurement of high temperatures the choice of a protection tube is nearly as important as the selection of the thermocouple. All thermocouples should be protected from reducing gases, sulfur, and metallic vapors. Only porcelain, quartz, and Pyrex-glass protection tubes should be used for platinum-rhodium thermocouples. Pyrex glass is usable up to 600 deg cent, quartz (fused silica) up to 1000 deg cent, and American-made porcelain (mullite) up to 1550 deg cent. In many cases it is advisable to protect these tubes by an outside or secondary protection tube made of materials such as chromel, nichrome, carborundum, graphite, alundum, nickel, iron, calorized iron, etc. All these tubes have their special uses. Base-metal thermocouples are ordinarily protected by metal protection tubes, but under severe conditions porcelain tubes are used alone or in combination with a metal tube. The thermocouple wires are usually insulated by two-hole porcelain insulating tubes.

AUTOMATIC RECORDERS. The types of pyrometers ordinarily used in the automatic recording of temperatures are (1) resistance thermometers, (2) radiation pyrometers, and (3) thermoelectric pyrometers. Of these the last has the greatest applicability. The type of curve ordinarily required is temperature versus time, in which case the instrument is equipped with a mechanism for periodically recording its indications upon a chart which moves with a uniform speed. Single- or multiple-point recorders operating on either the galvanometric or potentiometric principle are available. Multiple-point recorders are equipped with a commutator switch which automatically connects various thermocouples into the circuit successively.

STANDARD SAMPLES FOR CHECKING THERMOCOUPLE PYROMETERS. In order to enable the user of thermocouple pyrometers to check their indications, the National Bureau of Standards supplies standard samples of metals of certified melting points. Information concerning the samples which could be purchased from the Bureau in March, 1936, is given in Table XIII.

Table XIII. Standard Samples of Metals for Checking Thermocouple Pyrometers

Sample No.	Metal	Melting Point		Weight of Sample in Grams	Price per Sample
		Deg Cent	Deg Fahr		
42b	Tin	231.9	449.4	350	$2.00
43c	Zinc	419.5	787.1	350	2.00
44c	Aluminum	660.2	1220.4	200	2.00
45a	Copper	1083	1981.4	450	2.00
49a	Lead	327.35	621.2	1650	2.00

RADIATION PYROMETRY AND OPTICAL PYROMETRY. Radiation pyrometry and optical pyrometry are based respectively on the intensity of total radiation and of visible radiation from a hot body. Since different substances at the same temperature do not necessarily emit radiation of the same intensity or spectral distribution, this fact must be taken into account in the use of either radiation pyrometers or optical pyrometers. There is one form of radiator, the so-called black body, the radiation from which depends only on its temperature. A uniformly heated enclosure with a small opening, or even a furnace with not too large an opening, constitutes a good approximation to a black body. If the object, the temperature of which is to be measured, is inside

such a black body, or if the radiation comes from a deep cavity in the object, the radiation will depend only on the temperature.

FÉRY MIRROR TELESCOPE PYROMETER. In this instrument radiation of all wavelengths is brought to a focus, by means of a concave gold mirror, upon the hot junction of a minute thermocouple. The cold junctions of the couple are suitably screened from the direct radiation of the hot body. The concentration of heat at the hot junction develops an emf which may be measured by a potentiometer or galvanometer. In practice the galvanometer is usually calibrated to read temperature directly. Foster has transformed the Féry telescope into a fixed-focus pyrometer by placing the thermocouple and a small front diaphragm at the conjugate foci of the gold mirror.

THWING PYROMETER. In the Thwing pyrometer the reflecting mirror is replaced by an aluminum cone which by multiple reflections concentrates the radiation at its apex on one or more small thermocouples in series with a portable galvanometer. The instrument requires no focusing, the front diaphragm acting as a source. The object sighted upon must be large enough to cover the projection of the cone through this diaphragm.

PYRO RADIATION PYROMETER. In the Pyro radiation pyrometer, the radiation is focused by means of a quartz objective lens upon a thermocouple mounted in an evacuated bulb. A "receiving disc" fixed over the thermojunction can be seen in the field of view and must be fully covered by the image of the radiating source. The instrument is very compact and direct reading, the millivoltmeter being mounted in the telescope itself and graduated in temperature directly.

Precautions in the Use of Radiation Pyrometers. The mirrors or other reflecting devices must be kept bright and free from dust. With the Féry pyrometer, errors amounting to 100 deg cent have been observed, on account of ordinary accumulation of dirt upon the large gold mirror. Many radiation instruments require several minutes of exposure to the radiating source to indicate a maximum reading on account of the slow heating of the hot junction; others require less than 20 sec. The maximum indication should be accepted. Care must be taken that the source is large enough to "fill" the aperture of the pyrometer completely and that the pyrometer is focused. It is usually impossible to focus upon the back of a furnace through a very small peephole and obtain reliable results; the hole must be enlarged so that it does not cut into the cone of rays entering the instrument, or the pyrometer may be focused upon the hole itself. In the latter case the hole must be large enough to "cover" the thermocouple in the Féry pyrometer or the front diaphragms of the Thwing and Foster pyrometers. Variations in room temperature in general affect the hot and cold junctions nearly alike, so that very little error is introduced in the reading of a radiation instrument from this cause. It is desirable to employ the same ratio of diameter of source sighted on to sighting distance both in calibration and use of all radiation pyrometers.

Because of the uncertainty of the large correction required when sighting upon non-black bodies, radiation pyrometers are not suitable for measuring temperatures of materials in the open.

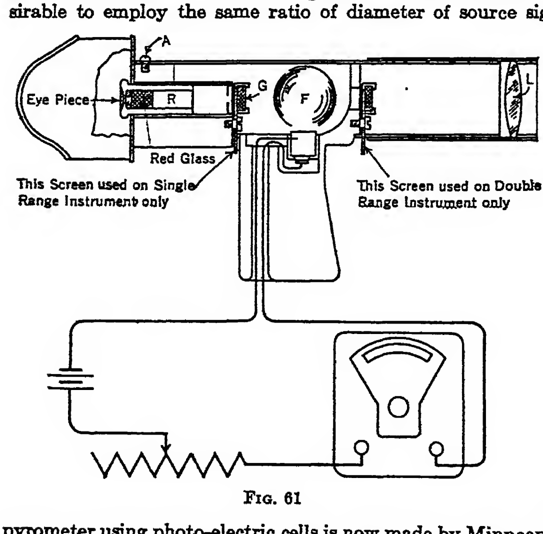

Fig. 61

OPTICAL PYROMETRY. An optical pyrometer is essentially a photometer for comparing the brightness of the red light (λ = approximately 0.65 μ) radiated by a body with that radiated by a comparison source such as an electric lamp. Temperatures on the international scale above 1063 deg cent are defined in terms of such measurements. An automatically recording optical pyrometer using photo-electric cells is now made by Minneapolis-Honeywell Regulator Co.

LEEDS & NORTHRUP OPTICAL PYROMETER. The filament F of a small electric lamp, Fig. 61, is placed at the focus of the objective lens L which superimposes

upon the lamp filament an image of the source to be measured. This image and filament are viewed through the eyepiece *R* which is equipped with a red glass screen. In making a setting the current through the lamp is adjusted until the filament and source appear equal in brightness.

The calibration of the lamp will remain remarkably constant if it is not operated above 1500 deg cent. For higher temperatures an absorption screen is provided between the lamp and objective, which may be thrown in or out of the field of view by a lever. This screen reduces the brightness of the source so that any temperature up to 1800 deg cent. can be matched by the lamp at 1400 deg cent or less. For higher temperatures a second absorption screen is used in combination with the first. Without the absorption screen the calibration gives a relation between lamp current and temperature. With the absorption screen, calibration gives a second relation between lamp current and temperature.

The F & F pyrometer is similar to the Leeds & Northrup instrument except that the current is kept constant at the "normal" value and an absorption wedge between source and lamp is adjusted until sufficient thickness of the wedge has been introduced to reduce the brightness of the source to that of the lamp at normal current. The wedge carries a pointer moving over a scale graduated directly in temperature. This instrument is made with two ranges, the same wedge being used alone or in combination with a fixed absorption glass.

EMISSIVITY CORRECTIONS FOR OPTICAL PYROMETERS. Optical pyrometers indicate the true temperature of black bodies only. A peephole in a furnace or a porcelain tube with the closed end immersed in molten metal or otherwise surrounded by the temperature to be measured may be considered as a black body. When an optical pyrometer is sighted upon objects in the open, corrections must be made in accordance with the following table:

Table XIV. Correction to Observed Temperature Using Red Light

Emissivity	Add Corrections below for the Following Observed Temperatures, deg cent						
	800	1000	1200	1400	1600	1800	2000
0.30	67	95	129	168	213	264	322
0.40	50	71	96	125	158	195	237
0.50	37	53	71	93	117	144	175
0.60	27	39	52	67	85	104	126
0.70	19	27	36	47	59	72	87
0.80	12	17	22	29	36	44	54
0.90	6	8	10	14	17	21	25
1.00	0	0	0	0	0	0	0

Values of the emissivity of a few of the more common materials are given in Table XV.

Table XV. Monochromatic Emissivity, E_λ, for Red Light

($\lambda = 0.65\mu$)

Material	Monochromatic Emissivity E_λ	Material	Monochromatic Emissivity E_λ
Silver	0.07	Nichrome, 900° C	0.90
Gold, solid	0.13	Nichrome, 1200° C	0.80
Gold, liquid	0.22	Cuprous oxide	0.70
Platinum, solid	0.33	Iron oxide, 800° C	0.98
Platinum, liquid	0.38	Iron oxide, 1000° C	0.95
Palladium, solid	0.33	Iron oxide, 1200° C	0.92
Palladium, liquid	0.37	Nickel oxide, 800° C	0.96
Copper, solid	0.11	Nickel oxide, 1300° C	0.85
Copper, liquid	0.15	Nickel, solid and liquid	0.36
Tantalum, 1100° C	0.60	Iridium	0.30
Tantalum, 2600° C	0.48	Rhodium	0.30
Tungsten, 1000° C	0.46	Graphite powder (estimated)l	0.95
Tungsten, 2000° C	0.43	Carbon	0.85
Tungsten, 3000° C	0.41	Porcelain	0.25 to 0.50
Nichrome, 600° C	0.95		

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NOTE: The abbreviations Benn, Chapman, Griffin, Macmillan, McGraw, Pitman, Van Nostrand, and Wiley denote the following publishers: Ernest Benn, Ltd., London; Chapman & Hall, Ltd., London; C. Griffin & Co., Ltd., London; Macmillan & Co , Ltd., London; McGraw-Hill Book Co., Inc., New York; Sir Isaac Pitman & Sons, Ltd., London; D. Van Nostrand Co., New York; and John Wiley & Sons, Inc., New York.

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NOTE: The abbreviations de Gruyter, Leiner, Oldenbourg, and Springer denote the following publishers: Walter de Gruyter & Co., Berlin and Leipzig; Oskar Leiner, Leipzig; R. Oldenbourg, Munich and Berlin; and J. Springer, Berlin.

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SECTION 6

PRINCIPLES OF ELECTROCHEMISTRY

BY

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REVISED BY

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ART.		PAGE
1.	Definitions	02
2.	Notation	04
3.	Chemical Equations	05
4.	Faraday's Laws	05
5.	Electrochemical Equivalents of the Elements	05
6.	Conductivity of Electrolytes	06
7.	Voltaic and Electrolytic Cells	07
8.	Polarization	07
9.	Reversible Electrodes—Depolarizers	08
10.	Contact Potentials; Electromotive Force	08
11.	Electromotive Force and Heat of Reaction	11
12.	Electrolytic Decomposition	13
13.	Theory of Solutions	15
14.	Theory of Electrolytic Conduction	18
15.	Endosmose or Electric Osmosis	25

PRINCIPLES OF ELECTROCHEMISTRY

Revised by Martin Kilpatrick and Mary L. Kilpatrick

Electrochemistry is the science which deals with the phenomena resulting from the direct transformation of electrical into chemical energy or the converse transformation of chemical into electrical energy. By general usage the term, especially as applied to industrial processes has come to include also those thermochemical phenomena which occur at temperatures produced in electric furnaces, although in such processes electrical energy frequently plays no other rôle than that of generating heat.

1. DEFINITIONS

The following terms are commonly used:

Element. An element is a substance which cannot be decomposed by the ordinary process of chemical analysis. (See Table of Electrochemical Equivalents, p. 6–06, and International Atomic Weights, Sect. 2.) At present 90 elements are known. In connection with the definition given, it should be added that the radioactive elements undergo spontaneous decomposition into other elements, and that recently certain of the elements have been decomposed by bombardment with alpha-rays, hydrogen nuclei, and neutrons.

Atoms and Atomic Weights. An atom is the smallest mass of an element that can enter into chemical combination with another element or with itself. As a standard, the atomic weight of oxygen has been arbitrarily chosen as 16. The combining or equivalent weight of an element is the weight which combines with 8 parts by weight of oxygen. The atomic weight is identical with, or a simple multiple of, the combining weight, depending on the number of atoms of the element which combine with one atom of oxygen. The number of equivalent weights of an element which constitute its atomic weight is determined from a study of the composition of the gaseous compounds of the element, or from a study of its specific heat. On the atomic theory the atomic weight is proportional to the mass of the atom.

Molecules and Molecular Weight. A molecule is the smallest mass of a substance which can exist and preserve its chemical properties. The molecular weight of an element or of a chemical compound is equal to the sum of the atomic weights of the atoms contained in the molecule, and may readily be calculated from the atomic weights when the molecular symbol of the compound is known. Thus the molecular weight of oxygen, O_2, is $2 \times 16.00 = 32.00$ as there are two atoms in the molecule. The molecular weight of silver nitrate, $AgNO_3$ is $107.88 + 14.01 + 3 \times 16.00 = 169.89$.

Radical. A radical is a combination of two or more elements which persists as a group in chemical reactions, e.g., the SO_4 in H_2SO_4 is a radical, as exemplified by the reaction $H_2SO_4 + Zn = ZnSO_4 + H_2$.

Formula Weight. The formula weight of an atom, radical, or molecule is the sum of the atomic weights of the elements of which it is formed. For example, the formula weight of an oxygen atom, O, is 16; the formula weight of an oxygen molecule, O_2, is 32; the formula weight of the radical NO_3 is $14.01 + 3 \times 16.00 = 62.01$.

Valence. An adequate consideration of valence would involve a discussion beyond the limits of this article. As a simple definition, the valence of an element may be taken as the number of hydrogen atoms, or atoms of an element such as chlorine which combines with hydrogen atom for atom, combining with an atom of the element in question. Thus from the formulas H_2O, HCl, KCl, $BaCl_2$ the valences of oxygen, chlorine, potassium, and barium are seen to be 2, 1, 1, and 2, respectively. The valence of a radical is similarly defined; for example, the formulas KNO_3 and H_2SO_4 show that the nitrate radical has a valence of 1, the sulfate radical, a valence of 2. Acids are classified as mono-, di-, or tribasic according to the number of replaceable hydrogen atoms which they contain (cp. HCl, H_2SO_4, H_3PO_4). Bases are classified as mono-, di-, or triacidic (KOH, $Ba(OH)_2$, $La(OH)_3$). Salts are classified as uni-univalent (KCl), bi-univalent ($BaCl_2$), uni-bivalent (K_2SO_4), bi-bivalent ($BaSO_4$), etc.

The valence of an element may vary, cp. the cupric compounds (CuO, $CuCl_2$, etc.)

in which the valence of copper is 2, and the cuprous compounds (Cu_2O, $CuCl$, etc.) in which the valence of copper is 1.

Equivalent Weight—Chemical Equivalent. The equivalent weight of an atom, ion, or radical is defined as its formula weight divided by its valence. The equivalent weight of an element or compound is also referred to as its "chemical equivalent." The equivalent weight of an acid, base, or salt is its molecular formula weight divided by the valence of the ion of higher valence. As hydrogen exhibits no other valence than unity its equivalent weight and atomic weight are identical. Similarly, the equivalent weight and molecular weight of hydrochloric acid, HCl, are each 36.47. Copper in copper sulfate, $Cu(SO_4)$, and in cupric chloride, $CuCl_2$, has a valence of 2; hence equivalent weights of these salts are one-half their molecular weights respectively.

In computations of electrochemical reactions it is frequently convenient to take as the unit of mass (or weight) of a substance, a mass in grams equal to the number expressing the atomic weight, ionic weight, molecular weight, or equivalent weight of the substance. The following names have been given these units.

Gram-atom or Gram-atomic Weight. The number of grams of an element equal to its atomic weight, e.g., one gram-atom of oxygen is 16 grams.

Gram-molecule or Mol. The number of grams of a substance equal to its molecular weight, e.g., one mol of silver nitrate, $AgNO_3$, is 169.89 grams.

Gram-equivalent. The number of grams of an atom, radical, or molecule equal to its equivalent weight. For example, a gram-equivalent of sulfuric acid is $\frac{1}{2}\,H_2SO_4 = \frac{1}{2}\,98.09 = 49.04$ grams.

Electrolyte, Electrolysis. When an electric current is passed through certain substances known as electrolytic conductors a chemical reaction occurs at the places where the current enters and leaves, and there is a transfer of matter through the substance. Fused salts, and aqueous solutions (or solutions in other ionizing solvents) of acids, bases, and salts, belong in the class of electrolytic conductors. The acid, base, or salt in question is called an electrolyte, and the process of the passage of a current through an electrolytic conductor is known as electrolysis. It is important to note that an electrolyte is ionized before the current is passed.

Electrodes. Electrodes are the conductors by which the current enters and leaves the electrolyte or its solution. The electrode at which the current enters is called the "anode," and that by which the current leaves is called the "cathode."

Ions. Anions and Cations. The constituents of an electrolytic conductor which conduct the current through the solution or the fused salt are called ions. They exist in the conductor before the passage of the current, and during the passage of the current the positively charged ions or cations travel toward the cathode, and the negatively charged ions or anions travel toward the anode.

The charge of an ion is proportional to its valence. Each individual univalent anion bears a charge of one electron; each bivalent anion, a charge of two electrons, etc. Each univalent cation has a deficit of one electron or a positive charge of magnitude equal to that of an electron; each bivalent cation has a deficit of two electrons, etc. The value of the electron, as determined by Millikan, is 4.774 ± 0.005 electrostatic units. See Handbook of Engineering Fundamentals, Eshbach, John Wiley & Sons, Inc., "Electron Theory."

To distinguish ions from neutral particles, small plus or minus signs are written after the symbol, the number of such signs being equal to the valence of the ion. Thus Cl^- represents a gram-atom of chlorine bearing a charge of an equivalent of negative electricity or N electrons, N being Avogadro's number. An equivalent of negative electricity is designated as $(-)$. Ca^{++} represents a gram-atom of calcium having a deficit of $2N$ electrons or $2(-)$.

Ionic Weight. The formula weight of an ion is called its ionic weight.

Gram-ion. The number of grams of an ion equal to its ionic weight, e.g., one gram-ion of hydrogen is 1.008 grams, and one gram-ion of SO_4 is 96.07 grams.

Electrochemical Equivalent. The electrochemical equivalent of an ion is the mass in grams of the ion liberated or deposited by one coulomb of electricity.

Electrochemical Constant or "Faraday." The electrochemical constant, denoted by F, and called the "faraday," is the number of coulombs required to produce one equivalent of chemical change. It is a constant for all ions. Its value is $96{,}500 \pm 10$ coulombs (International Critical Tables, vol. 1, p. 17).

Solvent, Solute, and Solution. A solution is formed if upon mixing substance A with substance B a homogeneous or one-phase system results. For dilute solutions, it is customary to refer to the substance present in great excess as the solvent, and to the other as the solute.

Osmotic Pressure. The osmotic pressure of a solution is the pressure which must

be exerted upon it in order that the vapor pressure of the solvent from the solution be the same as the vapor pressure of the pure solvent at the temperature in question. See section below on Theory of Solutions.

Specific Conductance (κ). The specific conductance of a solution is the reciprocal of its specific resistance, i.e., it is the conductance of a column of liquid 1 cm long and 1 sq cm cross-section. It is expressed in reciprocal ohms, or mhos, and is denoted by κ.

Concentration. The composition of a solution may be expressed in a number of ways, of which the more common are listed below.

(*a*) For a solution composed of n_1 gram-molecules of X_1, n_2 of X_2, . . . n_r of X_r, the mol fraction of the *i*th component is given as $N_i = \dfrac{n_i}{\sum_{j=1}^{r} n_j}$.

(*b*) The number of gram-molecules of X per liter of solution is called the molarity of X.

(*c*) The number of gram-molecules of X dissolved in 1000 grams of solvent is called the molality of X.

(*d*) The concentration of X may be expressed as the number of grams of X contained in unit volume of the solution.

(*e*) The number of gram-equivalents of X in unit volume of solution is called the equivalent concentration. The symbol η is used here to represent the number of equivalents per cubic centimeter of solution.

Dilution (ϕ). The dilution of a solution is the number of cubic centimeters of solution in which one gram-equivalent of solute is dissolved. It is denoted by ϕ. Hence $\phi = 1/\eta$ and $\eta = 1/\phi$.

Normal Solution. A normal solution is a solution containing one gram-equivalent of solute per liter. For such a solution $\eta = 10^{-3}$ or $\phi = 1000$.

Equivalent Conductance (Λ). The equivalent conductance of a solution at the dilution ϕ is the conductance which a volume (in cubic centimeters) of the solution containing one gram-equivalent of the solute would have, if placed between parallel plate electrodes 1 cm apart. It is denoted by Λ_ϕ or Λ_η, according as the dilution or concentration of the solution is given. Hence $\Lambda_\phi = \phi\kappa$, or $\Lambda_\eta = \kappa/\eta$.

The dilution or concentration of a solution should always be stated in connection with its equivalent conductance, otherwise the expression is indefinite.

Heat of Reaction. The heat of reaction is the heat energy given out or absorbed in a chemical reaction. It is usually expressed in calories. Ostwald recommends the use of a calorie equal to 100 times the mean gram-calorie for expressing thermochemical data, and this will be adopted in the present discussion. It will be denoted in this article by "cal."

Exothermic Reactions. Reactions in which heat is given out to the surrounding bodies.

Endothermic Reactions. Reactions in which heat is absorbed from the surrounding bodies.

2. NOTATION

For convenience of reference the symbols most frequently used are tabulated below.

κ = specific conductance in reciprocal ohms.
η = concentration, expressed as gram-equivalents of solute per cubic centimeter solution.
ϕ = dilution, expressed as cubic centimeters of solution per gram-equivalent solute.
Λ = equivalent conductance.
Λ_0 = equivalent conductance at infinite dilution.
α = degree of ionization.
E = electrical potential difference.
I = current strength.
r = resistance.
e = single potential difference, taken positive in direction of rise of potential.*
e_h = potential difference measured against a normal hydrogen electrode.
e_c = potential difference measured against a normal calomel electrode.
R = gas constant per mol.
F = the Faraday = 96,500 coulombs per gram-equivalent.
T = absolute temperature in degrees centigrade.
t = centigrade temperature.

* This convention is in agreement with that used elsewhere in this book, and with that employed in The International Critical Tables, see vol. 6, p. 332.

3. CHEMICAL EQUATIONS

A chemical equation, such, for example, as that representing the solution of zinc in a dilute hydrochloric acid solution (Aq = water),

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2,$$

is to be interpreted as follows: one gram-atom (65.37 grams) of zinc reacting with 2 mols ($2 \times 36.47 = 72.94$ grams) of hydrochloric acid in aqueous solution forms one mol ($65.37 + 2 \times 35.46 = 136.3$ grams) of zinc chloride in solution and one mol ($2 \times 1.008 = 2.016$ grams) of hydrogen gas. Since atomic weights are relative numbers, any unit of weight, as the pound or kilogram, may be substituted for gram in this statement.

If the reaction is intended to express not only the chemical change which takes place, but also the energy change involved, the " heat of the reaction " must be included, thus:

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2 + 342 \text{ cal.}$$

This thermochemical equation signifies that the intrinsic energy represented by the initial system, consisting of 65.37 grams of zinc and 72.94 grams of hydrochloric acid dissolved in enough water to form a dilute solution, is 342 calories (1 gram of water from 0 to 100 deg cent) greater than that represented by the final system, consisting of 136.3 grams of zinc chloride dissolved in water and 2.016 grams of hydrogen gas; in other words, when the substances in the initial system react and form the substances in the final system, an amount of heat energy is evolved equal to 342 cal. When energy is given up by a system to the surroundings (exothermic reactions), as in the above illustration, it is regarded as positive. If a reaction is accompanied by an absorption of heat energy (endothermic reactions), i.e., if the system takes up heat and thereby tends to cool the surroundings, the sign of the heat of the reaction is taken as negative.

4. FARADAY'S LAWS

The quantitative relations between the magnitude of an electric current and its chemical effect were discovered by Faraday in 1834 and are known as Faraday's laws. They may be stated as follows:

FIRST LAW. The amount of chemical change produced by an electric current is directly proportional to the total quantity of electricity which passes through the electrolytic cell. The amount of chemical change is independent of the voltage and intensity of the current, size of electrodes, and concentration of electrolyte, so long as the total *quantity* of electricity flowing through the circuit remains constant. (These factors, however, do affect the *ultimate products* of an electrolysis.)

SECOND LAW. A given quantity of electricity always decomposes *equivalent* weights of different electrolytes irrespective of their nature.

Faraday's laws are the expression of the results of direct experiment and have been tested to the limit of precision with which physical and chemical measurements are capable of being carried out at the present time. All evidence indicates that they hold rigidly, i.e., that they are exact laws of nature. In cases where exceptions have seemed to exist these have been shown to arise from secondary causes. See Kraus, *Trans. Am. Elec. Chem. Soc.*, 21, 119 (1912) for special cases of electronic and ionic conduction.

VALUE OF THE ELECTROCHEMICAL CONSTANT OR FARADAY. The determination of the number of coulombs required to deposit one gram-equivalent of an ion, i.e., the constant connecting the quantity of electricity and the mass of a substance liberated, involves two distinct investigations: first, the measurement of the number of coulombs which pass through a given electrolytic cell; and second, the determination of the amount of the chemical decomposition. For a résumé of work on this subject, see *Bull. Bur. Standards*, 13, 497 (1916). The Bureau of Standards gives 96,503 coulombs as the most probable value of the faraday, and recommends 96,500 as the best round value. The International Critical Tables, vol. 1, p. 17, gives $96{,}500 \pm 10$ coulombs.

5. ELECTROCHEMICAL EQUIVALENTS OF THE ELEMENTS

From the definitions of the electrochemical equivalent and the faraday, it follows that the electrochemical equivalent of any ion, expressed in grams per coulomb (one ampere per second), is equal to

$$\frac{\text{gram-equivalent of the ion}}{96{,}500}$$

Table 1 contains the values of the electrochemical equivalents of the more common elements which may be found useful in certain calculations. The electrochemical equivalent of any radical may be readily calculated from its formula; for example, the electrochemical equivalent of SO_4 is

$$(32.06 + 4 \times 16.00)/2 \times 96{,}500 = 0.0004978 \text{ gram or } 0.4978 \text{ milligram.}$$

Table 1. Electrochemical Equivalents of the More Important Elements

Based on the atomic weights of 1935 (*J. Am. Chem. Soc.*, 57, 787, 1935) and on $F = 96{,}500$ coulombs.

Element	Symbol	Atomic Weight	Valence	Milligrams Deposited by One Ampere in One Second	Grams Deposited by One Ampere in One Hour
Aluminum	Al	26.97	3	0.09316	0.3354
Antimony	Sb	121.76	3	0.4206	1.516
Arsenic	As	74.91	3	0.2587	0.9314
Barium	Ba	137.36	2	0.7117	2.562
Bismuth	Bi	209.0	3	0.7219	2.599
Bromine	Br	79.916	1	0.8282	2.982
Cadmium	Cd	112.41	2	0.5825	2.097
Calcium	Ca	40.08	2	0.2077	0.7474
Carbon	C	12.00	4	......	......
Cerium	Ce	140.13	3	0.4841	1.743
Chlorine	Cl	35.457	1	0.3674	1.323
Chromium	Cr	52.01	2	0.2695	0.9702
Chromium	Cr	52.01	3	0.1797	0.6469
Cobalt	Co	58.94	2	0.3054	1.099
Cobalt	Co	58.94	3	0.2036	0.7330
Copper	Cu	63.57	1	0.6588	2.371
Copper	Cu	63.57	2	0.3294	1.186
Fluorine	Fl	19.00	1	0.1969	0.7088
Gold	Au	197.2	1	2.044	7.358
Gold	Au	197.2	3	0.6812	2.452
Hydrogen	H	1.0078	1	0.01044	0.03755
Iodine	I	126.92	1	1.315	4.734
Iron	Fe	55.84	2	0.2893	1.042
Iron	Fe	55.84	3	0.1929	0.6944
Lead	Pb	207.22	2	1.074	3.866
Lithium	Li	6.940	1	0.07192	0.2589
Magnesium	Mg	24.32	2	0.1260	0.4536
Manganese	Mn	54.93	2	0.2846	1.025
Manganese	Mn	54.93	3	0.1897	0.6829
Mercury	Hg	200.61	1	2.079	7.484
Mercury	Hg	200.61	2	1.039	3.740
Nickel	Ni	58.69	2	0.3041	1.095
Nickel	Ni	58.69	3	0.2027	0.7297
Oxygen	O	16.000	2	0.08290	0.2984
Platinum	Pt	195.23	2	1.012	3.643
Platinum	Pt	195.23	4	0.5058	1.821
Potassium	K	39.096	1	0.4052	1.458
Silver	Ag	107.880	1	1.118	4.025
Sodium	Na	22.997	1	0.2383	0.8579
Strontium	Sr	87.63	2	0.4541	1.635
Sulfur	S	32.06	..	......	......
Thallium	Tl	204.39	1	2.118	7.625
Thallium	Tl	204.39	2	1.059	3.812
Tin	Sn	118.70	4	0.3075	1.107
Titanium	Ti	47.90	4	0.1241	0.4468
Tungsten	W	184.0	2	0.9534	3.432
Zinc	Zn	65.38	2	0.3388	1.220

6. CONDUCTIVITY OF ELECTROLYTES

SOLUTIONS. (See also article on Resistance and Conductance.) The specific conductance of an electrolytic solution varies between wide limits. It depends upon the nature of the solute and of the solvent, on the temperature, and on the concentration of the solution. The effect of these factors is discussed below. For numerical values see Kohlrausch and Holborn, Leitvermögen der Elektrolyte; Landolt-Börnstein, Tabellen; Tables Annuelles Internationales de Constantes; and the International Critical Tables.

FUSED ELECTROLYTES. Most inorganic salts conduct electrolytically when heated to a sufficiently high temperature to cause them to pass into the liquid state. For such electrolytes Faraday's, Ohm's, and Joule's laws hold as they do in the case of solutions. The conductance increases in general from 1 to 1 1/2 per cent per degree centigrade. An exhaustive résumé of all matters relating to fused electrolytes may be found in Lorenz's Elektrolyse der Geschmolzene Salze. On account of their enormous concentration (100 per cent) the specific conductance of fused salts is generally very high. This, together with the difficulty of obtaining a non-conducting chemically inert vessel of suitable shape to contain the salt and the difficulty of regulating high temperatures to a fraction of a degree, makes the measurement of the conductivity of fused electrolytes far more difficult than in the case of solution. Fused silica and natural quartz crystals have been successfully used for conductivity cells. See Jaeger and Kapma, *Z. anorg. Chem.*, 113, 27 (1920); Arndt and Ploetz, *Z. physik. Chem.*, 110, 237 (1924); Biltz and Klemm, *ibid.*, 110, 318 (1924); Lorenz, *Z. Elektrochem.*, 30, 371 (1924); and for further references, Taylor, Treatise on Physical Chemistry, vol. 1, pp. 624–628.

7. VOLTAIC AND ELECTROLYTIC CELLS

When a cell formed by two electrodes and one or more electrolytes gives out electric energy the cell is called a " voltaic " cell. All ordinary chemical batteries are voltaic cells. If across the terminals of a cell an external electromotive force, greater than the electromotive force developed within the cell, is impressed, then a current will flow through the cell in the opposite direction, and the cell will absorb electric energy. A cell which absorbs electric energy is usually referred to as an " electrolytic cell." The cells used in electrolytic refining and similar processes are electrolytic cells. An electrolytic cell may or may not have an electromotive force on open circuit, but may develop an electromotive force by virtue of the chemical changes or changes in concentration which take place due to the current forced through it from some external source. A voltaic cell becomes an electrolytic cell when the electromotive force impressed across its terminals exceeds (and opposes) its own electromotive force.

8. POLARIZATION

When an electric current passes through either a voltaic or an electrolytic cell, in general, there is developed at the electrodes of the cell, as a result of the chemical actions and changes in concentration which take place, a *back* or *counter electromotive* force, in addition to its open-circuit electromotive force (if any), and an *increase in the resistance* of the cell, in addition to the change in resistance due to the heating effect of the current. Both of these effects cause a decrease in the strength of the current through the cell. These two phenomena are said to be due to " polarization " and " transition resistance " respectively.

Even in the case where the electrolyte, as a whole, suffers no change in concentration, as in the electrolysis of copper sulfate between copper electrodes, a counter emf of polarization is produced in consequence of the difference in concentration of the copper ions in the neighborhood of the anode and cathode. Vigorous stirring of the electrolyte tends to reduce this concentration difference and the resulting polarization, but except for very low current densities, it cannot be completely eliminated. It is for this reason, in refining processes, that energy is required to transfer the metal from anode to cathode in addition to that necessary to overcome the internal resistance of the cell.

A transition resistance results from the formation of a film of poorly conducting material over the electrode and may or may not be present according to the character of the electrolysis.

MEASUREMENT OF EMF OF POLARIZATION AND TRANSITION RESISTANCE. As both transition resistance and polarization tend to cut down the current flowing through the cell, they cannot be distinguished by this effect alone. The transition resistance may be determined by measuring the ohmic resistance of the cell before and after the passage of the current by the usual alternating-current method. The existence of an emf of polarization in a cell which normally has no open-circuit emf may be qualitatively demonstrated by short-circuiting the cell through a galvanometer immediately after breaking the primary circuit; if the cell be polarized, a current which diminishes to zero will flow through it in the reverse direction. A voltmeter, or, better, an electrometer connected across the electrolytic cell, will indicate the polarization voltage the instant after the current is broken. This voltage will diminish as the polarization disappears, the rate depending upon the current through the voltmeter and the rate of diffusion of

the electrolytic products causing the polarization. Owing to the rapidity with which the emf of polarization falls off after the exciting cause is removed, special precautions must be observed in its measurement. One of the best methods is to connect the circuit containing the applied emf and electrolytic cell with a tuning-fork interrupter and electrometer so that at the instant the battery current is broken at each vibration of the fork, the circuit containing the cell and electrometer or voltmeter is closed, and vice versa. By this arrangement, the emf of polarization is measured during the fraction of a second that the battery circuit is open, and before it has had time to diminish sensibly. The polarizing current may also be regarded as practically constant.

It is frequently of importance to know, not the polarization of the cell as a whole, but the polarization at each electrode. This is obtained by measuring the drop in potential at each electrode against a normal hydrogen or calomel electrode while the impressed emf is acting on the electrolytic cell, or immediately after the current through the cell is interrupted.

9. REVERSIBLE ELECTRODES—DEPOLARIZERS

If the chemical and thermal actions which take place at a given electrode when a given quantity of electricity passes in one direction through a cell are exactly the reverse of the actions which take place when the same quantity of electricity passes through it in the opposite direction, the electrode and the solution with which it is in contact are said to form a "reversible electrode." Such an electrode does not polarize provided the concentration of the solution is kept constant.

There are two types of reversible metal-liquid electrodes, namely:

Electrodes of the First Type, consisting of a metal in contact with a solution of one of its own salts, e.g., copper in copper sulfate, zinc in zinc sulfate, etc. The two electrodes of a Daniell cell are of this type.

Electrodes of the Second Type, consisting of a metal in contact with a solution containing one of its difficultly soluble salts and a second soluble salt of some other metal, having the same anion. The difficultly soluble salt, called the "depolarizer," must be present in excess as solid. Such an electrode is mercury in contact with a solution of zinc sulfate containing mercurous sulfate in excess as solid; this is one of the electrodes of the Clark cell.

Reversible Gas Electrode. An electrode consisting of "platinum black" saturated with an atmosphere of hydrogen gas and dipping partially into a solution containing hydrogen ions is also a reversible electrode. This is a special form of electrode of the first type in which a gas by being occluded in platinum is made to play the rôle of a metal. Other "gas electrodes," e.g., chlorine, may be similarly prepared.

10. CONTACT POTENTIALS; ELECTROMOTIVE FORCE

The electromotive force of a voltaic cell, the back-electromotive force of an electrolytic cell, and the electromotive force of a thermocouple are all due to differences of potential which always exist at the junction of dissimilar substances. For a general discussion of this subject, see Langmuir, *Trans. Am. Electrochem. Soc.*, 29, 125 (1916); Bridgman, *Phys. Rev.*, 14, 306 (1919); and Sommerfeld and Bethe, Handbuch der Physik, vol. 24, pp. 333–622 (Springer, Berlin, 1933).

THE VOLTA EFFECT. When pieces of two different metals are brought into metallic contact, it is found that between points in the surrounding medium (gas or vacuum) immediately outside the two metals a difference of potential exists. When the surrounding medium is a perfect vacuum, the potential difference is known as the true Volta contact potential difference between the metals in question. The Volta contact potential difference K_{AB} may be thought of as made up of three parts: the potential difference between the ether and A, the potential difference at the junction of the metals A and B, and the potential difference between B and the ether.

Investigations on the thermo- and photo-emission of electrons have demonstrated that an amount of work w_A is required to expel an electron from the metal A; from measurements on the thermo- and photo-emission of electrons from A and B the Volta contact potential difference between A and B may be obtained. K_{AB} may also be measured directly by the condenser method. Considerable difficulty has been experienced in the experimental determination of K_{AB}, due chiefly to the fact that the adsorption of minute traces of chemically active gases upon the surface of the metals greatly affects the results. The following table, taken from the International Critical Tables, vol. 6, p. 57, gives values of K_{AB} for a number of pairs of conductors.

Table 2. Contact Potential of Miscellaneous Pairs of Conductors, Condenser Method, Room Temperature

$K_{AB} = V_A - V_B$, unit = 1 volt.

A	B	K_{AB}	A	B	K_{AB}
Al *	Fe	+0.87	$C + NH_3$ ¶	Cu	+0.079
Al *	Zn	+0.29	$C + H_2$ ¶	Cu	+0.096
Fe *	Zn	−0.60	$C + N_2$ ¶	Cu	+0.129
CuO *	Li	−1.52 †	$C + CO_2$ ¶	Cu	+0.130
		−1.11 ‡	C + NO ¶	Cu	+0.136
CuO *	Na	−2.52	$C + O_2$ ¶	Cu	+0.142
Cd §	Hg	−0.22	$C + O_3$ ¶	Cu	+0.155
Hg §	Sb	−0.26			

* In vacuum. † Fresh surface. ‡ Old surface. § Initial value in dry air, pressure 0.05 mm. Hg. K_{AB} varies with the time. ¶ Coconut charcoal saturated with the gas indicated.

THE PELTIER EFFECT. Heat is absorbed or evolved when a current passes though the junction of two metals, and reversal of the current causes the reversal of the heat effect. Conversely, if the two junctions of a closed circuit composed of metals A and B in series are kept at different temperatures, current flows around the circuit. Let the metals be so designated that current flows from A to B at the hot junction, and let P_{AB} be the heat absorbed when unit quantity of positive electricity flows from A to B at the temperature T. It may be shown thermodynamically that the temperature coefficient of the emf of the circuit is related to the Peltier heat P_{AB} by the equation

$$T\frac{dE_{AB}}{dT} = P_{AB}$$

where the subscripts indicate the direction of flow of the current at the hot junction.

P_{AB} is also known as the Peltier coefficient and as the Peltier electromotive force. The use of the term "Peltier emf," with the understanding that the heat absorbed is equal to the emf located at the junction between the metals, is open to criticism; see Bridgman, *Phys. Rev.*, 14, 318 (1919).

Numerous data for the computation of the Peltier heat and the thermoelectric emf of a bimetallic circuit are to be found in the International Critical Tables, vol. 6, pp. 213–229. For most pairs of metals P_{AB} is of the order of magnitude of a few millivolts; in the case of antimony and bismuth, however, it is approximately 0.03 volt. Fair agreement exists between the values of P_{AB} directly observed and the values computed from the equation

$$P_{AB} = T\frac{dE_{AB}}{dT}.$$

LIQUID-LIQUID POTENTIALS. When two dissimilar electrolytic solutions are brought into contact the phenomenon of diffusion takes place between them until a homogeneous mixture results. At the same time a difference of potential is produced between the solutions, the magnitude of which depends upon the velocity of migration and the relative concentration of the ions taking part in the diffusion, the charge which they carry, the absolute temperature, and the gas constant. Liquid junction potentials like metal-metal potentials are, in general, of small magnitude. Thus the potential difference between two solutions of potassium chloride, one solution having 10 times the concentration of the other, is only 0.0005 volt, and diminishes as the ratio of the concentrations approaches unity. Potassium chloride is often used as an intermediate electrolyte when it is desired to reduce the liquid-liquid potentials of a voltaic combination to a minimum because of the small potential to which it gives rise.

Liquid potentials are greatest between acid or between alkali solutions. Thus, between two hydrochloric acid solutions whose concentrations are in the ratio 10 to 1 at a temperature of 18 deg cent there is a potential difference of about 0.038 volt; between two sodium-hydrate solutions under the same conditions there is a potential difference of 0.034 volt.

METAL-LIQUID POTENTIALS. From what has been said regarding the magnitude of metal-metal and liquid-liquid potentials, it follows by the process of elimination that the main seat of the electromotive force of a voltaic cell must reside at the junctions between metals and liquids. The determination of the *absolute* value of the potential difference between a metal and the solution with which it is in contact is problematical. In practice electrode potentials are stated relative to that of the normal hydrogen electrode, which is by definition zero. Two normal electrodes are in common use, namely, the calomel electrode and the hydrogen electrode.

NORMAL CALOMEL ELECTRODE. The normal calomel electrode consists of mercury in contact with a normal solution of potassium chloride saturated with mercurous chloride, the latter being present in excess as a solid.

NORMAL HYDROGEN ELECTRODE. The normal hydrogen electrode consists of a strip of platinum coated with a thin deposit of platinum black and saturated with hydrogen gas at atmospheric pressure. The electrode is mounted partially surrounded by hydrogen gas and partly dipping into an acid solution in which the activity of the hydrogen ion (see below) is unity.

From a normal hydrogen to a normal calomel electrode at 25 deg cent there is a rise of potential (corrected for the liquid junction potential) of 0.281 volt, hence, if from a hydrogen electrode to any metal electrode the rise of potential is e_h, and if from a calomel electrode to any metal electrode the rise of potential is e_c, then

$$e_h = e_c + 0.281 \text{ volt}$$

The difference between any two metal-liquid potentials remains the same irrespective of the normal electrode to which they are referred.

SPECIFIC ELECTRODE POTENTIALS. Table 3, in which most of the values are taken from The International Critical Tables, vol. 6, p. 332, gives the potential drop at an electrode dipping into a solution in which the molal activity (see below) of the ions in equilibrium with the electrode is unity. "All molecular and ion species whose concentrations are not fixed by the nature of the phases present as shown by the reaction given are at the concentration at which their molal activities are unity, the activity coefficients being those given by Lewis and Randall (Thermodynamics and the Free Energy of Chemical Substances, New York, McGraw-Hill, 1923), except for HCl, for which see Randall and Young, *J. Am. Chem. Soc.*, 50, 989 (1928). All gases are present at a pressure of one atmosphere."

The electromotive force of a cell (with liquid junction potentials eliminated) is equal to the algebraic difference of the specific electrode potentials involved.

Table 3. Electrodes in Equilibrium with Aqueous Solutions in Which the Molal Activity of the Ions Indicated is Unity. The Electrode Assumes a Charge against the Solution of the Sign and Magnitude Indicated. T = 25 deg cent

Electrochemical Reaction	Volts	Electrochemical Reaction	Volts
$Li - (-) = Li^+$	−2.96	$2H^+ + 2(-) = H_2$	0.0
$K - (-) = K^+$	−2.92	$Cu^{++} + 2(-) = Cu$	+0.34
$Na - (-) = Na^+$	−2.72	$O_2 + 2H_2O + 4(-) = 4OH^-$	+0.41
$Mg - 2(-) = Mg^{++}$	−1.55	$I_2 + 2(-) = 2I^-$	+0.54
$Zn - 2(-) = Zn^{++}$	−0.76	$Fe^{+++} + (-) = Fe^{++}$	+0.75
$Fe - 2(-) = Fe^{++}$	−0.44	$Ag^+ + (-) = Ag$	+0.80
$Cd - 2(-) = Cd^{++}$	−0.40	$\frac{1}{2} Hg_2^{++} + (-) = Hg$	+0.80
$Co - 2(-) = Co^{++}$	−0.29	$Br_2 + 2(-) = 2Br^-$	+1.07
$Ni - 2(-) = Ni^{++}$	−0.23	$Cl_2 + 2(-) = 2Cl^-$	+1.36
$Sn - 2(-) = Sn^{++}$	−0.14	$Au^{+++} + 3(-) = Au$	+1.36
$Pb - 2(-) = Pb^{++}$	−0.12	$F_2 + 2(-) = 2F^-$	+1.9
$Fe - 3(-) = Fe^{+++}$	−0.04		
$H_2 - 2(-) = 2H^+$	±0.0		

Since the evaluation of individual ion activities involves the adoption of an arbitrary convention, specific electrode potentials are to be regarded as somewhat arbitrary.

VARIATION OF ELECTRODE POTENTIALS WITH CONCENTRATION. The change in the value of an electrode potential with change in the concentration of the ion with which the electrode is in equilibrium is given by the formula

$$e_1 - e_2 = 0.000198T/n \log_{10} a_1/a_2$$

$$e_1 - e_2 = 0.000198T/n \log_{10} m_1 \gamma_1/m_2\gamma_2$$

where e_1 is the electrode potential at concentration m_1, e_2 at concentration m_2, T the absolute temperature, n the valence of the ion, and $m_1\gamma_1 = a_1$, the activity of the ion (see below) at concentration m_1. The activity coefficient γ expresses the deviation from the gas laws; if γ is the same for solutions 1 and 2, the change in electrode potential may be obtained directly from the ratio of concentrations.

ELECTROCHEMICAL SERIES—NOBILITY OF THE ELEMENTS. The elements, as arranged in Table 3, constitute the "electrochemical series." Those elements for which e_h is negative are said to be less "noble" than hydrogen; those for which e_h is positive are said to be more "noble" than hydrogen. The alkali metals having the greatest tendency to form ions in water stand at one end while the "noble" metals, such as gold, platinum, palladium, having but a very slight tendency to form ions, are

at the other end. The halogens which go into solution as negative instead of positive ions stand at the lower end of the series. Other series have been given which differ from the above in that the elements are not compared under similar conditions in regard to the concentration of the solution with which they are in contact. Changing the electrolyte, e.g., to potassium cyanide, not only alters the numerical values of the potentials, but may completely change the order in which certain elements occur in the series.

Any metal if placed in a solution of a salt of a metal standing below it in the series will tend to replace it; thus, zinc precipitates iron, copper, silver, etc., from their solutions but will not displace the alkali metals from solutions of their salts. Any metal standing above hydrogen will tend to displace it from an acid solution with evolution of hydrogen. Metals below hydrogen in the series will not dissolve in acid by a simple replacement of hydrogen. From a chemical standpoint the adoption of the hydrogen electrode as a standard has the advantage over the calomel standard that it divides the metals into two groups according to their behavior towards acids.

The approximate electromotive force of a battery consisting of two metals dipping into normal solutions of their respective salts may also be computed at once from the table by taking the difference of the two corresponding electrode potentials. Thus a zinc-copper cell should have an electromotive force equal approximately to $E_{zn-cu} = (e_h)_{zn} - (e_h)_{cu} = -0.76 - 0.34 = -1.10$, the rise of potential being from the zinc to the copper in the cell. As a matter of fact, this combination, the Daniell cell, has an electromotive force of approximately 1.10 volts. Other conclusions to be drawn from the table will be pointed out below.

11. ELECTROMOTIVE FORCE AND HEAT OF REACTION

The electromotive force of a reversible cell maintained at constant temperature may be readily calculated from the first and second law of thermodynamics. By a "reversible" cell is meant a cell which does not polarize and which operates under conditions such that changes which take place within the cell constitute a thermodynamically reversible process. The discussion given below also applies, with a rough degree of approximation, to most commercial cells, which, as a rule, are not strictly reversible.

GIBBS-HELMHOLTZ EQUATION. In the case of a reversible cell, such, for example, as the Daniell cell,* shown in Fig. 1, the relation between the heat of reaction and the electromotive force of the cell is calculated as follows.

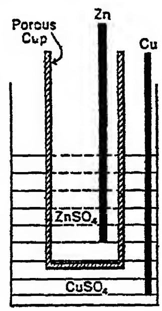

FIG. 1. Daniell Cell

From Faraday's laws, the quantity of electricity which must flow through the cell in order to deposit or liberate one gram-ion at either electrode is nF coulombs, where n is the valence of the ion of highest valence involved in the chemical reaction and F is the electrochemical constant (see above). Hence, the external work done by the cell (when there is no other external work than electrical work) is nFE joules, where E is the emf of the cell. If the cell is kept at the same temperature (T degrees, absolute scale) as the surrounding bodies, then the external work nFE done by the cell is the maximum external work it can do at this temperature T (see Vol. 1, Thermodynamics). Hence,

$$nFE = H' + nFT\frac{dE}{dT}$$

or

$$E = \frac{H'}{nF} + T\frac{dE}{dT}$$

where H' = the heat of reaction (heat evolved) in joules per gram-ion, corresponding to the chemical change which takes place.† H gives the change in intrinsic energy corresponding to the chemical changes which take place within the cell. This can be found by an independent calorimetric measurement, by causing the reaction to take place under such conditions (*constant volume*) that heat is the only form of energy produced.

TEMPERATURE COEFFICIENT OF ELECTROMOTIVE FORCE. From the above equation it follows that the energy developed by the cell is equal to the heat energy of the chemical reaction taking place within it plus a certain other quantity of energy equal

* The special construction of this cell renders it reversible.

† Since 1 large mean calorie (Ostwald calorie) = 418.5 joules and F = 96,500, this may be written, putting H = heat of reaction in large mean calories,

$$E = \frac{H}{2.036n} + T\frac{dE}{dT}$$

to $nFT\,dE/dT$. This may be either positive or negative, according as the temperature coefficient of electromotive force dE/dT is plus or minus, i.e., according as the electromotive force of the cell increases or decreases with the temperature. If the electromotive force on open circuit increases with the temperature of the cell, then on closed circuit the cell will tend to cool, i.e., its temperature will fall, and its electromotive force will likewise decrease, the energy $nFT\,dE/dT$ being derived from the heat energy within the cell (or surroundings). If the electromotive force on open circuit decreases with increasing temperature of the cell, then on closed circuit the cell will tend to heat up, i.e., its temperature will rise and its electromotive force will decrease. A cell for which dE/dT is other than zero can work at constant temperature only by absorbing heat from or giving out heat to the surroundings.

Only for the unique conditions that $dE/dT = 0$, or that $T = 0$, i.e., when the electromotive force is independent of the temperature or the cell works at temperatures approaching absolute zero, is $E = H'/nF$. By a curious chance it happens that the first of these conditions practically obtains in the Daniell cell considered above. The electromotive force of the cell is nearly independent of the temperature. Its observed value is $E = 1.10$ volts, while its value computed by the above equation, putting $dE/dT = 0$, is 1.09 volts.

THOMSON'S RULE. In computing the electromotive force of a battery from heats of reaction it is usually necessary to content oneself with a first approximation and neglect the second term in the above equation, since at present few data are available from which the value of dE/dT can, a priori, be obtained. The approximate equation

$$E = \frac{H'}{nF} = \frac{H}{230.6n}$$

is known as Thomson's rule. (H' is in joules, H in large mean calories.)

This rule also applies approximately to most commercial cells, although these are not strictly reversible. Of course, the heat of reaction used must be that corresponding to the actual chemical changes which take place in the cell.

NERNST'S THERMODYNAMIC THEOREM. Nernst, by assuming that the values of the intrinsic energy and free energy of a system not only become equal at the absolute zero but also that they approach equality asymptotically at this point, has been able to express the electromotive force (E) of a cell explicitly in terms of the thermal properties of the substances taking place in the reaction. So far as the new formula has been tested a satisfactory agreement has been found between theory and experiment. The principles involved are of fundamental importance and are fully discussed in Nernst's Silliman Lectures, Applications of Thermodynamics to Chemistry. See also Nernst's Theoretical Chemistry and Lewis and Randall's Thermodynamics, Chapter 31.

CONCENTRATION CELLS. Concentration cells may be divided into two classes: (*a*) those without liquid junction (without transference), and (*b*) those with liquid junction (with transference).

As an illustration of the former class the cell

$$\mathrm{Ag, AgCl} \mid \mathrm{HCl}(m_2) \mid \mathrm{H_2} \mid \mathrm{HCl}(m_1) \mid \mathrm{AgCl, Ag}$$

will serve. Here m_2 represents the concentration of hydrochloric acid in the more concentrated solution, m_1 the concentration of acid in the more dilute solution. The cell reaction is

$$\mathrm{HCl}(m_2) \rightarrow \mathrm{HCl}(m_1)$$

and if $\bar{F}_2$ and $\bar{F}_1$ are the partial molal free energies of hydrochloric acid in the two solutions the electromotive force of the cell is

$$E = (\bar{F}_2 - \bar{F}_1)/F$$

As an illustration of the latter class, the cell

$$\mathrm{Ag, AgCl} \mid \mathrm{HCl}(m_2), \mathrm{HCl}(m_1) \mid \mathrm{AgCl, Ag}$$

will serve. Let us suppose that to the left of the point A in the solution the concentration of the acid is constant and equal to m_2, and to the right of the point B, the concentration of the acid is constant and equal to m_1. Between the two points the concentration varies continuously. Then the electromotive force of the cell is given by the equation

$$E = -1/F\int_A^B t^+\,d\bar{F}$$

where t^+ is the transference number of the cation and $\bar{F}$ is the partial molal free energy of the acid. For a discussion of the general case, see Taylor, *J. Phys. Chem.*, 31, 1478 (1927). See also MacInnes and Beattie, *J. Am. Chem. Soc.*, 42, 1117 (1920), for the application of this method to the determination of transference numbers.

CARBON GENERATOR. The problem of converting the energy liberated in the combustion of carbon to carbon dioxide directly into electric energy has not yet been practically solved. The reaction, $C + O_2 = CO_2$, liberates a very large quantity of heat, namely, 961 large calories per gram-atom of carbon consumed. Of this, only about 10 per cent is converted into electrical energy through the agency of a steam engine and dynamo. The difficulties encountered in devising a commercial carbon generator are the following: (1) the velocity of the above reaction is small, except at high temperatures; (2) it is difficult to utilize a gas as an electrode, except through the agency of a conducting medium which occludes it, such as platinum black; (3) a satisfactory high-temperature electrolyte which will not deteriorate in the presence of CO_2 is difficult to obtain; and (4) carbon has a slight tendency to form ions.

12. ELECTROLYTIC DECOMPOSITION

Faraday's laws enable one to determine the amount of the substances primarily liberated or deposited at the electrodes of any electrolytic cell, but they do not enable one to predict the nature of the secondary chemical actions which may take place. The factors which determine what products will be formed in any given case are numerous. The chemical process occurring at the cathode is always of the nature of a reduction; at the anode, of an oxidation, these terms being used in their most general sense. It may be that the electrolyte, taken as a whole, undergoes no change, as is the case, for example, of a copper-sulfate solution when electrolyzed between copper electrodes. Here the chemical reaction at the cathode is the exact reverse of that at the anode. Generally, however, this is not the case; different chemical products are usually formed at the two electrodes, and appear either in the form of gas or as a solid precipitate, or remain dissolved in the electrolyte.

DECOMPOSITION VOLTAGE OF ELECTROLYTES. When a gradually increasing difference of potential is applied to the electrodes of an electrolytic cell, electrolysis does not in general begin at once but only after a certain minimum electromotive force is reached, even though the cell has no open-circuit emf. It should be noted that this condition does not apply to the case when the electrodes are such that the electrolyte as a whole undergoes no change as a result of electrolysis. The minimum voltage necessary to decompose a compound electrolytically is called its "decomposition voltage." It is influenced by a variety of factors and conditions.

Dilute solutions of most acids and bases have approximately the same decomposition voltage; this is due to the fact that it is the ions of water which are discharged at the electrodes. Salt solutions show considerable variation in decomposition voltage.

MEASUREMENT OF DECOMPOSITION VOLTAGE. Decomposition voltages have been experimentally determined in several ways. Certain investigators have taken the value necessary to produce visible electrolysis as the criterion; others have followed the polarization at the electrodes until it became constant and assumed this value as that at which electrolysis begins. Results obtained by the so-called ammeter-voltmeter method must be accepted with caution. In this method the value of the electromotive force applied to the cell is gradually increased from zero and plotted as abscissas and the corresponding currents through the cell plotted as ordinates. The resulting curves have the general form shown in Fig. 2.

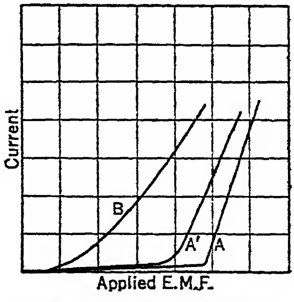

FIG. 2. Conductivity of Electrolytic Cells

The slope of these current-voltage curves depends upon the internal resistance of the cell, the size and the distance apart of electrodes. The critical voltage at which the current suddenly increases in value is, however, quite sharply defined with some electrolytes, e.g., silver nitrate between platinum electrodes, and other salts from which a metal is deposited. Below the critical voltage very little or no current passes through the cell and no visible decomposition at the electrodes is apparent. When the point A is reached, visible electrolysis begins. The sharpness of the bend in the curve and its course below A depend in large measure upon the sensitiveness of the galvanometer used for detecting the current, the size of electrodes and the tendency of the products of the electrolysis to go back into solution.

RESIDUAL CURRENTS. With a sensitive reflecting galvanometer, a steady deflection may be observed for days without any apparent electrolysis when an emf of a few

tenths of a volt (far below the " decomposition voltage ") is applied to platinum electrodes dipping into acidulated water. It might seem from this that Faraday's law is here violated. These slight currents are to be explained, however, by a diffusion of the products liberated slowly and in minute quantities at the electrodes, back into the solution. Currents produced in this manner are called " residual currents " and play an important rôle in the electrolysis of fused salts and probably also in conduction in solid electrolytes. The larger the residual current the less sharply marked is the decomposition point, see A'. For many fused salts the current-voltage curve is of the form shown in B, Fig. 2. It would be difficult to say from this curve at what voltage decomposition actually began.

OVERVOLTAGE. When a gas is evolved at either electrode the above relations are usually more complicated owing to the phenomenon known as overvoltage. Thus in the electrolysis of an acid between polished platinum or other metal electrodes, hydrogen gas is evolved, to liberate which the cathode potential must be raised to a value in excess of that required to liberate hydrogen at a reversible (platinum black) electrode.

This excess of voltage which is necessary to liberate a gas at any electrode over that required to liberate the same gas against a *reversible* gas electrode is called " overvoltage."

Its value depends on the character of the material of the electrode against which the gas is set free, upon the nature of the gas, and upon the current density employed. Moreover, the value obtained depends to a considerable extent upon the method of measurement used. In the earlier work on overvoltage a continuous current was employed; see Caspari, *Z. physik. Chem.*, 30, 89 (1899); Müller, *ibid.*, 50, 641 (1905); Tafel, *ibid.*, 65, 226 (1909). Most of the recent work has been done using an intermittent current; see Newbery, *J. Chem. Soc.*, 109, 1051, 1066 (1916); Glasstone, *ibid.*, 123, 1745 (1923); Knobel, *J. Am. Chem. Soc.*, 46, 2613, 2751 (1924). The overvoltages obtained by the former method are higher than those obtained by the latter.

Varying conditions make the usual reproducibility of overvoltage determinations not better than 0.05 volt. For tables showing the overvoltages of hydrogen, oxygen, and the halogens against various metals see the International Critical Tables, vol. 6, p. 339.

A study of the change of oxygen and hydrogen overvoltage with time has been made by Bowden and Rideal (see Bowden and Rideal, *Proc. Roy. Soc.*, A120, 59 (1928); Bowden, *ibid.*, A125, 446 (1929), A126, 107 (1929); Bowden and O'Connor, *ibid.*, A128, 317 (1930)).

ELECTROLYTIC SEPARATION OF METALS. The reason that two or more metals can be electrolytically separated from each other by a regulation of voltage follows from the preceding discussion. Consider a solution containing two salts, say silver and copper nitrates, each at normal concentration. If a gradually increasing emf be applied to inert electrodes dipping into this solution, electrolysis will begin when a voltage is reached sufficient to set free *simultaneously* any anions and cations present in the solution. Referring to Table 3, it will be seen that copper stands $0.80 - 0.34 = 0.46$ volt above silver in the electrochemical series, and hence silver can be deposited from the solution with emf nearly half a volt less than copper. No copper ions can separate until the applied emf has been increased 0.46 volt above that necessary to first separate the silver. The decomposition emf necessary will, of course, depend on the nature and concentration of the anions present. As electrolysis proceeds, the solution becomes weaker and weaker with respect to silver ions and the voltage necessary to separate them from the solution increases. If the electrolysis is continued until the silver remaining in solution has a concentration only 1/1,000,000 that at the start (the limit of analytical determinations), the change in voltage will be approximately 0.35 volt (see section above on Variation of Electrode Potentials with Concentration). This is still insufficient to bring the applied emf up to a value sufficient to permit the copper to deposit, and hence, for all practical purposes, silver and copper may be completely separated from each other. This process may be greatly accelerated by violently stirring the solution, as, for example, by the use of a rapidly rotating electrode.

ELECTRODE REACTIONS. Below are listed a number of reactions, of various types, by which the current is conducted from the solution to the electrode, or vice versa. The ultimate products are often quite different from the ions which conduct the current through the solution.

At the cathode:

$$2H^+ + 2(-) \rightarrow H_2$$

$$Cu^{++} + 2(-) \rightarrow Cu$$

$$8H^+ + 2SO_4^{--} + 2(-) \rightarrow SO_3^{--} + S + 5H_2O$$

$$Ag(CN)_2^- + (-) \rightarrow Ag + 2CN^-$$

$$4H^+ + PbO_2 + 4(-) \rightarrow Pb + 2H_2O$$

At the anode:

$$2Cl^- \rightarrow Cl_2 + 2(-)$$
$$4OH^- \rightarrow 2H_2O + O_2 + 4(-)$$
$$\begin{array}{l} COO^- \\ | \qquad \rightarrow 2CO_2 + 2(-) \\ COO^- \end{array}$$
$$2CH_3COO^- \rightarrow 2CO_2 + C_2H_6 + 2(-)$$

Which reaction is most likely to occur in any given case depends to some extent upon (a) current density, (b) temperature, (c) concentration of the solution, as well as upon the electrode potential.

In general, it may be said that at the cathode a reduction takes place, while at the anode oxidation (in its general sense) occurs. The intensity of these reactions depends upon the electrode potential at which the reaction takes place. This can be varied by the use of different electrode materials (thereby increasing the overvoltage). The efficiency of certain secondary reactions has also been found to depend upon the nature of the electrode (catalytic action). The efficacy of electrolytic reduction and oxidation in organic chemistry where the intensity of the reaction must be carefully regulated likewise rests on the above factors.

PASSIVITY. A metal is said to assume the "passive" state when it comports itself towards acids like a noble metal, i.e., becomes insoluble. Iron affords a very striking example of this phenomenon, although it is exhibited by other metals as well. Thus, if dipped in strong nitric acid, or if anodically polarized, it becomes passive. Several theories have been advanced to account for the phenomenon, but it is at present generally accepted that passivity is caused by the presence of a thin, insoluble film on the surface of the metal. It has been possible to isolate the film in some cases. For a review of the field see Desch, The Chemistry of Solids, Chapter 7 (Cornell Univ. Press). For résumé of theories see *Trans. Am. Elec. Chem. Soc.*, 29, 217 (1916); and see also the section on passivity in Foerster's Electrochemie wässeriger Losungen, Barth, Leipzig, 1921.

ALTERNATING-CURRENT ELECTROLYSIS. The decomposition of an electrolyte, and the solution of metal electrodes by electrolysis, can be effected under certain conditions by an alternating current as well as by a direct current. The chief determining factors are the velocity with which the chemical reaction takes place and the periodicity of the alternating current.

13. THEORY OF SOLUTIONS

To understand the theory of the various phenomena described above, a knowledge of the theory of solutions is necessary. Progress has been made in this field so rapidly that much of the material presented in textbooks is out of date. The following paragraphs will attempt to sketch the important advances.

SOLUTIONS OF NON-ELECTROLYTES. From the work of Raoult on the depression of the freezing point, the elevation of the boiling point, and the lowering of the vapor pressure of solutions, the general law was deduced that for a binary system

$$p_A = p_{A0} N_A$$

where p_{A0} represents the vapor pressure of pure solvent A and p_A its vapor pressure when present in a solution in which its mol fraction is N_A.

The law of Henry, as originally stated in 1803 is

$$c/p = k$$

where c is the concentration of the volatile solute, p its partial pressure, and k a constant characteristic of the substance. In terms of mol fractions this becomes

$$p_B = k'N_B$$

The third law found applicable to solutions of non-electrolytes is van't Hoff's law of osmotic pressure (1888).

OSMOTIC PRESSURE. The earliest recorded observations of the process of osmosis seem to have been those of Abbé Nollet in 1748, who found that, if a glass vessel was filled with alcohol, the opening covered with a bladder, and the vessel immersed in water, the bladder expanded and burst. In terms of a semi-permeable membrane the osmotic pressure may be defined as the equivalent of hydrostatic pressure produced when the solution and solvent, separated by a membrane permeable to the solvent, are in equilibrium, or as the equivalent of the excess pressure which must be imposed on a solution to prevent the passage into it of solvent through a semi-permeable membrane.

SEMI-PERMEABLE MEMBRANES. If a solution be placed at the bottom of a cylinder and carefully covered with a layer of pure solvent, the process of diffusion will begin at once and continue until a homogeneous dilute solution is formed. If the experiment be repeated with the single modification, that the pure solvent is separated from the solution by a piston consisting of a semi-permeable diaphragm which will permit the solvent to pass freely through it in either direction, but which will not allow the solute to pass through, the process of diffusion will take place as before, provided the piston is movable. Thus the piston will be forced back by the osmotic pressure of the solution while the solvent passes freely through it, thereby diluting the solution. Membranes possessing the property of semi-permeability are not only theoretically conceivable but may actually be prepared. Such a membrane is formed by the precipitate of copper ferrocyanide deposited within the walls of an unglazed porous cell when copper sulfate and potassium ferrocyanide solutions are allowed to diffuse into the cell from within and without respectively. This membrane is permeable to water, but impermeable to sugar and many other organic and inorganic substances, such as the copper and potassium salts from which it is formed. Most membranes, in fact, possess the property of semi-permeability to a limited degree.

MEASUREMENT OF OSMOTIC PRESSURE. To prevent the piston being forced up by the diffusing solution, a downward pressure P must be applied to it equal to that exerted upon it from below by the solution. The magnitude of P is, therefore, a measure of the osmotic pressure of the solution. Owing to experimental difficulties, it is impracticable to measure osmotic pressures in precisely the manner just described, but by slightly modifying the arrangement of the apparatus it may readily be done. Thus, if the solution is contained in a closed vessel, A, Fig. 3, the walls of which are made semipermeable by depositing a membrane of copper ferrocyanide within the pores of the cell, and this be immersed in the solvent B, the osmotic pressure will be exerted as before against the membrane from within. If the membrane could stretch like a rubber balloon, it would do so as the solution became diluted; but as it is restrained from doing this by the material of the walls of the cell on and in which it is deposited, dilution of the solution can take place by the solvent passing through the fixed membrane into the solution cell. This it will do unless the cell A is hermetically sealed. If an open manometer M of small bore be inserted in the top of A, the solvent as it passes into the cell will cause the solution to rise gradually in M until the hydrostatic pressure thus produced prevents further entrance of solvent. When equilibrium is established, the hydrostatic pressure gives a measure of the osmotic pressure of the solution then existing in the cell. As the entrance of an appreciable amount of solvent into the cell reduces the concentration of the solution, the pressure thus measured is less than that of the original solution; hence for quantitative work an open manometer is replaced by a closed mercury manometer.

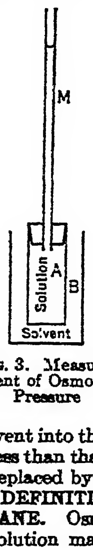

FIG. 3. Measurement of Osmotic Pressure

DEFINITION OF OSMOTIC PRESSURE WITHOUT REFERENCE TO A MEMBRANE. Osmotic pressure can also be defined in the following way. Let us consider a solution made up of the substance A and any number of other components. In a second vessel we have pure A. The partial pressure of A from the solution is less than from pure A at the same temperature, so that A would tend to distil from the pure substance into the solution. Distillation could be prevented by increasing the external pressure on the solution until the partial pressure of A from the solution was equal to that of pure A, or by decreasing the external pressure on the pure A until the vapor pressures were equal. The difference in total pressure on the two vessels is called the osmotic pressure.

The effect of external pressure on vapor pressure was first formulated by Poynting in 1881, and for dilute solutions is given by the equation

$$dp/dP = V_0/v$$

where P is the total applied pressure, p the vapor pressure of the solvent, V_0 the volume of one gram-molecule of solvent in the liquid state, and v the volume of one gram-molecule of solvent in the gaseous state. If $d\pi$ is the change in osmotic pressure corresponding to a change dp in the vapor pressure of the solvent, since $d\pi = -dP$

$$dp/d\pi = -V_0/v \quad \pi = (RT/V_0)\ln p_{A0}/p_A$$

Recently a new method of measuring osmotic pressure based upon the principles above has been devised; see Townsend, *J. Am. Chem. Soc.*, 50, 2958 (1928).

LAWS OF OSMOTIC PRESSURE IN NON-ELECTROLYTIC SOLUTIONS. The following laws were established by the early experiments on the osmotic pressure of dilute aqueous solutions of many organic substances, and of many solutions of non-electrolytes in organic solvents. These laws were the basis of van't Hoff's reasoning that the osmotic pressure of a solution was due to the impacts of the solute molecules on the semi-permeable membrane, the impacts of the solvent molecules being the same on both sides.

1. The osmotic pressure of a solution is directly proportional to its concentration (and therefore inversely proportional to the volume in which a given mass of the solute is dissolved).
2. It is directly proportional to the absolute temperature.
3. It is independent of the nature of the solute, being a function solely of the number of mols of substance dissolved in unit volume of solution.
4. The numerical magnitude of the osmotic pressure π of one mol of any substance dissolved in a volume v of solution at the absolute temperature T is identical with the gaseous pressure exerted by one mol of a perfect gas at the temperature T and occupying the same volume v.

The gas laws are therefore not only directly applicable to solutions, but the numerical value of the gas constant R is the same for each. Thus the combined laws of Boyle and Gay-Lussac for gases have the same form for solutions, namely,

$$\pi v = RT$$

where π = the osmotic pressure, v = the volume of the solution containing one mol of solute, and T = the absolute temperature.

Concentrated solutions of non-electrolytes deviate from these laws, just as highly compressed gases deviate from the gas laws.

THE LAWS OF DILUTE SOLUTION. DEVIATIONS THEREFROM. ACTIVITY. On the assumption that the vapors of solvent and solute obey the gas laws, any two of the following laws—Raoult's, Henry's, van't Hoff's—may be derived from the third. This may be done by means of Carnot cycles, or by the general principles and formulas of Gibbs; see Guggenheim, *J. Phys. Chem.*, 34, 1751 (1930).

Actual solutions of finite concentration show deviations from these ideal laws. In order to express the deviation Lewis introduced the idea of activity. He defined activity by the equation

$$\bar{F} = RT \ln a + B$$

where $\bar{F}$ is the partial molal free energy of the component in question, a its relative activity, and B, for a given solute and solvent, is a constant dependent on the temperature and pressure. In terms of this definition Raoult's law becomes

$$a_A = N_A$$

The activity can be determined from vapor pressure measurements, electromotive force measurements, etc. See Lewis and Randall, Thermodynamics.

SOLUTIONS OF ELECTROLYTES. THE THEORY OF ELECTROLYTIC DISSOCIATION. Arrhenius studied the conductivities of a large number of acids, bases, and salts in aqueous solution and noted that the specific conductivity did not vary directly with the concentration. When the data of Raoult on the lowering of the freezing point were published, Arrhenius put forward the theory of electrolytic dissociation. See Arrhenius, *Z. physik. Chem.*, 1, 631 (1887). He showed that α, the degree of dissociation of the electrolyte, calculated from the ratio of conductivities

$$\alpha = \Lambda/\Lambda_0$$

where Λ is the equivalent conductivity of the solution in question, Λ_0 the equivalent conductivity of an infinitely dilute solution, could be explained on the basis that salts, acids, and bases of the type RX dissociated into ions

$$RX \rightleftharpoons R^+ + X^-$$

He showed that i calculated from the equation $\pi v = iRT$ and from the lowering of the freezing point ($i = \Delta T/1.85$) could be related to α by the equation

$$i = 1 + (k - 1)\alpha$$

where k is the number of ions formed from each molecule.

Ostwald showed that the dissociation of weak monobasic acids in aqueous solution followed the mass law,

$$\alpha^2/(1 - \alpha)v = k$$

where α is the degree of dissociation and v the reciprocal of the total concentration.

It is to be noted in what follows that the experimental basis and subsequent support

of the idea of electrolytic dissociation were largely confined to work in aqueous solutions. The arguments in favor of electrolytic dissociation may be summarized as follows:

(1) Solutions of electrolytes show two independent sets of chemical properties, as if there were two independent constituents present. This fact is of course exceedingly important in analytical chemistry, particularly in qualitative analysis. Related to this is the fact that the heat of neutralization of a strong acid by a strong base, in dilute solution, is constant.

(2) Certain physical properties of solutions (specific volume, density, color, index of refraction, etc.) are additive with respect to the ionic components. For instance, in a colorless solvent, solutions of salts with one colorless ion and a common colored ion show, at high dilution, the same absorption spectrum.

(3) Solutions of electrolytes conduct electricity, and the equivalent conductance increases with dilution but approaches a limiting value.

(4) The abnormally high freezing-point depressions, boiling-point elevations, and osmotic pressures of solutions of electrolytes are accounted for by the theory.

(5) Those acids which give the most highly conducting solutions are also the most active chemically, that is, they react upon metals the most rapidly, and they show the greatest catalytic effect in such reactions as the inversion of sucrose.

Lewis gives as the principal weaknesses of the theory (see Lewis, *Z. physik. Chem.*, 70, 212, 1910):

(1) Lack of agreement in some cases between values for the degree of dissociation calculated from freezing points on the one hand and from conductivities on the other. Thus, for $N/2$ LiCl, $MgCl_2$, and $Ca_2Fe(CN)_6$, the values of α from freezing-point data are 94, 99, and 2 per cent, respectively, while from conductivities they are 71, 62, and 20 per cent.

(2) Additivity of properties of ions is observed even when considerable quantities of undissociated molecules are supposed to be present.

(3) The most vulnerable point in the theory is what has been called "the anomaly of strong electrolytes." There is not the slightest tendency to follow the dilution law in dilute solutions of strong electrolytes. In non-aqueous solutions the law of mass action is still less applicable.

It is realized that in the case of many electrolytes the physical properties could be explained on the basis that there were practically no molecules of the electrolyte present in the solution. This will be discussed after the presentation of the section on electrolytic conduction.

14. THEORY OF ELECTROLYTIC CONDUCTION

The external effects of a current flowing through an electrolyte cannot be distinguished from those produced by a current of the same strength conducted by a metal. Thus the magnetic effect of a current flowing through a helical glass tube filled with electrolyte is the same as that produced by the same current flowing through an equivalent circuit of an equal number of turns of copper wire. A current may be induced in a closed ring of electrolyte just as in a ring of metal. Ohm's Law and Joule's Law hold for conduction in electrolytes as well as in metals. The mechanism of the conduction in the two cases, however, is very different, as shown by the phenomena produced at the junction of two conductors in one of which the conduction is metallic or "electronic" and in the other of which it is electrolytic or "ionic."

EQUIVALENT CONDUCTIVITY. When a solution containing one equivalent weight of electrolyte is placed between parallel electrodes 1 cm apart and is diluted with solvent, the conductivity increases as the volume of the solution increases. Table 4 shows the increase in equivalent conductivity with dilution for a number of electrolytes.

For electrolytes like KCl, NaCl, $AgNO_3$, i.e., for strong electrolytes, the value of the equivalent conductivity in infinitely dilute silution, Λ_0, is determined by extrapolation. Extrapolation is best made by means of the square root law of Kohlrausch

$$\Lambda = \Lambda_0 - a\sqrt{c}$$

i.e., the equivalent conductivity, plotted against the square root of the concentration, yields a straight line in dilute solution. This was established as an empirical rule by Kohlrausch; Onsager and Debye and Hückel have given it a theoretical basis.

That increase in the concentration should cause a decrease in conductivity appears reasonable enough; it might be thought of as due to incomplete ionization, or to increase in viscosity of the solution and consequent decrease in the speed of the ions, or to retardation of the ions because of electrical effects. Of course, a combination of causes might be operating.

Table 4. Equivalent Conductance Λ of Typical Electrolytes Dissolved in Different Quantities of Water, at 18 Deg Cent

Concentration in Gm-equivalents per Liter = m = 1000η	Dilution in Liters per Gm-equivalent = ν = ϕ/1000	KCl	$NaCl$	KNO_3	$AgNO_3$	1/2 $CuSO_4$	1/2 H_2SO_4	HCl	CH_3COOH	KOH	NH_3
0.0001	10,000	129.07	108.10	125.50	115.01	109.95	......	(378)	107	......	(66)
0.0002	5,000	128.77	107.82	125.18	114.56	107.90	......	(378)	80	......	53
0.0005	2,000	128.11	107.18	124.44	113.88	103.56	(368)	377	57	......	38.0
0.001	1,000	127.34	106.49	123.65	113.14	98.56	361	376	41	(234)	28.0
0.002	500	126.31	105.55	122.60	112.07	91.94	351	375	30.2	(233)	20.6
0.005	200	124.41	103.78	120.47	110.03	80.98	330	373	20.0	230	13.2
0.01	100	122.43	101.95	118.19	107.80	71.74	308	369	14.3	228	9.6
0.02	50	119.96	99.62	115.21	......	62.40	286	366	10.4	225	7.1
0.05	20	115.75	95.71	109.86	99.50	51.16	253	358	6.48	219	4.6
0.1	10	112.03	92.02	104.79	94.33	43.85	225	351	4.60	213	3.3
0.2	5	107.96	87.73	98.74	......	37.66	214	342	3.24	206	2.30
0.5	2	102.41	80.94	89.24	77.5	......	205	327	2.01	197	1.35
1	1	98.27	74.35	80.46	67.6	25.77	198	301	1.32	184	0.89
2	0.5	92.6	64.8	69.4	......	......	183.0	254	0.80	160.8	0.532
3	0.33	88.3	56.5	(61.3)	......	......	166.8	215.0	0.54	140.6	0.364
5	0.2	....	42.7	....	......	......	135.0	152.2	0.285	105.8	0.202

Note.—The 1/2 before $CuSO_4$ and H_2SO_4 indicates 1 gram-equivalent = 1/2 mol.

For complete tables consult Landolt and Börnstein, Kohlrausch and Holborn, and Tables Annuelles Internationales de Constantes.

Because of the great increase of equivalent conductivity for electrolytes like NH_4OH and CH_3COOH when the solution is more dilute than 0.001 N, it is clear that, in order to obtain values of Λ from which to get Λ_0 by extrapolation, one would have to work with impossibly dilute solutions. The limiting value of the equivalent conductivity of a weak electrolyte is accordingly obtained in another way,

THE LAW OF INDEPENDENT MIGRATION OF IONS. Table 5 shows the limiting values of the equivalent conductivity for a series of potassium salts, and for a series of calcium salts, at 18 deg cent. The third and sixth columns of the table give the difference between the Λ_0 values of two salts with a common cation.

Table 5. Limiting Equivalent Conductivities of Some Salts of Potassium and Calcium

Salt	Λ_0	Difference	Salt	Λ_0	Difference
KCl.........	130.1		$CaCl_2$.........	116.5	
		− 1.5			− 1.5
KBr.........	131.6		$CaBr_2$.........	118.0	
		5.3			5.3
KNO_3........	126.3		$Ca(NO_3)_2$......	112.7	
		26.7			26.7
$KC_2H_3O_2$.....	99.6		$Ca(C_2H_3O_2)_2$...	86.0	
		1.1			1.1
KIO_3........	98.5		$Ca(IO_3)_2$.......	84.9	
		−34.1			−34.1
K_2SO_4.......	132.6		$CaSO_4$........	119.0	

Since the differences between the values of Λ_0 for KCl and KBr, and $CaCl_2$ and $CaBr_2$, are the same, although the individual values are quite different, Λ_0 must in each case be the sum of two constant quantities. That is to say, in infinitely dilute solution and at a given temperature, the current-carrying capacity of one equivalent of potassium ions is a fixed and constant quantity independent of the anion present. The limiting equivalent ionic conductance of the potassium ion is represented as l_{K^+}. The equivalent conductivity of an electrolyte at infinite dilution is equal to the sum of two constants, the equivalent conductances of anion and cation at infinite dilution,

$$\Lambda_0 = l_a + l_c$$

a law established by Kohlrausch and known as the law of the independent migration of ions.

This law immediately suggests a method of evaluating Λ_0 for electrolytes of the NH_4OH-CH_3COOH type, i.e., for weak electrolytes.

$$\Lambda_0 \text{ for } CH_3COOH = l_{H^+} + l_{CH_3COO^-}$$

$$\Lambda_0 \text{ for } CH_3COOH = \Lambda_0 \text{ for } HCl + \Lambda_0 \text{ for } CH_3COONa - \Lambda_0 \text{ for } NaCl.$$

From such a table of limiting equivalent conductivities as that given for the potassium and the calcium salts, it is evident that, if the absolute value of the limiting ionic conductance were known for one ion, all the other limiting ionic conductances might at once be calculated.

TRANSFERENCE NUMBERS OR TRANSPORT RATIOS. The current is carried through the solution by the simultaneous movement of the anions toward the anode, the cations toward the cathode. If the anions are faster moving than the cations, they carry the greater part of the current. For q coulombs of electricity passing, $q/96{,}500$ = number of equivalents of electricity passing = $N_t = N_a + N_c$, the number of equivalents carried by the anion, plus the number carried by the cation. The ratio N_a/N_t is called the transference number or the transport ratio of the anion, the ratio N_c/N_t is called the transference number or the transport ratio of the cation, in the solution in question. The sum of the transference numbers is unity.

The transference numbers are found from the changes in concentration which take place about the electrodes when a direct current is passed through the solution of an electrolyte. Consider the electrolysis of a solution of $AgNO_3$ between silver electrodes. When N_t equivalents of electricity have passed, N_t equivalents of silver have gone into the solution surrounding the anode, from the anode, and the same number of equivalents of silver have plated out from the solution surrounding the cathode, onto the cathode. N_c equivalents of silver have during this time moved out from the anode portion of the solution, and the same number have entered the cathode portion. The anode portion has gained $N_t - N_c$ equivalents of silver ion, and the cathode portion has lost the same number. The concentration of nitrate ion in the two portions has changed in the same

Table 6. Transport Ratios n_a of Anions in Aqueous Solutions at about 18 Deg Cent

(Values in parentheses are somewhat uncertain. From Le Blanc's Electrochemistry)

Gram-equivalents per liter Liters per Gram-equivalent	0.01 100	0.05 20	0.2 5	1 1	2 0.5	5 0.2
K{Cl, Br, I}, NH_4Cl	0.506	0.506	......	......	......	......
NaCl	0.604	0.604	......	......	......	......
LiCl	0.670	0.680	0.697	......	......	......
KNO_3	......	......	0.496	0.487	0.479	......
$NaNO_3$	......	......	0.614	(0.611)	(0.608)	0.585
$AgNO_3$	0.528	0.528	0.527	0.501	0.476	......
$KC_2H_3O_2$	......	......	0.33	(0.331)	(0.332)	0.335
$NaC_2H_3O_2$	......	......	(0.43)	(0.425)	0.421	......
KOH	......	......	0.736	(0.740)	......	......
NaOH	......	(0.81)	(0.82)	0.825	......	......
LiOH	......	......	0.85	(0.873)	......	......
HCl	0.167	0.165	......	......	......	......
HNO_3	0.170	0.170	0.170	......	......	......
1/2 $BaCl_2$	0.553	......	......	......	......	......
1/2 $CaCl_2$	(0.58)	(0.61)	(0.66)	0.686	(0.700)	0.737
1/2 $MgCl_2$	......	(0.63)	0.68	0.709	(0.729)	0.776
1/2 $CdCl_2$	0.570	0.570	(0.65)	(0.72)	0.745	0.865
1/2 CdI_2	0.558	0.606	0.86	......	......	......
1/2 $Ba(NO_3)_2$ at 25°	......	0.544	0.545	......	......	......
1/2 K_2CO_3	......	(0.39)	(0.41)	0.434	0.413	(0.380)
1/2 Na_2CO_3	......	(0.52)	(0.53)	0.548	(0.542)	......
1/2 K_2SO_4	0.505	0.512	......	......	......	......
1/2 Na_2SO_4	0.610	0.624	......	......	......	......
1/2 $CdSO_4$	0.616	0.635	0.672	0.746	......	......
1/2 $MgSO_4$	0.620	0.633	(0.66)	0.74	(0.76)	......
1/2 $CuSO_4$	......	0.625	0.657	......	......	......
1/2 H_2SO_4	0.176	0.175	......	......	......	......

Note.—The 1/2 before the various bivalent electrolytes indicates that 1 gram-equivalent = 1/2 mol.

way. Analysis of either electrode portion, together with the coulometer reading, yields directly N_c/N_t, the transference number of the silver ion in a silver nitrate solution of the concentration given. For uni-univalent salts in solutions less concentrated than 0.01 N there is very little change in the transference numbers of the ions with change in concentration. Table 6 gives the transference numbers of the anions of a number of electrolytes in aqueous solution at 18 deg cent.

Transference numbers may also be evaluated by measurement of the electromotive force of concentration cells with transference (see for instance MacInnes and Parker, *J. Am. Chem. Soc.*, 37, 1445 (1915); and MacInnes and Beattie, *ibid.*, 42, 1117 (1920)), and by the moving boundary method (see MacInnes, Cowperthwaite, and Blanchard, *J. Am. Chem. Soc.*, 48, 1909 (1926); a review of the problem of the determination of transference numbers is given by MacInnes and Longsworth, *Chem. Reviews*, 11, 172 (1932). For a more complete table of transference numbers, and a comparison of the results obtained by the different methods, see the International Critical Tables, vol. 6, pp. 309–311.

LIMITING IONIC CONDUCTIVITIES. Since N_c/N_t represents the fraction of the current carried by the cation, $\Lambda N_c/N_t$ is the equivalent conductance of the cation. The limiting value of $\Lambda N_c/N_t$, that is to say, the conductance of one equivalent of the cation in an infinitely dilute solution, is l_c. From conductivity data, and from transference data, the tables of limiting equivalent ionic conductivities are built up. Table 7 gives values of the equivalent conductivities of some typical ions at infinite dilution and 18 deg cent.

Table 7. Conductivities of Typical Ions at Infinite Dilution and 18 Deg Cent

(Values at $t°$ may be computed by the formula $l_t = l_{18}[1 + \alpha(t - 18) + \beta(t - 18)^2]$) (Kohlrausch)

Anions	l_{18}	α	β	Cations	l_{18}	α	β
F	46.6	0.0232	0.000094	Li	33.4	0.0261	0.000142
Cl	65.5	0.0215	0.000067	Na	43.5	0.0245	0.000116
Br	67.0	....	....	K	64.6	0.0220	0.000075
I	66.5	0.0206	0.000052	Rb	67.5	0.0217	0.000069
SCN	56.6	....	....	Cs	68.0	....	....
ClO_3	55.0	0.0207	0.000054	NH_4	64.0	0.0223	0.000079
BrO_3	46.0	....	....	Tl	66.0	....	....
IO_3	33.9	0.0233	0.000096	Ag	54.3	0.0231	0.000093
ClO_4	64.0	....	....	H	315.0	0.0154	−0.000033
IO_4	48.0	....	....	1/2 Ba	55.0	0.0239	0.000106
NO_3	61.7	0.0203	0.000047	1/2 Sr	51.0	....	....
MnO_4	53.4	....	....	1/2 Ca	51.0	....	....
OH	174.0	0.0179	0.00008	1/2 Mg	45.0	0.0255	0.000132
CHO_2	47.0	....	....	1/2 Zn	46.0	0.0256	0.000133
$C_2H_3O_2$	35.0	0.0236	0.000101	1/2 Cd	46.0	....	....
1/2 SO_4	68.0	0.0226	0.000084	1/2 Cu	46.0	0.0240	0.000107
1/2 Cr_2O_7	72.0	....	....	1/2 Pb	61.0	0.0244	0.000114
1/2 CO_3	60.0	0.0269	0.000155				
1/2 $(COO)_2$	63.0	....	....				

THE VELOCITY OF MIGRATION OF IONS. A charged body in an electric field is acted upon by a force equal to the product of the charge and the intensity of the field. If the field changes in one direction, only the force acting is

$$f = qdE/dx$$

where q is the charge of the particle, and dE/dx the gradient of the potential. The speed with which a small body moves through a medium of great frictional resistance is proportional to the force acting upon it. Therefore the speed u_c with which the cation moves through the solution is

$$u_c = kqdE/dx$$

The proportionality factor kq is written U_c and is the velocity of migration of the ion under unit potential gradient, or the *mobility*. Table 8 gives the velocity of migration of a number of ions.

The mobility of an ion of a given electrolyte is related very simply to the transference numbers, and to the equivalent conductivity (see Washburn, The Principles of Physical Chemistry, pp. 233–235). The former relationship is

$$N_c/(N_c + N_a) = U_c/U_c + U_a); \quad N_a/(N_c + N_a) = U_a/(U_c + U_a)$$

and the latter

$$\Lambda = F(U_c + U_a)$$

where F is the faraday.

Table 8. Velocities of Migration of Ions

Ion	Velocities in Cm per Sec per Volt per Cm			Ion	Velocities in Cm per Sec per Volt per Cm		
	0.1 Gram-equivalent per Liter, 18° C		Infinite Dilution, 18° C		0.1 Gram-equivalent per Liter, 18° C		Infinite Dilution, 18° C
	Observed	Calculated	Calculated		Observed	Calculated	Calculated
$\overset{+}{H}$........	0.0026	0.0028	0.003263	$\overset{+}{Ag}$.......	....	....	0.000562
$\overset{-}{OH}$.......	......	......	0.001802	$\overset{-}{SO_4}$.......	0.00045	0.00049	....
$\overset{-}{Cl}$........	......	......	0.000677	$\overset{-}{Cr_2O_7}$......	0.00047	0.00047	....
$\overset{+}{K}$........	......	......	0.000669	$\overset{+}{Ag}$........	0.00049	0.00046	....
$\overset{+}{NH_4}$......	......	......	0.000663	$\overset{+}{Ba}$........	0.00039	0.00037	....
$\overset{-}{NO_3}$......	......	......	0.000639	$\overset{+}{Ca}$........	0.00035	0.00029	....
$\overset{-}{ClO_3}$......	......	......	0.000570	....	....	....	....

THE CONDUCTANCE RATIO FOR STRONG AND FOR WEAK ELECTROLYTES. If the conductance ratio, Λ/Λ_0, is examined for a large number of strong and weak electrolytes, it is at once evident that the former class possess large values of the ratio Λ/Λ_0 as compared with the latter. For most uni-univalent salts, and for strong uni-univalent acids and bases, in aqueous solution, $\Lambda_{0.1\,N}/\Lambda_0$ is about 0.8. For bi-bivalent salts $\Lambda_{0.1\,N}/\Lambda_0$ is 0.4–0.5. Turning now to weak electrolytes, for NH_4OH and CH_3COOH in aqueous solution $\Lambda_{0.1\,N}/\Lambda_0 = 0.01$; for HClO and HCN, $\Lambda_{0.1\,N}/\Lambda_0 < 0.002$; and for the mercuric halides, $\Lambda_{0.1\,N}/\Lambda_0 < 0.001$.

In most textbooks at present in use the conductance ratios are regarded as measures of the degree of ionization, uni-univalent electrolytes of the KCl–$AgNO_3$ type (strong electrolytes) being said to be about 80 per cent ionized in 0.1 N solution. The weak electrolytes of the same valence type possess very much lower $\Lambda_{0.1\,N}/\Lambda_0$ values and are therefore considered much less ionized. There is far wider variation in the $\Lambda_{0.1\,N}/\Lambda_0$ ratio for weak electrolytes than for strong electrolytes of the same valence type. Examination of the ratio for a large number of uni-univalent salts shows that the deviation from the mean seldom exceeds 1 or 2 per cent, while changes in the conductance ratio for weak organic acids amount to hundreds of per cent.

It is obvious that the ratio $\Lambda_{0.1N}/\Lambda_0$ represents the degree of ionization of the electrolyte in 0.1 N solution only provided that in 0.1 N solution both ions move with the same speed as they do at infinite dilution.

Many of the Λ/Λ_0 ratios recorded in the literature are subjected to a viscosity correction. By Stokes's law, the speed with which a small particle moves through a medium of great frictional resistance is inversely proportional to the viscosity of the medium. Correcting for the effect of viscosity on the speed of the ions, the so-called " degree of ionization " becomes $\Lambda\eta/\Lambda_0\,\eta_0$ where η is the viscosity, rather than Λ/Λ_0.

APPLICATION OF THE MASS LAW TO STRONG AND TO WEAK ELECTROLYTES. There is another important difference in behavior between strong and weak electrolytes. For a uni-univalent electrolyte the dilution law of Ostwald (the law of mass action in terms of concentrations) runs

$$K_c = c_{A^+}c_{B^-}/c_{AB} = \alpha^2\,C/(1 - \alpha)$$

where α is the degree of ionization and C the stoichiometric concentration. This differs from the expression of the mass law by lack of the activity coefficient ratio, $f_{A^+}f_{B^-}/f_{AB}$. Now when α is represented by Λ/Λ_0 or, if there is a considerable change in the viscosity, by $\Lambda\eta/\Lambda_0\,\eta_0$, for weak electrolytes in aqueous solution fairly constant values are obtained for K_c. The classical example is perhaps acetic acid. As the concentration of acetic acid increases from 0.0001 to 1 M, K_c decreases from 1.87×10^{-5} to 1.74×10^{-5} (13 parts in 200, 7 per cent).

A strong electrolyte of the same valence type over the same concentration range

makes no pretense of obeying the dilution law. Nor does the inclusion of the ratio of activity coefficients

$$K_a = c_{A^+} c_{B^-} f_{A^+} f_{B^-} / c_{AB} f_{AB} = K_c f_{A^+} f_{B^-} / f_{AB}$$

yield a constant. Since the law of mass action is a thermodynamic law, it can only be concluded that the ratio Λ/Λ_0 does not represent with any accuracy the degree of ionization of strong electrolytes.

MODERN THEORIES OF SOLUTION. van Laar (*Arch. Teyler* (2) 7, I, 1 (1900)), Sutherland (*Phil. Mag.*, 14, 1 (1907)), and Bjerrum (*Z. Elektrochem.*, 24, 231 (1918)), from different points of view, suggested that the dissociation of strong electrolytes was much greater than appeared from the usual calculation on the basis of conductivity data. Milner (*Phil. Mag.*, 23, 551 (1912); 25, 743 (1913)) attempted to calculate quantitatively the effect of interionic forces, but owing to mathematical difficulties did not arrive at a strict solution of the problem.

In 1923 Debye and Hückel (*Physik. Z.*, 24, 185 (1923); 25, 97 (1924)) presented what appears to be a satisfactory solution of the problem. Briefly, their reasoning is as follows. Due to the Coulomb forces the "ionic atmosphere" surrounding any selected positive ion contains an excess of negative electricity. This causes an average electrical potential and an average electrical density different from zero to exist at any distance from the selected ion, the total potential and density being interrelated by the Boltzmann distribution principle and the Poisson equation. From this it is possible to calculate the increase in energy attending the removal of the n molecules constituting a gram molecule of any kind of ion, of valence z, from the "ionic atmosphere" (all other ions being simultaneously removed, their removal being attended by no other energy effects) by infinitely diluting the solution. Without following the mathematical derivation, it may simply be stated that they find that, for water as a solvent,

$$-\log_{10} f = 0.5z^2\sqrt{\mu}$$

where f represents the activity coefficient, z the valence of the ion, and μ the ionic strength defined by the equation $\mu = {}^1/_2\Sigma c_i z_i^2$. This limiting equation has been verified experimentally in the case of aqueous solutions.

Debye and Hückel extended the theory to the problem of conductivity also (*Physik. Z.*, 24, 305 (1923)). They showed that two factors cause a decrease in the mobility of the ions. The first is an "ion effect" due to the fact that the thermal equilibrium of the ions in the different configurations is disturbed by the migration of the ions, equilibrium being restored with finite velocity. The unsymmetrical arrangement of the ions creates an electric field which opposes the applied potential difference. The second effect arises from the fact that the ions tend to drag along solvent molecules according to the hydrodynamic laws for viscous media. About a given ion is found an excess of ions of opposite sign, so that the given ion has to move relative to a current of medium flowing in the opposite direction.

Taking into consideration the diminution in the mobility of an ion arising from these causes, Onsager (*Physik. Z.*, 27, 388 (1926); 28, 277 (1927)) derived the square root law of Kohlrausch

$$\Lambda = \Lambda_0 - a\sqrt{c}$$

and showed that a, the slope of the line, can be evaluated theoretically, the calculated and observed values of a being in good agreement.

In the field of weak electrolytes, the proper correction of the mobility for the effect of the "ionic atmosphere" has made possible the more accurate evaluation of the concentration of the ions. See, for example, Davies, *Phil. Mag.* (7), 4, 244 (1927), and Shedlovsky, Brown, and MacInnes, *Trans. Am. Electrochem. Soc.*, 66, 165 (1934).

To summarize: Owing to interionic forces the older method of determining the degree of dissociation of strong electrolytes is incorrect. All the evidence in the case of uni-univalent electrolytes seems to indicate that in aqueous solutions strong electrolytes of this type are practically completely dissociated. At the present time the corresponding dissociation constants are not known, because there is no physical property which is unquestionably due to undissociated molecules. In regard to weak electrolytes, correction of the mobility for the retardation caused by interionic forces leads to a more exact evaluation of the ionic concentrations.

EFFECT OF TEMPERATURE ON IONIZATION. The percentage to which a substance is ionized in solution may increase or decrease with the temperature. The sign and magnitude of the effect depend upon whether the ionization reaction is accompanied by an absorption or evolution of heat. Substances which dissociate with evolution of heat become less ionized with increasing temperature; substances which dissociate with absorption of heat become more ionized with rising temperature. Water, which is very

slightly ionized, belongs to the latter class. In general an increase of temperature from 0 to 100 deg cent produces a change in ionization of only a few per cent. Investigations by Noyes carried out at temperatures up to 360 deg cent have confirmed these predictions of theory. The effect of temperature on α may be computed through its effect on K by means of van't Hoff's thermodynamic relation

$$\frac{d \log K}{dt} = \frac{Q}{RT^2}$$

where K is the equilibrium constant of the ionization reaction, Q the heat of the reaction, R the gas constant, and T the absolute temperature.

NEGATIVE TEMPERATURE COEFFICIENTS OF ELECTRICAL CONDUCTIVITY. The fact that certain electrolytes, e.g., a phosphoric-acid solution, may have a negative temperature coefficient is readily explained in terms of the above relations. If the increase in the velocity of migration of the ions with rising temperature is more than offset by a diminution in the average number of free ions, resulting from a decrease in ionization, the conductivity of the solution will diminish. By combining solutions having positive and negative temperature coefficients in suitable proportions, electrolytes have been prepared which have nearly a zero temperature coefficient over quite a range of temperature. The following mixture, proposed by Manganini, has this property: 121 grams mannite, 41 grams boracic acid, 0.06 gram potassium chloride dissolved in sufficient water to make one liter. Its specific conductance at 18 deg cent is $\kappa = 0.00097$. Such a solution is well adapted for a liquid resistance just as manganine wire is adapted for resistance coils.

THE DEFINITION OF pH. Originally the term pH was introduced by Sörensen because of the form of the equation relating the free energy to the hydrogen-ion concentration, and $\log_{10} 1/c_{H^+}$ was called the hydrogen-ion exponent or pH. However, since the electrometric method was used in the determinations, and since the pH of the standard was based upon calculations of the hydrogen-ion concentration from conductivity data, the quantity measured was not that defined by the equation $pH = \log 1/c_{H^+}$. Accordingly a new definition, designated as p_aH, was later brought forward, p_aH being defined as $\log 1/a_{H^+}$ where a_{H^+} represents the hydrogen-ion activity. It is now realized that ionic free energies and activities cannot be evaluated without certain non-thermodynamic assumptions. The result is that pH's are really arbitrary values with the scale often varying from worker to worker, depending on: (1) the standard solution chosen, and the value assigned to it, and (2) the value of E_0 chosen for the cell used in referring to the standard. The cell employed is of the type

$$\text{Pt, H}_2 \mid \text{HCl, KCl} \mid \text{KCl} \mid \text{KCl, HgCl} \mid \text{Hg}$$

In order to determine E_0 it is necessary to evaluate the liquid junction potential.

The definitions of pH may be classified under three headings:

(1) $pH = -\log c_{H^+}$, the definition in terms of concentration.
(2) $pH = -\log 1.1a_{H^+}$, the "Sörensen" definition.
(3) $p_aH = -\log a_{H^+}$, the definition in terms of activity.

The second refers to pH's determined by use of Sörensen's value of the 0.1 N calomel electrode, or from the pH values of his standard buffer solutions, obtained therefrom. Most of the data in the literature are reported in terms of the second. There are certain advantages in the consistent use of the first definition. For a recent discussion see Kilpatrick and Kilpatrick, *J. Chem. Educ.*, 9, 1010 (1932).

THEORY OF LIQUID-LIQUID POTENTIALS. Nernst (*Z. physik. Chem.*, 4, 129 (1889)) was the first to show that in the case of electrolytes a difference of potential necessarily arises between two solutions in contact. The general theory has been worked out by Planck (*Ann. Physik*, 40, 561 (1890)) and by Henderson (*Z. physik. Chem.*, 59, 118 (1907); 63, 325 (1908)). Planck's formula applies to a junction in which the distribution of concentrations is maintained steady by artificial means, while that of Henderson may be described as a "continuous mixture layer."

Taylor (*J. Phys. Chem.*, 31, 1478 (1927)) and Guggenheim (*J. Am. Chem. Soc.*, 52, 1315 (1930)) have also considered a third type of junction which Guggenheim calls a "free diffusion" junction. The reader is referred to the papers of Taylor and Guggenheim for a discussion of the theory and the experimental realization of the conditions. Guggenheim finds that a reproducibility of 0.4 mv can be obtained with the Planck junction, and one of 0.2 mv with junctions of the "continuous mixture layer" and the "free diffusion" types. Guggenheim also points out that for a junction made at the end of a thin tube dipping into a wider one irregular fluctuations of several millivolts are obtained.

THE DEFINITION OF AN ACID. The usual definition that an acid is a substance containing hydrogen which in solution forms hydrogen ions, and a base a hydroxyl compound which forms hydroxyl ions in solution, was a natural result of the development of the

ideas of Arrhenius and Ostwald and the emphasis on aqueous solutions. A much more general and fruitful definition is that put forward by Brönsted (*Rec. trav. chim.*, 42, 718 (1923)) and Lowry (*Chem. & Ind.*, 42, 43 (1923)). Brönsted defines an acid by the formal equation

$$A \rightleftharpoons B + H^+$$

where H^+ represents the proton, A an acid of any charge whatsoever, and B the conjugate base of charge less by one than that of A. According to this definition the most common carrier of acid properties in aqueous solution, the hydrogen ion (H_3O^+), is only one of a number of acids.

$$H_3O^+ \rightleftharpoons H_2O + H^+$$
$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$$
$$NH_4^+ \rightleftharpoons NH_3 + H^+$$
$$H_2O \rightleftharpoons OH^- + H^+$$

Attention is also called to the fact that according to this definition the hydroxyl ion is only one of many bases.

The definition given applies to any solvent, emphasizes the acid, basic, amphoteric or inert nature of the solvent, and brings simplicity and order into the field. The definition has led to a number of important catalytic studies (see Brönsted, *Chem. Reviews*, 5, 284 (1928), and Kilpatrick and Kilpatrick, *ibid.*, 10, 213 (1932)).

15. ENDOSMOSE OR ELECTRIC OSMOSIS

If a current is passed along a film of water on a solid insulating surface, the water will be attracted to the negative electrode. Similarly an electric current flowing through a porous partition, or capillary opening, filled with conducting liquid, causes a flow of the liquid through the partition or opening. This phenomenon is known as endosmose or electric osmosis. For a review of the experimental and theoretical investigation of the phenomenon of endosmose see Schönfeldt, *Z. Elecktrochem.*, 39, 103 (1933).

The quantity of a given liquid which is carried through a given porous diaphragm in a definite time varies directly with the current strength and is independent of the area and thickness of the diaphragm. The quantity varies with the nature of the solution, being greater with liquids of high specific resistance and high specific capacity. The direction of flow is generally from positive to negative electrode, but under certain conditions the flow may be in the opposite direction.

APPLICATIONS OF ENDOSMOSE. Endosmose may be utilized to remove water from wet substances. Thus if a wooden box be equipped with perforated metallic plates at opposite ends and be filled with wet turf and current circulated through the turf from one end to the other, water will ooze out of the perforations of the cathode plate. Endosmose is utilized in tanning processes to accelerate the passage of tanning fluids through the hides. For a review of the industrial applications of endosmose see Schönfeldt, *Z. Elecktrochem.*, 38, 744 (1932).

ENDOSMOSE OF NEGATIVE FEEDERS. If electric conductors covered with a saturated braid or a number of such braids be made the cathode of a water bath and, say, 100 volts applied between anode and cathode, the braid will blister in a few minutes and the blisters will finally burst, grounding the conductor to the water. The same action goes on at lower voltages at a proportionately lower rate, but with equal certainty. No action is observable if the wire be made the anode instead of the cathode. Because of endosmose, insulation of low specific resistance should never be used on negative feeders. For the same reason it is not practicable to maintain a negative contact rail on electric railways, below earth potential, as the insulators soon become saturated with moisture and thereby become conductors (see Fortenbaugh, *Trans. A.I.E.E.*, 28, 1215, (1908)).

EFFECT OF ENDOSMOSE ON INSULATION MEASUREMENTS. When testing wires having insulation of low specific resistance, endosmose may further lower the resistance if the wire is negative to the water bath.

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SECTION 7

BATTERIES

PRIMARY BATTERIES

BY C. F. BURGESS

ART.		PAGE
1.	Classification	02
2.	Design	03
3.	Testing	03
4.	Installation	07
5.	Operation	07

LEAD-ACID STORAGE BATTERIES

BY J. LESTER WOODBRIDGE

ART.		PAGE
6.	General and Descriptive	08
7.	Cell Characteristics	13
8.	Installation	17
9.	Applications	18
10.	Care and Operation	24

ALKALINE-TYPE STORAGE BATTERIES

BY E. W. ALLEN AND F. BREHME

ART.		PAGE
11.	Theory and Electrochemical Reactions	24
12.	Design and Construction	25
13.	Electrical Characteristics	30
14.	Life	34

BATTERIES

PRIMARY BATTERIES

By C. F. Burgess

1. CLASSIFICATION

A *primary cell* is a device for producing electric current by electrochemical action. It is differentiated from a storage cell in that it is non-reversible and cannot be recharged efficiently.

A *primary battery* is a plurality of primary cells connected electrically. By common practice, however, the terms "battery" and "cell" are employed interchangeably.

ELECTRODES. In the primary cell, zinc is generally used as the negative electrode, or *anode*, which is attacked by the electrolyte. This zinc is metallically conductive, and to it is attached the negative terminal of the cell.

The positive electrode, or the *cathode*, is the conductive material to which the positive terminal is attached. The cathode must resist attack by the electrolyte. Some metals (platinum, gold) may be used, but conductive carbon rods have proved serviceable and are more economical.

ELECTROLYTE. The electrolyte is the solution, or liquid, which bridges the two electrodes and which conducts current while the cell is being discharged. A solution of sal ammoniac and zinc chloride in water is a typical electrolyte.

DEPOLARIZERS. The action of a primary cell depends upon the chemical action of the electrolyte upon the anode material, such as zinc. As a result of this chemical action, the anode goes into solution, releasing hydrogen at the cathode. This gas forms a film that lowers the electromotive force and raises the internal resistance of the cell so that no appreciable current can flow. This building-up of the products of reaction and consequent retarding of the desired processes is called polarization. To overcome this, a substance called a depolarizer is used. Certain metallic oxides are used as depolarizers, the most common being manganese dioxide, which is found abundantly in nature.

TYPES OF CELLS. It is by variations in construction that primary cells are given various characteristics and trade names. Thus, those cells which have a mobile liquid as electrolyte are called *wet cells*. In other cells the electrolyte is held or absorbed in a gelatinous or spongy material; these are non-spillable and are called *dry cells*. Of the great number of possible combinations of materials and methods of construction, the primary battery industry has settled upon the well-known " dry battery " as the major type.

WET CELLS. Less important, commercially, are the wet cells, exemplified by the Edison Lalande cell and the so-called air cell. In both of these, zinc serves as the anode, caustic soda solution as the electrolyte. In the Edison Lalande cell the cathode and depolarizer are a combination of copper and copper oxide. In the air cell the cathode is a porous carbon plate which is depolarized by the oxygen that it absorbs from the air.

DRY CELLS. The present day dry cell is an outgrowth of that combination of materials first suggested by Le Clanché. It uses zinc and carbon for electrodes, sal ammoniac and zinc chloride in solution as the electrolyte, manganese dioxide mixed with conductive carbon or graphite as the depolarizer. There is some polarization due to ammonia being formed, which zinc chloride is used to counteract.

The present primary battery industry has developed largely through empirical, cut-and-try methods—expediency, availability of materials, and cheap production being factors. In the early days of the electric era, the primary cell afforded the cheapest if not, in fact, the only practical source of electric energy. Today the cost of a kilowatt-hour from primary cells is 500 to 1000 times that of power from commercial circuits.

In spite of this wide discrepancy in unit power costs, the primary cell has an important place in the engineering and commercial world, because of certain important characteristics which it possesses: portability, availability at places remote from power circuits, minimum investment for a small generator, steadiness and uniformity of current, and flexibility in the combination of cells to produce batteries of any desired voltage.

The design of dry batteries has been influenced in large degree by the existing needs.

Thus there has been developed a battery known as a telephone battery to supply small energy drains over a long period of time, another type of battery called upon for ignition service, other types for flash-light, radio, and innumerable other uses.

The magnitude of the dry battery industry is indicated by the statistics for 1934, which show that about 500,000,000 cells were manufactured in the United States, representing a volume of business of about $30,000,000 at retail prices.

The uses for primary cells are:

(*a*) For portable lights, flashlights, electric lanterns.

(*b*) For electronic devices, including radio A, B, and C batteries.

(*c*) For telephone and hearing devices.

(*d*) For electromagnetic devices, including motors of both the rotating and vibrating types, fire alarms, signals, magnetic regulators, and the like.

(*e*) For spark or electrical discharges, such as in gas engine ignition, neon lights, ozonizers, and the like.

(*f*) For miscellaneous uses, which include electrolysis, electroplating, and medical uses.

2. DESIGN

Of all primary cells made, the dry battery type is by far the most important, having 95 per cent or more of the market. An excellent description of this type of cell is that published in Circular 79 of the Bureau of Standards, entitled, Electrical Characteristics and Testing of Dry Cells. The Bureau of Standards, in cooperation with the American Standards Association, has devoted much attention to an analysis of dry cells, and methods of use and tests. By them the various sizes have been designated by letter and dimensions, as shown in the following table. It is from cells of the dimensions given in this table that batteries using two or three cells in flashlights, to one hundred or more cells in radio batteries, are assembled.

Table 1. Constants of Tubular Cells

Types	Diameter		Length		Volume		Approximate Weight	
	Centimeters	Inches	Centimeters	Inches	Cubic Centimeters	Cubic Inches	Grams	Pounds
AA.......	1.27	0.5	4.77	1.875	6.8	0.418	14	0.030
A.........	1.59	0.625	4.77	1.875	9.4	0.579	27	0.059
B.........	1.9	0.75	5.41	2.125	15.4	0.949	34	0.075
C	2.39	0.94	4.6	1.812	20.7	1.27	41	0.09
CD.......	2.54	1.0	8.12	3.19	41.0	2.52	89	0.196
D........	3.18	1.25	5.73	2.25	46.0	2.83	100	0.22
F.........	3.18	1.25	8.73	3.44	69.0	4.25	156	0.34
G........	3.18	1.25	10.2	4.0	79.0	4.86	176	0.39
No. 6.....	6.35	2.5	15.25	6.0	435.0	26.8	1100	2.42

3. TESTING

The testing of dry cells for the determination of efficiency, life, and other characteristics is complicated by the variety of uses and requirements and by lack of manufacturing standards. From the Bureau of Standards' recommendations of tests designed for various classes of service and various types of batteries, Table 2 has been compiled, giving in a condensed form the adopted methods of testing.

As a means of comparing the merits of different makes of cells where the facilities for carrying out Bureau of Standards methods are not available, the so-called flash test will continue to be used extensively. This consists of measuring the momentary flow of current in amperes with the ammeter terminals connected directly to those of the cell. It has a definite usefulness in determining the uniformity of many cells of the same type and make, but unless properly interpreted, it is certain to be misleading. There are high-grade cells on the market that are specifically designed to give only a slight current on this test. In doing so the manufacturers are able to obtain increased storing qualities and greater life on the lighter drains more commonly employed in service.

Another quick test now in some favor among users is the continuous discharge of a cell through a flashlight bulb. This will show the value of the cell only for that use, and unless properly interpreted in connection with other tests will not indicate its value on intermittent or delayed service.

Table 2. Standard Dry Battery Tests

		Discharge Time	Resistance	Current Drain	End Point	Practical Use
		Flashlight Tests				
1.	4 ohm continuous	Continuously	4 ohms per cell	0.24 amp	0.75 volt per cell	Uniformity check
2.	Flashlight intermittent	5 min per day every day	12 ohms per 3 cells in series	0.28 amp	2.25 volts for 3 cells	Average city home use
3.	Heavy industrial	4 min every 15 min, 8 hr every day	12 ohms per 3 cells in series	0.29 amp	2.70 volts for 3 cells	Meter readers, watchmen, etc.
4.	Light industrial	4 min per hr, 8 hr every day	12 ohms per 3 cells in series	0 30 amp	2.70 volts for 3 cells	Farm use, campers, sports, etc.
5.	R.R. lantern intermittent	1 2 hr per hr; 8 hr per 6-day week	8 ohms per cell	0.15 amp	0.90 volt per cell	Railroad type lantern
6.	Heavy earphone intermittent	3 hr A.M. and 3 hr P.M. 6 days per week	15 ohms per cell	0.085 amp	1.0 volt per cell	Earphones
		Radio Tests				
7.	1250 ohms intermittent	4 hr per day, 6 days per week	83 1/3 ohms per cell	0.015 amp	17.0 volts for 15 cells	Average radio use
8.	1250 ohms continuous	Continuously	1250 ohms per 15 cells 83 1/3 ohms per cell	0.015 amp	17.0 volts for 15 cells	Police radio
9.	5000 ohms intermittent	4 hr per day, 6 days per week	5000 ohms per 15 cells 333 1/3 ohms per cell	0.0039 amp	17.0 volts for 15 cells	
10.	5000 ohms continuous	Continuously	5000 ohms per 15 cells 333 1/3 ohms per cell	0.0039 amp	17.0 volts for 15 cells	
		No. 6 Tests				
11.	10 ohms continuous	Continuously	10 ohms per cell	0.11 amp	0.90 volt per cell	Uniformity check
12.	Heavy intermittent	1 hr A.M. 1 hr P.M. 8 hr interval every day	10.5 ohms per 4 cells in series	0.44 amp	3.40 volts for 4 cells	Ignition service
13.	Light intermittent	4 min per hr 10 hr each day for 6 days; 4 min every other hr for 10 hr Sundays	20 ohms per 3 cells in series	0.194 amp	2.80 volts for 3 cells	Telephone service, doorbell, etc.
14.	Shelf	One yr at 70°	No drain			Depreciation due to storage

VARIATIONS IN PERFORMANCE. Since there is a variation in quality of cells depending upon their source, upon effects of shipment, and deterioration in storage, there may be wide variations in cells which reach the ultimate user. To determine the degree of this variation, an investigation has been conducted during the past year by an industrial laboratory. From dealers in New York, Chicago, Los Angeles, San Francisco, and Atlanta various popular brands of size D flashlight cells were purchased and sent to a central point for test by standard methods. The results given in Table 3 show the wide variation in quality of cells which are offered to the public.

Table 3. Variation in Performance of Commercial Cells

Test	Heavy Industrial	Light Industrial	Flashlight Intermittent
Time of Duration	½–3 days	4–24 days	100–190 days
Brand 1 (1)	300–360 min	800–900 min	900 min
Brand 2 (1)	220–250 min	480–500 min	730 min
Brand 3 and 4 (2)	160–210 min	300–440 min	500 min
Brand 5–18 (14)	130–200 min	200–300 min	650–740 min
Brand 19–23 (5)	100–180 min	140–280 min	600 min
Brand 24 (1)	160 min	390 min	
Brand 25–27 (3)	70–140 min	130–250 min	

Table 4 is a condensed compilation of many tests and is designed to show the performance which may be expected from the better-grade products available on the American market in 1934.

The results as shown in Table 4 were obtained under carefully controlled conditions as to age of cells (one week at start of test), temperature (70 deg fahr), and other particulars. Many factors affect the output of dry cells, some of the more important of which follow:

Age. At 70 deg fahr as much as 90 per cent of the original capacity is left in the size F cell at the end of a year. The amperage is not in this case a measure of capacity, and it usually falls off more rapidly than the actual service value. In general, smaller cells of the same grade will depreciate somewhat faster, but so many factors enter that it is impossible to set any definite figures for all sizes.

Heat. As the temperature increases, the rate of chemical action increases. Therefore, with increased temperature the following effects will be noticed:

(*a*) *Voltage* will rise in the order of 0.01 volt for each 13 deg fahr rise.

(*b*) The *amperage* will increase, and for a No. 6 cell this will be in the order of 1 amp for each 6 deg fahr.

(*c*) The capacity is nearly a straight-line function of the temperature of the battery during its discharge up to a certain maximum at above 100 deg fahr. The following table shows the capacities on continuous discharge at different temperatures, in percentage of that obtained at 70 deg fahr.

Temperature	−15° F	70° F	100° F	150° F
Size A on 333 ohms	27%	100%	127%	121%
Size D on 333 ohms	24%	100%	146%	114%
Size F on 83 ohms	13%	100%	140%	

The shielding of batteries against excessive temperatures is important. It must be borne in mind that when a cell is brought back to a high temperature from a lower one the characteristics for the higher are assumed, but if it has been overheated, then lowering the temperature will decrease the capacity still further.

(*d*) *Storage.* In that the rate of chemical action is increased, the local action within the cell will increase with rising temperature so that cells depreciate considerably faster. This is mainly due to increased local action, but in some cases also to the increased rate of loss of moisture. Dry cells should be stored in a cool place. Temperatures down to minus 10 deg fahr have not been found detrimental.

Type of Discharge. In a constant current discharge the same amperage is obviously drawn from the cell during all stages of its life. With constant resistances more current is drawn at the start and less at the end owing to the difference in voltage. Thus, as would be expected, on a constant resistance discharge the rate of change in voltage is greater at the start than at the finish. Even though the shape of the curve is changed, however, if the drain on constant resistance is considered as the average ampere value to any given end voltage, then the ampere-hours delivered to that voltage will be practically the same as though the cell had been discharged constantly at that amperage. The action of lamps is somewhat between these two types of discharge.

Table 4. Capacities of Various Cells

Cell	AA	A	B	C	CD	D	F	G	6
	Light Drain—Flashlight Type								Telephone
(A) Capacity, time	100 min	115 min	170 min	470 min	50 hr	950 min	1500 min	....	330 days
Ampere-hours	0.39	0.55	0.79	2.1	4.25	4.5	7.2	....	36
Test method	2	2	2	2	6	2	2	....	13
	Light Drain—"B" Battery Type								
(B) Capacity, time	....	180 hr	370 hr	....	....	1400 hr	2500 hr	3700 hr	....
Ampere-hours	....	0.69	1.2	....	....	5.45	9.85	14.5	....
Test method	....	10	10	....	....	10	10	10	....
	Heavy Drain								
(C) Capacity, time	70 min	60 min	110 min	130 min	380 min	450 min	1400 min	....	90 hr
Ampere-hours	0.29	0.25	0.46	0.54	1.6	1.9	5.8	....	40
Test method	1	1	1	1	1	1	1	....	12
Cell weight, grams	14	27	34	41	89	100	156	176	1100
Ampere-hours per gram (A)	0.028	0.020	0.023	0.051	0.048	0.045	0.046	....	0.033
Ampere-hours per gram (B)	....	0.025	0.035	....	....	0.054	0.063	0.082	....
Ampere-hours per gram (C)	0.021	0.009	0.014	0.013	0.018	0.019	0.037	....	0.036
Cell voltage and cubic centimeters	6.8	9.4	15.4	20.6	41	46	69	79	435
Ampere-hours per cubic centimeter (A)	0.057	0.058	0.051	0.10	0.10	0.098	0.10	....	0.083
Ampere-hours per cubic centimeter (B)	....	0.073	0.078	....	....	0.12	0.14	0.18	....
Ampere-hours per cubic centimeter (C)	0.043	0.027	0.030	0.026	0.039	0.041	0.084	....	0.092

Intermittent Drains. These can be related to continuous drains, but it must be remembered that each type of cell has a specific rate of discharge at which it delivers its maximum amount of power. If a cell is being run on a drain heavier than this optimum, then a lighter drain or an intermittent drain of fewer hours per day will increase the capacity of the cell. If, on the other hand, the cell is being discharged at a slower rate than its optimum, then decreasing the rate of drain or time of discharge per day will decrease the life of the cell. In general a factor can be determined for each type of cell. This is sometimes called "severity of service." It is the product of the percentage of total time during which the cell is discharged and the current drain. Except for very heavy drains, a cell being discharged at several rates but at the same severity of service will deliver the same energy.

Closed-circuit Voltage. On the heavier type of service considerably more power is obtained from the cell to lower voltage end points than to higher ones, but on lighter drains the current is delivered at relatively higher voltages so that when the end of the life of the cell is reached the materials are generally used up and the cell passes through the various lower voltages in a very brief period of time. In the former case, it can be figured roughly that the same period of time will elapse between the initial closed-circuit voltage and 1.25, 1.25 and 1.00, 1.00 and 0.75, and between 0.75 and 0.50. This rough slope of the discharge curve is fairly constant even for a wide range of temperatures.

4. INSTALLATION

Depending on the rate of discharge * desired and the point of maximum efficiency of the cell to be used, cells may be connected in parallel. Thus if the drain is too great for efficient use of one cell, the positive terminals of several cells may be connected together and the negative terminals connected, thus having the effect of one considerably larger cell. This does not merely increase the power obtained by the number of cells chosen, but by reason of the more efficient service the increase will be much greater than the mere additive values. If the drain is too light for one cell, then adding more in parallel will reduce the power delivered by each. When higher voltages are needed the cells are connected in series, plus to minus. Table 5 shows the life in ampere-hours obtained from various cells of good quality on continuous and intermittent drains.

Table 5. Variation in Ampere-hour Capacity

Cell Size—F	Capacity in Ampere-hours to 0.9 Volt						
	Milliampere drain	..	3	5	25	50	100
	Hours per day	3	9.1	9.0	10.1	10.1	8.1
	Hours per day	6	11.5	10.7	10.4	9.2	7.4
	Hours per day	24	12.6	11.2	6.9	5.5	4.2
Cell Size—B	Capacity in Ampere-hours to 0.9 volt						
	Millampere drain	..	1	3	10	25	50
	Hours per day	3	1.5	1.6	1.8	1.6	1.0
	Hours per day	6	2.0	1.9	1.7	1.4	0.92
	Hours per day	24	2.2	1.6	1.1	0.8	0.6

5. OPERATION

One important factor in the use of dry cells is that any cell if run down continuously far below its normal service end point will eventually leak. This leakage will occur either through the seal or through the zinc itself if the latter is thin enough to perforate in service. The solution discharged from a leaking dry cell is about 60 per cent zinc chloride in water. The effect of this on the materials used around dry cells will generally be serious. Even if a cell is open-circuited after it has been seriously run down, the tendency to leak will still remain. Thus if cells have been accidentally short-circuited or used continuously until they are dead, the only safe action is to remove them so that they can do no damage. In general, it will be found that cells delivering a high short-circuit amperage and greater service on heavy continuous discharge are more apt to leak. With the present state of the art there is very little danger from this source if cells are purchased from a reputable manufacturer and are properly treated.

* For Specifications see Circular of Bureau of Standards 79.

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LEAD-ACID STORAGE BATTERIES

By J. Lester Woodbridge

6. GENERAL AND DESCRIPTIVE

The action of a storage battery when discharging is similar to that of a primary cell except that the material of the electrodes which undergo the chemical reactions is not dissolved but remains in much the same mechanical condition as before discharge. When this requirement is fulfilled the material of the electrodes can be restored to its original condition by " recharging " or passing a current through the cell in the reverse direction to that of the discharge current. When this is done the electrical energy of the charging current is transformed into chemical energy and stored as such in the electrodes.

As there is no loss of material during the discharge of such a battery to expose fresh material to chemical action, it is necessary that the active material of the electrodes either be disposed in a very thin layer over a large surface or be porous to permit the use of the interior part of thick material.

LEAD-ACID BATTERY. A lead-acid cell consists essentially of positive plates of lead peroxide and negative plates of pure lead immersed in an electrolyte of dilute sulfuric acid.

(In accordance with common engineering parlance the terms positive and negative plates are employed in this article to denote respectively the plates which are connected to the positive and negative terminals of the external circuit on both charge and discharge. The term battery is employed to denote a *group* of two or more cells which constitutes an operating unit.)

CHEMICAL REACTIONS. The active elements of the lead-lead acid type of battery consist of lead peroxide (PbO_2) on the positive plate, sponge lead (Pb) on the negative plate, and dilute sulfuric acid (H_2SO_4) for the electrolyte.

Whatever the secondary reactions may be, it is agreed that the final result on discharge is the formation of lead sulfate ($PbSO_4$) on both the positive and negative plates, the SO_4 radical of the sulfuric acid combining with the lead of both plates to form this compound, resulting in the formation of some water (H_2O), with a consequent decrease in the specific gravity of the electrolyte. On charge the electric current splits up the lead sulfate ($PbSO_4$), returning the SO_4 radical to the electrolyte, oxidizes the positive plate to its original condition of lead peroxide (PbO_2), and reduces the negative plate to its original condition of sponge lead (Pb). This action may be represented as follows:

$$\underset{+\text{Plate}}{PbO_2} + 2H_2SO_4 + \underset{-\text{Plate}}{Pb} \rightleftarrows \underset{+\text{Plate}}{PbSO_4} + 2H_2O + \underset{-\text{Plate}}{PbSO_4}$$

This equation read from left to right is the equation of discharge; if read from right to left, it is the equation of charge. In practice, on charge, towards the end of charge some of the water (H_2O) is split up by the current into its component parts, hydrogen (H) and oxygen (O), the hydrogen being liberated at the negative plate and the oxygen at the positive plate. This occurs whenever the density of charging current is greater than can be utilized in decomposing the lead sulfate remaining in the plates.

Types of Lead-Acid Cells

Two general types of lead-acid batteries are in use, the Planté and the pasted plate, each type being subject to some modifications. The distinction between these types is based on the plate structure.

PLANTÉ TYPE. The Planté plate is essentially a pure lead plate having a highly developed surface presenting a large superficial area which is oxidized electrochemically until the entire surface is covered with a thin, porous layer of lead peroxide (PbO_2). This constitutes a positive plate. It may, however, be converted into a negative plate by reversal of the current which reduces the lead peroxide to pure lead in a porous form. Planté negative plates are little used at this time.

Planté Plates. Of the various types of Planté plates, among those most commonly in use may be mentioned the central-web type and the cast-lead having no central web. The main idea in both these methods of manufacture is to produce a large surface on which to form the active material.

Central-web Type. In the central-web type there is a solid sheet or "web" of pure lead on which the ribs are formed. This web prevents the circulation of the electrolyte through the plate. The ribs are formed from the original lead plate, either by rolling, spinning or cutting. In the rolling and spinning processes the plate is formed from a lead blank by means of a number of steel disks placed side by side and separated by small spacers on a shaft. The lead blank is passed between two sets of disks by forward and backward movements. The disks gradually work deeper into the plate and squeeze up lead into the spaces between the adjacent steel disks. In the cut type of plate the ribs are formed from the lead sheet by a tool, which at each stroke turns up one complete rib. The cutting edge works at an angle so that the finished ribs stand out from the surface. The ribs may incline upward from the central web and thus form pockets to hold the active material and prevent its falling away.

The disadvantage of the *web* type is the web itself. There is invariably a tendency towards unequal work on the two sides of the positive plate, this tendency being caused by difference in the plate spacing, unequal capacity of the negative plate on either side, inequality in the shape of the ribs, etc. Where the active material and the active surface of a plate are disposed in planes perpendicular to the face of the plate and extend *through* the plate, excessive action on one side simply works the plate a little further through from that side, the effect on the active material, however, being uniform throughout. With a plate provided with a central web, preventing such action, any inequality of work will charge or discharge one side more than the other, producing a tendency to buckle.

Cast-lead Type. The type of plate which has no central web is made by casting pure soft lead in a mold, casting having the advantage of allowing for distributing metal in the plate without limitations in manufacturing process. The plate as it comes from the mold consists of a great number of short vertical ribs running entirely through the plate and bound together by transverse ribs to give strength to the plate. In this manner a large surface can be obtained and in a plate having no central web the electrolyte can circulate through the plate, and the active material will be uniformly worked throughout even though the amount of work on the two sides of the plate be unequal. The best-known form of this plate is the " Tudor " positive.

Modified Planté or Manchester Positive Plate. The so-called Manchester positive plate is made by rolling corrugated strips of pure lead ribbon into rosettes or " buttons " which are forced into circular openings in a rigid grid of cast lead-antimony alloy. This grid is strong, stiff, and subject to very little electrolytic corrosion; it therefore provides a plate structure which is immune from the growth and buckling which frequently occurs with Planté plates composed entirely of pure lead. The pure lead buttons constitute the active part of the plate and function in every way similarly to the original Planté positive plate.

"Forming" of Planté Plates. In all Planté positives, after the ribs or corrugations have been formed on the lead blank or the pellets have been placed in the hard grid, the plates are assembled in a sulfuric acid bath containing some corrosive chemical, called a " forming agent," together with dummy lead plates. The forming agents used by various manufacturers are usually kept as trade secrets; the nature and method of using such agents determine largely the quality of the battery. To form the positive plates the dummies are connected as the cathodes, and a current is passed through the couple thus formed. The electrolytic action of this current causes lead peroxide (PbO_2) to be formed from the lead of the ribs. . The strength and duration of the current produce the desired thickness of lead peroxide on the ribs or pellets.

In recent years the Planté plate has largely disappeared from use in this country while the Manchester positive plate has continued in extensive use. In Germany the Planté positive plate has been used to the practical exclusion of the Manchester type. At the present time (1936) the offering of Planté positive plates has been more actively resumed by at least two manufacturers. It is therefore probable that the use of this type of plate in this country will increase.

The Planté and modified Planté types are used chiefly where space, weight, and first

cost are of secondary importance and where a battery more durable than the pasted plate type is desired.

PASTED PLATE TYPE. The pasted plate is made by applying to a rigid grid of cast lead-antimony alloy the active material in the form of a paste composed largely of lead oxides, usually litharge (PbO) or red lead (Pb_3O_4) or both, and a solution containing sulfuric acid or a sulfate which acts as a binder. The grid usually has the form of a double gridiron composed of a series of vertical ribs which extend through the plate, giving it strength and providing conductivity to carry the current to plate lugs. These vertical ribs are tied together by short cross-bars which are flush with the surface but extend only partly through the plate and are staggered on opposite sides. These short bars are usually horizontal, but one manufacturer places them diagonally making the so-called diamond grid. In either case the active material when applied to the grid is disposed in the form of corrugated strips or ribbons extending from the top to the bottom of the plate between the vertical ribs and held in place by the cross-bars.

As the active material has a depth equal to the thickness of the plate, composition of the paste must be such as to result in the necessary porosity while still maintaining sufficient strength and cohesion of the material to assure the desired life of the plates in service. The composition of paste used is varied for different services, and the variations are closely guarded trade secrets.

Plates of this type are very extensively used as both positive and negative, but in the composition of the paste red lead usually predominates for positive plates and litharge for negative plates.

After the grid is filled with the paste, the plate is dried to allow the paste to harden, then it is "formed" electrochemically. Positive plates are formed by immersing them in a forming bath and passing current through them connected as anodes which further oxidizes the lead oxides in the dried paste forming lead peroxide (PbO_2). Negative plates are formed by passing the current in the opposite direction which reduces the lead oxide to sponge lead. After the forming charge the plates are washed and dried and are ready for assembly.

Quite recently processes have been developed which permit the dried pasted plates to be assembled into cells and the plates to be formed while in the assembled cells. This process has been used only for cells which are manufactured in large quantities.

Modified Pasted Plate or Exide Ironclad. In the so-called Exide-Ironclad positive plate, the active material is prepared from lead oxides of composition similar to that used for pasted positive plates. This plate is therefore generally classed as a pasted plate although the plate structure differs entirely from the usual flat plates of that type. The frame of this plate consists of a series of parallel vertical core rods joined at each end to horizontal top and bottom bars all of lead-antimony alloy. The active material completely surrounds each core rod and is held in place by finely slotted rubber tubes. The core rod is thus in the center of a cylindrical pencil of active material where it is well placed to conduct the current to the top bars and the cell terminals. The effective surface of the active material exposed to the electrolyte is greater than in a flat plate type of equal superficial dimensions. The slotted rubber tube replaces the perforated rubber separators used with flat plates and acts more effectively to retain the active material in place as it softens with use and thus makes a plate having a longer useful life. Its first cost is greater than that of the usual flat plate type, but it is claimed by its manufacturer to give from 2 to 3 times the life in severe service.

This type of plate has come into very extensive use especially in motive power work, railway train lighting, and marine service.

ELECTROLYTE. The electrolyte used with the lead type of battery is always a dilute solution of sulfuric acid. The specific gravity of the electrolyte, when the battery is fully charged, varies from about 1.210 for stationary batteries to 1.300 for automobile ignition batteries. These values have been adopted as standard by all the leading manufacturers.

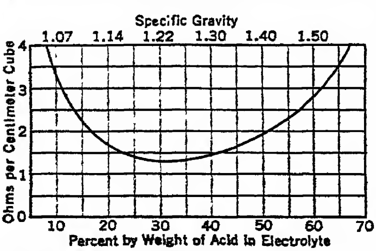

FIG. 1. Variation of Resistance with Specific Gravity

The proper specific gravity to use varies with the conditions. Fig. 1, showing the variation of the resistance of 1 cc of electrolyte with specific gravity shows that the resistance of dilute sulfuric acid is least at a specific gravity of from 1.224 to 1.240, this resistance increasing if the specific gravity be either increased or decreased. Numerous other conditions influence the selection of the proper specific gravity.

The curves in Fig. 2 show the specific gravity of various mixtures, both by weight and by volume, of one part of 1.840 specific gravity acid with from $^1/_{10}$ to 7 parts of water. There is also a curve showing the percentage, by weight, of 1.840 specific gravity acid in mixtures of various specific gravities. These curves are approximately correct at 60 deg fahr. Unless a compensating hydrometer is used in determining the specific gravity, allowance must be made for temperature variation, on the basis of an increase of one point (i.e., one one-thousandth) in gravity for each 3 deg fahr *decrease* in temperature, and vice versa; for instance, electrolyte that has a specific gravity of 1.210 at 70 deg fahr will have a specific gravity of 1.213 at 61 deg fahr, and 1.207 at 79 deg fahr.

Impurities in Electrolyte. The electrolyte should be of known purity, harmful ingredients such as iron, chlorine and nitrogen oxides being strictly limited. The various battery manufacturers issue exact specifications to the acid manufacturers, specifying the maximum amount of different impurities which the acid may contain. Electrolyte that is not approved by the company furnishing the battery should never be used.

Preparation of Electrolyte. In preparing the electrolyte, sulfuric acid, approved by the battery manufacturer, should be diluted with sufficient pure distilled water to bring the mixture to the required specific gravity. The acid should be poured into the water; *never pour the water into the acid.* If the water is poured into the acid, the heat formed by the mixture is sufficient to cause sputtering and damage may ensue.

The sulfuric acid manufacturing companies furnish electrolyte for battery work in such large quantities that they carry a stock of various standard mixtures. It will usually be found cheaper and more convenient to purchase the electrolyte ready-mixed than to purchase the concentrated sulfuric acid and prepare the mixture on the ground. The latter course, however, is sometimes adopted where the amount of acid used is considerable and where the item of freight saving is appreciable.

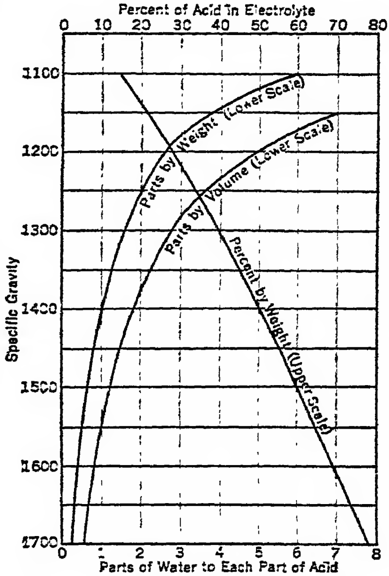

FIG. 2. Variation of Specific Gravity with Concentration

CONTAINERS. The containers for holding the battery plates and electrolyte may be of hard rubber or asphaltic compound, glass, or lead-lined wooden tanks depending on the size of the cells and on the nature of the service.

Rubber Containers. Hard rubber or composition containers are used when the cells must be portable or subject to mechanical shocks while in service. Typical examples of such cells are those used for automobile starting and lighting, electric street and industrial trucks, railway train lighting, submarine boat propulsion, etc.

In these batteries the plates are usually supported on ribs in the bottom of the jars, the ribs being of sufficient height to allow ample space for the accumulation of sediment, deposited owing to the gradual loss of material from the plates, without danger of its short-circuiting the plates. The chemical composition of the rubber compound is subject to the specifications of the battery manufacturer in order to prevent contamination of the electrolyte.

Glass Jars. Glass jar containers are used in preference to rubber containers whenever the service and size of the cells permit, on account of the greater ease of inspection and maintenance. The present tendency is toward the almost exclusive use of the cells assembled sealed and charged at the manufacturer's plant and shipped filled with electrolyte and ready for service as soon as installed and connected. This assembly is available in cells up to approximately 1000 amp-hr capacity at the 8-hr rate for pasted plate types and up to about 550 amp-hr for the formed plate types. Thoroughly annealed molded glass containers for from 1 to 3 cells are used for the smaller sizes and to a limited extent by some manufacturers for the larger sizes of single cells, but the internal strains which remain even after the most careful annealing and the higher cost of the molded jars have resulted in the much more extensive use of machine-blown jars for the larger cells. These jars are also thoroughly annealed.

In blown jar assemblies the plates are usually supported from the cover of the cell but in some cases separate supporting ribs are placed on the bottom of the jars. In molded jar assemblies ribs may be molded on the jar bottoms to support the plates.

Lead-lined Tanks. In the larger sizes of batteries lead-lined tanks are standard. Tanks are generally made from specially selected yellow pine, dovetailed together without the use of nails or other metallic fastenings. The upper edges should be slightly beveled inward so that the moisture will drain into the tank. The bottoms under the linings should be drained and ventilated; these bottoms are usually constructed of slats across the tanks, separated by a small spacing to provide drainage. To further facilitate drainage under the lining, the upper surface of these slats should be grooved crosswise. Before the tanks are lined they should be coated inside and outside with two coats of acid-resisting paint, and a third coat should be added outside during installation. The tanks should be lined with lead of 3 to 4 lb per sq ft, depending on their size. The lining should extend over the upper edge of the tank and a short distance down, but free from, the sides. Especial attention should be paid to the seaming of the linings. All seams should be burned with the hydrogen flame with pure lead, without the use of any flux. The lower corners of the lining should be reinforced by puddling with lead. The upper corner should be reinforced by burning on an additional thickness of sheet lead. Each tank should be so built that it is self-supporting without the use of any braces or reinforcements. If this is done, any tank in the battery can be removed and replaced without affecting the remaining tanks in any way. A poorly constructed wood tank is bound to cause trouble. Special attention should be paid to this detail in preparing specifications.

In cells of this type it is of course necessary to insulate the plates from the lead lining. The plates are therefore supported on vertical sheets of glass resting on the bottoms of the tanks. The tank linings under the glass sheets are reinforced. The glass sheets are ground smooth on the top and bottom edges, and the bottom corners are beveled at 45 deg to avoid injury to the lead lining during installation.

Cell Covers. Batteries in lead-lined tanks and in the old forms of open glass jar assemblies are usually provided with covers of double thick window glass which rest on the top of the cells. These covers serve the purpose of intercepting much of the fine spray thrown off from the surface of the electrolyte during " gassing " while charging and drain it back into the cell. They also serve to keep dirt out of the cells, this being quite essential when the air is likely to contain particles of metal or other matter injurious to the battery.

Both these purposes are served very much more completely with the sealed type of cell in either glass or rubber jars. In the blown-glass jar assembly where the plates are supported from the cover, the cover must be of sufficient strength to carry this weight even during shipment. It must also be impervious to the action of the electrolyte, and not affected by any temperatures which are likely to be encountered either during shipment or in subsequent service. Molded glass and hard rubber or similar compounds are materials most frequently used.

For sealed glass jar cells the covers are provided with a groove on the under side which fits over the edge of the jar, this groove being partially filled with a sticky plastic sealing compound which under the weight of the plates forms a tight seal between the cover and the jar. The joints between the cover and the cell terminal posts must be sealed to prevent creepage of electrolyte over the top of the cell. This is effected by sealing compound or by some form of gasket clamped between the cover and the terminal post by means of a threaded seal nut.

For rubber jar cells the covers are so shaped as to provide a V-shaped groove between the cover and the jar sides, which is filled with sealing compound. The seal between the terminal posts and the cover may be as described above for glass jar cells but in some types of automotive batteries this is accomplished by means of a lead thimble molded into the cover through which the terminal post passes. This thimble is then " burned " to the terminal post to complete the seal.

Separators and Separation. Since the positive and negative plates of the cells are assembled alternately, it is necessary that " separators " be provided to prevent their coming in contact with each other and so short-circuiting the cell. These separators must maintain their mechanical strength throughout the life of the battery and must be porous to allow diffusion of the electrolyte through them and the passage of the current through the electrolyte in their pores. Grooved wooden diaphragms, perforated hard rubber sheets, or a combination of both are ordinarily used. The woods most commonly used for separators include cedar, cypress, redwood, and Douglas fir. They may be cut as veneers, quarter sawed, or sliced. They are usually planed to the exact thickness required and to provide grooves for the circulation of the electrolyte and free passage of loosened material to the sediment space at the bottom of cell. Wood separators are treated chemically to

remove ingredients which when in contact with the sulfuric acid in the electrolyte will form acetic acid which is injurious to the plates.

The threaded rubber separator as used by the Willard Storage Battery Co. is composed of a semi-hard rubber sheet in which are imbedded cotton threads running transverse to the thickness of the separator to provide the required porosity.

Glass wool separators are used by the D. P. Battery Co. in England and the Gould Storage Battery Co. in this country in the so-called Kathanode battery. This form of separator is composed of fine glass fibers laid at different angles and cemented on the surface with a soluble cement to permit handling prior to assembly in the cell. It is placed against the positive plate where owing to its compressibility it comes into intimate contact with the plate and prevents or retards the softened material on the surface of the plate from falling to the bottom of the cell. It is used in conjunction with wood or with wood and perforated rubber separators.

The Electric Storage Battery Co. announced in 1934 a new type of separator under the trade name of Mipor. This separator is manufactured from rubber latex to which are added the sulfur required for subsequent vulcanization and certain other ingredients which cause the rubber content of the latex to partially coagulate into a jelly-like form which constitutes a three-dimensional network in which the water content of the latex is enmeshed.

When this jelly is sufficiently set it is vulcanized in the presence of water or saturated steam to prevent the collapse of the rubber mesh into the space occupied by the enmeshed water.

On completion of vulcanization and after drying, the material has all the characteristics of hard rubber except that it is highly porous. The pores are of microscopic dimensions (in the order of 0.00004 cm or 0.000016 in.) which prevents the penetration of even the smallest particles of active material, but the pores are so numerous as to permit ample diffusion of the electrolyte and ready passage of the electric current. The body of the separator being of hard rubber is unaffected by the electrolyte or by high temperature. This separator has a conductivity somewhat better than that of the corresponding wood separator.

7. CELL CHARACTERISTICS

A lead-acid cell has an open circuit voltage of approximately 2 volts regardless of the size of the cell. This voltage is, however, somewhat dependent on the specific gravity of electrolyte used, being approximately 2.06 volts per cell with 1.210 specific gravity and

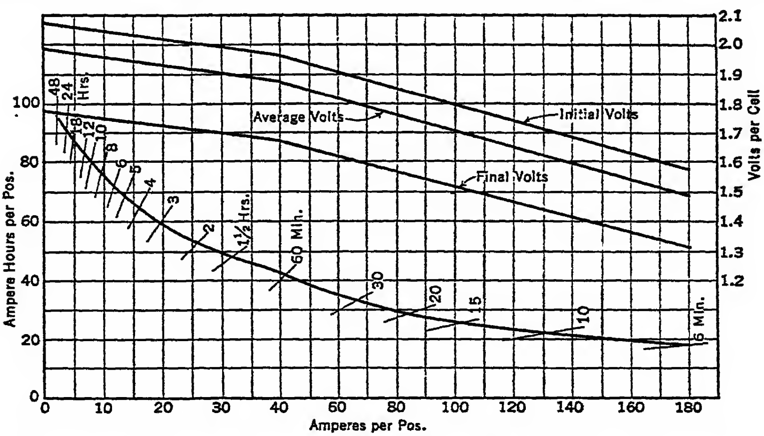

Fig. 3. Rated Disharge Characteristics of Lead-acid Cells, 1.210 Sp. gr., 77 deg fahr

2.10 with 1.280 specific gravity. A sufficient number of cells are connected in series to get the desired voltage of a battery.

The capacity of a cell or battery is usually expressed in amperes and time, as 20 amp for 8 hr, or as 160 amp-hr at the 8-hr rate, which is the same thing. The capacity desired determines the size of cells used in a battery. To define the capacity of a cell or battery completely it is also necessary to specify the minimum voltage permissible at the end of

the discharge period as 20 amp for 8 hr to a final voltage of 1.75 volts per cell. Manufacturers are usually prepared to furnish data giving the electrical characteristics of their different types of cells. Fig. 3 shows the usual form of such data. They are given in terms of amperes and ampere-hours per positive plate, and the voltages are for a single cell. These data are applicable to all sizes of cells using that size and type of plate.

In this country batteries are generally designated by a series of numbers and letters, as 60–EMGO–9, meaning 60 cells each having 9 plates, the letters being a code designating the size and type of plate and details of the cell assembly. As there is usually one more negative than positive plate in a cell and as the capacity is limited by that of the positive plate, a graph of the type given in Fig. 3 may be used for determining the capacity in ampere-hours and the initial, average, and final voltage of any size of battery of the designated type at any rate of discharge.

The capacity of the battery as given in ampere-hours is to be taken as the capacity which the manufacturer undertakes to deliver after the battery has been installed and is ready for an acceptance test. Practically all types of lead-acid batteries will increase their capacity during the first few months of service and will maintain this increased capacity during the greater part of their useful life.

The final voltage values as given define the minimum voltage at the end of a continuous discharge at any particular rate. It is true that some additional capacity can be obtained to lower final voltages, but the increase will not be great, as the voltage will fall quite rapidly if the discharge is continued. Although the final voltage values may be considered somewhat arbitrary, a definite value is necessary for test purposes, and those assigned meet all practical requirements.

The average voltages given are those which can be computed from frequent periodic readings during a continuous discharge at the specified current to the final voltage given. These values are used principally for determining the size of battery required to carry loads expressed in kilowatts and kilowatt-hours.

The initial voltages given are again somewhat arbitrary, as the instantaneous voltage that can be observed on closing a circuit may change quite rapidly during the first few seconds depending on what work the battery may have been doing for some little time prior to closing the circuit. The duration of this rapid change also depends on the rate of discharge. These transient voltages are seldom of engineering importance, and the values as given are substantially correct as representing the stabilized value which will exist after a short interval.

CAPACITY CHARACTERISTICS. The relation between capacity and rates *of continuous discharge* are given in graphs of the character shown in Fig. 3. The reduced capacity at higher rates of continuous discharge is due to depletion of acid in the electrolyte in the pores of the plates. During discharge the acid in the electrolyte combines with the active material of the plates and water is formed, thus reducing the strength of the electrolyte inside the plate. This effect is counteracted to a varying extent by diffusion between this weakened electrolyte in the plate and the stronger electrolyte outside the plate. At higher rates of discharge the electrolyte inside the plate is weakened faster than it can be replaced by diffusion. It is this depletion of acid in the pores of the plates that limits the capacity at high discharge rates, not any limitation in the plates themselves. If the discharge is interrupted, diffusion will of course continue, and this brings about the well-known recuperation of a battery during idle periods. The available capacity of a battery is therefore really a function of the total elapsed time of discharge rather than of the rate of discharge. The two are the same for a continuous discharge, but for intermittent discharge the capacity varies with the total elapsed time. Inasmuch as it is a lack of available acid that reduces the capacity at high rates of continuous discharge it is evident that the battery cannot be damaged by excessive discharge at such rates.

VOLTAGE CHARACTERISTICS. The initial, average, and final voltage lines as shown in Fig. 3 are straight lines showing a drop in voltage which is proportional to the current. This suggests that the drop is an IR drop and that the internal resistance of a cell can be calculated from these data. Although such a value can be used in some classes of calculations it is by no means of universal application. The true internal resistance of a cell is of little engineering importance, and in fact when measured by different methods in a laboratory widely varying results may be obtained. It is generally accepted that during a continuous discharge the internal resistance increases slowly during the first two-thirds of a discharge and increases at an increasing rate to a value at the end of a complete discharge approximately twice as great as at the start.

Fig. 4 shows the variation of voltage with time during continuous discharge at rates from the 24- to the 3-hr rate. The drop of voltage with time is due partly to the increase in internal resistance and partly to the reduction of electrolyte density in the pores of the plates (the latter effect is frequently referred to loosely as polarization).

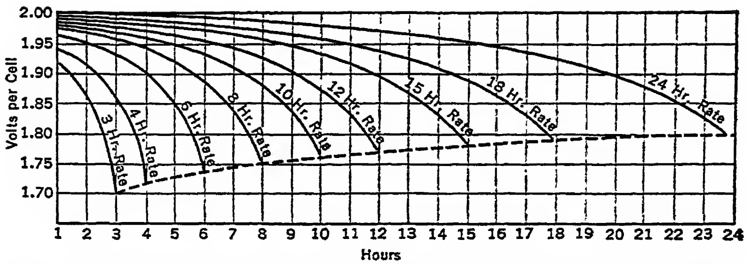

Fig. 4. Approximate Volt-time Curves for Batteries in 1200–1220 Sp. Gr. at 70°–80° F.

EFFECT OF TEMPERATURE ON CHARACTERISTICS. In data showing the characteristics of their different types of cells, manufacturers usually follow A.I.E.E. standards and base them on a temperature of electrolyte at the start of a test of 25 deg cent (77 deg fahr). Temperature affects both the internal resistance and the rate of diffusion of the electrolyte in such a way that both the capacity and voltage are lower at lower temperatures and higher at higher temperatures. It is impossible to make a general statement of the quantitative effects of temperature changes as they vary greatly with details of plate structure and assembly, and with the discharge rates. Fig. 5 gives data on one type of battery at two rates. The internal resistance of a battery, as determined from the momentary change in voltage with change in current, is approximately doubled by a drop in temperature from 25 deg cent (77 deg fahr) to 6 deg cent (20 deg fahr), this variation being in approximately a straight line between these limits.

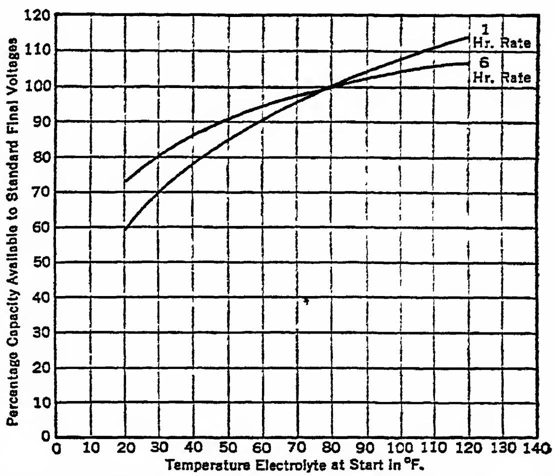

Fig. 5. Variation of Capacity with Temperature. Ambient Temperatures Constant at Values Shown. Capacities shown are to standard final voltages. Additional useful capacity will be obtained at the lower temperatures if discharges are continued to the knee in voltage curve

CHARGING CHARACTERISTICS. The electrical characteristics of a battery on charge are usually of little interest except in choosing charging equipment that will be both economical and convenient. A wide latitude as to charging rates is permissible with lead-acid batteries, the essentials being that the rates shall be such as to avoid excessive temperatures (about 45 deg cent — 113 deg fahr) and excessive gassing. These requirements demand that relatively low charging rates be used during the latter part of the charge. Manufacturers usually assign a rate which should not be exceeded at that time and which is generally called the "finishing rate." A general rule for determining the maximum permissible charging rate has been given as: The charging rate in amperes must not exceed the ampere-hours out of the battery unless the rate be the finishing rate or lower. This rule permits charging of an empty battery at rates too high to be convenient or economical but shows the wide latitude permissible between such charging and a constant current charge at the finishing rate which may require 16 hours or longer. Automatic charging may be done by the so-called modified constant voltage method which consists of a constant voltage charging source equal to approximately 2.6 volts per cell of battery and a fixed resistor between this source and the battery. Fig. 6 shows complete records of a battery charged in this manner. This method of charging is strongly recommended when batteries are to be charged frequently and in comparatively short time.

Constant current charging at the finishing rate or less is satisfactory if time is available.

When batteries are to be charged infrequently manual control of the charging current is frequently employed using a constant current at the finishing rate or a current about 2.5 times the finishing rate during the early part of the charge, then reducing the rate to the finishing rate. Fig. 7 shows the variation of voltage during such a two-rate charge.

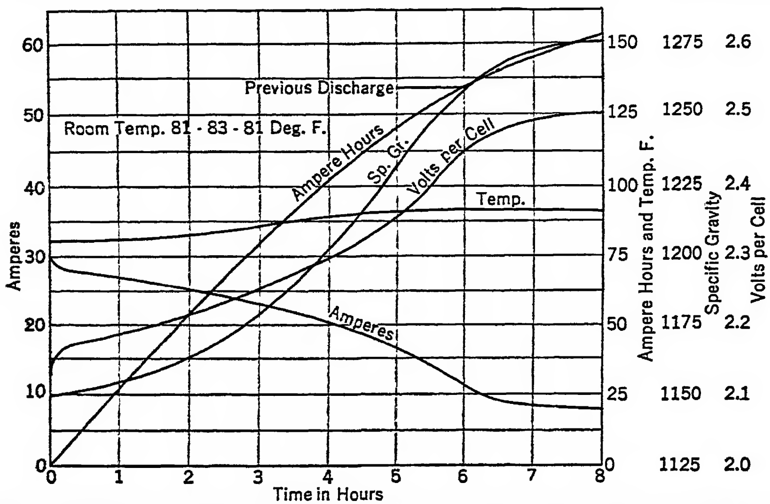

Fig. 6. Modified Constant Voltage Charge. 4 cells. 136 amp hr capacity in 6 hrs. Charged from 10.52 volt bus, 2.63 volts per cell, through fixed resistance of .0665 ohm. Ohm per cell = 0.0166

Trickle Charging is a term applied to maintaining a very low charge rate which is of the proper value to compensate for the internal losses which occur in an idle battery. It is employed to keep a battery in a fully charged condition over long intervals (see also Standby Batteries). The proper value of the trickle charge current depends on size and number of plates per cell, the specific gravity of the electrolyte, the temperature, and the age and condition of the battery. It is usually in the order of 0.1 per cent of the 8-hr rate for 1.210 specific gravity and temperature near 25 deg cent (77 deg fahr), but a more accurate value should be obtained from the manufacturer.

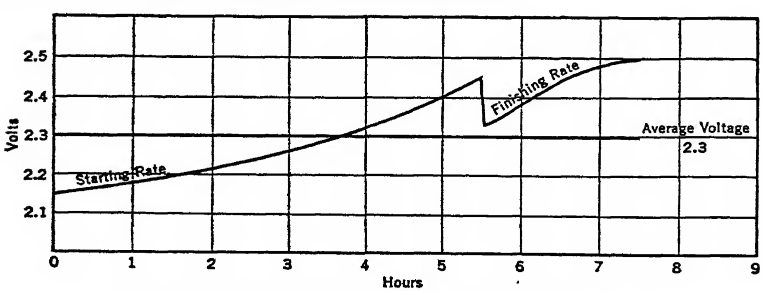

Fig. 7. Constant Current Charge. Characteristic Voltage Curve
Note: This curve is only approximate since temperature condition of the cell and other factors will cause variations from the factors shown.

Floating. A more satisfactory way to keep a battery in a fully charged condition is to maintain a constant voltage of the proper value across the battery terminals. This is known as "floating." When this is done, a low charging rate will automatically flow to the battery which will vary with varying conditions to accomplish the desired results. The proper value of the floating voltage varies with the specific gravity of the electrolyte. For 1.210 specific gravity the voltage should average 2.15 volts per cell and should not be

allowed to vary outside the limits of 2.10 and 2.20 for sustained periods. For 1.280 specific gravity the average should be 2.22 and the corresponding limits 2.17 and 2.27 volts per cell. If batteries are to be floated continuously in service, cells having low-gravity electrolyte should be used as they will have a longer useful life.

8. INSTALLATION

The essential features that require consideration in planning the details of all battery installations are: first, ample ventilation and moderate temperature; and second, accessibility of each cell for inspection and maintenance. There are other features depending on the type of battery and the nature of the service which will be discussed in later paragraphs. Ventilation is required to dissipate the oxygen and hydrogen which are given off from the battery in considerable quantities during the latter part of charge and to some extent at all times, in order to prevent the possibility of an accumulation of these gases in such proportions as form an explosive mixture and also to remove any acid spray which may be thrown into the air if open-type cells are used. It is also required to control the battery temperature, especially if there is any possibility that the battery may be subjected to high rates of charge or discharge. Accessibility to inspection and maintenance should be such as readily to permit the battery to receive such attention as is outlined under Care and Operation, page 7–24. Special attention should be given to ease of keeping electrolyte at the proper level (by adding water to the cells to replace evaporation), taking hydrometer readings, and keeping the battery clean and dry.

It is usually necessary for the battery manufacturer to make a complete detailed drawing of the battery exactly as it will be installed before he can prepare the necessary material for shipment. In the interest of both the user and the manufacturer it is very desirable that the latter be consulted in time to permit adequate provisions to be made.

GLASS JAR INSTALLATIONS. When batteries in sealed glass jars are of appreciable size they are usually installed in a separate battery room, although this is by no means a necessity. The vent plugs in the covers of modern batteries are so designed as to allow the escape of gases but to trap all electrolyte spray effectively with the result that there is no danger of escaping acid causing corrosion of any nearby metal surfaces. The batteries should be located in a clean, dry, ventilated place where they are not likely to be damaged. At least one side of each cell should be accessible from an aisle. The floor of the battery room should be of smooth concrete. The battery should be shielded from the direct rays of the sun by the use of diffusing glass in windows or skylights if necessary. The cells are mounted on wooden or steel racks whose upright members may rest on vitrified brick or tile. The racks may provide for one or more tiers of cells, but for the larger sizes of cells, single-tier racks are recommended. When the cells are arranged in more than one tier no cell should be located behind any of the uprights unless space is rigidly limited. Ample space (about 15 in.) should be allowed between the top of a cell cover and the bottom of the rack above to permit hydrometer readings.

RUBBER JAR INSTALLATIONS. Batteries assembled in rubber jars are generally used only when portability is necessary or where glass jars will not withstand the mechanical shocks to which the battery is likely to be submitted. The placing of portable batteries requires no special comment. In those installations where the battery is in motion it is necessary to provide against the battery's shifting its position and against the shocks due to sudden changes in motion. Springs have been tried and their use abandoned. Where the battery is necessarily mounted in a structure supported on flexible springs, as on a passenger automobile, the battery should be bolted solidly to the supporting structure. On electric street and industrial trucks where the body springs are relatively stiff the batteries are usually protected against jumping by some form of cleat or by holding-down bolts and against lateral or longitudinal shifting by positive mechanical stops. In locomotive and marine work no hold-downs are necessary. In no installation should the battery be wedged tightly in the compartment, as any pressure applied in this way is added to any other stresses that may be applied to the rubber jars due to weaving of the compartment or to mechanical shocks during service.

Compartments. Practically all rubber jar installations are made under conditions where the space required for the battery is also desirable for other purposes. This fact should not result in the battery's being installed in a manner which will prevent successful operation or cause excessive battery maintenance cost. Ventilation and accessibility are essential. Ample air inlets should be provided at the bottom of the compartments and outlets at the top to permit a free circulation of air. The trays should be separated from each other by at least $1/2$ in. to allow passage of air between them and to permit drainage when the battery is washed down. Rubber or porcelain spacing insulators are used for this purpose. Drainage should be provided for the wash water.

The size of the compartment should be such that no tray can move more than 1/2 in. in any direction. If blocking is required it should be placed next to the compartment walls, for if it is placed between trays, the inter-tray connectors may be subjected to strain.

In street and industrial truck service where the number of cells in series rarely exceeds 44 and where the vehicle is mounted on rubber tires no special insulation of the battery from the compartment is required, but in other services where the number of cells in series exceeds 16 and where the compartment is grounded, rubber or porcelain insulators are used between the trays and the bottom and sides of the compartment. For marine installations lead pans or water-tight lead-covered floors are sometimes provided beneath the insulated cells.

BATTERIES IN LEAD-LINED TANKS should always be installed in a specially prepared battery room, the details of which must be worked out jointly by the user and the manufacturer prior to preparation of the battery for shipment. The special features which may be required include forced ventilation system, which should be of the exhaust type with inlets near the floor at one end of the room and the exhaust outlet near the ceiling at the other end through a series of baffle plates to intercept any fine particles of electrolyte thrown into the air when the cells are gassing.

It is impossible in the confines of this section to give complete specifications for the floor construction, but such specifications based on many years of experience may be obtained from battery manufacturers. In general these specifications call for a smooth, level cement foundation surface on which one or more layers of acid-proof fabric are laid and protected by a bituminous compound mastic floor.

Provision must be made for supporting copper conductors to the battery terminals and to the end cells if the latter are used. All exposed conductors must be lead plated or painted with acid-resisting paint. Lead-covered cable in the battery room is not recommended. Supporting structure for end cell switches if used must be accurately installed, and all exposed metal must be protected against acid spray.

9. APPLICATIONS

The advantages of the storage battery as a device for storing energy, and especially electrical energy, may be enumerated as follows:

1. It is the lightest and most efficient device for storing energy in portable form, and it is therefore employed wherever energy is to be stored at one time and place for use at another.
2. Its energy is instantly available, no time being required to prepare for delivering the energy thus stored.
3. Its energy may be stored at any convenient rate and delivered at any other rate required.
4. It may be employed as a voltage transformer, storing electrical energy at one voltage and delivering it at another, by series-paralleling groups of cells.
5. The constancy of voltage of a storage battery is another characteristic which is often of pronounced importance.

Advantage is taken of one or more of these characteristics in the various commercial applications of the storage battery. The variety of applications covers such a wide range that only typical examples can be presented in this section. These examples are classified below under applications based primarily on (*a*) portability, (*b*) reliability of supply of electrical energy, and (*c*) economies effected, although in some cases there is an overlap of these classifications.

Applications Based on Portability

AUTOMOTIVE BATTERIES. Undoubtedly the largest single application of storage batteries is for the starting, lighting, and ignition of automobiles and buses. These batteries are charged automatically from a generator driven by the engine while it is running, and they supply power for the lights while the engine is shut down and for ignition and cranking when the engine is started. (For details of system as a whole see Sec. 17, Art. 28.)

The essentials for a battery for this service are ability to discharge at high rates with well-sustained voltage (low internal resistance), minimum effect from low temperature, light weight, small space, and as long battery life as is compatible with the other requirements. Pasted plate batteries are universally used. Three-cell (6-volt) batteries are the recognized standard for automobiles; 6 to 12 cell (12 to 24 volts) are used for buses. In order to reduce the weight and minimize the likelihood of freezing in cold weather, high-specific-gravity (nominal 1.280) electrolyte is used in temperate climates; lower specific

gravity (nominal 1.240) is recommended for tropical climates. The cells are generally assembled in 3-compartment rubber or asphaltic composition cases. The size of battery is usually determined by the cranking requirements, which depend on the size and number of cylinders of the engine and on the design of the cranking motor. The arrangement of the compartments in the container and the location of the battery terminals are subject to the requirements of each model of car. As the life of these batteries is considerably less than the life of the vehicle, leading battery manufacturers have provided the required variety of batteries to make replacement on any current model of automobile.

MOTIVE-POWER BATTERIES are used for the propulsion of electric street trucks, industrial trucks and tractors, and mine and industrial locomotives. These vehicles are used in short-haul or frequent-stop service which is inherent in the industrial applications cited. Long-haul, high-speed service is conceded to the gasoline truck in road work and to the trolley locomotive in mine work, but the economies which accrue with the use of battery-driven vehicles in frequent-stop, and therefore slow-speed, service seem to justify their use to a much greater extent than they have yet attained (1935).

Batteries in this service usually have sufficient capacity to perform a full day's work and are charged when the vehicle is out of service, usually at night. To a limited extent, exhausted batteries are replaced by charged ones at the end of a working shift and the vehicle kept in service for 2 or even 3 shifts a day. The essentials of a battery for this service are a minimum of space and weight, high electrical efficiency, mechanical strength to withstand road shocks, and long life in relation to the cost of purchase and maintenance. Pasted or modified pasted (Exide-Ironclad) plates are used to the total exclusion of Planté plates in this country, although the latter are used to some extent in Germany.

The cells are assembled in individual high-grade hard-rubber jars and these are in turn assembled in a number of hardwood trays arranged to suit the compartment in the vehicle. If the battery consists of more than 44 cells the trays are equipped with porcelain or rubber insulators to insulate the battery from the frame of the vehicle. The size of the battery is determined by the total amount of work to be performed and is usually specified by the truck manufacturer to conform to this work and to the characteristics of the motors he selects to drive the vehicle.

A user of motive-power batteries should exercise the same care in selecting suitable charging equipment as in selecting a suitable vehicle, and in properly charging and maintaining the battery as in inspecting and maintaining the running gear. The battery is likely to be subjected to greater hazards in the charging room than in service unless the charging equipment is properly selected and operated. Fully automatic charging equipment as described under Charging Characteristics (Modified Constant Voltage, page 7-15) is recommended by battery manufacturers.

RAILWAY TRAIN LIGHTING AND AIR CONDITIONING. Storage batteries are used in railway train lighting to supply current for lighting the cars when the train is standing or moving at slow speeds. When the train is moving above a certain speed (about 20 to 25 miles per hour) the lighting current is furnished by a generator mechanically driven from the car axle; this generator also recharges the battery automatically. In order to provide the constant voltage necessary for satisfactory lighting at variable train speeds, the generator voltage is controlled by an automatic voltage regulator. For details of these electrical systems, see Sec. 17, Art. 8.

Axle lighting systems for lighting only are generally designed for 32 volts and include 16 cells of lead-acid battery ranging in capacity from about 200 to 450 amp-hr. The much heavier loads due to air-conditioning equipment call for greater capacities up to 1000 amp-hr for 32-volt systems, and in some cases 64-volt or even 110-volt air-conditioning equipments have been adopted to reduce the size of conductors and facilitate the design of generators and control apparatus.

For the smaller sizes of batteries used for car lighting only, weight and space requirements have been considered to be of secondary importance and many Planté and modified Planté (Manchester) plates have been used and are still being maintained by many railroads although there is a tendency toward the use of flat plate and modified pasted plate (Exide-Ironclad) batteries to facilitate handling during service inspection and maintenance.

For the larger batteries required for air-conditioning, flat plate and modified pasted plates only are being used as space is not available on the car for the heavier types.

The cells are assembled in high-grade rubber jars, in hardwood trays of dimensions suitable for installing in battery boxes of dimensions standardized by the American Railway Association.

RAILWAY CONTROL BATTERIES are used to supply a continuous low-voltage source of energy for a variety of purposes on electric locomotives, on oil or gasoline-electric locomotives and rail cars, and on multiple-unit electric trains. On electric locomotives the batteries usually are operated in multiple with constant voltage motor-

generator sets to supply an unfailing source of energy for control circuits and marker and cab lights. They may also be used in connection with cab signals or automatic train control or with regenerative braking equipment when these features are embodied in the locomotive. On oil or gasoline-electric locomotives and rail cars they are also used for engine starting, air compressor operation, and generator excitation, and on the rail cars also for train lighting. On multiple-unit electric trains the present tendency is to equip each motor car with a battery and constant-voltage motor-generator for the operation of control circuits, marker and emergency car lighting, and for cab signals or automatic train control if employed. These circuits are paralleled throughout the train by means of " train lines."

The battery essentials for this service are similar to those for railway train lighting except where engine starting is required, in which case provision must be made for the high rate momentary discharges required for that purpose, and this feature will probably determine the size of the battery. Weight and space are of more importance than in train lighting owing to the larger amount of equipment required on each unit. For this reason the modified pasted plate (Exide-Ironclad) battery has found most general application. In all these applications the battery is kept practically fully charged at all times, receiving sufficient current from the generator to replace any discharge, usually of short duration, which may occur and also to replace any internal losses.

Applications Based on Reliability

STANDBY BATTERIES are installed to provide a reserve source of energy for use only in an emergency. They must therefore be kept in a state of full charge at all times. This may be done either by " floating " or by " trickle charging " (see Charging Characteristics, page 7–16).

STANDBY BATTERIES IN DIRECT-CURRENT DISTRIBUTION SYSTEMS are connected to the station bus at all times without the interposition of circuit breakers or fuses and thus are available at all times for instant use in any emergency. The batteries are so " floated " as to keep them fully charged at all times.

The middle point of the battery is connected directly to the neutral of the three-wire system: the positive and negative ends are connected to their respective buses through end cell switches by means of which the number of cells in use may be varied at will to meet the operating requirements. These switches permit the use at all times of the right number of cells to maintain the proper floating voltage across the cells in service regardless of variation in bus voltage. They also permit additional cells to be placed in circuit as may be required to control the voltage during emergency discharges. These switches are the only movable piece of apparatus used in connection with the battery at such times; their dependability is therefore of prime importance. They must offer no possibility of interrupting the discharge; they must be able to carry any currents that the battery can deliver for as long a time as the battery can deliver them; and they must be capable of being moved from point to point while carrying these currents without opening the circuit and without destructive arcing. Such switches are available having a continuous carrying capacity up to 10,000 amp and a 6-min carrying capacity of 40,000 amp. Two or more of these switches may be operated in multiple when more carrying capacity is required.

Charging. To charge these batteries after an emergency discharge, they are usually disconnected from the bus and charged from a motor-generator or rotary converter whose voltage is raised as may be required until its maximum voltage is reached, after which a booster generator is connected into the circuit to raise the voltage further to the value required to complete the charge. This booster is usually a three-unit motor-generator set having one motor which takes its power from the station bus and two generators one for each side of the battery, thus permitting independent control of the charge of the two halves to compensate for any inequality in discharge which may have occurred. The charging is done at a time when emergency protection is of minor importance.

Type of Battery. Direct-current distribution systems are largely confined to the more densely populated sections of the larger cities where real estate is very valuable. The space occupied by these batteries is therefore of great importance, and they are required to do very little work during their lifetime. Both these reasons indicate that the flat plate pasted type of battery should be used, and this type has been installed exclusively in recent years although many of the older modified Planté (Manchester) batteries are still in service.

Size of Batteries. As the purpose of these batteries is to prevent a failure of power on the distribution system they must be of sufficient capacity to carry the maximum load for which the station is designed for a long enough period to permit re-establishment of normal source of power. The length of this interval is largely a matter of judgment and varies with different stations, but 20-min protection for the maximum load is usually

considered to be sufficient. When the rate and duration of the discharge are known the size of cells required is easily determined from data similar to those shown in Fig. 3, page 7–13.

The number of cells is determined from the final voltage per cell at the maximum discharge rate and the minimum allowable voltage. The number of end cells must be sufficient to permit enough cells to be cut out of circuit to allow proper floating when the station bus voltage is at its minimum value. In order to keep the end cell switches as small as possible and to reduce the amount of copper required for connections between these switches and the end cells, it is now customary to use groups of 2 end cells each between switch contacts within the floating range and groups of 3 or 4 cells between contacts for those end cells which are employed only to maintain voltage during emergency discharges.

STANDBY BATTERIES FOR EMERGENCY LIGHTING AND POWER are installed to accomplish the same general purposes as the larger batteries used with central station direct-current distribution systems but the installations are so much smaller and the local conditions so different as to require very different treatment.

Emergency Lighting Batteries, in some places required by law or ordinance, are employed in theaters, schools, hospitals, banks, etc., to provide a second source of power which is automatically applied to selected circuits in case of failure of the normal power and thus maintain illumination in locations such as corridors and exits and in special locations such as operating rooms in hospitals, banking rooms and vaults in banks, etc. These batteries must be floated or trickle-charged from the normal source of power, but usually they are not operatively connected to the emergency circuits except in case of a power failure when an automatic switch disconnects the emergency circuits from the normal supply and connects them to the battery. In some cases automatic return to normal connections on return of power, and automatic recharging of the battery, are provided. Where the normal power is alternating current, the battery is trickle-charged by means of a small rectifier and recharged by a rectifier or motor-generator set. Where the normal power is direct current, the battery is usually trickle-charged from the main power supply in a manner similar to that described below under Emergency Power Batteries.

It is impossible within the limits of this section to discuss the details of the electrical connections used in this service as they must differ widely depending on the nature of the normal supply, that is, whether it is single-phase 2 or 3 wire, 2-phase 4 or 5 wire, or 3-phase 3 or 4 wire. These details have been worked out, however, and suitable apparatus is available for any of these conditions. Ordinarily no provision is made for the control of voltage during emergency discharges. A suitable number of cells is used, generally 60 for a nominal 115-volt circuit, and the voltage is allowed to fall as the discharge progresses. If a certain minimum voltage must be maintained a size of cell is selected large enough to meet the requirements.

Type of Battery. As these batteries are not subject to daily work, the pasted plate type has been most generally used, particularly where the installation has been required by law or ordinance, when the lower first cost has been the deciding factor.

Emergency Power Batteries are employed in industrial plants to insure continuity of power for such a period as is necessary to prevent loss of material in process in case of power failure, as in glass mills to clear the mill before the glass cools or in steel mills to right a ladle which is pouring molten metal, etc. The motors to be protected must be of direct-current types, although there is some prospect that the recent development in "inverters" may soon make this protection available for alternating-current motors. In direct-current service the batteries are trickle-charged in two multiple circuits through resistors from the normal supply circuit and recharged in the same manner through other resistors. In event of a power failure a low-voltage relay drops, making contacts which close a contactor connecting the two halves of the battery in series and trips a contactor which disconnects the protected circuit from the incoming power circuit. Two trickle-charge resistors are allowed to remain in circuit as the energy lost in them during the emergency is negligible; if, however, the battery is receiving a high rate charge, the circuit through the charging reistors is automatically opened. If the low-voltage relay will pick up on return of normal power, normal operation will be resumed automatically when the power returns. Connections for recharging the battery are usually made manually.

In some instances due to local conditions it is considered preferable to float and recharge the battery from a separate motor-generator set in order that the battery may be entirely isolated from shop circuit under normal conditions. In such a case a simple throw-over switch to disconnect the protected circuit from the shop supply and connect it to the battery is all that is required for automatic protection.

Type of Battery. Pasted plate batteries are ordinarily used for strictly emergency

protection, but in some cases where the battery is installed primarily for this purpose it may also be employed to perform some work during normal operation. For such applications see "Floated Working Batteries."

FLOATED WORKING BATTERIES. There are many electrical installations which justify the use of a standby battery for emergency protection but which also present opportunities for effecting economies if the battery is allowed to do some work during normal operation. These batteries are permanently connected to the circuits which they protect. During normal operation, an external source of power, usually a motor-generator set or rectifier, is also connected to the circuit and in multiple with the battery. The voltage across the circuit is maintained by the generator at an average value suitable for "floating" the battery. The generator delivers to the circuit the average power requirements, and the battery discharges automatically at times of high load and recharges automatically when the load is below the average. The total amount of the work done by the battery per day during normal operation is usually relatively small, and the battery is kept in a state of nearly full charge and therefore always available for emergency use. In some applications the battery may become appreciably discharged at times during normal operation; when this is contemplated the battery must have enough capacity to perform both services.

Type of Battery. In this service the life of the battery depends largely on the accuracy with which the "floating" voltage is maintained but in some cases on the total amount of work which the battery has to perform. If local conditions are such that the "floating" voltage may be erratic or if a considerable amount of work is required while floating, Planté, modified Planté (Manchester), or modified pasted (Exide-Ironclad) batteries are selected. If the floating voltage can be well maintained and the amount of work is small, flat pasted plate batteries are used.

CONTROL BUS BATTERIES are employed in practically all generating and substations to assure a continuous source of power in any emergency for operation of circuit breakers and other remote-controlled apparatus and also for emergency lights. The control bus is usually completely isolated from station buses so as not to be affected by any disturbance which may occur on them. The battery is permanently connected to this bus at all times, and a small motor-generator set (or rectifier) is provided to carry the continuous load, composed principally of indicating lights, and to keep the battery fully charged. The emergency lights ordinarily receive their energy from the "house" bus, but in case of failure of that bus the emergency light circuit is automatically disconnected from the house bus and connected to the control bus by a throw-over switch. The normal load on the control bus is thus very small, as the indicating lamps consume only about 0.1 amp or less for each oil circuit breaker, therefore a small motor-generator set can be used provided the battery is allowed to supply the high momentary loads required for operation of the circuit breakers. This is readily accomplished by employing a generator whose load-voltage characteristics are such as to present a sufficient droop in voltage at overload as to cause the battery to take practically all the excess demand. The design of circuit-breaker mechanisms has been standardized by N.E.M.A. for the operation of nominal 125-volt mechanisms within the limits of 130 to 90 volts for closing and 140 to 70 volts for tripping (for nominal 250-volt mechanisms these values are doubled). For 125-volt mechanisms 60 cells has been standardized as well as an allowance of 15 volts for copper drop between the battery terminals and the circuit-breaker mechanisms. The battery must therefore be of such a size as to be able to deliver the required loads with the voltage at its terminals not below 105 volts or an average of 1.75 volts per cell for 60 cells. Battery manufacturers publish in their trade catalogs the ratings of their cells to this voltage. The correct floating voltage for 60 cells, using 1.210 specific gravity electrolyte, is 129 volts, which is therefore the normal operating voltage of the control bus. The generator must therefore deliver this voltage when carrying the normal control bus load. In order to charge the battery after an emergency discharge, the generator voltage is raised from time to time as necessary but not above 140 volts, which is the maximum voltage permissible on the circuit-breaker mechanisms and indicating lamps. In order to prevent excessive overload on the generator when heavy demands are thrown on the bus for closing oil circuit breakers, the generator voltage must fall to 105 volts in order to permit the battery to take a large part of the increased load. Specially designed shunt-wound generators are extensively used for this purpose in manually operated stations where there are no prolonged changes in load on the control bus. In unattended stations where holding coils and relays may introduce prolonged changes in load, these changes introduce changes in the voltage of the control bus due to the drooping characteristics of the shunt generator which interfere with the proper floating of the battery unless artificial loads are introduced when these variable loads are disconnected. In order to avoid this complication, generators have been designed and are now available which have a very flat

voltage characteristic at all loads up to a predetermined point and a sharply drooping characteristic at higher loads. Such a generator maintains a favorable floating voltage during normal operation even under moderately variable loads but throws all excess load due to circuit-breaker operation onto the battery.

Size of Battery. In an emergency the battery must be able to carry the indicating lamp load throughout the duration of any loss of normal power. It is now common practice to consider a 12-hr interruption as possible in manually operated stations and 24 hr in non-attended stations. Emergency lighting protection is, however, usually provided for a period of 1 to 3 hr only. In addition to carrying these loads the battery must be able to deliver the current required for circuit-breaker operation *at any time* during the emergency. Trade catalogs give methods of approximating the size of battery required to meet any such desired conditions, but it is usually good practice to submit the conditions to manufacturers for their recommendations.

RAILWAY SIGNALING BATTERIES. Storage batteries are used extensively in the so-called a-c floating battery system for railway automatic signaling. The function of the battery is to supply a continuous source of energy for operation of the signaling devices in case of failure of either the power supply or the rectifier and also to maintain a constant voltage on the signal circuits under varying current demands. At each signal installation a battery is operated in multiple with a rectifier which is adjusted to maintain the proper voltage across the battery and to supply the average load required for operation of the necessary relays and signal lights. The general method of operation is similar to that described above. The rectifiers have a drooping characteristic which throws heavy demands for current on the battery which is automatically restored during times of lighter load. The batteries have sufficient capacity to carry the entire load between periods of inspection, if the normal supply of current in the rectifier should fail.

YACHT AND MOTORSHIP BATTERIES. Floated batteries on Diesel-engine-driven yachts and motorships are used in much the same way as has already been described. Their most important function is to assure a reliable supply of energy at all times for lights and for the operation of important electrically operated auxiliaries such as steering gears, winches, pumps, radio, etc. On the larger yachts they also furnish all necessary auxiliary power and light when the yacht is at anchor and the noise and vibration of a running engine are objectionable. They are also used to relieve the auxiliary engine generator sets of fluctuations in load caused by the simultaneous operation of the auxiliaries and so permit the use of smaller engine-generator sets and effect saving in cost, weight, and operating expenses as well as adding to safety and convenience. Fifty-six cell batteries are ordinarily used for nominal 115-volt installations, and the floating voltage is 125 volts. When necessary to recharge the battery the generator voltage is raised as may be required to not over 135 volts, which is permissible for the auxiliary motor equipment. To protect the lights and some of the more sensitive auxiliaries such as gyroscopic compass and radio equipment, automatic voltage regulators are employed in those circuits. The generators must be able to deliver full load at 112 to 135 volts, and, to assure stability of operation of the generators and battery in multiple, the generator voltage characteristic should under no operating condition show a rise in voltage with increasing load. With this condition met the voltage characteristic will necessarily be drooping at higher loads. This fact together with the drop in speed of the Diesel engine under overloads causes the battery to relieve the engine of excessive loads and permits the use of smaller auxiliary generating sets which operate at better load factors. Pasted or modified pasted plates in 1.280 specific gravity are generally used to economize in space and weight.

DRAWBRIDGE BATTERIES may be taken as typical of applications where protection is required against failure of the normal power supply and where the average load is low but short-time demands are heavy. The normal source of power may be from an internal-combustion engine generator set or from a public utility whose power bills include a maximum demand charge. In either case the battery supplies energy for the operation of the draw and so reduces the demand on the normal source of supply and improves the economy of operation as well as providing emergency protection. When the heavy load demands are fairly frequent, the battery is floated across the generator in the same manner as described under Yacht and Motorship batteries, above, except that the generator may have a more sharply drooping characteristic in order to throw a greater percentage of the load on the battery and thus maintain a more nearly constant load on the generator. If the heavy loads are sufficiently infrequent greater economy may be effected by cycle operation of the battery, that is, by using the battery alone for normal operation and running the generator only when it is necessary to recharge the battery.

TELEPHONE BATTERIES. The telephone companies rank among the largest users of storage batteries as they employ one or more batteries in practically every main and branch exchange. These batteries are always so operated as to leave a sufficient

reserve capacity to provide for continuity of service if the normal source of power should fail. They are, however, used in many cases as the sole source of power during intervals of light demand, this use depending on local operating conditions and the type of equipment employed. The batteries are floated at all times when not carrying the load or being charged.

CYCLED BATTERIES are used in some services where continuity of service is of such importance that it is desirable to have the circuit entirely isolated in order that it may not be affected by any external disturbances.

10. CARE AND OPERATION

The essential features of general care of storage batteries are simple and few, but they are *essential*. They are briefly as follows:

a. Keep the battery *clean* and *dry*.

b. Add water (which has been approved by the manufacturer) from time to time to keep the level of the electrolyte above the top of the separators.

c. Never add electrolyte or acid except to replace a loss known to be due to spillage. The added acid should then be of the same specific gravity as that in adjacent cells.

d. Never allow any "special solutions," powders, jellies, or any metals or other foreign matter to get into the cells.

e. Stop discharge, except in emergency, *before* the voltage falls too low for satisfactory operation.

f. Charge at rates low enough to keep the cell temperature below 45 deg cent (113 deg fahr), and while the cells are gassing never charge at rates higher than allowed by the manufacturer.

NOTE: All manufacturers furnish complete instructions for the care and operation of their different types of batteries. To follow these instructions systematically requires little time and work, which pay for themselves many times in prolonging the life of the battery and in assuring reliable and trouble-free performance.

Avoid excessive overcharge in ampere-hours, even if the rate of charge is low enough to avoid excessive temperature and gassing.

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ALKALINE-TYPE STORAGE BATTERIES

By E. W. Allen and F. Brehme

Alkaline storage batteries are so termed because of the fact that their electrolyte is an alkaline solution (either potassium or sodium hydroxide). Alkaline batteries of present commercial manufacture employ active materials of either the nickel-iron or the nickel-iron-cadmium combination. Such batteries are the Alconum, Alkum, Britannia, Deac, Edison, Jungner, Nife, and Saft, all of foreign manufacture except the Edison. An alkaline battery (Drumm) using the nickel-zinc principle has been in experimental use in Ireland for some years. The nickel-iron-alkaline storage battery was invented by Edison in 1901, this being the only alkaline battery of commercial importance in this country.

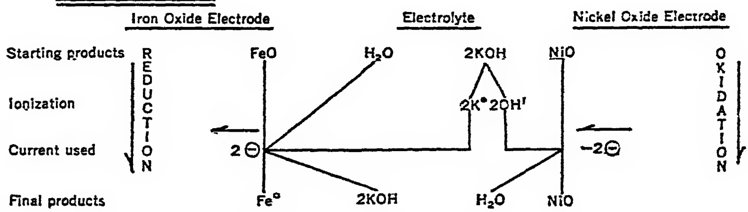
The more important uses to which Edison alkaline storage batteries are applied include: electric street trucks, electric industrial trucks and tractors, storage battery mine and industrial locomotives, railway carlighting and air conditioning, railway signal systems, multiple unit control, marine services, isolated lighting plants, emergency lighting, time clocks, police and fire alarm systems, oil circuit-breaker control, miners' safety cap lamps, airway beacons, and telephone service.

11. THEORY AND ELECTROCHEMICAL REACTIONS

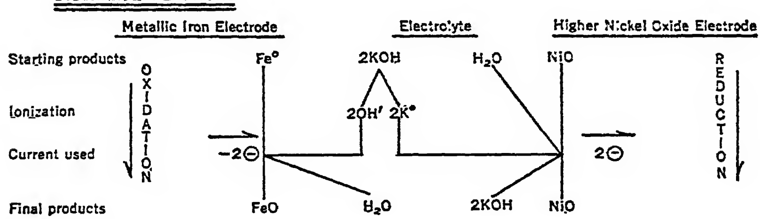
The fundamental principle of the Edison storage battery is the oxidation and reduction of metals in an electrolyte which neither combines with nor dissolves either the metals or their oxides, and which, notwithstanding its decomposition by the action of the battery,

is immediately reformed in equal quantity and is therefore a practically constant element without change of density or conductivity over long periods of time. The active materials being insoluble in the electrolyte, no chemical deterioration can take place. The chemical reactions in charging the Edison storage battery are: (1) the oxidation from a lower to a higher oxide of nickel in the positive plate, and (2) the reduction from ferrous oxide to metallic iron in the negative plate. The oxidation and reduction are performed by the oxygen and hydrogen set free at the respective poles by the electrolytic decomposition of water during charge. The charging of the positive plate is therefore simply a process of increasing the proportion of oxygen to nickel. The discharge of the cell is merely the reverse of this process, the hydrogen reducing the higher oxides of nickel to lower oxides and the oxygen oxidizing the iron to ferrous oxide. The electrolyte is an aqueous solution of potassium and lithium hydroxide and does not at any stage cause a disintegration or solution of the active materials. Its specific gravity does not change appreciably on charge or on discharge. The changes that occur when an Edison nickel-iron-alkaline cell is charged and subsequently discharged are represented by the following equations:

CHARGE REACTIONS



DISCHARGE REACTIONS



12. DESIGN AND CONSTRUCTION

POSITIVE PLATE. The positive plate consists of a nickel-plated steel grid holding nickel-plated steel tubes which contain the positive active material. When inserted in the tubes the active material is in the form of nickel hydrate, but this changes to an oxide of nickel after the formation treatment described later. In order to give the electrolyte free access to the active material the tubes have numerous perforations. To obtain improved conductivity the active material is alternated with layers of pure nickel flake at the time it is stamped into the tubes. The tubes are reinforced by eight seamless steel rings, equi-distantly spaced. The tubes are mounted on the positive grid and forced into permanent position under a pressure of 40 tons.

NEGATIVE PLATE. The negative plate is of similar construction to the positive except that a finely divided oxide of iron is employed as active material and is contained in rectangular perforated steel pockets instead of tubes. The pockets are forced into place under hydraulic pressure of 120 tons. The grid and pockets, like the positive grid and tubes, are of nickel-plated steel.

PLATE ASSEMBLY. The positive and negative plates are assembled into positive and negative plate groups (see Fig. 1). This is accomplished by passing a steel connecting rod through a hole in the top of each plate. Steel washers are employed to obtain proper spacing, the middle spacer being the base of the pole piece. A lock washer and nut are drawn up tight at each end of the connecting rod, binding the plate groups firmly together. All washers, nuts, connecting rods, and terminal posts, like the plates, are of nickel-plated steel.

The plate groups are assembled into complete elements by intermeshing the plates

so they are alternately negative and positive. The negative group always contains one more plate than the positive so that both outside plates are negative. Adjacent positive and negative plates are insulated from each other by the use of vertical hard-rubber pins running their entire length. Thin rubber sheets insulate the outside negatives from the sides of the container. The edges of the plates are insulated from the bottom and from the other sides of the container by hard-rubber frames. These frames also serve the

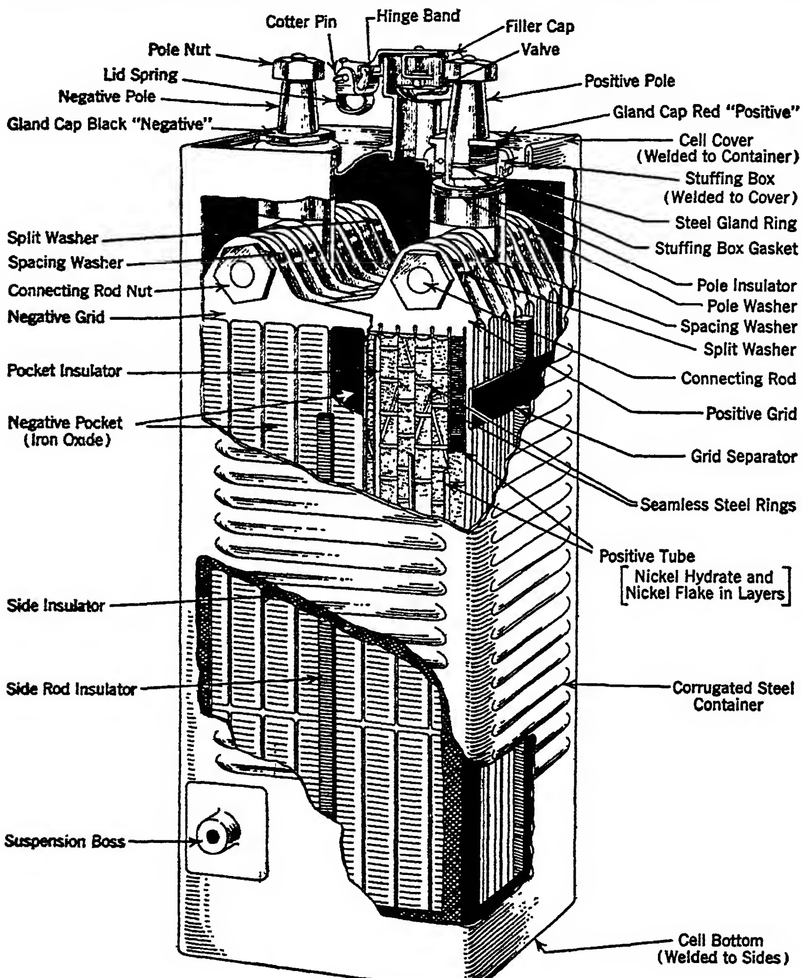


FIG. 1. Plate Assembly of Edison Storage Battery

purpose of separating the plates, holding them in correct alignment, and are so designed as to permit free circulation of the electrolyte.

CONTAINER. Caustic potash does not attack steel, and this fact is taken advantage of in the Edison nickel-iron-alkaline cell in order to obtain the strength, durability, and light weight of steel in the container as well as in the elements. The container, like the elements, is made of nickel-plated steel with all seams welded. For the larger sizes, cell containers are corrugated to increase strength and heat radiation. After the elements are inserted, the cell cover, also of nickel-plated steel, is welded in place. Soft rubber bushings, expanded by steel rings and hard-rubber gland caps, insulate the pole pieces

where they project through the cover of the cell and provide an air- and liquid-tight packing. The gland caps are colored red for the positive and black for the negative.

Projecting from the cell cover is the filler body on which is mounted a hinged filler cap. The filler cap is held positively in either a completely open or completely closed position by means of a flat steel spring. Suspended from the underside of the filler cap is a hard-rubber hemispherical valve which seats by gravity when the cap is closed. This valve, while permitting gas to escape, excludes the external air and reduces evaporation.

ELECTROCHEMICAL FORMATION. The electrolyte is then added and brought to a level of from $1/2$ in. to 3 in. above plate tops depending upon the type of cell. The electrolyte consists of a solution of potassium hydroxide in water. A small quantity of lithium hydroxide is added which increases the capacity and life of the cell. The electrolyte has a normal specific gravity of 1.200 to 1.210 at 60 deg fahr. Throughout the useful life of the cell, owing to spillage, loss through gassing and the introduction of impurities through flushing (i.e., addition of water to make up for losses due to evaporation and electrolysis) the electrolyte gradually weakens and may need renewal one or more times. When the specific gravity reaches a low limit of 1.160 at 60 deg fahr the electrolyte should be renewed. Standard renewal solution obtained from the manufacturer must be used.

When the assembly is completed and the cell filled with electrolyte, it is given several cycles of charge and discharge. This is known as the formation treatment and is for the purpose of stabilizing the electrochemical characteristics of the cell. It is continued until tests show normal operation and an excess over rated capacity. All cells are coated with a heavy insulating paint (Esbalite) after which a protective film of rosin petroleum jelly (Esbaline) is applied to the cell tops.

ANNEALING PROCESS. Before assembly all nickel-plated steel parts are subjected to high temperature in an atmosphere of hydrogen. During this operation the nickel plating is actually annealed to the steel, becoming an integral part of its surface.

BATTERY ASSEMBLY. Nickel-iron-alkaline cells are assembled into batteries in painted hardwood trays to facilitate handling (see Fig. 2). Steel bosses, previously spot welded to the sides of the cell containers, fit into hard-rubber buttons recessed in the tray slats. This type of assembly supports each cell in its proper place and insulates it from the tray and from adjacent cells. Tray assemblies are available in from 1 to 20 cells per tray, depending upon the size of the cells and the requirements of the installation. The usual application uses trays having somewhere between 3 and 8 cells.

The tops of the pole pieces are threaded. Immediately below they are tapered to

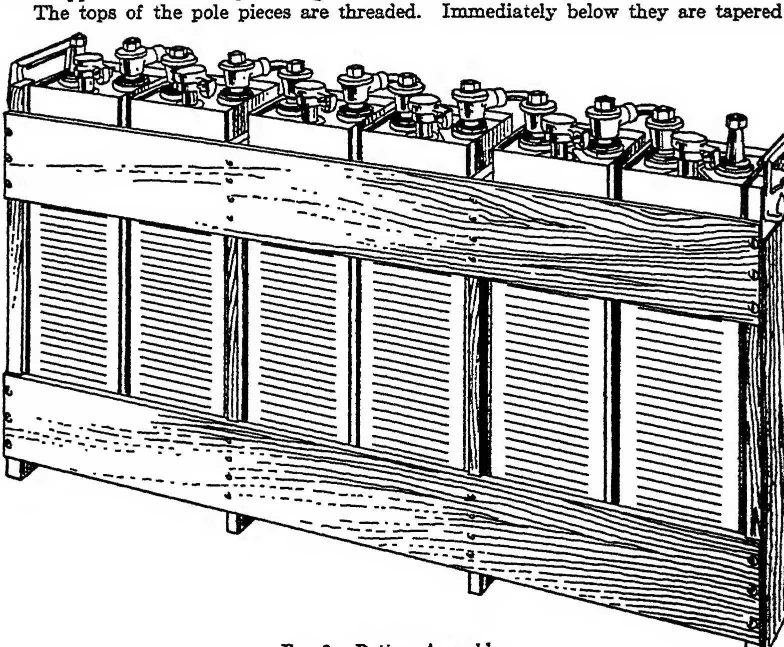


Fig. 2. Battery Assembly

fit the lugs of the intercell and intertray connectors. The connector lugs are steel forgings bored to fit the taper on the pole pieces and are swedged upon heavy copper connecting links. Connectors are secured by tightening down nuts and are removed by means of a small jack after the nut is loosened. All lugs, links, and nuts are nickel plated.

TYPES AND SIZES. The type of cell is determined by the size of the positive tubes and by the number of tubes per plate. The type is indicated by a letter, present manufacture including A, B, C, F, G, L, and N types. A, B, C, and F type positive plates have tubes $1/4$ in. in diameter and a little over 4 in. long. Tubes of this size have a normal discharge rate of $1/4$ amp per tube for 5 hr. The A type plate contains two rows of 15 tubes each, the B type plate contains one row of 15 tubes, the C type plate contains three rows of 15 tubes each, and the F type plate one row of 4 tubes. The N type plate contains one row of 7 tubes 3 in. long and $1/4$ in. in diameter. The G and L type positive plates have tubes $3/16$ in. in diameter and a little over 4 in. long. Tubes of this size have a normal discharge rate of $3/16$ amp per tube for 3 $1/3$ hr. Two rows of 20 tubes each are used in the G type positive plate and one row of 10 tubes in the L type positive plate.

The sizes of the various types of cells are determined by the number of positive plates. This is indicated by a number following the type letter. For example, a cell containing 4 A type positive plates is designated A-4; a cell containing 6 B type positive plates is designated B-6. Type and size designation is stamped on the cover of each cell. In duty such as railway carlighting and air conditioning where regular flushing is not always practicable, containers of extra height or both extra height and width are often used in order to increase the relative quantity of electrolyte. Such cells are designated as high and high wide types, the letters H and HW being added to the designation stamped on the cell covers: as A4H, A4HW, etc.

Table 1. Ratings and Weights

Cell Type	Normal Ratings *			Weights †	
	Amp-hr	Amp	Watt-hr	Standard Type	High Type
N2	11.25	2.25	13.5	1.94	(cell only)
B1, B1H	18.75	3.75	22.5	5.3	6.7
B2, B2H	37.50	7.5	45	6.0	7.2
B4, B4H	75.0	15	90	9.5	10.9
B6, B6H	112.5	22.5	135	13.0	15.2
A4, A4H	150	30	180	16.5	19.3
A5, A5H	187.5	37.5	225	19.6	22.5
A6, A6H	225	45	270	22.4	25.5
A7, A7H	262.5	52.5	315	25.8	28.6
A8, A8H	300	60	360	31.2	35.9
A10, A10H	375	75	450	38.1	43.8
A12, A12H	450	90	540	47.3	53.5
A14, A14H	525	105	630	56.6	60.8
A16, A16H	600	120	720	62.9	68.0
C4	225	45	270	24.3	......
C5	281.25	56.25	337	29.3	......
C6	337.5	67.5	405	34.4	......
C7	393.75	78.75	473	39.8	......
C8	450	90	540	45.5	......
C10	562.5	112.5	675	61.3	......
C12	675	135	810	71.0	......
L20	12.5	3.75	15	1.63	(cell only)
L30	18.75	5.625	22.5	2.4	(cell only)
L40	25	7.50	30	2.99	(cell only)
G4, G4H	100	30	120	12.5	14.5
G6, G6H	150	45	180	16.1	18.5
G7, G7H	175	52.5	210	20.8	23.0
G9, G9H	225	67.5	270	22.6	25.7
G11, G11H	275	82.5	330	29.0	33.4
G14, G14H	350	105	420	36.9	42.4
G18, G18H	450	135	540	49.5	56.0
G22, G28	550	165	660	62.9	68.0

* Ratings based on 5-hr discharge for A, B, C, and N types, and on 3 1/3-hr discharge for G and L types; average voltage 1.20.

† Pounds per cell, completely assembled, including trays, connectors, electrolyte, etc.

NOTE.—The letter "H" indicates high-type cells; these cells have the same characteristics as the standard-type cells but are built higher so as to hold more electrolyte. They are recommended for use where longer flushing intervals are desirable. Some of the A type sizes are available in an HW (high wide) container for railway carlighting service exclusively.

Table 2. Tray Dimensions (Standard Trays Without Skids)

Length and Width *

Length	N2†	L20	L30	L40	B1 B1H B2 B2H	B4 B4H	B6 B6H	G4 G4H
1 cell tray	2 15/16	2 3/8	2 13/16	3 1/2	3 5/32	4 11/32	5 17/32	4
2 " "	4 5/8	3 3/4	4 5/8	5 3/4	5	7 3/8	9 3/4	6 1/2
3 " "	6 5/16	5 1/8	6 7/16	8	6 27/32	10 13/32	13 31/32	9
4 " "	8	6 1/2	8 1/4	10 1/4	8 11/16	13 7/16	18 9/16	12
5 " "	10 1/16	7 7/8	10 1/16	12 1/2	10 29/32	16 27/32	22 25/32	14 1/2
6 " "	11 3/4	9 5/8	12 1/4	15 1/8	12 3/4	19 7/8	27	17
7 " "	13 3/16	11	14 1/16	17 3/8	14 19/32	22 29/32	31 19/32	20
8 " "	15 1/2	12 3/8	15 7/8	19 5/8	16 7/16	25 15/16	35 13/16	22 1/2
9 " "	17 3/16	13 3/4	17 11/16	21 7/8	18 21/32	29 11/32	40 1/32	25
10 " "	19 1/4	15 1/8	19 1/2	24 1/8	20 1/2	32 3/8	44 5/8	28
11 " "	20 15/16	16 7/8	21 11/16	26 3/4	22 11/32	35 13/32	....	30 1/2
12 " "	22 5/8	18 1/4	23 1/2	29	24 3/16	38 7/16	....	33

Widths

	N2†	L20	L30	L40	B1 B1H B2 B2H	B4 B4H	B6 B6H	G4 G4H
Width (with standard button)	3 1/2	3 5/8	3 5/8	4	6 1/16	6 1/16	6 1/16	6 1/8
Width (with extended button)	Extended button trays not available							6 21/64

Length	A4 A4H C4	G6 G6H	A5 A5H C5	G7 G7H	A6 A6H C6	G9 G9H	A7 A7H C7	A8 A8H C8 G11 G11H
1 cell tray	4 17/32	4 3/4	5 3/32	5 1/4	5 23/32	6	6 9/32	7 1/8
2 " "	7 9/16	8	8 11/16	9	9 15/16	10 1/2	11 1/16	12 3/4
3 " "	10 19/32	11 1/4	12 9/32	12 3/4	14 5/32	15	15 27/32	18 7/8
4 " "	14 1/8	15	16 3/8	17	18 7/8	20	21 1/8	24 1/2
5 " "	17 5/32	18 1/4	19 31/32	20 3/4	23 3/32	24 1/2	25 29/32	30 5/8
6 " "	20 3/16	21 1/2	23 9/16	24 1/2	27 5/16	29	30 11/16	36 1/4
7 " "	23 23/32	25 1/4	27 21/32	28 3/4	32 1/32	34	35 31/32	42 3/8
8 " "	26 3/4	28 1/2	31 1/4	32 1/2	36 1/4	38 1/2	40 3/4	48
9 " "	29 25/32	31 3/4	34 27/32	36 1/4	40 15/16	43	45 17/32	54 1/8
10 " "	33 5/16	35 1/2	38 15/16	40 1/2	45 3/16	48	50 13/16	....
11 " "	36 11/32	38 3/4	42 17/32	44 1/4	49 13/32	....	....	....
12 " "	39 3/8	42	....	48	....	....	....	....

Widths

	A4 A4H C4	G6 G6H	A5 A5H C5	G7 G7H	A6 A6H C6	G9 G9H	A7 A7H C7	A8 A8H C8 G11 G11H
Width (with standard button)	6 1/8	6 1/8	6 1/8	6 1/8	6 1/8	6 1/8	6 1/8	6 1/16
Width (with extended button)	6 21/64	6 21/64	6 21/64	6 21/64	6 21/64	6 21/64	6 21/64	6 17/64

Length	A10 A10H C10 G14 G14H	A12 A12H C12	G18 G18H	A14 A14H	A16 A16H G22 G22H
1 cell tray	7 3/8	7 3/8	7 3/8	7 7/16	7 7/16
2 " "	13 1/4	13 1/4	13 1/4	13 3/8	13 3/8
3 " "	19 5/8	19 5/8	19 5/8	19 13/16	19 13/16
4 " "	25 1/2	25 1/2	25 1/2	25 3/4	25 3/4
5 " "	31 7/8	31 7/8	31 7/8	32 3/16	32 3/16
6 " "	37 3/4	37 3/4	....	38 1/8	38 1/8
7 " "	44 1/8	....	....	....	....
8 " "	....	....	....	....	....
9 " "	....	....	....	....	....
10 " "	....	....	....	....	....

Table 2—*Continued*

WIDTHS

	A10 A10H C10 G14 G14H	A12 A12H C12	G18 G18H	A14 A14H	A16 A16H G22 G22H
Width (with standard button)	7 9/16	8 13/16	9 7/16	9 59/64	11 1/32
Width (with extended button)	7 49/64	9 1/64	9 41/64	10 1/8	11 15/64

HEIGHT *	Filler Cap Open	Filler Cap Closed
N2	8 7/16	7 7/16
L20, L30, L40 Regular Lid Valve	9 3/8	8 3/8
L20 (made with non-spill valve)	......	7 5/16
L30, L40 (non-spill valve)	......	8 1/4
B2, B4	10 13/16	9 3/8
B1H, B2H, B4H	12 9/16	11 1/8
B6	11 1/32	9 15/32
B6H	12 27/32	11 1/4
A4, A5, A7 G4, G6, G7, G9	15 29/32	14 5/16
A4H, A5H G4H, G6H, G7H, G9H	18 19/32	17
A6	15 31/32	14 3/8
A6H, A7H	18 21/32	17 1/16
A8, A10 G11, G14	16 13/32	14 3/4
A8H, A10H G11H, G14H	19 3/32	17 7/16
A12 G18	16 3/4	15 3/8
A12H G18H	19 15/32	18 1/16
A14, A16, G22	16 3/4	15 15/16
A14H, A16H, G22H	19 15/16	18 1/2
C4	21 23/32	19 23/32
C5, C6	21 23/32	20 1/16
C7, C8	21 23/32	20 1/2
C10, C12	21 23/32	20 31/32

* When figuring battery dimensions, it is good practice to add 1 in. to filler cap open height so as to insure sufficient clearance. Also for easy fitting, add 1/8 in. to length and width dimensions of each tray.

† For N2 cells, dual trays are supplied up to 20 cells; width is 7 in. and length is same as for single trays.

13. ELECTRICAL CHARACTERISTICS

Charging

Charging requires direct current. When only alternating current is available, a suitable motor-generator set, mercury-arc rectifier, or other current-rectifying device must be used. Specific gravity readings are no indication of the state of charge and are therefore not necessary except as an indication of when solution should be renewed. Although Edison batteries may be successfully charged at various rates it is first necessary to establish a standard or normal rate for reference purposes. The normal rate of charge has been chosen by the manufacturer in such a way that for an output of approximately the rated number of ampere-hours, when discharging at normal rate, the efficiency is higher than for a charge rate which is considerably higher or lower than the normal. The normal charge rate for the different types and sizes of Edison batteries is the same as the amperes shown in the tabulation given under the previous heading, Ratings and Weights.

CONSTANT CURRENT CHARGING. In constant current charging, the charge rate is held fairly constant throughout the charge, and this rate should approximate the normal rate. The battery voltage will rise from approximately 1.50 to 1.85 volts per cell during the charge, and adjustments will need to be made from time to time to maintain the current rate. A voltage of at least 1.85 volts per cell must be available at battery terminals in order to complete the charge at normal rate. Voltage regulation can be obtained by use of a variable resistance in series with the battery or by variation of the field rheostat of the charging generator if an individual motor-generator set is used. The length of time required to charge will depend upon the extent to which the battery was previously discharged, the battery being considered fully charged when the terminal voltage ceases to rise for a period of 30 minutes during charge with constant current flowing.

MODIFIED CONSTANT POTENTIAL CHARGING. Modified constant potential is taper current charging, the current during the first part of charge is higher than the

normal rate; during charge the rate decreases until at the end of charge the rate is less than normal. With this method a fixed and permanent resistance is inserted in series with the battery and no adjustments are made during charge. A charging line voltage of at least 1.84 volts per cell must be available and maintained throughout the charge, but higher voltages may be used. The higher the voltage the more closely will the taper of the charging rate approach the straight constant current method as a limit. When using the proper fixed resistance, the following charge rates will obtain for various charging voltages:

Line volts per cell	1.84	1.90	1.95	2.00	2.10	2.20
Initial rate (% normal)	165	155	147	140	128	124
Final rate (% normal)	65	70	74	78	84	86

In each case, the result will be to average normal rate for a complete charge. The length of charge required will be dependent upon the state of charge of the battery when it was placed on charge. The object is to put back into the battery 25 per cent more ampere-hours than have been taken out. The state of charge can be determined by the use of an ampere-hour meter (or by a special device made by the battery manufacturer known as an Edison charge test fork).

The following formula shows how to determine the proper fixed series resistance mentioned above: $V - (1.69 \times N)/A = R$, where V is the voltage of the charging line, N is the number of cells in series in the battery, A is the normal ampere rate of the battery, and R is the ohms fixed resistance required. Example: with a 45-volt charging line, what resistance is required to charge a 24-cell A8 Edison battery? Answer: 45 volts − (1.69 × 24 cells)/60 amperes = 0.074 ohm. It is to be noted that the series resistance thus calculated is the total resistance required between the line and the battery; the charging leads, switches, etc., will form part of the resistance, the balance to be added. The current-carrying capacity of the resistance should be sufficient to take care of the initial current rate which will flow into the battery at the beginning of charge. See table above.

BOOST CHARGING. Although not recommended as a regular practice occasional emergencies will require that the batteries be charged faster than ordinarily, which necessitates higher rates. Permissible ampere rates are: 1 hr at two times normal rate, or 1/2 hr at three times normal, or 1/4 hr at four times normal, or 5 min at five times normal. Frothing at the filler cap, or electrolyte temperatures in excess of 115 deg fahr, are indications that boosting has been carried too far and it should be discontinued at once.

LOW RATE CHARGING. Where discharge requirements are such that a low constant rate is used, or where intermittent or variable rates are such as to average a low value over a given period of time, then charge rates may be less than normal. In these cases, the charge rate may be as little as 1.2 times the average discharge rate. It must be remembered, however, that it will still be necessary to put back into the battery 25 per cent more ampere-hours than have been used on discharge. The length of time for a complete charge will thus be longer because the current is put in more slowly.

TRICKLE CHARGING. Edison batteries may be trickle-charged when used in such applications where trickle charging is ordinarily employed. The voltage required will normally be between 1.50 and 1.55 per cell depending upon the temperature, age of the battery, the rate used, and other factors. The proper trickling rate for any given installation can be approximated by the following formula: $(C \times 0.16) + (D \times 1.10)/(24 - H) = A$, where C is the rated ampere-hour capacity of the battery, D is the ampere-

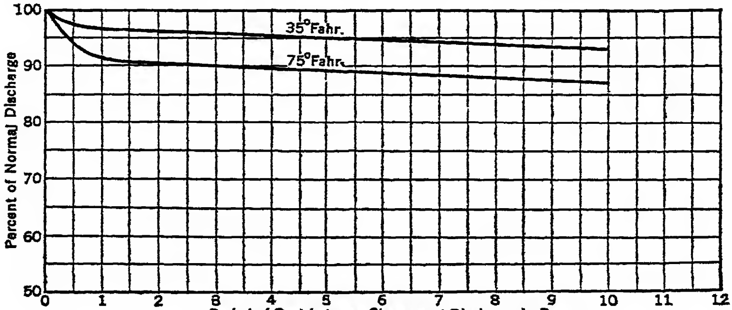


FIG. 3. Loss of Charge when Idle

hours used per day, H is the total hours of discharge per day, and A is the trickle charge rate in amperes.

RETENTION OF CHARGE. The curve of Fig. 3 shows the loss of charge when standing idle following a complete charge.

Discharge Characteristics

At the normal rating, Edison cells have an average voltage of 1.20 volts. However, like all other storage batteries, the voltage will be less when discharged at high ampere rates, and when discharged at low rates the voltage characteristics are better than normal. The curves of Fig. 4 show the discharge voltage characteristics of an Edison cell in good condition when discharged continuously at various rates. For instance, suppose that it is desired to know what voltage to expect from a 30-cell type A8 Edison battery when discharging at 120 amp and after it had been thus discharging for 1 1/2 hr. The ampere-

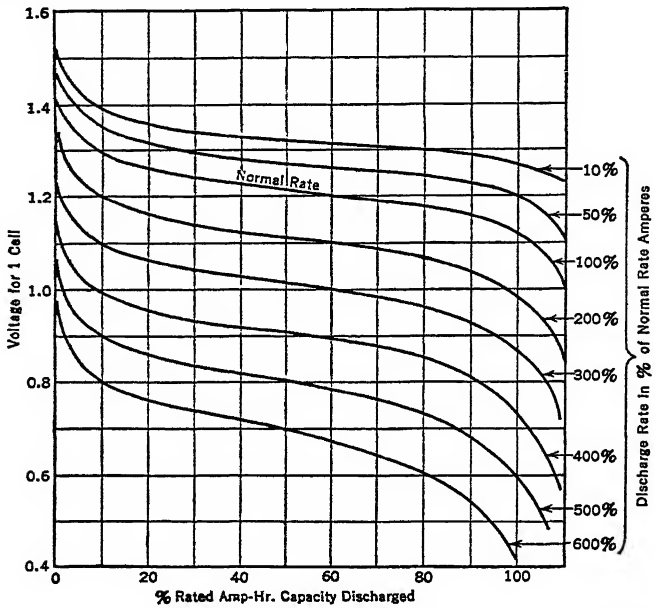


FIG. 4. Edison Alkaline Storage Battery Discharge Voltage at Battery Terminals (Note: Allowance has been made for losses in connectors and jumpers)

hours taken out would be ($120 \times 1\ 1/2$) 180, and since the rated capacity (see Table 1) of an A8 type cell is 300, then the cell would be in a condition of 60 per cent discharged. The 120-amp discharge rate is twice the normal (60 amp) rating of an A8; therefore by referring to the 200 per cent normal discharge curve at the point of being 60 per cent discharged the voltage is found to be 1.10 per cell; therefore a 30-cell type A8 Edison battery in good condition would deliver 33 volts at the end of 1 1/2 hr of discharge at a 120-amp rate.

CAPACITY. The capacity of a nickel-iron-alkaline cell depends upon the number and type of its positive plates. For ready reference the ampere-hour capacities are shown in Table 1. Normal watthour capacity is the normal ampere-hour capacity multiplied by the average voltage. The rated capacity is based on a 5-hr discharge to 1 volt per cell for A, B, C, F, and N types, and on a 3 1/3-hr discharge to 1 volt per cell for G and L types. These values are on the basis of discharge at normal rate. It will be noted from the above discharge curves that there is no appreciable difference in ampere-hour capacity at several times the normal discharge rate because the characteristics of the cell do not necessitate a limit to the final voltage. The cell may be discharged to zero voltage without injury.

During the first 125 duty cycles the capacity increases gradually above its initial capacity. This increase may be as great as 15 per cent above the rated capacity. After this the capacity gradually decreases. When the specific gravity of the electrolyte drops

to 1.160 at 60 deg fahr it is renewed, immediately following which another increase in capacity occurs. This is usually the point of maximum capacity during the life of the cell. In most services the useful life is considered at an end when the ampere-hour capacity drops to less than 80 per cent of rated. The Edison alkaline cell maintains its rated capacity for the greater part of its normal life.

EFFICIENCY. Although under laboratory conditions, ampere-hour efficiencies ranging up to 95 per cent can be obtained, the practical operating ampere-hour efficiency of an Edison alkaline battery is approximately 80 per cent, thus requiring an input of about 25 per cent more than has been taken out on discharge. The laboratory voltage efficiency ranges up to 80 per cent, but this has no meaning as far as practical operation is concerned because the voltage efficiency of a battery installation will depend upon the methods of charging used, the charging line losses, the efficiency of the charging current source, etc. For any given installation, the efficiency of battery operation can be computed by determining the watthour input required at the service meter as against the watthours obtained from the battery on discharge. However, in those services where alkaline batteries are applied, the cost of charging batteries is usually a relatively small item in the total cost

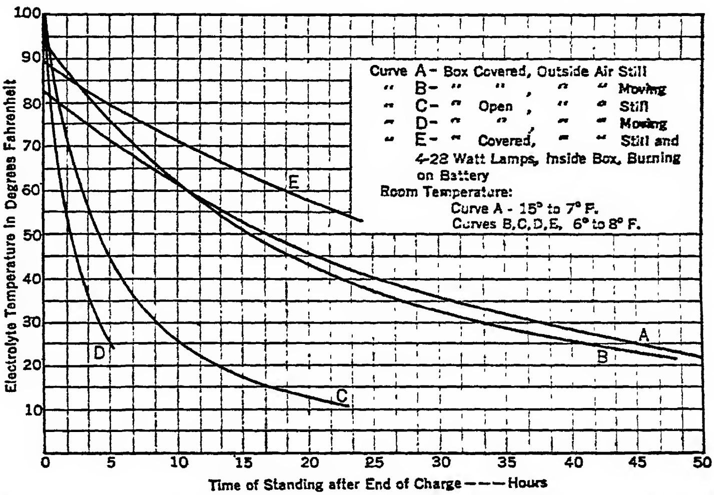


FIG. 5. Rate of Cooling of Electrolyte of Standard Battery of 40 Type A-6 Cells, Housed in a Regular Battery Box. Battery was charged at normal rate (45 amperes) with the air temperature at about 65° F. and then allowed to stand idle to cool under various conditions as noted above. Test made by Electrical Testing Laboratories

of battery operation, and a comparatively wide range in the efficiency of batteries makes but a slight difference in total operating cost.

TEMPERATURES. High temperatures are to be avoided as all batteries are damaged by high electrolyte temperatures. It is recommended that Edison alkaline batteries be operated at temperatures under 115 deg fahr. To assist in holding down electrolyte temperatures of Edison batteries used in heavy duty service, such as motive power, the battery compartments should have provisions for ventilation in warm weather. Evenly spaced slots or holes should be made at the top and in the bottom of the battery box; top ventilation is usually placed on the sides of the box as close to the top as possible. Depending on the severity of the work and on the climate, a certain range of ventilation has been found by experience to be satisfactory, and may be determined by the following formula: $F \times N \times C = A$, where F is a factor of 0.01 to 0.02 depending on severity of condition, N is the number of cells in the battery, C is the rated ampere-hour capacity of the battery, and A is the area in square inches of ventilation required. This amount of square inch ventilation should be applied to the top, and a like amount to the bottom of the battery compartment. Adequate ventilation will result in both better operating performance and longer battery life.

Low temperatures will not harm an Edison alkaline cell. At an electrolyte temperature of approximately -20 deg fahr the electrolyte will have become slushily crystallized but it will not freeze hard and cause injury. The capacity of the cell decreases as the

electrolyte temperature drops. The extent of the decrease varies considerably with the rate of discharge, being relatively small at low rates and large at high rates. At normal rate a reduction of electrolyte temperature from 115 to 50 deg fahr will reduce the capacity 10 per cent. Below 50 deg fahr the reduction in capacity is important. The electrolyte temperature of a working cell is always higher than the atmospheric temperature due to the heat generated by the passage of current. The amount of heat varies with the rate of charge and discharge, being relatively small at low rates and larger at high rates. In practice, electrolyte temperatures in a cell discharging at normal rate do not generally fall below 60 deg fahr. Under cold weather conditions battery compartments should be closed, and under extreme conditions it is sometimes good practice to insulate the compartment. Even though air temperatures may be very low, it takes a long time for electrolyte temperatures to drop to that of the outside air if the battery compartment is closed, as the curves of Fig. 5 show.

14. LIFE

The life of the cell is from 7 to 20 years, depending upon the service. The years of life depend upon the average percentage of a complete cycle occurring between charging and the number of these occurring per year as well as upon other factors including the kind of duty in which the cell is operated and the care it receives.

ACCIDENTAL CONDITIONS. Accidental conditions arising from use or from the care given the battery are generally without injury to the nickel-iron-alkaline cell. Short circuiting, accidental charging in the reverse direction, accidental overcharging, or standing idle for an indefinite period in a discharged condition does not cause permanent injury. Vibration and jolting are not generally a cause of injury. Permanent injury will result from prolonged overheating above 115 deg fahr and prolonged operation with electrolyte level below plate tops.

MAINTENANCE. The principal points in caring for Edison alkaline batteries are as follows:

1. Maintain solution at the proper level with distilled water in each cell as recommended in the manufacturer's instruction booklet 850X (obtainable from Thomas A. Edison, Inc., West Orange, N. J.).
2. Avoid electrolyte temperatures exceeding 115 deg fahr.
3. Keep clean and dry the cells, trays, and the surface on which trays are placed.

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SECTION 8

DIRECT-CURRENT MACHINES

BY

WALTER I. SLICHTER

DIRECT-CURRENT GENERATORS

ART.		PAGE
1.	Applications	02
2.	Definitions	02
3.	Ratings	03
4.	Parts of a Direct-current Machine	03
5.	Output Coefficient	04
6.	Electromotive Force Induced in Armature	05
7.	Armature Windings	05
8.	Flux per Pole	07
9.	Magnetic Circuit	08
10.	Shunt Field	10
11.	Armature Resistance	11
12.	Design of Commutator and Brushes	12
13.	Armature Reaction	13
14.	Commutation	16
15.	Regulation	17
16.	Losses	18
17.	Heating	19
18.	Special Machines	21
19.	Testing of D-c Machines	23
20.	Specifications for D-c Generators	25
21.	Installation	26
22.	Weight and Speed	27
23.	Examples of Design and Performance	27
24.	Electrostatic Generators	28

DIRECT-CURRENT MOTORS

ART.		PAGE
25.	Classification	30
26.	Ratings	31
27.	Principles	32
28.	Shunt Motor	33
29.	Series Motor	34
30.	Starting of D-c Motors	36
31.	Speed Control	37
32.	Use of Commutating Poles in Variable-speed Motors	38
33.	Installation and Erection	38
34.	Operation	39
35.	Specifications	39
36.	Performance, Weight, and Speed	41

DIRECT-CURRENT MACHINES

By Walter I. Slichter

DIRECT-CURRENT GENERATORS

1. APPLICATIONS

Direct-current, or, as it is sometimes called, continuous-current apparatus, was in very general use before the alternating-current type was introduced, and wherever the distance to which energy is to be transmitted is not a factor, there is no doubt that the d-c system is to be preferred. However, if energy is to be transmitted over long distances the a-c system with its transformer has unequivocal advantages. The result of these two conditions is that a compromise is adopted. The majority of the generating stations provide alternating currents and many of the motors use direct currents provided by rotary converters. Thus the d-c generators are becoming of less relative importance while the d-c motors maintain a very important position.

Any d-c generator may be used as a motor, and the principles of action and design are the same. The following paragraphs apply particularly to generators, and the special features of d-c motors are discussed under Direct-current Motors.

2. DEFINITIONS

The fundamental principle involved in the construction and operation of any type of dynamo-electric machine is the production of an electromotive force in one or more conductors by the relative motion of these conductors and a magnetic field. Such a machine may, as a rule, be used either as a generator or motor.

Shunt Machine. A shunt machine, either generator or motor, is one in which the entire field excitation is derived from a circuit of many turns and high resistance connected in "shunt" or multiple with the armature circuit. The characteristic of a shunt machine is poor regulation; that is, the voltage of a shunt generator decreases as the load increases. This is so marked that some shunt generators may be short-circuited, their terminal voltage dropping to zero, without resultant harm.

Series Machine. A series machine, generator or motor, is one in which the entire armature current flows through the field winding. Series generators are usually built to supply a constant current to an external circuit irrespective of the effective resistance of that circuit.

Separately Excited Machine. This type of machine is sometimes used in large stations where the exciting current is readily obtained from separately excited bus-bars and a voltage regulator is used.

Compound-wound Machine. This type of machine has on each field pole, in addition to its shunt winding, a few turns of thick wire which carry the load current and are known as the series winding. This winding causes the excitation to increase as the load increases and tends to keep the terminal voltage constant or even to increase it. If the field windings are proportioned to cause the voltage at full load to be higher than the voltage at no load the machine is said to be "over-compounded." A "flat-compounded" machine has the same voltage at full load and at no load; an "under-compounded" machine has a lower voltage at full load than at no load.

Short- and Long-shunt Connections. A compound-wound machine may be connected short shunt as in Fig. 1 or long shunt as in Fig. 2. The choice is merely a matter of convenience of station wiring.

Commutating Pole or Interpole Machines. These machines have small auxiliary poles alternately placed with respect to the main poles and excited by a few turns in series with the load. The effect of these poles is to improve the operation of the machine in the matter of commutation; see below.

Bipolar Machines. Small d-c machines are usually of the bipolar or two-pole type with a more or less inclosed frame of cylindrical shape.

Multipolar Machines. Large d-c machines are of the multipolar type, that is, have a large number of radial pole pieces.

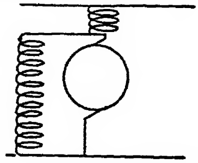

FIG. 1. Short-shunt Connection

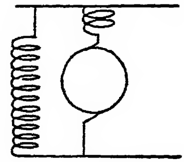

FIG. 2. Long-shunt Connection

Belt-driven and Direct-connected Types. Direct-current generators may be of either the "belt-driven" or the "direct-connected" type, as determined by the method of connecting to the driving unit. Belt-driven generators are characterised by a higher angular velocity than direct-connected.

3. RATINGS

The rating of a d-c machine is the output, together with such other characteristics as speed and voltage, assigned to it by the manufacturer. The continuous rating defines the load which can be carried for an unlimited period without causing any of the limitations, such as temperature, established by the Standardization Rules of the A.I.E.E., to be exceeded. See article on Standards.

VOLTAGE. The standard voltages for which d-c machines are built are:

80 volts for use on shipboard.

110–125 volts for lighting purposes and incidental small power motors, fan motors, and cooking utensils.

220 volts for three-wire systems with lighting and power combined.

500–600 volts for power alone and particularly for railway service.

1200–2400 volts for special railway service and heavy traction.

2000–6000 volts for series arc-light circuits fed by series machines.

SPEED. Machines are classified in commercial practice as high-speed, moderate-speed, and slow-speed. Table 1 shows roughly the suitable speeds and numbers of poles.

Table 1

Rating kw	High-speed		Moderate-speed		Slow-speed	
	Poles	rpm	Poles	rpm	Poles	rpm
0– 10	2	1500	4	1000	..	...
10– 50	4	1300	4	800	..	...
50–100	4	1100	4–6	700	6	300
100–300	6	850	6	500	8	200
300–500	6	750	8	400	10	150
500–up	6	650	8	300	12–up	100

4. PARTS OF A DIRECT-CURRENT MACHINE

The principal parts of a d-c machine are: the yoke, pole pieces, field coils, armature core, armature winding, commutator brushes, shaft, bearings, and mechanical frame or housing.

The yoke serves the double purpose of a mechanical support for the pole pieces or a framework for the whole machine and a path for the magnetic flux. It is always made of steel, sometimes of cast steel and more recently of forged steel bent to cylindrical form and welded to form a homogeneous magnetic ring.

The pole pieces, of forged or of assembled sheet steel, are usually bolted to the yoke and are formed with flaring projections at their inner ends to spread the flux over a considerable area of the armature and also to serve as a mechanical support to the field coils.

The pole pieces may be of solid steel or preferably built up of sheets of steel 0.025 in. thick and bolted together by insulated bolts passing through holes larger than the shanks of the bolts.

The pole pieces are usually wider at the face next to the air gap than in the main body in order to spread the flux over a greater area where it crosses the air gap to the armature. This enlarged section is called the pole shoe and may be integral with the

pole piece or a separate part bolted to it. The arc subtended by the pole shoe is usually from 60 to 70 per cent of the arc from the center of one pole to the center of the next pole.

The field coils consist of turns of copper wire wound upon a form which fits upon the pole piece. A current in these turns is the cause of the magnetomotive force which produces a flux in the magnetic circuit consisting of the yoke, pole pieces, and armature.

There may be two coils: a "shunt" of many turns of fine wire connected across the terminals of the machine and carrying current at all times; a "series" of a few turns of thicker wire connected in series with the armature and the load, and intended to strengthen the field as the load current increases.

The armature core is a hollow cylinder made up of many pieces of sheet steel, punched to shape and usually having slots punched around its outer periphery. The steel of the armature serves to carry the flux from one pole to the next, and the slots hold the various moving turns of wire known as the armature winding. The motion of this winding in the magnetic field is the cause of the generated emf, and a current in this winding is the cause of the force on the conductors which makes the armature move.

A general idea of the proportioning of the dimensions of an armature coil may be obtained from Table 2 showing usual practice.

Table 2

D = Arm. Diam., in.	Air Gap, in.	No. of Slots	Depth of Slots, in.	D = Arm. Diam., in.	Air Gap, in.	No. of Slots	Depth of Slots, in.
5	0.08	25– 40	0.6	30	0.15	80–150	1.50
10	0.10	30– 60	0.75	50	0.19	100–200	1.75
15	0.12	40– 80	1.00	100	0.25	150–300	2.50
20	0.125	60–100	1.25	150	0.35	200–400	3.25

The armature winding is an ingenious and special arrangement of the wires which gathers together in one path the conductors giving a voltage in the same direction, and thus multiplies the voltage of one conductor. There are many schemes of armature windings; those most common in d-c machines are described on pp. 8–05 to 8–08.

The commutator is peculiar to d-c machines. It causes a continual changing of the relative connection of the conductors inside the armature and the external circuit, so that, in spite of the fact that the voltage of each internal conductor is continually changing in value and direction, the external circuit receives a voltage of practically constant value and constant direction or polarity. The commutator consists of a large number of copper bars of a segmental cross-section, each insulated from its neighbors by mica and formed into a hollow cylinder.

The brushes are small slabs of carbon pressing and rubbing against the outer perimeter of the commutator, held in a definite place by "brush holders." Their function is to make a continuous electrical connection between the moving conductors of the armature and the stationary conductors of the external circuit, and to carry the current out from and back to the armature.

5. OUTPUT COEFFICIENT

The relation between the diameter and length of the armature and the output of any d-c generator (or the input of a d-c motor) are given by the following formula (see Principles Underlying Design of Electrical Machinery, by W. I. Slichter, Wiley):

$$D^2 L = \frac{610\, P\, 10^9}{\rho \sigma B N}$$

in which D = outside diameter of armature, inches.

L = length of armature, inches.

P = power output in kilowatts in case of generator.

P = power input in kilowatts in case of motor.

ρ = ratio $\dfrac{\text{pole arc}}{\text{pole pitch}}$, usually about 0.7.

Pole arc is the arc in inches covered by the pole face at the air gap.

Pole pitch is the arc from center to center of adjacent poles.

σ = ampere conductors per inch of armature periphery (see Table 3).

B = average flux density in the air-gap, lines per square inch.

N = revolution per minute at rating.

Ampere conductors per inch of periphery or specific loading, σ, is equal to the total conductors around the armature multiplied by the current in each conductor at rated load and divided by the perimeter of the armature.

The usual values for σ are given in Table 3.

Magnetic density in air gap, B, has values from 40,000 lines per square inch in small machines, 55,000 in medium, to 70,000 in large machines. Generally the density in the air gap is taken the same as the density at the pole face.

Table 3

Kw rating	For Continuous Rating	For Intermittent Rating
0–150	250–400	700
150–400	400–600	1000
400 and greater	600–900	1200

6. ELECTROMOTIVE FORCE INDUCED IN ARMATURE

Let Z = total number of armature conductors.
p = number of field poles.
m = number of parallel conducting paths between the positive and negative brush sets; that is Z/m is the number of armature conductors in series between positive and negative brush sets. (See below under Armature Windings.)
ϕ = total useful magnetic flux per pole.
N = revolutions of armature per minute.
f = frequency in cycles per second.

If a coil of wire be revolved about an axis in a magnetic field, as shown in Fig. 3, each length of conductor, or side of the coil, will pass entirely around the armature in $60/N$ seconds. The time taken for a conductor to pass through the magnetic field under each pole, or from a to b, is $\frac{60}{Np}$. Hence the average value of the emf induced in each armature conductor as it passes under each pole is $\frac{Np\phi}{60 \times 10^8}$ volts, since a cutting of 10^8 lines per second gives 1 volt. In an actual machine the conductors are uniformly distributed around the surface of the armature, therefore this is also the average voltage per conductor in each of the conductors between a and b at any instant. Since there are Z/m conductors in series between the positive and negative brush sets, the average value of the total emf between the brushes, when a suitable commutator (see below) is provided, is

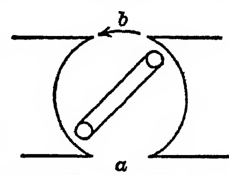

FIG. 3. Elementary Generator

$$E = \frac{p\phi ZN}{60 \times 10^8 m} \text{ volts}$$

The emf in each conductor alternates (i.e., passes through a complete cycle of positive and negative values) with a frequency of $f = pN/120$ cycles per second. Hence the formula for the average emf between brushes is

$$E = \frac{2f\phi Z}{10^8 m} \text{ volts}$$

The number of turns (S) in series between the positive and negative brushes is $Z/2m$, whence E may also be expressed as

$$E = 4f\phi S \times 10^{-8} \text{ volt}$$

7. ARMATURE WINDINGS

Though there is a large variety of types of armature windings, the practical man and even the designing engineer seldom meet types other than (1) the multiple-drum or lap winding and (2) the two-circuit series drum or wave winding. The other types may be used in a few special and exceptional machines, but a designer may work for many years without finding any necessity for using them.

MULTIPLE-DRUM OR LAP WINDING. This type of winding is very common in d-c machines, rotary converters, induction motors, and a-c generators. Its chief advantage is that it affords a very free choice in the number of coils and slots, and is very simple to lay out and connect. Its disadvantage is that it is not easily adapted to high voltages. Its chief characteristic is that there are always as many circuits in multiple and as many

studs of brushes as there are poles. The distinguishing feature in appearance is the direction of bending of the end connections, as represented by Fig. 4. The characteristic form of coils is shown in Fig. 7.

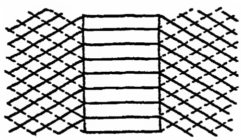

FIG. 4. Lap Winding

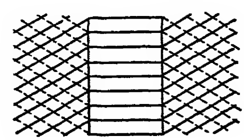

FIG. 5. Wave Winding

CONDITIONS FOR LAP WINDING. The customary simplex multiple-drum winding fulfills the condition expressed by the formula

$$C = pz = sb$$

where C = total number of coil-sides or bars.
p = number of poles.
z = average pitch * $= \dfrac{\text{front pitch} + \text{back pitch}}{2}$.
s = total number of slots.
b = number of coil-sides or bars per slot.

The total number of sides (C) is double the number of coils, as each coil has two sides. C must be a multiple of the number of slots (s) and of the number of poles (p).

The average pitch (z) must be an even number so that the front and back pitches may be different and odd $(z - 1)$ and $(z + 1)$, respectively. There are always two layers of coil-sides, top and bottom. One side of each coil lies in the top layer in one slot and in the bottom layer in another slot. The actual front and back pitches of a coil are always odd because they are made from a coil-side in the top layer (odd numbers) to one in the lower layer (even numbers) as in Fig. 6. Here the pitch is $14 - 1 = 13$.

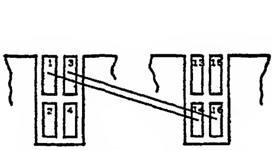

FIG. 6. Pitch of Connection

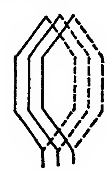

FIG. 7. Lap Winding

In order that the winding should progress continuously, the front pitch, or pitch of commutator connections, must differ by one coil (or two coil-sides) from the back pitch, or actual pitch of coils. The actual spread of a formed coil (Fig. 7) is the same front and back and is equal to the "back pitch."

The total number of coils, $Q = C/2$, is equal to the number of commutator segments and must be a multiple of the number of slots. In general if Q is a common multiple of the number of poles and the number of slots, a multiple-drum winding is possible. It is sometimes desirable that the number of slots should not be a multiple of the number of poles. In order to group the coils in poly-coils z should be a multiple of the coil-sides per slot.

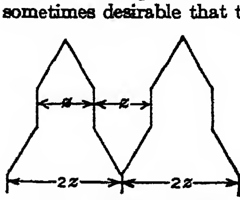

FIG. 8. Wave Winding

SERIES-DRUM OR WAVE WINDINGS, SIMPLEX. This type of winding is used in d-c armatures where the ordinary multiple drum would either give too low a voltage or require too many turns of fine wire in each coil. Its advantage is that it gives an armature that is better balanced magnetically and that only two brush studs are necessary. A greater number of studs may be used, however. There are always two circuits in multiple, regardless of the number of poles. The characteristic reverse bends of its face conductors are shown in Fig. 5. The characteristic form of the coils is represented in Fig. 8.

* Let the coil-sides be numbered consecutively from slot to slot, and let a point travel through the conductors in the order in which they are connected; if this point starting at conductor 1, say, passes through the point or commutator connection to conductor 8 and thus through the back connection to 3, then the front pitch is $8 - 1 = 7$ and the back pitch is $8 - 3 = 5$, and the average pitch is 6.

CONDITIONS FOR WAVE WINDING. The conditions to be fulfilled in laying out a series-drum winding are expressed in the formula

$$C = pz \pm 2 = sb$$

where C = total number of coil-sides or bars.
p = number of poles.
z = average pitch of end connections.
s = total number of slots.
b = number of coil-sides or bars per slot.

The total number of coil-sides (C) is double the number of coils and must also be a multiple of the number of slots. In the formula $+ 2$ is preferable to $- 2$, as the positive sign gives shorter end connections.

The average pitch z may be odd or even. If it is odd the front and back pitches are both equal to z. If it is even the front and back pitches must be $(z - 1)$ and $(z + 1)$. For a wire winding the back pitch must be one greater than a multiple of the coil-sides per slot b in order to fit the coils into the slots in groups.

The total number of slots (s) is very much restricted and is intimately connected with the number of poles unless the expedient of using a dead coil is used (see below).

The bars or coil-sides per slot b must be even and cannot be 4 or 8 in a 4-pole machine and cannot be 6 or 12 in a 6-pole machine, unless a dead coil is used.

USUAL ARRANGEMENTS for 4- and 6-pole machines with no dead coil are:

Four Poles. For $b = 6$, then z may be 25, 31, 37, etc., and usual values fulfilling all conditions are $s = 17, 21, 25$, etc. For $b = 10$, then z may be 47, 57, 67, etc., and s may be 19, 23, 27, 31, etc.

Six Poles. For $b = 4$, then z may be 17, 21, 25, etc., and s may be 26, 32, 38, etc.

USE OF DEAD COILS. By leaving one of the coils dead or out of circuit, a greater choice of slots and conductors is available. Four coil-sides per slot for 4 poles and 6 coil-sides for 6 poles may then be employed. In general $s \times b$ may be any number divisible by the number of poles. The formula is then

$$C = pz - 2 = sb - 2$$

Two bars or one coil are not connected in at all, and the number of commutator segments is equal to the number of active coils or one less than the total coils. The total number of active coils is $C/2$.

MULTIPLEX WAVE WINDINGS. It sometimes becomes desirable to provide a winding which has more than two circuits in multiple but not as many as the number of poles. In such a case a multiplex wave winding would be selected. These are windings in which the circuit passes completely around the armature more than once. If to do this we use two entirely separate electric circuits, we have a duplex doubly reëntrant winding, denoted by the symbol ○ ○. If, on the other hand, the winding closes on itself after passing twice around the armature, we have a duplex singly reëntrant winding, denoted by the symbol ◎. These types of windings are sometimes used when it is desired to have a number of circuits in parallel different from the number of poles. Thus we may have a 6-pole machine with 4 circuits in multiple which, with a given number of inductors, would give a greater voltage than a multiple-drum winding and lesser voltage than a series drum. This would be a duplex winding.

CONDITIONS FOR MULTIPLEX WINDING. The conditions are imposed by the formula

$$c = pz \pm 2\,m = sb$$

where the symbols have the same meaning as before and m is the number of multiple windings and $2\,m$ the number of circuits in multiple. If m and z are prime to each other we have a singly reëntrant winding. The greatest common factor of m and z gives the number of times the winding reënters, or in other words the number of independent windings.

The multiplicity and complexity may be carried to a very extreme limit. (See under Armature Windings in any standard textbook.) These multiplex windings are hardly ever used in the United States and England and only occasionally in Germany and France.

8. FLUX PER POLE

The value of the flux required to give the desired voltage is given by either of two formulas:

$$\Phi = \frac{E \times m \times 60 \times 10^8}{p\,Z\,(\text{rpm})} \qquad (a)$$

$$\Phi = \frac{E \times 10^8}{4\,f\,S} \qquad (b)$$

where Φ = flux lines per pole (useful).
E = the desired generated voltage.
m = number of parallel paths in the armature; see Armature Windings.
p = number of poles.
Z = total number of inductors on the armature.
rpm = revolutions per minute of the armature.
$f = \frac{p \times \text{rpm}}{2 \times 60}$ is the frequency induced in the armature.
$S = \frac{Z}{2m}$ = the turns of wire in series per path.

This flux flows from a north pole across the gap to the armature, divides so that one half goes to each of the adjacent south poles through the armature core then across the air gap, through a south pole, and from there through the yoke to the original north pole. Thus each pole piece carries two magnetic paths in parallel, while the armature core and field yoke carry one flux path.

In addition there is a leakage of flux from each pole piece, across the interpolar air space to the adjacent pole pieces. This flux never gets into the armature nor cuts the armature conductors. It is called the leakage flux (see explanation under A-c Generators). The ratio $\frac{\text{useful flux} + \text{leakage flux}}{\text{useful flux}}$ is called the leakage coefficient. It may be determined by the method given under A-c Generators or it may be assumed to have values as in Table 4.

Table 4

Rating in Kw	Leakage Coefficient
2.5	1.2–1.5
5	1.18–1.4
10	1.16–1.35
25	1.15–1.30
50	1.14–1.28
100	1.12–1.20
500	1.08–1.15

9. MAGNETIC CIRCUIT

It has been found advisable from experience to operate the various parts of the magnetic circuit (see Parts of a Machine) at certain conventional densities in order to keep the core loss and excitation within reasonable limits. The upper limits apply to a large or a slow-speed machine. Usual densities are given in Table 5.

Table 5

Part	Material	Lines per Sq In.
Field yoke	Steel	70,000–100,000
Field yoke	Cast iron	35,000– 70,000
Pole core	Steel	70,000–100,000
Air gap	Air	40,000– 70,000
Armature teeth	Steel laminations	90,000–125,000
Armature core	Steel laminations	60,000– 90,000

It is more convenient to analyse only one-half of the complete series circuit because the two halves are identical and the mmf for each half is supplied by each separate field coil. Thus, starting at a plane placed half-way between adjacent field poles, the main flux passes through the following parts in series: one-half the length in the yoke (i, Fig. 9), the pole piece (c), the air gap (g), the armature teeth (n), and one-half the length of the path in the armature core (k).

The number of ampere-turns required for excitation for any particular voltage or flux is calculated by means of a table as Table 6.

In the armature teeth and air gap, the total flux ϕ is equal to the useful flux per pole. In the armature core this flux is divided into two halves. The flux in the pole cores is greater than that in the air gap by the amount which leaks from pole tip to pole tip and from pole to pole.

By far the largest part of this leakage is between neighboring pole tips; thus it is reasonable to assume that the pole core carries the useful flux and the leakage flux, or the useful flux multiplied by the leakage coefficient.

Column 2 gives the total flux passing through each particular part.

Column 3 gives the cross-section of each part in square inches; the symbols refer to dimensions taken from a drawing similar to Fig. 9.

Column 4 calls attention to the fact that different materials have different permeabilities and magnetization curves.

Column 5 gives the density in lines or maxwells per square inch.

Column 6 contains the specific value of ampere-turns per inch for the particular material and density as taken from appropriate magnetization curves (see Sect. 2, §§ 15 and 16).

Table 6

1	2	3	4	5	6	7	8
Part	Flux	Cross-section	Material	Flux Density	Amp-turns per In.	Length of Path	No Load Amp-turns per Pole
Field yoke.....	$\nu\phi/2$	de	....	....	....	i	....
Pole core......	$\nu\phi$	$0.785\,f^2$ or fb	....	....	....	c	....
Air gap.......	ϕ	aLp	....	....	$0.313\,B$	Cg	....
Arm. teeth....	ϕ	$Tt_0\,l_i$	....	....	....	n	....
Arm. core...	$\phi/2$	hl_i	....	....	....	k	....

Net ampere-turns for flux: .....

Explanation of Table 6.

ϕ = useful flux per pole in maxwells.
ν = leakage coefficient.
a = pole arc in inches.
l_i = effective length of steel in armature core.
t_0 = effective width of tooth.
T = number of teeth carrying flux.
C = Carter coefficient.

Column 7 gives the length of the respective paths in inches, as taken from Fig. 9.

Column 8 gives the ampere-turns required for each part of the path, and is obtained by multiplying the respective values in column 6 by those in column 7. The sum of all values in column 8 is the excitation required in each field spool to establish the assumed value of useful flux. At no load this would correspond to a definite voltage as given by the formula in Art. 6.

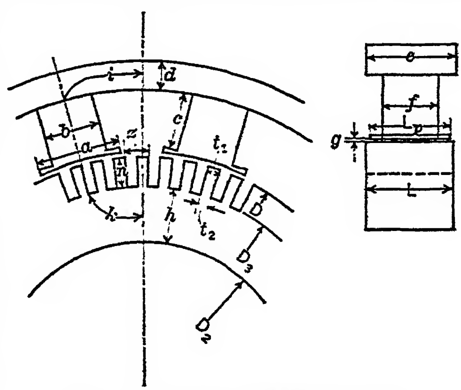

Fig. 9. Dimensions of Magnetic Circuit

Field poles are usually of laminated sheet steel, 25 mils thick, and the effective section of metal in them is usually 95 per cent of the actual cross-section, due to the insulating material between the steel sheets. The magnetic density in the pole shoe is usually so much lower than that in the pole core that it may be neglected. The purpose of a pole shoe is to make the area of the path of the flux across the air gap as large as possible and larger than the path in the pole core.

AIR GAP. The cross-section of the face of the pole shoe determines the cross-section of the path across the air gap. The pole shoe is usually rectangular in section, spans an arc of about 60 per cent of the pole pitch, but is about 0.5 in. less in length parallel to shaft than the armature core as the flux fans out at the ends. As a first approximation, the cross-section of the path of the flux in the air gap is taken the same as the area of the pole face.

If more accurate results are desired, it is necessary to consider the following effects:

(*a*) Fringing of flux at pole tips.
(*b*) Fringing of flux at the ends.
(*c*) Constriction of the flux due to air ducts in the armature.
(*d*) Constriction of the flux due to armature slots.

In many cases these four effects neutralize one another, (*c*) and (*d*) offsetting the favorable effect of (*a*) and (*b*). The effects (*a*) and (*b*) are simple to analyze, but (*c*) and (*d*) are more complex and are usually evaluated by means of the Carter coefficient.

The effect of the constriction of the flux in the air gap caused by the slots in the armature was analyzed mathematically by F. W. Carter in a paper in the *Transactions* of the Br.I.E.E. of 1901. The result is expressed in what is commonly known as the Carter coefficient, which is the ratio of the true reluctance across the gap with irregular distribution of flux to the simple geometrical reluctance (used in the text above). The Carter coefficient is a factor, greater than unity, by which the real length of the gap, g, may be

multiplied to get an effective length which may be used with the simple cross-section to get the correct value of the magnetomotive force required.

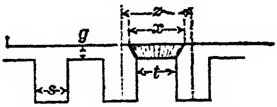

FIG. 10. Equivalent Length of Air Gap

In Fig. 10 let

g = actual or physical length of air gap.
g' = equivalent length of air gap.
l = length of iron core parallel to shaft.
s = width of slot at opening.
t = width of tooth face.
z = pitch of tooth and slot = $t + s$.
f = a numerical factor as in table.

The band of flux from any tooth spreads to a width greater than t and less than z. Let x represent this indefinite width; then let $x = t + fs$.

The apparent or simple reluctance of each band is $\frac{g}{l\,z}$, and the real reluctance is $\frac{g}{l\,x} = \frac{g}{l\,(t+fs)}$. The ratio of the real to the apparent is $\frac{z}{t+fs} = \frac{g'}{g} = C$, the Carter coefficient. The empirical constant, f, has been determined by Carter by mathematical analyses checked by tests and found to depend upon the ratio s/g, slot width divided by gap length, then

s/g	0	0.5	1	1.5	2	3	4	5	6	7	8	9	10
f	1	0.95	0.84	0.78	0.71	0.62	0.55	0.49	0.44	0.40	0.38	0.35	0.32

If the apparent density is obtained by means of the gross pole face area and the mmf found by means of the true gap, g, the correct mmf is found by multiplying this value by the Carter coefficient; or by multiplying the ampere-turns per inch for the apparent density by $g' = Cg$.

ARMATURE TEETH. At any given instant only a portion of the teeth carry flux. These are known as the "teeth under a pole" (T), and the number is slightly greater than those actually in the section subtended by the pole face, owing to the spreading at the pole tips. This spreading is such that the number of teeth under a pole is $T = \frac{a + 2g}{z}$. T may be a fractional number, as it is a time average.

The magnetic density in the teeth varies uniformly from a maximum at the root to a minimum at the face, and the mmf per unit length varies by a greater amount. Since the mmf required is the information desired it is usual to consider the "effective" section which gives average mmf. This is taken at a point one-third the distance from the narrowest end, thus the equivalent tooth section = $t_0 = \frac{t_1 + 2t_2}{3}$.

The effective length of iron (l_i) is the net length of magnetic material and excludes the space occupied by the ducts and by the insulation between laminations, thus = 0.9 (L-space of ducts). 0.9 is the stacking factor and applies to sheets having a thickness of 14 mils.

ARMATURE CORE. The length of the armature core between heads (L) is usually greater than the length of the pole piece by about 1/2 in. The core is made up of 14-mil steel, the same as the teeth, and the stacking factor is 0.9.

EXCITATION AT VARIOUS VOLTAGES, SATURATION CURVE. The relation between the actual voltage and the field mmf in ampere-turns or amperes for various voltages is plotted to give the "no-load saturation curve" and may be predetermined by repeating the foregoing calculation for different assumed voltages or may be determined by test (q.v.); see Fig. 25, p. 8–24.

10. SHUNT FIELD

The proper size of the wire in the shunt field coil is definitely set by three arbitrary quantities: the ampere-turns desired, the average length of a turn around the spool, and the voltage drop on the spool (see Slichter, Design of Electrical Machinery, Wiley).

Thus
$$q = \frac{\rho\, l\, F}{e}$$

where q = cross-section of proper size of conductor in circular mils.
ρ = 12.6 for copper at 75 deg cent—the resistance of 1 ft of copper wire of 1 cir mil area.
l = the average length of the turns on a shunt field in feet.
F = the ampere-turns per spool which are desired.
e = the voltage drop to be allowed on each spool for normal no-load conditions.

The proper number of turns, t, in the shunt field spool, is determined by the allowable current, or the allowable $I^2 R$ loss in the coil. This affects the efficiency of the machine and more particularly the heating of the coils. The greater the number of turns, the less will be the loss and the less will be the rise in temperature of the spools. Assigning a definite allowable value to this loss in watts or in field current, since watts per spool $= ei$, we have the fixed relation for the number of turns of the size of wire already chosen

$$t = \frac{eF}{\text{(watts)}}$$

The usual rise in temperature of shunt field coils of open d-c machines is about 60 to 80 deg cent per watt per square inch of outer cylindrical surface (omitting the end surfaces), and for a 40-deg rise on the spools, it is customary practice to allow from 0.5 to 0.6 watt per square inch of cylindrical surface.

In generators the allowable voltage per spool is taken 10 or 20 per cent below the rated voltage per spool to allow for a margin in emergencies.

An empirical method of determining the number of turns required is to set a density of 1000 amp per sq in. in the shunt field copper. This gives the current as $1000q = i_f$ and the number of turns as $F \div i_f = t$, where q is expressed in square inches.

11. ARMATURE RESISTANCE

By this is meant the resistance which the whole armature current meets in flowing from all the positive brushes to all the negative brushes, and is the resistance of two or more paths in parallel. In a series winding, there are only two paths in parallel. In a multiple winding, there are as many paths as there are poles. As all coils are alike and interchangeable, the average or mean length of a turn may be obtained from a drawing of one coil (see Fig. 11).

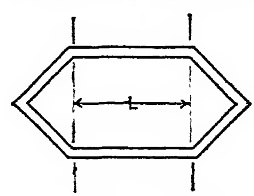

FIG. 11. Single Coil

The mean length of an armature turn is very closely approximated by assuming that the connections at each end have a length equal to a semicircle whose diameter is the winding pitch; thus, if the arc between the two sides of a coil is $\pi Dk/p$, where D = outside diameter of armature, k = the fractional pitch of the coils (if any) as a fraction, and p = number of poles, then the length of the end connections at one end is approximated very closely by $\pi^2 Dk/(2p)$, the length of the end connections at the two ends would be $\pi^2 Dk/p$, which may be written in round numbers $10Dk\ p$.

Then the whole length of a turn is m.l.t. $= 2L + 10Dk/p$ inches.

The resistance to the flow of the line current through all the parallel paths in the armature winding is

$$r_a = \frac{0.01\,N\,(\text{m.l.t.})}{12{,}000\,q\,m}$$

in which 0.0093 = resistance at 75 deg cent of a copper conductor 1000 ft long and 1 sq in. in cross-section.

N = number of turns in series in each path $= Z/(2\,m)$.

Z = total inductors on armature.

q = cross-section of conductor, square inches.

m = number of parallel paths.

If the cross-section of the conductor is circular and given in circular mils (cir mils) the resistance at 75 deg cent is

$$r_a = \frac{12.6\,N\,(\text{m.l.t.})}{12\,q\,m}$$

The voltage drop inside the armature is Ir_A, and the power lost in heat is $I^2 r_A$.

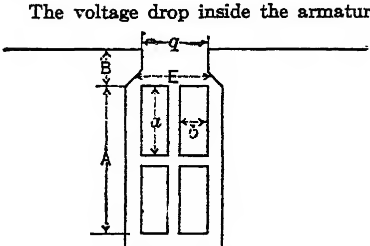

FIG. 12. Dimensions of Armature Coil

The rise in temperature of the winding T_W is given approximately by the relation (see Fig. 12)

$$T_W = \frac{100\,I^2 r_A}{(a+b)s(\text{m.l.t.})}$$

in which a = depth of one coil.

b = width of one coil.

s = total number of coils.

100 = rise per watt per square inch of coil surface with typical insulation used in low-voltage d-c machines.

12. DESIGN OF COMMUTATOR AND BRUSHES

The diameter of the commutator must be less than the diameter of the armature. The peripheral speed should be about 3000 ft per min and should not exceed 4000 unless a special construction is used. The number of segments should be such that the average volts per bar should be less than 15.

$$\text{Volts per bar} = \frac{(\text{Rated voltage}) \times (\text{Poles})}{\text{Number of segments}}$$

To avoid a weak construction the pitch of segments should not be less than 0.20 in. Of this amount about 30 mils is occupied by insulation.

DIMENSIONS OF BRUSHES. The width of the brushes is limited by the number of commutator segments which it is permissible to cover and the width of these segments. The number of segments covered is limited by the reactance voltage, as explained below. In general, the width of the brushes is from 2 to 3 times the pitch of segments. The area of the brush surface is such as to allow from 40 to 50 amp per sq in. of brush contact surface at full load. This surface is distributed equally over a number of studs equal to one-half the number of poles with a multiple-wound armature and may be all on one stud or disposed at pleasure with a series-wound armature.

TOTAL LENGTH OF COMMUTATOR. The total length of brushes per stud or the active length of the commutator is

$$l_c = \frac{2 \times (\text{Total surface of positive brushes})}{(\text{Width of brushes}) \times (\text{No. of studs})}$$

The total length of commutator is greater than this by about 0.5 in. to allow for clearances between brushes and at the ends. It is also customary to allow an extra space at the end of the commutator for insulation to prevent creepage. Commutators in general have a gross superficial area of 0.67 to 1.0 sq in. per amp of total current.

RESISTANCE OF BRUSHES AND BRUSH CONTACT. This is a complex and variable quantity partaking more of the character of the counter emf of an arc than a resistance. It is really a combination of a counter emf and a resistance in series. The magnitude of the loss is small compared with the other losses except in the case of low-voltage machines (80 volts or less) and it is therefore usually treated empirically and approximately.

The Standards of the A.I.E.E. state that a total drop of 2 volts in positive and negative brushes shall be assigned if pigtails are used and 3 volts if pigtails are not used. The total watts lost in all the brushes would therefore be the product of the total current by 2 or 3 volts, respectively.

Another method is to use the values for the resistance per square inch of each brush and brush contact as given in Table 7.

Table 7

Kind of Brush	Resistance of Each Brush, ohm				
	20 amp per sq in.	40 amp per sq in.	60 amp per sq in.	80 amp per sq in.	100 amp per sq in.
Hard	0.051	0.03	0.023	0.018	0.015
Medium	0.037	0.021	0.015	0.012	0.01
Soft	0.024	0.014	0.011	0.009	0.008

FRICTION OF BRUSHES ON COMMUTATOR. The value of this loss is

$$\text{Friction in watts} = \frac{(\text{No. of brushes}) \times (\text{Pressure per brush}) \times V_c \times k}{45},$$

where V_c = peripheral speed of commutator, feet per minute.
k = coefficient of friction = 0.25 for carbon and 0.14 for graphite.

The usual value of pressure on the brushes is from 1.5 to 2 lb per sq in. of contact surface.

RISE IN TEMPERATURE OF COMMUTATOR is

$$T = t \times \frac{(\text{Brush } I^2 r) + (\text{Brush friction})}{\text{Surface of commutator}}$$

where t has the value indicated in Table 8 and is the temperature rise in degrees centigrade for a total commutator loss of 1 watt per sq in.

Table 8

Peripheral Speed of Commutator, ft per min	Value of t
2000	15
3000	12
4000	10

13. ARMATURE REACTION (OR ARMATURE INTERFERENCE)

The armature reaction of a d-c machine has a very important influence on the commutation and regulation of both generators and motors. It is the effect of the mmf of the current in the armature conductors on the magnetic field set up by the field coils.

SEPARATE FLUXES BY FIELD AND ARMATURE CURRENTS. When current flows in the field coils and no current flows in the armature, a flux is set up following a path directly across the armature from pole to pole as in Fig. 13. On the other hand, when current flows in the armature and there is no current in the field a flux is set up in the armature across the axis of the poles, as in Fig. 14.

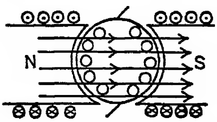

FIG. 13. Field Flux

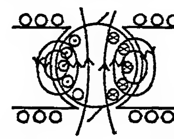

FIG. 14. Armature Flux

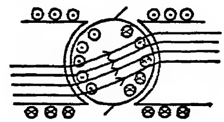

FIG. 15. Distorted Flux

RESULTANT FLUX. When both the field and the armature are excited, the flux is shifted around so that one tip of each field pole has its flux density increased and the other pole tip has its density decreased as in Fig. 15. If the brushes are at the geometrical neutral they are no longer on an axis at right angles to the flux; that is, they are no longer at the neutral point with respect to the resultant flux. With the brushes in this position the coil underneath a brush is cutting flux and generating a voltage, which is short-circuited by the brush and causes sparking. The brushes must, therefore, be moved a small angle in the direction of rotation in a generator (in the opposite direction in a motor) until they are on the neutral axis. The shift of the brushes aggravates the conditions, but nevertheless, unless the armature strength is too great, a position can be found in which a brush short-circuits a coil that is in the real neutral position and is inactive.

RELATIONS SHOWN VECTORIALLY. If certain approximations are made (see Sect. 3, Electrical Machines), the armature and field mmf's may be represented by vectors. In Fig. 16 F_1 and A_1 are the mmf's of the field and armature with the brushes on the geometrical or apparent neutral axis, and R_1 is the resultant of these. The brushes which are in line with A_1 are not at right angles to R_1. If the brushes are moved to an axis at right angles to R_1 as at A_2, then the resultant becomes R_2 and the desired conditions have not been attained. By moving the brushes still further to A_3, giving the resultant

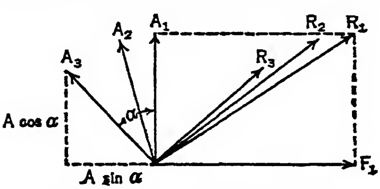

FIG. 16. Vector Relations of Mmf's

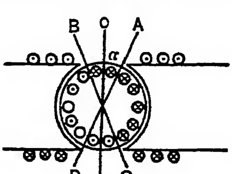

FIG. 17. Demagnetizing and Cross-magnetizing Turns

R_3, we are able to get R_3 and A_3 at right angles to each other. The stronger the armature flux as compared to the field flux the greater will be the angle through which the brushes must be moved.

When the brushes are moved through an angle α to the postion A_3 there is one component of the armature mmf, $A \sin \alpha$, directly opposed to the field mmf. Another component, $A \cos \alpha$, is at right angles to the field mmf. This is better understood by a reference to Fig. 17.

DEMAGNETIZING ACTION AND CROSS-MAGNETIZING ACTION. Let the movement of the brush from A_1 to A_3 be represented in Fig. 17 by the movement from O to B, or through the angle α. Then the conductors in the angle 2α, between A and B and between C and D, carry currents whose mmf directly opposes the magnetic strength of the field coils; these are known as back or demagnetizing conductors. The remainder of the armature conductors, in A–C and B–D, carry currents which give a mmf at right angles to the field and cause a distortion of the flux; these are known as cross-magnetizing conductors.

It should be noted that while all the conductors in the sections $A–B$ and $C–D$ subtending an angle $4\,\alpha$ constitute back ampere-turns, these are opposed to the strength of two poles. In the quantitative treatment in which the unit of design is a pole the turns of $2\,\alpha$ are used as the back ampere-turns per pole.

In practice the vectorial relations are not used because the space between the poles interposes a nullifying effect to the armature flux. In practice it is customary to call the effect of the conductors between A and B and between D and C the back-ampere turns and that of the conductors between B and V and between A and C the cross ampere-turns.

Quantitatively the armature reaction mmf per pole is the product of the turns in series per pole and the current in each conductor, thus:

Let Z be the total number of inductors on the armature; S the number of turns in series per path; I the total armature current; m the number of parallel paths; and p the number of poles. Then the armature reaction per pole is

$$AR = \frac{SI}{p} = \frac{ZI}{2\,mp}$$

If the brushes are on the geometrical neutral point all the armature reaction is cross-magnetizing. If the brushes are moved through an angle of α electrical degrees (one mechanical degree = $p/2$ electrical degrees as there are 360 electrical degrees for every pair of poles), then the back ampere turns per pole are $AR \times 4\alpha/360$ or $AR \times \alpha/90$. The number of cross ampere-turns is the difference between the total AR and the back ampere-turns.

For satisfactory performance there is an upper limit to the value of armature reaction per pole. Usual values are given in Table 9.

Table 9

Kw Rating	Armature Reactive Ampere-turns per Pole (AR)
0–100	1500–3500
100–300	3500–6000
300–500	6000–8000

The armature reaction ampere-turns per pole (AR) should not exceed certain values and should properly check those of Table 9.

If the (AR) comes out higher than advisable it is reduced by increasing the value of the flux.

EFFECT OF COMMUTATING POLES ON ARMATURE REACTION. The demagnetizing effect of armature reaction may be almost entirely overcome by placing the brushes in the neutral axis $O–O$ of Fig. 17. To prevent sparking under these conditions commutating poles must be used. The use of commutating poles does not in itself prevent armature reaction, but makes possible sparkless commutation when the brushes are set in such a position as to reduce armature demagnetization to a minimum.

STABILITY FACTOR. The stability factor is the ratio of the field ampere-turns for gap and teeth to the armature ampere-turns beneath the pole. The former are obtained from lines 3 and 4 of the tabulation for excitation and the latter are given by the formula

$$\frac{\text{Armature reaction ampere-turns per pole} \times \text{Pole arc}}{\text{Pole pitch}}$$

The field ampere-turns for gap and teeth must be greater than the armature ampere-turns, at the maximum current which will be used in the armature, by a ratio of 1.3 to 1.5 in a generator, and of 1.5 to 3 in a motor, if commutating poles are not used. If commutating poles are used this ratio is not important but is usually kept around unity.

The stability factor is adjusted to the value desired by using a greater length of air gap so that the field ampere-turns for gap are increased to the desired value.

EXCITATION UNDER LOADED CONDITIONS. When a generator carries a load, the armature reaction prevents the field coils from setting up as much flux as when there is no current in the armature. It is, therefore, necessary to have the field strength at any load greater than at no load by an amount sufficient to overcome or neutralize the armature reaction, or to provide special auxiliary poles for the purpose. There are three methods of estimating the effect of armature interference and overcoming it.

(*a*) It is common practice in the manufacture of d-c machines to provide a field coil of such ample capacity that it is sure to be more than sufficient to overcome armature interference and set up the flux desired. This field is then adjusted in practice by putting an adjustable resistance in series with the shunt field and a suitable resistance in multiple with the series field. For this practice it is usual to provide a total number of field ampere-turns per pole (F) equal to the net ampere-turns per pole at full load required for flux plus a number of field ampere-turns equal to 40 per cent of the armature reaction ampere-turns as calculated on the opposite page.

(*b*) The effect of armature reaction is divided into back ampere-turns and cross

ampere-turns. The excitation ampere-turns per pole to overcome the back ampere-turns (caused by shifting the brushes) is

$$A' = \frac{(\text{Armature reaction}) \times (\text{Brush shift in segments}) \times 2\,p}{\text{Total commutator segments}}$$

The excitation to overcome the effect of the cross ampere-turns depends upon many variables, but in machines designed in accordance with standard practice it bears a fairly uniform relation to the cross ampere-turns, depending upon the value of the ampere-turns in gap and teeth, as follows:

Cross ampere-turns = (Armature reaction ampere-turns) − (Back ampere-turns)

Field ampere-turns to compensate for cross ampere-turns is then $B' = K \times$ (Cross ampere-turns)

where K has values as shown in Fig. 18 (given by Cramp, see Bibliography). Thus the total field ampere-turns at full load is then equal to

$$F = (\text{Net ampere-turns for flux, Table 6}) + A' + B'$$

(c) It is possible by working with the saturation curve of the machine to actually determine the effect of the cross ampere-turns on the saturation in the teeth and pole face. This method is too elaborate and complicated to be discussed here and is of more value for educational than for practical purposes. It is discussed very fully in the books by S. P. Thompson and E. Arnold (see Bibliography, below).

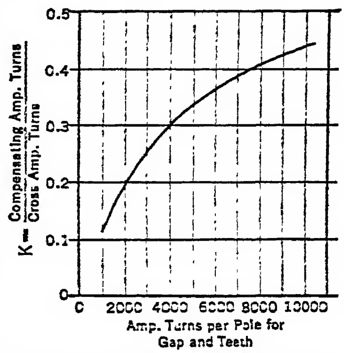

Fig. 18. Constant for Cross Ampere-turns

EXCITATION FOR VARIOUS LOADS. At no load the excitation required would be only that necessary to produce a flux corresponding to the rated terminal voltage. At any load corresponding to a current I the excitation in ampere-turns must be equal to the sum of: (1) that required to produce a flux corresponding to $E + r_a I$ at no load, where r_a is the total resistance of the armature between brushes, the brush resistance, and the resistance of the series field (see p. 8–09); (2) that required to overcome the back ampere-turns caused by I; and (3) that required to overcome the cross ampere-turns due to I.

SERIES FIELD. For a compound-wound machine the ampere-turns required in the series field are equal to the difference between the full-load ampere-turns and the no-load ampere-turns. As it is usual to shunt about 25 per cent of the current through a shunt resistance the number of series turns per pole for each coil is

$$t_1 = \frac{\text{Ampere-turns required}}{0.75 \times \text{Load current}}$$

The cross-section of each turn in square inches is taken equal to the full-load current divided by 1000.

This winding is usually wound in copper strap like a ribbon, the width being equal to the total length of field spool if it is wound under the shunt winding, or is arbitrarily chosen if the two coils are placed side by side. The thickness of each turn is the quotient of the cross-section divided by the width.

The coils are wound with an extra odd half turn to facilitate connection. Thus the depth of winding is

$$(\text{Thickness} + 0.016 \text{ for insulation})\ (t_1 + 1)$$

Allowance for a layer of insulation inside and outside adds about 0.375 in. to the depth of winding.

The mean length of turn l_f' is (see Fig. 9):

For square pole l_f' = (periphery of pole) + 4 (depth of winding), inches.
For round pole $l_f' = \pi\,(f +$ depth of winding).

The resistance per spool is

$$r_1 = \frac{0.0093\, t_1\, l_f'}{12{,}000\, q_1}$$

where q_1 is the cross-section of a turn in square inches.

The total drop in voltage in the series field is $0.75\, r_1 I$, which should be between 0.5 and 1 per cent of the rated voltage.

14. COMMUTATION

ARMATURE INDUCTANCE. Each coil of the armature carries a current flowing in one direction as it travels from a positive brush to a negative brush and in the opposite direction as the coil travels from a negative to a positive brush. Thus the direction of the current in each coil reverses during the time the commutator bars to which the coil is connected pass under the brush. To reverse the current in any circuit it is necessary to take out the energy stored up in the form of magnetic flux linked with the circuit, and put back an equal amount represented by a flux in the opposite direction. In doing this there is induced in the circuit a voltage which is proportional to the time rate of change of the flux and which opposes any change in the value of the current. This voltage is the emf of self-inductance of the coil.

METHODS OF IMPROVING COMMUTATION. To minimize the voltage of self-inductance it is necessary to keep the number of ampere-turns in each coil as low as possible and to cause the reversal to take place as slowly as possible. It is possible to neutralize the voltage of self-inductance by introducing an opposing voltage, which is accomplished in practical machines by giving the brushes a forward lead or by using interpoles.

FORWARD LEAD OF BRUSHES. The brushes are moved in the direction of rotation in a generator (opposite direction in a motor) away from the true neutral until the coil undergoing reversal is moving in a flux coming from an adjacent pole tip of such a density that it induces by rotation in that particular coil a voltage that opposes and neutralizes the voltage of self-inductance.

EFFECT OF COMMUTATING POLES ON COMMUTATION. Interpoles or commutating poles are placed between the main poles over the neutral space. These poles are excited by the load current until they give a flux of the proper direction and value to induce the desired neutralizing voltage. (See also above under Armature Reaction.)

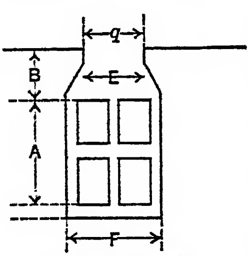

FIG. 19. Slot Diagram

INCREASING RESISTANCE IN PATH OF SHORT-CIRCUIT CURRENT also aids commutation. This changes the time constant of this circuit; that is, some of the stored energy of the magnetism is dissipated in I^2R loss in this resistance instead of in parking at the brush. The usual method of accomplishing this is to use carbon brushes which have a higher resistance of contact than metal brushes. Another method is to introduce a high resistance in the connection between the winding and the commutator segment.

REACTANCE VOLTAGE. The armature leakage flux per ampere-conductor may be resolved into three parts for convenience in analysis. These three fluxes are: P_s flux inside the slot itself, P_t flux from tooth tip to tooth tip, and P_c flux about the end connections.

Referring to the dimensions in Fig. 19 these fluxes are given by the expressions:

$$P_s = 3.2\,l\left(\frac{A}{3F} + \frac{B}{E}\right)$$

$$P_t = 2.35\,l \log_{10}\left(\frac{\pi m}{2q}\right)$$

$$P_c = 0.58\,l_c\left(\log_{10}\frac{2\,lc}{U c}\right)$$

where l = length of iron core.
m = interpolar distance between pole tips, Fig. 19.
q = width of slot opening at face.
l_c = length of end connection at one end.
U_c = perimeter of one coil of end connections.

All dimensions are in inches.

Also let t = turns in series between adjacent commutator segments
= turns per coil in a multiple winding
= $p/2$ times turns per coil in a series winding.

$$C = kt\frac{\text{(brush thickness)}}{\text{(segment pitch)}}.$$

k = 2 for a full pitch winding and approaches unity as the pitch decreases.

Then
$$L = 2\,Ct\,(P_s + P_t + Pc)10^{-8}\text{ henry}$$

Frequency of commutation,

$$f_c = \frac{12\,V_c}{120\text{ (brush thickness)}}$$

V_c = peripheral speed of commutator, feet per minute

$$I_c = \text{amperes per circuit in armature} = \frac{(I\text{ line})}{\text{no. of mult. cir.}}$$

Then assuming that the current in the short-circuited coil varies according to the sine law, the maximum value of the reactance voltage is

$$e_x = 2\pi f_c L I_c \text{ volts}$$

COMMUTATING POLES are used where either the armature reaction or the reactance voltage is very great. If the armature reaction at full load is greater than 80 per cent of the field ampere-turns at no load, then interpoles should be considered. If the reactance voltage is greater than 2.5 volts, interpoles should be considered.

The design of the interpoles may be made roughly in accordance with the practice in some of the large companies by making their widths equal to the widths of two slots and two teeth, and proportioning the winding to have 1.4 times as many ampere-turns as the armature reaction ampere-turns (AR). The axial length of the pole is frequently less than that of the armature; the gap density should be about 30,000, and the iron or steel of the pole should not be saturated when 1.5 times the full-load current is flowing through its windings.

In practice a shunt having inductance is connected across the terminals of the interpole winding and adjusted for sparkless commutation. When the interpoles are used on a compound-wound generator, it is possible to decrease the strength of the series coils to such a value as will produce the proper flux for the desired $E + r_a I$, letting the interpoles take care of the armature reaction. Series coils having 20 per cent as many ampere-turns as the armature will be satisfactory.

EXACT DESIGN OF COMMUTATING POLES. A more exact method starts with the value of the reactance voltage, e, and an assumed density in the air gap of B_1 = 30,000. The axial length of the interpole is then

$$L_1 = \frac{10^8 \times e}{k B_1 v_a}$$

where v_a = the peripheral velocity of armature in inches per second, and k = number of inductors in series in short-circuited coil = $2t$ (see formula for reactance voltage, above).

The width (a_1) of pole at face is made equal to or a little greater than twice the pitch of armature slots. The flux per pole in gap is $\phi_1 = B_1 L_1 a_1$.

The leakage coefficient is higher for interpoles than for the main poles and is in the neighborhood of 1.6 to 1.8.

The magnetic density is kept low in the pole core so that there will be margin for overload.

The necessary exciting ampere-turns to force this flux through the gap and to take care of the leakage and increased densities in the field and armature core are calculated. Let this value be A''.

Let the armature reaction ampere-turns per pole be B''.

Then the total ampere-turns on the interpoles should be $A'' + B''$.

The number of turns per pole is $(A'' + B'')$ divided by the full-load current of the machine, and the winding itself is laid out in the manner employed in laying out the series field.

15. REGULATION

This is arbitrarily defined as the ratio $\frac{E_0 - E}{E}$, where E_0 is the voltage at the terminals of the machine with no load connected, E is the voltage at the terminals when a load is connected, and the ratio is the regulation for that particular load. It is usually expressed as a percentage. The test is made by connecting a specified load to the generator, adjusting the excitation to get a specified voltage (with rising field current) and then opening the connection to the load without making any change in the shunt field rheostat, but usually holding the speed constant. Where generators are sold direct-connected to their own prime movers the speed is allowed to change in accordance with the governing of the prime mover. This is then the regulation of the set.

With shunt machines the voltage at full load is always less than at no load and the regulation is said to be a positive number. With compound-wound generators there may be over-compounding; that is, the voltage at full load is greater than that at no load and

the algebraic sign of the regulation would be negative. The regulation of a compound generator is determined with a falling field current; that is, the voltage is first raised to a value higher than normal and then brought down to the specified value before recording readings. This is because a compound generator frequently shows higher values of voltage at fractional loads than at no load, owing to the hysteresis loop.

16. LOSSES

CORE LOSS. The core loss consists of hysteresis and eddy-current losses in the armature core and teeth due to the reversal of the useful or main flux as the armature passes each pole. It is most readily calculated by using the values (C_h) of the hysteresis loss per cubic inch at one cycle per second for the specific magnetic density and (C_e) the eddy-current loss per cubic inch at one cycle per second for the specific density and thickness of sheet (usually 0.014 in., standard). These values are given for various qualities of steel (η and ϵ) in the curves in the article, Magnetic Materials (see Sect. 2, § 16).

$$\text{Total loss in core} = V_c\, C_h\, f + V_c\, C_e\, f^2$$

$$\text{Total loss in teeth} = V_t\, C_h\, f + V_t\, C_e f^2$$

where V_c and V_t = volume in cubic inches of the core and teeth respectively and f = frequency = poles × rpm/120.

C_h and C_e (see above) are taken for those densities which exist when the flux corresponding to the generated voltage at a specific load is maintained, i.e., $E + I\, r_a$ in a generator and $E - I\, r_a$ in a motor, where E is the terminal voltage and r_a the resistance of the whole armature circuit.

The prediction of the core loss from the design is one of the most unreliable calculations because the pulsation of the flux due to the passage of the teeth across the pole face introduces extra losses which are very difficult to predict except by very elaborate and complicated methods (see Bibliography). For convenience the value calculated as above is sometimes multiplied by an empirical factor of 1.2 to 1.5 to cover these extra losses.

FRICTION AND WINDAGE can be calculated by formula for any particular line of similar machines, but no method applicable in general can be given. Machines are designed so that the value of the friction loss very closely approximates those given in the table on the opposite page. The true friction varies directly as the speed, and the windage as the square of the speed. In shunt machines the speed does not change much with the load so this loss may be assumed constant in this class of machines (see Direct-current Motors).

BRUSH FRICTION LOSS. See p. 8–12.

STRAY POWER LOSS. This term is sometimes used to include both the core loss and friction and windage.

EXCITATION LOSS OR RI^2 IN SHUNT FIELD. Since in a self-excited machine the loss in the field rheostat is charged against the machine the total loss is equal to the terminal voltage multiplied by the current in the shunt field. The loss is usually constant and subject only to the arbitrary variations due to hand regulation.

RI^2 LOSS IN ARMATURE CIRCUIT, including the brushes, series field and connections, varies as the square of the armature current, and, therefore, approximately as the square of the load, and also irregularly to a small extent due to the variation of brush contact resistance with the value of the current. The temperature of the copper must also be considered, usually assumed as 75 deg cent. (See Resistance and Conductance.)

LOAD LOSSES, STRAY LOAD LOSSES. The mmf of the armature sets up a component of flux of its own whose magnitude is closely proportional to the armature current (because its path is purposely largely in air) which distorts the main flux and causes additional losses under load. These are: eddy currents in the armature conductors and in any masses of metal near the conductors, such as the pole faces, and increased hysteresis in the teeth and pole faces. This loss is zero with no armature current and increases rapidly with the armature current. Its rate of increase depends upon how much of it is hysteresis and how much is eddy loss.

The A.I.E.E. Standards say that for calculating the efficiencies of direct current generators or motors the load losses at any load shall be taken as 1 per cent of that output except for motors of 200 hp at 575 rpm and smaller, in which cases they shall be ignored. Some European standards say the load losses shall be taken as proportional to the square of the load. There is no recognized method of calculating or testing for them.

EFFICIENCY. The efficiency is the ratio of the output to the sum of the output and all losses. It is predetermined by estimating the value of each of the five losses described above for the particular load or output under consideration.

If P = the output in watts and the symbols A, B, C, D, and E represent respectively the various losses described above, then the efficiency for any load, P, is:

$$\text{Per cent efficiency} = \frac{100\,P}{P + A + B + C + D + E}$$

The efficiency is a maximum for that load at which the variable losses ($D + E$) are equal to the constant losses ($A + B + C$)—hence the desirability of keeping the constant losses small in a machine which is to be operated most of the time at partial load.

USUAL EFFICIENCIES AND LOSSES. An idea of a reasonable value for the efficiency of d-c machines of various sizes and the distribution of the losses is given in the following table. It must be remembered that the efficiency of a machine of any given size may vary throughout a wide range depending upon the speed, weight, and cost.

Rating, kw	Efficiency, per cent	Friction (total), per cent	Excitation, per cent	Core Loss, per cent	Arm. RI^2, per cent
1	80	6	6	4	4
5	84	5	4.2	3.2	3.6
10	86	4	3.6	3.0	3.4
20	88	3	3.0	2.8	3.2
50	90	2.6	2.2	2.2	3.0
100	91.4	2.3	2.0	1.7	2.6
200	92	2.2	1.8	1.6	2.4
500	93	2	1.6	1.4	2.0

17. HEATING

The rise in temperature of the field coils is discussed above under that heading, and a rough criterion for the heating of the armature coils is given under Armature Resistance.

It is desirable to determine more accurately the rise in temperature of the armature conductors, as the insulation on the windings will be injured if this temperature is excessive.

The temperature of the windings is affected not only by the heat due to $r_a I^2$ in the copper, but also by the heat from the core loss in the iron, and both losses must be taken into consideration in order to make an accurate prediction of the final temperature of the copper.

The following rational method takes into account the most important factors and, with a judicious choice of constants, gives reliable results. However, the drop in temperature due to conduction of heat inside the copper conductors and inside the iron core is neglected in the interest of simplicity and because it is small and of negligible effect except in very long machines such as turbo-generators.

All the heat loss in the armature is dissipated either at the surface of the iron (including that in ventilating ducts) or at the surface of the "end connections," sometimes called the "free ends" of the winding coils, as distinguished from the "imbedded portions." If the copper loss is great the copper will be hotter than the iron and some of the $r_a I^2$ will flow from the copper through the insulation in the slots to the iron and thence to the surface of the core iron. However, if the core loss is excessive or great compared with the copper loss the temperature of the iron will be higher than that of the copper and the direction of flow of heat energy will be reversed, some of the core loss passing from iron to copper and to the end connections which are cooled by air, either circulated by the rotation of the armature or by a system of forced ventilation or blowing.

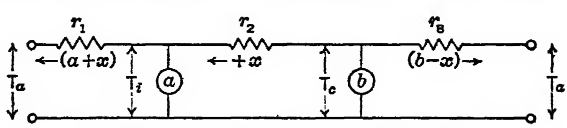

Fig. 20. Equivalent Circuit for Heat Losses

Thus there are two sources of heat, the core loss in the iron and the $r_a I^2$ in the copper, and two paths for the flow of the heat energy to the ventilating air, these paths being linked together by a third path between the copper and the iron. This may be represented by an electrical network as in Fig. 20.

Let a = core loss in watts.

b = armature copper loss in watts.

x = watts passing copper to iron, or vice versa.
r_1 = resistance of heat path from iron to ventilating air at surface of iron core and ducts.
r_2 = resistance of heat path from copper to iron through the insulation in the slots.
r_3 = resistance of heat path from copper to air through the insulation on the end connections. (This includes both the drop due to conduction through the insulation and due to convection at the surface.)

Then:
$$r_1 = t/A$$
$$t = \frac{170}{1 + 0.00025\, V}$$

V = peripheral velocity of the armature, feet per minute.
$$A = \pi DL + \pi/4\,(D^2 - D_2^2)\,(2 + 2d)$$

D = outside diameter of armature, inches.
D_2 = inside diameter of armature.
L = length of armature, inches.
d = number of air ducts in the armature core.
$$r_2 = \frac{k_1\, d_1}{U_s\, L\, s}$$

k_1 = specific heat resistance of the insulating material, 160 to 250. (See note.)
d_1 = thickness of insulation, in inches, between copper in the slot and iron of the core.
U_s = perimeter of slot insulation including two or more coils.
s = total number of slots.
$$r_3 = \frac{k_1\, d_2 + t}{U_c\, l_c\, Z}$$

d_2 = thickness of insulation on end connections.
U_c = perimeter of one coil at free part of end connections.
l_c = length of end connections or free part of a coil at one end of the armature.
Z = total number of coil sides = twice the number of coils.

Then by Kirchhoff's laws as shown by the circuit diagram of Fig. 20 there are two simultaneous equations:

Rise in temperature of the copper
$$T_c - T_a = (a + x)r_1 + xr_2$$
$$T_c - T_a = (b - x)r_3$$
where T_c is the actual temperature of the copper as measured by resistance and T_a is the temperature of the ventilating air.

From these equations:
$$x = \frac{br_3 - ar_1}{r_1 + r_2 + r_3}$$

The rise in temperature at the surface of the iron of the core will be:
$$T_i - T_a = (a + x)r_1$$

EXPLANATION OF THE CONSTANTS. t is the drop in temperature in degrees centigrade between the surface of the armature and the surrounding air ("ambient") for one watt per square inch. The value 170 is an empirical value to be used when the actual velocity of the air is not known but only the peripheral speed of the surface of the revolving part. If the true velocity of the air with respect to the surface, V, is known then the relation is:
$$t = \frac{130}{1 + 0.0005\, V}$$
(See Langmuir, A.I.E.E.)

k_1 is the drop in temperature per watt per square inch due to conduction through insulation having a thickness of one inch. For any other thickness, d_1 (inches) the drop is $k_1\, d_1$. k_1 is from 160 to 200 deg cent for varnished-cambric insulation and 250 for built-up mica insulation. The value is much affected by the treatment to exclude air pockets, and the lower values of the constant apply to insulation very carefully built up and heat treated.

d_2 is the thickness (inches) of the insulation on the end connections or free parts of the coils which is usually less than d_1 in the imbedded portion because of the "slot insulation" around that portion.

Usually four or six coil-sides are bound together in the slot to make a polycoil but are separated at the free ends, hence the reason for distinguishing between the number of slots and the number of coil-sides in the formulas for r_2 and r_3.

18. SPECIAL MACHINES

SERIES ARC-LIGHT GENERATORS. These machines are intended to give a constant value of current in the external circuit irrespective of the resistance or counter emf in that circuit. This is accomplished by causing the voltage impressed on the external circuit to decrease whenever there is tendency for the current to increase. The load usually consists of a number (20 to 50) of arc lamps connected in series, each taking about 40 volts. Each lamp requires the same value of current, and the total voltage is proportional to the number of lamps in use. If a lamp is no longer needed it is short-circuited by a switch and the current in the remainder of the lamps remains the same because the generator automatically reduces the voltage supplied. This regulation of the current is obtained by two or more of the following means: (1) high armature reaction which tends to weaken the field whenever the current increases; (2) movable brushes to vary the number of turns in series between brushes; (3) a series field arranged so that turns may be shunted, short-circuited, or cut out; and (4) a high degree of saturation in the magnetic circuit of the field so that the flux changes very slightly for considerable change of mmf.

The pole face density is usually quite high and the path of the flux in the field is of high reluctance, whereas the path of the armature flux is of low reluctance; thus there is much field distortion. The machines are usually wound for 2000 to 4000 volts. The distorting effect of armature reaction is usually relied upon to take care of sudden and small variations of the load, and large and lasting variations of the load are taken care of either by moving the brushes to another position giving either a lower or higher voltage, or by commutating or shunting some of the turns of the series field.

Brush Arc Machine. This was the first arc-light machine to be brought out. It consists of a ring armature having a winding of the open-circuit type. There are a number of spool-wound coils on the armature, diametrically opposite coils being connected in series to an independent pair of commutator segments. The brushes make a series connection of the various groups of coils, the coils of highest emf being connected in series, those of medium emf in multiple, and those of low emf being left out of circuit. The numerous field poles are on both sides of the ring armature. The two poles facing each other are of the same polarity so that the flux flows along the ring of the armature from one pole to the next pole on the same side of the ring. Regulation is obtained by shunting field turns and shifting the brushes for good commutation. A rotary oil pump is used as a regulator to move the handle of the field regulator and to move the brushes.

Thomson-Houston Arc Dynamo. This machine contains a spherical-shaped armature with a three-part open-circuit winding, the three windings being spaced at 120 deg on a ring armature and connected in "Y," the terminals going to the segments of a three-part commutator. The commutator has air spaces between segments and a jet of air to blow out any arcs between brushes and segments. There are four brushes on the commutator, connected in pairs. At some part of each revolution two coils are in multiple, at other positions only one. A relay moves the brushes so that as the load increases the positive and negative brushes move farther apart, thus giving more voltage. As one brush of a pair moves forward the other moves backward, thus keeping them symmetrical with respect to the neutral axis. The field structure is of a hollow, cage-like construction.

Wood Arc-light Machine. This generator has a closed-circuit Gramme ring armature with a commutator having a large number of segments. Regulation is obtained by moving the brushes so as to vary the voltage available and by the use of a high armature reaction balancing the field mmf.

HOMO-POLAR GENERATORS. This type of machine is also sometimes called "acyclic" and was formerly incorrectly called unipolar. Its method of operation is based on the principle of the Faraday disk, which consisted of a copper disk revolving about an axis and projecting between the poles of a magnet. By this rotation in a magnetic field an emf is set up between the axis and the periphery of the disk, and if brushes bearing on these two parts are connected to an external circuit a current will flow. The peculiar characteristic of a homo-polar machine is that each conductor always cuts the flux in the same direction, consequently the emf induced in it is always in the same direction and is not alternating as in the usual d-c machine. Thus no commutator is required.

This absence of a commutator is the feature which makes the homo-polar machine attractive. The commutator presents many difficulties in high-speed machines to be driven by steam turbines. It is for this application that recent attempts to develop a successful homo-polar machine have been directed. Instead of a commutator, collector rings with brushes are used to collect the current from the moving conductors. These collector rings, however, present difficulties in construction and operation on account of

the high peripheral speed at which they must run. The rings are subject to a considerable centrifugal force, and there is a tendency of the current to arc between the brush and the collector on account of the high rubbing speed.

Voltage. The voltage of such a machine is not only unidirectional as in all d-c machines but is really constant. But since there can be only a few conductors in series, the voltage generated is very low. The voltage generated per disk or inductor is

$$E = Blv\,10^{-8}$$

where B = magnetic lines per square centimeter, l = length of conductor in centimeters, v = velocity of conductor in centimeters per second.

This is more conveniently expressed,

$$E = \frac{NZ\phi\,10^{-8}}{60} \text{ volts}$$

where N = revolutions per minute, Z = conductors in series, ϕ = total flux traversing gap.

Radial and Axial Types. There are two types of homo-polar machines, the radial and the axial. The radial type (Fig. 21) is like the Faraday disk and consists of a disk revolving between the two poles of a cylindrical magnet. Brushes bear on the outer rim and the shaft to collect the current. The disk may be made of steel to reduce the reluctance of the magnetic path. The voltage of such a machine is limited to 10 or 15 volts, but the current may be very large. A variation of this type has two disks on the same shaft and the magnetic path so arranged that the voltage of the two disks may be added in series by brushes bearing on the peripheries of the two disks. The axial type (Fig. 22) consists of a cylindrical steel armature with copper bars in the surface, the whole revolving in a cylindrical field so arranged that the magnetic flux flows outward from the armature in a radial direction at all points. The several conductors on the armature are connected to slip-rings at both ends, and by means of brushes and stationary conductors, these conductors are connected in series. The voltage of such a machine may be from 40 to 50 volts per conductor.

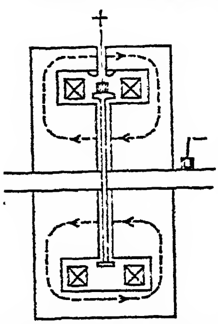

Fig. 21. Radial Type of Homo-polar Generator

Data on Large Axial Type Machine. In the *Trans. A.I.E.E.*, Vol. 24, Noeggerath describes a machine of the axial type rated at 300 kw for 500 volts at 3000 rpm. The armature has 12 conductors connected in series for 500 volts. The diameter of the armature is 19 in. and the length 12 in. The peripheral velocity is 15,000 ft per min. The armature is of cast steel and has 24 cast-steel collector-rings on it. The stationary conductors connecting the collector-rings together are placed in the face of the pole, and thus their mmf may be used to balance the armature reaction.

Excitation. The armature reaction of such machines is very high and has only a distorting effect. However, it weakens the field as the cross magnetization weakens one part more than it strengthens another due to saturation. By a proper arrangement of the movable connections between the collector-rings and stationary conductors a mmf may be set up which will strengthen the field and thus the machine may be compounded. These machines may be made self-exciting, but the resistance of the shunt fields must be very low in order that the machine may pick up on starting as the resistance in the brush contacts is high. The drop in voltage in each brush contact is about 0.8 volt at full load but is higher before the current flows. It sometimes requires from 10 to 20 times the normal voltage of brush contact to start the current.

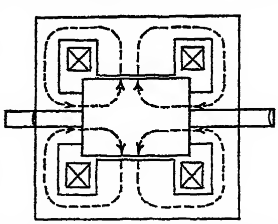

Fig. 22. Axial Type of Homo-polar Generator

Losses. Such a machine as described above has an efficiency of approximately 90 per cent at full load. The losses are made up principally of friction and $I^2 r_a$ in the brushes. The field $I^2 r_a$ is low as the air gap is small for mechanical considerations. The armature $I^2 r_a$ is almost negligible owing to the few turns. The eddy losses are low if the flux density is constant in one zone around the armature but the density may vary along an element of the cylinder. The total weight of the machine is about the same as that of the usual d-c machine but the proportion of copper is much lower.

THREE-WIRE DIRECT-CURRENT GENERATORS. The three-wire generators, used for supplying a three-wire d-c system and taking care of a reasonable unbalance of

the loads on the two legs, is really a double-current generator with collector rings as well as a commutator. It may have two, three, or four rings. The more phases, the less is the internal heating caused by the unbalanced current. The neutral of the distribution system is connected to the neutral or middle point of each of the "balance coils" or single-circuit transformers and the terminals of these coils are connected to the collector rings.

For a two- or four-ring connection the coils must have a tap at their middle point, but for a three-ring machine a Y connection may be used.

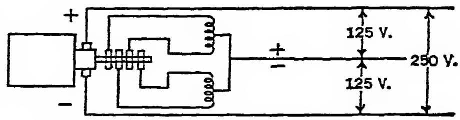

FIG. 23. Three-wire Direct-current Generator with Four Collector Rings and Two Coils

The unbalanced or neutral current divides among the coils inversely as the resistances, therefore equally. When the current enters the armature winding from the ring it divides again inversely proportionally to the resistance of the path to the brush whence the unbalanced component came originally. Thus the unbalanced current in any one coil of the armature is continually changing and reverses as the coil passes under a brush. The variation is a straight-line function, and this component is superimposed on the current in the armature due to the balanced portion of the load. A coil half-way between taps has a lower peak of varying current than a coil next to a tap as in a synchronous converter and hence less heating.

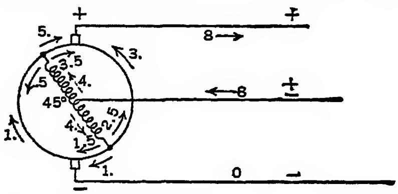

FIG. 24. Three-wire Generator. Path of Unbalanced Currents in a Two-ring Machine

Inside figures show the mathematical components of each 4 amp from coil terminal.
Outside figures show the actual currents in the armature coils at that instant.

An a-c magnetizing current of small magnitude is continually flowing from the collector rings through the coils whether the load is balanced or not. In some commercial machines the coils are placed inside the rotating armature, in which case only one collector ring is needed; but generally the coils are separate and external to the generator and encased in a structure like a transformer.

19. TESTING OF DIRECT-CURRENT MACHINES

Direct-current machines are judged by four characteristics: efficiency, regulation, commutation, and heating. It is necessary to subject a machine to actual full-load conditions to determine its heating and commutation, and it is desirable to do so to determine the regulation, but the efficiency is determined more accurately by the "separate loss method" which does not involve loading the machine.

The customary tests on d-c machines are (see also A.I.E.E. Test Code):

(1) Resistance measurements, cold and hot. (2) Saturation curve. (3) Core loss and friction test. (4) Load run for commutation and regulation. (5) Heat runs. (6) Compounding test. (7) Insulation test.

RESISTANCE MEASUREMENTS. The first measurements are made when the machine is at the same temperature as the room, that is, after it has been idle from 12 to 24 hours depending upon its size. This is in order that the relation between the resistance and temperature may be accurately known. The resistance of the shunt-field, series-field, and armature winding proper is measured.

RESISTANCE OF LAP WINDING. In measuring the resistance of the armature winding it is preferable to measure between two diametrically opposite points of the commutator of a multiple-wound armature. The effective resistance of the armature is calculated from this by the formula

$$r_a = \frac{4r'}{p^2}$$

where r' = resistance measured as above and p = number of poles.

RESISTANCE OF WAVE WINDING. In measuring two-circuit series-drum windings it is necessary to measure between two commutator segments separated by a distance equal to the periphery of the commutator divided by the number of poles. This will give the effective resistance of the armature.

BRUSH CONTACT RESISTANCE. The resistance of the brushes and brush contact is calculated from data such as given in Art. 12; a measurement made with the actual circuit is not very satisfactory. However, it is sometimes the practice to measure the resistance of the entire armature circuit from brush to brush by the drop in potential method for several values of current, the armature being at rest. This is not accurate, however, as the resistance of brush contacts depends upon the speed.

SATURATION CURVE. The saturation curve is not of immediate interest in determining the quality of a machine but is more particularly of interest to the designing engineer. It is made by driving the armature at the proper speed and supplying current to the shunt fields from a separate source of potential. As the current in the shunt field increases, the potential generated by the armature varies, and this is measured by a voltmeter.

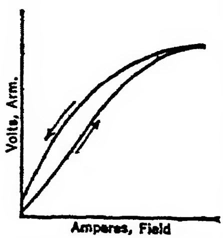

Fig. 25. Saturation Curve

On account of the existence of the hysteresis loop (see Magnetic Materials, Sect. 2, § 16), it is necessary to increase the field current gradually and never to reduce the value while changing from point to point until the maximum value has been reached. Then the field current is reduced, step by step, to zero. This gives two curves, one with "rising field current" and one with "falling field current." The curves plotted with volts as ordinates and field current or ampere-turns as abscissas show the typical knee of all saturation curves as seen in Fig. 25.

CORE LOSS AND FRICTION, OR STRAY POWER. These tests are made with the same arrangement as used in determining the saturation curve, and may be made at the same time. The machine to be tested is driven by a small motor having a capacity about 10 per cent of the rating of the machine under test. All the losses and constants of this motor must be known.

First the machine under test is driven at the desired speed with no current in the fields, and the power taken by the driving motor is noted. The mechanical output of the driving motor is calculated, and this gives the friction loss of the large machine.

Then the large machine is excited and the power to drive it is determined. This power represents the core loss plus the friction loss. Deducting the friction loss already determined, the values of the core loss for various values of excitation are found. See also Magnetic Materials.

The combined core loss and friction loss constitute the stray power loss.

LOAD RUNS. The machine is run and the excitation adjusted to give the proper voltage at no load. The load is then added and the terminal voltage noted. If E_0 = voltage at no load and E = voltage at full load, the regulation is $\dfrac{E_0 - E}{E}$.

If the machine does not have commutating poles it is necessary to make a preliminary test in order to set the brushes at that position which gives the best compromise in sparking at no load and full load. Commutation is judged by observing the action of the brushes at full load and 150 per cent load. The conclusions are a matter of judgment and experience, although degrees of sparking have been arbitrarily agreed upon and are represented in a chart or series of pictures.

HEAT RUNS are made by operating the machine at full load for a period of time until the temperatures of the various parts that can be noted during operation have become constant. The greater the capacity of the machine the longer will be the time necessary.

DEAD-LOAD AND PUMPING-BACK METHODS. Heat runs may be made either by the "dead load" method in which a resistance load, such as a water-rheostat, is connected to the terminals and full rated load power is required to drive the machine; or by the Hopkinson or "pumping-back" method in which two machines having similar characteristics are run together, one acting as a generator to supply electrical power to the motor which in turn drives the first by means of a belt or similar mechanical connection. For this test a "loss-supply" is required which may consist of a source of either electrical power or mechanical power. The amount of power required is from 10 to 20 per cent of the rating of one of the machines. The connections are shown in Fig. 26 for electrical loss supply, and Fig. 27 for mechanical loss supply.

THERMOMETER READINGS. During the heat run there are taken, at stated periods, readings of thermometers which show the temperature of the frame, field coils, bearings, and surrounding air. After the heat run the thermometers are placed on various parts of the machine, the bulbs being protected from radiation by small pads of cotton, and the following temperatures are noted:

Armature-core surface.
Armature-core ventilating ducts.
Armature conductors or winding.
Commutator surface.
Pole tips.
Field coils.
Bearings.
Frame.
Room.

The resistance of all electrical circuits should be measured and the average temperature of the copper calculated from the formula

$$= t_1 \frac{r_1}{r_0} (235 + t_0) - 235$$

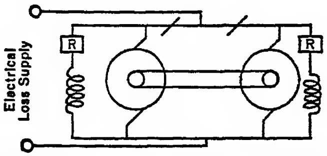

FIG. 26. Hopkinson Load Test, Electrical Supply of Losses

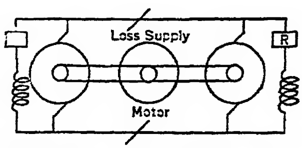

FIG. 27. Hopkinson Load Test, Mechanical Supply of Losses

where r_1 and r_0 are the hot and cold resistances respectively, and t_1 and t_0 the hot and cold temperatures respectively, 98 per cent conductivity copper being assumed. See article on Copper.

COMPOUNDING TEST. In order to adjust the current in the series field so that a compound-wound generator will give specified voltages at no load and full load, the machine is first operated at no load and the current in the shunt field is adjusted to give the desired no-load voltage. The load is then put on, and usually it will be found that at full load the terminal voltage is too great.

Strips of German silver or other resistance metal are then connected in multiple with the series field until by shunting current from the series field the voltage is reduced to such value as is desired. This shunt resistance is then made up in permanent form and connected in circuit. Before making the no-load adjustment it is desirable to overexcite the shunt field for a moment in order to overcome the hysteresis. See also Parallel Operation below.

INSULATION TESTS. Before any voltage is applied to a machine it is customary to measure the insulation resistance by means of a source of potential of 500 or 600 volts and a voltmeter reading to 600 volts. With the voltmeter in series the potential is applied between the conducting windings and to the frame of the machine.

$$\text{Insulation resistance, ohms} = \frac{E_0 - E_v}{E_v} r_v$$

where E_0 is the potential of the source, E_v is the reading of the voltmeter in series, and r_v is the resistance of the voltmeter.

The standards specify the proper values for various machines, but in general it should be more than 1/2 megohm. A lower value indicates moisture or a damaged winding.

After the heat runs it is customary to apply a high potential to test the insulation of the machine. This consists of applying an alternating potential between each electrical circuit and the frame of the machine for 1 minute. The magnitude of the potential depends upon the capacity and rated voltage of the machine under test and is definitely specified in the Standardization Rules of the A.I.E.E. (q.v.).

20. SPECIFICATIONS FOR DIRECT-CURRENT GENERATORS

The following memoranda are intended to assist in writing specifications.

PRINCIPAL CHARACTERISTICS AND CONDITIONS OF SERVICE. Service for which it is to be used, such as railway, lighting, etc. Voltage. Rated output, kilowatts. Speed.

STYLE AND DESCRIPTION, DETAILS OF CONSTRUCTION. Type, whether shunt or compound wound. Whether interpole. Details of speed and governing of prime mover and how generator is connected to prime mover, e.g., direct or belt driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. If field rheostat is to be supplied, its characteristics, including the effect upon the generator voltage of each step and of all steps: accessibility of armature. If belt driven, specify pulley details.

WORK TO BE DONE BY OTHER CONTRACTORS. See Alternating-Current Generators, under same heading.

PERFORMANCE AND TESTS. (See Standardization Rules of the A.I.E.E.). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at $^1/_4$, $^1/_2$, $^3/_4$, 1, and $1^1/_4$ loads. (Whether rheostat losses to be included in calculating efficiencies.) High potential tests of insulation. Requirements regarding effect of moisture upon insulation. When run at constant rated speed, the load may be varied from zero to a stated percentage of rated load without causing more than a stated variation of voltage, the field rheostat being kept constant.

21. INSTALLATION

In installing a d-c machine the following precautions must be observed:

1. For large machines with foundations the foundation bolts must be provided in accordance with drawings.
2. Bearings must be lined up and well cleaned before being filled with oil.
3. The armature must be properly centered so that the air gap is correct at all points. Taper wedges are used to measure the gap. The magnet frame should be bolted to the base.
4. The field coils must be properly connected. Test for polarity with a compass in order to make sure that no field coils are reversed. For a self-exciting generator there is one particular connection of the field to the armature for each direction of rotation.
5. The commutator must be smooth and polished; use sandpaper to polish the commutator, never use emery cloth.
6. The brushes must be properly and accurately spaced around the commutator. They must be sandpapered and fitted to the curvature of the commutator. The pressure on the brushes must be adjusted to the correct value which is usually 1.5 to 2 pounds per square inch of contact surface.
7. The machine must be thoroughly dried out by heating and the insulation measured as a check.

OPERATION. In starting up a single generator it is sometimes necessary to "charge" the field by separately exciting the shunt fields for a moment to set up residual magnetism.

To cause the machine to "pick up" or generate voltage by self-excitation it is necessary to cut out or short-circuit most of the resistance of the regulating rheostat connected in series with the shunt field. If the total resistance of the shunt-field circuit exceeds a certain critical value the machine will not "pick up," however much time is allowed.

The terminals of the shunt field must be connected to the terminals of the armature with a definite polarity depending upon the direction of rotation, otherwise the tendency will be to demagnetize the field rather than to build it up. This is usually found by trial and error.

PARALLEL OPERATION. In order to operate a power station under economical conditions it is necessary to have a number of machines whose aggregate capacity is equal to the maximum demand on the station. As the demand varies, the number of machines in operation is adjusted so that the machines running are operating at a load near their rating, and therefore, at a good efficiency.

SHUNT GENERATORS. In order to operate shunt generators in parallel, that is, feeding the same bus-bars, it is only necessary to adjust them all to the same polarity and voltage, connect them to the bus-bars, and adjust the division of load by strengthening the field of the underloaded machine if the voltage of the bus-bars is low. If the voltage of the bus-bars is high, weaken the field of the overloaded machine.

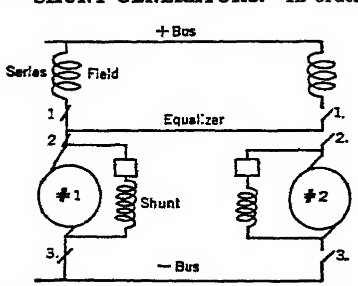

FIG. 28. Parallel Operation of Compound Wound Generators

COMPOUND GENERATORS, EQUALIZER CONNECTION. In order to operate compound-wound machines in parallel it is necessary to provide an equalizer connection which makes a common connection on all the machines at the point between the armature and the series field as shown in Fig. 28.

The function of the equalizer is to divide the load current at all times in the proper proportion between the series fields of the

different generators. This prevents the machines from acting as series generators or differential motors which would cause short circuits.

For compound-wound machines to operate successfully in multiple all machines connected to one set of bus-bars must have the same amount of compounding as well as the same voltage at no load. Owing to saturation in the magnetic circuit it is not always possible to make the compounding curve a straight line, i.e., the increase in voltage directly proportional to the load. It is, therefore, necessary to investigate the compounding curves of machines before they are operated in parallel. With unlike compounding curves one machine may become overloaded while another is underloaded, unless the field current of one machine is adjusted.

In connecting a machine in parallel with those in operation it is necessary to see that it has the proper polarity and voltage, that the equalizer circuit is made, and that the switches are closed in the order 1, 2, 3, as shown in Fig. 28. If any other order is used the effect is the same as having no equalizer.

In shutting down one machine, switch 3 must be opened first, and then 2 and 1.

22. WEIGHT AND SPEED

In Tables 10, 11, and 12 will be found the weight per kilowatt of d-c generators for usual or standard speeds. Table 10 gives the weight of a line of high-speed machines for

Table 10. Weight and Speed

Compound-wound D-c Generators for 125 and 250 Volts High Speed

Kw	Rpm	Weight, lb
25	1250	3,600
50	1180	5,200
100	1000	7,940
200	800	12,600
300	720	16,400
400	650	20,000
500	600	23,200

Table 11. Weight and Speed

Compound-wound D-c Generators for 125 and 250 Volts Moderate Speed

Kw	Rpm	Weight, lb
25	900	4,150
50	800	6,200
100	600	10,200
200	450	16,600
300	400	21,800
400	380	25,800
500	360	29,400

Table 12. Weight and Speed

Compound-wound D-c Generators for 125 and 250 Volts Slow Speed

Kw	Rpm	Weight, lb
25	520	5,500
50	360	9,250
100	260	15,600
200	190	25,600
300	160	34,400
400	140	42,000
500	120	51,000

the suitable speeds. Table 11 gives the same information for a line of moderate-speed machines, generators or motors, and Table 12 for a line of slow-speed generators designed for direct connection to steam engines.

23. EXAMPLES OF DESIGN AND PERFORMANCE

In Tables 13 and 14 will be found the essential data including mechanical, electrical, and magnetic characteristics of four examples of d-c machines. In these data are included all the factors necessary to determine the efficiency, regulation, commutation, and heating of the machines. Dimensions are in inches and weights in pounds. Performance data are deduced from tests.

Table 13. Mechanical Data on Typical D-c Machines

Dimensions in inches; weights in pounds

Type	1 Motor	2 Motor	3 Generator	4 Generator
Poles	2	4	6	8
Rating, hp or kw	3 hp	40 hp	500 kw	250 kw
Revolutions per minute	1200	1750	900	150
Volts	115	230	275	250
Current	23.4	146	1830	1000
Armature diam. out	7.13	12	31.5	45
Armature diam. in	1.63	4.25	19.5	32
Armature, total length	4.69	4.25	14	21.5
Air ducts	0	1 × 0.25	5 × 0.375	7 × 0.5
Slots, number	34	29	108	128
Slots, dimensions	0.95 × 0.22	1.625 × 0.36	1.12 × 0.36	1.32 × 0.52
Conductors per slot	28	10	2	6
Conductor, size	$d = 0.054$	$\frac{1}{32} \times \frac{5}{8}$	0.26 × 0.4	0.5 × 0.125
Conductors in par	4	2	6	8
Type winding	Drum	S.D.	M.D.	M.D.
Pitch of coils, slots	17	7	18	16
Air gap length	0.06	0.25	0.188	0.312
Pole arc, inches	6.83	6.5	11.5	13.25
Pole length, axial	4.25	4.25	13.5	21.
Magnet core, length	4.25	4.25	13.5	21.
Magnet core, width	3.94	4.25	8.5	8.13
Magnet, radial length	3.44	4.97	10	10
Yoke length	6	5	15	21
Yoke, radial depth	2.75	2.25	8	5
Shunt spool turns	2210	2160	900	372
Shunt conductor size	$d = 0.018$	$D = 0.032$	$d = 0.102$	0.144 × 0.156
Series spool turns	28	1.5	2.5	4.5
Series cond. size	0.14 × 0.13	1.25 × .047	2.5 × .75	9.25 × .075
Commutator diameter	5	7.5	20.75	30
Commutator length	3.13	3.75	13.5	14.5
Commutator segments	34	145	108	384
Studs × brushes	2 × 2	4 × 3	6 × 10	8 × 8
Dimensions each brush	0.75 × 1	0.5 × 1.25	1.25 × 1.25	0.75 × 1.25
Interpole arc	0.87	1.25	2	......
Interpole core	0.87 × 3	1.25 × 4.25	2 × 13	......
Interpole turns	143	29.5	4.5	......
Interpole cond	0.09 × 0.09	0.875 × 0.11	2.5 × 0.75	......
Interpole gap	0.75	0.312	0.25	......
Weight	250	2000	16,000	25,000

24. ELECTROSTATIC GENERATORS

In electrotherapeutic and x-ray work a unidirectional high-voltage current is desired and is often obtained by a kind of electric generator called a static generator. Two types of static generators are available for the purpose, the frictional machine and the influence machine.

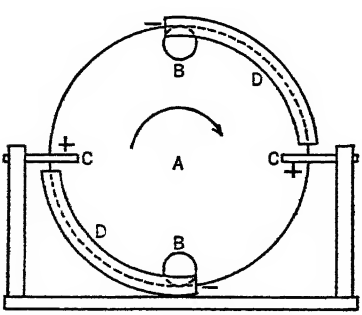

Fig. 29. Frictional Machine

FRICTIONAL MACHINE. In the frictional machine two dissimilar substances are rubbed together in some form of rotating apparatus as shown in Fig. 29. A rotating glass plate *A* is rubbed on both sides at *BB* by two pieces of leather which are greased and covered with the amalgam 1 Zn, 1 Sn, 2 Hg. The glass is electrified positively, and the charge is drawn off by the metal points *CC* which almost touch the glass. Two silk aprons *DD* cover the glass between the rubbers and the metal points, to prevent the leakage of the charge into the air. The rubbers are connected to ground to remove the negative charge which accumulates upon them.

INFLUENCE MACHINE. The influence machine generates an emf by electrostatic induction, the principle of the action being clearly shown (see Fig. 30) in a machine invented

Table 14. Electrical and Magnetic Data on Typical D-c Machines

Percentages are all in terms of rated or full-load values.

		1 Motor	2 Motor	3 Gen.	4 Gen.
Rating	....	3 hp	40 hp	500 kw	250 kw
Rated voltage	volts	115	230	275	250
Rated current	amperes	23.4	146	1830	1000
Flux per pole	10^6 maxw	1.12	1.37	8.8	13.5
Leakage coefficient	....	1.2	1.21	1.18	1.13
Excitation, no load	amp-turns	1040	2640	7300	6525
Excitation, full load	amp-turns	....	2840	8000	9140
Armature reaction	amp-turns	1390	2640	5500	6000
Stability factor	....	0.9	1.17	1.04	1.36
Excitation to balance arm. reaction	amp-turns	....	....	....	1150
Excitation at 110% voltage	amp-turns	1145	3200	9400	7800
Volts per bar	volts	6.8	6.4	15.2	5.2
Reactance voltage	volts	5 7	3.75	7	1
Brush friction	watts	60	216	3800	485
Friction, other	watts	100	580	7000	....
Core loss	watts	60	410	8300	2815
Armature resistance at 25 deg cent.	ohms	0.27	0.046	0.0016	0.006
Brush resistance at full load	ohms	0.044	0.014	0.00104	0.0026
Series field resistance	ohms	0.05	0.0019	0.00018	0.00137
Interpole field resistance	ohms	0.123	0.025	0.00034	....
Shunt field resistance	ohms	150	100	25.2	9.2
Friction loss, total	per cent	6	2.5	2	0.2
Core loss	per cent	2	1.2	1.56	1.
Shunt field loss	per cent	3	0.8	0.46	1.6
Armature copper loss	per cent	6.5	5.1	2.2	4.1
Efficiency at rating	per cent	82.5	89.5	93.8	93.1
Rise in temperature by thermometer:					
Armature	deg cent	....	50	44	18
Field	deg cent	....	25	24	15
Rise in temperature by resistance:					
Armature	deg cent	....	58	44	....
Field	deg cent	....	....	33	....

by G. Belli in 1831. Two spheres or disks AA, called carriers, and normally insulated from each other, are rotated about the shaft B. The two fixed plates CC are charged initially from some external source. When the carriers AA take the position shown in the figure, a momentary connection is made to the neutralizing wire E at the spring contacts DD. As a result of this connection between A and A, the carriers are charged by induction, the charge on each being opposite to that of the adjacent plate. When the carriers rotate still

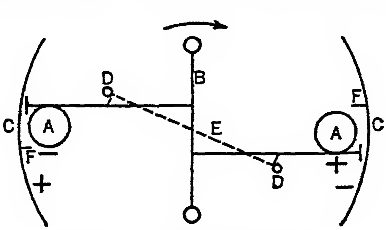

Fig. 30. Influence Machine

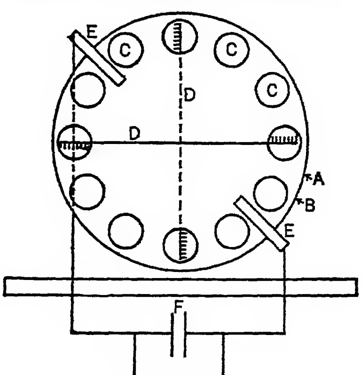

Fig. 31. Wimshurst Machine

further, the connections at DD are broken and the charges induced on the carriers become isolated. When the carriers are rotated through half a revolution from the position shown, they strike the springs FF and add their charges to the field plates CC.

WIMSHURST MACHINE. Many forms of influence machines based upon the above principles have been devised. The most successful designs were made by Toepler,

Holtz, and Wimshurst. The Wimshurst machine, shown in Fig. 31, has superseded most of the other forms because of its self-exciting powers and suitability for work in all conditions of atmosphere. Two glass disks, *A* and *B*, are rotated close together and in opposite directions. A certain number of tin-foil carriers *CCC*, are mounted upon the outside surface of each plate. Neutralizing conductors *DD* for each disk are placed at right angles to each other. The collecting combs *EE* are connected to a condenser *F* and a spark gap *G* across which the discharge takes place. If two diametrically opposite carriers on the back plate are charged positively and negatively, respectively, opposite charges will be induced upon the adjacent carriers on the front plate if they are connected by the neutralizing wire. The induced charges are isolated when this connection is broken and are drawn off at the collecting combs *EE*. In the Wimshurst machine induced charges in moving from the point of electrification to the collecting combs also serve to induce other charges on the back plate.

After a few revolutions of the disks, half of the carriers on each plate are charged negatively and half positively, giving a machine of reliable service and high power. Unlike the Holtz machine, the Wimshurst machine does not reverse its polarity when the distance between the terminals is made greater than the sparking distance. Large influence machines are constructed with several plates revolving in opposite directions and connected in parallel, the whole being contained in a glass case to protect the plates from dust. An eight-plate Wimshurst machine gives a spark of 8 in. and a twelve-plate machine will give a spark of 13 5/8 in. The plates in these machines are approximately 30 in. in diameter, and the machines give 6 sparks for each revolution of the disks.

VAN DE GRAAF GENERATOR. Very recently a new type of electrostatic generator has been developed which is designed to give 20 kw of direct current at 10 million volts. It consists of two hollow metal spheres, 15 ft in diameter, mounted upon insulating supports. Into each of these moves an endless traveling belt of silk. At the lower (ground) end the belt is electrified by friction or other means, and at the upper end a metallic brush draws off the charge and imparts it to the sphere. One sphere is charged positively and the other negatively. The spheres become charged to a very high potential and may be suddenly discharged by connection to a suitable spark gap, 40 ft at 7,000,000 volts. The device is installed at the Round Hill Laboratory of the Massachusetts Institute of Technology. It is being used in some very advanced experiments in physics, such as breaking down atoms.

For bibliography see p. 8–41.

DIRECT-CURRENT MOTORS

A motor is a dynamo-electric machine for converting electrical power into mechanical power; that is, it performs the converse function of a generator. Direct-current generators and motors are always interchangeable in function, although a machine which is designed specifically for a motor would probably not make a first-class generator, and vice versa. Motors of less than 5 hp are usually bipolar; larger machines are multipolar.

The applications of motors are treated in detail in a separate article, Industrial Applications of Motors (q.v.). The chief applications of d-c motors are the following:

Shunt Motor. Driving shafting, machine tools, blowers, reciprocating pumps; motor generators.

Series Motors. Railway and all other transportation work; hoists; cranes.

Compound Motor. Elevators, hoists, and machinery that must be started often.

Differential Motor. Very special applications of small units for peculiar speed conditions.

25. CLASSIFICATION

There are four types of d-c motors, differentiated by their characteristics and the connection of the exciting windings or circuits.

SHUNT MOTOR (Fig. 1). This motor has only one exciting winding, which is connected across the armature terminals and is thus in parallel or in shunt with the armature. The field winding consists of a large number of turns of fine wire on each pole, and usually the windings on all the poles are connected in series in one circuit. The current in the field depends upon the line voltage and upon the resistance of the field winding. The resistance of the field winding is purposely made high so that the field current will be

between 1 and 5 per cent of the full-load current of the motor. The characteristic of the shunt motor is a fairly constant speed for all reasonable values of load.

SERIES MOTOR (Fig. 2). This motor has only one exciting winding, which is connected in series with the armature so that all the current flows through the field as well as the armature. The field winding consists of a few turns of thick wire on each pole, and the windings on all poles are connected in series. The current in the field depends upon the load and is thus large with heavy load and small with light load. The resistance of the field winding is purposely made low so that the loss of voltage and power in that circuit will be small. The characteristics of a series motor are a speed varying with every change in load, high speed at light load and low speed at heavy load. The

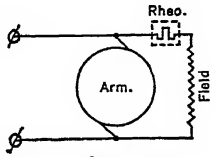

FIG. 1. Shunt Motor

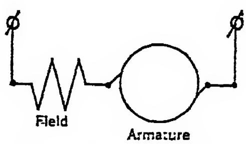

FIG. 2. Series Motor

efficiency is high throughout a wide range of speed. The speed will be dangerously high at no load; thus a series motor must always be connected rigidly to its load. Since the torque is high at low speeds this motor is particularly adapted to work requiring frequent starting.

COMPOUND (OR CUMULATIVE) MOTOR. This motor has both a series winding and a shunt winding on each pole, wound and connected so that the two windings assist each other in the production of magnetism. It is a combination of a shunt and a series motor designed to give the good starting qualities of the series motor and to avoid the danger of excessive speed at light loads. See also Industrial Applications of Motors.

DIFFERENTIAL MOTOR. This motor has a shunt and a series winding connected so that they oppose each other in the production of magnetism. The motor therefore has poor starting qualities, increases in speed with increase in load, but has no tendency to run at a dangerously high speed. The applications of this motor are very limited.

ENCLOSED VS. OPEN TYPE. These terms refer to the mechanical housing of the motor. The open type has all its parts freely exposed to the air and is therefore well ventilated. It is intended to be used indoors and in protected places. The enclosed type is intended to be used in exposed locations where there may be dampness or dirt. Special means must be provided to circulate the air inside the machine, but even then an enclosed motor is larger and more expensive than an open motor of the same capacity.

The relative capacities in output of open, semi-enclosed, and totally enclosed motors are shown by the accompanying data on one of a line of typical commercial motors. In general an enclosed motor weighs about 15 per cent more than an open motor of the same capacity in spite of the fact that it is allowed to operate at 15 deg cent higher temperature by commercial convention.

Table 1

Type	Output, hp	Temp. Rise, deg cent	Weight in pounds for 700 rpm and Given Temp. Rise
Open	10	40	970
Semi-enclosed	8	40	1000
Totally enclosed	5.75	55	1100

26. RATINGS

See Standardization Rules of the A.I.E.E. for limiting rise in temperature and for classification of types, Constant Speed, Multispeed, Adjustable Speed, and Variable Speed.

The National Electric Manufacturers Association, which includes representatives of most of the manufacturers of electric machinery in the United States, has standardized, for commercial practice, certain temperature ratings which are based on the A.I.E.E. rules. These regulations, which are for limiting temperatures as measured by thermometer with continuous duty, and the corresponding A.I.E.E. rules are:

Table 2

	Max. Rise on Commutator	Max. Rise on Other Parts
Open type	45° C	40° C
Protected type	55	50
Enclosed type	60	55
A.I.E.E. Rules	55	50

VOLTAGE AND CURRENT. Usual values of voltage for d-c motors are:

110–125 for small motors on lighting circuits.
220–250 for motors in factories, shops, etc.; on power mains or on the outside mains of a three-wire system.
500–600 for general railway work.
1200 for special railway installations.

The current required for any motor is found by the relation

$$\text{Current} = \frac{\text{Output in hp} \times 746}{\text{Efficiency} \times \text{Voltage}}$$

Usual values for the efficiencies of motors of various sizes are given below.

27. PRINCIPLES

The principles upon which a d-c motor operates are the same as those upon which a d-c generator operates (see Direct-current Generators). These principles are briefly as follows:

FORCE ACTING ON CONDUCTOR. A conductor of length l carrying a current I and placed in a magnetic field having a flux density B is acted upon by a force which is proportional to BlI, which force is in a direction at right angles to the direction of the magnetic flux and at right angles to the length of the conductor.

This in practical form gives the relation

$$T = \frac{p\phi ZI}{852\, m \times 10^6}$$

T = torque of an armature in pounds at 1-ft radius.
p = number of poles.
m = number of armature paths between brushes.
ϕ = flux per pole in armature (lines).
Z = number of active conductors or inductors on armature.
I = current taken by the armature from line.

This torque is exerted whenever a current flows and is independent of the speed. The core-loss and friction absorb some of the torque so that the torque at the pulley is slightly less than the value given by the formula.

COUNTER EMF IN CONDUCTOR. A conductor of length l moving with a velocity V in a magnetic field of density B has induced in it an electromotive force E proportional to BlV. In practice as soon as the armature starts to move a counter emf is induced in its conductors which has the value

$$E = \frac{p\phi Zn}{m \times 10^8}$$

when n = revolutions per second.

Thus as soon as the armature moves, this counter emf tends to stop the flow of current and the impressed emf must be increased to maintain the flow of current.

The relation between current and counter emf is given by the equation

$$E_i = E + IR$$

where E_i = impressed emf; E = counter or generated emf; and R = resistance of armature circuit.

In practice R is made as small as possible so that E and E_i are as nearly equal as possible.

REVERSING ROTATION. From a consideration of the equation for the torque it is evident that torque is proportional to the product ϕI, i.e., to the product of the flux in the armature by the current. If the direction of the current through the armature is reversed, that is, if I becomes negative, the product becomes negative and the torque is in the opposite direction. If the direction of the flux is changed (the armature current

being unchanged) the direction of torque is reversed. But if both ϕ and I are reversed the torque is not reversed. From this follows the rule for reversing the direction of rotation of any d-c motor; viz., change the direction of flow of current in either the field or armature winding but not in both.

SPEED CONTROL. From the equation for counter emf it follows that the speed is proportional to E/ϕ. That is, the speed varies directly as the counter emf and inversely as the flux. Thus to reduce the speed, decrease the counter emf by decreasing the emf impressed on the armature or increase the flux by increasing the field current. To increase the speed perform the converse. To decrease the emf impressed on the armature, a resistance may be inserted between the source of potential and the armature terminals. This is the customary manner of controlling the speed during the starting of motors. (See also below on Starting of D-c Motors.)

28. SHUNT MOTOR

Since the flux in the armature of a shunt motor is practically independent of load, the characteristics of the motor are: approximately constant speed for all reasonable variations of load, torque directly proportional to the armature current irrespective of speed, efficiency high throughout, a wide range of load but for only a small range of speed, see Fig. 3.

DESIGN OF SHUNT MOTOR. The method of design and calculation of shunt motors is the same as for d-c generators (see Direct-current Generators) except for the minor details noted below.

The armature reaction of a motor is in the opposite direction to that of a generator running in the same direction, and thus the field is distorted in the opposite direction. Hence, if the brushes are to be moved to assist commutation, they must be moved in a direction opposite to the direction of rotation of the armature.

The effect of armature reaction is to weaken the field. This causes a tendency to increase the speed and also causes bad commutation.

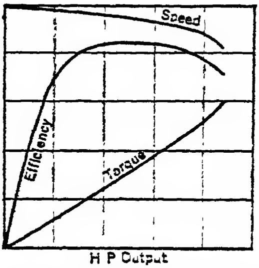

Fig. 3. Speed, Torque, and Efficiency Characteristics of a Shunt Motor

The stability factor of a motor must be greater than that of a generator because when the motor drops in speed as the load comes on, the field must be weakened to increase the speed. Hence the field is liable to be operated at an excitation less than normal.

TESTING OF SHUNT MOTORS. (See also Standardization Rules of the A.I.E.E.). The tests on shunt motors may be divided into two classes: (*a*) commercial, to determine the qualities and serviceability of particular motors; and (*b*) special, to determine the general characteristics and actions of a type of motor. Commercial tests are the following:

Resistance measurements.
Stray power test, including core loss and friction.
Input-output test for heating, efficiency and commutation.
Insulation test.

RESISTANCE MEASUREMENTS are made with the machine cold and later after the heat run. The resistances of the armature winding and field winding are measured, and the brush-contact resistance may be measured but is usually calculated (see Direct-current Generators). For any value of current the resistance losses (RI^2) are calculated from the measured hot resistances.

"STRAY POWER" TEST. The term "stray power" is applied to the lumped sum of the core loss and the loss due to friction, bearings, and windage. The stray power of a d-c machine can be determined approximately by impressing normal voltage on the field and letting the machine run as a motor without load, varying the voltage impressed on the armature from about 10 per cent above normal to about 10 per cent below normal. The speed, armature voltage, and armature current for each adjustment are observed. Then the stray power for any induced voltage is equal to the armature input less the corresponding RI^2, where R is the armature resistance and I the armature current. The value of stray power for any given load on the motor is then equal to the measured value corresponding to the same counter emf E, where E is calculated from the impressed voltage E_i by the relation $E = E_i - RI$.

If it is desired to determine the stray power more accurately by taking into account the effect of armature reaction, the field current may be adjusted so that the speed on

the above run is the same as the load speed. This gives a flux of the same average value as when the machine is under load.

CALCULATION OF EFFICIENCY FROM LOSSES. Let P = total output in kilowatts; R_a = resistance of armature, including brushes; I_a = armature current; I_f = field current; E_i = impressed voltage; S = stray power. Then the per cent efficiency is

$$\epsilon = \frac{100\,P}{E_i(I_a + I_f)} = \frac{100\,P}{P + E_i I_f + I_a^2 R_a + S}$$

INPUT-OUTPUT TEST WITH PRONY BRAKE. The input-output test may be made either by means of a prony or band brake on a pulley, or by using a d-c generator as a load. If the brake test is made the output is

$$\text{Watts} = \frac{PLN}{7.04}$$

where P = net pull in pounds, L = lever arm or radius in feet, N = revolutions per minute.

INPUT-OUTPUT TEST WITH GENERATOR AS LOAD. With large motors it is desirable to use a generator as a load in making an input-output test. In this case it is necessary to know the resistance of the generator armature circuit. It is also desirable to have the generator separately excited and to maintain a constant excitation throughout the entire test.

The input of the motor and the output of the generator, together with the speeds of both machines, are observed. A "counter-torque" test must also be made to determine the belt friction loss and the core loss and friction of the generator. This is performed by making two tests as follows:

(*a*) The motor input is observed when driving the unloaded generator at normal speed first through the regular leather belt and second through a light cotton belt. The difference in input to the motor in the first and second cases gives the belt-friction loss. As this loss is comparatively small, it may frequently be neglected.

(*b*) A regular stray power test (see above) is made on the generator when entirely disconnected from the motor. This gives the core loss and friction of the load machine (generator).

Then for any load during the load run the output of the motor under test is

$$P_1 = P_2 + R_2 I^2 + S + F$$

where P_1 = output in watts of motor, machine 1.
P_2 = output in watts of generator, machine 2.
$R_2 I^2$ = loss in armature winding of generator.
S = stray power of generator for speed and induced voltage at observed load.
F = belt-friction loss.

The ratio of this motor output to the electrical input as observed gives the efficiency of the motor.

HEAT RUN. From the input-output test it is also possible to determine the speed regulation, commutation features and heating. The heat run may also be made by "bucking" two machines as described in the article Direct-current Generators. Small motors will reach a constant temperature in a short time, and the heat run need only last 5 or 6 hours for a 100-hp motor. A thermometer is usually placed on the machine in a safe and accessible place and read every half hour until it indicates no further rise in temperature.

INSULATION TEST. The margin of safety on a 110- or 220-volt motor is usually so great that it is not necessary to make an insulation test. If the motor has been exposed to dampness it may be desirable to make the test after the motor has been thoroughly dried out. The method is indicated in the Standardization Rules of the A.I.E.E. (q.v.).

SPECIAL TESTS. As a special test there may be obtained a saturation curve of the machine and possibly the distribution of potential around the commutator. These are of particular interest in an adjustable-speed motor. In some of these motors with commutating poles there may exist some very high voltages between bars which are not evident except in the bar-to-bar potential test.

29. SERIES MOTOR

Since the flux in the series motor is produced by the load current, the flux increases with the current. The torque is proportional to the product ϕI and therefore increases more rapidly than the current. Thus four times full-load torque can be obtained with from two to three times full-load current.

The characteristics of the series motor are: increase of torque faster than increase of current, variation in speed inversely as the load, and high efficiency throughout a wide range of speed as well as load; see Fig. 4.

DESIGN OF SERIES MOTORS. In general the method of calculation is the same as for a direct-current generator (see Direct-current Generators). The special considerations are:

A series motor is usually designed to have a large output and low speed at the 1-hour rating. At any lesser output the speed will be higher, so the peripheral velocity must be quite moderate at the rated load and speed.

Since the speed is very nearly inversely proportional to the flux the speed curve depends on the shape of the saturation curve, to which very careful attention is paid in designing. By exactly fixing the flux for two extreme values of current the speed for these two values of current is fixed.

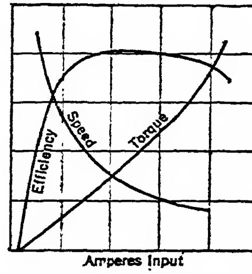

FIG. 4. Speed, Torque, and Efficiency Characteristics of a Series Motor

The relations between the speed and current of a series motor are shown by the formulas:

$$E_i = E + IR$$

$$E = \frac{p\phi Zn}{m\,10^8}$$

$$n = \frac{m(E_i - IR) \times 10^8}{p\phi Z}$$

where E_i = impressed voltage.
E = counter emf induced in armature.
R = total resistance of armature and field.
I = current taken by motor.
p = number of poles.
m = number of parallel paths between positive and negative brush sets.
Z = total number of conductors on armature.
n = speed of armature in revolutions per second.
ϕ = total flux per pole in maxwells.

Since the current in the field of a series motor is the same as that in the armature, the ratio of the turns in each is the same as the ratio of ampere-turns or mmf's. Thus if the mmf per pole of the field is to be 1.5 times that of the armature the number of turns will be 1.5 times the number of turns in series in the armature.

Since a series motor is usually an enclosed motor with a 1-hour rating its rise in temperature and rating are a direct function of the watts lost and the ability of the mass of the motor to store up this heat energy. In a 1-hour run the amount of energy radiated is only about 10 per cent of the amount stored in the mass. For a rise in temperature of 75 deg cent in 1 hour there should be about 0.4 lb of material for each watt of loss. This assumes reasonable provision in the construction of the motor for the transfer of the heat from the armature to the field and frame.

Much attention has been directed recently to the ventilation of these motors by drawing air from outside the motor by means of fan blades on the armature and by circulation of the air inside the motor through definite paths. This has considerably increased the weight efficiency of these motors.

In railway motors, which are the most general application of the series motor, commutating poles are very generally used, as this construction makes it possible to obtain a much greater momentary output from a motor of a given size (see Table 3).

Table 3. Design Constants of a Typical Series Motor for Railway Service

Rated horsepower, 1 hr.	50	Commutating pole turns	58
" voltage	600	Armature reaction, amp-turns per pole	3750
" current	80	Magnetic density in gap	34,000
" rpm	660	Amp × conds per inch, sigma	720
Number of main poles	4	Friction inc. gearing, watts	2400
" " commutating poles	2	Core loss	4000
Armature diameter × length	13.25 × 14	Copper loss	4400
" slots	25	Efficiency inc. gears, at rating	77.5
" conds over slot	30	Resistance of armature	0.322
" winding	S.D.	" " main field	0.190
Useful flux per pole	3.3×10^6	" " commutating field	0.139
Field turns per pole	60	Weight of motor, lb	2600

TESTING OF SERIES MOTORS. (See also Standardization Rules of the A.I.E.E.) To determine properly the speed and torque characteristics of a series motor an "input-output" test must be made, which involves subjecting the motor to actual load and overload conditions. This may be accomplished by running the motor with a prony brake as a load or with a direct-connected generator as a load (see above on Testing of Shunt Motors).

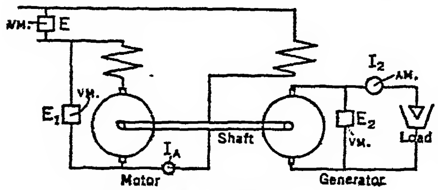

FIG. 5. Railway Motor Load Test

RAILWAY MOTOR TEST. When two similar motors are available the method used by the manufacturers of railway motors is most desirable. The two motors are direct connected, or geared to each other, and the electrical connections made as in Fig. 5. The test is run through by keeping constant rated voltage on the motor and regulating the load on the motor by changing the load on the generator.

As the two machines are operating under almost exactly the same conditions, their efficiencies are very nearly the same. Thus

$$\text{Efficiency of set} = \frac{E_2 I_2}{EI}$$

$$\text{Efficiency of each motor} = \sqrt{\frac{E_2 I_2}{EI}}$$

The speed and torque curves should be made for both directions of rotation of the armature as an incorrect brush setting will give results differing with the direction of rotation. The direction of rotation is changed by reversing the connections of either the field or the armature of the motor.

Commutation is observed during the speed and torque test.

The heat run is made with the same arrangement as the speed-torque test. In making the heat run the motor must start cold or at room temperature. The covers of the inspection openings of railway motors are customarily left open during the heat run.

LOSSES AND EFFICIENCY OF SERIES MOTORS. For a more accurate determination of the efficiency and losses the following special tests are made:

1. Resistance of armature, brushes and field. These tests are similar to those for a shunt motor, see above.

2. Core-loss test. On account of the variable speed and variable field of a series motor this test consists in repeating the usual core-loss test as described above for a generator at several different speeds. The field strength is varied step by step throughout the maximum range for each speed. Fig. 6 shows the curves for these different runs and the dotted line connects the points on the different curves that apply to the normal speed curve of the motor.

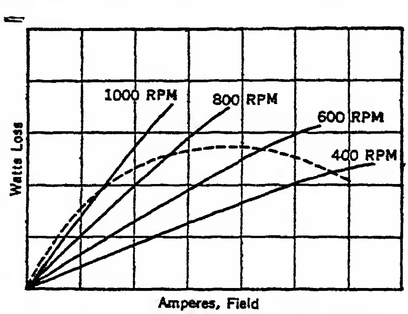

FIG. 6. Core-loss Curves of Series Motor

INSULATION TESTS. See Direct-current Generators and Standardization Rules of the A.I.E.E.

30. STARTING OF DIRECT-CURRENT MOTORS

A starting box, Fig. 7, or rheostat is always employed in starting d-c motors in order to reduce the voltage impressed on the motor when it is not running at a high enough speed to generate the proper counter emf.

Let E_t = line voltage.
I_0 = armature current at full load.
N_0 = speed at full load (shunt motor).
E_0 = counter emf at full load.
I = 1.5 I_0 usually accepted peak starting current.
r = resistance of the armature circuit.
R = resistance of starting box or rheostat.
E = counter emf at any other speed.

Then, in general:

$$\text{Current } \frac{E_i - E}{r + R} \quad \text{and} \quad N = \frac{N_0 E}{E_0}$$

The total resistance in the rheostat on the first step should be:

$$R_1 = \frac{E_i}{I} - r$$

the counter emf at the end of the first step

$$E_1 = E_i - I_0 (R_1 + r)$$

and the constant speed:

$$N_1 = \frac{N_0 E_1}{E_0}$$

For the second step the total resistance in the rheostat should be:

$$R_2 = \frac{E_i - E_1}{I} - r$$

At the end of the second step:

$$E_2 = E_i - I_0 (R_2 + r) \quad \text{and} \quad N_2 = \frac{N_0 E_2}{E_0}$$

Succeeding steps are determined until the value of R_2 decreases to zero. In general for $I = 1.5\, I_0$, we have $1.5\,(R_2 + r) = (R_1 + r)$ and $R_n = 2/3 R_{n-1} - 1/3\, r$.

Fig. 8 shows the sudden rise in current when the resistance is changed and the gradual decrease in current as the speed increases. The number of steps necessary depends upon the ratio of the maximum allowable instantaneous value of the current to the final constant value, upon the value of the armature resistance, and upon the inertia of the load.

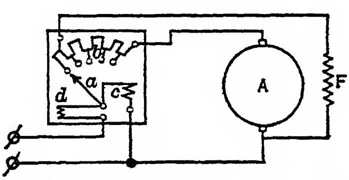

FIG. 7. Starting Box Connections for Shunt Motor

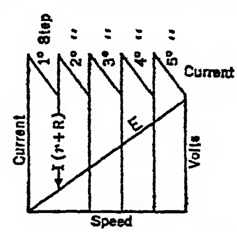

FIG. 8. Motor Current During Starting

STARTING BOX. (See also article on Rheostats.) A starting box usually contains the following features, as indicated in Fig. 7: (*a*) a means of opening and closing the circuit supplying all the current to the motor including the field current; (*b*) a set of resistance steps in series with the motor armature and a means of short-circuiting this resistance step by step; (*c*) a magnet coil connected across the motor terminals to open the circuit if the impressed voltage fails or falls below a specified value (low-voltage release); (*d*) a magnet coil carrying the main current to actuate a spring and open the circuit if the current exceeds a specified value (overload release). The usual connections of a starting box to the line and motor are shown in Fig. 7.

31. SPEED CONTROL

There are three methods of varying and controlling the speed of d-c motors, namely, potential, rheostatic, and field systems.

POTENTIAL CONTROL OR MULTIVOLTAGE SYSTEM. By means of several generators and several wires various definite voltages are made available, such as 240, 180, 120, etc. By connecting the motor to the 240-volt circuit, full speed is obtained; by connecting to the 180-volt circuit, $^3/_4$ speed is obtained, etc. The shunt field circuit is left connected at all times to a circuit of the proper voltage. A shunt motor with normal field excitation will be stable, that is, it will operate constantly, at the fractional speed. The efficiency will be good at the fractional speeds. A series motor controlled in this manner will be unstable, but for a given torque the speed will be roughly proportional to the voltage.

RHEOSTATIC CONTROL. A rheostat in series with the armature will reduce the voltage impressed on the armature by an amount proportional to the current, and thereby

reduce the speed. The speed is unstable with this arrangement, changing with every change of load, and the efficiency is poor.

FIELD CONTROL. By increasing the resistance in series with the field of a shunt motor the speed is increased due to the weakening of the field. If the motor has commutating poles to assure good commutation the speed may be varied in a ratio of 1 to 2, and even 1 to 3 in small sizes. The shunt motor is stable with this method of control and the efficiency is good. In a series motor the field may be shunted by a resistance to increase the speed but the motor is not stable and this practice is not to be recommended.

32. USE OF COMMUTATING POLES (INTERPOLES) IN VARIABLE-SPEED MOTORS

In motors intended to be operated over a large variation in speed, obtained by changing the field strength, and in motors which are to be subjected to heavy overloads, it is necessary to use commutating poles in order to obtain good commutation. In a motor without commutating poles the field strength must always be a certain percentage greater than the armature strength to prevent a shifting of the field flux and of the neutral point. Thus, if, in Fig. 9, F represents the distribution of field flux when existing alone and A represents a strong armature flux existing alone, then R shows the distribution of the resultant flux when both field and armature are excited.

It will be noticed that the neutral point has been shifted from XX at no load in a direction against rotation to YY at the load considered. The brushes would have to be shifted from XX at no load to a point ZZ beyond YY at load in order that they shall commutate a coil in a flux which is producing a voltage helpful to commutation.

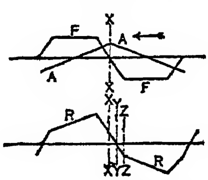

Fig. 9. Flux Distribution without Commutating Poles

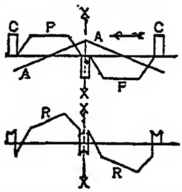

Fig. 10. Flux Distribution with Commutating Poles

RESULTANT FLUX WITH COMMUTATING POLES. If, however, commutating poles are placed between the main poles and excited with the armature current, they will maintain at the geometrical neutral a flux of the direction and value necessary to give good commutation. In Fig. 10, F and A represent the field and armature flux separately as before and C the commutating pole flux that would exist at full load. When at full load these fluxes are combined, there exists the resultant flux shown at R.

It will be noticed that there remains at the neutral point a small flux of the proper polarity and magnitude to provide an emf to reverse the current and give good commutation, and it is not necessary to move the brushes.

The commutating pole must be of the same polarity as the pole towards which the brush would have to be moved if there were no commutating poles. In fact, the principle of commutating poles is nothing more than bringing to the brush a part of the pole instead of moving the brush to the pole. Thus the polarity of the commutating pole is different during motor action from that during generator action. If the windings on the commutating poles are connected in series with the armature the conditions will be correct for either motor or generator action. See p. 8–17.

33. INSTALLATION AND ERECTION

In the installation and erection of a d-c motor there are certain features which must receive careful attention in order that the machine shall operate properly and not deteriorate with undue rapidity. Although this procedure varies with different motors according to their mechanical construction the following brief memorandum of points to be looked after will be found useful:

1. Base bolted down.
2. Bearings clean and filled with oil.
3. Bearings lined up.

4. Magnet frame bolted to base.
5. Field coils secured in place.
6. Field coils tested for open circuit, wrong connection, and polarity.
7. Armature in place.
8. Air gap adjusted by shimming.
9. Measure resistance of armature and field.
10. Measure insulation resistance.
11. Brushes properly fitted and spaced and pressure adjusted to about 1.5 to 2 lb per brush.
12. Commutator smooth and true.
13. Substantial connections of field circuit.
14. Field adjusted for correct direction of rotation.

The motor must be protected from moisture during shipment and if by accident it becomes damp it must be dried out before it is subjected to a voltage.

34. OPERATION

In the operation of a direct-current motor several factors should be considered.

CARE. All motors should be frequently inspected and the following points noted:

1. Bearings filled with proper amount of oil.
2. Brushes securely held in proper position.
3. Brushes fit properly.
4. Commutator smooth: danger of "high mica" or the insulation between commutator bars projecting above the bars.
5. Air gap true.
6. Commutator not worn in grooves.

TROUBLES. In the following paragraphs is given a concise list of the troubles that may be experienced in operating d-c motors and their causes as given by Crocker and Wheeler in Management of Electrical Machinery.

1. *Sparking at the Commutator.* Causes: Armature carrying overload. Brushes improperly spaced. Brushes not at proper position. Rough commutator. Poor brush contact. Internal short or open circuit. Field too weak. Unequal strength of poles. Vibration.

2. *Heating of Commutator and Brushes.* Sparking. Bearing trouble. Bad connections. Brush friction too great.

3. *Heating of Armature.* Overload. Internal short circuit, moisture or ground. Reversed coil. Excessive eddy currents.

4. *Heating of Field.* Internal short circuit.

5. *Heating of Bearings.* Bearings dry or dirty. Shaft out of true. Bearings out of line. Thrust due to belt. Unbalanced magnetic pull.

6. *Noise.* Armature not balanced. Brushes dry or not set at proper angle. Armature strikes.

7. *Speed Too Low.* Wrong voltage. Overload. Armature strikes. Bearing too tight.

8. *Speed Too High.* Wrong voltage. Field too weak.

9. *Motor Stops or Fails to Start.* Overload, open circuit, wrong connection.

35. SPECIFICATIONS

DIRECT-CURRENT MOTORS FOR INDUSTRIAL USE (*See also Specifications for Railway Motors.*) The following memoranda are intended to assist in writing specifications.

Principal Characteristics and Conditions of Service. Use to which motor is to be put, kind of load and method of drive. Voltage. Rating, horsepower. Speed.

Style and Description; Details of Construction. Whether to be open, semi-enclosed, or enclosed. Whether to be series, shunt, or compound wound; if shunt wound, whether shunt field rheostat is to be supplied; if compound wound, state whether cumulative or differential. Requirements regarding pulley or length of shaft. Whether rails are required. Whether starting rheostat is to be supplied, and if so, its general characteristics.

Performance and Tests. (See Standardization Rules of the A.I.E.E.) Temperature rises upon which ratings are to be based. Details of overload. Efficiency at 25, 50, 75, 100, and 125 per cent load; whether rheostat losses are to be included in calculating efficiencies. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Regulation; the supply voltage being constant, and the field rheostat fixed, a variation of load from zero

to ... per cent of full-rated load shall cause a variation of speed not greater than ... per cent. The shunt field rheostat to give speed variation of ... per cent in steps not greater than ... per cent and not less than ... steps, and to carry the current for any speed continuously without undue heating.

SERIES RAILWAY MOTORS. The following memoranda are intended to assist in writing specifications.

Principal Characteristics and Conditions of Service. General statement of the service, giving type of cars, whether current is direct or alternating, or both, etc. The motor shall be designed for normal operation at ... volts and shall operate safely at ... volts.

Style and Description; Details of Construction. The frame shall be designed so as to allow the easy removal of armature and field coils. It shall be provided with openings at both ends, and both above and below the shaft, which will enable the inside of the motor to be readily inspected and cleaned. Bearings shall be designed so that lubricant cannot enter the frame, and shall be so located that they may be easily emptied and cleaned. The diameter of the driving axle on which the motor is to be mounted shall be ... inches. Whether motor is to be of interpole type. Whether natural or forced ventilation.

Brushes and Brush Holder. The brush holders shall be readily removable through the hand holes. The springs holding the brushes against commutator shall not be relied on to carry current. The brushes shall be staggered or provided with adjustment parallel to the armature shaft so as to prevent the formation of ridges on the commutator.

Clearances. The minimum distance between motor frame and back of wheel flanges shall be ... inches, the minimum distance between bottom of motor and top of rail when tires are new shall be ... inches.

Gears and Gear Case (if any). Single or double reduction; what gear wheels shall be mounted on; material of wheel and pinion; description of teeth, whether cut or cast, and width of face of wheel or pinion; gear case material, how suspended, oil-tightness.

Suspension of Motors. General description and requirements, location of lugs on motor frames.

Data to be Furnished by Bidder. The armature will be bound with ... bands. Material and dimensions of the bands. Dimensions of openings in the frame. The brush holders will be adjustable so as to allow ... inch wear with uniform pressure on the brushes, after the diameter of the commutator has been reduced by ... inches. The current density in the carbon brushes will not exceed ... amperes per square inch at normal rated load. The gear ratio will be ...

Performance and Tests. (See also Specifications in article on Electric Locomotives.) Either the nominal rating and the continuous ratings at $^1/_2$, $^3/_4$, and full voltage should be specified, or the following data supplied:

Line voltage ....
Number of motors per car ....
Weight of loaded car, exclusive of motors and control equipment ....
Diameter of driving wheels ....
Schedule speed ....
Distance between stops ....
Duration of stops ....
Acceleration, miles per hr per sec ....
Retardation, miles per hr per sec ....

The engineer should also give a diagram of grades and curves.

Motor Characteristics. The bidder shall submit diagrams showing speed, tractive effort, efficiency, RI^2 losses, core losses, and any other information bearing on the performance of the motor. Requirements regarding the effect of moisture upon insulation.

Tests. The motor shall be tested at the manufacturer's works in the presence of the engineer's inspector. (In the case of new motor developments it is good practice to make the tests under service conditions; but for standard motors a stand test at the factory is sufficient.) A complete series of tests shall be made upon the first motor manufactured under this specification. These tests shall confirm all the statements made by the bidder in relation to operating characteristics. Should the motor fail to comply with any of these statements, the defects shall be corrected and any changes in construction or design which may be necessary to accomplish this shall be made at the contractor's expense. The first motor shall be submitted to a flashing test to determine the susceptibility of the motor to flash over on opening the maximum specified line voltage across the motor when running at maximum speed. After the acceptance of the first equipment, any other motors to be supplied under this specification shall be submitted to an approved stand test. The insulation of the armature windings, commutator and field windings, shall be subjected to stated alternating voltages (see Standardisation Rules of the A.I.E.E.) for a period of 1 minute.

36. PERFORMANCE, WEIGHT, AND SPEED

Usual values of the efficiency and losses, and also values of the weight and speed of shunt and series motors are given in the following tables:

Table 4. Performance of Shunt Motors

Hp	Full-load Efficiency, per cent	Field I^2R, per cent	Friction, per cent	Core Loss, per cent	Armature I^2R, per cent
0.5	70	5	10	5	10
1	79	4	8	4	5
2	82	3.5	7	3.5	4
5	85	3	5	3	4
10	87	2.5	4.5	2	4
20	88	2	4	2	4
25	89	2	3	2	4

Table 5. Weight and Speed of Shunt Motors

Hp	Moderate Speed		Slow Speed	
	Speed, rpm	Weight, lb	Speed, rpm	Weight, lb
0.5	2200	55	....	....
1	2000	105	1675	125
2	1750	200	1175	250
5	1100	470	925	600
10	850	880	700	1100
20	680	1550	550	1900
25	650	1900	500	2300

Table 6. Performance of Series Motors
Commutating Pole Railway Type

Hp*	Full-load Efficiency, per cent	Field I^2R, per cent	Friction, per cent†	Core Loss, per cent	Armature I^2R, per cent
40	79	6.5	5.3	2.2	7.0
50	82	5.0	5.0	2.1	5.9
75	84	4.5	5.0	2.0	4.5
100	85	4.3	5.0	2.0	3.7
150	86.5	3.5	5.0	2.0	3.0

* Horsepower for 75 deg cent rise in 1 hr.
† Friction includes loss in gearing.

Table 7. Weight and Speed of Series Motors
Commutating Pole Railway Type

Hp*	Speed, rpm	Lb, weight per hp †
40	850	50
50	800	48
75	725	40
100	700	35
150	650	30

* Horsepower for 75 deg cent rise in 1 hr.
† Weight includes cast-steel frame and gear pinion and gear case.

Design constants of some d-c motors are given on p. 8-28 under Direct-current Generators.

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SECTION 9

ALTERNATING-CURRENT MACHINES

BY

WALTER I. SLICHTER

ALTERNATING-CURRENT GENERATORS

ART.		PAGE
1.	Classification	02
2.	Design of Salient-Pole Generators	04
3.	Predetermination of Performance	08
4.	Single-phase Generator	15
5.	Design of Turbo-Generators	16
6.	Tests of A-c Generators	21
7.	Examples of Design and Performance	25
8.	Operation	25
9.	Specifications for A-c Generator	27

SYNCHRONOUS MOTORS

10.	General Principles	29
11.	Design and Calculations	31
12.	Tests of Synchronous Motors	34
13.	Installation and Operation	35

POLYPHASE INDUCTION MOTORS

ART.		PAGE
14.	Definitions and Principles of Operation	36
15.	Methods of Rating	38
16.	Design	39
17.	Testing of Induction Motors	47
18.	Calculation of Performance	49
19.	Methods of Starting	53
20.	Speed Control of Induction Motors	55
21.	Installation and Operation	56

SINGLE-PHASE INDUCTION MOTORS

22.	Calculation of Performance	58
23.	Methods of Starting	59

ALTERNATING-CURRENT COMMUTATOR MOTORS

24.	Design	61

ALTERNATING-CURRENT MACHINES

by Walter I. Slichter

ALTERNATING-CURRENT GENERATORS

In all dynamo-electric machines of the usual forms (excepting the homo-polar) the electromotive force induced in any one conductor or turn varies in value and alternates in direction. If the terminals of any coil or group of coils are brought out to an external circuit by means of revolving contacts, such as slip-rings, an alternating current will flow in the circuit. In order to obtain a direct or continuous current it is necessary to add a commutator or rectifier. Thus the most simple and elementary form of electric generator is an a-c generator, and the d-c generator is a special form adapted by the addition of a commutator to give a continuous or unidirectional current.

1. CLASSIFICATION

Alternating-current generators may be classified according to several different distinguishing characteristics. The following classification considers these characteristics in the order of their prominence.

Synchronous Generators. In the synchronous type the action of inducing the electromotive force results from the relative motion of the armature conductors and a constant magnetic field produced by exciting coils in which a continuous or direct current flows. The frequency of the alternating electromotive force depends directly on the number of field poles and the angular velocity of the revolving part.

Induction Generators. In the induction type the magnetic field is of the rotating polyphase type and is produced by polyphase alternating currents flowing in the same windings with the load current. The mechanical construction is usually that of an induction motor with a short-circuited polyphase winding on the revolving member. The frequency depends upon the characteristics of the external circuit. In practice the frequency is determined or " set " by a synchronous machine, either generator or motor, in the external circuit. This " frequency setter " is necessary to supply the exciting current of the induction generator; that is, by means of the frequency setter the power factor of the total load is adjusted to equal the inherent power factor of the generator.

A minority of the units of a station may be of the induction type with advantage as they will cause less disastrous effects in case of a short circuit on the system. Their instantaneous short-circuit current is less. A certain number of the units must be of the synchronous type to " set " the frequency and supply the exciting current for the induction machines. Consequently, if there is a large proportion of induction machines the synchronous machines operate at a poor power factor unless there is much capacitance effect in the load system, such as synchronous motors or line capacitance.

Revolving-field and Revolving-armature Types. In the revolving-field or revolving-armature type there are two, or some multiple of two, poles, each pole having its own coil for the d-c excitation, and the flux in each pole is to all intents and purposes constant in value. The relative movement of the poles and conductors causes the variation in flux interlinkages both in direction and intensity.

The early machines were constructed with an armature revolving inside a stationary field. As sizes and voltages increased it was found that a more effective use of the material could be obtained by making the armature the external member, and that the insulation of the high-voltage member was better preserved if that member was kept stationary.

At the present time all generators of large capacity (500 kw and greater) and for high voltages (600 volts and up) are made of the revolving-field type.

Inductor Type. In the inductor type the d-c excitation is concentrated in one (usually stationary) coil, and the variation in magnetic flux is obtained by revolving a spider with bare projecting poles which alters the reluctance of the magnetic path. Thus the flux threading any particular armature coil is always in the same direction, but varies in intensity or quantity.

There are no windings or insulation on the moving member of the inductor type,

which is therefore adapted to high speeds. The greater variation in the reluctance of the main magnetic path and the consequent variation in magnetic densities in the field structure of the inductor cause excessive eddy currents, hence the construction of this type has been abandoned for machines of large power.

Single Phase, Two Phase or Quarter Phase, and Three Phase. The single-phase generator is usually about 50 per cent heavier and more costly than a polyphase generator of the same rating. By changing the internal armature connections a polyphase machine may frequently be reconnected to be a three-, two-, or single-phase machine.

As transmission by three-phase currents is more economical of copper than by two-phase or single-phase currents, all power transmission lines are three-phase. Consequently unless there is some local condition requiring single- or two-phase, the three-phase generator is preferable.

Two-phase and three-phase generators of the same capacity and voltage strain are of practically the same dimensions, weight, and cost.

Separately Excited and Self-excited Types. For years it was attempted to develop a satisfactory and simple self-excited alternator, and some were very successful but not simple. The object was to obtain a constant voltage on the load by means of automatic self-excitation. However, with the perfection of the automatic voltage regulator the need for automatic self-regulation ceased, and there is now very little demand for self-excited alternators.

METHODS OF RATING. All a-c generators are rated in kilovolt-amperes (kva) and unless otherwise specifically stated are rated at that kva which they will give continuously with a rise in temperature not exceeding certain values depending upon the character of the insulating materials used; see Standardization Rules A.I.E.E.

Machines, or those parts of machines, insulated with mica, asbestos, or similar heat-resisting material, are allowed a rise in temperature of 70 deg cent as measured by thermometer, or 75 deg if measured by resistance. This includes most turbo-alternators, both armature and field, and the fields of many other alternators.

Machines, or those parts of machines, insulated with varnished cambric or impregnated cotton or linen, are allowed a rise of 50 deg cent as measured by thermometer, or 55 deg if measured by resistance. This includes the armatures of most slow and moderate speed machines.

Most machines have their fields designed sufficiently liberally to enable them to give their rated kva at 80 per cent power factor with rated voltage, and this is stated in the specifications. It should be noted that the heating of the *armature* depends upon the kva and not upon the kilowatt load. The lower the power factor of the load connected to the alternator, the greater the heating of the *field coils* for the same kva output.

VOLTAGE. Alternators are now built to generate voltages up to 22,000 and 30,000 volts between lines, either single phase or polyphase. Above that voltage the extra cost of the insulation and the danger of damage from discharges cause it to be less expensive to install transformers with a lower voltage alternator. Some engineers consider the limit of economical voltage of generators to be even lower than 22,000 volts.

FREQUENCY. The frequency depends upon the speed of rotation and the number of poles. If the speed of rotation of the revolving part is given in revolutions per minute, the frequency is

$$f = \frac{\text{rpm}}{60} \times \frac{\text{Number of poles}}{2}$$

In the early alternators it was found much more economical to run at high speeds, and thus high frequencies such as 133 and 125 cycles per second were customary; but, as systems increased in size and complexity, electrical difficulties arose as a result of these high frequencies, and the tendency has been to reduce the frequency till now we have 60, 50, 40. and 25 cycles per second as usual frequencies, of which 25 and 60 are standard in this country. In Europe 50 cycles is used instead of 60, and 15 or 16 cycles for railway work.

A frequency of 25 cycles is preferable where there is a very long transmission line, on account of the lower inductive voltage drop, and where there is much synchronous machinery, such as synchronous motors and rotary converters, as these are more stable and better adapted to parallel operation at low frequencies.

A higher frequency (60 cycles in the United States and 50 in Europe) is preferable for electric lighting, as the light is steadier. The higher frequency is also preferable where many transformers are used, as these are cheaper and more efficient at the higher frequencies.

PHASE AND LINE VOLTAGES AND CURRENTS. For moderate voltages, up to 3000 between lines, the method of connection is decided by such details as which

will give a convenient number for the conductors per slot for the required voltage, but for higher voltages the Y connection is usually chosen as it causes a lesser strain on the machine insulation for a given voltage between lines. See also Art. 7.

Single Phase. In a single-phase generator the voltage per phase is the same as the voltage between lines, and the current per phase is the same as the current per line, the product of voltage and current giving the volt-ampere rating of the machine.

Two Phase or Quarter Phase. In a properly designed machine of this type each phase supplies half the rating, thus the voltage and current per phase in the machine are respectively the same as the voltage and current per phase on the line. The current is equal to 1000 × (kva) ÷ 2 × volts per phase.

Three Phase. Machines of this type may be connected either Y or Δ. In a Y-connected machine the current per phase is the same as the current in each line and is equal to 1000 × (kva) ÷ $\sqrt{3}$ × (volts between lines), while the voltage per phase is (volts between lines) ÷ $\sqrt{3}$.

In a Δ-connected machine the line current is equal to 1000 × (kva) ÷ $\sqrt{3}$ × (volts between lines), and the current per phase is equal to (line current) ÷ $\sqrt{3}$, while the voltage per phase is equal to the voltage between lines.

2. DESIGN OF SALIENT-POLE GENERATORS

The procedure in designing an a-c generator for a given power output, voltage, and frequency to fulfil given requirements regarding regulation, efficiency, etc., is partly analytical and partly empirical. The data of five specific designs are given in the section on Examples of Design and Performance, Art. 7. The method of procedure is to lay out a preliminary design from rough calculations, calculate the performance of this design, modify the preliminary design where this calculation indicates, recalculate the performances, etc., until a design is arrived at which meets the given requirements.

DEFINITIONS. The following terminology is used in the discussion of the design of generators:

Pole Arc. The arc subtended by one pole face, measured along the periphery of the armature; in the following discussion it will be expressed in inches.

Pole Pitch. The arc measured along the periphery of the armature from the center of one field pole (N-pole, say) to the center of the next field pole (S-pole); in the following discussion it will be expressed in inches.

Ampere-conductors per Inch of Armature Periphery. The product of the number of conductors per inch measured along the periphery of the armature by the effective amperes flowing in each conductor.

Slot Pitch. The distance in inches measured along the armature periphery between the centers of two adjacent slots. The slot pitch is equal to the pole pitch divided by the number of slots per pole.

Coil Pitch. The number of slots spanned by a coil; that is, if a coil has one side in slot 1, and the other side in slot 7, the pitch of the coil is 7 − 1 = 6. A coil which spans a distance exactly equal to the pole pitch is said to have a "full pitch," but if it spans a lesser distance it is said to have a "fractional pitch." For example, if there are six slots per pole and a coil has one side in slot 1, and the other side in slot 5, it is said to have a fractional pitch of $^4/_6$ or $^2/_3$.

Leakage Factor. The ratio of the flux per pole which enters the armature core to the total flux (including the leakage flux) which would be produced by the field winding. See page 9–09.

PRELIMINARY CALCULATION OF MAIN DIMENSIONS. Let

P_0 = output in kilovolt-amperes (kva).
ρ = ratio; pole arc divided by pole pitch.
B = average magnetic flux density per square inch in air gap.
σ = ampere-conductors per inch of periphery.
V = peripheral velocity in feet per minute at gap.
D = diameter of armature in inches.
L = length of armature along shaft in inches.
N = revolutions per minute.
p = number of poles.
f = frequency in cycles per second.

Then the following relation holds for either a single-phase or a polyphase machine, other than turbo-alternators:

$$P_0 = \frac{\rho\sigma BVDLk_2 k_3}{144 \times 10^9} \qquad (1)$$

Five of the six design constants in the right-hand member of this equation are subject to choice; the equation then fixes the sixth constant. The choice of the values of the various constants should be based upon modern practice, an idea of which is given below in the following paragraphs. For values of the constants k_2 and k_3 see the section below on Predetermination of Performance.

For turbo-generators see section below on Design of Turbo-alternators.

Ratio of Pole Arc to Pole Pitch (ρ) is governed by two antagonistic phenomena; for the sake of small magnetic leakage and good form factor of emf wave a low value is desired; for the sake of low reluctance to the main flux and economy of material a high value is desired. The usual values are ranged from 0.6 to 0.7.

Magnetic Flux Density (B) in the air gap is limited by the exciting ampere-turns required to produce it, length of gap, and sometimes by the heating resulting from core loss. Usual values B are given in the accompanying table.

Cycles	Lines per Sq In
25	44,000 to 58,000
60	36,000 to 50,000

Ampere-conductors per Inch of Periphery (σ) is limited by the heating resulting from high current densities. If ventilation is good, insulation thin, or the slots very deep, as in armatures of large diameters, the values of σ may be high. Usual values of σ for continuous rating and for a rise in temperature of 50 deg cent are:

Ampere-conductors per Inch of Periphery

Diameter of Armature, in.	Less Than 2000 Volts	2000–7000 Volts
0–30	300– 400	....
30–100	400– 600	....
100–200	800–1200	600–1000
200 and up	....	800–1200

With special provisions for ventilation σ may be much greater, as, for example, in turbo-generators (q.v.).

The Peripheral Velocity (V) is determined by the mechanical design. Values run up to 6000 ft per min with no special features of mechanical design, from 6000 to 10,000 in salient-pole machines with special features, from 20,000 to 30,000 in turbo-generators with solid cylindrical rotors of high-grade steel.

The Diameter of Armature depends upon the peripheral speed and the revolutions per minute of the revolving part (N):

$$D = \frac{12\,V}{\pi N}\ \text{inches}$$

The Length of Armature (L)

$$L = \frac{144\,P_s\,10^9}{p\sigma BVDk_2\,k_3}\ \text{inches}$$

All the quantities on the right-hand side of the equation have been set by the preceding considerations, therefore this equation gives a reasonable value for the length.

Number of Poles (P) **and Revolutions per Minute** (N). This is determined by the frequency:

$$p = \frac{120\,f}{N}$$

Flux per Pole

$$(\Phi) = \frac{\pi\,DLB}{p}\ \text{maxwells}$$

Number of Turns in series per phase (S)

$$S = \frac{E\,10^8}{4.44\,f\,\Phi k_2\,k_3}$$

where E = voltage per phase.

This gives a complete, but preliminary and approximate, idea of the proportions of a machine for a given specification.

ARMATURE WINDINGS. Although there are many forms of armature windings which may be used, there are practically only two forms in general use in this country. These are the " chain winding " and the " lap winding."

Chain Winding (Fig. 1). This winding is characterized by having a number of coils equal to half the number of slots; that is, there is only one side of a coil in each slot. The coils may be either form wound and insulated, in which case slots with open faces are

required; or they may be wound by hand in place, in which case partly or entirely closed slots may be used.

There are at least two kinds of coils in each machine. These are characterized by the shape of their end connections, as these end connections lie some in one plane and some in another. If there is more than one slot per pole per phase, there must be four different shapes of coils, having two different pitches; the coils of any one phase per pole are placed concentrically. The chain winding is used in the older small high-voltage machines, 2000 volts or higher.

Lap Winding (Fig. 2). This winding contains a much larger number of coils, all of the same form and size. There may be two, four, six, etc., sides of coils in each slot,

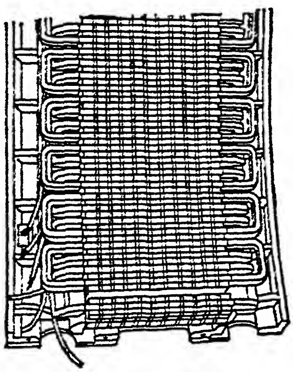

Fig. 1. Chain Winding

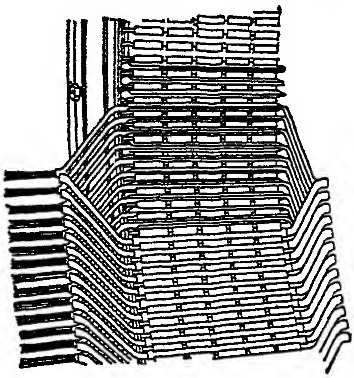

Fig. 2. Lap Winding

the coils being placed side by side two coils deep. This winding is similar to the multiple-drum winding of a d-c armature. With two coil sides per slot the slots must be open, but with more than two coil sides per slot the slots may be partly closed. This type of winding is very convenient if there are a large number of slots per pole.

The coils are connected in groups of two, three, etc., per pole per phase, depending upon the slots per pole per phase. Successive groups or poles of the same phase are connected in series or multiple by "pole connections."

In both forms of winding each coil may contain from one to many turns in series.

NUMBER OF SLOTS. The number of slots per pole on most polyphase machines is usually a multiple of the number of phases as three, six, etc., and this gives a simple winding, but more recently the practice has been introduced of making the number of slots per pole prime to the number of phases in order to provide a better wave shape.

The total number of slots on the armature must be a multiple of the number of phases in order to have equal voltages in the different phases. An example would be a three-phase, four-pole machine with a total of 42 slots or 10.5 per pole but 14 slots in each of the three phases. These would be connected with 14 coils in series in each phase in accordance with the following table:

Phase	Coils per Phase			Coils per Pole
	I	II	III	
Under pole 1...........	4	3	3	10
Under pole 2...........	3	4	4	11
Under pole 3...........	3	3	4	10
Under pole 4...........	4	4	3	11
Total..............	14	14	14	42

All the coils of each phase should be connected in series because the voltages of the separate groups differ both in magnitude and phase. But this winding for these four poles may be connected either in series or in multiple with the winding of another four poles in an eight- or twelve-pole machine (see accompanying table).

Size of Conductors and Slots. The size of each conductor is a compromise between a size which will give a reasonable current density in the conductor, and a size which will properly fill a reasonable slot (see discussion of magnetic circuit below). The current density in the copper conductor is from 2500 amp per sq in. in small machines to 2000 in medium and moderate voltage and 1600 in high-voltage machines.

The conductors are arranged in the slots so that the slots are usually about four times

as deep as they are wide. The size of the slots should be such as to allow a space for insulation over and above the space occupied by the conductors and such cotton covering as they may have. The space required for this extra insulation in straight slots is as follows:

For example, the necessary vertical depth of slot may be found in a machine insulated for 2200 volts with 2 coil sides per slot by multiplying the over-all diameter of cotton-covered wire, or depth of bar, by the number of conductors in a vertical row and adding 0.43 to the produce.

Voltage	Slot Factor	Insulation, in.	
		Vertical	Horizontal
500	0.5	0.350	0.140
2,200	0.42	0.430	0.180
4,400	0.38	0.75	0.250
6,600	0.35	1.00	0.32
13,000	0.30	1.50	0.55

Slot Factor. The ratio of cross-section of copper in a slot to cross-section of slot is known as the slot factor. Normal values of the slot factor for a 1000-kw machine are given in the accompanying table. For smaller machines the slot factor is slightly less and for larger machines slightly greater.

FIELD WINDING. The field winding is usually not calculated until a design of the armature and magnetic circuit has been adopted which will give the required regulation (see Art. 3). The field winding is designed for that value of excitation, ampere-turns per pole, which corresponds to that specified output which demands the most of the field, for instance, the highest value of kva output at the lowest power factor. The usual severe condition is to require rated kva output at an inductive power factor of 80 per cent. An anti-inductive load relieves the field of some of its duty. The correct cross-section of field conductor is definitely set by the relation:

$$q = \frac{0.01\,(\text{mlt})\,Fp}{12{,}000\,E_x} \quad \text{square inches}$$

mlt = mean length of a turn in the field coils, inches.
F = the specified excitation in ampere-turns.
p = the number of poles.
E_x = a specified d-c voltage on the field circuit.

The mean length of a field turn can be estimated from the dimensions of the magnet core. The value of F is determined as in preceding paragraphs. The value of E_x is usually conservatively assigned such as that the machine shall be capable of giving rated kva at 80 per cent power factor with 80 per cent of the rated exciter voltage. The number of field turns per pole is determined by the amount of power (watts) that can be dissipated by the surface of the coils or that it is desirable to dissipate from the point of view of efficiency. The more turns used, the less power is wasted at the assigned voltage. The lower limit of power or upper limit of turns is usually set by space limitations.

$$t = \frac{E_x F}{w}$$

where t = field turns per pole and w = allowable watts in the field, all poles.

The actual resistance of the field winding at 75 deg cent is:

$$R_f = \frac{0.01\,(\text{mlt})\,t\,p}{12{,}000\,q} \quad \text{ohms}$$

The actual excitation loss (not including that in the field rheostat) is $I_f^2 R_f$, in which I_f is the field current for a specified load, $I_f = F/t$. The heating of the field coils is determined by the outer cylindrical surface of the coils: $A = 2c(f + b + 4d)$ (see Fig. 3),

	Peripheral Speed, ft per min	Temp. Rise, °C per watt per sq in.
Revolving Field	1,000	55
	2,500	40
	5,000	25
	10,000	15
Revolving Armature	1,000	80
	2,500	60
	5,000	45
	10,000	30

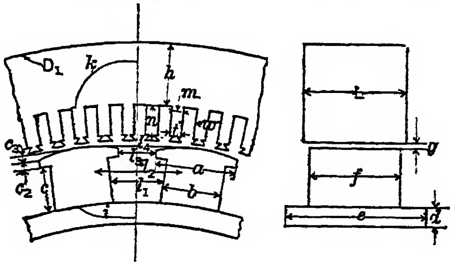

FIG. 3. Dimensions of Magnetic Circuit

and d is the depth of winding in a field coil. The watts per square inch of surface is given by $\frac{I_f^2 R_f}{p A}$.

Temperature Rise of Field Winding. The rise in temperature per watt radiated per square inch of surface of a field coil depends on the peripheral speed of the revolving field or revolving armature. The accompanying table gives the temperature rise in degrees centigrade per watt radiated per square inch. For other rates of radiation the temperature rise is proportional.

3. PREDETERMINATION OF PERFORMANCE OF SALIENT-POLE GENERATORS

The calculations for a quarter-phase or a three-phase machine are very similar and are given together in the paragraphs immediately following. The special features of a single-phase machine are described below. Examples of specific designs and the tested performance are also given below.

Magnetization Curve. The first step is the calculation of the magnetization curve, i.e., a curve showing the relation between the voltage per phase at no load (at normal speed) and the field ampere-turns. To do this the useful flux per pole corresponding to any given value of the no-load voltage is first calculated, and then the field ampere-turns per pole required to produce this flux are determined. A sufficient number of points to give a magnetization curve up to 20 per cent above rated voltage should be calculated.

Useful Flux (i.e., the flux cut by the armature conductors). Let

E = a given value of the terminal voltage per phase with zero armature current.
f = frequency in cycles per second.
S = number of turns in series per phase.
k_1 = a constant, depending on the shape of the pole shoe, which may be called the "pole shoe constant."
k_2 = a constant, depending on the distribution of the armature winding, which may be called the "winding-distribution constant."
k_3 = a constant, depending on the pitch of the armature coils, which may be called the "pitch constant."

Then the useful flux per pole entering the armature is

$$\phi = \frac{E \times 10^8}{4.44\, k_1\, k_2\, k_3\, f S} \tag{2}$$

Pole Shoe Constant (k_1). This constant is proportional to the form factor of the flux distribution around the periphery of the armature. In all modern machines the pole faces are so shaped that this distribution is practically a sine wave, and therefore $k_1 = 1$. This is usually done by making the air gap at the pole tips greater than at the center of the pole.

One method of doing this is to make the outline of the pole face a portion of a circle of such a radius (less than the radius of the armature in revolving field machines) that the gap at the tips is twice the gap at the center.

For very accurate predeterminations the distribution of the flux is carefully calculated. (See S. P. Thompson, Dynamo Electric Machinery, Vol. II, p. 206; C. A. Adams, *Trans. A.I.E.E.*, Vol. 33.)

Winding Distribution Constant (k_2). This constant allows for the fact that if there is more than one slot per pole per phase, the conductors in the various slots of one phase under a pole do not generate emf's of exactly the same phase. Since the slots pass under the pole consecutively the emf's of the conductors reach a maximum consecutively. These emf's must therefore be combined vectorially and not merely added together. k_2 is different in single-phase, two-phase, and three-phase machines.

Slots per Pole	Value of k_2		
	1 phase	2 phase	3 phase
1	1.000	.....	.....
2	0.707	1.000	.....
3	0.666	.....	1.000
4	0.65	0.925	.....
6	0.64	0.912	0.966
8	0.64	0.905	.....
9	0.64	.....	.....
12	0.64	0.90	0.960
18	0.64	0.90	0.960
24	0.635	0.90	0.958
∞	0.632	0.90	0.958

The table at the left gives the values of k_2 for uniformly distributed windings with equally spaced slots.

These constants apply to a winding uniformly distributed around the armature periphery. This is always the condition in a two- or three-phase machine. A single-

phase machine usually has its working winding irregularly distributed so the above constants are only of theoretical interest. (See special treatment of single-phase machines below.)

Pitch Constant (k_3). It is sometimes desirable to use coils having a fractional pitch particularly in machines of large pole pitch, in order to save copper and I^2R loss, and also in any machine to give a particularly good wave shape. When this is done each turn connects in series two conductors generating emf's which are not in phase and their resultant is therefore not as great as their arithmetic sum. The constant k_3 is the ratio of this resultant or vector sum to the arithmetic sum of the two emf's. The relation between k_3 and winding pitch expressed as a percentage of the pole pitch is given in the accompanying table.

Per Cent Pitch	k_3
100	1.00
83	0.97
80	0.95
75	0.93
67	0.87
50	0.71

Leakage Factor (ν). The leakage factor, i.e., the ratio of the total flux produced by the field to the flux which enters the armature, may be determined by calculating the permeance of the path of the armature flux and the permeance of the various leakage paths. The sum of the permeances of the main and leakage paths, divided by the permeance of the main path, is then the value of ν.

Referring to Fig. 3, the average radial depth of the air gap is approximately $1.25\,g$, and the reluctance of the gap is therefore $1.25\,g/af$. The reluctance to the useful flux of the iron part of the magnetic circuit is about 20 per cent of that of the gap in 60-cycle machines, and 40 per cent in 25-cycle machines. Hence the approximate value of the permeance of the path of the main flux is

$$P_0 = \frac{af}{1.5\,g} \text{ for 60 cycles} \quad \text{and} \quad P_0 = \frac{af}{1.75\,g} \text{ for 25 cycles.}$$

The flux emanating from or entering each side of a pole has a path l_1 inches long and cf square inches in cross-section. The permeance of this path to the plane midway between a pair of poles is $2\,cf/l_1$. There are two of these paths in multiple, one in each direction, from opposite sides of each pole. The total permeance of this path per pole is therefore $4\,cf/l_1$. If a uniform mmf acted on this path at all points the flux would be proportional to this permeance, but since the mmf varies from 0 at the yoke to the full mmf per pole at the pole shoe, the average mmf is one-half the mmf per pole. The leakage flux through this path is therefore proportional to the mmf per pole and to one-half this permeance. Hence the "effective permeance" of this path is

$$P_1 = \frac{2\,cf}{l_1}$$

The same reasoning applies to the flux which leaks out from the end surface bc (see Fig. 3) of the poles, giving as the effective permeance of this path

$$P_2 = \frac{2\,bc}{l_1 + b/2} = \frac{2\,bc}{l_2}$$

The leakage between the pole tips is due to the total mmf per pole, and the permeance of this path is therefore

$$P_3 = \frac{4\,c_2 f}{l_3}$$

The leakage from the faces of the poles at the chamfer is also due to the total mmf, and the permeance of this path is

$$P_4 = \frac{4\,c_3 f}{l_4}$$

The leakage factor is then

$$\nu = 1 + \frac{P_1 + P_2 + P_3 + P_4}{P_0} \tag{3}$$

Usual Leakage Coefficients

Salient-pole Machines

Pole Pitch, in.	Leakage Coefficient
6	1.4
12	1.3
18	1.25
24	1.2
36	1.15
42	1.1

Ampere-turns. The ampere-turns required to produce the useful flux ϕ may be calculated by the following systematized procedure. The symbols are: ϕ = useful flux per pole; ν = leakage factor; l = effective length of armature iron; s = pitch of slots at gap; D = diameter of armature at gap; D_1 = outside diameter of armature iron. Other dimensions as shown in Fig. 3. All dimensions are in inches.

Part	Flux	Cross-section, A	Flux Density = flux, A	Ampere-turns per Inch, m *	Length of path, λ	Total Ampere-turns $= m\lambda$
Field yoke	$0.5\ v\phi$	dc	......	......	i	......
Field pole	$v\phi$	$0.95bf$	......	......	c	......
Air gap	ϕ	af	......	......	See below	......
Armature teeth	ϕ	See below	......	......	n	......
Armature core	$0.5\ \phi$	hl	......	......	k	......

* The value of m is found from the B–H curves in the article on Magnetic Properties when the flux density has been calculated.

Usual Magnetic Densities

Part	Density
Air gap, 25 cycles	44,000 to 58,000
60 cycles	36,000 to 50,000
Armature teeth	90,000 to 100,000
Armature core	50,000 to 70,000
Field pole	90,000 to 100,000
Field yoke	70,000 to 80,000

Field Yoke. The field yoke carries only half as much flux as the pole piece, as the flux divides at this point. The material is usually cast or forged steel. Find the magnetic density as indicated, and refer to the proper magnetization curve to find the ampere-turns magnetizing force per inch for this density. The length of path is one-half the distance from the center of one pole to the center of the adjacent pole and is shown by i in Fig. 3. Find the total ampere-turns as indicated.

Field Pole or Magnet Core. This carries all the flux. The material is usually sheet steel of high permeability, but sometimes solid steel (formerly). The factor 0.95 is used for laminated steel poles; for solid poles this factor is of course unity. The length of this path is usually taken as c, the length of the space for the field spool. This is not strictly accurate, but the density in the pole shoe (c_1) is so low that the excitation required for this part is negligible.

Air Gap. Not all the flux in the pole piece crosses the gap, as some leaks across the interpolar space. The value of the flux in the gap is the same as the useful flux in the armature. For the area of gap section the cross-section (af) of the pole shoe is taken. The length of the path in the air gap is taken as the mean of the gap length. As the maximum length (at the pole tips) is usually made twice the minimum, and the outline of the pole face is made the arc of a circle, the average length is usually 1.25 times the minimum. The ampere-turns per inch are 0.313 × (density per square inch).

The minimum clearance between armature core and field poles in modern generators is given in the accompanying table.

Armature Diameter, in.	Minimum Air Gap, in.
40	0.15
80	0.20
120	0.30
160	0.40
200	0.50
240	0.60

Armature Teeth. The flux is confined at any one time to a certain portion of the teeth per pole known as "teeth under one pole." Since the flux spreads somewhat on leaving the pole piece it is logical to assume that it takes up a peripheral length equal to the pole arc plus twice the length of the air gap. If, therefore, this length is divided by the pitch of teeth at the periphery of the armature, the quotient is the average number of teeth carrying flux at any given instant. This figure may quite properly contain a *fractional* number of teeth. The teeth being wedge shaped or sectors of a circle, that cross-section (not the mean cross-section) which will give the average excitation must be chosen, since saturation increases more rapidly than the cross-section decreases. A good approximate value is found at a point one-third the distance from the minimum width towards the maximum width.

The effective cross-section of the teeth is then equal to this width multiplied by the product of the effective length of the core by the number of the " teeth under one pole." The effective length of core is the net length of iron in the core after deducting the space occupied by air ducts and insulation between sheets of steel. This latter is usually 5 per cent of the measurable length of iron. The effective length (l) = 0.95 × (total length of armature core less space occupied by ducts).

Armature Core. The flux divides again in the armature core, one-half the useful flux being in each section of the armature core. The core is made of annealed sheet punchings. The cross-section of core is equal to the radial depth of core back of the

slots multiplied by the effective length of iron in the core. The length of path is a little greater than one-half the pitch of poles at this radius, and is as shown at (*k*), Fig. 3.

Open-circuit Saturation Curve. A number of points, for various voltages from 50 up to 120 per cent of rated voltage, are calculated as above and a curve is plotted with volts per phase as ordinates and mmf in ampere-turns per pole as abscissas. This is the designer's curve. For commercial purposes the scales are volts between terminals and field current in amperes. The latter is obtained as on page 9–07.

ARMATURE RESISTANCE AND COPPER LOSS. The length of wire in one phase of the armature winding is calculated from the mean length of a turn and a number (S) of turns in series per phase. A turn consists of two straight portions imbedded in the slots, and the end connections at the two ends. A fairly accurate empirical method of figuring this is:

$$(\text{m.l.t.}) = 2L + \frac{10\,D_w\,k_w}{\text{poles}}\ \text{inches}$$

where L = total length of armature core.
D_w = pitch diameter of the winding along a circle at the center of the slots, i.e., diameter of the armature at the face plus the depth of a slot.
k_w = fractional pitch constant, i.e., coil pitch divided by the pole pitch.

Let $q \equiv$ the cross-section of one conductor (square inches) and $m \equiv$ the number of these conductors in parallel in one phase; then the resistance per phase of the armature at 75 deg cent is:

$$r = \frac{0.01\,S(\text{m.l.t})}{12{,}000\,q\,m}\ \text{ohms}$$

(0.01 is the resistance of a copper conductor of 1 sq in. cross-section and 1000 ft length at 75 deg cent.)

In a Y-connected armature the resistance between terminals is twice the resistance per phase, and in a delta connection it is two-thirds the resistance per phase. The total copper loss in the armature at I amperes per phase is $3\,I^2 r$ in a three-phase winding and $2\,I^2 r$ in a two-phase winding. Usual values are given in the table on page 9–14. The percentage Ir or resistance drop is $\frac{100\,Ir}{E}$. For reasonable heating the $I^2 r$ loss in each coil at rated load should be between 0.25 and 0.40 watt per square inch of actual outside surface of the coil itself. Surface = $(u + 2w) \times$ (m.l.t.). See Fig. 4.

ARMATURE LEAKAGE REACTANCE. The load current in flowing through the armature conductors sets up a local magnetic flux which interlinks with the conductors, thus producing inductance. The inductance, L, of any circuit is:

$$L = 3.2\,S^2\,P \times 10^{-8}\ \text{henrys}$$

where S = number of turns in series and P = permeance of the flux path in inches.

The corresponding reactance is $x = 2\,\pi f L$, where f is the frequency. This reactance causes a loss of voltage and a "dephasing" effect or lag of current behind the emf.

There are several parallel paths for this armature leakage flux, each path surrounding one or more conductors or slots.

1. That crossing the slot and passing through the windings. Here each line of flux interlinks a different number of conductors, and the average interlinkages must be calculated.

2. That crossing the upper part of the slot above the windings. All this interlinks all the conductors in one slot.

In the case of slots with full openings as usually employed in generators the effective permeance or equivalent flux per ampere-conductor for these two paths is (see Fig. 4):

$$P_s = 3.2\,L\left(\frac{u}{3w} + \frac{p}{w}\right)$$

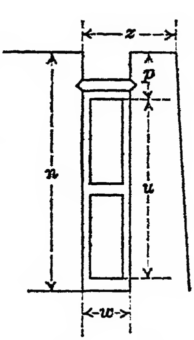

Fig. 4. Stator Slot Dimensions

3. That passing along the air gap from tooth-tip to tooth-tip. This is known as the "belt leakage" as it is caused by the combined mmf of the several slots in one phase belt.

$$P_t = \left(2.35\log_{10}\frac{\pi z}{2w} + A\right)L$$

where A is a constant depending upon s_1, the slots per pole per phase, as follows:

s_1.......	1	2	3	4	5	6	7	8	9	12	15	18
A.......	0	0.35	0.70	1.1	1.6	2.1	2.7	3.2	3.8	5.4	7.1	8.8

4. That surrounding a group of end connections of the coils per pole per phase and interlinking all the coils of this group. This is known as the "end connection leakage."

$$P_c = 0.58\, s_1\, l_c \log_{10} \frac{2l_c}{U_c}$$

for a double layer winding,

where l_c = length of end connections at one end = $1/2$ (mean length of a turn) $- L$.

$U_c = u + 2\, s_1 w$ = the perimeter of a coil.

Let the total permeance of the flux path around the group of conductors forming one side of a coil be $P = P_s + P_t + P_c$; then the reactance in ohms per phase is:

$$X = 2\,\pi f p s_1\, c^2\, k_2\, k_3\, P\, 10^{-8}$$

where p = number of poles.

c = effective conductors per slot $= \dfrac{\text{conductors per slot}}{\text{paths in parallel}}$.

f = frequency.

k_2 and k_3 = winding and pitch factors as on page 9–08.

s_1 = slots per pole per phase.

The reactance drop in volts is IX, where I is the current per phase. For purposes of comparison the "percentage reactance drop" is more significant. This is $100 \dfrac{IX}{E}$. It ranges between 10 and 20 per cent for 60-cycle generators and between 8 and 15 per cent for 25-cycle machines.

ARMATURE REACTION. When current flows through the armature the armature winding becomes the seat of a mmf which reacts on the field mmf and either distorts or diminishes the useful flux. If the current in the armature is in phase with the generated emf it causes a "cross magnetizing" force acting along an axis passing midway between any pair of poles.

As the phase of the current in the armature changes, the direction of the magnetizing force due to the armature mmf shifts, and a component is introduced either opposed to the field magnetizing force (for lagging current) or assisting the field magnetizing force (for leading current). In a polyphase generator the magnetizing force due to the armature mmf for a given armature current is constant in magnitude and has a fixed direction with respect to the field magnetizing force, depending on the phase of the current.

If the field iron had no polar projections or intepolar spaces, the direction of the armature magnetizing force would be such that the angle between this magnetizing force and a line perpendicular to the field magnetizing force would be equal to the phase angle between the induced voltage and current.

With the usual type of alternator having interpolar spaces the reluctance of the path offered to the armature mmf is intentionally much greater than that offered to the field mmf. The result of this is that the effect of armature reaction is minimized. The non-uniformity of the field iron, however, changes the relative directions of the two magnetizing forces and renders accurate calculations difficult. As a rule, however, the approximation resulting from the assumption of uniform distribution of field iron is sufficiently accurate. The error introduced by this assumption is on the safe side, since the armature reaction as thus calculated is greater than its actual value.

Armature Reaction Ampere-turns per Pole. Let S = number of turns in series per phase; I = effective value of armature current per phase; p = number of poles, and k_3 = pitch constant of the winding (see above under Magnetization Curve). Then the armature ampere turns per pole effective in producing armature reaction are

$$\frac{\sqrt{2}\, k_2\, k_3\, SI}{p} \quad \text{for two-phase machine}$$

$$\frac{1.5\,\sqrt{2}\, k_2\, k_3\, SI}{p} \quad \text{for three-phase machine}$$

As noted above, the armature reaction for a given effective value of the current is constant in magnitude. The armature reaction in single-phase generators is given in the discussion of these machines below.

The armature reaction ampere-turns of different machines at full rated load is greater for a high than for a low pole pitch, and is less for high frequencies than for low frequencies. The permissible armature reaction ampere-turns depend, of course, upon the desired regulation.

A reasonable value for armature reaction would be between 1500 and 5000 amp-turns per pole for a 25-cycle machine and between 1000 and 4000 for a 60-cycle machine.

SYNCHRONOUS REACTANCE. The effect of the armature leakage reactance and armature reaction upon the terminal voltage of a generator at given field excitation are of like character, since the voltage induced in the armature due to each of these causes is in quadrature with the current. The "synchronous reactance" of a generator is the equivalent reactance which would produce the same effect as the armature leakage reactance and armature reaction combined.

The synchronous reactance may be predetermined by finding the excitation ampere-turns (from magnetization curve) corresponding to a voltage equal to the drop due to leakage reactance and adding to these ampere-turns the armature reaction ampere-turns. The voltage from the magnetization curve corresponding to this sum divided by the armature current per phase gives the synchronous reactance per phase.

REGULATION. The regulation of a generator machine is defined as the ratio of the difference in terminal voltage at no load and at full load to the full-load voltage, the field excitation being kept constant at its full-load value. Expressed as a percentage the regulation is $100(V_0 - V) \div V$, where V is the full-load terminal voltage and V_0 the no-load terminal voltage at full-load field excitation.

Several different methods are employed for calculating the regulation. One known as the "electromotive-force method" uses the synchronous reactance and is very convenient but not very accurate, being pessimistic. It is used for qualitative methods and for predicting the curves of the general characteristics. The second is the "magnetomotive-force method," which employs the open-circuit saturation curve and the armature reaction and, though more accurate than the first method, is optimistic. It is the most rational and readily understood. The third method is the official method of the A.I.E.E. Standards (which see).

Electromotive-force Method. Let V = terminal voltage per phase at full load; R = a-c or effective resistance per phase in ohms; XI = volts per phase from magnetization curve corresponding to the field excitation required to send full-load current through armature on synchronous impedance test; I = full-load amperes per phase; $\cos\theta$ = power factor. Then the voltage per phase E at no load, corresponding to the full-load excitation, is the vector sum of V, RI and XI, or

$$E = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2}$$

The regulation is $100(E - V)/V$, and the full-load field ampere-turns F is taken from the point corresponding to E on the magnetization curve.

Magnetomotive-force Method. One way in which this method has been applied is the following: Let V = terminal voltage per phase at full load; R = a-c resistance per phase in ohms; I = full-load amperes per phase; $\cos\theta$ = power factor of the load; X = leakage reactance per phase in ohms. Calculate

$$V' = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2} \quad \text{and} \quad \theta' = \tan^{-1}\left[\frac{V\sin\theta + XI}{V\cos\theta + RI}\right]$$

taking θ' positive for I lagging behind V'. From magnetization curve find m = excitation ampere turns corresponding to V' and let n = armature reaction ampere-turns per pole for current I (see page 9–12). The total field ampere-turns is then

$$F = \sqrt{m^2 + n^2 + 2\,mn\sin\theta'}$$

Let V_0 = voltage per phase from the magnetization curve corresponding to this excitation F; then the regulation is

$$100\,\frac{V_0 - V}{V}$$

LOSSES. The losses in a synchronous alternating-current generator or motor are

(a) Friction, bearing and windage.
(b) Excitation or field copper loss.
(c) Core loss.
(d) Armature copper loss.
(e) Load loss.

Of these the first three are approximately constant for various loads, but the last two vary as the square of the load current.

Friction and Windage depend in magnitude upon the details of the physical or mechanical construction, the amount of induced ventilation, and speed. It is impossible to give a method of predetermining this quantity which will apply to various designs and makes. Each manufacturer has an empirical formula for each line of machines. This loss ranges from 2.5 per cent in 100-kva machines to 0.5 per cent in 10,000-kva machines. These values are for complete machines with their own bearings (two in number). Some machines are designed with only one bearing, the other bearing being a part of the prime

mover (hydraulic or steam) in which case the friction chargeable to the generator is less. Some machines with devices to produce ventilation, such as fan blades attached to the revolving part, have greater friction losses.

Core Loss. The core loss is made up of hysteresis and eddy-current losses. These losses are principally in the armature core and teeth, but if proper care is not taken there may be a considerable loss in the frame of the machine and the pole shoes.

It is a simple matter to calculate the magnitude of these losses in the armature core proper, as the frequency and flux density are definite in this part, but the losses in the teeth are due not only to the fundamental frequency and main flux, but also to pulsations due to the passage of pole tips past the teeth and the leakage flux of the armature. The total core loss is the sum of the hysteresis and eddy-current losses. See Magnetic Properties of Iron and Other Metals for curves of hysteresis and eddy-current losses and formulas for their calculation.

Excitation Loss. The calculation of the field current and of the resistance of the field winding is given above in the section on Field Winding. The total power required for excitation will be the product of this resistance and the square of the current.

If the machine is separately excited, which is usually the case, the losses in the field rheostat are, by convention of the A.I.E.E., not chargeable against the generator.

Armature Copper Loss. The calculation of the d-c resistance per phase of the armature winding is given above in the paragraph on Armature Resistance (page 9–11). The total armature copper loss is equal to the number of phases times this d-c resistance times the square of the current per phase.

Load Loss. When a current flows in the armature conductors a local flux is set up which will cause eddy currents in these conductors, if they are not well subdivided, and in the surrounding iron, as well as a hysteresis loss in the surrounding iron. The loss due to this load flux is called the "load loss."

The load loss is a function of the leakage flux and the subdivision of the conductors. A large number of turns of fine wire will involve a very small loss. If the conductors must be large, they may be made of stranded cable pressed to shape. It is almost impossible to calculate this loss, and very difficult to measure it. A rough method is to assume the resistance per phase increased by 15 to 25 per cent, as this loss, like the true copper loss, is proportional to the square of the armature current. The total amount of the loss is less than 1 per cent of the input in well-constructed machines.

EFFICIENCY. Let P = total output in kilowatts; R_a = resistance per phase of armature; I_a = armature amperes per phase; R_f = resistance of field winding; I_f = field current; q = number of phases; C = total core loss in kilowatts; and F = friction and windage loss in kilowatts. Then the per cent efficiency is

$$\frac{100\,P}{P + C + F + (1.15\,R_a qI_a^2 + R_f I_f^2) \times 10^{-3}}$$

This assumes the load loss equivalent to increasing the armature resistance (as calculated or measured by direct current) by 15 per cent. If the load loss is determined from the short-circuit core loss the formula for efficiency is

$$\frac{100\,P}{P + C + F + L + (R_a qI_a^2 + R_f I_f^2) \times 10^{-3}}$$

where L is the load loss in kilowatts for the specified current I_a.

The efficiency is a maximum for that load at which the constant losses are equal to the variable losses.

Customary values for the efficiency at full load and each of the losses at full load for various sizes of generators are given in the following table. These values are merely indications and vary with the frequency, voltage, speed, power factor, etc. A 60-cycle

Efficiency and Losses, Usual Values

Salient-pole Machines

Rating, kva	Efficiency, per cent	Friction, per cent	Excitation, per cent	Core, per cent	Armature,* per cent
100	91	2.5	2.5	2.4	1.6
500	94	1.1	1.5	2.2	1.2
1,000	95	0.9	1.0	2.1	1.0
2,000	96	0.6	0.7	1.8	0.9
3,000	96.5	0.6	0.6	1.7	0.8
5,000	97	0.55	0.4	1.6	0.5
10,000	97.2	0.5	0.35	1.5	0.45

* Copper and load loss.

low-voltage machine will be likely to have a better efficiency than a machine of the same rating for 25 cycles or high voltage.

HEATING. The rise in temperature of the field windings is as given on page 9–07. The rise in temperature of the armature winding and the core is determined by the method given for Heating under Direct-current Generators, page 8–19. Armatures of a-c machines are usually stationary and exterior to the revolving field structure; the armatures of d-c machines always revolve and are inside the field structure. Nevertheless, the design of the ventilation is the same in both cases. The air set in motion by the peripheral velocity of the field poles, and in some cases fan-blades on the poles, enters the radial air ducts in the armature structure, passes the coils where they cross the air ducts, and then cools the iron of the stator core. At the two ends the air from the revolving field fans the end connections, which have spaces between them for the purpose, and thus cools this portion of each coil. The velocity of this air can only be estimated, and the peripheral velocity of the external surface of the field poles is used as a criterion in the formulas for rise in temperature at the surface of coils and core. The armature or stator has air ducts every 2 or 2.5 in., and these are $^3/_8$ or $^1/_2$ in. wide and so arranged that the air flows radially outward from the air gap to the outside frame.

CHECKING CALCULATIONS. Substitution in the following formula gives an excellent check on the above calculations:

$$\frac{DL_f}{Q} = \frac{KA \times 10^9}{fB_g\,na}$$

where D = diameter of rotor at gap in inches.
L_f = length of pole face parallel to shaft.
Q = total kva of generator.
f = frequency in cycles per second.
n = armature reaction ampere-turns.
a = pole arc in inches.
A = diameter per pole in inches.
B_g = flux density, lines per square inch in air gap.
K = 22.5 for single-phase and 15.9 for two-phase or three-phase generators.

4. SINGLE-PHASE GENERATOR

Single-phase generators do not make as effective use of material as do polyphase generators, for the reason that the armature winding can occupy effectively only about one-half of the peripheral surface of the armature; if it occupies more than this there will be voltages generated in the windings which are so out of phase with each other that the resultant voltage is only from 63 to 70 per cent of the sum of all the voltages generated.

Therefore, if a polyphase generator is used as a single-phase machine with the same magnetic densities and the same copper densities, the output will be much less on account of the lesser voltage available. It is customary, therefore, to overload the magnetic elements of the machine somewhat to raise the voltage and thus reduce the overload on the copper which would be necessary to get the desired output. However, if both the iron and copper densities are increased until the machine gives as much output single phase as it is intended to give polyphase, there will be an increased heating. Thus, for the same heating, obtained by a readjustment of the iron and copper losses, a machine of a given first cost will give about 75 per cent as much output single phase as may be obtained polyphase.

In addition to the disadvantage of a single-phase generator that it cannot make use of all the periphery of the armature, it also labors under the disadvantage that its armature reaction is pulsating instead of constant, and this introduces an additional loss in the form of eddy currents.

By using two phases in series of a three-phase machine, or one phase of a two-phase machine, a fairly good single-phase machine is obtained. By arranging the winding slightly differently the same number and arrangement of inductors (or coil sides) can be connected up to give a simpler winding requiring no crossings of coils, that is, a winding "in one plane."

The formula for the calculation of the emf of a single-phase generator is

$$E = 4.44\, k_2\, k_3 f S \Phi\, 10^{-8},$$

where the symbols, with the exception of k_2, have the same significance as in the formula for polyphase machines, page 9–05. The value of k_2 depends upon the portion of the armature periphery (including the teeth between slots) occupied by the main winding. Let A = this fraction of the armature surface; then the corresponding values of k_2 are

given in the accompanying table. The value of A corresponding to a uniformly distributed winding is unity.

A	k_2
1	0.63
3/4	0.785
2/3	0.83
1/2	0.90
1/3	0.956

The average value of the armature reaction ampere-turns per pole of a single-phase machine is SI/p, where S = the number of turns, I = the armature current per phase, and p = the number of poles, but this quantity pulsates between 0 and twice the above value. The evil effects of this pulsating may be reduced by employing a short-circuited winding having its axis at 90 deg to the main field winding. A squirrel cage or "amortisseur" winding in the pole faces is frequently employed for this purpose.

Owing to this pulsating action the load losses are greater, and this should be taken into account in calculating the efficiency and regulation by considering the effective armature resistance as 1.15 to 1.5 times the d-c resistance.

The leakage reactance of a single-phase machine pulsates between a value equal to that obtained by the formula above in the paragraph on Armature Leakage Reactance, and a value two-thirds as great. Satisfactory results are obtained by multiplying the value obtained from the formula by 0.85 for the effective single-phase value. In this case the whole winding is considered as one phase.

5. DESIGN OF TURBO-GENERATORS

In turbine-driven generators the steam turbine is usually built by the same company that builds the generator, and the two machines are practically one unit. The steam turbine has developed to such an extent that for a given capacity in power it weighs less than a reciprocating engine, occupies much less space, costs less, and is more economical of steam and fuel. Good economy in steam turbines, however, is obtained only with high angular velocities. This has made it difficult to apply the turbine to useful purposes. The electric generator has shown itself to be the most suitable device to absorb the power of the turbine and make it available in subdivided form and at convenient speeds for general application. However, it required many years of experience to develop an electric generator which would operate successfully at the high angular velocities suitable for direct connection to the steam turbine. The principal difficulty was the design of a construction which would withstand the enormous stresses in the revolving member which resulted from the centrifugal force. Thus the design of the machine as a whole is largely a question of the type and construction of the rotor, which is the field.

NORMAL WEIGHTS AND SPEEDS. Most turbo-generators have much more copper on the fields and less on the armature than do slow-speed machines, although the total amount of copper is not far different in the two classes. The turbo-generator has much less magnetic iron on account of its high speed. It is quite normal in machines of 15,000-kva capacity to operate at 3600 rpm and machines of 100,000-kva at 1800 rpm. To do this involves the use of peripheral speeds of from 15,000 to 27,000 ft per min. At these high speeds the amount of material per kilowatt is much reduced. Increasing the speed tenfold reduces the weight of material to about one-quarter. This low weight is somewhat offset by the higher cost of construction necessary to withstand the enormous centrifugal forces. Most of the machines, even the large sizes, have two or four poles, or at most six poles. This involves the use of a pole pitch of 30 to 40 in. in 60-cycle machines, and 60 to 90 in 25-cycle machines. Since such large powers are concentrated in such small bulk, a great deal of energy in the form of losses must be dissipated in a small space. Thus special means of ventilation must be provided, such as numerous air ducts, fan blades on the rotor, or a separate blower, and a supply of clean, cool air.

CONSTRUCTION OF ROTOR. The design of a turbo-generator differs from that of the salient-pole type described above, because the revolving field of the turbo-generator consists of groups of coils placed in slots and distributed concentrically about the pole centers or cores, which resemble large teeth. The center of a pole occupies about one-third of the pole pitch and has no slots; see Fig. 5. The distributed field winding gives a peaked shape to the flux wave in the air gap, which is made to approach a sine form as near as practicable as shown in Fig. 6, which shows a developed cross-section of one pole. Thus the pole arc is the same as the pole pitch, but the maximum flux density is approximately 1.5 times the average.

The air gap is usually very large in order to provide a path for the large volume of ventilating air required. On account of the high peripheral speeds customary the diameter per pole is high.

The centrifugal force in the revolving member is from 1500 to 2500 lb for every pound of material near the periphery. A method for determining the centrifugal force in every

part is given in Electric Machine Design, by Gray, and in High-speed Dynamo-electric Machinery, by Hobart and Ellis.

The rotor must be of substantial and rigid construction, so that the critical speed of vibration is above the normal speed of operation. There are several methods of construction, prominent among which are:

a. A solid steel forging turned to shape with radial slots containing a distributed field winding. The shaft is in one piece with the field core; see Fig. 5.

b. A built-up structure consisting of steel disks about 3 in. thick held between the two end-plates by through-bolts, the radial or parallel slots being milled in the assembled structure.

c. Steel laminations with radial slots, assembled on a forged-steel shaft.

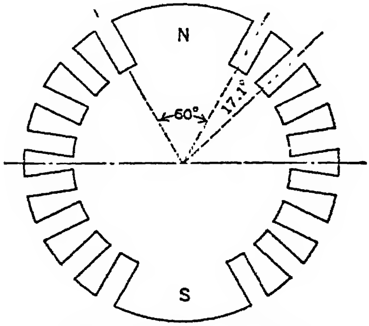

Fig. 5. Rotor Cross-section

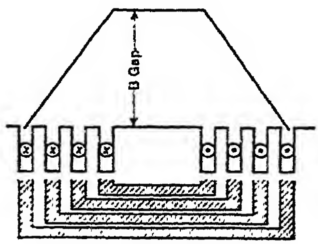

Fig. 6. Field Distribution

In these revolving fields the length is about the same as the diameter, and in large machines the length is greater than the diameter. The air gap is from 1 to 2 in. A uniform air gap is necessary to prevent noise and strains.

CONSTRUCTION OF STATOR. Since the pole pitch is large, a large number of slots per pole is used (12 to 24), and this gives a low armature self-inductance and high short-circuit current. Owing to the long pole pitch the end connections are long and subjected to considerable mechanical forces, as a result of the leakage flux. They must, therefore, be held securely in place by non-magnetic supports. For armature reaction in ampere-turns per pole values of 8000 to 20,000 are common, and the field winding is usually of a capacity of three times the armature ampere-turns. The regulation is poor (20 to 30 per cent). This defect is readily overcome by the use of automatic voltage regulators (see Section 12, Art. 18).

PROVISION FOR VENTILATION. The ventilation of these machines is a problem similar to that of air-blast transformers. Thus there must be provided: (*a*) sufficient

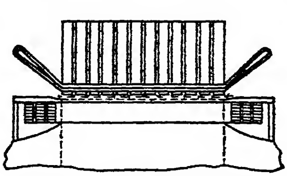

Fig. 7. Radial Ventilation

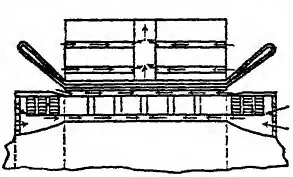

Fig. 8. Axial Ventilation

air to carry off the heat generated with a reasonable rise in temperature of the machine and the air; (*b*) ample duct capacity to prevent too high velocity of the air; (*c*) proper spacing of the ducts so that there is no part very far from an air duct; (*d*) proper precautions to prevent the air from carrying dirt and moisture into the machine.

One hundred cubic feet of air per minute at 20 deg cent will carry off 1 kw with a rise in the temperature of the air of 18 deg cent. Velocities of the air of 5000 to 6000 ft per min are common in the machine proper. The surface cooled will give off 4 or 5 watts per square inch with a rise of 35 to 45 deg cent above the cooling air.

There are two methods of ventilation (Figs. 7 and 8), the "radial" and the "axial." With the increase in size and therefore length of machines the axial becomes less desirable

and less used while the radial system is particularly adapted to long machines because of a refinement known as the multiple radial system (see Fig. 9).

In the simple radial method, air enters the air gap at both ends, flows along the gap, and then branches out radially through the air ducts in the core (3/8 in. wide spaced every 2 in.) at right angles to the shaft and goes to the extreme outer periphery of the stator. In the multiple radial the box frame outside the core is divided into air-tight compartments or large ducts, which are, alternately, for outgoing and incoming air. The middle third of the outside may be surrounded by an incoming duct which sends air radially inward through the core ducts to the air gap. There the stream divides into two parts, moves along the gap, and then radially outward through the two outer thirds of the core length. Air will also enter the air gap at the two ends and flow outward through the air ducts in the core near the ends. There may be any number of these multiple paths, depending upon the length of the core. In the axial method the air enters the air gap at each end and also many small ducts in the core back of the slots and parallel to the shaft. The air flows through these towards a large (3-in.) radial duct at the center of the machine, and then radially outward. The machine is totally enclosed in either case, and the air is propelled either by fan-blades on the rotor or by a separate motor-driven blower. The air is taken in at a definite entrance, usually outside the power house, and is expelled at a definite exit and is frequently used as preheated air for the fires under the boilers.

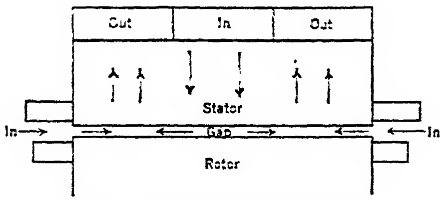

FIG. 9. Multiple Radial Ventilation

SUITABLE SPEED AND NUMBER OF POLES. For 25-cycle machines, 2 poles and 1500 rpm are used for all sizes including 160,000 kva. For 60 cycles, 2 poles and 3600 rpm are used up to 25,000 kva, 4 poles and 1800 rpm up to 200,000 kva. The data of five specific designs are given in Art. 7, page 9–26.

PRELIMINARY CALCULATION OF MAIN DIMENSIONS, Eq. (1), page 9–04, applies, but customary values of some of the constants are quite different for reasons stated above.

p is usually unity with distributed field windings.

B, the average density in air gap varies from 25,000 lines per square inch in 60-cycle machines to 32,000 in 25-cycle machines. Maximum density is 1.5 times the average.

V, the peripheral velocity, varies from 15,000 ft per min in small, low-frequency machines to 27,000 in large machines, figured at the surface of the revolving field.

σ, the ampere conductors of the armature per inch periphery of the field must be figured on the same diameter as V in order for the equation to check. σ has values from 1000 to 1800 (see table, page 9–05).

D = outer diameter of field, in inches, depends upon rpm and peripheral speed.

L = total length of field parallel to shaft, in inches.

k_2 and k_3 the same as on page 9–08.

THE ARMATURE WINDING. This winding is usually a double layer barrel with as many coils as slots and one, two, or three turns per coil. The coils usually have a pitch two-thirds that of a pole in order to shorten the end connections, save copper and space, and reduce the copper loss. It is of the lap winding type shown in Fig. 2. There may be as many circuits in multiple in each phase as there are poles, in order to reduce the amount of current per circuit. Each turn is usually given two twists so that it lies near the top of the slot on one side and near the bottom on the other side.

Connection of armature winding may be single-phase, two-phase, three-phase Y, or three-phase delta. For all three-phase machines for more than 6600 volts, the Y-connection is usually employed, for the reasons given on page 9–04.

General Proportions. The flux per pole depends upon the value chosen for B, the magnetic density in the gap (page 9–05); the required number of turns in series per phase (S) is determined by the volts per phase and the flux (page 9–05). There are usually 12 to 15 slots per pole in a small 60-cycle machine and as many as 24 in a large 25-cycle machine. Each slot contains two coil-sides and has two, four, or some even number of conductors divided in the two coils.

Size of Armature Conductors. A current density of 1500 amp per sq in. is common. On account of the high currents in large machines it is customary to divide the winding into multiple paths, each phase having a multiple path in each pole, thus a four-pole machine may have two or four paths in multiple in each phase. To reduce eddy-current

losses and facilitate bending, the conductors are usually made up of several strands of rectangular wire laid parallel and insulated from each other by cotton covering. This makes it possible to build up a conductor of convenient shape, but necessarily reduces the slot space factor.

Size of Armature Slots. The slot factor is low for machines of this type for three reasons: (1) high voltage is customary, (2) a considerable part of the slot is left vacant to serve as a passage for ventilating air, (3) the winding is placed deep in the slot to give sufficient reactance to limit the instantaneous short-circuit current. A typical slot would be 6 in. deep by 1 in. wide with the winding and wedge occupying only 5 in. of the depth. Usual depths of slots are shown in Table 6; slot factors and space allowed for insulation are given in Table 1. These are explained on page 9–07.

The Useful Flux in Armature is found as given on page 9–08, and has very high values as shown in Table 6. This is because the output per pole is so great. The leakage coefficient runs quite uniformly from 1.1 to 1.15 and is low because of the large pole pitch and distributed field winding.

Table 1. Slot Factor and Slot Insulation

Machine Voltage	Slot Factor	Insulation, inches	
		Depth	Width
2,200	0.35	1.25	0.18
4,400	0.35	1.50	0.25
6,600	0.33	1.50	0.32
13,000	0.27	2.00	0.55

Table 2. Usual Maximum Magnetic Densities

Part	25 Cycles	60 Cycles
Armature core...	90,000	80,000
Armature teeth..	100,000	85,000
Air gap.........	50,000	45,000
Rotor teeth.....	100,000	100,000
Rotor core......	90,000	90,000

THE MAGNETIC CIRCUIT is adjusted so that the magnetic densities are about as given in Table 2. In the stator and rotor teeth and in the air gap the density is not uniform, so that the maximum density (in the center of a field pole) is about 1.5 times the average density (see Polyphase Induction Motors). For a preliminary design this ratio may be assumed.

The Length of Air Gap is much influenced by its function in ventilation. For the electrical characteristic of the machine it is found desirable to have the length such that the ampere-turns excitation expended in the gap at no load and normal voltage are about equal to the armature reaction at rated load.

No Load Magnetization Curve. This is calculated as for induction motors (q.v.) on account of the distributed field winding and the peak wave of flux. The average densities are figured on a basis of full pitch pole arc and all stator teeth per pole carrying flux. The maximum density in gap, stator teeth, and rotor teeth is 1.5 times the average for a construction as in Fig. 5, with a pole center of 60 deg. With saturation in the teeth (over 110,000), or with specially wide pole centers, this constant becomes 1.45 or 1.40, respectively.

On account of the long air gap (long compared to slot openings and duct widths) the effect of slots in contracting the flux in the gap is not marked, so that the density in the air gap may be taken the same as at the surface of the rotor. Investigation shows that the increase in density in the air gap due to the slots is only from 2 to 4 per cent, and this is balanced by figuring the density at the surface of rotor instead of at the mean gap diameter. The section of rotor teeth is figured at a point one-third the distance from the bottom of the slots, and the total section includes the one large center tooth and several smaller teeth.

The average length of the path of the magnetic flux in the two cores is considerably less than one-half the pole pitch on account of the distributed flux. It is usually about 50 to 60 magnetic degrees.

The materials used are generally silicon steel for the stator and forged steel for the rotor. The tendency of the flux to pass through the slots instead of the teeth is not so great in these machines as in other types because, in the rotor, the flux can spread out with saturation, and in the stator, the densities are not very high. A magnetization curve between no load volts between terminals and ampere-turns per pole is usually plotted up to a voltage 120 per cent of rated voltage.

Rotor Slots. The rotor teeth must be carefully considered to determine whether there is sufficient material and strength at the root of the tooth to withstand the centrifugal force due to the masses of the tooth and the material in the slot. The force is $f = \frac{mv^2}{r}$, where m is the total mass, v the peripheral velocity at the radius of gyration, r. If $m = W/g$ in pounds, v = feet per second, and r = feet, then the force, f, is in pounds.

Approximately one-third of each pole pitch is left without slots or winding to form the center of the magnetic pole. This space is usually 60 electrical degrees between slot centers; thus in Fig. 5 there is a total of 16 slots for two poles, 8 slots per pole, and 4 coils per pole.

Table 3

Coils	Slots	Angle	1/2 Angle	Sin 1/2 angle
Coil in slots..................	4 and 5	60	30	0.50
	3 and 6	94.3	47.2	0.73
	2 and 7	128.6	64.3	0.91
	1 and 8	162.3	81.2	0.99
				3.13
Effectiveness of field, average..	... .	....	....	0.783

The slot pitch is determined by the fact that there are 8 slots and 7 teeth in 120 deg, thus the spacing is 120/7 or 17.15 deg. In some cases the slots are spaced as if there were to be 50 per cent more slots than are actually cut, in the example as if 24 slots at 15 deg were to be used but only 16 are actually cut, leaving smooth two spaces of 4 slots each. This would give a pole center of 75 deg and a lower ratio of maximum to average flux density in the air gap. The slots are very deep compared to their width; 6 to 1 is not unusual.

In some cases the slots have a narrow tunnel at the bottom underneath the winding, to carry ventilating air, and this air is let out along the length of the air gap by radial holes or air ducts, 3 to 6 in. apart, in the teeth.

ARMATURE LOSS. This is calculated and measured as in salient-pole machines (page 9–11) but the insulation used is of a higher grade and the copper is permitted to operate at a higher temperature so the losses are figured for a temperature of 100 deg cent. The mean length of a turn is longer because the straight portion of the coils projects from 1 to 2 in. beyond the core before it bends, thus adding 4 to 8 in. to the length of a turn. The total I^2R loss should be from 0.4 per cent in small machines to 0.2 per cent in a machine of 100,000 kva. The load loss in a 60-cycle machine is sometimes as great as the true resistance loss.

ARMATURE LEAKAGE REACTANCE. This is the same as in salient-pole machines, though lower in percentage because of the large number of slots per pole.

EXCITATION AT RATED KVA. Because the field winding is not concentrated in one coil per pole as in salient-pole machines it is not as effective in overcoming the armature reaction. The field acts as any distributed winding with fractional pitch, and the effectiveness of any one coil is proportional to the chord or to the sine of one-half the angle it subtends as shown in Table 3. The mmf to overcome the armature reaction is equal to the armature reaction divided by the effectiveness of the field which, usually has values around 0.8. Thus, letting n represent this mmf to overcome armature reaction, the excitation for any load at any power factor can be calculated by the method given on page 9–13.

FIELD COILS. Each of the several concentric field coils consists of 10 to 40 turns of flat wire wound on edge with about 5 mils of insulation between turns and about 100 mils of mica or asbestos insulation on the outside of the coil. These coils are placed deep in the slots, leaving fully 1/2 in. above the wedge for the ventilating air. The wedge is made very strong because it must hold the coil against enormous centrifugal forces. For this reason the field slot factor is low considering the voltage used, factors between 0.4 and 0.6 being usual. The field coils are the hottest part of the machine and, being insulated with mica or asbestos, can operate at temperatures from 100 to 150 deg cent with 125 as a common practice. The loss may be 0.9 watt per square inch of total coil surface for a rise of about 90 deg cent. The end connections are supported by a non-magnetic metal retaining ring, shrunk on tightly, to give an initial compression at standstill so there will be no movement of the coils between standstill and full speed. The fields are frequently proportioned to give rated voltage at rated kva output at 80 per cent inductive power factor with an exciter voltage of 200 out of 250, but this is a difficult specification to meet.

EFFICIENCY AND LOSSES. The losses are friction, excitation, core loss, armature I^2R loss, and load loss. The last two are usually combined and referred to as the armature short-circuit loss as it is obtained from the short-circuit test. The principal friction is that due to windage, and this is primarily the power required to ventilate the machine. The theoretical friction loss in ventilation is

$$\text{Kw} = 0.000273 \times (\text{cu ft per min}) \times (\text{pressure in oz})$$

and the actual loss is this amount divided by the efficiency of the blower and the motor. The usual pressure is 4 to 8 oz, or 7 to 14 in. of water. The excitation loss is the $I^2 R$ in the field as explained above and excludes the loss in the field rheostat. The core loss is proportional to the maximum densities in each part and to the quality of the material, which is usually silicon steel having $\eta = 0.001$ and $\epsilon = 0.00007$. Because of the long air gap and the simple flux wave the test values can be quite closely predicted by calculation. It will be noted in Table 4 that the friction loss is not only the greatest item but makes up almost one-half of the total loss.

Table 4. Usual Efficiencies and Losses of Turbo-generators

For non-inductive load

Kva	Per Cent Efficiency	Per Cent Friction	Per Cent Excitation	Per Cent Core Loss	Per Cent Armature
2,000	96	1.50	0.70	1.20	0.60
5,000	97	1.20	0.45	1.00	0.35
10,000	97.5	1.00	0.35	0.90	0.25
20,000	98	0.80	0.25	0.77	0.18
30,000	98.2	0.70	0.20	0.75	0.15
100,000	98.3	0.70	0.20	0.65	0 15

SYNCHRONOUS IMPEDANCE is very high on these machines and the steady short-circuit current correspondingly small. It is frequently the case that, with excitation for rated voltage and no load, the short-circuit current is less than the rated load current. Thus voltage regulation by external means is absolutely necessary, the inherent regulation being from 20 to 30 per cent at unity power factor and rated load.

6. TESTS OF ALTERNATING-CURRENT GENERATORS

(See also Standardization Rules of the A.I.E.E.) The principal tests are

Magnetization or saturation test.
Core-loss and friction tests.
Synchronous-impedance test.
Load-loss test.
Resistance measurements.
Heat runs.
Insulation tests.

Examples of test results are given below; typical test curves are given in Fig. 10.

MAGNETIZATION CURVE. The magnetization curve, popularly named the no-load saturation curve, shows the relation between the no-load voltage and the current in the field. Fig. 11 shows the connections for making this test. On account of the existence of the hysteresis loop (see article on Magnetic Properties of Iron), it is necessary, in running this test, to increase the field current gradually from point to point and never reduce the value at any step until the highest excitation has been obtained. The curve differs in shape from the magnetization curve of a closed sample of iron because the magnetic circuit of the alternator contains a considerable air gap, the magnetization curve of which is a straight line. Therefore, if the saturation curve of a machine contains a portion which is very straight, the indications are that the air gap is of considerable magnitude. If the machine is operated at very high magnetic densities, this is indicated by the fact that the point corresponding to rated voltage is found at a point on the curve where it is nearly horizontal.

The magnetization curve is usually plotted in terms of the volts between terminals. In comparing the observed and calculated magnetization curve it should be noted that the calculations are expressed in terms of the volts per phase.

CORE LOSS AND FRICTION. The open-circuit or true core-loss curve shows the core loss in watts for each value of the no-load terminal voltage. It is made by driving the generator at rated speed by means of a small motor, the efficiency of which is known, and varying the generator excitation. Fig. 11 shows the connections for this test. The voltage and mechanical power P (=input to motor multiplied by its efficiency) required for each value of excitation are noted. The power P_0 for zero excitation is the friction loss in the machine. The core loss for any excitation is $P - P_0$. The no-load saturation curve can be made at the same time as the core-loss test.

SYNCHRONOUS IMPEDANCE. The synchronous-impedance curve shows the relation between various values of field current, or excitation ampere-turns, and the

current that flows in the armature on short circuit. It is made by short-circuiting the armature through ammeters and operating at full frequency with various values of field current. Fig. 12 shows the connections for this test. Of course only fractional values of

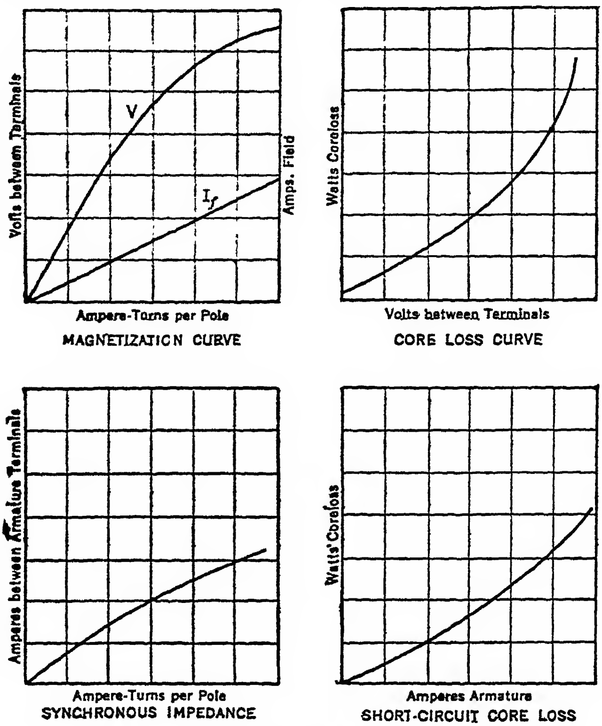

Fig. 10. Typical Test Curves

normal excitation are used, or the current in the armature would be so great as to cause damage.

The synchronous-impedance curve gives approximately the ampere-turns corresponding to armature reaction and the leakage reactance drop in the armature combined, i.e., the ampere-turns corresponding to the synchronous reactance. The approximation

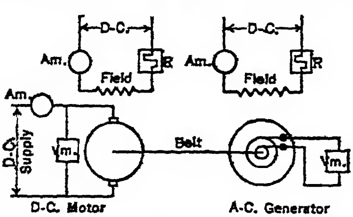

Fig. 11. Connections for Magnetization Curve, Core Loss and Friction Tests

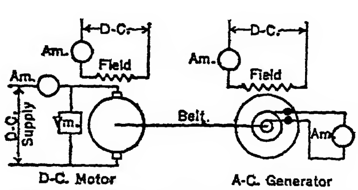

Fig. 12. Connections for Synchronous Impedance and Load Loss Tests

arises from the effect of the armature resistance and the low saturation or magnetization of the magnetic circuit under the short-circuit conditions.

STRAY LOAD LOSSES. These include iron losses, eddy-current losses in both copper and iron, due to leakage fluxes caused by and varying with the load current and also varying with the saturation in the iron. The load losses may be obtained at the same time and with the same set-up as the synchronous impedance test. The output of

the driving motor is accurately calculated, and from this is subtracted the friction of the machine under test (known) and the armature I^2R, calculated for the proper temperature, and the remainder is the load loss for the particular armature current.

The following is quoted from the Standardization Rules of the A.I.E.E.: Stray load losses shall be determined by operating the machine on short circuit and at rated load current. This, after deducting the windage and friction and I^2R loss, gives the stray load loss for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to specify a method for measuring them.

RESISTANCE MEASUREMENTS must be made when all parts of the machine are at some known temperature. They are also made after the heat run, when the machine is hot, to check the temperature as measured by thermometers.

Field Resistance is measured by the simple voltmeter-ammeter method.

Armature Resistance. The armature resistance between terminals is also measured by the simple voltmeter-ammeter method, by connecting two of the terminals to a source of direct current (the other terminal being free), the rotor (field or armature of course being at rest). For a single-phase or two-phase machine the resistance as thus measured is the resistance per phase. For a three-phase machine, let R_t = the resistance as thus measured and R_p = the resistance per phase. Then for a Y-connected armature $R_p = R_t \div 2$, and for a Δ-connected armature $R_p = 3R_t \div 2$. If the connection is not known, the calculation of the resistance drop and copper loss may be figured correctly, assuming it either Y or Δ connected, provided the resistance per phase and current per phase are both calculated on the same assumption regarding the connection. For a two-phase machine the resistance between terminals A and B and between B and C (B being the common terminals) should each be measured, and the average taken. For a three-phase machine the resistance between A and B, B and C, and A and C should be measured and the average taken.

CALCULATION OF REGULATION AND EFFICIENCY FROM TESTS. From the results of the preceding observations, the regulation and efficiency of the machine at various loads may be calculated by the methods given above, in the discussion of Predetermination of Performance, using the test data instead of the calculated quantities. See Standards of A.I.E.E. The synchronous-reactance ampere-turns may be taken as equal to the synchronous-impedance ampere-turns, since the resistance drop in the armature on short circuit and reduced excitation is seldom over 10 per cent of the synchronous-reactance drop, and as the two are in quadrature the synchronous-impedance drop differs from the synchronous reactance drop by less than 1 per cent. In using the magnetization curve and the synchronous-impedance curve one must keep in mind whether they are plotted in terms of voltage and current per phase or in terms of volts between lines and line current.

FULL-LOAD SATURATION CURVE. This curve shows the relation between the terminal voltage and field current with full-load current in the armature. It may be plotted from tests or calculated from the magnetization curve and the synchronous impedance in the same manner as the regulation is calculated.

ARMATURE LEAKAGE REACTANCE. There are three methods by which the leakage reactance may be determined:

(1) Synchronous-impedance Method. This gives only approximate results but is very generally used as a synchronous-impedance test is made on every generator. Let n be the field ampere-turns per pole required to force full-load current through the short-circuited armature. Let a be the calculated armature reaction ampere-turns per pole (see above), with rated current. Then $n - a$ is the excitation necessary to induce the leakage reactance voltage (IX) in the armature. From the open-circuit saturation curve find the voltage corresponding to $(n - a)$ ampere-turns. Reducing to volts per phase, if necessary, and dividing by the current per phase, the result is the leakage reactance X per phase in ohms.

(2) Full-load Characteristic Method. This method is also approximate. Let the excitation in ampere-turns per pole required to give rated voltage at full non-inductive load be q. Let the ampere-turns to give rated voltage without load be m. Then $n = \sqrt{q^2 - m^2}$ is taken as the synchronous-reactance ampere-turns, and the leakage reactance is calculated as described in the preceding paragraph.

(3) Inductance-measurement Method. This method gives exact values, but is not very often employed. From an external source of proper frequency, full-load current is sent through the armature of the machine to be tested, with the fields excited to the normal value but not rotating. The voltage drop across the armature terminals is measured for several different positions of the coils with respect to the poles, ranging through an arc of about one-half the pole pitch. Let Z = the voltage per phase divided by the current, R = the resistance per phase corrected for load loss (i.e., the effective

resistance as determined from the short-circuit core-loss curve); then the leakage reactance per phase is $X = \sqrt{Z^2 - R^2}$. This reactance will vary with the position of the coils and the maximum and minimum may be found by making this calculation for the different positions of the coils with respect to the fields.

HEAT RUNS are made at rated load and various other specified loads. The parts in which the rise in temperature is of interest are:

Armature core surface.
Armature core ventilating ducts.
Armature conductors.
Collector rings.
Both pole tips.
Field winding.
Bearings.
Frame.
Room.

The temperature of the field and armature winding should be measured both by thermometers and by the resistance method. In taking the temperature of a hot surface by thermometer a small pad of waste should be placed over the bulb after the thermometer is put in place. The pad should not be too large or it will prevent the normal radiation from the surface.

With large machines it is inconvenient and expensive to test under full-load conditions. For determining the heating without actually developing the full power of the machine several methods are available.

Reversed-field Method. The field circuit may be tapped and the full-load field current sent through a portion of the field coils in a direction opposed to that for normal operation. For example, in a 24-pole machine the current in 8 of the field coils may be directed in such a manner that they are reversed with respect to the remaining 16. The armature winding will therefore generate a voltage due to the 8 poles that are not neutralized and therefore of approximately one-third rated value. If the armature is short-circuited a current will flow due to this reduced voltage, and this current can be made to approximate closely the full-load value by making a proper division of the poles. If the machine is operated under these conditions the friction, excitation, core loss, and armature copper loss will be very nearly equal to their value under normal operating conditions, yet to drive the machine only an amount of power approximately equal to the sum of the losses is required.

Synchronous-motor Loading. If two machines are available they may be connected up as a generator and synchronous motor and the field excitation of the motor adjusted so that a current of full-load value, but having a large reactive component, will flow in the armature, the power factor being nearly zero. The power required to drive the generator at full-load excitation and at full-load current will then be small.

Direct-current Loading. Full-load losses may be simulated on a three-phase generator by connecting the three phases in delta and leaving the delta open at one point, to which a d-c ammeter and a source of direct current may be connected in series. The delta is first closed through an a-c ammeter and the triple-frequency current in the delta (see Alternating Currents) is measured. The d-c ammeter and source of direct current are then connected in series with the delta and the direct current increased until the sum of the squares of the local current of triple frequency and the direct current is equal to the square of the rated current per phase.

Open-circuit Short-circuit Method. Another ingenious method, advocated by Hobart, consists in alternating open-circuit and short-circuit tests, each under exaggerated conditions, so that at the end of each hour the total watthours lost in each part are equal to the watthours that would be lost in normal operation. For example, let the core and field copper loss combined be 4 kw, and armature copper loss 1 kw. If, now, the machine is operated for 20 minutes with armature short-circuited and the excitation adjusted to give 3 kw copper loss, and for 40 minutes on open circuit with excitation adjusted to give 6 kw core and field copper loss, then the loss per hour in the armature copper is 1 kwhr, and in the field winding and core 4 kwhr, the same as under normal conditions. For an exact division of the time between the open-circuit and short-circuit conditions, the core and field copper loss during the short-circuit run must be taken into consideration. After several hours of this relaying the machine will have reached a temperature corresponding to full-load operation.

INSULATION TESTS. The insulation of a machine is usually tested by applying a given voltage between the conductors and the part of the machine from which they are insulated. An insulation resistance test is also made.

Voltage Tests. During manufacture the coils are tested separately, and after the machine is assembled a voltage test between the completed windings and frame is made. After the heat run, while the machine is still warm, a third test of the insulation between windings and frame is made.

For this third test an alternating voltage is applied between armature and frame, according to the Standardization Rules of the A.I.E.E. (q.v.).

The voltage should be increased gradually, and the final value should be applied for 1 minute. In this test the machine acts as a condenser and therefore care should be taken that the frequency is not so high or the inductance of the supply circuit so great as to set up resonance. It is advisable to have considerable resistance in the supply circuit.

To prevent an uneven distribution of voltage, all the terminals of the winding should be connected together by fine wires and one terminal of the high-potential circuit connected to the common connection. The high potential should be measured by a spark gap connected in shunt to the machine.

INSULATION RESISTANCE. The insulation resistance is measured by connecting one terminal of a 500-volt d-c supply to the windings of the machine through a voltmeter with a 500 scale, and connecting the other 500-volt terminal to the frame. If the resistance of the voltmeter is R_v ohms and the voltmeter deflection x, the insulation resistance $R = \frac{R_v}{x}(500 - x)$. The required value is:

$$\text{Megohms} = \frac{\text{Rated voltage}}{\text{Rated kva} \times 1000}$$

7. EXAMPLES OF DESIGN AND PERFORMANCE

In Tables 5 and 6 will be found the essential data both of the mechanical and electrical features of ten representative alternators.

Table 5. General Dimensions and Design Constants of Salient-pole Generators
(Dimensions in inches)

Frequency	25	25	60	60	60
Poles	20	16	4	54	72
Kva	32,500	45,000	7 5	2,500	25,000
Power factor	100	100	100	100	100
Rpm	150	187.5	1800	133	100
Armature diameter at face	197	217	10	170	284
Armature, total length	65	68	6.5	20	48
Armature slots	300	288	72	648	432
Depth of slots	4.75	7	1.25	2.13	3.25
Peripheral speed, fpm	5,120	10,700	4,700	5,900	7,200
Ampere-conductors per inch	1,140	1,370	206	760	1,840
Armature reaction, ampere-turns	11,900	18,500	475	3,600	3,860
Flux per pole megalines	87	96	1.04	5.6	24
Average gap density kilolines per sq in.	65	62	35	48	60
Leakage reactance, per cent IX	8.3	11	6	8	16
Air gap length, in.	0.5	0.5	0.094	0.375	0.5
Excitation, no load ampere-turns	13,700	14,000	1,500	4,880	15,000
Excitation, rated load	20,800	25,000	1,825	7,200	16,300
Efficiency at rating, per cent	98	97.75	90	96	97.3
Weight, pounds	650,000	1,300,000	1,030	170,000	650,000
Pounds per kva	20	29	137	68	26

8. OPERATION

In the operation of an a-c generator the following factors should be considered:

PHASE CONNECTIONS AND GROUNDING. When a third harmonic is present in the emf wave of a three-phase generator, the triple-frequency emf's in the three phases (or windings) are additive when the three phases are connected in Δ, that is, the Δ forms a short circuit to the third harmonic emf's and a large triple-frequency current may therefore be set up in the windings irrespective of the load on the machine. On this account, large three-phase generators are usually Y-connected, since with this connection the third-harmonic emf's between any two terminals of the machine neutralize each other. However, when two or more Y-connected machines are operated in parallel with their neutrals grounded, a triple-frequency cross-current of considerable magnitude may be

Table 6. General Dimensions and Design Constants of Turbo-generators

Frequency	25	60	60	60	60
Poles	2	4	4	4	4
Kva	30,000	6,250	10,000	25,000	120,000
Power factor	100	80	80	80	95
Rpm	1,500	1,800	1,800	1,800	1,800
Diameter, rotor, in	50.5	38.7	45	45.2	55
Length rotor, in	112	52	50	106	283
Stator slots	72	72	72	72	84
Depth of slots, in	6.6	3.1	5.7	6.5	8.5
Peripheral velocity ft per min	19,800	18,200	21,000	21,300	26,000
Ampere-conductors per in	1,740	1,033	1,330	1,390	1,760
Armature reaction, ampere-turns	34,500	8,300	15,500	16,000	26,000
Flux per pole megalines	276	48.4	43	104	307
Average gap density kilolines per sq in.	31.4	30.5	24.5	29.5	25.1
Leakage reactance, per cent IX	9	18	15	12	14
Min air gap length, in	2.75	165	1.25	1.37	2.0
Rotor slots	44	48	40	36	40
Depth of slots, in	6.75	4.75	6	6.9	9
Excitation, no load ampere-turns	45,000	15,000	15,100	24,000	25,000
rated load and PF	66,000	17,000	33,000	44,000	51,000
Efficiency, rating, per cent	97.8	96.7	97.2	97.1	98.1
Total weight, lb	270,000	70,000	100,000	190,000	580,000
Pounds per kva	9	11	10	7.6	4.8

set up between the machines, unless the wave forms of the emf's of the various generators are exactly the same, which is practically never the case. To prevent such cross-currents with Y-connected machines with grounded neutral, it is the usual practice to ground the neutral of but one generator at a time. Provision must of course be made to shift this ground connection from one machine to any other, so that a ground connection can always be maintained irrespective of which machine or group of machines may be running.

DIVISIONS OF LOAD BETWEEN ALTERNATORS IN PARALLEL. The division of load between two or more alternators operating in parallel cannot be changed by altering their field excitation, as it can with d-c generators. Changes in the load taken by any alternator of a group can be effected only by admitting more or less steam to the driving engine or turbine (or water to a water-wheel). In order that the various alternators shall share the combined load properly, it is therefore necessary that the governors on the several prime movers give the same speed-load characteristics.

Although the field excitation has no effect on the distribution of the load among the alternators, it does affect the power factor of the load delivered by each machine. The excitation of each alternator should be so adjusted that it delivers its load at the same power factor as the others.

STARTING A SINGLE GENERATOR. Before a generator is started up its bearings must be inspected and cleaned and filled with oil if necessary. The machine is then brought up to the proper speed and the bearings again inspected to see that the oil-rings are running properly. The exciters or excitation circuit is then put in readiness and the rheostat in the alternator field circuit adjusted for maximum resistance. Before exciting the field the armature insulation must be thoroughly dry. If it is not the armature should be short-circuited through an ammeter and run for several hours at a partial excitation to give about rated current in the short-circuited armature. When the insulation is thoroughly dry the short circuit is removed and the excitation adjusted to give rated voltage at the armature terminals with correct speed. To shut down the machine the load is first removed by opening the circuit breaker; then the field rheostat is turned to maximum resistance, as is also the rheostat in the exciter field if there is an individual exciter. Then the field circuit is opened.

PARALLELING OF GENERATORS. Before connecting a generator to bus-bars to which one or more other generators are connected, the following conditions must be satisfied:

1. The frequency of the generator must be the same as that of the bus-bars.
2. The frequency of the generator, and therefore its speed, must be constant for an appreciable interval of time.
3. The voltage of the generator must be the same as the voltage of the bus-bars.
4. The generator and bus-bar voltage must be in phase.

If the two machines have not the same frequency or if the frequency is not constant, a condition will occur intermittently in which the two voltages are 180 deg apart or the two machines are in series on a short circuit, and a dangerous current will flow. If the voltages are not equal, a large "wattless" or reactive current may flow, and if the two

voltages are not in phase, a large power current will flow which will cause a mechanical shock. To indicate when these conditions are fulfilled any one of several "synchronizing" devices may be employed, as described in the article on Synchronizers and Synchroscopes.

Synchronizing with Lamps. The simplest method of synchronizing small machines is to use incandescent lamps as shown in Fig. 13. In Fig. 13A, the connections are such that the lamps remain dark when the above conditions are satisfied, whereas in Fig. 13B they will remain bright under these conditions. If the frequencies are wrong the lamps will flicker (the slower the flicker the nearer the two frequencies). If the voltages are wrong the lamps in 13A will glow slightly but steadily. Transformers should be used with the lamps in high-voltage machines.

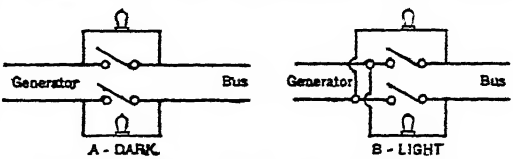

FIG. 13. Connections of Lamps for Synchronizing

HUNTING. Unless the angular velocities of two machines which are connected in parallel remain the same, either both constant or both varying together, a cross-current will flow, due to the phase displacement between them. This current will tend to drag ahead the machine which is lagging, but owing to the inertia of the rotating parts the machine which is at first lagging will "overreach" and become leading, and under certain conditions a cumulative seesaw action will be set up. When this takes place the machines are said to "hunt." The value of this current is proportional to the short-circuit current of the machines and to the angular displacement expressed in electrical degrees. In a machine having a large number of poles (40), a very small variation in angular displacement of the prime mover may cause a considerable (20-fold) phase displacement in electrical degrees.

To prevent hunting it is necessary to have:

1. A prime mover giving reasonably constant tangential effort and angular velocity. In general, the maximum variation in angular displacement between any machine and the bus-bars should not exceed 2.5 electrical degrees.

To secure this condition the machines should have constants such that they are not especially sensitive as pendulums to the strokes or impulses of the particular engines used.

2. A governor which is not too sensitive to slight and sudden variations in load, i.e., a damped governor.

3. A low-resistance drop (usually not over 10 per cent) in the connections between the machines. This refers particularly to groups of machines in power houses miles apart.

4. Short-circuited or "amortisseur" windings on the fields of the machines. Currents induced in these windings cause them to act as electrical brakes.

SHORT CIRCUITS. An alternator, when suddenly short-circuited, will deliver, for an instant, a current many times as great as will flow after conditions have become steady. This is due to the fact that it takes a definite period of time for the increased armature reaction to weaken the main magnetic field. During this period much damage may be done to windings and circuit breakers. This transient short-circuit current has a nominal or symmetrical effective value of $E \div X$ or the rated current multiplied by $\frac{100}{\%IX}$, but it may have a maximum instantaneous value of $2\sqrt{2}$ times this depending upon the part of the cycle at which the short circuit starts.

Use of External Reactance. Frequently, therefore, reactance or choke coils are connected in circuit with these machines to prevent a dangerous current flowing in case of sudden short circuit. See Reactance Coils.

Induction Generators on Short Circuit. Induction generators are free from this fault, since they lose their excitation almost immediately on short circuit, and are therefore becoming popular in large central stations.

USE OF IMBEDDED THERMOMETERS. In large generators thermocouples or resistance theremometers (see Pyrometers) are sometimes imbedded in the estimated hottest spot of the winding, and connected to a suitable indicating device to show at all times the maximum temperature of the machine. See also Standardization Rules of the A.I.E.E.

9. SPECIFICATIONS FOR A-C GENERATOR

The following memoranda are intended to assist in writing specifications.

Principal Characteristics and Conditions of Service. Service for which generator is to be used, such as a-c lighting, single-phase railway service, operating synchronous

converters for railway or lighting, etc. Voltage and number of phases. Rated output in kilowatts or kilovolt-amperes at stated power factor. Frequency and speed.

Style and Description; Details of Construction. Type of generator, revolving field, induction type, etc. Details of speed, governing of prime mover, and how generator is connected to prime mover, e.g., direct or belt-driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. Whether compensated. Whether exciter is to be supplied; if so, its characteristics. If field rheostat is to be supplied, its characteristics, including the effect upon the generator voltage of each step and of all steps; whether to be controlled by hand or automatically. Restriction of excitation current and voltage and requirements respecting carrying capacity of slip rings. Accessibility of armature. Whether imbedded thermometer coils shall be furnished. Windings shall be clamped securely to prevent any vibration of overhanging parts. Mechanical protection of armature conductors if exposed. If belt driven, specify pulley details; whether bed plate is desired.

Work to be Done by Other Contractors. Whether contractor is to furnish and install the following: main wiring, field wiring, field rheostat grids, dial plate, and chains. Point of division between engine and generator contracts.

Performance and Tests. (*See Standardization Rules of the A.I.E.E.*) Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. High-potential tests of insulation. Requirements regarding effects of moisture upon insulation. Requirements for parallel operation, i.e., whether the machine is to operate in parallel with similar machines or different ones. It is usual to specify that the terminal voltage shall vary according to a sine law. Regulation with 100 per cent power factor and normal speed; the load may be varied from zero to 150 per cent of rated load without causing more than a stated variation of voltage, the exciter field being kept constant.

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SYNCHRONOUS MOTORS

Any alternator will operate as a motor. If two synchronous alternators are connected in parallel to bus-bars supplying a load, and the driving power be removed from one prime mover, the alternator connected to this prime mover will continue to run at the same speed as before, taking power from the other alternator and driving its own prime mover or other apparatus coupled to it; i.e., this alternator acts as motor. The speed of such a motor depends solely upon the speed of the generator or generators supplying electric energy to it; it is therefore said to run in "synchronism" with the source of supply and is called a synchronous motor. The synchronous speed of a synchronous motor having p poles, when supplied with a current of a frequency of f cycles per second, is

$$N = \frac{120f}{p} \qquad \text{rpm}$$

If the load on such a motor increases, the speed will not decrease, unless the load reaches such an excessive value that the maximum output or "pull-out torque" is reached, when the motor will drop out of step and come to rest, the current taken increasing to short-circuit value and the torque decreasing to a negligible value.

DIFFERENCES BETWEEN AN ALTERNATOR AND SYNCHRONOUS MOTOR. The difference in construction between an alternator and a synchronous motor is that the latter has placed in the face of the field poles, a squirrel-cage winding, which is intended to give good starting torque and to prevent hunting while running. A synchronous motor usually operates better with a higher value of armature reaction than that of a well-designed generator of the same kilowatt rating. This increase in armature reaction is usually obtained in practice by operating the machine as a motor at lower voltage than that for which it would be operated as a generator. Thus a standard 2300-volt generator will operate very satisfactorily as a motor at 2080 volts, and as these are the natural values of the generated and delivered voltages, this characteristic of the synchronous motor fits in very well with customary distribution practice. Thus a standard generator may have a squirrel-cage winding added to its poles and become a good synchronous motor.

FIELD EXCITATION. Synchronous motors always require direct current for field excitation, and if a suitable d-c source is not available an exciter must be provided.

NUMBER OF PHASES. Synchronous motors may be single, two, or three phase. The single-phase motor is not self-starting and has a considerably lower efficiency than the polyphase motor. It is also more likely to hunt (see below) and be unstable, and is therefore far less desirable than a polyphase motor. The two-phase and three-phase motors are very similar in all their characteristics. There is a slight economy in the three-phase over the two-phase motor as there is in the three-phase over the two-phase generator (q.v.).

TERMINAL VOLTAGE. Since synchronous motors are usually built with a revolving field and a stationary armature, it is possible to insulate the armature winding for voltages as high as 13,000 and thus obviate the need of transformers in many cases.

ADVANTAGES AND DISADVANTAGES OF SYNCHRONOUS MOTORS. The advantages of synchronous motors as compared to induction motors are: higher efficiency, higher power factor, controllability of power factor, constant speed, higher voltage, lower cost. The disadvantages are: need of an exciter, possibility of hunting.

RELATIONS OF VOLTAGE AND CURRENT. The relations between line voltage and phase voltage are the same as in a-c generators (q.v.). The current in each line of a three-phase motor is

$$I = \frac{746\,P}{\sqrt{3}\,\epsilon E \cos\theta}$$

where P = horsepower output.
E = voltage between lines.
ϵ = efficiency at load assumed.
$\cos\theta$ = power factor (may be unity).

Usual values for efficiency are about the same as for a-c generators (q.v.).

APPLICATIONS. In order to transform from alternating to direct current, or from one kind of alternating current to another differing in frequency, potential, or phase relation, motor-generator sets, consisting of a synchronous motor direct connected to one or more generators, are often employed. By this means the potential of the secondary or distribution circuit is independent of the variation in potential of the primary circuit supplying power to the motor. In certain cases it is desired to take power from a 25-cycle circuit and supply power at 60 cycles for lighting purposes. Here a synchronous motor-generator set would be used. Such a set is frequently called a "frequency changer" (see Frequency Changer). In some applications of electric drive by induction motors one synchronous motor is installed for the purpose of making it take leading current in order to neutralize the lagging current taken by the induction motors. This effect is produced by over-exciting the fields of the synchronous motor. Such a synchronous motor is called a "rotary phase modifier" or "rotary condenser." It may incidentally be used to drive any machinery that does not require a large starting torque.

10. GENERAL PRINCIPLES

In any synchronous generator or motor when a current flows in the armature there is a loss of voltage proportional to IZ_0, where I is the current and Z_0 is a hypothetical quantity called the synchronous impedance, which includes the effect of the resistance, the leakage reactance and the demagnetizing effect of the armature current. This quantity is obtained by the synchronous impedance test (see below) and is expressed in complex quantities as $Z_0 = r + jx_0$ and in algebra $Z_0^2 = r^2 + x_0^2$, where r is the effective resistance per phase and x_0 is the synchronous reactance per phase. IZ_0 is therefore the drop in voltage per phase in the armature, and this voltage and the current differ in phase by an angle θ, where $\tan\theta = x_0/r$ and $\cos\theta = r/Z_0$. If a synchronous motor

is running and generating a counter emf e and is connected to bus-bars of voltage E, the current flowing in the armature will be proportional to the vector difference between E and e and inversely proportional to Z_0, all taken per phase.

VECTOR RELATIONS FOR MOTOR ACTION. In Fig. 1 let E represent the bus-bar or line voltage (per phase) impressed on the terminals of a synchronous motor. Let e represent the counter emf of the motor, and in this case assume $e < E$ and directly opposed to E. Then the difference between E and e will set up a current in the armature of the motor. IZ_0 will be the voltage, and the current will lag behind IZ_0 by an angle θ where $\cos\theta = r/Z_0$. Thus the motor takes a current I lagging behind the impressed voltage E.

Fig. 2 shows the relations when $e > E$, then IZ_0 will be reversed in phase as indicated.

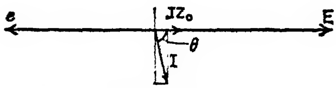

FIG. 1. Lagging Current

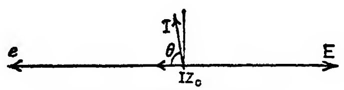

FIG. 2. Leading Current

I will always lag behind IZ_0 by the angle θ and will be found drawn upward. I lags behind e and IZ_0 but I leads the impressed emf E by an angle (180 deg $- \theta$). Thus when the field excitation is increased so that e tends to become greater than E, the machine takes a current leading with respect to the impressed emf. As in a synchronous generator the field excitation required for a given terminal voltage depends upon the phase relation of the external circuit or the load, so conversely in a synchronous motor the phase relation of the current into the armature at a given terminal voltage depends upon the field excitation and the load.

Fig. 3 shows the relations when e is more than 180 deg behind E, that is, e is behind the position it had in the preceding examples by an angle α. The vector resultant of E and e will be IZ_0 as shown, leading E. The current I will lag behind IZ_0 by the same angle θ, but is now almost in phase with E and lagging only slightly. Thus power ($EI\cos\phi$) is being sent into the machine and it acts as a motor, transforming electrical power into mechanical power. When the machine is running as in Figs. 1 and 2 the power is very small. If, however, a mechanical load is applied, the armature will drop back in position by a slight amount. This causes e to drop back in phase and the machine immediately draws power from the line.

If, as in Fig. 4, power is applied to make the armature move forward and cause e

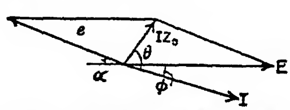

FIG. 3. Leading Current under Load Conditions

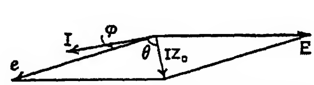

FIG. 4. Lagging Current under Load Conditions

to advance in the opposite direction, the resultant IZ_0 is thrown downward and I, lagging behind IZ_0 by θ, is thrown around almost in phase with e. The machine then becomes a generator, transforming the mechanical power applied into electrical power $eI\cos\phi$.

Synchronous Position. In the discussion of the action of a synchronous motor it is convenient to employ the term "synchronous position," by which is meant the position which any definite point on the revolving member occupies at the same period of each cycle of time. It is only necessary for the machine to change in synchronous position by a very slight angle α to cause a large energy current to flow. If α should become 90 deg, theoretically the power would become a maximum, and any increase in α means that θ becomes greater than 45 deg, the power decreasing and the machine falling out of step. When α becomes 180 deg a total emf equal to the sum of e and E is short-circuited by Z_0 and the current is enormous and the power factor low.

MAXIMUM TORQUE. If the speed changes for a short instant of time sufficiently to allow a point on the armature to drop back in synchronous position one-half the pitch of one pole (90 electrical degrees), the motor torque will increase from zero to the maximum available in the motor. Thus, for any torque less than the maximum the armature (or revolving field) need only change in speed sufficiently to drop back some distance less than one-half the pitch of a pole. If the load demands a torque greater than this maximum the armature will drop back more than 90 deg and will fall out of step and come to rest. In most synchronous motors the maximum torque is about 6 times normal torque.

11. DESIGN AND CALCULATIONS

The design and calculation of synchronous motors are very much like the design and calculation of a-c generators. There is a difference in the proportioning of certain details and there are certain features that are of importance in generators that are not important in motors (i.e., regulation) and the converse is also true.

ARMATURE REACTION. The armature reaction of a synchronous motor is expressed as $\frac{1.5\sqrt{2}\,SIk_2k_3}{p}$ for a three-phase machine and $\frac{\sqrt{2}\,SIk_2k_3}{p}$ for a two-phase machine, where S = turns in series per phase, I = full-load current per phase, k_2 and k_3 have the values given in the chapter on Alternating-current Generators and p = number of poles. The armature reaction of a motor is usually 30 to 50 per cent greater than that of a generator of the same rating and frequency. The higher value gives greater synchronizing torque per ampere and a better starting torque, and reduces the cross-currents between machines in case of hunting. The armature reaction ampere-turns at full load may be equal to the field ampere-turns at no load. Too great an armature reaction is objectionable, because it reduces the energy transfer between two machines and therefore reduces the synchronizing power, that is, the tendency of the machines to hold each other in step.

EXCITATION. The excitation of a motor is calculated in the same manner as that of a generator. The magnetic densities are usually a little less. The capacity of the field winding depends upon whether the synchronous motor is to be used as an ordinary motor or to regulate the power factor of a system by overexcitation.

LEAKAGE REACTANCE. The leakage reactance may be higher in a motor than in a generator as regulation is not of so great importance. However, too great a reactance will reduce the starting torque of the motor.

SHORT-CIRCUIT CURRENT AND SYNCHRONOUS IMPEDANCE. The short-circuit current of the motor depends upon the leakage reactance and the armature reaction. The short-circuit current and the synchronous impedance may be predetermined from the no-load saturation curve and the calculated leakage reactance per phase.

Let F = excitation in ampere-turns per pole for which it is desired to find the short-circuit current.
I = rated current per phase.
x = leakage reactance in ohms per phase.
E = voltage per phase due to F at no load.
S = turns in series per phase.
p = number of poles.

Then the ampere-turns synchronous impedance for full-load current is

$$\frac{2.12\,SI}{p} + \frac{IxF}{E}$$

and the short-circuit current with excitation F is

$$I_0 = \frac{EpF}{2.12\,SE + xpF}$$

The synchronous impedance at this excitation is $Z_0 = E/I_0$, and the synchronous reactance is $x_0 = \sqrt{Z_0^2 - r^2}$, where r is the resistance of the armature per phase.

EFFICIENCY AND LOSSES. The losses are predetermined as in a generator. They are: A = friction, B = excitation or field RI^2, C = core loss, D = armature RI^2; then

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + A + B + C + D}$$

PHASE CHARACTERISTICS OR V-CURVES. The phase characteristic is a curve showing the variation in armature current for any given load with varying field excitation. Fig. 5 shows the shape of the curves, which may be determined both by calculation and test. The phase characteristic for any particular load has the general

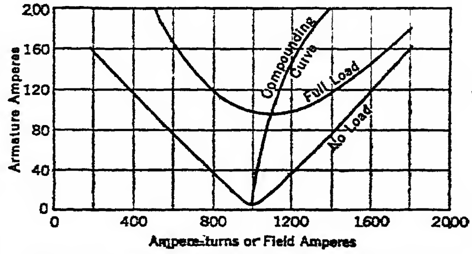

FIG. 5. Phase Characteristic of Synchronous Motor

shape of the letter V, and the group of such curves for various loads are frequently called the "V-curves" of the machine. There are two methods of calculating the phase characteristics, the electromotive force method and the magnetomotive force method.

Electromotive Force Method.

Let E = line voltage per phase.
e = motor counter emf per phase corresponding to the excitation to be used.
i = component of current in phase with e.
ei = mechanical output of armature.
i_1 = reactive component of current, positive for leading, negative for lagging.
r = armature resistance per phase.
x_0 = synchronous reactance per phase.

Then $E^2 = (e + ri - x_0 i_1)^2 + (x_0 i + ri_1)^2 = e_1^2 + e_2^2$.

E, r, and x_0 are constant, e is set by the value of the field current assumed, then i = (watts output)/e and i_1 remains the only unknown quantity. Solving for i_1, the total current $I = \sqrt{i^2 + i_1^2}$, and the power is $e_1 i - e_2 i_1$ (for i_1 lagging). If e is greater that E, then i_1 is leading, and the power factor is $(e_1 i + e_2 i_1)/EI$.

Magnetomotive Force Method.

Let E = line voltage per phase.
F = field ampere turns per pole for this voltage.
D = any value of field ampere turns per pole.
I = armature current per phase.
i = energy component of current = (power input) ÷ E.
p = number of poles.
ν = field leakage coefficient of machine.
S = turns in series per phase.
i_1 = wattless component of current for excitation D.

Then
$$pF = pD - 2.12\, Si_1 \nu$$
$$i_1 = \frac{pD - pF}{2.12\, S\nu}$$

and $I = \sqrt{i^2 + i_1^2}$ is the current in armature for power input Ei and for the excitation D assumed. If i_1 is negative $(F > D)$ then i_1 is lagging. If $D > F$ then i_1 is positive and leading.

ANGULAR LAG DUE TO LOAD (β). For any load on a synchronous generator there is an angular advance of the generated emf ahead of the terminal emf, and in a synchronous motor there is an angular lag of the generated emf behind the impressed emf. This phase displacement is accompanied by a shift in the synchronous position of the armature, which may be calculated and actually measured (see Tests of Synchronous Motors below).

To calculate this angle β expressed in electrical degrees (360 deg per pair of poles), let

E = line or terminal voltage per phase.
e = induced or counter emf per phase; may be taken from no-load saturation curve for given excitation.
r = resistance of armature per phase.
x_0 = synchronous reactance per phase.
I = current per phase.
ϕ = phase angle between E and I.

Then
$$e^2 = (E - Ir\cos\phi - Ix_0\sin\phi)^2 + (Ir\sin\phi - Ix_0\cos\phi)^2$$

and
$$\beta = \sin^{-1}\left[\frac{(Ir\sin\phi - Ix_0\cos\phi)}{e}\right]$$

or roughly
$$\sin\beta = \frac{I\cos\phi}{\text{Short-circuit current}}$$

The mechanical displacement of the armature for the load input $EI\cos\phi$ per phase is $2\beta/p$.

SYNCHRONIZING POWER. The synchronizing power of any synchronous machine is a measure of its ability to keep in step with others in parallel with it, generators or motors. It is expressed in the rate of change of power with respect to the displacement angle β, i.e., total kilowatts (all phases) per electrical degree (A.I.E.E. Standards).

$$\text{Synchronizing power, } P_s = \frac{\text{Power input}}{\beta}$$

where $\beta = \sin^{-1}\dfrac{\text{Power input}}{E \times \text{short-circuit current}}$, very closely.

A high resistance between machines reduces the synchronizing power as it reduces the impressed voltage. A reactance is not so bad. Increasing the excitation increases the short-circuit current and increases the synchronizing power.

HUNTING: NATURAL PERIOD. The rotating part of every synchronous machine acts like a pendulum, tending to swing ahead and behind its normal synchronous position. The mass of the armature (and its flywheel) acts like the mass of the pendulum, and the torque of the machine, being proportional to the displacement (β), corresponds to a spring or gravity acting on a pendulum. Such a combination has an "electromechanical period" of its own, and if the frequency of this period is in tune with any other pulsating force in the system, such as engine impulses, "hunting" or "surging" may occur.

The A.I.E.E. Standards give the natural frequency of hunting of any synchronous machine as:

$$F = \frac{266{,}500}{N}\sqrt{\frac{P_s f}{WR^2}}$$

in which F = cycles per minute.
N = revolutions per minute.
P_s = synchronizing power.
f = frequency of supply, cycles per second.
W = total weight (pounds) of whole revolving part.
R = radius of gyration of W in feet.

The formula shows that the greater the flywheel effect WR the lower will be the frequency. The greater the short-circuit current or excitation the greater will be the synchronizing power and hence the higher the natural frequency. The frequency may be decreased by connecting reactance coils in series with the machine between the bus-bars and the terminals of the machine. If F_0 is the frequency of any other pulsating force in the system the danger of hunting is greatest when

$$F/F_0 = 1, 2, 3, 4, \text{etc.}$$

A tendency to hunt is damped by solid pole pieces, bridges between poles, or, best of all, a squirrel-cage winding in the pole faces.

MAXIMUM OUTPUT. As the current which would flow during maximum output of a synchronous motor is so great that it would burn up the windings in case this output should last more than a fraction of a second, the value of the maximum output is of theoretical interest only. In practice the maximum output (for a given voltage) is only reached under two conditions: (1) when, owing to extraneous causes, the line voltage decreases to a fractional value, the maximum output decreasing as the square of the voltage; (2) when, owing to hunting or pulsation, the flow of energy into and out of the machine reaches excessive values momentarily. In one of these swings the power may reach the value of the maximum output or exceed it and the machine shut down. Although the power of the machine drops off gradually after the point of maximum output has been reached, the motor is unstable in this region and is more than likely to shut down when the condition is reached.

STARTING TORQUE. Synchronous motors may be started either as hysteresis * or induction motors. In the former case the motor requires a high voltage and takes a small current, but as the torque is very slight this method is seldom used in practice. When starting as an induction motor a high armature reaction is desirable and a low leakage reactance. A squirrel-cage winding in the pole face must be provided, and as in an induction motor this squirrel-cage winding must have a cross-section approximating a certain value. If the cross-section is too large, the currents will be excessive and the starting torque not the best. If the cross-section is too small, the currents will be small and the starting torque not the best. The cross-section of the squirrel-cage winding may be roughly predetermined by treating the machine as an induction motor, but this method is not accurate because the construction of the field renders the calculation of the leakage reactance inaccurate. It is thus much better determined empirically.

During starting and acceleration the armature flux sweeps past the field poles at high speed and induces a high voltage in the field windings. To protect these windings it is customary to break them up into several sections which are kept separate during starting and closed in series by the "field break-up switch" when synchronism has been closely approached. Salient-pole synchronous motors, if especially designed for starting conditions, will give 110 per cent of rated torque on starting and a "pull-in" torque of 110 per cent, with 3 to 4 times rated current. Starting torques of 2 or 3 times rated may be obtained by any of three special features:

* A piece of iron in a rotating field has a torque produced on it due to the hysteresis and eddy currents set up in it. Such an arrangement may be called a "hysteresis" motor; an open-circuited field of a polyphase synchronous motor therefore forms a hysteresis motor.

(*a*) A friction clutch between rotor and load.

(*b*) The "super-synchronous" motor in which both the primary and the field are rotatable on bearings, the field is normally the rotor but in starting is held stationary by the load. The primary, when voltage is applied, starts free and comes up to synchronous speed. Then a friction brake is applied to the primary; this reduces its speed, exerts a strong torque on the rotor, starts the load, and brings it up to speed as the primary comes to rest. This type will give 300 per cent torque in starting with 300 per cent current.

(*c*) The synchronous motor with polyphase wound rotor. The rotor is like that of an induction motor. It starts as a true induction motor with proper resistance in the secondary and has the usual starting characteristics of this type of motor. When the rotor approaches synchronism it is first completely short-circuited and then a direct current is introduced through one phase in series with the other two in parallel. This gives the d-c excitation.

PULL-IN TORQUE. In addition to the starting or stationary torque the "pull-in" torque is important in a synchronous motor.

It is the torque which the motor can exert to pull the rotor (and its load) into exact synchronism from the induction motor speed which is slightly less than synchronism (1 to 5 per cent). The lower the final resistance of the secondary the less is the slip and hence the easier is synchronization. Salient-pole rotors synchronize more easily than round rotors and usually synchronize before the d-c field is established; round rotors usually require d-c excitation before they synchronize. A very high load drag or considerable flywheel effect in the load make synchronization difficult as they require a great pull-in torque.

ROTARY PHASE MODIFIERS OR ROTARY CONDENSERS. Synchronous motors are sometimes used to improve the power factor and reduce the line current of an installation of a number of induction motors. If a factory has an installation of 100 kva of induction motors having an average power factor of 71 per cent and taking I amperes, then by installing a synchronous motor of 100 kva rating, designed to be overexcited, the power available will be doubled and the line current increased only by 41 per cent or to 1.41 I. Such a machine is called a "rotary condenser," and if it is rated at 100 kva it may give 71 kw of power and 71 kva to balance the reactive effect of the inductive apparatus. Other relations may be obtained in accordance with the principle of vector combinations.

Let P_1 = true power taken by the induction motors, kilowatts.
Q_1 = reactive power taken by the induction motors, kilovolt-amperes.
$L = \sqrt{P_1^2 + Q_1^2}$ = total kilovolt-amperes of induction motors.
P_2 = true power taken by rotary condenser, kilowatts.
Q_2 = reactive power taken by rotary condenser, kilovolt-amperes.
$K = \sqrt{P_2^2 + Q_2^2}$ = total kilovolt-amperes of condenser.

Then
$$\text{Line kva} = \sqrt{(P_1 + P_2)^2 + (Q_2 - Q_1)^2}$$

12. TESTS OF SYNCHRONOUS MOTORS

Certain tests on synchronous motors are the same as those made on an a-c generator; the methods of carrying out such tests are described in the article on Alternating Current Generators. The first five of the following tests are of this character:

1. **Resistance of armature and field circuits both cold and hot.**
2. **Saturation curve at no load and under special circumstances at full load.**
3. **Core loss.**
4. **Short-circuit or synchronous impedance.**
5. **Insulation tests.**
6. **Phase Characteristics or V-Curves** at no load, full load, and any other specified load. The machine is operated as a motor with the specified load kept constant throughout the run. The voltage and frequency impressed upon the motor are also kept constant. The current in the field is varied from the minimum at which the motor will operate to the maximum (from $^1/_4$ normal to 1 $^1/_2$ normal) and the variation in current input to armature noted. Readings are taken of load, volts armature, amperes armature, and amperes field. A curve is plotted with amperes armature as ordinates, and amperes or ampere turns per pole in field as abscissas. This gives the characteristic V-curves of the synchronous motor (see Fig. 5). The point of minimum current input for each load is very clearly shown. At this point the power factor is unity. At lesser values of field current the armature current is lagging and the power factor poorer. At greater values of field current the armature current is leading. The point of minimum current occurs at a higher value

of field current for the greater loads because the field excitation must be increased with the load to overcome the armature reaction due to the load current.

Compounding Curve. The curve connecting the points of minimum armature current in the group of V-curves is called the compounding curve for unity power factor.

7. Starting Tests. A low voltage is impressed on the armature and gradually increased until the motor starts. The field circuit is open in two or more places and a high-potential voltmeter connected across one section to determine the voltage induced in the field spools by the rotating magnetic flux. The test is repeated for several different initial positions of the revolving part, and a record is made of the time required for the machine to reach synchronism. The time at which synchronous speed is reached may be determined by the fact that the induced voltage in the field becomes zero. Readings are taken of volts armature, amperes armature, initial position, maximum volts field, time to reach synchronism. See also paragraph on Starting, below.

8. Armature Phase Position. The phase position of the armature discussed previously may be measured, although the item is of only theoretical interest and not of commercial importance. A synchronous motor is supplied with power from a special a-c generator, and on the end of the shaft of each machine is placed a contact-making disk, as shown in Fig. 6.

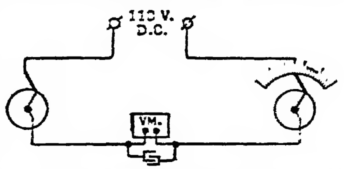

FIG. 6. Contact Method of Measuring Armature Phase Position

A voltmeter is connected in series with a source of direct current, the two disks, and the brushes pressing on the disks. The voltmeter reading is a maximum when the two brushes are in contact with the metal strips at the same time. The brush on the motor may be moved over a graduated scale or arc, so that its position may be varied and the actual angular movement measured. The brush on the generator remains stationary. As the load on the motor is increased it will be found necessary to move its brush in order to keep the voltmeter reading at its maximum value, and the angle β through which the brush has been moved for the change in load gives the phase position of the motor armature with respect to the generator armature. It will be found that the value of β is directly proportional to the load. The difference in phase of the electromotive forces in electrical degrees will be β multiplied by the number of pairs of poles of the motor. If hunting exists, the maximum angle of swing may be determined by moving the motor brush first one way as far as the effect may be noticed, and then in the other direction.

SPECIFICATIONS. Synchronous motors are rated in the same manner as synchronous generators, and the same heating limits and specifications apply. (See Alternating Current Generators.) It is customary to specify the value of the current taken by the motor in starting with no load other than the friction of its own bearings, or its own friction plus that of the machine to which it is connected, if it is part of a motor-generator set. It is also sometimes mentioned in the specifications that the motor will not hunt providing the total resistance drop between the generator and motor is less than some specified value (10 or 15 per cent).

13. INSTALLATION AND OPERATION

The precautions to be taken in installation are the same as for a-c generators (q.v.). Direct current must be provided for excitation. If a synchronous motor is operated on a polyphase system having unbalanced voltages it will take unequal currents in the different lines and tend to balance the voltages. These unequal currents, however, increase the heating somewhat for a given load.

STARTING. Provision must be made for a reduced voltage for starting the motor, either by means of taps on the transformers or by means of a starting compensator (q.v.). Large motors require two steps in starting, $^1/_3$ and $^2/_3$ of normal voltage. Small motors will start with one step at $^1/_2$ voltage. The field circuit is opened by the field "break-up" switch and a voltage applied to the armature. When the armature current has decreased from its large value at starting to a reasonable value the voltage is increased, step by step. When the motor has reached maximum speed the field is excited and the motor pulls into synchronism. The field current is then adjusted until the condition of minimum armature current is found or until the power-factor indicator records unity power factor. The load may then be applied.

If a synchronous motor is to be started often (several times a day) it is desirable to provide a special starting motor which brings the synchronous motor up to a speed a little above synchronism. Synchronizing must then be effected as in a-c generators. Such a

starting motor would require only about 30 per cent of the full-load current of the motor, and therefore have very little effect on the regulation of the system and not cause disturbance to the lights and other motors on the system. It usually requires less than a minute to bring a motor up to speed.

WEIGHTS. A standard line of synchronous motors for 60 cycles, 1200 rpm three-phase, 100 per cent power factor and for voltages up to 550 has weights (including base, pulley, exciter, and hand starter) approximately as follows:

Rating, hp	Weight in Pounds, Complete
25	1600
50	2500
100	3000
150	4000
200	4500

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POLYPHASE INDUCTION MOTORS

An induction motor may be either single, two, or three phase. Single-phase induction motors are treated in another chapter (q.v.). The induction motor is essentially a polyphase transformer with the secondary free to move; the electric energy transferred to the secondary is transformed by this motion directly into mechanical energy.

14. DEFINITIONS AND PRINCIPLES OF OPERATION

Certain terms used in connection with the induction motor can best be defined by a brief statement of the principles underlying its operation.

Primary and Secondary. By the primary of an induction motor is meant that part which receives energy by direct connection to the source of electric energy; the other member is called the secondary.

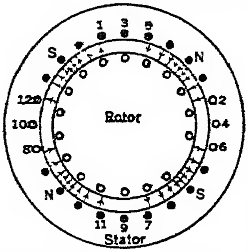

Fig. 1. Elementary Induction Motor

Stator and Rotor. That member of an induction motor which remains stationary, whether it be the primary or secondary, is called the "stator," and the revolving member is called the "rotor." In most machines the primary is the stator. A "squirrel-cage" rotor is one in which the conductors are straight bars of copper all connected together at each end of the rotor by copper rings.

Poles of an Induction Motor. In Fig. 1 is given a diagram of the primary winding of a two-phase four-pole induction motor. The small numbered circles represent the conductors forming the winding of one phase, the small black circles the conductors forming the winding of the second phase. The numbers opposite the circle give the order in which the current in phase 1 passes through the conductors, a cross indicating that the current goes down into the page and the open circles that the current is coming up.

The diagram is drawn to represent that instant at which the current in the second

phase is zero. At this instant the distribution of flux in the air gap will be roughly as indicated by the lines with arrows on them; that is, the flux will leave the stator iron in the two regions marked N and enter it in the two regions marked S. Consequently, the current in the winding of phase 1, which consists of 4 bands or groups of conductors, will produce 4 polar regions, or 4 poles, on the stator. As will be shown below, the combined effect of the two phase currents in the two windings is merely to cause a rotation of these polar regions. The " number of poles " is always equal to the number of bands of conductors into which the total winding of each phase is divided. The bands of conductors forming one phase usually overlap the bands of conductors forming the other phase, there being then two or more conductors per slot.

ROTATION OF MAGNETIC FLUX. In Fig. 1 is shown the distribution of flux in the gap when the current in phase 2 is zero; the curve marked A in Fig. 2 represents this same state of affairs, the cylindrical surface of the stator here being bent out into a plane, and the ordinates of the curve giving the value of the flux density in the gap at each point of this surface. The flux distribution is not a smooth curve as shown, but approximates such a curve.

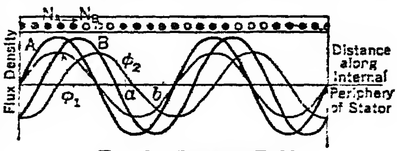

FIG. 2. Rotating Field

Next consider the case when the current in phase 1 has decreased to, say, 0.7 of its maximum value and the current in phase 2 has increased to 0.7 of its maximum value (this corresponds to 1/8 of the cycle). Then the flux distribution due to phase 1 remains in the same position as before but is reduced at each point of the gap to 0.7 of its maximum value, i.e., reduces to the curve marked ϕ_1. Similarly, the flux due to the current in phase 2 is similar in shape to the flux due to the current in phase 1, but its position is to the right of the latter by an amount equal to the width of one of the bands of conductors. The distribution of the flux due to the current in phase 2 is then as shown by the curve ϕ_2, the ordinates of which at the instant under consideration are 0.7 of their maximum values.

The resultant flux in the air gap at this instant is then the sum of the curves ϕ_1 and ϕ_2, namely the curve B. That is, the effect of the two fluxes due to the two phases is a resultant flux shifted forward, or moved around the gap, a distance equal to 1/8 the distance between successive north poles, but this resultant flux curve has the same shape and maximum value as before. This is strictly true only if the windings are distributed with absolute uniformity over the internal periphery of the stator. An extension of this analysis will show that the resultant flux remains constant in value at all times but travels around the air gap with a speed of

$$N = \frac{120f}{p} \text{ rpm}$$

where f is the frequency of the supply and p the number of poles.

This same result holds for a three-phase machine.

Synchronous Speed. The speed of rotation of the air-gap flux, namely the speed N given by the above formula, is called " synchronous " speed. At light loads the speed of the rotor is very nearly equal to this speed.

Slip. The slip of an induction motor is the ratio of the difference between the actual speed (N_1) of the rotor and synchronous speed (N) to the synchronous speed (N), i.e.,

$$s = \frac{N - N_1}{N}$$

The slip may be expressed as a fraction or as a percentage. The slip at standstill is unity; at no load it is very nearly zero. An induction motor driven at a speed higher than its synchronous speed has a negative slip; such is the case in an induction generator.

ELECTROMOTIVE FORCES IN SECONDARY. The emf's in the secondary of a polyphase induction motor are induced by the rotation of the flux produced by the currents in the primary windings, just as the emf's induced in the armature conductors of a generator are induced by the rotation of these conductors in the magnetic field set up by the field winding. In the generator only the conductors move, the field being stationary; but in the induction motor both the field and conductors move. In either case it is the *relative* motion of the field and conductors which determines the emf induced.

Let v be the linear speed at which the field moves, and let v_1 be the linear speed of the rotor conductors, and B the flux density at any particular conductor C at any instant. Then the emf induced in this conductor at this instant is $B(v - v_1)l = Blsv$, where s is the slip and l is the length of the conductor (see Section 3, Art. 49). The rotor emf is therefore proportional to the slip. As the rotor turns, the conductor C moves slower

than the rotating flux, and the state of affairs is just the same as if the flux remained at rest and the conductor moved through the field in the gap at a speed of $v - v_1 = sv$. Hence the emf induced in each rotor conductor is alternating, since the flux which it cuts varies from a positive maximum to a negative maximum, and a consideration of the relative speed of the conductor and the flux will show that the frequency of this induced emf is the frequency in the primary multiplied by the slip.

That is, the secondary electromotive force is proportional to the slip and has a frequency equal to the product of the slip and the frequency in the primary.

SECONDARY CURRENT AND TORQUE. The current set up in each rotor conductor by this emf will be practically in phase with this emf, since the rotor conductors have but a small reactance, particularly when the rotor is revolving at a speed near synchronism. Hence the current in any chosen rotor conductor at any instant is proportional to the emf in this conductor at that instant, which in turn is proportional to the flux density at this conductor at this instant. Since the force produced by a magnetic field on a conductor is equal to the product of the flux density by the current by the length of the conductor, the force acting at any instant on any rotor conductor will then be equal to

$$f = Bli = Bl\left(\frac{Blsv}{r}\right) = \frac{B^2 l^2 sv}{r}$$

where r is the resistance of the conductor. Since the current and the flux density both change signs at the same time (being in phase), the direction of this force will always be in the same direction and will consequently drive the rotor against whatever opposing force may exist.

Not only is the force on each conductor always in the same direction, but the *total* force acting on *all* the conductors is practically constant for a given value of the slip. Consider any two rotor conductors which are a distance apart equal to 1/4 the distance measured along the periphery of the rotor between successive north poles, for example, at a and b in Fig. 2. Then, assuming a sine-wave distribution of flux in the air gap, and calling B_m the maximum flux density, the flux density at a is $B_a = B_m \sin x$ and the flux density at b is $B_b = B_m \cos x$, where x is a function of the distance measured from some fixed point in the air gap. Then the total force on the two conductors at a and b is

$$\frac{l^2 svB_m^2}{r}(\sin^2 x + \cos^2 x) = \frac{l^2 svB_m^2}{r}$$

and is therefore constant, since B_m is a constant. Similarly for any other two conductors this same distance apart. Hence the total force on all the conductors is constant. On any practical machine the flux distribution is not an exact sine wave, and there is a slight pulsation in the total force, and therefore in the torque, but this pulsation is extremely small.

MAGNETIZING CURRENT. The currents set up in the secondary of an induction motor produce a rotating flux, which travels with the same speed with respect to the primary and in the same direction as the flux set up by the current in the primary, but the direction of this secondary flux at any point in the air gap is opposite to the direction of the primary flux. Hence the resultant flux when there is current in the secondary is equal to the difference of these two fluxes, and this difference remains practically constant irrespective of the secondary currents, just as the resultant flux in a transformer is practically independent of the secondary current. The primary current which would be necessary to produce this *resultant* flux is called the "magnetizing" current, and is very nearly equal to the current in the primary when the motor is running without load, in which case the current in the secondary is extremely small.

15. METHODS OF RATING

The Standardization Rules of the A.I.E.E. recommend that the rating of an induction motor should be the load in horsepower which it will deliver continuously at the shaft with a maximum rise in temperature of any part not exceeding 50 deg cent by thermometer.

Continuous Rating. The load (mechanical horsepower) which can be carried for an unrestricted period of time without exceeding any of the limitations as to temperature rise, etc., as established by the American Standards Association (Publication C50).

Short-time Rating. The load which can be carried for the time specified in the rating without exceeding these limitations.

In the absence of any specification as to the kind of rating the continuous rating is implied.

STARTING AND BREAK-DOWN TORQUE. In addition to the ability to carry its rated load without excessive heating and with reasonable constants, such as efficiency,

power factor, and slip, it is advisable to make sure that the motor is able to start such loads as must be brought up to speed with the motor, as good starting ability in an induction motor involves certain complications and expenses. This subject is treated at length in Art. 19, Methods of Starting. Another important point is that the motor should be able to carry momentary overloads without "breaking down" as it is called, which means gradually decreasing in speed to a standstill when the load is excessive. To be sure of this qualification we must know the maximum output of the motor, which should be at least 50 per cent greater than the rated output.

VOLTAGE. Motors may be wound for any voltage up to 13,000, but the great majority and all the small motors are wound for voltages of 110, 220, or 440 volts between lines.

FREQUENCY AND SPEED. Induction motors may be built for any frequency. The higher frequencies are satisfactory in those cases where the load never exceeds normal conditions. Lower frequencies, such as 25, are more favorable where frequent overloads are met with or large starting torques are required.

The speed of the rotor of an induction motor at normal loads approaches within 5 to 10 per cent of the synchronous speed. The synchronous speed is fixed by the frequency of the system and the number of poles of the winding so that for a given frequency of the supply circuit only certain speeds are available. Thus for 60 cycles we have

3600 for 2 poles; 1200 for 6 poles;
1800 for 4 poles; 900 for 8 poles, etc.

PHASE CONNECTIONS. Two-phase or quarter-phase motors are usually wound with independent phase windings. Three-phase motors are connected in Y or Δ, depending upon the convenience of the designing engineer.

In a single-phase and two-phase motor the voltage and current per phase are the same as the voltage between lines and current in line; in a Y-connected three-phase motor the current per phase is equal to the line current, and the voltage per phase is equal to the line voltage divided by $\sqrt{3}$; in a Δ-connected three-phase motor the current per phase is equal to the line current divided by $\sqrt{3}$, and the voltage per phase is equal to the line voltage.

CURRENTS TAKEN BY MOTORS.

Let P_0 = horsepower output.
I = current in each line.
ϵ = efficiency as a decimal fraction.
$\cos\phi$ = power factor as a decimal fraction.
E = voltage between lines (between one outside wire and the middle wire for three-wire two-phase line).

Then for

$$\text{Two phase: } I = \frac{373\,P_0}{\epsilon E \cos\phi}$$

$$\text{Three phase: } I = \frac{431\,P_0}{\epsilon E \cos\phi}$$

Usual efficiencies and power factors at full rated load for polyphase motors are as follows:

Table 1. Power Factors and Efficiencies

Horsepower	25 Cycles		60 Cycles	
	Efficiency	Power factor	Efficiency	Power factor
1	0.79	0.78	0.78	0.78
5	0.85	0.88	0.82	0.88
20	0.88	0.91	0.84	0.91
50	0.90	0.92	0.87	0.92
100	0.905	0.925	0.89	0.92
200	0.91	0.925	0.905	0.92

16. DESIGN

The methods of calculating two-phase and three-phase motors are practically the same. Most induction motors on single-phase circuits are made with polyphase windings, as the extra winding is necessary in starting.

The factors which must be considered in the design of an induction motor are the same

as those considered for a synchronous generator with the addition of the power factor. It is desirable to have a high power factor, but a high power factor requires a generous use of material, a small air gap, and a careful arrangement of windings. A high power factor at light load requires a small magnetizing current, and a high power factor at overloads requires a low value of leakage flux.

A small value of magnetizing current is obtained by using a small air gap and a large value of diameter per pole.

A low value of leakage flux is obtained by using a large value of diameter per pole and by subdividing the windings in a large number of slots.

GENERAL PROPORTIONS. In the discussion below the following symbols are employed:

E = volts per phase.
I = full-load current per phase.
I_m = magnetizing current per phase.
D = diameter of armature, inches.
L = length of armature core, inches.
L_1 = length of iron = (L − air ducts), inches.
l = effective length of iron, inches = 0.9 L_1.
p = number of poles.
f = frequency in cycles per second.
g = length of gap, inches.
B = average flux density in air gap, maxwells per square inch.
Φ = flux per pole, maxwells.
S = turns in series per phase.
N = revolutions per minute.
V = peripheral speed in feet per minute.
σ = ampere-conductors per inch of periphery.

General equation: $$\text{Kva input} = \frac{\sigma B V D L_1 k_2 k_3}{144 \times 10^9}$$

This is the same as for polyphase generators (see Art. 2, eq. 1) except that ρ is unity and L_1 the actual length of iron must be used for accuracy as there is very little spreading of the flux in the air gap. σ has values as given in Table 2. Usual values of B are 25,000 in 60-cycle motors and 30,000 in 25-cycle motors. V is determined by the mechanical construction chosen and usually is between 4000 and 6000 fpm. k_2 and k_3 are the winding constants as given on p. 9–08. L_1 is usually the desired unknown quantity. With $^1/_2$-in. ducts spaced 2.5 in. the relation is $L = (L_1 + {}^1/_2\, n)$, where n is the number of air ducts.

Table 2. Data on Induction Motors

Diameter, inches	Slots, number	Depth of Slots, inches	Gap, Radial, inches	Ampere-conductors per Inch
10	48–60	1.0 –1.4	0.02–0.03	350
20	60–90	1.25–1.8	0.03–0.04	500
30	72–120	1.5 –2.0	0.04–0.05	600
50	168–192	1.8 –2.12	0.06	850
100	300–420	3.0 –4.0	0.10	1000
150	260–450	3.5 –5.0	0.15	1200

The flux per pole is $\Phi = \dfrac{\pi BDL}{p}$, and the number of turns in series per phase is

$$S = \frac{\pi D \sigma}{2I \times \text{Phases}}.$$

The total number of "effective conductors" around the whole periphery of the primary is $Z = 2S \times$ Phases. These may be subdivided by the use of parallel conductors or parallel paths so that the real number of conductors is 2, 4, 6, etc., times this quantity. The winding of an induction motor is of the same general construction as is used in a-c generators (page 9–05). The multiple or lap winding is always used with two or four coil-sides per slot and two, three, or four slots per pole per phase. Thus several consecutively placed coils are connected in series in one phase belt, which may be connected either in series or multiple with the phase belts under other poles. Each coil may have as many turns in series as necessary for the voltage.

The three phases may be connected either in Y or in delta to suit the exigencies of the design. Fractional pitch is quite generally used to reduce the leakage reactance and the length of the end-connections, particularly in machines with a large diameter per pole, such as 25-cycle motors and high-speed machines with a small number of poles. A pitch of two-thirds is common in 25-cycle motors and five-sixths in 60-cycle machines.

The size of each conductor depends upon the current to be carried and the current density, which may be used as determined by heating. For a rise of 50 deg cent the values of current density given in Table 3 are usual in open motors having self-ventilation, like most industrial motors.

Table 3. Current Densities

Size of Motor	Voltage	Amperes per Sq In.
Small..........	Low...........	3000
Small..........	High...........	2000
Large..........	Low...........	2000
Large..........	High...........	1600
Large..........	Above 6000 ..	1000

In the United States the practice is to use open slots in the primary, the slots having a depth about four times the width and containing two coil-sides. In Europe, with lower labor costs, partly closed slots with overhung teeth are used extensively. In all cases the secondary slots are almost closed at the face and four-coil sides per slot are used for a definite winding to which external resistance is to be connected for starting, while one bar per slot is used for a squirrel-cage winding. These bars are well connected electrically to a ring at each end, sometimes of copper, sometimes of brass.

The size of the slots is determined by the size and arrangement of the conductors and the insulation, which consists of insulation on the individual conductors and on the coils, and in some cases of an insulating lining to the slots. This insulation occupies space as given in Table 4, the dimensions being total, on both sides of the conductors and coils, and also allows for the wedge which holds the coils in the slot. The width of open slots is from one-half to two-thirds of the slot pitch along the gap. The width of closed slots may be slightly greater than this.

Table 4. Insulation Allowances

Type of Slot	Voltage	Coil-sides per Slot	Allowance for Insulation	
			Vertical	Horizontal
			in.	in.
Straight......	500	2	0.30	0.08
Straight......	2200	2	0.45	0.15
Straight......	6000	2	0.65	0.25
Overhung....	500	4	0.40	0.15
Overhung....	2200	4	0.50	0.20

FLUX PER POLE.
$$\Phi = \frac{E \times 10^8}{4.44 fSk}$$

where k is the combined distribution constant of the winding as in Table 5.

Table 5. Distribution Constant k

Two Phase					Three Phase				
Slots per Pole	Per Cent Winding Pitch				Slots per Pole	Per Cent Winding Pitch			
	100	75	67	50		100	75	67	50
	k	k	k	k		k	k	k	k
2	1.00	....	....	0.71	3	1.00	....	0.87	....
4	0.93	0.85	....	0.66	6	0.97	....	0.84	0.69
6	0.91	....	0.79	0.64	9	0.96	....	0.83	0.68
8	0.905	0.84	....	0.64	12	0.96	0.89	0.83	0.68
12	0.90	0.836	0.78	0.63	18	0.958	0.885	0.83	0.68
Many	0.90	0.833	0.78	0.63	Many	0.958	0.885	0.83	0.68

PHASE CONNECTION. Whether a motor shall be delta or Y connected depends upon such minor details as the convenience of arranging the conductors in the slots. For instance, if for 110 volts and delta connection a desirable flux value and number of conductors would require 7 conductors per slot, a Y connection having 64 volts per phase and 4 conductors per slot could be substituted and would give a practicable winding; 64 volts per phase in Y connection gives 110 volts between lines.

EXCITING CURRENT. This current is made up of two components: the "magnetizing current" which lags the impressed voltage by 90 deg, and a component in phase with the impressed voltage which supplies the core loss. The "no load" or "running light" current consists of the magnetizing current and a small component to supply the friction loss. The magnetizing current is much the largest component, so that it is frequently stated that the running light current is the magnetizing current.

The distributed primary winding, which is the seat of the mmf, gives a peaked wave

shape of mmf in the teeth and gap which, with three phases and full pitch, takes the form shown at a in Fig. 3 at one instant and then one-twelfth cycle later takes the form shown at b, and changes from one to the other of these every twelfth of a cycle. At the same time the wave moves one-twelfth of a pair of poles along the air gap, the change from a to b being gradual. The maximum value varies between 1.72 and 1.5 times the average value with a mean of 1.61. If there were no saturation in the iron the flux would do the same but, owing to saturation in the teeth under the peak, the real flux wave is not as peaked, so that a reasonable value for the ratio of the maximum to the average flux density is from 1.57 to 1.5 and is usually taken at 1.57 although with extremely high densities in the teeth the ratio is 1.5. Customary values for the magnetic densities in the various parts are given in Table 6.

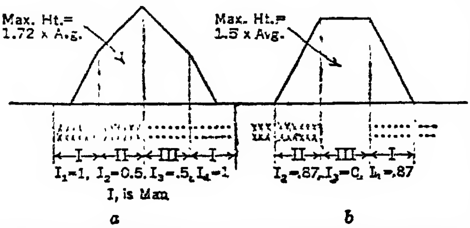

Fig. 3. Distribution of the Mmf of the Primary for the Two Limiting Conditions. Six Slots per Pole per Phase, Full Pitch

Table 6. Usual Maximum Flux Densities

	25 Cycles	60 Cycles
Stator core..........	70,000	50,000
Stator teeth.........	90,000	70,000
Air gap.............	48,000	40,000
Rotor teeth.........	80,000	60,000
Rotor core	90,000	80,000

MAGNETIZING CURRENT. The magnetizing current is calculated by determining the flux density in each part of the magnetic circuit and the ampere turns required for each part. This is most easily done by means of the following tabulation:

Magnetic Densities and Mmf's

(For dimensions refer to Fig. 4)

Part	Flux per Pole	Area	B_{max}	A.T. per Inch	Length Path	Total A.T.
Stator core........	$\phi/2$	$h_1 \times l$	$1 \times B_{avg}$	......	$\dfrac{\pi D_1}{2 \times \text{poles}}$	......
Stator teeth.......	ϕ	$t_0 \times l \times \dfrac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$	......	n_1	......
Air gap...........	ϕ	See below	$1.57 \times B_{avg}$	......	$C_1 C_3 g$	......
Rotor teeth........	ϕ	$t_3 \times l \times \dfrac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$	......	n_2	......
Rotor core.........	$\phi/2$	$h_2 \times l$	$1 \times B_{avg}$	......	$\dfrac{\pi D_2}{2 \times \text{poles}}$	......
					Total.....	....

Stator Core or Yoke. The flux divides in the core, one-half going each way. The maximum and average densities are practically the same and equal to ($\phi/2$ divided by the radial depth h_1 times the effective length l). The material is laminated steel of high permeability. The ampere-turns per inch length of path are obtained from a magnetization curve of the steel; see Magnetic Materials, Sect. 2. The length of path is indeterminate but closely approximates one-half of the pole pitch measured on the circle of diameter D_1. By multiplying the ampere-turns per inch obtained from the magnetization curve by the length of path, the ampere-turns required to send the flux through this path are obtained.

Stator Teeth. The average density in the teeth is first obtained. The effective area of one tooth is the area one-third the distance from the face to the root of the tooth as shown at t_0. This gives the average magnetizing force rather than the average density. The total cross-section of the path in the teeth is therefore $t_0 \times l \times$ (the number of teeth per pole). Owing to the peaked or sinusoidal space distribution of the flux the maximum density in the teeth is 1.57 times the average density. The ampere-turns required depend upon the maximum density. From the proper curve determine the ampere-turns per inch necessary to establish this density, and, by multiplying this quantity by the distance n_1 in inches, the ampere-turns for the stator teeth are found.

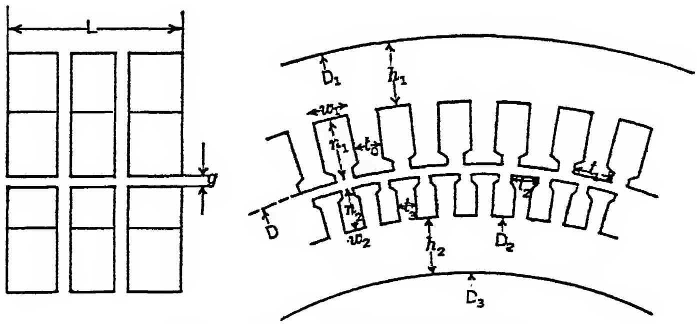

FIG. 4. Cross-section of Magnetic Circuit

The air gap in induction motors is very short, hence the flux is very definitely bunched at the faces of the teeth and this phenomenon assumes more importance than in any other type of machine. This is evaluated by the "Carter coefficient" which is explained on page 8–09 under D-c Generators.

Since, in an induction motor, there are slots in both the stator and rotor, there are two coefficients, C_1 and C_2. C_1 is for the stator and is based upon the stator slot opening and the stator slots per pole. C_2 is for the rotor and is based upon these characteristics of the rotor. The area of path is $\frac{\pi D L_1}{\text{Poles}}$, the maximum space density is $\frac{1.57\,\Phi}{\text{Gap area}}$. The ampere-turns per inch are max density $\times$ 0.313. The equivalent gap length is $C_1 C_2 g$, and the ampere-turns for the air gap are $1.57B \times 0.313\, C_1 C_2 g$.

Rotor Teeth. The calculation of the ampere-turns for the rotor teeth follows the same method as the calculation for the stator teeth.

Rotor Core. The same as for the stator core.

In a three-phase machine the magnetization is produced by the combined effects of the three windings. At the instant shown at a the magnetic density is a maximum, the current in one phase is a maximum, and the total mmf of the three windings is

$$\sqrt{2}I\,S + \tfrac{1}{2}\sqrt{2}I\,S + \tfrac{1}{2}\sqrt{2}I\,S = 2\sqrt{2}SI$$

where I is the effective value of the magnetizing current. Hence the magnetizing current in a three-phase machine is:

$$I = \frac{(\text{Ampere-turns per pole}) \times \text{Poles}}{2\sqrt{2}S \times k}$$

where k allows for the effect of fractional pitch and form factor as in Table 5.

Similarly for a two-phase machine:

$$I = \frac{(\text{Ampere-turns per pole}) \times \text{Poles}}{\sqrt{2}S \times k}$$

The form of the magnetizing current is not a true sine wave because of saturation and hysteresis but it is much nearer to a sine wave than the magnetizing current of a transformer since the motor has a considerable air gap in its magnetic circuit.

RESISTANCE PER PHASE OF PRIMARY WINDING. The resistance to direct current, or ohmic resistance, is given by the formula

$$r_1 = \frac{0.01\, S \times (\text{Mean length of turn})}{12{,}000\, an},$$

where S = turns in series per phase.
a = cross-section of one conductor in square inches.
n = number of conductors or circuits in parallel.

0.01 is the resistance at 75 deg cent of a conductor 1000 ft long and 1 sq in. cross-section. The mean length of turn is approximately (see Fig. 4)

$$2L + 10\frac{D}{p} \times (\text{Pitch as a fraction}) \qquad \text{inches}$$

Owing to the eddy currents set up in the primary conductors by the total flux and to eddy currents set up in the core by the leakage flux, the "effective" resistance of the primary winding is about 15 per cent greater than the value calculated by the above formula.

RESISTANCE PER PHASE OF SECONDARY WINDING. In a wound rotor the resistance per phase is found in the same manner as the resistance per phase of the stator winding. The secondary resistance reduced to primary is then equal to

$$\left(\frac{\text{Number primary turns per phase}}{\text{Number secondary turns per phase}}\right)^2 \times (\text{Actual effective secondary resistance}),$$

when the secondary has the same number of phases as the primary.

Squirrel-cage Secondary. The effective resistance of a squirrel-cage winding is best calculated on the assumption of equal ampere-turns in primary and secondary at rated load and determining the losses in the secondary for any given current in the primary.

$$c_1 = \text{Effective conductors per primary slot} = \frac{\text{Actual conductors}}{\text{Parallel circuits}}$$

$$\text{Ampere conductors per secondary slot} = \text{Rated primary current} \times c_1 \times \frac{\text{Primary slots}}{\text{Secondary slots}}$$

$$\text{Current density in secondary bars} = \frac{\text{Ampere conductors}}{\text{Area one bar}}$$

$$\text{Watts lost in all bars} = 0.825\ 10^{-6}\ (\text{Volume of bars})\ (\text{Amperes per square inch})^2$$

$$\text{Area of bars per pole} = (\text{Area one bar})\ (\text{Number of bars per pole})$$

$$\text{Current density in rings} = \frac{(\text{Current density in bars})\ (\text{Area bars per pole})}{4(\text{Area of one ring})}$$

$$\text{Watts lost in rings} = 0.825 \times 10^{-6}\ (\text{Volume of both rings})\ (\text{Density in rings})^2$$

The loss in watts in 1 cu in. of copper at 75 deg cent and at 1000 amp per sq in. is 0.825. For other metals this is proportional to the relative resistivity.

Resistance at 75 deg cent of secondary in terms of the primary for a three-phase motor

$$r_2 = \frac{\text{Total loss}}{3(\text{Rated primary current})^2}$$

For a two-phase motor the factor 2 takes the place of the 3 in this denominator.

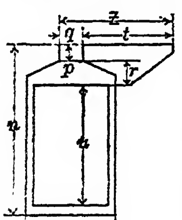

Fig. 5. Slot Dimensions

LEAKAGE REACTANCE. When the motor is loaded the currents in the secondary set up a counter mmf which causes part of the flux to pass along the air gap instead of into the secondary core. This flux does not interlink both members and is therefore a leakage or useless flux. It is proportional to the load currents and to the permeance of this path. As the greater portion of the path is in the air and is of high reluctance therefore that part of the path in the iron may be neglected. The flux in both the primary and secondary must be calculated.

There are three leakage flux paths to be considered in both primary and secondary. Referring to Fig. 5 the equivalent permeances or flux per ampere conductor are:

$$P_s = 3.2L\left(\frac{u_1}{3w_1} + \frac{2r_1}{w_1 + q_1} + \frac{p_1}{q_1}\right)$$

$$P_t = 3.2L\left(\frac{t_2 - q_1}{6g}\right)$$

$$P_c = 0.59 s_1\, l_c \log_{10} \frac{2l_c}{v_c} \text{ for a double layer winding}$$

$$P_1 = P_s + P_t + P_c$$

Primary leakage reactance per phase

$$x_1 = 2\,\pi f p s_1\, c_1^2\, P_1\, k\, 10^{-8} \text{ ohm}$$

Secondary leakage reactance per phase in terms of the primary winding:

$$x_2 = 2\,\pi f p s_2\, c_2^2\, P_2\, k\, 10^{-8} \times \left(\frac{c_1 s_1}{c_2 s_2}\right)^2$$

where g = length of gap in inches.
l_c = length of end connections at one end, in inches.

V_c = perimeter of a band of end connections per pole per phase, in inches.
c_1 and c_2 = the effective conductors per slot (see above).
s_1 and s_2 = the slots per pole per phase.
f = frequency.
L = total length.
k = winding distribution constant (see above).
P_1 and P_2 are for primary and secondary windings, respectively.

LOSSES IN INDUCTION MOTOR. The losses in an induction motor are:

Core loss.
Friction, bearing, and windage.
Primary copper loss.
Secondary copper loss.

The first two of these are approximately constant for all loads, and the last two vary as the square of the current per phase.

Core Loss. The distribution of the magnetic flux in an induction motor is quite irregular both in the core and in the teeth. The losses are therefore greater and their calculation more involved than in machines having uniform density in each part. In the primary the frequency of the passage of the secondary teeth introduces pulsations which increase the losses. In the secondary the frequency, being proportional to the slip, is so low that the core loss is negligible.

In order to avoid too lengthy and complex calculations use is made of empirical constants, by which the easily calculated losses are multiplied to derive the practical loss. The total loss consists of hysteresis and eddy-current loss in the primary core and primary teeth. The loss in watts per cubic inch for one cycle per second at any magnetic density is found by the curves given in the article on Magnetic Materials.

The losses are then calculated as follows:

Hysteresis Loss:
In primary core $= k_h C_1 f V_c$ watts,
In primary teeth $= k_h C_2 f V_t$ watts.
Eddy-current Loss:
In primary core $= k_e C_3 f^2 V_c$ watts,
In primary teeth $= k_e C_4 f^2 V_t$ watts,

where V = volume in cubic inches; f = frequency; k_h = empirical constant, 1 to 1.5; k_e = empirical constant, 3 to 4; C_1, C_2 = hysteresis loss per cubic inch for one cycle per second; C_3, C_4 = eddy-current loss per cubic inch for one cycle per second. η is usually 0.002 and $\epsilon = 0.0001$.

The higher values of k_h and k_e are to be used where open slots are used and where the frequency of the passage of the secondary teeth past one primary tooth is high. The core loss is the sum of all the above losses.

Friction and Windage Loss varies greatly with the style of motor and form of structure bearings, etc. The loss is made up of bearing friction and wind friction. The latter may be large purposely, as the motor may be designed with fan blades in order to circulate the air for the purpose of keeping the motor cool by ventilation. As each manufacturer has a formula which will calculate the friction loss correctly for machines built according to a particular plan, no general formula can be assumed. The nearest approach to an estimate is obtained from the percentage given in the table below.

Primary and Secondary Copper Losses. The calculation of the effective resistances of the primary and secondary windings is given above. The primary or secondary copper loss is equal to the product of the number of phases, the effective primary or secondary resistance per phase, and the square of the current per phase. The total copper loss is the sum of the primary and secondary copper losses.

POWER FACTOR. The power factor ($\cos \phi$) of an induction motor for a given current input (I) may be calculated roughly as follows:

Let $\frac{I_m}{I} + \frac{IX}{E} = a$. Then

$$\text{Power factor} = \frac{1}{\sqrt{1 + a^2}}$$

where I_m = magnetizing current per phase.
I = total current per phase.
$X = (x_1 + x_2)$ = total reactance per phase, the secondary reactance x_2 being reduced to the primary turns.
E = voltage per phase.

For a more exact formula for power factor and for the usual values of the power factor at rated load, see below.

EFFICIENCY. The efficiency (as a fraction) for a given current input is

$$\epsilon = 1 - \frac{F + C + m(r_1 + r_2)I^2}{mEI \cos \varphi}$$

where F = friction loss, in watts.
C = total core-loss, in watts.
m = number of phases.
r_1 = effective primary resistance per phase.
r_2 = effective secondary resistance per phase reduced to primary.
E = voltage per phase.
I = current per phase.
$\cos \phi$ = power factor.

The corresponding horsepower output is then

$$P_0 = \frac{\epsilon mEI \cos \phi}{746} \text{ horsepower}$$

Usual values of the efficiency are given above in the paragraph, Currents Taken by Motors. Usual values of the component losses for 60-cycle motors are given in the following table.

In 25-cycle motors the exciting current is usually greater than in 60-cycle motors and the IX drop less. Thus the power factor will be lower at light load and higher at overloads than in 60-cycle motors.

Table 7. Induction Motor Losses

Rating, hp	$\frac{I_m}{I}$	$\frac{XI}{E}$	$\frac{\text{Friction loss}}{\text{Input}}$	$\frac{\text{Core loss}}{\text{Input}}$	$\frac{r_1 I}{E}$	$\frac{r_2 I}{E}$
1	0.45	0.20	0.06	0.05	0.05	0.05
5	0.35	0.14	0.03	0.04	0.04	0.04
20	0.30	0.13	0.02	0.035	0.03	0.035
50	0.27	0.11	0.015	0.03	0.025	0.03
100	0.26	0.11	0.015	0.03	0.025	0.025
200	0.25	0.11	0.015	0.03	0.025	0.02

The efficiency of 60-cycle motors naturally tends to be higher than of 25-cycle motors. This quality is usually sacrificed by economizing in material and making the 60-cycle motors lighter and cheaper but of about the same efficiency as the 25-cycle motors.

Maximum Efficiency and Power Factor. Since induction motors frequently operate on an intermittent or variable load, it is desirable that the efficiency and power factor be high at fractional loads. It is therefore quite usual to design the motors so that the maximum efficiency comes at $^3/_4$ load and maximum power factor at less than full load. This is accomplished with regard to efficiency by making the core loss and friction small (the core loss is made small by using low flux densities) and with regard to power factor by making the magnetizing current small by using a small air gap.

SLIP AND SPEED. The slip for any given current is approximately

$$s = \frac{r_2 I}{E}$$

where the symbols are defined in the preceding paragraph.

The synchronous speed is

$$N = \frac{120 f}{p} \text{ rpm}$$

where f is the frequency of the supply and p the number of poles. The actual speed of the motor is then

$$N_1 = (1 - s) N$$

VALUES OF OTHER CHARACTERISTICS. The energy component of the no-load current is

$$I_e = \frac{(\text{Total core loss}) + (\text{Total friction loss})}{mE}$$

where m is the number of phases and E the volts per phase.

The total no-load current per phase is then

$$I_{00} = \sqrt{I_e^2 + I_m^2}$$

where I_m is the magnetizing current per phase.

The impedance per phase at standstill ("short-circuit" impedance) is

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}$$

where r_1 and r_2 are the effective values of the primary and secondary resistances per phase, the latter being reduced to primary turns, and x_1 and x_2 the primary and secondary reactances per phase, the latter being reduced to primary turns.

The current per phase at standstill ("short-circuit" current) is

$$I_s = \frac{E}{Z}$$

The starting torque in pounds at 1-ft radius is

$$T_s = \frac{7.06\, m r_2 E^2}{N Z^2}$$

where m, r_2, E, and Z are as defined above, and N is the synchronous speed in revolutions per minute.

The slip at maximum output is

$$s_m = \frac{r_2}{r_2 + Z}$$

The maximum output in watts is

$$P_m = \frac{0.5\, mE^2}{1.3\,(r_1 + r_2) + Z} \text{ watts}$$

This should be from 1.5 to 3 times the rating of the motor. A 25-cycle motor usually has a greater maximum output or overload capacity than a 60-cycle motor.

HEATING. Since in most induction motors the primary member is subject to both a core loss and a copper loss and is stationary, the heating of this member is very important. As the secondary contains only a copper loss and is usually revolving, its rise in temperature is generally much less than that of the primary.

The problem therefore consists of analyzing the flow of heat and drop in thermal potential in the primary as the energy lost in the windings flows to the iron of the core through the insulation in the slots and to the air around the projecting end connections. The core itself is maintained at a temperature above the air by the core loss. It is therefore necessary to calculate first the rise in temperature of the iron of the core caused by the core loss and that portion of the copper loss which is conducted to the core through the slot insulation. It is then necessary to calculate the rise in temperature of the copper above the iron and above the air around the end connections.

The laws for the flow of heat and difference in temperature in the primary of an induction motor are the same as in the armature of a d-c machine. The principal uncertainty is the velocity of the cooling air as it is induced by the intentional irregularities of the surface of the rotor which form small fan blades. However, the analysis given for d-c generators (page 8–19) is also used for induction motors.

17. TESTING OF INDUCTION MOTORS

(See also Standardization Rules of A.I.E.E.) Induction motors may be given either an "input-output" test at load under working conditions, from which the efficiency and power factor may be determined, or the motor may be given a no-load excitation and no-load short-circuit test, from which all the characteristics may be calculated. The latter method requires very little power and is preferable for large motors where it would be expensive to supply power and inconvenient to dissipate the energy.

EXCITATION TEST. In this test the machine is operated without load at constant frequency with the voltage varied through a range from 30 per cent above rated value down to as low a voltage as will cause the machine to rotate. The current in each phase is noted, and by means of two wattmeters the power required is noted. This test is sometimes also made with the machine operating single phase for the purpose of checking the core loss. The curves are plotted with voltage as abscissas and amperes and watts as ordinates. At very low voltages the core loss is negligible and therefore the watts input may be taken as friction loss. Around normal voltage the real core loss may be determined by subtracting from the input the small copper and the friction loss.

SHORT-CIRCUIT IMPEDANCE TEST. In this test the starting resistance, if there is any, is short-circuited and the armature is blocked to prevent rotation. A low voltage

is applied to the primary until the ammeter in the primary circuit shows a current of from 1 to 1.5 times the full-load value. Two watt-meters are used to read the power input. As the leakage flux, and hence the reactance, varies with the relative position of the rotor and stator teeth, it is customary to take readings with the rotor in several positions. Sometimes the rotor is allowed to revolve very slowly at, say, 2 to 3 rpm. From this test the impedance (Z) of the machine is obtained by dividing the volts per phase by the current per phase; the combined effective resistance (R) of primary and secondary is found by dividing the watts input per phase by the square of the current per phase. This latter is the effective value of the resistance, since the watts input includes all losses due to eddy currents. From these values the total reactance (primary and secondary) of the motor is $X = \sqrt{Z^2 - R^2}$. With the results of the two preceding tests and the measured resistances of the two members, the characteristics of the motor may be calculated either by the Steinmetz Method or the Circle Diagram. (See below under Calculation of Performance.)

STATIONARY TORQUE TEST (Fig. 6). This test may be made at the same time the impedance test is made; it consists in the measurement of the torque of the motor by means of a brake arm and spring balance. When the current of the machine has been adjusted to a suitable value the brake arm (to which a known weight has been attached to overcome bearing friction) and the spring balance are allowed to move downward through a small arc, during which the spring balance will register a pull (P_1) equal to torque (T) plus the weight (W) minus the friction (F), that is, $P_1 = T + W - F$. The spring balance and brake arm are then raised against the torque through the same small arc and the spring balance will then register a pull (P_2) equal to the torque plus the weight plus the friction, that is, $P_2 = T + W + F$. From these two readings the friction can be eliminated and the actual torque of the armature on the shaft is determined. This should be done at two or three equally spaced positions of the rotor, as the torque varies considerably with the position. The corrected torque of the machine is found by solving the two equations for T and multiplying T by the lever arm of the brake in feet; this gives the torque in pounds at 1-ft radius.

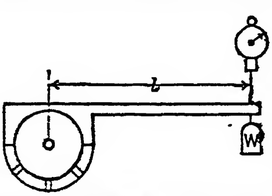

Fig. 6. Stationary Torque Test

LOAD TEST. To make this test on a small machine a prony brake is required: for a large machine a d-c generator may be conveniently employed as a load. The induction motor and generator are direct-connected if possible, otherwise belted together. All the constants of the generator are determined, so that its losses under any condition may be calculated. The motor is then allowed to drive the generator, the rated voltage being impressed upon the motor, and the generator is loaded by means of a rheostat or a water-box so that any load may be obtained. The voltage across each phase of the motor, the current in each phase, the total watts input (by two meters), speed, and, if possible, slip (by a slipmeter, see below) are all read. Care must be taken that correct voltage and frequency are supplied to the motor. The load on the motor is increased step by step to the maximum output point, which is easily known, since when it is reached a decrease in speed is not accompanied by any increase in output of the motor. The motor is still stable at the maximum output condition, since maximum torque occurs at a lower speed than maximum output.

After the load run the d-c generator is run as a motor with the same field strength as before, to determine the mechanical losses, first driving the induction motor at the proper speed, and then running alone at the same speed or speeds at which it ran in the load test. The no-load friction of the induction motor as previously determined in the Excitation Test being known, the increase in friction due to the belt and load is determined and half of it charged against the motor. Let

EI = the output of the d-c generator, in watts.
$I^2 R$ = the hot resistance loss in the d-c armature.
CT = the counter torque or stray power losses in the d-c generator plus one-half belt loss.
P_0 = watts input to induction motor.

Then efficiency of the motor $= \dfrac{EI + I^2 R + CT}{P_0}$.

From these tests, curves may be plotted for the speed, efficiency, power factor, current, and torque of the induction motor for any horsepower output.

Use of Stroboscope to Measure Slip. One type of slipmeter consists of a disk which is attached to the motor shaft and on which alternate sectors of black and white are shown,

preferably as many black sectors as there are poles on the motor. If this rotating disk is illuminated by means of an arc light or neon lamp supplied with the frequency impressed on the primary of the motor, the disk will appear to rotate at a speed proportional to the difference between synchronous speed and actual running speed. The number of these revolutions for 1 minute will be the slip in turns per minute, which may be translated into a fraction or percentage of synchronous speed.

HEAT RUNS. On machines of moderate size heat runs are advisable, but on large machines heat runs are expensive. It is not always necessary to make a heat run in order to know whether the motor is properly designed, as the losses can be accurately calculated from the excitation and short-circuit tests, and the machine may be run without load but with losses equivalent in value to the losses at full load. The usual heat run consists in operating the machine at a certain output for a period from 3 to 6 hours, depending on the size, measuring the resistance before and after the run, and taking temperatures by thermometer on the following parts after the run: primary winding, the iron surfaces in the air gap, secondary winding, bearings, frame.

A heat run which will indicate whether there is anything radically wrong with a motor consists in operating it for an hour or two with a voltage 15 per cent above normal, but without load.

INSULATION TESTS. High-potential tests are made on the primary in the manner described under Testing in the article on Alternating-Current Generators. The value of the high potential for the primary is chosen in accordance with its rated voltage from the Standardization Rules of the A.I.E.E. (q.v.). The potential applied to the secondary, however, has no relation to the rated voltage of the machine; as the working potential in the secondary is low, 1000 to 1500 volts is the usual range of testing potential for the secondary.

18. CALCULATION OF PERFORMANCE

The performance of an induction motor at any load may be determined directly from the load tests described above, or the performance at any load may be calculated from the excitation and short-circuit tests by either of the two methods given below. These methods are also applicable to the calculation of performance from the values of the constants calculated from the dimensions of the machine. The first method is given in detail by Steinmetz in his Elements of Electrical Engineering, and the Heyland Circle Diagram is given by McAllister in his Alternating Current Motors. The former is recommended where accuracy is desired and the latter (graphical) for the student desiring a general understanding of the relations.

STEINMETZ METHOD. This method is based upon the equivalent circuit of a transformer, as given in the article on Transformers. Let

E = impressed voltage per phase.

m = number of phases.

r_1, r_2, x_1, x_2 = resistance and reactance of primary and secondary, respectively, per phase and reduced to terms of primary.

s = slip as a decimal fraction (assumed).

g = primary no-load conductance = $\dfrac{\text{core loss}}{mE^2}$.

b = primary no-load susceptance = $\dfrac{I_m}{E}$.

Assume a slip s and calculate

$$a_1 = \frac{sr_2}{s^2x_2^2 + r_2^2}$$

$$a_2 = \frac{s^2x_2}{s^2x_2^2 + r_2^2}$$

$$g_1 = g + a_1$$

$$b_1 = b + a_2$$

$$c_1 = 1 + r_1g_1 + x_1b_1$$

$$c_2 = g_1x_1 - b_1r_1$$

Then

Counter emf per phase $\qquad e = \dfrac{E}{\sqrt{c_1^2 + c_2^2}}$

Current per phase $\qquad I = e\sqrt{g_1^2 + b_1^2}$

Total volt-ampere input to motor	$P' = mEI$
Watts input to motor	$P = me^2\,(g_1 c_1 - b_1 c_2)$
Watts output of armature	$P_0' = me^2 a_2\,(1 - s)$
Watts output of pulley	$P_0 = P'_0 -$ (friction in watts)
Efficiency	$\epsilon = \dfrac{P_0}{P}$
Power factor	$\cos\varphi = \dfrac{P}{P'}$

CIRCLE DIAGRAM OR GRAPHICAL METHOD. The method of the circle diagram is based on the fact that the electrical reactions in an induction motor (or transformer) may be represented without any great error by the reactions in two parallel inductive circuits as in Fig. 7. I_e and I_m represent the two components of the exciting current, which is assumed constant. R_m and X_m represent the total resistance and reactance (both assumed constant) of the motor and are in series with a variable resistance R_L representing the load.

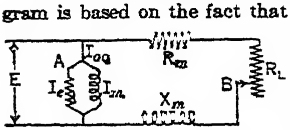

Fig. 7. Equivalent Circuit

The vector relations of the currents are as shown in Fig. 8, where

OE = the impressed voltage per phase
OM = the current I_{00} in A
MP = the variable current in B (not drawn)

Since the current represented by MP is equal to $\dfrac{E}{\sqrt{(R_m + R_L)^2 + X_m^2}}$, as R_L varies the point P will describe a circle through M, P and F, where $MF = \dfrac{E}{\sqrt{R_m^2 + X_m^2}}$. The total or resultant current will then be OP.

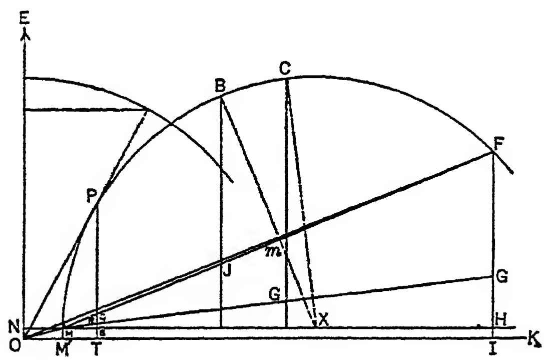

Fig. 8. Circle Diagram

Heyland Circle Diagram. The circle diagram upon which the following discussion is based is a modification of Heyland's transformer, or induction motor diagram.

Let (see Fig. 8):

OE = impressed voltage per phase.
OK be at 90 deg to OE.
OM = exciting current per phase, drawn in phase and magnitude.
OF = short-circuit current per phase, drawn in proper phase and magnitude.
IF = energy component of OF.

Join M and F; then MF is the secondary short-circuit current in terms of primary circuit.
Bisect MF at m.
Draw mX perpendicular to MF at middle point, intersecting NH at X.
With X as center and either XM or XF as radius, draw the semicircle MCF. This is the locus of the primary current.
Since $OE \times IF$ = watts input at standstill, draw HG such that $OE \times HG$ = primary I^2R at standstill.

Then $OE \times GF$ = secondary I^2R at standstill.
Draw GM. The vertical distance between GM and NH at any point gives a current which if multiplied by OE gives the power loss in primary.
Choose any point P on circle; then OP = current per phase.
Then $\cos \sphericalangle POE = \cos\phi$ = power factor.
MP (not drawn) = secondary current reduced to primary turns.

- $PT \times OE$ = power input to primary, in watts.
- $TS \times OE$ = no-load loss, in watts.
- $QT \times OE$ = total motor loss.
- $RT \times OE$ = total primary loss in watts.
- $RS \times OE$ = primary copper loss, in watts.
- $PR \times OE$ = secondary input, in watts.
- $QR \times OE$ = secondary copper loss, in watts.
- $QP \times OE$ = motor output, in watts.
- $OM \div OP$ = per cent magnetizing current.
- $M'T \div OP$ = per cent leakage reactance voltage.

These are all per phase

Draw XC perpendicular to MG and CG' perpendicular to NH.
Then $CG' \times OE$ = maximum torque in synchronous watts.
Draw XB perpendicular to MF and BJ perpendicular to NH.
Then $BJ \times OE$ = maximum output in watts.

- $QR \div RP$ = per cent slip (in case of induction motor).
- $OP \times OE$ = volt-amperes input.

Torque in pounds at 1-ft radius $T_s = (PR \times OE) \times m \times \frac{7.06}{N}$, where m = number of phases and N = synchronous speed in revolutions per minute.

CHARACTERISTIC CURVES. The observed or calculated values (by either of the above methods) of the slip, power factor, efficiency, apparent efficiency, current

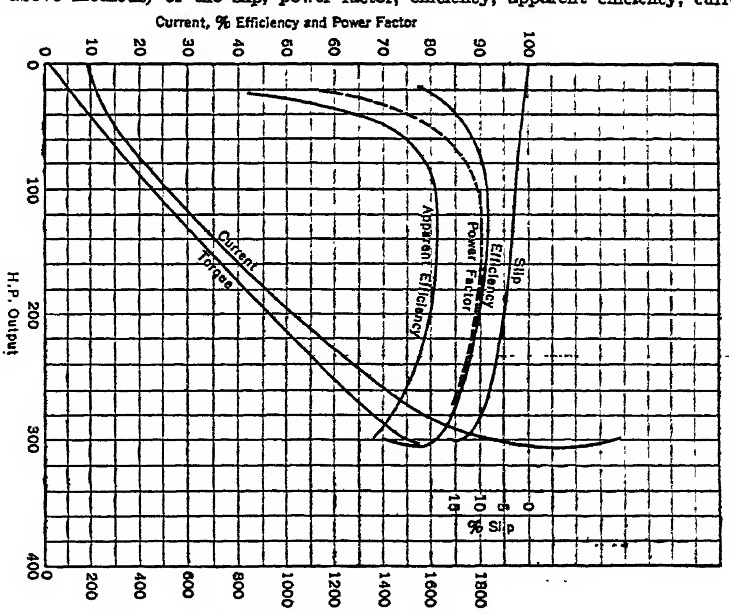

Fig. 9. Characteristic Curves of an Induction Motor

per phase, and torque may be plotted as ordinates with horsepower output at the shaft as abscissas. An example of the characteristic curves thus plotted is shown in Fig. 9. Usual values of the various quantities for different sizes of motors at rated load are given above. The "power factor" shows the relation between the true power input of the machine and the apparent power, called the "volt-amperes." A poor power factor does not involve any greater registration of the watthour meter or cost of energy to

Table 8. Induction Motors

Mechanical Data (Dimensions in Inches)

Type	1	2	3	4	5	6
	3-phase	3-phase	3-phase	3-phase	3-phase	3-phase
Number of poles	4	4	4	6	6	8
Rating in hp	5	15	30	5	10	20
Revolutions per minute	750	750	750	1200	1200	900
Primary volts between lines	440	220	220	220	440	220
Primary connections	Y	Y	Δ	Δ	Δ	Δ
Frequency	25	25	25	60	60	60
Stator						
Outer diameter punchings	17	21	28	17	18.25	26
Inner diameter punchings	12.06	15.07	19.07	12.094	11	19.07
Total length of iron	6	7.5	8	5	3.75	6
Number of ducts	0	0	0	0	0	0
Width of each duct	....	....	....	....	....	....
Total number slots	60	72	72	54	72	96
Depth of slot	1.25	1.25	1.44	1.56	1.5	1.25
Width of slot	0.32	0.34	0.39	0.50	0.3	0.33
Width of slot at face	0.32	0.34	0.39	0.25	0.3	0.33
Wires per slot	36	16	16	64	30	16
Size of wire	No. 14 B. & S.	No. 10 B. & S.	No. 8 B. & S.	No. 14 B. & S.	No. 15 B. & S.	No. 13 B. W. G.
Wires in multiple	1	2	2	2	1	2
Turns in series per phase	360	96	96	288	360	128
Per cent coil pitch	100	72	67	100	100	75
Rotor						
Outer diameter punchings	12	15	19	12	10.95	19
Inner diameter punchings	8	11	12	8.5	6	14
Total number slots	37	47	67	72	127	71
Depth of slots	0.56	0.47	0.47	0.94	0.5	0.47
Width of slots	0.56	0.56	0.56	0.34	0.12	0.56
Width of slots at face	0.063	0.063	0.063	0.19	0.02	0.063
Wires per slot	1	1	1	4	1	1
Size of wire or bar	0.5×0.45	0.35×0.5	0.35×0.5	0.34×0.11	0.35×0.09	0.35×0.5
Number in multiple	1	1	1	4	1	1
Cross-section each ring, sq in	1 3	0.94	2	....	0.198	1.2
Resistance relative to copper	2	2	2	....	1	2
Air gap on one side	0.03	0.035	0.035	0.047	0.025	0.035

Table 9. Induction Motors

Electrical Data (All Quantities Per Phase)

Type	1	2	3	4	5	6
Volts per phase, E	254	127	220	220	440	220
Current per phase at rating	6.6	38	42	8.1	7.3	31.2
Flux per pole, megalines	0.636	1.2	2.07	0.29	0.48	0.645
Magnetizing current I_m	2.1	9	13	2.64	2.4	12.3
Friction, watts	80	175	530	150	295	390
Core loss, watts	150	390	1240	180	210	760
Primary resistance at 60° C, ohms	2.86	0.193	0.13	1.15	2.78	0.213
Second. resistance at 60° C, ohms	1.76	0.18	0.17	1.11	3.5	0.21
Short-circuit current	36	212	362	29.4	26	205
Y = (Short-circuit current) ÷ I_m	17	23.6	28	11.2	10.8	16.6
Reactance per phase	5.52	0.55	0.54	8	5.05	1.10
Efficiency at rating	0.83	0.853	0.883	0.83	0.86	0.875
Power factor at rating	0.905	0.91	0.917	0.845	0.90	0.83
$I_m \div I$	0.32	0.24	0.31	0.33	0.33	0.39
Friction ÷ P_0	0.018	0.013	0.02	0.033	0.034	0.023
Core loss ÷ P_0	0.033	0.029	0.049	0.04	0.024	0.044
$IR_1 \div E$	0.074	0.058	0.025	0.042	0.046	0.03
$IR_2 \div E$	0.046	0.054	0.032	0.041	0.058	0.03
$IX \div E$	0.143	0.165	0.103	0.29	0.084	0.156
Slip, per cent	0.047	0.042	0.032	0.03	0.038	0.03

operate the motor, but it does involve poor regulation of voltage in the system as a whole and larger capacity of wiring, transformers, etc. The "apparent efficiency" is equal to the product of the power factor and efficiency, or is equal to the ratio of output in watts divided by input in volt-amperes. Its value determines the actual capacity of the lines and transformers supplying the motor.

EXAMPLES OF DESIGN AND PERFORMANCE. In the accompanying tables are given the essential data for both mechanical and electrical features of six three-phase induction motors. The list of items will be found useful as a guide in collecting data on various machines. Performance data are deduced from tests.

19. METHODS OF STARTING

In order to start an induction motor of any size without injurious heating, either a resistance must be connected in the secondary circuit or the voltage impressed on the primary must be reduced. The two general methods of starting induction motors are known as "potential control" and "rheostatic control." The same methods are used for speed control (see below).

STARTING BY "POTENTIAL CONTROL" METHOD. This method consists of reducing and regulating the voltage impressed on the primary, usually by means of a starting compensator or auto-transformer which provides one or two fractional voltages. In order to make use of this method the secondary must be of higher resistance than with other methods of starting. For this reason, and because there is no need of making any change in the windings, a squirrel-cage rotor winding is customarily used with this method of starting. This winding is made up of one bar per slot, and all bars are connected at both ends to rings. To start the motor the primary is connected to taps on the compensator which give a voltage of $1/2$ to $2/3$ the rated voltage if it is a small motor, and $1/3$ to $1/2$ if it is a large motor. A small motor may be brought up to full speed on this voltage, but a large motor may require an intermediate step. The connections are shown in Fig. 10. It is customary to adjust the motor resistance and starting voltages to give the relations in Table 10.

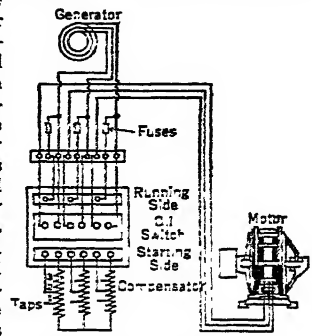

Fig. 10. Potential Control

BOUCHEROT WINDING. This type of winding for the rotors of induction motors is a device designed to give good torque at starting by means of high secondary resistance and inherently to reduce this resistance as the motor accelerates, so that at full speed the secondary resistance will be low, the slip small, and the efficiency high.

The device consists of two independent windings on the secondary (usually squirrel cage): one, a high-resistance winding of brass or German silver bars is placed near the periphery of the rotor in the usual form of slots; the other winding, of low resistance (large copper bars), is placed in slots like tunnels below the bottom of the surface slots. A narrow slit or shaft extends from the bottom of the outer slot to the top of the inner slot.

Table 10. Starting Conditions

Voltage on Motor, per cent	Current in Line, per cent	Starting Torque, per cent
33	75	22
50	175	50
66	300	88
100	700	200

The outer winding acts like any high-resistance winding, giving good starting conditions. The inner winding is so highly inductive that at standstill, when the secondary has full frequency, it has a very high reactance, carries very little current, and contributes but little to the action of the motor. At speeds near synchronism the secondary frequency is very low (of the order of one cycle per second), the reactance is low, and the inner low-resistance winding carries most of the current; consequently under running conditions the motor has practically the same characteristics as one having a single low-resistance secondary.

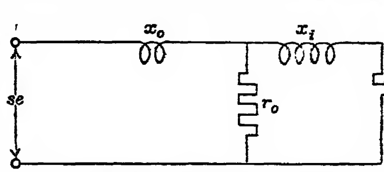

Fig. 11. Boucherot Double Squirrel-cage Winding

Boucherot Double Squirrel-cage Winding.

r_i = resistance of inner winding, low.
x_i = reactance between two windings, high.
r_0 = resistance of outer winding, high.
x_0 = reactance between outer winding and primary.

$$Z_i = r_i + jx_i$$

$$Y_i = \frac{r_i - jx_i}{Z_i^2}$$

$$Y_i + Y_0 = Y_i + \frac{1}{r_0} = \frac{r_i r_0 - jx_i r_0 + Z_i^2}{r_0 Z_i^2}$$

$$Z_{i0} = \frac{r_0 Z_i^2}{r_i r_0 - jx_i r_0 + Z_i^2}$$

$$Z_2 = Z_{i0} + jx_0 = \frac{r_0 Z_i^2 + jx_0 r_i r_0 + x_i r_0 x_0 + jx_0 Z_i^2}{r_i r_0 - jx_i r_0 + Z_i^2} = r_2 + jx_2$$

≡ the equivalent resistance and reactance of the combined windings for substitution in all calculations of characteristics.

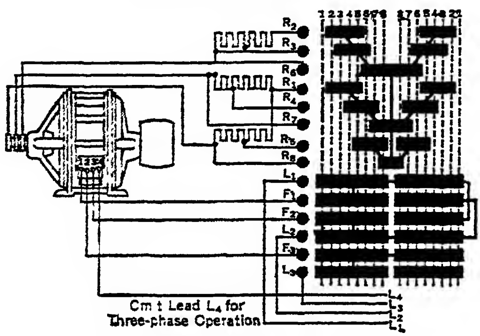

Fig. 12. Rheostatic Control

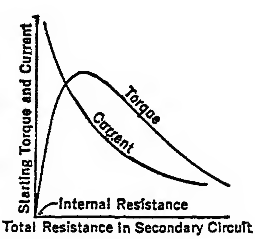

Fig. 13. Starting Resistance, Torque, and Current

STARTING BY "RHEOSTATIC CONTROL" METHOD. Better apparent torque efficiency, that is to say, more torque for a given current, is obtained by inserting in the secondary circuit a much greater resistance than can be left permanently in circuit. This is accomplished by having a special starting resistance connected in series with the armature winding and a switch for short-circuiting the resistance either step by step or as a whole, as the motor speeds up (see Fig. 12). There are two practical methods of doing this:

The first is intended to be used only when the torque required at starting is not very great, in which case the starting resistance may be small and located inside the armature spider. The switch lever is so arranged that the resistance can be short-circuited in steps while the armature is revolving. This obviates the need of collector rings and external connections.

The second method consists in bringing the three terminals of the secondary winding to collector rings. From brushes bearing on these rings conductors lead to external resistances with steps or taps so that the resistance may be short-circuited gradually. This scheme is used where a large starting torque (greater than full-load torque) is required. It may be used also for speed control as shown in Fig. 12.

The proper value of resistance per phase in the secondary is determined by the relation

$$\text{Torque in pounds at 1 ft} = \frac{E^2 r_x}{Z^2} \times \frac{7.06\, m}{\text{rpm}} = \frac{7.06\, m r_x E^2}{N Z^2}$$

where E = primary voltage per phase.
r_x = total secondary resistance per phase in terms of primary.
$Z = \sqrt{(r_1 + r_x)^2 + X^2}$, where r_1 is the resistance per phase of primary and X the total reactance per phase of both primary and secondary.
m = number of phases.
N = synchronous speed in revolutions per minute.

The relation between starting torque and total resistance of the secondary is shown by the curve in Fig. 13.

20. SPEED CONTROL OF INDUCTION MOTORS

The speed of an induction motor may be controlled in five ways:

(a) By varying the potential applied to the primary of motor having a suitable permanent resistance in the secondary.
(b) By varying the resistance in the secondary circuit.
(c) By changing the connections of the primary winding in a manner to change the number of poles.
(d) By varying the frequency of the applied voltage.
(e) By connecting the secondary of one motor to the primary of another, called the "concatenation" method of control.

POTENTIAL CONTROL OF SPEED. This method is an elaboration of the potential-control method of starting. A suitable resistance or auto-transformer reduces the impressed voltage to the fractional value desired (two-thirds is the usual ratio). The auto-transformer wastes less power than a resistance but is more expensive. Because of the reduced voltage, the flux and torque-per-ampere are reduced and more current will be required for a given torque. The induction motor should have a very large resistance in the secondary, which is preferably of the squirrel-cage type. This resistance gives the motor a speed characteristic such that its full-load speed is some 10 per cent less than that of a normal motor. As the load is increased the speed may fall to about 30 per cent of the no-load value without the motor breaking down or falling out of step, which in the normal motor usually takes place at about 80 per cent of the full-load speed. This motor is not stable in speed as each slight change in the load will cause a change in the speed more or less inversely proportional to the load. The advantages of this method of control are the simplicity of the connections and devices. The disadvantages are the greatly increased heating in the motor itself with the decreased speed. Thus the motor must be larger than if other speed-control methods are used. Table 11 shows the efficiencies obtained.

RHEOSTATIC CONTROL OF SPEED. With this method (an elaboration of the method of the same name for starting) the secondary or rotor must have a definite winding (which costs more than the squirrel-cage winding used in the preceding method) with slip rings and brushes to lead out the current. The friction and resistance losses due to these brushes decrease the efficiency of the motor a slight amount. The action of this method is based on the principle that in an induction motor the drop in speed for any given torque is proportional to the resistance of the secondary circuit.

Assuming a motor which has at full speed a net resistance of the secondary proper of 1 ohm, the speed-torque curve would be as shown for $R = 1$ in Fig. 14. This motor would have a slip of 5 per cent for full-load torque and of 25 per cent for maximum torque, a torque at starting of 80 per cent of full-load torque and a very large starting current. If by some means this resistance is doubled the speed-torque curve would be as shown for $R = 2$, which shows a slip of 10 per cent for full-load torque and of 50 per cent for maximum torque. For $R = 5$, $R = 8$, etc., there would be other speed curves. By starting with a resistance of 8 ohms a torque of about 200 per cent of full-load torque would be obtained at starting with about twice full-load current. By allowing the motor to follow this curve until the torque has dropped to the full-load value, the motor would reach 60 per cent of synchronous speed, or 40 per cent slip. Then by reducing the resistance in steps the torque and speed would follow the heavy zigzag line until the motor reached full speed.

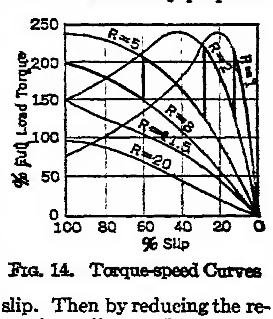

Fig. 14. Torque-speed Curves

With this method of control the torque per ampere remains practically constant as in a shunt motor regulated by resistance in the armature circuit. The efficiency varies directly with the speed as shown in Table 11.

The advantages of this method are the higher efficiency and particularly the smaller losses in the motor itself. The losses are in the rheostat. The disadvantages are the necessity of collector rings, brushes, controllers, etc. The motor is not stable at any fractional speed but the speed will change with every change of load.

CHANGE OF POLES TO CONTROL SPEED. By a proper design of the windings an induction motor may be made to operate with either 4 or 8 poles, 6 or 12 poles, or even 4 or 6 poles or 6 or 8 poles. This is accomplished by a throw-over switch to which

taps from the windings are brought. In this arrangement the pitch of the primary winding must be made a compromise between the proper value for the different numbers of poles, and therefore the constants of the motor are not as good as those of a standard motor. It is also necessary to use a squirrel-cage armature, since this is suitable for any number of poles without change of connection. This type of motor operates advantageously only at the two speeds corresponding to the two arrangements of poles and is stable at each of these speeds. If a wider range is desired the potential-control scheme may be combined with it. The speeds and efficiencies with this scheme of control are given in the following table. At half-speed the efficiency is almost double that obtained with the other methods, but the losses in the motor are greater than with the rheostatic control.

Table 11. Comparison of Methods of Speed Control

(For constant torque equal to torque at full load)

Nominal Speed	Potential			Rheostatic			Change of Poles		
	Speed	Volts	Eff.	Speed	Volts	Eff.	Speed	Volts	Eff.
No load	1.00	100	....	1.00	100	....	1.00	100	....
Full speed	0.89	100	81	0.96	100	86	0.96	100	86
Three-quarter speed	0.67	66	59	0.72	100	65	....	....	....
Half speed	0.45	57	37	0.48	100	43	0.48	100	74
Quarter speed	0.22	56	17	0.24	100	22	....	....	....

CHANGE OF FREQUENCY METHOD OF CONTROL. The speed of an induction motor at any load varies directly with the frequency of the supply circuit. If two circuits from two alternators of different frequencies are provided the motors may be connected to one circuit for one speed and to the other circuit for another speed. This method of speed control requires as many separate generators and circuits as the number of speeds desired. It is, therefore, costly and not widely used.

CONCATENATION CONTROL OR CASCADE CONTROL. If two motors have definite windings in both the primary and secondary and are rigidly connected to a common shaft, they may be operated at a fractional speed corresponding to a number of poles equal to the sum of the number of poles on the two motors. With the concatenation control the primary (stator) of motor 1 is connected to the supply circuit, the secondary (rotor) of 1 is connected to the primary (rotor) of 2, and the secondary (stator) of 2 is connected to a resistance which is eventually short-circuited. When the two motors have the same number of poles, which is the usual commercial condition, motor 1 transforms half the power into mechanical power at the shaft at *half speed* and the remainder into electrical power at half frequency. Motor 2 receives this electrical power and transforms it into mechanical power at this same speed. If the number of poles is different, the speed in rpm of the combination is

$$N = \frac{120 f}{p_1 + p_2}$$

These simple relations are exact only on the assumption of no losses and the secondary of 2 short-circuited.

The objections to this method of control are that the first motor has to carry the magnetizing current of both motors, thus having a low power factor. The first motor must receive the power for both motors; therefore, it must be the larger of the two and specially designed or there is an inefficient use of material in the second motor. If the two motors are alike (the usual commercial condition) the torque of the two motors in concatenation is not as great as that of the two motors in parallel.

The efficiency at fractional speed is, however, better than with the rheostatic method of control. Only two "free-running" speeds are available with two motors having the same number of poles. This scheme has been adopted frequently for the speed regulation of induction motors used on electric locomotives in foreign countries. In such applications it is frequently the custom to allow one motor to be idle at the higher speed.

21. INSTALLATION AND OPERATION

Induction motors are usually built, even in large sizes, as a unit including the bearings, which are usually a part of the end frame of the motor proper. They may be either direct-connected or belted to their load, but the latter method is more general, since each induction motor can be built for only a certain definite speed corresponding to a certain number of poles. The smaller motors need no foundation, and in fact are fre-

quently attached to the wall or to the ceiling, the bearings and end shield being made in such manner that they may be turned through 90 deg or 180 deg so that the oil rings will operate properly under these conditions. In most motors, reasonable ventilation, free from dust and dirt, must be available. For certain applications, such as cement mills or mines, the motors are built totally enclosed and may then be even submerged in water. In this case, of course, a motor of a given rating is larger and more expensive than one of the open type.

Small motors are designed to start merely by closing the main switch. With larger motors, if the starting switch is at the proper position, potential may be applied to the motor and the starting resistance gradually cut out by moving the switch.

Induction motors are very sensitive to variation in the impressed voltage. A decrease in impressed voltage from 100 to 80 would cause the maximum output and maximum starting torque to decrease from 100 to 64 and roughly would cause a proportional increase in the heating for a given load.

Unequal voltages in the different phases also cause a decrease in the maximum output and an increase in the heating for any given output. An unbalancing of 25 per cent in voltage would double the heating effect at full load, i.e., would give the same heating as an overload of 50 per cent.

CARE IN STARTING. Before starting the motor for the first time it is desirable to make sure that the starting device is in operating condition and in the proper position, in order that the motor should not become injuriously heated. Attention should be paid to having the wiring so proportioned as to carry the starting current without an excessive drop in voltage (see above).

FAULTS. Some of the more common faults occurring in induction motors, together with their signs and remedies, are the following.

Secondary Open-circuited. The motor will not start and will not take a current greater than the exciting current. The cause is probably due to the starting resistance not being connected in.

One Phase of Secondary Open-circuited. The motor has a tendency to remain at half synchronous speed although the current is apparently normal. If the armature is blocked it will be found that the current in the three phases will be unbalanced.

One Phase of Primary Open. The motor will not start and the current will be unbalanced.

One Phase of Primary Reversed. The currents in the primary will be very much unbalanced when the motor is running and the starting torque will be very slight.

Short-circuited Coil in Primary. There will be humming when potential is applied to the motor and excessive local heating around the short-circuited coil.

Vibration. Vibration due to mechanical unbalancing is chiefly noticeable at high speeds and particularly in high-speed machines. If the vibration is due to magnetic unbalancing it is probably caused by inequality in the air gap at different portions of the circumference and with different positions of the armature. This may be detected by measuring the air gap with taper wedges at various points around the circumference, first with the armature in one position then in several other positions.

SPECIFICATIONS FOR INDUCTION MOTORS. The following memoranda are intended to assist in writing specifications. See also article on Specifications.

Principal Characteristics and Conditions of Service. Use to which motor is to be put; kind of load and method of drive. Voltage and number of phases. Rating, horsepower. Frequency and speed.

Style and Description; Details of Construction. Whether to be open, semi-enclosed, or enclosed. Requirements regarding pulley or length of shaft. Whether rails are required. Method of starting; compensator, external resistance or internal resistance; whether motor is to be run at speeds other than full speed. Whether the starting devices are to be supplied.

Performance and Tests. (See Standardization Rules of the A.I.E.E.). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation.

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SINGLE-PHASE INDUCTION MOTORS

A two- or three-phase induction motor may be operated as a single-phase machine after it is brought up to speed. Under these conditions it operates at a lower efficiency, at lower power factor, and with a lower maximum output than it would have as a polyphase motor. The slip for a given output is less in a single-phase than in a polyphase motor.

LOAD AND VOLTAGE RATING. On account of the poorer operating characteristics and particularly on account of the lower maximum output or maximum torque, it is necessary to rate the motor at a lower capacity. When a three-phase motor is operated single-phase with the same voltage between lines, its maximum output will be approximately 40 per cent of the three-phase maximum output.

For best conditions, such as best distribution of losses and ratio of rated to maximum output, it is customary to use a three-phase motor, to reduce the rated output of the motor, and to increase the rated terminal voltage in a definite ratio. Thus, if P be the rated output, in watts, of a given motor when operating on a three-phase circuit having a voltage between lines equal to E, then it would be advisable to operate the motor single-phase with a voltage between lines equal to 1.3 E, and to assign the motor a rating of 0.67 P to 0.75 P.

This will result in a distribution of losses in the motor quite similar to that which obtains during three-phase operation. The maximum output as a single-phase motor (at the higher voltage) will be about 67 per cent of the maximum output of the three-phase motor. The efficiency and power factor will be reasonable.

If the voltage of the single-phase supply circuit must be the same, the winding of the motor is changed to give about 75 per cent as many turns in series per phase as for normal three-phase operation.

EXCITING CURRENT AND POWER FACTOR. At a given voltage between terminals the volt-amperes input at no load for excitation are practically the same for single-phase and polyphase operation. Thus the no-load current of a single-phase motor is considerably greater than when operating polyphase. The increase in the applied voltage or the decrease in the number of turns makes a still greater increase in the magnetizing current. Thus the power factor of a single-phase motor is very poor at light loads and not very good at rated load.

22. CALCULATION OF PERFORMANCE

To predetermine the characteristics of a single-phase motor it is calculated as a three-phase motor for the same voltage between lines as the single-phase circuit. The magnetizing current, core loss, resistance per phase, and reactance per phase are all calculated as usual. The motor primary may be connected either delta or Y, but for purposes of calculation it is desirable to pro-rate the constants of a Y-connected motor on the assumption that it is delta connected.

To pro-rate the resistance and reactance per phase of primary and secondary, multiply the values for a Y-connection by 3 to obtain the values for the equivalent delta constants. The voltage per phase is presumed to increase in the ratio 1 to 1.73, while the voltage between lines remains the same.

The single-phase magnetizing current is found by dividing the volt-amperes excitation for three-phase conditions by the single-phase voltage. The core loss in watts remains practically the same single-phase, and the energy component of the single-phase exciting current is equal to core-loss watts divided by rated voltage.

The resistance and reactance of the primary of a single-phase motor are taken the same as the equivalent delta values per phase.

The resistance and reactance of the secondary of a single-phase motor are taken as $1/_3$ of the three-phase equivalent delta values.

These values are then substituted in the formulas of the Steinmetz method (see Motors, Polyphase Induction) and the characteristics calculated for several assumed values of slip. The only difference in calculation is that the torque in synchronous watts is

$$T = e^2 a_1 (1 - s)$$

and the output of the armature is

$$P = e^2 a_1 (1 - s)^2$$

It is of course understood that in a single-phase motor the output calculated for one phase is also the total output of the motor.

23. METHODS OF STARTING

A single-phase motor has no torque at standstill. It must be started by some device such as a phase-splitting device. It may be started in either direction, and as soon as it starts to rotate a slight torque develops which increases with the speed. When this torque has reached a great enough value to overcome the friction and inertia the special starting apparatus may be disconnected and the motor will continue to accelerate to its proper speed.

STARTING OF SMALL MOTORS. Small motors may be started without auxiliary electric circuits by giving the armature a spin by hand, after which (if there is no load) they will accelerate. Certain small motors are designed with a loose pulley which is clutched at a predetermined speed of the armature by means of a centrifugal governor. These motors are provided with "shading coils" or a small external phase-splitting device, to give them just enough starting torque to overcome their own friction.

STARTING OF LARGE MOTORS. A phase-splitting device is generally used; either a reactor and resistor or a condenser and resistor may be employed. Commutator devices are also used.

Use of Reactance Coil and Resistor. The connections employed are shown in Fig. 1. This device consists of a resistance and a reactance connected so as to advance the phase of the emf impressed on one circuit of the motor and retard the phase of the emf on another circuit, while the line voltage is impressed on the third winding. This may be accomplished either by connecting the resistance and reactance across the line terminals and in multiple with the motor windings as shown in Fig. 1A, or by connecting the resistance and reactance in series with the respective windings as shown in Fig. 1B.

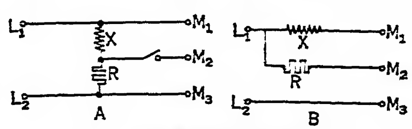

FIG. 1. Phase Splitting Circuits

In these figures L_1 and L_2 represent line terminals, R is the resistance coil, X the reactance coil, and M_1, M_2 and M_3 the three motor terminals. The former method gives a greater starting torque but takes a greater current in proportion to the torque. The latter method is more efficient. The motor must have sufficient resistance in its secondary circuit to give good starting characteristics as a three-phase motor.

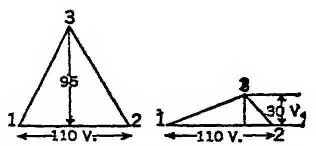

FIG. 2. Split Phase Vector Diagram

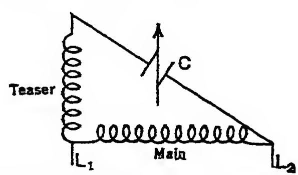

FIG. 3. Principle of Connection of a Capacitor Motor

The principle of this device is that the voltages between the outside terminals and the middle point are out of phase with each other and form a somewhat flattened vector triangle similar to that of an unbalanced three-phase system. The ratio of the starting torque obtained with such a device to the normal starting torque with balanced three-phase voltages is the same as the ratio of the altitude of the triangle of vector voltages to the altitude of the equilateral triangle. Thus in Fig. 2 the ratio of altitudes is 30/95 = 0.316; thus the single-phase starting torque would be 31.6 per cent of the three-phase starting torque.

Capacitor Motor. If a condenser of the proper capacitance (see Fig. 3) is substituted for the resistance the starting torque will be increased and the efficiency and power factor

improved, but this is much more expensive and frequently involves the use of an auto-transformer across the condenser to put a higher voltage across the condenser and reduce the required capacitance. Many small fractional horsepower single-phase motors use this principle by connecting a condenser (and auto-transformer) in series with a quadrature winding in the motor and connecting this combination in parallel with the main winding. Usually a centrifugal switch is used to cause the auto-transformer to put a high voltage across the condenser in starting and a lower voltage while running, thus giving a high starting torque by momentarily overloading the condenser, and normal conditions for running. Such motors have a good power factor and cause little radio interference because there is no commutator. They are used for refrigerators, oil burners, etc.

Use of Commutator. Another method of starting single-phase motors involves the use of a commutator which permits the motor to start as a repulsion motor (which has good starting qualities; see A-c Commutator Motors), and after the motor has reached a considerable speed the brushes are removed from the commutator and a short-circuited squirrel-cage winding comes into play.

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ALTERNATING-CURRENT COMMUTATOR MOTORS

There are several different types of alternating-current commutator motors designed to operate on single-phase circuits, but they differ chiefly in the electrical connections employed. They may be divided into two general classes, series motors and repulsion motors. While these motors differ in their connections and in slight details in their

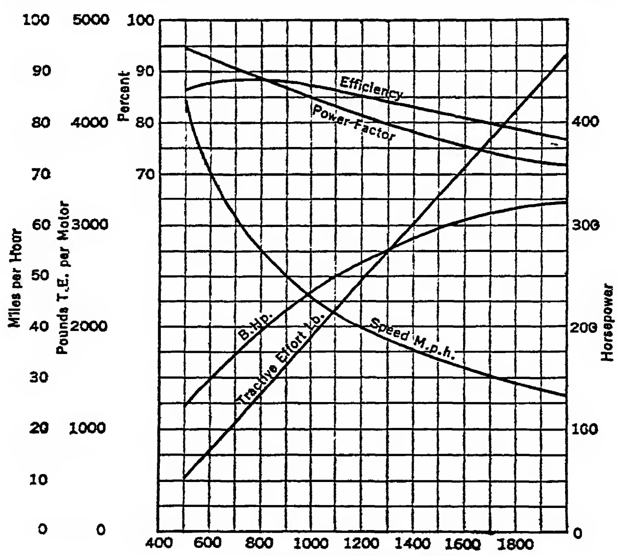

FIG. 1. Characteristic Curves of Series Compensated Single-phase Motor for 25 Cycles

characteristics, they all have the general characteristics of the d-c series motor, that is, increasing torque with decreasing speed and a high efficiency over a considerable range of speed. Alternating-current commutator motors with shunt motor characteristics are also used to a limited extent abroad.

GENERAL CHARACTERISTICS. The torque of any single-phase a-c commutator motor is constant in direction, but pulsating in value, and its average value is proportional to the product of the effective value of the flux and the effective value of the armature current. The direction of the torque may be changed by changing the direction of the current in the field with respect to the armature, or vice versa. The power factor increases with increase of the speed and therefore decreases with increase of load. The efficiency, though not as good as that of a d-c motor of the same rating is, however, fairly high. The motors have in addition to the losses common to d-c motors a core loss in the field, increased core loss in the armature, increased commutation loss, and increased RI^2 loss in special windings. In Fig. 1 are given the characteristic curves of the a-c compensated series motors used on the single-phase locomotives of the New York, New Haven & Hartford Railroad.

APPLICATIONS. The most general application of a-c commutator motors in large sizes is in railway and hoisting work; see Sections 16 and 17. The same principles of operation are made use of in the devices for starting single-phase induction motors (see Art. 23), the motor being brought up to speed as an a-c commutator motor and then by a change of connections made to operate as a single-phase induction motor.

24. DESIGN

The salient features in the design of the various types of a-c commutator motors are described briefly below.

STRAIGHT A-C SERIES MOTOR. Since the torque of an ordinary d-c series motor does not change in direction when the current through both the field and the armature reverses simultaneously, any d-c series motor will develop a unidirectional torque when connected across a-c mains. However, when an ordinary d-c series motor is thus used the power factor of the load taken by it is very low, there is a large eddy-current loss in the field structure, and violent sparking occurs at the commutator. To make a series motor practicable for a-c service, the field structure must be laminated in order to avoid eddy currents and the field coils must have only a few turns to avoid too great self-inductance, and the consequent low power factor. The greater tendency to spark in the case of the a-c series motor is due to the *alternating* field flux which interlinks the coils short-circuited by the brushes, thus inducing in these coils a relatively large emf not present when the motor is operating on a d-c circuit. This difficulty can be avoided to a certain extent by designing the motor for a small field flux and with but a few turns in series in each armature coil. In general, therefore, single-phase commutator motors are built for low voltages, such as 200, with one turn per coil, multiple-wound armatures, and with the armature ampere-turns per pole about four times the field ampere-turns per pole. The effect of the armature ampere-turns can be neutralized by a "compensating" winding described below. The field flux is practically limited to that value which will give 4 volts per turn in the short-circuited armature coil, as this is about the limit that may be commutated with carbon brushes.

Resistance Leads. As an additional means of preventing sparking, high-resistance leads are frequently used between the commutator segments and the armature coils. These leads are made of a high-resistance metal strip bent back and forth several times, and imbedded in the armature slots along with the armature conductors proper. On account of the dissipation of heat in these leads a lower current density must be used in the armature conductors proper than is used in the case of d-c motors. The arrangement of the leads is such that the main or useful current passes through two high-resistance leads in multiple as it enters the armature, while the undesirable short-circuited current passes through two high-resistance leads in series. It should be noted, however, that at any instant there is current only in those leads connected to coils which are being short-circuited at this instant.

Compensating Winding. To obviate the high armature reaction in an a-c series motor and at the same time to improve the power factor, a "compensating" winding is usually employed. This consists of a distributed winding imbedded in slots in the pole faces and connected usually in series (Fig. 2) with the main field winding and armature in such a manner that the current through it sets up a magnetomotive force which practically neutralizes the effect of the armature ampere-turns. When the compensating winding is connected in series with the field and armature, as shown in Fig. 2, the motor

is said to be "conductively compensated." An "inductively compensated" motor has this winding short-circuited upon itself and the current in it is induced from the armature by transformer action. Inductive compensation is not operative on d-c circuits, but is as satisfactory as conductive compensation for a-c operation.

THOMSON REPULSION MOTOR. This motor has a stationary structure or field with a completely distributed winding, which may be wound for any voltage. In this is placed a low-voltage armature designed with all the refinements necessary for single-phase commutator work. The brushes bearing on this commutator are short-circuited upon themselves and are so placed that the line connecting the positive and negative brushes makes an angle α with the neutral axis of the field. The field turns lying within the angle $(90 - \alpha)$ induce a current in the armature winding by transformer action, and the field turns lying within the angle α constitute the "exciting" turns and set up the necessary flux to produce the driving force. The arrangement is equivalent to the circuits shown in Fig. 3, although actually there is but a single field winding. This motor then acts exactly like the combination of a transformer and a series motor in one structure. It may be reversed by changing the position of the brushes or by shifting the points of connection of the external circuit to the field or stator winding. This motor operates particularly well near synchronous speed as then it has practically a rotating magnetic field and no excessive commutation difficulties, but at starting and at low speeds the commutation is not as good as that of the compensated series type.

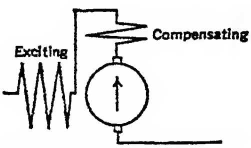

Fig. 2. Compensated Series Motor

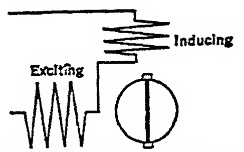

Fig. 3. Thomson Repulsion Motor

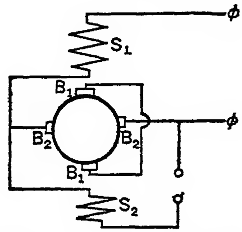

Fig. 4. Repulsion-starting; Induction-running Motor

WAGNER REPULSION-STARTING, INDUCTION-RUNNING MOTOR (Fig. 4). The stator has two windings, the main or primary S_1 is connected in series with the exciting brushes B_2 on the armature as in the series motor. This gives the starting characteristics of the series a-c motor. The armature has two sets of brushes. The main pair B_1 in line with the primary flux is short-circuited, thus giving compensation as in the compensated series motor. The other pair of brushes B_2 is connected in series with the primary and across a "power factor compensating coil" S_2, wound on the stator in the same axis as the primary. A switch in this circuit is left open when starting, but is closed by a centrifugal governor just before synchronous speed is reached. Closing this switch short-circuits this pair of brushes through the compensating coil and makes the armature a two-phase rotor, and thus eliminates series-motor action and makes the action that of a true single-phase induction motor. The compensating coil being in circuit causes the anti-inductive effect in this coil to react on the primary as in the compensated repulsion motor, and give a good power-factor or leading current. By adjusting the number of turns or current in this coil S_2 the power factor may be regulated.

To prevent the motor from running away in case the centrifugal switch should fail to act, and to relieve the brushes under load, a regular squirrel-cage winding is placed in the bottom of the armature slots underneath the commutated winding and separated from it by magnetic separators to magnify the magnetic leakage and render the squirrel-cage winding ineffective at starting. This motor is very satisfactory in practice, has a starting torque 1.5 times full-load torque for a starting current of 3 times full-load current. The power factor is anti-inductive at light loads, unity at full load, and inductive at over-loads. The efficiency at full load is about 80 per cent in the 5- and 10-hp sizes. Reversal of rotation is effected by changing the connections to the exciting brushes. Small motors may be thrown directly on the line for starting. Large motors have a series resistance or reactance to limit the starting current to a reasonable amount.

REPULSION-INDUCTION (R. I.) MOTOR OF THE GENERAL ELECTRIC CO. This motor is shown diagrammatically in Fig. 5. It has two windings on the stator, the main or primary S_1 being connected directly and solely across the line as in any induction motor. The armature has two equally spaced sets of brushes displaced a

small angle (20 deg) from the axis of the primary flux, thus obtaining the repulsion motor starting effect. The main brushes B_1 are short-circuited and the auxiliary brushes B_2 connected permanently in series with the "power factor compensating coil" S_2 on the stator. The method of operation is best understood by considering the motor as being a repulsion motor and a single-phase induction motor on the same shaft. At starting the repulsion-motor torque is great and the induction-motor torque is zero. Just below synchronism the induction-motor torque is great and repulsion-motor torque small. Above synchronism the repulsion-motor torque still exists but the induction-motor torque has become negative (generator). Thus at no load the repulsion motor pulls the induction motor above synchronism until the positive torque equals the negative torque and friction. At full load the speed is nearly synchronous. The compensating coil introduces an anti-inductive effect described above, and by varying the number of turns included, the power factor may be made unity at any load. The efficiency is about 80 per cent in a 5-hp motor. Reversal of rotation is effected by moving the brushes or, if to be done often, by introducing a special coil in the primary whose connection is changed for reversal. An ordinary starting box is used to limit the current to about three times full-load current for 1.5 times full-load torque.

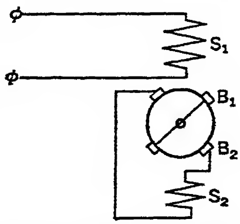

FIG. 5. Repulsion-induction (R.I.) Motor

THE "SCR" MOTOR. The "SCR" motor of the General Electric Co. is a single-phase repulsion-induction motor in that it starts as a repulsion motor and runs at almost constant speed as an induction motor. It is available for operation on 60 cycles, has a starting torque of 200 to 300 per cent of rated, and a starting current of 175 to 200 per cent of rated. It has a good efficiency (80 per cent) and high power factor (93 per cent).

There are two brush studs per pair of poles, but there is no centrifugal switch or change of connection between starting and running, the transition being accomplished by the change in reactance of the squirrel-cage winding. The stator, primary, is wound as a repulsion motor, the winding having two components with respect to the brushes, an exciting and an inducing winding, but both in series with the supply. The rotor carries a d-c multiple drum winding near the surface, and this is connected to a commutator in the usual manner. The slots are of the "Boucherot" type (see Induction Motors) with a deep slot below the outer slots. In these lower slots is a squirrel-cage winding. The leakage of flux between the two windings is carefully proportioned so that at starting and low speeds (high slip frequency) the leakage reactance of the squirrel cage is so great that it is practically inactive, and at high speeds (low slip frequency) the squirrel cage carries most of the load as an induction motor and limits the speed. At no load the repulsion action carries the speed above synchronism, but the motor is stable at this speed and drops about 6 per cent at full load.

THE "BTA" OR SCHRAGE MOTOR. The "BTA" or Schrage motor of the General Electric Co. is a polyphase adjustable-speed motor with a speed range of about 3:1 in which the speed is changed by shifting the relative position of two sets of brushes on the commutator. It gives a starting torque of 140 to 250 per cent of rated with a starting current of 125 to 175 per cent with the brushes in the position for lowest speed. At the high-speed setting its efficiency is 65 to 70 per cent and power factor 90 to 100 per cent; at the lowest-speed setting the efficiency is about 30 per cent and power factor 65 to 70 per cent.

The rotor has two windings: one, the primary, is a usual polyphase winding connected by slip rings to the supply; the other, the "adjusting" winding, is placed in the same slots as the primary and is an ordinary d-c winding connected to the commutator. The stator carries the secondary which is an ordinary three-phase winding with both ends of each phase brought out and each end connected to a separate individual brush stud. Thus there are six brush studs per pair of poles, arranged alternately on two brush yokes, the three beginnings of windings to one yoke and the three ends to the other yoke. The yokes may be moved with respect to each other, which varies the number of commutator segments included in the circuit of each phase of the secondary. The voltage generated by motion of the adjusting winding is of slip frequency and is of zero frequency at synchronism because then the flux of the primary is at standstill in space. The frequency of the voltage of the adjusting winding is the same as that of the induced voltage in the secondary, and the two may be added or subtracted.

With the two brushes of one phase winding on the same commutato segment that winding is short-circuited and the motor acts as an ordinary induction motor and runs just below synchronous speed. With the brushes moved apart one way a voltage is

added to that induced in the secondary by slip, the secondary current is increased, and the motor runs faster. Conversely, if the brushes are moved apart in the other direction a voltage is subtracted from the secondary induced voltage and the motor runs slower. By moving both yokes together the phase of the adjusting voltage is changed, and this must be done in the first place to get the two voltages in proper phase for the best power factor. The direction of rotation is changed by interchanging two of the power supply leads. The motor is started with full voltage. This type of motor is used for blowers, pumps, and compressors.

THE FYNN-WEICHSEL MOTOR. The Fynn-Weichsel Polyphase Induction-Synchronous Motor of the Wagner Co. starts as an induction motor but when once brought up to speed runs as a self-excited synchronous motor whose principal features are its good power factor and its ability to correct power factor. The rotor carries two windings, the true polyphase primary connected to the supply by slip rings and a d-c winding connected to a commutator on which bear two brushes per pair of poles. The stator carries the secondary, which is a two-phase winding with the terminals brought out for connection to a starting resistance as in all induction motors with phase-wound secondaries. But the stator winding consists of two parts in quadrature, and on one of them is impressed the voltage from the brushes on the commutator. As in

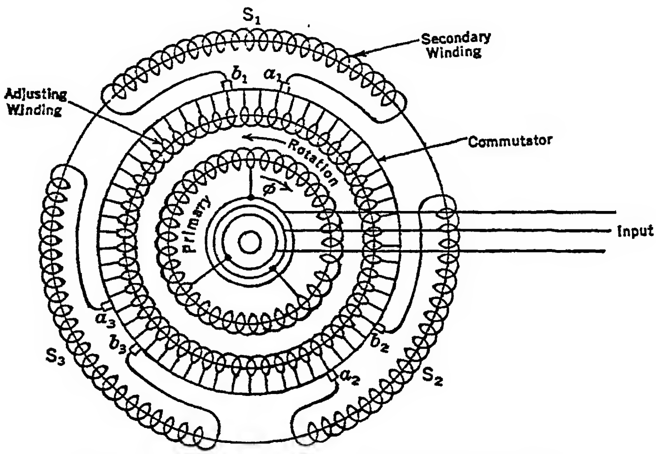

FIG. 6. Bipolar Schrage or B.T.A. Motor (for Two Poles)

the BTA motor this voltage is of slip frequency, hence of zero frequency at synchronism. This voltage is impressed upon one of the stator windings and gives a definite self-excited constant field at synchronism as in the shunt d-c motor. Thus we have a synchronous motor whose primary is revolving and whose field is excited by direct current and is stationary. At synchronism the other part of the stator winding is inactive. The efficiency in a 15-hp motor ranges from 80 to 90 per cent and the power factor may be made anti-inductive. The starting torque and current are like that of any polyphase induction motor started with resistance in the secondary.

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SECTION 10

TRANSFORMERS

BY

WALTER I. SLICHTER

ART.		PAGE
1.	Classification	02
2.	Transformer Principles	04
3.	Transformer Connections	07
4.	Three-phase Transformers	10
5.	Design	10
6.	Predetermination of the Performance	12
7.	Cooling of Transformers	14
8.	Testing Single-phase Transformers	15
9.	Testing of Three-phase Transformers	18
10.	Examples of Performance	18
11.	Specifications for Transformers	19
12.	Installation of Transformers	19
13.	Operation of Transformers	21
14.	Auto-transformers and Compensators	21
15.	Examples of Transformer Design	23

TRANSFORMERS

By Walter L. Slichter

The electrical transformer, commonly called the static transformer, is a piece of stationary apparatus used to transform a-c energy at one voltage to some other higher or lower voltage. The single-phase transformer consists of two electrical circuits, usually of a large number of turns, interlinked with a common magnetic circuit of iron. Since the power is approximately the same in both windings, the currents in the two windings are inversely proportional to the voltages in the windings. The polyphase transformer is essentially two or more single-phase transformers made into a single piece of apparatus, but so designed that at least a part of the magnetic circuit is common to all the phases. A three-phase transformer has three high-tension and three low-tension windings arranged on a single iron core; see Figs. 1 and 2.

TERMINOLOGY. The winding by which the energy enters the transformer is logically the "primary," and the one by which the energy leaves the transformer is called the "secondary." Since either winding of the transformer may be connected to the source of energy, these terms are not definite unless the manner of connection is also stated. When referring to the transformer as a separate piece of apparatus the terms "high-tension" winding and "low-tension" winding are used to distinguish the two windings, the high-tension winding being the one with the greater number of turns. When the high-tension winding is connected to the source of supply it is the primary, and the transformer is said to be used as a "step-down" transformer; when the low-tension winding is connected to the source of supply, the high-tension winding is the secondary, and the transformer is said to be used as a "step-up" transformer.

1. CLASSIFICATION

Transformers may be classified according to their operating characteristics, their construction, or the method of cooling.

Constant-potential and Constant-current Transformers. Transformers may be either constant potential, such as are intended to give an approximately constant potential on the secondary side, or constant current, intended to give an approximately constant current on the secondary. Both types are intended to operate on a supply circuit of a constant potential.

Series Transformers are connected in series with the main circuit and receive a variable voltage and current in the primary. The secondary circuit is closed through a path of low impedance, and thus the secondary current will be proportional by the ratio of turns to the load current flowing in the primary or supply circuit. They are generally used to supply low-reading ammeters and wattmeters from circuits carrying very heavy currents.

Auto-Transformers or Compensators, sometimes called single-circuit transformers, consist of one electric circuit interlinked with the magnetic circuit and a tap brought off from some part of the winding. The voltage between this tap and either terminal of the electric circuit will be a fraction of the total voltage, and thus a fractional voltage may be secured from this piece of apparatus. It is customary to proportion the windings on each side of the tap in accordance with the current to be carried. Auto-transformers are generally used where the ratio of voltages is quite near to unity as they can then be constructed with much less copper than the regular transformer. See Article 14 on Auto-transformers.

Potential Regulators are a form of transformer in which the voltage of one member may be varied from zero to a fixed maximum either by changing the direction of the magnetic flux or by changing the phase of the emf of the secondary with respect to that of the primary.

CORE AND SHELL TYPES. Two methods of arranging the electric and magnetic circuits are in use, the corresponding construction being designated as the "core type" or the "shell type." At present a large number of lighting transformers of small and medium capacity are constructed in a manner which is a composite of the core and shell types.

Core Type. The single-phase core-type transformer consists of a single magnetic circuit interlinked with two electric circuits, each consisting of a group of coils as shown in Fig. 1. The three-phase core-type transformer is also shown in Fig. 1.

Shell Type. In the shell type of transformer each electric circuit is interlinked with two magnetic circuits having a common path inside the coils but branching outside of the coils as shown in Fig. 2.

Distributed Core Type (Fig. 2*a*) is a compromise between shell and core type and is much used for small lighting transformers because it economizes in iron and distributes the heat. It consists of the two electric circuits wound concentrically about one central core which divides at the top and bottom into four radiating magnetic circuits surrounding the coils on four sides. It is shown in Fig. 2*a* in plan view.

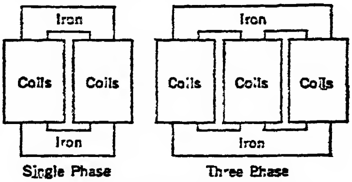

FIG. 1. Core Type Transformer

COMPARISON OF CORE AND SHELL TYPE. The core type of construction is best adapted for high-voltage low-capacity transformers, and the shell type for low-voltage high-capacity transformers. This arises from the fact that the most economical disposition of material in the core type demands a large number of turns and small cross-section of iron while in the shell type a large cross-section of iron and small number of turns may be used to advantage. In the shell type the coils are usually wound in flat "pancakes," this type of construction being particularly well suited to the use of heavy copper ribbon or straps. In the core type the coils usually consist of two or more spools, long in comparison with their diameter. The use of the core type is increasing both in number and size of transformers as it is found possible to arrange the coils so they will better withstand the mechanical strain due to short circuits.

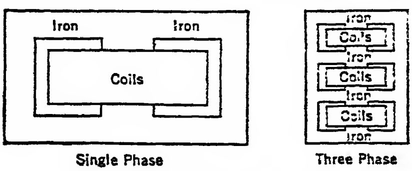

FIG. 2. Shell Type Transformers

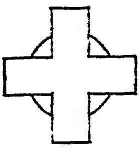

FIG. 2*a*. Distributed Core Type

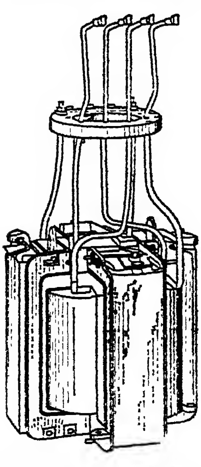

FIG. 3. Core and Coils of Distribution Transformer with Distributed Core

CLASSIFICATION ACCORDING TO METHOD OF COOLING. (See also below.) Transformers may be subdivided into classes in accordance with the method used for dissipating the heat due to their internal losses. As a transformer is a very compact piece of apparatus the problem of carrying away the heat is very important, and various ingenious means have been devised for the purpose.

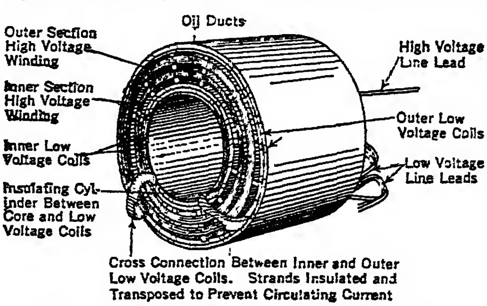

FIG. 4. Details of Coil Construction, Core Type

Air-cooled. This type has no special means of cooling but relies upon the natural circulation of air. It is used only in transformers of very small sizes, such as instrument transformers.

Self-cooled, Oil-immersed. In this type of transformer the core and windings are submerged completely in oil in a tank, and the windings and core are subdivided by ducts in order that the oil may circulate and carry off the heat from the internal parts. The heat is carried to the surface of the tank, which contains the transformer, and from there dissipated to the surrounding air. The tank is specially designed to provide large air-cooling surfaces.

Water-cooled, Oil-immersed. Similar construction to that of the self-cooled type but in addition a coil of pipe carrying running water is submerged in the oil.

Forced-oil-cooled. Transformers artificially cooled by circulation of oil are used when the size is too great for the self-cooled oil type and no cooling water is available. The oil is circulated through external coils or tanks which give a greater cooling surface.

Air-blast Transformers. These are designed with many special air passages through which currents of air are forced by means of a blower, the heat being carried off to the atmosphere in this manner.

Forced-air-cooled, Oil-immersed Transformers have large radiating coils or radiators external to the main tank through which the oil circulates, and these radiators are cooled by a forced circulation of air as in the radiator of an automobile.

METHODS OF RATING. The Standardization Rules of the A.I.E.E. specify that the continuous rating of a transformer is that output in kilovolt-amperes available at the secondary terminals, at rated volts and frequency, without exceeding a temperature rise of 55 deg cent as measured by resistance. In oil-cooled transformers the rise by thermometer should be the same; in air-blast transformers the rise by thermometer may be 60 deg cent.

The iron core of transformers takes a very long time to reach a constant temperature; that of oil-cooled transformers in particular may require from 10 to 12 hours.

2. TRANSFORMER PRINCIPLES

(See also Section 3.) The essential features of a transformer consist of a primary winding having a number of turns interlinked with a magnetic circuit, and a secondary winding also interlinked with the same magnetic circuit, as shown in Fig. 5.

SIMPLE THEORY, NEGLECTING LEAKAGE REACTANCE. In the following discussion the assumption will first be made that all the flux links both primary and secondary windings; the effect of the leakage flux will be discussed later.

In Fig. 5, S_1 is the primary winding, M the magnetic circuit (of iron), and S_2 the secondary winding. If an alternating current is caused to flow in S_1 it will set up a flux in M which at any instant is proportional to the current i_1. Thus

$$\phi = \frac{4\pi S_1 i_1}{10\,\mathcal{R}}$$

where S_1 is the number of turns, i_1 the instantaneous value of the current, and $\mathcal{R}$ the magnetic reluctance of the path in M.

Thus ϕ alternates with i_1 and since it interlinks with S_2 it will induce a voltage in S_2 at any instant equal to

$$e_2 = -S_2 \frac{d\phi}{dt} 10^{-8} \quad \text{volt}$$

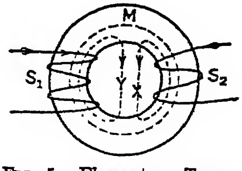

FIG. 5. Elementary Transformer

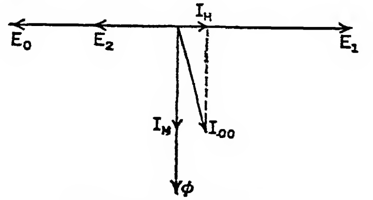

FIG. 6. No-load Relations

in which S_2 is the number of turns. The negative sign means that any current due to e_2 will tend to diminish ϕ.

NO-LOAD CONDITIONS. When there is no current in the secondary, the vector relations are as shown in Fig. 6. Let E_1 be the voltage impressed upon S_1; then the maximum value of the alternating flux will be

$$\phi = \frac{10^8 E_1}{4.44 f S_1}$$

where f is the frequency of alternation of E_1. Strictly, the numerator is $10^8 (E_1 - r_1 I_{00})$, where I_{00} is the exciting current and r_1 the resistance of the primary, but the term $r_1 I_{00}$ is practically negligible. This flux will lag 90 deg behind E_1 or 90 deg ahead of the counter emf E_0 which it induces in S_1. The true magnetizing current I_m will be in phase with the flux, and the hysteresis component of current I_H will be in phase with E_1. Hence the total no-load current I_{00} will assume some phase a little less than 90 deg behind E_1.

The flux ϕ will induce in the secondary turns S_2 an emf 90 deg behind the flux, hence 180 deg behind E_1 or in phase with the primary counter emf. The effective value of this emf is

$$E_2 = 4.44 f S_2 \phi \, 10^{-8}$$

RESISTIVE LOAD ON SECONDARY. When the secondary is closed through a non-inductive external circuit a current will flow. This current will tend to demagnetize the iron, that is, to reduce the flux and hence the counter emf of the primary. This action, however, allows the current in the primary to increase until the secondary mmf is balanced and there is left an excess mmf in the primary just large enough to give sufficient flux to induce the counter emf E_1'. The vector relations are as shown in Fig. 7, again neglecting the leakage flux.

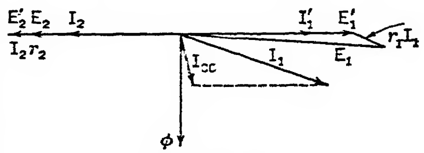

FIG. 7. Full Load, No Leakage

LEAKAGE REACTANCE. Because part of the lines of induction set up by the currents in the two windings pass through the air space (X in Fig. 5), the mmf's set up by the secondary current and the load current in the primary (the current I_1' in Fig. 7) cannot neutralize each other, since the leakage fluxes established by the two currents are in the same direction, and not in opposition as are the fluxes in the iron. The primary leakage flux is in phase with the total primary current and the secondary leakage flux is in phase with the secondary current; these leakage fluxes are therefore not in phase with the useful flux (i.e., the flux which links both primary and secondary). The result is that the leakage fluxes cause a decrease in the secondary voltage and also a shifting of its phase with respect to the primary voltage.

Since the two leakage fluxes are in phase with the total currents in the primary and secondary respectively the voltages induced by the alternation of these fluxes are in quadrature with the currents. The quotient of the voltage induced in the primary by the alternating primary leakage flux divided by the primary current is called the "primary leakage reactance," and the quotient of the voltage induced in the secondary by the alternating secondary leakage flux divided by the secondary current is called the "secondary leakage reactance." These reactances are practically constant, since the major portion of the leakage path is in the air.

COMPLETE VECTOR DIAGRAM OF TRANSFORMER. Fig. 8 shows diagram-

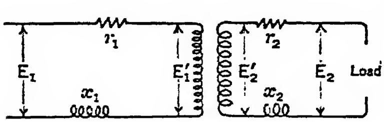

FIG. 8. Diagram of Circuits

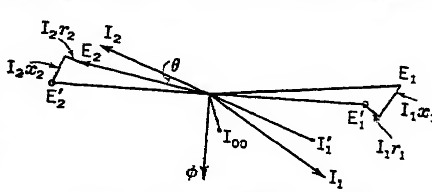

FIG. 9. Complete Vector Diagram

matically the primary and secondary windings of the transformer and Fig. 9 the vector diagram.

E_2 = the secondary terminal emf.
$\cos\theta$ = power factor of load.
I_2 = secondary current, lagging by angle θ behind E_2.
$I_2 r_2$ = secondary resistance drop in volts; in phase with I_2.
$I_2 x_2$ = secondary reactance drop in volts, due to $1/2$ of leakage flux, at 90 deg to I_2.
E_2' = induced emf in the secondary (hypothetical).
E_1' = emf necessary to overcome counter emf in primary; opposed and proportional to E_2' by ratio u.
u = ratio of transformation = S_1/S_2.
I_1' = primary load current; opposed and proportional to I_2 by ratio $1/u$.

ϕ = mutual or useful flux.
I_{00} = primary no-load current.
I_1 = total or resultant primary current.
$I_1 r_1$ = primary resistance drop; parallel to I_1.
$I_1 x_1$ = primary reactance drop, at 90 deg to I_1.
E_1 = the required voltage on the primary, that is, the impressed emf.

"EQUIVALENT" CIRCUIT OF TRANSFORMER. In practice it is not possible to measure the primary and secondary reactances separately. They must be measured together and treated as one quantity, called the total reactance x_t of the transformer, which may be expressed in terms of either the primary turns or the secondary turns.

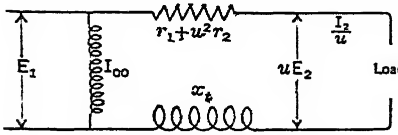

FIG. 10. Equivalent Circuit

A simple approximate method which is sufficiently accurate for all practical purposes is based on the equivalent circuits shown in Fig. 10. This method is used in calculating the characteristics of a transformer from constants obtained from tests.

Let E_2 = terminal emf of secondary.
$\cos\theta_2$ = power factor of load.
I_2 = secondary current.
u = ratio of transformation = S_1/S_2.
$E_2' = uE_2$.
$I_2' = \frac{I_2}{u}$.
R = total resistance in terms of primary = $r_1 + u^2 r_2$.
X = total reactance in terms of primary.
I_M = primary magnetizing current.
$I_H = \frac{\text{core loss}}{uE_2}$, to a close approximation.
E_1 = primary impressed emf.
I_1 = total primary current.
$\cos\theta_1$ = power factor of transformer with load, i.e., power factor at primary terminals.
P_1 = watts input at primary terminals.
η = per cent efficiency.

Then

$$I_1 = \sqrt{(I_2'\cos\theta_2 + I_H)^2 + (I_2'\sin\theta_2 + I_M)^2}$$
$$E_1 = \sqrt{(E_2'\cos\theta_2 + RI_1)^2 + (E_2'\sin\theta_2 + XI_1)^2}$$
$$P_1 = E_2 I_2 \cos\theta_2 + R(I_2')^2 + E_2' I_H$$
$$\cos\theta_1 = \frac{P_1}{E_1 I_1}$$
$$\eta = \frac{100\,P_2}{P_1}$$
$$P_2 = E_2 I_2 \cos\theta_2$$
$$\text{Per cent regulation} = \frac{100\,(E_1 - uE_2)}{uE_2}$$

CONSTANT-CURRENT TRANSFORMERS. These are used for supplying current to arc lamps connected in a series circuit which requires from 4 to 10 amp and about 2000 volts. The principle of operation for this type of transformer is based on the existence of the leakage or useless flux which causes a repelling force between the primary and secondary windings. One winding is arranged so that it may move with respect to the other. As the current in the windings increases, the repelling force increases and the windings move apart. This allows more flux to leak or pass between the windings and less to thread the secondary. Therefore the voltage induced in the secondary decreases as the current tends to increase. A counterweight is attached to the movable winding to regulate the amount of movement. The current in the primary remains fairly constant, but the power factor decreases with decreasing load on the secondary so the power in the primary is approximately proportional to the output of the secondary.

SERIES TRANSFORMERS operate under conditions similar to those in a constant-potential transformer with short-circuited secondary. Under operating conditions the flux is small as the only voltage to be induced in the secondary is that required to overcome the impedance drop in that winding and the instrument which it supplies. The only counter

emf in the primary is that due to impedance drop. The series transformer must be operated with the secondary short-circuited by low impedance. If the secondary circuit is open there is no counter mmf to balance the primary turns, which, being in series with the main line, carry a current irrespective of the secondary circuit. The primary ampere-turns would then set up a magnetic flux of considerable magnitude. As the transformer is not designed for this condition, the density would become very high in the magnetic circuit of small cross-section, and the transformer is likely to burn up from the heat due to core loss.

3. TRANSFORMER CONNECTIONS

The various transformer connections which are commonly used in lighting and power services are described below; see also Article 13 on Operation.

Single-phase System with Three-wire Secondary (Fig. 11). Standard practice in residence lighting with the a-c system involves grounding the neutral wire on the low-tension side, the primary side not being grounded. Lamps or motors operating at 110

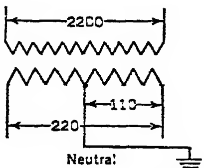

Fig. 11. Single-phase, Three-wire

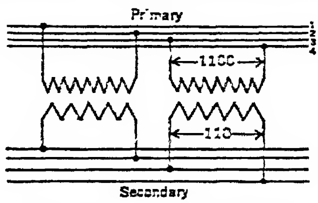

Fig. 12. Two-phase, Four-wire

volts are connected between the neutral and either side. The maximum potential between any secondary and ground is 110 volts, but if either of the outside wires becomes grounded it constitutes a short circuit on that half of the transformer.

Two-phase or Quarter-phase Four-wire System (Fig. 12). The standard two-phase or quarter-phase system is essentially two independent single-phase systems which are usually independent electrically throughout. When the two phases are not electrically connected inside the generator, either wire of one phase may be connected to either wire of the other phase, without any flow of current resulting. In certain two-phase generators, however, the windings are interconnected, in which case any interconnection of the wires coming from the generator will cause a flow of current through this connection.

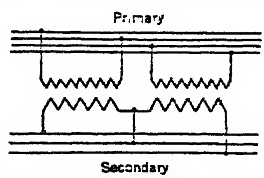

Fig. 13. Two-phase, Three-wire

Two-phase Three-wire System (Fig. 13). This connection is occasionally used for the distribution of power in small systems. There is a possibility of a slight saving in copper, but the chances of unbalanced voltage and bad regulation, particularly with an inductive load, render it objectionable.

Three-phase Y and Δ Connections (Figs. 14 and 15). Transformation in a three-phase system with either three independent single-phase transformers or with a three-phase transformer, having three primary coils and three secondary coils on one iron core,

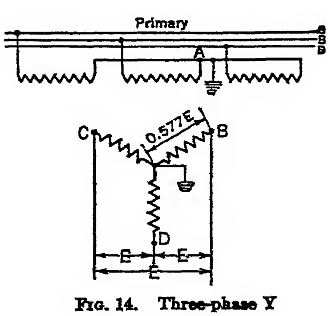

Fig. 14. Three-phase Y

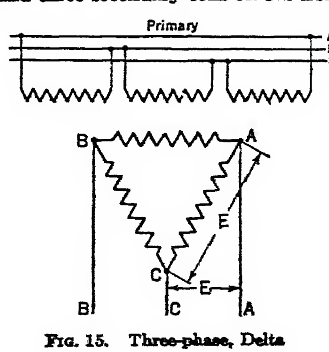

Fig. 15. Three-phase, Delta

is accomplished by connecting the primary either in Y or in Δ and the secondary either in Y or in Δ. With Y or Δ connections there are the following relations between voltage per transformer winding and voltage between lines, and between current in transformer windings and current in lines.

Connection	Volts bet. Lines	Volts per Winding	Current per Line	Current per Winding
Y	E	$0.58\,E$	I	I
Δ	E	E	I	$0.58\,I$

The power in the three transformers in any case is $3 \times 0.58 \times EI = 1.73\,EI$.

There are four combinations of these connections which may be used on any bank of transformers. These connections are given in the following table, and also the ratio of the voltage between lines for the low-tension and high-tension sides for a given ratio (u) of transformation in each individual transformer connection.

Connection		Low-potential, volts bet. lines	High-potential, volts bet. lines
Low Pot.	High Pot.		
Δ	Δ	E	uE
Y	Y	E	uE
Δ	Y	E	$1.73\,uE$
Y	Δ	E	$0.58\,uE$

In a-c secondary distribution networks it is standard practice to use a Y connection on the low-potential secondaries and bring out a conductor from the neutral point, thus making a four-wire system. Between any "outer" and the neutral, 125 volts, single-phase, is available for lamps and small devices; between the three outers there is 208 volts, giving a true three-phase, 208-volt system for larger motors.

PARALLEL CONNECTION OF THREE-PHASE BANKS. If several banks of transformers in the same system are connected in parallel on one side, then to connect the other sides in parallel the connections must be such that the voltage between any two lines on this side will have the same phase in all the banks. From this relation result the following rules:

With ΔΔ on one bank, the other bank must be ΔΔ or YY.
With YY on one bank, the other bank must be ΔΔ or YY.
With ΔY on one bank, the other bank must be ΔY or YΔ.
With YΔ on one bank, the other bank must be YΔ or ΔY.

Even when these relations are satisfied a short circuit will result unless the three phases of each bank are connected in the proper sequence. This can be readily determined by the polarity test described below in Article 9 on Testing.

RELATIVE ADVANTAGES OF Y AND Δ CONNECTIONS. There has been much discussion as to the relative advantages of Y- and Δ-connected transformers for high-tension transmission and as to the value of a grounded neutral. The general and fundamental arguments may be summed up as follows:

With a ΔΔ system the advantages are that if one transformer becomes disabled the system may be operated from the other two, operating on open delta (see below). If the load is unbalanced the voltages do not become unduly unbalanced. Resonance cannot occur. On the other hand, each transformer must be insulated for full line voltage as there is no neutral to ground, and if one line becomes grounded the voltage strain on the rest of the system becomes 1.73 times the normal strain, and this strain may extend to the low-tension winding and the generator which is connected to the transformers.

With YY-connection there is a very unstable neutral and a possible excessive voltage strain on one phase, unless the neutral is grounded.

With a ΔY arrangement for step-up transformers, the neutral on the high-tension side may be grounded. The voltage strain on any transformer is then limited to 58 per cent of the line voltage. There is a possibility of operating the remaining two transformers, if one becomes damaged, by using the neutral as a third conductor. However, there is also the possibility of resonance under certain circumstances and the danger of causing disturbances in nearby telephone and telegraph circuits. Any accidental ground on the system makes a definite short circuit on one phase.

The arrangement of Y Δ for step-up transformers is not desirable on account of the unstable neutral with unbalanced load, but is permissible with a balanced load or with a good connection from neutral of transformers to neutral of generator. This connection, however, is frequently used for step-down transformers particularly when connected to a balanced load.

GROUNDED NEUTRAL VS. UNGROUNDED NEUTRAL. In any system without a grounded neutral there are numerous possibilities of disturbances resulting in a high-voltage strain on the various parts of the insulation of the system. With the high voltages

now in use for transmission systems these disturbances may give a great deal of trouble by breaking down the insulation, as it is not always possible to employ a large margin of safety and lightning arresters do not always protect from these disturbances. On the other hand, if the neutral is grounded, most of these disturbances will merely result in an excessive current, and if the circuit breakers are installed in the proper places they will open the circuit, so that the only adverse result will be a temporary interruption of service.

The choice is then between a system with ungrounded neutral and a large margin of safety in the insulation, and a system with grounded neutral, moderate insulation, and the possibility of occasional interruption of service.

EFFECT OF HIGHER HARMONICS. Many alternators generate an emf whose wave shape contains higher harmonics (see Section 3, Art. 8), and the magnetizing current of transformers operating at high magnetic densities has a distorted wave shape which contains a prominent third harmonic. If these harmonics are present in a Y-connected machine they cause the voltage between each line and the neutral to vary so that there exists an unstable neutral, and the voltage strain on any one phase is indeterminate. If the neutrals of two pieces of apparatus in which these harmonics exist are grounded or joined together, a high-frequency current will flow in the neutral connection. If on the other hand these pieces of apparatus are Δ-connected a third harmonic current will circulate inside the apparatus. This may be measured by connecting an ammeter in the Δ between two phases. This current heats the apparatus, does no profitable work, and is therefore to be avoided. High-frequency current in the line will cause high voltages to occur at certain points if a large amount of capacitance is present, as is the case in any underground line, even of moderate length, and also in long overhead lines. All possible means both in the design of the generators and the connection of of the apparatus should be taken to prevent the occurrence of, or to diminish the effect of, these higher harmonics.

THREE-PHASE OPEN Δ OR V CONNECTION (Fig. 16). This system consists in omitting one transformer from the delta connection, and is used to save expense, particularly in temporary installations or in new installations where the load is not great at first but is expected to increase in time. Thus the purchase and installation of the third transformer are postponed until the load requires it. This connection can only be recommended for low voltages, such as 2300, as it is likely to produce dangerous potentials due to

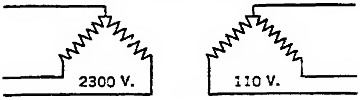

Fig. 16. Three-phase, Open Delta

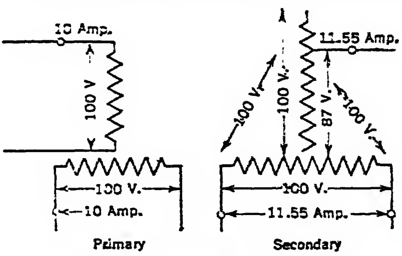

Fig. 17. Two-phase to Three-phase

electrostatic unbalancing. The regulation and efficiency are also poor, as one phase of the load receives its power from two transformers in series. The aggregate capacity of the two transformers should be 15 per cent greater than the load.

TWO-PHASE TO THREE-PHASE TRANSFORMATION. The Scott or T-connection, the standard method of transforming from two-phase to three-phase, consists of two transformers which on the two-phase side may be connected in the normal two-phase manner either independently or interlinked. On the three-phase side one transformer has a tap at the middle point and the other a tap giving 87 per cent of full transformer voltage. Fig. 17 shows the method of connecting and the currents in primary and secondary with balanced loads. Since the total power is $2\,E_2 I_2 = \sqrt{3}\,E_3 I_3$, it follows that $I_3 = \dfrac{2}{\sqrt{3}}\dfrac{E_2}{E_3} I_2$, or the three-phase current is 16 per cent greater than it would be for straight single- or two-phase transformation. Thus one of the windings of one transformer must carry 16 per cent more than its share of the volt-amperes of the load and is overloaded in that proportion.

In commercial practice it is customary for the sake of interchangeability to make the capacity of both transformers 16 per cent greater than the load and to put both a 50 per cent and 87 per cent tap on both transformers. For this connection of transformers each half of the main transformer winding must be distributed over both legs of the core in order to prevent flux (and therefore voltage) unbalancing. If this pre-

caution is taken both the primary and secondary voltages are balanced if the load is balanced. Any unbalancing of the secondary causes a like amount of unbalancing on the primary; thus if this connection is used to transform from three-phase to two-phase and all the load comes on one phase of the secondary it will also come on one phase of the primary.

4. THREE-PHASE TRANSFORMERS

Considerable space, wiring, and first cost may be saved by the use of a three-phase transformer in place of three single-phase transformers. This saving is warranted in large installations, but not in small installations where the convenience of having one interchangeable single-phase transformer as a spare for several banks of three transformers each is an important item.

Core of Three-Phase Transformer. Three-phase transformers may be of the core type as shown in Fig. 1, in which case all the magnetic paths are of the same cross-section, or of the shell type shown in Fig. 2. In the latter type the horizontal cross-pieces and outside vertical pieces may have one-half the cross-section of the central vertical core, providing the coil on the central leg is connected with a definite polarity with respect to the other two coils in order to have the fluxes differ in phase by 60 deg.

Operation of Damaged Three-phase Transformer. In a shell-type three-phase transformer if the primary and secondary are delta-connected it is possible to operate with only two phases in open delta at reduced capacity in case of trouble in the third phase of the system. This is accomplished by separating the third phase entirely from the system and short-circuiting both the primary and secondary windings.

5. DESIGN

(See also Article 7, p. 14 on Cooling of Transformers.) The methods of design for single-phase and polyphase transformers are similar, the chief difference being that of the magnetic circuit. In the design of polyphase transformers each leg is treated as an independent single-phase transformer. The number of turns is adjusted to the voltage per phase, and the cross-section of the conductors to the current per phase. All legs are wound alike, and the phases are connected as described in the preceding section.

The a-c transformer is the most efficient piece of electrical apparatus. Its efficiency at full load is usually better than 95 per cent and frequently better than 98 per cent. For this reason it is very small in volume and light in weight for a given output compared to other pieces of electrical apparatus, and hence the chief difficulty in design is the necessity of providing sufficient surface and a proper means to carry off the heat developed.

The difference in mechanical construction between the core and shell type is that the winding of the core type usually consists of two or more long spools (in which the length is considerably greater than the diameter) surrounding each leg of the transformer. The shell type consists usually of large flat "pancake" coils, laid alongside of each other on the central core with proper separating devices to permit ventilation. See Figs. 1 and 2 (p. 03).

DETERMINATION OF FLUX (ϕ) AND NUMBER OF PRIMARY TURNS (S_1). In order to estimate a desirable value of the flux to be used a factor C is employed which is defined by the relation

$$C = \frac{\phi}{S_1 I_1} \tag{1}$$

The proper value of this factor C depends upon the type, capacity, and voltage of the transformer, and the relative weights of copper and iron.* Usual values are given in the accompanying table.

Values of C

Form of Transformer	Voltage	Core Type Value of C	Shell Type Value of C
Natural draft	0–6000	55–70	500–700
	6000 up	70–75	500–700
Oil-cooled	0–10,000	75–100	
	10,000 up	100–150	
Air-blast or water-cooled	0–10,000	110–160	600–1000
	10,000 up	160–240	600–1000

* A discussion of this factor C will be found in Arnold's Transformers and in S. P. Thompson's Dynamo Electric Machinery, Vol. II.

Volts per Turn. The same criterion may be expressed in a more convenient empyrical form as "volts per turn":

$$e = \text{volts per turn} = 4.44 f \phi\, 10^{-8}$$

Usual values of volts per turn for 60 cycles in commercial practice in the United States are given in the accompanying table.

For 25-cycle transformers the volts per turn are about 50 per cent greater.

By this criterion, reasonable values for the flux and the primary and secondary turns may be readily estimated.

Usual Values of Volts per Turn

Rating in Kva	Volts per Turn	
	Core Type	Shell Type
1	0.5	1.2
10	2.5	3.75
50	5.5	8.5
100	7.0	12.0
500	13.0	26.0
1000	18.0	36.0

CROSS-SECTION OF CORE (A) AND MAGNETIC FLUX DENSITY (B). The core section is determined by the magnetic density which it is desirable to use, which in turn is determined by the core loss. It is found that in average practice a loss of 1 watt per pound of iron can be dissipated without excessive rise in temperature. The corresponding flux densities in iron having a thickness of 14 mils are given in the following table. These values are approximate only, since the quality of the steel used is exceedingly variable. It should also be noted for a transformer intended for supplying a load of low power factor that the iron loss should be less than 1 watt per pound for the best distribution of material.

Values of Flux Density

Size of Transformer	Kind of Steel	Lines per Sq In.	
		25 Cycles	60 Cycles
Small	Ordinary transformer sheet	50,000	40,000
Small	Special; silicon-steel	70,000	60,000
Large	Ordinary transformer sheet	75,000	65,000
Large	Special; silicon-steel	90,000	75,000

Construction. The usual arrangement of the windings and core of a core-type transformer are shown in Fig. 18, and of a shell type in Fig. 19. Usual values for the magnetic densities B in the iron are given above, and for the current density in the copper in the following table.

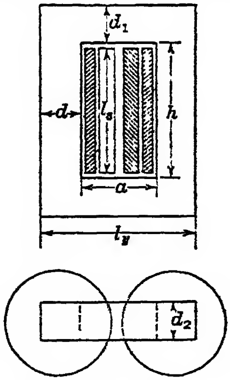

Fig. 18. Core Type

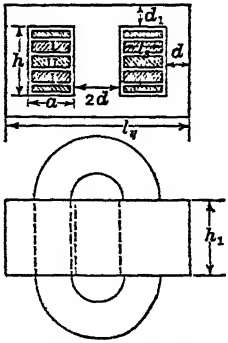

Fig. 19. Shell Type

Current Density

Condition of Transformer	U = Amperes per Sq In.
Poorly cooled....................	850–1200
Ordinary oil-cooled, air blast, etc.....	1100–1600
Large, well-cooled................	1500–1900

INSULATION OF WINDINGS. The insulation of each turn usually consists of the cotton covering if the coils are wound with wire, or mica paper if the coils are wound with strip copper. This is proportioned to withstand the potential between

turns as given above. The voltage between conductors on adjacent layers may be equal to twice the voltage per layer. To prevent breaking down between layers, a layer of Fuller board is used as a separator, and this should project beyond the windings at the ends to prevent creepage. The maximum voltage between layers should be kept below 400, and to accomplish this it is customary to limit the voltage per coil to about 5000 volts. Between the windings and the core a layer of pressboard and sometimes wood is placed, while between primary and secondary windings a layer of pressboard and micanite may be used. For very high voltages the end turns of the high-tension winding for about 75 ft from the terminals is given a special insulation to withstand the sudden high potentials which occur when there is a sudden change in the potential applied to the transformer. These high potentials result from the distributed capacity of the transformer, between the high-tension winding and the core, frame, and other winding.

TERMINAL BUSHINGS. At the point where the high potential leads pass through the case there is a very great dielectric stress which must be taken care of by the use of a proper kind of insulation and a proper disposition of the insulation to prevent a concentration of the dielectric flux at a few points. For voltages below 40,000 a porcelain bushing is customarily used.

For higher voltages it is necessary to supply a large creepage distance and to have the surface submerged in oil to prevent corona effect. This is accomplished in one form of bushing, known as the "condenser type," by surrounding the terminal with layers of insulation and putting sheets of tinfoil between the layers. This arrangement is equivalent to a series of condensers. By properly proportioning the area of the successive layers of tinfoil the drop in potential across the insulation is kept uniform. The whole terminal is enclosed in an oil-filled casing. Another form of bushing known as the "oil-filled type" consists of a long cylinder of composition insulation which surrounds the lead and is filled with oil. The cylinder is divided into compartments to keep the oil properly distributed, and disks or collars project outward from the outside to increase the creepage distance.

END COILS OF SHELL TYPE. With the flat coils customarily used in the shell-type transformer the subdivision of the windings is usually such that there is a half coil of the low-potential winding at each end of the winding space as in Fig. 19. This is to reduce the leakage flux. In order to reduce the leakage flux further all coils may be divided into halves with a ventilating duct between halves. The space between a primary and secondary coil is then reduced to that necessary for the insulation.

6. PREDETERMINATION OF THE PERFORMANCE

From the above calculations a drawing to scale of the transformer may be laid out. The next step is to calculate its performance, i.e., to predetermine the values of the efficiency, regulation, and temperature rise. This last feature is treated in the following article on the Cooling of Transformers.

MAGNETIZING CURRENT (I_M). The final flux density will probably differ a few per cent from the value assumed earlier in the calculation. The cross-section of core is usually proportioned to give the same magnetic density in all parts, as this condition gives minimum core loss for a given weight of iron.

The mean length of path in the iron is measured or calculated. If H (found from the magnetization curve of the iron, see article on Magnetic Properties of Iron) is the ampere-turns per inch for the given density the magnetizing current of the transformer will be

$$I_M = \frac{H \times \text{(length path)}}{\sqrt{2}\, S_1}$$

Sometimes it is desired to allow for the minute air gaps at the joints of the punchings. Arnold finds that under practical conditions each joint represents a gap of 0.002 in. Thus if there are n joints (usually 4) there should be added to I_M an amount

$$\frac{0.313 \times 0.002\, Bn}{\sqrt{2}\, S_1}$$

CORE LOSS. The core loss consists of hysteresis and eddy current losses in the steel punchings. These losses have been considerably reduced in recent years by improvements in the manufacture and quality of the steel (see article on Magnetic Properties of Iron).

$$\text{Core loss} = C_1 f V + C_2 f^2 V \text{ watts}$$

where f = frequency in cycles per second.

V = volume of iron in cubic inches.

C_1 = watts per cu in. per cycle per sec of hysteresis loss for the particular value of B and of the particular material.

C_2 = watts per cu in. for one cycle per sec for eddy current loss for the value of B and the particular material.

Values for C_1 and C_2 are given in the article on Magnetic Materials in Section 2 on pp. 2–41 to 2–47.

COPPER LOSS. The mean length of turn of both primary and secondary windings is obtained from a sketch to scale. The d-c or ohmic resistance of each member at 60 deg cent is obtained by substituting the proper values in the formula

$$R = \frac{0.0093\, lS}{12{,}000\, an}$$

where l = mean length of turn of primary or secondary respectively in inches.
S = number of turns in series.
a = cross-section of conductor in square inches.
n = number of conductors or coils in multiple.

To allow for eddy currents caused by the leakage flux, the above resistance is increased by 15 per cent to give the effective resistances r_1 and r_2. The total copper loss is then $I_1^2 r_1 + I_2^2 r_2$.

TRANSFORMER REACTANCE. As explained in Art. 2 on Transformer Principles, the leakage of flux between the primary and secondary windings of a transformer causes a component of voltage in each member which is out of phase with the current; if the current lags very much this voltage may have a considerable component opposed to the useful voltage and cause a loss of voltage or poor regulation. It is, therefore, necessary in designing to estimate the amount of this flux and calculate the voltage it would produce. In practice empirical formulas are usually used, but a logically deduced formula is desirable as it is more easily adapted to unusual cases.

The phase of the current in the secondary coil is nearly opposed to that in the primary and may be assumed to be exactly opposed without any great error. The result as shown in Fig. 20 is that all these ampere conductors, both primary and secondary, tend to set up a flux in the same direction in the space between the windings and to a lesser degree in the windings themselves. A part of the flux in the intervening space interlinks with the primary turns and another part with the secondary turns. In addition the flux in the windings themselves interlinks with some of the turns. The flux in each part is proportional to the ampere turns producing it and to the permeance of the path. In this case, since the path is in air, the permeance is the area divided by the length, where the length is the average length of the flux lines.

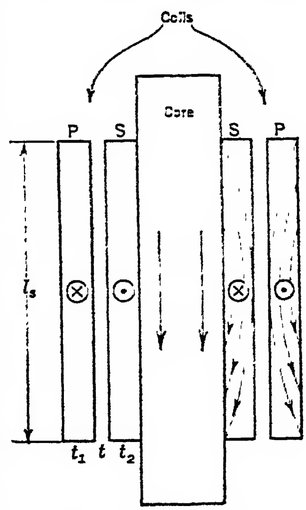

FIG. 20. Leakage Flux

The leakage flux passes between the windings and one part closes its path in the iron inside the inner coil and the other part in air outside the outer coil (in the core type). The reluctance of the path between the two coils is large compared to that of the other two portions, because the inside path has a high permeability and the outside path has a large cross-section. The reluctance of the path between the coils is therefore accurately (and easily) calculated and a constant used to allow for the rest of the path.

In order to calculate the inductance it is necessary to have a cross-section of the windings showing their thickness, length, and arrangement, as in Figs. 18, 19, and 20. Two cases must be considered:

(A) Where there are as many primary coils as secondary coils, and all coils are full size (usual core type).

(B) Where there is a "half" secondary coil at each extremity of the group of windings (the usual arrangement in the shell-type transformers).

Arnold's Method of Calculating Reactance. Arnold calculates the inductance of a single primary coil and its secondary mate or mates, and multiplies this by the number

of primary coils in series or divides by the number in multiple. This gives the total "short-circuit" inductance of the transformer in terms of the primary voltage or turns.

Referring to Figs. 18, 19, and 20 for the core and the shell type, respectively, let the various quantities be represented as follows, all dimensions being in inches:

q = number of primary coils in series.
p = number of primary coils in parallel.
s_1 = number of turns in one primary coil.
s_2 = number of turns in one whole secondary coil.
l_s = height of coils in cylinder type.
l_s = depth of coils in flat type.
t_1 = thickness of a whole primary coil.
t_2 = thickness of a whole secondary coil.
t = distance between coils.
U_1 and U_2 = mean length of primary and secondary turns respectively.
U_m = average of U_1 and U_2.
k = an empirical constant, varying from 0.95 with flat coils to 1.06 with cylinder coils.

Reactance of Core Type. (Fig. 20). In a core-type transformer with cylindrical coils and an equal number of full-sized primary and secondary coils, the permeance of the path of the flux in the duct which interlinks with a primary coil is $\frac{tU_1}{2\,l_s}$ and the permeance of the path in the winding itself is $\frac{t_1 U_1}{3\,l_s}$.

The inductance of one primary coil is

$$L_1 = \frac{3.2\,ks_1^2\,U_1}{l_s}\left(\frac{t_1}{3}+\frac{t}{2}\right)10^{-8}$$

The inductance of one secondary coil is

$$L_2 = \frac{3.2\,ks_2^2\,U_2}{l_s}\left(\frac{t_2}{3}+\frac{t}{2}\right)10^{-8}$$

Reducing the latter to terms of the primary turns by multiplying by $\frac{s_1^2}{s_2^2}$, adding together, and multiplying by $2\,\pi f$, the total reactance of the transformer in terms of the primary is

$$X = 2\,\pi f\,\frac{3.2\,qks_1^2\,U_m}{pl_s}\left(\frac{t_1+t_2}{3}+t\right)10^{-8}$$

Reactance of Shell Type. If, as in the shell-type transformer shown in Fig. 19, there is a half secondary coil at each end to give an increased intermixing, then

$$X = 2\,\pi f\,\frac{3.2\,qks_1^2\,U_m}{2\,pl_s}\left(\frac{t_1+t_2}{6}+t\right)10^{-8}$$

7. COOLING OF TRANSFORMERS

It is necessary to keep the temperature of the various parts of a transformer within such limits that the materials of which it is constructed do not become damaged and deteriorate too rapidly. When subjected to too high a temperature the insulating materials disintegrate and lose their mechanical and dielectric strength, and the iron deteriorates in its magnetic qualities so that the core loss becomes greater for a given density. This so-called aging of the iron may cause an increase of as much as 50 per cent in the loss in the iron. The effect varies with the character of the iron and is practically negligible in silicon steel.

MAXIMUM RISE IN TEMPERATURE. The maximum rise in temperature above a room temperature of 25 deg cent at which the various materials of a transformer should be operated are given in the table.

Material	°C
Iron	70–75
Cotton	60
Paper	70
Mica, asbestos	90

To guard against this possibility of damage the Standardization Rules of the A.I.E.E. recommend that the windings shall not increase in temperature more than 55 deg cent, as measured by resistance, above the surrounding air, and the other parts shall not increase more than 55 deg cent as measured by thermometer.

MEANS OF DISSIPATING HEAT. The problem is to provide a path of low thermal resistance by which the heat energy may pass to the surrounding air. To

accomplish this it is necessary, first, to provide sufficient surface in the subdivided transformer to transfer the heat to the cooling agent, air or oil, without too great a difference in temperature; second, so to subdivide the transformer that no part of the iron is more than 1 in., and no part of the copper more than 3/8 in., from a cooling surface; third, to provide a sufficient quantity of the cooling agent, air, oil, or water, to carry away the heat at the same rate as it is generated; and fourth, to provide sufficient surface on the containing case or tank to transfer the heat from the internal oil to the external air without too great a difference in temperature.

CALCULATION OF EXPOSED SURFACE OF TRANSFORMER. The practical method of estimating the rise in temperature consists in calculating the drop in temperature in the successive media through which the heat must pass to be dissipated. The first step is to calculate the exposed surface of the transformer core and core ducts and of the coils and coil ducts.

For a rise in temperature of 55 deg cent above the surrounding air the allowable watts loss per square inch should be approximately:

	Coil Surface	Core Surface
Oil-immersed self-cooled..	0.40	1.5
Air-blast...............	0.75	1.0
Forced-oil-cooled.......	1.00	1.75
Oil-immersed water-cooled	0.85	1.75

Naturally air-cooled transformers (instrument transformers) should have from 4 to 6 sq in. of surface for each watt of loss.

OIL-COOLED TRANSFORMERS. The transformer is submerged in oil in a tank so that the level of the oil is from 2 to 3 in. above the top of the transformer. The quantity of oil is from 6 to 10 lb per kva rating, or from 250 to 500 lb per kw loss. The oil ducts should be 1/4 in. wide and run vertically. If the tank is smooth there should be from 4 to 8 sq in. of tank surface (not including top or bottom) for each watt loss. For sizes greater than 25 kw it is customary to use fluted or corrugated sides to the tank. In this case there should be from 6 to 10 sq in. of radiating surface per watt loss because the air does not circulate as rapidly in the grooves as over a smooth surface and consequently radiation is poor.

For a rise of 55 deg cent of the transformer there is an average rise of 30 deg of the oil. The maximum rise of the oil is 1.3 to 1.5 times the average.

AIR-BLAST TRANSFORMERS. This type of transformer is usually set over a large air duct in the floor which is supplied by a blower with air at a pressure of 1/2 to 1 oz (3/4 to 1 1/2 in. of water). The air enters the transformer at the bottom and is divided into two streams, one passing vertically through the windings and the other transversely through the iron. There is a damper or valve for each stream, so that the proper amount of air is provided to each part independently. Ducts 1/2 in. wide are provided every 3 to 4 in. A liberal amount of air for 55 deg cent rise is 150 cu ft per min per kw loss The air is expected to rise from 15 to 20 deg cent in its passage through the transformer. The theoretical quantity of air in cubic feet per minute is

$$Q = \frac{1.65 \text{ watts}}{T_a}$$

where T_a is the rise in temperature of the air, which is usually about half as great as the rise of the transformer. See Installation, page 10-19.

WATER-COOLED TRANSFORMERS. The cooling coils are suspended in the tank in the oil near the top, usually above the transformer proper, and carry a continuously circulating stream of water. It is customary to provide a gallon of water per minute for each kilowatt of loss and 1 sq in. of surface of water-coils per watt loss. This assumes a temperature drop of 20 to 25 deg cent in the insulation and oil, 2 to 3 between oil and water, and 20 to 25 in the water.

8. TESTING SINGLE-PHASE TRANSFORMERS

The customary commercial tests on transformers and the best order of making them are: cold resistance, polarity, ratio and checking of taps, impedance, core loss and exciting current, parallel run, heat run, insulation tests. The efficiency and regulation are calculated from the results of these tests.

Oil-cooled transformers should never be tested or subjected to potential unless they have been filled with oil from which all moisture has been removed.

Examples of test results are given below.

COLD-RESISTANCE MEASUREMENT. The cold resistance must be very carefully made as it is used as a basis of calculating the temperature after the heat run. After the transformer has been standing in one place long enough for all its parts to reach

the same temperature as the surrounding air, a direct current of 10 to 15 per cent of the rated current of the coils is sent through the windings and the drop measured with a voltmeter. At the same time the temperature of the windings is measured by a thermometer.

TEST OF POLARITY. This test gives information necessary for connecting several transformers in a bank and have them operate in parallel or on polyphase circuits. Direct current is sent through one winding of a transformer and a d-c voltmeter connected across the other winding. If the current is stopped, the voltmeter will give a deflection either positive or negative. For similar connection and direction of current on all the transformers of a bank the deflection should be of the same sign. A small current should be used as otherwise the throw of the voltmeter needle may be sufficient to bend it.

RATIO OF TURNS. With no load on a transformer the ratio of voltages is the same as the ratio of the turns. Thus, with a known voltage applied to the low-tension winding, the ratio of turns of the two main windings and of the sections between taps can be checked up by connecting across the other terminals another voltmeter of proper range (using a potential transformer if necessary).

IMPEDANCE TEST. This test is important in order to calculate the regulation and in order to determine whether transformers will run in parallel with each other. One winding of a transformer is short-circuited, and a voltmeter, ammeter, and wattmeter are connected in the circuit of the other winding. A voltage of 1 to 8 per cent of the rated voltage of this winding and of the proper frequency is impressed. The voltage is regulated so that readings are taken at values of current from 50 to 125 per cent of the rated current of the winding. The wattmeter reading will be in the neighborhood of 2 per cent of the rating of the transformer. The total impedance of the transformer will be $Z = E/I$. The total effective resistance will be $R = \text{Watts}/I^2$, which will include the effect of eddy currents, and the total reactance will be $X = \sqrt{Z^2 - R^2}$. This reactance cannot be separated into primary and secondary reactance. The results of this test are usually plotted in two curves, one between volts and amperes and the other between volts and watts.

CORE-LOSS AND EXCITING-CURRENT TEST. The alternator supplying the power for this test should give a sinusoidal emf wave, as any distortion in the shape may cause a variation of 5 to 10 per cent in the core loss. A peaked wave gives a lower core loss than a sine wave. For this test the high-tension winding is left open and rated voltage at the proper frequency is impressed on the low-tension winding. A voltmeter, ammeter, and wattmeter are connected in the low-tension circuit, the voltmeter having a range including the rated voltage of the transformer and the ammeter a range of approximately 15 per cent of the rated current of the machine. Readings are taken with a voltage of 50 per cent to 125 per cent of the rated voltage of the transformer. Both ampere-volt and watt-volt curves are then plotted. If extreme accuracy is desired, the RI^2 loss should be substracted to give true core loss.

Separation of Eddy and Hysteresis Losses. In certain special cases it is desired to investigate the iron of a transformer by separating the hysteresis from the eddy-current loss. Let

W_1 = the core loss at normal voltage and frequency.
W_2 = the core loss at half voltage and half frequency.
W_e = eddy-current loss at normal voltage and frequency.
W_h = hysteresis loss at normal voltage and frequency.

Then
$$W_e = 2\,W_1 - 4\,W_2$$
$$W_h = 4\,W_2 - W_1$$

PARALLEL RUN TEST. The test for polarity and ratio having shown nothing wrong in the transformers, they are connected two at a time, the low-tension windings being connected in parallel to a generator and the high-tension windings in parallel with each other with an ammeter of about 10 per cent the capacity of the transformer connected in one lead between them. The voltage of the alternator is gradually increased from zero and the current in the ammeter noted. This current should not be greater than 5 per cent of the rated current of that winding at rated voltage. If a transformer has double windings in either member the same test should be made on these windings in parallel.

HEAT RUN. The heat run is made at the rated load of the transformer and sometimes at an overload of 25 or 50 per cent, depending upon the guarantees. The run may be made by connecting the transformer to a load such as a water rheostat (q.v.), but as there are other equally good methods, avoiding the waste of so much energy, this dead load is seldom used. The other methods require two or three transformers to be tested simultaneously.

Two Transformers "Bucking" (Fig. 21). The low-tension windings of two transformers are connected in multiple to a voltage of rated value and frequency. The high-tension windings are connected up so that their emf's oppose or "buck" each other, and are in series with an adjustable source of emf, of rated frequency, having a value of 2 to 5 per cent of the rated voltage of the windings. This "loss-supply" may be either an alternator and transformer, or a potential regulator. The voltage of this source is adjusted so that full-load current circulates in the high-tension windings and thereby induces full-load current in the low-tension windings. It should be realized that, although only 2 to 5 per cent of the rated voltage is needed to send the current through the primaries, yet each primary is generating its rated voltage and if there should be a ground anywhere a dangerous potential strain or shock might result.

Three-transformer Arrangement (Fig. 22). Three single-phase transformers may

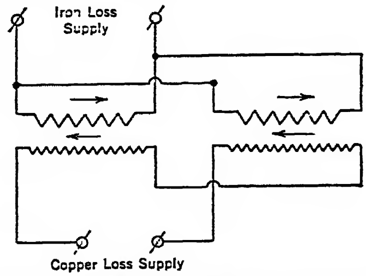

FIG. 21. Connection for Heat Run by Bucking

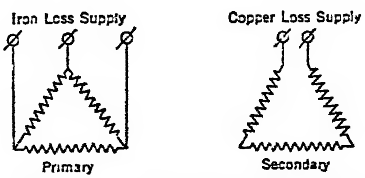

FIG. 22. Heat Run by Open Delta Method

be connected with their low-tension windings in delta to a three-phase source of supply of proper voltage and with the high-tension windings also connected in delta with one corner open into which the "loss supply" is connected preferably by means of an auxiliary transformer to isolate the dangerous potential of the primaries. The "loss-supply" voltage must be adjusted so that the desired current flows in the primary winding.

Time for Heat Run. The time required for a heat run may be considerably shortened by operating at an overload for a short while, or, if the transformer is an air-blast transformer, by operating without the blast until a reasonably high temperature has been attained. Then the proper load is adjusted and the run continued until all temperatures remain practically constant, that is, do not rise more than 1 deg cent in 2 hours. The voltage should be cut off once every hour and a resistance measurement quickly made to determine the temperature of the windings.

Temperature Rise by Resistance is calculated from the formula

$$T = (234.5 + t)\left(\frac{R_h}{R_c} - 1\right)$$

where T = temperature rise in degrees centigrade.

t = temperature in degress centigrade, by thermometer, at which the cold resistance is measured.

R_c = resistance in ohms at temperature t.

R_h = "hot" resistance in ohms at end of heat run.

The number 234.5 is the reciprocal of the resistivity temperature coefficient (referred to 0 deg cent), of copper, i.e., the reciprocal of 0.00427.

Temperature Rise by Thermometer. In an air-blast transformer it is desirable to measure by thermometer the temperature of the incoming air, of the outgoing air (from iron and coils), of the primary windings, and of the secondary windings. Spirit thermometers and not mercury thermometers should always be used for measuring the temperature of transformer windings.

In an oil-cooled transformer the temperature, by the thermometer, of the tank at top and bottom and of the oil in two or three places near the top should be taken.

INSULATION TESTS. To test the insulation between turns and sections of coils double the rated voltage per section is impressed on each section for 1 minute. This test should be made at a high frequency, preferably at double the rated frequency. After this one and a half times normal voltage at rated frequency is applied for 5 minutes to discover the effects of the double voltage test. These voltages should be applied and removed gradually.

High-potential Test on Complete Transformer. To test the dielectric strength of the insulation as a whole the following high-potential test is made. Connect both terminals of the high-tension winding to one terminal of the high-potential transformer. Ground both ends of the secondary winding to the core and frame, and connect to the other terminal of the high-potential transformer. Adjust a needle gap to arc at the desired test voltage and increase the voltage gradually until the gap arcs over. Decrease the voltage till the arcing ceases, and hold the voltage as near as possible to the arcing point for 1 minute. The voltage should then be decreased gradually. The proper testing voltages and spark-gap adjustments are given in the Standardization Rules of the A.I.E.E. (q.v.).

EFFICIENCY AND REGULATION. The efficiency and regulation may be calculated as described above under Equivalent Circuit of Transformer, page 10–06 but it is the more usual practice to determine the efficiency by means of the separate losses:

Let P_2 = secondary output in watts.
A = core loss in watts for that voltage on primary which gives rated voltage on secondary with load.
B = I^2R in both windings as obtained in the impedance test and for the current corresponding to P_2

Then
$$\text{Efficiency} = 1 - \frac{A + B}{P_2 + A + B}$$

9. TESTING OF THREE-PHASE TRANSFORMERS

In testing three-phase transformers the same methods are followed as with single-phase units. The only difference is in testing for polarity; owing to the mixing of the magnetic circuits special care must be exercised. The direct current must be sent in one direction through one primary phase and in the opposite direction through the other two, so that they will not neutralize one another. With a voltmeter similarly connected on the primary and secondary of each phase in turn, break the direct current; if the connections are right the voltmeter on the secondary will deflect in the same direction as the steady deflection on the primary.

PARALLEL RUN. For the parallel run of two three-phase transformers, connect their low-potential sides in multiple to a source of three-phase potential. Connect together the primary terminals No. 1 of both transformers and bring the pairs of terminals No. 2 close together so they may be connected by a small fuse wire. If no spark is noticeable when the fuse wire spans the connection, then the No. 2 terminals may be permanently connected. The same procedure is followed with the No. 3 terminals.

10. EXAMPLES OF PERFORMANCE

Usual values of the performance characteristics of transformers are given below.

Exciting Current ranges from 2 to 6 per cent of full-load current in lighting transformers of the core type and from 5 to 10 per cent in power transformers.

Core loss ranges from 0.5 to 1.25 per cent of the rated output.

Total I^2R ranges from 0.75 per cent of the rated output in large sizes to 2 per cent in small sizes.

Total Reactance drop ranges from 1.25 to 5 per cent of the voltage, being less for the shell type than for the core type. The value depends largely on the purpose of the transformer, methods of construction, and opinion of the designer.

Efficiency and Regulation. The variation of the efficiency with the load of two small 60-cycle transformers for lighting purposes is shown in Fig. 23. The efficiency and regulation at full load of a line of these transformers are shown in Fig. 24. These

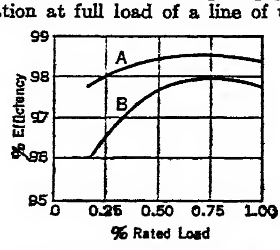

FIG. 23. Efficiency of 60-Cycle Transformers
A = 50 kva; B = 10 kva

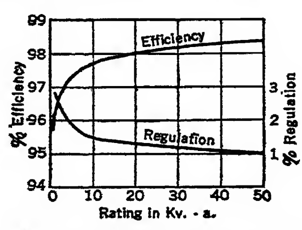

FIG. 24. Efficiency of a Line of 60-Cycle Transformers

transformers are small and of the core type. Larger transformers would have even better characteristics.

11. SPECIFICATIONS FOR TRANSFORMERS *

The following memoranda are intended to assist in writing specifications.

Principal Characteristics and Conditions of Service. Service for which transformer is to be used, e.g., operating synchronous converters, induction motors, lights, etc. High- and low-tension voltages at normal load. Taps for obtaining different voltages. Rating in kilovolt-amperes and in kilowatts at stated power factor. Frequency.

Style and Description: Details of Construction. Whether oil-, air-, or water-cooled. Style and location of terminals. Where line surges are likely to occur, it is usual to specify, for large transformers, that the end turns, say 10 per cent of the total turns at each end, shall have extra heavy insulation.

Work to be Done by Other Contractors. Who is to supply and install floor framing and supports; high- and low-tension wiring, wiring and supports for delta- or star-connections, if to be used three-phase.

Performance and Tests. (See Standardization Rules of the A.I.E.E.) Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load with stated transformation ratio. High-potential tests of insulation. Requirements regarding effect of moisture and heat on insulation, such as the following: The transformers shall contain no material which will be permanently injured by moisture or by an occasional temperature of 95 deg cent, provided that this temperature is not maintained at any one time for a period greater than 3 hours. The transformers shall be capable of operating continuously at 80 deg cent, without the insulation being damaged thereby. Regulation with rated non-inductive load. Regulation with load of rated kilovolt-amperes at stated power factors, say 100 per cent and 90 per cent. State formula by which regulation is to be calculated. Reactance between primary bus and secondary terminals when transformers are to operate compound-wound synchronous converters. (The required reactance is usually specified by the manufacturers of the synchronous converter.) Amount of air or water for cooling, in cubic feet per minute at stated pressure. After the transformer has been in service for 1 year, its efficiency at full load shall be not lower than the above guaranteed amount by more than a stated percentage and after 2 years its efficiency shall be not lower than the guaranteed amount by not more than a stated percentage.

12. INSTALLATION OF TRANSFORMERS

Transformers require no special foundations but merely a level floor of sufficient strength to carry the weight. Provision should be made for an electrical connection from the tank of the transformer to the ground.

OIL- AND WATER-COOLED TRANSFORMERS. The greatest enemy to successful operation of transformers in general, and high-voltage transformers in particular, is moisture, in that 0.1 per cent of moisture in oil renders it unfit for use. This may result either from rain or dripping water falling into the transformer or onto some of the parts, or from the condensation of the moisture in the atmosphere on the various parts. For this reason all parts of a transformer must be very carefully inspected, cleaned, and dried before the transformer is put into service. If the operating potential is 15,000 volts or less an inspection will tell whether the transformer should be dried, but for voltages greater than 15,000 the drying operation should be carried out in every case.

Methods of Drying. The best method of drying is to send a current of dry air at 90 deg cent through and around the transformer. This should continue for 24 hours in all transformers, for 72 hours in high-voltage transformers of reasonable capacity, and longer in special cases. Another method of drying is to short-circuit one winding of the transformer and to apply to the other member a voltage of 1 to 2 per cent of the rated voltage of that winding so that a current of $^1/_5$ to $^1/_3$ of the rated value flows through the winding. This current should be adjusted so that a spirit thermometer placed on the low-tension coils shows a temperature of 80 deg cent and not greater.

Preparation of Oil. No potential should be applied to any oil-cooled transformer unless it is supplied with a proper amount of oil as the oil forms the essential insulating medium as well as cooling medium. The oil should be tested before using. Oil is considered in good condition when it will withstand 22,000 volts between disks 1 in. in diameter and 0.1 in. apart. Transformers for 40,000 volts or less will operate satisfactorily

* By W. A. Del Mar.

when this dielectric strength has dropped to 16,500 volts, but when this condition has been reached the oil should be dried, purified, and strained. Transformers for a voltage greater than 40,000 should not be used with oil which breaks down under 20,000 volts in the above-described test apparatus. A special apparatus for drying and purifying the oil is on the market (see *Gen. Elec. Bulletin* 4134).

The transformer tanks should be filled by pouring the oil through a fine cloth strainer and allowing the oil to settle 12 hours before using. Rubber tubing should not be used for carrying the oil as the sulfur in the rubber will eventually cause trouble. The cover of the transformer should prevent any water dripping into the transformer but there should be a free exit allowed for gases which may gather. If the transformer stands in a moist atmosphere a special "breather" containing calcium chloride should be employed. In all transformers the level of the oil should be well above the top of the transformer proper and this should be noted 2 or 3 days after the original filling to see whether the transformer windings have absorbed sufficient of the oil to lower the level a dangerous amount.

As the oil used for insulation in transformers undergoes a slow chemical deterioration in the presence of oxygen many oil-cooled transformers are sealed tight or closed with a special device which prevents the entrance of fresh air as the tank "breathes" owing to the expansion and contraction of the oil with change of temperature. Some transformers have the space above the oil filled with an inert gas such as nitrogen. In all events there should be some device to prevent the entrance of moisture.

Certain small sizes of oil-cooled transformers are designed to be suspended from cross-arms on poles or the sides of buildings or to be installed in manholes in a subway. These transformers are especially protected against the weather.

Precaution Against Overheating. It is most important to be assured of the proper circulation of the cooling medium as the temperature of the transformer will rise quickly to an excessive value in case of a failure of the cooling medium. In this case the load must be reduced and kept at such a value that the temperature of the oil at the top does not exceed 80 deg cent. If at any time the oil reaches an excessive temperature there will be a tendency for a deposit to form on the transformer and coils, which will interfere with the proper cooling. An inspection should be made occasionally for this purpose and the deposit removed. The oil should be sampled and tested each week for the first month and every 6 months thereafter. In taking samples of the oil great care should be exercised that the vessel in which the sample is contained is perfectly dried. The temperature of the oil in a self-cooled transformer should never exceed 80 deg cent, and in a water-cooled transformer 65 deg cent. Oil-cooled transformers must be in a well-ventilated compartment in which the air should not be more than 5 deg above the outside atmosphere. An inspection should be made from time to time to see that there is no condensation on the inside walls of the tank.

Precaution for Multiple Operation. Transformers should never be connected in multiple on both the primary and secondary sides unless it is known that the polarity is correct and that the ratio and regulation of the two transformers are the same.

AIR-BLAST TRANSFORMERS; DUCTS AND BLOWER SET. Air-blast transformers should be placed over an air duct of sufficient size to permit the required current of air to flow at a velocity of less than 500 ft per min. The duct should be of non-combustible material and should have smooth sides in order to offer very little resistance to the current of air. Air is supplied to this duct by means of motor-driven blowers of capacity to supply the proper quantity of air for a group of transformers at pressures from $1/2$ to $1\frac{1}{2}$ oz. Roughly the rating of the blower motor in horsepower is equal to

$$\frac{\text{(Cubic feet of air per minute)} \times \text{(pressure in ounces)}}{1200}$$

The air enters the transformer at the bottom, flows vertically through the coils, and passes out at the top, and also flows transversely through the iron, passing out at the side. Separate dampers at each outlet are provided in order to regulate the two currents independently. The dampers are regulated so that the outgoing air has a temperature 20 deg above the incoming air.

Since the transformers are so dependent on the air blast for their operation and safety, it is customary to provide a reserve blower set.

Location. Care must be exercised to protect the transformers from moisture and dirt and particularly, since they are open at the top, to place them where water and dirt cannot drop in the open top. All terminals of the air-blast type of transformer are usually brought out below so that the primary and secondary cables may be laid in the air duct and all connections and inspection may be made from below. It is therefore desirable that the air duct be sufficiently large to enable a man to move about therein.

Drying and Cleaning. In putting the transformer in operation all moisture must be removed before applying potential. This may be accomplished by running the blower set and forcing air through the transformers, which will be sufficient if the air is dry. Another method is to short-circuit one winding of the transformer and apply a low voltage of from 1 to 2 per cent of the rated voltage to the other winding, so that 75 per cent of full-load current flows through the windings. This is maintained for 24 to 36 hours. All transformers of this type should be cleaned once a month by means of compressed air at 20 lb pressure.

Measurement of Temperature. In determining the temperature of the windings only that calculated from a resistance measurement is dependable, as the coils are so thickly wrapped in insulation that a thermometer will not show the true temperature.

13. OPERATION OF TRANSFORMERS

Single-phase transformers may be used singly or in parallel on single-phase circuits, and in various combinations on polyphase circuits as described above. The polarity should be determined before making the connections.

PARALLEL OPERATION ON SINGLE-PHASE CIRCUITS. Single-phase transformers are very generally operated with the primaries in parallel on the supply circuit and their secondaries in parallel on the load circuit. In order successfully to operate in this manner two or more transformers, the transformers must have: (1) the same ratio of transformation; (2) the same regulation; and (3) the same value of X/R, where R and X are the total resistance and reactance respectively of any one of the transformers. Transformers divide the load inversely proportional to their reactances, provided the ratio of reactance to resistance is the same in both cases. If transformers are purchased to operate in parallel with others, proper provision must be made in the design of the transformers, and this fact should be stated in the specifications.

In general, a 50-kw transformer will have about twice the resistance and twice the reactance of a 100-kw transformer of the same commercial line, and therefore these transformers will divide the load in proportion to their capacity. But this is not likely to be true of transformers of different lines or made at widely different times.

If transformers not satisfying the above conditions are to be run in parallel, an impedance coil having a proper resistance and reactance may be connected in the circuit of one of them, thus establishing the necessary conditions for the proper distribution of load, provided the ratios of transformation are the same.

TRANSFORMERS ON POLYPHASE CIRCUITS. See Art. 3 on Transformer Connections.

14. AUTO-TRANSFORMERS AND COMPENSATORS

An auto-transformer or single-circuit transformer, also called a "compensator," consists of a transformer having the usual iron core but only one electrical circuit instead of two. This circuit is tapped at various points as shown in Fig. 25, and the primary and secondary circuits, while independent outside the transformer, unite in the same winding in the transformer. If an alternating voltage is impressed across the points ab, a magnetizing current will flow in the winding setting up an alternating flux which will link every turn and induce therein an alternating voltage. The voltage between any two taps, as ac, is proportional to the number of turns between the taps; thus any ratio of voltages may be obtained. If the secondary ac is loaded a current will flow in the primary and the primary and secondary currents will flow in the two parts of the winding as indicated in the figure.

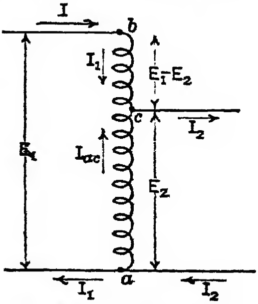

FIG. 25. Auto-transformer

VOLTAGE AND CURRENT RELATIONS. Let N_1 be the total number of turns between a and b, N_2 the turns between a and c, E_1 the voltage across the terminals a and b, and E_2 the voltage across the terminals a and c. Then, neglecting the resistance and reactance of the winding,

$$E_2 = \frac{N_2}{N_1} E_1$$

Let I_1 be the current entering the terminal b, I_2 the current in the *external* circuit connecting a and c, and I_{ac} the current *in the transformer winding* between a and c. Then, neglecting the resistance and reactance of the winding and the magnetizing current,

$$I_2 = \frac{N_1}{N_2} I_1$$

$$I_{ac} = I_2 - I_1 = \frac{N_1 - N_2}{N_2} I_1$$

The current in the turns between b and c is

$$I_{bc} = I_1$$

Auto-transformer Versus the Two-circuit Transformer. For $N_2 = {}^1/_2 N_1$ the current in the turns between a and c would be just equal, neglecting the exciting current, to the current in the turns from b to c (but opposite in direction). Consequently for a 2 to 1 transformation but one winding of an ordinary two-circuit transformer could be used, provided a tap was available at the middle point of this winding, and the rated output of the transformer could be obtained without the current in this winding exceeding its rated value. Since under these conditions there would be no current in the second winding, the heating of the transformer would be less than it would be were the transformer used as an ordinary two-circuit transformer, and therefore a greater output could be obtained without exceeding the nominal temperature rise.

In general, for the same power input into the connected load an auto-transformer requires but $\frac{E_1 - E_2}{E_1}$ of the copper required for a two-circuit transformer, where E_1 is the high-tension and E_2 the low-tension voltage. The higher the ratio of transformation the less the saving in copper. There is also a serious objection to an auto-transformer of high ratio of transformation, in that an accidental ground on the high-tension lead, b in Fig. 25, would establish a high voltage between the low-tension leads and the ground, which may cause a dangerous shock to a person touching either low-tension lead or may cause other damage. This may be partially prevented by grounding the common point of the high- and low-tension circuits, the point a in Fig. 25, in which case an accidental ground on the high-tension side would produce a short circuit, which would open the circuit breaker in the primary circuit, *provided the resistance between the two grounds is low and the circuit breaker operates properly.* These two provisions, however, may not always be realized even though reasonable care is taken, and it is therefore not considered good practice to use an auto-transformer of high ratio of transformation, except in special cases where economy of space is an important factor and danger from shock can be guarded against, e.g., on a-c locomotives.

RATING OF AN AUTO-TRANSFORMER. In commercial practice the rating of an auto-transformer in volt-amperes is taken equal to the difference between the high- and low-tension voltages multiplied by the rated current in that part of the winding, bc in Fig. 25, across which this voltage exists. The capacity of an auto-transformer for a given work bears to the capacity of a two-circuit transformer for the same purpose the ratio $\frac{E_1 - E_2}{E_1}$. For example, to step down the voltage of the supply mains from 500 to 400 volts and supply a load of 100 kva would require an auto-transformer having a rating of $\frac{500 - 400}{500} \times 100 = 20$ kva as against a two-circuit transformer having a rating of 100 kva. The weight, dimensions and cost of an auto-transformer are very nearly the same as for an ordinary two-circuit transformer having the same voltage and kva rating.

APPLICATIONS. Auto-transformers are used chiefly where the required change in voltage is small, e.g., for motor starters and for balancing the voltage between two or more circuits. The smaller the ratio of transformation the greater is the gain in cost and efficiency resulting from the use of an auto-transformer instead of a two-circuit transformer. Auto-transformers are also used to provide a neutral for the Edison three-wire system; see Distribution Circuits. In single-phase railway work single-phase auto-transformers are used on the locomotives for transforming the trolley voltage (from 3000 to 6000 volts) to the motor voltage (about 500). This is a rather high ratio of transformation for an auto-transformer, but as the saving in weight is very important and as one terminal is grounded, it has proved satisfactory for this service; see Electric Locomotives.

Auto-transformers are frequently constructed three-phase (and occasionally two-phase). These are generally used for starting polyphase motors by providing a lower voltage for the starting period. They are also occasionally used in high-tension distri-

bution. If the voltage applied to an a-c motor is reduced to one-half, the motor starting current will be one-half and the volt-amperes one-quarter. Thus the starting current in the line will be one-quarter. A three-phase auto-transformer may be either Y- or delta-connected. (See Section 3, Art. 35). The delta connection cannot be made to give a balanced voltage less than half that of the higher voltage, as may be readily seen from an inspection of Fig. 26.

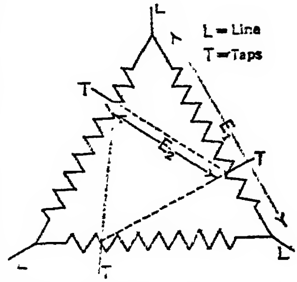

FIG. 26. Delta-connected Auto-transformer

DESIGN. The design of an auto-transformer is quite similar to that of a two-circuit transformer (q.v.), and the leakage reactance is calculated in the same way. For low-leakage reactance the primary and secondary coils must not be entirely separated, but each must be divided into sections and intermixed to as great an extent as possible. Having found the magnetizing current, core loss, resistance and reactance in the usual manner, the calculations follow the same methods as for power transformers. In general, the efficiency at unity power factor is given by the equation

$$\text{Efficiency} = \frac{E_2 I_2}{E_2 I_2 + (r_1 - r_2) I_1^2 + r_2 (I_2 - I_1)^2 + \text{Core loss}}$$

where r_1 is the total resistance of the winding measured between the high-tension terminals and r_2 the resistance measured between the low-tension terminals. The efficiency is better than that of a two-circuit transformer of the same kva rating.

15. EXAMPLES OF TRANSFORMER DESIGN

Column	1	2	3	4	5
Form of cooling	Oil	Oil	Oil	Oil	Air
Type of core	Distrib.	Core	Distrib.	Distrib.	Shell
Frequency	60	60	60	60	25
Rating, kva	5	10	20	50	75
High-tension voltage	2200	2200	2200	6600	2500
High-tension amperes	2.3	4.58	9.16	7.6	30
Low-tension voltage	220	220	220	230	320
Low-tension amperes	23	45.8	91	219	234
Design constant, C	205	125	222	98	1550
Volts per turn, e	1.67	1.83	3.45	3.6	11.4
High-tension winding					
Total series turns	1320	1200	640	1836	219
Coils in series	2	2	2	2	3
Coils in multiple	1	1	1	1	1
Conductor size	No. 16	No. 14	(0.105×0.09)	(0.085×0.085)	(0.34×0.08)
Resistance 25	11	6.5	1.43	5.85	.49
Low-tension winding					
Total series turns	132	120	64	64	28
Coils in series	2	2	4	4	4
Coils in multiple	1	1	1	2	1
Conductor size	No. 5	No. 4	2(0.23×0.155)	4(0.415×0.113)	6(0.34×0.09)
Resistance 25	0.08	0.047	0.0204	0.0053	0.0095
Flux density, kilo-maxwells	58	85	63	57.2	67.5
Core length, inches	4	3	4.6	5.125	8
width	3	3	4.6	5.125	21
Window height	6	6.75	9.2	15.8	5.5
width	2	4.25	3.1	6	8
Magnetizing current	0.03	0.52	0.43	0.125	1.82
Core loss, watts	40	133	164	900	1520
No-load current, %	1.3	12	4.8	1.6	6.9
Core loss, %	0.8	1.3	0.8	1.7	1.96
H.T. copper loss, %	1.4	1.3	0.59	0.7	0.57
L.T. copper loss, %	1.0	0.95	0.81	0.5	0.67
Efficiency at rating	96.8	96.5	97.8	97.1	96.8
Total IX drop, %	2	3.1	2.9	4	1.36
Average amperes per square inch, U	1000	1400	1100	1100	1200

Columns 1, 3, and 4 are for the typical distribution transformers with distributed core for use in house-to-house distribution.

Column 2 is a simple core-type transformer.

Column 5 is for a transformer, one of three, for supplying a synchronous converter for 600 volts direct current for railway work.

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SECTION 11

CONVERTERS AND RECTIFIERS

BY

WALTER I. SLICHTER

ENERGY CONVERTERS

ART.		PAGE
1.	Motor-generators	02
2.	Dynamotors	02
3.	Synchronous or Rotary Converters	03
4.	Principles of Converter Action	04
5.	Design of Synchronous Converters	08
6.	Tests and Operation of Converters	10
7.	Motor-Converters	12

FREQUENCY CONVERTERS

8.	Induction-synchronous Type	13
9.	Synchronous-synchronous Type	14
10.	Induction Type	16

PHASE CONVERTERS AND BALANCERS

ART.		PAGE
11.	Phase Converter	17
12.	Phase Balancer	17
13.	Phase Shifter	18

RECTIFIERS

14.	Mercury-arc Rectifier	18
15.	Rectifier with Grid Control	21
16.	Hot-cathode Mercury Rectifiers	21
17.	Three-electrode Hot-cathode Mercury Rectifiers	21
18.	Tungar Rectifier	21

CONVERTERS AND RECTIFIERS

By Walter I. Slichter

ENERGY CONVERTERS

1. MOTOR-GENERATORS

A motor-generator set is a combination of a motor and a generator having separate fields and armatures but mounted on the same shaft with common base and bearings. Combinations of various types of motors and generators are used; some of the more important combinations and their applications are described below.

Direct-current Motor Driving Direct-current Generator. These sets are used when it is desired to convert low-voltage direct current into high-voltage direct current, or vice versa; they are used in preference to a dynamotor (q.v.) when good regulation is desired in the secondary circuit.

Direct-current Motor Driving Alternating-current Generator. These sets are used for converting direct into alternating current. See also Art. 3 on Synchronous Converters.

Induction Motor Driving Direct-current Generator. The induction motor may be wound for potentials as high as 13,000 volts, and the transformation from alternating current at this voltage to direct current may be made without the use of transformers. Since the induction motor has a decreasing speed with increasing load the d-c generator must be compounded to give good regulation; with proper compounding excellent regulation may be obtained. The induction-motor-generator set is sometimes used in preference to a synchronous converter when the service requires specially good regulation. However, the efficiency of such a set (about 85 per cent at rated load) is less than that of a synchronous converter, even if no transformers are required by the motor-generator set. The motor-generator set also occupies from 50 to 80 per cent more floor space, weighs from 30 to 50 per cent more, and costs from 25 to 50 per cent more than a synchronous converter, in spite of the fact that they are designed to operate at the highest practicable speeds.

Synchronous Motor Driving Direct-current Generator. This combination is preferable in many instances to the preceding, because it operates at constant speed and because the field of the synchronous motor may be adjusted to make use of line compounding or to compensate for low power factor in other apparatus on the circuit. Provision must be made for the d-c excitation for the synchronous motor. If the d-c generator is wound for too high a potential a special exciter must be provided. Since there is no load on the set at starting the synchronous motor may be started in the usual manner.

2. DYNAMOTORS

A dynamotor is a d-c device combining both motor and generator action in one magnetic field. It has an armature having two separate windings and two separate commutators, one at each end of the armature. Either winding may be used as the motor winding, and the other as the generator winding. Such a machine performs the same function in a d-c circuit that a power transformer does in an a-c circuit, i.e., serves as a means of transforming high-voltage direct current into low-voltage direct current, or vice versa.

PERFORMANCE CHARACTERISTICS. The device corresponds to two machines in which there is only one core loss, one friction loss, one excitation loss, but two losses due to rI^2 in the two armature circuits. It is therefore more efficient than a motor-generator set, but less efficient than one machine. Since the currents in the two windings flow in opposite directions their resultant magnetic effect is zero. The machine has therefore no armature reaction (except for the slight amount due to the current to overcome the losses in the machine). It is not subject to the troubles of field distortion and bad commutation that occur in either motors or generators. It is impossible to compound a dynamotor, since any increase in field strength intended to increase the voltage of the generator would decrease the speed of the motor by the same amount and no change would result. The

ratio of the two voltages is therefore fixed by the number of turns, and only varies from this by the loss due to rI drop in both windings. These two drops are additive, and therefore the regulation of such a machine is not very good.

APPLICATIONS. Dynamotors are used largely to give large currents to start other motors, or to give low voltages or a fractional voltage in a multivoltage system for speed control. The motor of the combination may be wound for the line voltage and the generator for any fraction of the line voltage. Thus the combination supplies a large current at a low voltage, which will give a good starting torque in motors connected to it with a reasonable consumption of power. They are used as equalizers or " balancers " in three or multiwire circuits, but are not as desirable for this work as motor-generator sets with compound-wound machines, on account of their poor regulation. They are also used to supply a low voltage for such purposes as telephone and telegraph systems and the low voltage and large currents for electrolytic work.

3. SYNCHRONOUS OR ROTARY CONVERTERS

Since in general it is more economical to transmit electrical energy in the form of alternating currents and in many cases more convenient to utilize it in the form of direct currents, some means of converting from one form of electrical energy to the other is desirable. For this purpose synchronous converters and motor-generator sets (q.v.) are available. Synchronous converters are also called " rotary converters."

A synchronous converter is a machine very similar to a d-c generator in which certain commutator segments, or the conductors connected to them, are connected to 2, 3, 4 or 6 collector rings, as the case may be. When the movable member is caused to rotate, the voltage between any two collector rings is alternating. Such a machine, when driven by an engine or motor, may be operated as an a-c generator or as a " double-current " generator giving alternating current from its collector rings and direct current from its commutator. If the collector rings are connected to a source of alternating currents the machine will run as a synchronous motor and direct current may be obtained from the brushes on the commutator; i.e., the machine, with but one set of windings, acts simultaneously as an a-c motor and a d-c generator. It has therefore the friction, core loss, and excitation loss of one machine instead of two, and since the motor and generator currents flow in the same winding and during at least the major part of each cycle are in opposite directions, they more or less balance each other and the armature rI^2 loss is much less than in either a motor or generator alone.

Synchronous Converter vs. Motor Generator. A converter is much more efficient and weighs and costs less than a motor-generator set of the same capacity. It also occupies less space. However, since only one winding is used, there is a definite relation between the emf's of the a-c and d-c terminals. The maximum value of the alternating wave bears a definite relation to the direct emf (see below). It is therefore necessary to supply the converter with a voltage of the same order as the direct voltage, and this involves the use of transformers, if a high-voltage transmission line is used to supply the converter. Motors operating at voltages as high as 13,000 volts can be used in motor-generator sets.

Relative Efficiencies. The efficiency of a converter is in the neighborhood of 93 per cent and of the transformers 97 per cent, thus the efficiency of the combination is about 90 per cent. The efficiency of a synchronous motor is in the neighborhood of 93 per cent and of a d-c generator 92 per cent, thus the combination motor-generator set has an efficiency of 85.5 per cent. If the supply voltage is greater than 13,000 volts, transformers will also be needed for the motor-generator set and the net efficiency would then be 83 per cent.

Synchronous Converter vs. Rectifiers. A converter differs from a rectifier (q.v.), since the former gives a direct emf of constant and uniform value and the latter gives a pulsating unidirectional voltage and current. In the former the energy is stored in the form of magnetism for an instant whereas in the latter there is no magnetic field and no storage of energy. A rectifier will not work satisfactorily on an inductive d-c circuit, but a converter will.

APPLICATION OF CONVERTERS. The most common application of synchronous converters is in electric railway work. The great majority of motors for electric traction are d-c series type, operating at 500 to 600 volts. The energy for these motors must be transmitted over long distances, which requires a high-voltage a-c transmission line and converters to link the d-c distribution with the a-c transmission. Converters have also been developed for high-voltage d-c traction systems, these converters giving 1500 volts direct current. Two such converters may be connected in series for 3000-volt d-c distribution (*Elect. J.*, 12, p. 154, Apr. 1915). Synchronous converters are also used for lighting and power service and for electrolytic work.

TERMINOLOGY. The following terms are used to describe certain characteristic features of the various kinds of synchronous converters.

Phases and Rings. A single-phase converter has two collector rings, and each ring is connected to the windings by as many equally spaced taps as there are pairs of poles. The taps for the two rings alternate at equal spaces. A single-phase converter is therefore a two-ring converter.

A three-phase converter has three rings and three equally spaced taps (one for each ring) for every pair of poles. A four-phase or quarter-phase converter has four rings and four taps for every pair of poles. A six-phase converter has six rings and six taps per pair of poles.

Shunt and Compound-wound Converters. A converter may be shunt or compound wound, depending upon the service for which it is intended. The series winding is intended to make the converter take leading current when the load increases and thus automatically increase the voltage at the a-c terminals, but the ratio of the a-c *terminal* voltage to the d-c voltage remains unaltered.

Converter with Series A-c Booster. A synchronous converter with series booster is frequently used when exact hand regulation of the d-c voltage is required, as for lighting circuits. It is merely a converter with a separately excited a-c generator on the same shaft, and each phase of the armature of this booster is in series with one phase of the converter and the line. This generator acts as a booster and raises the voltage impressed upon the armature of the converter.

Inverted Converter. Sometimes a converter is operated to convert from direct to alternating current. It is then called an "inverted converter." The machine will operate satisfactorily in this manner, but its speed depends upon the nature of the a-c load. An inductive load in the a-c circuit causes the armature to demagnetize the fields, with a resultant increase in speed. It is therefore dangerous to operate an inverted converter on an inductive load unless it is provided with a speed-limit device. This effect does not occur when the machine is operating as an a-c motor, since its speed is fixed by the frequency of the supply circuit.

Cascade Converter or Motor Converter. This is a combination of an induction motor and converter connected in series or concatenation (see Motor-converters p. 11–12). The converter receives half the power in mechanical form from the shaft and half the power inductively, in the form of alternating current at half frequency, from the secondary of the induction motor. By this means the steadiness of a 30-cycle converter is obtained in a 60-cycle unit.

Split-pole Converter. The "split-pole" or "regulating-pole" converter is designed to give a variable ratio of alternating emf to direct emf for operation in parallel with storage batteries and similar purposes. The field poles are divided into sections the excitation of which may be controlled independently. The effect of the split pole is primarily to produce a third harmonic in the flux distribution curve, which increases or decreases the *total* flux per pole and therefore the d-c voltage, but does not change the a-c counter emf between slip rings, since these rings are connected to the armature winding at points 120 electrical degrees apart. Split pole converters give a voltage variation of about 20 per cent.

Commutating-pole Type. In most large converters, 500 kw and above, particularly for 60 cycles and high voltage, 600 volts, it is customary to provide commutating poles to improve commutation and reduce the cost by making it possible to run with a higher value of "volts per bar." See D-c Generators.

RATING AND PERFORMANCE. The standards of the A.I.E.E. specify a maximum rise of 50 deg cent on any part of the windings and 60 deg on the commutator for continuous operation. Twenty-five-cycle converters will give momentarily an output of three times their normal rating without damage or injurious sparking at the commutator, but 60-cycle converters are more sensitive to overloads.

4. PRINCIPLES OF CONVERTER ACTION

In this section are briefly treated those features of a synchronous converter, in which the converter differs in action from an a-c or d-c generator; see table on page 11–07 for a summary of the voltage, current, and capacity relations.

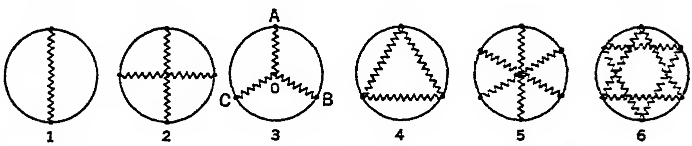
CONNECTIONS AND VOLTAGE RATIOS. The ratio of voltage on the a-c side to that on the d-c side depends upon the number of rings and type of connection employed.

Two-ring Converter. The two collector rings are connected by taps to the same winding as the commutator; hence the alternating emf has the same values as the direct emf at the instant that the taps pass the brushes. As this is also the maximum value of the

alternating emf, the effective value (i.e., the value to be indicated by a voltmeter) will be 0.707 times the maximum, or 0.707 times the direct emf. Fig. 1 shows in a simple manner the connection of a single-phase converter. The external circle represents a two-pole armature winding and inside is shown the supply transformer connected to two taps diametrically opposite each other. The voltage across this transformer would be 0.707 E, where E is the voltage between the positive and negative brushes on the d-c side.

Four-ring Converter. If two additional collector rings are connected to conductors spaced half way between the former taps, there results the quarter-phase converter shown in Fig. 2. The voltage across each supply circuit or transformer is the same as before, but the two voltages will differ in phase by 90 deg.

Three-ring Converter. If three collector rings are connected to taps spaced 120 deg apart the emf between adjacent collector rings will be the vector sum of AO and OB in Fig. 3. Since AO and OB each equal $0.5 \times 0.707\ E$ the voltage AB will be $\sqrt{3} \times 0.5$



FIGS. 1–6. Transformer Connections and Vector Relation of Voltages in Synchronous Converter

$\times\ 0.707\ E = 0.612\ E$. Thus in a three-ring or three-phase converter the voltage between adjacent taps is 0.612 times the direct voltage, and the connections of transformer are as in Figs. 3 and 4.

Six-ring Converter, Diametrical and Double Delta Connections. A six-ring converter may be connected diametrical as in Fig. 5 in which case the voltage of each transformer will be $0.707 \times E$ and the result will be like the combination of three single-phase groups. A six-ring or six-phase converter may also be connected " double delta " as shown in Fig. 6, which is similar to the combination of two groups of three-phase delta transformers. In both cases the voltage between adjacent taps of a six-phase converter is 0.355 E.

***n*-Ring Converter.** In general the voltage between taps of any converter having n equally spaced taps per pair of poles is

$$E_{ac} = \frac{E \sin \frac{\pi}{n}}{\sqrt{2}}$$

CURRENT RATIOS. To determine the ratio of the continuous current I to the alternating current I_3 per collector ring in a three-phase converter, for example, assume the d-c output equal to the a-c input with unity power factor; then

$$\sqrt{3}\ E_3 I_3 = EI$$

and, from preceding paragraph, $E_3 = 0.612\ E$

where E_3 = a-c voltage between lines; I_3 = alternating current per line; E = direct voltage; I = direct current. From these two relations

$$I_3 = 0.94\ I$$

The ratios of currents for other converters are obtained similarly, and are given in the table on page 11–07.

In actual practice the current on the input side must be greater than that given by these relations, in order to supply the losses in the converter, and the alternating current will also vary inversely as the power factor, which is taken as unity in the table.

RESULTANT COIL-CURRENT. In any machine acting simultaneously as a motor and a generator the two currents must flow in opposite directions, and the current in any particular conductor will be the difference between the two. In any particular coil the direct current is constant in amount and direction from the instant the commutator segment connected to this coil passes the positive brush to the instant it passes the negative brush, and conversely from negative to positive brush. In any coil midway between the a-c taps the alternating current is a maximum when this coil is half way between brushes (for unity power factor), and the current falls to zero as the coil reaches the interpolar position. Therefore, for the period of time that the direct current in a coil remains constant in amount and direction there is also in it a variable current changing from zero to a maximum and back to zero again. The net or resultant current will therefore have a wave shape

and frequency somewhat as shown at R in Fig. 7. The heating in this particular coil will therefore be proportional to the product of the resistance of the coil by the square of this current, and it is readily seen that the power lost is less than that due to either the direct or alternating current alone.

A coil situated very near one of the a-c taps will carry a direct current subject to the same law as before mentioned, but the alternating current in this coil is the same as that in the middle coil and has its maximum value when the middle coil is at the middle of the poles and not when this particular end coil is at the middle of the pole. The alternating

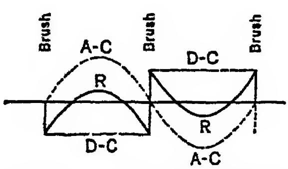


Fig. 7. Current in Coil Midway between Taps

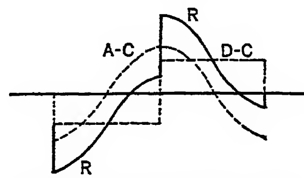
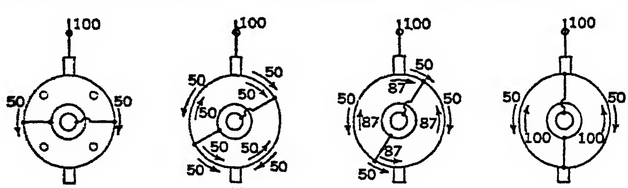


Fig. 8. Current in Coil at Tap in Two-ring Converter

current in this coil may therefore reach its maximum value soon after the coil has passed a brush, as shown in Fig. 8, and the resultant of the two currents is greater than that in the middle coil. The farther a coil is situated from the middle coil of a group, the greater is the phase displacement of its alternating current, the greater is the value of the resultant current, and the greater is the heating effect. Thus although the heating effect in a rotary converter winding is less than that of a d-c machine having the same output it is different in each and every coil, is minimum in the central coil (when the power factor is unity), and is maximum at the coil nearest the tap, and the greater the angle between taps the greater is the total heating effect.

Distribution of Heat Losses in Armature Winding. In Figs. 9 to 12 inclusive, representing a two-ring converter, the numbers outside the circles represent the direct current in the winding (corresponding to 100 amp in the external d-c circuit and unit power factor on a-c side) and the numbers inside the circles the instantaneous alternating current in the winding. It will be seen that the resultant current in a coil midway between taps, Figs. 9 and 12, never exceeds 50 amp. A coil 30 deg from the middle, Fig. 10, has a maximum



Figs. 9–12. Component Currents in a Two-ring Converter

current of 100 amp. A coil 60 deg from the middle, Fig. 11, has a maximum current of $50 + 87 = 137$ amp, and a coil 90 deg from the middle (tap coil) may have a current of $100 + 50 = 150$ amp. The heating of the armature winding is therefore not uniformly distributed, though the conduction of heat from one part of the winding to another tends to equalize the temperature rise.

If the converter operates at a power factor different from unity, the position of minimum resultant current is no longer at the middle coil, but is moved one way with leading current and the opposite way with lagging current. Thus one end coil has improved heating conditions, and the middle coil and the other end coil have much worse heating conditions, the result being that the heating as a whole has increased and is more non-uniformly distributed than with unity power factor.

The power lost in each individual coil of a converter and the effect of the power factor on this loss is very well shown in Figs. 13 and 14, taken from a paper by J. E. Woodbridge (*A.I.E.E.*, 1908). In Fig. 13 the curved line shows the relative rI^2 loss in a coil having any position throughout 120 deg of one phase of a three-phase converter when the power factor is unity. The curve shows the ratio of the loss compared to the loss in the same machine acting as a d-c generator of the same capacity. It will be noted that the middle coil has a loss of 22 per cent of that of the generator and the end coils 120 per cent, and

that the average loss is 57 per cent. For a power factor of 0.966, representing a phase displacement of current of 15 deg, the loss in individual coils ranges from a maximum value of 180 per cent at one tap to a minimum value of 22 per cent in the coil shifted 15 deg to one side of the middle coil. The other end coil has a loss of a little over 80 per cent of the generator loss. The average value has been increased to 65 per cent.

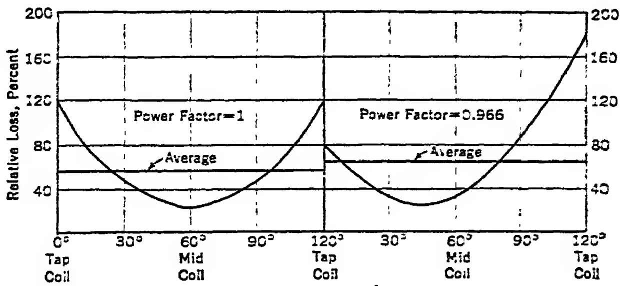


FIG. 13. Relative rI^2 in Individual Coils of Three-ring Converter

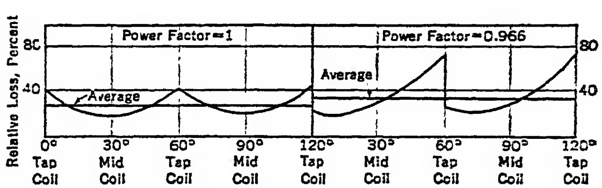


FIG. 14. Relative rI^2 in Individual Coils of Six-ring Converter

In Fig. 14 the same ratios are shown for a six-phase converter, in which the winding of one phase is distributed over only 60 deg. Consequently the conditions are better and the maximum loss due to low power factor is less.

DEPENDENCE OF OUTPUT UPON NUMBER OF PHASES. As a result of these conditions we have the following relations of the capacitance of a given armature with various numbers and connections of taps, the capacitance being based on an equal total amount of rI^2 loss.

It should be remembered that there are other losses besides coil losses in the converter, and that therefore practical figures are slightly different from those given in Table 1.

FIELD EXCITATION. The variation of the field excitation of a synchronous converter has much the same effect as in the case of a synchronous motor (see Synchronous Motors); that is, if its field is underexcited the armature will draw a lagging current which assists the field and sets up the necessary flux, whereas if the field is overexcited the armature will draw a leading current which opposes the field and reduces the flux. By this means the converter may be made to take either a leading or lagging current. A leading current flowing over a line having inductance tends to raise the voltage at the receiving end of the line.

Table 1.—Voltage, Current, and Output Ratios

	D-c Generator	Converters					
		2-ring	3-ring	4-ring	6-ring diametrical	6-ring double delta	12-ring
D-c volts	100	100	100	100	100	100	100
A-c volts between lines	...	71	61.2	71	71	61.2	71
A-c volts between rings	...	71	61.2	50	35	35	18
D-c amperes	100	100	100	100	100	100	100
A-c amperes in line	...	141	94	71	47	47	24
A-c amperes in winding	...	71	55	50	47	47	45
Relative RI^2 loss	100	137	55	37	26	26	20
Relative output:							
Unity power factor	100	85	134	165	197	197	224
87 per cent power factor	. .	...	99	115	129	129	135

Line Compounding. If there is sufficient leading current and sufficient inductance in the line, the voltage at the receiving end may be greater than at the sending end in spite of the resistance of the line. This is sometimes called "line compounding." The relation between the voltages at the two ends of the line may be expressed as follows:

Let E = voltage to neutral at sending end.
V = voltage to neutral at receiving end.
I_1 = component of line current in phase with V, i.e., the power component of the line current.
I_2 = component of line current at 90 deg to V, i.e., the *leading* reactive component of the line current.
r = resistance of one line.*
x = inductive reactance of one line.*

Then, noting that I_2 is to be taken positive when leading,

$$E^2 = (V + rI_1 - xI_2)^2 + (xI_1 + rI_2)^2$$

Use of Series Field. In practice a series field is added to the converter and the shunt excitation is adjusted so that the armature current is lagging at no load. As the load increases the series field increases, the adjustment being such that at about $^3/_4$ load the proper excitation for unity power factor is given. Hence at all loads over $^3/_4$ the field will be overexcited, the current will be leading, and the voltage will be raised or compounded.

5. DESIGN OF SYNCHRONOUS CONVERTERS

DESIGN. The design of a synchronous converter is very similar to the design of a d-c generator (q.v.), except for certain special conditions due to the fact that the frequency is fixed by the frequency of the supply system, and that greater latitude is allowed in the choice of the nominal value of the d-c armature reaction and copper density, because the real value of these quantities is the difference between their nominal d-c values and their a-c values.

SPEED AND NUMBER OF POLES. The revolutions per minute and the number of poles are definitely related to the frequency of the supply circuit, in cycles per second, in accordance with the formula:

$$120 \times (\text{Frequency}) = (\text{Number of poles}) \times (\text{rpm})$$

The choice of the number of poles usually depends upon the commutator.

COMMUTATOR. The design of the commutator is usually the limiting feature and therefore the first to be considered. This is particularly true of either high-frequency (60-cycle) machines or of those in which the d-c voltage is 600 or over. Three factors must be considered in the design to secure successful commutation and life of the commutator, namely, peripheral speed, voltage between bars, thickness of bars. If the peripheral speed or the voltage between bars is too high commutation will be bad. If the commutator bars are too thin the commutator will not retain its shape and commutation will be bad. These three limiting factors are very closely related and in a high-voltage machine give very little choice. The diameter of the commutator depends upon the voltage and frequency, and its minimum value is given by the three following relations. Let

s = pitch of commutator bars in inches. This ranges from 0.15 in small machines to 0.20 in high-voltage or high-frequency machines, to 0.40 in liberally designed machines. These values include the width of bar and about 0.03 in. insulation between bars.
V = peripheral speed of commutator in feet per minute. This ranges from 3500 in liberally designed low-voltage, 25-cycle machines to 5000 in 60-cycle machines, and is extended to 6000 under compulsion.
e = average volts per bar = machine voltage divided by the number of bars between brush studs. Normal values are from 8 to 16. The maximum voltage between bars is about 1.57 times this.
f = frequency in cycles per second.
E = voltage between d-c terminals.
p = number of poles.
n = total number of commutator bars.
d = diameter of commutator in inches.

Then $$s = \frac{Ve}{10Ef}, \qquad n = \frac{pE}{e}, \qquad d = \frac{sn}{\pi}$$

* r and x should include respectively the resistance and reactance not only of the line wire, but also of the transformers and reactance coils through which the line current passes.

ARMATURE AMPERE-TURNS AND CURRENT DENSITY. Since the d-c and a-c armature reactions are opposed to each other the nominal value of the d-c armature reaction ampere-turns may be chosen much higher than in generators, and values of 4000 to 8000 ampere-turns per pole are in common practice. The nominal or apparent value of the ampere conductors per inch of periphery varies from 500 to 900, and the apparent current density (d-c) in the armature copper from 2500 to 5000 amp per sq in.

DIAMETER OF ARMATURE. The diameter of the armature per pole varies from 3.5 to 6 in., and the diameter is usually from 6 to 8 in. greater than the diameter of the commutator, the difference depending upon the possibility of making a good mechanical construction of the end connections. The number of slots in the armature and the number of segments in the commutator must be a multiple of both the number of phases and the number of poles.

ARMATURE WINDING. The winding of the armature is usually of the multiple drum type, although the series winding may be used. The turns per pair of poles are divided by equally spaced taps (to the slip rings) into as many groups as there are phases. The number of turns in series between brushes is adjusted for the proper direct emf and a reasonable value of flux per pole. The alternating emf bears a definite relation to the direct emf, depending upon the type of connection (see above), and also to some extent upon the shape and length of pole arc.

FLUX DENSITY IN AIR GAP. The air gap or pole-face flux density usually has a value ranging from 40 to 60 kilolines per square inch as in generators.

DAMPING COPPER IN POLE FACE. In the pole face of every converter a squirrel-cage winding should be provided to assist in starting and to prevent hunting. The total cross-section of copper per pole ranges approximately from 1/10 to 1/5 of the armature copper per pole, and the end rings must be of reasonable cross-section compared to the bars. The joints between bars and end rings must be carefully made.

SHUNT AND SERIES FIELDS. The determination of the length of armature and length of commutator, and the design of the field, follow the same laws as the design of these parts for a d-c generator. The proportioning of the series field results from the considerations given above under Field Excitation, the problem being that a certain amount of leading current is required at a given load and to make the armature take this leading current the field excitation must be increased by a certain number of ampere-turns, from which the number of turns in the series field can be determined.

EQUALIZER CONNECTION. In large multipolar machines it is customary to connect to a common ring all commutator bars which are at the same potential; these "equalizer connections" avoid local cross-currents in the armature from flowing through the brushes and causing bad commutation. These cross-currents are caused by unequal or uneven air gap.

SHAFT, BEARINGS, ETC. Since the transfer of energy is in the conductors themselves, there is no mechanical torque other than that to overcome friction and core loss.

Table 2.—Efficiency and Losses of Converters

Kw rating	500	300
Frequency	25	60
Core loss	1.00	1.75
Armature rI^2	0.55	0.60
Shunt field rI^2	0.70	0.60
Brush rI^2	0.40	0.40
Bearing friction	0.55	1.50
Brush friction	0.30	0.65
Efficiency	96.50	94.50

Therefore the shaft, bearings, and mechanical housing of a rotary converter are quite light and present no particular mechanical difficulties in mechanical design. For the same reason converters do not require very elaborate foundations. Machines of less than 1000-kw capacity are usually supplied with a base and are complete in one piece; larger sizes are supplied with foundation plates.

EFFICIENCY AND LOSSES. The efficiency and distribution of losses of a typical 25-cycle and 60-cycle converter are shown in the Table 2. All values are in per cent of input at full load.

EXAMPLES OF DESIGN. In Table 3 are given design data on four representative converters of different capacities.

Table 3—Synchronous Converters

Design Data

Item	Unit	1	2	3	4
Poles		4	6	6	6
Rating	kw	500	500	100	300
Speed	rpm	750	500	1000	1200
D-c volts	volts	600	600	440	600
Frequency	cyc per sec	25	25	50	60
Number of phases		6	6	3	3
Armature reaction	amp-turns	15,000	6720	7400	3500
Nominal σ*	amp cond	1500	710	675	475
Flux density in gap	kilolines	58	52	23	53
Armature diameter	inches	25.6	36	21	28
Slots per pole		24	24	65/6	21
Armature length	inches	11	17.25	7.1	10.5
Commutator diameter	inches	19.7	28	16.5	20
Number of segments		288	288	194	252
Periph. speed of commutator	feet per minute	3800	3650	4300	6280
Volts per bar	volts	8.3	12.5	13.6	14.3
Pitch segments	inches	0.17	0.306	0.266	0.25
Armature diameter per pole	inches	6.4	6	3.5	4.75
Nominal U†	amperes	6000	3600	2630	3400

* σ = ampere conductors (d-c) per inch periphery.
† U = amperes (d-c) per square inch.

6. TESTS AND OPERATION OF CONVERTERS

TESTS. The following are the usual tests made on converters to determine the efficiency, regulation, and heating and to show any defects in construction.

1. Resistance of Armature, Shunt Field, and Series Field. The armature resistance is usually measured between points on the commutator diametrically opposite and the equivalent resistance calculated from this value by the equation

$$\text{Equiv. resistance} = \frac{4 \times (\text{Diametrical resistance})}{(\text{Number of poles})^2}$$

To obtain the true rI^2 in a converter armature this equivalent resistance is multiplied by the square of the external direct current and by a constant as given in Table 4.

Table 4.—Loss Factors

	Theoretical	Commercial
Single-phase	1.39	1.47
Three-phase	0.56	0.59
Quarter-phase	0.37	0.39
Six-phase	0.26	0.27

These values hold only if the converter is operating at unity power factor. For any other power factor the reactive component of the alternating current per phase must be found and the square of this component times the resistance per phase of the armature gives the additional rI^2 loss due to the lesser power factor.

2. No-load Saturation Curve. The no-load saturation curve, as in a-c generators and d-c generators (q.v.) is usually plotted between commutator voltage and ampere-turns of shunt field.

3. Core loss as in a-c and d-c generators (q.v.).

4. Phase Characteristic at no load and at full load as in synchronous motors (q.v.).

5. Synchronous Impedance as in a-c generators (q.v.).

6. Starting Tests to determine current and voltage necessary to start the converter on a-c time to reach full speed, and voltage induced in field windings as in synchronous motors (q.v.).

7. Heat Run. This may be made either with a resistance for load or two similar converters may be tested in parallel by the Hopkinson or "pump back" method; see Transformers. In addition to a source of d-c power of the rated voltage of the converter to supply the losses, either a d-c booster or an a-c potential regulator is needed to adjust the load.

8. Insulation Tests. See Alternating-current Generators and Standardization Rules of the A.I.E.E.

9. Pulsation Test. A synchronous converter is very sensitive to any change in impressed voltage or frequency. A sudden change in either of these factors will cause a pulsation which will start the machine hunting, as discussed in the article on Synchronous Motors. This hunting may increase until the machine falls out of step or flashes over. To

determine whether a converter has a dangerous tendency of this kind, a test is made in which two similar machines are supplied with power from a common generator. Between each converter and the common connection a resistance is placed in each a-c line, having a value that will give with full-load current a drop in voltage of 15 per cent of the rated voltage of the machine. Thus there is 15 per cent rI drop between each converter and the generator and 30 per cent between the two converters. The two machines are operated at no load and at rated voltage with the shunt fields adjusted for minimum input. The voltage across the commutators is observed for any periodic variation. Then the field of one machine at a time is varied from half normal to twice the normal value, and any indications of periodic variations of the direct voltage noted. If the machines have a proper damper winding in the pole faces, they should not develop any dangerous hunting, even under the above unfavorable conditions.

SPECIFICATIONS. The following memoranda are intended to assist in writing specifications.

Principal Characteristics and Conditions of Service. Use to which converter is to be put, such as railway, lighting, motor driving, or battery charging. Whether it is to convert from alternating to direct current, or vice versa. Voltages and number of phases. Nominal rating in kilowatts or institute rating in kilowatts. Frequency and speed.

Style and Description; Details of Construction. Whether interpole; whether shunt or compound wound, or whether there is a split-pole field winding; whether rheostat is to be supplied for shunt field; if so, its characteristics. Proposed method of starting, and whether starting apparatus is to be supplied. Whether a speed-limit device is required; if so, the shunt field rheostat shall have sufficient resistance to speed the machine for testing the speed-limit device, the latter being set at 15 per cent over rated speed. Whether an end play device (or oscillator) is desired, and if so, what type or types are acceptable.

Work to be Done by Other Contractors. Whether the synchronous converter contractor is to furnish and install the following: Main wiring, field wiring, field-rheostat grids, dial plate and chains, starting panels, starting rheostat or motor generator, foundations.

Performance and Tests. Temperature rises upon which nominal and Institute ratings are to be based. Overload capacity (see Standardization Rules of the A.I.E.E.). Commutation limits. Efficiency at 25, 50, 75, 100, 125, and 150 per cent nominal load. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Converters shall operate in parallel without "hunting" from no load to stated (say 200 per cent) overload, provided drop in high-tension lines due to resistance between any converter and any other synchronous apparatus in the system does not exceed a stated value (say 20 per cent), and provided that the phase variation does not exceed a stated value (say 2.5 deg). What voltage regulation is required, how it is to be obtained, and whether it is to be hand or automatic. If the converter is shunt wound, state voltage-regulation requirements. What per cent reactance between a-c bus and converter is required?

OPERATION. In the operation of synchronous converters the points mentioned below should receive special attention.

Transformer Connections. The usual methods of connecting transformers to supply converters are shown diagrammatically in Figs. 1 to 6 and are discussed above in the section on Voltage Ratios and also in the article on Transformer Connections. Of the three-phase to six-phase connections the choice must be made with some care, as all connections are not equally good for each specific use of the converter.

Methods of Starting. The several methods of starting converters are as follows:

Alternating-current Starting, which is the same as the starting of motors; see Synchronous Motors.

Direct-current Starting. The machine is started as a d-c shunt motor and synchronized on the a-c side after it is up to speed. This requires less power, but takes more time and more skill in order to synchronize. To secure maximum starting torque, the a-c side should be disconnected from the transformers, which otherwise act as a shunt to a part of the starting current (Newburg and Smith, *Elec. J.*, 15, p. 24, January, 1918).

Starting with Auxiliary Motor. This involves the extra cost and extra continuous loss of the auxiliary induction motor. It is no more efficient than starting by direct current and requires the same amount of time. By the use of suitable relays the starting and synchronizing may be made entirely automatic (Wensley, R. J., *El. Ry. J.*, 53, p. 948, May 17, 1919).

Combination Alternating- and Direct-current Starting. The machine is started up with direct current, then disconnected from the d-c mains and connected to a low-voltage tap of the a-c supply, and brought up to full speed. This method is more economical of power and time but requires more starting apparatus than either the a-c or d-c methods.

Field Break-up Switch. All converters are supplied with a switch to open the field in

several places to avoid the strain of the high potential induced in the field during starting and to reverse the direction of current in the field after the machine is up to speed in order to reverse the polarity if it should not be right. This is usually a double-throw switch with several poles.

End-play Device. In order to prevent the brushes from wearing grooves in the commutator and collector rings a device is mounted upon one end of the shaft to move the shaft endwise back and forth periodically. In small machines this is a mechanical device consisting of a ball running between two warped surfaces. In large machines it consists of an electromagnet which periodically pulls out the armature a short distance. The magnetic pull of the main field poles pulls the armature back.

Speed-limiting Device. In case a converter should be disconnected from the main a-c generating circuit and still remain connected so that it would tend to operate from the d-c side there is danger of its speed becoming dangerously high. To avoid this a centrifugal governor is placed on the shaft and arranged to operate the main d-c switches of the converter electrically.

VOLTAGE REGULATION. There are several methods of regulating the voltage delivered by the commutator of a converter.

Compound-wound Converter with External Reactance. This method is automatic and will give about 10 per cent variation in voltage. It is standard practice for railway work; see also Railway Substations.

Shunt-wound Converter with Synchronous Booster. A synchronous generator is carried on the same shaft as the converter and connected in series between the transformers and the collector rings of the converter. The method is good but expensive. G. A. Juhlin (*Inst. El. Eng. J.*, 55, p. 241, April, 1917) gives a detailed discussion of the relative advantages of reactance and booster regulation.

Shunt-wound Converter with Induction Regulator. A large variation in voltage is possible, but this method does not respond to quick changes. It is quite generally used in lighting work.

Shunt-wound Converter and Taps on the Transformers. The voltage ratio of the transformers may be varied. This is usually accomplished by connecting the line to different taps on the primaries, which involves opening the circuit or short-circuiting a portion of the transformer at each change.

Regulating or Split-pole Converter. With this type of converter the voltage regulation is gradual and normally accomplished by hand, but by the addition of an automatic voltage regulator may be made automatic.

WEIGHTS AND SPEEDS. The weights and speeds of a commercial line of 60-cycle compound-wound converters for 250 volts direct current are given in Table 5. All have commutating poles.

Table 5.—Typical Rotary Converters

Poles	Kw	Rpm	Phases	Weight, pounds
6	100	1200	3	3700
6	150	1200	6	4370
6	200	1200	6	5650
6	300	1200	6	6800

7. MOTOR-CONVERTERS

A motor-converter is a combination of an induction motor and synchronous converter, with the secondary of the motor and the armature of the converter mounted upon the same shaft and connected together electrically without slip rings. The induction motor receives all the a-c power, transforms a part of it into mechanical power delivered to the shaft, and also acts as a transformer delivering the rest of the power in electrical form at a lower frequency from its secondary to the armature of the converter. It operates like two induction motors in " concatenation " or " cascade." The object is to obtain the steadiness of a 30-cycle converter on a 60-cycle circuit.

The speed of a motor converter depends upon the supply frequency and varies inversely as the sum of the number of poles in both machines. Thus if a combination of a 6-pole motor and 6-pole converter be operated from a 60-cycle circuit the speed of the armature will be 600 rpm. The primary of the induction motor will operate at 60 cycles, but the secondary will supply the armature of the converter with 30 cycles. Thus the converter may be built with the good design constants of a 30-cycle machine.

This combination is larger than a converter but smaller than a motor-generator set, and its efficiency is lower than that of a converter. It is used somewhat in Europe but not much in the United States.

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FREQUENCY CONVERTERS

Frequency converters are motor-generator sets in which both machines are designed for operation with alternating current, but for two different frequencies, thus a 25-cycle motor may drive a 60-cycle generator. They are used for three general classes of work which may be typified as follows:

(*a*) To supply 60-cycle current for lighting from a 25-cycle power circuit. The 60-cycle current is much more satisfactory for lighting than the 25-cycle current.

(*b*) To supply 25- or 15-cycle current for single-phase railway from a 50- or 60-cycle power system.

(*c*) To tie together two large power stations which started independently with different frequencies and, as a result of growth and changed economic conditions or ownership, find it desirable to be able to exchange energy either for purposes of evening up the demand or to meet the emergency of an accident to the generators of one of them. Most of the large power stations in a given neighborhood, even when owned by different corporations, are tied together to help each other out in case of trouble in one of them. For this type of service it is desirable that the frequency converters be "reversible," that is, either machine may be the driving motor.

Although several types have been proposed and discussed on paper, only two types have attained, or are likely to attain, any great commercial importance. These are the induction-synchronous and the synchronous-synchronous types. The former is not ordinarily reversible; the latter is.

8. INDUCTION-SYNCHRONOUS TYPE

An induction motor direct connected to a synchronous a-c generator (either single-, two-, or three-phase at either end or any combination thereof) may be designed, by choosing a suitable relation between the number of poles on each machine, to receive power at any frequency and deliver power at any other frequency. Thus a 10-pole motor operating on a 25-cycle circuit driving a 24-pole generator at 300 rpm will deliver approximately 60 cycles. This combination has the disadvantage that an increasing load causes decreasing speed and therefore decreasing frequency in the load circuit. This can be taken care of in some cases by choosing a ratio of poles which will give a nominal ratio of 25 to 62.5 cycles, for instance, and by added resistance in the secondary of the induction motor, regulate the speed at all loads so that the generator gives 60 cycles. Where the secondary system or load is an independent circuit without other generators there is no objection to allowing the frequency to vary slightly from the nominal value.

If there are to be two frequency converters in parallel supplying the load the adjustment of the division of load may be accomplished either by suitably designing the resistances of the secondary of the two induction motors so that they have the same slip, or by having an adjustable resistance connected in these secondary circuits of the motors. The great advantage of the induction-synchronous type is that, owing to the characteristics of the induction motor, a variation of 1 or 2 per cent in the frequency of either of the two systems will not cause a serious overload on the set, as is the case with the synchronous-synchronous type.

9. SYNCHRONOUS-SYNCHRONOUS TYPE

This consists of a synchronous motor driving a synchronous a-c generator. The constant-speed characteristic of a synchronous motor makes it particularly suited as the driving motor of a frequency converter, as this insures a constant ratio of frequencies irrespective of the load on the set.

Such sets are more generally used than any other type, particularly for linking together two systems whose frequencies are fixed by the presence of other generators. However, any considerable variation in the frequency of one system will place a considerable overload on the set and may cause it to pull out of step. Consequently, it is advisable that the capacity of the set be not too small in proportion to the capacity of the system. The synchronous motor usually has an amortisseur or squirrel-cage winding in its pole faces in order to assure good starting ability and to reduce hunting (see Synchronous Motors). This type of set is reversible, that is, will transfer energy either way between the systems which it ties together. The reversal of the flow of energy is accomplished by moving one of the stationary armatures in its cradle as described below.

In order that a synchronous-synchronous set may operate properly in parallel and divide the load proportionately with other synchronous apparatus on the two systems to which the motor and generator are respectively connected, the number of poles of both motor and generator must be carefully chosen. Thus to change from 25 to 60 cycles per second, the highest speed which allows an exact transformation of frequency is 300 rpm, and this requires 10 poles on the 25-cycle machine and 24 poles on the 60-cycle machine.

DIVISION OF LOAD ON SYNCHRONOUS SETS. The parallel operation of two or more sets of this type involves certain complicated considerations. In the first place the division of the load depends not only upon the voltage regulation of the individual machines constituting a set, but upon the mechanical position of the armatures on the shaft. Two sets built apparently the same may not divide the load equally because of inaccuracy in the placing of the keyways, etc. To avoid this trouble it is customary to mount the stationary member of one machine of each set movable in a cradle, so that the angular phase position of this member (usually the stationary armature) with respect to the base, may be adjusted. The two sets to be operated in parallel are loaded, and the stationary armature of one machine is rotated in the cradle until the load on the two machines divides properly.

SYNCHRONIZING FREQUENCY CONVERTERS. The synchronizing of such sets is difficult, and several complicated conditions must be satisfied. The theory is quite involved and is discussed at length in a paper by J. B. Taylor on Parallel Operation, *Trans. A.I.E.E.*, Vol. 25, p. 113, from which the attached table is taken. The essential requirement is that it is necessary to synchronize not only the motor with its supply circuit, but also the generator with its load circuit. Thus, when the motor has been synchronized properly, it may be found that the generator is 180 deg out of phase with the load circuit. To bring the generator into phase it is necessary to cause the motor to "slip" one or more poles successively until the generator comes into proper phase; this can be done by reversing the field of the motor one or more times, depending upon the number of poles on the generator and on the motor. The necessity for more than one reversal is due to the fact that with a large number of poles on both motor and generator there are only a few combinations in which the poles of the motor and the generator of one set match up with the poles of motor and generator of another set. Thus with a 10-pole motor and 24-pole generator (the most common arrangement) when the motor slips one pole or 180 electrical degrees of its circuit, the generator slips $24/10 \times 180$ or 432 electrical degrees of its circuit. It is necessary to make the motor slip 5 poles or 5×180 deg to make the generator slip the first even multiple of 360, that is, $5 \times 432 = 2160$ or 6×360 deg. In this case it might be necessary to make the motor slip 4 poles before getting the right combination. Five reversals would reproduce the original conditions, and, after that, more reversals would repeat the cycle of possible combinations. Table 1 shows the possible combinations for various commonly used sets.

In practice, this difficulty is overcome by the use of a special synchronizing indicator

(see Synchronizers) having two hands on the same dial; one hand shows the phase relation of each member of the set with respect to its particular external circuit. It is necessary to synchronize the set when both hands are not only stationary but point to the zero position. If only the motor were properly synchronized it would be found that both hands were stationary, the motor hand pointing to zero but the generator hand pointing to some other position.

Table 1.—Possible Combinations for Synchronous-synchronous Frequency Converters

Cycles		Poles		Speed	Kw	Chances of Correct Synch.	Field Reversing Switches	
Motor	Gen.	Motor	Gen.	Rpm	Capacity		Motor	Gen.
25	60	10	24	300	Max	1 in 5	Yes	Yes
25	62.5	4	10	750	600	1 in 2	....	Yes
25	62.5	8	20	375	Max	1 in 2	....	Yes
40	60	4	6	1200	150	1 in 2	....	Yes
40	60	8	12	600	3000	1 in 2	....	Yes
40	60	12	18	400	Max	1 in 2	....	Yes
25	50	4	8	750	600	Certain	....	....
25	50	8	16	375	Max	Certain	....	....
33.3	60	10	18	400	Max	1 in 10	Yes	Yes
30	60	6	12	600	1000	Certain	....	....
30	60	8	16	450	Max	Certain	....	....

Explanation of Table. The first two columns show the frequencies of the two circuits to be tied together; the third and fourth columns, the number of poles on motor and generator, respectively; the fifth column, the speed of the set. The sixth column, the maximum capacity in kilowatts in which such sets are available in commercial practice ("max" means as large as desired). The seventh column states the chances of the set coming in right the first time, and indicates the maximum number of trials that might be necessary before the right combination is found: thus in the first line, four trials might have to be made to secure proper synchronizing. The last two columns tell whether it is necessary to have field reversing switches on each member. These are used to make the motor slip a pole and to reverse the polarity of the generator. The use of reversing switches on both members of a set reduces the number of trials necessary to get proper synchronizing.

Synchronous-synchronous sets are started by means of a squirrel-cage winding in the field poles which makes the motor start (at a reduced voltage) as an induction motor and pull into step without synchronizing.

By providing ample capacity in the motor, particularly in field copper, these sets may be used for power-factor correction of the motor circuit by overexcitation of the motor.

The efficiency of these sets is quite high, but is of course the product of the efficiencies of the two machines. The machines may be wound for high voltage, thus eliminating the need of transformers for voltages of 13,000 or less. Two-bearing enclosed sets are quite popular in order to economize in space and in friction loss.

APPLICATION. For the supply of power to single-phase railways at 25 cycles from a 60-cycle three-phase transmission network it is customary to use a 24-pole, three-phase synchronous motor running at 300 rpm at 60 cycles, driving a 10-pole single-phase synchronous generator giving 25 cycles. Several of these are in use in Philadelphia, notably a 30,000-kw single-phase generator driven by a 36,000-kva three-phase motor. The stator of the 60-cycle machine may be shifted through 24 mechanical degrees, equivalent to 288 electrical degrees. A shift of 90 deg is used in load control, and the balance, 188 deg, is reserved for synchronizing. A 70-hp motor is used to do the shifting. The single-phase generator would rate at 61,000 kva if operated three-phase. This is the price paid to minimize the effects of the pulsating single-phase armature reaction.

A special type of frequency converter makes use of the principle of "frequency splitting" by splitting the 60 cycles into $25 + 35 = 60$. The fraction $^{25}/_{60}$ of the power is transformed electrically, and $^{35}/_{60}$ is transformed mechanically. This makes possible a reduction in the rating and size of one of the machines to $^{35}/_{60}$ of the total power transformed.

A typical set, of which many are in use (Hell Gate, New York), consists of a 14-pole, 35,000-kw, 60-cycle induction motor which would tend to run at 514 rpm. The rotor is phase wound with collector rings. The other machine is a 10-pole 20,400-kw, 25-cycle synchronous machine for 300 rpm. The two are on one shaft and forced to run at 300 rpm by the synchronous machine (see Fig. 1). At 300 rpm the induction motor has a slip of 214 rpm which gives a voltage at 25 cycles in the secondary, and this is fed to the 25-cycle

system. In addition the induction motor delivers power to the shaft and thus drives the synchronous machine. The power converted electrically is 35,000 × 214/514 = 14,600 kw, and the power converted mechanically is 35,000 × 300/514 = 20,400 kw, which is the rating necessary for the synchronous machine (neglecting losses).

Because it is difficult to design the winding of the induction motor rotor for high voltage, transformers are interposed between it and the 25-cycle system. The flow of energy is reversible, depending upon the relative phase angles of the two machines, and the transfer of power may be controlled by moving the stator of the synchronous machine in its cradle. The set is started by an auxiliary induction motor, synchronized with one system, and then, by moving the stator of the adjustable machine through a small angle, it is synchronized

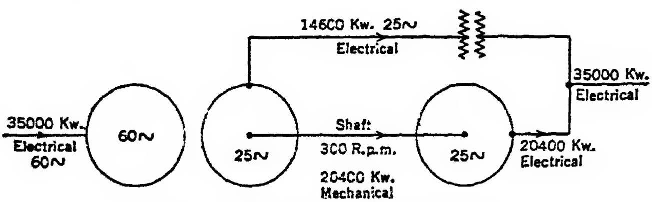

Fig. 1. Induction Frequency Converter

with the other system and the load adjusted as desired. The synchronous machine may be used for power-factor correction. Such sets must be of large capacity as they tie the two systems together rigidly and any shock on one is transferred to the other and the set is greatly overloaded in trying to hold the two systems together.

10. INDUCTION TYPE

When the stator of an induction motor is excited from a supply circuit having a frequency of f_1 cycles per second and the rotor of this motor is driven by another motor in the *opposite* direction to that in which it would rotate owing to the currents in its stator winding, the frequency of the current induced in the rotor winding is

$$f_2 = \frac{N_0 + N}{N_0} \cdot f_1$$

where N_0 = synchronous speed of the motor and N = actual speed at which its rotor is driven. This combination of two motors (the driven motor really acts also as a generator) may therefore be used as a frequency changer.

Neglecting the losses in the driven motor, the electrical input into its stator is

$$\frac{f_1}{f_2} \times \text{(Output of set at frequency } f_2\text{)}$$

and the mechanical output of the driving motor is

$$\frac{f - f_1}{f_2} \times \text{(Output of set at frequency } f_2\text{)}.$$

This combination is less expensive than the usual motor-generator set, but it has the disadvantage of poor regulation, as every change in the potential of the supply circuit is transmitted to the receiving circuit; it is therefore but seldom used.

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PHASE CONVERTERS AND BALANCERS

A phase converter is a machine for converting single-phase power into polyphase power, or vice versa. A phase balancer is a machine for balancing the load on one or more polyphase generators when these generators are supplying an unbalanced load to the circuit to which they are connected.

11. PHASE CONVERTER

The phase converter is a single-unit two-phase induction motor, or a two-phase synchronous motor with an extra heavy squirrel cage.

SINGLE-PHASE TO POLYPHASE CONVERSION. The Norfolk and Western Electric Locomotives receive single-phase power through a single trolley. Three-phase induction motors are used to drive these locomotives. The connecting link between the single-phase and polyphase system is a two-phase induction motor operated as a phase converter. The connections to change from single-phase to an approximately balanced three-phase system are shown in Fig. 1.

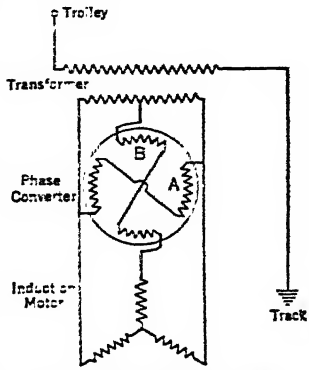

Fig. 1. Connections of Phase Converter

The machine is driven by power supplied to phase *A*. Phase *B* corresponds to the teaser coil of a transformer T-connection. This phase has an emf induced in it of the proper magnitude and phase position to make the voltages across the three phases approximately balanced.

The phase converters on the Norfolk and Western are started by single-phase commutator motors. The operating speed of the converter is high, permitting a design of minimum weight for a given output. The converter in addition to furnishing power to polyphase motors may be used to drive auxiliary apparatus. Polyphase motors receiving their power from phase converters may be used for regenerative braking. The rated capacity of a two-phase balancer should be one-half of the usual polyphase power required.

POLYPHASE TO SINGLE-PHASE CONVERSION. It is quite common to supply the single-phase power for railways from existing three-phase systems, and a phase converter may be used for this purpose; but so many difficulties arise from any electrical interconnection between a single and a polyphase system that experience has shown the superiority of a motor-generator set for this purpose. The set consists of a three-phase synchronous motor driving a specially designed single-phase generator. As in most commercial uses a conversion of frequency from 60 to 25 is also necessary, a frequency converter (q.v.) is the usual solution.

12. PHASE BALANCER

In many three-phase distribution circuits the loads are not balanced on the three phases because small single-phase loads are connected on the various phases. The unbalance changes from time to time so a simple segregation of the single-phase load is not possible. The phase balancer was introduced about 1915 to correct this condition but, as the problem of unbalancing has become less with the increase in size of generating systems, no new ones have been put in service for some time and the device is only of theoretical interest. It makes use of the principle of symmetrical components, according to which any unbalanced three-phase system of currents may be resolved into two balanced three-phase systems of opposite phase rotation or sequence (see Section 3, Art. 37).

The phase balancer puts into the system the small balanced three-phase currents of opposite or negative phase sequence to those of the generators, leaving the generators to supply the other and larger balanced system or component. The shunt balancer is the more usual as it may be made to take care of changing conditions automatically. It consists of a large and a small synchronous machine on the same shaft with their respective phases in series but reversed in phase connection.

For details the reader is referred to the bibliography and particularly to the article by Heningsen.

13. PHASE SHIFTER

This is officially known as a three-phase transformer for phase angle control. It consists of a three-phase static transformer with shunt and series coils in one member, usually the secondary, the series coils being arranged to give voltages in quadrature to the respective shunt coils. The series coils have taps so that the amount of quadrature voltage may be varied in steps from about -10 to $+10$ per cent. This gives a variation in phase angle of -6 to $+6$ deg. Its purpose is to shift the phase angle of the secondary with respect to the primary. It is used as a tie between two systems operating from independent generating stations, and its purpose is to regulate and control the power and direction of energy flow through it, thus dividing the load on two systems.

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RECTIFIERS

The term rectifier is applied to any piece of apparatus for transforming alternating into direct current, or vice versa, by electric conduction alone without the storage of energy in a magnetic field. There are many types which, in order of practical importance, are:

1. Mercury arc with a cold (pool type) cathode and gaseous conduction.
2. Mercury vapor with a hot cathode and gaseous conduction ("Phanotron").
3. Mercury vapor with a hot cathode and gaseous conduction with a grid for control, three electrode ("Thyratron").
4. Thermionic, with a hot cathode and pure electron conduction in a high vacuum ("Fleming Valve").
5. Copper oxide.
6. Electrolytic.
7. Crystal.
8. Vibrating reed.
9. Rotating commutator.

The first three are used in power work, the fourth to eighth inclusive are used only in communication, particularly in radio. The last is not very satisfactory.

APPLICATIONS. Mercury-arc rectifiers find their chief use in the supply of direct current to railways at 600, 1500, or 3000 volts. To a lesser degree they are used in charging storage batteries. They are at their best at high voltages, where their efficiency is much higher. The other rectifiers are used to supply direct current to the plates of the tubes used in radio transmitting and receiving sets which usually take power from an a-c supply system.

14. MERCURY-ARC RECTIFIER

The operation of this rectifier depends upon the fact that a tube containing mercury vapor under a low pressure and having one electrode of mercury, and the other of some other conductor, graphite, offers a very high resistance to a current tending to flow through the tube from the mercury to the other electrode, but has a very low resistance to a current flowing in the opposite direction, provided the current is once started by forming an arc in the tube.

CONNECTIONS FOR SINGLE-PHASE OPERATION. The connections employed in practice are shown in Fig. 1. The complete equipment consists of a source of alternating current *HG*, the rectifier tube *AA′*, two reactances *E* and *F*, and the load represented as a storage battery *J*. The rectifier tube is an exhausted glass vessel in which are two graphite electrodes (anodes *AA′*) and one mercury cathode *B*. Each anode is connected to a separate side of the a-c supply, and also through one-half of the main reactance to the negative side of the load. The cathode *B* is connected to the positive side. There is also a small

starting electrode *C* connected to one side of the a-c circuit through a resistance and used for starting the arc. When the rectifier tube is rocked so as to form and break a bridge of mercury between the cathode *B* and starting anode *C* a small arc is formed. This produces mercury vapor in the tube, and the arc immediately jumps to one or the other of the main anodes and alternates on these during regular operation.

MODE OF OPERATION. To analyze the operation, assume an instant when the terminal *G* is positive and *H* negative. The positive current will then flow from anode *A′* to cathode *B*, through the load *J* to *D*, and through reactance *F* back to *H*. The current cannot jump from *A′* to *A* on account of the high counter emf of the arc. The small arrows surrounded by circles show the path of the current during this half cycle. During the next half cycle the terminal *H* is positive and the current flows to *A* through the tube to *B*, through the load *J*, reactance *E*, and back to *G*. The small arrows show the path of the current during this half cycle. Hence during a whole cycle the cathode *B* is continuously negative, but first one anode is active and then the other. If the voltage and current should become zero coincidentally the arc would become extinguished and operation cease. Hence the reactances *E* and *F* are introduced. At the end of the first half cycle described, when the line voltage drops to zero, the inductance of *F* maintains the current and a local circuit is formed through *A*, *B*, and *F*, which maintains the arc until the voltage at *G* has risen to a value which is sufficiently high to maintain the arc.

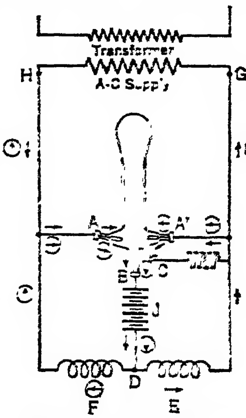

FIG. 1. Mercury Vapor Rectifier

The rectifier thus makes use of both half waves, or the entire alternating current, and the result is a uniform pulsating unidirectional current. On account of the reactance in the circuit, this current in the load never falls to zero and, in fact, with sufficient reactance, may be made very nearly constant. But this extreme is not always desirable, as it distorts the current wave in the a-c supply circuit.

ARC RECTIFIERS ON POLYPHASE CIRCUITS. All rectifiers of real power rating are operated from a three-phase supply and the transformers are connected to impress six-phase voltages on the rectifier and, in the case of great power, twelve-phase. The six-phase rectifier has six working, or power, anodes (graphite) and one cathode, a pool of mercury insulated from the tank. There are also usually a "starting" anode and an auxiliary "exciting" anode. The connections for such a rectifier are shown in Fig. 2 on the following page.

The tank is of steel, insulated from the ground, and with the electrical connections to the several electrodes carried through the tank walls by special "insulating seals," in some cases of porcelain and in others of a special glass (see Bibliography) in which the different materials have the same heat-expansion coefficient. This tank is surrounded by another tank, and between them circulates the water for cooling purposes. The inner tank is maintained at a vacuum of 1 to 5 microns, and the upper limit of successful operation is 10 microns (0.010 mm). The vacuum is maintained by two pumps in series, a mercury-condensation pump for fine adjustment and a mechanical rotary pump for coarse work. The latter runs only occasionlly.

The rectifier is started by closing the switches and causing the starting electrode to dip into the mercury pool and then immediately be withdrawn. This starts an arc which ionizes the mercury and causes the arc to spread to the exciting anodes and then to the six power anodes. The exciting electrodes are small electrodes placed near the cathode and connected to the supply through a limiting reactance so that they will not take much current but will maintain the arc at times of light load. The voltage on the d-c side is theoretically equal to the maximum instantaneous value of the a-c voltage from anode to cathode, but in practice there is a drop of 15 to 25 volts lost in the arc itself which with the loss in the transformers and reactors means a total loss of voltage of 18 to 30 volts irrespective of the rated voltage and almost irrespective of the current. This voltage has a ripple in it which becomes less and less marked as the number of anodes is increased, being almost negligible except to telephone practice with 12 anodes. Reactances in series with the d-c load are used, and sometimes these are fitted with wave filters to avoid telephone interference. The secondaries of the transformers are always connected in Y, and with the six-phase type a double Y is used with the two neutrals joined by a reactance which sometimes is in the form of an "interphase transformer" which can be designed to prevent an excessive rise in voltage at very light loads and thus give the device a better regulation.

In a simple rectifier there is a considerable drop in voltage between no load and 10 per cent of rated load, above which load the voltage does not drop off very much. There are three ways of overcoming this: (a) holding a false resistance load on the rectifier; (b) using saturation in the interphase transformer; and (c) using grids or third electrodes to control the voltage (see below).

Parallel operation on the a-c side offers no difficulties whatever, but proper division of the d-c load depends upon using one of the three expedients just mentioned.

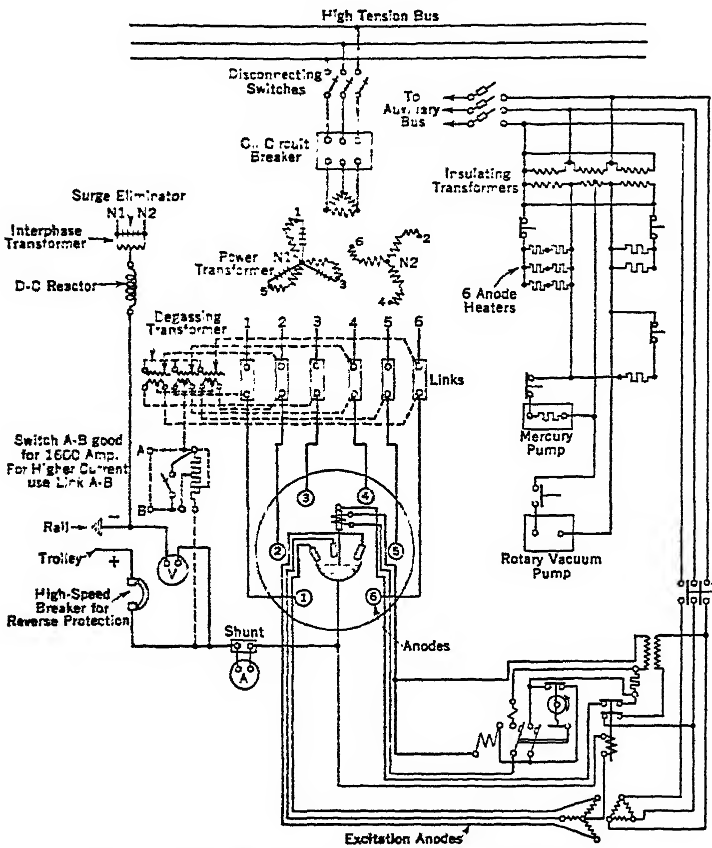

FIG. 2. Connections for 6-phase, 600 volt Rectifier (G. E.)

EFFICIENCY. The efficiency of a 600-volt rectifier ranges from 93 to 94 per cent from 25 per cent load to 150 per cent load, and of a 1500-volt rectifier from 95 to 96 per cent.

This is the overall efficiency including the losses in all the auxiliaries, transformers, pumps, reactors, and excitation. The efficiency of the rectifier alone depends only upon the delivered voltage and the arc drop thus: Eff. = 600/600 + 25 = 0.96 at all loads.

POWER FACTOR. This ranges from 92 to 95 per cent; it is caused by wave distortion and by the reactive power of the transformers. It cannot be regulated as in converters.

CAPACITIES AND OVERLOADS. Ratings up to 3000 kw at 625 volts are in common use. Rectifiers will stand very great overloads for short periods of time. For instance, guarantees are: 150 per cent load for 2 hr, 200 per cent for 5 min, and 300 per cent for 1 min. If a short circuit, known as an "arc back," does occur inside the rectifier, it is not

likely to cause any permanent damage; the rectifier may be started up again and immediately put back into service.

COMPARISONS. A complete rectifier outfit costs about 10 per cent more than a converter outfit and takes about 50 per cent more floor space, but it requires very modest foundations, gives no trouble from vibration, and operates as well on 60 cycles as on 25, which is not true of a high-voltage converter. Many are in operation, unattended, in automatic substations as 60 on the Independent Subway System of New York City.

15. RECTIFIER WITH GRID CONTROL

The addition of a grid in front of each anode and the application of a voltage of the proper value and phase give the rectifier some of the characteristics of the " three-electrode tube " (see Communication) in that the magnitude of the output voltage and current may be controlled. Also it is possible to reverse the flow of energy and change from direct to alternating current, thus making an " inverter." These have been proposed for railways on which the locomotives are designed for regeneration on down grades.

In a gaseous tube the grid can prevent the current at its anode from starting until such time as the grid is properly excited, but, after the current has started, the grid cannot stop it because so much ionized gas is present, and so the current will continue till the end of the half cycle or wave. Thus the output can be limited but not increased.

16. HOT-CATHODE MERCURY RECTIFIERS

In these the cathode is caused to emit electrons by heat applied to a suitable metal, and the tube contains a very small amount of mercury in the form of vapor. This will operate as a rectifier with smaller values of current than the pool type but with about one-half the voltage drop, therefore with a much higher efficiency at low voltages such as 125 and 250. They are available in sizes for 100 amp continuously. They are known under the trade name of Phanotron and have been applied to supplying 125–250 volts direct current on the Edison three-wire distribution.

17. THREE-ELECTRODE HOT-CATHODE MERCURY RECTIFIERS

These have a hot cathode, a cold anode, a grid, and a small amount of mercury vapor like the preceding, and though restricted to smaller currents than the pool-type rectifier are susceptible to more accurate control of the voltage and current by the grid. They are known by the trade name of Thyratron and may be used for converting from alternating to direct current at any frequency and from direct to alternating current. They may thus be used to convert from one frequency to another. It has been proposed to convert a-c power at the power station to d-c power at high voltage, transmit at high-voltage direct current and then convert back to alternating current at any desired frequency at the receiving end, thus obviating the difficulties of power system instability (see Bibliography).

18. TUNGAR RECTIFIER

This rectifier acts on the principle of the Fleming valve (see Communication), and consists of a heated tungsten filament for one electrode, and a tungsten plate for the other, both placed in a glass bulb containing argon gas. If the plate is made positive to the filament a thermionic current will flow across the intervening space in the vacuum, but if the filament is positive to the plate no current will flow. By combining two tungar rectifiers with an auto-transformer, as is done with the two halves of the mercury-vapor rectifier, both half waves may be rectified. This rectifier is limited to small current (15 amp) and is used chiefly for charging small groups of storage batteries at 60 volts or less. The voltage drop in the arc is about 15 volts for a valve in good condition.

The other rectifiers available, Fleming valve (hot cathode and high vacuum), copper oxide, electrolytic, crystal and vibrating types are used only in radio and communication and are treated in that volume.

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SECTION 12

SWITCHING, CONTROL, AND PROTECTION

BY

STEPHEN Q. HAYES

ART. CIRCUIT ELEMENTS PAGE

1. Definitions.......... 02
2. Fundamental Ideas.......... 02
3. Switches.......... 03
4. Fuses.......... 06
5. Resistors.......... 09

CIRCUIT BREAKERS

6. Carbon Breakers.......... 11
7. High-speed Air Breakers.......... 13
8. De-ion Breakers.......... 14
9. Miscellaneous Air Breakers.......... 15
10. Elements of Oil Circuit Breakers.......... 16
11. Operating Mechanisms.......... 20
12. Relays.......... 23
13. Selection of Oil Circuit Breaker.......... 25

CONTROLLERS AND REGULATORS

14. Motor-starting Devices.......... 31
15. Drum Controllers.......... 33
16. Control Panels.......... 34
17. Control Rheostats.......... 34
18. Generator Voltage Regulators.......... 36
19. Induction Voltage Regulators.......... 39

ART. PROTECTION PAGE

20. D-c Circuit Protection.......... 42
21. A-c Circuit Protection.......... 43
22. Lightning Protection.......... 45

STATION EQUIPMENT

23. D-c Switchboard Instrument Equipment.......... 49
24. A-c Switchboard Instrument Equipment.......... 49
25. Miscellaneous Meters.......... 50
26. Panels and Desks.......... 51
27. D-c Switchboards.......... 55
28. A-c Switchboards.......... 56
29. Bus-bars.......... 56
30. Cables.......... 59

STATION STRUCTURES AND LAYOUT

31. Open Construction.......... 60
32. Semi-enclosed Construction.......... 61
33. Enclosed Construction.......... 61
34. Segregated Phase.......... 62
35. Metal-clad Switchgear.......... 64
36. Station Layout.......... 69
37. Automatic Operation.......... 73
38. Load Dispatcher Systems.......... 74

SECTION 12

SWITCHING, CONTROL, AND PROTECTION

By Stephen Q. Hayes

CIRCUIT ELEMENTS

1. DEFINITIONS

The subjects of switching, control, and protection are very closely related in that they all are concerned with the opening and closing of circuits and employ the same equipment in so doing. The distinctions between them arise from the types of circuits dealt with and the reason for changing the condition of the circuit.

Switching is usually employed in connection with the operation of circuits involved in the development and distribution of electric light and power.

Control is the term applied to describe the operation of equipment to open or close circuits of industrial motors, furnaces, and similar devices for converting electrical power to mechanical motion, or for heating and similar purposes.

Protection is the term applied to opening a circuit for the purpose of preventing harm to any of the component parts.

Switchgear is an assembly of coordinated individual apparatus employed for the rendering of electrical service, namely, for the control and distribution of electrical energy. It comprises switchboards and switching equipment.

Switchboards are usually understood to be the panels, control desks, pedestals, or similar supports on which the meters, relays, control handles, instrument switches, etc., are mounted.

Switching Equipment is usually interpreted to mean the circuit breakers, switches, and similar devices in the main circuit together with their related current and potential transformers, bus-bars, connectors, etc. The term "switchgear" is often used as an alternative expression.

2. FUNDAMENTAL IDEAS

Switching operations are a necessary element in any system of electrical circuits. One of the important parts of switching is the opening and closing of the circuit which is generally called "circuit-breaking." Apparatus for accomplishing this varies from a very simple device to open low-voltage circuits and small currents, to complicated large breakers to interrupt large power circuits connected to immense sources of power. The problems involved are correspondingly varied. In most cases both the closing and opening of the circuit are performed by the same devices, and the requirements of each function must be considered in the design of the apparatus.

Circuit Breaking may be defined as the interruption of the continuity of an electrical circuit in which energy is flowing. In general, the same apparatus—whether it be switch, controller, contactor, air breaker, or oil breaker—is used not only to "break" the circuit but, when desired, to restore the continuity of the circuit or to "make" the circuit.

INTERRUPTION OF DIRECT CURRENT. The initial effect of attempting to interrupt the flow of direct current by a circuit breaker is an arc when the contacts begin to open. This arc persists until it is interrupted by the further opening of the circuit breaker to a point where the voltage required for the maintenance of the arc is greater than the available voltage. The current then reduces to zero and the arc is extinguished. This completes the opening of the circuit.

The type of d-c load is important, since some kinds of load are more difficult to open than others. Incandescent lamps, heating equipment, and similar resistances, having little or no self-induction, may be switched off easily. Batteries on charge and shunt wound motors, owing to their counter electromotive force, give an opposing voltage independent of the current so that the switch when opening has to break only the small actual

voltage needed to force the current to flow through the internal resistance of the battery or motor. While the shunt field winding of such motors has a high self-induction, the field circuit remains closed on the armature circuit so that its energy does not discharge back into the switch or breaker. Series motors, however, lose their counter electromotive force on the opening of the switch or breaker, and their full energy discharges back into the switch. This partly accounts for the vicious arcing that usually occurs when opening a d-c railway circuit, where the load is largely made up of series motors.

INTERRUPTION OF ALTERNATING CURRENT. This operation is similar to that for direct current, but there is this fundamental difference, that the current has a zero value with each alternation which facilitates the breaking. When opening an alternating current of unity power factor, such as lighting or heating load, the current passes through zero simultaneously with the voltage and the arc is extinguished as the current passes through zero. When the dielectric strength across the gap exceeds the arc voltage, current must cease to flow.

Theory and experiment indicate that, in a very short interval of time, embracing this moment of zero current, the medium containing the arc returns from its *momentary* condition of a comparatively good conductor carrying current with a low voltage drop to its *normal* condition of a comparatively good insulator, supporting the full generated voltage of the circuit, with passage of little current. It is this rapid transition, at the moment of zero current, from the state of a highly conducting arc to the state of an insulating non-ionized gas which is important for the extinction of the arc in an alternating-current switching device.

If the system voltage is high and the contact separation of the circuit-breaking device is small, the arc re-establishes itself. If, at the next passage of current and voltage through zero, the contact separation has increased sufficiently, the arc will not re-establish itself and remains extinguished, thus definitely interrupting the circuit.

An ideal breaker is one which always extinguishes the arc at the first zero point of the current wave and at the same time introduces a sufficient insulating gap to prevent re-establishment of the arc.

OIL. Submerging the circuit-breaker contacts in oil does not entirely prevent an arc at the contacts, but it cools the contact and the arc so that the extinction and re-establishment voltages in oil may reach a value many times that in air. The switching energy liberates heat, decomposing and carbonizing the oil and causing a gas bubble at the contact. Drops of carbonized-oil residues float in the arc space and act as de-ionizing centers for the ions. These de-ionizing centers or nuclei consist of volumes of relatively cool un-ionized gas arising from the decomposition of the oil and mixed turbulently into the arc space. It seems to certain engineers that the principal cause of the arc-interrupting capacity of the oil circuit breaker is its action as a gas blast switch, the gas blast arising from the decomposing oil. If the arc is not submerged deep enough in the oil the hot gases in rising to the surface are not sufficiently cooled and may ignite the oil at the surface and explode the arc gases that may have accumulated above the oil. An alternative theory developed by other engineers suggests that the successful operation of an oil circuit breaker is due to the scavenging action of an oil blast that sweeps away the arc products.

3. SWITCHES

A switch is a device for making or changing connections in an electric circuit. It must, when closed, carry its rated current without excessive drop, or excessive heating; it must take care of overloads met in practice; it must be designed to prevent, or render harmless, any arcs that are formed when being opened; it must, when open, insulate all live parts for the maximum potential of the circuit.

KNIFE SWITCHES. The use of knife switches is rather limited in circuit-breaking, as an arc will occur between the switch blade and the break jaw. Permissible arcing, without affecting the performance of the switch, determines its arc-breaking capacity. The more rapidly the switch is opened, the less the damage from the arc. Also the arc is more vicious and difficult to control on d-c circuits than on a-c circuits of the same voltage and class of service. For this reason, it has been found that a knife switch with dimensions satisfactory for opening a 250-volt d-c circuit can be safely used to open the circuit on a 550-a-c line. Hand-operated switches in d-c circuits are usually limited to 600 volts. When used to break current, quick break attachments are *recommended* for all switches rated for over 250 volts d-c; they *must be* provided for those rated at or above 200 amp and 500 volts d-c, or 800 amp and 250 volts a-c.

The current-carrying capacity of a switch or circuit breaker is determined by its permissible temperature rise. The capacities usually assigned for such devices are based

on the current which they can carry continuously, the temperature rise not exceeding 30 deg cent in the current-carrying parts.

The non-uniformity of current distribution in conductors of a-c circuits increases as the currents become larger and also increases with the frequency. For 25-cycle circuits a lesser amount of current would cause a given temperature rise than on direct current and on 60-cycle circuits a still smaller current would cause the same temperature rise, so that, for the larger capacities, current ratings are frequently given for d-c, 25-cycle, and 60-cycle service. The following table of amperes for 30-deg rise is characteristic of knife-switch ratings for different conditions.

D-c	25-cycle	60-cycle	D-c	25-cycle	60-cycle
1200	1100	1100	3000	2500	2200
1600	1400	1200	4000	3400	2800
2000	1800	1600	6000	4400	4000

Fig. 1 shows some of the features of a two-pole single-throw 200-amp rear-connected switch typical of the normal American design of knife switches.

Various modifications in the standard knife switch are made to allow for starting of d-c motors, to provide auxiliary contacts for the discharge resistance in the field circuit, and to take care of other special conditions.

STARTING SWITCH. Where it is necessary to start a synchronous converter from the d-c end or to start a d-c motor of large capacity under relatively easy starting conditions, a multipoint knife switch of the type indicated in Fig. 2 can be utilized to advantage for cutting out steps of a starting resistance. The switch illustrated has four sets of contact jaws of such length that the switch blades make contact with each set in succession. Each switch has four blades, a construction that allows ample ventilation and reduces the depth of the switch from the switchboard. A ratchet device, with pushbutton in the handle, has been provided on the larger sizes, to prevent large machines being started too quickly by throwing the switch through all the positions without stopping at any one.

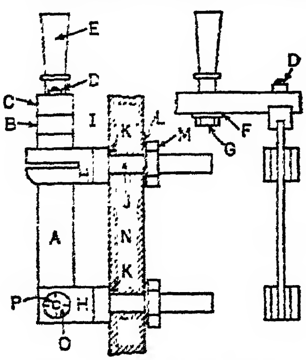

FIG. 1. 200-ampere Low-jaw Knife Switch

A, copper blade, hard drawn, 98% conductivity; *B*, blade block pinned and sweated to *A*; *C*, Micarta cross bar with milled slots; *D*, screw fastening cross bar to blade block; *E*, handle bolted to cross bar; *F*, washer for handle bolt; *G*, nut for handle bolt; *H*, jaw blades pinned and sweated in blade block; *I*, blade block with milled slots; *J*, studs electrobrazed to blade blocks; *K*, dowel pins locating switch studs; *L*, washer for switch stud; *M*, nut for switch stud; *N*, base for switch; *O*, slotted spring washer; *P*, hinge bolt.

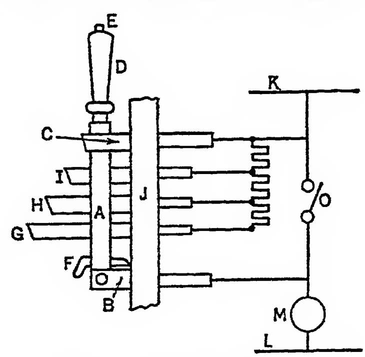

FIG. 2. Typical Starting Switch

A, switch blades; *B*, hinge jaw; *C*, main break jaw; *D*, handle; *E*, push button to operate ratchet; *F*, ratchet operated by push button; *G*, *H*, *I*, intermediate break jaws; *J*, switch base; *K*, positive bus; *L*, negative bus; *M*, armature of d-c machine; *O*, short circuiting switch.

FIELD SWITCH. Another modification of the knife switch that is frequently employed in opening and closing field circuits is the field discharge switch, Fig. 3. This is usually a two-pole switch with an auxiliary jaw and blade for cutting in a discharge resistance across the field terminals before the circuit is opened by the switch. By connecting this discharge resistance across the field, the electromagnetic energy in the winding is dissipated in the resistor instead of causing undue strain on the insulation of

the field, as would be occasioned by attempting to open the field circuit suddenly without any protective resistance.

FIELD TRANSFER. Where it is necessary to transfer the field circuit of a machine with its rheostat without opening the circuit, a transfer switch can be utilized to advantage. This switch usually has blades of the rocker type and makes connection from the middle contacts with the upper set of contacts before opening on the lower set, thus transferring the circuit through the rheostat and field between excitation buses without opening the field circuit, with but a momentary paralleling of the excitation buses.

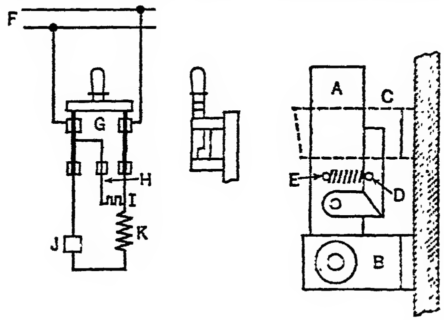

Fig. 3. Field Switch—Quick Break Attachment

A, switch blade; *B*, hinge jaw; *C*, break jaw; *D*, quick-break attachment; *E*, spring to actuate quick-break attachment; *F*, excitation bus bars; *G*, auxiliary switch blade; *H*, auxiliary switch jaw; *I*, discharge resistance; *J*, field rheostat; *K*, field of generator.

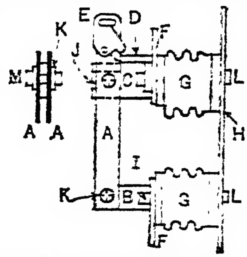

Fig. 4. Disconnecting Switch, Standard Duty

A, double blade embracing switch posts *B* and *C*; *B*, hinge post, sweated and pinned, or riveted, in block *F*; *C*, break post, slotted at *J*, attached at *B*; *D*, stationary part of latch; *E*, movable part of latch, pivoted on blade *A*; *F*, switch blocks bolted to inserts in porcelain insulators *G*; *G*, corrugated porcelain pillars with cemented-in inserts; *H*, steel base for switch; *I*, bolts for attaching blade blocks *F* to inserts in insulators *G*; *J*, slot in break post *C* for cross bolt *M* of switch blades *A*; *K*, slotted spring washers; *L*, bolts for attaching steel base *N* to inserts in insulators *G*; *M*, cross bolt for switch blades *A*.

Where there is a possibility that there may be a considerable difference in voltage between the two sources of d-c supply to the rheostat and field circuit, field transfer switches are provided with auxiliary jaws and limiting resistors so that, in the middle position, when the field circuit with its rheostat is connected to both sources of excitation, a certain amount of resistance is placed between these two sources to limit the interchange of power.

DISCONNECTING SWITCHES. Another frequent use for knife switches is to employ them as disconnecting switches for the purpose of isolating oil circuit breakers, lightning arresters, and other devices in the high-tension circuit to permit the more ready inspection of such equipment. These disconnecting switches are not supposed to open the circuit when any appreciable amount of current is flowing, although certain modifications, provided with a suitable arcing horn, can open the charging current in the transmission line, the magnetizing current of transformers, or a relatively moderate amount of power at high voltage.

For indoor use up to 2500 volts, the disconnecting switches are usually mounted on bases of ebony asbestos, marble, or similar material; for higher voltages it becomes necessary to use porcelain pillars which usually are mounted on metal supports. A typical switch is illustrated in Fig. 4. Many of the earlier disconnecting switches were made without latches and depended solely on friction to maintain their blades in the closed position, but it was found that the repulsion effect due to very heavy short circuits frequently caused such switches to blow open, and latches are now usually provided for the more important installations.

OUTDOOR DISCONNECTING SWITCHES. These differ from the indoor principally in the use of shell-type insulators instead of the pillar type, the use of non-rusting bolts and fittings, and frequently the provision of a sleet hood for the outdoor switch jaws or some special form of contact designed to crush the ice formation as the switch closes. Most disconnecting switches are operated by means of hook sticks, where single-pole operation is satisfactory and the switches can be mounted within ready reach of the attendant. Gang operation, either manual or electrical, is used in many installations, particularly at higher voltages.

For outdoor service, the disconnecting switch can be supplied for inverted mounting, upright mounting, or sidewall mounting. Arcing horns are provided when the air-break

switches are required to open an appreciable amount of current. If gang operation of the outdoor disconnecting switches is desired, disconnecting switches of the type shown in Fig. 5 can be utilized to advantage as plain disconnects for isolating the apparatus, opening all phases at one time. This switch comprises several complete poles all operated by a single mechanism, the tilting insulators on all phases being mounted on a common square shaft. Modifications of the tilting switch are made for voltages up to 73 kv by employing the multi-unit type of built-up porcelain supports with base, mechanism, contacts, and other features suitable for the clearance needed at such a voltage.

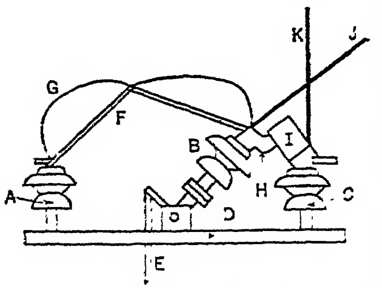

Fig. 5. Gang-operated Disconnect Switch

A, hinge-jaw insulator; *B*, moving insulator; *C*, break-jaw insulator; *D*, base; *E*, operating mechanism; *F*, Hinged-switch mechanism; *G*, shunt to hinge-mechanism; *H*, movable main contact; *I*, stationary main contact; *J*, movable arcing horn; *K*, stationary arcing horn.

INSTRUMENT SWITCHES. Switches of the drum type (see page 12–33) have practically superseded all other types for use in connecting one instrument to any of several circuits and for making the multipoint connection required in synchronizing generators. These switches are so arranged that the turning of the switch handle operates a shaft on which are mounted various contact-making segments. These segments make or break contacts with stationary fingers mounted on an insulated base.

CONTROL SWITCHES. Drum-type switches are designed for the control of electrically operated switches and circuit breakers, rheostats, engine and turbine governors, feeder potential regulators, etc. In general, control relays are operated directly from such control switches in order to handle such heavy operating currents as may be met with in the case of switches and circuit breakers (see Arts. 11 and 12). These control switches are essentially multicircuit double-throw switches. They are usually provided with large pistol-grip handles, and have a spring return mechanism which causes the switch to return automatically to the off position when released from the operating position. They usually have a mechanical marker to indicate the last operation of the switch. Suitable indicating lamps can be used in conjunction with control switches to obtain electrical indication of the position of circuit breakers or other devices.

Very compact control switches, occupying a panel space 2 $^3/_8$ in. square have been developed primarily for miniature switchboards. Indicating lamps $^3/_4$ in. diameter outside of fittings are also available, and very compact instruments are used to round out the design which permits controlling circuits 6000 ft. away over telephone cables.

4. FUSES

Metal strips or wires which open electric circuits by melting or fusing when the current reaches a predetermined value are called fuses. They were used in the earliest electric plants to furnish automatic protection for feeder and generator circuits. Their use is now confined largely to low-voltage distribution circuits and to the protection of small pieces of apparatus such as motors and small-capacity transformers.

All fuses have an inherent time element feature owing to the fact that the current must heat the fuse metal up to its melting temperature. This time varies with the size and type of fuse, the large ones in general taking longer to reach their fusing temperature than the smaller ones, on account of their thermal capacities. There are three designs of fuses in general use, namely, the open or link, the expulsion, and the enclosed types. Enclosed fuses were developed to replace the open fuse; they comprise, in the broad sense, types using links enclosed in an insulating container for safety. Enclosed fuses include various types of expulsion fuses, De-ion fuses, powder-filled fuses, oil-immersed fuses, liquid spring-operated fuses, etc.

OUTDOOR EXPULSION FUSES. Fuses of the type indicated in Fig. 6 are available for voltages up to 115 kv and currents up to 200 amp. The interrupting capacity of this line of fuses range from 7000 amp at 4500 volts to 400 amp at 115,000 volts. These fuses are so placed, in open-end tubes, that the fuse, when melting, generates arc gases. It was originally thought that the gas expansion caused a strong blast through the tube, blowing out the arc and so clearing the circuit. Later investigation seems to prove that the arc suppression is due to de-ionizing action similar to that described in connection with the De-ion circuit breaker (see Art. 8).

In applying such fuses, care must be taken to fuse them to such capacity that they will blow on short circuits but not on overloads, as gradually increasing overloads do not volatilize the metal fast enough to secure the proper explosive action.

De-ion power fuses are now available in current ratings up to 200 amp, for voltages of 7500, 15,000, and 23,000 volts with a three-phase interrupting capacity of 325,000 kva at 7500 volts, 500,000 kva at 15,000 volts, and 600,000 kva at 23,000 volts. This fuse is

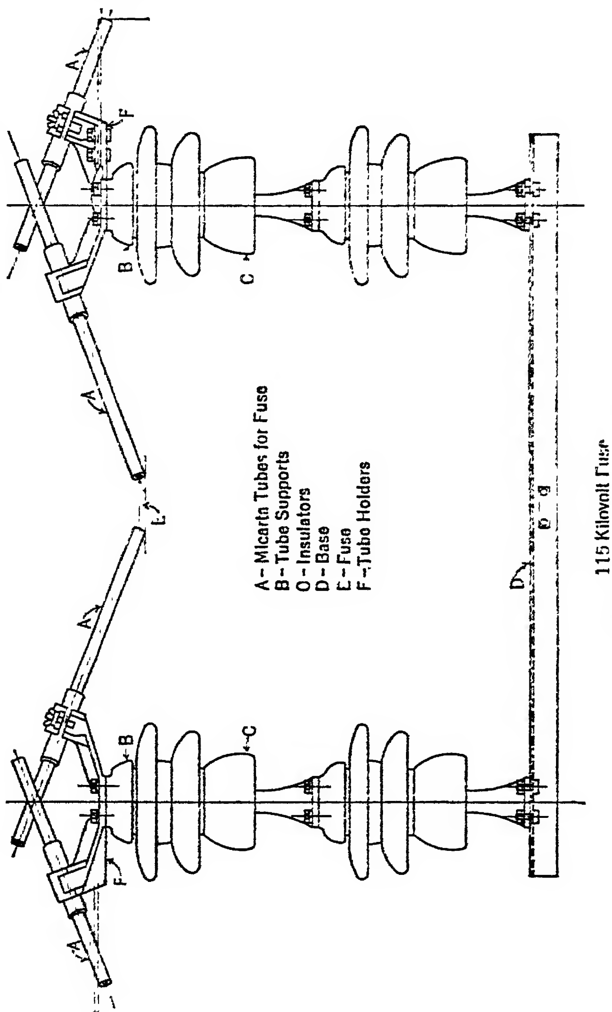

FIG. 6. 115 Kilovolt Fuse

arranged for outdoor service but may be used indoors without change. It is equally effective at rupturing high or low current, and its speed of operation is very fast. The fuse can be renewed at the installation; it is of the dry type, the de-ion chamber being lined with inert boric acid in molded dry form. The material of the wall of any fuse determines the nature of the gases expelled from the tube into the atmosphere, and the boric acid lining gives off an inert boric oxide and water vapor; the water in the form of vapor provides one of the most efficient de-ionizing media known.

CARTRIDGE-TYPE ENCLOSED FUSES. These fuses were so made at first that it was necessary to return them to the manufacturer to have the cartridge refilled after the fuse had blown, and this practice is still followed on ratings of 200 amp or larger. Certain standard dimensions and types of contacts have been adopted for various sizes up to 600 amp at 250 and 600 volts. Up to 60 amp the ferrule type of contact is used as shown in Fig. 7, and blade contacts shown in Fig. 8 are used for fuses from 61 to 600 amp.

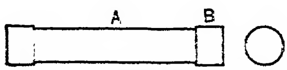

Fig. 7. Enclosed Fuse with Ferrule Contacts

A, fuse tube, fiber; *B*, metal ferrule contacts.

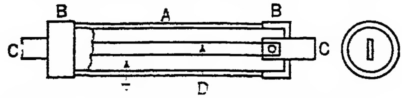

Fig. 8. Enclosed Fuse with Blade Contacts

A, fuse tube, fiber; *B*, metal ferrules; *C*, blade contacts; *D*, fuse strip; *E*, filler, gas-absorbing material.

Rupturing capacity up to 250-volt and 600-volt cartridge fuses, as determined by test, shows that on large systems the circuit characteristics should be such as to limit the maximum overload current passing through the fuse to approximately 10,000 amp at the rated d-c voltage.

REFILLABLE FUSES. These were developed to permit the user to place a new fuse in the old cartridge. Two general classes of renewable fuses have been developed, one in which the renewal element is a bare link without any powder filling, and one that has the fusible element enclosed in a powder-filled tube. Some of the features of one type of enclosed fuse are shown in Fig. 9, employing the drop-out type of bare link fuse.

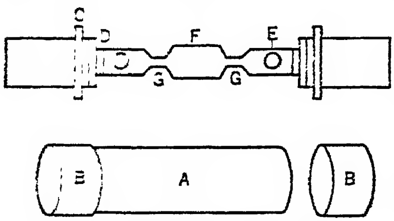

Fig. 9. Renewable Cartridge Fuses

A, hard fibre tube with threaded ends; *B*, removable metal caps vented; *C*, removable washers arranged for venting; *D*, fixed washers arranged for venting; *E*, fuse attachment bolt; *F*, drop out link of fuse; *G*, melting points of fuse.

High-voltage cartridge fuses have been applied successfully for several years, giving excellent protection for power circuits of low capacity. They are particularly useful for opening short circuits within their rupturing capacity, but it has been found that the fuses of this type for 4500 volts and above will not open low overloads at their rated voltage, as the gradual heating of the fuse material tends to reduce the de-ionizing effect of the powder filler, and the arc-gas pressure, as a result, is not sufficiently high. This effect somewhat limits their field of usefulness, but they can be employed satisfactorily on potential-transformer circuits where low-voltage fuses will take care of usual overload conditions. The high-voltage cartridge fuse has a current rating of two or three times full load of the potential transformer, and will blow only on a transformer breakdown or a defect in the wiring, giving essentially short-circuit conditions. Although these fuses have inherently a relatively high interrupting capacity

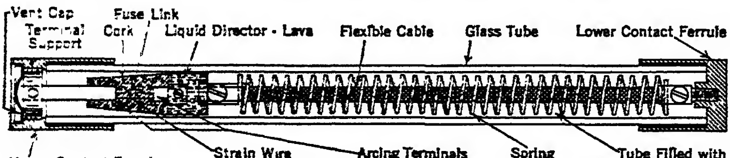

Fig. 10. Schweitzer–Conrad Fuse Construction

they are frequently used in stations where the short-circuit current would exceed the fuses' rupturing capacity. For such cases, current-limiting resistors are employed, these being designed to be placed in series with the potential-transformer fuses where the short-circuit amperes in the station or power source exceed the interrupting capacity of the fuses.

LIQUID FUSES. Liquid fuses, such as the so-called carbon tetrachloride and oil-immersed fuses, have a relatively high first cost but meet a demand for overload and

short-circuit protection, on the higher-power circuits, better than the cartridge type. In the first type of fuses (cf. Fig. 10), available from 7500 to 132,000 volts, a glass tube is used and the fuse is immersed in a liquid having a freezing point of —65 deg fahr. Carbon tetrachloride is non-inflammable and one of the best fire-extinguishing mediums known, with a very high dielectric strength. The fusible element is in two parts: one is of low resistance and low mechanical strength to carry the normal current: the other is a high-resistance element of high tensile strength attached to a spiral spring. When the low-resistance element melts, the current is shunted to the high-resistance element, and its melting permits the spring to separate the contacts rapidly and open the circuit. These fuses are not refillable in the field, and this drawback and their higher price frequently lead to the use of other types such as the cartridge or the expulsion type.

OIL-IMMERSED FUSES. These are made at present only up to 15,000 volts, but they meet a demand for a fuse with relatively high rupturing capacity and refillable features. Owing to the necessity for an oil tank, oil terminals, and bushings, their cost is comparable to that of an oil breaker. They are usually arranged with the equivalent of a movable cover carrying the fuse in suitable clips so that the fuse can be safely handled. When the cover is closed the fuse is automatically connected to the main terminals under oil. This device is particularly suitable for subway applications, but most people prefer to pay a little more to obtain an oil circuit breaker, or are satisfied with a cheaper type of fuse.

5. RESISTORS

Resistors (see also Section 4, Arts. 1–4) are used for starting and regulating the speed of series-, shunt-, and compound-wound d-c motors, and wound rotor a-c motors with either manual or magnetic controllers. Starting and speed regulation are obtained on the d-c motors by varying the resistance in series with the motor armature and on the a-c service by varying the resistance in series with the motor secondary. Resistors are also used for grounding the neutrals of generators or transformer banks. They are designed to pass enough current, if a line wire becomes grounded, to trip out the highest set circuit breaker. Where a number of generators or transformer banks are operated in parallel it is customary to ground only one generator neutral at a time to avoid circulating currents. Resistors are designed to comply with the A.E.S. and when correctly applied will operate with an average temperature rise not exceeding 350 deg cent.

Resistors for d-c motors have one or more frames usually connected in series. All standard a-c wound rotor motors whether for two- or three-phase circuits have their secondaries wound for three phase. The resistors for each phase usually consist of 1, 2, 3, or multiples of 3, frames of tubes or grids. Secondary resistors for a-c motors are designed *for star connection. Resistors for manual controllers may be connected with all three* secondary phases closed or one secondary phase open on the first point of the controller. A lower value of torque is obtained with the single-phase connection. Resistors for magnetic controllers are connected with all three phases closed in the secondary on the first point.

TYPES OF RESISTORS. For motor control service, resistors are made of various classifications, depending on percentage of full-load current and the length of time that they are to be maintained in service, and are built in various forms.

Flat Units for resistance elements consist of a molded flat core of vitreous material on which the resistance wire is wound; the surface is then coated with a special cement and baked. Thus the resistance material is protected from injury and made proof against moisture. This type is frequently used in connection with small field rheostats.

The plate-type resistance element utilizes coils attached to a circular base of insulating material. The coils are either covered with insulating, heat-conducting cement and baked, or are enclosed in a ventilated iron case. Suitable contacts and a switch arm are provided.

Cylindrical Units usually have a core of asbestos tubing or sometimes a metal tube coated with an insulating material such as enamel. A wire of low temperature coefficient is used. The tube with the winding is covered with an insulating compound, and porcelain bushings or metal rings are placed on the end. The units are then thoroughly baked and mounted rigidly on a frame or in a suitable case. The coating protects the resistance material from mechanical injury, forms a good conductor of heat, and in case of a burnout prevents appreciable arcing and unwinding of the wire.

Edgewound Resistors like that in Fig. 11 consist of a special non-corrodible alloy ribbon wound edgewise around a porcelain insulator which in turn is reinforced by a flat steel support fastened through it. Both ends of the steel support are notched so that any unit can be removed from the frame without disturbing any others. Because of its unique

construction the resistor tube is almost unbreakable. Shattering of any section of the porcelain tube has no effect whatever upon its operation as the ribbon requires only a fractional part of the support which it receives. The construction of the edgewound resistor provides excellent radiation, eliminating the possibility of localized heating.

Cast Grids are available in the different types indicated in Fig. 12. The L type is designed especially for panel and wall mounting; the 5-in., 8-in., and 14-in. grid plates are intended for floor or angle iron mounting. If the current-carrying capacity required is known, the grid having the proper cross-sectional area can be selected from standard designs. This line of grids covers a range of capacity of 18 to 346 amp per grid. By paralleling units any desired current rating can be obtained.

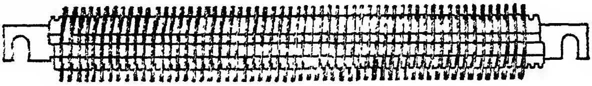

FIG. 11. Edgewound Resistor

Before being assembled the grids are dipped in aluminum paint, which serves as a protective coating against corrosion both during storage and after the grids have been placed in service.

The grids are mounted on three mica-insulated tie rods which are suspended between end frames that may be tightened without disturbing the length of the resistor. The end frames for the L-type, 5-in., and 8-in. grids are galvanized bent sheet steel; the 14-in. grids are of cast iron. The general construction is indicated in Fig. 13.

Where cast grids are used to ground the neutral of a high-tension transformer bank on an outdoor installation it is desirable to have the grids suitable for use without any covering. Various schemes have been tried, but the heating of the resistance tends to crack off the weatherproofing. It is quite possible that some nickel-iron alloy or stainless-steel compound will be found capable of standing the weather without deterioration, even when the grids are heated by the passage of ground current to a fault.

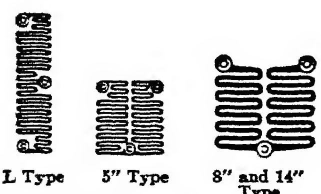

FIG. 12. Cast Grid Resistor Construction

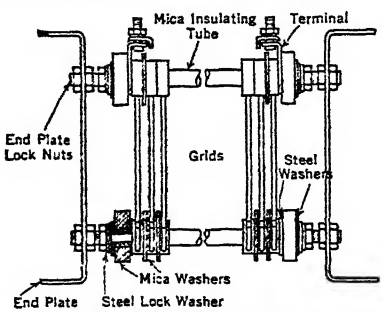

FIG. 13. Grid Resistor Assembly

CAPACITY OF RESISTORS. Wire wound resistors mounted directly on the stationary contact studs are used for the smaller sizes of rheostats; edgewound non-breakable and non-corrodible resistors are used on the intermediate size, and grid resistors on the largest size.

Capacity of grid resistors depends largely upon the ventilating space. The frames should never be stacked more than 4 ft high, and, when space is available, each frame should be separated from the next by approximately the width of the end frame. Frames may be mounted on the floor, platform, or wall, but the grids in all cases must be in a vertical position.

For high voltages, particularly for temporary testing in connection with generating equipment, water rheostats of different types have been evolved, consisting of two or more electrodes dipping down into the water by varying amounts. In some a continuous flow of water is provided; in others the water is maintained in a barrel or other container.

FIELD DISCHARGE RESISTORS. Resistors are used for protecting the field windings of large motors and generators. When the field switch is opened, the voltage induced in the winding may be sufficient to puncture the insulation, and to avoid this a discharge resistor is connected in parallel with the field at the time the circuit is opened. This resistor allows the energy to be dissipated without increasing the voltage to a dangerous

degree. A suitable field switch with a discharge clip should be provided. The resistance of the resistor should usually be slightly greater than that of the field with which it is to be used.

CURRENT-LIMITING RESISTORS. Resistors are used in series with potential transformer fuses to limit the current, in case of short circuit, to such values as the fuses can interrupt safely. This means that potential transformer fuses when in series with the current-limiting resistor can be applied to any system, irrespective of kva rating, as long as the voltage rating is not more than that of the fuses or resistors. The resistors are of the wire type and consist of coiled nickel-chromium wire wound in grooves on a heavy wet-process porcelain tube. An iron casting is the means of support for these resistors and is cemented to each end of the porcelain tube. There is no covering surrounding this resistor, thus allowing easy inspection and radiation of the heat without the danger of burning or charring covers. These resistors are frequently mounted in connection with the potential transformer fuses (Fig. 14), and they have resistances varying from 4.5 ohms for 2500-volt service to 225 ohms for 25,000-volt service.

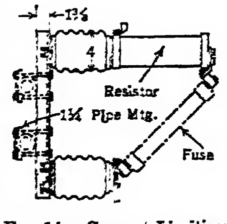

FIG. 14. Current-Limiting Resistor and Fuse

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CIRCUIT BREAKERS

Circuit breakers are devices used to open electric circuits by the mechanical separation of contacts that are normally held closed by a latch, toggle, or similar mechanism. Usually they are built to operate automatically under predetermined conditions. Two main types of circuit breaker are in use in the United States: one in which the circuit is opened in air, known generally as the air-break circuit breaker; and the other where the circuit is opened in oil, known as the oil-break circuit breaker. These two terms are usually shortened to "air breaker" and "oil breaker."

6. CARBON BREAKERS

The majority of air breakers protect their main contacts from burning by the use of auxiliary carbon contacts which take the final arc so that an air breaker is usually a carbon circuit breaker. Supplementing the pair of copper contacts, which carry the current when the breaker is closed, a pair of carbon contacts is provided in parallel with the copper ones, and the circuit is transferred from the copper to the carbon by having the copper contacts open first, leaving the carbons to take the final arc. Carbon is used for the final contacts because, under the action of the arc, it burns clean whereas copper contacts will either pit or form globules which would adhere to the surface. Also the comparatively high resistance of the carbon is advantageous.

The *main elements* in the normal carbon breaker comprise the two stationary contacts, to which the leading-in and leading-out terminals are attached, and the movable contacts, which bridge between the two stationary ones. The stationary contacts consist of blocks of copper against which the moving contacts are pressed, when the breaker is closed. For the smaller sizes of breakers, the round studs are electro-brazed to this copper block, the size of stud depending upon the capacity of the breaker. The movable-contact elements of air circuit breakers and many oil circuit breakers are laminated brushes in a U-shape built up from thin strips of copper. The ends of each lamination make direct contact with the stationary blocks.

The main features of a carbon-break circuit breaker are shown in Fig. 1. The main brush is laminated and is made by giving the extra-hard copper-strip laminations a set to a smaller radius than they have in the finished brush. This causes the innermost lamination of the brush to press by spring action against the inside block, and each of the other laminations in a similar manner presses against the lamination in front of it. Each lamination therefore makes an independent contact at each end on the stationary contact blocks. Owing to the elliptical form of the brush, the ends of the laminations bear with heavy contact pressure against the contact blocks. The laminations have a substantial

sliding motion relative to the contact blocks during the opening and closing of the breaker. This sliding contact breaks up the oxide film that may have accumulated.

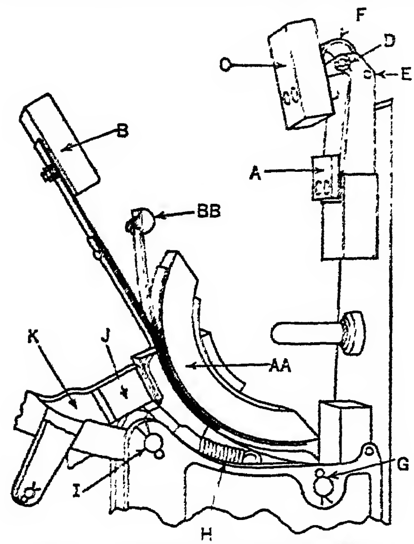

Fig. 1. Main Features of Air Breaker

AA, main brush; *BB*, arcing tip; *A*, copper arcing plate; *B*, moving carbon; *C*, stationary carbon; *D*, pin; *E*, carbon holder; *F*, copper shunt; *G*, pivot rod; *H*, springs; *I*, phosphor-bronze pin; *J*, phosphor-bronze clip; *K*, projection on toggle mechanism for large breakers.

The *magnetic blowout* is used in some circuit breakers to assist in rupturing the arc by blowing it out into a much longer path (cf. Fig. 2). When the contact pieces *A* and *B* are separated, the arc that is drawn is extended by the magnetic field *C* into some such shape as *D*. For heavy d-c railway work the magnetic blowout breaker has been used extensively. In such a breaker the final arc occurs on auxiliary contacts, usually placed in an intense magnetic field. For the higher voltages, such as 3000 volts direct current, and for heavy currents, the plain carbon breaker becomes impractical because of the tremendous short-circuit arcs, and some sort of blowout device or De-ion structure is necessary.

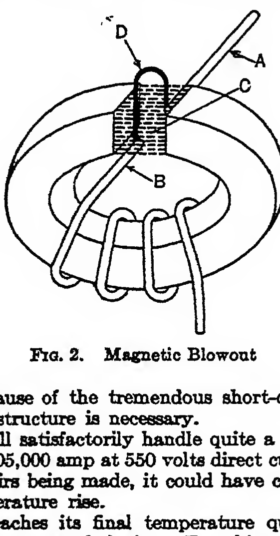

Fig. 2. Magnetic Blowout

Carbon breakers without a magnetic blowout will satisfactorily handle quite a range in load. In a test, a 4000-amp breaker interrupted 105,000 amp at 550 volts direct current and was in such good shape that, without any repairs being made, it could have carried full-load current continuously under normal temperature rise.

RATING. Inasmuch as the circuit breaker reaches its final temperature quickly with steady current load, it is necessarily a maximum rated device. For this reason, current ratings given in manufacturers' publications for all carbon circuit breakers are maximum rated, based on the allowable temperature rise that is reached after a continuous run of approximately one hour or more at the rated current.

Air circuit breakers are suitable for service in either d-c or a-c circuits, but the question of heating and the difficulty of handling heavy a-c currents particularly on 60 cycles, without a high inductive drop, places a natural limitation on the a-c equipment. For circuits of 3000 amp and above it becomes almost essential to interleave the conductors of various phases to reduce the inductive drop to a minimum, and an effort is usually made to keep the current in a-c circuits below this figure. For the few cases where this current limit must be exceeded, carbon breakers are available with several sets of brushes spaced well apart for ventilation. For 25-cycle service air breakers are available up to 10,000 amp, and for 60-cycle service up to 7000 amp, this last breaker having a d-c rating of 14,000 amp.

RANGE. A typical calibration range for automatic overload trip is approximately 80 to 160 per cent of the 30-deg cent rise ampere rating. Breakers can readily be set to trip at any point within this range. There are often special conditions in circuits which require certain features of functioning on the part of the breakers in addition to their normal action of opening at a predetermined load. These features can usually be added as attachments. The following are some of the more important ones, their function being practically denoted by their name: shunt trip, inverse time limit, undervoltage trip, reverse-current trip, field-discharge attachment, lock-out, and signal lights.

OPERATING MECHANISM. As a rule, carbon breakers are held closed automatically by a toggle, trigger or latch. Various trip mechanisms are constructed to dis-

engage the holding device and to permit the breaker to open. Manual closing is the ordinary method employed, but electrical operation is used where power operation is required. Ordinarily a cylindrical solenoid magnet as shown in Fig. 3 is furnished, located beneath the breaker, or a motor is employed. The solenoid with d-c excitation is standard with most companies because it is simpler in construction, more reliable in operation, and more easily kept in repair; the d-c solenoid is much lower in cost than the a-c solenoid so that alternating current is used only when no direct current is available. The adaptation of the Rectox rectifier to circuit-breaker control has permitted the furnishing of a rectifier at each solenoid and the employment of an a-c control circuit running to the breaker, rectification by means of the Rectox device and supplying direct current to the solenoid.

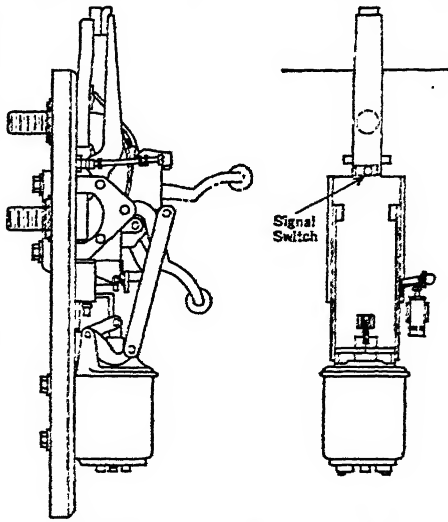

Fig. 3. Solenoid-operated Air Breaker

MOUNTING. Carbon breakers are mounted on panels, walls, or pedestals. The smaller breakers are usually placed on panels with combinations of other equipment needed on the switchboard. Medium-sized breakers are also panel mounted, but the larger ones are often pedestal mounted. The pedestal mounting is used particularly where heavy service conditions are encountered; it is especially desirable for high-speed breakers on account of the vibration caused by their quick trip action.

7. HIGH-SPEED AIR BREAKERS

High-speed breakers were developed for use in connection with heavy d-c railway electrification, particularly at 1200, 1500, and 3000 volts, in order to have a breaker that

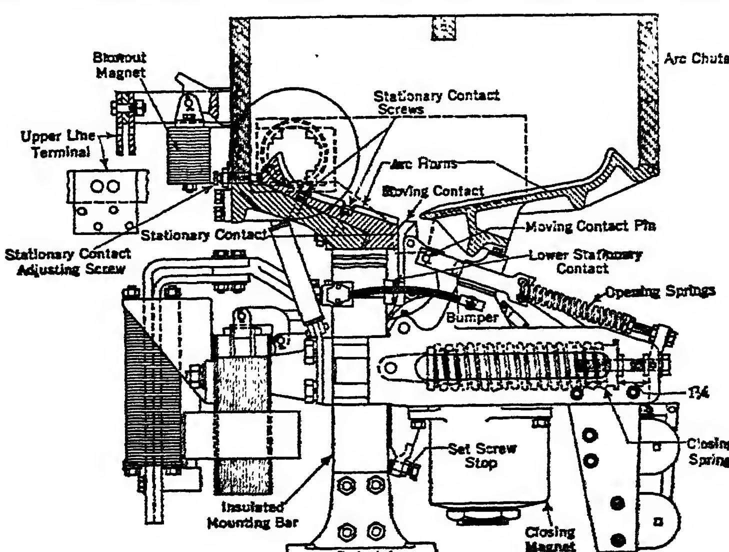

Fig. 4. Sectional Elevation of a High-speed Breaker, Showing Arrangement of Parts

would open the circuit in a few thousandths of a second to prevent a flashover on the commutators of d-c machines. The main idea of the high-speed breaker is to interrupt high-value, fast-rising current in the minimum time. To assist in obtaining a high-speed action, the usual auxiliary arcing contact is omitted and the arc is drawn on the main current-carrying contact. As shown in Fig. 4 the design of the breaker utilizes a movable contact, made of solid copper, bridging two stationary contacts, and held closed by the action of the electromagnet. Inside of the breaker the path of the line current, as indicated in the schematic diagram, Fig. 5, is divided into two parallel circuits, *A* and *B*. Circuit *A* is stacked with iron punchings. Circuit *B* is comparatively non-inductive and passes through the magnetic circuit of the breaker holding-in coil in such a manner that the flux due to the load current neutralizes the flux of the holding magnet.

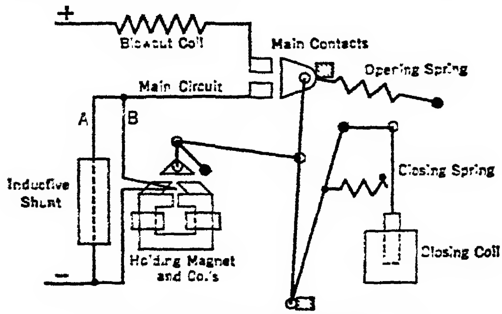

FIG. 5. Diagram of High-speed Breaker

Under normal-load conditions, the holding-coil flux predominates, and the breaker is held closed against the action of strong springs. The currents in the two parallel paths vary inversely as the resistance of the paths. Under short-circuit conditions, one path being inductive, the current divides inversely as the impedance of the paths. Therefore the non-inductive path *B* will carry a much larger proportion of the short-circuit current, and the holding coil flux will be reduced, allowing the breaker to be opened instantly at the high rate of acceleration caused by the action of the powerful springs, which were kept in tension by the holding magnet. Tests on the high-speed breaker indicate that its speed of action depends somewhat upon the characteristics of the machine with which it is being used, owing to the difference in rate of current rise, under short-circuit conditions. For short-circuit current values of 4000 amp upward and with current rises at the rate of 3,000,000 amp per sec and upwards, this breaker may be expected to limit the current rise in 10/1000 sec. This type of breaker has also been adapted for trolley wire protection in 12,000-volt 25-cycle railway electrification.

8. DE-ION BREAKERS

For several years, research work has been carried on in the development of an oilless circuit breaker suitable for a-c service indoors in order to eliminate the hazards incident to oil fires, gas explosions, etc. This has resulted in the De-ion circuit breaker which replaces a single long arc by a large number of short arcs in series and provides a means of extinguishing the short arcs.

The fundamental principle of this breaker is based on the laws of conduction of electrical energy in a gas, such as air, where, under certain conditions, the air may change in a very short space of time from a momentary condition of being a comparatively good conductor carrying current at a low voltage, to its normal condition of a comparatively good insulator supporting the full generated voltage of the circuit with the passage of very little current. It has been found that the thin layer of air immediately adjacent to the cathode regains its insulating qualities almost instantly after the current reaches zero, while the remainder of the arc path builds up its dielectric strength at a much slower rate. As the current wave in an a-c arc approaches zero, and for a short time immediately following zero, the factors producing new ions in the arc paths have practically ceased their activity. Electrons are expelled from that portion of the air space immediately adjacent to the cathode so that a thin layer of air immediately adjacent to the cathode becomes de-ionized in an exceedingly short space of time. The first 250 volts, peak value, or about 175 volts, root mean square value, are borne almost entirely by this cathode layer, and the ability to withstand this voltage is attained in a fraction of a microsecond.

Fig. 6 shows, in a diagrammatic manner, the main features of the De-ion circuit breaker. There are two main stationary contacts supported in, but insulated from, the supporting framework. There is a main moving brush contact controlled through the operating mechanism linkage which is insulated, and which is operated from the solenoid and has

all the usual attachments for electrical closing and tripping. Mechanically and electrically connected to the main moving brush by means of the supporting conductor is the moving arcing contact provided with a horn which, in the closed position, makes contact against the stationary arc-tip. This stationary arcing contact is connected through the main current blow-in coil and the lead to the upper stationary contact. In the upper part of the breaker are de-ionizing plates and certain magnets.

When the De-ion breaker is opened while carrying current, the main moving brush leaves the main contact slightly before the moving arcing contact parts from the stationary contact, so that momentarily all the current that has previously been flowing from the main contacts flows from the lower main contact through a flexible shunt to the moving brush, through the connections to the moving arcing contact, through the stationary arcing contact, and through the main blow-in coil and the lead to the upper main terminal. The arc is immediately transferred by the action of the blow-in coil to the main arcing horns and up the inclined surface of these into the de-ionizing chamber immediately above. This single long arc between terminals, when blown into the de-ionizing chamber, is immediately broken up into a number of small arcs between the de-ionizing plates. At about a dozen places the radial field coils are in shunt connection across gaps and draw currents proportional to the drop across these gaps. With the reactance of the coils in shunt with the arcs across the short plates, these arcs become unstable and are extinguished. The radial coils are then in series with the arcs in the adjacent main de-ionizing chambers and carry the full series current. The radial field coils are so connected that they will cause the arc to be driven in a rotating manner rapidly around inside the chamber. This de-ionizing chamber comprises a series of de-ionizing plates, magnetic plates, and radial field coils suitably mounted.

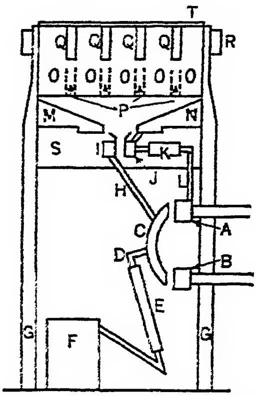

FIG. 6. Diagrammatic Sketch of De-ion Breaker

A and *B*, main stationary contacts; *C*, main moving contact; *D*, operating mechanism linkage; *E*, operating mechanism insulation; *F*, operating solenoid with attachments; *G*, supporting framework; *H*, connection of main moving contact to arcing contact; *I*, moving arcing contact with horn; *J*, stationary arcing contact with horn; *K*, blow-in current coil; *L*, connection from *K* to *A*; *M*, front main arcing horn; *N*, rear main arcing horn; *O*, de-ionizing plates; *P*, guide plates; *Q*, radial field coils; *R*, steel cylinder; *S*, steel plates in magnetic blow-in circuit; *T*, magnetic blow-in return circuit.

The stator plates clamped together give the effect of a larger number of very short gaps in series, each of these having its own cathode and anode and each gap having theoretically an insulating value of about 175 volts root mean square.

The De-ion circuit breaker is most effective as an a-c device. The de-ionizing chamber is applicable to d-c circuits up to a certain voltage limitation, and the advantages to be gained by such applications are not at the present time outstanding ones. The de-ionizing principle has been extended into several different classes of a-c circuit interrupters. Industrial contactors operating on this principle on voltages up to 550 volts have been in continuous service, and the de-ionizing principle has been made applicable to breakers up to 25,000 volts.

Small De-Ion Circuit Breakers are now available in ratings up to 600 amp at 250 volts direct current and 600 volts alternating current. They are made as one-, two-, or three-pole units in sizes from 15 amp up, corresponding to the ratings of the wires they are to protect. The trip mechanism is actuated by a thermal device calibrated to trip at 25 per cent overload. On the larger breakers an additional trip of the magnetic type is available. These breakers will trip satisfactorily in a circuit capable of developing 10,000 amp on short circuit. With the small breakers, the circuit voltage cannot actually force this amount of current through the breaker, but with the larger ratings the current can actually reach this figure. These breakers are more compact than the usual air-break type, they are completely enclosed, they cannot be held closed on overload, and they will stand repeated short-circuit tests within their rating of 10,000 amp.

9. MISCELLANEOUS AIR BREAKERS

The use of compressed gas for interrupting high-voltage circuits has been experimented with for many years, but it was not until adequate testing facilities were available in the larger manufacturing works that real progress was made.

The A.E.G. of Germany developed breakers, suitable for voltages up to 100 kv, that depended on a blast of air for blowing out the arc. The high-voltage circuit was made at the top of a tube through a terminal connected by the moving contact to the lower terminals of the bottom end of the tube. Operation was generally secured by an air cylinder fed from the same source of air pressure which gives arc rupture, and the mechanical operation of the device was so arranged that the movement of contact parts and valve actions assured air pressure simultaneously with or just previous to the parting of the contacts, so that there was no possibility of drawing a high-voltage arc without the necessary pressure for rupture being present. Air pressures up to about 225 lb per sq in. were usually available from the air reservoir.

For small stations involving only a few breakers carbon dioxide obtained from bottles of carbonic acid was used, in this way eliminating the special air compressor. The carbon dioxide was better as an arc-extinguishing means than the compressed air. A number of breakers with an arc kva rupturing capacity up to 500,000 were put in service for different voltages.

A live pot circuit interrupter using water was developed by the Siemens Schuckert Company of Germany, and breakers for service up to 500,000 kva rupturing capacity have been built. This breaker is provided with a fluid chamber divided into two parts, the inner chamber being filled with water or other liquid and the outer chamber being a water-storage space and air space to allow expansion of the gas when it passes through the throat bushing of the inner chamber. Thus pipes in the outer chamber are provided for leading the expanded moist gases safely away from the vicinity of the arc rupturing space. Arc interruption is secured by adiabatic expansion of the gases through the throat bushings into the outer chamber. This action causes a decrease in temperature and corresponding moisture condensation in the arc stream which reduce the movement of the electrons and increase the dielectric strength of the arc path. In accordance with one possible theory offered in explanation of the arc extinction this decrease in temperature is accompanied by corresponding condensation of moisture on the electrons thereby increasing their inertia and making breakdown of the space between the contacts impossible.

Various types of vacuum breakers have been built by different makers. In theory a vacuum breaker relies on absence of air or gas pressure and so is dielectrically opposed to other switching devices which use a blast of gas either generated by the arc itself or supplied from an outside source for arc rupture.

The Siemens Schuckert Company have done some development work on a circuit breaker using tanks full of resistance liquids. The contacts within these tanks are normally held together when the circuit is closed and are drawn apart when the breaker is open, introducing resistance progressively until the current is reduced to a minute value which is opened by the moving contact rising above the top of the liquid.

10. ELEMENTS OF OIL CIRCUIT BREAKERS

An oil circuit breaker may be defined as a device, other than a fuse, constructed primarily for interruption in oil of a circuit under infrequent and abnormal conditions. This definition excludes oil motor-starting devices with low interrupting capacities intended for frequent service. A description of a typical oil circuit breaker may be summarized by stating that it usually comprises one or more metal tanks, practically filled with insulating oil, having metal tops with insulating bushings through which pass conductors of copper rods which are attached to stationary contacts located below the surface of the oil. Movable contacts, operated from an external or internal mechanism, bridge the stationary contacts and complete the circuit when the breaker is closed, and are displaced from the bridging position when the breaker is opened, thus interrupting the circuit. Auxiliary contacts, readily renewable, are usually provided to take the final arc when opening, and to protect the main contact from burning.

Oil quenches the arc, when the contacts are drawn apart, by its cooling effect, and this quenching is most effective when the current passes through zero. The use of oil also permits smaller distances between live parts, and between live parts and ground, owing to its high dielectric strength. For this reason, an oil circuit breaker is usually of smaller dimensions than a corresponding air circuit breaker.

CONTACTS. Contacts of oil circuit breakers have two very important duties. When closed, they must carry the full-load current of the breaker with a small temperature rise, and they must carry for a few seconds, without deterioration, whatever amount of short-circuit current the system can pass through them. When opening or closing under load or short-circuit conditions, the main current-carrying parts must be so protected that they will not be burned or scarred sufficiently to prevent carrying full-load current again

with low temperature rise when the breaker is reclosed. On small circuit breakers the necessary protection is obtained by the special shape of the contact. On large circuit breakers the arcing contacts are separate from the main contacts.

Main Contacts of three kinds are in general use in American oil circuit breakers, namely, the solid butt type, the wedge and finger type, and the laminated butt type. Arcing contacts are generally provided to protect the main contacts from burning. They are mostly of the wedge and finger type, a lever roll-in type, and the bayonet type with plain break or quick break action. To insure arc extinction, five different schemes are in general use, namely, plain break, multibreak, explosion chamber, high speed, and De-ion grid. The General Electric Co. uses the plain break, oil blast or impulse, and explosion-chamber form. The Condit Co. uses the plain break and the multibreak form. The Pacific Electric Co. and the Kellman Co. use the multibreak contact. The Westinghouse Co. uses plain break and high-speed contacts and the De-ion grid.

Butt Contacts may be plain or laminated. Plain butt contacts consist of properly shaped stationary members engaging moving members, one of the members being resiliently supported by means of springs. This type is adaptable to low current breakers and is most frequently found on the high-voltage breakers. The laminated butt contacts are made in several forms, the elliptical form being the earliest and utilizing copper strip laminations. The application of the semi-elliptic brush is now limited to breakers of relatively low current-interrupting capacity or short-time rating because of the tendency of the magnetic forces to deform the brush. Various types of reverse brush contacts have been designed to overcome this difficulty. For heavier currents, non-welding alloys, plated on or inset, are used.

Wedge and Finger Contacts are shown in Fig. 7; they comprise one or more wedges,

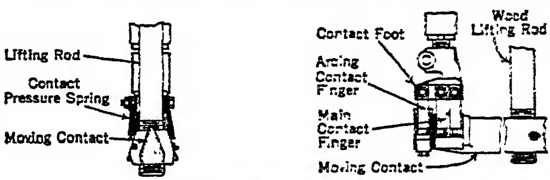

Fig. 7. Wedge and Finger Contact

usually on the movable element, which enter between the contact fingers when the breaker is closed. These contact fingers, which are arranged in pairs, are so constructed that they conform themselves to the wedge surfaces and present at all times a uniform contact pressure over the contact area.

Magnetic Blowout Effect in circuit breakers at voltages of 15,000, and below, has been found by experience and test to be very pronounced with heavy currents. Full advantage is taken of the magnetic loop (comprising the studs and moving contacts at each pole) for the maximum blowout action in all plain break breakers.

De-ion Grids, introduced in 1929 by the Westinghouse Co. for use on high-voltage breakers, furnish a method for keeping the oil in contact with the arc. The schematic relationship of the arc and the elements of the grid structure is shown in Fig. 8. The interrupting element or stack of grid plates, from which its name is derived, is attached at the end of the high-tension bushing. The grid is made up of a number of similar units, each unit in turn being made up of plate elements of insulating and magnetic material. Each individual grid plate carries a slotted opening, all of these registering in the complete grid to form a single deep and relatively narrow groove extending throughout its length. This groove is closed at one end of the grid but open for its entire length at the other end. As the contacts part on opening, the movable element passes downward through the groove and on to the end of the stroke well out of the grids, leaving an ample space for a layer of clean oil between the contact surfaces and the bottom of the grids to insure adequate insulation in the open position. The arc produced by parting of the contact is drawn and extinguished in this narrow groove, closed on all sides except for the opening at the inner end necessary to permit contact movement and the openings at the top and bottom of the grid. As the contact moves downward, towards the open position, the arc is drawn down through the air gap of the iron plate giving rise to a magnetomotive force, in the iron circuit and across the air gap, of one turn times the current in the arc. As the contact continues downward the same effect is produced in the next succeeding iron plate, and so on until the arc is extinguished. The arc is drawn between the parting contacts in the vertical, relatively narrow, deep groove formed by slots in the several plate elements of the grid and closed at the outer end. Plates of magnetic material are spaced at intervals through-

out the grid and so arranged as to form a partial return circuit and to provide a magnetic field when the arc is drawn through the air gap. This magnetic field operates to move the arc quickly toward the closed end of the groove. The grid being submerged in oil, the groove is filled with oil which cannot escape except through small openings at top or bottom, and which, for the short interval of time the arc exists, is for all practical purposes solidly entrapped. The arc moving towards the closed end of the slot is in effect being forced against a solid wall of oil throughout its entire length, resulting in a high rate of decomposition of the oil with an accompanying continuous and adequate supply of fresh un-ionized gas. This gas cannot escape, except through the arc stream, since the arc fills the open end of the groove. The driving power behind the arc forces the gas back through the arc stream, diluting it with un-ionized gas while current is flowing; the fresh gas acts as the de-ionizing surface after the current zero is reached. Numbers of tests

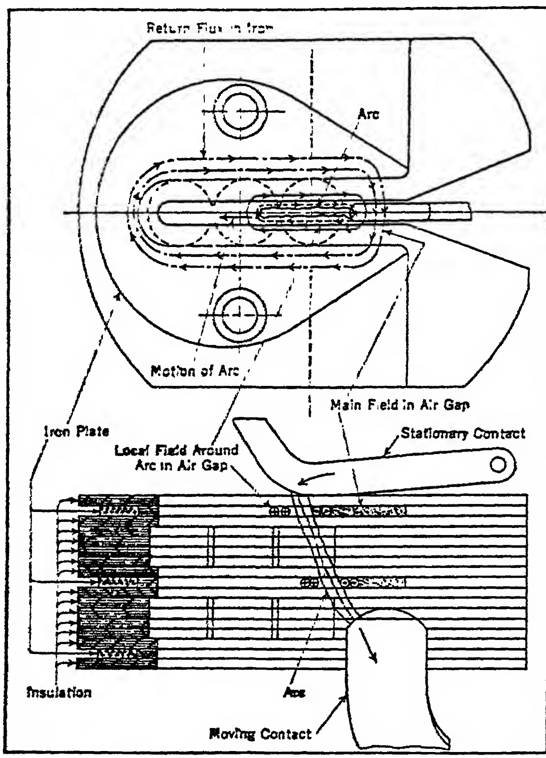

FIG. 8. Di-ion Grid Structure

have been made on large power systems which verify in actual practice the functioning of these breakers.

A porcelain tank breaker with greatly reduced oil volume is one of the promising lines of development for high-voltage breakers.

TANKS. The main purpose of breaker tanks is to hold the oil that submerges the contacts. Incidentally they dissipate the heat and enclose the working parts of the breaker. The type of tank is governed by considerations of the duty to be performed and of cost. Rectangular tanks are used for small breakers of moderate rupturing capacity. Circular tanks with dome-shaped bottoms are used for the largest-capacity low-tension oil circuit breakers, as this construction gives the strongest type of tank capable of withstanding the stresses frequently encountered when interrupting large amounts of power.

Mounting. Breakers of smaller size are built for mounting on pipe or angle iron frames. The frames are attached to the tops, and the tanks are supported by the tops to

which they are fastened. Tanks can be lowered for inspection or adjustment by special devices. Tanks of the largest sizes, principally for high-voltage service, are built for floor mounting, and access to the inside of the tank is usually secured by placing a manhole at the top.

Live Tanks. Although most oil circuit breakers are of the dead tank construction so that the tank may be grounded, the live tank construction indicated in Fig. 9 has been employed for many years on certain lines of breakers. The general design of the breaker has a form of inverted explosion chamber with main contacts in air. The explosion chamber consists of a steel cylindrical oil tank with insulated linings supported on a porcelain post, set in a base with clamp fittings and mounting bolts. Inside the oil tank is a system of baffle plates held in place and supported from the oil tank top. Separation chambers above the explosion chamber contain a quantity of quartz pebbles through which the gas has to pass in escaping. The oil condensed drains through the perforated disk and exhaust opening in the breaker top into the explosion chamber.

Mufflers are used in many breakers for scavenging the gases which are developed when the breakers are opened. The mufflers allow the gases to escape but prevent the throwing of the oil. These mufflers are usually arranged in the form of a labyrinth.

Tank Tops are not merely covers for oil receptacles; they also carry insulating bushings for the breaker studs and contacts, and often they carry the breaker mechanism and perform other functions. Where the leads through the bushings in the top carry heavy currents at 60 cycles, the possible heating of these tops has to be investigated very carefully. The heating of the tank tops is caused by hysteresis and eddy-current losses. To avoid these losses, breakers for the heavier currents utilize bronze inserts around the bushings or all-bronze tops for currents in excess of 1200 amp at 60 cycles.

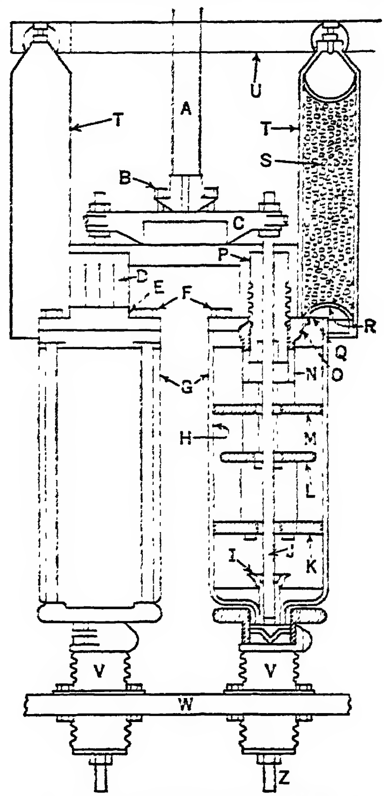

FIG. 9. Typical Live-tank Breaker

A, wooden lift rod; *B*, clamp attaching cross bar to lift rod; *C*, metal cross bar; *D*, main moving finger contact; *E*, main stationary wedge contact; *F*, tie rods; *G*, steel tank; *H*, tank lining; *I*, conical wedge arcing contact; *J*, rod for arcing contact; *K*, lowest baffle; *L*, middle baffle; *M*, top baffle; *N*, insulating ring; *O*, insulating top; *P*, insulating bushing; *Q*, exhaust opening; *R*, perforated disc; *S*, quartz pebbles; *T*, separation chambers; *U*, exhaust piping; *V*, foot insulator; *W*, base; *Z*, terminal rod.

BUSHINGS. Bushings are required in the normal design of top-connected oil circuit breakers for insulating, from the metal top, the copper studs that support the stationary contacts and carry the current to these contacts. These terminal bushings range in voltage requirements from 2.5 to 287 kv or higher and in current from about 200 to about 6000 amp. Up to 23 kv for small amounts of power, porcelain bushings can be employed; for higher voltages, oil-filled or condenser bushings are used.

Oil-filled bushings shown in Fig. 10 consist in general of metal tubes at the center and series of short insulating cylinders separated by radial disks to increase creepage distances. These cylinders and disks taper gradually from the flange toward the end. Oil is admitted through the top of the visual gage at the top of the bushing and can be removed through the oil drain at the bottom.

Condenser Bushings used on the high-voltage Westinghouse breakers are usually of the design shown in Fig. 11. The condenser bushing is made by rolling, pressing, and baking onto a conductor alternate layers of Micarta paper and metal foil. The bushing is thus made of concentric cylinders of insulation with a layer of metal foil between them. The area of the foil and the thickness of the Micarta cylinders are controlled to give suita-

ble capacitances and uniform voltage drops from the center to the surface. In the typical condenser bushing illustrated, the part outside of the tank is covered in a weather casing of porcelain, the space between the bushing and the casing being filled with an insulating compound. A portion of the bushing inside the tank under the oil and near the contact is protected by a porcelain arc shield. Bushing-type current transformers are employed with breakers of the high-voltage design; the copper conductor of this bushing forms a primary winding, the secondary winding and the iron constituting the bushing transformer itself. Because of the voltage gradient provided by the condenser bushing, it is possible to make a connection between the last metal foil, the one nearest to ground, and the tank and secure economically a reliable potential device to use in operating synchroscopes and certain relay coils.

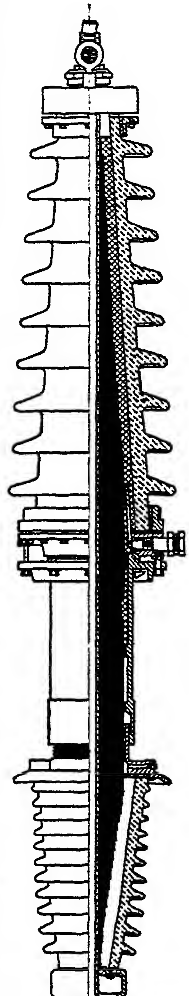

FIG. 11. Condenser Bushing

11. OPERATING MECHANISMS

Various methods are employed for operating oil circuit breakers. The smaller oil breakers are operated manually. For voltages up to 2500 volts, currents up to 1200 amp, and station capacities under 3000 kva, direct control is usual with the breakers mounted directly on the panel or panel framework. For higher voltages, heavier currents, or larger kilovolt-ampere capacity, the breakers are mounted apart from the panel, and connections between the panels and breakers are made by rods, bell cranks, and levers. With smaller sizes of manually operated remote-control circuit breakers, the latches and their release coils, by which automatic operation is secured, are mounted directly on the cover plate of the panel. The handle is in two parts to form a trip-free feature to prevent the breaker's being held in the closed position on overloads.

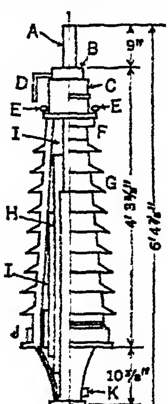

FIG. 10. Oil-filled Bushing. *A*, line terminal; *B*, metal cap; *C*, oil gage; *D*, air breathing tube; *E*, lifting eyebolts; *F*, top casting; *G*, porcelain shell; *H*, insulating barriers; *I*, oil space; *J*, bottom casting; *K*, oil drain; *L*, cable sleeve; *M*, cable.

Fig. 12 shows a single-handle cover plate such as is used with moderate-capacity breakers with two trip coils. In the larger sizes, the automatic latching details are located at the circuit breaker, thus taking the strain of the latched load off the panel and remote-control bell cranks and levers, and also reducing the mass to be accelerated at the time of automatic opening of the breaker. In American practice, mechanical remote control is practically always through a system of operating rods (usually pull), bell cranks, and levers.

Electrical Operation in some form is used when the distance between the switchboard and switching devices makes the application of hand-operated breakers questionable, or when the physical size of the apparatus is too great for convenient manual operation. Solenoid operation is the method usually employed in the majority of American plants; a typical solenoid mechanism is indicated in Fig. 13. This mechanism can be mounted in any desired location with reference to the breaker itself, and the pipes connected from the mechanism to the breaker can be attached to either the down-pull rod end or the up-pull rod end. All the newer solenoid mechanisms include a mechanical trip free feature which furnishes free opening of the breaker without the restraint of the inertia of the core. Owing to the greater simplicity of the d-c solenoid type of mechanism, it is normally employed, the current for this operation being obtained from a storage battery or through a rectifier like the Rectox from an a-c source of power in preference to using a motor mechanism.

Motor Mechanisms of the flyball type are used by various builders in installations where only a-c control circuits are available and, in certain cases, for d-c operation. Fly-

balls are used as they furnish a convenient means of translating rotary to linear motion and they permit the motor to get nearly up to speed before imposing any load on it, thus obviating the necessity of a motor of abnormally high starting torque. The mechanism

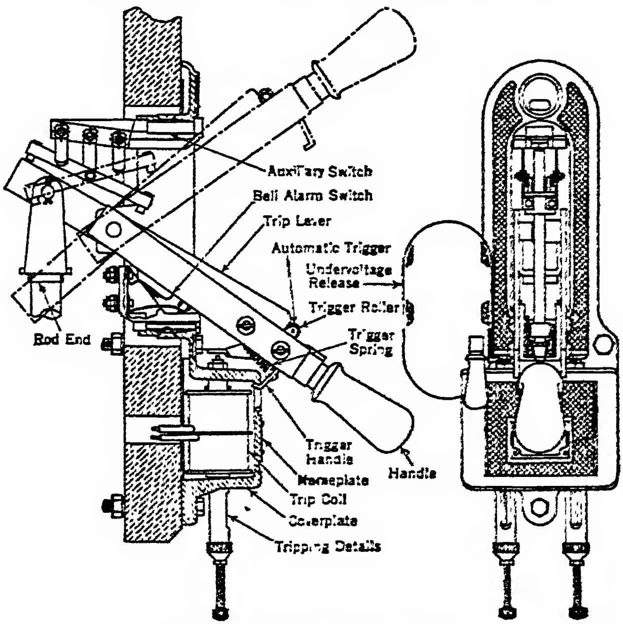

FIG. 12. Single-handle Cover Plate

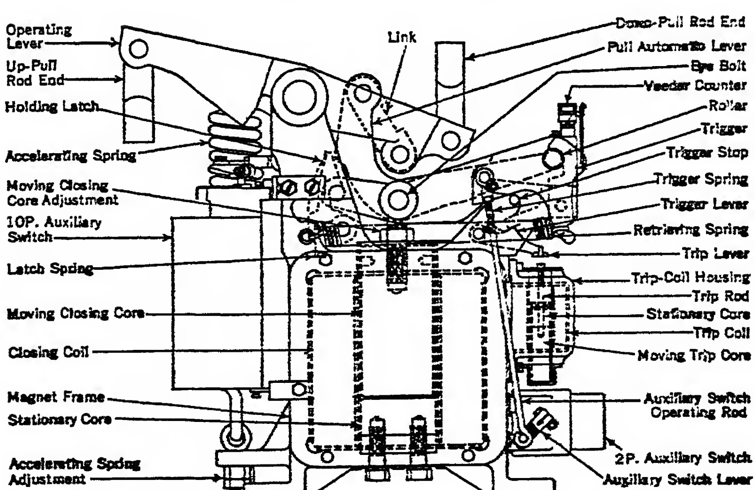

FIG. 13. Solenoid-operated Mechanism

consists of a motor-driven system of flyballs, sleeve shaft, and operating levers, mechanically trip free in action, for closing the breakers; a hardened-steel latch maintaining it in the closed position; a tripping device to disengage the latch automatically when the

tripping coil becomes energized; and an accelerating spring to insure positive fast opening of the breaker contacts upon release of the latch.

Outside Mechanism. The arrangement shown in Fig. 14 has been utilized in the older and smaller breakers as affording a simple and cheap method of operation. With this type of construction the steel lifting rods attached to the outside mechanism pass up through the top of the tank, and any gas formed in the breaker tank during operation passes out through the vent in the top provided for this steel lifting rod. This type of construction gives entire satisfaction for moderate capacity breakers and moderate

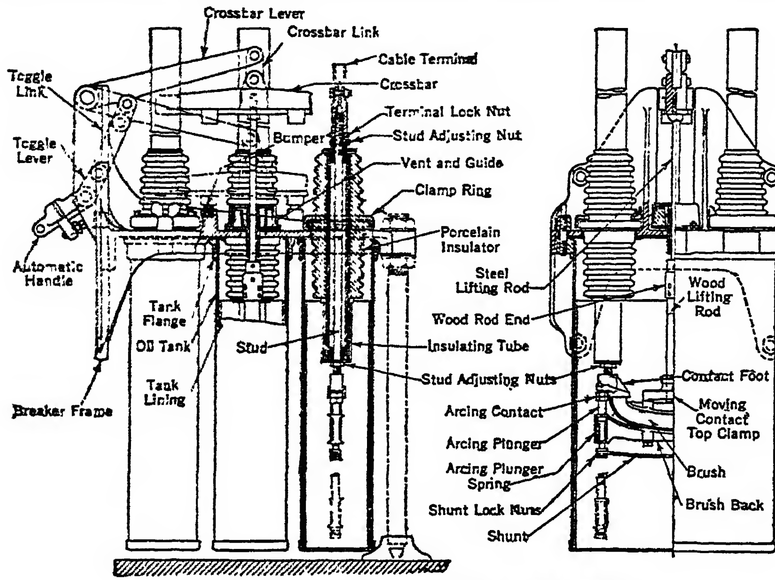

Fig. 14. Typical Oil Breaker with Outside Mechanism

voltages, but it is not so well adapted to breakers of higher rupturing rating and higher voltage as is the inside mechanism.

Inside Mechanism. With the modern design of breakers of moderate and large rupturing capacity the tops provide space for the installation of the breaker mechanism inside the breaker chamber, without appreciably increasing the overall height of the breaker structure. The only outside mechanical connection is then through a rotating shaft extending through a reamed and bushed hole in the side of the breaker top. This design removes the levers entirely from the vicinity of the live contact terminals, thus greatly increasing their electrical clearances to the grounded parts outside the breaker. It also eliminates the necessity of carrying the moving-contact lifting rod through the breaker top, avoiding the possibility of unbalanced forces on the lifting rods, and moreover leaves the outside of the breaker structure with a neat, trim appearance. Fig. 15 shows the arrangement of the inside breaker mechanism of typical design.

CONTROL OF OPERATION. Control energy for electrically operated circuit breakers and similar apparatus and switchgear installations is a very important matter. The amount of energy required for the closing of solenoid-operated breakers ranges from approximately 5 kw for the smaller breakers to 50 kw for the larger ones. The time demand is only 10 to 15 cycles, and usually a storage battery is furnished to supply the energy. In most electrical power plants an operating voltage having a nominal rating of 110 to 125 volts is used, but electrical equipment can be made suitable for 220–250 volts or any other desired voltage.

It is usually found desirable to segregate various control circuits. Where there is a natural grouping of breakers ordinarily a separate control circuit is provided for each group. Each circuit should be protected by its own switch and fused for not less than three times the maximum current needed for closing the largest breaker. This fusing will give protection in case of a short circuit in the control system and yet assure a supply of current under normal operating conditions. Individual circuits, usually at the breaker

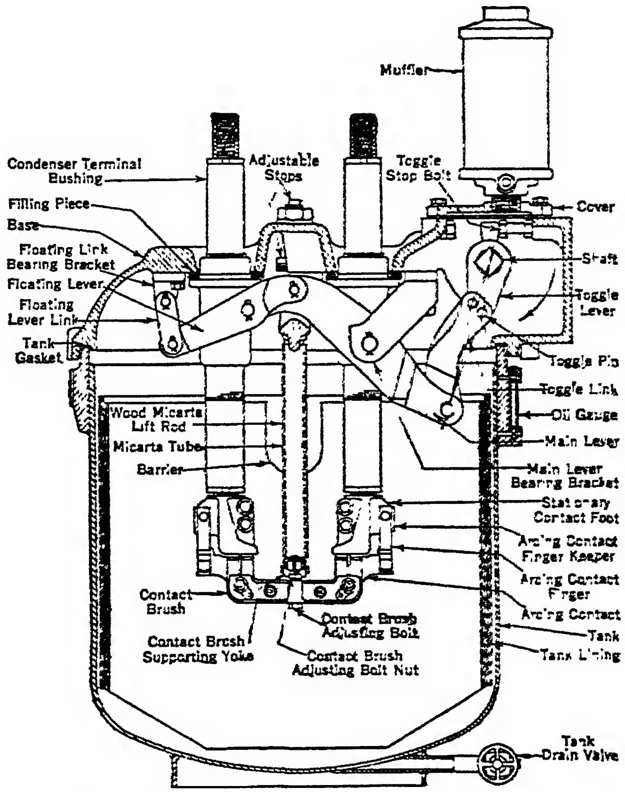

Fig. 15. Typical Oil Breaker with Inside Mechanism

points, should be fused at about half the current rating of the closing coil circuit, owing to the fact that the time of closing is extremely short and the closing current seldom reaches the full-load amount before it is cut off by the signal switch on the oil circuit breaker.

12. RELAYS

Relays are provided for use in connection with breakers to give breakers certain characteristics that cannot readily be taken care of in the breaker design. These relays may provide protection against overload, overcurrent, reverse current underload, overvoltage, undervoltage, reverse power, reverse phase connection, or open circuit on one phase, and can be made instantaneous or inverse time limit with an adjustable minimum tripping point. Various relays can be suitably interlocked so that breakers will function in certain predetermined sequences. Relays have three types of applications: service protection, apparatus protection, and automatic control.

Service Interruption is the most important factor to be guarded against by the protective relay. This may be done either by automatically sectionalizing the electrical system so as to disconnect the disabled section, or by automatically restoring the service after it has been accidentally interrupted.

Apparatus Protection. Another use for protective relays is to protect apparatus either by disconnecting it from service in case of failure or on the occurrence of abnormal conditions, or by functioning to remedy wholly or partially the abnormal condition. In cases where a short circuit or ground occurs in the winding of a machine such as a generator or transformer, it is usually necessary to have the apparatus disconnected.

D-c Relays for overload can be built either with series coils in the form of a solenoid

or with a magnetic circuit to slip over a stud or bus-bar, or they can be operated from a shunt. By the addition of a voltage coil, a d-c reverse power relay can be arranged.

A-c Relays for overcurrent of modern design operate on the induction principle, the contacts being closed by the rotation of a disk. It is necessary to have a certain quantity of current flowing in the relay winding before the disk will begin to rotate. After it has once been started, the speed of rotation is directly proportional to the current flowing in the winding until such a current reaches an excessively high value. This gives the relay an inverse time characteristic, or in other words, the greater the current flowing in the relay winding, the sooner the relay contact will close. The design of the relay and the action of the damping magnet are such that the time of action is definite for any given current value.

Directional Relay. This is usually obtained by means of combination of an overcurrent relay and a wattmeter element. The relay is made as a single-phase device, and the current element is very similar to the standard overcurrent relay and has identical characteristics. The wattmeter element operates on a very small percentage of normal voltage. The contacts of the overcurrent element are connected in series with the contacts of the wattmeter element, and the contacts of the wattmeter will close only when the power flows in one direction, so the relay as a unit operates only when power is flowing in one given direction.

Impedance Relays. On large, complicated transmission systems, the method of sectionalizing by means of overcurrent and directional relays is often difficult and sometimes even impossible to apply. This is due to the necessity of employing such a large number of relays that the timing sequence cannot be kept within safe limits. For such a system a special relay has been developed consisting of an overcurrent element of normal design with special voltage element so constructed that its action in relation to the closing of the relay contact depends on the impedance between the relay and the fault. In other words, the voltage element acts in opposition to the overcurrent element, and when the relay is a considerable distance from the fault, the voltage on the relay is comparatively high, thus taking a long time for the relay to close its contacts. On the other hand, the relays near the fault will have a very low voltage imposed on the voltage element, thus causing the relay to operate in much less time because of less opposition being offered to the action of the overload element by the voltage element. Thus the relay that is nearest the fault operates first without any special attention being given to time settings.

Balanced Relays. Balanced current protection is very often desirable when two or three similar transmission lines are used to connect two stations or two different points in a system. Balanced current protection on such lines means that each line normally carries approximately the same current, and when this condition does not exist, the line carrying the greatest current must be in fault.

Carrier Current Relays can be used to advantage in a few very special cases where long lines at high voltage and large power demands are to be controlled, as the lines from Boulder Dam to Los Angeles. The scheme involves the use of coupling condensers on the high-tension lines so that the power lines themselves can be used for carrying control circuits of 50- to 150-kc frequency, and these radio-frequency currents are utilized for the control of relays of various types at each end of each line. Under certain conditions, an impulse is set up at one end of the line by the action of certain relay devices, and this impulse is communicated to the other end as a 50- to 150-kc signal sent over the main transmission lines. These high-frequency circuits permit the operation of relays of all kinds to give the protection desired.

Pilot Wire Relaying can be carried out where the distances are small and the importance of the relaying sufficiently great to warrant running "pilot wires" between stations where parallel lines between them have to be protected at each end. This normally involves balancing the current that flows into a tie circuit against that which flows out so that if these two currents are the same the relays will not operate. In other words, these function only in the case of a fault on the tie lines between stations; they will not function in the case of a fault external to the section being protected.

High-speed Relays are becoming available to speed up the functioning of a protective system and to energize the tripping circuit of an oil circuit breaker in the minimum time to get full advantage of the high speed of action of some of the latest breakers. Various schemes are used to obtain the high speed on different types of relays.

Ground Protection can be obtained from overcurrent, differential, or balanced relays by operating these from suitable arrangements of transformers so that the current in the ground leg and not in the phase legs will act on the relays. Impedance relays are not usually employed to advantage for ground protection owing to the high impedance often found in the ground circuit.

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13. SELECTION OF OIL CIRCUIT BREAKER

The smallest non-automatic oil circuit breakers are essentially knife switches whose jaws and blades are located under oil with an external operating handle. One type is built up to 60 amp and 4500 volts, and one for heavier currents at lower voltages. The smallest automatic oil circuit breakers available for indoor service wall mounting have capacities up to 200 amp 2500 volts. The smallest type suitable for switchboard mounting is made in capacities of 400 amp 5000 volts. The next larger size is built in capacities of 400 and 600 amp at 7500 volts, 800 amp at 2500 volts. These smaller breakers have a single rectangular tank enclosing all the contacts. A larger breaker is built with each pole of the breaker as a separate unit with its own frame and tank. One mechanism is used to operate all poles. Other indoor breakers are available having individual tanks per pole for service up to 23,000 volts. Practically all breakers of 50,000 kva rupturing capacity and above are oil tight and equipped with arc control and extinguishing devices.

Single Circular Tank Three-pole Breakers during the last few years have been made by many manufacturers; they are particularly suitable where relatively high rupturing capacity is to be obtained in a relatively small space. One design of breaker has tanks ranging in diameter from 16 to 32 in., current ratings 600 to 3000 amp, breakers being suitable for 15 or 7 1/2 kv. At 7 1/2 kv, rupturing capacity ranges from 8000 to 50,000 amp, and for 15 kv on the three larger diameter tanks, the capacities range from 6000 to 20,000 amp. The essential feature of all these breakers is the enclosing of the three phases in a single breaker chamber. This single round tank provides the most economical form in space requirements for increasing the body of oil, and the single circular tank represents the most economical form of material used as well as the shape best suited to the bearing of heavy internal pressures. The structural details of these breakers differ slightly, but their general arrangement is approximately the same except for modifications in the contacts and other detail features.

Indoor Oval Tank Breakers are available in ratings of 600, 1200, and 2000 amp, all rated normally at 15,000 volts. The line of oval tank breakers have rupturing capacities of 4000, 6000, and 10,000 amp at their rated voltage. In the complete line of oval tank breakers the mechanisms are arranged so as to have all moving parts inside the breaker. These breakers have mechanically trip-free mechanisms and condenser type bushings, and each breaker unit is vented to a common muffler.

Round Tank Breakers with one tank per pole are available where greater rupturing capacities are desired. These breakers use circular tanks, usually with each pole in a separate masonry compartment or steel cubicle. This type of indoor oil circuit breaker is adapted for the control of circuits up to 6000 amp at 15,000 volts, and 3000 amp at 23,000 volts. The main brushes are of the parallel-path design for most sizes. Although these various round tank breakers are normally utilized as completely assembled three-pole breakers, they are adaptable for segregated phase construction with the breaker unit arranged either vertically or horizontally.

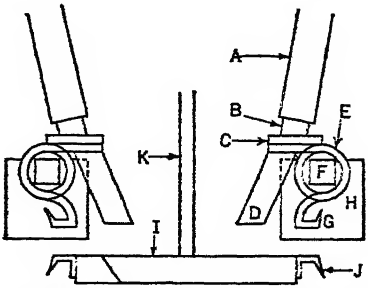

FIG. 16. High-speed A-c Oil Circuit Breaker. *A*, bushing; *B*, contact stud; *C*, contact foot; *D*, laminated brush stationary contact; *E*, series blow-out coils; *F*, magnetic circuit; *G*, stationary arcing tip; *H*, arc chute; *I*, moving contact-aluminum; *J*, moving arcing tip; *K*, insulating lift rod.

High-speed Railway Oil Circuit Breaker of a typical design has most of its interesting features indicated in Fig. 16. The breaker is essentially a single-pole outdoor round tank oil circuit breaker provided with a magnetic blowout for the arc with a contact designed to start quickly and to move rapidly. The main features of this breaker are

indicated on the drawing. The breaker with its actuating relay is designed to operate so that the short-circuit current on a 25-cycle railway system will be limited to not more than two half cycles, for currents of 6000 amp upward.

Outdoor Breakers are available having various current and voltage ratings and various assigned rupturing capacities. These breakers are for use on heavy-duty a-c circuits of large capacity of moderate or high voltages; they can be used for indoor or outdoor service up to 287 kv, the lower voltage ratings being available in capacities of 6000, 1200, and 2000 amp as frame-mounted breakers, Fig. 17, and those of the higher voltages for 600 and

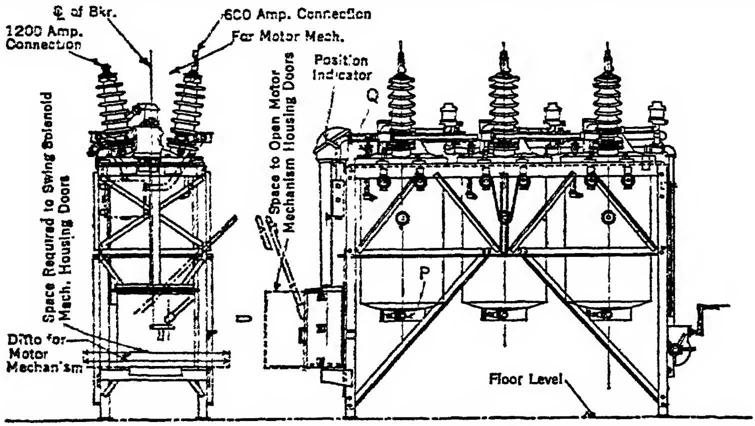

Fig. 17. Frame-mounted Oil Circuit Breaker

1200 amp as floor-mounted breakers, Fig. 18. Rupturing capacities up to 3,500,000 arc kva are available.

The selection of an oil circuit breaker for application to an electrical system or circuit requires a knowledge of the characteristics of the breaker and the characteristics of the system or circuit. Breakers are usually classified according to their rated voltage, rated current, etc., at a definite frequency, interrupting capacity, 5-sec and momentary current-carrying capacity. Systems may be classified according to their normal operating voltage, normal current, normal frequency, and short-circuit characteristics.

RATING. Tables 1 and 2 give the latest available data on this subject (N.E.M.A. tabulation):

Table 1. Schedule of Ratings for Indoor Oil Circuit Breakers as of Oct. 1, 1935

Rated Voltage	Interrupting Capacity, kva	Current Rating, amp	Rated Voltage	Interrupting Capacity, kva	Current Rating, amp
5,000	20,000	400	15,000	1,000,000 (M)	3,000
5,000	50,000 (M)	2,000	15,000	1,500,000 (M)	1,200
7,500	25,000	600	15,000	1,500,000 (M)	2,000
7,500	50,000 (M)	1,200	15,000	1,500,000 (M)	3,000
7,500	100,000 (M)	600	15,000	1,500,000 (M)	4,000
7,500	350,000 (M)	3,000	23,000	500,000 (M)	600
15,000	50,000 (M)	600	23,000	500,000 (M)	1,200
15,000	100,000 (M)	600	23,000	500,000 (M)	2,000
15,000	100,000 (M)	1,200	23,000	1,000,000 (M)	*
15,000	100,000 (M)	2,000	23,000	1,500,000 (M)	†
15,000	150,000 (M)	600	23,000	2,500,000 (M)	†
15,000	150,000 (M)	1,200	34,500	500,000	600
15,000	250,000 (M)	600	34,500	1,000,000 (M)	1,200
15,000	250,000 (M)	1,200	34,500	1,000,000 (M)	2,000
15,000	250,000 (M)	2,000	34,500	1,500,000 (M)	1,200
15,000	500,000 (M)	600	34,500	1,500,000 (M)	2,000
15,000	500,000 (M)	1,200	34,500	1,500,000 (M)	3,000
15,000	500,000 (M)	2,000	34,500	2,500,000 (M)	2,000
15,000	1,000,000 (M)	1,200	34,500	2,500,000 (M)	3,000
15,000	1,000,000 (M)	2,000			

* Use breaker of same rating. † Of 34,500 volts class.

Note: "M" Ratings applicable to indoor oil circuit breakers and either indoor or outdoor metal-clad switchgear. For outdoor oil circuit breakers for 46 kv and above see below.

It should be noted that most of the smaller breakers are hand-operated distant control whereas the larger ones are electrically operated distant control. In every case these are the non-automatic breakers, that are made automatic by the use of proper relays and current transformers.

Oil circuit breakers and other switching equipment are rated in rms volts based on a dielectric test in accordance with the A.I.E.E. Rules that all indoor breakers be tested

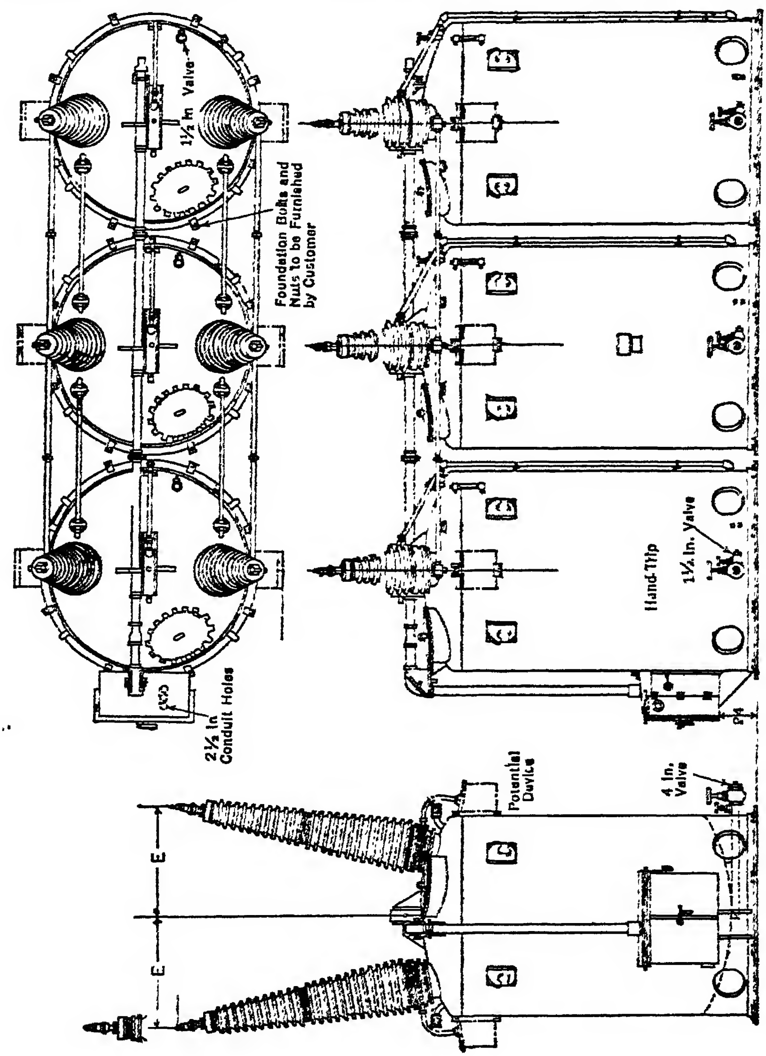

FIG. 18. Floor-mounted Oil Circuit Breaker

at 2 1/4 times rated voltage plus 2000 volts. Outdoor breakers will meet this same dry test and will meet a wet test of 2 times rated voltage plus 1000 volts.

Altitude affects arcing distances in air. For definite voltages these distances increase with the decrease in atmospheric pressure while the cooling effect diminishes. Standard ratings of all oil circuit breakers are assigned to altitudes of 1 km (3300 ft) above sea-level and less; for higher altitudes standard listed breakers must be de-rated according to tables published by the manufacturers.

Table 2. Schedule of Ratings for Outdoor Oil Circuit Breakers as of Oct. 1, 1935

Rated Voltage	Interrupting Capacity, kva	Current Rating, amp	Rated Voltage	Interrupting Capacity, kva	Current Rating, amp
15,000	50,000	600	34,500	1,500,000	3,000
15,000	100,000	600	46,000	150,000	600
15,000	175,000	600	46,000	500,000	600
15,000	500,000	600	46,000	1,000,000	1,200
15,000	500,000	1,200	69,000	250,000	600
15,000	500,000	2,000	69,000	500,000	600
15,000	1,000,000	1,200	69,000	1,000,000	600
15,000	1,000,000	2,000	69,000	1,000,000	1,200
15,000	1,000,000	3,000	69,000	1,500,000	600
15,000	1,500,000	2,000	69,000	1,500,000	1,200
15,000	1,500,000	3,000	69,000	2,500,000	1,200
15,000	1,500,000	4,000	69,000	2,500,000	2,000
23,000	150,000	600	115,000	500,000	600
23,000	500,000	600	115,000	1,000,000	600
23,000	500,000	1,200	115,000	1,500,000	600
23,000	500,000	2,000	115,000	1,500,000	1,200
34,500	150,000	600	138,000	750,000	600
34,500	500,000	600	138,000	1,500,000	600
34,500	500,000	1,200	138,000	1,500,000	1,200
34,500	500,000	2,000	138,000	2,500,000	1,200
34,500	1,000,000	1,200	161,000	1,500,000	1,200
34,500	1,000,000	2,000	161,000	2,500,000	1,200
34,500	1,000,000	3,000	230,000	2,500,000	1,200
34,500	1,500,000	1,200	345,000	3,500,000	1,200
34,500	1,500,000	2,000			

Interrupting Ratings are usually based on the highest rms current at specified voltage which the breaker can interrupt and close against a short circuit, followed immediately by the reopening of the breaker, that is without purposely delayed action; allowed to remain open for 15 sec for the larger "oil-tight" breakers; or 2 sec for the smaller "non-oil-tight" breakers; and then closed against the short circuit and followed immediately by the reopening of the breaker, that is, without purposely delayed action. The breaker should perform its rated duty under these conditions without emitting flame and, at the end of the duty cycle, must be in substantially the same mechanical condition as at the beginning, but the interrupting capacity may be materially reduced.

Speed of Operation has been given a great deal of attention in the past few years, and breakers are now available that will open a 60-cycle circuit in 3 cycles, and breakers have been built to open a 25-cycle railway circuit in 1 cycle on the 25-cycle basis, at 25 to 100 per cent short-circuit current and at nominal voltage.

Impulse Tests have been applied to high-voltage breakers the same as to other high-voltage equipment so that proper coordination has been obtained between the flashovers of transmission-line insulators, lightning arresters, bushings for transformers and circuit breakers, supports for the high-tension wiring, and other parts subject to high-voltage surges.

The effects of short circuits on the breakers may manifest themselves as strains on the tanks, fittings, mechanism, contacts, bushings, etc., caused by electromagnetic stresses, reduction of insulation value, and reduced ability to carry current. Excessive current may burn or weld the contacts, take the temper out of brush contacts or backing springs —in effect, cut down the carrying capacity of the breaker for subsequent service.

SHORT-CIRCUIT CALCULATIONS. Satisfactory calculations of short-circuit characteristics of a system can be made on simple systems rather readily, but on complicated ones the calculations are very involved. For the simpler cases, an approximation sufficiently close for practical purposes can generally be made by neglecting the effect of resistance and capacitance and using the reactance only, along with suitable time decrement curves. These curves show graphically the test results of short-circuiting a-c synchronous machines, and are available in the publications of circuit-breaker manufacturers.

Reactances of generators, transformers, etc., may be estimated where the actual data for a proposed system are unknown, but certain preliminary calculations are to be made. The reactance in generators varies from about 8 per cent for a small high-speed unit to about 30 per cent for large slow-speed water-wheel machines, and that in the transformers varies from about 3 to 15 per cent of their ratings. A fair figure to use in a hydroelectric system with step-up transformers is a combined reactance of generators and transformers of roughly 33 per cent so that the current in a symmetrical short circuit on the high-

tension side of the step-up transformers of the generating station is approximately three times full-load rating. In a relatively long transmission line the reactance may be in the nature of 10 per cent, the reactance of the step-down transformers 7 or 8 per cent, so that a symmetrical short circuit on the low-tension side of the transformers of the substation will probably be about twice the full-load rating, assuming the capacity of the substation to be the equivalent of the generating station.

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CONTROLLERS AND REGULATORS

The function of control apparatus when used with industrial motors of various kinds is to enable the latter to take care of various features not directly incorporated in the motor design. The functions of control apparatus are some or all of the following:

To limit the current during the acceleration of the motor.
To limit the torque during acceleration.
To change the direction of rotation of the motor.
To limit the load on the motor.
To disconnect the motor on failure of voltage.
To regulate the speed of rotation.

To start and stop the motor at fixed points on the cycle of operation, or at the limit of travel of the load.

To stop the motor.

To protect the operator from injury.

The simplest of the control apparatus are those intended only for starting the motors; the more complicated devices also provide for speed regulation, reversing the direction of rotation, and other features.

The starting and speed regulating can be accomplished by means of switches, contactors, or similar devices. The switches can be the plain knife switches described in Art. 3 (p. 12–03), or they can be in the form of face plates with contact arms, drum controllers, or contactors.

Contactors are modified forms of air circuit breakers and are used principally in industrial work for motor control and in automatic stations. In general, a contactor like Fig. 1 might be considered as a circuit breaker normally requiring some auxiliary source of

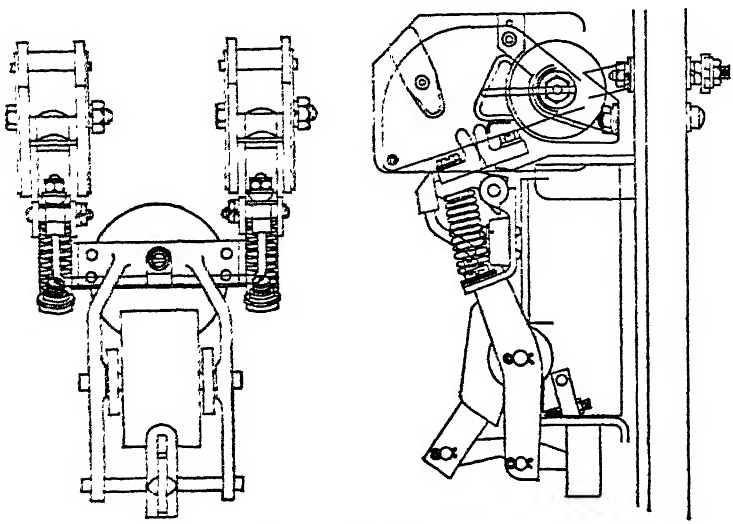


Fig. 1. Typical Contactor

power, like a solenoid, to hold it in the closed position. Latching contactors can be supplied.

Contactors are designed to withstand the severe requirements of general industrial service and are rated on a continuous current-carrying capacity basis. They will carry a 25 per cent overload for 1 hr and will successfully rupture four times their rated current. Low-voltage release is an inherent characteristic of magnetic control when used with any sort of two-wire master switch. The normal design of contactor consists of an insulating base with a contactor mounted on it. The contactor illustrated in Fig. 1 has copper-to-copper contacts which roll into engagement and is provided with the arc-splitter construction of the magnetic blowout. The circuit is closed and opened on the extreme tip of the contacts, and the burning, due to hot spots when closing and rupturing the arc when opening, is in this way localized. As the contactor closes, the area of contact is quickly transferred from the burned tips to the clean surface at the heel. What actually takes place is that the moving contact rolls down along the surface of the stationary contact. The tendency to stick or weld at the instant of closing, which characterizes most forms of copper-to-copper contact, is completely eliminated by the powerful leverage introduced by this rolling movement. Also a slight sliding or wiping action takes place which serves to prevent overheating of the contacts due to an accumulation of dust or oxidized scale. The arc splitter and the magnetic blowout lengthen and cool the arc incident to opening the circuit. The resultant burning is in this way materially reduced, and experience has shown that the useful life of the contact is prolonged. Contactors are available in various current ratings from 50 to 1600 amp and can be made single pole or multipole. Wherever

possible direct current is utilized to operate the contactors but modifications are available utilizing alternating current for this service.

Network Protectors are special types of contactors or air breakers, motor or solenoid-operated, with associated relays designed to connect transformers to an a-c network when the conditions are correct and to disconnect them when conditions are wrong. To withstand high temperatures, high-pressure butt contacts are used, employing silver main contacts as silver maintains a much better contact surface over a long time than copper, silver oxide being a much better conductor than copper oxide.

Network protectors of the air-break type are utilized chiefly in connection with low-voltage three-phase four-wire systems of 115–199 or 120–208 volts. Special types of oil circuit breakers have been developed as network protectors for use on three-phase four-wire 2300–4000 volt systems. These network systems themselves are considered in the section of this handbook dealing with Distribution.

14. MOTOR-STARTING DEVICES

STARTING SWITCH. Motors up to 1 hp, whether d-c or single-phase, can usually be started by closing a switch that connects the motor directly to the source of power. Automatic protection against overload can be provided by fuses or circuit breakers, the latter sometimes being incorporated in the design of the main switch. So-called sentinel breakers and similar types can be furnished with current rating suitable for any motor from $1/20$ up to 1 hp. This device is essentially a small combination switch and circuit breaker in a Bakelite case and is provided with a bimetallic thermal element. A momentary overload will not cause the breaker to trip. If, however, a load heavy enough to damage the motor should continue, the thermal element will cause the breaker to trip before the motor is harmed. The handle automatically moves to the central position to indicate that the breaker is open. After the cause of the overload has been removed, the breaker is reset by moving the handle to the off position and then to the on position. It cannot be held on the on position while the injurious overload condition exists.

FACE-PLATE STARTERS. Fig. 2 shows a typical starter for use with series, shunt,

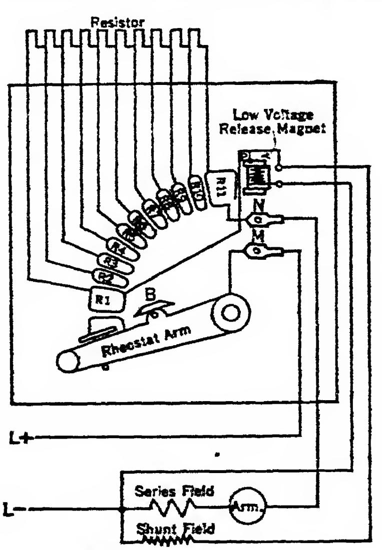


Fig. 2. Face-plate Starter

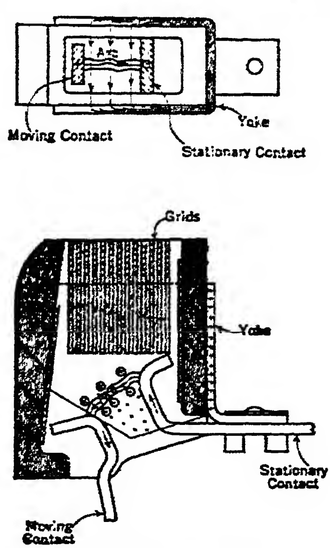


Fig. 3. De-ion Chamber of Contactor

and compound d-c motors in non-reversing service and for single-phase repulsion motors up to 10 hp. This control is applicable to starting duty only. It provides low-voltage protection to the motor. Provision is made by spring return to prevent the arm being left in an intermediate position on the contacts, which in time would endanger the resistors. In case of power failure the arm returns to the off position. The motor is started by moving the rheostat handle slowly to the right. When the arm reaches the full "run" position,

To start and stop the motor at fixed points on the cycle of operation, or at the limit of travel of the load.

To stop the motor.

To protect the operator from injury.

The simplest of the control apparatus are those intended only for starting the motors; the more complicated devices also provide for speed regulation, reversing the direction of rotation, and other features.

The starting and speed regulating can be accomplished by means of switches, contactors, or similar devices. The switches can be the plain knife switches described in Art. 3 (p. 12-03), or they can be in the form of face plates with contact arms, drum controllers, or contactors.

Contactors are modified forms of air circuit breakers and are used principally in industrial work for motor control and in automatic stations. In general, a contactor like Fig. 1 might be considered as a circuit breaker normally requiring some auxiliary source of power, like a solenoid, to hold it in the closed position. Latching contactors can be supplied.

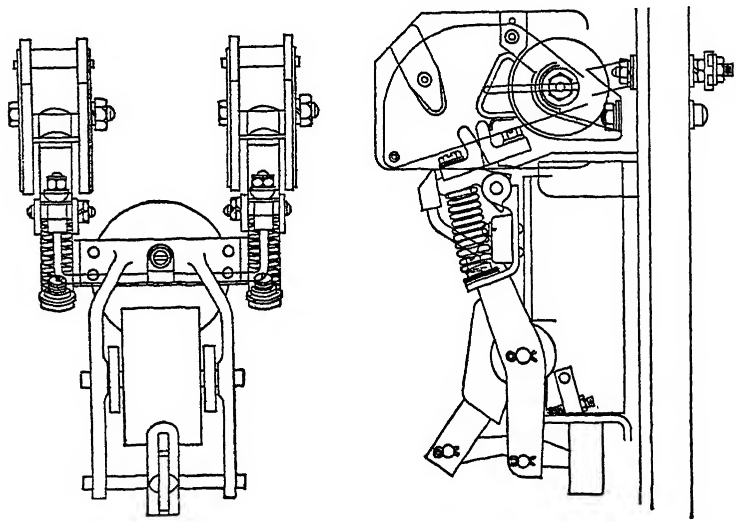


Fig. 1. Typical Contactor

Contactors are designed to withstand the severe requirements of general industrial service and are rated on a continuous current-carrying capacity basis. They will carry a 25 per cent overload for 1 hr and will successfully rupture four times their rated current. Low-voltage release is an inherent characteristic of magnetic control when used with any sort of two-wire master switch. The normal design of contactor consists of an insulating base with a contactor mounted on it. The contactor illustrated in Fig. 1 has copper-to-copper contacts which roll into engagement and is provided with the arc-splitter construction of the magnetic blowout. The circuit is closed and opened on the extreme tip of the contacts, and the burning, due to hot spots when closing and rupturing the arc when opening, is in this way localized. As the contactor closes, the area of contact is quickly transferred from the burned tips to the clean surface at the heel. What actually takes place is that the moving contact rolls down along the surface of the stationary contact. The tendency to stick or weld at the instant of closing, which characterizes most forms of copper-to-copper contact, is completely eliminated by the powerful leverage introduced by this rolling movement. Also a slight sliding or wiping action takes place which serves to prevent overheating of the contacts due to an accumulation of dust or oxidized scale. The arc splitter and the magnetic blowout lengthen and cool the arc incident to opening the circuit. The resultant burning is in this way materially reduced, and experience has shown that the useful life of the contact is prolonged. Contactors are available in various current ratings from 50 to 1600 amp and can be made single pole or multipole. Wherever

possible direct current is utilized to operate the contactors but modifications are available utilizing alternating current for this service.

Network Protectors are special types of contactors or air breakers, motor or solenoid-operated, with associated relays designed to connect transformers to an a-c network when the conditions are correct and to disconnect them when conditions are wrong. To withstand high temperatures, high-pressure butt contacts are used, employing silver main contacts as silver maintains a much better contact surface over a long time than copper, silver oxide being a much better conductor than copper oxide.

Network protectors of the air-break type are utilized chiefly in connection with low-voltage three-phase four-wire systems of 115–199 or 120–208 volts. Special types of oil circuit breakers have been developed as network protectors for use on three-phase four-wire 2300–4000 volt systems. These network systems themselves are considered in the section of this handbook dealing with Distribution.

14. MOTOR-STARTING DEVICES

STARTING SWITCH. Motors up to 1 hp, whether d-c or single-phase, can usually be started by closing a switch that connects the motor directly to the source of power. Automatic protection against overload can be provided by fuses or circuit breakers, the latter sometimes being incorporated in the design of the main switch. So-called sentinel breakers and similar types can be furnished with current rating suitable for any motor from $^1/_{20}$ up to 1 hp. This device is essentially a small combination switch and circuit breaker in a Bakelite case and is provided with a bimetallic thermal element. A momentary overload will not cause the breaker to trip. If, however, a load heavy enough to damage the motor should continue, the thermal element will cause the breaker to trip before the motor is harmed. The handle automatically moves to the central position to indicate that the breaker is open. After the cause of the overload has been removed, the breaker is reset by moving the handle to the off position and then to the on position. It cannot be held on the on position while the injurious overload condition exists.

FACE-PLATE STARTERS. Fig. 2 shows a typical starter for use with series, shunt,

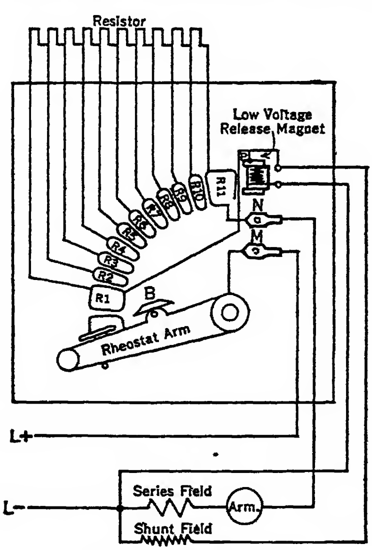


Fig. 2. Face-plate Starter

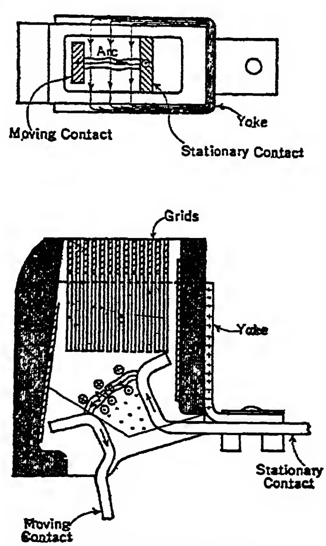


Fig. 3. De-ion Chamber of Contactor

and compound d-c motors in non-reversing service and for single-phase repulsion motors up to 10 hp. This control is applicable to starting duty only. It provides low-voltage protection to the motor. Provision is made by spring return to prevent the arm being left in an intermediate position on the contacts, which in time would endanger the resistors. In case of power failure the arm returns to the off position. The motor is started by moving the rheostat handle slowly to the right. When the arm reaches the full "run" position,

it is held in place by a magnet which has its coil connected across the line. The handle is provided with a spring which prevents it from remaining in any position except off or run. It can remain in the run position only when power is on; in case of power failure the holding magnet releases and the spring returns the arm to the off position.

LINE STARTERS utilizing contactors are used to start compound-wound d-c motors directly across the line in those applications where the high starting torque attained by this method of starting will not injure the driven machine. A thermal overload relay is provided to protect the motor against continuous overload condition.

Line starters with De-ion arc quenchers or similar devices are available for across-the-line starting of squirrel-cage and wound-rotor induction motors ranging in size from $^1/_2$ to 200 hp for voltages up to 600. These devices are particularly suited for starting motors driving machine tools, textile machinery, woodworking machines, pumps, stands, and other machines where remote control with complete protection to the operator, motor, and machine is desired. These are essentially magnetic starters and for the larger capacities De-ion arc quenchers (see Art. 8) like Fig. 3 are furnished.

Where necessary oil-immersed line starters can be provided for such locations as mines, cement mills, chemical plants, and oil refineries. They can also be used in high altitudes where the operation of the standard types would be affected by low barometric pressure. Reversing line starters are available, utilizing two three-pole contactors mounted back to back, one for the forward and one for the reverse direction of rotation. These are mechanically interlocked to prevent closing both at the same time.

MAGNETIC STARTERS. For d-c motors, magnetic starters are available for non-reversing or reversing service, constant or adjustable speed with dynamic braking in the off position, and with thermal overload protection. Magnetic starters are available for use with wound-rotor induction motors. These have a primary contactor and a number of secondary contactors, the latter short-circuiting the various amounts of resistance.

Magnetic starters can be furnished for use with squirrel-cage induction motors where the line or load conditions require reduced-voltage starting and where the advantages of remote control are desired. These starters may be operated either from a push button or from automatic master switches such as float switches or pressure regulators. To start a motor it is merely necessary to press the start push button on an automatic master switch to operate. This closes the circuit through a contact in the timing relay. This relay energizes the starting contactor, or contactors, which close and connect the motor to reduced voltage from the auto-transformer thus limiting the starting current to an allowable value and permitting smooth starting. The time that the motor is connected to the auto-transformer is governed by the setting of the accelerating relay. After a definite time adjustable up to 30 sec, the timing relay opens the circuit of the starting contactor and closes the circuit to the running contactor which connects the motor directly to the line. The motor is therefore accelerated at the most rapid permissible rate, independent of the operator and without damage to the motor or driven machinery. These starters are arranged to provide either low-voltage release or protection depending on conditions. With these larger magnetic starters the auto-transformers are usually provided with 50, 65, and 80 per cent taps so that the starting voltage may be easily adjusted to suit the particular starting conditions.

TIME STARTERS. Time starters are remote-controlled magnetic starters adapted for adjustable-speed d-c motors up to 10 hp, 230 volts. They provide three points of definite time limit acceleration, permitting safe and accurate starting of the motor under full or partial load. A clock escapement mechanism permits changing the accelerating time to suit the application requirement accurately and definitely. The device consists essentially of a three-point accelerating contactor, mechanically connected to an ingenious clock escapement mechanism. Definite time limit acceleration is obtained in starting by delayed action of the accelerating contacts brought about by a clock escapement mechanism.

AUTO-STARTERS. An auto-starter is a reduced-voltage non-reversing auto-transformer type starter for use with squirrel-cage induction motors. This device is furnished with overload protection by an inverse time limit overload relay and low-voltage protection in case of power failure. Low-voltage starters for use up to 50 hp 60 cycle, 30 hp 25 cycle are designed for operation without the immersion of the contacts in oil. Where needed, however, or for the larger starters, oil immersion of the contacts is normally used. Overload protection is provided by a thermal relay for the smaller sizes, this relay depending for its operation on the fact that a bimetal trip arm bends under the influence of the heat from the heater. Heating elements are available in capacities up to 95 amp. When the starter rating requires the heater to carry more than 95 amp, a relay provided with parallel heaters can be furnished. Low-voltage protection is provided by a relay which holds the operating handle in the running position as long as normal voltage is maintained but

releases it and disconnects the motor from the line whenever the voltage fails. A time delay low-voltage device can be furnished providing a time delay adjustable from 1/2 to 2 1/2 sec, so that, if the voltage returns to normal after having failed within the time setting of the attachment, the auto-starter will not drop out and the motor will continue to run.

For larger motors, using auto-transformer starting, a double-throw breaker is provided with special moving and stationary contact arrangements which provides three poles in the run position and five poles in the start position. The additional poles for the start position are utilized to connect in the auto-transformers that are disconnected in the running position as shown in Fig. 4.

With manually operated reduced voltage starters, connections can be made for either

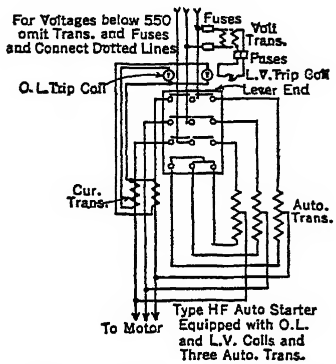


FIG. 4. Auto-starter Connections

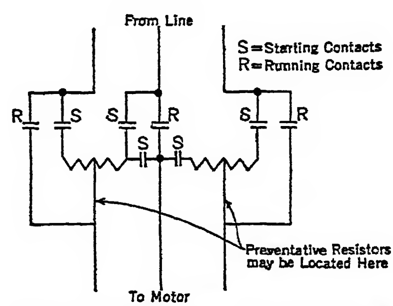


FIG. 5. Auto-starter Connections from Line

the open transition starting or the closed transition starting, these connections being indicated in Fig. 5.

15. DRUM CONTROLLERS

Drum reversing switches for d-c motors are available for use principally with small d-c motors. These are sometimes provided with ballast resistors or starting resistors.

For larger motors reversing drum switches of the cam contactor construction are available, being used particularly for controlling series-, shunt-, or compound-wound motors in general reversing service on cranes, bridges, roll and transfer tables, and in similar applications where starting and speed control are desired by means of resistance in series with the armature. These drum switches are designed for frequent and severe service. The operating principles of the cam-type switch are similar to those of magnetic contactors except that the contacts are closed by cams instead of magnets. A heavy helical spring opens the contacts. The shape of the cam gives a quick break and quick make action to the contacts regardless of how slowly the handle is moved. The contacts are the same as those used on magnetic starters and have the same rolling action when opening or closing. The arcing can take place only at the contact tips, leaving the heel clean and smooth for carrying the current. Individual magnetic blowouts with arc splitters are provided to cool and stretch the arc under the influence of magnetic field and to rupture it quickly. These magnetic blowouts are used on all 550-volt drum switches.

Drum starters, either reversing or non-reversing, can be furnished for small polyphase squirrel-cage induction motors to connect them directly to the line. These devices are used extensively where the air may become charged with explosive gases or highly inflammable particles of dust or lint, as in textile mills, flour mills, and woodworking plants. With these starters the contacts are usually immersed in oil. Similar across-the-line reversing-drum controllers can be furnished where the oil immersion of the contact is not required.

Drum controllers are available for 230-volt motors from 1 to 35 hp for machine tool service and similar duties where it is desired to have complete control of the motor from one operating handle. They are designed for starting and controlling the speed of adjustable-speed shunt-wound motors having speed ranges of 1 1/2 to 1, 2 to 1, 3 to 1, or 4 to 1. They are built for reversing service and can be provided with an arrangement for dynamic braking.

Non-reversing drum controllers can also be used to advantage for the secondary control

of wound rotor induction motors. The controller may be used for either starting or speed-regulating duties depending upon the resistors employed. A separate switch for the primary motor circuit, which includes the advantages of low voltage and overload protection, is required for use with these drum controllers.

16. CONTROL PANELS

D-c protective panels are available for use with manual controllers to provide overload and low-voltage protection. The contactor can be provided with a thermal overload relay operated by heaters selected according to the motor ratings. In addition to overload protection, low-voltage protection is provided by this device. The wiring is so arranged that the protective panel remains inoperative until the drum is again returned to the off position. This insures starting the motor with all resistance in series with the armature. Panels can be provided to furnish protection of this type for any number of motors up to 7. Overload protection is provided by relays with oil dashpots giving inverse time limit action. All overload relays are gravity-reset, making it necessary only to return all master switches and drum controllers to the off position and to press the start button to start again after an overload has tripped open the line contactors.

For the larger sizes of wound rotor motors ranging up to 500 hp 220 volts, 1000 hp 440–6600 volts, control panels are frequently employed, one panel containing the oil circuit breaker for use in the primary circuit and the other panel containing the drum controller for use in the secondary circuit of the motor. The motor is started by closing the primary oil circuit breaker with all the resistance in the secondary circuit. The secondary drum is then gradually rotated to its successive contact positions, each movement of the drum short-circuiting a portion of the resistor and thus allowing the motor to accelerate. The drum is finally moved either to the run position or to some speed position depending upon whether the duty is for starting or for speed regulation. The motor is stopped by returning the drum to the off position, by tripping of the primary breaker or by use of a stop push button in series with the undervoltage trip of the primary breaker. In all cases the drum must be returned to the off position before the motor can be started.

17. CONTROL RHEOSTATS

On a-c or d-c systems, where rotating machines are used, it is possible, in most cases, to vary the voltage, speed, power factor, or other characteristics of the machine by adjusting the field excitation. This can be done either by hand control of the field rheostat or by automatic control of the main field or exciter field by means of a faceplate regulator or by a carbon-pile or vibrating type of voltage regulator.

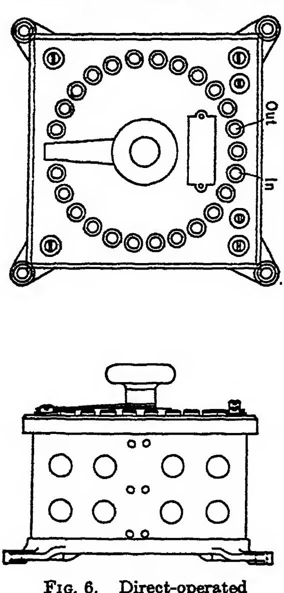


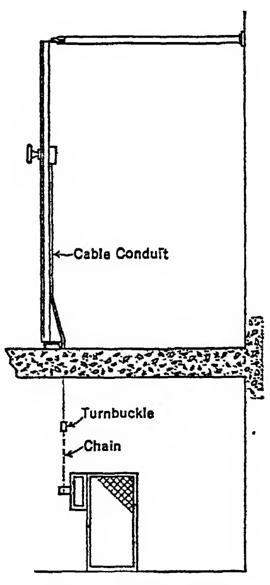
FIG. 6. Direct-operated Field Rheostat

FIELD RHEOSTATS. When adjusting the current in the shunt field or separately excited fields of a-c and d-c generators and motors, it is customary to use field rheostats made up of faceplates and resistors of suitable design. The faceplate comprises a series of contacts arranged in a circle mounted on a slate or marble base and provided with movable contact arms. With small units direct operation of the rheostat by means of a hand wheel and shaft is frequently possible as indicated in Fig. 6. In moderate-capacity plants, sprocket chain and wire rope transmission is customarily employed connecting together the hand wheel of the switchboard and the faceplate near the resistors, as indicated in Fig. 7. In very large stations where electrical operation is applied to the oil circuit breakers, the field rheostat faceplates are usually made motor operated. The faceplate and resistors of field rheostats are usually assembled on an angle- or strap-iron frame protected by grillework. Remote-control rheostats are used to permit mounting the faceplate adjacent to its grid resistors and for reducing the length of cable connections from fields to rheostats and exciters, particularly in large machines. By having these resistors properly located, the heat from them can be readily carried away without discomfort to the operator.

The number of steps for a field rheostat depends on the closeness with which adjustments of field currents are to be made and on the range desired. They usually have 30 to 70 divisions of resistance, but double this number may be obtained in the case of two or more plates by staggering the levers.

To select the proper rheostat it is necessary to know the resistance of the field, the minimum field current required, and the voltage of the circuit. For ordinary conditions, a rheostat resistance equal to the resistance of the generator field is satisfactory. Machines which are regulated by automatic voltage regulators frequently require a rheostat resistance of two to four times that of the generator field. The resistance per step is also tapered so that at the " all resistance in " end of the rheostat the resistance per step is higher than the value at the " resistance out " end. A logical choice of resistances per step would cause each step to make a fixed percentage change in the field current.

The current-carrying capacity of the steps of the rheostat is tapered so that the capacity at the high-capacity end is two to three times that when the resistance is all inserted. For small values of current the resistance element is usually imbedded in a vitreous enamel compound which attaches it to the plate of the rheostat. Only the contacts extend through the enamel, which acts as a mechanical support for the resistance element. For currents above 60 amp, grid or some other high-capacity type of resistor can be furnished connected electrically to a faceplate, the faceplate in turn being operated by a sprocket mechanism.



Rheostat on Floor below Switchboard

Fig. 7. Sprocket-operated Field Rheostat

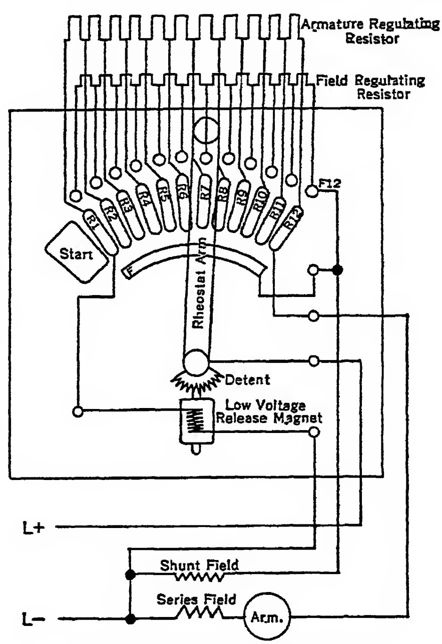


Fig. 8. Starting and Speed-control Rheostats

With those rheostats which contain distinct resistance elements per unit, the terminals of the elements are brought out to the metal contacts usually arranged in a circle on an insulating fireproof plate. The contact of the switch arms for currents up to 25 amp per contact arm consists of an ordinary straight finger contact brush; for currents up to 350 amp solid sliding plungers with evenly faced surfaces held by springs against the contact segments are used in the switch arms. For higher currents the laminated brush has been found more satisfactory.

SPEED-REGULATING RHEOSTATS. Starting and speed-regulating rheostats are available for wound rotor induction motors to be used in the secondary circuits with line starters employed as primary switches. These rheostats are used where it is desirable to operate the motor at less than full speed by having resistance connected into the secondary

wiring. Resistors, capable of carrying the load current continuously, are provided, usually giving about 29 points of speed control.

Speed-regulating rheostats with or without combination starting rheostats can be furnished for regulating the speed of shunt- or compound-wound d-c motors in nonreversing service, where a 50 per cent speed reduction through armature control or field control is desirable. Where the starting and field control are combined in a single device like Fig. 8, the rheostat has two arms, one of which cuts out the armature starting resistors and the other cuts in the field resistors. When the arm reaches the position where all the starting resistors are cut out, the armature contact arm is held in place by a magnet which leaves the field contact arm free to move, regulating the speed of the motor.

The field contactor is so interlocked with the armature contact arm that the field resistance can be cut in for speed regulation only when the armature contact arm is in the run position. It must always be returned with the armature arm to the off position. This prevents the motor from starting without full field strength on it.

MOTOR-OPERATED RHEOSTATS. These are usually provided with limit switches that will be thrown over by the movement of the rheostat arm when it reaches either extremity of its travel. This limit switch shuts power off from the motor when the connections of the motor are such as to tend to move the rheostat arm beyond the limiting point. They allow connections to be so made that the motor can move the arm back, away from the limiting point. When the rheostat is first placed in service the connections should be carefully checked up to see that the limit switches function properly to permit the desired connection to the motor.

18. GENERATOR VOLTAGE REGULATORS

The earliest a-c plants with poorly regulating generators were able to maintain proper voltage only by depending on the switchboard operator continually to adjust the voltage by means of the rheostat. To reduce the amount of adjusting, generators were made with very good inherent regulation, but they were expensive to build and their windings were difficult to brace against the effects of their heavy short-circuit currents, so the trend of generator design returned to generators of high reactance and poor inherent regulation as soon as a satisfactory regulator had been developed for maintaining the a-c voltage at its proper value under conditions of varying load.

To maintain proper voltage on a-c generators, regulators are available that adjust the field current either of the generator itself or of its exciter. In order to maintain practically constant voltage on the a-c and d-c generators, or to have these machines compound automatically to take care of feeder drop, field regulators of various kinds have been designed.

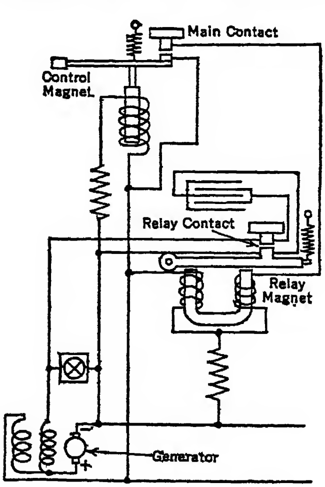


FIG. 9. D-c Controlled D-c Regulator

Two types of voltage regulators have been developed to meet various problems of adequate voltage regulation satisfactorily. One is the well-known vibrating type of voltage regulator, and the other the rheostatic type of voltage regulator. Another development is the carbon-pile regulator.

VIBRATING REGULATORS. These are made in two principal forms, one using a d-c vibrating magnet and the other an a-c vibrating magnet relay as the anti-hunting control device. Either design of vibrating regulator depends on the rapid opening and closing of a circuit that shunts the field rheostats and thus changes the resistance in the field circuit of the generator to be regulated. For d-c service the regulator usually works upon the main generator field and for a-c service upon the field of the exciter. In both cases the rheostat is so adjusted that when in circuit it tends to lower the voltage considerably below normal, and when the rheostat is short-circuited the generator voltage rises. The regulator automatically closes the shunt circuit when the voltage drops to a predetermined value and opens it when the voltage rises above that value.

The regulator with the d-c vibrating magnet for the control of a d-c generator consists essentially of a main control magnet (Fig. 9) whose winding is connected across the gen-

erator terminals, and a differentially wound relay magnet. When the effect of the potential winding increases because of a rise in generator voltage, the contact of the main control magnet is opened and in turn one winding of the relay magnet is de-energized. Thus the relay contact is opened and the short circuit removed from the generator rheostat.

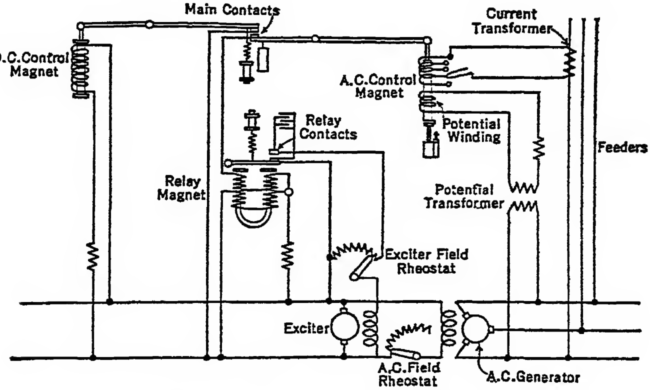


FIG. 10 D-c Controlled A-c Regulator

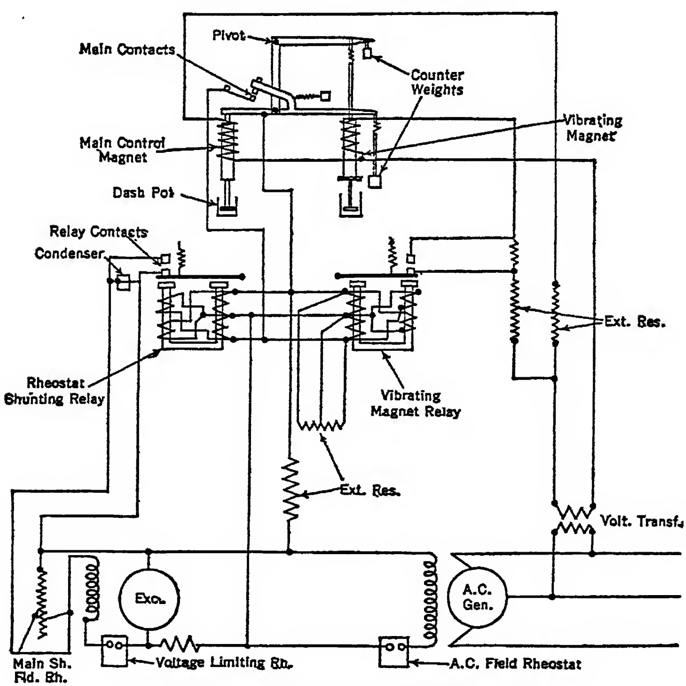


FIG. 11. Regulator with A-c Vibrating Relay Control

When the voltage drops, the main contact is closed and the differentially wound relay magnet acts to short-circuit the field rheostat. The relay contacts are shunted by a condenser to reduce sparking.

The regulator with the d-c vibrating magnet for the control of a-c generators works on the exciter field as shown in Fig. 10. The main contacts with this type of regulator are

acted on by two sets of control magnets, one connected across the exciter bus and tending to move the main contacts farther apart as the exciter voltage rises and the other acted on by a-c potential coils and current coils. When the main contact closes it energizes the relay magnet, thus closing the relay contact, short-circuiting the exciter rheostat, and raising the voltage of exciter and generator.

The use of the exciter voltage as one of the main control elements prevents the generator voltage overshooting, for, as the exciter voltage rises to bring up the generator a-c voltage, the d-c control tends to pull the main contacts apart and so reduce the voltage again. The compensating current winding of the a-c solenoid is provided with a dial switch to give any desired amount of compensation for the line drop in the feeder circuit in which the current transformer for the current coil is located.

The regulator with the a-c vibrating magnet connected like Fig. 11 momentarily short-circuits and reintroduces a portion of the resistance in the exciter circuit, and the average voltage delivered by the exciter will depend on the relative portion of the time that this part of the field resistance is in circuit. The main control magnet has its core attracted upwards and its core stem connected to a floating lever which is pivoted to the bell crank of the vibrating magnet. The two magnets are energized from the same voltage transformer that actuates the main contacts.

CARBON-PILE REGULATORS. A coil, core stem, lever system, carbon disks, and inverted air dashpot constitute a carbon-pile regulator. The coil has a current winding for parallel operation with another regulator, or line drop compensator, and a voltage winding. Pressure on the disks is supplied by a main spring, and is released by the pull of the core on the lever system. By varying the pressure on the carbon piles the resistance is adjustable through rather wide limits, and by automatically controlling the pressure the voltage is automatically controlled. The carbon-pile regulator connected like Fig. 12 is applied exactly as a field rheostat. On any generator the field current increases with the load or voltage according to the saturation curve of the generator. The total field resistance must decrease as the field current increases. If the range of resistance and current required between no load and full load does not exceed certain values, a carbon-pile regulator can be employed to advantage and is usually cheaper and almost as fast in response as the vibrating type.

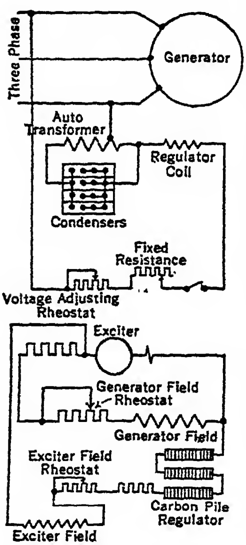


FIG. 12. Carbon Pile Regulator

RHEOSTATIC REGULATOR. The faceplate type regulator differs from the vibrating type in that there is a gradual cutting in or cutting out of field resistance, instead of the short-circuiting of a portion of resistance, for a greater or shorter length of time. The exciter rheostatic type provides a motor-operated rheostat in the exciter field circuit.

ELECTRONIC REGULATORS. Regulators utilizing the new electronic tubes of various designs are available for numerous applications where the energy available to operate the regulating equipment is of such low magnitude that the application of the conventional electromechanical regulators cannot be considered. Most electromagnetic regulators require approximately 100 volt-amperes for operation whereas electronic regulators can be designed to operate from considerably less than 1 microwatt. Owing to this reduction in control energy requirement brought about by the electronic tube the entire art of regulator design has been modified to obtain extremely high sensitivity and a quick response characteristic. The cost of the tubes and their upkeep militate somewhat against the use of the electronic regulator, and the application of this type of regulator can be considered as a special one.

D-c Machine Voltage Regulators of the electronic type have been used in applications where high sensitivity and quick response action are of importance. In most industrial applications of d-c voltage regulators a sensitivity of 1 per cent is entirely satisfactory, and this can be obtained with standard electromechanical voltage regulators at lower cost than the electronic type which can be used to obtain a sensitivity of 0.1 per cent.

A-c Machine Voltage Regulators of the electronic type of various designs have been available for several years and have given very satisfactory results. With electronic regulators a sensitivity of 0.1 per cent has been obtained for testing lamps and in other applications where a very close margin of voltage must be obtained. For ordinary industrial service such a high sensitivity is not needed, and electromechanical regulators are

used; also a simplified electronic type with one control tube and two power tubes has been developed.

QUICK RESPONSE EXCITATION is usually provided by a combination of faceplate and vibrating-type regulators utilized with exciters of proper characteristics. When a sudden demand is made on the excitation circuit a single relay or group of relays will short-circuit a portion of the exciter field resistance and the faceplate regulator will follow it up until the arm of the faceplate regulator is in such a position that the excitor is delivering the voltage required for the new conditions.

To obtain this quick response regulation various devices are utilized by different manufacturers. Among the most successful is a " pilot exciter " to excite the main exciter, often eliminating the main generator field rheostat. This " pilot exciter " is specially designed for very quick response and will momentarily impose on the field of the main exciter a very high voltage, enabling " field forcing " either up or down to be obtained. Another scheme is to split the exciter field into parallel paths with a permanent resistance in series with them. By this means an exciter response is obtained in 3 cycles on a 60-cycle circuit. For example, the normal voltage of the " pilot exciter " may be 250 with a maximum " ceiling voltage of 350," and each of the split fields of the main exciter might normally require only 60–80 volts except for the external resistance. Putting 250–350 volts on these fields momentarily will build up the exciter voltage very rapidly. The control of the regulator of the " pilot exciter " may involve a " torque motor element," " phase sequence network," or other device of similar nature. The use of a rectifier to place a high voltage on the exciter fields momentarily is another means of getting quick action.

APPLICATION OF VOLTAGE REGULATORS depends on several factors, entirely independent of the size and design of the regulator itself. It is not only necessary that the regulator be properly designed, but it is also essential that the exciters, generators, and prime movers possess characteristics that will harmonize with each other and will assist in keeping the voltage at the desired value under rapidly changing load conditions. Regulators are also used to maintain proper frequency, to control loads, to permit automatic synchronizing, etc. By using two current transformers, " cross-current compensation " can be obtained and speed or other characteristics controlled for industrial service.

RANGE. The standard a-c generator voltage regulators are adapted for voltage regulation of a-c generators requiring a nominal excitation range of either 45–135 volts or 60–150 volts, that is, a voltage range of 1 to 3 or 1 to 2 1/2.

With the broad range system of regulation, full automatic voltage regulation can be obtained for all ranges of excitation within the limits of the exciter. The standard regulator can be made broad range by energizing the relays from a separate source of direct current, such as a small motor-generator or a storage battery. The broad range system of regulation is directly applicable to synchronous condensers for maintaining voltage at the receiving end of a transmission line by adjusting the wattless load of the synchronous condenser either lagging or leading as required.

A standard vibrating regulator cannot be used if the minimum field voltage required is near or below the residual of the exciter. This is often the case where synchronous condensers are used for line regulation owing to the wide operating range required and the fact that synchronous condensers are usually designed for low field voltage with zero power factor lagging in order to keep the size, and consequently the cost, of the machine as low as possible. The extended broad range type which has been designed for such applications is a combination of a vibrating regulator and a rheostatic regulator. Within the stable limits of the exciter, the vibrating part of the regulator will function in the ordinary manner. When it is necessary to reduce the voltage across the generator field to a value that is below the residual point of the exciters, the rheostat part of the regulator comes into play and inserts some resistance in the field of the generator, necessitating an increased exciter voltage.

19. INDUCTION VOLTAGE REGULATORS

On a-c circuits containing no generating equipment or other rotating apparatus, voltage regulation is usually obtained by means of induction regulators, changing taps on transformers, or some combination of these two schemes. Where two systems, or two portions of the same system, are tied together for power interchange, voltage regulation at the tie point is usually necessary to regulate the interchange of wattless kva at such a point, and many interesting arrangements have been made for carrying out this voltage regulation.

The first type of regulator was a transformer with many taps and provision for

connecting the feeder to any tap. This could be done by switches of various kinds, and the natural development was to arrange the contacts in the form of a ring on a suitable face-plate and to provide a movable arm for connecting the feeders with any of the taps.

Induction regulators are made for single- or three-phase service and arranged for hand operation and motor operation, for motor operation controlled from a distant point, or for complete automatic operation by means of relays.

SINGLE-PHASE REGULATOR. Fig. 13 is in effect a two-winding transformer with the secondary winding arranged for connection in series and the primary winding arranged for connection directly across the line. With the transformer thus connected, a voltage will be induced in the secondary that will add to or subtract from the feeder voltage, according to the connections used. The current in the primary produces a magnetic field that induces a voltage in the secondary. The portion of this field passing through the secondary winding, and consequently the voltage induced in that winding, depend upon the angular position of the secondary with respect to the direction of the primary field. The induced voltage is a maximum when the axis of the coils coincide, zero when the coils are at right angles to each other, and maximum in the opposite direction when the axis of the coils coincide but with primary coils reversed in position. This induced voltage in the secondary therefore adds to or subtracts from the feeder voltage by a value varying from maximum regulation to zero according to the position of the coils.

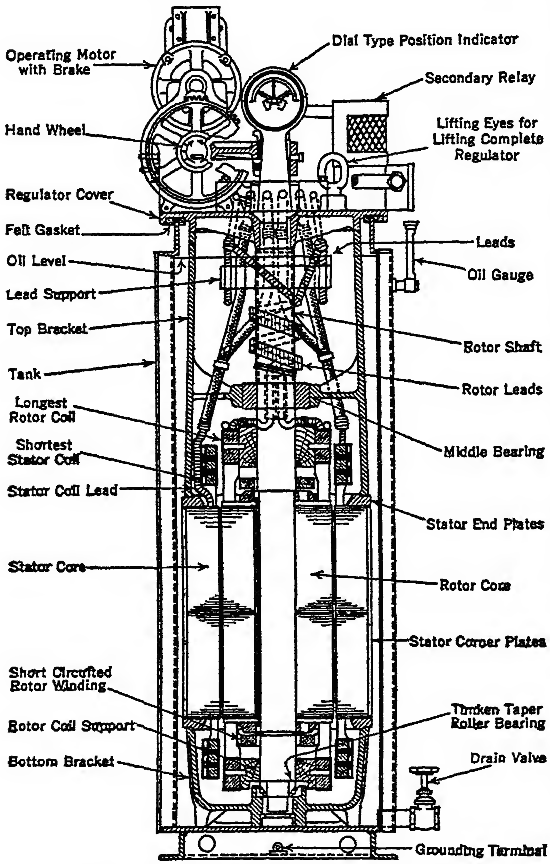


FIG. 13. Single-phase Induction Regulator, Self-cooled

The single-phase regulator adds or subtracts its voltage directly and not vectorially relative to the voltage from the main circuit so that the use of a single-phase regulator does not cause any phase displacement by its use.

POLYPHASE REGULATOR. Fig. 14 resembles somewhat a vertical shaft, phase wound, polyphase motor, immersed in oil in a suitable tank. The regulator primary is wound with a distributed winding of the same number of phases as there are phases in the feeder to be regulated, and each phase of the regulator is connected across a separate phase of the feeder. The secondary winding is made up of the same number of separate windings as the primary, and each of these separate windings is connected in series with one of the feeder wires. The primary sets up a magnetic flux of constant value which induces a constant voltage in each of the secondary windings. The induced voltage of the secondary is combined vectorially with that of the feeder. As the position of the rotor is changed, the phase angle between the feeder voltage and the secondary voltage changes, the feeder voltage is either increased or decreased.

Induction regulators can be arranged either for indoor or outdoor service and can be

self-cooled, air-blast cooled, or water-cooled, depending upon their size and local condition.

CONSTANT-CURRENT REGULATORS are a special type of transformer with movable coils that change a constant potential source of supply into a constant-current feeder. These are used chiefly for series lamp circuits, but they have other purposes.

AUTOMATIC OPERATION. If desired for either single or three-phase regulation, automatic operation can be obtained by the action of a voltage relay either with or without a compensating device. This relay acts in conjunction with the motor on the regulator so that, as the load comes on, or the bus voltage drops, the motor will turn the regulator in such a direction as to increase the voltage. By means of a compensator, which can be set for certain ohmic and certain inductive drops, the voltage at the point of distribution can be maintained constant, independent of the amount or power factor of the load, if the total drop is within the range of the regulator.

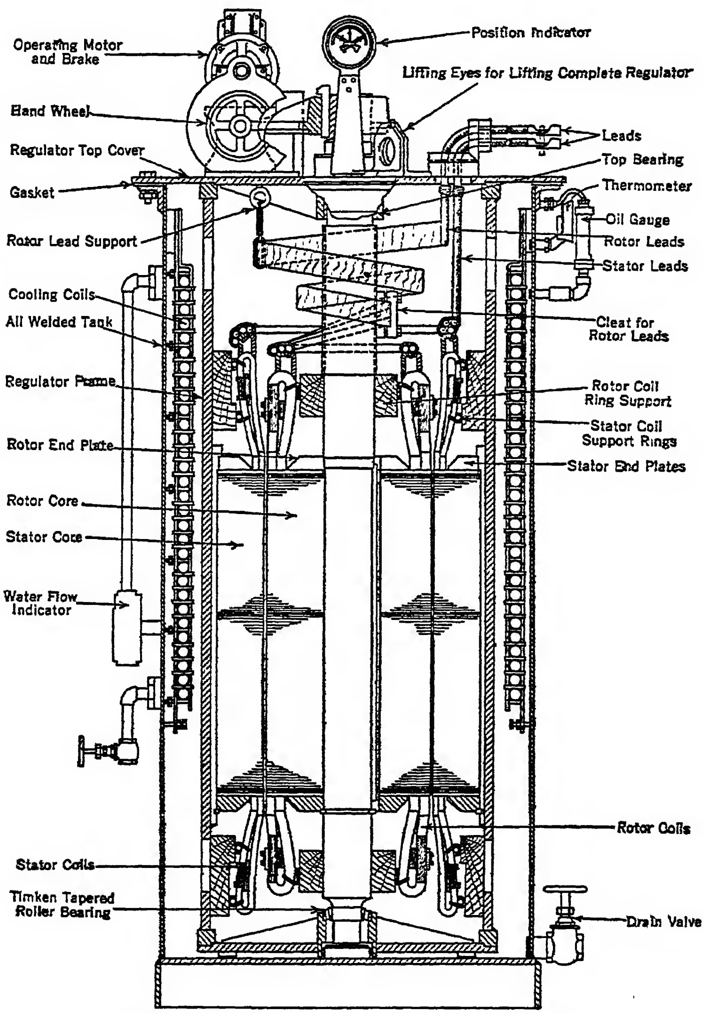


FIG. 14. Polyphase Induction Regulator, Water-cooled

FEEDER VOLTAGE REGULATORS of the electronic type utilize a reactor or impedance in the form of a two-winding transformer. The primary of this transformer is connected in series with the circuit to be controlled while the secondary is connected across the anode and cathode of the tube. The tube varies the effective impedance of the reactor. The action involves the reflection of varying impedance from the tube back into the primary. The tube can be considered as a switch to short-circuit the secondary, the time of remaining closed being varied.

STEP-INDUCTION REGULATOR. For very large capacities, a combination of taps on a step-down transformer or an auto-transformer with an induction regulator is sometimes used and is called a step-induction regulator. By designing the transformer with a voltage between the taps not greater than that of the induction regulator, it is possible to get a smooth voltage variation over the entire range of voltage covered by taps from the highest to the lowest.

Regulating the voltage of high-voltage transmission lines is now possible by means of taps in the step-up or step-down transformers through which the ratio is changed without disconnecting the transformers from service or interrupting the load. Various schemes of tap changing are available, and their details are considered in connection with the part of this handbook dealing with transformers.

PROTECTION

The fundamental problem in all protective schemes is to differentiate accurately between normal and abnormal conditions and to distinguish positively, under abnormal conditions, between faulty apparatus and line and the remainder of the system. Then the faulty apparatus or line must be disconnected from the remainder of the system with the least possible disturbance. In general, the function of the circuit breaker, acted on by a trip-coil or protective relay, is to disconnect the faulty apparatus or line from the remainder of the system before trouble can spread, and this separation should be made with the least possible interruption to service.

20. D-C CIRCUIT PROTECTION

For the automatic protection of d-c circuits, air circuit breakers or fuses are usually employed. For the protection of two-wire generators, single-pole circuit breakers are used on large panels; either fuses or breakers are used on smaller panels. For the protection of three-wire generators, circuit breakers with equalizer contacts are used on the large boards and four-pole circuit breakers on the smaller switchboards, as it is necessary to open up the positive equalizer and negative equalizer circuits as well as the main positive and main negative circuits under most conditions.

GENERATOR PROTECTION. In considering the type of protection for generators the following abnormal conditions should be taken into account: sustained overload, excessive temperature, and internal faults. In the usual attended station it is not present-day practice to provide automatic protection against sustained overloads and excessive temperature but to allow the operator, warned by instrument readings and alarms, to take the necessary steps to correct these conditions. In non-attended stations, relays perform the necessary duties.

FEEDER PROTECTION. For the protection of lighting feeder circuits up to 600-amp capacity, enclosed fuses were formerly used but are rapidly being replaced by small air or De-ion breakers. For feeders of capacities greater than 600 amp, circuit breakers are used.

Single bus railway panels provide overcurrent protection in one side of the circuit only, namely, in the positive side, opposite the series field. This protection is sufficient for synchronous converters having overcurrent protection on the a-c side and for motor-driven generators having overcurrent protection on the motor circuit.

Two-wire lighting and power panels provide automatic overcurrent protection in only one side of the circuit, namely, in the positive side opposite the series field. Three-wire lighting and power panels provide complete automatic protection in positive and negative circuits.

21. A-C CIRCUIT PROTECTION

In modern a-c generating stations, it is not considered good practice to provide overload protection for generators as the windings of such machines are so thoroughly braced that they will withstand momentarily a short circuit with full voltage maintained on the field. The objection to the use of overload protection in the generator circuit is that such devices might cause the machine to be disconnected from the bus-bars and to shut down the plant without any real necessity. The heating of the generator windings has to be watched carefully by means of good temperature indicators.

Generator failures usually occur in the armature circuit. The usual protection for armature faults is the use of differentially connected overcurrent relays or biased current differential relays. Essentially the scheme is to balance the current entering the apparatus against the current coming out. Any leakage of current to another phase or the ground, if of sufficient magnitude, will upset the balance and send the current through relays which, when actuated, trip the circuit breakers and other devices and disconnect the faulty apparatus by tripping the generator main breakers, the generator neutral breaker, and the generator field switch.

FIELD AND EXCITER CIRCUITS. It is standard practice not to supply automatic protection in exciter or field circuits. The sudden opening of the field circuit of an a-c generator caused by the operation of a fuse or breaker in the field or exciter circuit might cause far greater damage, by puncturing the insulation of the a-c generator, than would result from the overloading or even short-circuiting of an exciter.

FEEDER PROTECTION. Automatic overload protection for the feeder circuits of a-c systems may be provided by means of either fuses or circuit breakers depending on the capacity, voltage, and application of the system. Instantaneous trip breakers, within their breaker capacity ratings, are satisfactory for feeders supplying a lighting load or a steady power load. Breakers in circuits subject to high fluctuating overload should be equipped with inverse time limit devices so that breakers will not trip out on load fluctuations of normal character.

TRANSFORMER PROTECTION. Transformers can be protected by straight overcurrent relays, but it has become general practice to provide differential current balance protection by using current transformers in the high-tension and low-tension main transformer windings. It is sometimes difficult to obtain correct ratios for current balance with standard current transformers, but relays with suitable taps are available to take care of this condition.

SYNCHRONOUS CONVERTER PROTECTION. Protection for the a-c side of a synchronous converter is provided on the high-tension side of the step-down transformer by an instantaneous overload oil circuit breaker tripped from current transformers. The breaker is also equipped with low-voltage release and auxiliary switch. A low-voltage trip, instantaneous overload carbon circuit breaker is provided for the d-c side. The speed limit switch furnished with and mounted on the converter opens upon overspeed and causes both a-c and d-c breakers to trip simultaneously. Reverse current relays are also provided on the d-c panel arranged to open the a-c breaker upon reversal of d-c power which in turn opens the d-c breaker.

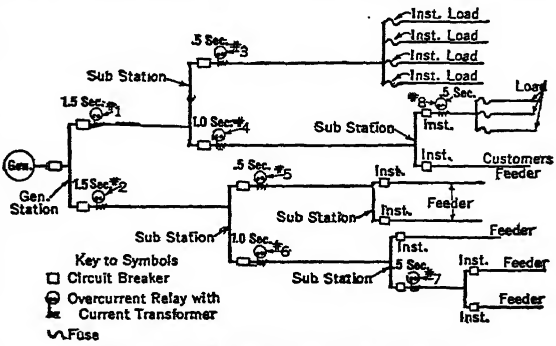


Fig. 1. Typical Radial Transmission System

RADIAL DISTRIBUTION. The simplest system of distribution is where there is a single source of power, with a number of feeders leaving the generator bus-bar, each feeder in turn being subdivided into a number of small feeders. This is termed the radial distribution system shown in Fig. 1. The protection of such a system against short circuit may be secured by overcurrent protective devices. The smaller branches may be disconnected automatically from the remainder of the system by the blowing of a fuse or the operation of instantaneous circuit breakers. The circuit breakers near the generator

should be equipped with definite time relays. A time interval between the successive relays long enough to assure a reasonable margin of safety above that required for a circuit breaker to operate is allowed. Relays at the generator bus are given the higher time settings. The setting of the other relays is decreased as the distance away from the generator increases till the most remote relay has an instantaneous setting.

RING OR LOOP SYSTEM. This system is shown in Fig. 2; it is simply a continuous transmission line running through a series of substations and finally terminating at its starting point. This forms one of the best ways of securing uninterrupted service with a minimum expense for feeders. The number of substations which may be thus tied together depends upon their geographical location and the relay protection desired. With the normal overcurrent and reverse power relays, the number of stations is limited, by reason of the fact that the required time interval necessary between successive relays adds up to an unsafe value. By the use of the impedance relays, this difficulty is largely removed.

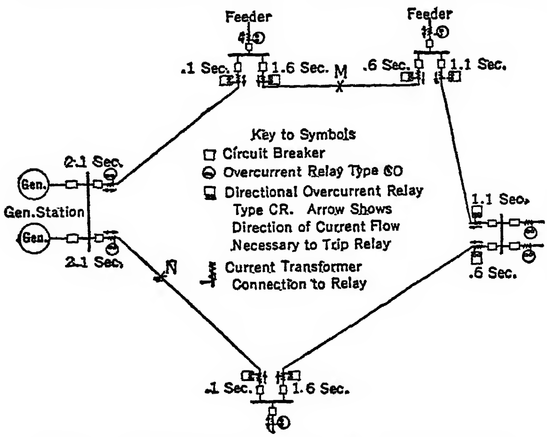


Fig. 2. Diagram of Ring or Loop System of Distribution

On a simple loop system having only one source of power, directional overcurrent relays with definite time limit characteristics may be used for protection against short circuit. The time setting of each successive relay must be increased by an amount sufficient (usually 0.5 sec in practice) to allow time for the circuit breaker in the preceding substation to open. The directional relays applied to each substation are fixed so that they will trip only when excess power is flowing away from the substation bus.

Network Systems both primary and secondary are dealt with in the section of this handbook dealing with Distribution. Suitable relays and switching devices are available for these networks.

Balanced Relays can take care of parallel circuits between a generating station and receiving stations as shown in Fig. 3. Under normal conditions, the current in each of the parallel feeders will be the same; and since the relays have a higher impedance than the current transformers, the current from the latter will circulate through all of them in series without any flowing through the relays.

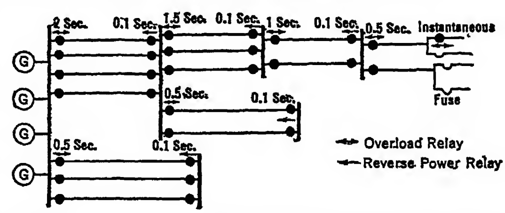


Fig. 3. System whereby Lines Run To and Through Several Sub-stations

Inasmuch as the action of the balanced current relay is practically instantaneous in clearing faulty lines, it makes an ideal application for protection for parallel lines between generating stations. On other parallel feeders, it is very often advisable to use overcurrent relays in connection with balanced current relays, thus securing single line protection in addition to the quick balance protection when both lines are in service.

SYMMETRICAL COMPONENTS. One of the recent developments in connection with a-c circuits is the use of "symmetrical components" (see Sect. 3, Art. 37), permitting the convenient analysis of unbalanced loads and single-phase short circuits in polyphase systems. For a three-phase system the components consist of positive, negative, and zero phase sequence sets of current and voltage vectors which may be combined to give the actual currents and voltages in each phase for any condition of unbalance, thus facilitating proper relay settings. The components themselves may often be of use, and can be measured with the aid of certain "networks." The positive phase sequence components

of current and voltage are a measure of the balanced load and become equal to the actual currents and voltages under balanced conditions. The negative phase sequence components are a measure of phase unbalance, and a relay responding to the negative sequence current may be used for protecting a machine against internal faults on unbalanced operation. The zero phase sequence components are a measure of ground unbalance. The zero sequence component of current is a measure of the ground or " residual " current and is often used to energize relays protecting against ground faults. The zero sequence voltage is likewise a measure of " residual " voltage, which is often used with the " residual " current for directional ground relay operation.

HIGH-SPEED RELAYS. The time delay introduced by the usual relay is advantageous only because of the reduced circuit-breaker duty resulting from the decrement in short-circuit current. The disturbance to the system produced by a short circuit will be considerably reduced if a faulted section or piece of equipment is instantaneously isolated from the remainder of the system; and where stability or inductive interference with neighboring circuits is a factor, high-speed fault clearing is of the greatest importance. As definite time delay may no longer be utilized to select the faulted section, relay systems operating upon other principles are required. High-speed overcurrent, directional overcurrent, balanced current, differential, and distance (impedance or reactance) relays have been developed which when used with high-speed circuit breakers are capable of isolating a faulted section in one-fourth of a second or even less. A set-up of the system on an a-c calculating table enables the proper selection of relays, current transformers, etc., to be selected to obtain the best scheme of relaying.

22. LIGHTNING PROTECTION

All electrical systems are subjected to transient or short-duration overvoltages in some degree. These overvoltages arise from various causes, and their intensity varies over a wide range. Since the major cause of the ultimate failure of electrical apparatus is insulation failure and since any overvoltage hastens this failure, a benefit will always be derived from the use of static protective equipment. The decision in any specific case as to whether or not such equipment shall be used, and as to the type to be selected, is dependent on the relation between the cost of the equipment, initial and maintenance, and the savings by the use of equipment, both in apparatus and service.

The factors which control the application of static protective equipment are so varied and some of them so uncertain quantitatively that there are no exact rules to follow. Each case is an individual problem which must be solved individually by experience on the actual system, or on similar systems, similarly located, taking into account not only the prevalence of trouble but also the specific conditions such as value of apparatus, expense of repairs, expense of service maintenance, and results of failure of service.

GROUND WIRES. A very effective protection for an overhead, high-voltage transmission line can usually be obtained by the use of an overhead ground wire. This is a grounded wire which is run above and parallel to the main wires of an overhead line. This acts as a partial screen and is one of the oest means for protecting such a line from lightning disturbances. The protection is not perfect, however, since the wire forms only a partial screen.

INSULATION COORDINATION has been adopted a great deal in recent years to get the proper breakdown values for the insulation of the transmission line, the circuit breakers, transformers, and other important pieces of apparatus in a step-up or step-down transforming station. Suitable insulation levels against breakdown under impulse voltages should be established. The insulation strength of the transformers is made the highest, followed by the transformer bushings to insure the arcing over of the bushings before any internal breakdown of transformer occurs. The insulation strength of oil circuit breakers is made a little lower than that of the transformers, and the oil circuit breaker bushings will arc over before puncture or circuit-breaker failure. The insulation of the transmission line is made a little weaker than that of the oil breakers so that in case of a lightning stroke near the station there will be an arc-over of the line insulators before there is trouble elsewhere.

LIGHTNING ARRESTERS. At the step-up and step-down transforming stations where there are valuable banks of transformers to be protected, or where there is an extended distribution system to protect, it has been found well worth while to install lightning arresters to provide a shunt path to ground in case of overvoltages arising from any causes.

Arresters are of two main classes, those with resistance characteristics and those with valve or counter electromotive force characteristics. The resistance characteristic class

includes the various multigap types, the horn gap and resistance type, circuit-breaker type, and the magnetic blowout. In these, the current flowing after the gap is broken down is proportional to the applied voltage. The valve or counter electromotive force class includes the electrolytic, auto-valve and oxide film types. The latest developments of arresters are the " thyrite " resistance type and " porous block " valve designs.

The simplest resistance type of arrester comprises a spark gap so set that excess voltage will cause the gap to arc over, allowing the charge due to the voltage to pass to ground. There is combined with the gap some means of suppressing the power arc which follows and which tends to continue after the abnormal voltage has ceased. An ideal lightning arrester should take no current at the ordinary operating potential, but at any potential much higher than ordinary there should pass enough current to limit the abnormal potential to some safe value. When the abnormal potential ceases, the arrester should stop taking current from the line. The closer an arrester approaches this ideal the better the arrester.

Horn Arresters like Fig. 4 have been employed on high-voltage circuits to give a certain amount of protection at a relatively low price. One or more horn gaps are connected in series with a resistance to ground. When a surge has caused the horn gap to arc over, the resistance limits the power current that tends to follow. The magnetic effect of this limited current with the tendency of air heated by the arc to rise is usually sufficient to cause the arc to rise up the horns and blow itself out. The resistor in the ground circuit can be made in the form of resistance rods or in the form of a jet or column of water. The latter form of resistance is usually made of earthenware tubes containing a mixture of water and glycerin, and the resistance is selected to allow about 1 amp to flow at the time of a discharge.

Gap Arresters. For lower voltages, various designs have been employed that utilized the " non-arcing metal " discovered by A. J. Wurts. This metal, a type of brass, when arranged in the form of knurled cylinders with a small space separating them, would discharge the excess voltage without allowing the power current to follow, provided the amount of power available was small and certain other conditions were met.

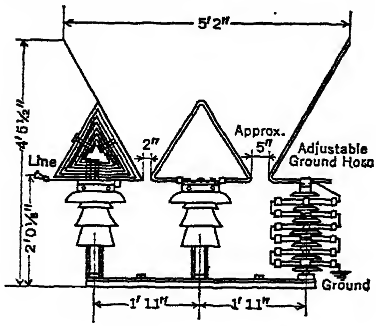


Fig. 4. Horn Type Arrester

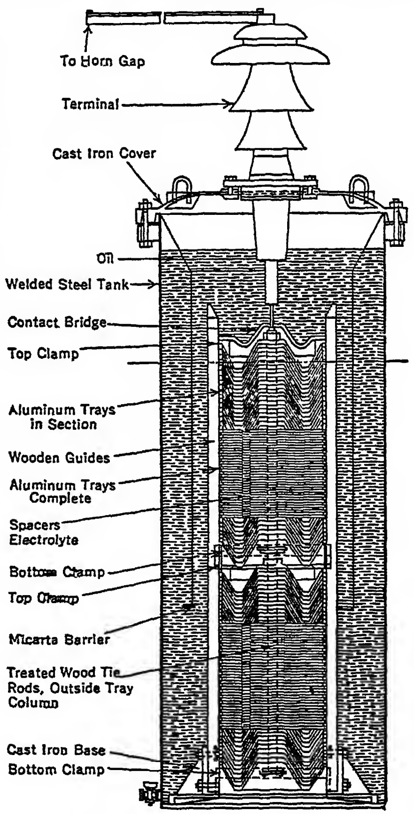


Fig. 5. Electrolytic Arrester

Electrolytic Arresters. These were the first " valve-type arresters " that were suitable for high voltages and large power systems. This type of arrester consisted of nested conical or cup-shaped aluminum trays like Fig. 5 filled with a liquid electrolyte and immersed in oil in a steel tank. The liquid electrolyte formed an insulating film on the trays, and this permitted only a very small current to flow at normal voltage. The film

broke down at high voltage, discharging the line, but the film sealed up quickly on the cessation of the abnormal voltage and shut off the power current. A horn gap was usually provided in series with this set of trays or aluminum cells. Periodic charging was needed to maintain the films in good condition. Although they were relatively expensive, they were used to a very large extent until the introduction of the "oxide film" and "autovalve" designs that gave about the same protective value at a lower cost, and without attendance.

Oxide Film Arresters, Fig. 6, depend for their functioning on the fact that certain dry chemical compounds, such as lead oxide, can be readily changed from a very good conductor (litharge) by the application of a slight degree of heat, such as would be caused by the passage of a lightning discharge, to a practical insulator (lead peroxide). The litharge was assembled in porcelain containers between metal covers with a film of insulating lacquer, and enough of these assemblies were placed in series to withstand the normal voltage. An overvoltage would puncture the lacquer film, and the power current that tended to follow the discharge would heat up a small part of the litharge (lead oxide), changing it to peroxide whose insulating properties sealed off the follow current. For the more moderate voltages on distribution circuits the lead oxide was made up into pellets the size of sugar pills, and these were assembled in a tube with metal caps making essentially a number of small pellets in series and parallel to form the arrester. This oxide film arrester was the standard of the General Electric Co. until it brought out its latest design.

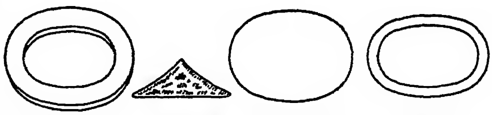


Fig. 6. Oxide-film Arrester Parts

Thyrite Arresters, brought out early in 1930, depend on the non-linear relation between voltage and current in the material, so that at heavy surge current the resistance of the arrester is low enough to limit high surge voltages to an appreciable degree. The Thyrite material is not really a valve-type arrester, so it is necessary to use a series gap which is housed in each 11.5-kv container. Because a single gap is unable to break an arc of the magnitude often met, a series of small gaps is used. At line voltage the resistance is high enough to limit the power current to values that can be interrupted at the zero of the cycle by the multiple gap. No discharge occurs up to a very definite breakdown voltage. After breakdown, the voltage across the gap is very low. The arrester utilizes blocks $^3/_4$ in. thick, 6 in. in diameter for each 1000 volts, employing 11 in series for 11.5 kv. A number of the 11.5-kv containers are assembled in series for higher voltages.

Autovalve Arresters were first brought out in 1922 by the Westinghouse Co. in the "mica disc" form, Fig. 7, utilizing the property of a glow discharge that occurs at a definite voltage of about 350 volts between disks that are very close together and are normally separated by a thin mica washer so proportioned that the discharge occurs at the inner edge of the disk. Later designs eliminated the mica washers. The composition of the disk is such as to furnish enough resistance to limit the power current that might tend to follow to such a low value that the discharge would not change from a "glow discharge" to an "arc discharge." For line arresters and general distribution circuits a series of these disks was used in a porcelain tube with an air gap in series with them. For station service to provide greater discharge capacity, four sets of disks in parallel were used, located in porcelain insulators, provision being made for connecting the groups in series.

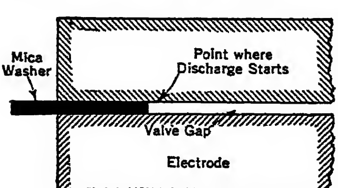


Fig. 7. Mica-spaced Autovalve Arrester Parts

Porous Block Arresters are the latest developments in the line of valve-type arresters (see Fig. 8). The autovalve element consists of one or more porous blocks in series, fabricated from a mixture of ceramic material and conducting particles into a uniform structure. The blocks contain a large number of pores through which the lightning discharge passes. A block 1 in. thick is rated at 3000 volts rms value, and the blocks are made of 4 $^1/_2$ or 2-in. diameter depending on the class of service for which they are intended. An enclosed multiple gap is used in series with the block. The principle of operation depends on the fact that the voltage necessary to maintain a discharge can be raised to a high value by confining the discharge to narrow passages within insulating walls, and in this arrester the discharge takes place through myriads of pores. The pore

equipment as compact as possible. These instruments are only 4 in. wide and 4 1/4 in. high and permit a very compact arrangement on a switchboard. These instruments have been made as small as is practicable while still retaining a long, easily read scale. All conventional indications may be supplied in this form of instrument such as voltage, current, watts, and frequency.

Instrument Transformers are used for two reasons: first, to protect station operators from contact with high-voltage circuits; and second, to permit the use of instruments with a reasonable amount of insulation and a reasonable current-carrying capacity (see also Section 5). The function of instrument transformers is to deliver to the instrument a current or voltage which shall be always proportional to the primary current or voltage and which shall not exceed a safe potential above ground. Generally the secondary of a voltage transformer is designed for about 115 volts and the secondary of the current

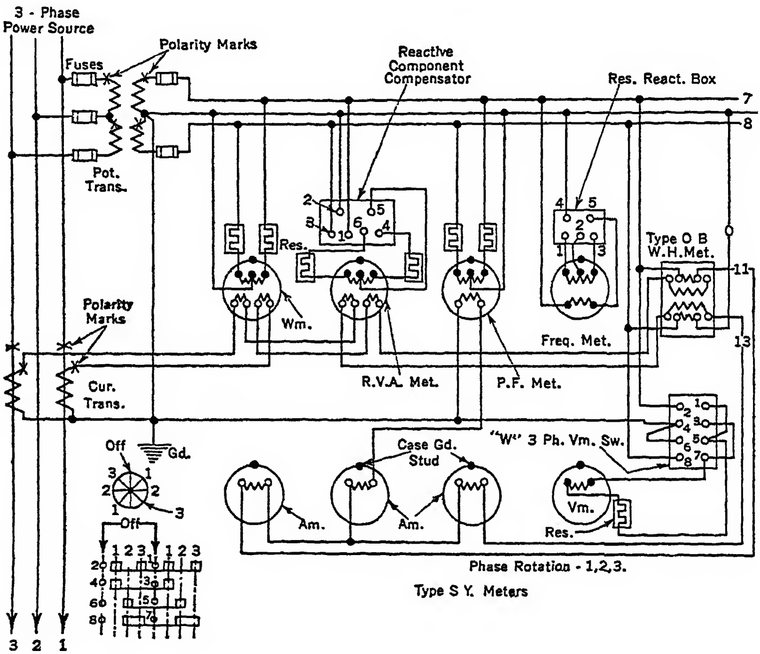


Fig. 1. Metering Equipment on Three-phase, Three-wire Circuit

transformer for 5 amp. Fig. 1 shows typical connections from instrument transformers to instruments of various kinds on a three-phase three-wire circuit.

Grounding. All instrument transformers should be grounded on the secondary side as an extra precaution against danger from the high voltage in case the insulation should be punctured by lightning or other abnormal stresses. In polyphase groups, any point of the secondary may be grounded, but it is preferable to use a neutral point or a common wire between two transformers.

25. MISCELLANEOUS METERS

BUSHING POTENTIAL DEVICES provide means of obtaining low voltages suitable for instrument use without the expense or hazard of the high-voltage potential transformer. The component parts of the device are mounted in a sheet-steel protective housing and are provided to connect to a tap near the grounded end on a special bushing of an oil breaker. The equipment within the housing consists of an enclosed spark gap, a low-voltage potential transformer, and a reactor and a condenser. The limitations on the burden of these devices depend on the primary voltage and are 66 kv 7 volt-amperes, 110 kv 15 volt-amperes, 132 kv 20 volt-amperes, 154 kv 27 volt-amperes, 187 kv 50 volt-

amperes, 220 kv 60 volt-amperes. These burdens are based on 60-cycle service and must be reduced proportionately for other frequencies. These condenser taps can also be used on porcelain or oil-filled bushings having condenser characteristics. Also a separate device may be provided having condensers in series to ground, lower ones being tapped for potential. The capacity of these devices are up to 120 volt-amperes at 287 kv 60 cycles.

Ground Detectors are not usually mounted on switchboards but are employed to indicate grounds on circuits that are normally ungrounded. On ungrounded two-wire d-c systems a voltmeter with a central zero may be employed connected between ground and the neutral point of a resistance across the d-c circuit. For low-voltage circuits lamps can be used as in Fig. 2. For a-c circuits of 2300 volts and above electrostatic ground detectors can be employed. These may be made as single-phase instruments or as three phase, connected to the circuits through condensers as shown in Fig. 3. Neon lamps can often be used as ground detectors connected directly to the lines or to condensers or to a string of suspension insulators as shown in Fig. 4. Ground detectors of the tube type coupled with suitable resistors can be used.

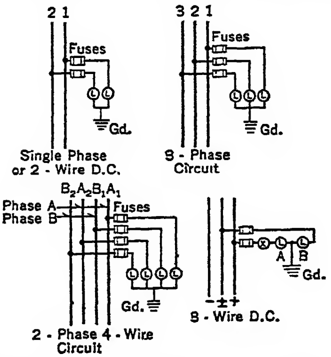


FIG. 2. Diagrams of Ground Detector Equipments - Using Lamps Only. For 125–250 volts use three 110-volt lamps; for 250–500-volts use three 220-volt lamps

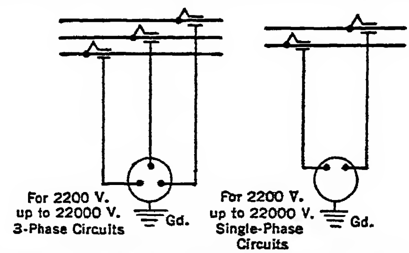


FIG. 3. Diagrams of Electrostatic Ground Detectors

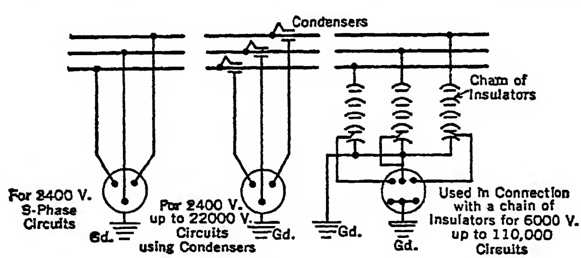


FIG. 4. Diagrams of Electrostatic Glow Meters

26. PANELS AND DESKS

Type of current, voltage, and similar factors determine the class of switching device which should be used, and this, in turn, determines the class of switchboard to be installed. The desired operating conditions for an installation are factors in the selection of either mechanically or electrically controlled apparatus. The allowable space may determine the type of equipment.

The switching arrangement should not offer an unnecessary risk to the men who have to operate the switching apparatus, particularly under conditions arising from electrical failures of lines and apparatus. Simplicity should be the guiding spirit in the layout of the switching equipment. Satisfactory operating results cannot be obtained from poorly arranged layouts. Coincident with simplicity there should be a reasonable degree of flexibility which has for its object the providing of such a degree of continuity of service as may be commensurate with the class of load.

Direct-control Boards are those like Fig. 5 where all the apparatus is mounted, either directly or partly, upon the panels and the remainder on the panel-supporting framework. This switchboard for 2400 volt, three-phase service controls two a-c generators, with vibrating regulator voltage control, exciters, two power feeders, rectifier circuits for arc lights, and a-c synchronous motor-generator for supplying direct current.

Manual Remote-control Boards are those on which only the lighter pieces of apparatus are mounted. The main circuit breakers and their associated apparatus are supported on suitable framework at a reasonable distance from the panel board. The oil circuit breakers

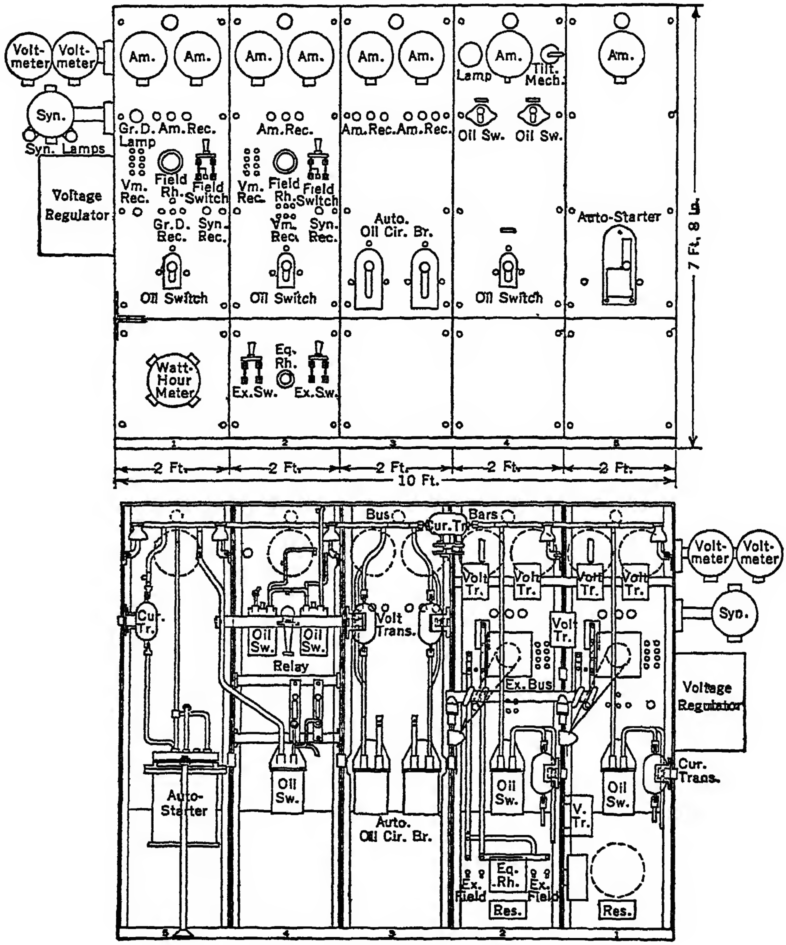


FIG. 5. Direct-control Switchboard

or other switching devices are operated by suitable operating rods and links attached to the handles on the front of the panels, like Fig. 6.

Electrical Remote-control Boards are those which use electrically operated circuit breakers and other electrically operated switching devices located apart from the panels and operated either by means of manual control switches or automatically through suitable relay equipments.

LIMITS. The direct-control switchboard is usually limited to a station capacity not exceeding 3000 kva, a maximum circuit capacity a-c 1200 amp, and a maximum voltage of 2500, although in special cases these limits may be exceeded. The manual remote-

control switchboards are usually limited to approximately 25,000 kva, the limitations really arising from the distance between the location of the switchboard panels and its correlated oil switching devices and from the physical effort required to operate switching devices through systems of bell cranks and connecting-rods. For systems of more than 25,000 kva capacity the short-circuit conditions usually require breakers of such a size that manual closing is apt to prove difficult. The electrical remote-control switchboards have practically no limitations and are used to control any amount of power up to the maximum concentration encountered in large power systems.

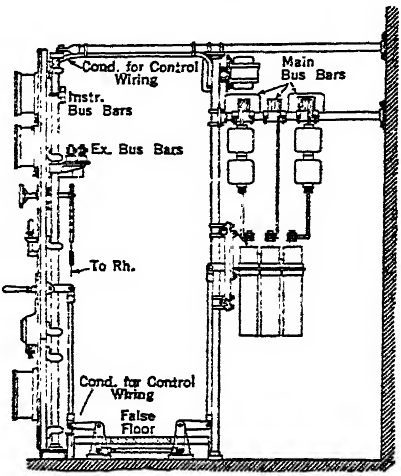


FIG. 6. Manual Remote-control Switchboard

ELECTRICALLY OPERATED SWITCHBOARDS. These boards usually take one of three general forms. Namely, the panel board, the combination control desk and elevated instrument board, or the combination pedestal and board. Various standard arrangements for electrically controlled equipment are shown in Fig. 7. *a* illustrates a typical vertical panel board with the instruments, control switches, relays, and similar devices mounted on the face of the panel. *b* shows the arrangement of a control desk with the control switches placed on the desk and the instruments mounted on the wall in front of the operator. *c* shows the control desk where there are only a comparatively few meters, these being set flush in the face of the desk. *d* shows the modification of the desk arrangement with the meters on a small slab or bracket extending up from the horizontal slab of the desk. *e* shows the control desk arrangement with vertical panels forming the back of the desk and the vertical panels containing indicating meters. *f* is a further modification of the control desk arrangement with vertical panels containing the indicating meters and a complete switchboard at the rear to contain the recording meters, relays, and similar devices. With this arrangement, a self-supporting control desk is provided. *g* shows the so-called gallery type of desk with the meters located on a framework supported from the horizontal slab of the desk at such a height that the operator standing at the control desk can look above the edge of the desk and below the meter panel to observe, from the switchboard gallery, the machine which he is controlling. *h* is a modification of the gallery type of control desk. *i* is a modified arrangement of control desk using a separate instrument frame supported on ornamental pillars, these pillars as a rule being arranged to form the support of a gallery railing. *j* shows the combination utilizing a gallery-type control desk for the generators and a vertical panel switchboard for the feeders. *k* shows a combination control desk and panel board, the generator breakers being controlled from the desk, the generator instruments being on the vertical panel, and all the

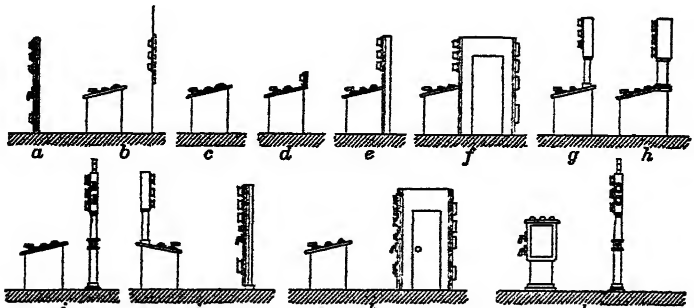


FIG. 7. Forms of Control Switchboards for Electrically Operated Switchgear

feeders being controlled by the vertical panels. The recording meters, graphic meters, and relays are placed on an auxiliary board back to back with the feeder board. *L* shows an arrangement of control pedestals and instrument posts. Where it is desired to have a very compact arrangement the control desk has many advantages.

PANEL GROUPING. Switchboard panels are normally grouped into a continuous board to facilitate more ready control by the station attendant and to also facilitate short connections between the wiring and bus-bars between the various panels.

For a long while, black finished slate was considered the standard material for most switchboards, and it still is used where knife switches, carbon breakers, or similar live equipment is employed. Where live parts can be mounted away from or back of the panel, the present tendency is to use steel wherever it can be employed to advantage. Where slate or marble panels are used, the standard switchboard frames are made up of angle iron, Fig. 8, or gas pipe construction, Fig. 9. For the smaller panels, the frame consists of vertical sections of gas pipes resting in floor flanges and supporting the necessary panel brackets to which the panel is bolted. Pipe frame is also used for many of the larger panels.

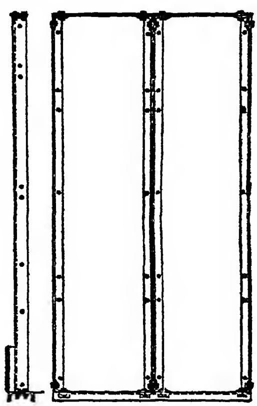


FIG. 8. Angle Frame

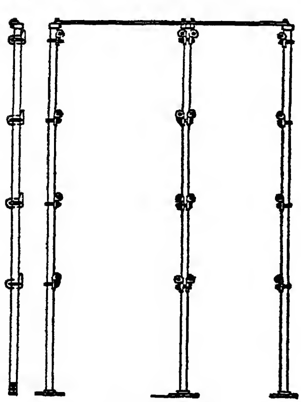


FIG. 9. Pipe Frame

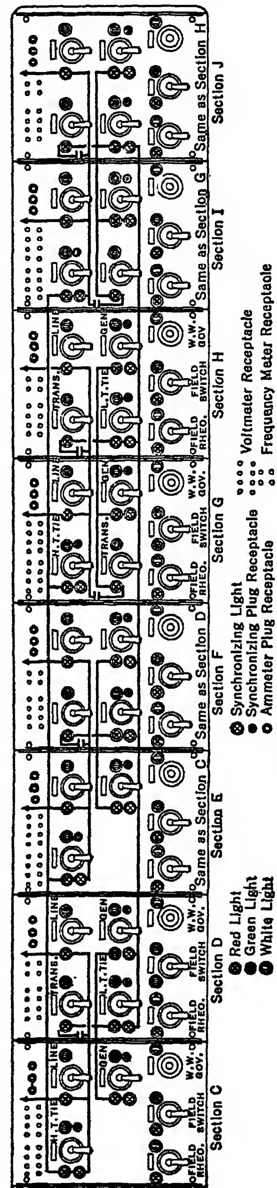


FIG. 10. Miniature Bus on Control Desk

One of the recent developments is to utilize steel panels up to 90 in. high made of stretcher level steel 1/8 in. thick with the sides bent 3 to 6 in. at right angles to the face and then 1 in. parallel to the face so that the plan view of the panel resembles a very narrow letter C. The material used is the same steel as is employed in metal furniture and presents a very fine appearance. Sufficient stiffness is obtained by the shape of the steel so that no auxiliary framing of any kind is required.

Adjacent panels are bolted together through the web of the steelwork in the same manner as angle iron framework on switchboards. The final bend in the steel panel is a 1-in. projection parallel to the face of the switchboard, and this projection is utilized for the purpose of attaching suitable steel brackets spanning the panels and used for the support of wiring and other details.

The same type of construction has been extended to cover the field of control desks, housing back of board breakers, miniature boards, etc. The advantages are fine appearance, permanence, safety, light weight, more complete factory assembly, and low cost.

Miniature Control Switchboards have been designed to meet the growing need for smaller and more compact centralized control boards on which the various control switches and instruments are concentrated in full view and under the control of one operator. A complete set of miniature switchboard devices has been developed to provide a much smaller switchboard capable of doing everything that is ordinarily performed with the conventional but larger switchboard. The miniature switchboard is particularly well adapted to control electric generating stations and their associated transmission systems and for operating distribution and switching substations. Owing to a substantial saving in space, better supervision because of smaller size, and lower installation cost, the miniature switchboard is finding increased favor. Remote control over extended distances up to approximately 6000 ft is economically possible, by using telephone cable with interposing relays and auxiliary transformers for the instruments, these transformers being about the size of bell-ringing transformers. The simplest form of miniature switchboard makes use of the modern stretcher-leveled steel panels, but more elaborate arrangements such as desks may be used.

Miniature Bus is extremely desirable for the proper and efficient operation of electrically operated panel boards or control desks of any size. It furnishes the operator a bird's-eye view, as it were, of the station wiring, since the miniature bus is a skeleton or single-wire diagram of all main circuits of the station with devices for indicating the relative location of all circuit breakers, disconnecting switches, power transformers, and feeder circuits. Miniature bus, Fig. 10, showing the top of a linear control desk is usually made of polished copper strap run along the top of the desk or, in a vertical board, on the face of the panel. Frequently, when a station used for power transmission supplies various voltages, different colors are adopted for the low-voltage and high-voltage miniature buses so as to distinguish them more readily.

27. D-C SWITCHBOARDS

D-c switchboards cover a wide field and include in their range every application of direct current. The larger boards are used for d-c railway systems and the lighting or power systems of large industrial plants, hotels, central stations, etc. The smaller generator and feeder panels like Fig. 11 are intended primarily for light and power systems of small industrial plants, small hotels, and central stations of small capacity; the battery charging panels are designed for controlling the charging of storage batteries used in lighting service and hence on electric vehicles.

Battery Charging in garages, small plants, etc., usually employs a switchboard consisting of a generator, or an incoming line, section; meter equipment; and one or more charging sections where the direct current is to be used for battery charging.

Small D-c Panels. The next larger size d-c panels utilize slabs 48 in. high and pipe framework, and these switchboards are particularly adapted to welding service and to the control of one to three generators in small industrial plants or central stations operating d-c two-wire systems of 250 volts or less. The capacity of a single generator panel is limited to 600 amp, and that of a complete switchboard composed of these panels to 1500 amp, with the number of panels limited to 6. For greater capacities, 90-in. switchboard panels are recommended.

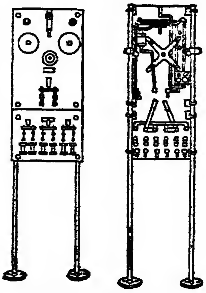


Fig. 11. Small D-c Switchboard

Mining Switchboards suitable for substation service in mining installations controlling motor-generator sets are usually arranged with very simple connections.

Large D-c Panels. For the control of d-c generators, the d-c end of synchronous converters and d-c feeders for 250 volts, two-and three-wire light and power service, and 600-volt railway service, panels with a total height of 90 in. have been standardized by various switchboard builders. These 90-in. panels are usually provided with two or three sections with the carbon breakers mounted on the top sections. Electrical operation is fairly common for high-voltage a-c boards using oil circuit breakers; it is not used so frequently for direct current or low-voltage alternating current with carbon breakers, but there are some cases where it is also used to advantage for that class of service. Probably the place where electrical operation is used most frequently for d-c service is for the control of exciters and field circuits in a generating station where electrical control is employed for a main a-c circuit and the exciter and field switchboards are electrically controlled from the generator switchboard. Newer tendency is to mount heavy low-voltage live apparatus, such as large air circuit breakers, away from the control board.

28. A-C SWITCHBOARDS

A-c switchboards may be divided into the following three classes depending on the mounting and method of operation of the apparatus: direct control, distant mechanical control, and electrically operated switchboards.

For moderate-capacity low-voltage a-c service, switchboards are particularly designed for the control of from one to three generators in small industrial plants and central stations operating a-c systems below 500 volts. The capacity of the single generator panel is limited to 600 amp, and that of a complete switchboard, composed of these panels, to 1800 amp with the number of panels limited to 6. For greater capacities 90-in. panel switchboards are preferable.

Remote-control A-c. Hand-operated remote-control a-c switchboards are applicable where the simplicity of connections or accessibility desired cannot be obtained with panel-mounted apparatus, where the station's capacity or voltage is so high as to make it desirable to mount switching equipment apart from panels, and where the station arrangement permits the use of manually operated remote-controlled oil circuit breakers. Electrically operated remote-controlled switchboards are applicable where the equipment must be remote controlled but where manually operated switchboard apparatus is not suitable. With remote-control switchboards, either manually or electrically operated, the most interesting features of design center in the arrangement of the breakers, bus-bars, and connections in the structure, and these are considered in the chapter on Station Structures and Layout. Similarly, metal-clad switchboards are described in Art. 35.

29. BUS-BARS

In order to provide facilities for utilizing the current developed in an electrical generating station to the best advantage, it is customary to have one or more sets of circuits into which the various generators deliver all their current and from which the various feeders draw all their current. These common circuits were originally known as " omnibus " bars, which term was later shortened to " bus "-bars.

D-C BUS-BARS. With shunt-wound d-c machines, it is necessary to have a positive bus and a negative bus; if two machines are run in series on a three-wire system, a neutral bus is also needed. For compound wound d-c generators on a two-wire system, an equalizer bus is required. For railway service, with ground return, the feeder connects only to one bus, usually the positive, the other bus, the negative, being grounded, and the equalizer bus merely running between the machines. The equalizer bus is normally made half the capacity of a main d-c bus, but the cable connections from the generator equalizer should be of full capacity.

EXCITER BUS, on panel switchboards for a-c service, ordinarily extends across the exciter panels and the a-c generator panels, and if it is used exclusively for the exciting current the capacity need not exceed the total current required for the generator fields. Fig. 12 shows the usual arrangement of panels and bus-bar systems for moderate-voltage a-c switchboards with the bus-bars on the rear. In the third arrangement shown the station load consists of two parts, lighting and power. A tie switch is provided to permit parallel operation of the two halves of the station when desirable. The feeder panels are placed at the extreme ends of the board to permit addition to feeders without disturbing the rest of the board, lighting feeders being taken out at one end and power feeders at the other. The switchboard and exciters are located in the center of the station with the a-c generators symmetrically arranged on either side. The last arrangement shown consists of double-throw feeder circuits and single-throw generator circuits. This system may be desirable

because of the difficulty of parallel operation where one set of generators differs in characteristics from the other, because of different voltages on the two bus-bars, or because one is alternating and the other direct current. Dotted lines in the main bus-bars indicate an arrangement for double-throw generator circuits; in the exciter bus-bars they indicate that arrangement is made for parallel operation of all exciters, and in the instrument and synchronoscope bus-bars they indicate that arrangement is made for using either set of instruments for the station as desired.

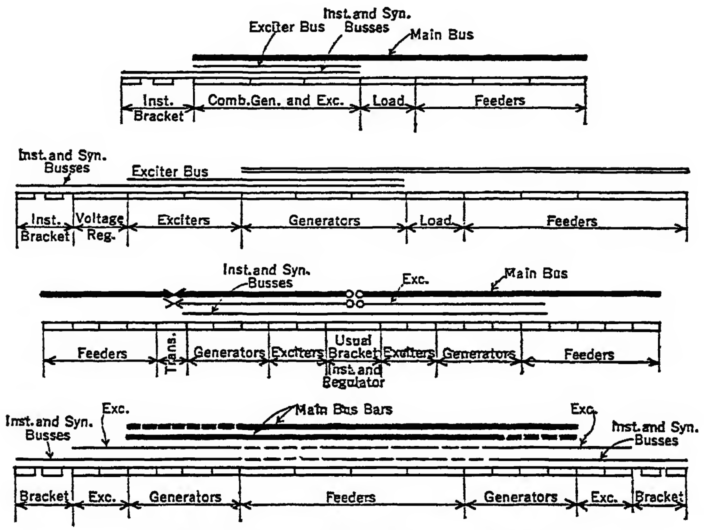


FIG. 12. Arrangement of Panels and Bus-bars

A-C BUS-BARS. For single-phase a-c systems there are two buses, for two-phase systems usually four buses, and for three-phase usually three buses.

Where there is only a single set of bus-bars, either in d-c or in a-c stations, the connections are said to be arranged on the single-bus system; when the connections can be made to either of two sets of bus-bars, the system is spoken of as double-bus, and if the connections can be made to both sets of bus-bars, instead of only to either set, the system is spoken of as the selector system.

CAPACITY OF BUS-BAR. The amount of bus copper required for a switchboard

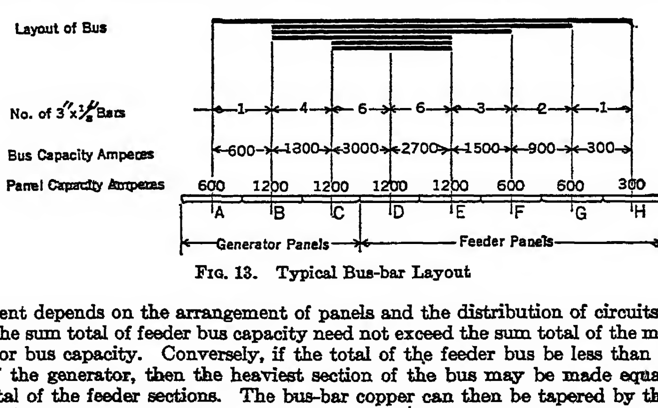


FIG. 13. Typical Bus-bar Layout

equipment depends on the arrangement of panels and the distribution of circuits. Obviously, the sum total of feeder bus capacity need not exceed the sum total of the maximum generator bus capacity. Conversely, if the total of the feeder bus be less than the sum total of the generator, then the heaviest section of the bus may be made equal to the sum total of the feeder sections. The bus-bar copper can then be tapered by the use of laminated bus-bars as indicated in Fig. 13. This construction reduces the amount of bus-bar conductor to a minimum and permits making extensions easily.

High-voltage conductors, with their corresponding relatively small currents, frequently use tubing for bus-bars and connections as this has many advantages over rods, wire, or straps, these advantages being principally increased stiffness for the same amount of

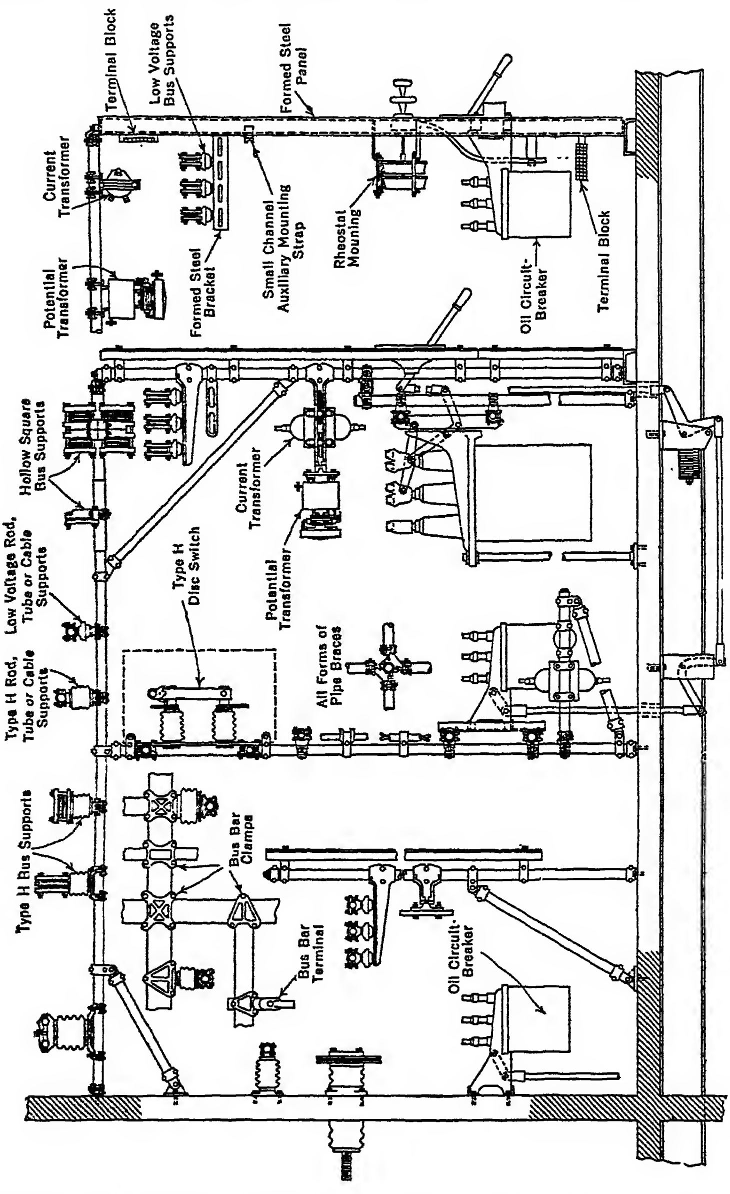


Fig. 14. Switchboard Details and their Application

material, large and effective radiating surface, and the facility of making connections by flattening the tubing at the point desired and bolting the tubing together at such points. In other cases pipes are threaded and standard screwed fittings are used. Tubing of approximately 1-in. outside diameter is not apt to be troubled by brush discharge or

corona effect that is sometimes noted with small wires or straps having sharp edges when used on extremely high-voltage circuits. In many cases standard iron tubing is employed. Corona effects on high-tension conductors may be reduced by using the following minimum sizes of conductors: 35 kv No. 6, 50 kv No. 4, 73 kv No. 1, 115 kv 4/0.

CURRENT DENSITY. The amperes allowable per strap in the main connections and bus-bars will vary according to the conditions of installation and service. For a-c switchboards, for capacities requiring but one copper strap, a 2 by $^1/_4$ in. strap will carry 550 amp and a 3 by $^1/_4$ in. strap will carry 850 amp, with a 30-deg cent rise, at frequencies not greater than 60 cycles. For bus capacities above 2500 amp 60 cycles, or 4000 amp 25 cycles, the arrangement and capacity of panels in order to obtain the most suitable bus-bar layout must be carefully studied. In general, for bus capacities greater than those mentioned, the maximum temperature rise of the copper will exceed 30 deg cent owing to unequal distribution of current in the buses. As a rule, a suitable number of 3 by $^1/_8$ in. or 4 by $^1/_4$ in. straps is used for bus-bar capacities up to 4000 amp, 6 by $^1/_8$ in. straps for bus-bar capacities above 4000 up to 8000 amp, and 10 by $^1/_4$ in. straps for bus-bar capacities above 8000 amp direct current. Above 4000 amp alternating current, copper or aluminum channels are used effectively and economically. The real limits arise from the oxidizing points of the material used for contacts. Provision is made to use vaseline or non-oxidizable films between contact surfaces, silver plating, welding, or brazing. An important consideration is the pressure at the joints.

BUS-BAR STRESSES. In the larger generating stations, on account of the tremendous values of short-circuit currents resulting from the size and number of generators represented in present-day station practice, close attention must be given to the adequacy of the bus-bar supports. Various curves and formulas have been deduced for the purpose of calculating the mechanical strain on bus-bar supports at the instant of short circuit. Careful consideration must be given to obtain adequate mechanical strength and thermal capacity for the current transformers and all apparatus in the circuit under short-circuit conditions.

Supports of bus-bars, connections, etc., are often made as in Fig. 14, which shows a number of applications of brackets and fittings of various kinds for pipe and angle structures, using clamps with cemented type of supports with insulators of various kinds, bus-bar clamp terminals, methods of mounting oil circuit breakers, current and potential transformers, fuses, and similar details. For low-voltage supports cutouts of slate, marble, asbestos, etc., can be used.

MAIN CONNECTIONS. Connections between switches, circuit breakers, current transformers, and bus-bars, where heavy currents are to be carried, frequently use copper straps, and the same material is supplied for the bus-bars. The exact amount of current to be carried for a given rise depends somewhat on local conditions, ventilation, etc., and whether the bus is being used for d-c, 25-cycle, or 60-cycle service, and the temperature rise is not the same for different parts of the bar. A typical test of the average conditions for 60-cycle service at 25-deg cent rise, indicated that one bar 3 by $^1/_8$ in. would carry 650 amp; two bars, 1150 amp; three, 1500; four, 1800; five, 2000; six, 2160, showing that, owing to skin effect, lack of ventilation, etc., permissible current density falls off rather rapidly as the number of bars increase.

ELECTRICAL OPERATION. When electrical operation is employed, no main connections on the back of the switchboard panels are required, but usually a very considerable amount of small wiring has to be done for interconnecting the control switches, indicating lamps, instruments, relays, etc., and various methods of running this fine wiring have been developed. The method used depends largely on the number of wires involved and the amount of space available at the rear of the panel.

30. CABLES

For the connections between generators, transformers, feeder circuits, and the switching gear, it is occasionally possible to use bare copper conductors, although in most cases, particularly for the connection between the generators, the low-tension side of step-up transformers, and their switchgear, insulated wire or cables are better adapted to the actual arrangement of the station. Under certain conditions of very heavy currents where bus and connections are relatively long, aluminum can frequently be used to advantage in place of copper.

The carrying capacity of insulated cables and wires is determined by the maximum temperature at which it is safe to operate them with insulation of various kinds (see Section 14, Arts. 58 and 60). The permissible maximum temperature with rubber insulation is lower than with varnished cloth, and that in turn is lower than with certain other types

of insulation. On account of the unequal distribution of alternating current in large conductors, the large stranded cables will have a higher temperature rise with alternating than with direct current. For this reason, it is recommended that single-conductor stranded cables, unless built with a hemp core, be limited to 1,000,000 cir mils for all a-c circuits. When a cable of 1,000,000 cir mils is used on 60 cycles, the carrying capacity is approximately 10 per cent less than that assigned to d-c service.

Single-conductor a-c cables should not be placed in an iron conduit owing to the hysteresis loss. Three conductor cables carrying separate phases of a three-phase circuit can usually be placed in such a conduit without any difficulty arising. If the currents are small, such as those from current and potential transformers, the hysteresis loss in iron conduit is inappreciable.

For three-phase generator leads where the current is small enough to permit the use of standard three-conductor cables, these are to be preferred to three single-conductor cables. It is not practical to make three-conductor cables larger than 750,000 cir mils; therefore single-conductor cables in parallel are recommended for larger capacities.

Multiple-conductor cables are used with electrically operated switchboards, as the instruments and control switches are usually located some distance from meter transformers, circuit breakers, rheostats, and other accessories, and it is necessary to use connecting leads of varying lengths. Each individual conductor is insulated for 600-volt service and is covered with braid having an identifying color. The insulated conductors are assembled and covered with a layer of tape and an outer braided covering or lead sheath. The outer covering of the cable selected depends upon the nature of the installation.

STATION STRUCTURES AND LAYOUT

In the earliest plants the switchgear was scattered around the station or possibly assembled on the walls, so that no floor space was allotted to it that could be used for other purposes. The next step involved placing the apparatus on a panel switchboard near the wall where little room was taken up by it. As stations grew, more space had to be devoted to the switchgear, until in the modern high-voltage, large-capacity plants one portion of the building, or in some cases a separate building, is assigned to the switchgear and is designed especially for its proper housing.

Direct-control switchgear has the switching apparatus mounted directly on the panels and has no structure of the type considered in this article.

Distant-control switchgear is often used in stations that distribute at the generator voltage where, owing to the size of the breakers, the voltage employed, the capacity of the system, or for other reasons, it is not advisable to mount the switchgear directly on the switchboard panels. For this service the breakers and other apparatus are mounted in structures.

The following apparatus must usually be considered in choosing a satisfactory arrangement for distant control: circuit breakers; bus-bars and connections; rheostats; instrument transformers; fuses for potential transformer primaries and for main wiring, when employed; and disconnecting switches.

31. OPEN CONSTRUCTION

Structure arrangements adopted may be of open, semi-enclosed, or entirely enclosed structures. For small stations of 3000 kva or thereabouts the open arrangement is usually employed as masonry structures would hardly be warranted.

Wall Mounting is used when a simple, cheap equipment with wall-mounting breakers is furnished by the switchboard builder, with only the connections to the circuit breakers and bus-bars and bus-bar supports and terminals, wall braces and brackets necessary to adapt the circuit breaker to wall mounting supplied.

Frame Mounting. Each bus and circuit-breaker structure section with frame-mounting breakers consists of 1 1/4-in. pipe framework, together with necessary mounting brackets and supports for the equipment consisting of oil circuit breakers, disconnecting switches, instrument transformers, bus-bars, and connections. In place of pipe, welded structural steel shapes can be used. Fig. 1 shows a typical arrangement of a small oil circuit breaker arranged for distant mechanical control and mounted on a pipe frame structure with disconnecting switches and bus-bars, the structure being located immediately back of the switchboard panel. Disconnecting switches are used between the bus and oil circuit

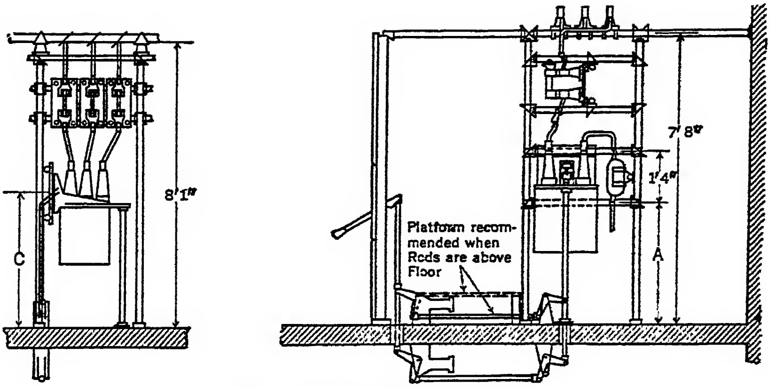


Fig. 1. Typical Open Bus Structure

breaker, and other sets can be used for disconnecting the oil circuit breaker from the outgoing feeder circuit. The breaker illustrated has all three poles in the same tank and hung from a common frame. This design is suitable for various types of breakers of moderate rupturing capacity.

32. SEMI-ENCLOSED CONSTRUCTION

A semi-enclosed structure is employed if the circuit breakers are of the type having one supporting frame for all poles, and it is considered advisable because of frequent heavy overloads and short circuits to enclose them in cells. Fig. 2 shows a typical masonry construction utilizing a moderate-capacity three-pole oil circuit breaker with each pole in a separate tank, and all three poles on the same framework. The breaker is placed in a masonry compartment that can be completely closed in, so that there will be no likelihood that trouble which starts in one breaker compartment may spread to adjacent ones.

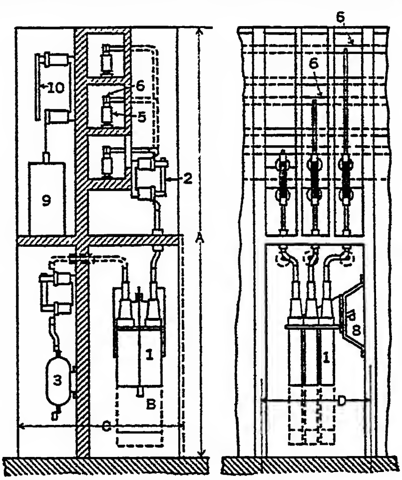


Fig. 2. Typical Semi-enclosed Bus Structure

33. ENCLOSED CONSTRUCTION

In large-capacity a-c plants of 13,200 volts or less, with generators connected directly to the bus, the amount of current that can be concentrated on a short circuit is very great and every precaution has to be taken to prevent trouble spreading if it ever starts. For this reason, it has become customary to employ masonry or metal compartments and cellular construction for the oil circuit breakers and bus-bars in stations of this character. The main idea of the masonry cellular scheme is to provide an insulating fireproof barrier between leads of opposite polarity in heavy-capacity plants. Usually the vertical walls of the circuit-breaker and bus-bar structures are built of brick or concrete, and the horizontal shelves between the bus-bars are ordinarily made of concrete, soapstone, slate, or marble. In some instances, the bus-bar structures have been made of asbestos lumber, transite, or similar material.

Masonry Structures for bus-bar work are made semi-enclosed or entirely enclosed. In the former case, the wall of the structure which separates the horizontal bus-bars, and the vertical connections, are made practically continuous. The back of the bus-bar shelves is built into this wall, and pilasters, properly spaced, support them in the front. A modi-

fication of this scheme uses a continuous wall instead of pilasters as a support of the front of the shelves. The bus-bar, connections, etc., are almost completely enclosed except for openings provided with doors at the bus contacts, etc. Fig. 3 shows a typical arrangement of oil circuit breakers located in a concrete compartment with the bus bars placed above the breakers. With this type of construction each pole of the breaker is in a separate masonry cell. The operating mechanism for the breaker is located on the metal covering above the cells containing the breaker poles. A rectangular opening is left in the back wall of the structure, and the breaker leads, in the form of bare copper strap, are taken through this opening. The back breaker stud connects through a disconnecting switch to the outgoing circuit; the front studs connect through a second disconnecting switch and a series transformer to the bus-bars located in masonry compartments above the breaker. With this type of construction, doors are provided for the front of the breaker structure, closing in the breaker itself, and the use of doors is optional for the rear of the structure.

Limitations. Masonry structures are satisfactory for voltages up to approximately 15 kv. For higher voltages open construction is usually preferable. The masonry con-

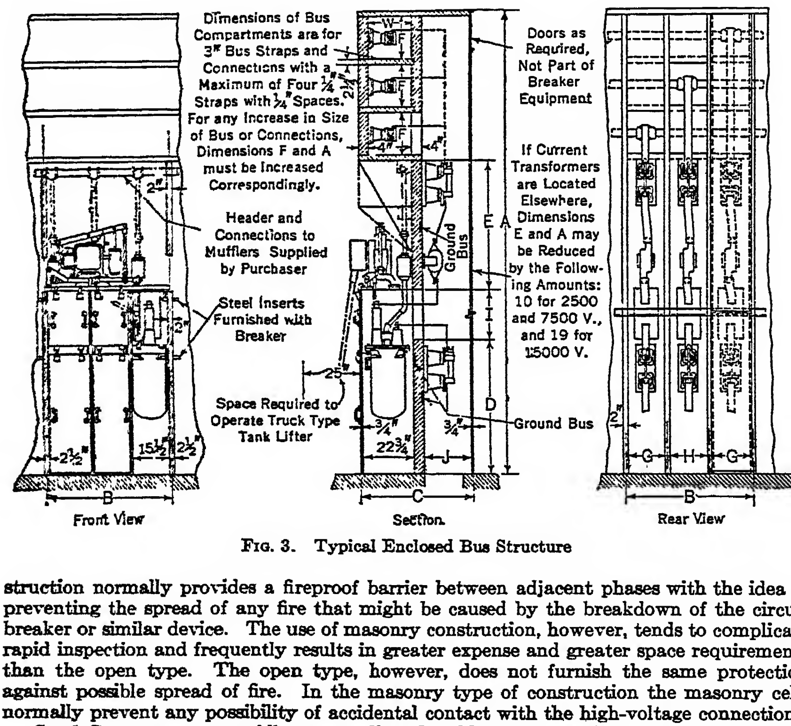


Fig. 3. Typical Enclosed Bus Structure

struction normally provides a fireproof barrier between adjacent phases with the idea of preventing the spread of any fire that might be caused by the breakdown of the circuit breaker or similar device. The use of masonry construction, however, tends to complicate rapid inspection and frequently results in greater expense and greater space requirements than the open type. The open type, however, does not furnish the same protection against possible spread of fire. In the masonry type of construction the masonry cells normally prevent any possibility of accidental contact with the high-voltage connections.

Steel Structures are rapidly superseding the older forms as the complete switchgear can then be a factory-built device and any trouble after erection is usually a phase-to-ground fault instead of a phase-to-phase short circuit. This metal-clad construction is considered in Art. 35.

34. SEGREGATED PHASE

Most of the older installations utilizing structure-mounted breakers had the three poles of each three-phase circuit breaker located side by side with the three bus-bars side by side vertically or horizontally in the same structure. A later modification utilized in some of the more important installations, with a large number of feeders fed from the generator bus, adopted a segregated phase layout. All the breaker poles in the *A* phase are placed together in a single corridor with the *A* bus above them. Similarly the *B* poles and *B* bus are together and the *C* poles and *C* bus together. The three corridors containing

the three independent collections of breaker poles and buses when placed on the same level gave a horizontal segregated phase arrangement such as employed in the Hellgate

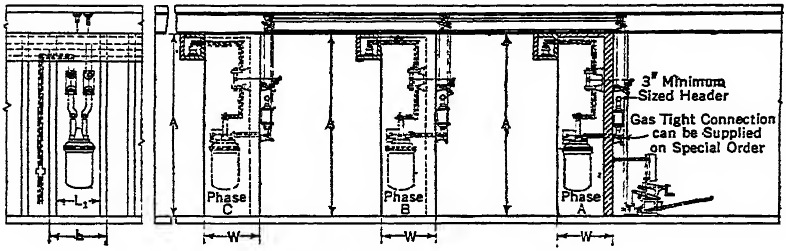


FIG. 4. Typical Horizontal Segregated Phase Structure

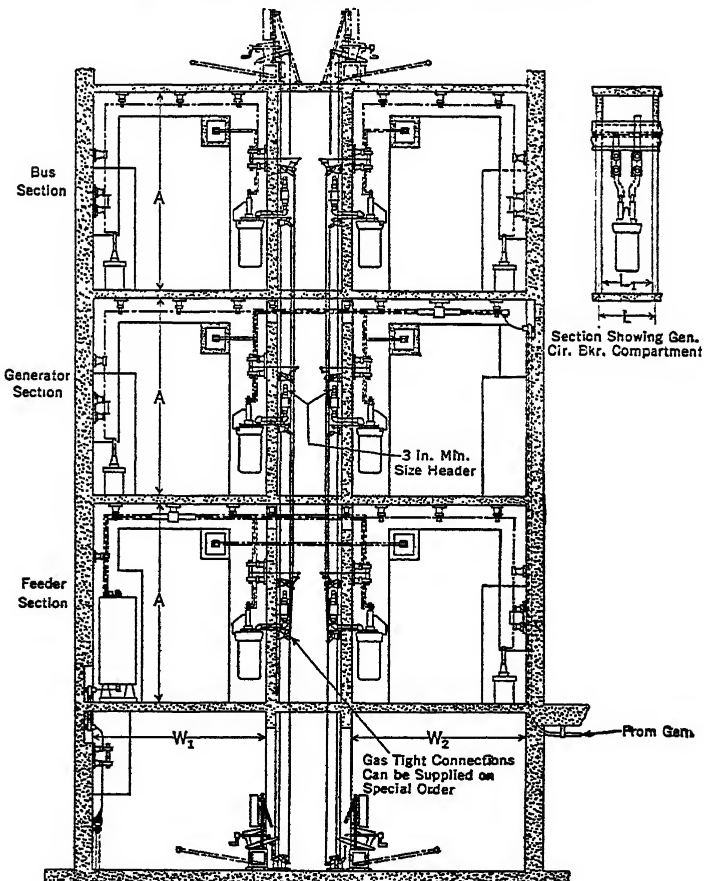


FIG. 5. Typical Vertical Segregated Phase Structure

Station in New York; if they are placed vertically one above the other on separate floors, the scheme is known as the vertical phase segregation.

Horizontal Segregated Phase arrangement is shown in Fig. 4 with the *A* phase at the right, the *B* phase in the middle, the *C* phase on the left, each phase being in its own

separate room or corridor on the same floor. A single mechanism run horizontally overhead operates the three poles of the same breaker simultaneously. All six disconnecting switches associated with the breaker are mechanically operated by a suitable mechanism, and interlocks are provided between the operating mechanism and the breakers and the disconnecting switches to prevent improper manipulation of the disconnects.

Vertical Phase Segregation is shown in Fig. 5 with the *A* phase on the top floor illustrating a bus section, *B* phase on the middle showing a generator section, and the *C* phase at the bottom showing a feeder section. On the top or bottom floor are placed the mechanisms for operating the oil circuit breakers and disconnecting switches, these mechanisms being suitably interlocked. On each of the other floors are placed the oil circuit breakers and buses. The advantage claimed for the segregated phase layout is the practical impossibility of obtaining a phase-to-phase short circuit on the buses, unless simultaneous grounds occur on different phases.

35. METAL-CLAD SWITCHGEAR

Metal-clad switchgear includes indoor cubicles, outdoor switchhouses, truck-type roll-in switchgear, and vertical lift switchgear. Steel panels and desk sections have been considered in previous articles; cubicles and switchhouses will be considered in Art. 36; and truck-type roll-in and verticle-lift switchgear in this article. With the truck-type roll-in switchgear each oil circuit breaker is permanently mounted on its own truck and is rolled into the housing where it makes horizontal connection, through suitable primary devices, to the bus-bars and leads. With the verticle-lift construction the breakers do not always have individual trucks but are placed one at a time on a transport truck, moved into the housing, and lifted vertically into engagement, through suitable primary devices, to the bus-bars and leads. The breaker is locked in position in a suitable support, so that the transport truck may be removed from the housing and used with other breakers. The latest design provides a raising and lowering mechanism in each housing, and the handling carriage is a " dolly " transfer truck.

Metal-clad switchgear with the factory-built and completely assembled features is suitable for installation in any station and for any breaker rupturing capacity, and may be adapted to either indoor or outdoor service. Designs are completed and have been constructed for indoor service up to 22 kv and outdoor service up to 33 kv, but equipments can readily be supplied for higher voltages.

The more general use of metal-clad switch gear is up to 22-kv indoor, 33-kv outdoor, up to 2,500,000-kva rupturing capacity of breakers, but this type of construction is applicable up to higher voltages outdoors. The main advantages of metal-clad switchgear arise from the isolation of the equipment, the possibility of safety interlocks, the interchangeability of spare breaker units, etc.

Complete Switchboards for generating stations, substations, industrial plants, and other locations can be supplied with the safety enclosed design of the vertical lift or removable truck type. Panels are available for the control of feeder circuits, generators, and motors. For generators and motors of large capacity the exciter and field controls are located on a stationary panel at the side of the generator or motor control housing. For moderate-size machines, where the field current does not exceed a few hundred amperes, it is possible to mount the field ammeter and switch on the truck.

TRUCK PANELS. The typical truck-type panel, as the term is generally applied, consists of two elements, the truck or movable element and the housing or stationary element. The former carries the oil circuit breaker, instrument transformers, breaker-operating mechanism, instruments and meters, and movable disconnecting devices. The housing carries the bus-bars, stationary disconnecting devices, cables, and the tracks for removable element.

Housings are extremely rugged structures which will not be distorted during shipment or when inserting or withdrawing the circuit breakers on their own trucks or by means of the transport truck or while interrupting maximum short circuits. The sheet steel used in their construction is not less than $^1/_8$ in. thick. Access to bus-bar supports, primary contacts, cable terminals, and the like is provided by removable plates; and hinged doors are provided on the vertical-lift type opposite the voltage transformer fuses, and similar devices. For the higher voltages using oil-immersed potential transformers, the fuses are attached to the hinged covers of the potential transformer compartments in such a way that they are immersed in oil and connected in circuit when the cover is in the closed position. With cover open, fuses are disconnected from circuit and may be safely handled. Plugs are also used, preferably on large units.

Exhaust pipes are supplied for conducting gases away from the muffler connection of

the oil circuit breaker, and a leak-proof coupling device joins the exhaust pipe and the muffler when the breaker is in the operating position. Terminals for small wiring are provided so that it will be necessary only for the user to connect incoming secondary or control wiring to the terminal points in the housing. A heavy copper ground connection is bolted to the circuit-breaker frame and is carried to the ground bus in the stationary housing through a copper plate, on the circuit breaker of the vertical-lift type or the truck of the roll-in type, and a finger type contact mounted on the side of the housing. Flexible copper shunts provide a positive ground connection to the circuit breaker or truck unit. Additions can be made to metal-clad switchgear installations in a very simple manner by additional housings and extending the bus through these housings.

Manually Operated Truck-type Switchboard is arranged to carry the instruments and relays as well as the operating handles of the circuit breakers. This truck-type switchboard takes the place of the open-type vertical switchboard where the breakers are directly on the rear of the panel or in a structure a short distance back of the board. The truck-type board eliminates the customer's work of assembling a pipe structure with breaker, disconnecting switches, and barriers (if used), and the installation of conduits, cables, and operating rods.

Electrically Operated Truck-type Board is similar to the manually operated board and

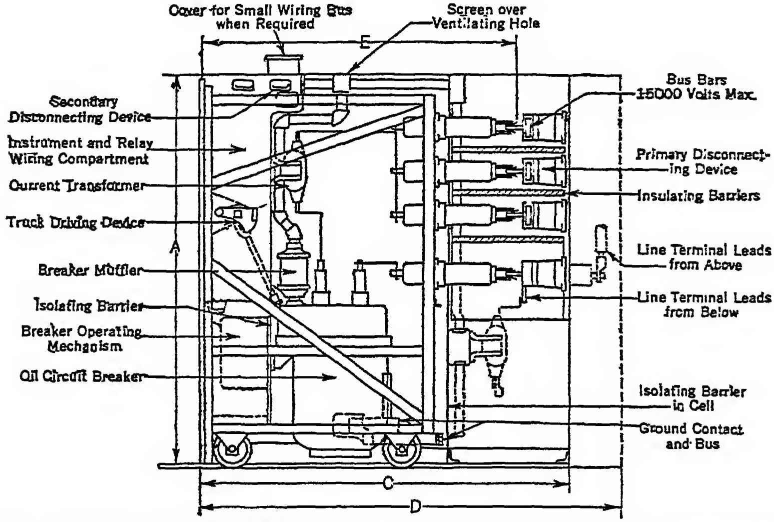


FIG. 6. Outline Drawing of Truck-type Switchgear with Solenoid Operated Oil Circuit Breaker

takes the place of the usual separate structure and panel board. The general arrangement of such an equipment is shown in Fig. 6.

Electrically operated breakers and relays located on a truck and a separate control board for instruments and control switches is an arrangement where the breakers, bus-bars, disconnecting switches, current and potential transformers are all located on a factory-built steel structure instead of being placed in a masonry or pipe frame structure.

Where trucks are controlled from separate panel boards, their housings include two rows of contact shoes, one for the operating position and the other for the disconnected position of the truck. This feature is provided so that the operation of electrically closing and tripping the circuit breaker may be tried out, while the truck is in the disconnected position, and isolated from the bus-bars, in the same manner as though the breaker were mounted in a masonry structure and isolated by means of disconnecting switches.

Disconnecting Devices. The primary disconnecting devices between housing and truck take the place of a six-pole gang-operated disconnecting switch mechanically interlocked with the oil circuit breaker for a three-phase equipment. The stationary disconnecting devices are usually front connected, the buses being mounted directly in front of the insulator. Adapter plates are used to keep the jaws of the disconnecting devices for different bus capacities in the same location relative to the moving element. This insures complete contact between the movable and stationary members of the devices.

Fig. 7 shows a typical primary disconnecting device. The blade member of the disconnecting device is located in a horizontal plane at a fixed height above the rail in the housing. The finger member is located at a corresponding height on the truck. The fingers are on the truck so that they will be dead when accessible for inspection. The fingers are mounted in pairs and are securely bolted to an insulator block in a thoroughly flexible manner. The fingers connect to the block by means of flexible shunts, and flat steel springs are used for holding them in position as well as for furnishing high contact pressure. The two fingers

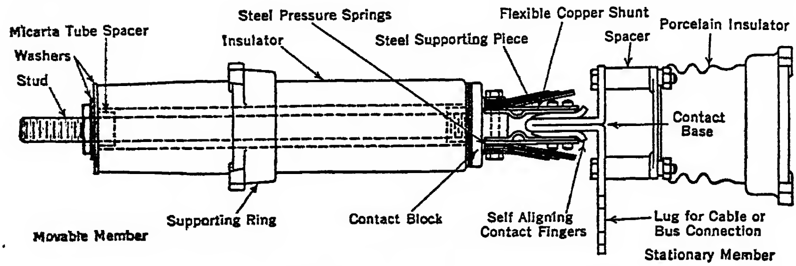


FIG. 7. Primary Disconnecting Contacts for Trucks

forming a pair are assembled so as to have a slight initial pressure, one against the other, and when the blade is in service between the fingers extremely high contact pressure results.

In metal-clad equipment the control and secondary circuits must also be broken, and these pass through secondary disconnecting devices so arranged that proper contact between movable and stationary parts is made not only in the operating position of the oil circuit breaker but also in the disconnected position to permit checking breaker operation and testing of secondary circuits while the equipment is not connected to the bus but is still in the housing.

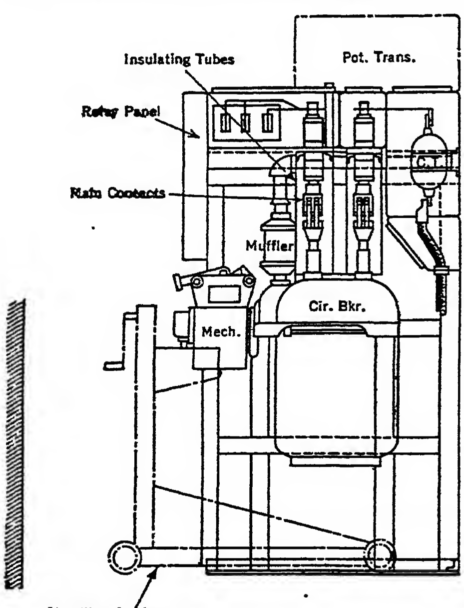


FIG. 8. Indoor Lift-up Switchgear

The secondary disconnecting devices employed in some trucks utilize finger contacts, and contact shoes mounted on insulated blocks threaded over a rectangular steel rod. Contact pressure between fingers and shoes is maintained by means of heavy helical steel springs under the fingers. The insulating pieces have barriers so that short circuits between the fingers or between shoes are impossible.

VERTICAL-LIFT SWITCHGEAR. In addition to the well-known truck-type equipment with horizontal draw-out features, another form of metal-enclosed switchgear has been developed as the vertical-lift type with completely metal-clad buses and connections. This switchgear, Fig. 8, really takes the place of the separately mounted oil circuit breakers, disconnecting switches, bus-bars and connections, current and potential transformers, formerly supplied with an electrically operated switchboard.

The use of single-pole units each with its own operating solenoid is very suitable for three-phase four-wire grounded neutral systems where the individual phase circuits may be opened or closed independently. These individual pole units can, however, be electrically connected together and operated from the same control switch and relays to func-

tion as a three-pole breaker, with the three poles adjacent and the three buses in the same metal bus holder. If preferred, they may be kept entirely independent to obtain the benefit of the segregated phase arrangement where all the poles connecting to the A phase are arranged in one row, those to the B phase in a second row, those to the C phase in the third row. These three rows may be parallel or consecutive, as best fits in with the general arrangement desired at the point of installation. With indoor installations the parallel rows may be arranged for horizontal or vertical phase segregation.

Transport Truck. When a breaker is to be installed in its housing it is brought to the housing on the transport truck, built of structural-steel members. arc welded together. The breaker is lifted from the truck to its operating position in the housing by means of screws and bevel gears, hand operated for the smaller units, motor operated for the larger ones. The wheels on the transport truck are either of the castor or swivel type to permit moving the transport truck in any direction after the breaker has been withdrawn from the housing.

Disconnecting Devices. The breaker studs are provided with rugged primary disconnecting devices, Fig. 9, of the blade and finger types, self-aligning, carefully located by means of jigs and gages to insure interchangeability. The secondary disconnecting devices of the multicontact train coupler type have their stationary members secured to the removable breaker element. The movable members are fastened by flexible cable connectors to the terminal block compartments in the housing so that the breaker may be lowered to the disconnect position and tested without disturbing the control connections.

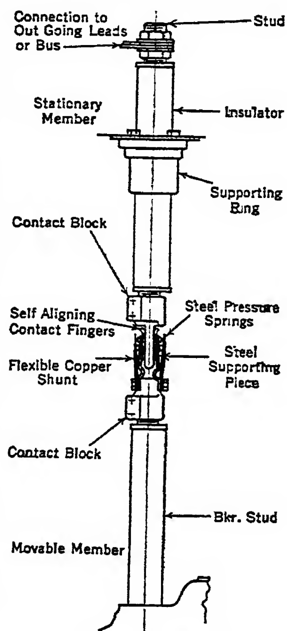


FIG. 9. Primary Disconnecting Contact for Lift-up Switchgear

Buses are thoroughly insulated by tape or tubing and supported throughout by Micarta or similar supports permitting a closer grouping of live parts and smaller clearances to metal casings, and practically eliminating maintenance. Connections to the primary disconnecting devices, including the joints, are also insulated. All buses and connections are enclosed in metal compartments. The compartments may be filled with insulating compounds such as oil, petrolatum, or asphaltum, or may be left unfilled. Most frequently, insulated bus located in air is used.

OUTDOOR METAL-CLAD. A modified form of metal-clad switchgear for outdoor construction has been furnished to certain customers who desired segregated phase layout with draw-out breakers of relatively high rupturing capacities. With this modified arrangement, Fig. 10, each pole of the breaker is on a separate truck similar to the normal truck-type design, but the buses and main connections are enclosed in non-magnetic metal containers and are immersed in an insulating fluid. The breakers in question are single-pole round tank units, having a rated interrupting capacity of 42,900 amp at 15,000 volts, 23,000 amp at 25,000 volts, or an arc rupturing kilovolt-ampere rating of 1,000,000.

OUTDOOR VERTICAL-LIFT. This equipment is a modification of the corresponding indoor equipments with the individual poles arranged to be carried on a transfer truck and then elevated into such a position that one stud connects to the bus and the other to the supply or feeder circuit. At the same time the breaker-operating mechanism and the mufflers for the exhaust gases from the tanks are suitably coupled, and the connections to the control, instrument, and relay wiring are completed. Suitable interlocks are provided, and means are furnished for testing out the secondary circuits. The compartments containing the high-tension bus-bars and connections are usually arranged so that they may be filled with oil or insulating compounds, and proper provision is made for the current and potential transformers.

COMPARATIVE COSTS. First cost of the metal-clad switchgear covers a complete, factory-assembled equipment and is naturally somewhat greater than the cost of switching equipment which is shipped unassembled. It includes the necessary assembly labor. However, the purchaser is saved the cost of building cells with inserts and getting it properly connected and adjusted for successful operation.

Labor cost of assembling, inspecting, and testing equipment in the factory will be

less than the cost of equivalent work done in the field because the factory men have better facilities and are thoroughly experienced in doing such work, particularly with their own apparatus.

Installed cost should always be analyzed by the purchaser when considering the pur-

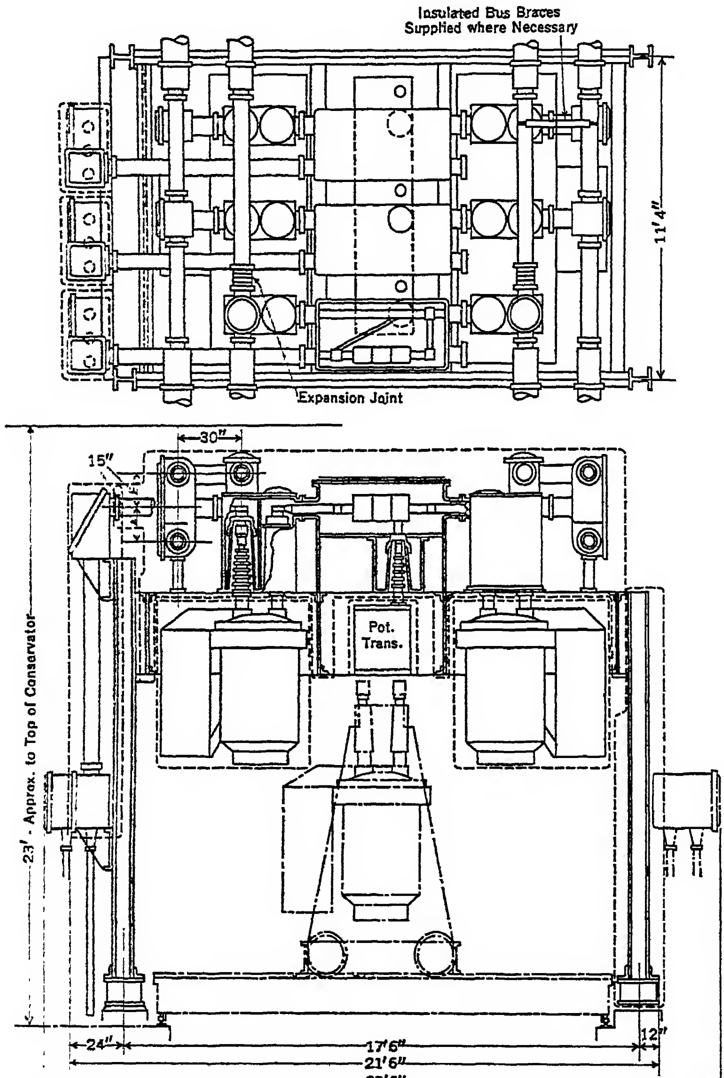


Fig. 10. Outdoor Lift-up Unit.
Dotted lines indicate division points for shipment. Each bay is assembled and shipped filled with oil

chase of switching equipment, and, unless he can design and build structures and install equipment in them for less than the manufacturer, the purchaser can well afford to buy completely assembled equipment.

36. STATION LAYOUT

While the breaker and bus structures in certain plants can be considered independently of the rest of the equipment in the station, it is usual in large plants to give careful consideration to the effect of the switchgear arrangement on the entire design of the station.

Certain features of the arrangement work out best for a location at the end of the building, others for a location at the side of the building or in the separate switchhouse.

The circuit-breaker and bus-bar arrangement in the larger plants often affects to a large extent the general design of the station. In stations that do not contain any prime movers the designer usually has a free hand in determining the best location of switching equipment relative to the feeders, transformers, and rotating machinery which are controlled.

Trend of American design during recent years has been to locate transformers and high-tension switchgear out-of-doors, but there are a few cases where, owing to local conditions, indoor equipment is utilized. It may be advantageous to consider a few typical drawings, some showing indoor equipment and others outdoor arrangements.

SYNCHRONOUS CONVERTER STATION. Fig. 11 shows the sectional view of a synchronous converter substation containing 1000-kw six-phase converters with air-blast

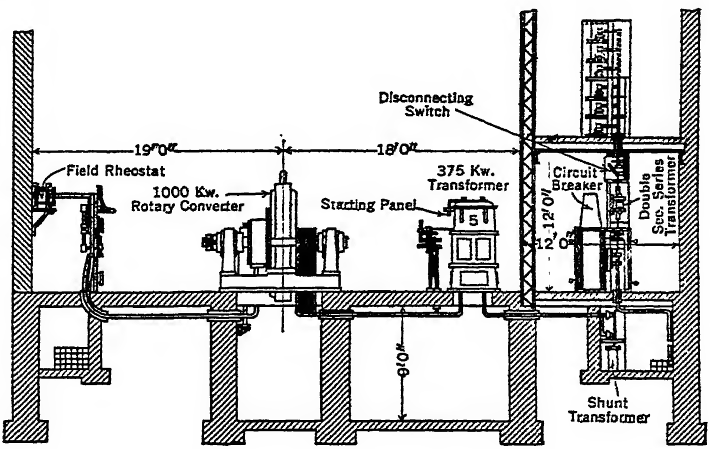


FIG. 11. Synchronous Converter Sub-station

transformers fed from 13,200-volt underground circuits. The incoming leads from the cable ducts pass through an oil breaker and disconnecting switches to the bus-bars located on a gallery, and from the bus-bars back through other disconnecting switches and breakers to the high-tension terminals located at the bottom of the air-blast transformers. The low-tension leads from the transformer go to the starting panel and then to the a-c end of the rotary converter. The d-c end of the converter is connected by suitable cables to the d-c switchboard and placed near the left-hand wall. With such an arrangement the wiring of the station is kept very straight and simple and the space is utilized to the best advantage.

Portable Substations for interurban electric railways have often been made of the semi-outdoor construction type utilizing an outdoor self-cooled three-phase transformer with the converter and low-tension switchgear inside the cabin of the car.

Rectifier Stations have been supplied to certain railways, in which rectifiers have been installed to convert from alternating to direct current for railway service.

HYDROSTATIONS. Fig. 12 shows a sectional view through the Holter plant of the Montana Power Co. installed a number of years ago but still typical of an indoor installation of switchgear. The plant contains four 16,000-hp vertical-shaft waterwheels with 12,000-kva 6600-volt generators and four three-phase transformers stepping up to 110 kv.

STEAM POWER PLANTS. These plants usually have arrangements of transformers and high-tension switching equipment that do not differ very much from the corresponding arrangements in hydroelectric plants except that usually a number of galleries are avail-

able along one side of the generating station and these galleries frequently can be utilized to advantage for the transformers and switching.

On the top floor of the electrical galleries near the center of the building is located the control room with the control desk, instrument panels, feeder switchboard, and similar equipment.

STATION AUXILIARIES. A very important problem in steam station design is the arranging of the switching for the station auxiliaries such as the condenser pumps, boiler supply pumps, draft fans, and coal-handling apparatus. This equipment is often the last decided upon; adequate space is rarely available, and provision for suitable structures is difficult to make. Metal-clad switchgear with the complete factory-built containing structure is frequently the best answer to this problem.

Indoor Cubicles, Fig. 13, are designed to give the advantage of complete factory-

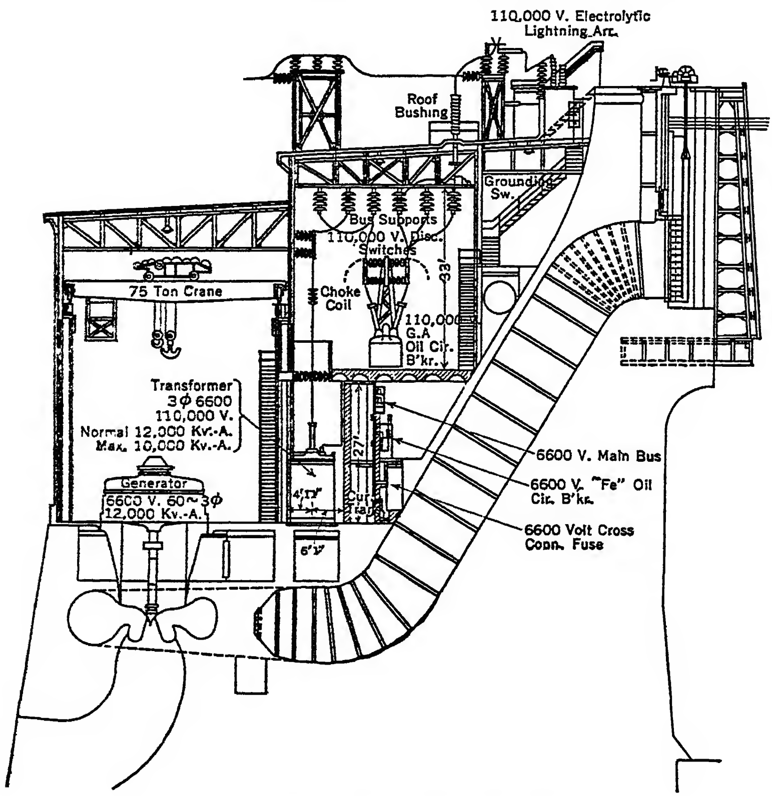


FIG. 12. Sectional View of Water Power Plant

assembled, steel-enclosed, safety-pipe switching equipment at a minimum expense. These units are for either manually, a-c motor-, or d-c-solenoid-operated oil circuit breakers and may be equipped with automatic reclosing equipment, if electrically operated, which will reclose the circuit breaker a definite number of times on a predetermined schedule.

These cubicles are for the control and protection of feeder circuits against overload and short circuits.

They are finished products which have been completely assembled, tested, and inspected before shipment, and they are ready for operation as soon as the incoming and outgoing lines are connected, bus assembled, joints inspected and taped, and the breaker tanks filled with oil.

The cubicles are constructed of structural-steel members and $^1/_8$-in. thick open-hearth

steel sheets securely welded together to form a rigid structure unit that will not warp from changing temperatures or handling.

The lower section of the better-grade cubicles is divided into two compartments, one for housing the oil circuit breaker and instrument transformers and the other for housing the breaker-operating mechanism and control wiring. Between these two compartments is a vertical steel barrier which completely separates the high-tension equipment from the breaker mechanism and secondary control wiring, thus assuring perfect safety to attendants or maintenance men.

Cubicles up to and including 42-in. widths have single hinged doors; those of greater width have double doors hinged at each side. The framework for supporting the oil circuit breaker consists of steel angles and shapes securely welded to the sides of the cubicle and to the foundation angles. The circuit breakers are located at one side of the cubicles to allow room for incoming cables from below.

Separate superstructures constructed of formed sheet steel are bolted to the upper section of the cubicles and are of the same width and depth as the cubicles to which they

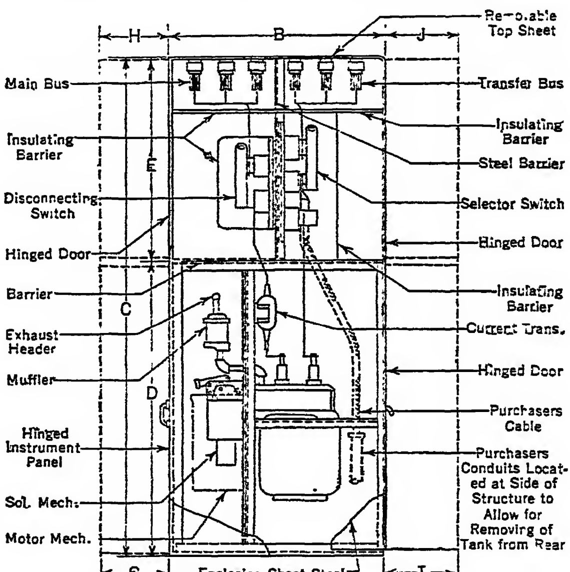


FIG. 13. Indoor Cubicles for Double or Main and Transfer Buses—7500 volts

are attached except in equipments where double-throw or selector-type disconnecting switches are used. In these latter cases it is necessary for the superstructure slightly to overhang the lower section in the rear only.

In the superstructures are mounted the disconnecting switches, bus supports, and bus copper. Hinged doors permit ready access to and convenient operation of the disconnecting switches, and removable sheet-steel covers are provided for access to the bus, busbars, and connections. Insulating barriers are used where necessary between the buses and disconnecting switches, and sheet-steel barriers are provided between the main and transfer buses or double buses where such arrangements are supplied.

The superstructures are suitable for enclosing one bus and either one or two sets of disconnecting switches, or main and transfer buses and two sets of disconnecting switches. Double bus arrangements can also be provided for.

OUTDOOR STATIONS. Fig. 14 shows a typical layout of outdoor transformer and switchyard. This drawing has in most places two sets of dimensions, one giving the dimensions for a 154-kv and the other for a 110-kv installation. The steelwork for the tower construction has been shown in merely typical form, and the dimensions and spacings are more than ample. The drawing gives a fairly clear idea of one manner in which the transformers and switching equipment could be arranged to advantage for outdoor service.

At the time when this drawing was prepared, estimates were made as to the approximate

cost of indoor and outdoor layouts, comparing the space required in the building for housing indoor transformers and switchgear with the cost of the steelwork erected and the extra cost of making the apparatus suitable for outdoor service. Both for 110 kv and 154 kv the calculations indicated that the outdoor equipment would cost approximately 13 per cent less than the building and indoor equipment.

When considering the question of outdoor stations three different types of equipment present themselves, namely: outdoor switchhouses and metering equipment; outdoor substation with air break switches; outdoor substations with oil circuit breakers.

Outdoor metering equipment had been designed for those feeder installations where

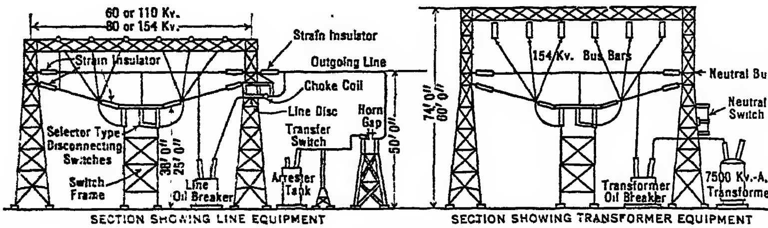
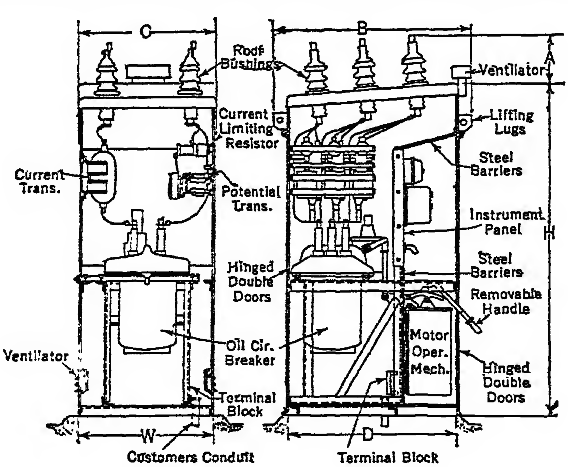


FIG. 14. Transformer and Switchyard—154 kv

metering is the only requirement or where an air break disconnecting switch is being used. The equipment is usually pole mounted and consists of an accessible housing for the high-tension connections and instrument transformers to be located near the top of the pole, together with a meter cabinet placed at a convenient height above the ground for meter readings. The salient features of outdoor substation structures are to take advantage of the available space and to locate the apparatus in such a way as to facilitate its inspection and adjustment without danger to the operator.

OUTDOOR SWITCHHOUSES usually include both circuit-breaker and watthour-



Rear View - Side View

FIG. 15. Outdoor Switchhouse

meter equipment properly housed and protected for mounting in exposed locations. They are used for the control of outdoor distributing substations which supply power to small towns, farming communities, manufacturing plants, and numerous similar installations where the connected load is not large enough to warrant the expense for substation buildings with indoor apparatus. Fig. 15 is a typical switchhouse designed to stand on its own base and is of sufficient height to prevent accidental contact with the incoming and outcoming leads. Access to the apparatus is gained through large hinged doors at both front and rear of the house.

Outdoor Cubicles are essentially steel houses suitably designed to contain the oil circuit breakers, disconnecting switches, instrument transformers, bus-bars, connections, and wiring and may be considered as a combination of switching equipment, bus structure, and containing building. Incoming and outgoing connections can be made by underground cable or by overhead leads to weatherproof bushings in the same manner as for the individual outdoor switchhouses—as they are the logical development of the same idea to take care of many circuits in place of one.

Outdoor Substations with air break switches are provided for use as small-capacity transforming stations operated from high-voltage transmission and distribution circuits where the use of oil circuit breakers is hardly warranted by the value of the apparatus installed and the importance of the circuit being controlled. For this class of service, air break switches, usually with arcing horns, have been provided. The capacity of a station of this type is limited by the safe rupturing capacity of the fuses, and for ordinary installations a capacity of 7500 kva should not be exceeded.

Outdoor Stations with Oil Circuit Breakers are furnished for larger capacities and higher voltages and more important circuits. The arrangement of any outdoor station has to be determined from local conditions, and circuits to be controlled.

37. AUTOMATIC OPERATION

There has recently been a great development in unattended stations, which are either automatic or operated by supervisory control from the dispatcher's headquarters.

The field of application for automatic substations and hydroelectric generating stations is rapidly broadening. In general, it may be said that any substation which would require the presence of an operator, if manually operated, can be made automatic. If there are not more than two or three machines, saving in operation will usually result.

EQUIPMENT. Automatic substation equipment has been designed to duplicate in every way the manual operation of substation apparatus, without the attention of an operator. Starting and shutting down of the station are functions of the load demand. In addition, many protective devices uncommon to the average substation give absolute protection, which is free from the human element.

Automatic Equipment for Railway Service has usually been designed for full automatic operation; that is, the starting impulse is received automatically, when the trolley voltage drops below a predetermined value. The station also stops automatically when the load demand falls below a predetermined minimum, for a continuous period of time, that can be adjusted from 3 to 30 minutes.

Protective Equipment is exceptionally complete. Every effort has been expended to make the substation equipment simple and compact without the sacrifice of any protective or automatic feature. Each device of an automatic station has a number marked directly on the switchboard and on the wiring diagram. The American companies furnishing automatic substation equipment have agreed on a system of numbering so that the same devices, or similar devices performing certain functions, are assigned definite numbers, and an operating engineer familiar with the connections and equipment of one manufacturer can readily understand the corresponding equipment of another manufacturer.

Synchronous Condensers are used for power-factor correction and for voltage maintenance in terminal substations supplied from power transmission circuits. Automatic equipment is available for starting such condensers, connecting them to the power system and adjusting their field circuits to obtain the maximum corrective benefit from the use of the synchronous condenser. When conditions are such that the condenser is no longer needed it will be automatically disconnected from the circuits.

TYPES OF STATIONS. For railway service, automatic substations have been built for a-c self-starting synchronous converters up to 3000 kw 600 volts direct current. Equipment also can be readily supplied for motor-started converters and for 1200- or 1500-volt d-c service with one 1200- or 1500-volt machine or two 600- or 750-volt machines in series. Automatic equipment has lately been furnished for use on 3000-volt railway service and for mercury-arc rectifier stations.

Mine Service. Automatic equipment has been standardized for motor-generator sets up to 300 kw, 25 or 60 cycle, 2300-volt alternating current, 275- or 550-volt direct current. This capacity is usually the maximum required, but larger capacities can be readily taken care of by substituting larger circuit breakers and contactors. Automatic synchronous converter substation equipment of like capacities can also be supplied.

Three-wire automatic stations have been provided for d-c systems, usually supplied by motor-generator sets or booster converters to allow for proper variation in the d-c voltage by means of suitable voltage regulators.

Hydroelectric Stations can be furnished with automatic switching equipment for the control of waterwheels and generators. These stations may be arranged to start in any of the following manners:

1. By means of push-button control from a distant station.
2. When the frequency of the line is reduced.
3. When the water head exceeds a certain height.
4. When the load on other generators exceeds their rating.
5. Through supervisory control equipment operated by the dispatcher, located at some distant point.

Automatic control equipment can be applied to almost any hydroelectric generating station regardless of size, type, or head of water. Either induction generators or synchronous generators may be used, the latter being preferable on most systems. With synchronous machines, either the self-synchronizing method or the automatic synchronizing method of starting is used. The self-synchronizing method is employed if the relative size of the generator is small compared to the adjacent sections of the power system. When the waterwheel is within 10 per cent of normal speed, the generator, provided with a damper winding, can be thrown on the system and started, and the field circuit will be closed when the machine is up to speed. It pulls into step like any self-starting synchronous motor under similar conditions.

If the generator is large compared to the adjacent parts of the system, or if it does not have a damper winding, an automatic synchronizing equipment is provided, which regulates the waterwheel speed and closes the breaker at the first favorable point of synchronism.

Protection is given by suitable relays against troubles in bearings, governor, exciter, etc., or overspeed, overvoltage, and similar conditions.

Shutting down operation in an automatically controlled hydroelectric station is controlled by protective devices, operating on either the lockout relay or a master relay; or the station is shut down by hand by opening a knife switch. The only immediate action resulting is the de-energization of the governor solenoid. This causes the gates to close, thus shifting the load from this unit to other parallel units of the system. The generator is still connected to the line with normal excitation delivering load in proportion to the diminishing gate opening. As the gates reach a point slightly below the no-load running position, the shunt trip coil, on the oil circuit breaker, opens the breaker. As the gates reach the zero gate opening position, brakes, if used, are applied to stop the machine.

A-c Substations. Where automatic operation is desired in a-c substations without attendants, the main feature desirable is that all transformer and feeder circuits should be normally closed ready for the delivery of power. In case of a feeder overload, the feeder breaker should trip out after a short time interval and should try several times to have service restored on that feeder. For this service periodic reclosing relays are used. These are made in various forms and are arranged to energize one or more circuits several times in succession with regular time intervals between each energization. Also multi-unit transformers are sometimes used to switch in and out automatically on load demand.

38. LOAD DISPATCHER SYSTEMS

In any large power system with several generating stations, many substations, and possible interconnections with other systems, it is customary to center the responsibility and authority for the best operation of the system in the hands of the system operator or load dispatcher. It is necessary for him to know, at all times, what stations are connected to the line, what machines are in operation, what load is being carried, what power resources are available, and what is apt to be the power demand at any hour of the day and any day of the year.

Supervisory Control Equipment has been created to meet a demand for equipment which will automatically give the load dispatcher visual indication of the conditions of his power equipment and at the same time provide him with a means of controlling his power apparatus. The automatic station equipment protects itself during starting, running, and stopping conditions. There are certain emergency conditions, however, over which the automatic station equipment has no control. This might be illustrated by the case of a fire in a certain section of the city where the electrical circuits are supplied from automatic stations so that it would be necessary for the dispatcher to open feeder circuits supplying this district.

For supervisory control equipment the audible, code visual, and synchronous visual types of equipment are available. The latest developments along these lines are the "Visicode supervisory control" and "Polaricode" for high-speed operation. Each system has its own field of application.

Audible Type using a dispatcher's cabinet is usually employed for those applications where low cost and extreme simplicity are the prime requisites. It is suited to the distant control of small hydraulic generating stations, small switching and distributing stations and small interurban railway stations.

Visicode Supervisory Control is the latest development combining simplicity of design with minimum line wire cost, as all the controlling and answer-back signals are handled over two wires. Entire equipment is normally at rest, and any number of apparatus units can be controlled and supervised, with a very compact equipment. Up to 20 points the dispatcher equipment can be mounted on a single panel 16 by 90 in., and the substation equipment placed on a similar panel. From 21 to 90 points take two panels at dispatcher's office; two at substation for 21–30; three for 31–60; and four for 61–90. Greater numbers can be handled if desired by using more panel space.

Synchronous Relay Visual type supervisory control is an all relay system embodying the principle of step-by-step synchronous selectors. The equipment consists of relay cases mounted on steel panels, the relays forming the only moving parts. One of these panels is located in the dispatcher's office and the other in the station to be controlled. The control keys and signalling lamps in the dispatcher's office may be mounted in any convenient arrangement to fit into the general plan.

At the substation there are a number of interposing relays which serve to relay the signals from the relay panel to the power equipment. These are usually mounted on the supervisory switchboard. All apparatus is normally at rest.

Selective Remote Metering may be accomplished over the same wires used for the supervisory control equipment by merely assigning a position for that feature and by pulling out the stop key associated with that position so that drive circuits will be stopped, thus connecting the remote metering transmitters with the supervisory lines at the substation and, at the same time, connecting the meter of the proper type to the lines at the dispatching station and giving the dispatcher an indication of the current or voltage to be metered.

SECTION 13

POWER STATIONS AND SUBSTATIONS

STEAM POWER STATIONS
BY
F. S. BENNETT

ART. PAGE
1. Planning Steam-electric Stations..... 02
2. Heat Balance...................... 03
3. Selection and Type of Apparatus..... 06
4. Piping and Valves.................. 08
5. Arrangement of Apparatus.......... 11
6. Cost of Power Stations.............. 12
7. Bibliography...................... 14

HYDROELECTRIC POWER STATIONS
BY
R. A. HOPKINS AND A. G. CHERRY

8. Selection of Site.................... 14
9. Types of Developments............. 15
10. Elements of a Development.......... 17
11. Waterwheels...................... 19
12. Governors........................ 21
13. Generators....................... 21
14. Auxiliary Power Supply............ 24
15. Ice Prevention.................... 26
16. Costs............................ 28
17. Bibliography...................... 28

INTERNAL-COMBUSTION POWER STATIONS
BY
W. A. SLOAN

ART. PAGE
18. Internal-combustion Engines......... 29
19. Power Plants...................... 32
20. Economics........................ 33
21. Bibliography...................... 36

SUBSTATIONS
BY
W. I. SLICHTER

22. Classification of Substations......... 36
23. Substations for Light and Power Systems........................... 37
24. Railway Substations................ 39
25. Bibliography...................... 46

POWER STATION CIRCUITS
BY
R. A. HOPKINS

26. Main Electrical Connections......... 46
27. Excitation........................ 52
28. Station Wiring.................... 53
29. Bibliography...................... 56

STEAM ELECTRIC POWER STATIONS

F. S. Bennett

1. PLANNING STEAM-ELECTRIC STATIONS

The factors affecting the planning are: location, capacity, size of units, and steam pressure and temperature.

Data on power station equipment will be found in Kent's Handbook for Mechanical Engineers; on structural details, in Merriman's Handbook for Civil Engineers; and on buildings in Kidder-Parker's Architects' and Builders' Handbook.

LOCATION. This factor divides as follows:

1. Real estate values.
2. Location of market.
3. Water supply (quantity and temperature).
4. Cost of intake and outlets for water.
5. Prevailing direction of winds (should be away from residential sections).
6. Railroad and water transportation facilities.
7. Local restrictions on transmission lines.
8. Area of real estate available for coal storage and ash disposal.

CAPACITY. The capacity is determined by:

1. Water supply.
2. Market, present and future.
3. Interconnections to existing systems.

SIZE OF UNITS. The steam and electric units should be carefully studied for the following:

1. Present requirements.
2. Future growth.
3. Type of service (industrial or utility).
4. Expected load curve of station.
5. Spare capacity.

The industrial power plant requires careful study, as small units generally prevent expansion to larger units because of building limitations. One solution is to build in units with little or no building to hamper the next unit which may be larger or smaller.

Central stations are now so interconnected electrically that each one is a potential source of emergency service for the others. The size of units, therefore, does not need to be determined by the spare capacity or the load curve, which may be adjusted by making certain stations for base load. The other factors are more important.

STEAM PRESSURE AND TEMPERATURE. This factor is entirely one of fuel economy. The selection is affected by:

1. Cost of fuel.
2. Load curve (total output and daily output).
3. Reliable apparatus available.
4. Water supply.

A curve, Fig. 1, gives an idea of the gain to be made from increasing the steam pressure. The factor of reliable apparatus affects this curve, as higher initial temperature and higher reheats should be used to get the most out of the higher pressures. Fig. 2 gives data on actual performances of stations using turbines and regenerative feedwater heating. The binary cycle of mercury is just coming into service and may radically affect the choice of steam pressure and temperature.

The present limit of commercial use is 1450 lb gage and 900 deg fahr total temperature.

The industrial plant which has a steam demand for manufacturing purposes can generally employ economically the highest steam pressure. There are, however, some detrimental factors for high pressures and temperatures in industrial plants. A steam pressure of 1450 lb, 900 deg fahr temperature does not lend itself readily to rapid changes

in load and it requires longer to start after shutdown. It requires scale-free pure water. The industrial plant usually has all these handicaps. A pressure of 400 to 600 lb and

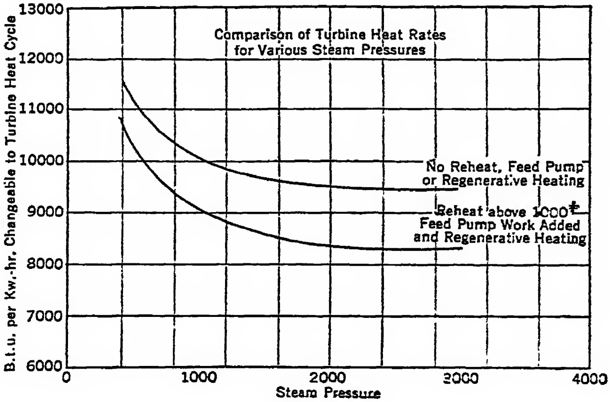


Fig. 1

temperature not above 750 deg fahr have proved successful. Future development may extend this range.

Fig. 2 is based on actual station performances except for 2500-lb steam. This tabulation indicates that the greatest loss is still in the heat not available in a steam turbine

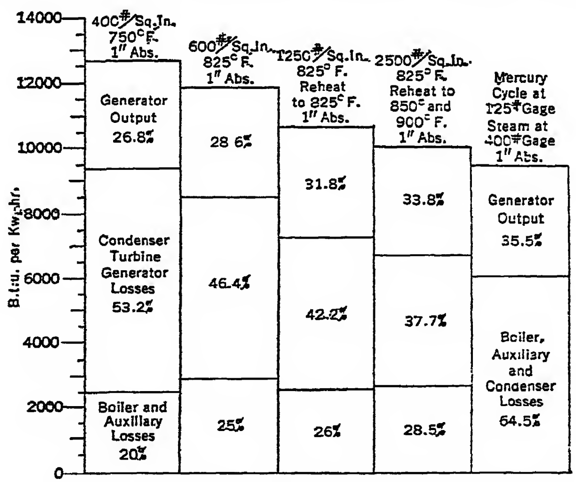


Fig. 2

cycle. These figures are useful in making a preliminary study of economy, but may be slightly altered by efficiencies of individual equipment. They must be modified for losses, such as boiler banking and partial loads of generating machinery.

2. HEAT BALANCE

The distribution of heat in a power station is very necessary to fix the requirements for apparatus.

The distribution of the heat delivered to the turbine has become a very complicated affair owing to regenerative feedwater heating, deaeration of feedwater, and evaporation of boiler make-up water.

FEEDWATER HEATERS. Fig. 3 gives a basis for selecting the number of feedwater heaters and their location in the steam pressure range of the machine. It should be noted that practical difficulties in building a high-pressure machine to extract steam at high initial pressures may be encountered. A simple calculation will show the greater

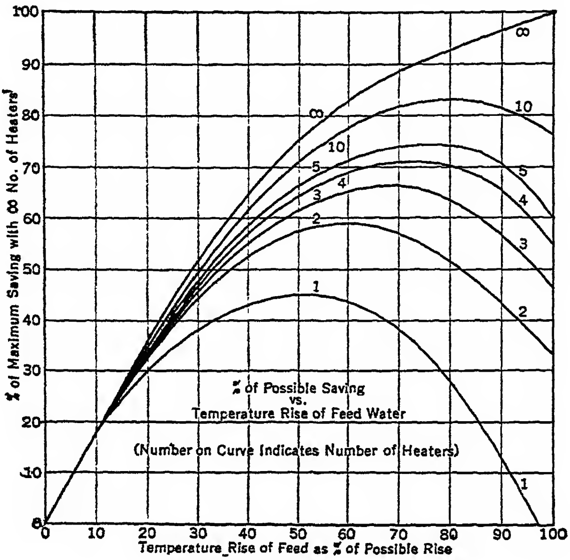


Fig. 3

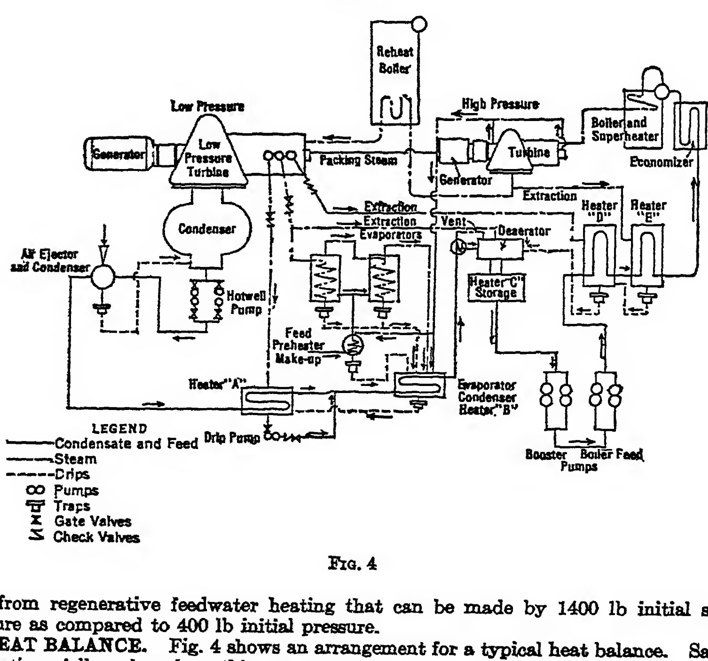


Fig. 4

gain from regenerative feedwater heating that can be made by 1400 lb initial steam pressure as compared to 400 lb initial pressure.

HEAT BALANCE. Fig. 4 shows an arrangement for a typical heat balance. Sample calculations follow, based on this arrangement.

Calculations Turbine Heat Cycle: 80,000-kw unit
Conditions: 1422 lb gage, 825 deg fahr total temperature, 1 in. mercury absolute boiling point.

Kw output	48,985	81,330
Throttle pressure, lb gage	1,422	1,422
Throttle superheat, deg fahr	235	235
Btu per lb at throttle	1,387	1,387
Throttle flow, lb per hr	362,000	652,000
Throttle w.r.	7,390	8,016
Btu per kwhr charged to cycle	9,010	9,000
Condenser flow, lb per hr	271,700	460,920
Condenser w.r.	5,543	5,669
Btu per lb in exhaust l.p.t.	1,024.0	1,011.4
H.p.t. exhaust pressure, lb abs	231	405
Btu per lb in exhaust of h.p.t.	1,239.6	1,277.8
Flow through reheater, lb per hr	328,180	582,900
L.p.t. inlet pressure, lb abs	215.0	377.5
Btu per lb at l.p.t.	1,438.0	1,431.9
Superheat at l.p.t inlet	437	386
Generator losses, h.p. unit	576 kw	640 kw
Generator losses, l.p. unit	1,362 kw	1,618 kw
Total packing leakage, lb per hr	6,000	10,700
Btu per lb in packing steam	1,333.9	1,359.9
Evaporator condenser pressure, lb abs	7.27	11.77
Pounds per hour steam to air ejector	2,000	2,000
Temperature condensate from hotwell	79.3	79.2
Temperature condensate from air ejector cond	87.4	84.2

Heater A

Shell pressure, lb abs	4.56	7.56
Heater pressure, lb abs	4.07	6.80
Btu per lb in extracted steam	1,123.0	1,113.0
Saturated steam temperature at heater	153.8	175.5
Condensate flow through heater	273,700	462,920
Pounds per hour steam extracted	15,200	38,360
Temperature condensate leaving	143.8	165.5
Temperature rise due to return of drips	0.7	1.0

Evaporator Condenser Heater B

Pressure in heater, lb abs	7.27	11.77
Saturated temperature in heater	178.5	201.0
Condensate flow, lb per hr	294,900	511,980
Temperature entering heater	144.3	166.3
Temperature leaving heater	168.5	191.0

Deaerator Heater C

Shell pressure, lb abs	25.9	44.3
Heater pressure, lb abs	23.3	39.9
Btu per lb in extracted steam	1,238.4	1,231.0
Saturated temperature at heater	236.3	267.2
Condensate flow to heater	294,900	511,980
Extracted steam, lb per hr	14,680	27,820
Temperature leaving heater	235.3	266.2
Condensate leaving heater	362,000	652,000

Heater D

Shell pressure, lb abs	100.3	173.1
Heater pressure, lb abs	90.2	155.8
Saturated temperature steam at heater	320.5	361.5
Btu per lb in extracted steam	1,359.8	1,354.0
Condensate flow through heater	362,000	652,000
Pounds per hour steam extracted	24,300	51,700
Temperature leaving heater	310.5	351.5
Evaporator condenser pressure, lb abs	7.27	11.77
Pounds per hour steam to air ejector	2,000	2,000
Temperature condensate from hotwell	79.3	79.2
Temperature condensate from air ejector cond	87.4	84.2

Heater A

Shell pressure, lb abs	4.56	7.56
Heater pressure, lb abs	4.07	6.80
Btu per lb in extracted steam	1,123.0	1,113.0
Saturated steam temperature at heater	153.8	175.5
Condensate flow through heater	273,700	462,920
Pounds per hour steam extracted	15,200	38,360
Temperature condensate leaving	143.8	165.5
Temperature rise due to return of drips	0.7	1.0

Calculations Turbine Heat Cycle: 80,000-kw unit—*Continued*

EVAPORATOR CONDENSER HEATER B		
Pressure in heater, lb abs	7.27	11.77
Saturated temperature in heater	178.5	201.0
Condensate flow, lb per hour	294,900	511,980
Temperature entering heater	144.3	166.3
Temperature leaving heater	168.5	191.0
DEAERATOR HEATER C		
Shell pressure, lb abs	25.9	44.3
Heater pressure, lb abs	23.3	39.9
Btu per lb in extracted steam	1,238.4	1,231.0
Saturated temperature at heater	236.3	267.2
Condensate flow to heater	294,900	511,980
Extracted steam, lb per hr	14,680	27,820
Temperature leaving heater	235.3	266.2
Condensate leaving heater	362,000	652,000
HEATER D		
Shell pressure, lb abs	100.3	173.1
Heater pressure, lb abs	90.2	155.8
Saturated temperature steam at heater	320.5	361.5
Btu per lb in extracted steam	1,359.8	1,354.0
Condensate flow through heater	362,000	652,000
Pounds per hour steam extracted	24,300	51,700
Temperature leaving heater	310.5	351.5
HEATER E		
Shell pressure, lb abs	231	405
Heater pressure, lb abs	207.9	364.5
Btu per lb in extracted steam	1,239.6	1,277.8
Saturated temperature steam at heater	385.2	435.8
Condensate flow through heater	362,000	652,000
Pounds per hour steam extracted	26,120	60,500
Temperature leaving heater / Final feedwater temperature	375.2	425.8

To make these calculations the following are required: an up-to-date steam table, Mollier diagram with expansion lines of the turbine, pressure flow curves for the extraction points, exhaust loss of the machine, efficiency of the generator, and packing leakage. The machine designer must furnish all these data but the steam table. (See Kent's Handbook for Mechanical Engineers.)

The heat distribution from the fuel to the turbine is generally very simple so far as station design is concerned. The cost and load factor fix the heat recovery beyond the boiler. For pressures up to 600 lb, the boiler takes out 75 to 80 per cent of the heat and an air preheater gets an additional 10 per cent. The preheater limit is to keep the exit flue gases above the dew point which varies from 250 to 300 deg fahr. Exit gases below these values form corrosive acids which rapidly destroy the heat-recovery equipment.

Pressures above 600 lb require economizers to keep the preheated air temperature to a safe value.

3. SELECTION AND TYPE OF APPARATUS

The selection of suitable apparatus is largely a matter of experience. The following general rules should be applied:

1. Make a diagram showing services and connections of other apparatus.
2. Write a specification defining services and capacities.
3. Determine the most suitable manufacturers by records of past performance in other plants, or by other suitable methods.
4. Interview the manufacturers' representatives, requesting data on a definite set of requirements.

DIAGRAM. The preparation of a diagram allows the engineer to complete the picture of the requirements and relations of the various pieces of apparatus and paves the way to make a more complete specification.

SPECIFICATIONS. It is generally undesirable to try to make a specification cover the manufacturing details. The writer usually knows less than the manufacturer and quite often radically increases the cost without any gain in utility or reliability.

The specifications, however, should define the services required of the equipment, the capacities (normal and maximum), the auxiliaries to be furnished, the foundations, erection, and other work to be performed by the contractor.

The types of suitable apparatus are governed by so many factors that only a general discussion of the principal apparatus can be given with a few definite rules.

BOILER TYPES. Steam boilers for 250 lb pressure and above are of two general types: bent-tube multiple-drum and header single-drum. Modern developments have made both suitable for the highest commercial pressure services. The choice is largely a matter of personal opinion.

TYPE OF FUEL. The method of burning fuel pulverized or on a stoker is subject to much discussion. However, several facts are available. Low melting ash coal and high-volatile coals can be burned pulverized at better efficiencies and with higher capacities. Higher overall efficiencies for wider ranges of capacities can be maintained. Pulverized coal requires a larger furnace volume per unit of coal burned than stokers. High-ash-content coals can be handled better with the present development in pulverized-coal burning.

BOILERS. The furnaces are of two general types: a solid refractory wall or a water-cooled wall. The solid refractory wall is definitely limited in capacity. This capacity is rated in Btu released by the fuel per cubic foot of furnace volume. A maximum value for a solid wall is 35,000 Btu and an average 20,000. The completely water-cooled furnace has a maximum value of 60,000 Btu and an average value of 30,000. There are various combinations of solid and water-cooled walls with corresponding rates of heat release.

The water-cooled wall is generally divided into one having bare tubes and one having protected tubes. The bare-tube wall is more satisfactory for base load boilers and dry-ash removal. The wall with tubes protected with refractory or metal blocks maintains better combustion conditions with light and fluctuating loads and is more satisfactory for liquid-ash removal. The metal blocks firmly fastened to the tubes have been the most satisfactory for developing the maximum capacities.

The retention of ash and its disposal present a problem with pulverized fuel. The high furnace temperature obtainable with the covered water-cooled wall furnishes a partial solution. The ashes are melted and collected in the furnace bottom. This molten pool acts as a trap for catching additional ash. This slag bottom furnace catches from 30 to 50 per cent of the ash. The dry bottom furnace catches only from 5 to 25 per cent.

The disposal is most successfully accomplished by a water jet system. The liquid ash is run out on a spout and hit by a jet of water which breaks it up into fine particles. These drop into a metal trough and are carried along by jets to a sump. A pump handling a mixture of this fine ash and water delivers it to a waste space. This system can be applied to a dry bottom furnace or a stoker installation.

BOILER ROOM AUXILIARIES. The auxiliaries of the boiler room are important and need careful study. The supply of pure feedwater for steam pressures above 250 lb is very essential. The make-up supply should be evaporated and the main supply deaerated.

Evaporators are well standardized. The essential features required are easy cleaning and suitable provision for expansion.

The deaerator is usually of the direct contact heating type—either pressure, vacuum, or combination. The capacity should always be adequate to cover the maximum, and not the average, requirements.

The boiler feed pumps are almost universally of the centrifugal type except for capacities of 50 gal per min or less. The essential features are good thrust bearings, heavy castings, and large shafts. The most rugged driving unit is an induction motor. Generally the spare unit is a steam turbine drive.

INSTRUMENTS. The modern station requires indications of apparatus performance. The instrument has taken the place of the experienced guess of the operator. The following list is about the minimum required to operate one modern boiler.

1. Meter to indicate and record steam pressure, steam temperature, steam flow (integrate), and air flow.
2. Meter to indicate and record feedwater pressure, temperature, and flow (integrate).
3. Meter to indicate and record gas temperatures (usually four pens).
4. Meter to indicate and record water level in boiler drum.
5. Meter to indicate air and gas pressures (usually six to eight separate indicators).
6. Meter to indicate and record temperature of coal and air mixture and mill differential pressure (pulverized fuel only).
7. Meter to record boiler gas and air temperatures, including preheater and economizer.
8. Indicating gages for special requirement of boiler, such as spare pump, head tank pressure, etc.

The turbine room also requires instruments for measuring the following quantities:

1. Inlet steam pressure.
2. Vacuum.
3. Barometric pressure.
4. Generator air temperature.
5. Turbine stage pressure.
6. Turbine speed.
7. Generator output.

These instruments are the minimum; generally several special ones are required for the particular installation. In addition, a signal outfit is required.

TURBINE-GENERATOR. The selection of a turbine-generator requires a determination of its capacity, average possible vacuum, steam temperature and pressure, and speed. It is useless to buy a machine for high vacuum and be able to obtain it for only 10 per cent of the year. A water temperature curve over as long a time as possible should be used to fix the best working vacuum. American practice has largely been single-shaft machines for 25 and 60 cycles at 1500 and 1800 revolutions for capacities above 10,000 kw. European practice has tended to multiple-shaft, 3000-rpm speed, for 50 cycles. The developments for high steam pressures, 1200 and higher, have been to higher speeds and multiple-shaft machines. The purchaser is interested only as he may obtain better prices, efficiency, or reliability. The statistical records of both American and European plants indicate much greater reliability for slower-speed single-shaft machines, but slightly better efficiency for the multiple-shaft high-speed machines. The capacity chosen requires data on probable future plant developments. The tendency in the past has been for too small a capacity, especially in industrial plants.

CONDENSERS. The features that must be decided are tube length, size of tubes, material, single or double water pass, and divided water boxes.

The tube length is often a factor of available space. Generally speaking, it is desirable to keep the tube length short, say 12 to 14 ft for 5000- to 10,000-kw machines, 16 to 18 ft for 15,000- to 25,000-kw machines, and 20 to 30 ft for larger machines. One-inch tubes are desirable where the water is dirty, and $^3/_4$- or $^5/_8$-in. are used only where the water is clean. The merits of single or double water pass arrangements are dependent upon the amount of water available and the pumping head exclusive of the condenser.

The condenser auxiliaries have become very nearly standard—steam jet air-removal apparatus with surface coolers and constant-speed motor-driven centrifugal circulating pumps. The large units have at least two pumps, one of which may be used with cold water, in this way saving power input. The condensed steam is handled by constant-speed motor-driven centrifugal pumps.

4. PIPING AND VALVES

The increase in pressure and temperature of steam power plants has made it necessary to study very carefully the expansion stresses and flange construction. The general arrangement tends toward unit systems, particularly in large plants. This is due to

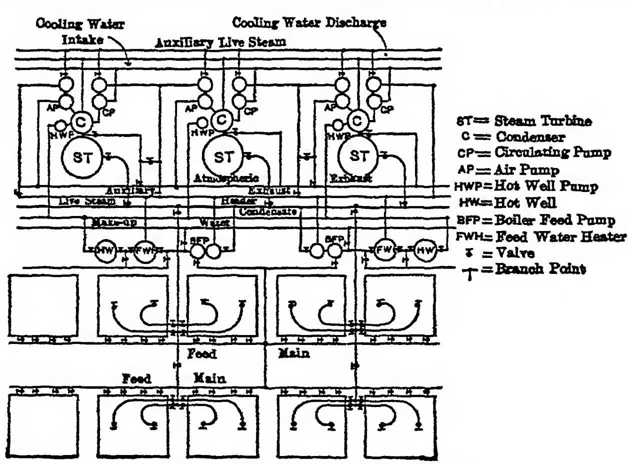


FIG. 5. Parallel Method of Connection

increased pressure and temperature which make it difficult to handle the large headers required.

PIPING ARRANGEMENTS. The parallel system of piping is shown in Fig. 5. This system has many merits and should be used where the capacities and pressure do not require headers beyond the limits of safety.

The unit system (Fig. 6) is used for larger systems and particularly for high pressures, such as 400 to 1200 lb. The majority of operators like to interconnect these units to

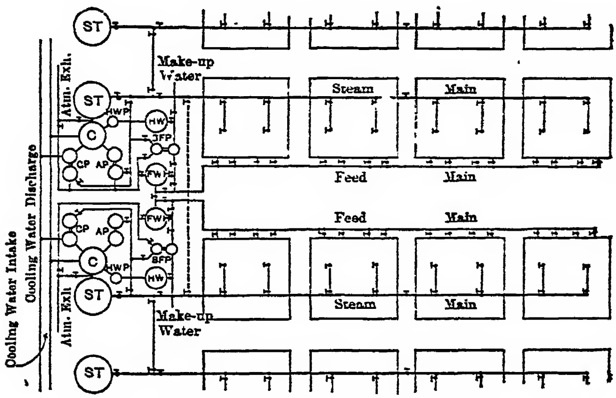


Fig. 6. Unit Method of Connection

obtain more flexibility. This interconnection does not provide for header capacity to carry the capacity of both units connected.

EXPANSION PROVISIONS. The theory underlying the standard forms of pipe bends is well covered by the A.S.M.E. paper by W. H. Shipman, 1930. See Kent's Mechanical Engineers' Handbook, Section 5, vol. 2 of this series.

FLANGES AND WELDED JOINTS. The question of the type of flange joint that should be used has been the subject of much discussion, and many patents and inven-

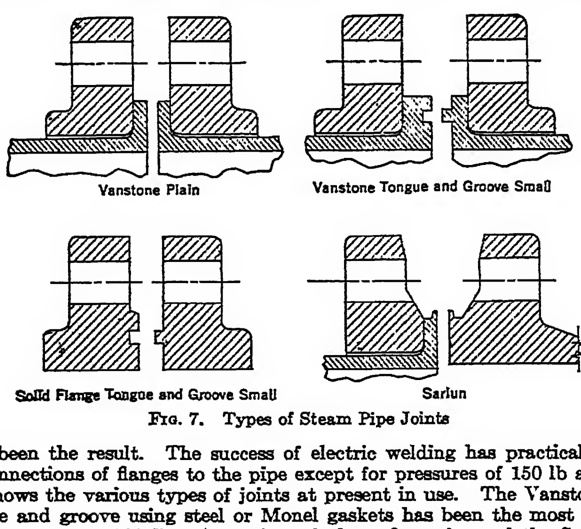


Fig. 7. Types of Steam Pipe Joints

tions have been the result. The success of electric welding has practically eliminated threaded connections of flanges to the pipe except for pressures of 150 lb and less.

Fig. 7 shows the various types of joints at present in use. The Vanstone type with small tongue and groove using steel or Monel gaskets has been the most successful for pressures from 300 to 1200 lb. A number of plants have favored the Sarlun type for 900- and 1200-lb pressures.

The electric arc welded joint has been developed so that the joint at the weld is the equal of the pipe. Fittings and valves have been developed so that a complete job can

be done. This type of joint will supersede all the others for the higher pressures. Fig. 8 shows typical forms of welded joints, valves, and forged fittings. These simplify the piping system and save the cost of expensive bends.

PIPE SIZES. The pipe size is fixed by the allowable drop in pressure. Steam pipes usually have **vapor velocities** from 7000 to 11,000 ft per min. Lower velocities generally represent too large an investment. The usual velocity for a water pipe is from 5 to 8 ft per sec. Condenser hot-well suction pipe may be run at 4 ft per sec to obtain suitable pump suctions and syphon effects. Gas and air pipes are usually run near the lower limit for steam, 5000 to 7000 ft per min.

VALVES. Steel valves, either cast or forged, are required above 600 deg fahr temperature. Globe valves should be used wherever the medium will be throttled. Gate valves should be used for stop valves because their pressure loss is low. The material of the seats and disks of a valve is very important. The requirements are: very hard non-

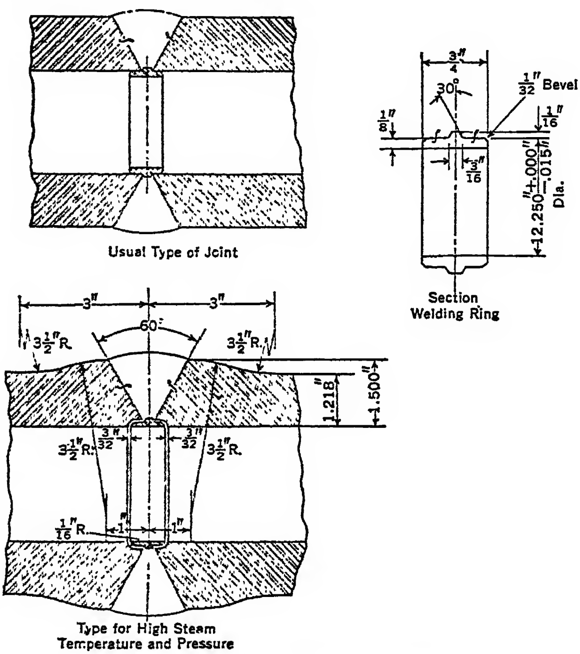


Fig. 8. Types of Welded Pipe Joints

abrasive surfaces free from warping and cracks. These seats and disks should be renewable and capable of refitting.

Motor operation allows valves to be installed in otherwise inaccessible places. The motor operation is a valuable feature for relatively quick operation and shutting off of the lines in case of accident.

A safe rule is to by-pass all valves 6 in. and larger for all pressures above 100 lb. Stop valves should be placed on all pipe lines where they leave the header. Drains are very important on steam lines. They should take care of both the live and dead conditions of the pipe lines.

PIPE INSULATION. Steam and hot water lines require insulation from the heat-loss standpoint; cold water lines from condensation. The general type of insulation is 85 per cent magnesia for temperatures up to 500 deg fahr and special high-temperature type for higher values. There are many other types, such as air cell, hair felt, rock wool, and aluminum foil. These all have a place, but require special study to justify their application. The thickness of insulation should be the subject of a cost study. Quite often the facts are not available for this study, and insulation is arbitrarily selected. The following table can serve as a guide.

Temperatures 500 to 800 Deg Fahr

Pipe Size	Thickness Higher Temperature Type	Thickness 85 Per Cent Magnesia	Thickness Cement
18 to 10 in. inc.	1 1/2 in.	2 in.	1/2 in.
9 to 6 in. inc.	1 1/2 in.	1 3/4 in.	1/2 in.
5 to 4 in. inc.	1 in.	2 in.	1/2 in.
3 in. and smaller	2 in.	....	1/2 in.
Temperatures 500 to 300 Deg Fahr			
18 to 10 in. inc.	....	2 1/2 in.	1/2 in.
9 to 6 in. inc.	....	2 in.	1/2 in.
5 to 4 in. inc.	....	1 1/2 in.	....
3 in. and smaller	....	1 1/2 in.	....

The covering of the insulation depends upon the location. Indoors, where no water or high-moisture air is present, canvas can be used, sewed on where appearance must be considered and pasted on where appearance is not important. Wet places should have two layers of roofing felt wired on and painted with asphalt between layers.

5. ARRANGEMENT OF APPARATUS

The arrangement of power station apparatus is largely dependent upon the kind of service, water conditions, soil conditions, and individual ideas. A few typical considerations will be given.

1. Natural light, as far as possible, should be provided for.
2. Ventilation should leave no dead pockets.
3. Apparatus should be accessible for repairs.
4. Piping should be laid out before the major apparatus is fixed in location.
5. Picture drawings should be made to allow the plant to be visualized.
6. Future expansion of the plant should be provided for by drawing studies of the largest and smallest units.
7. It is desirable to group all the feed pumps and service water pumps so that a common operator can control them.

GENERATING ROOM. There are two general schemes for arranging a turbine electric generating room: units parallel to the boiler axis, and units perpendicular to the boiler axis. The parallel units fit best where there is a multiplicity of boilers. The perpendicular units are best arranged for a single boiler or unit scheme. The parallel units save on building and crane spans. These are a factor with large, long turbine units.

The well-designed generator room sets the generating units on an island foundation and leaves the basement well lighted and accessible to crane service.

The modern turbine and condenser are usually connected solidly on the exhaust, and expansion is provided for by supporting the condenser on springs, so as to give very little uplift on the turbine.

The switching equipment and transformers have all gone from the generating room and are arranged outdoors, with only cable connections and disconnecting switches at the machine. A convenient and general arrangement is to have these switches and the current and potential transformers under the generator foundation.

The general practice is to water-cool the ventilating air for the generator by means of finned tube surface coolers. The ventilating air is circulated in a closed duct system through these coolers and the generator by means of fans on the rotor of the generator or separate motor-driven fans. This system excludes dirt and limits the fire hazard.

The use of steam temperatures from 500 to 1000 deg fahr has created a considerable fire hazard from the lubricating oil. Storage tanks, filtering systems, circulating pipes, and everything containing oil should be kept as far as possible from the steam lines. Flanges should be welded on, and as few joints as possible made. If flanges are used, they should have not less than four bolts, and in general it is desirable to construct them to the 400-lb steam standard.

BOILER ROOM. The modern boiler room should be well lighted, well ventilated, and free from dust and smoke. These features are obtained by making the fuel-handling equipment dust-tight and the operating floor at the lowest level with no intervening floors, access to the upper part being by galleries and stairs. Good ventilation can be obtained where there are no floors and usually natural light. The general tendency is to

make boilers large and expect continuous service with few interruptions for repairs. The making of boiler rooms clean and cool has removed the reason for walling off the generator room. The present tendency is to make the boiler and turbine room one. This saves space and cost and tends to reduce the number of operators required. Another step that has followed the combination of the turbine and boiler room is partial elimination of the building structure. A notable example is the General Electric Co. Outdoor Station at Schenectady, N. Y.

AUXILIARY EQUIPMENT. The auxiliary equipment, such as boiler feed pumps, house service pumps, and feedwater treating, should be concentrated as much as possible, leaving the piping simple and compact. This concentration helps to keep the number of operators small, as the control of these can well be combined with the condensing equipment by locating them adjacent. The general practice is to make the normal units motor-driven and have a few spare steam units for emergency and starting.

6. COST OF POWER STATIONS

The cost of a power station is variable because of the differences in land values, cost of condensing water supply, ideas of designers for arrangement of apparatus, and building construction.

Fig. 9 is based on an average of the cost, per kilowatt of installed capacity, of 60 large stations; it fairly represents the cost at the time (1925 to 1929). The cost per kilowatt, of smaller-capacity plant is shown in Fig. 10.

The majority of central stations today (1936) are below 400 lb and 750 deg fahr temperature. These stations have heat rates per kilowatt-hour from 20,000 to 14,000 Btu.

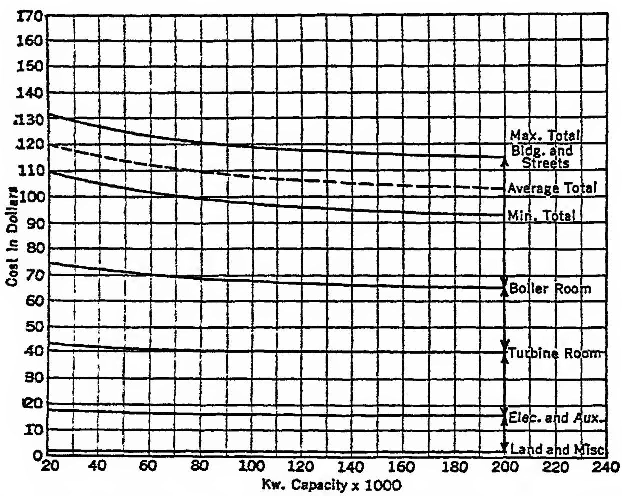


FIG. 9. Typical Costs of Steam Power Stations from 20,000 to 200,000 kw Capacity

These can be improved to 12,000 Btu by adding new boilers for 1200 lb, 900 deg fahr temperature, and new turbines to reduce this pressure to the existing station pressure. The continuity of service can be safeguarded by the existing boilers without spare high-pressure equipment. The cost of such a change is $90 to $100 per kw of the additional capacity. These costs are estimated, as very few stations have been changed in this manner up to 1936.

The most economical change that can be made is to superimpose the mercury cycle. This cycle generates mercury vapor from the fuel; the vapor passes through a turbine and generates electric power; the exhaust from the mercury vapor turbine heats water as it condenses and makes steam, which is superheated in the mercury boiler and then discharged into the station header. This cycle will give a station heat rate of 9000 Btu per kwhr and is estimated to cost $90 to $100 per kw of additional capacity. There are three commercial installations, and the success of these would undoubtedly mean the general use of this cycle instead of the 1200-lb superimposed capacity.

The industrial plant presents a large field for increase in economy by superimposing higher pressure or the mercury cycle. Fig. 11 has been prepared on the 1200-lb cycle to show the effect of initial pressure on the cost of additional capacity. The curve is based on 300,000 lb of steam per hour, 250 lb exhaust discharge pressure, and 750 deg total temperature. It is evident that 1200 lb is somewhere near the economical. There is no experience beyond this pressure, so that the exact performance can only be estimated.

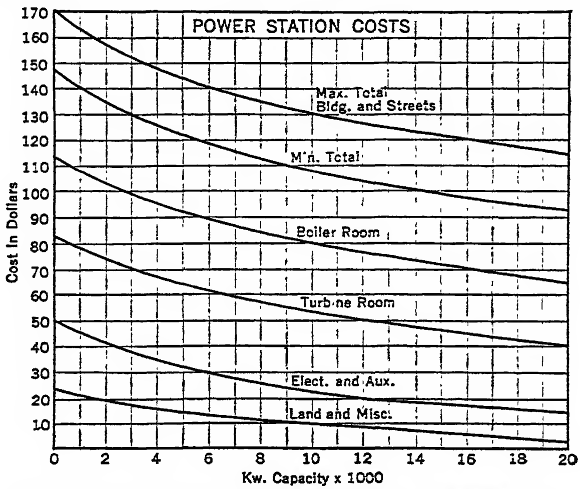


FIG. 10. Typical Costs of Steam Power Stations up to 20,000 kw Capacity

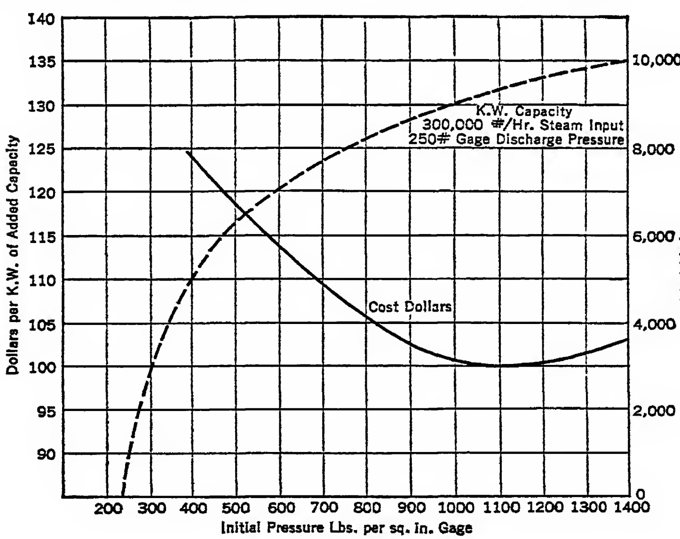


FIG. 11

The operating costs of a plant are variable and tend to decrease as the plant capacity increases. These costs are usually divided into operating labor, maintenance and supplies, fixed costs, and fuel. These costs are usually expressed as mills per kilowatthour. The cost, therefore, is a function of the total kilowatthours produced per year. This is expressed as load factor or the ratio of actual kilowatthours to maximum possible kilowatthours that can be generated per year. The total cost ranges from eight (8) to ten (10)

mills for small plants and from five (5) to seven (7) mills for large plants. These costs are divided—three (0.3) to five (0.5) tenths mills for labor; two (0.2) to four (0.4) tenths mills for supplies and repairs; two and one-half (2.5) to three and one-half (3.5) mills for fuel, and three (3) to five (5) mills for capital charges. These costs may be larger if fuel is above $5 per net ton or the load factor is below 35 per cent.

7. BIBLIOGRAPHY

See Art. 29 of this section, and bibliographies of Sections 4 and 8–12.

HYDROELECTRIC POWER STATIONS

By R. A. Hopkins and A. G. Cherry

8. SELECTION OF SITE

In the selection of a water-power site, approximate methods are used for preliminary studies, but in the final consideration of an important power site there is no substitute for an exhaustive study of its potentialities.

AVAILABLE POWER AND ENERGY. Analysis of the available power and energy includes the study of hydrological conditions (rainfall, runoff, and stream flow), head limitations, facilities for storage and pondage, and the conversion of the available flow and head into electrical energy.

The United States Geological Survey, the United States Weather Bureau, and many of the individual states publish much pertinent information on stream flow, climatology, geology, flood flows, and similar subjects.

When records of stream flow are not available for the vicinity of the particular site under consideration they may be obtained or approximated by one or more of the following methods:

1. Establishment of a gaging station at the site.
2. Deduction from local rainfall statistics.
3. Comparison with flow of adjacent or similar watersheds where records are available.

Stream flow data usually are summarized by the preparation of **hydrographs**, for which daily averaged flows are plotted chronologically, and **flow duration curves**, for which flow quantities are plotted against percentage of time of occurrence. Where storage is a factor, use frequently is made of the **mass diagram** for which the summation of daily or monthly flows is plotted chronologically for the entire period under study. The slope of the mass curve at any point represents the rate of flow, and from this curve the amount of storage required to regulate the flow to a definite minimum may be determined graphically. Particular attention should be given to data for years of maximum and minimum stream flow.

The **head** to be developed is subject to headwater and tailwater limitations. Headwater elevation is usually controlled by the cost of reservoir lands, hydraulic structures, and water rights, and by limitations due to the backwater influence of water impounded by the dam. The range of **headwater elevation** is influenced by the type of spillway, the reservoir operation, and flood flows. The range of **tailwater elevations** is usually established by the natural river levels at the lower end of the tailrace. In the case of overlapping developments the backwater influence of the lower development controls the tailwater levels of the upper development.

From the various combinations of headwater and tailwater conditions a **duration of head curve** may be plotted, which shows gross or net head plotted against time.

On the amount of **storage** that can be economically provided depends the extent to which the stream flow can be equalized, and the amount of installed capacity which can be justified. A run-of-river plant has no storage, and the stream flow must be used as it occurs. Sometimes, storage can be provided economically in a separate storage reservoir, particularly if the same storage would serve a series of developments on the same stream.

After the stream flow, head, and storage conditions have been analyzed, the available power and energy for dry, average, and wet years can be computed, and its conversion into electrical energy and absorption by available markets can be studied.

AVAILABLE MARKET. Since hydroelectric plants usually feed into power systems, it is necessary to analyze the power market to determine whether the power and energy available can be satisfactorily utilized. As a basis for this study the following information regarding the market or power system should be compiled:

1. Seasonal generating capacities of the steam and hydro stations in the system.

2. Annual or seasonal energy and capacity available to the system through interconnection with other systems.

3. Transmission line data, including estimated transmission losses to distribution points.

4. Data to predict probable future capacity and energy requirements of the system for several years.

Based upon these data, estimates of monthly system peak loads and energy requirements should be prepared for the future period under consideration. The estimates are usually based on data and records kept by the power company for previous years, and the future growth is predicted in accordance with past growth modified to suit general business tendencies, acquisition, or development of new market, and similar factors.

ABSORPTION OF POWER AND ENERGY BY THE MARKET. The power and energy available are next studied with particular reference to the market data to see whether they will satisfy the future system requirements on any reasonable operating basis.

Usually these system requirements are for additional capacity to satisfy additional peak load requirements, additional energy to provide additional kilowatthours on the base of the load, or a combination of these two. When additional system capacity is the major requirement and the stream flow at the site under study fluctuates widely, a relatively large amount of storage should be provided. Where the system energy cost is comparatively high, energy may become more important than capacity, in which case the justification of a reservoir would depend upon the need of increasing the natural flow during low water periods. Even on large rivers the absolute prime power is so small that sites are rarely subject to economical development on the basis of prime capacity alone. Such sites are economically justified only when they can be developed for large capacities and operated during much of the year on system peak loads, or at low load factors. Pondage is essential in all cases where capacity value is of importance.

PHYSICAL CHARACTERISTICS OF THE SITE. Generally, a satisfactory, though not necessarily economical, type of dam can be constructed on about any kind of foundation, but it is imperative that the engineer have the most complete knowledge possible of all foundation conditions. Data on foundations are usually obtained from core or wash borings or from test pits.

It is also necessary to investigate the geology and topography of the entire reservoir area in order to discover possible future sources of leakage to adjoining watersheds or to the stream below the dam.

Each site should also be considered for its practical advantages for construction, including particularly such items as accessibility and transportation facilities, possibilities for stream diversion during construction, and local supplies of materials, such as sand, gravel, lumber, etc.

9. TYPES OF DEVELOPMENTS

HIGH-HEAD PLANT. The high-head plant (more than 850 ft) requires a relatively small quantity of water in proportion to its capacity. The reservoir is usually a natural lake or a basin formed by low impounding or diversion dams. The water is conducted from the reservoir to the power station by canals, flumes, tunnels, pipe lines, or a combination of two or more of these types of waterways. The waterways are relatively small. The water wheels are usually of the impulse type, direct-connected to horizontal-shaft generators. Often two wheels per unit are used, one at each end of the generator shaft.

MEDIUM-HEAD PLANT. The medium-head plant (850 to 50 ft) may have the power house at a distance from the reservoir, or adjacent to or constructed integral with the dam. A large reservoir is common with this type of development. Where the total head is less than 500 ft, most of the head is usually developed by the height of the dam. When the power station is at a distance from the dam the connecting waterways are usually penstocks, although sometimes canals or flumes are used to conduct the water from the main reservoir to a high-level forebay closer to the station. The waterways are larger with the lower heads. The water wheels are usually of the Francis, reaction type, direct-connected to vertical-shaft generators.

LOW-HEAD PLANT. The low-head plant (less than 50 ft) usually has the power station located at the dam, either behind a forebay or constructed as an integral part of the dam. The dam is low and the reservoir small, providing only limited storage or pondage. The waterways are relatively large. The water wheels are usually of the high-speed, propeller type, with or without adjustable blades, depending on the range of head and the operating conditions. The wheels are usually direct-connected to vertical-shaft generators. Owing to the importance of conserving the head, particular attention must be given to the design of the intake, draft tubes, and tailrace. Fig. 1 shows a typical low-head plant.

EFFICIENCY. The maximum overall efficiency of a hydroelectric plant from reservoir water level to bus-bar will vary from 75 to 88 per cent, depending on length of waterways, head, and type of turbine. Medium-head plants with Francis type turbines will have the highest maximum efficiency, low-head plants with propeller type turbines slightly lower,

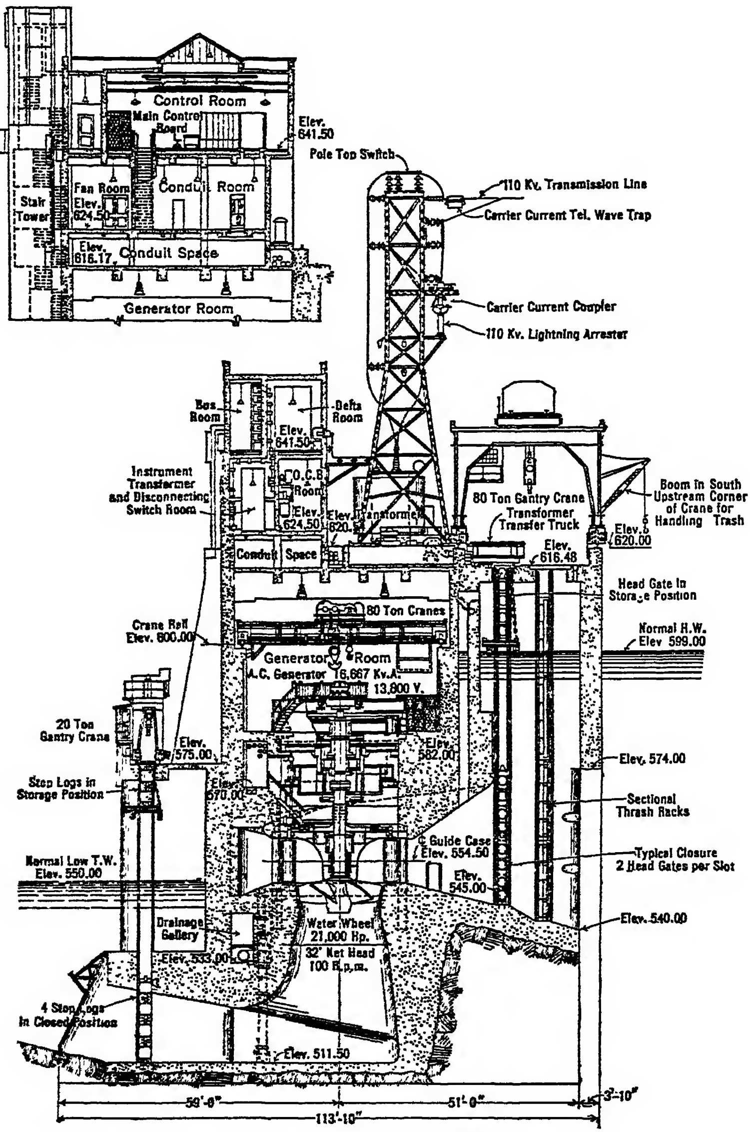


FIG. 1. Typical Low-Head Plant

and high-head plants with impulse wheels the lowest. The average overall efficiency of these plants will be somewhat less than 75 to 88 per cent and will vary over a wide range, depending on the method of plant operation, the variation in head, and the amount of wasted water.

PUMPED STORAGE DEVELOPMENT. The pumped storage development is becoming economical in special cases for peak load service. Many such plants are in successful operation in Europe and in America. An outstanding development of this type, the Rocky River Development of the Connecticut Light and Power Co., was completed in 1929. In this type of development, tidewater or water from a low-level stream or reservoir is pumped to a high-level artificial or natural reservoir during off-peak hours, the stored water being used to provide system peak capacity. Off-peak energy available from other steam and hydro plants in the system is used for the pumping operation. The maximum overall efficiency of a pumped storage plant from bus-bar ingoing to bus-bar outgoing will vary from 60 to 70 per cent. In Germany the Herdecke plant on the Ruhr River, with an installation of four 48,000-hp units under 534-ft head, develops an average overall efficiency of 65 per cent and a maximum of 68 per cent.

10. ELEMENTS OF A DEVELOPMENT

THE RESERVOIR. The chief function of the reservoir is to store water during periods of excessive stream flow in order that the surplus water may be used to advantage during periods of deficient flow.

The construction and operation of a reservoir are subject to local and federal regulations pertaining to items which involve public health and the conservation of natural resources, such as reservoir clearing, navigation facilities, fish propagation, and mosquito control.

Water stored in reservoirs is subject to losses from natural causes such as evaporation and leakage.

THE DAM. The choice of type of dam to be used in connection with the development of a particular site depends on many conditions, chief of which are the character of the foundation, the topography of the site, the height of the dam, and the availability of materials for construction. The various types of dams in common use are:

a. **Solid Masonry Dam.** Usually of mass or cyclopean concrete, but sometimes of stone masonry. May be of gravity or arch type or a combination. Requires rock foundation of unquestioned soundness and durability.

b. **Hollow Masonry Dam.** Usually of reinforced concrete but occasionally built of precast concrete slabs. Includes such special types as slab and buttress, multiple arch, and multiple dome. Ordinarily suitable for rock foundation of poorer quality than required for a solid masonry dam, although if high bearing stresses are used, the best rock foundation is necessary.

c. **Earth Dam.** Built of selected soils with or without core walls of concrete, wood piling, or steel sheet piling. Cores of sluiced material or puddled clay sometimes used. Suitable for soft foundations where adequate materials are available.

d. **Rock Fill Dam.** Built of rock fill either with concrete or timber deck or with a core wall. Suitable for rock foundation, where material is available, at less cost than concrete.

e. **Timber Dam.** May be of timber frame and deck construction or of rock-filled timber cribs. Seldom practical for heights exceeding 20 ft.

The **Spillway Section** of the dam, and sometimes all of the dam, is designed to discharge surplus and flood water. That portion not designed as a spillway is usually called a retaining or impounding section. The discharge capacity of the spillway is established so as to provide a margin of safety over the largest recorded flood. The spillway may be built with a free crest or with a controlled crest. The discharge over a free-crest spillway is controlled only by the water level in the reservoir. The discharge over a controlled-crest spillway is regulated by flashboards or crest gates.

Crest Gates of the following types are commonly used for gate-controlled spillways: sliding, roller (fixed wheels), Stoney (roller trains), radial or Taintor, drum, tilting, bear trap, and rolling. Gates may be operated by individual hoists or by traveling hoists.

Water may be discharged past the dam also through siphons or various types of sluice gates and valves.

Accessories sometimes required in connection with the construction of a dam include such structures as navigation locks, fish ladders, and log chutes.

WATERWAYS. Under this heading are classified all structures and equipment between the reservoir and the turbines, including the forebay, intakes, and water conduits.

The **forebay** is the pool adjacent to the intake. Many developments have no definite forebay, but when water is brought to the intake by canal or flume it is usually necessary to expand the section at the intake in order to reduce the velocity.

The **intake** is located at the upstream end of the conduits leading to the turbines and contains structures and equipment necessary to control the flow entering the conduits. The intake is usually provided with racks or screens to keep trash and ice from entering the conduits and with gates or valves to close off the waterways. Where the dam is of

earth or rock fill, intake towers isolated from the dam are generally used. In cold climates the intake is frequently housed to prevent freezing.

Trash Racks, in front of the intake, are usually constructed of round-edged vertical bars spaced from 1 1/2 to 8 in. on centers depending on the size and type of waterwheel runner. Water velocity at the bars is usually between 1.5 and 2.5 ft per sec.

Intake Gates of the following types are in common use: sliding, Stoney (roller trains), roller (fixed wheels), Sirnit (fixed wheels unequally spaced, resting in recesses in guide rail), caterpillar, radial or Taintor, cylinder (usually for circular intake towers). Intake valves of the following types are also used: rectangular butterfly (horizontal shaft), circular butterfly (horizontal or vertical shaft), needle or Johnson, rotary. The circular butterfly, the needle and the rotary valves and the cylinder gates are suitable for use under high heads; the remaining types are rarely used for heads exceeding 100 ft. Permissible velocities at the headgate openings vary from 2 to 10 ft per sec.

Artificial Water Conduits may be classified as: (1) open conduits; canals or flumes; (2) closed conduits; tunnels or pipes. Frequently a combination of two or more types of conduit will be found to be practical as well as economical. Long conduits are usually constructed in two portions: (1) the **high-level conduit,** which is designed for relatively low pressure and follows the topography or hydraulic gradient to a suitable location near the power house, from which point (2) the **penstock** conveys the water to the turbine, being designed for the necessary higher pressures. A **surge tank** or regulator is located at the junction of the high-level conduit with the penstock to prevent excessive changes in pressure during fluctuations in the flow.

Canals may be lined or unlined, depending upon the character of the ground through which they pass, the shape of cross-section, and the water velocity. Ordinary soils will stand velocities up to 2.5 feet per sec without erosion. Canals may be lined or paved with wood, rock, brick, concrete, or gunite.

Flumes may be of wood, concrete, or steel. In section they may be rectangular or semicircular, or they may be semicircular or semi-oval with vertical sides. The wooden flume is seldom used for permanent construction on account of its short life and high maintenance cost. In cold climates long flumes or canals are subject to ice troubles. (See Ice Prevention.)

The **tunnel** is the most permanent, but usually the most expensive, form of conduit. Whether designed as a flow or a pressure tunnel it is usually lined for additional structural safety, to minimize head losses by providing a smooth-surfaced and uniform section, and to reduce losses of water by leakage. Tunnel linings are usually of reinforced concrete, although brick linings have occasionally been used.

Pipe Lines may be of wood, concrete, or steel, although concrete pipe lines are rarely used except for very short sections.

Wood-stave Pipe is used extensively for conduits under moderate heads in locations where the topography is favorable. It is relatively cheap in first cost, and if continuously filled has fairly long life. It is particularly adaptable for use in inaccessible locations. Two types of wood-stave pipe are in general use: (1) machine-banded pipe, and (2) continuous wood-stave pipe. The former is used for heads up to 500 ft and in diameters up to 42 in. The latter is built up in place and has been constructed up to a diameter of 16 ft.

Steel Pipe Conduit is usually constructed with either riveted or welded joints, certain advantages being inherent in each type. For penstocks under very high heads, forged-steel pipe has been used successfully. Steel pipe when properly painted and maintained has a long life. In the design of a steel pipe line or penstock, careful attention must be paid to such details as supporting saddles, expansion joints, and anchorages. Permissible velocities in steel pipe are as high as 20 ft per sec for high heads, but for ordinary heads usually range from 8 to 12 ft per sec. Maximum economical velocity will generally depend on the length of conduit and the load and head conditions.

TAILRACE. The tailrace is the channel or waterway that conducts the water from the discharge end of the turbine draft tubes back to the natural river channel. Except under very favorable topographical conditions it is rarely economical to obtain additional head by extensive tailrace excavation. Permissible tailrace velocities usually vary from 3 to 5 ft per sec, although somewhat higher velocities are permissible for the higher head plants.

Impulse wheels discharge above tailwater level, and the tailrace therefore has no important effect on their operation other than to reduce maximum water level and permit lower setting of the water-wheel.

SUBSTATION. Because the hydroelectric station is usually at a distance from the load-center, a step-up substation and high-voltage transmission lines are usually required. The substation is sometimes constructed as a part of the power station. This design is particularly advantageous at high-head plants in mountainous regions where level ground area is limited.

The **outdoor substation** is usually preferred where space is available. The cost of

foundations and steel structures for the outdoor substation is considerably less than that of the housing and barriers for the indoor station, and the cost of outdoor equipment is only slightly more than that of indoor equipment. The outdoor substation may be located on the roof of the power house, on the dam, or on the ground. It should be as close as practicable to the power house in order to reduce the length of main and control cables and to afford convenient access by the attendants. The substation should be adequately fenced to exclude all unauthorized persons.

MISCELLANEOUS PERMANENT STRUCTURES AND FACILITIES. Various miscellaneous structures and facilities which may be required in connection with a hydro-electric development include:

a. Permanent railroad connection for handling equipment and supplies.
b. Permanent highway connection for easy access to plant.
c. Operators' quarters (houses, clubhouse, water supply, sewage system, lighting).
d. Parking space for visitors and garages for employees.
e. Office building (occasionally provided for important stations).
f. Storage building.
g. Boats and boathouse for reservoir patrol.
h. Gage houses (for automatic recording of water levels).

The extent of these facilities will vary with the size and importance of the development and with its distance from settled communities.

11. WATERWHEELS

Waterwheels are classified broadly in three general types: impulse, Francis, and propeller. Ordinarily impulse wheels are used for high heads, Francis wheels for intermediate heads, and propeller wheels for low heads. There are no well-defined limits separating the head ranges for these three types of wheels. Generally, impulse wheels are used for heads above 850 ft, Francis wheels for heads from 850 to 50 ft, and propeller wheels for heads below 50 ft.

The relation of the waterwheel capacity to the generator capacity should be based on the expected variations in head, flow, and load requirements, since the efficiency of the waterwheel varies considerably with load and head variations. The waterwheel capacity is sometimes selected so that at normal head the "best gate" capacity, that is, the most efficient load, coincides with the rated generator capacity. If capacity under heads lower than normal is important the waterwheel may be of sufficient capacity to deliver full generator capacity at a head lower than normal.

Generally, it is economical to use the smallest number of units which can be operated to use the water economically with respect to the load requirements. However, at run-of-river plants, it may be desirable to install units of different capacities in order to use the water most economically under varying conditions of flow and head. At low-head plants a similar result may be obtained by the use of propeller runners with adjustable blades.

While it is desirable to use waterwheel speeds as high as practicable in order to keep down generator size and cost, it is also necessary to maintain a balance between head and speed to obtain smooth operation and avoid serious pitting of the runners. Wheels are selected on the basis of their specific speed, which may be defined as the revolutions per minute at which a homologous runner would operate while developing 1 hp under 1-ft head. The relation between specific speed and actual speed is expressed by the formula:

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

in which N_s = specific speed.
N = speed of runner in revolutions per minute.
P = capacity of runner in horsepower.
H = head in feet.

It will be seen that high-head runners have low specific speeds while low-head wheels have high specific speeds.

Creager and Justin in Hydro-electric Handbook give the following empirical formula for determining the maximum safe specific speed to avoid pitting of the runners of Francis wheels for various heads:

$$\text{Max} \quad N_s = \frac{5050}{H + 32} + 19$$

Speeds based on this formula are frequently exceeded in practice since a small amount of pitting is not objectionable, and even in cases of considerable pitting the saving in first

cost of equipment due to the higher speed may more than offset the cost of repairing the pitted runner.

The speeds derived from the specific speed formula are subject to correction to conform to the nearest synchronous speed which is determined from the equation:

$$S = \frac{120f}{p}$$

in which S = synchronous speed in revolutions per minute.
f = frequency in cycles per second.
p = number of poles of generator. always an even number.

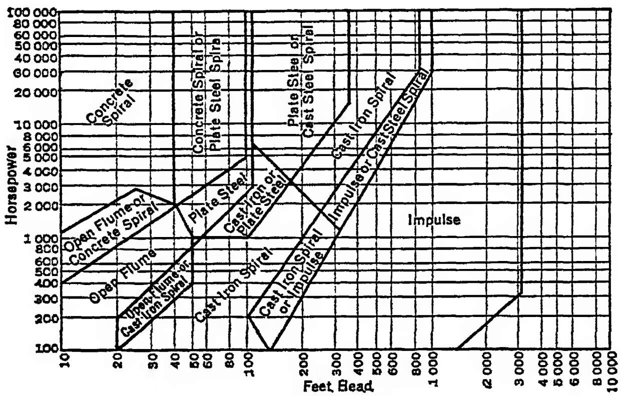
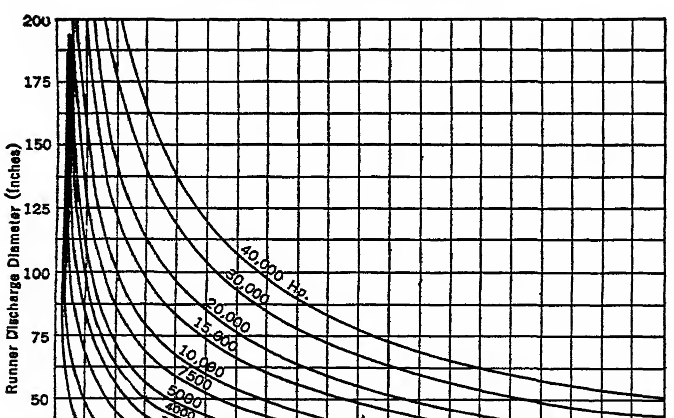


Fig. 2. Type of Unit and Setting



(*Courtesy of Messrs. Creager & Justin*)

Fig. 3

The selection of the type of unit and setting appropriate for various conditions of head and power is shown in Fig. 2.

For vertical-shaft units of the Francis or propeller type, the dimensions of the setting are in general a function of the discharge diameter of the waterwheel runner, and Fig. 3

gives these discharge diameters for various combinations of head and power at normal specific speeds. Various overall dimensions of the turbine setting with reference to the discharge diameter of the runner are shown in Fig. 4. This figure shows a concentric

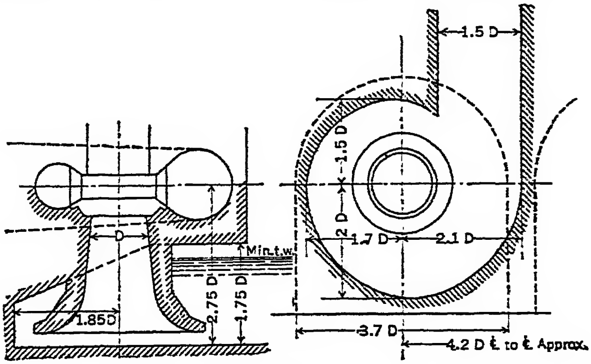


FIG. 4

draft tube. A draft tube of the elbow type would usually require some additional depth and length.

12. GOVERNORS

An approximate formula for the governor capacity required for a particular unit is:

$$C = \frac{50\,P}{\sqrt{H}}$$

in which C = governor capacity in foot-pounds.
P = capacity of unit in horsepower.
H = head in feet.

For large-sized, high-head units this value should be decreased somewhat, while for low-head, open flume units it should be slightly increased.

The governor flyballs may be directly geared to the main shaft or they may be mounted on the governor and driven by one of the following means: shaft and bevel gears from main shaft, belt drive from main shaft, or alternating current motor drive from transformer on main generator leads, from slip-rings on direct-connected exciter or from small alternating current generator on main shaft.

Governors are almost invariably operated by oil pressure, although in the past various other liquids have occasionally been used. The oil pressure required for operation is usually 150 lb per sq in. or less, and the capacity of the oil-pressure system is a function of the governor capacity and the number of units. Generally an accumulator or pressure tank is provided for each unit or pair of units, and uniform pressure during operation is maintained by use of a substantial air cushion in the top of each tank.

13. GENERATORS

TYPE. Waterwheel generators are usually revolving-field, synchronous type with salient field poles. There appears to be no immediate limit to the capacity that can be built. The speed is rarely over 720 and is sometimes as low as 60 rpm. Vertical-shaft generators are much in the majority. The vertical-shaft construction, while dictated by the waterwheel, is at the same time a distinct advantage in the design of a large-diameter, low-speed generator. Generator voltages up to 24,000 volts are feasible at the present time, but as transformers are nearly always used between the generators and the transmission line, there is usually no reason for exceeding 15,000 volts.

Runaway Speed. The runaway speed of a waterwheel with gates open, governor out of action and no load on the generator, is from 160 to 200 per cent of rated speed. When full load is suddenly dropped, with the governor in action, set to close the gates in from 2 to 5 sec, the speed will rise to 120 to 140 per cent of rated speed. A shorter time setting of the governor would cause excessive pressure in the penstock and wheel case due to inertia of the moving water. Furthermore, it is not feasible to use a quick-closing valve

ahead of the gates to trip closed in case of governor trouble, for this, too, would build up excessive pressure. As a consequence of this inability to shut off the input suddenly, as can be done with a steam turbine, the waterwheel generating unit may attain a speed of 140 per cent in case of loss of load and 200 per cent in case of accident.

The **flywheel effect** of the generator is depended upon largely to limit the speed variation with sudden load changes, since the governor action must be relatively slow, as explained above. A large flywheel effect not only gives closer speed regulation but in some cases is desired to improve system stability. Generators of standard design usually have sufficient flywheel effect for this purpose. In exceptional cases where it is not sufficient, it can be increased, at considerable cost, by increasing the diameter or massiveness of the rotor or by adding a flywheel.

STABILITY. A long transmission line, when lightly loaded or open at the far end, accepts low power factor, leading current from the generator, and this current in the armature windings tends materially to increase generator voltage. The generator field excitation may be reduced as the armature reaction increases, so as to maintain normal voltage, but if the line is long and the generator not sufficiently large, the field excitation would have to be not only reduced to zero, but even reversed in order to avoid excessive voltage. Operation with reversed field excitation is not considered good practice as the generator is likely to be unstable, lose synchronizing power, slip a pole, and build up excessive voltage. The charging of a line with two generators has also been found difficult because the machines do not always parallel easily with weak excitation. A generator of normal design can be expected to deliver satisfactorily a charging kva at zero power factor leading equal to about 75 per cent of its rated kva without instability and without developing more than rated armature voltage. If this is not sufficient to meet conditions, special generator design, involving stronger field excitation and less armature reaction, must be resorted to, although the size and cost of the machine may be considerably increased by these changes.

CONSTRUCTION. The flame-cut, welded and bolted steel frame construction, now conventional, is more reliable, mechanically, than the former cast-iron construction, is of lighter weight, can be fabricated in less time, and is more readily adapted to the ventilating and construction requirements of the turbine room. In some cases the generators have been designed for partial exposure to the weather, omitting the conventional high turbine room and serving the machines with an outdoor gantry crane.

Fig. 5 shows a cross-section through a typical vertical waterwheel-driven generator of the **overhung,** or **umbrella** type. This particular type, distinguished by having one combination guide and thrust bearing just below the rotor hub and carried by the lower bracket, offers advantages of less overall height and simpler bearing layout than the formerly more common type having two guide bearings, one below and one above the rotor, and a thrust bearing at the top of the machine, supported by the upper bracket. It has a disadvantage, for higher speeds, in the free length of shaft above the bearing. The combination guide and thrust bearing simplifies the lubrication. The heat is removed by water-cooling coils. The oil level gage and the thermometer are essential. In some overhung designs the rotor arms are sloped to bring the center line of the field more nearly into the plane of the guide bearing and further to reduce the total height of machine. The rim of the rotor is sometimes made structurally separate from the arms so that the arms are relieved of all centrifugal forces from the rim, thereby greatly simplifying the structural design. The rotor is sometimes covered with thin steel plates to reduce windage.

VENTILATION AND FIRE PROTECTION. Three general methods of **generator ventilation** are in common use. (1) Air may be taken from the room, passed through the generator, and discharged into the room by means of the fan action of the rotor and without external ducts. This allows no control of generator-room air condition and is used only in the smaller stations. (2) Air may be taken from the room or from outdoors, passed through the generator, and discharged to the room or outdoors by means of the fan action of the rotor, assisted if necessary by external motor-driven fans, and by means of external ducts and dampers. Air is often taken from a point over the tailrace where clean, moist air is available. The ducts should be short, direct, and designed to minimize friction. They are often built into the building structure. (3) Air may be recirculated through the generator and a water-cooled heat exchanger by means of the fan action of the rotor and a compact, closed duct system. Cooling water pumps are generally needed. This method has the advantage of constantly clean air, small duct space, minimum fire hazard, and minimum noise. The advantage of clean air is indicated in one case of open ventilation where a test showed 11 per cent cooler generator operation after the air passages had been cleaned. Closed ventilation has the disadvantage of requiring water pumps and of not tempering the generator-room air.

Fire Protection by means of perforated water pipes arranged to spray the windings is applicable to either system of ventilation. When carbon dioxide or other similar systems are used with open ventilation, self-closing, electrically tripped doors may be used to confine the gas in the machine in case of fire, but for these latter systems of fire protection the closed ventilating system is to be preferred.

Brakes are required on the waterwheel driven generator to bring the unit to rest. Without brakes a comparatively slight leak at the turbine gate would cause the machine to rotate indefinitely after being shut down. For vertical units the brakes act against

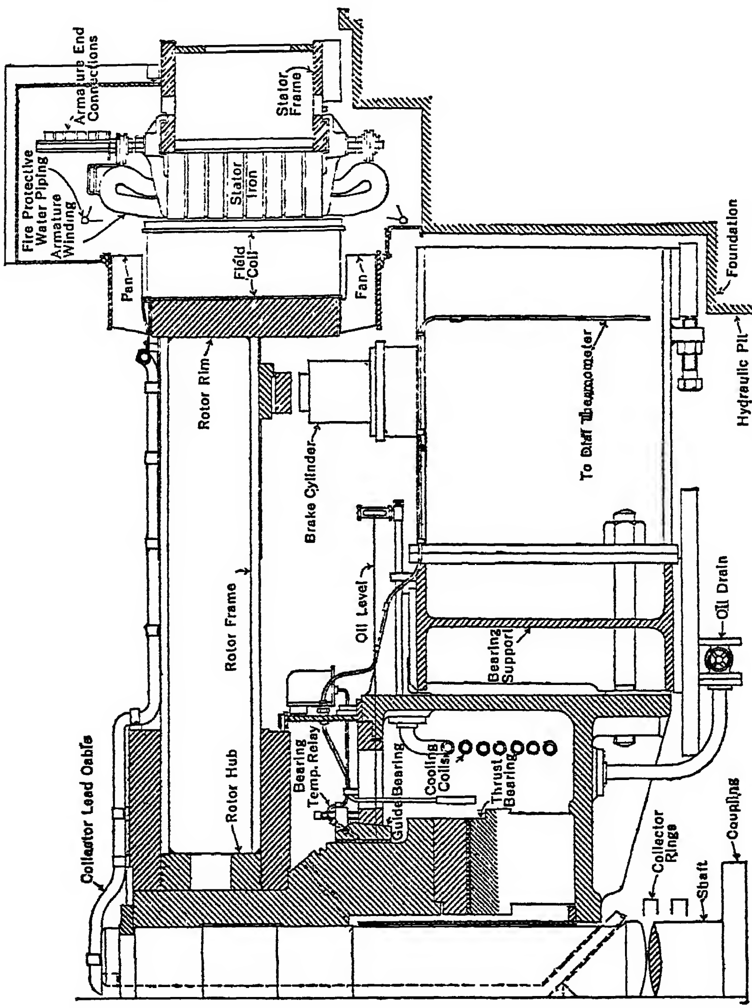


FIG. 5. Cross-section of Typical Overhung Generator

a plane surface on the lower face of the rim of the rotor. They are sometimes designed to be used also as hydraulic jacks to raise the rotating element off the thrust bearing to a position where it can be blocked for inspection. For horizontal machines the brake consists of two or more shoes in contact with a brake pulley. Brakes may be operated by magnets, oil, water, air, or mechanical linkages. The most common medium is air for braking and oil in the same cylinders for jacking.

14. AUXILIARY POWER SUPPLY

The auxiliaries of the hydroelectric station are as a rule driven by electric motors. In some cases the oil pumps and exciters are driven by the main units for maximum reliability. When oil pumps and exciters are motor driven the importance of the service justifies duplication in the source of supply and in parts of the distribution system. Normal and emergency auxiliary power is usually obtained from one or more of the following sources:

Transformer on station bus.
Transformer on generator leads.
Line from another station.
Auxiliary generator driven by main waterwheel.
Auxiliary generator driven by auxiliary waterwheel.
Auxiliary generator driven by fuel.

The transformer on the station bus has low maintenance and operating cost although it requires one or more heavy-duty oil circuit breakers, which in some cases may constitute an important item of initial cost. It is reliable during normal operation, but is not independent of main system disturbances. In many stations both the normal and the emergency auxiliary power are supplied by transformers on the bus.

The transformer on the generator leads is simple and efficient, and usually the oil circuit breakers can be omitted, thus reducing the cost. On the other hand, a larger number of small transformers is usually required than with transformers on the bus, and if this is the case the cost and efficiency may be less favorable. The scheme is reliable during normal operation but is subject to main system disturbances. Each generator may drive its own auxiliaries, independent of the others, but if this is done some common source must be provided for the several auxiliaries which are common to all units and for driving oil pumps and exciters while starting up. The common bus may be fed from one live generator at a time by means of automatically selecting switches, or from the bus through a transformer.

The line from another station is not likely to be reliable as a normal supply, but may be used to advantage as an emergency source. If it can be depended upon when needed it may be of great value when the station is shut down and the transmission lines are de-energized.

The auxiliary generator driven by the main waterwheel, or shaft generator, as it is sometimes called, is fairly simple and efficient but is quite expensive in case of a slow-speed main shaft. It is also more expensive to maintain and operate than transformers. It is reliable and independent of electrical disturbances on the main system but is subject to speed variations. In cases where exciters are driven by this source, there is a tendency for a system disturbance to become intensified owing to speed variation of the exciters.

The auxiliary generator driven by an auxiliary water wheel, or house generator, as it is sometimes called, is the most expensive in first cost, maintenance, and operation of all schemes mentioned. Its efficiency is less than transformer or shaft generator efficiency. It may in some cases be subject to disturbance from ice and trash. It has the decided advantage of being entirely independent of the main system and always available whether the main units and transmission lines are energized or not.

The auxiliary generator driven by fuel is sometimes favorable as an emergency source for use when the station is shut down and disconnected from its transmission lines. In some cases, only the supply to certain important motors such as those on spillway gates is supplemented by a small fuel-driven emergency auxiliary generator.

A source that can be depended upon when the station is shut down is essential if advantage is to be taken of the hydroelectric station's inherent quick-starting ability to support steam stations which require a much longer starting period. Head gates must be operated and in most cases governor and lubricating oil pumps must be started before the main units can be started. In some instances it is imperative that the station be started when the transmission lines are not energized. Power for this purpose may be taken from an auxiliary line from an outside source, or from a waterwheel-driven auxiliary generator.

Voltages commonly used for auxiliary power systems are 2200, 550, 440, and 220 volts. The most economical voltages to use are indicated by an analysis of the wiring and the control equipment, taking into account the locations of the transformers, buses, and motors. In many of the larger stations it is found economical to use 2200 volts for the large motors and one of the lower voltages for the smaller motors.

Among the many possible auxiliaries which may require electric power supply are the following: Head gates, sluice gates, trash rack rakes, navigation locks, forebay ice prevention, trash rack heating, gate guide heating, filler gates, fish protection. Lubricating oil pumps, governor oil pumps, lubricating and insulating oil handling and filtering pumps, house service water pumps, generator cooling water pumps and fans, air compressors, transformer oil and water pumps and blowers. Elevators, incline railway, car pullers, turbine room crane, gantry cranes, transformer hoists, building ventilating fans, water coolers, water heaters, space heaters, drying ovens, machine shop. Exciters, battery charg-

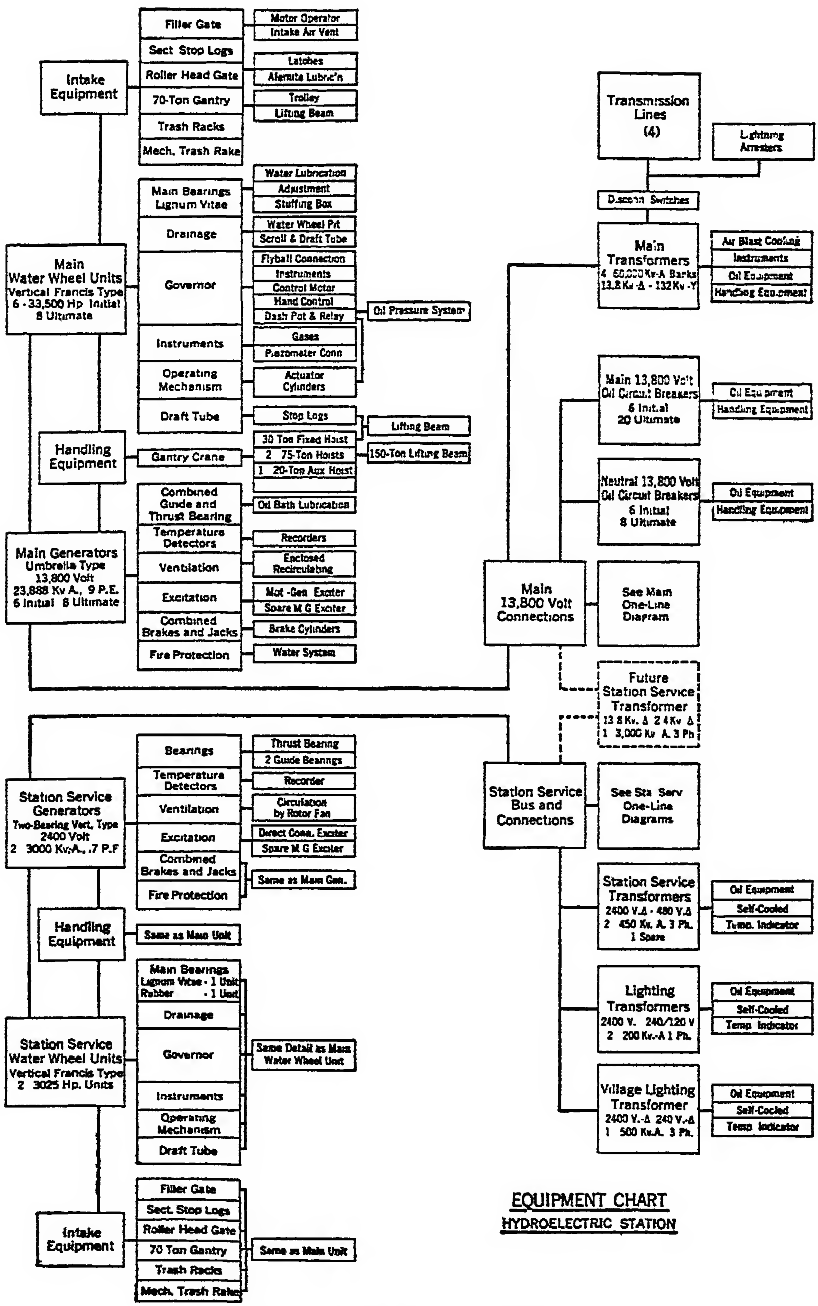


Fig. 6. Typical Equipment Chart

ing, telephones, signals, clocks, water level recorders and indicators, power receptacles, transmission line ice melting, local lines, village light and power. Lighting for power house, switch yard, grounds, storehouse, locks, employees' cottages, display signs.

It is sometimes found advantageous to diagram the various elements of main and auxiliary equipment in order to express their relation to each other in a manner similar to the better-known electrical single-line diagrams. A typical equipment chart of this type is shown in Fig. 6. This chart shows the general relation of the main equipment and principal auxiliaries. In practice, a chart of this kind is supplemented by other charts in greater detail and by flow diagrams for the various piping systems. It is also cross-referenced to the electrical single-line diagrams. These charts are useful in studying the equipment requirements and arrangement and for checking over the completed station arrangement prior to and during preliminary operation.

Excitation

POWER. The amount of excitation required by a 60-cycle, three-phase, synchronous, salient-pole, waterwheel-driven generator of normal design, operating at rated load and voltage and 0.8 power factor, may be roughly estimated by the following empirical formula:

$$\text{kw} = 10\sqrt{\frac{K}{S}}$$

where kw = excitation in kilowatts.
K = generator capacity in kilovolt-amperes.
S = generator speed in revolutions per minute.

Voltages used for the excitation system are 125 volts for smaller systems and 250 volts for larger systems.

Exciter drive should be from sources separate from the main power so that main power disturbances will not be reflected into the excitation system. At least one exciter should be driven by a prime mover to provide for starting up the station, unless a reliable source of motor drive can be assured when all main units are shut down.

15. ICE PREVENTION

The operation of hydroelectric plants located in cold climates requires definite provision for combating ice during the winter seasons. Sometimes ice is encountered in such large quantities that it must be cut and sluiced away, involving considerable expense and waste of water. Except in extreme cases, however, a plant can be kept operative by a small expenditure of energy properly applied at vital points if adequate provision is made in the original design.

Gates should be protected from excessive pressure of sheet ice, and their sills, guides, and seals should be kept free of ice sufficiently to allow operation. This has been accomplished by applying small amounts of steam or electric heat inside the gates and at imbedded parts of the guides which were made hollow for this purpose. To secure economical heating it is very important that the downstream side of the gates, exposed to the cold air, be thoroughly housed with heat-insulating material and that the heat inside the gates be evenly distributed by fans, chimneys, and other means. The data in Table I are from N.E.L.A. reports of successful installations.

Table I

Installation	1	2	3	4
Size of gate	50 × 19.5	50 × 33	40 × 24	30 × 36
Kw in gate proper	35	36	45	36
Kw at guides and seals	10	16	16	12
Total kw	45	52	61	52
Watts per sq ft	46	31.5	63	47.5

Flashboards should be protected against surface ice. The most effective method appears to be to maintain an open channel clear of ice along the upstream face about 3 or 4 ft wide. This has been accomplished by placing electric heaters or low-efficiency lamps in reflectors just above the water surface, covering them with canvas or heavy paper. Installations of this type have been reported using from 6 to 17 kw per 100 ft of flashboards.

Air Bubbler Systems have been used to maintain an open channel clear of ice along the upstream faces of gates and flashboards. A bubbler system consists of perforated pipes or groups of small nozzles installed along the lower, upstream face of the gates or flashboards, and supplied with air under pressure, which rises and creates a circulation of

warmer water from the bottom of the pond to the surface. The data in Table II are taken from N.E.L.A. reports of installations of this type.

Table II

Installation	1	2	3	4	5	6	7
Diameter of orifices, in	0.0314	0.0314	0.062	0.0156	0.08	0.062	0.0156
Spacing of orifices, ft	5	5	10	3	4	3	3
Depth of orifices, ft	5	15	18	3	12	8.5	12
Air pressure, lb per sq in	80	80	10	60	6.5	22	20
Cu ft per min of free air per 100 ft of channel	34	34	6	7	17.5	25	7
Kilowatts per 100 ft of channel including compressor and motor losses	3.4	3.4	0.25	0.55	0.7	1.44	0.5

Surge Tanks of small size are likely to freeze, especially during periods of steady load. Satisfactory performance in cold weather has been obtained by insulation alone or by insulation with steam or electric heat. When heat is applied in the air space between tank and insulating sheathing, part of it is obviously lost through the sheathing, and when applied directly to the water, part is obviously lost by exchange of water between tank and penstock. The most economical application, therefore, depends upon local conditions. It is thought by some that conditions generally favor applying the heat directly to the water. In a few cases, freezing has been prevented by anchoring a wood pole in the tank in such position that its motion breaks up any surface ice which might start to form.

Penstocks, if exposed to the weather and designed for low velocity, may partially freeze, causing loss of head, and in thawing again, the ice may pass on in sufficient quantities to clog the turbine gates. The usual precaution against this trouble is to insulate or bury the penstock. It is an advantage, in this connection, to take the water from the lower part of the reservoir so as to secure warmer water.

Trash Racks must be protected from sheet ice, the pressure of which might injure the structure, and from frazil ice, which tends to adhere to the bars and clog the passages. Protection from sheet ice is afforded in some designs by a curtain wall in front of the rack extending down well below the surface of the water. Frazil ice is usually the greatest of all ice hazards. Although it generally appears for only a short time in early winter and occasionally in the spring, it is likely to clog the racks at these times sufficiently to curtail the station output seriously. At one plant not more than 25 per cent of the available water could be used during winter months before rack heating was installed. Only where the pond or forebay is of large size and the intake is deep, can complete immunity from frazil ice be assured. Trouble from frazil ice is prevented in some cases where the waterwheels have large passages, such as with the propeller-type wheel, by raising the trash racks during the frazil ice flow, allowing the ice to pass through the plant. Air bubbler systems have been installed in front of the racks, designed to carry the frazil ice up to the surface where it can be sluiced off. In one plant a mechanical rake of special design is used successfully to clear racks of frazil ice. The method most extensively used consists of heating the trash rack bars. It is found that the bars need be raised only a fraction of a degree above the freezing temperature to prevent the frazil sticking to them. If the top of the rack extends above the surface of the water, and is enclosed, heat can be applied at this point by means of warm air fans or steam coils, and some heat will be carried down in the rack by conduction through the bars. This method has proved successful in moderately cold locations. In a number of installations, some of which are located in places where the climate is severe, the racks are heated by passing electric current through them. The energy required to keep the racks clear, in watts per square inch of bar surface exposed to the water, has been reported as 0.87, 0.88, and 0.89, respectively, from three installations which have successfully passed through one or more ice seasons. The power factor has been found to vary from 0.72 to 0.83. The impedance of the rack

Table III

Installation	1		2	
	Volts	Ohms	Volts	Ohms
A-c 60-cycle tests	15	0.078	44	0.264
	29	0.068	121	0.197
	34	0.059	160	0.163
D-c test	....	0.018	.....	0.042

is subject to wide variation depending upon the quality of steel, size, and shape of bars, spacing of bars, and arrangement of bars in series-parallel circuits. It is also probably dependent to a considerable extent upon the voltage, the frequency, and the water resistance. Tests upon trash racks installed in 1930 and 1931, respectively, provide the data in Table III.

16. COSTS

The cost of hydroelectric developments probably varies more widely than that of any other class of projects. Such items as land requirements for reservoir and plant, water rights, reservoir clearing, relocation of roads and railroads, the length and height and type of dam, the necessity for and layout of tunnels, pipe lines, forebays, tailraces vary greatly for each development. The cost per kilowatt of the initial installation will usually be higher than the cost per kilowatt of the ultimate installation, where the plant capacity is increased over a period of years, since such items as the reservoir, dam, and headworks must be completed in connection with the initial development.

Accurate cost figures of hydroelectric developments are difficult to obtain for publication since the complete cost of a development includes not only the costs of the physical structures and machinery, but also the costs of preliminary investigations, surveys, foundation exploration, land acquirement, federal and state licenses, financing, and other, less tangible items. For the above reasons no attempt will be made to give overall costs of developments beyond the broad statement that the cost per kilowatt will range from about $90 under exceptionally favorable conditions or $125 for more general conditions for a large plant to $250 for a small plant constructed under disadvantageous conditions. Unit costs for individual items of construction will likewise vary over a wide range depending on location, distance from material supply sources, prevailing wage scales, and so forth.

The procedure suggested for obtaining a preliminary cost estimate for the development of a proposed site is to prepare a preliminary design of the project and base the estimate on this preliminary design and the best information available on local unit costs of structures and equipment.

A suggested outline for the preparation of a complete project estimate is as follows:

1. **Preliminary and General Expense.** Includes cost of preliminary investigation and exploration, organization and legal expenses, franchises, federal and state licenses, financing, and interest and taxes during construction.

2. **Engineering.** Includes studies of economical development of the site, preparation of general and detailed plans, specifications for structures and equipment, purchase, inspection and expediting of equipment and material, and general consulting services.

3. **Hydraulic Work.** Includes land and water rights, surveys and borings, reservoir basin clearing, relocation of railroads and highways, removal of buildings, river diversion, cofferdams, dams, dikes, headworks, forebay structures, waterwheel settings, intake gates, crest gates, stop logs, hoists and handling equipment, canals, flumes, tunnels, pipe lines, penstocks, surge tanks, turbine equipment, tailrace.

4. **Electrical Work.** Includes generating and excitation equipment, transformers, switch gear, auxiliary power supply, station wiring, electrical control, signal and telephone systems.

5. **General.** Includes power station building, station yard, private roads, railroad serving development, auxiliary equipment, shop equipment, auxiliary buildings, permanent operators' quarters, preliminary operation and tests.

6. **Substations.** Includes land, substation buildings, equipment foundations, switching structures, electrical equipment switch gear, wiring, auxiliary equipment, preliminary operation and tests.

7. **Overhead Transmission Lines.** Includes land, right-of-way, clearing, fencing, foundations, steel and wood towers, conductors, insulators, communication and maintenance facilities, preliminary operation and tests.

17. BIBLIOGRAPHY

See Art. 29 of this section and bibliographies of Sections 4 and 8–12. Also Kent's Handbook for Mechanical Engineers, Merriman's Civil Engineers' Handbook.

INTERNAL-COMBUSTION POWER STATIONS

By W. A. Sloan

18. INTERNAL-COMBUSTION ENGINES

Applications

The Diesel engine is an excellent prime mover for electrical generation in plants up to about 10,000 kw. It has the advantages of low fuel cost, short periods for warming up, and no standby losses, and it requires little water. Small sizes are about as efficient to operate as large ones, whereas with steam engines or turbines the steam rates are often twice as great in small units as in large. However, unlike the steam turbine, the plant floor area and cost increase nearly proportionally with capacity, which fixes an economic limit of from 5000 to 10,000 kw for Diesel generating stations at present.

Except where a large supply of by-product gas is available, as in steel mills, the Diesel engine is the present choice of engine in the small power field. It is used by public utilities, municipalities, hotels, and factories.

Types of Engines

PRINCIPLE. The internal-combustion engine is a prime mover whose action depends upon the heating of air by burning fuel within the cylinder. This fundamental principle is the same regardless of the fuel, which may be a combustible gas, vapor, or oil. Pulverized coal has been used successfully but is in the experimental state. The combustion causes a rapid rise in temperature and pressure in the products of combustion, which expand behind a piston moving it forward. (Rotary gas engines or gas turbines are still [1936] in the experimental state.) In its simplest form the internal-combustion engine is similar to a reciprocating single-acting steam engine, the reciprocation of the piston in the cylinder being changed to rotation of the shaft by means of a crank and connecting rod.

CLASSIFICATION. There are two distinct methods of applying the fundamental principle of operation of internal-combustion engines, so that, with reference to the thermodynamic cycle, engines may be classified as either explosion or Diesel.

In the **explosion type**, air which has been mixed with a suitable quantity of fuel in gaseous or vaporized form is moderately compressed in the cylinder and is then ignited, giving a rise in pressure which is almost instantaneous. The ignition is accomplished by an electric spark. In this class belong all gas and gasoline engines and some oil engines. The compression must be so regulated that the temperature of ignition is not reached before the spark jumps, and the mixture of fuel and air must be of such proportions that it will burn rapidly. (See Section 17, Art. 17.)

The **Diesel engine** compresses air only, to a predetermined temperature above the ignition point of the fuel. The fuel is then forced into the cylinder in a finely atomized state and ignites spontaneously without explosion, by reason of the heat resulting from the compression of the air. In the pure Diesel the fuel is injected into the cylinder by means of a blast of high-pressure air, and the timing of injection is such that there is no rise in pressure during combustion. The development of engines using mechanical means of injecting fuel into the cylinder (commonly called solid-injection Diesel) gives rise to a cycle somewhat different from the constant-pressure combustion of the pure Diesel. The recognized definition of a Diesel now is: an engine in which the fuel, injected after compression is practically completed, is ignited solely by the heat resulting from the compression of the air supplied for combustion.

There is one type of solid-injection engine, commonly called **semi-Diesel**, which has lower compression than the Diesel, so that the temperature is below that required for ignition, and in which sufficient additional heat is supplied by a hot plate or bulb forming part of the combustion chamber. The plate or bulb must be heated for starting, after which the heat of combustion will keep it at the required temperature, unless the load is too light. These engines are less efficient than those having compression ignition and are rough running in multicylinders because of the uncertainty of ignition.

Any of the thermodynamic cycles for internal-combustion engines may be mechanically performed in an engine giving an impulse in one end of each cylinder in either one or two revolutions of the crankshaft.

An engine requiring two revolutions of the crankshaft to complete its working cycle is called a four-cycle engine, having reference to the four piston strokes in two revolutions. The following sequence of operations takes place during four consecutive strokes: (*a*) inspiration of a mixture of gas and air during an entire stroke, or air only in the case of a

Diesel; (*b*) compression during the second (return) stroke; (*c*) ignition or injection causing combustion at or near the dead center and expansion during the third stroke; (*d*) expulsion or exhaust of the products of combustion during the fourth (return) stroke.

In the two-cycle engine the working cycle is completed in one revolution of the crankshaft or two strokes of the piston. Starting with the combustion, at or near dead center, the piston moves toward the crankshaft with the gases expanding behind it. Toward the end of this first stroke the piston uncovers an exhaust port and the burned gases escape. Shortly after, an inlet port opens, admitting a mixture of fuel and air (air only in the case of a Diesel) from a reservoir in which it has been slightly compressed. The inlet and exhaust ports close early in the return stroke, and during the remainder of this second stroke compression occurs. In some designs admission is accomplished through mechanically operated valves in the cylinder head. Large engines have separate compressors to furnish the small pressure required for forcing in the air or mixture, but many small engines have a closed crankcase which is used as the reservoir, and the piston displacement itself furnishes the pressure.

When the design is such that combustion occurs on only one side of the piston it is a single-acting engine; if on both sides, a double-acting. The majority of internal-combustion engines are single-acting, the double-acting principle being restricted mostly to very large sizes. The decision between single- and double-acting engines depends on the cost of production. When plain uncooled pistons can be used, the single-acting is cheaper; but when it becomes necessary to water-cool the pistons, it is generally desirable to use the double-acting principle.

Power and Rating

POWER. In contrast to steam prime movers the internal-combustion engine has a definite limit of power. The power depends upon the weight of air that can be passed through the engine in a given time and upon the efficiency with which it can be burned. This is evident, since, however much air is present in the cylinder, sufficient fuel can always be admitted to combine with it, but further addition of fuel will be useless unless there is sufficient oxygen for its combustion.

The power developed in one end of a single cylinder of an internal-combustion engine may be calculated by the following formula:

$$\text{ihp} = \frac{P\,L\,A\,N}{33{,}000}$$

where ihp = indicated horsepower.
P = indicated mean effective pressure in pounds per square inch.
L = length of stroke in feet.
A = area of cylinder bore in square inches.
N = number of impulses per minute;
= revolutions per minute for two-cycle engines;
= revolutions per minute/2 for four-cycle engines.

The mean effective pressure can be obtained from an indicator diagram and varies with the load. At full load it depends on the kind of fuel and the amount of compression.

A portion of the power developed in the cylinder is absorbed in engine friction so that the available power at the shaft is less than that developed in the cylinder and is called the brake horsepower, being that measured by a dynamometer or prony brake. Mechanical efficiency is the ratio of brake to indicated horsepower and varies from 70 to 90 per cent at full load, depending on the size and type of the engine.

RATING. Internal-combustion engines are rated on their brake horsepower usually with an allowance for overload. This overload allowance may vary from 10 to 20 per cent of rating, and the characteristic of limitation of power in such an engine makes it desirable to know either the maximum power or the overload capacity when selecting an engine for a definite service. A good criterion for the selection of an engine is the brake mean effective pressure (bmep) on which it was rated. The brake mean effective pressure is really the product of indicated mean effective pressure and mechanical efficiency and may run as high as 100 lb per sq in. or above in ordinary engines. For continuous service it is well to select four-cycle engines rated on about 70 to 80 lb per sq in. brake mean effective pressure, and two-cycle engines, with crankcase compression, rated on 40 to 50 lb sq in. The brake mean effective pressure may be calculated, when the engine dimensions are known, by putting the brake horsepower in the formula for indicated horsepower thus:

$$\text{bhp} = \frac{P_1\,L\,A\,N}{33{,}000}$$

where bhp = brake horsepower.

P_1 = brake mean effective pressure in pounds per square inch.

Other terms same as in formula for indicated horsepower.

While internal-combustion engines are mechanically somewhat limited as to size, engines have been built up to 7000 brake horsepower, the majority being of less than 3000 brake horsepower. A few years ago piston speeds of 700 to 800 ft per min were considered high for stationary engines, but recent designs are utilizing speeds as high as 1500 ft per min. This naturally helps to reduce the weight per brake horsepower.

The power of an internal-combustion engine being limited by the weight of air taken into the cylinder in a given time, altitude will have an effect on the power. Manufacturers rate their engines for sea-level operation, and allowance should be made for decrease in capacity at altitudes above 2000 ft. Altitudes of 10,000 ft will reduce the brake horsepower about 30 per cent, and other altitudes have proportional effect.

EFFICIENCY AND ECONOMY. The internal-combustion engine uses a cycle theoretically more efficient than the steam engine or turbine and consequently should give a more economical engine. Unlike the steam engine, the internal-combustion engine does not become appreciably more economical as the size is increased. Although in small powers it is far more economical than the steam engine, in very large powers it does not retain this advantage to anything like the same extent, particularly in view of recent developments of the regenerative and reheat steam cycles. Full-load test figures show thermal efficiencies above 20 per cent, referred to brake horsepower, for gas and gasoline engines, and above 30 per cent for Diesel engines.

In an explosion engine the efficiency is entirely dependent upon the amount of compression, but with the Diesel the length of time required for combustion has a decided influence. Although theoretically the explosion-type engine gives a higher efficiency for equal compressions, this advantage cannot be obtained in practice, since an explosion engine compresses a mixture which would pre-ignite if the compression were too high, while the Diesel compresses only air.

The fuel consumption of internal-combustion engines may be expected to vary somewhat with the kind of engine and the fuel, but Table I gives an idea of what might be expected on the average, in terms of Btu in the fuel.

Table I. Average Consumption in Btu per Bhp-hr

Per cent of rated load	100	75	50	25
Gas and gasoline engines	10,500	12,000	14,700	20,000
Diesel engines	8,550	8,800	9,500	11,500

The losses in an internal-combustion engine are the heat carried away in cooling water, that carried away in the exhaust gases, friction, and radiation. For explosion engines the amounts vary considerably with compression and somewhat with speed. For a Diesel the full-load heat balance will not vary much from that given in Table II.

Table II. Average Full-load Heat Balance Diesel

Brake horsepower	30%
Cooling water loss	32
Exhaust gas loss	28
Friction, compressor, radiation	10
In fuel	100

LUBRICATION. Owing to the high temperatures that prevail in the cylinder of the internal-combustion engine, the question of proper lubrication is a serious one. Cylinder oil should be of a high grade, free from acids, and composed of hydrocarbons that leave no residue after combustion. Only mineral oils, therefore, are suitable for the purpose.

The amount of oil required per horsepower-hour varies with the character of the installation and the method of operation. With a properly designed and operated lubricating system the average consumption of lubricating oil would be about 0.0005 gallon per hour per rated brake horsepower for four-cycle engines and possibly twice as much for two-cycle.

COOLING. Internal-combustion engines must be cooled by circulating water through jackets surrounding the heated parts. The quantity of water which must be circulated depends upon the initial and final temperatures.

Inlet water temperatures are usually from 80 to 90 deg fahr. In general the outlet

temperature should not be above 160 deg fahr or there may be danger of breaking down the film of lubricating oil on the cylinder walls or warping of valves. In order to avoid excessive heat stresses in large engines the temperature is usually not allowed to go above 120 deg fahr, but for engines of medium size it may run as high as 130 to 140 deg fahr. If the water contains foreign matter the temperature must be kept low to avoid precipitation and formation of scale, or a water-softening plant must be used.

Excessive circulation resulting in low final temperatures increases the fuel consumption. It is particularly undesirable with gasoline engines as it may cause precipitation of part of the fuel. Very low cooling water temperatures increase the viscosity of the lubricating oil, resulting in an increase of piston friction.

Sometimes a desirable grade of water is available in sufficient amount to allow it to be wasted after circulation through the jackets, but where water is scarce or expensive, cooling towers or spray ponds are generally employed for cooling before recirculation.

The quantity of water to be circulated depends on the thermal efficiency of the engine, the proportion of heat loss to the jackets, and the rise in temperature through the engine. The following table gives the quantity required in gallons per hour per brake horsepower for a wide range of conditions.

Gallons of Cooling Water per Hour per Brake Horsepower

Thermal Efficiency, %	Jacket Loss, %	Temperature Rise, deg fahr				
		30	35	40	45	50
15	30	20.4	17.4	15.3	13.6	12.2
	35	23.7	20.3	17.8	15.8	14.2
	40	27.1	23.2	20.3	18.0	16.2
20	30	15.3	13.1	11.5	10.2	9.1
	35	17.8	15.3	13.4	11.9	10.7
	40	20.4	17.4	15.3	13.6	12.2
25	30	12.2	10.5	9.1	8.2	7.3
	35	14.2	12.2	10.7	9.5	8.5
	40	16.3	13.9	12.2	10.9	9.7
30	30	10.2	8.8	7.6	6.8	6.1
	35	11.9	10.2	8.9	7.9	7.1
	40	13.5	11.7	10.2	9.1	8.2

19. POWER PLANTS

SELECTION OF UNITS. Division of the capacity into two or more units allows the operator to meet the load curve with units loaded to somewhere near the point of best efficiency. In addition it makes the plant more reliable, as at least partial service can be maintained in case of accident to one of the units.

High rating for a given engine lowers the cost per horsepower but shortens the life. It is better, therefore, to select engines of rather conservative rating in speed and brake mean effective pressure. The design should be as simple as possible. Details of design, workmanship, and materials as proved by the engine's record and the manufacturer's reputation are more important than the type of engine. The specifications should be drawn up by experts and contain only such points as are essential to the purchaser, such as weight and space limitations, giving the manufacturer the choice of type and details of design. It should be remembered that, though bids may be tabulated, quality cannot be tabulated, and the cheapest engine may not give the most economical service.

PLANT ARRANGEMENT. It is common practice to set the units on parallel lines. The average plant, having from two to four units, when so arranged, is a nearly square building. When steam engines are replaced by Diesels and the old building used, the arrangement will naturally depend on the shape of the old plant and engines may have to be placed with their center lines in line. Ample clearance must be allowed for the dismantling of engine, generator, and exciter.

FOUNDATIONS are a very important point in the installation of Diesel engines, as the mass must be sufficient to absorb the vibration; engines located in basements of hotels, department stores, and similar places, should be insulated from the rest of the building. In certain localities silencers are required in place of ordinary mufflers for the exhaust. The air for combustion should be free of dust and dirt.

The oil storage should be located outside the plant, either above or below ground

depending on the local conditions. The capacity of the oil storage is determined by the maximum rate of fuel consumption of the plant and the longest expected time between deliveries. The oil may be handled directly from the storage tank to the engine, or an overhead day tank may be placed inside the building.

20. ECONOMICS

INVESTMENT COSTS AND FIXED CHARGES. The average unit cost of representative Diesel electric generating plants will be found in Figs. 1 and 2, in installed capacity up to 10,000 kw. The costs are divided into four different accounts, land being omitted, since it is recognized that location would have more effect on this item than the others. The building costs are for substantial but not elaborate structures. The figures given are for each item erected, including its foundations, but no allowance has been made for overhead. Overhead costs would cover administrative expense, financing, engineering, etc., and usually runs about 10 to 15 per cent of the costs as shown.

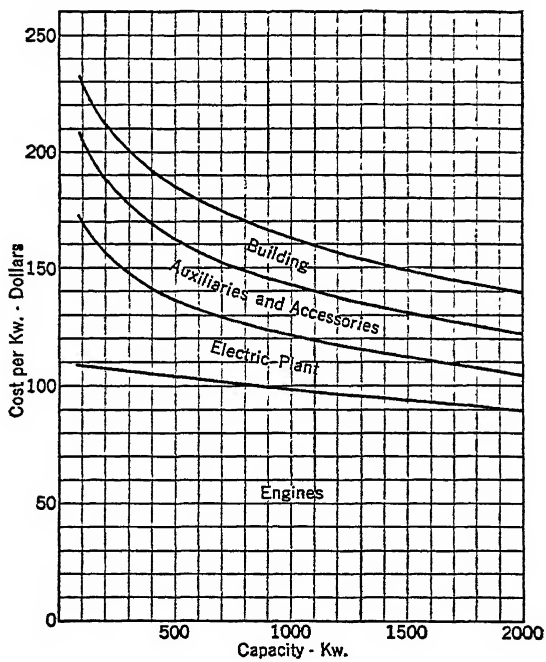


FIG. 1

The first costs of operating a generating plant which must be considered are the fixed charges on the investment. These charges include interest, depreciation, insurance, and taxes. Interest may be considered as constant for all the individual parts which make up the capital investment, but there is a difference in depreciation, insurance, and taxes between the individual parts of the plant. However, for the purpose of a rough estimate, the following average figures may be used. These figures are the yearly cost in per cent of the capital investment.

Fixed Charges

Interest	6%
Depreciation	6%
Insurance and taxes	1.5%

OPERATING COSTS. The operating costs of Diesel electric generating plants will include the cost of fuel and lubricating oil, attendance, cost of engine and other plant

repairs, supplies, and miscellaneous, including water. These items will vary with the character of load and method of operation, and the factor which they appear to follow is the plant running capacity factor, which is defined as follows:

$$\text{Plant running capacity factor, per cent} = \frac{\text{Plant output in gross kwhr} \times 100}{\text{Total rated kwhr of individual units}}$$

The expression "rated kwhr" refers to the kilowatt rating of an engine-generator set multiplied by the number of hours operated. For example, if a unit having a rating of 200 kw was operated 4000 hr, the rated kwhr equals 800,000, no matter what the actual output may have been. Total rated kilowatthours of individual units may be

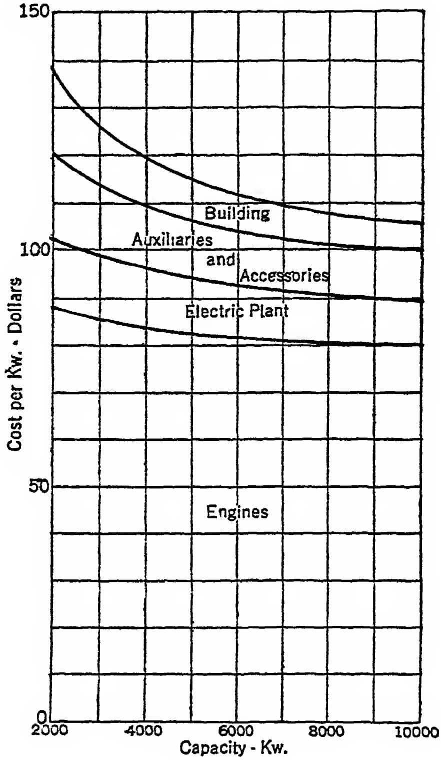


Fig. 2

estimated by fitting the units to the expected load curve and calculating the hours of operation required for each unit.

Estimating the cost of fuel and lubricating oil involves a knowledge of the quantities to be used and the price per gallon. Fig. 3 gives the fuel and lubricating oil economics of Diesel electric generating plants in gross kilowatthours per gallon of each oil, with respect to the plant running capacity factor. These curves were taken from the most recent report of the A.S.M.E. subcommittee on Oil-Engine Power Cost and represent actual performance of 120 Diesel plants for the year 1933. This same report shows that the cost of fuel oil used in these plants varied from 1.6 to 7.3 cents per gallon, the majority of cases being from 4 to 5 cents. The cost of lubricating oil varied from 21.5 to 81.8 cents per gallon, the majority of cases being from 50 to 60 cents.

The same report was analyzed for the other operating costs, and these are shown in Figs. 4 and 5, in cents per gross kilowatthour generated against plant running capacity factor. Fig. 4 gives the attendance including superintendence. Fig. 5 gives the combined cost of all repairs, supplies, and miscellaneous, including water.

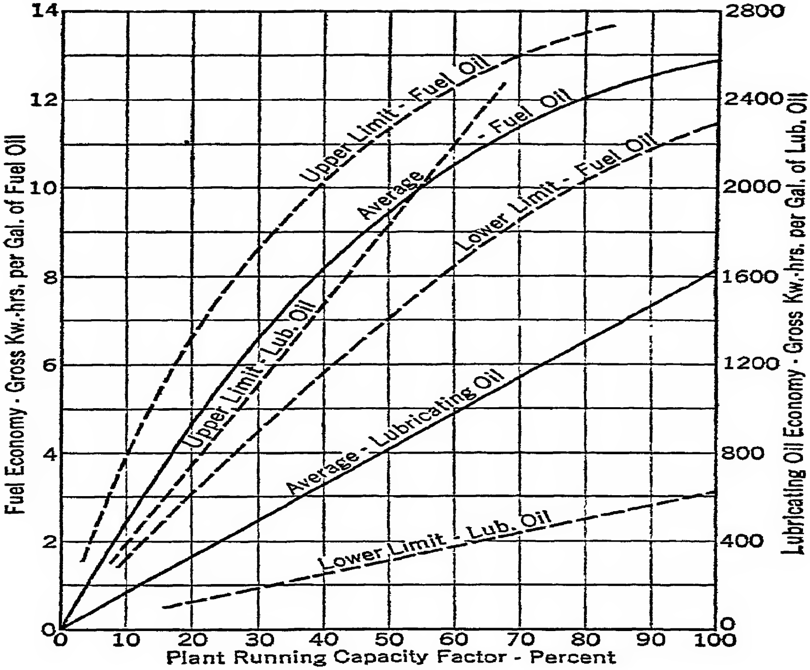

Fuel Economy - Gross Kw.-hrs. per Gal. of Fuel Oil
Lubricating Oil Economy - Gross Kw.-hrs. per Gal. of Lub. Oil
Upper Limit - Fuel Oil
Average - Fuel Oil
Lower Limit - Fuel Oil
Upper Limit - Lub. Oil
Average - Lubricating Oil
Lower Limit - Lub. Oil
Plant Running Capacity Factor - Percent

FIG. 3

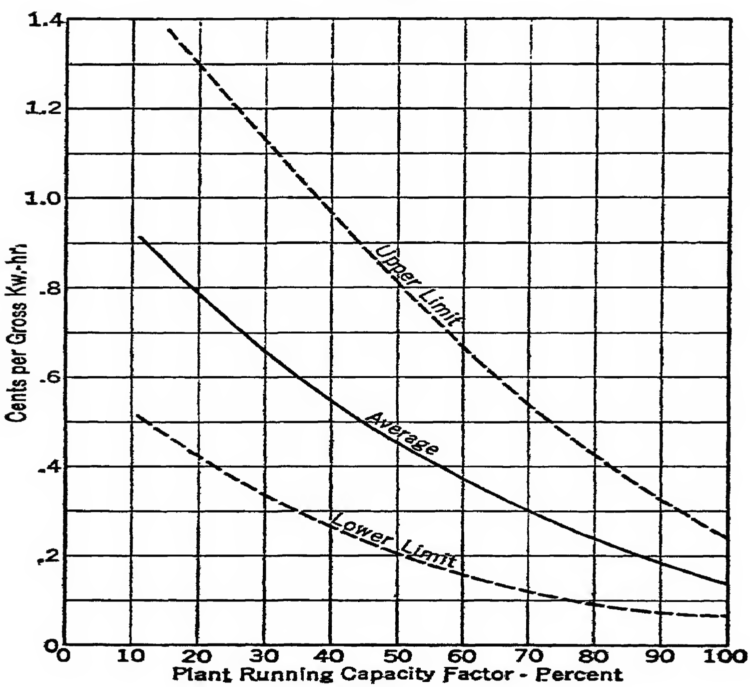

Cents per Gross Kw.-hrs.
Upper Limit
Average
Lower Limit
Plant Running Capacity Factor - Percent

FIG. 4

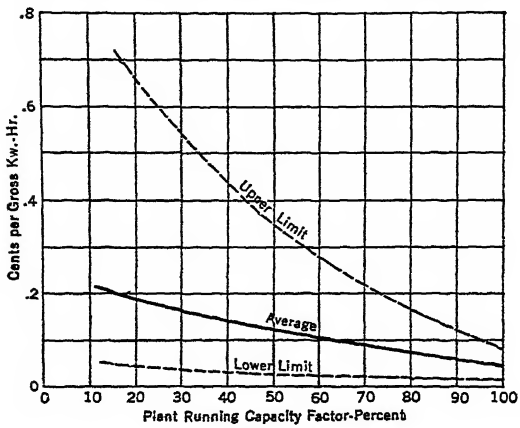


FIG. 5

The data in Figs. 3, 4 and 5 are all on the basis of gross kilowatthours generated. The net kilowatthour output is found by subtracting the power used for plant auxiliaries and station lights from the total gross output of the plant. From 1 to 10 per cent of the gross output may be used in the plant, from 2 to 4 per cent representing a good average.

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See also Art. 29 of this section for general bibliography on Power Stations.

SUBSTATIONS

By W. L. Slichter

22. CLASSIFICATION OF SUBSTATIONS

A substation is an aggregation of electrical apparatus for the purpose of control, regulation, subdivision, and transformation or conversion of electrical energy. It is the connecting link between two or more sections of a transmission or distribution system.

Most substations perform two or more of these functions.

FUNCTIONAL CLASSIFICATION. Substations may be classified according to their functions as follows:

1. Interconnecting two or more transmission lines for tying together two or more sources of power. This may be a simple "switching station" or it may involve the transformation of voltage if the lines operate at different voltages. The flow of energy may be either way.

2. Transforming voltage by means of transformers from some higher voltage to a lower one, usually interconnecting the high-voltage transmission line with a primary or secondary distribution system. If it feeds directly into the low-voltage (220-volt) distribution system it is called a "distribution substation" and the energy flows only one way.

3. Regulating the voltage or power factor by means of synchronous condensers, induction regulators, or tap-changing transformers.

4. Converting from alternating current at one voltage to direct current at a lower voltage for the supply of direct current to railways or customers.

5. An industrial substation receiving power at some high voltage and transforming to some convenient voltage or converting to some other form to suit the particular needs of some large industrial establishment.

CLASSIFICATION BY INDOOR AND OUTDOOR TYPES. Substations may be of the indoor or outdoor type, depending upon the degree of protection from the weather.

The tendency is to put the transformers, power switches, synchronous condensers, and frequency changers outdoors and the main remote control or master switchboard indoors. Converters, rectifiers, and storage batteries are usually placed indoors. For an outdoor station the bus-bars, disconnecting switches, lightning arresters, fuses, and line insulators are supported well above the ground on a truss structure of angle irons like a bridge; the transformers and oil switches, if any, are placed upon concrete slabs on the ground. For smaller stations, up to 1000 kva and for voltages up to 37,000, horn gap switches are used in place of oil switches, but the load is usually opened by the switches on the low-tension side of the transformers.

CLASSIFICATION BY CONTROL. Substations may be of the manually controlled type, in which operators are always in attendance; of the automatic type, in which all operations are performed by automatic features; or of the remote control or supervisory type, in which most operations are initiated by an operator at a distant station.

23. SUBSTATIONS FOR LIGHT AND POWER SYSTEMS

Location of Substations

SWITCHING OR INTERCONNECTING SUBSTATION. The location of a switching or interconnecting station is dictated by the engineering features of the system rather than by economics. Generally it is at one or both ends of a transmission line: at the generating end to interconnect the various generators to several outgoing lines, at the receiving end to tie one or more transmission lines to branches serving various parts of a city or district. Usually a switching station is also a transforming station, and the physical and mechanical construction of the two are similar.

DISTRIBUTING SUBSTATION. The location of a distributing station is governed by economics. The greater the number and the closer together the stations, the greater will be the first cost and maintenance of the stations, but the investment in copper in the distributing mains will be less and the voltage regulation at the customers' premises will be better. The number, location, and capacity of distributing stations are usually judged by the maximum power demand of the customers per square mile, the diversity factor of the load, and the length and cost of the low-voltage feeders to the customers. In the larger cities it is customary to have two classes of distributing substations: (*a*) those in which the voltage is stepped down from some high voltage (66,000) to some moderate voltage (4400 to 6600) and (*b*) those in which this moderate voltage is stepped down to 220 and 125 volts to serve the customers. The latter are usually placed in a vault in some cellar or in the street, have no attendants, and usually contain transformers with tap-changing control and a network protector. In the suburbs and in smaller cities the usual arrangement is to have a substation to transform from the high voltage to 2200 volts, distribute at 2200 volts, and change to 220 and 125 volts by numerous small transformers, usually installed on the poles, each serving from 5 to 10 customers.

TRANSFORMER SUBSTATIONS. A transformer substation may serve to interconnect a high-voltage (220-kv) transmission line with one or more secondary transmission lines (66 or 110 kv) which in turn supply the low-tension distribution circuits, or it may supply these distribution circuits directly. Between the main transmission line and the domestic customer there may be several voltages and several substations, for instance, 220 kv to 66 kv, 66 kv to 4000 volts, and 4000 to 220 volts. At each of these junctions there may be a substation.

Primary Substations. These substations usually transform from 13,000 or 33,000 volts three-phase to 4000 or 2300 volts three-phase, and are fed either on the "radial" system or the "primary network" system. In the former each substation is connected directly to the power station by one or more feeders in parallel, preferably taking different routes, and is called a terminal substation. In the network system the various substations are interconnected with each other as well as with the power station so that power may be routed to any particular substation through one or two other substations. In this case the transmission line is usually sectionalized at this station, each section being brought into the station and connected to the others and to the transformers by oil switches and protective devices. This is called a "tap" substation, and the elementary connections are as in Fig. 1. See also Section 14.

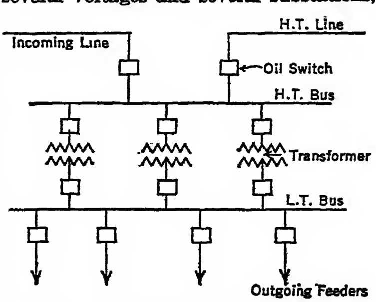

FIG. 1. Transformer Substation Diagram

These substations are located as near as possible to the center of gravity of the load, which runs from 3000 kva per square mile in the residential section of large cities to 30,000 kva per square mile in industrial sections. It is not usual to build any one substation for more than 30,000 kva, but to increase the number of substations and shorten the length of the low-tension distribution. The principal operation in such a station is to cut out the feeder when it fails and substitute a reserve feeder, and, as this is easy to accomplish by automatic means, the trend is toward the use of automatic substations because the saving in labor reduces the operating expenses by a greater amount than the increase in fixed charges and maintenance. When a circuit breaker opens it is automatically reclosed two or three times after suitable intervals, and if the trouble has disappeared it remains closed. If, after the third closing, the trouble has not disappeared, and the breaker opens again, it stays open, and an indicator in the headquarters of the power dispatcher shows which particular breaker is open.

In modern practice these lighting substations, if of the indoor type, are frequently constructed with the **truck** type switching equipment in which each panel, including its circuit breaker is on wheels and may be disconnected from the buses and moved out from its place for inspection and repair. This can be done only when the circuit breaker is open and the panel is dead so the mechanism of the truck serves as a protective interlock and disconnecting switch. For voltage regulation, tap-changing or polyphase induction regulators are used where the load is balanced and single-phase regulators on each of the outgoing lines to neutral where the load is a single-phase residence lighting load which is likely to be unbalanced. These automatic substations are inspected once a day by a trained inspector, and all troubles are reported to a repair gang. An indoor transformer station for 13,000 volts requires about 2.5 cu ft per kva, and an outdoor station from 0.25 to 0.4 sq ft per kva.

Distribution Substations. These usually transform from 6600 or 13,000 volts three-phase to a special 208-volt three-phase, four-wire system which gives 208 three-phase for motors of any considerable size and 120 volts single-phase from any wire to neutral for lighting and small motors and appliances. The low-tension side is usually connected into a "secondary network" by which any customer may receive power from more than one substation. Because of this it is necessary to provide a network protector in each substation to prevent a flow of energy from the low-tension network back into the high-tension mains in case of a fault in the latter or in the transformer. The network protector is controlled by a relay which not only opens the secondary when the flow of energy is wrong, but holds it open as long as the relative magnitude and phase of the voltages across it are such that, if connection were re-established, the flow of energy would still be wrong. This type of substation is used in large cities, and the whole equipment is usually placed in a vault under the street or in a cellar and is operated without attendants.

Equipment

A typical substation would contain: lightning arresters, disconnecting switches, oil circuit breakers, high-tension bus-bars, transformers, oil circuit breakers, disconnecting switches, low-tension bus-bars, voltage regulators, outgoing feeders and their switches metering instruments, protective devices, and a communication system.

TRANSFORMERS. The transformers are usually of the oil-cooled type (except where local fire regulations prohibit) to avoid the use of blowers and other moving auxiliary machinery. They may be of the single-phase type with three to a bank, or of the three-phase type, depending upon costs. A station of large capacity with several banks would probably have three-phase transformers whereas a small station would use single-phase transformers with only one transformer as a spare for any bank.

SWITCHING EQUIPMENT. The latest practice favors the use of the minimum amount of switching equipment consistent with reliability of operation and protection. The use of modern relays makes it possible for one switch to perform a multiplicity of functions: short circuits, time overload, reverse current, and grounds. The metering equipment depends upon local conditions, but it is the custom to have a volt, ampere, watthour, and power-factor meter on each outgoing feeder.

COMMUNICATION. Communication with the main power house, power dispatcher, and other substations may be provided in several ways: leased lines, special private line, carrier current telephony over the power wires or supervisory control, which again may be by private line or by carrier currents.

Synchronous Condenser Substation

A synchronous condenser substation will probably have the usual switching and transforming apparatus and the condensers in addition. Condensers are synchronous

motors with a very generously designed field so that they may be greatly overexcited. Overexcitation causes the condenser to draw a leading (anti-inductive) component of line current which, in passing through the inductive reactance of the transmission line or added lumped inductance, causes a rise in voltage at the receiving end. Conversely, underexcitation causes a lagging current and a lowering of voltage. By operating underexcited at light loads and overexcited at heavy loads the delivered voltage may be kept at a specified value in spite of line loss. Thus

$$E = e + (i + ji_1)(r + jx)$$

where $E \equiv$ voltage per Y phase at the sending end.
$e \equiv$ voltage per Y phase at the receiving end.
$i \equiv$ component of load current in phase with e.
$ei \equiv$ power per phase delivered.
$i_1 \equiv$ reactive component of current delivered.
$r + jx =$ impedance of one line of transmission.

E, e, and ei may be set, and i_1 will be the reactive current required of the condenser, positive for leading and negative for lagging. If the line voltage is very high, greater than 100,000, the distributed capacity of the line may cause a charging current of sufficient magnitude to help considerably.

Synchronous condensers are usually provided with a complete automatic starting equipment, including auto-transformer with 50 per cent starting tap, interlocks between speed, field closing and final closing switch.

Synchronous Converter Substations

Stations for converting from alternating to direct current are generally used for railway service and are discussed under that heading. However, in lighting service by 120-240 volts, direct current, shunt-wound converters are used instead of compound-wound. The voltage regulation or compounding is obtained by means of series boosters on the same shaft as the converter. A series booster is a three-phase generator with its phase windings independent and each in series with one of the three leads which supply the collector rings of the converter. The field of the booster is separately excited and hand controlled so that any degree of voltage control may be obtained. The Edison system substations in the large cities usually contain a large storage battery arranged in two groups to serve as a reserve to the 250-125 volt three-wire d-c distribution.

24. RAILWAY SUBSTATIONS

(See also Synchronous Converters; Energy Requirements for Railways; Switchboards; Switchgear Equipment; Transformers.) Substations are used for electric railways when the length of the road is so great that the whole road cannot be supplied by one power station at the voltage required by the motors without either an excessive drop in voltage or a prohibitive amount of copper or both. Practically all electric railways require substations. In practice there are two types of substations: (1) for transforming from alternating to direct current, and (2) for transforming from high-voltage to low-voltage alternating current.

Location and Capacity

The location, capacity, and number of substations involve not only the cost of the copper required for distribution and the cost of the substations, but also the distribution of traffic and special local conditions. The fundamental economics of the subject are expressed by the general theorem that the cost of operating the substations plus the cost of interest and fixed charges on investment in substations and line copper shall be a minimum. This is explained by the fact that if to any given arrangement an additional substation be added with the proper rearrangement of the spacing, the amount of copper used in the distributing system may be considerably decreased on account of the lesser distance between substations. If the saving in interest on the value of the copper is greater than the cost of operation of this new substation plus the interest and fixed charges on the first cost of the substation, the change is warranted.

Allowable Voltage Drop. The distance between substations depends directly upon the allowable loss in voltage and the amount of copper in the trolley and varies inversely as the load. The allowable loss of voltage is a matter of the special conditions of each road. In city roads a drop in voltage of 8 per cent with average load and 15 per cent with maximum load is frequently observed. In interurban roads a drop of 12 per cent with average load and 30 per cent with maximum load is customary.

The two conditions which determine the allowable drop in voltage are the effect

on the lights and the effect on the control circuit of the car, each of which factors requires a lesser drop in voltage than the actual operating characteristics of the motors.

Distance between Substations. For the service usually found in practice Table I shows the average distance between substations.

Table I

Volts	Type	Miles between Substations	
		Single-track Road	Double-track Road
600	Direct-current, trolley	10	15
600	Direct-current, 3d rail	13	19
3,300	Single-phase, trolley	17	23
1,200	Direct-current, trolley	19	25
1,200	Direct-current, 3d rail	38	43
6,600	Single-phase, trolley	45	50
11,000	Single-phase, trolley	..	70

Where the length of track receives its power from only one direction the allowable distance for a given drop in voltage is one-quarter the distance allowable between substations.

Capacity Required. The capacity of the substation depends upon the traffic and the size of the cars, since if many cars are operating it is probable that only a few will be starting simultaneously and it is during starting that the cars demand the maximum amount of power. It is, therefore, necessary to determine the number of cars which will be located on the section of track supplied by one substation and the frequency of starting. In interurban service the number of cars on a given section is given by the formula:

$$X = \frac{2 \times (\text{Length of section}) \times (\text{Number of cars per hour})}{\text{Schedule speed in miles per hour}}$$

The average load on the substation is equal to the average power demand of each car times the number of cars on the section divided by the efficiency of the distribution system. In a complex system the local conditions must be studied to determine how many of these cars are likely to be starting at once. In other than city roads a load factor of 0.3 to 0.5 may be used to determine the maximum load on a substation, and in single-track interurban roads it is customary to assume one car starting and one car running. See Energy Requirements for Railways, Section 17.

Converter and Motor-Generator Substations

Substations may convert alternating into direct current by means of synchronous converters, mercury rectifiers, or motor-generator sets. Converters have been in vogue longest and are most common. They are particularly adapted for operation on 25-cycle alternating current and delivering 600 volts direct current. At 60 cycles they do not operate as well as the other types but are still satisfactory. For delivering 1200 volts or higher they are not as desirable as the motor-generator or the rectifier because of commutation difficulties. However, the converter has the advantage that it readily lends itself to power-factor and voltage control. For 1200 to 3000 volts, the rectifier is more efficient, because the principal loss is due to the counter emf of the arc, which is about 20 to 30 volts irrespective of the delivered voltage. The rectifier has a higher all-day efficiency than the other types because of the small losses at light load.

EQUIPMENT. A standard a-c to d-c railway substation usually contains the following pieces of apparatus: converters or motor-generators, transformers, reactances, blowers, cables, and switching equipment.

Arrangement of Apparatus. (See also Switchgear Equipment.) It is customary to arrange the apparatus in a substation so that the current travels in as nearly as possible a straight line across the station. Thus, with the incoming line on one side there are, in the order mentioned: the lightning arresters, oil switches, transformers, a-c switchboard, reactance coils, converters, d-c line panel. A typical arrangement for a small station is shown in Fig. 2; see also article on Switchgear Equipment.

Floor Space Required. For substations with all apparatus on one level the floor space required is about 0.20 sq ft per kw. In cities where real estate is expensive, it may be desirable to put the oil switches and lightning arresters in a gallery on the upper level and the transformers on a floor above, but this involves more expense for attendance.

Converters. (See Synchronous Converters for data regarding converters.) Compound-wound commutating-pole converters are used where the load is variable to provide

automatically any desired voltage regulation. The converter by means of its series winding is made to take a leading current which, in passing through the reactance

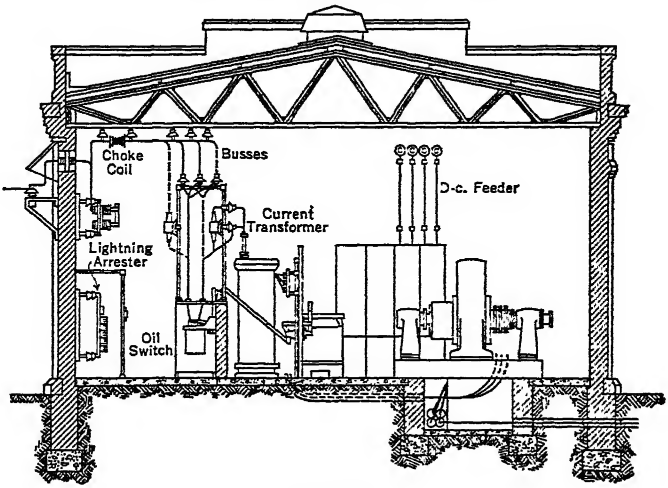

Fig. 2 (a)

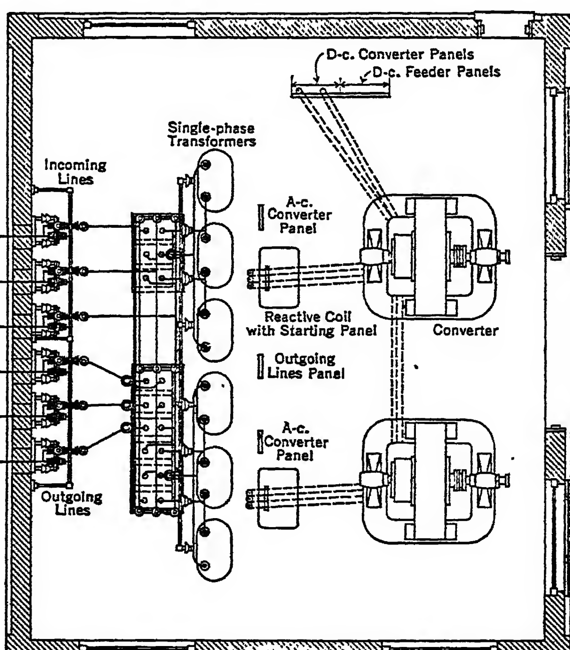

Fig. 2 (b)
Typical Arrangement of a Small Substation

of the line, the transformers, or an additional reactance introduced for the purpose, tends to neutralize the line drop. (See Transmission Lines.) In city service where a

large number of cars are operating on one section, the load is fairly constant and shunt-wound converters are generally used.

Both three-phase and six-phase converters are used, the three-phase for the smaller and the six-phase for the larger capacities. In a three-phase converter for a direct emf voltage of 600 at no load, the transformers should supply the converter with 370 volts between rings or between lines. In the six-phase converter it is customary to use the diametrical connection, in which each transformer secondary supplies 430 volts to the converter, giving 600 volts at the commutator. Each converter for railway work is customarily supplied with a "speed-limiting" device on one end of the shaft and an "end-play" device on the other end. (See Synchronous Converters.)

Transformers. (See Transformers.) The transformers used in railway substations may be either of the oil-cooled, air-blast, or water-cooled type. The oil-cooled type is used where the expense of the complications for air blast or water cooling are not warranted. Air-blast transformers may be used for voltages up to and including 33,000; the objections to them are the necessity of providing a pit, air ducts, and blower to supply the ventilation. Water-cooled transformers (which are oil-insulated transformers with water circulating in a special coil submerged in the oil) are built in sizes from 500 kw upwards and for all voltages. Their use depends upon the availability of water for cooling purposes. The usual aggregate capacity of transformers for the various sizes of synchronous converters is about the same as the converter capacity.

The transformers may be either of the single-phase or three-phase type. For small or moderate installations the single-phase type in banks of three is preferable on account of the economy of maintaining only one single-phase transformer as a spare for a whole station. In railway work it is customary to connect the secondary of the transformers in delta for three-phase, because of the possibility of operating at reduced output on open delta in case of failure of one transformer. The primary windings of the transformer are usually connected Y with grounded neutral. It is common practice to provide transformers for railway work with four $2\,^1/_2$ per cent taps on the high-potential winding, in order to use similar transformers in all substations and yet make allowances for the difference in the line drop between the power station and the various substations. Either $^1/_3$- or more generally, $^1/_2$-voltage taps are provided on the low-potential side for starting the converters.

Blowers for Air-blast Transformers are usually driven by three-phase induction motors receiving power from the low-tension side of the transformers. The amount of air required per minute per kilowatt rating of each transformer ranges from 3 cu ft in the large sizes to 5 cu ft in the small sizes. This air is supplied at a pressure of from $^3/_8$ to 1 oz per sq in. The blowers must be capable of supplying this amount of air with an allowance of about 10 per cent for leakage in the air-blast chamber. It is customary to provide two blower sets each capable of supplying air for all the transformers in the station and maintaining one as a reserve unit. In a very large station three blower sets are sometimes provided, two of which are together capable of supplying the service and the third is a reserve. A rough idea of the size of the motor necessary to drive a blower for a given purpose may be obtained from the following formula:

$$\text{Horsepower} = \frac{(\text{Cu ft air per minute}) \times (\text{Pressure in ounces})}{1200}.$$

Reactances for Voltage Regulation. The voltage at the d-c bus-bars is regulated automatically by means of line compounding which consists in adjusting the shunt-field excitation of each converter so that the converter takes lagging current at no load, operates at unity power factor at about $^3/_4$ load, and takes leading current at all loads greater than $^3/_4$ load. To accomplish this compounding, it is necessary that there should be a certain amount of reactance between the power-station bus-bars and the converters. (See Synchronous Converters.) There is seldom enough reactance in the transmission line for this purpose, so that additional reactance is inserted either by the use of special transformers having considerable leakage reactance, or by means of reactance coils. The usual reactance coil has a capacity in kilovolt-amperes equal to 15 per cent of the kilowatt rating of the converter, i.e., with full-load current passing through the reactance the voltage measured across its terminals would be 15 per cent of the voltage to neutral on the a-c side of the converter.

Reactance coils are either oil cooled or air blast depending upon the transformers used and are generally built with the three circuits in one unit, with the starting switches for the converters mounted upon the frame. For six-phase converters a three-phase reactance coil is used, but since the current per wire is one-half that of the current of a three-phase converter of the same rating, the reactance per line must be twice as great for the six-phase converter.

Switchboards. (See also Section 12.) The following switchboard panels are standard for converter substations.

1. Incoming a-c line panel.
2. Outgoing a-c line panel.
3. High-tension a-c converter or rectifier panel.
4. D-c converter or rectifier panel.
5. D-c feeder panel.
6. Equalizer and negative panel (on the converter).

Where a substation is tapped off a transmission line at an intermediate point it is good practice to bring the transmission line into the substation, interpose control switches, and then carry the circuit out of the substation on to the next substation. For this reason, in all but terminal substations, it is customary to provide both an incoming and an outgoing a-c line panel. In connection with the a-c panel of the switchboard there are a line switch, lightning arrester, choke coil, current transformers, and main oil switch, by means of which potential may be removed from all transformers. Between the transformers and the converter are the starting switches, reactance coil, and possibly measuring devices.

Single-pole switchboard panels are used for the direct current, the positive main bus-bar being the only one on the board. The negative terminals of the converters are connected with switches to the negative or ground return bus-bar, which is frequently located beneath the converter. The series field is connected on the negative side, and the equalizer, series-field-shunt, and field break-up switches are frequently placed on the machine itself. The equipment of a standard d-c converter panel comprises one of each of the following:

Carbon break circuit breaker with overload and low-voltage release, the latter interconnected to the speed-limit device.
Illuminated dial ammeter with shunt.
Field rheostat.
Two-point receptacle.
Single-pole main switch.
Single-pole double-throw station lighting switch.
Watt-hour meter.

Crane. Where ground space is limited it is good economy to provide a crane in order that the various pieces of apparatus may be lifted over each other when they are taken apart for repairs, as otherwise considerable space must be left to move them about to and from the entrance.

Methods of Starting Converters. There are several methods of starting converters, as is explained under Synchronous Converters. Starting from the a-c end as an induction motor is most desirable for railway work, as it avoids the necessity of synchronizing and requires less time. The ability to start a machine quickly and get it on the line in the shortest possible time is very important in railway work, and is an advantage inherent in this method of starting. Three-phase converters are started, by means of suitable starting switches, from $1/2$ voltage taps on the transformer secondaries and take approximately full-load current from the line. Six-phase converters are started from $1/2$ voltage taps and take $3/4$ full-load line current. Since 60-cycle converters usually take a greater starting current than 25-cycle converters and are usually operated on a system supplying power for other purposes, where voltage disturbances are objectionable, special means of starting 60-cycle converters are frequently employed.

The usual method of starting a synchronous converter from a-c supply is as follows:

1. Open all switches except main negative (on machine).
2. Close a-c line switch feeding buses.
3. Close high-tension transformer switch.
4. Close starting switch on low-voltage taps.
5. When converter reaches synchronism as shown by low frequency of swings of d-c voltmeter, close equalizer switch.
6. Close switch between series field and the shunt to series field.
7. Correct polarity if necessary by throwing shunt-field break-up switch to reverse position, leaving it closed only momentarily, then throw to running position.
8. Connect a-c terminals of converter to full voltage of transformers by throwing starting switch to running position.
9. Close d-c circuit breaker.
10. Adjust field rheostat.
11. Close main d-c switch.
12. Adjust for correct division of load, power factor, and voltage.

(See also below under Automatic Substations.)

LOAD FACTOR AND EFFICIENCY. The load factor (q.v.) of a converter substation is usually low, from 30 to 50 per cent, i.e., the load on the station is relatively light except during the morning and afternoon rush-hours, 7 to 9 A.M., and 5 to 7 P.M. respectively. The all-day efficiency of the station itself, or the ratio of the kilowatthours output to the kilowatthours input, is less than the efficiency at maximum or rated load; but the overall efficiency between the a-c generators in the power house and the cars is about constant throughout the day, since the efficiency of a transmission line increases with decrease of load, thus offsetting the low light-load efficiencies of the transformers and converters. In general practice in interurban 600-volt railways the maximum and all-day efficiencies of the various apparatus are approximately as given in the following table:

Apparatus	Full-load Efficiency, Per Cent	All-day, Efficiency, Per Cent
Step-up transformers	98	97
High-tension line	95	98
Step-down transformers	97	94
Converters	90	88
Low-tension distribution	85	88
Overall, a-c generator to motors	69	69

AUTOMATIC SUBSTATIONS. To decrease the cost of operation, by eliminating the need of attendants, substations are now equipped so that the entire control and operation are accomplished mechanically and automatically. When the voltage on the adjacent section of the trolley falls below a specified value (450 volts), a contact-making voltmeter closes a control circuit which starts a motor-driven controller operating a large gang of switches and contactors. These switches automatically and consecutively accomplish the various starting operations. The converters are started from the a-c side so there is no synchronizing, but the polarity of the d-c end is regulated, the field excitation adjusted at the proper time, and the several converters connected in parallel. Protection against overload, low-voltage, and overspeed are provided.

When the load falls below a specified value, a contact-making ammeter closes a circuit which opens all switches and shuts the converters down. A large number of these automatic substations have been installed containing up to three converters per station and in sizes up to 2000 kw units of converters.

Rectifier Substations

APPLICATION. The great majority of the older 600-volt railroads use synchronous converters, but many of the newer 600-volt roads and the 1500-volt roads use rectifiers. Some roads using 3000 volts (e.g., St. Paul) have motor-generator sets with two 1500-volt generators in series electrically, both driven by one synchronous motor; others (e.g., Lackawanna) use rectifiers for 3000 volts direct current.

TRANSFORMERS. Mercury-arc rectifiers are usually constructed with six anodes and are supplied by three-phase transformers with double secondaries giving six phases. Larger sizes may be constructed with twelve anodes and supplied from three-phase transformers with specially designed secondaries. Ideally the voltage on the d-c side is equal to the peak voltage of the a-c side ($\sqrt{2} \times$ the rms value), but owing to the drop in the arc and the transformers d-c voltage obtained is less by some 30 or 40 volts. Six-hundred volt rectifiers have an overall efficiency at rated load of about 94 per cent, including the losses in the auxiliaries. For 1500 volts the efficiency is about 96 per cent and for 3000 volts it is 97 per cent.

EFFICIENCY. The efficiency at $1/4$ load is usually above 92 per cent as compared to 88 per cent in a converter. The power factor is from 92 to 95 per cent (higher at overloads) within the working range. The cause of the low value is primarily wave distortion; it is not due to a lagging current of fundamental frequency. This wave distortion, common to all rectifiers, may cause interference in nearby telephone circuits, in which case the offending harmonics may be suppressed by a special wave filter.

AUXILIARY APPARATUS. Each rectifier requires the following auxiliary apparatus: transformers, vacuum control and pumps, temperature control and water circulation, ignition and excitation equipment, heater for starting, voltage control for compounding if required, usual overload protection, and protection against arc-back (Fig. 3).

Many rectifier substations are entirely automatic in their operation and require no attendants, as in the New York Independent Subway. Since there are no moving parts very light foundations may be used.

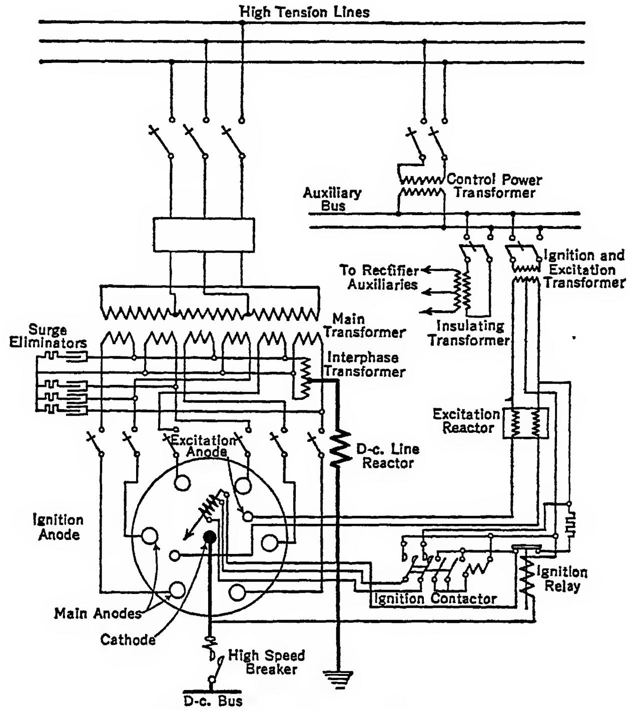

Fig. 3. Rectifier Substation Diagram

SPACE. A rectifier substation usually requires slightly more space for a given power than a converter substation, but as there are no moving parts very light foundations may be used and there is no vibration or noise. They may be adapted for regeneration on the train by a special arrangement to provide for the flow of energy from the d-c to the a-c side. Rectifier substations cost about 30 per cent more than converter substations.

PORTABLE SUBSTATIONS. Circumstances frequently occur under which the traffic on a certain branch of a railway is very heavy for only a few weeks in the year, and possibly for only two or three days in the year. Moreover traffic may be exceptionally heavy on one branch for one short period and on another branch at another period. In such cases as these it would be very expensive to provide on each branch substations having a capacity to meet the heavy demand. To meet such conditions " portable " substations are used. These consist of what are practically large steel furniture cars, in which are installed one synchronous converter and the necessary transformers and control devices. Side tracks are provided at points on the branch lines, and when needed this portable substation is hauled to the place desired and its high-tension terminals are connected to the high-tension transmission line, provision for which must be made in building the line, and its d-c terminals are connected to the trolley and the rail respectively. Such a substation may be of great convenience and value for interurban roads where summer parks, circuses, and athletic games occasionally render traffic conditions difficult. The car is limited in its size by the standard clearance outlines of the roads over which it must pass. It usually weighs, with all its equipment, about 75 tons. Apparatus having a capacity of 500 kw may be installed in such a car, and the primary voltage may be as high as 33,000 volts. (See also, Section 12 on Switchgear Equipment.)

A-c Single-phase Substations

For a single-phase railway operating at a high voltage on the trolley (11,000 or 12,000) it is not necessary to place substations as near together as in d-c systems. However, even with 12,000 volts, there is a limit to the length of road which may be supplied from one feeding point. For instance, on the New Haven or Pennsylvania with heavy traffic on four

tracks, it is necessary to step-up the voltage at the power house to 44,000 or 88,000 and step-down again to trolley voltage at intervals of 10 to 20 miles. A substation for this purpose need contain only transformers and protective devices and usually has neither a building nor attendants, the circuit breakers being reset automatically or by remote control from a supervisory station. Some roads use a three-wire system with auto-transformers by which the trolley wire may serve also as one of the transmission wires. See Fig. 4.

If power is purchased by the railway from existing 60-cycle commercial systems a frequency converter substation is used, as the motors on the cars and locomotives will not operate with 60 cycles but must have 25 cycles and the commercial power companies will not allow a single-phase load to be connected to the same lines as supply the other customers. A frequency converter consists of a 60-cycle, three-phase synchronous motor driving a 25-cycle single-phase (in this case) generator, the usual combination being a 24-pole 60-cycle machine and a 10-pole 25-cycle machine, both running at 300 rpm. A synchronous frequency changer ties the two systems together rigidly so that the slightest change in frequency or phase in one system reacts upon the other system. One of the two machines has its stationary part (stator) in a cradle so that it may be rotated a few degrees in order to adjust the load. In modern practice these frequency converters are totally enclosed and self-ventilated, and they require no building or attendant.

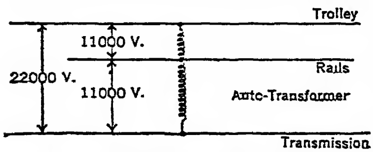

FIG. 4. Single-phase Railway Distribution by Auto-Transformers

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See also Art. 29 of this section.

POWER STATION CIRCUITS

By R. A. Hopkins and A. G. Cherry

26. MAIN ELECTRICAL CONNECTIONS

FUNDAMENTAL REQUIREMENTS. The scheme of electrical connections to be used for generators, transformers, transmission lines, and station auxiliaries largely determines the physical arrangement of the electrical bay, the equipment, and the wiring. (See Section 12.)

Efficient Operation of the station at all loads should be provided for by suitable grouping and switching of generators, transformers, and lines. The efficiency of a steam turbine-generator varies considerably with the load, and the efficiency of a waterwheel generator varies considerably with both the load and the head. For this reason, if the load is not constant it may be necessary to switch the units from hour to hour to secure greatest economy. Switching of transformers is less important because transformer efficiency is relatively high at all loads. The lines have small loss at light load, and for this reason, as well as to safeguard continuity of service, they are usually left in service at all loads. An exception to this rule is found in long, multiple-circuit lines where one or more circuits are usually disconnected at light load to avoid overloading transformers and generators with reactive kilovolt-amperes.

Retirement of generators, circuit breakers, and other moving equipment at regular intervals for inspection and maintenance should be provided for. Where interruption of station output cannot be permitted the connections must provide for the substitution of spare equipment during this routine maintenance.

Faults occurring in any part of the equipment or connections should be promptly detected and isolated with minimum disturbance to other circuits and minimum loss of load. This requires a certain amount of busing of generators and lines so that the portion of load carried by the faulted part is automatically transferred without interruption. The relay scheme should be so arranged that the retirement of a breaker for inspection does not impair any of the relay protection.

Synchronizing must be provided for. Low-voltage circuit breakers were formerly preferred for synchronizing on account of their quicker action as compared with high-voltage breakers. High-voltage breakers inherently required somewhat longer closing time on account of their longer contact travel. The time of closing of modern breakers is so well controlled and of such consistent duration, however, that synchronizing is now feasible at all voltages up to 220 kv.

Short Circuits and System Stability must be carefully investigated. (See Sections 3 and 14.) The possibility of severe short circuits requires the use of heavy bus supports, current transformers, and other equipment to withstand the mechanical stresses and of heavy circuit breakers to interrupt the current, all of which contribute materially to the cost of the installation. Low stability may cause serious rejection of load during system disturbances. Reactance in generators, transformers, and circuits reduces short-circuit severity but also lowers the stability. The amount of reactance to be used in generators, transformers, and main circuits, therefore, must be a compromise. **Sectionalizing** of the station, as opposed to busing, greatly reduces short-circuit severity, but nevertheless some degree of busing is usually demanded in order to equalize the load among the transformers and generators and to transfer load from a faulted machine. For the small and medium-size station the busing is usually provided as required, and the breakers and other devices are designed for the resulting short-circuit currents. In very large stations, solid low-voltage busing might result in short-circuit currents beyond the ratings of available breakers and other equipment, and in this case some form of high-reactance busing may be used, such as bus reactors, double-primary transformers, and double-circuit generators. These devices insert reactance between main circuits, but not directly in them, thus limiting short-circuit current without impeding the delivery of normal station output. The use of high-speed relays and fast breakers to improve stability increases breaker duty but does not affect mechanical stresses. Some of the other means of increasing stability, such as flywheel effect, damper windings, quick response excitation, quick-acting governors, and sensitive voltage regulators, may at the same time increase both mechanical stresses and breaker duty. Therefore, the studies of short circuits and stability should be conducted jointly.

Charging Current for long transmission lines at light load must be provided. Under extreme conditions this may require the connecting of two generators to one line at light load although one generator is used per line for normal load conditions. It has been found much more satisfactory, however, to be able to charge a line with a single generator.

ELEMENTARY CONNECTIONS. A few elementary connections, which may be modified and combined to meet local requirements most completely, form the basis of all connection schemes. The more useful of these elementary connections are illustrated in Fig. 1 and analyzed as follows:

Fig. 1 (*a*) shows the **single, low-voltage bus.** Any number of generators may be operated to secure best station efficiency at all loads, but the transformers cannot be operated this way unless all lines are in parallel to the same substation bus. Only the bus sectionalizing breakers can be retired for inspection without curtailing the output, and this can be done only by sectionalizing the station. A bus fault will shut down at least one generator, transformer, and line. A generator fault will not cause immediate loss of load if the bus is operated united, that is, with sectionalizing breakers closed. A transformer fault will cause immediate loss of a line. Short-circuit stresses and breaker duty are high when the bus is united. Auxiliary power may be taken from two or more sections of the bus. A large number of lines cannot be accommodated without an expensive transformer layout. Air break switches may be used in the lines since interrupting duty can be handled by the transformer low-voltage breakers. The scheme is simple, compact, and inexpensive.

Fig. 1 (*b*) shows the **single, high-voltage bus.** Generators and transformers can be operated for best station efficiency provided their individual best efficiencies are at the same load. Only the bus sectionalizing breakers can be retired without curtailing the output, and this can be done only by sectionalizing the station. A bus fault will shut down at least one generator, transformer, and line. A generator or transformer fault will not cause immediate loss of load if the bus is operated united. Short-circuit stresses and breaker duty are high when the bus is united. Auxiliary power cannot be taken from the bus without using expensive, high-voltage transformers. Any number of lines may be

accommodated. Air break switches cannot be used, as every breaker shown must be used for interrupting. Generator breakers may be omitted. The scheme is simple but, as compared with the single low-voltage bus, requires more space and the breakers are more expensive.

Fig. 1 (*c*), the **double bus**, is shown for low-voltage, but may be used also for high-voltage. Some stations have both. Selector breakers are shown for the transformers and selector switches are shown for the generators, but either construction may be used for either location. The principal advantages of the double bus are that either bus

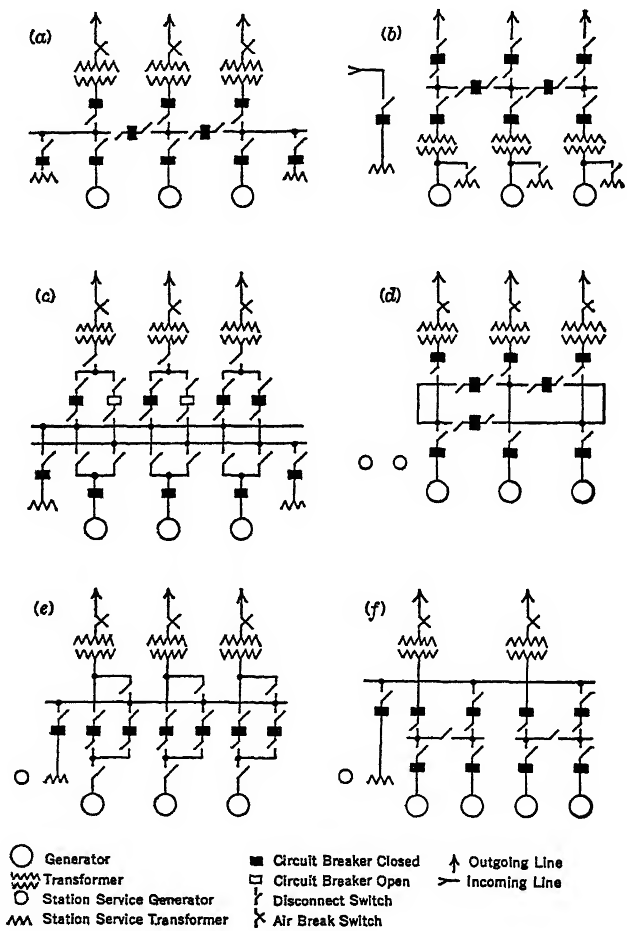

Fig. 1

with its breaker may be retired for inspection without curtailing the output or sectionalizing the station and that both buses may be operated isolated from each other with any desired grouping of generators and lines on each bus. If selector breakers are used throughout and both buses are operated in parallel, with all breakers closed, the double bus has the further decided advantage that a bus fault will not interrupt station output. Bus sectionalizing breakers are sometimes used to localize bus faults. The two buses may be tied together by closing both breakers of any two-breaker circuit, and they must be so tied together while transferring any of the circuits not having selector breakers.

Fig. 1 (*d*), the **ring bus**, is shown for the low-voltage bus but may be used also for the

high-voltage bus. Some stations have both, and in a few stations double ring buses are used. The principal advantages of the ring bus are flexibility of connections, ability to retire any bus section or sectionalizing breaker for inspection, and ability to isolate any bus section by relay action without sectionalizing the station or curtailing the output.

Fig. 1 (*e*), the **transfer bus**, sometimes called **synchronizing bus**, may be used for either the low-voltage or the high-voltage bus, and the switching facilities may be arranged in many ways other than that shown. The bus may be used to tie the station together and to transfer load from one circuit to another, but is not essential to the delivery of the station output as are all buses previously discussed. The bus or any breaker may be retired without curtailing the output. A bus fault does not cause loss of load unless load is being transferred from one circuit to another. A bus fault would shut down the auxil-

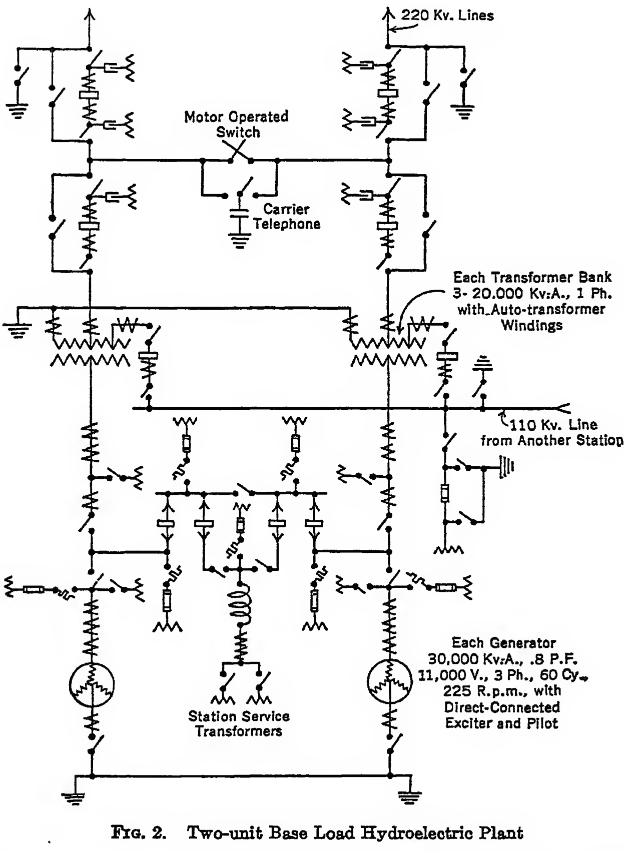

FIG. 2. Two-unit Base Load Hydroelectric Plant

iaries, however, unless a second source of auxiliary power supply were available for automatic relay. If current-limiting reactors must be used, this arrangement provides a good place for them, that is, between each generator and the transfer bus. With the reactors thus located, the station output is not delivered through the reactors under normal operation, and short-circuit current from one main circuit to another must pass through two reactors in series. Auxiliary power may be taken from the low-voltage transfer bus. The scheme may be made simple, compact, and inexpensive.

Fig. 1 (*f*), the **group bus**, is of particular value where several generators or several lines are to be connected to a transformer.

ACTUAL CONNECTIONS. Actual schemes of connections taken from the single-line diagrams of well-known recent power stations are illustrated and analyzed as follows:

Fig. 2 illustrates a two-unit base load hydro plant with two transformers and two lines

feeding a large power system. A smaller station feeds the system through the same transformers and lines. Breakers between generators and transformers are omitted. The low-voltage bus and breakers between main circuits are for providing auxiliary power and for synchronizing the two generators with each other before connecting them to the 220-kv lines. Any breaker may be retired for servicing without reducing station output. Each 220-kv breaker has a by-pass air-break switch so that the breaker can be serviced without opening the circuit. An interlock scheme is used to transfer the relay protection to the other breaker in series while one breaker is being serviced. The motor-operated tie switch provides flexibility in operating. Generators and transformers are protected individually by differential relays. Excitation is provided by direct-connected main and

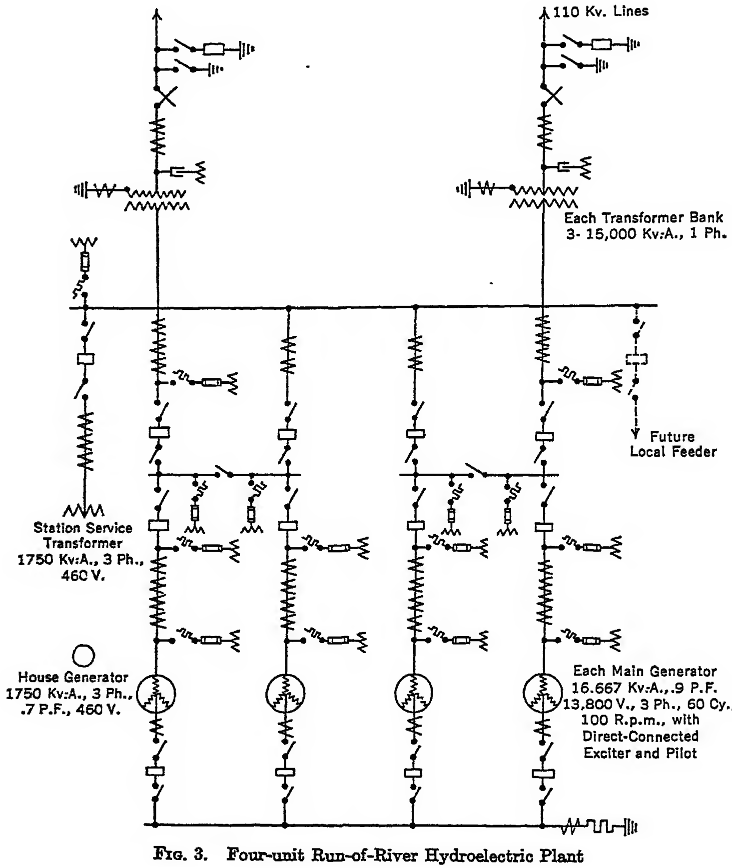

Fig. 3. Four-unit Run-of-River Hydroelectric Plant

pilot exciters on each main unit and a spare exciter which can be driven by a prime mover or by an induction motor.

Fig. 3 illustrates a four-unit run-of-river hydro plant with two transformers and two lines feeding an extensive transmission system. The main connections include two sectionalized low-voltage group buses and a low-voltage transfer bus. The number of generators in service can always be adjusted to the load to secure best station efficiency. Duplication of circuit breakers for purpose of maintenance is not considered necessary as sufficient servicing time is available during light load periods. Station service and also future local feeders are provided for by the transfer bus. A waterwheel-driven house generator also supplies station service.

Fig. 4 illustrates a four-unit peak load hydro plant with four transformers and two lines

feeding a large power system. No low-voltage bus is provided. Each generator circuit is thoroughly isolated from the others. The flexibility of two high-voltage transfer buses is obtained by the use of only one and one-half breakers per circuit. Any breaker or either bus can be retired for servicing without interrupting the output. A bus fault will not interrupt the output. Station service is taken normally from an outside source, but in case of failure of this cource, it is automatically selected from one of the generators. Stability is obtained by the use of high-speed circuit breakers, low-reactance generators, large flywheel effect, specially designed damper windings, and quick-response excitation.

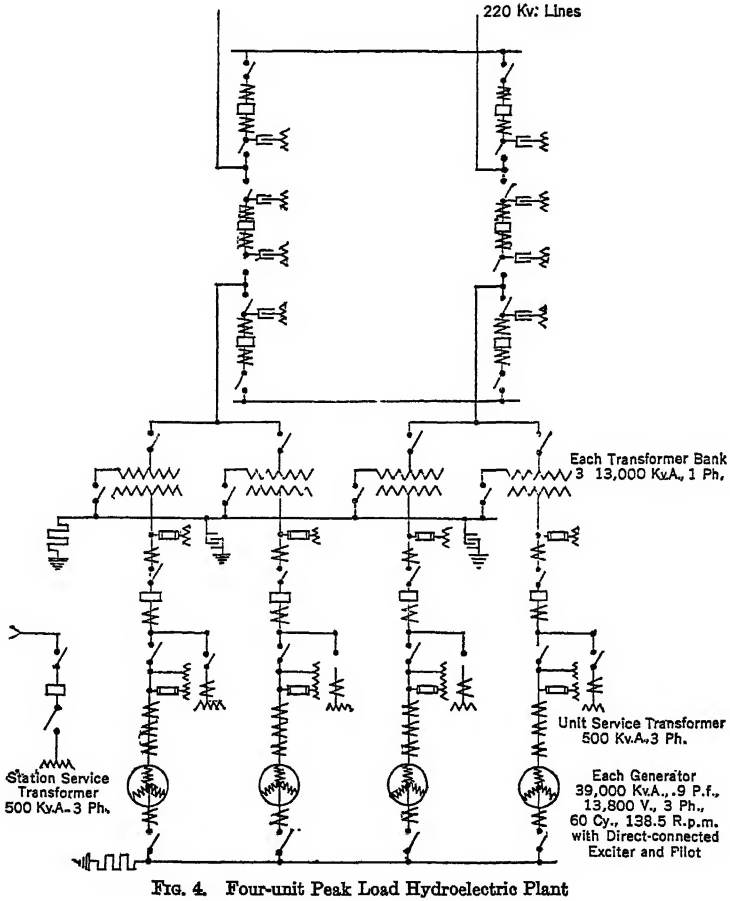

FIG. 4. Four-unit Peak Load Hydroelectric Plant

Fig. 5 illustrates a large steam plant with two or more main units, each connected through its transformer bank to the high-voltage double bus. The generator breakers are in duplicate to facilitate maintenance. The spare transformer is connected by means of removable links. Each main generator has a direct-connected auxiliary generator and also an emergency steam-driven auxiliary generator, the latter being arranged for starting and connecting to the auxiliary bus in less than 15 seconds. All unit auxiliaries are normally fed from the unit itself but in emergency may be fed from any other unit or from an outside source. The boilers are arranged in groups corresponding to the generating units, and the piping and wiring correspond in degree of flexibility. The greater importance of the auxiliaries in the steam plant than in the hydro plant demands considerably greater flexibility and duplication.

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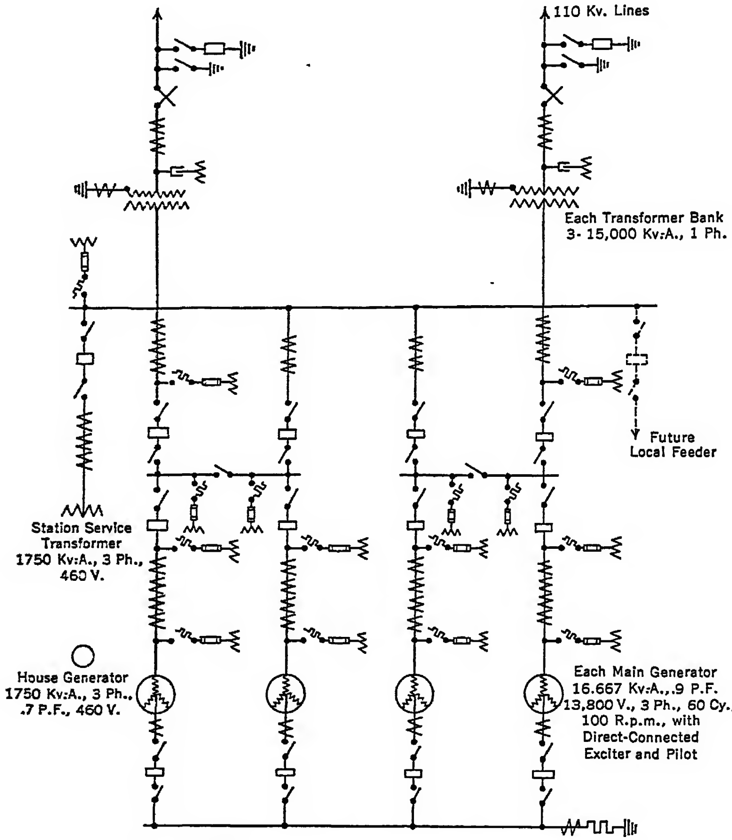

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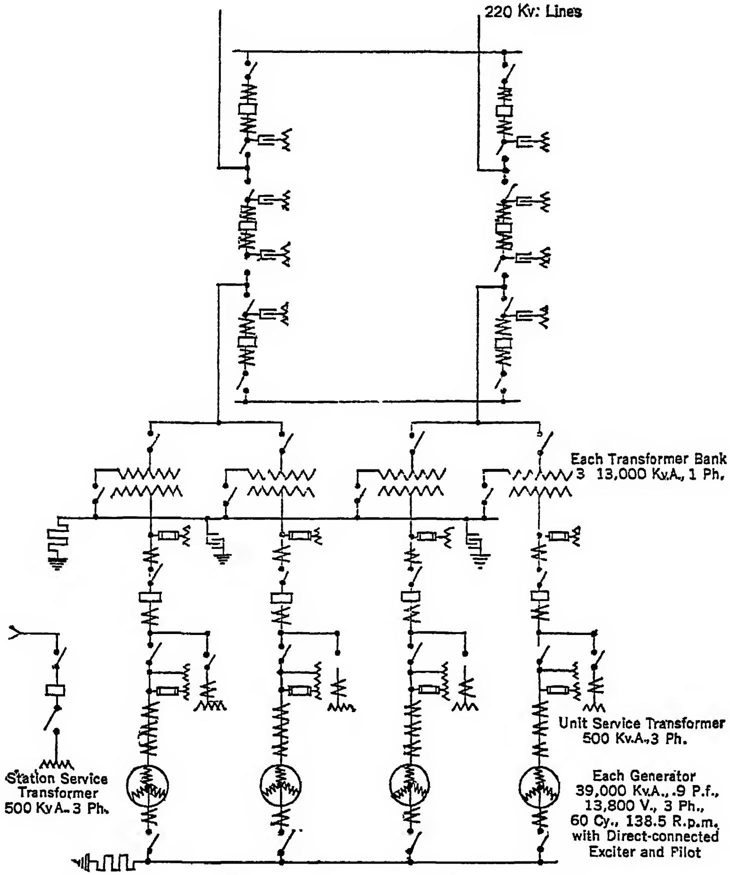

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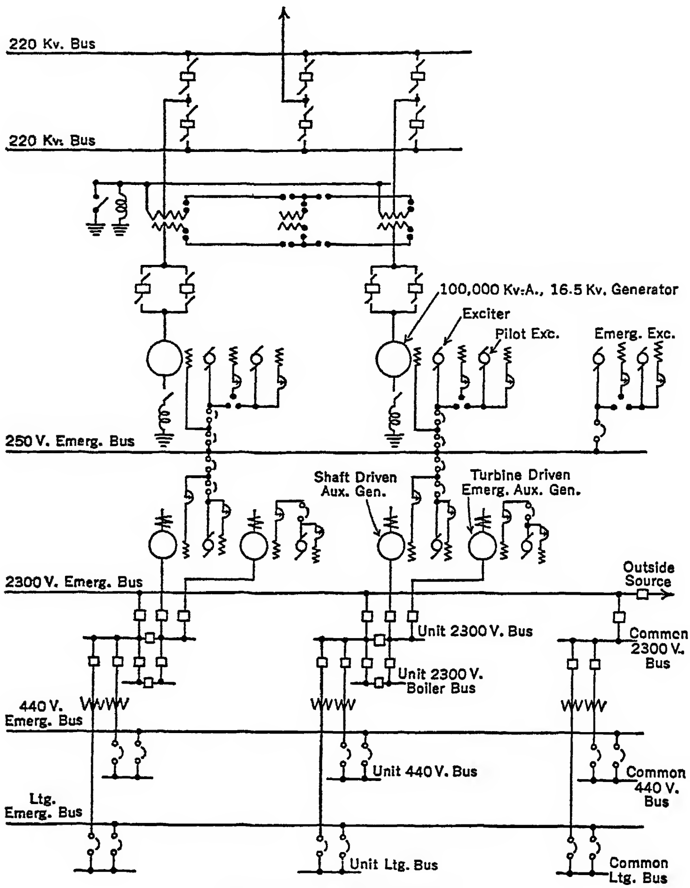

Fig. 5. Large Steam Plant

27. EXCITATION

CENTRALIZED SYSTEM. The centralized excitation system consists essentially of a group of exciters connected to an excitation bus which serves all the generators. It has the advantage of using a minimum number of large exciters, resulting in better exciter efficiency and fewer machines to maintain. The principal disadvantage is that a fault anywhere on the excitation system is likely to affect all the generators. To partially reduce this hazard the system may be divided into two parts, each serving about half the generators. One spare exciter can be made to serve either part of the system. The exciters for the centralized system may have motor drive or prime-mover drive or a combination of the two.

INDIVIDUAL EXCITATION. The individual excitation system consists of an individual exciter for each generator. A spare exciter must be provided with a bus to allow it to serve any generator. This system has the advantage that a fault in any individual excitation circuit will not affect any other generator, and for this reason alone the individual system is usually chosen for stations having a moderate number of large-capacity generators. A slight disadvantage lies in the large number of small exciters with their lower efficiency and higher maintenance cost. The main exciters for the individual system may be driven by the main units or by motors. The spare exciter may be motor driven or prime-mover driven or both.

CONTROL AND WIRING. Control and wiring for the excitation system should be simple and reliable. Overload circuit breakers are not used since disturbances on the a-c system frequently reflect back through the generator fields causing surges in the excitation system which might open automatic breakers causing loss of excitation and a-c output. Either reverse current circuit breakers or speed-limit switches are used for exciters operating in parallel with each other, to disconnect the exciter from the bus in case it should start to motorize. Short circuits, however, on the a-c system in some cases cause reverse current in the excitation system rendering reverse current circuit breakers more of a hazard than a safeguard, unless they are given high current settings. The generator field circuit should never be opened, when energized, except by means of a field switch or circuit breaker provided with an auxiliary contact and resistor to discharge suitably the energy stored in the highly inductive field winding. The **excitation wiring** should be short, direct, of ample capacity, and insulated for 8 to 10 times rated exciter voltage.

28. STATION WIRING

CIRCUITS. Station wiring circuits should be as short and direct as possible. Ample space should be allowed for adequate spacing of conductors. Substantial supports and thorough protection from mechanical injury and moisture must be provided. For heavy-current circuits ventilation is necessary. All exposed wiring must be properly guarded to safeguard employees. These requirements demand a close coordination with the building design to the end that the wiring facilities may as far as possible be built into the structures.

MAIN, LOW-VOLTAGE WIRING. Rated current is always a controlling factor and, for the larger stations, requires extremely careful consideration. Heating and mechanical effects of short-circuit currents are also of vital importance in the design of these circuits. Two general types of wiring have come into use: open wiring, having conductors supported on insulators and partially enclosed in structures; and conduit wiring, using insulated cables in conduits.

Open Wiring is generally used for the main low-voltage buses and sometimes also for the circuits. Its advantages over conduit wiring are obvious for use in buses and connections to current transformers, oil circuit breakers, and such devices located close to the bus. For the circuits to generators, transformers, and feeder outlets, open wiring is often used if the circuits are very short and the necessary space is available, but for longer circuits it is justified only with very heavy current ratings where special arrangements of conductors and ventilation are necessary.

The **conductor** for open wiring may be of tube, bar, or cable. **Tube** has the advantage of most economical shape of cross-section both for a-c carrying capacity and for mechanical stiffness. It has the disadvantage that all taps, bends, and splices must be made by accurately machined fittings. **Bar** has the advantage over tube that bends in one plane can be made by bending the bar, and other bends, taps, and splices may be either clamped or bolted, and soldered if desired, without depending upon factory-machined parts. Bar also has the advantage over tube and cable that large radiating surface is inherent in the shape of cross-section and good ventilation may be obtained by using spacers between the bars of each conductor. Bar has the particular advantages for use in buses that the bus may be tapered to suit the loads at various sections and that bars may be added as station capacity is increased. **Expansion joints** are required for long runs of either bar or tube. **Cable** has the particular advantage for circuits longer than standard tube and bar lengths that it provides continuous copper from end to end without need of splices.

Insulators for open wiring are usually of glazed porcelain with metal bases bolted to the structure and with tops provided with suitable clamps surrounding the conductor. For alternating current, the conductor clamps are made of non-magnetic metal. For heavy short-circuit conditions, two or more porcelain insulators are sometimes used at each support, arranged so that all mechanical stresses from the conductor will be delivered to the structure through porcelain in compression.

Enclosures for open wiring are usually of concrete, brick, tile, soapstone, gypsum, asbestos, steel, or other fire-resisting, structural material having sufficient mechanical strength. The structure should be of ample size to provide suitable clearances between phases and from phase to ground, should have strength to withstand safely the mechanical forces of short circuits, and should surround the circuit sufficiently to protect it from mechanical injury. Doors or grilles should be provided over all openings to a distance of at least 6 ft above the floor to prevent any person coming in contact with the conductors. The structure or the room in which it is located should be rat-proof. When more than one circuit occupies a room, it is sometimes considered an added protection to enclose each

circuit behind fireproof doors designed to prevent the spread of flame and ionized gases. Enclosed circuits should be well ventilated to prevent excessive temperature rise. Reinforcement and other magnetic material should be kept away from and should not form loops around conductors carrying heavy current as the heating caused thereby might crack the structure. Grounding of insulator bases to ground cables paralleling the circuit, though not universal practice, is commonly used by some operating companies. It provides the equivalent of a partial metallic shield around the circuit, thereby avoiding stray ground circuits.

Design of open-wiring buses and circuits should be based on (1) temperature rise of not over 30 deg cent with rated current; (2) mechanical strength at least twice the maximum stresses indicated by short-circuit analyses; and (3) insulator wet flashover strength of at least two and one-half times working voltage and clearances suitable for working voltage.

Conduit Wiring is commonly used for the low-voltage circuits except those of very heavy current rating. It is less expensive and occupies much less space than open wiring. The conductor may be soft annealed copper cable of standard stranding, rope core, hollow core, or three-phase sector design. Flexible stranding should be used if the conduit includes difficult bends. Insulation may consist of varnished cambric, high-grade rubber, or other insulating compounds having suitable characteristics under the temperature and humidity conditions in the conduits.

Conduits for the low-voltage wiring may be of rigid steel, brass, aluminum, fiber, transite, or tile, or of flexible steel or bronze. Rigid conduits are used as far as possible, the flexible types being used only where required by vibration or expansion joints in the building, or by connections to movable machinery. Steel conduit is the most universally accepted for both concealed and exposed construction on account of its superior strength and low cost. For sizes larger than 1½ in., however, fiber conduit is generally less expensive for concealed construction, but it cannot be used exposed. For circuits of such capacity that each phase must occupy an individual conduit, fiber is generally used for concealed construction and brass or aluminum for exposed work, since steel conduit would cause hysteresis losses under these conditions. Heavy current capacity circuits that must be run exposed are sometimes made up of a number of paralleled smaller circuits, each in a steel conduit. This construction has the advantage that skin effect and proximity effect are practically neutralized, but great care must be taken to make all the paralleled circuits of the same length and to equalize them thoroughly with heavy buses and tight connections at each end. Tile duct is used occasionally for power station wiring, but has the disadvantage that bends are difficult to make. Tile finds its greatest usefulness in straight duct lines. Tile and fiber must both be concealed in concrete for strength and protection.

Installation of the Conduit System in most cases must proceed with the building structure. Careful planning is necessary to avoid unnecessary bends and offsets. It generally costs less to imbed the conduits in the structure even at the expense of slightly thicker floors and walls in certain panels than to support the conduits exposed after the structure is in place. Conduits imbedded in concrete structures should have at least 2 in. clear space between them to allow the concrete to flow into place. Large groups of conduits should be installed in floor fills or wall fills, independent of the building structure. Wherever steel conduits are imbedded in cinders or cinder concrete they should be protected from the acid of the cinders by a coating of cement or other suitable material. Joints in conduits should be made water-tight and each run should be drained and vented if possible, to prevent the accumulation of condensed moisture. All metal conduits should be thoroughly grounded. Before cables are pulled, the conduits should be thoroughly cleaned and dried.

Pull boxes should be provided for long conduit runs or runs with difficult bends. A safe rule for the average case is to allow between pull boxes not over 50 ft with bends equivalent to two 90-deg bends, or not over 100 ft with bends equivalent to one 90-deg bend, or not over 200 ft without bends. Pull boxes should be of substantial, fireproof construction and of such size and shape that the cables will not have to be bent during installation to less inside radius than 6 times their outside diameters if rubber insulated or 7 times if cambric insulated or 16 times if paper insulated. The cables of these important main low-voltage circuits should be supported in the boxes by porcelain insulators, and where they enter the boxes they should be protected by fiber sleeves inside the conduit ends.

MAIN HIGH-VOLTAGE WIRING. Voltage is the chief concern in the design of this wiring. Rated current and short-circuit current should be investigated, but they do not usually control the design. Open wiring is almost universally accepted for these buses and circuits, whether located indoors or outdoors.

Two General Types of buses and connections are used: the **rigid type,** consisting of

copper tube or steel pipe supported on porcelain insulators; and the **flexible type**, consisting of stranded copper or aluminum cable supported on strain insulators. Combinations of the two types sometimes meet the conditions to best advantage. The general arrangement of the wiring and structures is materially affected by the mounting of the disconnecting switches, that is, whether with insulator stacks vertical or horizontal. Vertical stacks are generally preferred for the higher voltages on account of their greater stability; but for lower voltages, where either vertical or horizontal stacks are permitted, a more compact station can usually be arranged with disconnecting switches mounted with stacks horizontal.

Steel Structures and Supports for the flexible type of wiring are of two general types; trussed structures, suitable for the lower voltages, where spans are comparatively short; and dead-end towers, used generally for higher voltages where the bays are too large for economical trussing. For the trussed structures, the steel members may be solid rolled sections for short spans but are usually latticed. The structures should be designed for the stresses of the pull-off lines as well as those of the station buses and connections. Steel structures should be galvanized after fabrication and assembled with non-rusting bolts; or, if plain steel and riveted connections are used, the structure should be designed to facilitate routine painting.

AUXILIARY POWER AND LIGHT WIRING. The importance of these circuits justifies the highest grade of wiring available, but the voltage, normal current, and short-circuit current requirements are less severe than with the main low-voltage wiring. Conduit wiring of the general types described above is commonly used. Any open wiring should have flame-proof covering and should have insulators adequate for the short-circuit stresses.

CONTROL AND SIGNAL WIRING. This wiring should be of highest quality, and the layout should be as simple as possible to assist in quickly tracing circuits and in making extensions and additions. Color coding and systematic tagging of wires greatly assist the tracing of connections. All wires should be stranded to avoid breakage from vibration, and they should be of a design able to resist flooding or condensed water in the conduits. Current transformer secondaries and shunt leads should be of large size when the circuits are long.

Conduit Wiring is generally used. Each circuit or group of associated circuits should have a separate conduit to give utmost segregation and protection. The conduit system is generally so elaborate, particularly near the switchboards, that it is impracticable to attempt to install it in the structural slabs. The conduits are usually exposed, or concealed in a floor fill. With the floor-fill construction the structural slab is dropped from 6 to 12 in. below finished floor grade, leaving a space in which the conduits may be installed. The space is later filled around the conduits with cinder concrete or other suitable material, and the floor finish applied on top of the fill.

At the main switchboards, where most of the control wiring terminates, great care should be exercised to avoid congestion, to provide maximum flexibility for cross-connections between panels and for future work. At the same time, adequate separation and segregation of circuits should be afforded so that any fault that might develop will be confined to the circuit on which it originates. For large installations it has become usual practice to provide a **terminal room** below the switchboard room, where terminal boards, cross-overs, fuses, and other facilities can be systematically arranged and properly safeguarded.

SWITCHBOARD WIRING. This is sometimes done at the factory and sometimes at the job. The wire should have a flame-proof covering, and the insulation and covering should be so applied that it will not be injured by abrupt, right-angle bends. It should be applied neatly in vertical and horizontal runs and should be firmly supported in such a manner that each wire is accessible. To facilitate working on bench boards it is desirable to provide an opening in the floor below the bench with a platform at suitable height below floor grade so that the workman may stand below the bench or sit on the edge of the floor and conveniently reach all terminals. The wiring of gage boards containing water piping should be carefully barriered from the piping.

STATION GROUNDING SYSTEM. This includes a number of low-resistance ground connections, usually consisting of driven rods, buried plates, and water pipes all interconnected by heavy copper cables, to which the building steel and all electrical ground buses are securely connected. (See Section 14, Art. 95.)

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SECTION 14

POWER TRANSMISSION AND DISTRIBUTION

SYSTEMS

By W. A. Del Mar

ART.		PAGE
1.	Constant-potential or Multiple System	03
2.	Constant-current or Series System	04
3.	Direct-current Systems	05
4.	Alternating-current Systems	05
5.	Combined Direct- and Alternating-current Systems	06
6.	Relative Voltages and Weights of Copper Required for Various Systems	07
7.	Standard Voltages	07
8.	Efficiency of Distribution	08
9.	Regulation	08
10.	Voltage Control	09
11.	Balancing of Load	10
12.	Bibliography	11

ELECTRICAL DESIGN OF TRANSMISSION AND DISTRIBUTION SYSTEMS

By W. A. Del Mar, J. H. Palmer, F. W. Peek and R. A. Philip

13.	Definitions and Fundamental Relations	12
14.	Electrical Design of Direct-current Lines	12
15.	Electrical Design of Alternating-current Lines	16
16.	Factors Which Affect the Voltage and Power Loss in A-c Lines	18
17.	Methods of Calculating Voltage and Power Loss in A-c Lines	19
18.	Stability of Transmission Lines	28
19.	Corona	32
20.	Bibliography	36

CAPACITANCE AND INDUCTANCE TABLES

By W. A. Del Mar

21.	Capacitance	37
22.	Inductance	44
23.	Bibliography	46

POLE AND TOWER LINES

By Howard Enos

24.	Mechanical Design	54
25.	Wood Poles for Overhead Lines	78
26.	Steel Towers and Poles	93
27.	Cross-arms	102
28.	Insulators for Overhead Lines	105
29.	Insulator Pins and Hardware	112
30.	Conductor Installation	114
31.	Inspection of Pole and Tower Lines	117
32.	Bibliography	117

UNDERGROUND CONDUITS

By W. A. Del Mar

ART.		PAGE
33.	Terminology	119
34.	Use of Conduits	119
35.	Types of Ducts	120
36.	Splicing Chambers	121
37.	General Design of Conduit Lines	125
38.	Rodding and Wiring	126
39.	Specifications for Conduit Lines	127
40.	Installation of Conduit Lines	129
41.	Maintenance of Conduit Lines	130
42.	Bibliography	132

DISTRIBUTION

By W. A. Del Mar

43.	Distribution Circuits	132
44.	Overhead Distribution	136
45.	Underground Distribution	142
46.	Bibliography	142

BARE WIRES AND CABLES

By W. A. Del Mar

47.	Dimensions, Weights, and Resistances	143
48.	Strength, Elasticity, and Expansion Coefficients of Wires	161
49.	Cables for High Voltages	163
50.	Skin Effect and A-c Resistance	165
51.	Current-carrying Capacity of Bare Wires and Cables	166
52.	Tests of Bare Wires and Cables	167
53.	Specifications for Copper Wire and Cable	167
54.	Bibliography	171

INSULATED WIRES AND CABLES

By W. A. Del Mar

55.	Terminology	171
56.	Design and Construction	173
57.	Applications of Various Types	175
58.	Design	175
59.	Tests	186
60.	Current-carrying Capacity	192
61.	A-c Resistance and Reactance	215
62.	Flexibility	216
63.	Single-conductor Cables for Three-phase Systems	216
64.	Expansion and Contraction in Impregnated-paper Cables	217
65.	Ionization-free Cables for High Voltages	217
66.	Wire and Cable Specifications	218
67.	Installation	219
68.	Cable Joints	220
69.	Fault Location	223
70.	Weights of Insulated Wires and Cables	225
71.	Bibliography	226

WIRING OF BUILDINGS

By G. W. Zink

72.	General Requirements	226
73.	Systems of Wiring	227
74.	Standard Voltages for Lighting and Power	229
75.	Standard Frequencies	229
76.	Authorities Governing the Installation of Wiring	229
77.	Methods of Wiring	230
78.	Service Entrance Cable	238
79.	Vertical Riser Cables	238

ART. PAGE
80. Wire and Wiring Accessories........ 239
81. Wires and Cables.................. 240
82. Layout of a Wiring System....... ... 241
83. Grounding Practice for Interior Wiring Systems..................... 242
84. Residence Wiring.................. 243
85. Wiring of Large Buildings.......... 244
86. Designs and Plans....... 244
87. Wiring of Buildings for Miscellaneous Devices........................ 256
88. Bibliography...................... 257

GROUNDING OF ELECTRIC CIRCUITS AND ACCESSORIES

By R. F. Abele

89. Purposes of Grounding............. 258
90. Electric Shock from Grounded and Ungrounded Circuits............. 258
91. Fire Hazard...................... 259
92. Arcing Grounds.................... 259
93. Relaying.......................... 260
94. System Grounding.................. 260
95. Station Grounding................. 261
96. Pole Line Grounding............... 261
97. Cable Sheath Grounding............ 261
98. Ground Connections....... 262
99. Bibliography...................... 267

ELECTROLYSIS OF UNDERGROUND STRUCTURES

By W. A. Del Mar

100. Stray Currents.................... 267
101. Corrosion Efficiency and Passivity... 268
102. Distribution of Earth Currents...... 268
103. Structures Affected by Corrosion.... 269
104. Electrolytic Surveys............... 269
105. Mitigation of Electrolysis........... 272
106. Bibliography...................... 273

Note: Chapters by R. A. Philip in the second edition were based on those in first edition by R. A. Philip, H. F. Thomson and E. A. Ekern.

POWER TRANSMISSION AND DISTRIBUTION

SYSTEMS

Revised by W. A. Del Mar
Previous Revision by R. A. Philip

Circuits designed for transmitting relatively large amounts of power from one fixed point to another are called transmission lines; those for delivering small amounts at numerous points are called distribution circuits. Transmission lines usually have no or few branches, while it is characteristic of distribution circuits to have many branches. The various systems of transmitting and distributing electric energy for light and power may be classified under two general heads, viz., constant-potential or multiple systems and constant-current or series systems, and each of these may be subdivided into d-c and a-c systems.

1. CONSTANT-POTENTIAL OR MULTIPLE SYSTEM

In this system, which is the one principally used for electrical distribution, the voltage between conductors is kept as constant as practicable and the current varies as the load changes.

Direct or alternating current can be used equally well for certain purposes, principally those for which the heating effect of the current is used, including the lighting of incandescent lamps, cooking, and heating. For certain purposes, where the current effects chemical or physical changes, such as charging storage batteries and electroplating, direct current is essential; for other purposes, such as operating arc lamps, it is better. For motive power, the direct current is most favorable where acceleration, variable speed, and adjustable speed are desirable, whereas alternating current gives best results where uniform unvarying speed is desired. While the fields of the two kinds overlap to such a large extent that either kind can be used for general distribution, it is found that it is often more advantageous to use both. For low-voltage, underground distribution, direct current is advantageous because the heavy currents can best be carried on single conductor cables; no subway transformers and no small, high-tension fuses are required.

Alternating current may be supplied either directly from the generators in the power station or from the secondaries of transformers in substations. Direct current may be supplied either directly from d-c generators or from converter substations supplied with high-voltage alternating current.

LIGHT AND POWER CIRCUITS. For a load consisting of both electric lamps and electric motors the same circuit may be used throughout for both classes of service or entirely or partially independent circuits may be employed. The use of the same circuit throughout for light and power service has the advantages of reduced number of wires, transformers, and meters, reduced weight of wire, and reduced capacity of transformers and meters; the use of separate circuits for light and power has the advantage of requiring less capacity of feeder regulators and, sometimes, of less weight of wire for the same perfection of regulation on the lighting circuits. It also simplifies the problem of balancing the phases in polyphase distribution. In the business districts of many cities the Edison three-wire d-c system is used, in which case light and power are equally supplied from the same circuit, the motors as a rule being connected to the outside wires. In business districts where the lighting is done by alternating current the lighting is frequently on single-phase circuits and the power on either separate polyphase circuits or on 500-volt d-c circuits. In residential districts, where the lighting is done by alternating current, small motors are usually put on the same services as the lights, whereas larger motors are put on separate meters, services, and transformers, but the same primaries are used for both services. In factory districts when the power is the predominating load the incidental lighting is sometimes taken off the same services, but generally there are separate meters and transformers. Usually the same polyphase primaries are used for power and light

with separate transformers and secondaries; in these cases the lighting transformers are commonly distributed as equally as convenient between the several phases, in order to balance the load. See Section 15, Art 2, for effects of voltage variations on lamp performance.

TWO-WIRE SYSTEM. The simplest multiple system is the two-wire system, where all devices are connected directly in multiple. This system is used very extensively for d-c light and power, from isolated plants, for power circuits (usually 500 volts), and railway circuits from central stations. It is also used for single-phase a-c distribution for both primary and secondary circuits.

THREE-WIRE SYSTEM. The three-wire system, Fig. 1, is obtained by replacing the outgoing wire of one two-wire system and the return wire of a second two-wire system by a single wire, called the neutral. The voltage between the outside wires is then double the voltage between the neutral and either outside wire. For example, 110-volt lamps may be connected between the neutral and either outside wire, and 220-volt motors may be connected between the two outside wires, and both the lamps and motors be supplied with their rated voltage. The neutral wire carries a current which depends only upon the *difference* in the loads on the two sides of the system and their distribution. As a rule the neutral of a three-wire main is made equal in cross-section to each outside wire. With perfectly balanced load the three-wire system with all three wires of the same size results in a saving in copper of 62.5 per cent as compared with a two-wire system supplying the same load at the same regulation.

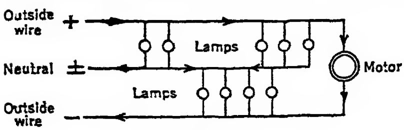

Fig. 1.

The three-wire system is used very extensively for d-c light and power distribution from central stations, also for large isolated plants and for alternating current for lighting on the secondary circuit. Three-wire systems are usually 110 volts on each side of neutral, or 220 volts between outsides, though there are several systems using 220 volts on each side and 440 volts between outsides.

The Edison three-wire distribution system, as used in the business sections of large cities, consists of a set of interconnected three-wire mains supplied by two-conductor or three-conductor feeders from one or more power houses or substations. The different feeders feed into the same set of mains at different points. At the power house or substation end the feeders are often all supplied from a common bus, though where there is a great difference in the length of the feeders there are sometimes two buses which are run in multiple when the load is light but separated when it is heavy, the short feeders on one called the low bus, and the long feeders on the other called the high bus, because of the relative voltages. As the feeders from the several substations connect to interconnected mains, the bus voltage in each is raised or lowered in accordance with the drop in its own feeders. All mains which connect feeding points supplied from a given bus are made large enough to allow large equalizing currents to flow through them, without requiring any great drop in the mains, in order to equalize the voltages at these points.

2. CONSTANT-CURRENT OR SERIES SYSTEM

The lamps or other devices are connected in series, and the current through them is kept constant, the voltages varying automatically to increase or decrease the energy delivered. It is the principal system used in the United States for street lighting, especially in the less-congested areas, but it is now little used for any other purpose. The current used for street lighting is usually 6.6 amp at voltages from 2200 to 11,000.

SOURCE OF CURRENT. Alternating current is usually obtained from a constant-current transformer on a constant potential a-c circuit.

CUT-OUTS, BY-PASSES, AND TRANSFORMERS. Switching out of lamps on series circuits is accomplished by short-circuiting them; this leaves the lamps charged to the potential of that point in the circuit. To make them safe to handle, "absolute cut-outs" are used which also disconnect both conductors to the lamp, leaving the circuit closed.

To avoid the excessively high voltage which would occur when the circuit opens, due to lamps burning out, an automatic by-pass is provided in multiple with the lamps, sometimes consisting of a piece of paper which punctures on a moderate rise of voltage, or of a choke coil which takes but little current at normal lamp voltage. Sockets for series incandescent lamps are arranged to close the circuit automatically when the lamp is withdrawn.

Transformers, one at each lamp, are sometimes used to insulate the secondary circuit

containing the lamp from the primary or for reducing the current for low-current lamps used in series with lamps taking higher current.

ADVANTAGES AND DISADVANTAGES OF SERIES LIGHTING. Constant-current or series systems have the advantage that low-voltage lamps may be used on high-voltage circuits without the expense, losses, or complication of transformation. They have the disadvantage that the lamps are dangerous to handle, the efficiency is low at light loads, and it is impracticable to distribute any large amount of power on a single circuit. In the series system, the current and consequently the loss in the conductor are the same irrespective of the load; in the multiple system, the current is proportional to the load, and the watts lost in the line therefore vary as the square of the load and the per cent loss directly as the load. For this reason the series system is not an economical one where the load varies and averages much below full load, which is the case with most commercial loads. For street lighting where all the lamps are turned on and off at once the efficiency at partial loads is of no importance. In constant-current systems the resistance of the circuit does not affect the uniformity of the voltage on the lamps at different parts of the circuit; with constant-potential systems it does. For a scattering load of lamps, such as street lamps, a uniform light can therefore be obtained from the several lamps, with a much smaller weight of copper and fewer wires, by using the series than by using the multiple system.

THURY SYSTEM. In Europe the constant-current or series system, with direct current of extra high voltage, has been used for long-distance power transmission under the name Thury sytem. The current is obtained by connecting several generators in series, and is utilized by a number of motors also in series. The advantages are simple switchboards, no transformers, and minimum strain on line insulators, the last due first to the fact that in direct current the effective voltage is as high as the maximum voltage, and second, that in a constant-current system the working voltage remains at its maximum value only during the short period when the load is also a maximum. Among the disadvantages are the necessity for insulating frames of generators and motors, need of speed governors on motors, and the necessity of converting the current by moving machinery in every case where it is used for lighting and in most cases for power. The Thury system is not used in the United States.

3. D-C SYSTEMS

Direct current has the following advantages for lighting: (1) safety, since the lines are in no way associated with high-voltage conductors; (2) freedom from power factor, reactance, and skin effect, which results in superior voltage regulation in heavily loaded low-voltage circuits; (3) the direct availability of the storage battery as a reserve and a load regulator; (4) the self-exciting and self-regulating features of d-c generators; (5) the superiority of d-c motors for adjustable speed service and for the operation of elevators and cranes; and (6) the marked superiority of d-c arc lamps. Direct current is generally used in isolated plants and in congested city districts because of the greater ease with which good voltage regulation is maintained.

4. A-C SYSTEMS

The simplest form of a-c system is the single-phase system, for which the connections are exactly the same as for the d-c systems. Both two-wire and three-wire circuits are used, the former for primary circuits and small secondary circuits and branches, and the latter for large secondary circuits. The principal use of single-phase circuits is for electric lighting and auxiliary uses, such as heating or cooking, and fan motors. For delivering large amounts of power a two-phase or three-phase system is generally used.

TWO-PHASE SYSTEMS. In this system there are two single-phase currents having a difference in phase of 90 deg, or a quarter of a cycle. These currents may be distributed on the three-wire, four-wire, or five-wire system.

Three-wire Two-phase Systems. Each single-phase current has a separate outgoing wire but unites in a common return wire. Each two-phase motor has two circuits, each connected between an outside wire and the return wire. The voltage between the two outside wires is 41 per cent greater than between outside wire and return wire, and the current in the return wire is 41 per cent greater than in each outside wire. This is an unsymmetrical system and has the disadvantage that even a balanced load will cause a distortion and unbalancing of the delivered voltage of the two phases because of the unsymmetrical drop in the common return wire.

Four-wire Two-phase System. Each of the two single-phase circuits has a complete, independent two-wire circuit. There are two variations: first where the circuits are

insulated from each other, in which case a cross between either wire of one circuit with either wire of the other will change the voltage stress to ground, but will not affect the delivered voltage or cause a short circuit; second, when the neutrals of the two circuits are connected. In the latter case, from each wire of one circuit to either wire of the other circuit, the voltage is 71 per cent of the voltage between wires of the same phase. The four-wire system with insulated phases is probably the most extensively used of the two-phase systems.

Five-wire Two-phase System. This is a modification of the four-wire system, with interconnected neutral in which the common neutral is extended as a fifth wire. Lamps may be connected from each of the four wires to the neutral. The five-wire system may be considered as two three-wire single-phase systems, one for each phase, with a common neutral wire.

THREE-PHASE SYSTEMS. In this system there are three single-phase alternating currents with a phase difference of 120 deg, or of one-third of a cycle. These currents may be distributed on the three-, four- or six-wire systems.

Three-wire Three-phase System. Each single-phase current has a separate outgoing wire; the three return currents neutralize so that no return wire is required. The three wires are necessarily interconnected, the voltages are usually the same between any two and the currents equal in each of the three conductors, provided the loads on the three phases are equal, i.e., provided the load is balanced. When equally loaded the voltage drops in the three conductors are equal and symmetrical. This is the most extensively used of the three-phase systems.

Four-wire Three-phase System. This is a modification of the three-wire system in which a neutral wire is extended as a fourth wire. Lamps or transformers may be connected from each of the three wires to the neutral, which carries only the unbalanced current, due to the differences in loading of the three phases. The voltage between the three outside wires is 73 per cent greater than from each outside wire to neutral.

Six-wire Three-phase System. If to a three-wire, three-phase system, three wires are added, one with voltage midway between that of each pair of outside wires, lamps may be divided into six groups, between the three outside wires and the three adjacent middle wires. The result is the same as though there were three single-phase three-wire circuits, one for each phase, with the six outside wires combined in pairs giving three common outside wires in place of the three pairs. When connected in this way, the three middle wires cease to be neutrals, as between the three there is a three-phase voltage equal to one-half of that between the three outside wires.

SIX-PHASE SYSTEM. This is used for circuits in the interior of substations (q.v.), such as from transformers to rotary converters, but is not used for distribution.

FREQUENCIES IN USE IN THE UNITED STATES. At present the two frequencies in most general use and adopted as standards in new work are:

60 cycles per second, used by the majority of companies operating a-c lighting systems.

25 cycles per second, generally used for a-c railway work or where the alternating current is to be converted into direct current before final use.

USE OF TWO FREQUENCIES. When the bulk of the load is direct current, but a small though important part is a-c lighting, two frequencies, 25 and 60 cycles, are sometimes used. Sometimes the two frequencies are generated by separate prime movers, though sometimes all current is generated at 25 cycles and the 60-cycle current obtained from frequency changers. (See Section 11, Motor-Generators.)

5. COMBINED D-C AND A-C SYSTEMS

Three-phase transmission and d-c distribution are readily combined but require either rotary or electronic conversion equipment. (See Section 11.) Electric railways, in particular, have commonly used rotary converters for 500-600 volt systems, but the demand for higher voltages on overhead trolley systems led to the use of motor-generators and electronic converters. (See Section 13, Art 24.)

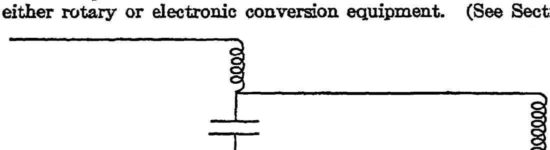

Fig. 2

A new system of transmission is being developed comprising conversion of constant-potential a-c power to constant-current d-c power, transmission of constant current d-c power, and reconversion to constant-potential a-c power.

A constant potential applied to a reactor and condenser in series may be made to yield constant current to a circuit derived from the condenser as shown in Fig. 2. This is known as the **Boucherot circuit.** (Steinmetz, Alternating Current Phenomena.)

Constant alternating current is obtained from such a circuit, transformed up, and rectified by means of inverters with grid-controlled vapor-discharge tubes. At the receiving end, the operations are reversed.

The system has the advantage that the d-c transmission line may be short-circuited without damage to line or equipment.

A description of an experimental line for 10 amp at 15,000 volts is given by Willis, Bedford and Elder, *J.A.I.E.E.*, January, 1935.

6. RELATIVE VOLTAGES AND WEIGHTS OF COPPER REQUIRED FOR VARIOUS SYSTEMS

The comparison of the various systems with respect to the weight of copper required, as tabulated below, is based upon the assumptions (1) that the energy delivered, (2) that the energy loss, and (3) that the maximum voltage strain on insulation between any wire and ground are respectively *the same* for all systems.

When the neutral or middle point of the system is grounded, then the maximum voltage strain on the insulation to ground is the maximum instantaneous value of the voltage between any outside wire and neutral. In the case of an alternating sine wave of voltage the maximum instantaneous value is equal to $\sqrt{2}$ times the effective value determined by a voltmeter. When the neutral is not grounded, an accidental ground on any leg will throw full line voltage across the insulation between any other leg and ground.

On this basis of comparison, assuming a sine wave of voltage and 100 per cent power factor for the a-c systems, the relative voltages between outside wires and the relative weights of copper are as follows:

Relative Voltages and Relative Weights of Copper

System *	Relative Voltages between Outers		Relative Weights of Copper *	
	Grounded neutral	Insulated neutral	Grounded neutral	Insulated neutral
	Per cent	Per cent	Per cent	Per cent
Direct-current (2-wire)	100	50	100	400
Single-phase (2-wire)	71	35.5	200	800
Two-phase (4-wire)	71	35.5	200	800
Three-phase (3-wire)	61	35.5	200	600

* Neutral wire not included; when neutral is added increase the figures given in the ratio of weight of neutral to combined weight of the outside wires for the system in question. For example, the addition of a neutral wire equal in size to either outside wire to a d-c 2-wire system with grounded neutral gives a relative weight of copper of $100 \times 1.5 = 150$ per cent.

EFFECT OF POWER FACTOR. To correct the above tabulation for a-c systems in cases where the power factor is less than unity, divide the relative weight of copper given by the square of the power factor (e.g., for a single-phase two-wire system with insulated neutral and 70 per cent power factor the weight of copper will be $800/0.70^2 = 1630$).

7. STANDARD VOLTAGES

Unless otherwise specified, the "voltage" of a polyphase system refers to the potential difference between phases.

VOLTAGES COMMONLY USED. The voltages most used are:

Direct current:

100 to 125 volts	Lighting, small power, and field excitation.
200 to 240 volts	Power.
500 to 600 volts	Power, electric urban railways.
1200 to 1500 volts	Interurban electric railways.
2400 to 3000 volts	Electric railway trunk lines.

Alternating current:

110–125 volts	Secondary distribution.
2,500 volts 4,500 volts 7,500 volts 15,000 volts	Primary distribution.
23,000 volts 34,500 volts 46,000 volts 69,000 volts 138,000 volts	Transmission.

The above voltages are the standard ones which are only approximated by the actual voltages in use. See also chapter on Distribution.

8. EFFICIENCY OF DISTRIBUTION

The ratio of the energy which is registered by the customers' meters, or which would be so registered if all customers had meters, to the energy supplied by the generator to the bus-bar in the generating station, may be called the overall efficiency of distribution, using the word distribution in a large sense to include transmission. The losses may be divided into several kinds: line loss, transformer loss, converter loss, meter loss and error, leakage, and unaccounted for. For some purposes it is useful to consider each loss from two points of view: (1) as a loss of energy, and (2) as a loss of power at full load; the first may be called the energy loss and the second the capacity loss. The corresponding efficiencies are usually designated as all-day or energy efficiency and the full-load or capacity efficiency, respectively.

FIXED AND VARIABLE LOSSES. The total energy loss consists of two components; (1) a fixed loss independent of load, including the core loss of transformers, the core loss, excitation, friction, and windage of rotating converting apparatus the loss in the shunt coils of meters, dielectric loss, the copper loss in constant-current circuits, and the loss in the arc of mercury-arc rectifiers for constant-current circuits; (2) a variable loss proportional to the square of the current, including the copper loss of constant-potential circuits and of transformers, the armature copper loss of rotating converting apparatus. The effect of the fixed loss on the per cent efficiency depends on the load factor of the load; that of the variable loss depends both on the load factor and the shape of the load curve.

REPRESENTATIVE LOSSES. For a lighting system, the full-load losses in primary feeders, primary mains, transformers, secondary mains, services, and meters may be expected to be as much as 17.5 per cent of the power generated, and the daily energy loss 33.3 per cent of the energy generated, giving 82.5 per cent capacity efficiency and 66.7 per cent energy efficiency respectively.

EFFECT OF NATURE OF LOAD. In making estimates of the efficiencies of particular systems, the effect of the following items should be considered: (1) relation of transformer capacity to maximum loads, (2) load factor, (3) shape of load curve, (4) power factor, (5) diversity factor.

9. REGULATION

An ideal constant-potential distribution would have one uniform, unvarying voltage, and would be said to have perfect regulation. The greater the variation from such constancy the poorer, i.e., the greater numerically, the regulation. The regulation is usually specified in percent variation, either above or below a standard mentioned. The standard is usually either the nominal voltage desired or the actual average voltage obtained.

EVIL EFFECTS OF POOR REGULATION. The evil effects of high voltage are short life of electric lamps, excessive speed of d-c motors, excessive exciting current of induction motors, and burning out of motors and other devices; on the other hand, low voltage greatly diminishes both the candlepower and efficiency of electric lamps, decreases the maximum power of motors, and increases the current which a motor will take for a fixed horsepower output. As electric lamps are much more sensitive to change of voltage than motors, separate circuits are often used for lighting and power, the former having devices for regulation which are omitted from the latter.

The following figures give roughly the quantitative effect of voltage variation between 5 per cent below and 5 per cent above normal:

Each per cent decrease in voltage decreases candlepower of tungsten incandescent lamps.................................3.5 per cent
decreases torque of induction motor......................... 2 per cent
Each per cent increase in voltage decreases the life of tungsten incandescent lamps*.. 10 per cent
increases the magnetizing current of induction motors........... 2 per cent

ORDINARY LIMITS OF REGULATION. Roughly, the maximum voltage variation at the lamps on a lighting system should never exceed 5 per cent, i.e., the regulation above or below normal should never be greater than 2.5 per cent, and should be as much less as is economically feasible; the voltage variation on power systems is usually about 10 per cent (5 per cent above or below normal) and is sometimes considerably more.

CALCULATION OF REGULATION. In order to calculate the variation in voltage at any receiving device or group of such devices from no load to full load, the voltage at the generating or substation or feeding point being assumed constant, the impedance drops in all parts of the distribution system must be calculated and properly combined. The various parts of the system to be considered are the house wiring, service wires (or leads from street mains to the house), secondaries, distribution transformers, primary mains, primary feeders, substation transformers, transmission lines, and power station transformers.

In making such calculations account should be taken of the fact that the loads or currents in the several parts of the system seldom have their maximum values at the same time. The maximum drop in house wiring will not occur simultaneously in all houses, nor will the maximum service drop occur together on all services, etc. Furthermore, the maximum house wiring drop for a given house may not occur at the same time as the maximum drop on the secondary mains, transformer or primary mains to which it is connected.

Effect of Line Reactance. The regulation of an a-c system of unity power factor will be poorer than that of a d-c system of the same copper efficiency (see tabulation above) because of additional drop due to line reactance.

Effect of Lagging Power Factor. A lagging power factor usually makes the regulation of an a-c system worse than it would be for unity power factor and therefore much worse than for a d-c system of the same copper efficiency.

Effect of Leading Power Factor. A leading power factor usually makes the regulation better than it would be for unity power factor and may give even better regulation than can be obtained from a d-c system of the same copper efficiency.

Effect of Currents in Neutral. Any current in the neutral wire of a balanced system produces a drop which tends to unbalance the voltage of the two sides. In the case of a three-wire d-c or single-phase system *1 per cent drop in neutral produces 2 per cent difference in the voltages on the two sides.* Voltage drop in the neutral therefore affects more seriously the regulation than an equal amount in the outside wires. If the currents in the neutral all have the same direction, say toward the station, the voltage drops in the various parts of it will be cumulative, and though the drop in each section may be small and no current may actually reach the station, the aggregate effect may be serious. The individual loads on the two sides of the neutral should be connected so that the unavoidable neutral currents flow alternately in each direction, thus causing drops in alternate directions thereby neutralizing each other over the total length of the circuit.

FEEDER NEUTRAL IN THREE-WIRE SYSTEMS. When the "feeder and main" system is used with a three-wire system, two-wire feeders should be used only on the outside wires, and only a single feeder neutral be used. That is, with respect to the neutral, the "tree" system gives better regulation for the same weight of copper than the feeder and main system, but with respect to the outside wires the latter system gives the better results.

10. VOLTAGE CONTROL

The more common methods of controlling the voltage at feeding points when the feeder and main system of distribution is used are the single bus system, the high and low bus system, and feeder regulators.

SINGLE BUS SYSTEM. All the feeders may be connected to a single bus and the bus voltage be raised as load increases so as to compensate for average drop of all the

* Average for 5 per cent increase in voltage; the first per cent increase in voltage decreases the life 13 per cent. (See Section 15, Art. 2.)

feeders on it. This method gives excessive voltage on such feeders as are comparatively short and low voltage on those that are long. Usually maximum voltage is carried in the evening during the lighting peak and lower voltage during the day, giving very poor regulation on power circuits having maximum load during the day and low load at night. This method is used on most small d-c and a-c distribution systems.

HIGH AND LOW BUS SYSTEM. The bus may consist of two parts which may be separated, and operated at different voltages, feeders of greater average drop connected to the "high" (voltage) bus and the others to the "low" (voltage) bus. The voltage of each bus is raised or lowered in accordance with average drop of feeders on it as in the single bus system. This method is used extensively on the Edison three-wire d-c systems, as noted above. The unequal drop of the several feeders on same bus is equalized by heavy interconnection through mains.

FEEDER REGULATORS. A feeder may have a separate regulator adjustable to compensate for its own drop. This method is sometimes used on d-c railway feeders and very extensively on a-c lighting feeders. A feeder or other distribution circuit which contains a voltage regulator is frequently referred to as a "boosted circuit." Boosted circuits are not usually interconnected but may be if the boosting is of the same nature in each. This system is the more flexible and economical for extensive distribution systems, since each feeder is independently regulated and may be proportioned in cross-section with regard to economy rather than inherent voltage regulation. Single-phase regulators are preferred for close control if the loads are subject to unbalancing.

CONTROL OF POLYPHASE SYSTEMS. In a polyphase system the load may be as equally divided among the phases as possible and the voltage regulated with respect to any one phase taken as representing the average of all; or the lighting load may be connected on a single phase and the voltage regulated for this phase alone. The former method is more common, as it permits of full output of all the phases of the generator being used and gives a more equal voltage on the several phases of polyphase motors.

TRANSFORMERS AS OUTSIDE BOOSTERS. An auto-transformer or an ordinary transformer may be connected to a feeder at any point and used as a "booster" to raise or lower the voltage. In an ordinary transformer the primary and secondary are connected in series thus converting the transformer into an auto-transformer; see Auto-Transformers. Such boosters are not adjustable and have a bad effect on the regulation as they give excessive voltage on the boosted part of line at light load.

To cut such a booster out of service without taking it off the circuit the primary coil must be open-circuited and the secondary short-circuited. *Caution:* the main circuit must be open before cutting out booster, because if the primary is opened while current is flowing in the secondary the booster becomes a step-up transformer and may give a dangerously high voltage on the primary. If, on the other hand, the secondary is short-circuited first, a destructive short-circuit current may flow through it and the primary, and when the primary is opened a dangerous arc will form.

USE OF TRANSFORMERS OF DIFFERENT RATIO. Transformers are also made with taps so that a uniform secondary voltage can be obtained with a varying primary voltage. The plan is not a good one, because if the difference in ratio is correct for uniform voltage at full load it gives unequal voltages at light load. It also has the very serious disadvantage that the haphazard changing of transformer ratios by ignorant linemen to compensate for dim light, due perhaps to lamps already blackened by excessive voltage, makes it impossible to carry out any systematic plan for securing the best average regulation for the system as a whole.

11. BALANCING OF LOAD

In three-wire d-c or single-phase systems and in all polyphase systems supplying single-phase load it is necessary to balance the load approximately between the two sides or the several phases as the case may be. Unbalanced load has two bad effects: (1) it loads the two sides or phases of the system unequally, making it impossible to get full output out of the lightly loaded side or phase without overloading the other; (2) it makes the regulation of the system worse by causing high voltage on the lightly loaded side or phase and low voltage on the other.

The first difficulty is not serious, because through the conduction of heat from the loaded to the unloaded coils of transformers and generators the machine capacity is not reduced in proportion to the unbalance; for moderate unbalancing, say up to 10 per cent greater load on one side or phase than on the other, it is doubtful if any appreciable effect could be discovered. As the total load usually consists of a great number of small parts, a very little foresight in dividing the load in the first place between the sides or phases, and in suitably distributing subsequently connected load, will give a balance

good enough for all practical purposes. When polyphase alternators were first installed for supplying existing single-phase lighting circuits, it was supposed to be important to have a close balance of load between the phases, and early switchboards were therefore provided with transfer switches for throwing single-phase feeders from phase to phase. It was later found that, instead of being necessary to transfer the load from phase to phase, following the diurnal or annual variations of load, the circuits could stay on the same phase indefinitely and that transfer switches were unnecessary and undesirable. The time when rebalancing of this kind is necessary is usually when new circuits are established, at which time the changes of connections are best made by changing the taps to the bus-bars.

MOTOR-GENERATOR BALANCER. On d-c three-wire systems the difference in current on the two sides may be balanced without taking the neutral current to the generator by a balancer consisting of two similar machines mechanically coupled and electrically connected in series. Each machine is wound for the voltage of one side of the system (110 volts for a 110/220-volt system); the common connection between the machines is connected to the neutral and the other two terminals to the two outsides; see Fig. 3 (the field windings are omitted for clearness). The unit acts as a motor-generator (q.v.), whichever machine happens to be on the light side being the motor for the time being and the other the generator. By strengthening the field of the one on the side where the voltage is low and weakening that of the other, so as to keep the same total voltage, the voltages on the two sides as well as the currents may be balanced if necessary.

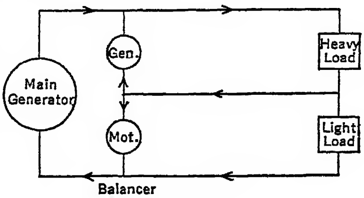

FIG. 3

The output of the balancer generator and the input of the balancer motor are practically equal to each other, neglecting the losses in the machines, and each is equal to

$$\frac{P_1 - P_2}{2}$$

where P_1 is the load on the heavily loaded side of the system and P_2 the load on the lightly loaded side. For example, if the load on one side is 110 kw and that on the other side 90 kw, the load on each unit of the balancer is 10 kw.

It would, however, be unsafe to use a balancer as small as such calculations for the normal unbalancing would indicate as correct, because in case of a short circuit on one side of the system, or of loss of a large amount of load on one side, caused, say, by the blowing of fuses, the balancer would be dangerously overloaded. This may not only destroy the balancer, but may burn out many lamps and motors on one side owing to excessive voltage. In small systems, say up to 100-kw total capacity, the balancer should be able to operate momentarily with any loading up to the capacity of the main generator, and in large systems should have capacity sufficient to burn off a short circuit on one side of the system without creating an unduly high voltage on the lightly loaded side.

DYNAMOTOR AS A BALANCER. A smaller and cheaper balancer is obtained by using a dynamotor (q.v.), which is a single machine with two windings on one armature and two commutators. The armatures are connected in series and balance the currents, but as there is only one field the voltages cannot be balanced.

A-C BALANCING COIL. On a-c single-phase three-wire systems it is usual to obtain the neutral from the middle point of the coil of the transformer supplying the current. It can, however, be obtained at any point of the two-wire circuit without going back to the transformer by using a balance coil. A balance coil is a transformer with two similar windings connected in series across the circuit and with the neutral wire connected to the common point between the windings. Whichever coil is on the lightly loaded side acts as the primary and the other as the secondary, thereby balancing the circuit by transferring one-half of the difference in load from the heavily loaded side to the lightly loaded side. Such coils, which are essentially auto-transformers with a 2:1 ratio, may be used in various ways: to obtain the neutral for an unbalanced three-wire circuit from a two-wire circuit; to supply a 110-volt load from a 220-volt circuit; to supply a large 110-volt load from a 110/220-volt circuit without connecting to the neutral. In practice balance coils are not much used as the neutral can more cheaply be obtained from the transformer.

12. BIBLIOGRAPHY

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ELECTRICAL DESIGN OF TRANSMISSION AND DISTRIBUTION SYSTEMS

Revised by W. A. Del Mar

13. DEFINITIONS AND FUNDAMENTAL RELATIONS

Previous Revision by R. A. PHILIP

The following definitions and relations apply to all types of transmission and distribution lines.

GENERATOR END AND LOAD END. By the generator end of the line is meant the end which is connected to the source of power (either directly or through transformers), and by the load end is meant the end which is connected to the load or substation which is supplied with power over the line.

PER CENT POWER LOSS (Q). By per cent power loss as used in this chapter is meant the percentage ratio

$$Q = 100 \cdot \frac{\text{Total power lost in the line}}{\text{Total power delivered at load end}} \tag{1}$$

Hence, if P is the power delivered, then the total power supplied to the line and load is

$$P_0 = P\left(\frac{100 + Q}{100}\right) \tag{2}$$

PER CENT VOLTAGE LOSS (D). By per cent voltage loss as used in this chapter is meant the percentage ratio *

$$D = 100 \cdot \frac{(\text{Voltage at generator end}) - (\text{Voltage at load end})}{\text{Voltage at load end}} \tag{3}$$

Hence calling E the voltage at the load end, the voltage at the generator end is

$$E_0 = E\left(\frac{100 + D}{100}\right) \tag{4}$$

The per cent voltage loss allowed under various conditions is discussed in detail in Article 9; the allowable voltage loss is usually between 2 and 20 per cent, the most common figure, for transmission lines, being 10 per cent.

In a d-c line with a single load at its far end, the per cent power loss and the per cent voltage loss are always equal, but in an a-c line the per cent voltage loss may be either greater or less than the per cent power loss, depending upon the constants of the line and the power factor of the load, or may even be negative, i.e., there may be an actual rise of voltage at the load end above the voltage at the generator end; see below.

EFFICIENCY OF TRANSMISSION. By the efficiency of transmission is meant the percentage ratio

$$100 \cdot \frac{\text{Power output of line at load end}}{\text{Total power input to line at generator end}} \tag{5}$$

The per cent efficiency is related to the per cent power loss Q as follows:

$$\text{Per cent efficiency} = \frac{10{,}000}{100 + Q} \tag{6}$$

14. ELECTRICAL DESIGN OF D-C LINES

Previous Revision by R. A. PHILIP

Two types of problems arise: (1) given a definite line with known constants, what is the power loss and voltage loss for a given load; and (2) to transmit a given amount of power a given distance with a given allowable loss, what will be the size and weight of the conductor required? In the following paragraphs are given the necessary formulas for the several cases. (See also Section 17.)

TWO-WIRE LINE; CONCENTRATED LOAD AT FAR END. Let

E = volts between wires at the load end of the line.

$P = \dfrac{EI}{1000}$ = kilowatts taken by load.

* In an a-c line the difference in the numerator of this ratio is the algebraic difference between rms values of the two voltages, not the vector difference.

$I = \dfrac{1000\,P}{E}$ = amperes taken by load.

l = length of each line wire in feet.

r = ohms per 1000 feet of conductor; see tables in chapter on Bare Wires and Cables.

$R = \dfrac{rl}{500}$ = total resistance of line (both conductors), in ohms.

The following relations then hold:

$$\text{Total kilowatts lost} = p = \frac{RI^2}{1000} = \frac{rlI^2}{500{,}000} = \frac{2\,rlP^2}{E^2} \tag{7}$$

$$\text{Total volts lost} = v = RI = \frac{rlI}{500} = \frac{2\,rlP}{E} \tag{8}$$

$$\text{Per cent power loss} = Q = \frac{100\,p}{P} = \frac{rlI^2}{5000\,P} = \frac{200\,rlP}{E^2} \tag{9}$$

$$\text{Per cent voltage loss} = D = \frac{100\,v}{E} = \frac{rlI}{5\,E} = \frac{200\,rlP}{E^2} \tag{10}$$

$$\left.\begin{matrix}\text{Resistance of each con-}\\ \text{ductor per 1000 ft}\end{matrix}\right\} = r = \frac{500\,v}{lI} = \frac{QE^2}{200\,lP} = \frac{DE^2}{200\,lP} \tag{11}$$

CALCULATION OF SIZE AND WEIGHT OF CONDUCTOR FOR CONCENTRATED LOAD. From the value of r calculated from any one of the relations given in eq. (11), the size of wire may be found from the tables in the chapter on Bare Wires and Cables; the next larger size of wire (next smaller gage number) should usually be chosen when the calculated resistance lies between that of two commercial sizes. The wire selected must also have sufficient current-carrying capacity; see Arts. 51 and 60. For outside lines, however, the current-carrying capacity will in general be ample unless the allowable voltage loss is excessive. For an outside overhead line a wire smaller than No. 6 A.W.G. (or B. & S.) gage is seldom used, chiefly on account of its lack of mechanical strength.

Let w = weight per 1000 ft of the wire finally selected; then

$$\text{Total weight of conductor in pounds} = W = \frac{wl}{500} \tag{12}$$

DIRECT CALCULATION OF TOTAL WEIGHT OF CONDUCTOR (W); TWO-WIRE LINE. For preliminary estimates it is sometimes convenient to calculate the total weight of conductor directly, without reference to a wire table. The total weight of conductor for a two-wire line with concentrated load at its end is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E}\right)^2 \text{ pounds} \tag{13}$$

where P is the power taken by the load, l the length of the line (length of each wire), E the voltage at the load, Q the per cent power loss (= per cent voltage drop for 2-wire d-c line), and K a constant depending upon the material of the conductor and the units in which P, l, and E are expressed, as given in Table I.

Table I. Values of K in Formula 13

Material	E in volts, l in feet, P in kilowatts	E in kilovolts, l in miles, P in kilowatts
Copper (98 per cent conductivity)	13.5	380
Aluminum (61 per cent conductivity)	6.5	185
Any material of specific gravity δ having a conductivity of c per cent at 20° C	$\dfrac{141\,\delta}{c}$	$\dfrac{3940\,\delta}{c}$

NOTE: The values of K given for copper and aluminum are about 5 per cent greater than their theoretical values to allow for stranding, higher working temperature, etc.; 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20 deg cent.

Example: Two-wire D-c Line, Concentrated Load. A load of 100 kw is to be transmitted over a two-wire line to a motor operating at 230 volts, the motor being 1000 ft from the power-house switchboard. For a 10 per cent power loss or voltage drop in the line, the approximate total weight of copper required is, from eq. (13),

$$W = \frac{13.5 \times 100}{10}\left(\frac{1000}{230}\right)^2 = 2550 \text{ lb}$$

From eq. (11) the resistance per 1000 ft is

$$r = \frac{10 \times (230)^2}{200 \times 1000 \times 100} = 0.0264 \text{ ohm per 1000 ft}$$

The nearest even circular mil size is 400,000 cir mils (stranded), which has a resistance of 0.0270 ohm per 1000 ft at 77 deg fahr (see Bare Wires and Cables), and a weight of 1240 lb per 1000 ft. From eq. (12) the total weight of conductor is then

$$W = \frac{1240 \times 1000}{500} = 2480 \text{ lb}$$

This wire, if bare, weatherproofed, or insulated with paper or varnished cambric, will safely carry the required current of 100,000/230 = 435 amp, but if rubber insulated and mounted indoors, a larger wire should be required, viz., 600,000 cir mils, according to the National Electric Code (see Arts. 51 and 60).

CALCULATION OF TWO-WIRE D-C LINE IN TERMS OF VOLTAGE AT GENERATOR END. When the volts E_0 at the generator end are given instead of the volts E at the load end, the calculations for a line of given total resistance of R ohms with a concentrated load of P kilowatts at the load end may be made in the same manner as above by first finding the volts E at the load by the formula

$$E = \frac{E_0}{2}\left[1 + \sqrt{1 - \frac{4000\, RP}{E_0^2}}\right] \tag{14}$$

For an efficiency of transmission of less than 50 per cent, the sign before the radical should be − instead of +, but an efficiency of less than 50 per cent practically never occurs in power transmission. It is of interest to note that for an efficiency of 50 per cent $P = E_0^2 \div (4000\, R)$ which is the maximum power which can be delivered at the far end of the line for a given impressed voltage at the generator end.

When E has been calculated by this formula (14), the formulas (7) to (13) above may be applied directly.

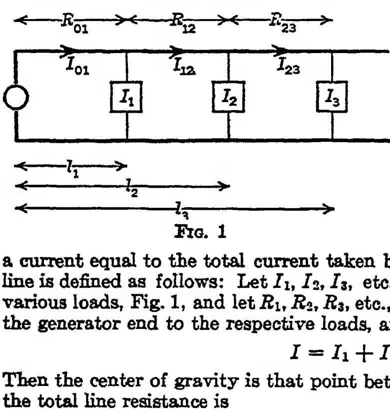

FIG. 1

TWO-WIRE LINE; DISTRIBUTED LOAD. When a line supplies a number of loads at different distances from the generator end, the voltage loss to the far end of the line is the same as would be produced by a load concentrated at the "center of gravity" of the line and taking a current equal to the total current taken by all the loads. The center of gravity of the line is defined as follows: Let I_1, I_2, I_3, etc., be the currents taken from the line by the various loads, Fig. 1, and let R_1, R_2, R_3, etc., be the total line resistances (both wires) from the generator end to the respective loads, and put

$$I = I_1 + I_2 + I_3 + \ldots \tag{15}$$

Then the center of gravity is that point between which and the generator end of the line the total line resistance is

$$R_g = \frac{R_1 I_1 + R_2 I_2 + R_3 I_3 + \ldots}{I} \tag{16}$$

When the line conductor has the same cross-section throughout its length, then the center of gravity is at the distance,

$$l_g = \frac{l_1 I_1 + l_2 I_2 + l_3 I_3 + \ldots}{I} \tag{16a}$$

from the generator end, where l_1, l_2, l_3, etc., are the distances of the respective loads from the generator end.

The total voltage loss to the far end of the line is then

$$v = R_g I = \frac{r l_g I}{500} \tag{17}$$

where the distance l_g is in feet and r is the resistance of the conductor per 1000 ft, the second relation in (17) holding only when the conductor has the same cross-section throughout its length.

The total kilowatts lost in the line are

$$p = \frac{1}{1000}\left(R_{01} I_{01}^2 + R_{12} I_{12}^2 + R_{23} I_{23}^2 + \ldots\right) \tag{18}$$

where, referring to Fig. 1, R_{01} = total resistance (both wires) from 0 to 1, R_{12} = total resistance (both wires) from 1 to 2, etc., and $I_{01} = I_1 + I_2 + I_3 + \ldots$ = the current in the line from 0 to 1, $I_{12} = I_2 + I_3 + \ldots$ = the current in the line from 1 to 2, etc. When

the cross-section of the line conductor is the same throughout its length eq. (18) may be also written

$$p = \frac{r}{500{,}000}\left(l_{01} I_{01}^2 + l_{12} I_{12}^2 + l_{23} I_{23}^2 + \ldots\right) \tag{18a}$$

where the distances l_{01}, l_{12}, l_{23}, etc., are as shown in Fig. 1, and are measured in feet, and r is the resistance of the line conductor per 1000 ft.

Calculation of Size and Weight of Conductor for Distributed Load. For a conductor of the same cross-section throughout, the required resistance per 1000 ft for a given voltage loss of v volts to the end of the line may be calculated from the formula

$$r = \frac{500\, v}{l_g\, I} \tag{19}$$

where l_g, expressed in feet, and I are given by formulas (16a) and (15). The size and weight of conductor can then be found by reference to the wire tables in the chapter on Bare Wires and Cables.

When the loads are far apart and the smaller loads are farthest from the generator, it is sometimes advisable to use different sizes of conductors for the various portions of the line. For a given voltage loss v to the end of the line, the minimum weight of conductor is obtained when the volts lost per unit length of conductor in each section of the line is proportional to the square root of the current in this portion of the line. For minimum total weight of conductor then, referring to Fig. 1, the resistance per 1000 ft *of wire* for the section between 1 and 2, say, must be

$$r_{12} = \frac{1}{\sqrt{I_{12}}} \cdot \frac{500\, v}{l_{01}\sqrt{I_{01}} + l_{12}\sqrt{I_{12}} + l_{23}\sqrt{I_{23}} + \ldots} \tag{20}$$

where the lengths are in feet; and similarly for the other sections. The weight of wire for each section may then be found by reference to the wire tables in the chapter on Bare Wires and Cables, and the total weight W can then be computed. Vice versa, for a line proportioned in this manner the voltage loss to the end of the line for a given total weight of copper will be a minimum.

A line proportioned in this manner, however, does not give minimum power loss for the total weight of conductor used. For a given total weight of conductor the total power loss will be a minimum when the power loss per unit length of conductor in each section of the line is directly proportional to the current in this section, i.e., when the voltage loss per unit length in each section is the same, which means that the weight per unit length of each section must be proportional to the line current in this section. Whence letting W be the total weight of the conductor in pounds, then the power loss will be a minimum when the section from 1 to 2, say, has a weight in pounds per 1000 ft of

$$w_{12} = I_{12} \frac{500\, W}{l_{01} I_{01} + l_{12} I_{12} + l_{23} I_{23} + \ldots} \tag{21}$$

where the lengths are in feet; and similarly for the other sections. The size of wire for each section may be obtained from the wire tables in the chapter on Bare Wires and Cables. When the sizes as found by the two formulas (20) and (21) differ considerably, the choice will depend upon which is the more important, minimum power loss or minimum voltage loss.

THREE-WIRE D-C LINE. When a three-wire circuit is exactly balanced, i.e., when the loads between each of the two outer wires and the neutral are the same and are connected to the neutral at the same point or points, no current flows in the neutral wire. The formulas given above for a two-wire line then apply directly to the case of a balanced three-wire line, noting however that the E (= volts between wires) in these formulas is to be taken as the volts between the *outer* wires, and that the weight as calculated by the above formulas is the weight of the two outer wires. The neutral wire is usually made equal in size to each outer wire, but when only slight unbalancing is expected it is sometimes made smaller. When the neutral is made equal in size to the outer wire the total weight of the three conductors will be 50 per cent more than that given by formula (13), when E in this formula is taken equal to the volts between the outer wires.

The exact calculation of the voltage loss and power loss when the loads on the two sides of the system are different and are connected at different points is somewhat complicated, but can always be effected by the application of Kirchhoff's laws for an electrical network; see Section 3.

15. ELECTRICAL DESIGN OF A-C LINES

Previous Revision by R. A. Philip

As a rough guide in fixing upon a preliminary design, the following facts should be noted; complete formulas for the various calculations required are given later.

1. A power loss of approximately 10 per cent of the delivered power is usually allowed.
2. A line voltage of approximately 1000 volts per mile of line is common practice for long-distance lines not over 150 miles in length; that is, for a 10-mile line a line voltage of 10,000 volts would be employed; for a 100-mile line a line voltage of 100,000 volts would be used. The maximum line voltage at present (1936) employed is 238,000 volts, and the maximum distance of transmission is 350 miles.
3. On the basis of 1000 volts per mile of line, unity power factor at the load, a 10 per cent power loss, and copper at 10 cents per pound, the cost of the copper required for a three-phase line is $2.66 per kilowatt delivered, and for a single-phase or two-phase four-wire line $3.45 per kilowatt delivered.

TOTAL WEIGHT OF CONDUCTOR FOR A-C LINES. The size and total weight of the conductor required for any conditions † of length, power delivered, power factor, line voltage, and power loss may be calculated as follows:

Let E = voltage between wires at the load end of the line.
P = total power taken by all phases of the load.
$\cos\phi$ = power factor of load, as a decimal.
l = length of line (= length of each line wire).
Q = allowable total power loss in per cent of delivered power.

Then the total weight of all conductors is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E\cos\phi}\right)^2 \text{ pounds} \tag{22}$$

where K is a constant depending upon the number of phases and wires, the material of the conductor, and the units in which the various quantities are expressed, as given in Table II.

Table II. Values of K in Formula 22

Material and Units	Single-phase or Balanced 4-wire 2-phase *	Balanced 3-wire 3-phase
Copper (98 per cent conductivity):		
E in volts, l in feet, P in kilowatts	13.5	10
E in kilovolts, l in miles, P in kilowatts	380	280
Aluminum (61 per cent conductivity):		
E in volts, l in feet, P in kilowatts	6.5	4.9
E in kilovolts, l in miles, P in kilowatts	185	140
Any material of specific gravity δ having a conductivity of c per cent at 20° C:		
E in volts, l in feet, P in kilowatts	$\frac{141\,\delta}{c}$	$\frac{106\,\delta}{c}$
E in kilovolts, l in miles, P in kilowatts	$\frac{3940\,\delta}{c}$	$\frac{2950\,\delta}{c}$

Note.—The values of K given for copper and aluminum are taken about 5 per cent greater than their theoretical values to allow for stranding, higher working temperatures, etc.; 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20 deg cent.

* For a 3-wire 2-phase system with the middle conductor having a cross-section equal to $\sqrt{2}$ times the cross-section of either outer wire multiply these constants by 0.85, taking for E the voltage (volts or kilovolts) between the middle wire and either outer wire.

COMMERCIAL SIZE OF CONDUCTOR AND CORRESPONDING TOTAL WEIGHT. Formula (22) takes no account of the available commercial sizes of wire. These sizes differ successively by approximately 25 per cent in cross-section. The weight can also be determined by calculating the resistance of the required conductor per 1000 ft or per mile, and taking from the wire tables in the article on Bare Wires and Cables, the nearest commercial size. Neglecting the charging current, the required resistance per unit length of wire is

$$r = \frac{K_1\,Q(E\cos\phi)^2}{lP} \text{ ohms} \tag{23}$$

† These formulas are based on the assumption that the charging current is negligible in comparison with the load current, which condition is practically realized in all but the longest high-voltage lines; formulas for power loss taking the charging current into account are given later.

where K_1 is a constant depending on the number of phases and wires and the units in which the other quantities are expressed, as given in Table III.

From the wire table the corresponding weight (w) per unit length of conductor having a resistance nearest to that calculated by formula (23) is obtained; the total weight of conductor, including all wires, is then

$$W = K_2 wl \quad \text{pounds} \tag{24}$$

where K_2 is a constant depending upon the number of phases and wires and the units in which w and l are expressed, as given in Table III.

Table III. Values of K_1 and K_2 in Formulas (23) and (24)

Units	Single-phase	Balanced 4-wire 2-phase *	Balanced 3-wire 3-phase
E in volts, l in feet, P in kilowatts, r in ohms per 1000 feet and w in pounds per 1000 ft. . .	$K_1 = 0.005$ $K_2 = 0.002$	$K_1 = 0.01$ $K_2 = 0.004$	$K_1 = 0.01$ $K_2 = 0.003$
E in kilovolts, l in miles, P in kilowatts, r in ohms per mile and w in pounds per mile. . . .	$K_1 = 5$ $K_2 = 2$	$K_1 = 10$ $K_2 = 4$	$K_1 = 10$ $K_2 = 3$

* The values of K_1 given in this column when used in formula (23) will give the resistance per 1000 ft or per mile of either outer wire in a 3-wire 2-phase system; the middle wire should, for the same energy loss per pound, have a cross-section 41 per cent greater than either outer, but when commercial sizes (A. W. G.) are used, either a wire one gage number smaller (25 per cent greater cross-section) or two gage numbers smaller (60 per cent greater cross-section) may be used; in the first case the corresponding value of K_2 is 0.81 times the values given in this column and in the second case 0.90 times the values given in this column.

Current per Wire; Heating of Line Conductors. The size of wire as determined from formula (23) must be ample to carry the required current without overheating. Heating of the line conductors is seldom a limitation in outside overhead lines, but for inside wiring or underground cables the temperature rise may set a limit to the size of wire which may be used. It is therefore always wise to determine the current which the conductor must carry, and make sure that the wire is sufficiently large not to overheat; see articles on Bare Wires and Cables, Insulated Wires and Cables, for tables of current-carrying capacity under various conditions. (Arts. 51 and 60.)

The current per line wire in amperes may be calculated from the following formulas, in which E *is the kilovolts* between wires at the load end, P *the total kilowatts* (all phases) delivered to the load, and $\cos\phi$ the power factor of the load as a fraction.

$$\left.\begin{aligned} &\text{Single-phase:} && I = \frac{P}{E\cos\phi} \\ &\text{Two-phase,* 4-wire, balanced:} && I = \frac{P}{2E\cos\phi} \\ &\text{Three-phase, 3-wire, balanced:} && I = \frac{P}{\sqrt{3}E\cos\phi} \end{aligned}\right\} \tag{25}$$

Example: Calculation of Weight and Size of Conductor for a Three-phase Line. A load of 20,000 kw is to be transmitted by means of an overhead three-phase line of copper wire to a substation 50 miles away operating at 60,000 volts between wires, the frequency being 25 cycles per second, and the power factor of the load 80 per cent with the current lagging; a power loss of 10 per cent of the delivered power to be allowed. From formula (22) the required total weight of copper is

$$W = \frac{280 \times 20{,}000}{10}\left(\frac{50}{60 \times 0.8}\right)^2 = 607{,}000 \text{ lb.}$$

From formula (23) the required resistance per mile of conductor is

$$r = \frac{10 \times 10(60 \times 0.8)^2}{50 \times 20{,}000} = 0.231 \text{ ohm per mile}$$

The nearest commercial size is 250,000 cir mils (stranded), which has a resistance of 0.228 ohm per mile at 77 deg fahr (see Bare Wires and Cables), and a weight of 4080 lb per mile. From formula (24) the total weight is then

$$W = 3 \times 4080 \times 50 = 612{,}000 \text{ lb}$$

* E is here the volts between the two wires of the same phase. This formula also gives the current in each outer of a balanced 2-phase 3-wire line, E being the kilovolts between either outer and the middle wire; the current in the middle is $\sqrt{2}$ times the current in each outer.

The current corresponding to the given load is, from formula (25),

$$I = \frac{20,000}{\sqrt{3} \times 60 \times 0.8} = 241 \text{ amp.}$$

which will give a negligible temperature rise in the wire. See Current-carrying Capacity in article on Bare Wires and Cables, Art. 51.

CALCULATION OF SIZE AND WEIGHT OF CONDUCTORS FOR A GIVEN PER CENT VOLTAGE LOSS. The voltage loss in an a-c line depends not only upon the resistance of the line, but also upon the line reactance, and in long lines upon the electrostatic capacity of the line. It is therefore impossible to express directly in a simple formula the size or weight of the wire in terms of the voltage loss. The most practical method of making such calculations is to assume first that the per cent power loss is equal to the given per cent voltage loss, and calculate the size by formula (23); then using this size of wire calculate the per cent voltage loss by the formulas given below. If this calculated voltage loss differs appreciably from the given voltage loss, choose the next larger or smaller size of wire (accordingly as the calculated loss is greater or less than the given loss) and recalculate the voltage loss, and so on, until the proper size of wire has been found.

16. FACTORS WHICH AFFECT THE VOLTAGE AND POWER LOSS IN A-C LINES

Previous Revision by R. A. Philip

Owing to the inductance and electrostatic capacity the per cent voltage loss in an a-c line is not so easily calculated as the voltage loss in a d-c line, and is in general different from the per cent power loss by an amount dependent upon the inductance and capacity of the line, the frequency, and the power factor of the load.

DETERMINATION OF LINE CONSTANTS. The four fundamental line constants are the resistance (r) and inductance (L) of the line conductors per unit length and the capacitance (C) and leakage conductance (G) per unit length. For all but the shortest transmission lines the mile is usually the most convenient unit of length, and this unit will be used throughout the remainder of this article unless distinctly stated otherwise. Tables of resistance, inductance, and capacitance both per mile and per 1000 ft are given respectively in the chapters on Bare Wires and Cables and Capacitance and Inductance Tables. From the inductance and capacitance per mile or per 1000 ft may be calculated for any given frequency the reactance $x\ (=2\pi fL)$ and the capacitance susceptance * $b\ (=2\pi fC)$ for the corresponding unit of length. In the last two chapters just mentioned are given full tables of reactance and of capacitance susceptance per mile for frequencies of 60 and 25 cycles per second; dividing the numerical values given in these tables by 5.28 will give the corresponding quantity per 1000 ft. For any other frequency of f cycles per second, multiply the numerical values given in the tables for 25 cycles by the ratio $f/25$; i.e., for 40 cycles the reactance is 1.6 times the reactance for 25 cycles.

Allowance for Skin Effect in Conductors. The skin effect is practically negligible at 25 cycles for all copper conductors smaller than 1,000,000 cir mils, and at 60 cycles for all copper conductors smaller than 450,000 cir mils. The corresponding limiting sizes for aluminum are about 30 per cent larger. The skin effect is quite appreciable in copper or aluminum cables with a steel core; it is usual to neglect the conductivity of the steel core entirely in calculating the resistance of such cables. See Arts. 50 and 61.

Apparent Resistance and Reactance in Unsymmetrical Arrangements of Wires. When the three wires of a three-phase line are so arranged that they form the three edges of an equilateral prism the reactance of each wire is the same as for one wire of a two-wire line. However, when the wires are arranged all in one plane, as is frequently done, the unequal mutual induction sets up a reactive electromotive force in each outer wire which is not in quadrature with the current in this wire; see eq. (25) in Art. 22. As a result, both the apparent resistance and the apparent reactance of each outer wire are different from its true resistance and reactance. Let r = the true resistance per mile of each wire in ohms, x = reactance per mile of each wire in ohms, as given in the tables under Inductance and Inductive Reactance; and f = frequency in cycles per second. Then the apparent resistances and reactances per mile of the three wires, No. 2 being the middle wire, are:

$$\left.\begin{array}{lll} r_1 = r + 0.00121f & r_2 = r & r_3 = r - 0.00121f \\ x_1 = x + 0.00070f & x_2 = x & x_3 = x + 0.00070f \end{array}\right\} \quad (26)$$

* The capacitance susceptance to neutral as given in the tables (i.e., in micromhos per mile) is equal to the charging current in amperes per mile per million volts between wire and neutral; the charging current for any other voltage to neutral is in proportion.

The changes in the apparent resistances do not indicate any change in the power dissipated as heat in the wires but a transfer of energy from one wire to the other by the magnetic field surrounding the wires. These relations assume sine-wave currents equal in effective value and differing in phase by exactly 120 deg. The assumption that the currents are exactly balanced cannot be strictly true, since the inequality in the apparent resistances and reactances of the three wires tends to unbalance the system, but the values just given may be taken as a fair approximation when the voltage loss in the line is not over 10 per cent, say. When the line wires are transposed these mutual-inductance effects are eliminated from the line as a whole, though the apparent impedances of the three wires in any one "exposure" of the transposition will be different; the transpositions, however, keep the currents balanced.

Leakage Conductance. The leakage current, even at very high voltages, is usually negligible in power transmission lines, but for telephone lines, the leakage is much greater, on account of the large number of small insulators used, and has a very appreciable effect on both the attenuation and distortion of the voice currents.

When the voltage is sufficiently high on a power line to cause the formation of corona an appreciable leakage current passes from one wire to the other. Even for a sine-wave voltage this leakage current is by no means sinusoidal, since its instantaneous values are practically zero except during the peak of the voltage wave, and consequently the corona loss cannot be accurately represented by a constant leakage conductance. Roughly, however, calling p_c the average value of the corona loss from each wire in watts per mile, corresponding to the given line voltage, the leakage conductance to neutral in micromhos per mile due to the corona may be taken equal to

$$g_c = \frac{p_c}{V^2} \tag{27}$$

where V is the effective (rms) kilovolts to neutral.

RISE OF VOLTAGE AT LOAD END OF LINE ON OPEN CIRCUIT. In every a-c transmission line the voltage at the load end when this end is open is higher than at the generator end, although in short low-frequency lines this rise is inappreciable. In overhead lines for which the product

$$(\text{Cycles per sec}) \times (\text{Length of line * in miles}) < 10{,}000 \tag{28}$$

this no-load rise as a percentage of the delivered voltage is, to a close approximation when the resistance of the wire is less than that of a No. 0 B. & S. copper wire, equal to $\left(\frac{fl}{4000}\right)^2$, where f is the frequency in cycles per second, and l the length of the line in miles. For example, in a 25-cycle line 160 miles long this no-load rise is 1 per cent of the delivered voltage; in a 60-cycle line of the same length the no-load rise in voltage is 5.8 per cent of the delivered voltage. The relation expressed by the above formula under the conditions stated is independent of the value of the delivered voltage and of the size and spacing of the wires, at least for all practical cases.

This rise, which is due to the charging current taken by the line, may be looked upon as present at all loads, but when the load is appreciable the voltage drop, due to the load current, unless leading, more than offsets this voltage rise. A leading current may increase the rise in voltage at the load end as the load comes on.

SYNCHRONOUS CONDENSERS (OR PHASE MODIFIERS) TO MAINTAIN CONSTANT VOLTAGE AT LOAD END. By making the line current at the load end of the line lead the line voltage by the proper phase angle, it is possible to compensate entirely for the change in the load voltage which normally takes place as the load current increases. An overexcited synchronous motor connected in parallel with the load is sometimes used for this purpose, as described in detail in the article on Synchronous Motors. A synchronous motor so used is commonly called a synchronous condenser, since the current taken by it leads the voltage impressed on its terminals.

17. METHODS OF CALCULATING VOLTAGE AND POWER LOSS IN A-C LINES

Previous Revision by R. A. Philip

The absolutely rigorous calculation of an a-c line requires that the distributed nature of the inductance and capacity be considered, i.e., that the line be considered as made up of an infinite number of sections such as shown in Fig. 4. However, simpler approximate methods may be employed for nearly all power lines such as are now used, giving results

* Length of each conductor.

sufficiently accurate for all practical purposes. The accuracy of these approximate methods depends primarily upon the frequency and length of the line; the less the value of the product of these two quantities the simpler is the method which may be employed. Accurate calculation of even a short a-c line with distributed load can be effected only by rather complicated network equations; see under Kirchhoff's laws in Section 3.

LIMITATIONS OF APPROXIMATE METHODS OF A-C LINE CALCULATIONS. The approximate methods of calculating a-c lines, in the order of their simplicity, may be designated as (1) the simple impedance method; (2) the single end-condenser method; (3) the middle condenser or " T " method; and (4) the split condenser or " π " method; the " π " method is also called the " U " method. For short low-voltage lines, the simple impedance method, which entirely neglects the charging current, is usually sufficiently accurate. The single end-condenser method takes the charging current into account in a manner usually sufficiently accurate for all but the longest high-voltage lines. The " T " and " π " methods are still more accurate, but are not given herein, as for exact calculations the rigorous method given below should be used. In fact, it is always well to check an approximate solution by this exact method.

FUNDAMENTAL ASSUMPTIONS ON WHICH FORMULAS GIVEN BELOW ARE BASED. All the formulas given below are based on the assumptions of pure sine-wave currents and voltages and a perfectly balanced system. By using the voltage to neutral instead of the voltage between wires, the formulas are also put in such shape that they may be applied directly to either a single-phase, a two-phase four-wire line, or a three-phase three-wire line, a two-phase four-wire line being considered as two separate single-phase lines. The fundamental idea in this method of treatment is that each line wire is considered as a separate circuit. The return wire shown in the various diagrams is therefore to be considered as having no impedance. This method of treatment of *balanced* polyphase circuits is strictly accurate (see Alternating Currents), but when the system is not balanced the circuit must be treated by the more general methods of network calculations; see Section 3. Also, when the voltage and current waves are not sinusoidal, each harmonic must be treated separately as described in Section 3, Art. 8.

Simple Impedance Method

This method is based upon the assumption that the electrostatic capacity of the line may be neglected entirely, and that the line may be considered simply as an impedance in series with the load, this impedance having a resistance equal to the total resistance of the line conductor and a reactance equal to the total inductive reactance of the line conductor. Fig. 2 is a diagram of the circuit, and Fig. 2a is a complete vector diagram of

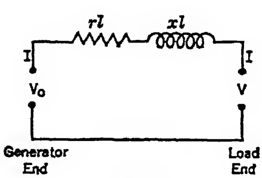

Fig. 2

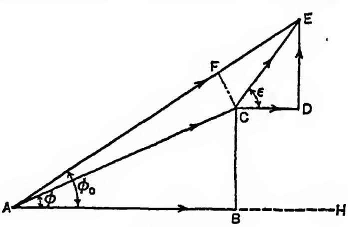

Fig. 2a

the current and voltage. When the wires are unsymmetrically arranged transpositions are assumed

$$\overline{AH} = I \qquad \overline{BC} = V \sin \phi \qquad \overline{CE} = zlI$$
$$\overline{AC} = V \qquad \overline{CD} = rlI \qquad \overline{AE} = V_0$$
$$\overline{AB} = V \cos \phi \qquad \overline{DE} = xlI \qquad \overline{FE} = V_0 - V$$

Let V = volts to neutral at load end of the line (= volts between wires divided by 2 in case of a single-phase line, and volts between wires divided by $\sqrt{3}$ in the case of a three-phase line.

I = amperes per wire; see formula (25) above.

l = length of each line wire in miles.

$Z = \frac{V}{lI}$ = " equivalent " impedance of the load per mile of line.

$\cos \phi$ = power factor of the load at end of line.

$\sin\phi = \sqrt{1 - \cos^2\phi}$ = the reactive factor of the load; $\sin\phi$ is to be taken positive for a lagging and negative for a leading current.

V_0 = volts to neutral at the generator end of the line.

r = conductor resistance per mile of line in ohms; see tables in chapter on Bare Wires and Cables.

x = conductor reactance per mile of line, in ohms; see tables in chapter on Capacitance and Inductance.

$z = \sqrt{r^2 + x^2}$ = conductor impedance * per mile of line, in ohms.

Q = per cent power loss, as a percentage of delivered power.

D = per cent voltage loss, as a percentage of delivered voltage.

From the vector diagram it is evident that the voltage at the generator end is

$$V_0 = \sqrt{(V\cos\phi + rlI)^2 + (V\sin\phi + xlI)^2} \tag{29}$$

which may also be written

$$V_0 = V\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2} \tag{30}$$

where r and x are per mile of line.

The current at the generator end is the same as at the load end.

The per cent power loss is

$$Q = \frac{100\,rlI}{V\cos\phi} = \frac{100\,r}{Z\cos\phi} \tag{31}$$

and the per cent voltage loss is

$$D = \frac{100(V_0 - V)}{V} = 100\left[\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2} - 1\right] \tag{32}$$

The power factor at the generator end is

$$\cos\phi_0 = \left(\frac{100 + Q}{100 + D}\right)\cos\phi \tag{33}$$

RELATION BETWEEN IMPEDANCE DROP AND VOLTAGE LOSS. The total impedance drop, which is zlI volts, should be carefully distinguished from the voltage loss, which is $v = V_0 - V$ volts. The vector diagram, Fig. 2*a*, will make the difference clear. For a given impedance drop of, say, A per cent, the voltage at the load end of the line may be anything from A per cent less than the voltage at the generator end to A per cent greater than the voltage at the generator end. The determining factor is the difference between the power-factor angle (ϕ) of the load and the power-factor angle $\left(\epsilon = \tan^{-1}\frac{x}{r}\right)$ of the line; only when $\epsilon - \phi = 0$ are the voltage loss and impedance drop the same. When $\epsilon - \phi$ is greater than 90 deg (which may occur for a leading current, since ϕ is then negative) the voltage at the load end will in general be higher than at the generator end although the impedance drop in the line may be very large. As a fair approximation, when the impedance drop is less than 20 per cent, that is, when z/Z is less than 0.2, the percentage voltage loss may be written

$$D = \frac{100\,z}{Z}\cos(\epsilon - \phi) \tag{34}$$

z and Z being the impedances of the line and load respectively, and ϵ and ϕ the power-factor angles of the line and load respectively.

EXAMPLE OF CALCULATION BY SIMPLE IMPEDANCE METHOD. Take the case of a three-phase, 60-cycle line 50 miles long, the wires being No. 0000 A.W.G. (or B. & S.) stranded copper spaced symmetrically with 6 ft between centers, and let the load be 15,000 kw at 60,000 volts between wires and at a power factor of 80 per cent with lagging current. The voltage to neutral is then $60{,}000 \div \sqrt{3} = 34{,}600$, and the current per wire $15{,}000 \div (\sqrt{3} \times 60 \times 0.8) = 180$ amp. The resistance of each wire per mile

* A convenient way of calculating an expression of the form $\sqrt{a^2 + b^2}$ is to write it $a\sqrt{1 + \left(\frac{b}{a}\right)^2}$ or $b\sqrt{1 + \left(\frac{a}{b}\right)^2}$ according as a is greater or less than b; the expression under the radical will then always lie between the numbers 1 and 2, and no difficulty will be experienced with decimal points. When b/a is less than 0.3, then the expression $\sqrt{a^2 + b^2} = a + \frac{b^2}{2a}$ with an error of less than 0.1 per cent; and when a/b is less than 0.3, then $\sqrt{a^2 + b^2} = b + \frac{a^2}{2b}$ with an error of less than 0.1 per cent. The error in the approximate expressions diminishes very rapidly as the ratio of b/a or a/b, as the case may be, decreases, being only 0.02 per cent when the ratio is 0.2.

is 0.269 ohm at 77 deg fahr, and the reactance 0.728 ohm. The equivalent impedance of the load per mile of line is $Z = 34{,}600 \div (50 \times 180) = 3.84$. Cos $\phi = 0.8$ and $\sin\phi = \sqrt{1 - 0.8^2} = 0.6$. Whence

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.84 \times 0.8} = 8.76 \text{ per cent}$$

$$\text{Per cent voltage loss} = D = 100\left[\sqrt{\left(0.8 + \frac{0.269}{3.84}\right)^2 + \left(0.6 + \frac{0.728}{3.84}\right)^2} - 1\right] = 17.5\%$$

$$\text{Power factor at generator end} = \cos\phi_0 = \left(\frac{100 + 8.76}{100 + 17.5}\right) \times 0.80 = 74.1 \text{ per cent}$$

$$\text{Per cent impedance drop} = \frac{100\sqrt{0.269^2 + 0.728^2}}{3.84} = 20.2 \text{ per cent}$$

Single End-condenser Method

This method assumes that the total current at the load end is equal to the actual load current plus (vectorially) the charging current which would be taken by a single condenser shunted across the line at the load end, the capacity of this condenser being taken equal

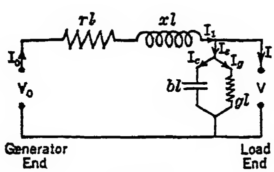

FIG. 3

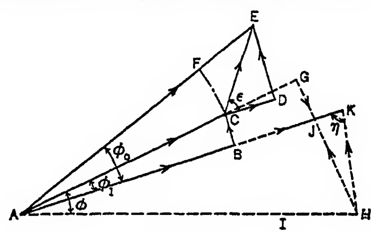

FIG. 3a

$\overline{AH} = I$

$\overline{AC} = V$

$\overline{AG} = I\cos\phi$

$\overline{GH} = I\sin\phi$

$\overline{HJ} = blV \times 10^{-6}$

$\overline{JK} = glV \times 10^{-6}$

$\overline{HK} = ylV \times 10^{-6}$

$\overline{AK} = I_1 = I_0$

$\overline{AB} = V\cos\phi_1$

$\overline{BC} = V\sin\phi_1$

$\overline{CD} = rlI_1$

$\overline{DE} = xlI_1$

$\overline{CE} = zlI_1$

$\overline{AE} = V_0$

$\overline{FE} = v$

to the total capacity of the line. This method gives too low a voltage at the generator end by approximately the same amount that the straight impedance method gives it too high, and also gives the power loss too low by approximately the same amount that the straight impedance method gives it too high. By averaging the losses obtained by the two methods a close approximation to their true values is obtained.

Fig. 3 is a diagram of the circuit, and Fig. 3a is a complete vector diagram of the voltage and current; voltages are shown by full lines and currents by dotted lines. The diagrams and formulas are for the general case of a line with leakage, but for nearly all practical cases the leakage may be neglected.

The effect of the electrostatic capacity of the line is to change both the numerical value and the phase angle of the line current. Or the condenser and the load may be looked upon as forming together an equivalent load taking a current I_1 at a power factor $\cos\phi_1$ differing from the actual current and power factor of the load. Let

V = volts to neutral at the load end of the line.
I = actual amperes per wire at the load end.
$\cos\phi$ = actual power factor at the load end.
$\sin\phi = \sqrt{1 - \cos^2\phi}$ = actual reactive factor at the load end.
l = length of each line wire in miles.
b = capacity susceptance to neutral per mile of line, in micromhos, see tables in Article 21.

g = leakage conductance to neutral per mile of line in micromhos, usually taken equal to zero in power lines, as explained above.

$y = \sqrt{g^2 + b^2}$ = dielectric admittance to neutral per mile of conductor, in micromhos. Note that for no leakage $y = b$.

The total leakage current, total charging current and total exciting current of the line are then respectively

$$I_g = glV \times 10^{-6} \qquad I_c = blV \times 10^{-6} \qquad I_e = \sqrt{I_g^2 + I_c^2} \tag{35}$$

The total line current, i.e., the resultant of the actual load current and the exciting current, is

$$I_1 = I\sqrt{\left(\cos\phi + \frac{I_g}{I}\right)^2 + \left(\sin\phi - \frac{I_c}{I}\right)^2} \tag{36}$$

On the assumptions of this method of calculation, I_1 is also the current at the generator end.

The power factor of the equivalent load formed by the actual load and the condenser is then

$$\cos\phi_1 = \frac{I\cos\phi + I_g}{I_1} \tag{37}$$

and the reactance factor of this equivalent load is

$$\sin\phi_1 = \frac{I\sin\phi - I_c}{I_1} \tag{38}$$

The formulas given above for the straight impedance method are then directly applicable, using for I, $\cos\phi$, and $\sin\phi$ in those formulas the values of I_1, $\cos\phi_1$, and $\sin\phi_1$ just calculated; i.e., the straight impedance method is to be applied not to the actual load but to the equivalent load formed by the actual load and a condenser having an admittance equal to the total admittance of the line.

EXAMPLE OF CALCULATION BY SINGLE END-CONDENSER METHOD. Three-phase line, 50 miles long, No. 0000 A.W.G. stranded copper, 6 ft between centers, frequency 60 cycles, load 15,000 kw, 60,000 volts between wires at load end, 80 per cent power factor at load. This is the same example as used above for the straight impedance method. Then $V = 34{,}600$, $I = 180$, $\cos\phi = 0.8$, $\sin\phi = 0.6$, $l = 50$, $b = 6.03$, $g = 0$, $r = 0.269$, $x = 0.728$. Then,

$$\text{Charging current} = I_c = 6.03 \times 50 \times 34{,}600 \times 10^{-6} = 10.4 \text{ amp}$$

$$\text{Resultant current} = I_1 = 180\sqrt{(0.8)^2 + \left(0.6 - \frac{10.4}{180}\right)^2} = 174 \text{ amp}$$

$$\text{Power factor of equivalent load} = \cos\phi_1 = \frac{180 \times 0.8}{174} = 0.828$$

$$\text{Reactive factor of equivalent load} = \sin\phi_1 = \frac{180 \times 0.6 - 10.4}{174} = 0.561$$

$$\text{Impedance of equivalent load per mile of line} = Z_1 = \frac{34{,}600}{50 \times 174} = 3.98$$

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.98 \times 0.828} = 8.16 \text{ per cent}$$

$$\text{Per cent voltage loss} = D$$

$$= 100\left[\sqrt{\left(0.828 + \frac{0.269}{3.98}\right)^2 + \left(0.561 + \frac{0.728}{3.98}\right)^2} - 1\right] = 16.30 \text{ per cent}$$

$$\text{Power factor at generator end} = \cos\phi_0 = \left(\frac{100 + 8.16}{100 + 16.3}\right) \times 0.822 = 76.4 \text{ per cent}$$

The per cent power loss and voltage loss obtained by the straight impedance method neglecting the line capacity are respectively 8.76 and 17.6 per cent. As noted above, the single end-condenser method gives these losses too low (for inductive loads) by approximately the same amount that the straight impedance method gives them too high, whence closer approximations to the true losses are: per cent power loss = $(8.16 + 8.76) \div 2 = 8.46$ and per cent voltage loss = $(16.3 + 17.6) \div 2 = 17.0$.

CALCULATION OF EFFECT OF SYNCHRONOUS CONDENSER. Formulas (36) to (38) apply directly to the calculation of the effect of a synchronous condenser at the end of the line taking a current having an in-phase or energy component equal to I_g and a quadrature leading component equal to I_c. Fig. 3a then represents the vector relations of the currents and voltages, the vector JK being the in-phase component of the current taken by the synchronous condenser and HJ the quadrature component.

Exact Calculation of A-C Lines of Any Length and Frequency *

The charging current (and also the leakage current) for any element of a transmission line passes through only that portion of the line conductor between the given element and the generator end. The exact determination of the line current, voltage, and power at any point therefore requires that this fact be taken into account; in other words the capacity and leakage of the line are distributed and not lumped as assumed in the above methods of calculation. In Fig. 4 are shown three successive elements of one wire of a line, a return or neutral of zero impedance being also shown to complete the circuit. This method of treating separately each wire of a line is fully explained above.

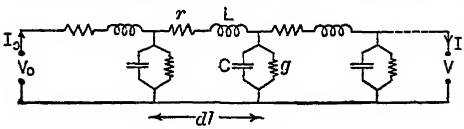

FIG. 4

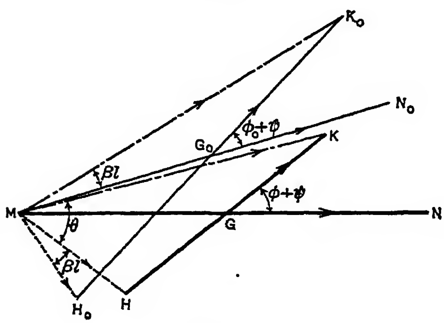

FIG. 4a

$\overline{MN} = I$	$\overline{MK_0} = A\epsilon^{\alpha l}$
$\overline{HK} = YV$	$\overline{MH_0} = B\epsilon^{-\alpha l}$
$\overline{MK} = A$	$\overline{MN_0} = I_0$
$\overline{MH} = B$	$\overline{H_0K_0} = YV_0$

In order to make clear the physical meaning of the various terms employed, the general solution is first given in terms of instantaneous values. Following this are given the working formulas (1) in terms of exponentials, viz., ϵ^x and ϵ^{-x}, (2) in terms of real hyperbolic functions, and (3) in terms of hyperbolic functions of complex angles.

GENERAL EQUATIONS OF TRANSMISSION LINE. Let l = the distance in miles of any point along a transmission line, measured from any arbitrarily chosen point (say from the load end); let i = the instantaneous current in amperes in the conductor at this point, taken positive in the direction *opposite* to that in which l is measured (i.e., positive when *toward* the load end); let v = the potential drop in volts between this point and the neutral; and let r = the conductor resistance in ohms per mile, L = the inductance of the conductor in henrys per mile, C = the capacity to neutral in microfarads per mile, and g = the leakage conductance to neutral in micromhos per mile. The following relations then hold at any point along the line, t being time in seconds measured from any arbitrarily chosen instant:

$$\frac{dv}{dl} = ri + L\frac{di}{dt} \tag{39}$$

$$10^6\frac{di}{dl} = gv + C\frac{dv}{dt} \tag{40}$$

If the circuit is composed of two or more sections of different constants (e.g., an overhead section and an underground section, or a circuit formed by a step-up transformer, transmission line, and a step-down transformer) then a similar set of equations holds for each section of the circuit, the constants r, L, g, and C being in general different for the several sections.

The complete solution of any two equations of the form given by (39) and (40) consists of an infinite series of terms for both v and i, corresponding terms in the two series having the following values:

$$i = \sqrt{2}\epsilon^{-(u-s)t}[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1) + B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2)] \tag{41}$$

$$v = \frac{\sqrt{2}}{Y}\epsilon^{-(u-s)t}[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1 - \psi) - B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2 - \psi)] \tag{42}$$

PHYSICAL INTERPRETATION AND NAMES GIVEN TO THE VARIOUS CONSTANTS. The constant ω in eqs. (41) and (42) is equal to $2\pi f$, where f is the frequency of the oscillation represented by these equations. In the most general case any change in

* This division is abstracted from the lecture notes of Dr. H. Pender.

the circuit conditions, such as closing or opening a switch, or a lightning stroke in the vicinity of the circuit, may set up an infinite number of oscillations of different frequencies, their frequency being determined by the initial conditions at the instant the change is made. The current and voltage set up by each oscillation is represented by a set of terms of the form given by (41) and (42), and the resultant current and voltage will be respectively the sum of all the current terms and the sum of all the voltage terms. The oscillation of any given frequency, however, may be considered separately, as it is uninfluenced by the presence of the other oscillations. Moreover, in the case of a composite circuit, consisting say of a step-up transformer, transmission line, and step-down transformer, if an oscillation of frequency f is set up in one part of the circuit it will also appear in all other sections of the circuit, though it may be greatly damped in these sections and therefore produce no appreciable effect.

Attenuation Constant (α), Wavelength Constant (β), Wavelength (λ), and Velocity of Propagation (U). Referring to eqs. (41) and (42), each oscillation sets up in each section of the circuit two waves, each of which has a wavelength $\lambda = 2\pi/\beta$; the constant β is therefore called the wavelength constant; it is a function of the frequency and of the constants r, L, g, and C of the circuit. The two waves travel along the line in opposite directions each with a velocity $U = \omega/\beta$; in a composite circuit this velocity U is in general different for each section of the circuit. One wave may be looked upon as the incident and the other as the reflected wave. The amplitude of each wave diminishes by the factor ϵ^{α} as the wave travels unit distance; the factor ϵ^{α} is called the attenuation factor, and the constant α is called the attenuation constant. The attenuation constant is a function of the frequency and the constants r, L, g, and C of the circuit; see below.

Surge Admittance (Y) and its Power-factor Angle (Ψ). The constant Y, which is equal to the quotient of the amplitude (or rms value) of the incident current wave by the incident voltage wave, is called the surge admittance, and its reciprocal is called the surge impedance; it is a function of the frequency and the constants r, L, g, and C of the circuit; see below. The constant ψ, which is equal to the angle by which the incident current wave leads the incident voltage wave, is called the power factor angle of the surge admittance; it is a function of the frequency and the constants r, L, g, and C; see below.

Amplitude Constants (A and B) and Phase-angle Constants (θ_1 and θ_2). The constants are equal to the amplitudes of the incident and reflected current waves at the point from which the distance l is measured, and the constants θ_1 and θ_2 give the phase of these two waves at this point ($l = 0$) at the instant from which time is measured ($t = 0$). Note that the incident current wave at $l = 0$ leads the reflected current wave by the angle $\theta = \theta_1 - \theta_2$. The determination of these constants for steady-state conditions is given below.

Natural Damping Constant (u), Energy Transfer Constant (s), and Composite Damping Constant ($u - s$). In the general case of a natural oscillation in a composite circuit the amplitude of each wave diminishes in unit time by the factor $\epsilon^{(u-s)}$; this factor is called the composite damping factor, and the constant $(u - s)$ is called the composite damping constant. The composite damping constant, like the frequency f, is the same for all sections of a composite circuit. In the case of a line of uniform constants throughout, not connected to any terminal apparatus, it can readily be shown that $s = 0$, in which case the amplitude of the oscillations diminishes in unit time by the factor ϵ^{u}; this factor is therefore called the natural damping factor, and the constant u, which for a section having the constants r, L, g, and C per unit length, is equal to

$$u = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) \tag{43}$$

is called the natural damping constant. Any section of a composite circuit for which the actual damping $\epsilon^{(u-s)}$ is less than the natural damping ϵ^{u} must receive energy from some other section; consequently a positive value of s for a given section means that energy is transferred into this section from some other section of the circuit. Similarly, a negative value of s for a given section means that energy is transferred from this section to some other section. The constant s may therefore be called the energy transfer constant. Since the voltage and current in the circuit cannot increase indefinitely the energy transfer constant s can never have a positive value greater than u.

Since the composite damping constant $(u - s)$ is the same for all sections of a circuit, it follows that the transfer of energy from one section to another by oscillations in a composite circuit will always be into the section in which u is the larger from the section in which u is the smaller. Neglecting the leakage conductance g, this means that energy will be transferred into section 1 from section 2, when r_1/L_1 is greater than r_2/L_2, that is, energy is transferred from the section of the larger time constant (L_2/r_2) to that of the smaller time constant (L_1/r_1). When the resistances are small, this means that in the limiting

case all the energy ($= {}^1/_2\, L_2 I^2$) of the magnetic field of the second section may go into electrostatic energy ($= {}^1/_2\, C_1 V^2$) in the first section, producing therefore a very high voltage at the junction point when the inductance L_2 of the second section is large compared with the capacity C_1 of the first section. This accounts for the very high voltages sometimes set up during switching operations at the junction point of an overhead line with an underground line, or in a transformer connected to a long overhead line.

STEADY-STATE CONDITIONS IN A TRANSMISSION LINE. From the above discussion it is evident that, when a sufficient time (usually a small fraction of a second) has elapsed after any change in the circuit conditions, the only terms left in the general equations of a transmission line for a given impressed sine-wave voltage of frequency f are those for which $s = u$, viz.,*

$$i = \sqrt{2}\left[A\epsilon^{\alpha l} \sin(\omega t + \beta l) + B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta)\right] \tag{44}$$

$$v = \frac{\sqrt{2}}{Y}\left[A\epsilon^{\alpha l} \sin(\omega t + \beta l - \psi) - B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta - \psi)\right] \tag{45}$$

The effective value of the current at any point is then equal to the sum of two vectors having the lengths $A\epsilon^{\alpha l}$ and $B\epsilon^{-\alpha l}$, the former leading the latter by the angle $(2\beta l + \theta)$, and the effective value of the voltage is equal to the difference of these same two vectors divided by Y. The phase angle between the voltage and current is equal to the phase angle between the sum and difference of the A and B vectors less the angle ψ.

Notation for Steady-state Conditions. These relations are clearly shown in the vector diagram, Fig. 4*a*, which is a complete vector diagram of a transmission line with distributed capacity and leakage. The four constants α, β, Y, and ψ are constants of the line, independent of the load, and are expressed in terms of the ordinary line constants as follows: Let

f = frequency in cycles per second.

r = conductor resistance per mile, in ohms; see tables in Article 47 of this section.

$x = 2\pi f L$ = conductor reactance per mile, in ohms, corresponding to the impressed frequency f; see tables in chapter on Inductance and Inductive Reactance.

$z = \sqrt{r^2 + x^2}$ = conductor impedance per mile, in ohms.

g = leakage conductance to neutral per mile of line, in micromhos. For power lines g is usually taken equal to zero.

$b = 2\pi f C$ = capacity susceptance to neutral per mile of line, in micromhos, corresponding to the impressed frequency f; see tables in Article 22 of this section.

$y = \sqrt{g^2 + b^2}$ = dielectric admittance per mile, in micromhos; when $g = 0$, then $y = b$.

$\alpha = 10^{-3}\sqrt{\dfrac{yz - bx + gr}{2}}$ = the attenuation constant; for r and g small compared with x and b this reduces to $\dfrac{10^{-3}}{2}\left(r\sqrt{\dfrac{C}{L}} + g\sqrt{\dfrac{L}{C}}\right)$.

$\beta = 10^{-3}\sqrt{\dfrac{yz + bx - gr}{2}}$ = the wavelength constant; for r and g small respectively compared with x and b this reduces to $2\pi f \times 10^{-3}\sqrt{LC}$, which for an overhead line equals approximately $\dfrac{2\pi f}{180{,}000}$.

$Y = 10^{-3}\sqrt{\dfrac{y}{z}}$ = surge admittance; for r and g small respectively compared with x and b, this reduces to $10^{-3}\sqrt{\dfrac{C}{L}}$. The reciprocal of the surge admittance is called the surge impedance.

$\psi = \tan^{-1}\sqrt{\dfrac{yz - bx - rg}{yz + bx + rg}}$ = the power-factor angle of the surge admittance, taken positive for $gx < br$ and negative for $gx > br$. For r and g small compared with x and b, then $\psi = 28.7\left(\dfrac{r}{x} - \dfrac{g}{b}\right)$ degrees.

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* For steady-state conditions time may be counted from any arbitrarily chosen interval, i.e., θ_1 in eq. (41) and (42) may be put equal to zero, and for convenience $-\theta$ may be used for θ_2.

$U = \frac{2\pi f}{\beta}$ = velocity of propagation in miles per second; for a frequency f sufficiently high to make r negligible compared with x, and g negligible compared with b, this reduces to $\frac{10^3}{\sqrt{LC}}$, which for an overhead line with wires far apart is equal to the velocity of light in air, viz., 180,000 miles per second, approximately.

$\lambda = \frac{2\pi}{\beta} = \frac{U}{f}$ = wavelength of each wave in miles; for a frequency f sufficiently high to make r negligible compared with x, and g negligible compared with b, this reduces to $\frac{10^3}{f\sqrt{LC}}$, which for an overhead line is equal approximately to $\frac{180{,}000}{f}$ miles.*

In the vector diagram and in the formulas given below, let

l = length of the line in miles.
I = effective (rms) value of the amperes per wire at the load end.
V = effective (rms) value of the volts to neutral at the load end.
ϕ = the power-factor angle at the load end, i.e., $\cos\phi$ is the power factor at the load end. ϕ is taken positive for a lagging and negative for a leading current.
I_0, V_0, ϕ_0 = corresponding quantities at the generator end.

Solution by Vector Diagram (Fig. 4*a*). Having calculated the constants α, β, Y, and ψ, and knowing the current I, voltage V, and power-factor angle ϕ of the load,

Lay off $\overline{MN} = I$ as the base line, and bisect it at G.

At the angle $(\phi + \psi)$ ahead of $\overline{MN}$ lay off the line $\overline{HK}$ equal in length to YV, so that it is also bisected by G.

Then measure off $\overline{MK} = A$ and $\overline{MH} = B$.

Lay off at the angle 57.3 βl deg ahead of A the line $\overline{MK_0}$ equal in length† to $A\epsilon^{\alpha l}$.

Lay off at the angle 57.3 βl deg behind B the line $\overline{MH_0}$ equal in length to $B\epsilon^{-\alpha l}$.

Bisect H_0K_0 at G_0.

Then the line $\overline{MN_0} = 2\overline{MG_0}$ is equal to the current at the generator end; the line $\overline{H_0K_0}$ divided by Y is equal to the voltage at the generator end; the angle between $\overline{G_0N_0}$ and $\overline{G_0K_0}$ less the angle ψ is the power-factor angle at the generator end.

Note that the voltage at the load end if drawn in the diagram would be at the angle ψ behind the vector $\overline{GK}$, and at the generator end would be at the angle ψ behind $\overline{G_0K_0}$.

Algebraic Solution for Steady-state Conditions. The vector diagram may be solved algebraically as follows: Calculate first the constants A, B and θ from the formulas:

$$A = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 + 2\,YVI\cos(\phi + \psi)} \tag{46}$$

$$B = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 - 2\,YVI\cos(\phi + \psi)} \tag{47}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi + \psi)}{I^2 - (YV)^2}\right] \tag{48}$$

Note that $\sin\theta$ has the same algebraic sign as the numerator of this fraction and $\cos\theta$ has the same algebraic sign as the denominator of this fraction; this fixes the quadrant in which θ lies.

Put
$$A_0 = A\epsilon^{\alpha l}, \quad \text{and} \quad B_0 = B\epsilon^{-\alpha l} \tag{49}$$

The current, voltage, and power-factor angle at the generator end are, then, expressing all angles in degrees,

$$I_0 = \sqrt{A_0^2 + B_0^2 + 2\,AB\cos(114.6\,\beta l + \theta)} \tag{50}$$

$$V_0 = \frac{1}{Y}\sqrt{A_0^2 + B_0^2 - 2\,AB\cos(114.6\,\beta l + \theta)} \tag{51}$$

$$\phi_0 = \tan^{-1}\left[\frac{2\,AB\sin(114.6\,\beta l + \theta)}{A_0^2 - B_0^2}\right] - \psi \tag{52}$$

Note that the quadrant in which $(\phi_0 + \psi)$ lies is determined by the algebraic signs of the numerator and denominator of the fraction in the brackets, just as in the case of the angle θ.

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* The above formulas for z, y, α, β, Y, ψ, U, and λ also hold for the transient or free oscillations in a single circuit, and also for each section of a composite circuit, provided in these formulas $r_1 = r(u - s)L$ is substituted for r and $g_1 = g \div (u - s)C$ is substituted for g.

† See Fund. of Eng., Section 1, Table 17, for values of ϵ^x and ϵ^{-x}, where x is any number.

Solution of Steady-state Conditions in Terms of Hyperbolic Functions.* The above expressions for the current, voltage and power factor at the generator may also be put in the form,

$$I_0 = I\sqrt{\frac{\cosh(2\,\alpha l + \gamma) + \cos(114.6\,\beta l + \theta)}{\cosh\gamma + \cos\theta}} \tag{53}$$

$$V_0 = V\sqrt{\frac{\cosh(2\,\alpha l + \gamma) - \cos(114.6\,\beta l + \theta)}{\cosh\gamma - \cos\theta}} \tag{54}$$

$$\phi_0 = \tan^{-1}\left[\frac{\sin(114.6\,\beta l + \theta)}{\sinh(2\,\alpha l + \gamma)}\right] - \psi \tag{55}$$

where θ and γ are given by the formulas

$$\gamma = \tanh^{-1}\left[\frac{2\,YVI\cos(\phi + \psi)}{I^2 + (YV)^2}\right] \tag{56}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi + \psi)}{I^2 - (YV)^2}\right] \tag{57}$$

The other quantities are as above defined. Note that θ is the same angle as given by eq. (48), and the quadrant in which it lies is to be determined as described in the note under (48). Also note that the constant γ given by (56) may be expressed in terms of A and B, given by (46) and (47), by means of the formula

$$\gamma = \log_\epsilon\left(\frac{A}{B}\right) = 2.302\log_{10}\left(\frac{A}{B}\right) \tag{58}$$

Formulas for Open Circuit and Short Circuit at Load End. When the line is open at the load end, $I = 0$, whence, from eq. (46) to (48), $A = {}^1/_2\,YV$, $B = {}^1/_2\,YV$, and $\theta = 180$ deg (since the denominator of the fraction is negative). When the line is short-circuited at the load end, $V = 0$, and $A = {}^1/_2 I$, $B = {}^1/_2 I$, and $\theta = 0$ deg (since the denominator of the fraction is positive). The current, voltage and power-factor angle at any point along the line may then be found in either case by substituting these values in eq. (50) to (52), which reduce to the simple hyperbolic forms:

On Open Circuit	On Short Circuit
$I_0 = YV\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\,\beta l)}$	$I_0 = I\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\,\beta l)}$
$V_0 = V\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\,\beta l)}$	$V_0 = \frac{I}{Y}\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\,\beta l)}$
$\phi_0 = -\tan^{-1}\left[\frac{\sin(114.6\,\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$	$\phi_0 = +\tan^{-1}\left[\frac{\sin(114.6\,\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$

Solution of Steady-state Equations in Complex Hyperbolic Functions. Expressing all quantities other than the length l in symbolic notation, the equations of the transmission line, for *steady-state conditions only*, may be written

$$\dot{I}_0 = \dot{I}\,\frac{\sinh(l\sqrt{\dot{y}\dot{z}} + \dot{B})}{\sinh\dot{B}} \tag{59}$$

$$\dot{V}_0 = \dot{V}\,\frac{\cosh(l\sqrt{\dot{y}\dot{z}} + \dot{B})}{\cosh\dot{B}} \tag{60}$$

where the symbols other than $\dot{B}$ have the same meanings as above, except that they are all expressed as complex quantities, and

$$\dot{B} = \tanh^{-1}\left(\frac{\dot{I}}{\dot{V}}\sqrt{\frac{\dot{z}}{\dot{y}}}\right) \tag{61}$$

The real part of $\sqrt{\dot{y}\dot{z}}$ is the attenuation constant and the imaginary or "j" part is the wavelength constant.

18. STABILITY OF TRANSMISSION LINES

By John H. Palmer

General Principles and Definitions

This article should be read in connection with Section 3, Arts. 57–64 incl., where a mathematical treatment of the subject is given. In particular, Art 57 gives definitions reported by the Standards Committee of A.I.E.E.

* Tables of hyperbolic functions are given in Fund. of Eng., Section 1, Table 17.

Causes of Instability

Some of the common causes of instability of a power system are as follows:

Transient instability:

1. Line switching.
2. Sudden changes in load.
3. System faults.

Steady-state instability:

1. Overloading.
2. Highly inductive loads with subsequent low power factor of the system.

Let us assume a simple power system consisting of a generator with its prime mover supplying power to a synchronous motor over a two-circuit transmission line.

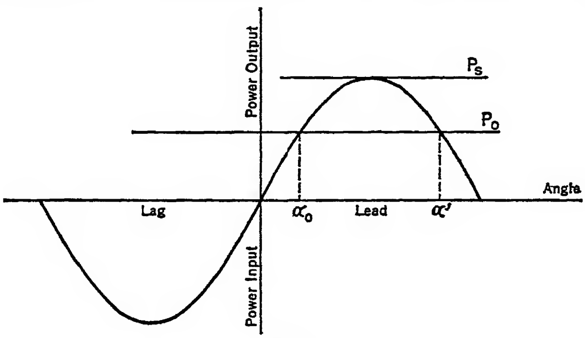

Fig. 5

For a steady-state condition there will be a phase angle between the internal emf's of the generator and motor. This angle is a function of the load and usually varies as shown in Fig. 5.

Let us now assume a mechanical power input to the generator represented by P_0. For this input α_0 is the angle between the generator and motor emf's for the steady-state equilibrium condition. P_0, the input to the generator, however, intersects the curve at another point α'. This, also, is a point of equilibrium, but at this point the system is unstable. For instance, assume that for some reason there is a slight increase in the angle α, beyond α'. This (from the curve) will cause a decrease in power output, and since P_0 is constant, the generator will accelerate, which will cause the angle α to increase. This in turn will cause a further drop in output so that the effect will be cumulative and the motor will drop out of step. By similar reasoning it will be seen that a decrease in α from α' will have the same effect. Similar analysis of the diagram at the angle α_0 shows that this is a point of stable equilibrium. It is now evident that P_s is the maximum power which can be transmitted over this system under stable, steady-state conditions. In other words, it is the stability limit of the system. However, with power input of P_0, a transient swing of the angle α from α_0 may extend as far as α' without causing instability. Under this condition, even on the descending side of the curve, the power output will be greater than the input, slowing down the generator and causing the angle to return to the stable position at α_0.

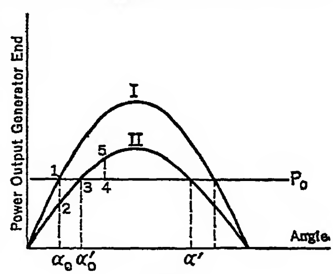

Fig. 6

LINE SWITCHING. Let us now consider again the same transmission line and determine the effect of opening one line at either end. The power-angle diagram for the system at the generator end is shown in Fig. 6, curve I representing conditions before switching, and curve II, conditions after switching.

Then if we assume the power input to the generator to be P_0, the original steady-state angle α will be at α_0. After the switching operation a new position of equilibrium will be reached at α_0'. When the system is operating at α_0 the opening of the line reduces the power that will be taken over the system at that angle by the vertical distance between

the power input curve and the new output curve II. The distance between points 1 and 2 represents the power consumed in accelerating the generator rotor. Because of this acceleration the angle α will increase along the new curve toward point 3. As it approaches point 3 the accelerating force will decrease and the velocity will increase until at point 3 the accelerating force will be zero and the velocity a maximum. As a result the rotor will overrun the position of equilibrium at 3 toward point 5, causing a retarding force to be set up which will increase as 5 is approached. At point 5 the velocity will be normal, but the angle α and the output will be too great for equilibrium, producing a maximum retarding force tending to bring conditions back to point 3. The rotor will again overrun the position of equilibrium, causing oscillation about the final position 3. The amount of overswing will be such that the area 1, 2, 3 equals area 3, 4, 5 except for the losses, which produce the necessary damping action to stop this oscillation. For this case, the system is stable since the overrun does not carry the angle beyond α'.

SUDDEN CHANGES IN LOAD. This differs from switching operations in that the circuit constants remain substantially the same and the input and output vary, whereas with switching, the input and output are substantially the same and the circuit constants are changed. Considering again the two-circuit power system, let us assume that the load is increased as indicated in Fig. 7, from P_0 to P_1. If this increase is sudden the prime-mover governor will not respond immediately and the increase in power will have to be supplied by the kinetic energy of the system, slowing it down.

But in this case the motor will slow down faster than the generator, so that the relative effect will be the same as an increase in speed of the generator, increasing the angle α. This slowing down, however, will cause the governor of the prime mover to act and restore the speed to normal by increasing the power input to the system, to correspond to the new power output. There will be some oscillation of the system as described under Line Switching, but if stable it becomes steady at the angle α_0', corresponding to the new load. The same conditions of stability apply as before.

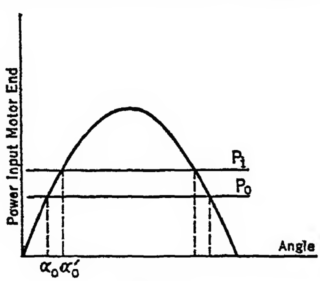

FIG. 7

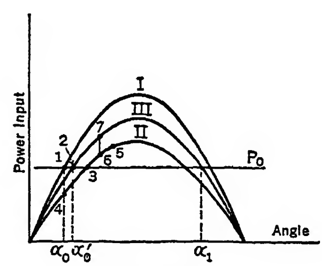

FIG. 8

SHORT CIRCUITS. Short circuits present considerably more complications than switching operations and changes in load, because three distinct networks are involved:

(*a*) The original condition prior to the short circuit.

(*b*) A second condition during the short circuit.

(*c*) A third condition after the short circuit has been cleared, usually by a switching operation.

The second condition is what makes the short circuit radically different from switching operation and load changes. The short circuit may increase or decrease the power input according to whether it is a high- or low-resistance fault. A high-resistance fault is usually in the form of a failure to ground in a grounded neutral system, and there is a certain fault resistance for which the power input will be a maximum.

Low-resistance Fault. Fig. 8 shows the power-angle diagrams for a low-resistance fault, in which curve I shows the condition before short circuit occurs; curve II, the condition during short circuit; and curve III, the condition after the fault is cleared.

Point 1 at angle α_0 represents the point of equilibrium before short circuit occurs. Immediately when the fault is applied, the operating point will jump to 4. This will cause the generator to speed up to try to reach the point of equilibrium 3 on curve II. An oscillation will then be set up between points 4, 3, and 5 as previously described, during which time the fault will be cleared, say at point 6. Conditions will then jump to point 7, curve III. This will cause a second oscillation about 2, and the system will soon become stable at 2, angle α_0', provided it does not swing beyond the maximum stability angle α_1.

High-resistance Fault. Fig. 9 shows the power-angle diagram for a high-resistance fault, with the three curves, I, II, III, representing the same respective circuit conditions as in Fig. 8. Here again the same sequence of events will take place as described for a low-resistance fault. However, in this case the electrical power input to the line is greater than the mechanical input at point 4 and will be supplemented for a short time by the inertia of the generator. This will cause a large oscillation to start about the new point of equilibrium 3 during short circuit, from 4 to 5, changing the angle from leading to lagging as shown.

The area 1, 3, 4 represents the energy given up by the generator in moving from 4 to 3 during which time the electrical input to the system is greater than the mechanical input P_0. The generator starts giving up its kinetic energy immediately, and the motor starts giving it up as soon as the angle becomes lagging. It will be noted that at point 10 the electrical power input to the system becomes negative, indicating that the *generator* has started to operate as a motor, taking energy from the *motor* as well as from its prime mover. Thus the energy given to the generator by the *motor* during the swing down to point 5 is represented by area 10, 5, 11. The energy supplied to the system by the prime mover during this swing is represented by area 1, 9, 11, 12. Of this energy, 1, 3, 10, 12 would be furnished to the motor and fault while 3, 9, 11, 10 would be used in accelerating the generator. The energy given up by the motor (10, 5, 11) would also be used in accelerating the generator. Suppose now that during the oscillation about point 3 a circuit breaker opens at point 6 clearing the line on which the fault has occurred. Conditions will immediately jump to point 8 on curve III. The generator with the load removed will accelerate very rapidly with conditions following curve III up to point 2 where it will have a surplus of kinetic energy equal to area 2, 7, 8. If this area is greater than the area enclosed between P_0 and curve III, the generator will overrun the second equilibrium point at α_1 and instability will occur. If this area 2, 7, 8 is smaller, the system will be stable.

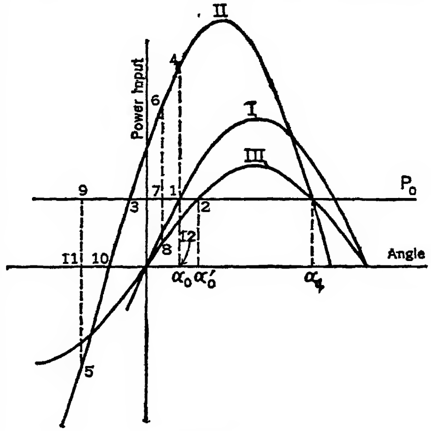

FIG. 9

From the above it will be seen that fault resistance is of great importance in the consideration of short circuits.

Assumptions Made. The preceding discussion has been given in an attempt to present a general picture of the phenomena of stability. Certain simplifying assumptions have been made which in practice will require modification; for example, the mechanical inputs and outputs of machines are assumed constant. When actual values are known as a function of time, suitable corrections can be made. Also, it is assumed above that the internal voltages of the machines are constant. The effect of this may be corrected for by drawing power-angle curves of different voltages.

Methods Used to Increase Stability

The above method of visualizing the phenomena of stability also aids in an understanding of the various methods which have been used to increase stability. These methods are as follows:

1. The use of synchronous condensers at strategic points along the line for improvement of power factor and voltage regulation, and for supplying power to the line under transient conditions.
2. Improved inherent regulation of machines.
3. The use of static capacitors in series with the line.
4. Increased speed of excitation.
5. Faster-acting prime-mover governors.
6. High-speed relays and circuit breakers to reduce the oscillations subsequent to the opening of a breaker.
7. Taking account of ground resistances, under condition of high fault current, in designing the system.

The above discussion of power-angle relations is largely from C. L. Fortescue's paper on Transmission Stability, A.I.E.E., 1925, p. 984.

Mechanical Analogies

Another excellent method of visualizing transmission-line characteristics is by means of mechanical analogies. See Mechanical Analogy to the Problem of Transmission Stability, by S. B. Griscom, *Elec. J.*, May, 1926.

The following is based on this article. Referring again to the above transmission system, the vector diagram under certain conditions might be as shown in Fig. 10, assuming

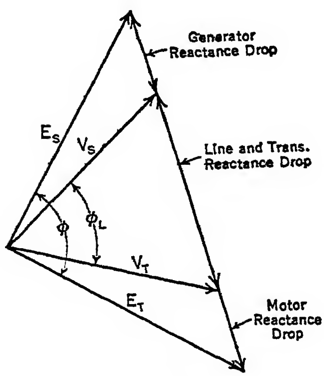

FIG. 10.

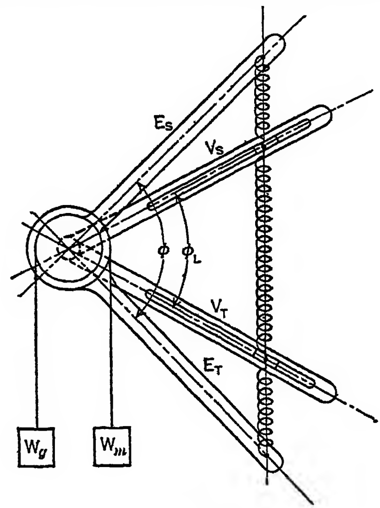

FIG. 11

a transmission line of pure reactance and zero resistance. Here:

E_s = internal emf of the supply generator.
V_s = terminal voltage of the generator.
V_t = terminal voltage of the motor.
E_t = internal emf of the motor.

This may be represented mechanically as shown in Fig. 11.

The arms represent the corresponding vectors shown in Fig. 11 with the spring representing the reactance drop.

The moment produced by W_g = the power input to the generator.

The moment produced by W_m = the power output of the motor.

Now, assuming constant internal voltages; if W_m increases slowly so that the generator governor can follow the change in load and keep the system in equilibrium, then W_g will increase correspondingly and the angles ϕ and ϕ_L will increase, lengthening the (reactance drop) springs. This will also decrease the terminal voltages V_s and V_t as shown. This will continue until angle ϕ reaches 90 deg, after which the mechanical system will become unstable and will collapse. This is due to the fact that while the force exerted by the springs is still increasing with the angle, the moments produced by this force acting on the arms at E_s and E_t starts to decrease after passing the 90-deg angle. The same will be true of the electrical system. Instability occurs when the internal emf's of the generator and motor become 90 deg out of phase.

Many other effects can be demonstrated with similar analogies, and this is very thoroughly discussed in the above reference.

19. CORONA

Original Author, F. W. Peek
Revisions by W. A. Del Mar

If a potential difference is established between smooth parallel wires, or between concentric cylinders, and gradually increased, a voltage is finally reached at which a

hissing noise is heard. If it is dark, a pale glow, called the electric "corona," will be seen to surround the wires. When wattmeters are inserted in the line a loss is noticed to start at this critical voltage. There is also noted the characteristic odor of ozone, and the air around the wires becomes ionized. These phenomena are referred to as "corona effects."

The corona starts at the conductor surface, because the voltage stress per unit distance, or "voltage gradient," is highest there. The breakdown or corona extends out to a point where the stress is below the breakdown point of air. If the wires are close together, so that the stress is fairly uniform, the breakdown immediately extends from conductor to conductor without corona, and the phenomenon is called a "spark-over." The corona does not extend to the other conductor when the separation is large compared to the wire diameter, because it reaches a point in space where the gradient is below the breakdown gradient of air. Increasing the voltage after the corona point is reached causes the corona to extend until finally a spark occurs from conductor to conductor. The power loss increases very rapidly with increase in voltage above the critical point.

Corona is caused by either a-c or d-c voltages, and starts at approximately the same maximum stress. At the critical voltage, corona occurs only at the crest of the alternating wave. As the voltage is increased above the critical point, the loss extends over a greater portion of the wave. In the a-c corona the eye sees a superposition of the corona caused by the plus and minus half-cycles of the a-c waves. If the effects of the half-waves are viewed separately, it is noticed that a reddish haze surrounds the wire while it is negative, whereas the surface of the wire glows bluish-white while it is positive. Positive and negative d-c corona have exactly the same appearance as the positive and negative corona of the corresponding a-c half-waves. When a negative wire becomes oxidized, reddish tufts appear. If there are rough spots on the wire the corona starts at these points at a lower voltage. At points the positive corona extends out as a bluish-white spray; the negative corona appears as a red tuft.

Corona is also caused by transient voltages. Corona produced by voltages lasting less than a millionth of a second can be seen by the eye and distinction can be made between a positive and negative half-wave.

CORONA VOLTMETER. For a given conductor arrangement and air density corona always starts at the same maximum voltage. Use has been made of this fact in the corona voltmeter (Whitehead and Isshiki, J.A.I.E.E., May, 1920.) For this purpose, a wire in the center of a cylinder is generally used. In one form, the voltage is applied and the air density varied until the corona appears. The voltage is then readily calculated or read from a scale. A tapered wire has also been used. The starting point can be detected by the glow, the noise, the odor, or the conducting ionized air.

LAWS OF CORONA-FORMATION. The laws of corona have been quite definitely worked out. The chief factors affecting corona formation are:

For a given spacing and air density the corona starting voltage is lowered by decreasing the wire diameter.

Increasing the spacing increases the corona starting voltage, but the effect of changing the diameter is relatively much greater.

Decreasing barometric pressure decreases air density and decreases the corona starting voltage.

Increasing temperature decreases air density and decreases the corona starting voltage.

Dirt, water, etc., on the conductor surfaces lower the corona starting voltage by increasing the stress.

The *apparent strength* of air is not constant in irregular fields but greater at the surface of small conductors than large ones. The strength at the start of corona is always constant, however, at a distance from the conductor which is a known function of the radius, known as the **energy distance.** This corresponds to the strength in a uniform field of 76 kv per in. (maximum value in case of a-c voltage), which seems to mean that the actual strength of air is 76 kv per in. In order, however, that a finite thickness only may be brought to this stress in an irregular field, the stress at the conductor surface must be higher. At a given spacing the corona always starts at a lower voltage on small wires than large ones, because, although the *apparent strength* of air at the small conductor surface is greater, the stress produced by a given voltage is relatively much higher for the smaller wires.

Corona does not start at exactly the same voltages when the wires are alternately plus and minus. The difference is at most a few per cent and is greater for small wires than large ones.

For perfectly clean, smooth wires of uniform diameter, there is no loss until the *visual critical voltage* is reached, when the loss assumes a *definite value* and increases as a function of the ratio of the applied voltage and the *disruptive critical voltage.* The *visual critical*

voltage takes into account the *apparent strength* of the air, which is a function of the wire diameter; the *disruptive critical* voltage corresponds to the constant strength of air for uniform fields, which is about 76 kv per in., maximum value. The *visual critical voltage* is always higher than the *disruptive critical voltage. Owing to irregularities there is always a loss below the visual critical voltage.*

Corona loss increases with increasing frequency being proportional to the frequency, between 25 and 60 cycles.

The formulas for calculating corona on transmission lines are given below.

Corona on Transmission Lines

It is of great importance in the design of high-voltage transmission lines to know the various factors that affect the corona formation and to be able to estimate accurately the starting voltage and loss for a given line.

The various characteristics may be calculated from the following formulas based on the work of F. W. Peek, as modified by Wm. S. Peterson, A. Nuttall, V. Siegfried, J. S. Carroll and B. Cozzens, Trans. A.I.E.E., 1933, vol. 52, pp. 55–63.

Let e = rms value of the kilovolts to neutral ($= 1/\sqrt{3}$ kilovolts between wires for three-phase; $= 1/2$ kilovolts between wires for single-phase).

e_0 = disruptive voltage in kilovolts to neutral at which the voltage gradient at the surface of the conductor is equal to the air breakdown value.

t = temperature of the conductor in degrees fahrenheit.

b = barometric pressure, in inches.

$\delta = \dfrac{17.9b}{459 + t}$ = specific gravity of air, referred to air at 77 deg. fahr and 29.9 in. barometric pressure.

r = radius of conductor, in inches.

$g_0 = 53.6$ = disruptive critical gradient, kilovolts per inch, rms value.

$g_v = g_0\,\delta\left(1 + \dfrac{0.189}{\sqrt{\delta r}}\right)$ = visual critical gradient, kilovolts per inch, rms value.

s = distance between conductor centers, in inches.

f = frequency, cycles per second.

Then, for smooth round conductors, the disruptive critical voltage, rms value of kilovolts to neutral, is

$$e_0 = 2.302\, m\, g_0\, r\delta^{2/3} \log_{10}\left(\frac{s}{r}\right) \tag{62}$$

where m is the roughness factor and has the following values.

Condition of Conductor	m
New, unwashed conductors	0.67 to 0.74
Washed with a grease solvent *	0.912 to 0.93
Scratch-brushed	0.885
Buffed	1.00
Dragged and dusty	0.72 to 0.75
Weathered (5 months)	0.945
Weathered wire at low humidity	0.92
Same, at night, as low as	0.78
For general design	0.87 to 0.90

* It is found that die grease greatly increases corona loss.

For stranded conductors having 12 or more strands in the outer layer, the disruptive critical voltage, rms value of kilovolts to neutral, is

$$e_0 = \frac{123.4\,\delta^{2/3} m\left[\log_{10}\dfrac{s}{cr_1} + (n-1)\log_{10}\dfrac{2s}{(d - 2\,cr_1)}\right]}{\dfrac{1}{cr_1} + (n-1)/(d - 2\,cr_1)} \tag{63}$$

where r_1 is the radius of an individual strand, n is the number of strands in the outside layer, and c is given by

$$c = \left[1 - \frac{\sin\left(\dfrac{\pi}{2} + \dfrac{\pi}{n}\right)}{\left(\dfrac{\pi}{2} + \dfrac{\pi}{n}\right)}\right] \tag{64}$$

For **stranded conductors** having 6 outside strands,

$$e_0 = \frac{123.4\,\delta^{2/3}\,rm\left[\left(\log_{10}\frac{s}{r}\right) + 0.0677\right]}{1.37} \tag{65}$$

The **visual critical voltage,** rms value of kilovolts to neutral, is given approximately by

$$e_v = 2.302\, m_v\, g_v\, r \log_{10}\left(\frac{s}{r}\right) \tag{66}$$

where m_v, for polished wires, $=0.72$ for local corona and $=0.82$ for decided corona all along cable.

The **power loss in fair weather,** in kilowatts per mile of single conductor, is

$$p = \frac{0.0000337\, fe^2F}{\left(\log_{10}\frac{s}{r}\right)^2} \tag{67}$$

where F is a function of $\frac{e}{e_0}$, having the values given in Table IV.

Table IV. Empirical Corona Loss Function, F

$\frac{e}{e_0}$	F	$\frac{e}{e_0}$	F	$\frac{e}{e_0}$	F	$\frac{e}{e_0}$	F	$\frac{e}{e_0}$	F
0.6	0.011	1.4	0.32	2.2	8.4	6.0	22	12	28
0.8	0.018	1.5	0.92	2.4	9.8	7.0	24	14	29
1.0	0.036	1.6	2.70	3.0	10.5	8.0	26	16	29
1.2	0.083	1.8	5.1	4.0	16.5	9.0	27	18	29
1.3	0.140	2.0	7.0	5.0	19 0	10.0	28	20	30

Methods of Increasing Size of Conductors. For equal conductivity an aluminum conductor has about a 25 per cent greater diameter than a copper conductor, and, therefore, approximately 25 per cent higher critical voltage. The advantage of aluminum may be still further increased by the addition of a steel cable core. Aluminum, however, is more easily nicked or scratched than copper, during installation, thereby losing some of this advantage.

Copper cables of large diameter are now made with tubular strands, or with ordinary strands on a twisted I-beam core, or in annular form with interlocked or keyed strands. See Art. 49, chapter on Bare Wires and Cables.

ARRANGEMENT OF CONDUCTORS. In the above formulas it has been assumed for three-phase lines that the conductors are so arranged that they form the edges of an equilateral prism. When the conductors are not so arranged, but are placed symmetrically in a plane, corona will start at a lower voltage on the center conductor than on the outside conductors. The actual critical voltage on the center conductor will be approximately 4 per cent lower, and on the outside conductors 6 per cent higher, than for the equilateral prism arrangement with the same spacing.

VOLTAGE VARIATION ALONG LINE. In practice, owing to the drop in voltage along the line, the corona loss will be different at different points on the line. This may be allowed for by calculating the loss per mile at various points, and plotting a curve with loss per mile as ordinates and length in miles as abscissas. The area of this curve then represents the total loss.

If an insulated system is operating near the critical corona voltage, and one conductor becomes grounded, the corona loss will be quite high.

THE CORONA LIMIT OF HIGH-VOLTAGE TRANSMISSION LINES. SAFE AND ECONOMICAL VOLTAGES. It will generally be found that it is safe and economical to operate a line up to, but not above, the fair-weather value of the disruptive critical voltage, that is, up to the value of e_0 given by eq. (62), (63) or (65), for average barometer and summer temperatures. This will give loss during storms, but as storms do not extend over the whole line at one time, generally it will not be serious. During cold weather the critical voltage will be higher and storm loss less.

Besides the loss of energy, corona loss may be undesirable from another standpoint, viz., since the loss occurs only on part of the wave, it may introduce harmonics if it is excessive.

Table V gives the factors by which these voltages must be multiplied to give the corresponding disruptive critical voltage at various elevations. This correction factor

is equal to the ratio of the barometric pressure at the given altitude to the barometric pressure at sea level.

Table V. Approximate Altitude Correction Factors at 25 Deg Cent

Altitude		Correction Factor	Altitude		Correction Factor
Feet	Meters		Feet	Meters	
0	0	1.00	5,000	1525	0.87
500	152	0.99	6,000	1830	0.86
1000	305	0.97	7,000	2135	0.84
1500	459	0.96	8,000	2400	0.82
2000	610	0.95	9,000	2745	0.80
2500	765	0.94	10,000	3050	0.77
3000	915	0.93	12,000	3660	0.74
4000	1220	0.91	14,000	4270	0.70

Special Calculations. For small conductors the loss, in kilowatts per mile of conductor, is given by the expression,

$$p = \frac{390}{\delta}(f + 25)\sqrt{\frac{r + \frac{0.93}{s} + 0.016}{s}}\,(e - e_d)^2 \times 10^{-5} \qquad (68)$$

where $$e_d = 2.302\, m\, g_d\, r \log_{10}\left(\frac{s}{r}\right)$$

and $$g_d = g_0\delta\left[1 + \frac{0.189}{\sqrt{\delta r}}\frac{1}{(1 + 1480\, r^2)}\right]$$

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CAPACITANCE AND INDUCTANCE TABLES

By W. A. Del Mar

21. CAPACITANCE

See also Section on Electricity in Eshbach, Fundamentals of Engineering.

Overhead Wires

SINGLE ROUND WIRE PARALLEL TO THE GROUND.

H = height of wire above ground,
d = diameter of wire, in same unit as H.

The capacitance $= \dfrac{7.354 \times 10^{-3}}{\log_{10} \dfrac{4H}{d}}$ microfarads per 1000 ft (1)

$= \dfrac{38.83 \times 10^{-3}}{\log_{10} \dfrac{4H}{d}}$ microfarads per mile (2)

TWO PARALLEL ROUND* WIRES.

D = distance apart, center to center.
d = diameter of wire, in same unit as D.

The exact capacitance †

$= \dfrac{8.467 \times 10^{-3}}{\cosh^{-1} \dfrac{D}{d}}$ microfarads per 1000 ft (3)

When D is greater than $10\,d$ the following formulas for the capacitance *between wires* may be used instead of the above with an error of less than 0.1 per cent:

$= \dfrac{3.677 \times 10^{-3}}{\log_{10} \dfrac{2D}{d}}$ microfarads per 1000 ft (4)

$= \dfrac{19.41 \times 10^{-3}}{\log_{10} \dfrac{2D}{d}}$ microfarads per mile (5)

The *capacitance to neutral* in all cases is $C_0 = 2C$. Tables of capacity to neutral for various sizes of wires and various spacings are given in the tables below. Note that these tables and the above formulas are strictly applicable to ordinary overhead lines only when the distance from the wires to other conductors, particularly the earth, is large compared with their distance apart. However, the effect of the earth is usually small in most practical cases (see below), and the formulas and tables give a very fair approximation to the actual capacitances.

The capacitances of standard strands given in the following tables are calculated by the same formula as for smooth round wires using for the diameter d the diameter of the strand; see *Bare Wires and Cables.* The values as thus calculated are therefore not exact, but the error is probably less than 3 per cent for all practical cases.

Effect of the Earth on the Capacity of Two Overhead Wires. If both wires are at the same height, the effect of the earth is to increase the capacitance by an amount equal to the increase in capacitance which would result from decreasing the distance between the wires from the actual distance D to the "equivalent" distance D'. For D small compared with the height H, which is usually the case, the equivalent distance D' is practically equal to D, and the effect of the earth is therefore negligible.

$$D' = \frac{D}{\sqrt{1 + \left(\dfrac{D}{2H}\right)^2}} = \text{“equivalent” distance apart} \qquad (6)$$

* When the wires are far apart compared with the linear dimensions of their cross-section, the second group of formulas also applies approximately to wires of any shape of cross-section provided d is taken equal to the perimeter of the cross-section divided by π, i.e., equal to the "equivalent" diameter of the cross-section.

† Taking into account the non-uniform distribution of the charge on each wire; see Pender and Osborne, *Elec. World,* 1910, Vol. 56, p. 667.

THREE-PARALLEL WIRES.

Equilateral Triangle Arrangement.

D = distance apart, center to center.
d = diameter of wire in same units as D.

The normal capacitance between any two of the wires with the third wire insulated is

$$\frac{3.677 \times 10^{-3}}{\log_{10} \dfrac{2D}{d}} \quad \text{microfarads per 1000 ft} \tag{7}$$

which is the same as the capacitance between two parallel wires by themselves; see above.

Equilateral Triangle Arrangement with Balanced Three-phase Voltages. For sine-wave voltages between the wires equal in effective value to V and differing in phase by 120 deg, the following relations hold between the *effective* values of the voltages and charging currents for each of the three wires.

$$I = 2\,\pi f C_0\, V_0 \tag{8}$$

where $V_0 = \dfrac{V}{\sqrt{3}}$ = voltage to neutral.

$$C_0 = 2C_{12} = \text{capacity to neutral}$$

when the C's are in microfarads and the charging current in microamperes.

The charging current for any wire is 90 deg ahead of the voltage drop from that wire to the neutral. For the same voltage V between wires in a single-phase system as in a three-phase balanced system, the charging current per wire in the three-phase system with the equilateral triangle arrangement of wires is $2/\sqrt{3} = 1.155$ times the charging current per wire in the single-phase system.

For any other arrangement of wires and for an unbalanced three-phase system, the general equations in Eshbach, Fundamentals of Engineering may be used.

Effect of the Earth on the Capacity of Three Overhead Wires. Ordinarily the formulas given above neglecting the effect of the earth are sufficiently accurate for all practical purposes; compare with the effect of the earth on a two-wire line.

Cables

SINGLE-CONDUCTOR CABLE. Round Wire in Concentric Sheath.

D = inside diameter of sheath.
d = diameter of conductor (both in same units).
K = specific inductive capacity of dielectric.

The capacitance of a length of cable long compared to its diameter

$$= \frac{7.354 \times 10^{-3}\, K}{\log_{10} \dfrac{D}{d}} \quad \text{microfarads per 1000 ft} \tag{9}$$

Round Wire in Eccentric Sheath.

a = radius of wire.
b = inside radius of sheath.
c = distance between center of wire and center of sheath (all in same units).

The ratio of the capacitance eccentric, to the capacitance, concentric,

$$= \frac{\cosh^{-1}\left(\dfrac{b^2 + a^2}{2\,ab}\right)}{\cosh^{-1}\left(\dfrac{b^2 + a^2 - c^2}{2\,ab}\right)} \tag{10}$$

TWO-CONDUCTOR CABLE.

D = inside diameter of sheath.
a = distance between centers of wires.
d = diameter of wires (all in same units).

Assuming the entire space between conductors and sheath to have uniform specific inductive capacity K, the capacitance from conductor to conductor

$$= \frac{3.677 \times 10^{-3}\, K}{\log_{10}\left(\dfrac{2a}{d} \cdot \dfrac{D^2 - a^2}{D^2 + a^2}\right)} \quad \text{microfarads per 1000 ft} \tag{11}$$

THREE-CONDUCTOR CABLE.

D = inside diameter of sheath.
a = distance between centers of wires.
d = diameter of wires (all in same units).

Assuming the entire space between conductors and sheath to have uniform specific inductive capacity K, the capacitance between one conductor and any other one will be

$$\frac{3.677 \times 10^{-3} K}{\log_{10} \frac{2a\rho}{d}} \quad \text{microfarads per 1000 ft} \tag{12}$$

where

$$\rho = \sqrt{\frac{(3D^2 - 4a^2)^3}{(3D^2)^3 - (4a^2)^3}}$$

That is, the capacitance is the same as that of two parallel wires by themselves but at a distance ρa between centers instead of the actual distance a.

Relations between capacitances of various combinations of conductors and sheath will be found in *Alternating Currents*, by Alex. Russell, and in *Electric Cables*, by W. A. Del Mar.

Capacitance to Neutral* of Smooth Round Wires

Microfarads per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.01199	0.006608	0.005192	0.004618	0.004282	0.003884	0.003643	0.003477
000	0.4096	0.01099	0.006317	0.005013	0.004477	0.004161	0.003783	0.003555	0.003396
00	0.3648	0.01016	0.006055	0.004847	0.004344	0.004045	0.003688	0.003470	0.003319
0	0.3249	0.009458	0.005812	0.004692	0.004218	0.003936	0.003597	0.003390	0.003245
1	0.2893	0.008855	0.005587	0.004546	0.004100	0.003833	0.003511	0.003313	0.003174
2	0.2576	0.008332	0.005381	0.004408	0.003988	0.003735	0.003428	0.003239	0.003107
4	0.2043	0.007455	0.005010	0.004157	0.003781	0 003553	0.003274	0.003102	0.002980
6	0.1620	0.006753	0.004688	0.003933	0.003595	0.003388	0.003134	0.002975	0.002863
8	0.1285	0.006177	0.004406	0.003732	0.003426	0.003238	0.003005	0.002859	0.002755
10	0.1019	0.005693	0.004155	0.003551	0.003273	0.003100	0.002886	0.002751	0.002655
12	0.08081	0.005277	0.003931	0.003386	0.003132	0.002974	0.002776	0.002651	0.002562
14	0.06408	0.004921	0.003730	0.003235	0.003003	0.002858	0.002675	0.002558	0.002475
16	0.05082	0.004611	0.003549	0.003099	0.002885	0.002750	0.002580	0.002472	0.002394

Size of Wire, A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.003351	0.003171	0.003043	0.002947	0.002806	0.002706	0.002542	0.002436	0.002361
000	0.003276	0.003103	0.002981	0.002889	0.002753	0.002657	0.002498	0.002396	0.002323
00	0.003204	0.003039	0.002922	0.002833	0.002702	0.002610	0.002456	0.002358	0.002287
0	0.003135	0.002977	0.002864	0.002779	0.002653	0.002564	0.002416	0.002320	0.002251
1	0.003069	0.002917	0.002809	0.002727	0.002606	0.002520	0.002376	0.002284	0.002217
2	0.003006	0.002860	0.002756	0.002677	0.002560	0.002477	0.002338	0.002249	0.002184
4	0.002887	0.002752	0.002656	0.002582	0.002474	0.002396	0.002266	0.002182	0.002121
6	0.002777	0.002652	0.002563	0.002494	0.002392	0.002319	0.002197	0.002118	0.002061
8	0.002676	0.002559	0.002476	0.002412	0.002317	0.002248	0.002133	0.002059	0.002004
10	0.002581	0.002473	0.002395	0.002335	0.002245	0.002181	0.002073	0.002002	0.001951
12	0.002493	0.002392	0.002319	0.002262	0.002178	0.902118	0.002016	0.001949	0.001900
14	0.002411	0.002316	0.002247	0.002194	0.002115	0.002058	0.001961	0.001898	0.001852
16	0.002334	0 002245	0.002180	0.002130	0.002056	0.002002	0 001910	0.001850	0.001806

* The capacity *between* wires equals one-half the values given in this table.

Capacitance to Neutral* of Smooth Round Wires

Microfarads per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.06332	0.03490	0.02741	0.02438	0.02261	0.02051	0.01924	0.01836
000	0.4096	0.05802	0.03336	0.02647	0.02364	0.02197	0.01998	0.01877	0.01793
00	0.3648	0.05366	0.03198	0.02559	0.02293	0.02136	0.01947	0.01832	0.01752
0	0.3249	0.04995	0.03069	0.02477	0.02227	0.02078	0.01899	0.01790	0.01713
1	0.2893	0.04676	0.02951	0.02400	0.02165	0.02024	0.01854	0.01749	0.01676
2	0.2576	0.04400	0.02842	0.02328	0.02106	0.01972	0.01810	0.01710	0.01640
4	0.2043	0.03937	0.02645	0.02195	0.01997	0.01876	0.01729	0.01638	0.01573
6	0.1620	0.03566	0.02475	0.02077	0.01898	0.01789	0.01655	0.01571	0.01512
8	0.1285	0.03262	0.02326	0.01971	0.01809	0.01710	0.01587	0.01510	0.01455
10	0.1019	0.03006	0.02194	0.01875	0.01728	0.01637	0.01524	0.01453	0.01402
12	0.08081	0.02787	0.02076	0.01788	0.01654	0.01570	0.01466	0.01400	0.01353
14	0.06408	0.02599	0.01970	0.01709	0.01586	0.01509	0.01412	0.01351	0.01307
16	0.05082	0.02434	0.01874	0.01636	0.01523	0.01452	0.01362	0.01305	0.01264

Size of Wire, A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.01769	0.01674	0.01607	0.01556	0.01482	0.01429	0.01342	0.01286	0.01246
000	0.01730	0.01639	0.01574	0.01525	0.01454	0.01403	0.01319	0.01265	0.01227
00	0.01692	0.01604	0.01543	0.01496	0.01427	0.01378	0.01297	0.01245	0.01207
0	0.01656	0.01572	0.01512	0.01467	0.01401	0.01354	0.01275	0.01225	0.01189
1	0.01621	0.01540	0.01483	0.01440	0.01376	0.01330	0.01255	0.01206	0.01171
2	0.01587	0.01510	0.01455	0.01413	0.01352	0.01308	0.01235	0.01187	0.01153
4	0.01525	0.01453	0.01402	0.01363	0.01306	0.01265	0.01196	0.01152	0.01120
6	0.01467	0.01400	0.01353	0.01317	0.01263	0.01225	0.01160	0.01118	0.01088
8	0.01413	0.01351	0.01307	0.01273	0.01223	0.01187	0.01126	0.01087	0.01058
10	0.01363	0.01306	0.01264	0.01233	0.01186	0.01152	0.01094	0.01057	0.01030
12	0.01316	0.01263	0.01224	0.01194	0.01150	0.01118	0.01064	0.01029	0.01003
14	0.01273	0.01223	0.01187	0.01159	0.01117	0.01087	0.01036	0.01002	0.009777
16	0.01232	0.01185	0.01151	0.01125	0.01085	0.01057	0.01008	0.009768	0.009536

* The capacity *between* wires equals one-half the values given in this table.

Capacitance to Neutral* of Standard Stranded Conductors

Microfarads per 1000 FEET of each conductor of a single-phase or of a symmetrical three-phase line

Size of Cable, C.M. or A.W.G.	Diam. of Strand, inches	Inches between Conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	....	0.0105	0.00725	0.00617	0.00558	0.00492	0.00454	0.00428
750,000	0.998	....	0.00959	0.00683	0.00586	0.00533	0.00472	0.00437	0.00414
500,000	0.814	0.0245	0.00856	0.00630	0.00547	0.00501	0.00447	0.00415	0.00394
350,000	0.681	0.0181	0.00783	0.00591	0.00517	0.00476	0.00427	0.00398	0.00378
250,000	0.575	0.0147	0.00725	0.00558	0.00492	0.00454	0.00409	0.00383	0.00364
0 000	0.528	0.0135	0.00699	0.00542	0.00480	0.00444	0.00401	0.00376	0.00358
000	0.470	0.0122	0.00666	0.00523	0.00465	0.00431	0.00390	0.00366	0.00349
00	0.418	0.0112	0.00637	0.00504	0.00450	0.00418	0.00380	0.00357	0.00341
0	0.373	0.0103	0.00610	0.00488	0.00437	0.00407	0.00371	0.00349	0.00333
1	0.332	0.00958	0.00586	0.00472	0.00424	0.00396	0.00361	0.00341	0.00326
2	0.292	0.00891	0.00561	0.00456	0.00411	0.00384	0.00352	0.00332	0.00318
4	4.232	0.00790	0.00520	0.00429	0.00389	0.00365	0.00336	0.00318	0.00305
6	0.184	0.00712	0.00486	0.00405	0.00369	0.00348	0.00321	0.00304	0.00293

Capacitance to Neutral* of Standard Stranded Conductors—*Continued*

Size of Cable, C.M. or A.W.G.	Feet between Conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.00410	0.00383	0.00365	0.00351	0.00331	0.00317	0.00295	0.00281	0.00271
750,000	0.00396	0.00371	0.00354	0.00341	0.00322	0.00309	0.00288	0.00274	0.00265
500,000	0.00378	0.00355	0.00339	0.00327	0.00310	0.00298	0.00278	0.00266	0.00257
350,000	0.00363	0.00342	0.00328	0.00316	0.00300	0.00289	0.00270	0.00258	0.00250
250,000	0.00351	0.00331	0.00317	0.00307	0.00292	0.00281	0.00263	0.00252	0.00244
0 000	0.00345	0.00326	0.00312	0.00302	0.00287	0.00277	0.00260	0.00249	0.00240
000	0.00337	0.00318	0.00306	0.00296	0.00282	0.00272	0.00255	0.00245	0.00237
00	0.00329	0.00312	0.00299	0.00290	0.00276	0.00267	0.00251	0.00240	0.00233
0	0.00322	0.00305	0.00293	0.00284	0.00271	0.00262	0.00247	0.00237	0.00229
1	0.00315	0.00299	0.00288	0.00279	0.00266	0.00257	0.00242	0.00233	0.00226
2	0.00308	0.00292	0.00281	0.00273	0.00261	0.00252	0.00238	0.00229	0.00222
4	0.00295	0.00281	0.00271	0.00263	0.00252	0.00244	0.00230	0.00222	0.00215
6	0.00284	0.00271	0.00261	0.00254	0.00244	0.00236	0.00223	0.00215	0.00209

* The capacity *between* conductors equals one-half the values given in this table.

Capacitance to Neutral* of Standard Stranded Conductors

Microfarads per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of Cable C.M. or A.W.G.	Diam. of Strand, inches	Inches between Conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	....	0.0554	0.0383	0.0325	0.0294	0.0260	0.0240	0.0226
750.000	0.998	....	0.0506	0.0361	0.0309	0.0281	0.0249	0.0231	0.0218
500,000	0.814	0.134	0.0452	0.0333	0.0289	0.0264	0.0236	0.0219	0.0208
350,000	0.681	0.0955	0.0413	0.0312	0.0273	0.0251	0.0225	0.0210	0.0200
250,000	0.575	0.0776	0.0383	0.0295	0.0260	0.0240	0.0216	0.0202	0.0192
0 000	0.528	0.0713	0.0369	0.0286	0.0253	0.0234	0.0212	0.0198	0.0189
000	0.470	0.0644	0.0352	0.0276	0.0245	0.0227	0.0206	0.0193	0.0184
00	0.418	0.0590	0.0336	0.0266	0.0238	0.0221	0.0201	0.0189	0.0180
0	0.373	0.0544	0.0322	0.0258	0.0231	0.0214	0.0196	0.0184	0.0176
1	0.332	0.0506	0.0309	0.0249	0.0224	0.0209	0.0191	0.0180	0.0172
2	0.292	0.0470	0.0296	0.0241	0.0217	0.0203	0.0186	0.0175	0.0168
4	0.232	0.0417	0.0275	0.0227	0.0205	0.0193	0.0177	0.0168	0.0161
6	0.184	0.0376	0.0256	0.0214	0.0195	0.0184	0.0169	0.0161	0.0154

Size of Cable, C.M. or A.W.G.	Feet between Conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.0216	0.0202	0.0193	0.0185	0.0175	0.0168	0.0156	0.0148	0.0143
750,000	0.0209	0.0196	0.0187	0.0180	0.0170	0.0163	0.0152	0.0145	0.0140
500,000	0.0200	0.0188	0.0179	0.0173	0.0164	0.0157	0.0147	0.0140	0.0135
350,000	0.0192	0.0181	0.0173	0.0167	0.0159	0.0153	0.0143	0.0136	0.0132
250,000	0.0185	0.0175	0.0168	0.0162	0.0154	0.0148	0.0139	0.0133	0.0129
0 000	0.0182	0.0172	0.0165	0.0160	0.0152	0.0146	0.0137	0.0131	0.0127
000	0.0178	0.0168	0.0161	0.0156	0.0149	0.0143	0.0135	0.0129	0.0125
00	0.0174	0.0165	0.0158	0.0153	0.0146	0.0141	0.0132	0.0127	0.0123
0	0.0170	0.0161	0.0155	0.0150	0.0143	0.0138	0.0130	0.0125	0.0121
1	0.0166	0.0158	0.0152	0.0147	0.0141	0.0136	0.0128	0.0123	0.0119
2	0.0162	0.0154	0.0149	0.0144	0.0138	0.0133	0.0126	0.0121	0.0117
4	0.0156	0.0148	0.0143	0.0139	0.0133	0.0129	0.0122	0.0117	0.0114
6	0.0150	0.0143	0.0138	0.0134	0.0129	0.0125	0.0118	0.0114	0.0111

* The capacity *between* conductors equals one-half the values given in this table.

25-Cycle Capacity Susceptance to Neutral* Smooth Round Wires

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	9.948	5.483	4.306	3.830	3.552	3.222	3.023	2.884
000	0.4096	9.115	5.241	4.158	3.714	3.451	3.139	2.949	2.817
00	0.3648	8.430	5.024	4.020	3.602	3.356	3.059	2.878	2.752
0	0.3249	7.847	4.821	3.891	3.499	3.265	2.983	2.812	2.691
1	0.2893	7.346	4.636	3.770	3.401	3.180	2.913	2.748	2.633
2	0.2576	6.912	4.465	3.657	3.309	3.098	2.844	2.686	2.576
4	0.2043	6.185	4.155	3.448	3.137	2.947	2.716	2.573	2.471
6	0.1620	5.602	3.888	3.263	2.982	2.811	2.600	2.468	2.375
8	0.1285	5.125	3.654	3.096	2.842	2.686	2.493	2.372	2.286
10	0.1019	4.722	3.447	2.946	2.715	2.572	2.394	2.283	2.203
12	0.08081	4.378	3.261	2.809	2.598	2.466	2.303	2.199	2.126
14	0.06408	4.083	3.095	2.685	2.492	2.371	2.218	2.122	2.053
16	0.05082	3.824	2.944	2.570	2.393	2.281	2.140	2.050	1.986

Size of Wire, A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	2.779	2.630	2.525	2.444	2.328	2.245	2.108	2.020	1.957
000	2.718	2.575	2.473	2.396	2.284	2.204	2.072	1.987	1.928
00	2.657	2.520	2.424	2.350	2.242	2.165	2.038	1.956	1.896
0	2.602	2.470	2.375	2.305	2.201	2.127	2.003	1.924	1.868
1	2.547	2.419	2.330	2.262	2.162	2.089	1.972	1.895	1.840
2	2.493	2.372	2.286	2.220	2.124	2.055	1.940	1.865	1.811
4	2.396	2.283	2.203	2.141	2.052	1.987	1.879	1.810	1.760
6	2.305	2.199	2.126	2.069	1.984	1.924	1.822	1.756	1.709
8	2.220	2.122	2.053	2.000	1.921	1.865	1.769	1.708	1.662
10	2.141	2.052	1.986	1.937	1.863	1.810	1.719	1.661	1.618
12	2.067	1.984	1.923	1.876	1.807	1.756	1.672	1.617	1.576
14	2.000	1.921	1.865	1.821	1.755	1.708	1.628	1.574	1.536
16	1.935	1.862	1.808	1.767	1.705	1.661	1.584	1.535	1.498

* The susceptance *between* wires equals one-half the values given in this table.

25-Cycle Capacity Susceptance to Neutral* Standard Stranded Conductors

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}

Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of Cable, C.M. or A.W.G.	Diam. of Strand, inches	Inches between Conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	....	8.70	6.01	5.10	4.62	4.08	3.77	3.55
750,000	0.998	....	7.94	5.67	4.85	4.41	3.91	3.63	3.42
500,000	0.814	21.0	7.10	5.23	4.54	4.14	3.71	3.44	3.27
350,000	0.681	15.0	6.48	4.90	4.29	3.94	3.53	3.30	3.14
250,000	0.575	12.2	6.01	4.63	4.08	3.77	3.39	3.17	3.02
0 000	0.528	11.2	5.79	4.49	3.97	3.67	3.33	3.11	2.97
000	0.470	10.1	5.53	4.33	3.85	3.57	3.24	3.03	2.89
00	0.418	9.26	5.28	4.18	3.74	3.47	3.16	2.97	2.83
0	0.373	8.54	5.06	4.05	3.63	3.36	3.08	2.89	2.76
1	0.332	7.94	4.85	3.91	3.52	3.28	3.00	2.83	2.70
2	0.292	7.38	4.65	3.79	3.41	3.19	2.92	2.75	2.64
4	0.232	6.55	4.32	3.57	3.22	3.03	2.78	2.64	2.53
6	0.184	5.90	4.02	3.36	3.06	2.89	2.65	2.53	2.42

25-Cycle Capacity Susceptance to Neutral* Standard Stranded Conductors—*Continued*

Size of Cable, C.M. or A.W.G.	Feet between Conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	3.39	3.17	3.03	2.90	2.75	2.64	2.45	2.32	2.25
750,000	3.28	3.08	2.94	2.83	2.67	2.56	2.39	2.28	2.20
500,000	3.14	2.95	2.81	2.72	2.57	2.47	2.31	2.20	2.12
350,000	3.02	2.84	2.72	2.62	2.50	2.40	2.25	2.14	2.07
250,000	2.90	2.75	2.64	2.54	2.42	2.32	2.18	2.09	2.03
0 000	2.86	2.70	2.59	2.51	2.39	2.29	2.15	2.06	1.99
000	2.79	2.64	2.53	2.45	2.34	2.25	2.12	2.03	1.96
00	2.73	2.59	2.48	2.40	2.29	2.22	2.07	1.99	1.93
0	2.67	2.53	2.43	2.36	2.25	2.17	2.04	1.96	1.90
1	2.61	2.48	2.39	2.31	2.22	2.14	2.01	1.93	1.87
2	2.54	2.42	2.34	2.26	2.17	2.09	1.98	1.90	1.84
4	2.45	2.32	2.25	2.18	2.09	2.03	1.92	1.84	1.79
6	2.36	2.25	2.17	2.10	2.03	1.96	1.85	1.79	1.74

* The susceptance *between* conductors equals one-half the values given in this table.

60-Cycle Capacity Susceptance to Neutral* Smooth Round Wires

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	23.87	13.16	10.33	9.191	8.524	7.732	7.253	6.922
000	0.4096	21.87	12.58	9.979	8.912	8.283	7.532	7.076	6.760
00	0.3648	20.23	12.06	9.647	8.645	8.053	7.340	6.907	6.605
0	0.3249	18.83	11.57	9.338	8.396	7.834	7.159	6.748	6.458
1	0.2893	17.63	11.13	9.048	8.162	7.630	6.990	6.594	6.319
2	0.2576	16.59	10.71	8.777	7.940	7.434	6.824	6.447	6.183
4	0.2043	14.84	9.972	8.275	7.529	7.073	6.518	6.175	5.930
6	0.1620	13.44	9.331	7.830	7.155	6.745	6.239	5.923	5.700
8	0.1285	12.30	8.769	7.430	6.820	6.447	5.983	5.693	5.485
10	0.1019	11.33	8.271	7.069	6.515	6.171	5.745	5.478	5.286
12	0.08081	10.51	7.827	6.741	6.236	5.919	5.527	5.278	5.101
14	0.06408	9.798	7.427	6.443	5.979	5.689	5.323	5.093	4.927
16	0.05082	9.176	7.065	6.168	5.742	5.474	5.135	4.920	4.765

Size of Wire, A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	6.669	6.311	6.058	5.866	5.587	5.387	5.059	4.848	4.697
000	6.522	6.179	5.934	5.749	5.482	5.289	4.973	4.769	4.626
00	6.379	6.047	5.817	5.640	5.380	5.195	4.890	4.694	4.550
0	6.243	5.926	5.700	5.531	5.282	5.105	4.807	4.618	4.483
1	6.111	5.806	5.591	5.429	5.188	5.014	4.731	4.547	4.415
2	5.983	5.693	5.485	5.327	5.097	4.931	4.656	4.475	4.347
4	5.749	5.478	5.286	5.139	4.924	4.769	4.509	4.343	4.222
6	5.531	5.278	5.101	4.965	4.762	4.618	4.373	4.215	4.102
8	5.327	5.093	4.927	4.799	4.611	4.475	4.245	4.098	3.989
10	5.139	4.924	4.765	4.648	4.471	4.343	4.124	3.985	3.883
12	4.961	4.762	4.614	4.501	4.336	4.215	4.011	3.879	3.781
14	4.799	4.611	4.475	4.369	4.211	4.098	3.906	3.778	3.686
16	4.645	4.467	4.339	4.241	4.090	3.985	3.800	3.683	3.595

* The susceptance *between* wires equals one-half the values given in this table.

60-Cycle Capacity Susceptance to Neutral* Standard Stranded Conductors

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}
Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of Cable, C.M. or A.W.G.	Diam. of Strand, inches	Inches between Cables, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	....	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	....	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0 000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.18	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of Cable, C.M. or A.W.G.	Feet between Cables, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0 000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

* The susceptance *between* conductors equals one-half the values given in this table.

22. INDUCTANCE

SELF-INDUCTANCE OF A SINGLE STRAIGHT ROUND WIRE, RETURN NEGLECTED.

Let l = length of wire, in centimeters.
d = diameter of round wire, in centimeters.
$r = \frac{d}{2}$ = radius of round wire, in centimeters.
L' = inductance, in millihenrys.

Then for a round wire the self-inductance is

$$L' = 2\left[l \log_\epsilon \frac{l + \sqrt{l^2 + r^2}}{r} - \sqrt{l^2 + r^2} + \frac{l}{4} + r\right] \times 10^{-6} \tag{13}$$

$$= 2l\left[\log_\epsilon \frac{2l}{r} - \frac{3}{4}\right] \times 10^{-6}, \text{ approximately} \tag{14}$$

Where the permeability of the wire is μ and that of the medium outside is unity, and the frequency is low, eq. (2) appears in the form

$$L' = 2l\left[\log_\epsilon \frac{2l}{r} - 1 + \frac{\mu}{4}\right] \times 10^{-6} \tag{15}$$

This last formula is of theoretical interest only, as the value to assign to μ is doubtful and when such wires are used even for currents of moderate frequencies, the skin effect is appreciable; see Article 50.

The self-inductance of a round wire may be considered as made up of two parts, viz., the inductance due to the flux *external* to the wire, and that due to the flux *within* the wire. The first or "external" inductance is

$$L_e = 2l\left[\log_\epsilon \frac{2l}{r} - 1\right] \times 10^{-6} \tag{16}$$

and the "internal" self-inductance is

$$L_i = l\frac{\mu}{2} \times 10^{-6} \tag{17}$$

both being in millihenrys.

SELF-INDUCTANCE OF A HOLLOW TUBE OF CIRCULAR SECTION, RETURN NEGLECTED. The *external* inductance is the same as for a solid wire, i.e., eq. (4), taking for r the external radius of the tube. The *internal* inductance of the tube, putting r_2 = external radius and r_1 = internal radius, is

$$L_i = 2\,\mu l\left[\frac{r_1^4}{(r_2^2 - r_1^2)^2}\log_\epsilon \frac{r_2}{r_1} - \frac{1}{4}\,\frac{3r_1^2 - r_2^2}{r_2^2 - r_1^2}\right] \times 10^{-6} \tag{18}$$

The term in the square brackets is always less than ¼, i.e., the internal inductance of a hollow tube is always less than the internal inductance of a solid wire; see eq. (5). In the limit, where $r_1 = r_2$ (tube with infinitely thin walls), the *internal* inductance is zero.

MUTUAL INDUCTANCE OF TWO PARALLEL STRAIGHT WIRES OR BARS. See Fig. 1. Same notation as above, and in addition let D = distance between centers of the two wires, in centimeters.

Then the mutual inductance between the two wires AC and BD is

$$M = 2\left[l\log_\epsilon \frac{l + \sqrt{l^2 + D^2}}{D} - \sqrt{l^2 + D^2} + D\right] \times 10^{-6} \tag{19}$$

$$= 2l\left[\log_\epsilon \frac{2l}{D} - 1 + \frac{D}{l}\right] \times 10^{-6}\text{, approximately} \tag{20}$$

when the length l is great in comparison with D.

Eq. (7), which is an exact expression when the wires have no appreciable cross-section, is not an exact expression for the mutual inductance of two parallel cylindrical wires, but is not appreciably in error even when the section is large and D is small if l is great compared with D.

Eq. (7) is also applicable, with a practically negligible error, to bars of rectangular section, and in fact to the mutual inductance between any two parallel conductors of any section and external to each other, e.g., between an overhead wire and a rail, the distance D being the distance between the centers of gravity of the two sections.*

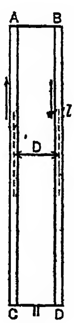

Fig. 1

MUTUAL INDUCTANCE BETWEEN A TUBE AND AN INTERIOR WIRE. Using the same notation as for eq. (6) above, the mutual inductance in this case is

$$M = 2l\left[\log_\epsilon 2l - \frac{r_2^2\log_\epsilon r_2 - r_1^2\log_\epsilon r_1}{r_2^2 - r_1^2} - \frac{1}{2}\right] \times 10^{-6} \tag{21}$$

TOTAL INDUCTANCE OF A TWO-WIRE TRANSMISSION LINE. For a length so great that d and D are negligible compared with l, this total inductance per wire for equal round wires is

$$L = 2l\left[\log_\epsilon \frac{2D}{d} + \frac{\mu}{4}\right] \times 10^{-6} \tag{22}$$

where d is the diameter of each wire. The total inductance of the line per unit length of *wire* of unit permeability is

$$\left.\begin{aligned} L &= 0.01524 + 0.14037 \log_{10}(2\,D/d) \quad \text{millihenrys per 1000 ft} \\ &= 0.08047 + 0.74113 \log_{10}(2\,D/d) \quad \text{millihenrys per mile} \end{aligned}\right\} \tag{23}$$

where D and d may be expressed in any units of length provided they are both expressed in the *same* units. To obtain the total inductance of both wires multiply by *twice* the

* Accurately, the geometrical mean distance between the two areas; see *Bull. Bur. Stand.*, 1912, Vol. 8, pp. 125 and 126. For round wires, solid or tubular, the geometrical mean distance between them is exactly the distance between their centers.

length of the line. The formulas given in eq. (23) also apply approximately to stranded wires, provided d is taken as the diameter of the solid wire having a cross-section equal to that of the copper in the stranded wire; i.e., the inductance of a No. 0000 stranded wire on a given spacing is approximately the same as that of a No. 0000 solid wire on the same spacing.

Tables of L and the corresponding reactances for 25 and 60 cycles for various sizes of wires and various spacings are given below.

The above formula is not applicable to conductors at close spacing when the frequency is high; see F. B. Silsbie, *Elec. World*, Vol. 68, p. 125, July 15, 1916.

INDUCTANCE DROP IN A THREE-WIRE TRANSMISSION LINE. The value of L to be used in the following equations is that to be derived from eq. (23).

Equilateral Triangle Arrangement. For a sine-wave current of effective value I and frequency f, the inductive drop in each wire per unit length of line has the effective value

$$V = 2\pi fLI \tag{24}$$

and leads the current in this particular wire by 90 deg, irrespective of whether the load be balanced or not, the only condition being that no current returns to the generator through any other conductor than the three line wires.

Three Parallel Wires in the Same Plane. Let the three wires of the three-phase system have equal diameters d, let No. 2 be the middle wire, and let Nos. 1 and 3 be at equal distances (D between centers) from No. 2 and on opposite sides of No. 2. Under these conditions, considering unit length of line, put

L = same values as in (23), taking D as the distance between either outer and the middle wire,

and M = 0.04225 millihenry per 1000 ft
= 0.2231 millihenry per mile

Then

$$\left.\begin{aligned} V_1 &= 2\pi f[LI_1 - MI_3] \\ V_2 &= 2\pi f I_2 L \\ V_3 &= 2\pi f[LI_3 - MI_1] \end{aligned}\right\} \tag{25}$$

TOTAL INDUCTANCE OF A CONCENTRIC CABLE. The total inductance of a concentric main per unit length of *cable* is

$$L = 2\left[\log_\epsilon \frac{d_2}{d_1} + \frac{d_3^4}{(d_3^2 - d_2^2)^2}\log_\epsilon \frac{d_3}{d_2} - \frac{d_3^2}{2(d_3^2 - d_2^2)}\right] \times 10^{-6} \text{ millihenry per cm.} \tag{26}$$

where d_1 = diameter of internal conductor, assumed solid; d_2 = internal diameter of outer conductor; and d_3 = external diameter of outer conductor.

INDUCTANCE OF OVERHEAD WIRES WITH EARTH RETURN. This case is not susceptible of a definite solution, since the inductance depends upon the distribution of the return current in the earth. When the current returns through one or more rails immediately below the wire, the leakage current to the earth may be neglected and the wire and rails treated as linear conductors, applying the formulas given above. When there is no metallic return circuit, an approximate solution may be obtained by considering the earth as equivalent to the " images " of the overhead wires in the plane of the earth's surface, i.e., considering the return circuit as consisting of the same number of wires as there are overhead, these fictitious return wires being the same distance below the earth as the actual wires are above it. The value of the inductances as thus calculated can never be greater than the actual inductances but will usually be slightly less than the actual values.

The *external* self-inductance of a rail is practically the same as that of a round wire, taking for r the perimeter of the rail divided by 2π; see eq. (4). The *internal* self-inductance depends upon the permeability μ and the frequency of the current. The internal inductance, however, is small compared with the external inductance and for approximate calculations may be neglected. See Electric Railways, Section 17.

EFFECT OF FREQUENCY ON INDUCTANCE. See article on Skin Effect.

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Self-inductance of Solid Non-magnetic Wires *

Millihenrys per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, cir mils or A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.5750	0.1245	0.1667	0.1915	0.2090	0.2337	0.2512	0.2648
750,000	0.8660	0.06627	0.1332	0.1755	0.2002	0.2178	0.2425	0.2600	0.2736
500,000	0.7071	0.07863	0.1456	0.1879	0.2126	0.2301	0.2548	0.2724	0.2860
350,000	0.5916	0.08950	0.1565	0.1987	0.2235	0.2410	0.2657	0.2832	0.2968
250,000	0.5000	0.09976	0.1667	0.2090	0.2337	0.2512	0.2760	0.2935	0.3071
0000	0.4600	0.1048	0.1718	0.2141	0.2388	0.2563	0.2810	0.2986	0.3122
000	0.4096	0.1119	0.1789	0.2211	0.2459	0.2634	0.2881	0.3057	0.3193
00	0.3648	0.1190	0.1860	0.2282	0.2529	0.2705	0.2952	0.3127	0.3263
0	0.3249	0.1260	0.1930	0.2353	0.2600	0.2775	0.3022	0.3198	0.3334
1	0.2893	0.1331	0.2001	0.2423	0.2671	0.2846	0.3093	0.3269	0.3405
2	0.2576	0.1402	0.2072	0.2494	0.2741	0.2917	0.3164	0.3339	0.3475
4	0.2043	0.1543	0.2213	0.2635	0.2883	0.3058	0.3305	0.3481	0.3617
6	0.1620	0.1685	0.2354	0.2777	0.3024	0.3199	0.3447	0.3622	0.3758
8	0.1285	0.1826	0.2496	0.2918	0.3165	0.3341	0.3588	0.3763	0.3899
10	0.1019	0.1967	0.2637	0.3060	0.3307	0.3482	0.3729	0.3905	0.4041
12	0.08081	0.2109	0.2778	0.3201	0.3448	0.3623	0.3871	0.4046	0.4182
14	0.06408	0.2250	0.2920	0.3342	0.3590	0.3765	0.4012	0.4187	0.4323
16	0.05082	0.2391	0.3061	0.3484	0.3731	0.3906	0.4153	0.4329	0.4465

Size of Wire, cir mils or A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2760	0.2935	0.3071	0.3182	0.3358	0.3494	0.3741	0.3916	0.4052
750,000	0.2847	0.3023	0.3159	0.3270	0.3445	0.3581	0.3828	0.4004	0.4140
500,000	0.2971	0.3146	0.3282	0.3393	0.3569	0.3705	0.3952	0.4127	0.4263
350,000	0.3080	0.3255	0.3391	0.3502	0.3678	0.3814	0.4061	0.4236	0.4372
250,000	0.3182	0.3358	0.3494	0.3605	0.3780	0.3916	0.4163	0.4339	0.4475
0000	0.3233	0.3408	0.3544	0.3656	0.3831	0.3967	0.4214	0.4390	0.4526
000	0.3304	0.3479	0.3615	0.3726	0.3902	0.4038	0.4285	0.4460	0.4596
00	0.3374	0.3550	0.3686	0.3797	0.3972	0.4108	0.4356	0.4531	0.4667
0	0.3445	0.3620	0.3756	0.3867	0.4043	0.4179	0.4426	0.4601	0.4737
1	0.3516	0.3691	0.3827	0.3938	0.4114	0.4250	0.4497	0.4672	0.4808
2	0.3586	0.3762	0.3898	0.4009	0.4184	0.4320	0.4568	0.4743	0.4879
4	0.3728	0.3903	0.4039	0.4150	0.4326	0.4462	0.4709	0.4884	0.5020
6	0.3869	0.4045	0.4181	0.4292	0.4467	0.4603	0.4850	0.5026	0.5162
8	0.4011	0.4186	0.4322	0.4433	0.4608	0.4744	0.4992	0.5167	0.5303
10	0.4152	0.4327	0.4463	0.4574	0.4750	0.4886	0.5133	0.5308	0.5444
12	0.4293	0.4469	0.4605	0.4716	0.4891	0.5027	0.5274	0.5450	0.5586
14	0.4435	0.4610	0.4746	0.4857	0.5033	0.5169	0.5416	0.5591	0.5727
16	0.4576	0.4751	0.4887	0.4998	0.5174	0.5310	0.5557	0.5732	0.5868

* The inductances given in this table also apply, with a practically negligible error (about 1 per cent), to ordinary stranded wires of the *same cross-section.*

Self-inductance of Solid Non-magnetic Wires *

Millihenrys per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, cir mils or A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.3036	0.6572	0.8803	1.011	1.103	1.234	1.327	1.398
750,000	0.8660	0.3499	0.7035	0.9266	1.057	1.150	1.280	1.373	1.445
500,000	0.7071	0.4152	0.7688	0.9919	1.122	1.215	1.346	1.438	1.510
350,000	0.5916	0.4726	0.8262	1.049	1.180	1.272	1.403	1.496	1.567
250,000	0.5000	0.5267	0.8803	1.103	1.234	1.327	1.457	1.550	1.622
0000	0.4600	0.5536	0.9072	1.130	1.261	1.353	1.484	1.577	1.648
000	0.4096	0.5909	0.9445	1.168	1.298	1.391	1.521	1.614	1.686
00	0.3648	0.6282	0.9818	1.205	1.335	1.428	1.559	1.651	1.723
0	0.3249	0.6654	1.019	1.242	1.373	1.465	1.596	1.688	1.760
1	0.2893	0.7029	1.057	1.280	1.410	1.503	1.633	1.726	1.798
2	0.2576	0.7402	1.094	1.317	1.447	1.540	1.671	1.763	1.835
4	0.2043	0.8148	1.168	1.392	1.522	1.615	1.745	1.838	1.910
6	0.1620	0.8894	1.243	1.466	1.597	1.689	1.820	1.912	1.984
8	0.1285	0.9641	1.318	1.541	1.671	1.764	1.894	1.987	2.059
10	0.1019	1.039	1.392	1.615	1.746	1.839	1.969	2.062	2.134
12	0.08081	1.113	1.467	1.690	1.821	1.913	2.044	2.136	2.208
14	0.06408	1.188	1.542	1.765	1.895	1.988	2.118	2.211	2.283
16	0.05082	1.263	1.616	1.839	1.970	2.062	2.193	2.286	2.357

Size of Wire, cir mils or A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	1.457	1.550	1.622	1.680	1.773	1.845	1.975	2.068	2.140
750,000	1.503	1.596	1.668	1.726	1.819	1.891	2.021	2.114	2.186
500,000	1.569	1.661	1.733	1.792	1.884	1.956	2.087	2.179	2.251
350,000	1.626	1.719	1.791	1.849	1.942	2.014	2.144	2.237	2.309
250,000	1.680	1.773	1.845	1.903	1.996	2.068	2.198	2.291	2.363
0000	1.707	1.800	1.872	1.930	2.023	2.095	2.225	2.318	2.390
000	1.744	1.837	1.909	1.967	2.060	2.132	2.262	2.355	2.427
00	1.782	1.874	1.946	2.005	2.097	2.169	2.300	2.392	2.464
0	1.819	1.911	1.983	2.042	2.135	2.206	2.337	2.430	2.501
1	1.856	1.949	2.021	2.079	2.172	2.244	2.374	2.467	2.539
2	1.894	1.986	2.058	2.117	2.209	2.281	2.412	2.504	2.576
4	1.968	2.061	2.133	2.191	2.284	2.356	2.486	2.579	2.651
6	2.043	2.135	2.207	2.266	2.359	2.430	2.561	2.654	2.725
8	2.118	2.210	2.282	2.341	2.433	2.505	2.636	2.728	2.800
10	2.192	2.285	2.357	2.415	2.508	2.580	2.710	2.803	2.875
12	2.267	2.359	2.431	2.490	2.582	2.654	2.785	2.877	2.949
14	2.341	2.434	2.506	2.565	2.657	2.729	2.860	2.952	3.024
16	2.416	2.509	2.581	2.639	2.732	2.804	2.934	3.027	3.099

* The inductances given in this table also apply, with a practically negligible error (about 1 per cent), to ordinary stranded wires of the *same cross-section.*

25-Cycle Reactance of Solid Non-magnetic Wires*

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, cir mils or A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.04770	0.1032	0.1383	0.1588	0.1733	0.1939	0.2085	0.2196
750,000	0.8660	0.05497	0.1105	0.1456	0.1661	0.1807	0.2011	0.2157	0.2270
500,000	0.7071	0.06523	0.1208	0.1558	0.1763	0.1909	0.2115	0.2259	0.2372
350,000	0.5916	0.07425	0.1298	0.1648	0.1854	0.1998	0.2204	0.2350	0.2462
250,000	0.5000	0.08274	0.1383	0.1733	0.1939	0.2085	0.2289	0.2435	0.2548
0000	0.4600	0.08697	0.1425	0.1775	0.1981	0.2126	0.2331	0.2477	0.2589
000	0.4096	0.09283	0.1484	0.1835	0.2039	0.2185	0.2389	0.2536	0.2649
00	0.3648	0.09869	0.1542	0.1893	0.2097	0.2243	0.2449	0.2594	0.2707
0	0.3249	0.1045	0.1601	0.1951	0.2157	0.2302	0.2507	0.2652	0.2765
1	0.2893	0.1101	0.1661	0.2011	0.2215	0.2361	0.2565	0.2712	0.2825
2	0.2576	0.1163	0.1719	0.2069	0.2273	0.2419	0.2625	0.2770	0.2883
4	0.2043	0.1280	0.1835	0.2187	0.2391	0.2537	0.2741	0.2887	0.3001
6	0.1620	0.1397	0.1953	0.2303	0.2509	0.2653	0.2859	0.3004	0.3117
8	0.1285	0.1515	0.2071	0.2421	0.2625	0.2771	0.2975	0.3122	0.3235
10	0.1019	0.1632	0.2187	0.2537	0.2743	0.2889	0.3093	0.3239	0.3353
12	0.08081	0.1749	0.2305	0.2655	0.2861	0.3005	0.3211	0.3356	0.3469
14	0.06408	0.1866	0.2422	0.2773	0.2977	0.3123	0.3327	0.3473	0.3587
16	0.05082	0.1984	0.2539	0.2889	0.3095	0.3239	0.3445	0.3591	0.3703

Size of Wire, cir mils or A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2289	0.2435	0.2548	0.2639	0.2785	0.2898	0.3103	0.3249	0.3362
750,000	0.2361	0.2507	0.2620	0.2712	0.2858	0.2971	0.3175	0.3321	0.3434
500,000	0.2465	0.2609	0.2723	0.2815	0.2960	0.3073	0.3279	0.3423	0.3536
350,000	0.2554	0.2701	0.2814	0.2905	0.3051	0.3164	0.3368	0.3514	0.3627
250,000	0.2639	0.2785	0.2898	0.2990	0.3136	0.3249	0.3453	0.3599	0.3712
0000	0.2682	0.2828	0.2941	0.3032	0.3178	0.3291	0.3495	0.3642	0.3755
000	0.2740	0.2886	0.2999	0.3090	0.3236	0.3349	0.3554	0.3700	0.3813
00	0.2800	0.2944	0.3057	0.3150	0.3294	0.3407	0.3613	0.3758	0.3871
0	0.2858	0.3002	0.3115	0.3208	0.3354	0.3466	0.3671	0.3818	0.3929
1	0.2916	0.3062	0.3175	0.3266	0.3412	0.3525	0.3730	0.3876	0.3989
2	0.2975	0.3120	0.3233	0.3326	0.3470	0.3583	0.3789	0.3934	0.4047
4	0.3092	0.3238	0.3351	0.3442	0.3588	0.3701	0.3906	0.4052	0.4165
6	0.3210	0.3354	0.3467	0.3560	0.3706	0.3818	0.4023	0.4169	0.4281
8	0.3327	0.3472	0.3585	0.3678	0.3822	0.3935	0.4141	0.4286	0.4399
10	0.3444	0.3590	0.3703	0.3794	0.3940	0.4053	0.4257	0.4404	0.4517
12	0.3561	0.3706	0.3819	0.3912	0.4056	0.4169	0.4375	0.4520	0.4633
14	0.3678	0.3824	0.3937	0.4030	0.4174	0.4287	0.4493	0.4638	0.4751
16	0.3796	0.3942	0.4055	0.4146	0.4292	0.4405	0.4609	0.4755	0.4869

* The reactances given in this table also apply, with a practically negligible error (about 1 per cent), to ordinary stranded wires of the *same cross-section.*

60-Cycle Reactance of Solid Non-magnetic Wires *

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of Wire, cir mils or A.W.G.	Diam. of Wire, inches	Inches between Wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6089	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6300	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6582	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of Wire, cir mils or A.W.G.	Feet between Wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6762	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8038	0.8528	0.8878	0 9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

* The reactances given in this table also apply, with a practically negligible error (about 1 per cent), to ordinary stranded wires of the *same cross-section*.

Inductance ‡ per Mile of Single Conductor, Aluminum Steel Core Cable

Multiple Layer Conductors †—All Current Densities

Circular Mils or A.W.G. (B. & S.) Aluminum	Number of Wires		Copper Equivalent CM or A.W.G. Based on Copper 97% Alum. 61%	Inductance in Millihenries per Mile of Each Conductor, of a Single-phase, Two-phase or Three-phase Circuit, Distance D between Centers of Conductors §													
	Al.	St.		3 ft	4 ft	5 ft	6 ft	8 ft	11 ft	13 ft	15 ft	17 ft	19 ft	23 ft	25 ft	30 ft	35 ft
1,590,000	54	19	1,000,000	1.30	1.40	1.47	1.53	1.62	1.72	1.78	1.82	1.86	1.90	1.96	1.99	2.05	2.10
1,510,000	54	19	950,000	1.31	1.41	1.48	1.54	1.63	1.73	1.79	1.83	1.87	1.91	1.97	2.00	2.06	2.10
1,431,000	54	19	900,000	1.32	1.42	1.49	1.55	1.64	1.74	1.79	1.84	1.88	1.92	1.98	2.00	2.06	2.11
1,351,500	54	19	850,000	1.33	1.42	1.50	1.56	1.65	1.75	1.80	1.85	1.89	1.93	1.99	2.01	2.07	2.12
1,272,000	54	19	800,000	1.34	1.43	1.51	1.56	1.66	1.76	1.81	1.86	1.90	1.94	2.00	2.02	2.08	2.13
1,192,500	54	19	750,000	1.35	1.44	1.52	1.57	1.67	1.77	1.82	1.87	1.91	1.95	2.01	2.03	2.09	2.14
1,113,000	54	19	700,000	1.36	1.45	1.53	1.59	1.68	1.78	1.84	1.88	1.92	1.96	2.02	2.05	2.10	2.15
1,033,500	54	7	650,000	1.38	1.47	1.54	1.60	1.69	1.79	1.85	1.89	1.93	1.97	2.03	2.06	2.12	2.17
954,000	54	7	600,000	1.39	1.48	1.55	1.61	1.70	1.81	1.86	1.91	1.95	1.98	2.04	2.07	2.13	2.18
900,000	54	7	566,000	1.40	1.49	1.56	1.62	1.71	1.82	1.87	1.91	1.96	1.99	2.05	2.08	2.14	2.19
874,000	54	7	550,000	1.40	1.49	1.57	1.62	1.72	1.82	1.87	1.92	1.96	2.00	2.06	2.08	2.14	2.19
795,000	54	7	500,000	1.42	1.51	1.58	1.64	1.73	1.84	1.89	1.94	1.98	2.01	2.07	2.10	2.16	2.21
795,000	26	7	500,000	1.41	1.50	1.58	1.63	1.73	1.83	1.88	1.93	1.97	2.01	2.07	2.09	2.15	2.20
795,000	30	19	500,000	1.40	1.49	1.56	1.62	1.71	1.81	1.87	1.91	1.95	1.99	2.05	2.08	2.14	2.19
715,500	54	7	450,000	1.43	1.53	1.60	1.66	1.75	1.85	1.91	1.95	1.99	2.03	2.09	2.12	2.18	2.23
715,500	26	7	450,000	1.43	1.52	1.59	1.65	1.74	1.85	1.90	1.95	1.99	2.02	2.08	2.11	2.17	2.22
715,500	30	19	450,000	1.41	1.51	1.58	1.64	1.73	1.83	1.89	1.93	1.97	2.01	2.07	2.10	2.15	2.20
666,600	54	7	419,000	1.45	1.54	1.61	1.67	1.76	1.86	1.92	1.96	2.00	2.04	2.10	2.13	2.19	2.24
636,000	54	7	400,000	1.45	1.55	1.62	1.68	1.77	1.87	1.92	1.97	2.01	2.05	2.11	2.14	2.19	2.24
636,000	26	7	400,000	1.45	1.54	1.61	1.67	1.76	1.87	1.92	1.97	2.01	2.04	2.10	2.13	2.19	2.24
636,000	30	19	400,000	1.43	1.53	1.60	1.66	1.75	1.85	1.90	1.95	1.99	2.03	2.09	2.12	2.17	2.22
605,000	54	7	380,500	1.46	1.55	1.63	1.68	1.78	1.88	1.93	1.98	2.02	2.06	2.12	2.14	2.20	2.25
605,000	26	7	380,500	1.45	1.55	1.62	1.68	1.77	1.87	1.93	1.97	2.01	2.05	2.11	2.14	2.20	2.25
556,500	26	7	350,000	1.47	1.56	1.63	1.69	1.78	1.89	1.94	1.99	2.03	2.06	2.12	2.15	2.21	2.26
556,500	30	7	350,000	1.45	1.55	1.62	1.68	1.77	1.87	1.93	1.97	2.01	2.05	2.11	2.14	2.19	2.24
500,000	30	7	314,500	1.47	1.56	1.63	1.69	1.79	1.89	1.94	1.99	2.03	2.06	2.13	2.15	2.21	2.26
477,000	26	7	300,000	1.49	1.59	1.66	1.72	1.81	1.91	1.97	2.01	2.05	2.09	2.15	2.18	2.23	2.28
477,000	30	7	300,000	1.48	1.57	1.64	1.70	1.79	1.90	1.95	2.00	2.04	2.07	2.13	2.16	2.22	2.27
397,500	26	7	250,000	1.52	1.62	1.69	1.75	1.84	1.94	1.99	2.04	2.08	2.12	2.18	2.20	2.26	2.31
397,500	30	7	250,000	1.51	1.60	1.67	1.73	1.82	1.93	1.98	2.03	2.07	2.10	2.16	2.19	2.25	2.30
336,400	26	7	0,000	1.55	1.64	1.71	1.77	1.86	1.97	2.02	2.07	2.11	2.14	2.20	2.23	2.29	2.34
336,400	30	7	0,000	1.53	1.63	1.70	1.76	1.85	1.95	2.01	2.05	2.09	2.13	2.19	2.22	2.28	2.33
300,000	26	7	188,700	1.57	1.66	1.73	1.79	1.88	1.99	2.04	2.09	2.13	2.16	2.22	2.25	2.31	2.36
300,000	30	7	188,700	1.55	1.65	1.72	1.78	1.87	1.97	2.02	2.07	2.11	2.15	2.21	2.23	2.29	2.34
266,800	26	7	000	1.59	1.68	1.75	1.81	1.90	2.00	2.06	2.10	2.14	2.18	2.24	2.27	2.33	2.38
Single Layer Conductors *—Current Density 0 Ampere per Square Inch																	
266,800	6	7	000	1.59	1.68	1.76	1.81	1.91	2.01	2.06	2.11	2.15	2.19	2.25	2.27	2.33	2.38
0,000	6	1	00	1.74	1.84	1.91	1.97	2.06	2.16	2.22	2.26	2.30	2.34	2.40	2.43	2.48	2.53
000	6	1	0	1.79	1.88	1.95	2.01	2.10	2.20	2.26	2.30	2.34	2.38	2.44	2.47	2.53	2.58
00	6	1	1	1.82	1.92	1.99	2.05	2.14	2.24	2.30	2.34	2.38	2.42	2.48	2.51	2.57	2.61
0	6	1	2	1.86	1.95	2.02	2.08	2.18	2.28	2.33	2.38	2.42	2.45	2.52	2.54	2.60	2.65
1	6	1	3	1.89	1.99	2.06	2.12	2.21	2.31	2.37	2.41	2.45	2.49	2.55	2.58	2.63	2.68
2	6	1	4	1.93	2.02	2.09	2.15	2.24	2.34	2.40	2.44	2.48	2.52	2.58	2.61	2.67	2.72
3	6	1	5	1.96	2.05	2.12	2.18	2.27	2.38	2.43	2.48	2.52	2.55	2.61	2.64	2.70	2.75
4	6	1	6	1.99	2.08	2.16	2.21	2.31	2.41	2.46	2.51	2.55	2.59	2.65	2.67	2.73	2.78
5	6	1	7	2.02	2.12	2.19	2.25	2.34	2.44	2.50	2.54	2.58	2.62	2.68	2.71	2.77	2.81
6	6	1	8	2.06	2.15	2.22	2.28	2.38	2.48	2.53	2.58	2.62	2.65	2.72	2.74	2.80	2.85
Single Layer Conductors *—Current Density 600 Amperes per Square Inch																	
266,800	6	7	000	1.64	1.73	1.80	1.86	1.95	2.06	2.11	2.16	2.20	2.23	2.29	2.32	2.38	2.43
0,000	6	1	00	1.80	1.89	1.96	2.02	2.11	2.21	2.27	2.31	2.35	2.39	2.45	2.48	2.54	2.59
000	6	1	0	1.82	1.92	1.99	2 05	2.14	2.24	2.30	2.34	2.38	2.42	2.48	2.51	2.56	2.61
00	6	1	1	1.85	1.94	2.02	2.07	2.17	2.27	2.32	2.37	2.41	2.45	2.51	2.53	2.59	2.64
0	6	1	2	1.88	1.97	2.04	2.10	2.19	2.30	2.35	2.40	2.44	2.47	2.53	2.56	2.62	2.67
1	6	1	3	1.91	2.00	2.07	2.13	2.22	2.33	2.38	2.43	2.47	2.50	2.56	2.59	2.65	2.70
2	6	1	4	1.94	2.03	2.10	2.16	2.25	2.36	2.41	2.46	2.50	2.53	2.59	2.62	2.68	2.73
3	6	1	5	1.97	2.06	2.13	2.19	2.28	2.38	2.44	2.48	2.52	2.56	2.62	2.65	2.71	2.76
4	6	1	6	1.99	2.09	2.16	2.22	2.31	2.41	2.47	2.51	2.55	2.59	2.65	2.68	2.74	2.78
5	6	1	7	2.03	2.12	2.19	2.25	2.34	2.45	2.50	2.55	2.59	2.62	2.68	2.71	2.77	2.82
6	6	1	8	2.07	2.16	2.23	2.29	2.38	2.49	2.54	2.59	2.63	2.66	2.72	2.75	2.81	2.86

See end of table for reference notes.

Inductance ‡ per Mile of Single Conductor, Aluminum Steel Core Cable—*Continued*

Single Layer Conductors *—Current Density 1200 Amperes per Square Inch

Circular Mils or A.W.G. (B. & S.) Aluminum	Number of Wires		Copper Equivalent CM or A.W.G. Based on Copper 97% Alum. 61%	Inductance in Millihenries per Mile of Each Conductor, of a Single-phase, Two-phase or Three-phase Circuit, Distance D between Centers of Conductors §													
	Al.	St.		3 ft	4 ft	5 ft	6 ft	8 ft	11 ft	13 ft	15 ft	17 ft	19 ft	23 ft	25 ft	30 ft	35 ft
266,800	6	7	000	1.85	1.94	2.01	2.07	2.16	2.27	2.32	2.37	2.41	2.44	2.50	2.53	2.59	2.64
0,000	6	1	00	1.87	1.96	2.03	2.09	2.18	2.29	2.34	2.39	2.43	2.46	2.52	2.55	2.61	2.66
000	6	1	0	1.90	2.00	2.07	2.13	2.22	2.32	2.38	2.42	2.46	2.50	2.56	2.59	2.64	2.69
00	6	1	1	1.93	2.02	2.09	2.15	2.24	2.35	2.40	2.45	2.49	2.52	2.58	2.61	2.67	2.72
0	6	1	2	1.95	2.04	2.11	2.17	2.26	2.36	2.42	2.46	2.50	2.54	2.60	2.63	2.69	2.74
1	6	1	3	1.96	2.05	2.13	2.19	2.28	2.38	2.43	2.48	2.52	2.56	2.62	2.64	2.70	2.75
2	6	1	4	1.97	2.07	2.14	2.20	2.29	2.39	2.45	2.49	2.53	2.57	2.63	2.66	2.72	2.76
3	6	1	5	1.99	2.08	2.15	2.21	2.30	2.41	2.46	2.51	2.55	2.58	2.64	2.67	2.73	2.78
4	6	1	6	2.00	2.10	2.17	2.23	2.32	2.42	2.48	2.52	2.56	2.60	2.66	2.69	2.74	2.79
5	6	1	7	2.05	2.14	2.21	2.27	2.36	2.46	2.52	2.56	2.60	2.64	2.70	2.73	2.79	2.84
6	6	1	8	2.08	2.17	2.25	2.30	2.40	2.50	2.55	2.60	2.64	2.68	2.74	2.76	2.82	2.87

† By multiple layer conductors is meant conductors with two or more layers of aluminum over the steel core.

* By single layer conductors is meant conductors with one layer of aluminum over the steel core.

‡ The inductance values of the table are based upon actual tests on various sizes of cable at various current densities at one foot spacing. The inductances at other spacings were calculated from those at one foot spacing by means of the fundamental inductance formula.

The inductance L' for any spacing D' not given in the table is equal to the inductance L at the next smaller spacing D given in the table plus the quantity $0.74113 \log_{10} D'/D$. Thus $L' = L + 0.74113 \log_{10} D'/D$. Or the inductance in millihenries to be added to that at the next smaller spacing may be taken from table below.

D'/D	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
$L+$	.016	.031	.045	.059	.072	.084	.097	.108	.120	.131
D'/D	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
$L+$	.141	.151	.161	.171	.180	.189	.198	.207	.215	.223

§ For any three-phase arrangement of conductors $D = \sqrt[3]{ABC}$, where A, B and C are the conductor spacings.

60-Cycle Reactance ‡ per Mile of Single Conductor Aluminum, Steel Core Cable

Multiple Layer Conductors †—All Current Densities

Circular Mils or A.W.G. (B. & S.) Aluminum	Number of Wires		Copper Equivalent CM or A.W.G. Based on Copper 97% Alum. 61%	60-Cycle Reactance X in Ohms per Mile of Each Conductor of a Single-phase, Two-phase or Three-phase Circuit (See Footnotes), Distance D between Centers of Conductors §													
	Al.	St.		3 ft	4 ft	5 ft	6 ft	8 ft	11 ft	13 ft	15 ft	17 ft	19 ft	23 ft	25 ft	30 ft	35 ft
1,590,000	54	19	1,000,000	.492	.527	.554	.576	.611	.650	.670	.686	.701	.716	.739	.749	.771	.790
1,510,500	54	19	950,000	.495	530	.557	.579	.614	.653	.673	.690	.706	.719	.742	.752	.775	.793
1,431,000	54	19	900,000	.499	.533	.561	.583	.618	.656	.676	.694	.709	.722	.746	.756	.778	.797
1,351,500	54	19	850,000	.502	.537	.564	.586	.621	.660	.680	.697	.713	.727	.749	.759	.783	.800
1,272,000	54	19	800,000	.506	.541	.568	.590	.625	.663	.684	.701	.716	.730	.752	.763	.785	.804
1,192,500	54	19	750,000	.510	.544	.572	.594	.628	.667	.688	.705	.720	.734	.757	.767	.789	.808
1,113,000	54	19	700,000	.514	.549	.576	.598	.633	.671	.692	.709	.724	.737	.761	.771	.793	.812
1,033,500	54	7	650,000	.518	.553	.580	.602	.637	.676	.696	.714	.729	.742	.765	.775	.798	.816
954,000	54	7	600,000	.523	.559	.585	.607	.642	.681	.701	.719	.734	.747	.770	.780	.803	.821
900,000	54	7	566,000	.527	.561	.588	.611	.646	.684	.705	.722	.737	.751	.774	.784	.806	.825
874,000	54	7	550,000	.518	.563	.590	.612	.647	.686	.706	.723	.739	.752	.775	.785	.808	.826
795,000	54	7	500,000	.534	.569	.596	.618	.653	.692	.712	.729	.745	.758	.781	.791	.813	.832
795,000	26	7	500,000	.532	.567	.594	.616	.651	.689	.710	.727	.742	.756	.779	.789	.811	.830
795,000	30	19	500,000	.526	.561	.588	.611	.645	.684	.704	.722	.737	.750	.774	.783	.806	.824
715,500	54	7	450,000	.541	.576	.602	.625	.660	.698	.719	.736	.751	.765	.788	.798	.820	.839
715,500	26	7	450,000	.538	.573	.600	.622	.657	.696	.716	.733	.748	.762	.785	.795	.817	.836
715,500	30	19	450,000	.533	.568	.595	.617	.652	.691	.711	.728	.743	.757	.780	.790	.811	.831
666,600	54	7	419,000	.545	.580	.607	.629	.664	.703	.723	.740	.756	.769	.792	.802	.824	.843
636,000	54	7	400,000	.547	.582	.610	.631	.667	.705	.725	.743	.758	.772	.795	.805	.827	.846
636,000	26	7	400,000	.546	.580	.607	.630	.664	.703	.723	.741	.756	.769	.792	.803	.825	.843
636,000	30	19	400,000	.540	.575	.602	.624	.659	.697	.718	.735	.751	.746	.787	.797	.819	.838
605,000	54	7	380,500	.551	.585	.613	.625	.670	.708	.729	.746	.761	.775	.798	.808	.830	.849
605,000	26	7	380,500	.548	.583	.610	.632	.667	.706	.726	.744	.759	.773	.796	.806	.828	.846
556,500	26	7	350,000	.553	.588	.615	.638	.672	.711	.731	.748	.764	.777	.800	.811	.832	.851

See end of table for reference notes.

60-Cycle Reactance ‡ per Mile of Single Conductor Aluminum, Steel Core Cable—*Continued*

Multiple Layer Conductors †—All Current Densities

Circular Mils or A.W.G. (B. & S.) Aluminum	Number of Wires		Copper Equivalent CM or A.W.G. Based on Copper 97% Alum. 61%	60-Cycle Reactance X in Ohms per Mile of Each Conductor of a Single-phase, Two-phase or Three-phase Circuit (See Footnotes), Distance D between Centers of Conductors §													
	Al.	St.		3 ft	4 ft	5 ft	6 ft	8 ft	11 ft	13 ft	15 ft	17 ft	19 ft	23 ft	25 ft	30 ft	35 ft
556,500	30	7	350,000	.548	.582	.610	.632	.667	.705	.726	.743	.758	.772	.795	.805	.827	.846
500,000	30	7	314,500	.554	.589	.616	.638	.673	.712	.732	.749	.765	.778	.801	.811	.834	.853
477,000	26	7	300,000	.563	.598	.625	.647	.682	.720	.741	.758	.773	.787	.810	.820	.842	.861
477,000	30	7	300,000	.557	.592	.619	.641	.676	.715	.735	.752	.768	.781	.804	.814	.836	.855
397,500	26	7	250,000	.574	.609	.636	.658	.693	.732	.752	.769	.780	.798	.821	.831	.853	.871
397,500	30	7	250,000	.568	.603	.630	.652	.685	.726	.746	.763	.779	.792	.813	.826	.847	.866
336,400	26	7	0,000	.584	.619	.646	.668	.703	.742	.762	.779	.794	.808	.831	.841	.863	.882
336,400	30	7	0,000	.578	.613	.640	.662	.697	.736	.756	.774	.789	.802	.825	.835	.858	.887
300,000	26	7	188,700	.591	626	.653	.675	.710	.748	.769	.786	.801	.813	.838	.848	.870	.889
300,000	30	7	188,700	585	620	.647	.669	.704	.743	.763	.780	.796	.809	.832	.842	.864	.883
266,800	26	7	000	.598	633	.660	.682	.717	.756	.776	.793	.802	.822	.845	.855	.877	.896
Single Layer Conductors *—Current Density 0 Ampere per Square Inch																	
266,800	6	7	000	.600	.635	.662	.684	.719	.757	.778	.795	.810	.824	.847	.857	.879	.898
0,000	6	1	00	.657	.692	.719	.741	.776	.815	.835	.852	.868	.881	.904	.914	.937	.955
000	6	1	0	.673	.708	.735	.757	.792	.831	.851	.869	.884	.897	.920	.930	.953	.971
00	6	1	1	.688	.722	.750	.772	.806	.845	.866	.883	.898	.912	.935	.945	.967	.986
0	6	1	2	.701	.736	.763	785	.820	.859	.879	.897	.912	.925	.948	.958	.981	.999
1	6	1	3	.714	.748	.776	.798	.832	.871	.892	.909	.924	.938	.961	.971	.993	1.01
2	6	1	4	.726	.760	.788	.810	.845	.883	.904	.921	.936	.950	.973	.983	1.01	1.02
3	6	1	5	.738	.773	.800	.822	.857	.895	.916	.933	.948	.962	.985	.995	1.02	1.04
4	6	1	6	.751	.785	.812	.835	.869	.908	.929	.946	.961	.975	.998	1.01	1.03	1.05
5	6	1	7	.763	.798	.825	.847	.882	.921	.941	.958	.973	.987	1.01	1.02	1.04	1.06
6	6	1	8	.777	.811	.838	.861	.895	.934	.955	.972	.987	1.00	1.02	1.03	1.06	1.07
203,000	8	7	127,700	.616	.651	.678	.700	.735	.774	.794	.811	.826	.840	.863	.873	.895	.914
203,200	16	19	127,800	.592	.627	.653	.676	.711	.749	.769	.787	.802	.815	.838	.849	.871	.890
211,300	12	7	132,900	.601	.635	.662	.685	.719	.758	.779	.796	.811	.824	.847	.858	.880	.898
190,800	12	7	120,000	.607	.641	.668	691	.725	.764	.785	.802	.817	.831	.854	.864	.886	.904
176,900	12	7	111,200	.611	.646	673	.695	.730	.769	.789	.806	.821	.835	.858	.868	.890	.909
159,000	12	7	100,000	.618	.653	.679	.702	.737	.775	.795	.813	.828	.841	.865	.875	.897	.916
134,600	12	7	84,600	628	.662	.690	.712	.748	.785	.806	.823	.838	.852	.875	.885	.907	.926
110,800	12	7	69,700	.639	.674	.701	.723	.759	.797	.817	.835	.850	.863	.887	.897	.919	.938
101,800	12	7	64,160	.645	.679	.706	.729	.763	.802	.823	.840	.855	.869	.892	.902	.924	.943
80,000	8	1	50,310	.712	.747	774	.796	.831	.870	.890	.907	.923	.936	.959	.969	.992	1.010
Single Layer Conductors *—Current Density 600 Amperes per Square Inch																	
266,800	6	7	000	.618	.652	.679	.702	.736	.775	.796	.813	.828	.842	.865	.875	.897	.915
0,000	6	1	00	.677	.712	.739	.761	.796	.835	.855	.872	.888	.901	.924	.934	.956	.975
000	6	1	0	687	.722	.749	.771	.806	.845	.865	.883	.898	.911	.934	.944	.967	.985
00	6	1	1	.698	.733	.760	.782	.817	.855	.876	.893	.908	.922	.945	.955	.977	.996
0	6	1	2	.708	.743	.770	.793	.827	.866	.886	.904	.919	.932	.955	.966	.988	1.01
1	6	1	3	.720	.754	.782	.804	.838	.877	.898	.915	.930	.944	.967	.977	.999	1.02
2	6	1	4	.730	.765	.792	.814	.849	.888	.908	.926	.941	.954	.977	.987	1.01	1.03
3	6	1	5	.741	.776	.803	.825	.860	.898	.919	.936	.951	.965	.988	.998	1.02	1.04
4	6	1	6	.752	.786	.814	.836	.871	.909	.930	.947	.962	.976	.999	1.01	1.03	1.05
5	6	1	7	.765	.800	.827	.849	.884	.923	.943	.960	.975	.989	1.01	1.02	1.04	1.06
6	6	1	8	.779	.814	.841	.863	.898	.937	.957	.975	.990	1.00	1.03	1.04	1.06	1.08
203,000	8	7	127,700	.627	.662	.689	.711	.746	.785	.805	.822	.837	.848	.874	.884	.906	.925
203,200	16	19	127,800	602	.638	.664	.687	.722	.760	.780	.798	.813	.826	.849	.860	.882	.901
211,300	12	7	132,900	.611	.646	.673	.696	.730	.770	.789	.807	.822	.835	.858	.869	.891	.909
190,800	12	7	120,000	.618	652	.679	.702	.736	.775	.795	.813	.827	.841	.864	.875	.897	.915
176,900	12	7	111,200	.622	.657	.684	.706	.741	.780	.800	.817	.832	.846	.869	.880	.901	.920
159,000	12	7	100,000	.628	.664	.690	.713	.748	.786	.806	.824	.839	.852	.876	.886	.908	.927
134,600	12	7	84,600	.639	.673	.700	.723	.757	.796	.817	.834	.849	.863	.886	.896	.918	.937
110,800	12	7	69,700	650	.685	.712	.734	.769	.808	.828	.846	.861	.874	.898	.908	.930	.949
101,800	12	7	64,160	.656	.690	.717	.740	.774	.813	.834	.851	.866	.880	.903	.913	.935	.953
80,000	8	1	50,310	.724	.758	.785	.808	.843	.881	.901	.919	.934	.947	.970	.981	1.003	1.022
Single Layer Conductors *—Current Density 1200 Amperes per Square Inch																	
265,800	6	7	000	.697	.732	.759	.781	.816	.855	.875	.892	.907	.921	.944	.954	.976	.995
0,000	6	1	00	.705	.739	.766	.789	.823	.862	.883	.900	.915	.929	.952	.962	.984	1.00
000	6	1	0	.717	.752	.779	.802	.836	.875	.895	.913	.928	.941	.964	.975	.997	1.02
00	6	1	1	.727	.762	.789	.811	.846	.884	.905	.922	.937	.951	.974	.984	1.01	1.03
0	6	1	2	.734	.768	.796	.818	.852	.891	.912	.929	.944	.958	.981	.991	1.01	1.03
1	6	1	3	.740	.774	.802	.824	.858	.897	.918	.935	.950	.964	.987	.997	1.02	1.04
2	6	1	4	.744	.779	.806	.828	.863	.902	.922	.940	.955	.968	.991	1.00	1.02	1.04
3	6	1	5	.749	.784	.811	.833	.868	.907	.927	.944	.960	.973	.996	1.01	1.03	1.05
4	6	1	6	.755	.790	.817	.839	.874	.913	.933	.950	.966	.979	1.00	1.01	1.03	1.05
5	6	1	7	.771	.806	.833	.855	.890	.929	.949	.967	.982	.995	1.02	1.03	1.05	1.07
6	6	1	8	.785	.819	.846	.869	.903	.942	.963	.980	.995	1.01	1.03	1.04	1.06	1.08

See end of table for reference notes.

60-Cycle Reactance ‡ per Mile of Single Conductor Aluminum, Steel Core Cable—*Continued*

Single-layer Conductors *—Current Density 1200 Amperes per Square Inch

Circular Mils or A.W.G. (B. & S.) Aluminum	Number of Wires		Copper Equivalent CM or A.W.G. Based on Copper 97% Alum. 61%	60-Cycle Reactance X in Ohms per Mile of Each Conductor of a Single-phase, Two-phase or Three-phase Circuit (See Footnotes), Distance D between Centers of Conductors §													
	Al.	St.		3 ft	4 ft	5 ft	6 ft	8 ft	11 ft	13 ft	15 ft	17 ft	19 ft	23 ft	25 ft	30 ft	35 ft
203,000	8	7	127,700	.674	.709	.736	.758	.793	.831	.852	.869	.884	.898	.921	.931	.957	.972
203,200	16	19	127,800	.650	.685	.711	.734	.769	.807	.828	.845	.860	.874	.897	.907	.929	.948
211,300	12	7	132,900	.659	.693	.720	.743	.777	.816	.837	.854	.869	.883	.906	.916	.938	.956
190,800	12	7	120,000	.665	.699	.726	.749	.783	.822	.843	.860	.875	.889	.912	.922	.944	.962
176,900	12	7	111,200	.669	.704	.731	.753	.788	.827	.847	.864	.880	.893	.916	.926	.949	.967
159,000	12	7	100,000	.676	.711	.737	.760	.795	.833	.854	.871	.886	.900	.923	.933	.955	.974
134,600	12	7	84,600	.686	.720	.748	.770	.805	.843	.864	.881	.896	.910	.933	.943	.965	.984
110,800	12	7	69,700	.697	.733	.759	.782	.817	.855	.875	.893	.908	.921	.945	.955	.977	.996
101,800	12	7	64,160	.703	.737	.765	.787	.821	.860	.881	.898	.913	.927	.950	.960	.982	1.001
80,000	8	1	50,310	.754	.789	.816	.838	873	.911	.932	.949	.964	.978	1.001	1.011	1.033	1.052

† By multiple layer conductors is meant conductors with two or more layers of aluminum over the steel core.

* By single layer conductors is meant conductors with one layer of aluminum over the steel core.

‡ The table values were derived from the equation $x = 2\pi fL$, in which x is the reactance in ohms, L is the inductance in henries per mile of single conductor and f is the frequency. The reactance at any other frequency than 60 cycles is $f/60$ times the table values.

The reactance x' at any spacing D' not given in the table is equal to the reactance x at the next smaller spacing D given in the table plus the quantity $0.2794 \log_{10} D'/D$.

Thus $x' = x + 0.2794 \log_{10} D'/D$. Or the reactance in ohms, to be added to that at the next smaller spacing may be taken from table below.

D'/D	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
$x+$	.006	.012	.017	.022	.027	.032	.036	.041	.045	.049
D'/D	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
$x+$	.053	.057	.061	.064	.068	.071	.075	.078	.081	.084

§ For any three-phase arrangement of conductors $D = \sqrt[3]{ABC}$, where A, B and C are the conductor spacings.

POLE AND TOWER LINES

By Howard Enos

(Some of the original material by R. A. Philip and C. M. Spofford has been used.)

24. MECHANICAL DESIGN

In designing a pole or tower line the following factors, in addition to the purely electrical characteristics, must be considered: (1) character of the route, (2) clearances, (3) grade of construction, (4) type of supporting structures, (5) type of insulators, (6) conductors, (7) mechanical loading, (8) joint use by other utilities, (9) right-of-way. The requirements imposed by these several conditions are noted below, together with tables, formulas, and curves useful in making the necessary calculations.

Character of the Route

Usually the lower-voltage lines are run along streets and highways, wherever possible, in order to reach customers more easily and to make the lines accessible for maintenance. The higher-voltage transmission lines are more often run across country on private right-of-way in order to obtain the most direct route, as well as to avoid buildings and low-voltage lines, and to obtain adequate space for towers. Lines which are run along streets and highways must be designed to give proper clearances from buildings, other lines, railroad tracks, and trees. Poles must be so located as not to obstruct the public use of streets and driveways or to be subject to damage from traffic.

Urban Distribution Lines usually require close spacing of poles on account of the necessity of keeping the span length of service wires from pole to house as short as possible. Poles are usually set on property lines to avoid obstructing the front of each lot and to avoid overhanging, with the service wires, property adjacent to the building served. Normal span lengths for urban distribution lines vary from about 100 to about 175 ft, the average length being about 125 ft. Poles are usually set from 6 in. to 1 ft inside the curb when along streets, but in alleys it is often necessary to set the poles within the traveled space.

The restrictions mentioned above do not exist, for the most part, in respect to **rural distribution lines.** Much longer spans are possible, and the length of span is determined largely by economic considerations. Spans of 200 to 300 ft are common, and many lines are built with spans as long as 600 ft. Rural customers are relatively far apart. In the eastern part of the United States the average spacing is about 4 customers to the mile. In parts of the country where large farms are the rule, customers may be spaced from 1 to 2 miles apart. Poles from which services are taken to farm buildings, as for urban lines, must be set close to the building served. Rural lines, on account of long-span construction, require nearly as much engineering design from the mechanical standpoint as higher-voltage transmission lines. Owing to the wide spacing of customers, rural lines, for the greater portion of their length, consist of primary circuits only. On account of the relatively small electrical loads small conductors are used. With small conductors mechanical loading of the lines is low, in spite of the long spans. This results in the possibility of using small poles, especially when bare, high-strength conductors are used. Wood poles are used almost exclusively for rural distribution lines.

Higher-voltage transmission lines on private right-of-way are usually built with long spans, and the type of terrain covered by the line largely determines the type of construction to be used. In fairly level country the height of towers is determined mainly by the length of span, the ground clearances required, and economic considerations. In mountainous territory the valleys may often be crossed by single spans of more than a mile in length using fairly low towers. Under such conditions the towers must be specially designed for wide spacings of conductors and for heavy transverse and longitudinal stresses. Lines on private right-of-way often require special consideration of side-hill construction.

Clearances

The following clearances for conductors must be considered: to ground, tracks, buildings, trees, conductors and structures of another line, other conductors on the same structure, the structure itself, guy wires and other equipment on the structure, and the edge of the right-of-way.

Space does not permit the tabulation here of clearances for all the above conditions for all voltages used for overhead lines. The National Electrical Safety Code gives clearances which are considered good practice. For higher-voltage transmission lines on private right-of-way the clearance to ground should be 20 ft or more, depending on the voltage. The clearance to any grounded portion of the structure, to trees, or other structures, or to the edge of the right-of-way should exceed by a safe margin the arcing distance across the insulator string when the string is deflected the maximum amount by wind pressure. Consideration should also be given to the behavior of lightning flashovers as affected by the presence of grading shields or rings.

Horizontal separations between conductors for high-voltage transmission lines depend on the conductor size and material, the span length, the voltage, and whether pin-type or suspension insulators are used. Opinions of engineers vary as to exact values of spacing to be used for a given set of conditions. The following spacings represent average practice. For 33-kv lines on pin-type insulators: 4 to 6 ft for spans of 150 to 500 ft, respectively. For 33-kv lines on suspension insulators: 6 to 10 ft for spans of 300 to 1000 ft, respectively. For 132-kv lines: 10 to 17 ft for spans from 500 to 1500 ft, respectively. For 220-kv lines: 20 to 26 ft for spans from 800 to 1500 ft, respectively.

Mr. P. H. Thomas gives the following formula for computing the minimum horizontal spacing for conductors (*A.I.E.E. Trans.*, 1928):

$$S = Cd\frac{D}{w} + A + \frac{L}{2} \tag{1}$$

where S = horizontal spacing in feet; C = a constant depending on conductor material ($C = 4$ for copper and $C = 3.5$ for A.C.S.R. usually) and character of terrain; d = per cent sag; D = diameter of conductor in inches; w = horizontal loading lb per foot; A = the arcing distance of the line voltage in feet; L = the length of suspension insulator string in feet.

Vertical separations should be about the same as stated for horizontal separations, but consideration should be given to unequal sags due to unequal ice loading. Also the conductors of a vertical configuration should be offset away from the vertical planes through the other two conductors from 1 to 3 ft, depending upon the span length and the voltage. Horizontal separations between conductors of different circuits should be somewhat greater than between conductors of the same circuit to avoid involving both circuits in the same case of flashover. Horizontal spacings for conductors on steel towers determined by the required clearances to the tower are usually adequate.

TELEPHONE CIRCUITS FOR POWER LINES. When the line voltage does not exceed 66,000 volts, the telephone circuits are usually carried on the same poles or towers as the power circuits, being placed below the power conductors. A separate line of wooden poles on the same right-of-way is occasionally employed when the voltage is higher than 66,000.

Where the telephone wires are on the same supporting structure as the power wires, sufficient clearance between the power and telephone circuits must be allowed to make the telephone line accessible for repairs and also to prevent the two circuits touching under abnormal conditions. On wood pole lines of short span (100 to 125 ft) the vertical clearance at the poles or towers ranges from 4 ft for 22,000 volts to 6 ft for 66,000 volts. On long-span lines greater spacing is necessary to allow for safe clearance in the middle of the longest span due to the change in the sag of the power and telephone wires under all conditions of unequal ice loading and all variations of side deflection due to wind.

Telephone wires are ordinarily of copper, though copper-clad steel or A.C.S.R. is sometimes used for long spans. For spans up to 125 ft No. 10 B. & S. copper may be used (though No. 8 is preferable); for longer spans larger sizes (usually No. 8 or No. 6 or even No. 4) are necessary to allow for sleet load. A spacing of 12 in. between wires may be used for 125-ft spans, but a wider spacing is necessary for longer spans. Where inadequate spacing is used the telephone lines will frequently become crossed by the wind, unless they are strung with little sag, in which case they are overloaded and broken by sleet. With wide spacing and large sag higher poles must be used.

Carrier Current Communication. It is impracticable to carry telephone circuits on the structures of long-span high-voltage transmission lines, and it is very expensive to provide separate wood pole telephone lines. As a result, many power companies now use the power circuit conductors themselves as the telephone circuit. This is accomplished by modulating a high-frequency voltage by the voice current and impressing the modulated carrier voltage upon the transmission circuit by capacity coupling. At the receiving end the transmission line is capacity coupled to the receiving apparatus which demodulates the carrier voltage, and the resultant is a reproduction of the original voice current, which operates an ordinary telephone receiver or a loud speaker. The apparatus is quite similar to ordinary radio transmitting and receiving equipment, and provision can be made for either one-way or two-way conversations.

For additional data on this subject see the volume on Communication Engineering.

GROUND WIRES. Grounded cables or wires are placed above the transmission line circuits to protect the latter from lightning discharges (see Lightning Protection). They are usually grounded at each supporting structure except where short spans are used. The same care must be exercised to obtain clearances between conductors and ground cables or wires, as outlined above under telephone circuits. For tower lines having flexible towers and single-circuit towers having conductors arranged in a horizontal plane, two ground cables are preferable, but for double-circuit tower lines either one or two may be used. As a general rule a line drawn through the ground cable and any conductor should not make an angle of more than 45 deg with the vertical. See chapter on Grounding, Art. 96.

Grade of Construction

The criterion for the strength requirements of a line is known as the "grade of construction." In the National Electrical Safety Code the grades are designated by the letters, A, B, C, D, E, and N, grade A being the highest and requiring the greatest strength. The grade to be used depends upon the type of circuit, the voltage, and the surroundings of the line. For example, a power line of any voltage crossing over a main track of a steam railroad requires grade A construction, but under certain other conditions may require as low as grade N. Regulatory bodies of various states have set up similar requirements.

Type of Supporting Structures

Wood poles are used to support lines of any voltage. Steel structures of various types are used to a limited extent to support low-voltage lines but are largely used for higher-voltage transmission lines. Reinforced-concrete poles are also used to a limited extent where good appearance is a factor and where the expense can be justified.

Wood poles have the advantage of increasing the insulation value of a line against lightning and therefore are coming into greater use for high-voltage transmission lines as an economical means of minimizing flashovers. Wood poles are used with pin-type insulator construction for low-voltage distribution lines and low-voltage transmission lines. Single wood poles are used with suspension-type insulator construction up to

about 66 kv. Single-circuit lines of higher voltage are common using two-pole H-frame construction. Such construction has been used even for extremely long spans. With wood pole structures the stresses which cannot be sustained by the poles themselves are sustained by proper guying. (See Article 25 on Wood Poles.)

Steel structures are used in various forms such as: tubular poles, square latticed poles, and wide-base towers. Tubular steel poles are used primarily for low-voltage distribution lines and as trolley supports where good appearance is a factor, although they are being largely superseded by smoothly finished wood poles. Square latticed steel poles are used to some extent for medium-voltage transmission lines on narrow right-of-way and where high lateral strength is required which cannot be obtained by the use of wood poles without guying.

Wide-base steel towers are used for long-span lines on private right-of-way where considerable height is necessary to provide ground clearance for the conductors. Such towers usually have square or rectangular bases, although triangular-base towers have been used to some extent for single-circuit medium-voltage transmission lines. Wide-base towers are usually classified as suspension towers, angle towers, and strain or dead-end towers. Suspension towers are used in the straight sections of a line and are designed mainly to withstand stresses in a transverse direction resulting from wind pressure on ice-coated conductors and the tower itself. In a longitudinal direction they are designed to carry a certain amount of unbalanced stress due to broken conductors and wind loading. Certain torsional stresses due to unequal conductor loadings must also be taken into account. Angle towers are designed to withstand both the balanced stresses due to the angle in the line and also any unbalanced stresses due to wind pressures and unequal conductor loading as well as possible broken conductors. Strain towers are used at dead-end points in the line and must be designed for full lateral stresses the same as suspension towers, also for the full longitudinal stresses of all the loaded conductors and for torsional stresses due to broken conductors. Semi-strain towers are sometimes used for situations where the longitudinal stresses are intermediate between those of suspension and strain towers.

The great majority of towers for lines up to 154 kv are double circuit for economic reasons. Even though the second circuit may not be added until later, the additional cost for double-circuit towers above that for single-circuit towers is so small as usually to be justified. Such towers commonly have each circuit arranged in a vertical configuration on opposite sides of the tower. For lines of 220 kv or higher, single-circuit towers are nearly always used, although the use of double-circuit towers may be economically desirable under certain conditions. The cost of double-circuit towers for such voltages is not far different from the cost of separate lines. Single-circuit towers for the extreme high voltages invariably have the conductors arranged in a horizontal configuration.

After all the mechanical and electrical features of the line have been considered, the final choice of the type of structure to be used is determined by the overall economics of the problem. The selection should be made on the basis of minimum annual cost, modified by the other factors involved. (See article 26 on Steel Towers.)

Type of Insulators

Pin-type insulators are used almost exclusively for straight line work for voltages up to 15 kv and quite generally for that purpose up to 22 or 33 kv. Suspension or disk-type insulators are used for dead-ending lines of any voltage, although small conductors of low-voltage lines are often dead-ended on double arm construction using pin-type insulators. Suspension insulators are used for tangent and angle construction for practically all lines above 33 kv and sometimes for 22- and 33-kv lines. Pin-type insulators are unsatisfactory and expensive for the higher voltages. (See also article on Insulators.)

Conductors

Conductor materials for overhead lines include: copper, bronze, aluminum, steel, copper-clad steel, and steel-reinforced aluminum. All-aluminum and all-steel conductors have a limited application in modern overhead construction owing to the low strength of aluminum and the tendency of steel to corrode. High strength for long-span construction is obtained by the use of bronze, hard-drawn copper, copper-clad steel, or steel-reinforced aluminum. Conductors are practically always stranded in sizes larger than No. 2 A. W. G. Tubular copper conductors have been used on high-voltage lines. Steel-reinforced aluminum conductors are always stranded. The steel reinforcement forms the core and may be either solid or stranded. The aluminum wires are concentrically stranded around the steel core. Where large-diameter conductors are required to raise the corona

limit of the line, they may be obtained by the use of annular conductors or aluminum cables with steel-core reinforcement. The mechanical characteristics of conductor materials are given in Table I below. (See chapter on Conductors.)

Table I. Mechanical Characteristics of Conductor Materials

Item	H. D. Copper	Aluminum	Steel
Ultimate strength, in lb per sq in..	60,000–65,000	25,000–50,000	60,000–80,000
Yield point, in lb per sq in........	30,000–35,000	11,000–14,000	35,000–40,000
Modulus of elasticity, in lb in. units.	16×10^6	9×10^6	22×10^6–28×10^6
Coefficient of linear expansion per deg fahr......................	9.6×10^{-6}	12.8×10^{-6}	6.6×10^{-6}
Weight in pounds of a 1-ft length having a cross-section of 1,000,000 cir mils (stranded).............	3.09	0.92	2.67

See also chapter on Bare Wires and Cables.

The maximum stress in conductors should not exceed the yield point (usually called "elastic limit"). The elastic limit and modulus of elasticity of a conductor are not, however, fixed under all conditions. They are reduced by fluctuating stresses. Probably the working limit for fluctuating stress should not be over 50 to 70 per cent of the ultimate strength for uniformly increased stress, depending upon the material.

If a sample of wire is stretched in a testing machine with a maximum stress exceeding any previous stress on the wire, but within the working limit, and the stress is then gradually removed, a new stress-strain curve is obtained which is a straight line practically parallel to the tangent to the original stress-strain curve at the origin, but displaced along the axis of elongation. If the stress is again increased the stress-strain values will lie on the curve obtained under the previous reduction of stress, and, if the previous maximum stress is not exceeded, upon reduction of stress the stress-strain values will lie on the same straight line. Under such conditions the conductor will have a constant value of modulus of elasticity. The practical application of these principles may be carried out by two methods. One of these is to prestretch the conductor to the maximum stress which it may be expected to sustain in operation. If this method is used (which is often impracticable), sag-tension calculations are based on stress-strain diagrams for different temperatures obtained by stressing the conductor to the expected maximum stress and then reducing the stress to zero, and stress-length curves with different loadings together with length-sag curves. Conductors are then strung to the required sag, which sag they will always have under the same temperature condition. A more usual method is to calculate the stringing sags on the basis of the stress-strain diagram under initial stress application. These sags will be less than those which will exist for the same temperature after the conductor has been mechanically loaded. Therefore, for checking ground clearances the final sags should be calculated from the stress-strain diagram obtained by stretching to maximum allowable stress. It is not necessary to go to such refinements for lines having short spans and small sags. For further discussion of this subject see an article by Theodore Varney, entitled A.C.S.R. graphic method for sag-tension calculations, published by the Aluminum Co. of America. In the case of composite conductors such as A.C.S.R. the virtual modulus of elasticity and virtual coefficient of expansion are not the same as the values for either of the component materials. For example, for the composite cable of A.C.S.R. the virtual modulus of elasticity is:

$$M_{as} = M_a R_a + M_s R_s \tag{2}$$

and the virtual coefficient of expansion is:

$$A_{as} = \frac{A_a M_a R_a}{M_{as}} + \frac{A_s M_s R_s}{M_{as}} \tag{3}$$

where M is modulus of elasticity, A is coefficient of expansion, R is fraction of total cross-section, represented by each material; and subscripts a and s indicate aluminum and steel, respectively.

In a composite cable the stress may be transferred entirely to one material at extremely high temperatures and to the other material at extremely low temperatures. At high temperatures the material having the lower coefficient of expansion takes the stress, and at low temperatures it is taken by the material having the higher coefficient of expansion. This transfer of stress is not abrupt because of mechanical stretching of the material which tends to take more than its share of the stress. However, when the combined

rate of stretching and expansion of one material exceeds the combined rate of the other the transfer becomes complete. Data for checking this condition should be obtained from stress-strain diagrams of the individual materials. Further information on this subject may be obtained from the chapter on Wires and Cables and also from the article by Theodore Varney mentioned above. Fig. 1 shows typical stress-strain diagrams for composite cables and homogeneous cables. Curve 1 is the initial stress curve for A.C.S.R.; curve 2 is the final stress curve for the composite cable (on reduction of stress); curve 3 is the initial stress curve for the steel portion; curve 4 is the final stress curve for the steel portion; curve 5 is the initial stress curve for the aluminum portion; and curve 6 is the final stress curve for the aluminum portion. Curve 7 is the initial curve for a homogeneous

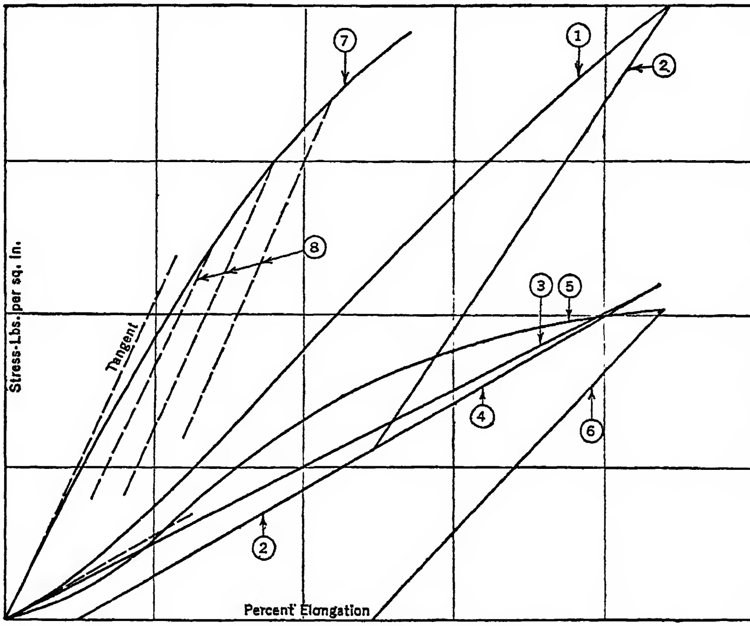

FIG. 1. Typical Stress-strain Curves

cable, and curves 8 are final stress curves obtained by reduction of stress after reaching the several maxima.

Mechanical Loading

The term mechanical loading refers to the external conditions which produce mechanical stresses in the line conductors and supports. It also includes the weight of the conductors and structures themselves. The external conditions referred to include: the temperature range to be expected, the collection of ice on the conductors and structures, and the variations in wind pressure on conductors and structures. The map shown on Fig. 2 is usually taken as a basis for determining the heaviest loading conditions for a line in a certain locality. The loadings corresponding to the terms heavy, medium, and light are as follows:

Heavy: $^1/_2$ in. ice, 8 lb wind pressure per sq ft, 0 deg fahr.
Medium: $^1/_4$ in. ice, 8 lb wind pressure per sq ft, 15 deg fahr.
Light: no ice, 12 lb wind pressure per sq ft, 30 deg fahr.

When local conditions are known to vary from the above, such variations should be taken into account.

TEMPERATURE RANGE. The maximum and minimum air temperatures which have been observed in any locality for a period of years can be obtained from the records of the U.S. Weather Bureau, or from similar records for other countries. The minimum air temperature recorded (which may be as low as −40 deg fahr in some of the northern states) will be the minimum temperature which the conductor may be expected to reach.

However, since the conductors are exposed to the direct rays of the sun, they will reach a maximum temperature in the summer considerably in excess of the Weather Bureau records, which give the temperatures in the shade.

Another important temperature which should be determined is that of the wire when coated with ice. As noted below, a sleet storm is usually followed by a fall in temperature, and although the ice forms at 32 deg fahr, the wire may reach a much lower temperature while the ice is on it.

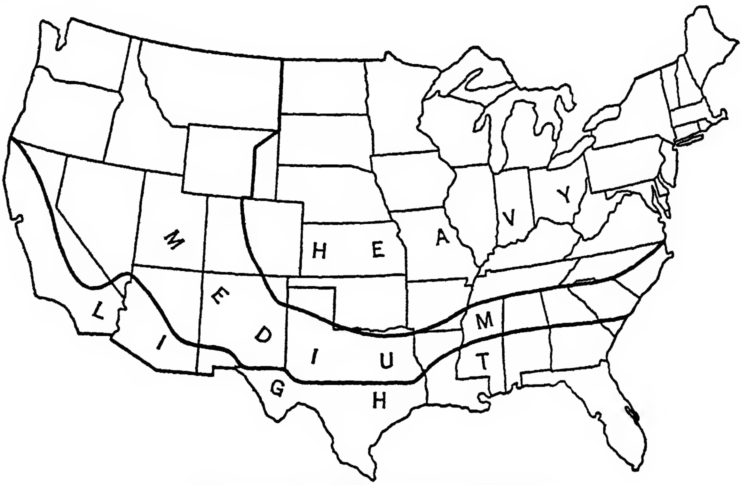

Fig. 2. Loading Map for Overhead Lines

The following temperature ranges have been used in the design of certain lines:

	Maximum	Minimum
Eastern Canada	+120 deg fahr	−40 deg fahr
Mississippi Valley	+120 deg fahr	−20 deg fahr
Southern California	+140 deg fahr	+10 deg fahr

COLLECTION OF ICE ON WIRES. Investigation of the records of the Weather Bureau leads to the conclusion that sleet and ice storms are generally followed by falling temperatures and high winds, and transmission lines should be designed to meet these conditions. Records indicate that under favorable conditions ice and sleet will collect on wires and cables to the same amount in any climate where freezing temperatures are obtained. Mild, moderate, and cold climates differ in the frequency with which conditions are favorable. In general, sleet storms are most frequent in the moderate climates, since precipitation takes place more often at freezing temperatures. Destructive sleet storms occur in the eastern part of the United States at least as far south as Atlanta. One-half inch thickness of solid ice on wires and cables is generally assumed in designing transmission lines, but thicknesses of one-quarter and three-quarters inch are also assumed in the more favorable and unfavorable localities, respectively.

Ice and sleet generally collect quite uniformly on wires throughout their length. The collection is sometimes in the form of icicles but more often is egg-shaped in cross-section, with the wire in the small end of the section. It frequently falls off non-uniformly in sections.

Clear solid ice weighs 57 lb per cu ft or 0.033 lb per cu in, but sleet or frozen snow such as often collects on wires weighs much less, sometimes as little as 8 lb per cu ft.

WIND PRESSURE. Wind pressure is a subject upon which little exact information exists, although many experiments have been made and much study has been given to the subject by engineers and scientists. Among the unsettled questions are:

(*a*) The relation between pressure and velocity.
(*b*) The variation of pressure with size and shape of exposed plane surfaces.
(*c*) The direction and intensity of pressure upon non-vertical surfaces.
(*d*) The intensity of pressure upon non-planar surfaces.

(*e*) The total pressure upon a number of parallel bars or other members placed side by side.

(*f*) The decrease of pressure upon leeward surfaces.

(*g*) The lifting power of the wind.

Relation between Indicated (U. S. Weather Bureau) Wind Velocity and Actual Velocity. The indications of the anemometers used by the U. S. Weather Bureau do not give the *actual* wind velocity, but give values considerably higher than the actual velocities, as shown in Table II:

Table II. Relation between Indicated and Actual Wind Velocity

Indicated Velocity, mi per hr	Actual Velocity, mi per hr	Indicated Velocity, mi per hr	Actual Velocity, mi per hr
10	9.6	60	48.0
20	17.8	70	55.2
30	25.7	80	62.2
40	33.3	90	69.2
50	40.8	100	76.2

In the U. S. Weather Bureau reports the indicated and not the actual wind velocities are given. However, as the anemometers used give the *average* velocity for several minutes, the instantaneous velocities due to sudden gusts may be considerably greater than the indicated velocities; the indicated velocity probably more nearly represents the " gust " velocity than the actual average velocity. In all calculations of maximum wind pressure it is therefore recommended that the *indicated* velocity be used.

The Weather Bureau records give no indication of the " gust " velocities which may occur during the 5-min periods, and which may greatly exceed the average velocity. Tests with a Dines pressure tube anemometer have shown that the extreme maximum is about 50 per cent greater than the average for short periods.

The extreme maximum wind velocity observed in Chicago in the whole 36-year period from 1873 to 1910 was 84 miles per hour (uncorrected) in February, 1894. A velocity of 76 miles per hour (uncorrected) was observed once in November, 1898, and a velocity of 72 miles per hour (uncorrected) was observed seven times. During the 10-year period from 1894 to 1903 the maximum wind velocity in a few other representative localities was as follows, all velocities being the observed or uncorrected velocities: Bismarck, N.D., 72; Eastport, Me., 78; Buffalo, N. Y., 90; New York City, N. Y., 78; Galveston, Tex., 84; Savannah, Ga., 76; Salt Lake City, Utah, 60. All the maxima range between 60 miles and 90 miles per hour. (See N.E.L.A. Overhead Systems Reference Book for more complete data.)

Relation between Pressure and Velocity. The pressure varies about as the square of the velocity, the results given by different experimenters for the pressure due to a *normal wind on a plane surface* ranging from

$$P = 0.004V^2 \quad \text{to} \quad P = 0.0032V^2 \tag{4}$$

where P = pressure in pounds per square foot.
V = actual wind velocity in miles per hour.

The former of these values is for small flat surfaces; the latter represents the results of unusually careful experiments by Stanton (see *Minutes of Proc. Inst. Civil Engineers*, Vols. 156 and 171) upon the intensity of pressure on plates varying in size from 25 to 100 sq ft and is probably more nearly correct than the higher value. In the Stanton formula the values are reduced to correspond to a temperature of 60 deg fahr and an atmospheric pressure of 14.7 lb per sq in., i.e., barometric pressure of 30 in.

The influence of size and shape of exposed surface is an important question and is not well understood, although it is known that the resultant pressure on a large surface may be taken as less per square foot than that on a small surface, since the maximum intensity of the wind is due to gusts of comparatively small cross-section.

Formulas for Pressure on Plane Surfaces When Wind is Not Normal. The pressure upon vertical plane surfaces may be taken as normal to the surface and equal in intensity to the assumed wind pressure. Upon surfaces which are not vertical, the pressure is usually considered to be normal to the surface but lower in intensity than upon vertical surfaces. The variation in pressure with respect to the slope is not well understood, and a number of empirical formulas are in use, among which are the Duchemin formula

$$P_n = P\frac{2\sin i}{1+\sin^2 i} \tag{5}$$

and the Hutton formula

$$P_n = P\,(\sin i)^{(1.84 \cos i - 1)} \tag{6}$$

where P = intensity of normal pressure upon the vertical surface.
P_n = intensity of normal pressure upon the given surface.
i = angle made by surface with the horizontal.

The following theoretical formula results from the assumption that the wind always blows in horizontal lines, and that if the pressure be resolved into normal and tangential components, the tangential component may be neglected:

$$P_n = P \sin^2 i \tag{7}$$

This formula gives lower values than the empirical formulas and probably gives too low results since it makes no allowance for the reduction in pressure on the leeward side which is known to occur, and which may in part be attributed to the influence of the tangential component. It should also be noted that the wind does not blow uniformly in horizontal lines but may deviate considerably from the horizontal.

The values given by these three formulas are tabulated for comparison, using an assumed value of 30 lb per sq ft for P. In the absence of further experience upon this phase of wind pressure it would seem wise to use one of the empirical formulas instead of the theoretical one. The Hutton formula is used quite generally by structural engineers in England and the United States.

Pressure on Non-Planar Surfaces. The pressure upon non-planar surfaces is important in the case of chimneys, standpipes, and other similar objects.

Upon the same assumptions as made in the preceding paragraph it may be demonstrated that theoretically the pressure on a cylinder is two-thirds of the total pressure on a plane diametrical section. This value is quite generally used. The pressures thus obtained lack experimental proof but are probably more nearly correct than the pressure obtained by the same method upon plane surfaces.

Effect of Reduction of Pressure on Leeward Side. The pressure upon the windward side of an exposed surface is a function of the density and velocity of the air currents. The pressure on the leeward side is also a function of the shape of the surface, and has been shown by numerous experiments to be less than the static pressure of the air current. The resultant total pressure upon a surface is in consequence a function not only of the direct pressure on the windward side, but also of the pressure on the leeward side, which in turn is a function of the form of the surface. No algebraic formula can be given which will indicate the pressure on surfaces of varying shape with any considerable degree of precision.

Wind Pressure on Wires. H. W. Buck (*Trans. Int. Elec. Cong., St. Louis*, 1904, Vol. 2, p. 318) gives the following formula for the pressure due to a normal wind on a stranded wire:

$$P = 0.0025V^2 \tag{8}$$

Table III. Wind Pressure in Pounds per Square Foot

P = 30 lb per sq ft

Angle i, degrees	Theoretical	Duchemin	Hutton
	$P \sin^2 i$	$P \frac{2 \sin i}{1 + \sin^2 i}$	$P\,(\sin i)^{(1.84 \cos i - 1)}$
5	0.0	5.2	3.9
10	0.9	10.1	7.3
15	2.0	14.6	10.5
20	3.5	18.4	13.7
25	5.3	21.5	16.9
30	7.5	24.0	19.9
35	9.9	25.8	22.6
40	12.4	27.3	25.1
45	15.0	28.3	27.0
50	17.6	29.0	28.6
55	20.1	29.4	29.7
60	22.5	....	....
65	24.6	Above 60 deg	Above 60 deg
70	26.4	use 30 lb	use 30 lb
75	28.0		
80	29.1		
85	29.7		
90	30.0		

where P is the pressure in pounds per square foot of projected area of wire and sleet, if any (length times diameter), and V is the velocity in miles per hour. This formula is based upon tests made on a 950-ft span at Niagara Falls; the wind velocities were measured by a U. S. Weather Bureau anemometer corrected to give actual average velocities.

Practical Rules for Wind Pressure Allowance. The many uncertainties connected with wind pressure make worthless the attempts to specify with precision its magnitude and direction. In the lack of additional information and further theoretical studies there seems to be no reason for deviating from the common rules which have been used for many years with satisfactory results.

For wind pressure on roofs and buildings it is common practice to allow 30 lb per sq ft acting horizontally upon the sides and ends of buildings, or on the vertical projection of roofs. It is also very important to figure the wind stresses on the steel frame considering it as an independent structure without walls, floors, or partitions, since failures often occur in erection.

Steel Towers. The National Electrical Safety Code requires the stated wind pressure for a given loading area to be increased 60 per cent for flat surfaces. In the case of latticed structures the wind pressure is taken on 150 per cent of the actual exposed area of one lateral face to allow for pressure on the opposite face. For grade A, B, and C construction the wind pressure is taken as 3 times the specified wind pressure for the loading area in computing the strength of the tower without conductors.

Vibration of Conductors. Under certain critical conditions of temperature and tension in conductors light intermittent winds across the line may start vertical vibration of the conductors. If conditions are exactly right the initially small vibrations are increased in amplitude by resonance until they have in some cases reached sufficient magnitude to break conductors or wreck structures. Probably some vibration occurs in all overhead conductors under the critical conditions. In short spans they do not, as a rule, reach dangerous amplitudes. It was not until long-span construction became common that these effects were noticed or correctly diagnosed. The vibrations tend to fatigue conductors at the point of support due to the rapid bending in opposite directions. The rigidity of clamping of the conductor at the support contributes to the fatigue of the conductor material. Mechanical damping devices attached to the conductors near the supports are often used to prevent the resonant condition as well as to reduce the rigidity of the support.

VERTICAL AND TRANSVERSE FORCES ON A SUSPENDED WIRE. The resultant force acting on one foot of a suspended wire is in general made up of three components, viz.:

c = weight of the conductor (including insulation, if any) per foot length, in pounds.
i = weight of the ice coating per foot length of the conductor, in pounds.
h = wind pressure per foot length of the conductor, in pounds.

The weight of the conductor per foot length may be taken directly from the tables in the articles on Bare Wires and Cables. Let d be the diameter in inches, and let t be the thickness of the ice coating; then the weight of the ice coating per foot length of the conductor is

$$i = 1.24\,t(d + t) \tag{9}$$

Let p be the wind pressure per square foot of projected area; then the wind pressure per foot length of the conductor, i.e., the horizontal component of the resultant force, is

$$h = \frac{p(d + 2t)}{12} \tag{10}$$

The vertical component of the resultant force per foot length of conductor, which is equal to the resultant force for no wind, is

$$v = c + i \tag{11}$$

Values of v and h for various ice thicknesses and various sizes of wires are given in Table II. v and h being known, the resultant force w for any combination of wind and ice loads is readily determined by the formula

$$w = \sqrt{v^2 + h^2} \tag{12}$$

Calculation of Sag and Tension

A wire or cable suspended between towers takes the form of a catenary curve. In transmission-line practice the maximum deflection or sag is always small compared to the span; that is, the curve is very flat. The shape of such a flat catenary curve does not differ appreciably from a parabola and, as the approximate parabolic formulas are much simpler than the more exact catenary formulas, they are used instead. The flatness

Table IV. Vertical, Transverse and Resultant Loadings

Pounds per lineal foot

Size of Conductor	Diameter overall, in.	Area of Conductor, sq in.	Weight of— Conductor +0.5in. of ice= heavy lb per ft	Conductor +0.25in of ice= medium lb per ft	Conductor only= light lb per ft	Transverse Force on Conductor with Ice Covering (if any) Heavy	Medium	Light	Resultant Force on Conductor Due to Weight and Wind Heavy	Medium	Light
Bare solid copper:											
A. W. G. No.											
12	0.081	0.0051	0.381	0.122	0.020	0.721	0.387	0.081	0.815	0.406	0.084
10	.102	.0082	.406	.141	.031	.735	.401	.102	.840	.425	.107
8	.128	.013	.440	.168	.050	.752	.419	.128	.872	.451	.137
6	.162	.021	.491	.207	.079	.775	.442	.162	.918	.467	.180
4	.204	.033	.564	.268	.126	.803	.470	.204	.986	.540	.240
3	.229	.041	.612	.308	.159	.820	.486	.229	1.023	.576	.279
2	.258	.052	.672	.359	.201	.839	.506	.258	1.075	.620	.327
1	.289	.066	.744	.421	.253	.860	.526	.289	1.137	.674	.384
0	.325	.083	.832	.498	.319	.884	.550	.325	1.214	.742	.456
00	.365	.104	.943	.596	.405	.910	.577	.365	1.310	.829	.545
000	.410	.132	1.075	.714	.509	.940	.607	.410	1.428	.937	.653
0000	.460	.166	1.237	.861	.640	.974	.640	.460	1.574	1.073	.788
Bare stranded copper:											
A. W. G. No.											
6	.18	.021	.505	.216	.083	.787	.454	.180	.935	.503	.198
4	.23	.033	.580	.275	.126	.820	.487	.230	1.005	.559	.262
3	.26	.041	.634	.320	.161	.840	.507	.260	1.053	.599	.306
2	.29	.052	.696	.372	.204	.860	.527	.290	1.106	.645	.355
1	.33	.066	.775	.440	.259	.887	.554	.330	1.178	.707	.420
0	.37	.083	.867	.519	.326	.914	.580	.370	1.260	.779	.493
00	.41	.104	.979	.618	.413	.940	.607	.410	1.357	.866	.582
000	.46	.132	1.116	.740	.519	.974	.640	.460	1.481	.978	.693
0000	.52	.166	1.287	.892	.652	1.014	.680	.520	1.638	1.122	.834
Cir. Mils											
250,000	.57	.196	1.436	1.025	.770	1.047	.714	.570	1.777	1.294	.958
300,000	.63	.236	1.630	1.201	.928	1.087	.754	.630	1.960	1.418	1.121
350,000	.68	.275	1.815	1.370	1.081	1.121	.787	.680	2.133	1.580	1.277
400,000	.73	.314	1.992	1.539	1.234	1.154	.820	.730	2.308	1.744	1.434
450,000	.77	.353	2.177	1.705	1.388	1.181	.847	.770	2.477	1.904	1.587
500,000	.81	.392	2.355	1.870	1.541	1.207	.874	.810	2.646	2.064	1.741
1,000,000	1.15	.785	4.112	3.521	3.086	1.434	1.100	1.150	4.355	3.822	3.293
T.B.W.P. solid copper:											
A. W. G. No.											
12	.21	.0051	.476	.178	.035	.807	.474	.210	.937	.506	.213
10	.25	.0082	.519	.208	.053	.834	.500	.250	.987	.542	.255
8	.26	.013	.547	.234	.075	.840	.507	.260	1.003	.558	.270
6	.32	.021	.622	.289	.112	.880	.547	.320	1.078	.619	.339
4	.38	.033	.711	.370	.164	.920	.587	.380	1.163	.694	.414
3	.41	.041	.760	.405	.200	.940	.607	.410	1.213	.730	.456
2	.44	.052	.840	.474	.260	.960	.627	.440	1.276	.768	.511
1	.47	.066	.919	.540	.316	.980	.647	.470	1.344	.843	.566
0	.50	.083	1.029	.640	.407	1.000	.667	.500	1.435	.924	.645
00	.53	.104	1.143	.745	.502	1.020	.687	.530	1.532	1.013	.730
000	.62	.132	1.326	.900	.630	1.080	.747	.620	1.711	1.170	.872
0000	.65	.166	1.482	1.047	.767	1.100	.767	.650	1.846	1.286	1.005
T. B. W. P. stranded copper:											
A. W. G. No.											
2	.444	.052	.857	.486	.270	.961	.630	.444	1.289	.796	.520
1	.518	.066	.961	.567	.328	1.012	.679	.518	1.396	.884	.613
0	.620	.083	1.120	.694	.424	1.080	.747	.620	1.557	1.020	.751
00	.662	.104	1.245	.806	.522	1.109	.775	.662	1.667	1.118	.843
000	.734	.132	1.421	.960	.654	1.157	.823	.734	1.832	1.331	.983
0000	.785	.166	1.599	1.122	.800	1.191	.857	.785	1.994	1.412	1.121
Cir. mils											
250,000	.862	.196	1.832	1.331	.985	1.241	.908	.862	2.213	1.611	1.309
350,000	.978	.275	2.264	1.727	1.345	1.319	.986	.978	2.620	1.988	1.663
500,000	1.108	.392	2.894	2.316	1.894	1.406	1.072	1.108	3.217	2.552	2.194
750,000	1.343	.589	3.968	3.317	2.822	1.563	1.229	1.343	4.265	3.538	3.125
1,000,000	1.531	.785	4.937	4.228	3.674	1.688	1.355	1.531	5.218	4.439	3.980

Table IV. Vertical, Transverse and Resultant Loadings—*Continued*

Pounds per lineal foot

Size of Conductor	Diameter overall, in.	Area of Conductor, sq in.	Weight of—			Transverse Force on Conductor with Ice Covering (if any)			Resultant Force on Conductor Due to Weight and Wind		
			Conductor +0.5in. of ice= heavy lb per ft	Conductor +0.25in of ice= medium lb per ft	Conductor only= light lb per ft	Heavy	Medium	Light	Heavy	Medium	Light
Bare solid steel:											
Stl. W. G. No.											
8	.162	.0206	.482	.198	.070	.775	.442	.162	.912	.484	.176
6	.192	.0290	.528	.235	.098	.795	.462	.192	.955	.518	.216
4	.225	.0400	.586	.283	.135	.817	.484	.225	1.006	.560	.263
Bare stranded steel:											
1/4-inch	.250	.0352	.584	.275	.119	.833	.500	.250	1.018	.570	.277
5/16-inch	.312	.0606	.711	.380	.205	.875	.542	.312	1.126	.661	.374
3/8-inch	.375	.0832	.826	.476	.282	.917	.583	.375	1.234	.753	.469
7/16-inch	.437	.1204	.991	.622	.408	.958	.625	.437	1.379	.882	.598
1/2-inch	.500	.1443	1.111	.722	.489	1.000	.667	.500	1.495	.983	.699
9/16-inch	.562	.1922	1.312	.904	.652	1.042	.708	.562	1.675	1.149	.861
5/8-inch	.625	.2356	1.498	1.071	.799	1.083	.750	.625	1.849	1.307	1.014
Solid bare copper-covered steel:											
A. W. G No.											
10	.102	.0082	.402	.138	.029	.735	.401	.102	.838	.425	.106
8	.128	.013	.437	.163	.046	.752	.419	.128	.870	.450	.136
6	.162	.021	.485	.201	.073	.775	.442	.162	.914	.485	.178
4	.204	.033	.554	.257	.116	.803	.470	.204	.975	.535	.235
Stranded bare-copper-covered steel:											
5/16-inch	.306	.0571	.710	.382	.209	.871	.538	.306	1.124	.659	.370
3/8-inch	.384	.091	.882	.529	.332	.923	.590	.384	1.276	.792	.508
7/16-inch	.432	.114	.998	.630	.418	.955	.622	.432	1.381	.885	.601
1/2-inch	.486	.144	1.139	.755	.526	.991	.658	.486	1.510	1.001	.716
9/16-inch	.546	.182	1.313	.910	.663	1.031	.698	.546	1.670	1.147	.859
3 No. 10	.220	0245	.534	.233	.087	.813	.480	.204	.973	.534	.221
3 No. 8	.276	.0386	.620	.301	.138	.851	.517	.256	1.053	.598	.290
3 No. 6	.349	.0618	.748	.406	.220	.899	.566	.324	1.170	.697	.391
Stranded aluminum, bare:											
A. W. G. No.											
2	.293	.052	.554	.230	.062	.862	.529	.293	1.020	.577	.300
1	.328	.066	.592	.258	.079	.886	.552	.328	1.065	.609	.337
0	.368	.083	.637	.290	.099	.912	.579	.368	1.113	.647	.380
00	.414	.104	.692	.331	.125	.943	.610	.414	1.170	.693	.432
000	.464	.132	.756	.379	.158	.976	.643	.464	1.234	.746	.489
0000	.522	.166	.832	.437	.198	1.015	.682	.522	1.312	.810	.557
Bare stranded aluminum, steel-reinforced:											
A. W. G. No.											
4	.250	.0383	.523	.213	.058	.834	.500	.250	.984	.544	.257
2	.316	.0608	.598	.268	.092	.878	.544	.316	1.062	.607	.329
1	.355	.0766	.647	.305	.117	.904	.570	.355	1.112	.646	.374
0	.398	.0967	.704	.348	.147	.932	.599	.398	1.168	.693	.424
00	.447	.1219	.772	.401	.185	.965	.632	.447	1.236	.748	.484
000	.501	.1537	.853	.465	.232	1.001	.668	.501	1.315	.814	.552
0000	.564	.1939	.954	.547	.294	1.043	.710	.564	1.414	.896	.636
Cir. mils											
336,400 30 × 7	.741	.3260	1.297	.834	.527	1.161	.828	.741	1.741	1.175	.909
397,500 30 × 7	.806	.3850	1.422	.945	.622	1.204	.870	.806	1.863	1.285	1.018
477,000 30 × 7	.883	.4619	1.593	1.093	.747	1.255	.922	.883	2.028	1.430	1.157
556,500 30 × 7	.953	.5391	1.761	1.240	.871	1.302	.969	.953	2.190	1.574	1.292
636,000 54 × 7	.977	.5904	1.722	1.194	.818	1.317	.984	.977	2.168	1.547	1.274
715,500 54 × 7	1.036	.6348	1.861	1.314	.921	1.357	1.024	1.036	2.303	1.666	1.386
795,000 54 × 7	1.093	.7053	1.998	1.434	1.023	1.395	1.062	1.093	2.437	1.784	1.497
1,192,500 54 × 19	1.338	1.0554	2.651	2.012	1.526	1.558	1.225	1.338	3.075	2.356	2.030

of the curve allows of some further simplifications even in the parabolic formulas, viz., (1) the tension is considered uniform throughout the span, the slight excess of tension at the ends over that at middle being neglected; (2) the change of length of the wire due

to elastic stretch or temperature expansion is taken as equal to the change of length of a wire equal in length to the horizontal distance between the points of support.

Notation Used in Sag-tension Formulas. The following notation, listed alphabetically, will be used throughout the discussion of sag and tension.

A = cross-section of the conductor (actual metal cross-section) in square inches = circular mils ÷ 1,273,000.

a = coefficient of linear expansion of the conductor per degree fahrenheit; see Table I above.

D = deflection, in feet, of the lowest point of the conductor from the line through supports when suspended from two points of support at the same elevation and at a distance L apart. (D is measured in the direction of the resultant transverse force.)

e = difference in elevation of the two points of support, in feet.

F = longitudinal horizontal component of the stress in the conductor, in pounds. (The resultant stress in the wire at the insulator is equal to $\sqrt{F^2 + H^2 + V^2}$, where V is the weight of the conductor and ice from the insulator to the lowest point of the span, and H is the total wind pressure on half the length of span; H and V in this expression are usually negligible compared with F.)

h = wind pressure in pounds per foot length of conductor assumed perpendicular to the vertical plane through the two points of support; see eq. (10) and Table IV above.

L = length of span in feet, i.e., the horizontal distance between the two points of support in feet.

l = length in feet of the arc of the curve in which the conductor hangs, i.e., the length of stretched conductor between the two points of support.

M = modulus of elasticity of the conductor in pound-inch units; see Table I above.

$S = \dfrac{vD}{w}$ = sag of the lowest point of the conductor below the horizontal line through the points of support; for no wind $S = D$.

T_0 = maximum allowable tension in the conductor in pounds per square inch of its cross section; T_0 is usually taken as one-half the ultimate strength of the conductor; see Table I.

v = vertical force in pounds on a one-foot length of the conductor, including the weight of conductor and the weight of the ice, if any, on it; see eq. (11) and Table IV above.

$w = \sqrt{v^2 + h^2}$ = resultant load in pounds on a 1-ft length of the conductor.

$Z = \dfrac{hD}{w}$ = side swing, in feet, of the middle point of the conductor, measured perpendicularly to the vertical plane through the two points of support.

The various symbols with the subscript "0" will be used to designate the values of the various quantities under the conditions of maximum assumed loading; see paragraph above on Maximum Loading.

FUNDAMENTAL EQUATIONS OF A WIRE SPAN. As noted above, a perfectly flexible wire suspended between two points of support hangs in a catenary. The assumption that the wire hangs in a parabola instead of in a catenary is sufficiently accurate for all practical calculations of wire spans, the error in the sag calculated on this assumption being less than 2 per cent of its true value when this sag is less than 0.06 times the length of the span (e.g., less than 60 ft in a 1000-ft span), and the error in the length of the wire calculated on this assumption being less than 0.002 per cent of its true value for the same limiting conditions. The formulas given below are all based on the assumption of a parabola.

Deflection, Sag and Side Swing. For a given length of span L, loading w, and stress F, the deflection D for the points of support at the same elevation is given by the relation

$$D = \frac{wL^2}{8F} \tag{13}$$

When there is no wind this is also equal to the vertical sag, that is $S = D$. When there is wind, w is greater than the vertical loading v, and the vertical sag for the points of support at the same elevation is

$$S = \frac{vD}{w} = \frac{vL^2}{8F} \tag{14}$$

D in eq. (14) has the value given by eq. (13). When one point of support is at an elevation

e above the other, then the vertical sag of the lowest point of the conductor below the lower point of support is

$$S' = S\left(1 - \frac{e}{4S}\right)^2 \tag{15}$$

where S is given by eq. (14). The horizontal distance of the lowest point of the conductor from the lower point of support is

$$L' = \frac{L}{2}\left(1 - \frac{e}{4S}\right) \tag{16}$$

The side swing Z of the middle point of the conductor, which is the point which is deflected the maximum distance from the vertical plane through the two points of support, is

$$Z = \frac{hD}{w} = \frac{hL^2}{8F} \tag{17}$$

D in eq. (17) has the value given by eq. (13).

Length of Stretched Conductor. The length of conductor between the two points of support for a given length of span L, loading w, stress F, and difference of elevation e, is

$$l = L + \frac{8}{3}\frac{D^2}{L} + \frac{e^2}{2L} \tag{18}$$

where D has the value given by eq. (13), that is, D is the deflection for the same length of span, loading, and tension, but for the points of support at the same elevation.

Effect of Changes in Loading and Temperature. When the loading or the temperature changes, the stress in the conductor will change to some new value, F_0 say, and the deflection will change to some new value, say D_0. Let the new loading be w_0 and the new temperature t_0, the initial temperature being t; also let a be the coefficient of linear expansion, M the modulus of elasticity, and A the cross-section of the conductor in square inches. Then, when the points of support remain fixed, the following relation must hold

$$\frac{8}{3L^2}(D^2 - D_0^2) = a(t - t_0) + \frac{1}{MA}(F - F_0) \tag{19}$$

The D's in this equation are the same as given by eq. (13) for a loading of w and w_0 respectively and stresses of F and F_0, respectively. Note that eq. (19) is independent of the difference in elevation of the two points of support; also that the two sets of symbols, with and without the subscripts, refer to *any* two sets of conditions.

STRESS-TEMPERATURE AND STRESS-DEFLECTION CHARTS. In order to apply the above equations to the calculation of deflections and stresses under various temperature and loading conditions, eqs. (13) and (19) may be written as follows:

$$D = \frac{wL^2}{8F} \tag{20}$$

$$D^3 = D\left[\frac{L^2 a}{2.66}(t - t_0) + D_0^2 - \frac{F_0 L^2}{2.66\,MA}\right] + \frac{3\,wL^4}{64\,MA} \tag{21}$$

From solutions of these equations under various conditions, curves showing the relation between deflection and temperature and between stress and temperature may be plotted. Such curves provide stringing charts and show maximum and minimum sags from which clearance templates may be constructed.

For an individual span the process of plotting sag curves for various temperatures is quite simple. The first step is to decide upon the maximum allowable tension (F_0), the temperature (t_0) at maximum tension, and the maximum expected mechanical loading, i.e., thickness of ice coating and wind pressure. Using eqs. (9) to (12), inclusive, or Table IV, the resultant force on the conductor (w_0) may be determined. Inserting these values in eq. (20), the deflection (D_0) for maximum tension at temperature (t_0) may be computed. The second step is to apply the above values to eq. (21). Six to eight temperatures should be selected, ranging from the lowest to the highest temperatures to which the line may be exposed. It should be remembered that the temperature of a conductor carrying current and exposed to the sun may rise as much as 30 deg fahr above the ambient air temperature. Eq. (21) should be solved for each of the selected temperatures to obtain the corresponding values of deflection (D). These values of (D) may then be plotted against the corresponding values of temperature, giving a stringing curve and also values of maximum and minimum sags. By the use of eq. (20), the tensions (F) may be determined for each value of deflection and temperature. Plotting the values of (F) thus determined against temperature provides a stringing tension curve.

For any temperature the corresponding stresses and deflections may be taken from the curves. Conductors strung in accordance with these values will not have the maxi-

mum allowable stress or sag exceeded if the maximum loading or temperature selected are not exceeded.

The stresses computed are independent of any difference of elevation of supports. If there is a difference of elevation of supports the sags below the lower support may be determined by the use of eq. (15), and the horizontal distance of the lowest point of the conductor may be obtained by using eq. (16).

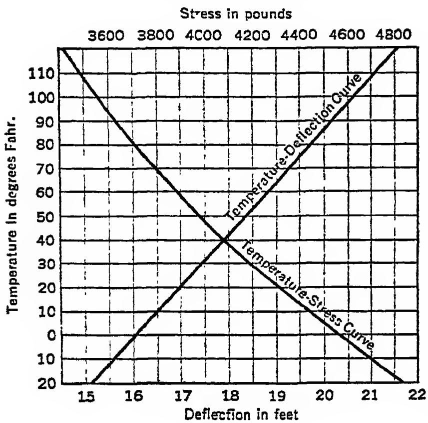

FIG. 3

Fig. 3 shows an example of stress-temperature and deflection-temperature curves for a 300,000-cir-mil 19-strand copper conductor on an 800-ft span, with the maximum allowable tension (T_0) at 30,000 lb per sq in., and maximum loading (w_0) equal to 1.807 lb per ft, due to 1/2-in. ice thickness and 6 lb per sq ft wind pressure, at 0 deg fahr.

For example, if the cable is strung at 70 deg fahr, then it should be given a deflection (vertical sag) of 19.25 ft, provided the points of support are at the same elevation, or at a stress of 3800 lb, this stress being independent of the difference of elevation of the two points of support. If there is a difference of elevation of 30 ft, say, between the two points of support, then from eq. (15), the cable should be drawn up until the sag of the lowest point below the lower point of support is

$$S' = 19.25\left(1 - \frac{30}{4 \times 19.25}\right)^2 = 7.17 \text{ ft}$$

and the stress will then be 3800 lb.

DIRECT CALCULATION OF CHANGE IN DEFLECTION AND STRESS WITH LOADING AND TEMPERATURE.* When for F in eq (19) is substituted its value from eq. (20) there results a cubic equation in D, eq. (21). This cubic equation can be solved directly by means of Tables V(*a*) and V(*b*) given herewith (H. Pender, *Elec. World*, Vol. 66, p. 344, 1915). The procedure is as follows: using the same notation as above, calculate

$$b_1 = \frac{1000\,F_0}{MA}$$

$$b_2 = \left(\frac{w_0 L}{0.155\,F_0}\right)^2$$

$$b_3 = 1000\,a(t - t_0)$$

$$b = \frac{1000\,wL}{MA}$$

$$B = \text{numerical value of } [b_1 - (b_2 + b_3)]$$

$$x = \frac{b}{B\sqrt{B}}$$

Take y from table. Then the deflection is

$$D = yL\sqrt{B} \tag{22}$$

and the stress is

$$F = \frac{wL^2}{8D} \tag{23}$$

Note that b_1 may be greater or less than $(b_2 + b_3)$; in either case B is to be taken *positive* and equal to the numerical value of $b_1 - (b_2 + b_3)$. Also note that:

b_1 is the elongation, in feet per 1000 ft, of a straight wire when subjected to a stress of F_0 pounds.

b_2 is the number of feet per 1000 ft of wire, by which the length of the wire at t_0 degrees and loading w_0 exceeds the horizontal distance between the points of support (when these are at the same elevation).

* From lecture notes by Dr. H. Pender.

b_3 is the elongation, in feet per 1000 ft of wire, due solely to a change in temperature from t_0 to t degrees.

b is the elongation, in feet per 1000 ft of wire, due to a stress equal to wL(= its own weight plus the total ice and wind, if any, at the temperature t).

Table V(a). Values of y in Terms of x

$b_1 > (b_2 + b_3)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0013	0.0025	0.0036	0.0047	0.0057	0.0067	0.0076	0.0084	0.0092
0.1	0.0099	0.0106	0.0113	0.0119	0.0125	0.0130	0.0135	0.0140	0.0145	0.0150
0.2	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0178	0.0182	0.0185	0.0188
0.3	0.0191	0.0194	0.0197	0.0200	0.0203	0.0206	0.0209	0.0212	0.0215	0.0218
0.4	0.0221	0.0224	0.0227	0.0229	0.0231	0.0233	0.0235	0.0237	0.0239	0.0241
0.5	0.0243	0.0245	0.0247	0.0249	0.0251	0.0253	0.0255	0.0257	0.0259	0.0261
0.6	0.0263	0.0265	0.0267	0.0269	0.0271	0.0273	0.0275	0.0277	0.0279	0.0281
0.7	0.0283	0.0285	0.0287	0.0289	0.0291	0.0293	0.0295	0.0297	0.0299	0.0301
0.8	0.0303	0.0304	0.0305	0.0306	0.0307	0.0308	0.0309	0.0310	0.0311	0.0312
0.9	0.0313	0.0314	0.0315	0.0316	0.0317	0.0318	0.0319	0.0320	0.0321	0.0322

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.0327	0.0340	0.0352	0.0363	0.0373	0.0383	0.0392	0.0401	0.0410	0.0418
2	0.0426	0.0434	0.0442	0.0449	0.0456	0.0463	0.0470	0.0477	0.0484	0.0490
3	0.0496	0.0502	0.0508	0.0514	0.0520	0.0526	0.0532	0.0537	0.0542	0.0547
4	0.0552	0.0557	0.0562	0.0567	0.0572	0.0577	0.0582	0.0587	0.0591	0.0595
5	0.0599	0.0603	0.0607	0.0611	0.0615	0.0619	0.0623	0.0627	0.0631	0.0635
6	0.0639	0.0643	0.0647	0.0651	0.0655	0.0659	0.0663	0.0666	0.0669	0.0672
7	0.0675	0.0678	0.0681	0.0684	0.0687	0.0690	0.0693	0.0696	0.0699	0.0702
8	0.0705	0.0708	0.0711	0.0714	0.0717	0.0720	0.0723	0.0726	0.0729	0.0732
9	0.0735	0.0738	0.0741	0.0744	0.0747	0.0750	0.0753	0.0756	0.0759	0.0762

x	0	1	2	3	4	5	6	7	8	9
10	0.0765	0.0789	0.0812	0.0834	0.0855	0.0875	0.0894	0.0913	0.0931	0.0949

Table V(b). Values of y in Terms of x

$b_1 < (b_2 + b_3)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0194	0.0200	0.0205	0.0210	0.0215	0.0220	0.0224	0.0228	0.0232	0.0236
0.1	0.0239	0.0242	0.0245	0.0248	0.0252	0.0255	0.0258	0.0261	0.0263	0.0266
0.2	0.0269	0.0272	0.0274	0.0277	0.0279	0.0281	0.0283	0.0286	0.0288	0.0290
0.3	0.0292	0.0295	0.0297	0.0299	0.0300	0.0302	0.0304	0.0306	0.0308	0.0310
0.4	0.0312	0.0314	0.0316	0.0318	0.0320	0.0322	0.0323	0.0325	0.0327	0.0329
0.5	0.0330	0.0332	0.0333	0.0335	0.0337	0.0338	0.0340	0.0341	0.0343	0.0345
0.6	0.0346	0.0348	0.0349	0.0350	0.0352	0.0353	0.0354	0.0355	0.0358	0.0359
0.7	0.0360	0.0361	0.0362	0.0363	0.0365	0.0366	0.0368	0.0369	0.0370	0.0371
0.8	0.0372	0.0373	0.0374	0.0375	0.0377	0.0378	0.0379	0.0380	0.0381	0.0383
0.9	0.0384	0.0385	0.0386	0.0387	0.0388	0.0390	0.0319	0.0392	0.0393	0.0394

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.0395	0.0406	0.0416	0.0425	0.0434	0.0443	0.0451	0.0460	0.0467	0.0475
2	0.0482	0.0490	0.0497	0.0504	0.0510	0.0516	0.0522	0.0528	0.0534	0.0539
3	0.0544	0.0560	0.0566	0.0571	0.0577	0.0581	0.0586	0.0591	0.0596	0.0590
4	0.0595	0.0600	0.0604	0.0609	0.0613	0.0617	0.0621	0.0625	0.0629	0.0633
5	0.0637	0.0641	0.0645	0.0649	0.0653	0.0657	0.0660	0.0663	0.0667	0.0670
6	0.0673	0.0677	0.0681	0.0685	0.0689	0.0692	0.0695	0.0698	0.0701	0.0704
7	0.0708	0.0711	0.0714	0.0717	0.0720	0.0723	0.0726	0.0729	0.0732	0.0735
8	0.0738	0.0741	0.0743	0.0747	0.0749	0.0751	0.0754	0.0757	0.0760	0.0762
9	0.0765	0.0768	0.0771	0.0773	0.0776	0.0779	0.0781	0.0784	0.0787	0.0789

x	0	1	2	3	4	5	6	7	8	9
10	0.0792	0.0817	0.0840	0.0861	0.0881	0.0900	0.0919	0.0937	0.0954	0.0970

Example. To find the sag at which a No. 0 B. & S. stranded copper wire must be strung at 60 deg fahr on a 400-ft span so that the wire will have a factor of safety of 2 at 0 deg fahr when loaded with ice 0.5 in. thick all around the wire and with a wind pressure of 8 lb per sq ft of projected area. The breaking strength of the wire is 4980 lb, cross-section 0.083 sq in., modulus of elasticity 16×10^6, and coefficient of expansion 9.6×10^{-6}.

The data and calculations are then as follows:

$$F_0 = 2490 \qquad M = 16 \times 10^6$$
$$w = 0.323 \qquad L = 400$$
$$w_0 = 1.26 \qquad t - t_0 = 60$$
$$A = 0.083 \qquad a = 9.6 \times 10^{-6}$$

Then

$$b_1 = \frac{1000 \times 2490}{16 \times 10^6 \times 0.083} = 1.878$$

$$b_2 = \left(\frac{1.26 \times 400}{0.155 \times 2490}\right)^2 = 1.705$$

$$b_3 = 1000 \times 9.6 \times 10^{-6} \times 60 = 0.576$$

$$b = \frac{1000 \times 0.323 \times 400}{16 \times 10^6 \times 0.083} = 0.0975$$

Noting that b_1 is less than $(b_2 + b_3)$,

$$B = 1.705 + 0.576 - 1.878 = 0.403$$

$$x = \frac{0.0975}{0.403\sqrt{0.403}} = 0.382$$

and, from Table V(b), $y = 0.0308$. Whence

$$D = 0.0308 \times 400\sqrt{0.403} = 7.82 \text{ ft}$$

This is the sag at 60 deg fahr with no ice or wind.

The sag at 0 deg fahr with no ice or wind is found in exactly the same way, noting that for these conditions $b_3 = 0$, and b_1 is greater than $(b_2 + b_3)$, giving for B the value

$$1.878 - 1.705 = 0.173$$

and for x the value

$$\frac{0.0975}{0.173\sqrt{0.173}} = 1.355$$

and y from Table V(a) is then 0.0367, and

$$D = 0.0367 \times 400\sqrt{0.173} = 6.10 \text{ ft}$$

A fairly quick slide-rule method of solving the cubic equation in D is given here. Though not as accurate as the method outlined above, it may serve a useful purpose for many calculations. The combination of eqs. (20) and (21) may be put in the following form:

$$D^3 = D\frac{Q}{K} + \frac{wL^2}{8K} \tag{24}$$

where

$$\frac{Q}{K} = D_0^2 + \frac{L^2 a}{2.66}(t - t_0) - \frac{F_0}{K} \tag{25}$$

$$K = \frac{2.66\,MA}{L^2} \tag{26}$$

and are constant for a given set of conditions.

Having calculated $\frac{Q}{K}$ and $\frac{wL^2}{8K}$, set the index of scale C to a tentative value of D on the D scale of the slide rule. Calculate D^3 using the A and B scales, and $D\frac{Q}{K}$ using the C and D scales. Usually this can be done without changing the setting. Mental addition of the right-hand side of the equation will determine how closely the equation balances. Two or three trial values for D will result in a very close approximation. One or two more trials with accurate addition will determine a value of D which is accurate enough for many purposes.

Many graphical methods for sag and tension calculation have been developed. The Thomas method and the Pender and Thompson charts are covered in detail in the N.E.L.A. Overhead Systems Reference book, and the method developed by Theodore Varney for A.C.S.R. is given in an article published by the Aluminum Co. of America. A convenient set of tables for sag calculations based upon the catenary was presented by Mr. J. S. Martin

before the Engineers Society of Western Pennsylvania in 1922. These tables have been recently republished by the Copperweld Steel Co.

STRESS AND DEFLECTION IN SPANS OF UNEQUAL LENGTH. By stringing cables according to the stress determined for each particular length of span, the maximum allowable tension will be reached in all spans under maximum loading conditions. Under other loading conditions the tension will be unequal where span lengths are not the same. This is shown in Fig. 4, which is plotted for the same conditions as stated in the discussion of Fig. 3, but for spans of 600 to 900 ft in length. Fig. 5 shows the corresponding deflections.

The unequal stresses on the two sides of the insulator will tend to bend the tower or insulator pin, but any motion of the point of support will tend to equalize the stresses. When suspension insulators are used, the stresses are practically equalized, since the

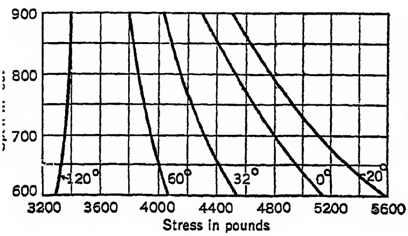

FIG. 4

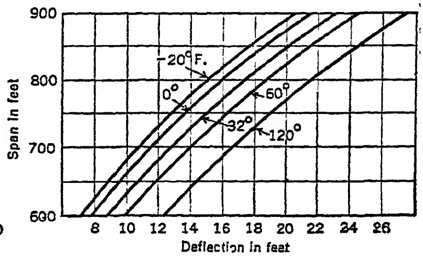

FIG. 5

insulator is free to move. It should be noted that the motion of the insulator necessary to equalize the stresses is small. When suspension insulators are used the cable is therefore strung at a tension corresponding to the ruling span. The ruling span is usually taken as the average span plus two-thirds of the difference between the average span and the maximum span.

To plot stringing curves for unequal spans under the ruling-span method it is necessary first to plot a temperature-stress curve for the ruling span by the method outlined above in the paragraph on Stress-Deflection-Temperature Charts. By the use of eq. (12), deflections corresponding to several span lengths at several stringing temperatures are computed, using stresses (tensions) taken from the stress-temperature chart for each temperature considered. From these values it is possible to plot a span-deflection curve for each temperature, which may be used as a stringing chart.

When the cable is thus strung the tension under minimum temperature and maximum loading will usually exceed the assumed tension in the shorter spans and be less than the assumed tension in the longer spans. Similarly, under maximum temperature conditions, the longer spans will have a sag in excess of the value calculated on the assumption of the maximum allowable tension being reached with maximum loading, and the shorter spans will have a sag less than the calculated value. The actual stresses and deflections can be calculated by the method given in the following paragraph.

CALCULATION OF STRESSES IN UNEQUALLY LOADED SPANS.* When suspension insulators are used, any tendency of the stresses in two adjacent spans to become unequal will produce such a deflection of the insulator, in the direction of the span with greater stress, as will establish equilibrium in the line. This state of affairs will occur (1) when adjacent spans carry unequal ice loads, (2) when the wire on one side of the insulator breaks, and (3) to a slight extent with changes in temperature when the adjacent spans are unequal in length, as noted above. The following method of calculating the "equilibrium" stresses in the wires and corresponding sags is applicable to all cases of initially unbalanced stresses, irrespective of their cause. The method may also be used to calculate the stress and sag in spans supported on pin insulators, provided the moment of bending of the pin and of the pole or tower is known or can be calculated.

Change of Stress Due to Change in Length of Span. When the length L of the span (i.e., distance between points of support) increases by λ inches, due to a horizontal displacement of the insulators (without slipping of the wire), the stress in the wire is increased by the same amount as would be produced by a fall in temperature of

$$t - t' = \frac{\lambda}{12\,aL} \quad \text{or} \quad t' = t - \frac{\lambda}{12\,aL} \tag{27}$$

* From lecture notes by Dr. H. Pender.

degrees fahrenheit, where t is the actual temperature and t' may be called the "equivalent" temperature corresponding to the change in length λ. In this equation λ is the actual increase in the distance between the points of support in inches, a the temperature coefficient of linear expansion per degree fahrenheit, and L the original length of the span in feet. For example, in an 800-ft span of copper wire an increase of 1 in. in L corresponds to a drop of temperature of $1 \div (12 \times 9.6 \times 10^{-6} \times 800) = 10.85$ deg, and an increase in length of λ inches corresponds to a drop of temperature of $t - t' = 10.85\,\lambda$ deg.

Hence the stress-temperature chart for any given length of span, see Fig. 3, may be used directly to determine the stress in the wire after any change in the length of the span, due to the deflection of the insulator. For example, consider an 800-ft span of 300,000-cir-mil bare copper conductor, at 32 deg fahr, without ice or wind, initially stressed to 4170 lb and let the length of the span be increased 4 in. as the result of the deflection of the insulators by this amount. This increase in length of span will then give the same stress in the wire as would be produced if the temperature fell from 32 deg to $t' = 32 - 4 \times 10.85 = -11.4$ deg. The point on the stress-temperature curve (Fig. 3) corresponding to a temperature of -11.4 deg gives the new stress, viz., 4700 lb.

Horizontal Pull of Insulator. Referring to Fig. 6, let m = the horizontal distance in inches (measured along the span) which any insulator is deflected from the vertical, taken positive when to the right, say, and negative when to the left. Let x = the length of the insulator string in inches, i.e., distance from point of attachment to tower to point of attachment to wire; V = total weight of wire and ice between the lowest point of the wire in the span to the left of the insulator and the lowest point of the wire in the span to the right of the insulator plus one-half the weight of the insulator; H = total wind pressure on the length of wire between the middle points of the two adjacent spans, plus half the wind pressure on the insulator; and put $W = \sqrt{V^2 + H^2}$. Then the horizontal component of the pull of the insulator toward the left along the line of the span is *

$$P = \frac{m}{\sqrt{x^2 - m^2}} \cdot W \tag{28}$$

For example, consider two adjacent spans of 300,000-cir-mil copper, each 800 ft long and with points of support at the same elevation. Let the span to the left be free of ice and let the one to the right have a 1/4-in. ice coating; assume the insulator to be 60 in. long and to weigh 100 lb. Then for no wind $H = 0$, $V = 1.19 \times 400 + 0.915 \times 400 + 100/2 = 892$ lb. Whence for deflections of the insulator of less than 12 in. the horizontal pull of the insulator is $P = (892 \times m) \div 60 = 14.9m$, or 14.9 lb per in. deflection.

STRESSES IN A SERIES OF SPANS WHEN POINTS OF SUPPORT ARE NOT FIXED. Referring to Fig. 6, let the left-hand end of span 1 be anchored, and assume the insulator at the right-hand end to be deflected a horizontal distance of m_1 inches, due, for example, to a change in the loading on the succeeding spans (or to a change in temperature when the spans are of unequal length). From eq. (27) calculate the "equivalent" temperature t'_1 corresponding to this change in length, and from a stress-temperature curve, corresponding to the assumed loading w_1 of this span find the stress on this curve corresponding to a temperature of t'_1 degrees; call this stress F'_1.

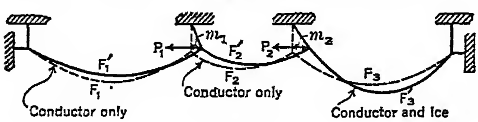

FIG. 6

Next calculate the transverse and vertical loads on the insulator, viz., H_1 and V_1, and the resultant load $W_1 = \sqrt{V_1^2 + H_1^2}$, as explained above. Then from eq. (28) or (29) calculate the horizontal pull P_1 of the insulator. The stress in the second span, assuming the value of m_1 chosen at the start is correct, must then be

$$F'_2 = F'_1 + P_1 \tag{30}$$

t'_2 is then determined from F'_2 on the stress-temperature curve. From eq. (27) the corresponding increase in the length of span 2 must then be

$$\lambda_2 = 12\,aL_2(t - t'_2) \tag{31}$$

where L_2 is the length of the second span. The corresponding deflection of the insulator at the right-hand end of span 2 must then be

$$m_2 = m_1 + \lambda_2 \tag{32}$$

always taking the insulator deflection positive when to the right, say.

* When m is less than 20 percent of x this may be written, with an error of less than 2 percent,

$$P = \frac{m}{x} W \tag{29}$$

Using the values of λ_2 and m_2 thus found, calculate the λ_3 and m_3 in exactly the same manner as λ_2 and m_2 were calculated, and similarly for the succeeding spans until the next anchor tower is reached. For the anchor tower at the right-hand end of the nth span, say, the deflection of the insulator must be zero, viz.,

$$m_n = 0 \tag{33}$$

If m_n as calculated comes out greater than zero, then the assumed value of m_1 is too great; if m_n comes out less than zero the assumed value of m_1 is too small. By calculating m_n for two or three assumed values of m_1, and plotting m_n as ordinates against m_1 as abscissas, the correct value of m_1 will be where this curve crosses the axis of abscissas. Using this correct value of m_1, the stresses and deflections in each span may then be accurately calculated by the process just given. The complete process is best shown by an example.

Example. Consider three spans between anchor towers (Fig. 6), all of the same length, 800 ft, and all supports at the same elevation, 300,000-cir-mil copper being used for the conductor. Let the temperature be 32 deg fahr, and let the middle span have a $^1/_4$-in. ice coating but the other two spans have no ice on them; also assume no wind. The stress-temperature chart given in Fig. 3 then applies directly, provided the wires are strung in accordance therewith. Assume that each insulator weighs 100 lb and has a length of 60 in. Then for an increase of λ inches in the length of any span, the "equivalent" temperature is, from eq. (27),

$$t' = 32 - 10.85\lambda$$

or if the equivalent temperature rise t' is known

$$\lambda = 0.092(32 - t')$$

The horizontal pull of any insulator for a deflection of m inches (small compared with the length of the insulator) is, from eq. (29),

$$P = 14.9\,m$$

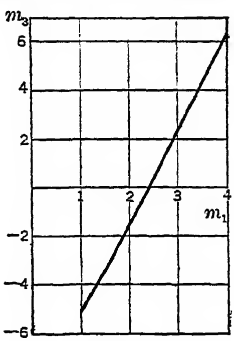

Fig. 7

In the following table are given the calculations for assumed values of m_1 of 1, 2, 3, and 4 in., and in Fig. 7 are plotted the corresponding calculated values of m_3 against m_1. It is seen that the relation between m_3 and m_1 is practically a straight line cutting the horizontal axis at $m_1 = 2.4$, which is therefore the correct value of m_1. The calculations for $m_1 = 2.4$ in. are given in the last column of the table. Hence the stresses and deflections in the two end spans (without ice) are $F'_1 = F'_3 = 4460$ lb and $D'_1 = D'_3 = 16.3$ ft respectively, and the stress and deflection in the middle span loaded with $^1/_4$ in. of ice are $F'_2 = 4496$ lb and $D'_2 = 21.1$ ft respectively.

These calculations will be facilitated, if many are to be made, by plotting stress-temperature curves for each loading assumed. The equivalent temperatures corresponding to calculated stresses are then taken directly from the curves. If only a few calculations are necessary eqs. (20) and (21) may be employed.

m_1	= assumed	1	2	3	4	2.4
t'_1	$= 32 - 10.85\,m_1$	21.1	10.3	−0.6	−11.4	6
F'_1	From Fig. 3 ($w = 0.915$)	4300	4410	4550	4700	4460
P_1	$= 14.9\,m_1$	15	30	45	60	36
F'_2	$= F'_1 + P_1$	4315	4440	4595	4760	4496
t'_2	($w = 1.19$)	105	90	74	55	84
λ_2	$= 0.092(32 - t'_2)$	−6.72	−5.33	−3.86	−2.12	−4.78
m_2	$= m_1 + \lambda_2$	−5.72	−3.33	−0.86	1.88	−2.38
P_2	$= 14.9\,m_2$	−85	−50	−13	28	−36
F'_3	$= F'_2 + P_2$	4230	4390	4582	4788	4460
t'_3	($w = 0.915$)	26	13	−3	−16	6
λ_3	$= 0.092(32 - t'_3)$	0.55	1.75	3.22	4.42	2.39
m_3	$= m_2 + \lambda_3$	−5.17	−1.58	2.36	6.30	0.01

Location of Towers and Determination of Clearances

The height of towers is determined so as to give some specified minimum clearance from conductor to ground for some length of span chosen as a nominal standard on the basis of level ground. In practice the ground is rarely level, and the towers are actually located to conform to the irregularities of the ground. In locating towers of a given

height the spans are made as long as possible consistent with maintaining the ground clearance. The irregularities of the ground are ordinarily advantageous and permit of slightly longer spans on the average than could be obtained with the same height of towers on level ground.

Profile and Plan of Right-of-way. In order to locate towers properly it is necessary to have a profile of the right-of-way. Profiles are conveniently plotted on standard ruled profile section paper to a vertical scale of 20 ft to the inch and a horizontal scale of 200 ft to the inch. Three profiles are desirable, one along the center of the tower line and one

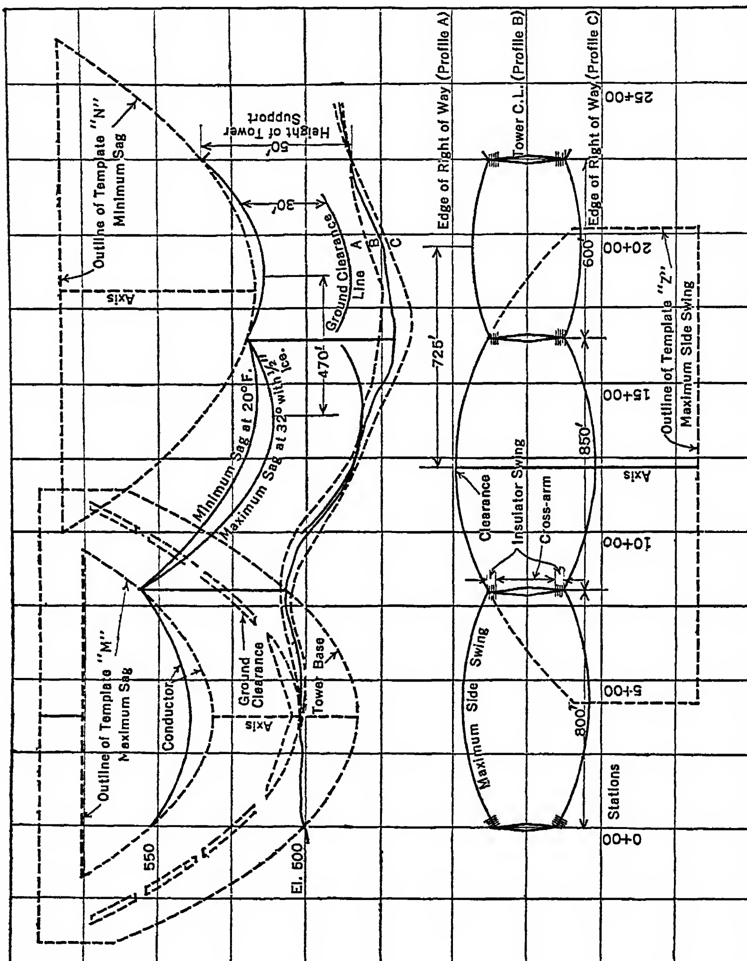

Fig. 8

on each side, say at each edge of right-of-way, as shown on Fig. 8 at *A*, *B*, and *C*. The two side profiles indicate the amount and direction of the slope of the ground across the line which must be allowed for in determining ground clearance and foundation or tower extensions.

A plan of the right-of-way is of course also necessary for determining the construction at angles in the line, and the clearances from the conductor to the edge of the right-of-way when the conductor is deflected horizontally by the wind. Such a plan is shown at the bottom of Fig. 8.

TEMPLATES FOR LOCATING TOWERS. Three templates are required, one for ground clearance with maximum sag, marked M in Fig. 8, one for uplift at times of minimum sag, marked N, and one for maximum side swing, marked Z. These are cut from thin celluloid and are to the same horizontal and vertical scales as used for the profile and plan of the right-of-way.

Since the curvature of the catenary or parabola in which the wire hangs depends only on the tension and loading and not on the length of the span or on the difference in elevation of the points of support, all spans having the same tension and loading can be drawn (for any one predetermined scale) from a single template, irrespective of their lengths or of the differences in elevation of the points of support. However, when the elevations of the points of support are not the same, the lowest point of the curve is shifted from the middle of the span toward the lower support, but the axis of the curve remains vertical.

Construction of Maximum Sag Template M. The maximum sag is found from the deflection-temperature curve, Fig. 3, and may be the deflection corresponding to the maximum temperature, and conductor only, e.g., 21.4 ft in Fig. 3, or may be the deflection corresponding to 32 deg fahr and the maximum ice loading. Wind will increase this deflection but will not increase the *vertical sag*. Call S_m this maximum sag. Then the equation of the maximum sag template, or template M, is

$$y = \left(\frac{4\,S_m}{L^2}\right) x^2 \tag{34}$$

where L is the length of the particular span for which the maximum sag is S_m, and the origin of the curve is its lowest point and the axis of y is vertical; all dimensions are in feet. Three parabolic curves are given by this template; the top curve represents the position of the cable and is drawn on the basis of the average span under the maximum load; the middle curve is a similar parabola below the upper curve a distance equal to the minimum allowable clearance to ground; the bottom curve is another similar parabola below the upper curve by a distance equal to the height of cable above ground at the support.

Instead of solving eqs. (34), (35), and (36), use may be made of the multipliers in Table VI which gives the percentage of the sag at the center of the span for various percentages of the span. Level supports are assumed. If supports are not level, use a value for span length equal to twice the horizontal distance from the upper support to the point of maximum sag.

Table VI

Percentage of Span Length	Percentage of Sag at Center of Span	Percentage of Span Length	Percentage of Sag at Center of Span
5 or 95	19.5	30 or 70	84.4
10 — 90	36.5	35 — 65	91.3
15 — 85	51.0	40 — 60	96.0
20 — 80	64.0	45 — 55	98.9
25 — 75	75.0	50 — 50	100.0

Construction of Minimum Sag Template N. The minimum sag is also found from the temperature-deflection chart, and is usually the deflection corresponding to the minimum temperature and conductor only, e.g., 15.1 ft in Fig. 3. Call this sag S_n; then the equation of the minimum sag template, or template N, is

$$y = \left(\frac{4\,S_n}{L^2}\right) x^2 \tag{35}$$

Construction of Maximum Side-swing Template Z. The maximum side swing occurs at time of maximum wind pressure and may be at maximum temperature or may be at 32 deg fahr when covered with ice. In the latter case the side swing depends on the shape (circular or elliptical) of the ice covering and its specific gravity. For a circular covering of solid ice one particular thickness (usually but not necessarily the maximum thickness) gives the greatest side swing. Calling the maximum side swing Z_m, then the equation of the side swing template, or template Z, is

$$y = \left(\frac{4\,Z_m}{L^2}\right) x^2 \tag{36}$$

LOCATING TOWERS BY MEANS OF TEMPLATE M. Choose a starting point, as shown for example in Fig. 8, at station 0 + 00, elevation 500.0 ft for the first tower location. The template M is then placed over the profile and shifted until its axis is vertical and the lower curve is at station 0 + 00, elevation 500.0 ft, and the middle curve is tangent to the ground profile as shown. The proper location for the second tower is at the point where the lower curve again intersects the ground profile, or at station 8 + 00, eleva-

tion 506.0 ft, in the example. The operation is then repeated for the next tower. Adjustments in length of span are usually necessary to meet local conditions, in order to avoid locating towers in roads or swamps and to bring towers at angle points. Adjustments which increase the ground clearance are of course allowable.

The position of the conductors with maximum sag may be drawn on the profile from the top curve of the template.

UPLIFT ON INSULATOR; USE OF TEMPLATE *N*. An insulator sustains the weight of the lengths of conductor from the insulator to the lowest point of the span on each side. If the conductor leaves the insulator horizontally on one side, the lowest point of that span is at the insulator, which then sustains no weight due to that span. If the conductor has an upward inclination where it leaves an insulator, it is exerting an uplift equal to the weight of a length of conductor extending from the insulator along the span produced in the reverse direction to the lowest point of the parabola. Where the conductor has a downward inclination on one side and upward on the other side of the insulator, there will be a weight or uplift on the insulator equal to the difference between the weight of conductor on one side and uplift on the other.

Suspension insulators when used hanging downward to sustain weight are incapable of resisting uplift. Where uplift occurs the conductor may be dead ended or may be be tied down or weighted down.

The method used for locating towers ordinarily precludes uplift under the loading which gives maximum sag, but uplift may occur when the loading is less. To determine this the minimum sag is drawn on the profile with template *N*. The minimum sag curve is drawn between points of support, keeping the axis of the parabola vertical as before.

SIDE SWING OF SUSPENSION INSULATORS. Let l_1 = the length of conductor between the lowest point in the span to the left of the insulator and the lowest point in the span to the right of the insulator, and let l_2 = the distance between the middle points of these two spans, both in feet. Also let v = the weight of the conductor and ice per foot length, and h = the wind pressure per foot length (see Table IV). Then the vertical pull on the insulator is vl_1 and the transverse horizontal force is hl_2. Also let v_1 = the weight of the insulator and h_1 = the total wind pressure on it. Then the insulator is deflected sidewise from the vertical by approximately the angle

$$\theta = \tan^{-1}\left[\frac{hl_2 + 0.5\,h_1}{vl_1 + 0.5\,v_1}\right] \tag{37}$$

Usually the weight of the insulator and the wind pressure on it are negligible compared with the weight and wind pressure on the conductor, in which case

$$\theta = \tan^{-1}\left(\frac{hl_2}{vl_1}\right) \tag{37a}$$

When the points of support are at the same elevation $l_1 = l_2$ and

$$\theta = \tan^{-1}\frac{h}{v} \tag{37b}$$

Calling X the length of the insulator in feet, then the transverse horizontal deflection of the insulator is $X \sin\theta$ ft.

For example, consider the side swing of the third insulator (from the left) in Fig. 8. Then $l_1 = 470$, $l_2 = 850/2 + 600/2 = 725$, $v = 1.61$ (for 300,000-cir-mil conductor with $^1/_2$ in. of ice), and $h = 0.82$ (for wind pressure of 6 lb per sq ft). Whence, neglecting the weight of the insulator and the wind pressure on it,

$$\theta = \tan^{-1}\frac{0.82 \times 725}{1.61 \times 470} = 38°$$

If the insulator is 5 ft long, the transverse horizontal deflection is then $5 \sin 38° = 3.1$ ft.

If the angle of swing as thus determined is excessive the cables and insulators will be lifted up into the cross arms at times of low temperatures and high winds. The remedy is the same as in case of direct uplift.

SIDE CLEARANCE; USE OF TEMPLATE *Z*. Where a right-of-way of definite width is obtained it is necessary to determine whether the conductor will swing beyond the edge of the right-of-way. Therefore after the towers have been located by the use of the profile they should be marked on the plan and the side swing marked in from template *Z*, as shown in Fig. 8. In determining side swing, the swing of the insulator (if suspension type) must be allowed for, as well as the side swing of the conductor. Adequate margin should be allowed between the extreme position of conductor and the edge of the right-of-way, so that a safe clearance will be preserved from any structures

erected adjacent thereto. Where extraordinarily long spans must be used, an adequate extra width of right-of-way should be obtained in the first place.

LOSS OF CLEARANCE BETWEEN CONDUCTORS DUE TO UNEQUAL ICE LOADING. Where one span is loaded with ice and the one immediately below it is not, the clearance is reduced. This condition may sometimes arise due to the ice falling off the lower wire before it falls off the upper wire. Where the wires are directly over each other the normal clearance must be great enough to prevent the crossing of wires under these conditions. For ice loading without wind, clearance under unequal loading is most easily obtained by offsetting the wires horizontally for the required clearance instead of increasing the vertical clearance. However, to prevent crossing of the unequally loaded wires when deflected by wind pressure, this horizontal offset must be considerable, as the clearance must then be obtained between the wires in their inclined positions.

If the conductors to which Fig. 3 refer are normally 10 ft apart vertically on an 800-ft span, the sag would be 17.5 ft without ice at 32 deg fahr, 19.0 ft with $1/4$ in. ice, and 20.9 ft with $1/2$ in. ice at 32 deg fahr. Consequently, if two cables are used one above the other, and ice should form on the upper but not on the lower, then, assuming fixed points of support (pin insulators), the clearance would be reduced by 3.4 ft for $1/2$ in. ice, and 1.5 ft for $1/4$ in. ice, making the clearances 6.6 ft and 8.5 ft respectively, instead of 10 ft.

Where suspension insulators are used the reduction of clearance from unequal ice loading is greater. If one span is loaded with ice and the adjacent spans of the same wire are not loaded, the sag of the loaded span will be increased because the insulators will swing toward that span. Similarly if one span is unloaded and adjacent spans are loaded, the sag will be decreased. The minimum clearance occurs where only one span of the upper wire is loaded and is immediately over the only unloaded span of the lower wire. The actual reduction is readily calculated by the method given above in the paragraph on Calculation of Stresses in Unequally Loaded Spans. The amount of reduction depends on the number of spans between anchor towers, and the distance from the anchor towers at which the unbalanced loading occurs. For a 300,000-cir-mil copper cable on 800-ft spans at 32 deg fahr with unequal loadings on sections of one, two, three, and five spans (see Fig. 9) the assumed conditions of unequal loading and the loss of clearance are as follows:

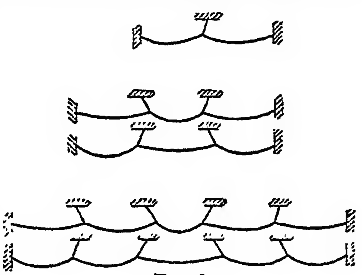

Fig. 9

Number of Spans between Anchor Towers	1	2	3	5
Upper conductor; $1/4$ in. ice on spans Nos.:*	1	1	2	3
Lower conductor; $1/4$ in. ice on spans Nos.:*	....	2	1, 3	1, 2, 4, 5
Sag of middle span of upper conductor, feet	19.0	20.5	21.1	21.6
Sag of middle span of lower conductor, feet	17.5	15.9	15.5	15.2
Loss of clearance in middle span, feet	1.5	4.6	5.6	6.4

* The other spans assumed to have no ice load.

STRESSES AND DEFLECTIONS DUE TO BROKEN CONDUCTORS. When a conductor breaks in a span supported by suspension insulators, the insulators adjacent to the broken span swing up into line with the cable, throwing increased slack into the unbroken part of the cable equal to the length of the insulator. This slack divides between the unbroken spans, increasing the deflection of each. The stresses and deflections of the unbroken spans may be determined by the method given above for Calculation of Stresses in Unequally Loaded Spans, calling the span in which the break occurs span 1.

Joint Use of Poles with Other Utilities

When supporting structures of power lines are used jointly by other utilities, such as telephone or other communication systems, additional factors are introduced into the problem of line design beside those needing consideration in the case of power lines alone. Often a higher grade of construction is necessary, and attention must be given to the required separations between the conductors and equipment of the two utilities. When the poles are used jointly by power and communication utilities the separations required are generally greater than necessary between attachments of the same or different

power utilities. Usually contracts are entered into between the joint-use parties which set forth in detail the construction specifications to be followed.

Right-of-way

The right-of-way should be as short and straight as practicable. Its width will depend largely on the type of line to be constructed. A width of 50 ft is generally considered adequate for a single pole line with moderately short spans. For a double circuit tower line even with fairly long spans a width of 100 ft is generally adequate. With exceptionally long spans and extremely high voltages widths of as much as 200 ft may be necessary. In the case where two parallel pole or tower lines are to be constructed on the same right-of-way the distance between the center lines of the two lines should be added to the width of right-of-way required for a single line. The towers for the two lines should be placed directly opposite each other in order to reduce the width of right-of-way required.

The direction of the line will often need to be shifted to avoid buildings or other obstructions. In such a case consideration should be given to the maximum side swing of the conductors in the wind, and adequate clearance should be provided between the conductors and such obstructions under the worst condition of side swing. In computing the side swing, allowance should be made for swing of suspension insulators.

When it is necessary that the right-of-way cross railroads, roads, or other lines the length of crossing should be reduced to a minimum by making the crossing at as near right angles as practicable. Rights-of-way through swamps often require expensive road building and expensive tower foundations, though small swamps (up to about 1000 ft across) can often be crosssed in a single span. Steep side hills require extra expense for foundations and tower extensions, and introduce a hazard of injury to tower from sliding earth, rocks, trees, or snow. A right-of-way through forests requires expensive clearing.

It is usually advantageous to own and fence the right-of-way. However, when the right-of-way passes through farm lands, it is sometimes advantageous not to fence it in, but to have it cultivated and kept free from brush. Instead of purchasing a right-of-way, it is often sufficient to obtain easements covering the location of towers and suspension of wires. Easements and right-of-way agreements should include the right to remove and trim trees under and adjacent to the line. The right-of-way should be passable (or at least accessible) for patrolling as well as for construction.

The first step in selecting a route for a line is to lay out the possible routes on accurate maps. Topographic maps are of considerable assistance in this work. Rough reconnaissance surveys are made to determine the most suitable route. Aerial photographic surveys are very valuable especially in mountainous country. The next step is to obtain the right-of-way either by outright purchase or by obtaining easements. This is followed by sending out a party to make an accurate survey, including plans and profiles showing buildings, obstructions, natural features and property lines. (See Location of Towers.)

25. WOOD POLES FOR OVERHEAD LINES

Basis for Selection of Poles

The length of pole required for a particular location is determined by the following factors:

1. Amount of vertical space required for wires and equipment.
2. Clearance required above ground or obstructions for wires and equipment.
3. Sag of conductors.
4. Depth pole is to be set in the ground.

The size (class) of pole is determined by the strength required to sustain the mechanical loading imposed upon it. The strength of an unguyed pole is usually determined by its circumference at the ground line. This dimension determines the resisting moment of the pole when bending as a cantilever. For a guyed pole the resisting moment at the point of guy attachment must be sufficient to withstand the bending stresses imposed at that point. The top of the pole must also be of adequate diameter to permit the attachment of cross-arms without unduly weakening the pole near the top. Four and one-half inches is about the minimum top diameter suitable for cross-arm construction.

The basis for the selection of kind and quality of pole timber is principally economic. Long life is very desirable provided that it can be obtained at a low annual cost and provided that the line is expected to be more or less permanent. Long life for poles is obtained by the use of durable wood species and usually by preservative treatment methods applied to both durable or non-durable species. In order to obtain the quality of pole timber desired purchase specifications are required, and the specifications should be enforced by careful inspection of the poles before, during, and after the preservative treatment.

Methods of Specifying Pole Dimensions

A wood pole is usually specified by its total or "nominal" length and by its class. The word class refers to the dimensional classifications set up by the specifications of the American Standards Association. In those specifications classes numbered from 1 to 10 inclusive are provided for poles of the wood species principally used. Classes 1 to 7 inclusive specify the minimum top circumference for each class and the minimum circumference at 6 ft from the butt end for each nominal length in each class. Class 1 provides the largest ground line circumference, and Class 7 the smallest. Classes 8 to 10 inclusive specify minimum top circumferences only. Table VII shows dimensions for three wood species.

The dimensions specified for each class of Classes 1 to 7 inclusive are so selected that for any one wood species, any pole of a certain class, regardless of the length, will support the same conductors located in the same manner relative to the top of the pole (including wind and ice loading on conductors and pole) with the same fiber stress (within close limits) in the pole. This permits the same class pole to be used throughout a line if the size and arrangement of the conductors do not change, but a different class may be necessary on account of changes in grade of construction, angles, etc. Poles of different species but of the same class will support the same mechanical loading. The following tables are the A.S.A. standards for classes of poles.

Taper of Poles. The taper of various kinds of poles, specified as the difference, measured in inches, between two circumferences 10 feet apart, is given as follows in *Forest Service Bull.* 84: chestnut (Maryland), 3.8 to 4.0; northern white cedar (Michigan), 5.2; western yellow pine (California), 4.0; lodgepole pine (Montana), 3.0; loblolly pine (Texas), 2.4; western red cedar (Washington), 3.5. Trees grown upon a high elevation have a greater taper in the trunk than trees grown lower down.

Timber, to be desirable for use as poles, should have the following qualities: straightness, small taper, low weight, few knots, good resistance to decay, lack of serious defects, high strength, and softness enough to permit the spikes of a climber to enter readily. No pole timber is perfectly resistant to decay, but any timber can have its life greatly prolonged by preservative treatment. Treatment methods are discussed later in this article.

UNCERTAINTIES IN NAMES OF TIMBER TREES. The terms cedar, pine, etc., used in describing poles and cross-arms, and even the apparently more exact terms, such as white cedar, yellow pine, etc., each cover several kinds of trees and have different meanings in different localities.

At least eight pines (of the thirty-five native ones) are in the market, some of which so closely resemble each other in their minute structure that they can hardly be told apart; and yet they differ in quality and should be used separately, although they are often mixed or confounded in the trade.

Forestry Bulletin 10 states: "'Yellow pine,' is applied in the trade to all the southern lumber pines; in the Northeast it is also applied to the pitch pine; in the West it refers mostly to bull pine. 'Yellow longleaf pine,' 'Georgia pine,' chiefly used in advertisement, refers to longleaf pine."

Timbers Ordinarily Used for Poles and Cross-arms

The principal timber trees used for poles and cross-arms are briefly described below in accordance with the names used in the trade.

Chestnut grows throughout the Appalachian Mountain region. A blight in recent years has reduced the supply of satisfactory chestnut timber to an almost negligible amount. The sapwood is very thin, usually from about $1/8$ to $3/8$ in. in thickness. It is not so straight as cedar and is likely to be knotty. It is slightly stronger and heavier than cedar. At the present time it is mostly used for poles in the locality where it is cut.

Northern White Cedar has its principal source of supply in the region of the Great Lakes. It is used mainly in the northeast quarter of the United States for small poles. Its large taper and spreading butts make it undesirable for long poles. The sapwood varies from $1/2$ to 1 in. in thickness and is usually thicker at the top than at the butt. Frequently, the butts are decayed at the center. It is very slow in growth, requiring about 190 years for a 30-ft pole.

Southern White Cedar grows mainly in the southern swamps and is somewhat less durable than the northern cedar. It is used very little for poles as its sapwood decays very quickly.

Red Cedar is a small to medium-sized tree scattered through the forests, or, in the West, sparsely covering extensive areas (cedar brakes). The red cedar is the most widely distributed conifer of the United States, occurring from the Atlantic to the Pacific and

Table VII. American Standards Association Standards for Dimensions of Wood Poles

Creosoted Southern Yellow Pine Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Distance from Butt (Feet)	Minimum Circumference at 6 ft. from Butt (Inches)							No Butt Requirement	No Butt Requirement	No Butt Requirement
16	3 1/2					21.5	19.5	18.0			
18	3 1/2			26.5	24.5	22.5	21.0	19.0			
20	4	31.5	29.5	27.5	25.5	23.5	22.0	20.0			
22	4	33.0	31.0	29.0	26.5	24.5	23.0	21.0			
25	5	34.5	32.5	30.0	28.0	26.0	24.0	22.0			
30	5 1/2	37.5	35.0	32.5	30.0	28.0	26.0	24.0			
35	6	40.0	37.5	35.0	32.0	30.0	27.5	25.5			
40	6	42.0	39.5	37.0	34.0	31.5	29.0	27.0			
45	6 1/2	44.0	41.5	38.5	36.0	33.0	30.5	28.5			
50	7	46.0	43.0	40.0	37.5	34.5	32.0	29.5			
55	7 1/2	47.5	44.5	41.5	39.0	36.0	33.5				
60	8	49.5	46.0	43.0	40.0	37.0	34.5				
65	8 1/2	51.0	47.5	44.5	41.5	38.5					
70	9	52.5	49.0	46.0	42.5	39.5					
75	9 1/2	54.0	50.5	47.0	44.0						
80	10	55.0	51.5	48.5	45.0						
85	10 1/2	56.5	53.0	49.5							
90	11	57.5	54.0	50.5							

Chestnut Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Distance from Butt (Feet)	Minimum Circumference at 6 ft. from Butt (Inches)							No Butt Requirement	No Butt Requirement	No Butt Requirement
16	3 1/2					22.5	21.0	19.5			
18	3 1/2			28.0	26.0	24.0	22.0	20.5			
20	4	33.5	31.5	29.5	27.0	25.0	23.0	21.5			
22	4	35.0	33.0	30.5	28.5	26.5	24.5	22.5			
25	5	37.0	34.5	32.5	30.0	28.0	25.5	24.0			
30	5 1/2	40.0	37.5	35.0	32.5	30.0	28.0	26.0			
35	6	42.5	40.0	37.5	34.5	32.0	30.0	27.5			
40	6	45.0	42.5	39.5	36.5	34.0	31.5	29.5			
45	6 1/2	47.5	44.5	41.5	38.5	36.0	33.0	31.0			
50	7	49.5	46.5	43.5	40.0	37.5	34.5	32.0			
55	7 1/2	51.5	48.5	45.0	42.0	39.0	36.0				
60	8	53.5	50.0	46.5	43.5						
65	8 1/2	55.0	51.5	48.0	45.0						
70	9	56.5	53.0								

Western Red Cedar Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Distance from Butt (Feet)	Minimum Circumference at 6 ft. from Butt (Inches)							No Butt Requirement	No Butt Requirement	No Butt Requirement
16	3 1/2					23.0	21.5	19.5			
18	3 1/2			28.5	26.5	24.5	22.5	21.0			
20	4	34.5	32.0	30.0	28.0	25.5	23.5	22.0			
22	4	36.0	33.5	31.5	29.0	27.0	25.0	23.0			
25	5	38.0	35.5	33.0	30.5	28.5	26.0	24.5			
30	5 1/2	41.0	38.5	35.5	33.0	30.5	28.5	26.5			
35	6	43.5	41.0	38.0	35.5	32.5	30.5	28.0			
40	6	46.0	43.5	40.5	37.5	34.5	32.0				
45	6 1/2	48.5	45.5	42.5	39.5	36.5					
50	7	50.5	47.5	44.5	41.0	38.0					
55	7 1/2	52.5	49.5	46.0	42.5	39.5					
60	8	54.5	51.0	47.5	44.0						
65	8 1/2	56.0	52.5	49.0	45.5						
70	9	57.5	54.0	50.5	47.0						
75	9 1/2	59.5	55.5	52.0	48.5						
80	10	61.0	57.0	53.5	49.5						
85	10 1/2	62.5	58.5	54.5							
90	11	63.5	60.0	56.0							

Table VII. American Standards Association Standards for Dimensions of Wood Poles—*Continued*

Northern White Cedar Poles

Class		1	2	3	4	5	6	7	8	9	10
Min. Top Circ. (Inches)		27	25	23	21	19	17	15	18	15	12
Length of Pole (Feet)	Ground Line Distance from Butt (Feet)	Minimum Circumference at 6 ft. from Butt (Inches)									
16	3 1/2					26.0	24.0	22.0	No Butt Requirement	No Butt Requirement	No Butt Requirement
18	3 1/2			32.5	30.0	28.0	25.5	23.5			
20	4	39.5	37.0	34.0	31.5	29.0	27.0	25.0			
22	4	41.0	38.5	36.0	33.0	30.5	28.0	26.0			
25	5	43.5	41.0	38.0	35.5	32.5	30.0	28.0			
30	5 1/2	47.5	44.5	41.5	38.5	35.5	32.0	30.5			
35	6	50.5	47.5	44.0	41.0	38.0	35.0	32.5			
40	6	53.5	50.0	46.5	43.5	40.0	37.0				
45	6 1/2	56.0	52.5	49.0	45.5	42.0					
50	7	58.5	55.0	51.5	47.5	44.0					
55	7 1/2	61.0	57.5	53.5	49.5	46.0					
60	8	63.5	59.5	55.5	51.5						

from Florida to Minnesota, but attains a suitable size for lumber only in the southern and more especially in the Gulf States and is seldom used for poles.

The term juniper is commonly used by telephone men for southern white cedar; the term also is applied to red cedar. Juniper poles come from Virginia, the Carolinas, and other South Atlantic States.

Western Red Cedar is light, straight, and durable, and is one of the principal pole timbers used at present. The main sources of supply are in northern Idaho, western Washington, and British Columbia. Its durability is greatly increased by treating the butts to a point about 1 ft above the ground line with creosote. (See Preservative Treatments.) It does not have as great strength as southern pine, and it does not stand impact very well. The sapwood is somewhat subject to dry rot in certain localities.

Cypress is a large deciduous tree, occupying much of the swamp and overflow land along the coast and rivers of the southern states. Cypress is usually considered a durable wood, and the heartwood is, in fact, one of the most durable of our native species. The sapwood, however, decays quickly, and this seriously weakens the pole. The width of the sapwood on pole-size trees is from 3/4 to 1 1/4 in. Cypress frequently is too large for use as a pole and has greater value for lumber. Even when its general diameter is small enough the butt will often be so big that it adds too much weight.

Southern Yellow Pine is a trade term covering the longleaf, shortleaf, loblolly, slash, and pond pines. These species are strong, straight, and symmetrical, but are subject to quick decay in the form of poles unless treated with preservative throughout their length. There is a plentiful supply of these pines, and they easily reproduce, with rapid growth.

Longleaf Pine is a large tree which forms extensive forests and furnishes the hardest and strongest pine lumber in the market. Coast region from North Carolina to Texas. The longleaf pine is strikingly heavy, hard, and resinous, and usually very regular and narrow ringed, showing little sapwood, and differing in this respect from the shortleaf pine and loblolly pine, which usually have wider rings and more sapwood, the latter excelling in that respect.

Shortleaf Pine resembles loblolly pine; often approaches in its wood the Norway pine. The common lumber pine of Missouri and Arkansas, North Carolina to Texas and Missouri.

Loblolly Pine is a large tree; it forms extensive forests; wider-ringed, coarser, lighter, softer, with more sapwood than the longleaf pine, but the two often confounded. This is the common lumber pine from Virginia to South Carolina and is found extensively in the southern states, from Virginia to Texas.

Norway Pine is a large tree; it never forms forests; usually scattered or in small groves, together with white pine; largely sapwood and hence not durable. Minnesota to Michigan; also in New England to Pennsylvania. The Norway pine, which may be confounded with the shortleaf pine, can be distinguished by being much lighter and softer. It may also, but more rarely, be confounded with heavier white pine, but for the sharper definition of the annual ring, weight, and hardness.

Western Yellow Pine is used for poles to a limited extent in certain parts of the Southwest, where the high cost of more durable pole timbers makes it necessary to find a cheaper substitute. The life of this timber, untreated, is very short. In the upper part of the San Joaquin Valley of California, where a study of this species was made, untreated pine

poles last only two or three years; but since the wood when not exposed to the soil is fairly durable, it is believed that a butt treatment with a good wood preservative will result in a pole that will give good service. A butt-treated pine pole costs considerably less than an untreated cedar pole in this locality.

Lodgepole Pine is cut to a limited extent for poles. It grows at high altitudes in the Rocky Mountains. It decays quickly in contact with the soil, but is durable when not so exposed. The tree grows tall and straight, with very little taper, and makes a well-shaped pole. In certain parts of the West, where there are large bodies of fire-killed lodgepole that remain standing for many years, sound and thoroughly seasoned, conditions for effective treatment are excellent. If given a butt treatment, this dead timber makes a durable pole, and in many localities the cost of the pine pole plus the cost of the treatment is less than that of the Idaho cedar untreated. The sapwood of pole-sized timber may be an inch or an inch and a quarter thick.

Douglas Fir is a conifer found largely in the North Pacific coast region and principally in Oregon. It is light, strong, and straight grained. It is very durable when not in contact with the soil and therefore particularly adapted for use as cross-arms.

Terms Describing Part of a Pole

Knots are the heartwood of branches extending transversely through the sapwood of the trunk outwardly from the central heartwood.

Annular Rings are the concentric rings added yearly under the bark as the tree grows. Each ring is composed of two rings called respectively spring wood and summer wood.

Spring Wood and Summer Wood are differentiated by their density and color. The spring wood is relatively porous and usually is light in color; the summer wood is denser and darker in color. In conifers the summer wood is a reddish yellow on account of its resin content. In most woods the summer wood is thinner than the spring wood.

Pith. The pith of a tree is the central core about which the annual rings are formed. It goes through the tree from top to bottom and branches into the limbs. The pith is quite thick, usually $^1/_8$ to $^1/_5$ in. in Norway pine and in the southern species, though much less so in white pine, and is very thin, $^1/_{15}$ to $^1/_{25}$ in., in cypress, cedar, and larch. The pith of the tree is the weakest part on account of the many knots which it invariably and necessarily contains.

Sapwood. The sapwood of a tree is a zone of wood next to the bark, 1 to 3 or more inches wide and containing 30 to 50 or more annular rings (in coniferous trees). It is of lighter color than the inner, darker part of the log which is the heartwood. Sapwood changes to heartwood as the tree grows.

The width of the sapwood is small for longleaf and white pine and great for loblolly and Norway pines. In old trees of longleaf pine the sapwood forms about 40 per cent of the merchantable log, in the loblolly and in all young (coniferous) trees the bulk of the wood is sapwood.

Sapwood, being the normal condition of the outer rings of a tree, is not a "defect" in poles, where the whole cross-section of the tree (except bark) is used. Being weaker and more liable to decay it is considered a "defect" in pins and cross-arms, which are better if made from the heartwood only.

Defects in Wood Used for Poles and Cross-arms

The following are the defects in timber which are frequently referred to in specifications for poles and cross-arms.

Cup-shakes. These are cracks extending circumferentially at one or more places, caused by the separation of the annual rings.

Dote. This is a speckled stain found in beech, American oak, and other timber, due to incipient decay. It is produced by imperfect seasoning or by exposure for a long period to a stagnant atmosphere.

Heart-shakes. These are splits or clefts occurring in the center of the tree. They are common in nearly every variety of timber and are very serious when they twist in the length, as they interfere with the conversion of the tree into boards or scantlings. They sometimes divide the log in two for a few feet from the end.

Star-shakes. When several heart-shakes occur in one tree they are called star-shakes from the appearance produced by their radiation from the center.

Wind Cracks. Shakes or splits on the sides of a balk (a log which has been squared off) of timber, caused by shrinkage of the exterior surface, are called wind cracks.

Dry Rot. Dry rot is a special form of decay in timber caused by the growth of a fungus which spreads over the surface like a close network of threads, white, yellow, or brown, and causes the inside to perish and crumble. Causes which render timber favorable to the

growth of this fungus are: large proportion of sapwood; felled at wrong season when full of sap; cutting down in the spring or fall of the year instead of in midwinter or midsummer, when the sap is at rest; stacked for seasoning without sufficient air spaces being left; fixed before thoroughly seasoned; painted or varnished while containing moisture. (Six preceding definitions from Carpentry and Joinery, by Paul N. Hasluck.)

Sound Knot. A sound knot is one which is solid across its face and as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece.

Loose Knot. A loose knot is one not firmly held in place by growth or position.

Pith Knot. A pith knot is a sound knot with a pith hole not more than $^1/_4$ in. in diameter at the center.

Encased Knot. An encased knot is one which is surrounded wholly or in part by bark or pitch. Where the encasement is less than $^1/_8$ in. in width on both sides, not exceeding one-half the circumference of the knot, it shall be considered a sound knot.

Rotten Knot. A rotten knot is one not as hard as the wood it is in.

Pin Knot. A pin knot is a sound knot not over 2 in. in diameter.

Spike Knot. A spike knot is one sawn in a lengthwise direction. The mean or average width shall be considered in measuring these knots.

Pitch Pocket. A pitch pocket is an opening between the grain of the wood containing more or less pitch or bark.

Pitch Streak. A pitch streak is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well defined streak, or where the fiber between grains—that is, the coarse-grained fiber, usually termed " spring wood "—is not saturated with pitch, it shall not be considered a defect.

Wane. Wane is bark, or lack of wood from any cause, on edges of timber.

Shakes. Shakes are splits in timber which usually cause a separation of the wood between annual rings.

Checks. Checks are splits in timber which usually cause a separation of the wood across annual rings. (Last twelve definitions are those used in the timber-test work of the Forest Service in describing defects. *Forest Service Circular 88*, Revised.)

Wind-shake. A crack or incoherence in timber produced by violent winds while the timber was growing.

Wind. A turn or bend. A piece of timber is out of wind when it is perfectly straight or flat.

Warped. Twisted out of shape by seasoning.

Cat-faces. Old wounds, partially overgrown, leaving a long, narrow, dead surface exposed.

Insect Damage is caused by the boring of various insects and is indicated by small holes or bumps on the pole surface.

Crook is an offset in the axis of the pole. The axis above the crook may be parallel to, coincident with, the axis below the crook, or it may be curved.

Sweep is produced by a curving axis. It may be in one plane and one direction, or in two planes (double sweep), or in two directions in one plane (reverse sweep).

Volume and Weight of Poles

A quick way to find the approximate volume of a pole is to multiply the area of the circle at the center of gravity by the length of the pole. The formula for the volume, considering a pole as a frustum of a cone, is

$$v = \frac{\pi}{1728}(d_1^2 + d_1d_2 + d_2^2)h \tag{38}$$

where v = volume in cubic feet; d_1 = diameter at butt in inches; d_2 = diameter at top in inches; h = length of pole in feet.

The weight of a pole may be found by multiplying its volume in cubic feet by its weight per cubic foot. Table VIII gives the weight per cubic foot.

Seasoning

Poles of chestnut and cedar should be seasoned because it increases their resistance to decay, increases their strength, and decreases their weight. The strength of partially seasoned timber, other things being equal, increases as the amount of moisture it contains decreases. Thoroughly seasoned timber of small sizes is sometimes three or even four times as strong as the same timber when green.

Seasoning of poles reduces their weight, commonly from 16 to 30 per cent, and even more for some species, with a corresponding decrease in the cost of transportation. Thor-

Table VIII. Weight per Cubic Foot of Poles

Kind of Pole	When Cut		When Seasoned	
	Weight,* pounds per cubic foot	Moisture, per cent of dry weight	Weight,* pounds per cubic foot	Moisture, per cent of dry weight
Southern white cedar	38.9	88	25.0	21
Chestnut (N. C.)	56.5	101	43.2	54
Chestnut (N. J.)	51.8	85	42.2	50
Chestnut (Pa.)	54.0	92	40.7	45
Chestnut (Md.)	56.4	86	44.9	48
Northern white cedar	34.2	90	22.9	27
Western red cedar	42.4	133	23.5	29
Western yellow pine	66.6	154	30.3	16

* Including contained moisture.

ough seasoning is essential if the poles are to be treated with preservatives. The percentage of moisture in a pole when cut varies with the season when cut as shown in Table IX.

In general, poles cut during the spring and summer lose weight most rapidly. Poles cut during autumn and winter lose weight less rapidly, but more regularly. Too rapid seasoning may be detrimental to the timber by causing excessive checking. Shrinkage of poles during seasoning is very slight and does not exceed 1 per cent on the circumference.

Table IX. Moisture Content When Cut, Percentage of Dry Weight

Kind of Pole	Spring	Summer	Autumn	Winter
Southern white cedar (N. C.)	68	77	87	88
Chestnut (N. C.)	97	91	95	101
Chestnut (N. J.)	81	83	81	85
Chestnut (Pa.)	89	92	88	88
Chestnut (Md.)	83	84	85	86
Northern white cedar (Mich.)	77	82	79	90
Western red cedar (Cal.)	...	133	...	...
Western yellow pine (Cal.)	149	147	145	154
Southern yellow pine	...	52	...	...

The time in months required for poles cut at different periods of the year to season to approximately air-dry weight is given in Table X.

Table X. Time Required for Seasoning

Kind of Pole	Spring	Summer	Autumn	Winter	Moisture Content * Seasoned
	Months	Months	Months	Months	Per cent
Chestnut (Md.)	5	4	8	7	55
Southern white cedar (N. C.)	3	3	8	5	26
Northern white cedar (Mich.)	12	9	7	6	37
Western red cedar (Cal.)	..	..	..	..	43
Western yellow pine (Cal.)	5	3	9	6	25

* The average amount of moisture remaining in the poles after seasoning as above in percentage of the weight of the dry wood.

Roofing

If the top of the pole is left flat, rainwater will more easily penetrate the top of the pole, following the grain, causing early decay of the pole top. Poles are accordingly "roofed" by cutting the top to form an inclined surface. Either a single cut is made at an angle of 15 to 30 deg, or two such planes at an angle of 30 to 45 deg are cut to meet in a horizontal ridge. Roofs are sometimes painted, or coated with a bituminous substance, or covered with zinc, lead, or copper caps.

Preservative Treatments

Practically all wood species when used as poles are subject to decay either where in contact with the earth or throughout the length of the pole. Untreated cypress, pine, and juniper poles last only from 5 to 10 years; untreated cedar and chestnut poles last from 15 to 20 years under favorable conditions. Very few reliable data are available to

show the full value of preservative treatments in prolonging the life of poles. Pole service records available do show, however, that increases in life of at least 10 years, on the average, may be expected as a result of the best preservative treatments. From an economic standpoint such treatments are justifiable since they result in:

1. Increase of pole life.
2. Possibility of using smaller poles, as less allowance need be made for decay.
3. Possibility of using wood species not naturally durable.

Cedar and chestnut poles decay mainly in contact with soil, and therefore the butt trearment method is principally used for those species. The pines are subject to decay throughout their length, and therefore must be full length treated. Poles should not be painted with oil paints as these prevent evaporation of moisture and promote decay.

Cause of the Decay of Timber. Decay of wood is due to low forms of plant life called fungi. The germs of decay are not inherent in the wood. The wood-destroying fungi start from the outside, either from adjacent rotten wood or by spores, which correspond to seeds, being carried by the wind and deposited on the surface. While the fungi from these spores begin at the " surface " of the wood, this surface must be understood to include all holes or cracks which the spores may enter.

Fungi require for their growth and development air, heat, moisture, and food. Warmth, preferably between 60 and 100 deg fahr, favors decay. Cold retards it, and temperatures above 150 deg fahr prevent it. Under water or deep under the surface of the ground where the air is excluded, decay does not take place. Ordinarily wood which is seasoned until it is air-dry does not contain sufficient moisture to support the growth of fungi.

Preservatives. The best method of checking the growth of fungi is to poison their food, which is the wood itself. Such poisonous substances are called preservatives and are injected into the wood by various methods. Only two preservatives are extensively employed in the United States at the present time. These are creosote and zinc chloride. The latter is mainly used for preserving lumber, seldom for poles or cross-arms. Carbolineum, coal-tar solution, and crude petroleum have a limited application. Solutions of various arsenic salts have also been used, but their value has not been fully determined.

Creosote Oil is a distillation by-product of coal tar and is produced in the manufacture of coke and illuminating gas. It is sometimes known as dead oil of coal tar. The American Wood-Preservers' Association has specifications for three grades of creosote oil. That known as Grade 1 is mainly used in the preservation of poles. Creosote is a mixture of chemical compounds and consists principally of liquid and solid aromatic hydrocarbons, tar acids, and tar bases. It has a specific gravity of not less than 1.03, and it has a continuous boiling range from 200 to 325 deg cent. In Grade 1 creosote oil the amount of residue at several distillation temperatures is strictly limited. It is believed that oils boiling below 200 deg cent will not remain in the wood to preserve it, but that the fractions above 200 deg cent are admirably suited to the purpose.

Zinc Chloride is the zinc salt of hydrochloric acid. Wood preservative treatment with this salt is accomplished by either the pressure-tank process or by soaking the wood. Either method uses a water solution of the salt, the solution generally being not above 5 per cent in concentration. Since the salt is water-soluble it is subject to leaching if the wood becomes wet. Such leaching, of course, quickly reduces the preservative effect. Therefore, wood treated with zinc chloride should be used only in dry locations such as building interiors. For some preservative purposes zinc chloride is mixed with creosote or gas tar distillate or solution.

Carbolineum, like creosote, is derived from the distillation of coal tar but at higher distillation temperatures. It has good preservative properties but is somewhat more expensive than creosote.

Water-gas Tar is a by-product in the manufacture of water-gas and is somewhat similar to the tar obtained in the manufacture of coal gas. The oil obtained by the distillation of water-gas is known as **water-gas tar distillate** and is quite similar to creosote oil, although its toxic properties are not considered as great as those of creosote. A mixture of 60 per cent of the distillate with 40 per cent of refined or filtered water-gas tar is known as **water-gas tar solution.**

METHODS OF TREATMENT. The methods of applying the preservatives to the pole are the brush treatment, open-tank treatment, and pressure-tank treatment.

The brush treatment is applied to a part of the butt at the ground line, the open-tank treatment to the whole butt, and the pressure-tank treatment to the whole pole.

The brush treatment is least expensive and gives the least protection and the pressure tank is most expensive and gives the most protection.

Open-tank Treatment is used for poles which are durable except where in contact with the soil. It is used mainly for cedar and chestnut poles. In order to promote the penetration of the preservative the section of the pole to be treated is usually incised.

The wood is punctured by machines in a definite pattern of short, narrow incisions to a depth of $^3/_8$ to $^1/_2$ in. The butts of the poles are immersed in creosote oil at a temperature of about 230 deg fahr for not less than 6 hours. The depth of immersion is at least 1 ft more than the height of the ground line above the butt end. After the hot bath the butts are immersed in a cold creosote bath to the same depth. The temperature of the cold bath is not allowed to exceed 150 deg fahr. The length of time in the cold bath must not be less than 2 hours. All the outer bark and practically all the inner bark must be removed before the treatment.

Pressure Treatment is used for all poles, such as pine, which decay quickly even in the above-ground section. It is a full-length treatment, in which small cars are loaded with poles and run into horizontal cylinders which may be made pressure-tight. Pressure treatments are classified as full-cell and empty-cell processes. In the full-cell process the wood cells are left practically full of preservative at the end of the treatment. In the empty-cell process the wood cells are practically emptied of preservative before the completion of the treatment.

In the full-cell process the poles are subjected first to a vacuum to draw out as much moisture as is practicable, then without breaking the vacuum the preservative is introduced into the tank at a temperature between 165 and 200 deg fahr. The pressure is then raised to 100 lb per sq in. or more and maintained until the maximum penetration and injection which are practicable has been obtained. Lastly the tank is quickly drained of preservative and a quick high vacuum applied to remove the preservative from the surface of the poles. The net final retention of preservative is usually specified at from 10 to 20 lb per cu ft of pole volume. To insure long life of the pole, the entire sapwood content should be penetrated by preservative.

In the Reuping empty-cell process, air pressure is applied of sufficient amount to assist properly in the final emptying of the wood cells. Following the application of the air pressure, and without reducing it, the preservative is introduced into the tank at a temperature between 165 and 200 deg fahr and under sufficient pressure to obtain the desired penetration. After the pressure application is completed the cylinder is quickly drained and a vacuum is applied and maintained until the net retention of preservative is reduced to the specified amount. In the Lowry empty-cell process the application of initial air pressure is omitted, thus permitting lower pressures while the preservative is in the cylinder; however, higher vacuum is needed at the end to remove the excess preservative. The net final retention of preservative is usually specified as from 6 to 10 lb per cu ft of pole volume. Complete penetration of the sapwood is also desirable in this process. It should be noted that in all these pressure processes excessive temperatures, pressures, and vacuums tend to injure the timber.

Specifications for Poles

The American Standards Association has prepared specifications covering dimensions and quality of timber for pole use. These specifications are widely used, although individual purchasers often prepare their own. Space does not permit the incorporation of these specifications here.

Forces Acting on a Pole

A pole is subject to the following forces:

1. Vertical forces due to weight of pole, wires, sleet, etc., and to downward pull of guys.
2. Lateral horizontal forces due to wind across line on pole, wire, sleet, etc.
3. Longitudinal horizontal forces due to unbalanced pull of wires.
4. Torsional forces due to unbalanced pull of wires.

A pole is strong as regards the vertical forces but weak for horizontal forces, and the cross-arms are weak for the torsional forces. The theory of good line work is, therefore, first to reduce the horizontal and torsional forces as much as possible by balancing the stresses, and second to convert remaining unbalanced horizontal stresses into vertical stresses on the pole by the use of guys.

In practice the lateral horizontal force of the wind is one which cannot ordinarily be provided for by guys. Calculations for strength of poles, when made, are ordinarily limited to the effect of side wind.

BREAKING OF POLE BY CROSS WIND. The principal forces tending to break a pole are wind pressures on pole and conductors when the wind blows transversely. These tend to break it by cross bending.

Let M_1 = moment of the wind on the pole.
M_2 = moment of the wind on the wires.
M = moment of resistance of the pole.

Then the condition that the pole shall not break is that

$$M_1 + M_2 < M \tag{39}$$

The calculation of M_1, M_2, and M is given below.

Moment of Wind on Pole (M_1). Moment at ground level due to wind pressure on pole is

$$M_1 = \frac{P_1 H_1^2(D_1 + 2D_2)}{72} \tag{40}$$

M_1 = moment at the ground in pound-feet.
P_1 = wind pressure in pounds per square foot of projected area of pole.
H_1 = height of pole in feet.
D_1 = diameter of pole at ground in inches.
D_2 = diameter of pole at top in inches.

The maximum bending moment due to horizontal forces at the top of the pole is ordinarily assumed to be at the ground level; it is really a little below ground level and opposite the center of pressure of resistance furnished by the ground.

Moment of Wind on Wires (M_2). Moment at ground level due to wind pressure on the wires is

$$M_2 = \frac{P_2 H_2 nd(S_1 + S_2)}{24} \tag{41}$$

M_2 = moment at the ground in pound-feet.
P_2 = wind pressure in pounds per square foot of projected area of wires.
H_2 = height of wires above ground in feet.
n = number of wires.
d = diameter of wires (including ice) in inches.
S_1 and S_2 = lengths of adjacent spans in feet.

Where wires are of different diameters or at different levels the formula is to be applied to each size and each level separately and moments summed.

Table XI. Fiber Stresses and Breaking Loads

Kind of Timber	Maximum Working Stresses, N.E.S. Code, lb per sq in.	Test Results		
		Fiber Stress at Rupture, lb per sq in.	Breaking Load, lb	Elastic Limit, Lb per sq in.
Arborvitae	....	4250 *	....	2600 *
Cedar:				
Red, western	5600	6065 ¶	1310 ‡	....
Red, western	....	....	2215 §	....
Red, western	....	....	1930 §	....
Oregon	....	....	3040 §	....
White, northern	3600	3621 ¶	2659 †	....
Chestnut	6000	6480 ¶	3240 †	....
Cypress	4800	7110 *	....	4430 *
Douglas fir	7400	....	....	....
Pine:				
Lodgepole	6600	5130 *	1430 ‡	3080 *
Southern, yellow	7400	8026 ¶	....	....
Longleaf	....	8630 *	....	5090 *
Shortleaf	....	7710 *	....	4360 *
Yellow, Cal	5000	5180 *	....	3180 *
Redwood	4400	....	....	....
Spruce, Engelmann	....	....	1405 ‡	....

* *Forest Service Cir.*, 213; green, clear pieces.
† L. W. Winchester, *Elec. World*, March 16, 1911; 29- to 31½-ft poles set in ground 4 to 6 ft, load applied 22 to 26 ft above ground.
‡ *Forest Service Cir.*, 204; 25-ft poles, 7 in. top, force applied at top.
§ Pac. Tel. & Tel. Co.; 25- to 35-ft poles, 6 to 9 in. top, force applied at top.
¶ American Standards Association.

Moment of Resistance (M). The moment of resistance or strength of a circular pole for bending as a cantilever is

$$M = 0.000264 fC^3 \tag{42}$$

M = maximum allowable moment of resistance at the ground line, in pound-feet.
f = maximum allowable fiber stress, in pounds per square inch.
C = circumference of the pole at the ground line, in inches.

Fiber Stress (f) and Actual Tests of Strength. Table XI gives the value of the fiber stress for various kinds of timber and the actual breaking load from tests of a number of poles.

Weakest Point of a Pole. A pole is approximately a truncated cone in shape. For a bending force applied at one end such a cone is weakest at the point where the diameter is $^3/_2$ the diameter at the point (near the small end) where the force is applied. A pole with 8-in. diameter at the cross-arm is, therefore, weakest where it is 12 in. in diameter and may be expected to break at this point provided this point is above the place where maximum bending occurs. If it is less than 12 in. in diameter at the point of maximum bending then the break may be expected here. This rule must be considered approximate as it neglects the fact that the pole is not homogeneous, i.e., outer annual rings are sapwood and inner are heartwood, and also neglects effect of knots, etc. In practical working calculations the weakest section is taken at the ground line, since that point tends to become weaker than any point above ground as a result of its greater moisture content and because of its greater tendency to decay.

STRESS DUE TO ANGLE IN LINE. If there is an angle in the line an additional stress is imposed upon the supporting structure at the angle point due to the tensions in the conductors. If the conductors in the adjacent spans have equal tensions (t) and the angle of departure of the line is (a), the resultant force on the structure will be:

$$F = 2\,t \sin \frac{a}{2} \text{ pounds} \tag{43}$$

and the angle between the direction of the resultant and the direction of either span will be: 90 deg $- \frac{a}{2}$. If the tensions in the adjacent spans are not equal the resultant force will be:

$$F = \sqrt{t_1^2 + t_2^2 - 2\,t_1 t_2 \cos a} \tag{44}$$

and the angle (b) between the resultant and the span in which the tension is (t) is obtained from:

$$\cos b = \frac{F^2 + t_1^2 - t_2^2}{2\,Ft_1} \tag{45}$$

Attachment of Cross-arms to Poles

Wooden cross-arms are attached to wooden poles:

1. By gaining the pole, see below.
2. By one or two bolts.
3. By one or two cross-arm braces.

The forces at the point of attachment which these fastenings must resist are:

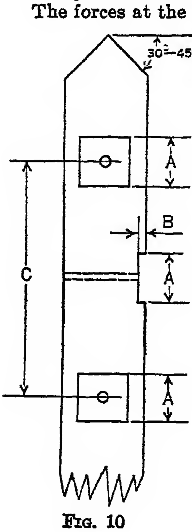

FIG. 10

1. A force vertically downward, equal to weight of cross-arm, pins, insulators, and wire (including sleet).
2. A horizontal force parallel to axis of arm, equal to pressure of wind blowing across line on wires.
3. A horizontal force at right angles to axis of arm: (a) toward pole or (b) away from pole and equal to difference in pull of wires on two sides of arm.
4. A couple in a vertical plane parallel to arm, equal to difference in moments of weight on the two ends of arm.
5. A couple in a horizontal plane parallel to arm, equal to difference in moments of wire pull on the two ends of arm.
6. A couple in a vertical plane at right angles to arm, equal to difference in moments of wire pull (caused by pin leverage) in the two directions.

GAINING. A gain is a notch cut in the side of a pole to receive a cross-arm. The width (vertical dimension) of the gain should be just large enough for the cross-arm. The depth of gain varies from $^1/_2$ to 1 in. With gains shallower than $^1/_2$ in. the cross-arm has insufficient support below and the flat bearing surface at the back is inadequate unless the pole is of larger diameter than usual. Deep gains greatly weaken the top of the pole especially when double arms are used. Poles which require full-length preservative treatment should be drilled, gained, and roofed completely before treatment.

Cross-arm Construction

On ordinary straight-line construction single cross-arms are generally used. If conductors are carried at more than one level a single arm is used at each level and all cross-arms are placed on the same side of the pole. If the pole has any sweep the arms are placed on the concave side to leave the convex side clear for climbing. At four-way corners single arms are used with alternate arms at right angles to each other. At two-way corners, dead-ends, angles, and where the line stresses are not balanced longitudinally, double arms are used. In alleys and at other places to avoid obstructions, alley arms are often used.

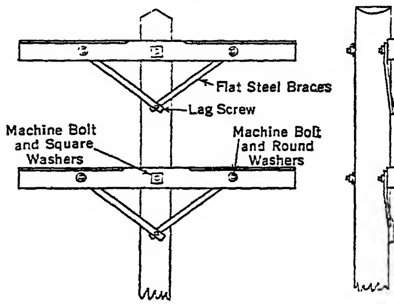

Fig. 11

The size of cross-arms depends on the size of conductors carried and upon the voltage of the circuits. The spacing of arms depends upon the voltage of the circuits carried and upon the length of spans.

The accompanying figures show various types of wood pole construction. Fig. 11 shows single-arm construction for low- and medium-voltage lines. Fig. 12 is an example of double-arm construction at a dead-end in the line. Fig. 13 shows typical construction for a four-way corner. Fig. 14 is an example of alley-arm construction. Fig. 15 shows a typical medium-voltage transmission line pole for straight line construction using suspension insulators. Fig. 16 is the type of construction sometimes employed for corners on single-circuit transmission lines. Two poles of this type are sometimes used for corners in double-circuit lines of vertical configuration. Fig. 17 is an example of construction largely used with single circuit wood pole transmission lines of high voltage.

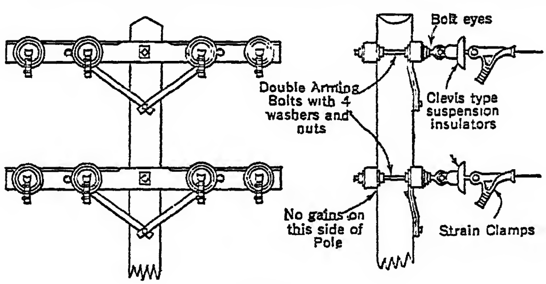

Fig. 12

Hardware

Hardware for wood pole lines is usually of steel which is hot-dip galvanized. The N.E.L.A. has prepared specifications, covering material, dimensions, and galvanizing, for the commonest items of hardware. Specifications of the A.S.T.M. also cover material and galvanizing. Hardware items include: machine bolts, double-arming bolts, lag screws, square and round washers, eye bolts, clevises, guy plates, hooks and shims, hub plates, pole steps, and cross-arm braces. Most items are manufactured in a wide range of sizes to cover the varied conditions to be met in wood pole construction. Wood braces with steel connectors are often used on transmission lines to minimize phase-to-phase flashovers.

Setting Poles

Table XII gives the average depth of setting recommended by the National Electrical Safety Code. In soft or swampy soil, poles should be braced by cribs or swamp braces and guyed if necessary. Unguyed poles at angles in the line should be set from 6 in. to a ft deeper.

In ordinary firm soil pole holes are dug by hand, using digging bars and long-handled, spoon-shaped shovels, or by hole-digging machines. Such digging machines are earth

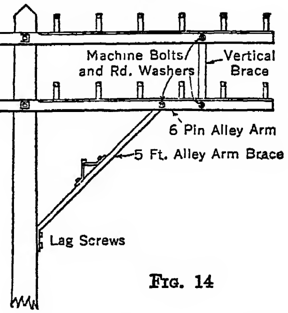

FIG. 14

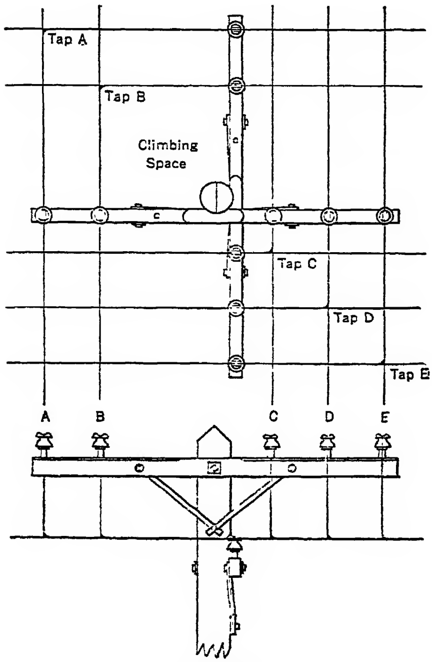

FIG. 13

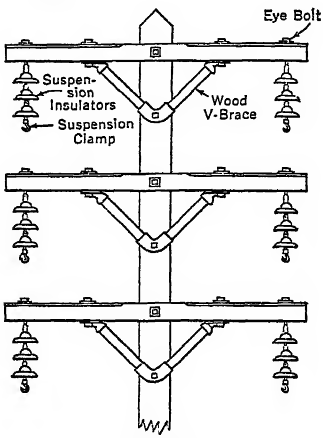

FIG. 15

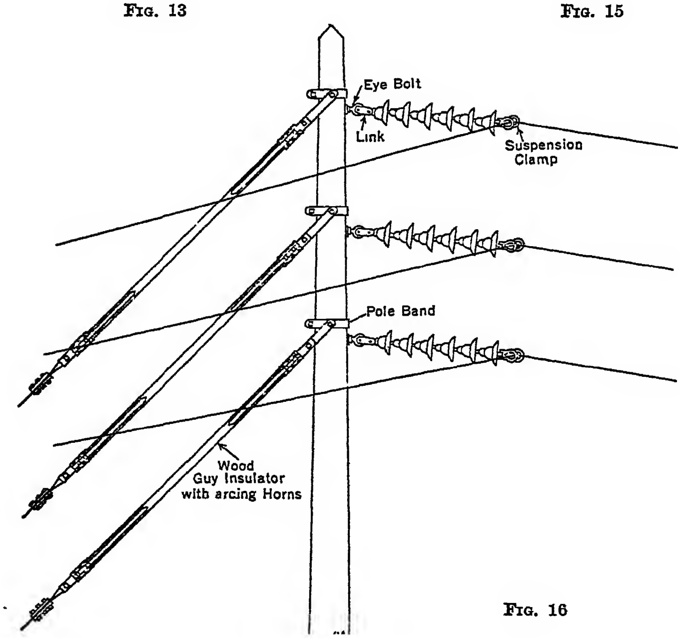

FIG. 16

Table XII. Average Depth of Pole Setting

Length of Pole, ft	Ft in Soil	Ft in Rock	Length of Pole, ft	Ft in Soil	Ft in Rock
20	5.0	3.0	55	7.0	5.0
25	5.0	3.5	60	7.5	5.0
30	5.5	3.5	65	8.0	6.0
35	6.0	4.0	70	8.0	6.0
40	6.0	4.0	75	8.5	6.0
45	6.5	4.5	80	9.0	6.5
50	7.0	4.5	..	...	...

augers driven by gasoline engines and mounted on trucks, trailers, or tractors. The motive power of the truck or tractor is generally used to drive the auger. Caterpillar tractors are especially suitable for this work in rural territory as they are able to travel over uneven or soft ground without difficulty. Pole holes should be the same diameter at the top as at the bottom and should be only enough larger than the pole butt to allow the use of tamping bars to tamp the backfill thoroughly around the pole.

In swampy soil or quicksand a barrel or split caisson is required to prevent the sides of the hole from caving in. As the hole is dug, the barrel or caisson is gradually pushed down; the barrel may be left in the hole after the pole is set, but the caisson may be withdrawn to be used again. Another successful method employs a high-pressure water jet. In this method the hole is started with shovels and the pole is set up in position and held by hand lines. A hose with a long pipe nozzle is laid alongside the pole, and the water under high pressure is started. The jet loosens the soil and brings it to the surface in suspension. By reason of its weight the pole drops slowly into the hole. Eighty-foot poles have been set 40 ft deep by this method in extremely deep marshes.

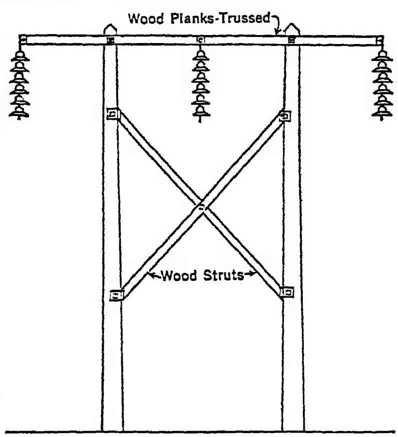

Fig. 17

Explosives are often used for setting poles in soft or sandy soils, or in rock. In soft soils a charge of dynamite is laid at the bottom of a small pipe driven to the required depth of hole. The pipe is then withdrawn and the pole is set on the surface directly over the charge and held upright by hand lines. When the charge is exploded the pole drops into the hole and the soil drops back around the pole. When the pole is to be set in rock a small hole is drilled to the required depth and the charge laid at the bottom. The small hole is backfilled and tamped. Firing the charge opens a hole to take the pole. Experience in such work is necessary to determine the size of charge and the proper depth to open a hole no larger than necessary for the pole.

A pole may be raised and set in the hole by a hoisting rig such as a gin pole or a derrick mounted on a truck or tractor. Poles may be set by three to five men using pike poles. After the pole is set in the hole it is supported in proper position by pike poles while the backfill is made and tamped.

Guying of Poles

Whenever a pole is not strong enough to withstand the bending stresses imposed on it by unbalanced forces it should be guyed. Guys should be strong enough to take the entire stress in the direction in which they act, the pole acting only as a strut. Guys consist usually of stranded steel wires, together with means of attaching to poles and anchors, and also include strain insulators where they are required.

STRESSES IN GUYS AND ANCHORS. To compute the tension in a guy wire use the formula:

$$T = \frac{M}{H \sin a} \text{ pounds tension} \tag{46}$$

where M is the bending moment applied to the pole in the plane containing the pole and

the guy, in pound-feet; H is the height above ground of the point of attachment of the guy to the pole in feet; a is the angle which the guy makes with the vertical.

GUY WIRE. Stranded steel wires used for guys are usually heavily galvanized, but in locations where corrosion is excessive stranded copper-clad steel wires are often used. Mild steel is common for lightly loaded lines, but Siemens-Martin or high-strength steel is used for many important or heavily loaded lines. Table XIII gives the characteristics of various types of guy wires.

Table XIII. Mechanical Characteristics of Stranded Steel Cables

Nominal Size, in.	Diameter, in.	Area, sq in.	Breaking Load, lb		
			Ordinary	Siemens-Martin	High-strength
5/8	0.625	0.2356	12,720	15,900	26,500
9/16	.562	.1922	10,380	13,000	21,620
1/2	.500	.1443	7,790	9,740	16,230
7/16	.437	.1204	6,500	8,130	13,540
3/8	.375	.0832	4,490	5,620	9,360
5/16	.312	.0606	3,270	4,090	6,820
1/4	.250	.0352	1,900	2,380	3,960

ATTACHMENT TO POLES AND ANCHORS. Guys are attached to poles by wrapping the end of the wire twice or more around the pole and clamping the free end to the main part of the guy by means of one or more guy clamps. Steel shims or plates are placed between the wrappings of the guy wire and the pole. Steel hooks are also used to prevent the guy from slipping down the pole. Hooks, shims, and plates are attached to the pole by nails or lag screws. An alternative method is to attach the guy wire to an eye-bolt or patented attaching device which is bolted through the pole. Anchors sunk in the ground are provided with long anchor rods extending slightly above the ground surface and terminating in an eye. The free end of the guy is passed through the eye of the anchor rod, bent back parallel to the main portion of the guy, and clamped to it. The loop is protected by a guy thimble where it bears on the anchor rod eye.

STRAIN INSULATORS. Strain insulators are placed in guys to prevent the lower part of the guy from becoming electrically energized by contact of the upper part of the guy with conductors or by leakage. Two strain insulators are used whenever a single insulator would not effectively prevent a hazard to the public or to linemen. No guy insulator should be located less than 8 ft from the ground. Insulators are not necessary where no part of the guy is within 8 ft from the ground or where the guy is permanently and effectively grounded.

Two types of strain insulators are used. One type is made of porcelain and the other is of wood with steel connecting parts. Porcelain insulators are usually designed so that the porcelain is in compression with the guy wire interlinked. In this type the porcelain may fail and allow the guy wire loops to come together, making the insulation ineffective without a mechanical failure of the guy. Such insulators are used only in connection with low- and medium-voltage lines. The National Electrical Safety Code requires guy insulators to have a mechanical strength equal to that of the guy in which they are installed, a dry flashover value of twice the line voltage, and a wet flashover not less than the line voltage. Wood insulators are now used extensively with high-voltage wood pole transmission lines. They are made of treated wood with steel connecting parts at each end and are provided with arcing horns to prevent flashovers from burning the wood. Generally the wood portion is long compared with its cross-sectional dimensions, and usually the cross-section is rectangular. The wood is used in tension, and of course a failure of the insulator causes a mechanical failure of the guy.

GUY ANCHORS AND STUBS. Guys may be anchored to the earth, to other poles, and occasionally to trees or buildings. When guys are anchored to the earth it is necessary to bury in the earth some object to which the guy is to be connected. Such an object, known as an anchor, must be so designed as to press against a sufficient volume and weight of earth to resist completely the force due to the tension in the guy. A common form of anchor is the so-called " dead-man " or log anchor. It usually consists of a log or section of a pole buried in a trench. An anchor rod is inserted through a diametrical hole bored through the middle of the log and is held in place by a plate and by nuts on the end of the rod. A transverse trench is provided running from the surface of the ground, at the proper angle with the ground surface, down to the bottom of the trench dug for the log. After the anchor is in the proper position the trenches are, of course, backfilled and tamped.

A large volume of undisturbed earth is provided for the anchorage by this method, and it is therefore useful for anchoring heavy guys.

Many forms of patented anchors are available. These are usually made of steel or malleable iron. Some are designed to screw into the earth, and some are placed or driven into small holes and afterwards expanded to increase the bearing area. Such patented anchors are made in a wide range of sizes to be used with small or large guy wires and light or heavy loads.

Guy stubs are short poles to the tops of which guys from line poles are attached when the guy is required to clear the ground by a considerable distance. An anchor guy is usually run from the top of the guy stub to the ground, in which case the stub is raked a sufficient amount so that it will act only as a strut. Sometimes guy stubs are made self-supporting by giving them considerable rake and by burying a log just below the surface of the ground and another log at the bottom of the stub on the opposite side for the stub to bear against. Fig. 18 shows a self-supporting guy stub and a common form of log anchor.

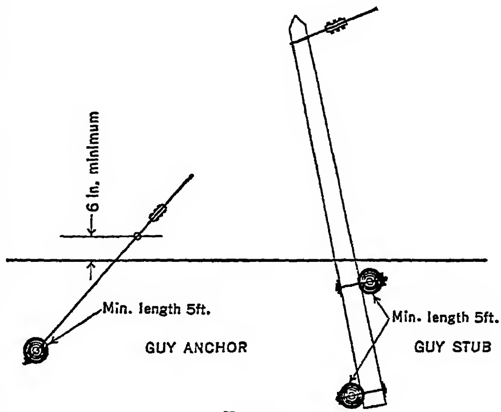

FIG. 18

Repairing Decayed Poles

If poles have been weakened at the ground line by decay the strength may be restored by cutting off the decayed butt and resetting the pole, thus reducing its height by 6 to 8 ft. If reduction of height is not permissible, the pole may be stubbed by setting along side of it a short pole or stub extending a few feet above ground to which the old pole or the undecayed part above ground is bolted or otherwise fastened. This does not look well and is unsuitable for city distribution lines but has been used for transmission lines. Another method is to reinforce the decayed pole by a sleeve of concrete (usually reinforced) extending above and below the decayed portion.

Several patented devices are also available. Some of these act as sockets for the upper portion of the pole which has been cut off at or above the ground line. Another method leaves the pole intact but T-shaped steel bars are driven into the ground close beside the pole and are clamped tightly to the pole by curved steel rods.

26. STEEL TOWERS AND POLES

Steel structures are often used instead of wood poles for supporting electrical circuits. They have the advantages over wood structures that they can be made as large as necessary and that they may be made self-supporting without the use of guys. They have the disadvantage of not having an inherent insulating value, and, therefore, when steel structures are used the line insulators must be selected with that fact in mind. They do, however, have the advantage of being less susceptible to damage by grass, brush, or forest fires.

General Features of Design

Towers are composed of two parts, the tower proper and the foundation. Towers are usually built of standard structural-steel shapes. Angles are used for most members. Channels are used for the larger members of some towers, usually the cross-arms, or for posts of flexible towers. Flat pieces are sometimes used for the minimum-sized bracing of light towers. Round rods are used for tension members in some types. The principal members of a tower are the corner posts or legs, which are vertical or approximately vertical, and are usually the heaviest members of the tower proper, and the horizontal and diagonal web members which connect the posts together in vertical planes which constitute the sides or faces of the tower.

The spread of a tower at the base is generally between one-fourth and one-fifth of the height. The greatest economy in cost of tower plus foundation usually requires a little wider base than that which gives the least cost for the tower taken alone.

Towers are usually designed for either one or two three-wire circuits, usually with one or two ground wires above and sometimes with a telephone circuit below. Where two circuits are on one tower they are generally located on opposite sides to reduce the hazard of repairing the line. (See Fig. 19.) The three wires of a circuit are occasionally arranged to form an equilateral triangular prism, but frequently lie in a single plane which is usually horizontal for single-circuit towers, and vertical for two-circuit towers. (See Figs. 20 and 21.)

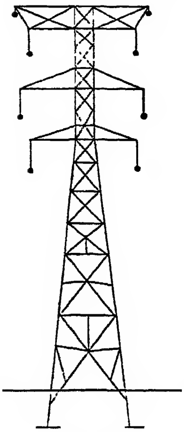

FIG. 19

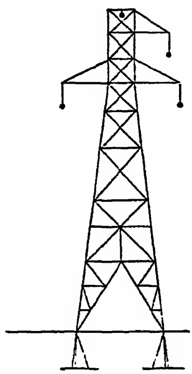

FIG. 20

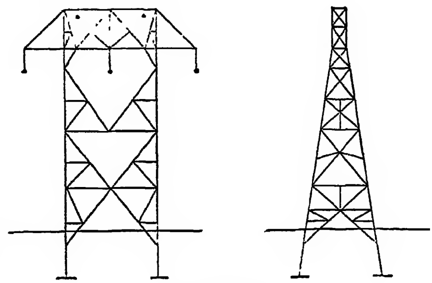

FIG. 21

Steel poles are commonly of the tubular type or of the square latticed type. Tubular poles serve mainly to support trolley conductors and feeders on city streets but latticed poles are used to support distribution lines and low- or medium-voltage transmission lines. Latticed steel poles have narrow bases; that is, the taper of the pole is small. The vertical members are usually angles and the lacing on each face is either flat steel bars or small angles with one leg turned inward. They may be designed for one or more low-voltage circuits arranged in horizontal or triangular configuration. Single-circuit high-voltage lines may be arranged in triangular configuration, but double-circuit high-voltage lines are always arranged in vertical configuration with one circuit on each side of the pole.

TYPES OF TOWERS. Two general types of towers have been used; the flexible and the rigid. Flexible towers are more or less obsolete at the present time. The general form of such a tower is a flat A-frame having good lateral strength but none in the direction of the line. Strength in that direction is usually attained by attaching a ground wire rigidly to the top of each A-frame. Such structures are not adequate for important or high-voltage lines, and they have been largely superseded by wood pole structures.

Rigid Towers are usually triangular, square, or rectangular. The square tower is probably the most common. Square towers usually have the four faces framed with the same size members (even though the stresses in the longitudinal and lateral faces rarely figure the same), because of the economy of manufacture and erection which results from the simplicity. This feature has an advantage in design in that the torsional stresses are more simply determined. Rectangular (including square) towers have the disadvantage that the unequal settlement of the foundations may produce high internal stresses not allowed for in the design. Triangular towers avoid internal stresses from unequal foundation settlement, but present difficulties in the joining of standard structural shapes, and stresses in them are difficult to calculate. (See Fig. 22.)

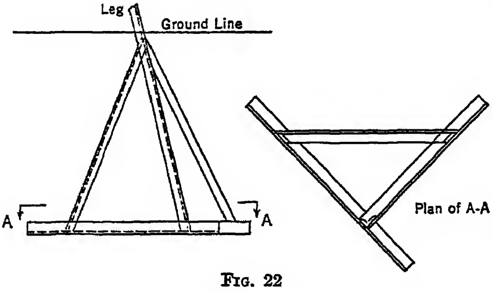

FIG. 22

CONNECTIONS OF MEMBERS. The members of a tower are usually connected by bolts. By using no rivets the members may be fabricated in quantities, compactly

bundled, easily handled even in rough country, and erected by less skillful labor; also the galvanizing can be done after all shop work is completed. All the bolts of a tower should be of one diameter ($^5/_8$ in. is suitable for the members generally used) and of as few different lengths as possible. Bolt holes should be slightly larger than the bolts ($^1/_{16}$ in. is a usual amount). By designing bolted connections so that friction between the surfaces develops the full compressive strength of members, the play in the bolt holes with changing compression and tension in the members is eliminated.

If the unit shearing stress is made two-thirds of the unit tensile stress in the members connected, and the unit bearing stress is made twice the unit shearing stress, a bolted connection will be not less than 10 per cent stronger than the members connected.

CLEARANCE BETWEEN CONDUCTORS AND TOWER. Clearance from conductor to tower is usually based on that existing at maximum sideswing of the insulator string, toward the tower members, due to wind pressure on the conductors. The sideswing is usually taken at 60 deg, but sometimes, under favorable conditions, at only 45 deg from the vertical. The sideswing should be calculated for maximum wind pressure both with and without ice. Reduced clearance of the conductor itself from the corners of the tower should also be considered. The angle of swing will be greater for small or light-weight conductors than for large or heavy ones. Clearance from the cross-arm should be considered as well as from the vertical portion of the tower. If arcing rings or arc-suppressing devices are used on insulator strings, clearances from their live parts should be determined. Flashovers ordinarily are due to lightning potentials impressed on the conductors; therefore, arcing distances should be selected on the basis of the characteristics of impulse flashovers. The flashover distance to the tower, through air, should be at least 10 per cent greater than that over the insulator string, with its protective equipment, if any.

FOUNDATIONS FOR TOWERS. Structural steel, mass (unreinforced) concrete, reinforced concrete, or piles may be used for tower foundations. Rock footings are also used in special locations.

Structural-steel Foundations are cheap and easily transported. They usually consist consist of grillages of structural-steel I-beams with tie members consisting of I-beams or channels. (See Fig. 23.) These grillages are either buried at a proper depth in the earth

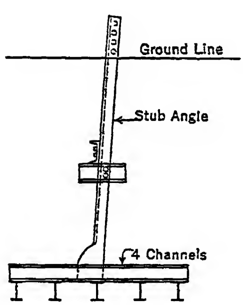

FIG. 23

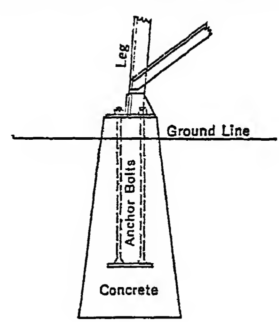

FIG. 24

or surrounded by a mass of concrete. They should have sufficient horizontal area so that they will have adequate bearing surface to withstand downward pressures and carry sufficient of earth to withstand possible uplift forces. The corner leg members are usually carried down to the grillage and connected to it in a proper manner to transmit the vertical and horizontal stresses involved.

Concrete Foundations have an advantage over structural steel in that they can more easily be varied in depth, spread, etc., to accommodate themselves to local conditions of soil. This is especially advantageous where boulders or irregular ledges interfere with the use of a standard-sized foundation. (See Fig. 24.)

Mass-concrete foundations are advantageous in those cases where it is necessary or desirable to have a foundation of such weight as to withstand much uplift with little reliance on the holding power of the earth. The towers may be conveniently attached to anchor bolts imbedded in the foundation, or the leg member may extend down to the bottom of the concrete. To avoid tension in the concrete the bolts must extend to the bottom, with proper plates for distributing the stress. The anchor bolts and plates then become a crude system of reinforcement.

Reinforced concrete foundations are durable and require less material than mass concrete, thereby facilitating transportation.

Piles are used under or for foundations in very marshy ground where the holding power of other foundations is unreliable.

Rock Footings for towers standing on ledges may consist of anchor bolts grouted into holes drilled in rock and extending through level bearing plates grouted to the rough rock surface at the proper elevation.

Forces Acting on Towers

The stresses in towers are caused by: (1) the weight of tower, insulators, clamps, cables (conductors, ground wires, telephone wires) and ice loads on them; (2) the wind pressure on above;* (3) the unbalanced tension in cables when dead ended or broken on one side; (4) the unbalanced resultant due to cable tension at angles in the line; (5) the loads imposed when erecting towers, stringing wire, or repairing line.

A careful study should be made of all the combinations of these loads which are possible or probable. Often no single combination can be found which will produce the maximum stress in all tower members, and therefore several combinations must be used to determine the design.

In a square anchor tower carrying six wires, three on each side, the maximum stress may be expected in the corner posts when all six wires are pulling in the same direction; the maximum stress in the web members will probably be produced by three wires pulling on one side in one direction and the three on the other side in the opposite direction. In the first case the tower is subject to a bending stress and in the second to a torsional stress. In each case the stresses due to weight and wind are to be superimposed. The wind may act as a force along the line or across the line, but generally its longitudinal effect is negligible while its lateral effect is important.

STRESSES IN TOWER MEMBERS.—The stresses in the several members of a tower are usually determined graphically from the assumed loadings by means of stress diagrams; see section on Trusses in Kent's Handbook for Mechanical Engineers. In most designs the distribution of stress is not fully determinate.

Fundamental Assumptions. Certain assumptions are, however, commonly made which give a determinate distribution for the purposes of design. Among these assumptions are:

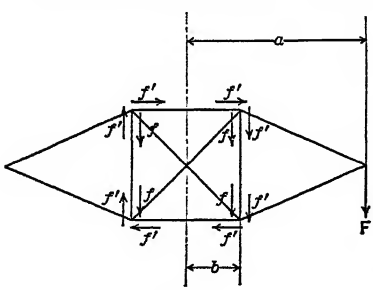

FIG. 25

1. An unbalanced stress on the tower (say a broken wire pulling on one side) can be resolved into an equal stress at the axis of the tower and a torsional moment.
2. The equivalent stress at the axis of a rectangular tower can be considered as balanced between the two faces parallel with it, each face taking one-half the stress.
3. The torsional moment can be considered as divided between all four faces of a rectangular tower.
4. If the tower is square each face takes one-fourth of the torsion.

The above relations may be expressed as follows (see Fig. 25):

Let F = unbalanced force in pounds applied at end of cross-arm.

a = distance, in feet, from *end of cross-arm* to axis of tower.

b = distance, in feet, from *side of tower body* to axis of tower.

Then

$f = \frac{F}{4}$ = balanced force in pounds applied at each corner post equivalent to F in bending effect on body of tower. (47)

$f' = \frac{Fa}{8b}$ = torsional force in pounds applied at each corner post equivalent to F in twisting effect on body of tower. (48)

* The wind pressure on a tower is assumed to be uniformly distributed per square foot of surface against which the wind blows, one-half of it consequently being on the windward side of the windward face and the other half on the windward side (inside) of the leeward face. For simplicity in calculation this uniformly distributed force is replaced by a series of concentrated forces, one at each panel point equivalent to the total distributed force extending over a half panel above and a half panel below the panel point. By panel point is meant a point of intersection of principal members, for example, of horizontal members with vertical members; and by panel is meant the section of a side between panel points, a panel usually being bounded by two vertical and two horizontal members.

5. In a tower framed with a double system (i.e., diagonals in duplicate and suitable for compression as well as tension) each system may be considered as taking one-half of the stress as far as possible.

Approximations Made in Calculating Tower Stresses. The stress diagrams are usually simplified by employing certain approximations:

(*a*) Faces of towers are usually battered so that they deviate slightly from a true vertical plane, but the stress diagrams usually neglect this inclination and are based on the vertical projection of the face.

(*b*) Where the face of a tower does not lie in one plane (i.e., has a change of batter as occurs frequently at bottom cross-arm where a prismatic cage joins a pyramidal base) the change of inclination is neglected and the diagrams are based on a single vertical projection as before.

Subject to the limitations of the assumptions and approximations given above, the four faces of the tower can be regarded as four cantilevers, supported at the base and loaded at the top, which are independent except that the four corner posts are each common to two faces and must contain the resultant of both stresses.

Where a face of a tower or any part of a face has any considerable inclination the above approximations may not be used without danger of serious error.

Unstressed Members. A tower usually contains members no stress in which is shown by the stress diagram, viz.:

1. Diagonal members in a horizontal plane do not usually appear in the stress diagram when located below the lowest cross-arm. These members play an important part in the distribution of torsion among the faces. In a rectangular tower the torsion will usually redistribute between the four faces at each level where there are horizontal diagonals; therefore the failure of the stress diagram to show stresses in them may be taken to indicate that the assumed distribution of torsion is not quite correct rather than a true absence of stress.

2. Redundant members are braces which carry no determinate stress but perform the important function of supporting the compression members which do carry stress. The unit stress allowable in a compression member diminishes as the unsupported length increases. The weight of compression members is therefore diminished by dividing their unsupported length by braces applied at one or more intermediate points.

Unit Stresses in Towers. Towers are designed for certain combinations of loads. If the tower were designed to withstand all possible combinations of loads with each load at its maximum possible value, the tower would be uneconomical as its overload capacity under conditions which would occur on the average would be out of reason. Therefore, towers are designed for average loading conditions, and unit stresses are used which will take care of a reasonable overload. The term "factor of safety" is no longer used in reference to tower design since it is misleading; the term "possible overload" is used in its place.

For ordinary transmission towers the following unit stresses in pounds per square inch are generally used:

Axial tension on net section	20,000
Axial compression on gross section (not over 15,500, $\frac{L}{R}$ not over 150)	$20{,}000 - 85\frac{L}{R}$
Axial compression on gross section $\left(\frac{L}{R}\text{ exceeding }150\right)$	$15{,}500 - 55\frac{L}{R}$
Shear on bolts or rivets	13,500
Bearing on bolts or rivets	27,000
L = unsupported length (inches)	
R = least radius of gyration (inches)	

For river crossing or other important structures the following unit stresses are generally used:

Axial tension on net section	18,000
Axial compression on gross section (not over 14,000, $\frac{L}{R}$ not over 150)	$18{,}000 - 80\frac{L}{R}$
Axial compression on gross section $\left(\frac{L}{R}\text{ over }150\right)$	$13{,}500 - 50\frac{L}{R}$
Shear on bolts or rivets	12,000
Bearing on bolts or rivets	24,000

The use of the above stresses results in a possible overload capacity of 90 to 96 per cent for ordinary transmission towers and about 120 per cent for river crossing towers.

Eccentricity in Stresses at Joints. As tower members are ordinarily connected together the stresses in them are slightly eccentric, thereby preventing the full strength of the members being developed. The eccentricity should be eliminated or reduced as much as possible by having the center of gravity of the several members at each connection meet at one point as exactly as possible.

Forces Acting on Tower Foundations

Foundation stresses are of two classes: first, the foundations resist the tendency of the tower to slide and overturn due to the external forces on it considering the tower as a self-contained structure; second, the foundations resist certain stresses which would be internal tower stresses were the tower framed as a complete self-contained structure, but which become external stresses because the ground is depended on for the function of certain omitted members. The weight of the tower can evidently be reduced by thus substituting the ground for certain members, but the size of foundation is thereby increased. The amount of these latter stresses depends on the outline and framing of the tower, and their effect should not be overlooked in determining loadings on foundations.

The magnitude and direction of the forces acting on the tower foundations may be illustrated by taking the case of a rectangular tower, and considering a transmission line which runs north and south.

Let a = width (feet) of base of tower (east and west).

b = length (feet) of base of tower (north and south).

W = total weight (pounds) of tower, insulators, fittings and one span of all the wires, including ice load, if any.

W' = total weight (pounds) of any unbalanced load, such as a wire c feet off center.

F = resultant force (pounds) of the wind on the tower and a complete span of all the wires (with ice coat, if any), acting at a distance of d feet above the foundation, wind assumed blowing across line from west to east.

P = pull (pounds) of any unbalanced force toward the south applied at a distance of e feet above the foundation and f feet to the west of axis of tower, as for example, a dead-ended wire or when a wire on the north side of the tower is broken.

Then, assuming that the forces divide equally among the four foundations and that the torsional forces are in a circumferential direction, the relations given in Table XIV hold. These assumptions are reasonably correct for a tower with the four legs joined at the bottom with a horizontal strut in each of the four faces and with horizontal ties across the diagonals, unless the framing which is usually provided in the other faces is inadequate. Probably few towers in use fully meet these requirements. Therefore, there are usually additional stresses of large magnitude due to the foundations performing the function of missing or inadequate members, as pointed out in the notes appended to Table XIV.

RESULTANT FORCES ON FOUNDATIONS. From the above relations the resultant force on each of the four foundations may be found. In general there will be on each foundation; (1) a downward pressure, (2) a direct uplift, and (3) a horizontal overturning force, producing a tendency to slide and an uplift on one side of the foundation and a downward pressure on the other side.

Downward Pressure. The downward pressure usually is of little importance in determining the size of the foundation as a foundation large enough for uplift and overturning is unnecessarily safe against downward pressure.

Direct Uplift. The uplift is very important, as the weight of tower and foundation is rarely sufficient to provide more than a small fraction of the holding-down power required. The excess uplift is usually resisted by the earth in which the foundation is buried. Not only is the weight of the earth directly over the foundation effective but there is an additional resistance due to friction or cohesion of the earth which may be several times greater. These forces are usually computed on the assumption that they are equivalent to the weight of the earth in a frustum of an inverted cone or pyramid covering the foundation and extending to the surface of the earth. The face of this cone is usually taken as making an angle of 30 deg with the vertical.

Horizontal Overturning Force, Sliding, and Indirect Uplift. The horizontal overturning force is also important. Its effect on the base may be resolved into two components, one a horizontal force tending to slide the foundation and the other a moment tending to rotate the base about a horizontal axis. The resistance of the earth to these forces is an obscure subject, especially if the foundation is of irregular shape. The following discussion which neglects several favorable elements may be considered as a conservative view of the earth resistance.

Table XIV. Forces on Tower Foundations

Magnitude of Force on Each of the Four Foundations	Direction at Each Foundation			
	N. E. corner	S. E. corner	S. W. corner	N. W. corner
$\frac{W}{4}$	Down	Down	Down	Down
$\frac{cW'}{a}$	Down	Down	Up	Up
$\frac{F}{4}$ (Note 1)	East	East	East	East
$\frac{dF}{a}$	Down	Down	Up	Up
$\frac{P}{4}$ (Note 2)	South	South	South	South
$\frac{eP}{b}$	Up	Down	Down	Up
$\frac{afP}{4(a^2+b^2)}$ (Note 3)	North	North	South	South
$\frac{bfP}{4(a^2+b^2)}$ (Note 3)	West	East	East	West

NOTES: Where there are no struts between the bottoms of the legs, and especially where the bottom panel is framed on the single system, both of which conditions are usual:

1. The force of the wind F will give a greater force than $F/4$ in an easterly direction on the two west foundations and a correspondingly less force on the other two, and will in addition produce four new forces tending to force the legs apart on the compression side and draw them together on the tension side.
2. Similarly the pull of the wire P will give a greater force than $P/4$ in a southerly direction on the two north foundations and a correspondingly less force on the other two, and will in addition produce a westerly force at the N. E. and S. W. corners and an easterly force at the S. E. and N. W. corners.
3. Similarly, the torsional forces due to P will increase in magnitude and change in direction, and the unbalanced pull P may develop new forces tending to raise two diagonally opposite legs and depress the other two.

The resistance to sliding may be considered as due to the friction of the bottom of the base on the earth, and it may further be assumed that any base large enough to resist uplift will also furnish sufficient friction to prevent sliding. The arm of the overturning force is then the same as the height of the foundation (bottom of base to top where tower is attached), and the overturning moment is equal to the horizontal force multiplied by this arm. The resisting moment may be considered as due entirely to the vertical reaction of the earth on the top and bottom of the foundation. These vertical pressures may be taken as varying uniformly from zero at a horizontal neutral axis through the middle of the base, to a maximum at the edges of the base most remote from the axis. The moment of resistance is calculated as for a beam subject to bending and having a cross-section identical with the area of the base. It may be assumed that any unit pressure allowable on the uplift edge will be amply safe on the opposite edge where the pressure is downward, so that calculations for uplift only are necessary. The maximum allowable unit stress on the uplift edge may be taken as equal to the average unit resistance to uplift of the whole foundation, determined as described above under Direct Uplift.

Limiting Conditions. Usually the most severe condition that a tower foundation is required to meet consists of a combination of uplift and overturning. For this condition the unit stress of uplift proper must be added to the maximum unit stress of uplift due to overturning and the sum must be within the allowable average unit resistance to uplift.

STRENGTH OF FOUNDATIONS. (See also Kent, on Strength and Elasticity.) A foundation is subject to stresses from the tower tending to move it and from the resistance of the earth preventing motion. The foundation should, of course, be strong enough not to break when subject to these opposing forces. As the points of application of the resistance of the earth and the magnitude of the unit stresses transmitted by the earth at any point are subject to great uncertainty, the foundation should be designed for strength for the distribution of earth resistance which is most severe, considering for example that, while the holding power is calculated on a uniformly distributed earth resistance, it may be developed in practice by concentrated pressure from stones or timber located near the outer edge of the base.

IMPORTANT POINTS REGARDING DESIGN OF FOUNDATIONS. The following are important conclusions which follow from the above discussion:

1. The inverted cone theory of resistance to uplift gives a calculated resistance which increases at a rapid rate with the depth (eventually increasing approximately as the cube of the depth). It would be unsafe, however, to apply the theory for foundations differing much from those of usual dimensions, say for depths much exceeding 6 ft and for foundations where the spread of base was much less than the depth to which it is buried.

2. The foot of the tower (top of foundation) should be brought as close to the surface of the ground as possible to reduce the overturning moment.

3. The tower diagonals of the bottom panel of the tower should intersect the corner posts as low as possible, because this is the actual point of application of the overturning force to the foundation, and if this intersection is above the foundation this extra length must be added to the arm of the overturning moment.

4. By inclining the axis of the foundation approximately in line with the inclined tower leg (i.e., to bring it as near as possible into line with the resultant of the horizontal overturning force and the vertical pressure or uplift) a more economical use of material may be made to resist the combined uplift and overturning.

Testing of Towers

Since towers are generally indeterminate structures, new designs are usually tested. Test loads proportional to the loads specified for the design are applied until the required factor of safety has been proved or failure occurs. Towers are usually mounted on a rigid base for testing. This gives the strength which the tower would develop on an "ideal" foundation. In practice the tower must be expected to develop somewhat less strength, as unequal movement of the foundations will ordinarily overstrain certain members. Test loads may be applied by means of weights suspended directly at proper points for vertical loads and applied by means of pulleys attached to a tower-testing structure ("test tower") for horizontal loads. This method of application makes the determination of the test loads easy, but it is inconvenient to have the weights fall any considerable distance if the tower is tested to failure.

TESTING OF FOUNDATIONS. Foundations are occasionally tested, but the test is usually for determining the holding power of the soil rather than the strength of the foundation. For the former purpose the test result will depend largely on the character of the soil and its condition (dry, wet, or frozen). Tests for holding power are necessary only for uplift and overturning forces. In testing it is important that the testing machine should not press down on the surface of the soil near the foundation. The machine should rest on the ground outside of the base of an inverted cone of angle of 45 deg from the vertical and enveloping the base of the foundation under test.

Specifications, Contracts, and Proposals

Different manufacturers of towers use different details of construction, so that specifications written for the purpose of obtaining proposals should contain only conditions and requirements. These should state the loadings, possible overload, and maximum allowable stresses, and should show outline dimensions of the tower as determined from clearances required.

The specifications should state whether towers are bolted or riveted, galvanized, or painted, shipped assembled or partly assembled, tested or not tested, etc., besides containing the usual structural-steel specifications; see Steel.

The proposals should show the arrangement and sizes of members and typical details of connections.

Contracts are oftentimes let on the basis of furnishing an approximate number of towers when the exact number required cannot be determined in advance, and a unit price per pound is included to cover extensions, special foundations, and modifications that may be required.

The galvanizing of towers and parts is usually specified to pass the A.S.T.M. specification for galvanized steel (see Galvanizing for Iron or Steel). All members of towers that are not too heavy, so that there is danger of their buckling in the process, may be specified to be galvanized by the hot-dip process.

Installation and Erection

Towers are generally shipped disassembled, with each piece marked to show the type of tower of which it is a part, also with a part number to show its location in the tower. Similar parts are usually bundled together with the proper number of each for an individual tower. A tower schedule should be drawn up, showing, for each tower number; the type of structure; shipping point; extensions, if any; adjacent spans; and angle in line, if any. A bill of material for each different kind of tower in the line should be provided. These schedules will aid materially in the distribution and checking of material.

The staking gang follows the right-of-way clearing gang, staking the tower locations

accurately according to the survey. Closely following the staking gang comes the material distribution crew, and after them the foundation gang.

PREPARATION OF FOUNDATIONS. Foundations should be set with their bases below the frost line. The hole should not be excavated deeper than necessary, so that the foundation may rest on undisturbed earth. If any of the several foundations of a tower rest on loose backfill, unequal settlement may be expected which may greatly weaken the tower. The hole should not be larger than necessary, as the backfill will be less effective than undisturbed earth in resisting uplift. The resistance of concrete foundations can sometimes be increased by digging a hole smaller in diameter than the base and undercutting it at the bottom; with structural-steel foundations large stones can sometimes be placed against the steel to increase its resistance to motion, either vertical or lateral. The backfill should be well tamped in place, and especial care should be used if it is probable that the foundation will be subject to heavy stress before the earth has had time to settle. On sloping ground, filling may be required on the low side to give the designed weight of earth against uplift. In water or mud the floating power of hydrostatic pressure beneath the foundation must be allowed for. Where towers are raised on extensions the increased foundation stresses due to increased moment must not be overlooked.

The several foundations of a tower should be set accurately by template both as regards spacing and elevation so that towers stand truly vertical and have no initial stresses due to distortion.

ERECTION OF SUPERSTRUCTURE. The method of erection depends largely upon the type of tower and its location. Short, light towers, under favorable conditions, may be completely assembled lying on the ground. Such a tower may be raised into position by the use of an A-frame or gin pole. A line is connected to the tower slightly above its center of gravity and run over the top of the gin pole or A-frame and through proper rigging to the source of power. Sufficient guys and hand lines should be provided to control the position of the tower while being raised.

Heavy towers must be assembled piece by piece, although panel sections may be assembled on the ground and raised into position by means of booms, floating gin poles, or other suitable rigging. Individual pieces are raised to position by hand lines or tackle attached to the leg members.

With bolted connections the nuts should be prevented from working loose by checking threads on bolts, riveting over the end of the bolt, or by lock-nut or washer.

Each tower member should have a number which should be shown on the erection drawings and marked on the corresponding member of each tower.

The members should be cut, bent, and punched to template so that the parts will be interchangeable and the assembled tower will fit the foundation prepared for it.

A clean-up gang should follow the wire stringing crew to inspect and tighten bolts, pick up left-over material, and make sure everything is in proper condition. This gang should work with the inspecting engineer who makes the final inspection.

Maintenance of Towers

All bolted connections on towers should be carefully watched and kept tight, and the galvanizing or paint should be inspected regularly and towers must be repainted before any deterioration from rusting occurs. Foundations are the most likely source of trouble in operation. These should be kept properly backfilled and should be watched for unequal settlement of legs.

Dimensions, Weights

Data on these items for a number of towers are given in Table XV.

Table XV. Data on Typical Steel Towers

Use	Conductors		Number of Ground wires	Voltage, kv	Height, feet		Base, feet		Normal Span	Maximum Angle, degrees	Weight, pounds
	Number	Size, circular mils			To Lower Cross-arm	Overall	Across Line	Along Line			
T	6	336,400 ACSR	1	132	64.0	95.0	18.6	18.6	1060	5	10,900
L	6	336,400 ACSR	1	132	64.0	96.6	22.0	22.0	1060	15	13,600
A	6	336,400 ACSR	1	132	64.0	100.0	22.0	22.0	1060	45	20,000
T, L, A	3	336,400 ACSR	1	132	45.0	53.0	23.0	17.0	3000	45	13,000
T, L, A	3	397,500 ACSR	1	132	45.0	55.0	23.0	17.0	4000	45	15,000
T	3	795,000 ACSR	2	220	71.1	79.6	33.0	33.0	1100	1 1/2	13,110
A	3	795,000 ACSR	2	220	71.1	79.6	33.0	33.0	1100	60	30,910
*	*	*	*	*	*	55.0	2.5	2.5	*	*	2,640

* Latticed steel pole, not including arms, max. bending moment = 165,000 lb-ft, weight includes 6-ft length in ground. T-suspension tower; L-angle tower; A-dead-end tower.

Table XV—*Continued*

Data on Foundations for Above Towers

Type	Number per Tower	Depth below Ground, ft	Area, in.	Weight Each
Steel	4	9	45 × 60	406
Steel	4	10	45 × 60	403
Steel	4	12	72 × 76	1082
Steel	4	9	72 × 74	665
Steel	4	11	60 × 82	843
Steel	4	7	....	392
Steel	4	7	....	2675

27. CROSS-ARMS

Cross-arms are usually of wood though sometimes of steel. "Buck-arms" or "reverse arms" are cross-arms attached to a pole at right angles to the principal arms, and are used for taking off wires at right angles to the line, either at the junction of intersecting lines or at services. "Double arms" are pairs of cross-arms attached to opposite sides of a pole so as to act as one compound arm. Double arms are used to increase the strength of an arm and to permit the use of two pins and two insulators for supporting a single wire where additional strength is required.

Cross-arms are used principally for supporting pins, insulators, and wires, though lightning arresters, transformers, switches, and other miscellaneous appliances are often mounted on them, usually for the purpose of keeping the pole free of incumbrances so that it will be more easy to climb.

When city distribution lines are located in alleys it is common to locate poles next to the property line. Where it is not permissible to let arms overhang private property special arms may be used which extend on one side of the pole only. These must be well braced and should not be used for dead-ending wires. Another type of construction is obtained by locating two poles on opposite sides of the alley and putting special cross-arms across the alley between them.

Forces on and Stresses in Cross-arms

The forces which a cross-arm resists are:

(*a*) Vertical forces due to weight of pins, insulators, wires (with sleet), and accidental loads due to linemen standing on arms, etc.

(*b*) Transverse horizontal forces due to wind pressure on the wires (with sleet) at right angles to the line.

(*c*) Longitudinal horizontal forces due to the pull of the wires where the pull is unbalanced. Unbalanced pull is usually due to an angle in the line, the ending of the wire at the arm, a change in the size of wire at arm, and an unequal tension in the spans on the two sides of the arm.

The principal internal stresses produced in an arm from these forces are:

1. A bending force in a vertical plane due to vertical forces (*a*).
2. A bending force in a horizontal plane due to horizontal forces (*c*).
3. A twisting force about the longest axis of the arm due to the "pin leverage" of the horizontal forces (*c*). The pin leverage is the distance from the center of the wire to the axis of the arm.

Of these stresses the most destructive is probably the twisting stress which tends to split the arm in a vertical plane through the pinholes and along the grain of the wood. On this account the pin and insulator should be no taller than necessary, and the pin should extend completely through the arm to give the best distribution of bearing pressure.

The vertical and horizontal bending stresses are of some importance and may be computed by the usual beam formula. Data of tests on strength of cross-arms for these stresses are given below.

STRENGTH TESTS OF CROSS-ARMS. (Forest Service Cir. 204.) Tests made on 3 1/4 in. by 4 1/4 in. by 6 ft, 6-pin air-dried cross-arms with vertical load distributed equally at each pin hole gave the results in Table XVI.

The tests showed that for ordinary use the strength, for vertical loads of 6-pin arms, need not be considered in calculations of line construction, except in the rare case of abrupt change of the grade of the line.

Table XVI

Kind of Wood	Average Maximum Load, lb	Maximum Crushing Strength, lb per sq in.
Longleaf pine, 75 per cent heart	10,180	8950
Longleaf pine, 100 per cent heart	9,780	8940
Shortleaf pine	9,260	7300
Longleaf pine, 50 per cent heart	8,980	5425
Shortleaf pine, creosoted	7,650	5770
Douglas fir	7,590	7080
White cedar	5,200	4700

Cross-arm Classifications

Cross-arms are classified generally as (1) telephone arms, (2) power distribution arms, and (3) transmission arms. They are more specifically designated by stating the number of pin positions, overall length, cross-sectional dimensions, and whether standard arms, alley arms, or other special types.

Requirements for Cross-arms

The dimensions of cross-arms standardized by the N.E.L.A. are shown in Table XVII.

Table XVII

Use of Arms	Number of Pins	Width, in.	Depth, in.	Length, in.
Distribution—Std	2	3 1/4	4 1/4	36
Distribution—Std	4	3 1/2	4 1/2	67
Distribution—Std	6	3 1/2	4 1/2	96
Distribution—Std	8	3 3/4	4 3/4	120
Distribution—Alley	6	3 1/2	4 1/2	96
Transmission—Std	2	3 3/4	4 3/4	84
Transmission—Std	4	3 3/4	4 3/4	120

Transmission cross-arms suitable for use with suspension insulators are usually special and have cross-sectional dimensions 4 in. by 5 in. or 5 in. by 7 in., or larger if necessary.

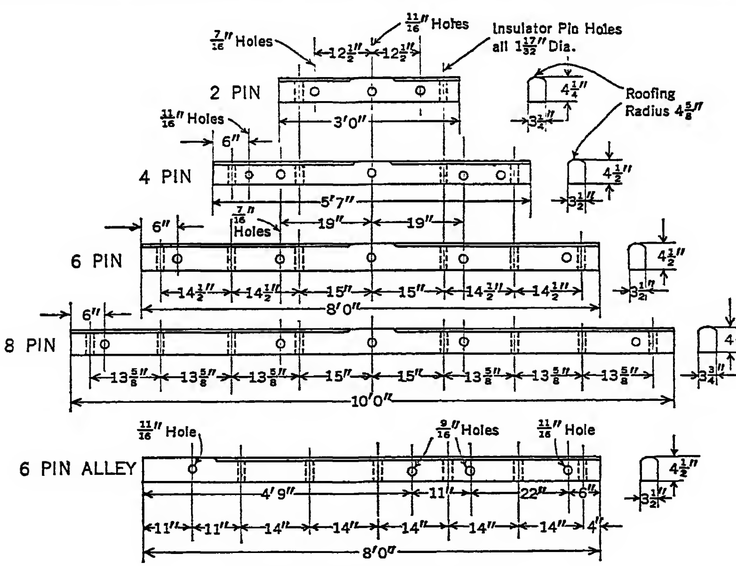

FIG. 26. Standard Cross-arms

Fig. 26 shows dimensions and drilling for N.E.L.A. standard cross-arms. The strength of double arms, with spacing bolts, in the direction of the line should not be considered as more than 30 per cent greater than that of a single arm.

DISTANCE BETWEEN POLE PINS. The requirements for climbing space determine this dimension. If telephone cross-arms are used on joint poles with power cross-arms above them, the pole-pin spacing must be the same as required for the power cross-arms. On power cross-arms the pole-pin spacing is required to be: 24 in. for voltages up to 300, 30 in. for voltages up to 7500, 36 in. for voltages from 7500 to 15,000, and more than 36 in. for voltages above 15,000.

DISTANCE BETWEEN OTHER PINS. For low-voltage power distribution cross-arms the pin spacing called for by the N.E.L.A. Standards is 14 $1/2$ in. The 4-pin medium-voltage transmission cross-arm standardized by the N.E.L.A. has a pin spacing of 36 in. On long-span construction even on low-voltage lines it may be necessary to have greater conductor spacings. This is often accomplished by using longer arms with more pin positions, but omitting intermediate pins. Pins not used should be left out of the arm.

DISTANCE FROM END PIN TO END OF ARM. This is 4 in. for low-voltage power arms and 5 in. for medium-voltage transmission arms. These distances are chosen to minimize splitting of the arm by pin leverage.

PINHOLES. Pinholes are specified to be of a diameter which will give a close fit for the pin. N.E.L.A. standard wood pins have a maximum diameter at the shank of 1 $1/2$ in. Pinholes for such pins are to have nominal diameters of 1 $17/32$ in. and are tested with a steel gage having plugs of 1 $1/2$ in. and 1 $9/16$ in. The hole must take the 1 $1/2$-in. plug without forcing, but not the 1 $9/16$-in. plug. Holes for steel pins must take a testing plug $1/16$ in. smaller than the diameter of the pin shank without forcing.

BOLT HOLES. For fastening the arm to the pole one $11/16$-in. hole is placed at the center of the arm for a $5/8$-in. machine bolt. For fastening flat cross-arm braces to the arm two $7/16$-in. holes are placed each side of the middle of the arm 38 in. apart. These holes are drilled to take $1/2$-in. machine bolts. Bolt holes are tested by steel gages and must take a test plug $1/16$ in. smaller than the bolt diameter without forcing.

SPECIFICATIONS. The points to be covered in specifications for cross-arms are as follows:

Material. Usually Douglas fir or creosoted yellow pine.

Quality of Material. Should be free from loose or unsound knots, loose heart, rot, shakes, wane, or wormholes. Checks should not exceed 12 in. in length, $3/4$ in. in depth, or $1/16$ in. in width. Grain should be parallel with the axis of the arm within 5 deg. Knots larger than $1/4$-in. diameter should be avoided. Large pitch pockets are not desirable. Sapwood should not be allowed in yellow pine arms nor sapwood in excess of 25 per cent in Douglas fir arms. Excessive warp should not be allowed.

Dimensions, Pinholes, etc., as noted in previous section.

Seasoning. Cross-arms should be thoroughly air-dried or kiln-dried before manufacture. When arms are dried after manufacture the pinholes become elliptical owing to shrinkage across the grain.

Preservative Treatment

Yellow pine cross-arms are usually pressure treated with creosote. Since the heartwood of yellow pine is difficult to penetrate with preservative, the treating process should be carried out so as to obtain the maximum penetration without injuring the timber. (See Preservative Treatment of Poles.)

Cross-arms should not be painted, especially with paints containing oil, although creosote stains may be used. Paint seals the pores of the wood and prevents evaporation of moisture. Retained moisture promotes fungus growth, and painted arms soon rot internally.

Use of Specifications

Where the maximum overall economy can be obtained by using the best quality of material standard specifications should be strictly adhered to, but local conditions may make it advisable to use cheaper material in some cases. Under such conditions the requirements may be relaxed somewhat, but consideration should be given to the probable life or failure of the material.

Storage

Cross-arms held in storage should be stacked on creosoted skids. Each layer should be placed at right angles to that below it. Plenty of air space should be left around each arm for ventilation. Each stack should be roofed to drain off rain, and it should be protected from the direct sun. Cross-arms should not be stored in heated buildings; open sheds are best.

28. INSULATORS FOR OVERHEAD LINES

Insulators for overhead lines may be classified as pin-type, suspension, and strain insulators. Pin-type insulators are used for low- and medium-voltage lines. Suspension insulators are used for all voltage lines. Strain insulators are used in guys and for dead-ending low-voltage lines. The general features of design common to all classes of line insulators will be first considered.

Design of Line Insulators

The insulating materials principally used for line insulators are: wet-process porcelain, dry-process porcelain, and glass. Wet-process porcelain is used for this purpose to a far greater extent than dry-process porcelain. Wet-process porcelain has greater resistance to impact and is practically impervious to moisture without glazing; dry-process porcelain is not. However, dry-process porcelain, if well made, has a somewhat higher crushing strength. Dry-process porcelain is therefore used only for the lowest-voltage lines, and even there to only a limited extent.

Prior to the development of high-grade wet-process porcelain, glass was used extensively for all lines up to about 44 kv. The glass available at that time was fragile and subject to breakage due to temperature changes combined with the effect of internal strains. Improved porcelain quickly eliminated glass from line use except for low-voltage signal lines. Recently, however, great advances have been made in the art of glass manufacture resulting in the development of glass insulators which are quite tough and which have low internal strains. The use of these insulators is therefore increasing for low- and medium-voltage lines.

An insulator must be designed to stand extreme and sudden temperature changes, sleet and rain, as well as smoke, dust, and often special conditions such as salt fogs, salt-water sprays, and chemical fumes, without deterioration from chemical action, breakage from mechanical stresses, or electrical failure. The design of high-tension insulators is a process of compromise between requirements often antagonistic in nature.

PROPERTIES OF PORCELAIN AND GLASS. Table XVIII gives some of the more important characteristics of porcelain, glass, and Pyrex.

Table XVIII

Property	Glass	Porcelain	Pyrex
Tensile strength, lb per sq in	7600–12,100	6000–8500	3000
Crushing strength, lb per sq in	12,100–50,000	44,000–60,000	120,000
Modulus of elasticity, lb per sq in	7–12 ($\times 10^6$)	10–15 ($\times 10^6$)	9×10^6
Coefficient of expansion per deg F	4.4–4.92 ($\times 10^{-6}$)	1.82–3.7 ($\times 10^{-6}$)	1.8×10^{-6}
Coefficient of expansion per deg C	7.9–8.83 ($\times 10^{-6}$)	3.3–6.6 ($\times 10^{-6}$)	3.2×10^{-6}
Weight, lb per cu in	0.09–0.125	0.08–0.085	0.0815
Puncture strength, kv per in	1800–3000	316–695	2800+
Dielectric constant	6.0–8.0	6.15	4.48

SHAPE. The insulating surfaces should conform to the flow lines of the electrostatic field; the surfaces of the rain sheds or petticoats should conform to equipotential surfaces. The leakage resistance per shell for multipart pin-type insulators should be about equal; the plane of mechanical rupture should not coincide with the plane of electrical stress, and the unit should have approximately equal capacitances per shell.

Thickness to Resist Puncture. The insulating material must be thick enough to resist puncture by the combined working voltage of the line and any probable transients whose time lag to spark over is great. If this thickness is greater than desirable from a manufacturing standpoint, two or more pieces are used to give the proper aggregate thickness. The thickness of a porcelain part must be so related to the distance around it that it will arc over before it will puncture. The ratio of puncture strength to arc-over voltage is the factor of safety of the part, or of the insulator, against puncture.

This ratio should be high to give sufficient margin to protect the insulator from puncture by the transients before mentioned.

LEAKAGE AND ARCING DISTANCES. The leakage and the dry and wet arcing distances are criteria of the effectiveness of insulators in their insulating function. These terms are defined in the following paragraphs and are illustrated in Fig. 27. Table XIX gives values for these factors for representative insulators.

Leakage distance is the shortest distance between conductor and pin, or between cap and pin, measured along the surface of the insulating material. This distance is roughly

proportional to the amount of voltage required to cause leakage current to flow over the surface of the insulator. It does not, however, take into account the varying width of leakage path nor the conductivity of the dirt or moisture film which may be deposited on the surface.

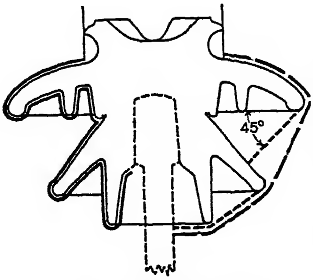

Fig. 27

Wet arcing distance is the shortest distance between conductor or cap, and pin, measured partly over the surface of the insulating material and partly through the air. Where, in any portion of the path to be measured, the striking distance through air between any two points is less than the length of path over the surface between those points, the air striking distance is measured for that portion of the wet arcing distance in place of the distance over the surface. The wet arcing distance is a fairly good measure of the voltage required to arc over the insulator when exposed to rain.

Dry arcing distance is the shortest distance between conductor or cap, and pin. It is measured entirely through the air, and is a fairly good measure of the voltage necessary to arc over the insulator when it is perfectly dry.

Table XIX. Characteristics of Typical Insulators

Type	Dry Flashover, kv	Leakage Distance, in.	Dry Arcing Distance, in.	Wet Arcing Distance, in.
Low-voltage pin-type	50	4.75	3.12	1.37
Low-voltage pin-type	70	9.00	4.50	2.25
Medium-voltage pin-type	95	12.12	6.25	4.00
Medium-voltage pin-type	170	35.50	14.37	9.50
Suspension 10 in. disc	80	12.00	8.00	4.12
Suspension 12 in. disc	95	12.50	9.20	4.30

FREE ARCING. The porcelain must extend beyond the charged conducting connections (i.e., tie wire or cap at the top and pin at the bottom) sufficiently so that the distance between the connections through the air around the porcelain is greater than the arcing distance of the maximum voltage to be carried. The arcing distance required for a given voltage may be determined roughly (but only very roughly) from the tables of arcing distances between needle points; see article on Spark Gap. The greater radius of curvature (compared with needle points) of such metal parts as the insulator pin decreases the potential gradient at the terminals. Also the porcelain has a much greater specific inductive capacity than the air, and its proximity to the arcing path disturbs the electrostatic field through the air. Surface charges on the porcelain because of surface leakage or corona also modify the field.

Free arcing is the property of arcing-over along a line which does not touch the porcelain body from the point where the arc leaves the metal cap to where it strikes the metal pin. Where the arc touches any part of the porcelain the great heat fractures the porcelain in a few seconds; hence the desirability of designing the insulator so that it is free arcing. A properly designed insulator will arc over as a whole before any individual part (i.e., shell or unit) arcs over. In many defective designs the insulator will fail by some parts arcing over, thereby increasing the voltage on others which then fail by puncture or arcing over.

SPREAD OF PETTICOATS OF PIN INSULATORS. In two concentric shells the two surfaces which lie opposite to each other are at different potentials except where they are cemented together. The difference in potential between two points on opposite surfaces is greater the further they are removed from the joint. Unless the shells diverge correspondingly so as to increase the distance between the shells as the potential increases the air will break down and part of the leakage surface will be short-circuited by a corona discharge. This divergence is shown in Fig. 32 where the top is a disk made slightly convex to shed water and the inner shells are cone shaped.

MINIMUM HEIGHT OF PIN. Pin insulators mounted on metal cross-arms should be provided with metal pins which have sufficient length above the cross-arm to insure that flashover will take place to the pin rather than the arm, otherwise the full flashover value of the insulator will not be obtained. The distance to the arm from the lowest skirt should never be less than the shortest distance from the skirt to the pin even with wood arms. On the other hand, the pin should not be longer than necessary on account of the increase in bending moment on the pin with increased height.

COLOR AND GLAZING. Brown, slate and white are the common colors used in glazing porcelain. Brown is the most common color since it is more of an aid in determining faults. Slate-colored glazing of the same color as galvanizing on towers makes insulators a less conspicuous target for malicious destruction.

Glaze is somewhat similar in composition to porcelain. The ingredients are mixed with water and held in suspension while the unfired insulator is dipped in the mixture. Upon firing, the glaze takes on a glass-like consistency which retards the collection of dirt and allows the dirt to be washed off easily by rain.

CEMENTING OF INSULATORS. Porcelain insulator parts are cemented together with neat Portland cement which is carefully selected for quality, strength, fineness, and absence of metallic particles. The insulators after cementing are placed in compartments or tanks and slowly cured under automatically controlled humidity and temperature conditions. The surfaces to be cemented are left unglazed and are either corrugated or sanded to obtain good bond.

PINHOLES. The pinholes in pin-type insulators are made in various forms to accommodate various forms of pins. Insulators for pins with full-size threads are made with the threads molded in the porcelain or have threaded sheet-metal thimbles cemented into unthreaded sockets. Some pins have malleable-iron thimbles, and insulators for use with these pins have the pin sockets unglazed, but with either plain or corrugated surfaces for cementing to the thimbles. Some insulators for this purpose have the sockets glazed but with sharp porcelain grains held to the surface by the glaze. Special flexible cements are often used for thermal relief.

FAULTS IN INSULATORS. The more common faults in porcelain are porosity, folds and flaws in molding, and the development of checks and hair cracks in process of drying, incomplete and non-uniform glazing, warping, air bubbles, conducting impurities, under- and over-firing, and chipping of edges. Only 50 to 75 per cent of molded shapes ordinarily pass final test, and even fewer of the more difficult shapes. Inspection and testing are essential to eliminate faults in both design and manufacture.

Distribution Line Insulators

Insulators for power lines from 115 to 17,000 volts are included in this classification. Both pin-type and suspension insulators are used for lines within this voltage range. For secondary lines carried on racks, spool insulators are used.

PIN-TYPE INSULATORS. The smallest insulator in this class is the side-groove rounded-top type. The side groove is $^3/_8$ in. in radius, the overall height is 3 $^9/_{16}$ in., and the skirt diameter is 3 $^1/_4$ in. The dry flashover voltage is about 50 kv, and the wet flashover voltage is about 20 kv. This insulator is suitable for voltages up to 5 kv, although it has some disadvantages in not being able to accommodate large wires and in not having a top groove.

All the other insulators in this class are provided with both top and side grooves. The most-used insulator of this class is suitable for lines up to 7500 volts. It has top and side

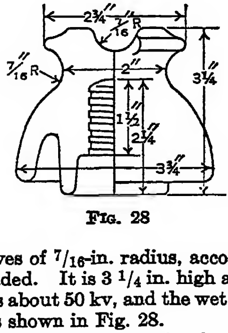

FIG. 28

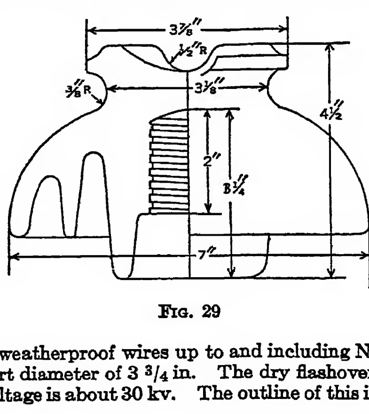

FIG. 29

grooves of $^7/_{16}$-in. radius, accommodating weatherproof wires up to and including No. 4/0 stranded. It is 3 $^1/_4$ in. high and has a skirt diameter of 3 $^3/_4$ in. The dry flashover voltage is about 50 kv, and the wet flashover voltage is about 30 kv. The outline of this insulator is shown in Fig. 28.

The larger pin-type insulators of this class are similar in form to the insulator described in the preceding paragraph. All dimensions are increased to provide the increased flashover values and leakage distances required for the higher voltages. An example of an insulator

suitable for 17-kv lines is shown in Fig. 29. This insulator has a dry flashover of 90 kv and a wet flashover of 50 kv.

All the pin-type insulators of this class are made for pins with 1-in. diameter tops, but the larger sizes are also made for 1 3/8-in. pins.

SUSPENSION INSULATORS. In this voltage class, suspension insulators are used mainly for dead-ending lines or for angle construction, although in a few cases lines of 15 to 17 kv have been built using suspension insulators throughout. The ones most used are of the cap and pin (or disk) type. The insulator disks have a diameter of 6 to 10 in. The caps are malleable iron or steel and are provided with clevis tops for connecting to the cross-arm. The pin takes the form of a hook, eye, or ball, depending upon the method of connection desired. Still other variations of cap and pin designs are available for special purposes.

The top of the insulator disk is provided with a hollow projection in the shape of a truncated cone. This projection is cemented into the hollow cap, and the pin is cemented into the hollow portion of the cone. Fig. 30 shows a typical suspension insulator for low-voltage lines. It has a dry flashover of 50 kv and a wet flashover of 30 kv and is suitable for use on lines up to 7500 volts.

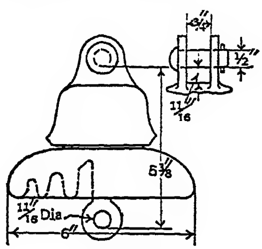

Fig. 30

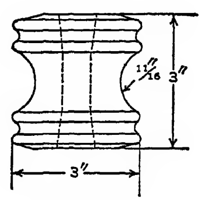

Fig. 31

SPOOL INSULATORS. Insulators to be used on secondary racks having vertical rods for insulator supports are of the spool type. These are available in two standard sizes, one for light racks and one for heavy racks. Fig. 31 shows a typical insulator for heavy racks. The smaller size spool is similar in shape with a height of 2 1/8 in. and a diameter of 2 1/4 in. The groove radius is 1/2 in.

Transmission Line Insulators

For transmission lines above 17 kv both suspension insulators and pin-type insulators are used although the pin-type insulators are now mainly confined to lines below 45 kv.

PIN-TYPE INSULATORS. Pin-type insulators for high-voltage transmission lines must necessarily be made up of two or more parts cemented together. This construction has often resulted in radio interference due to lack of a good bond between the cemented parts, and in early failure due to the effect of temperature changes. It is not now considered good practice to use pin-type insulators having more than two parts. Fig. 32 shows a pin-type insulator suitable for use on lines of 35 kv. The dry flashover of this insulator is about 125 kv, and the wet flashover is about 85 kv.

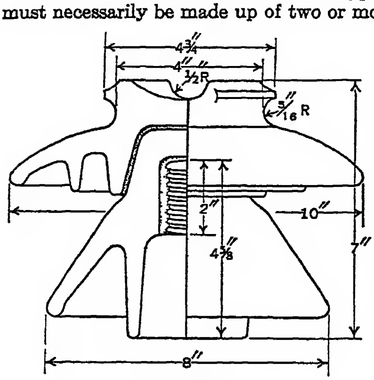

Fig. 32

SUSPENSION INSULATORS. Suspension insulator units are usually connected together in strings of two or more for high-voltage transmission lines to provide sufficient insulation. The construction of these units is similar to that described under the subject of Distribution Line Insulators. Ten-inch disks are used almost exclusively, although high-strength 12-in. disks have been used under special conditions. Suspension insulators are designed to give spacings between individual units of 4 3/4 in. to 7 in., to meet varying insulation requirements. For most purposes, however, the 5 3/4-in. spacing is satisfactory. Suspension insulators are made with different

mechanical strengths. It is therefore possible to select insulators to meet almost any conditions, including the economic requirements of a given line.

The suspension-type insulator is always used in tension, the connections at the two ends being made so that the insulator is free to swing in any direction; the insulator takes

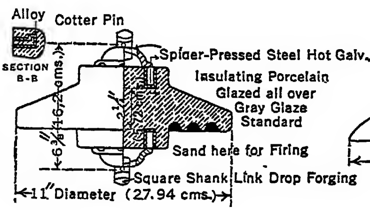

FIG. 33

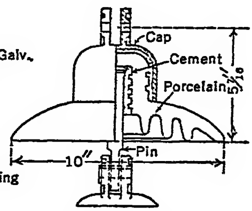

FIG. 34

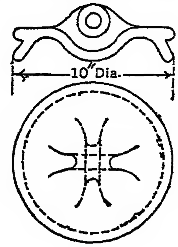

FIG. 35

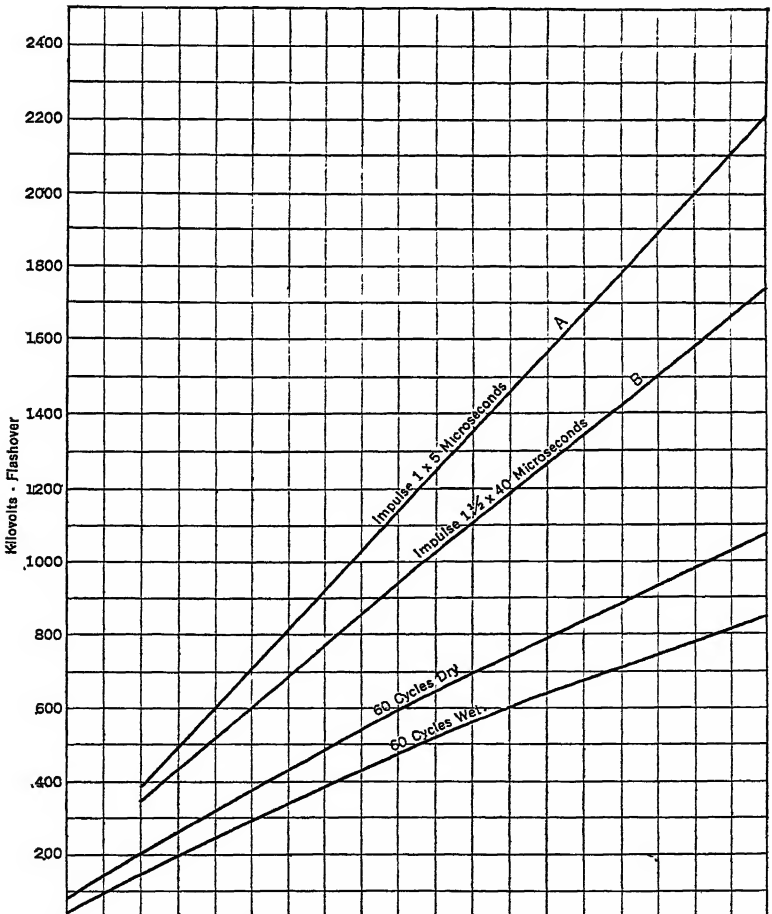

FIG. 36

such a position that its axis coincides with the direction of the mechanical stress. This type is used hanging below the cross-arm with axis vertical as a suspension insulator for sustaining the weight of cable at points where there is little horizontal force, and also with axis approximately horizontal as a " strain " or " dead-end " insulator at points where the horizontal force predominates.

Electrical Characteristics of Insulator Strings

The potential per unit required to flash over a suspension insulator string composed of units of the same design decreases with an increasing number of units as shown in Fig. 36. This is due to an unequal potential gradient. The potential gradient for various insulator strings is shown in Fig. 37. If grading shields or arcing rings are used in connection with

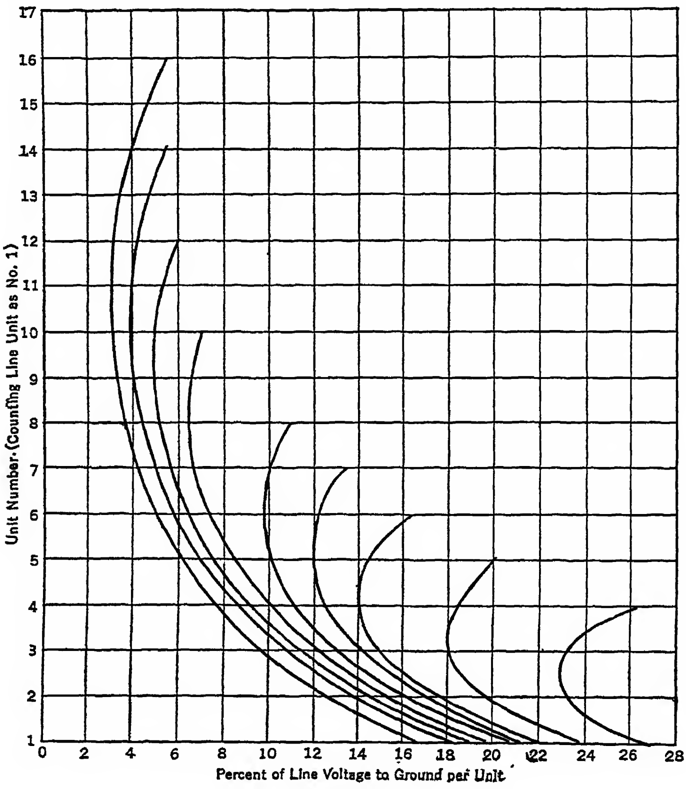

FIG. 37

an insulator string the voltage gradient is changed and made more uniform. A typical effect of adding grading rings is shown in Fig. 38.

The impulse flashover characteristics of typical insulator strings is shown in Fig. 36. Curve *A* gives the crest voltage for flashover of a positive wave of 1 × 5 microseconds (i.e., reaching crest value in 1 microsecond and dropping to $^1/_2$ crest value in 5 microseconds from the start of the wave). Curve *B* gives flashover values for a positive wave of 1 $^1/_2$×40 microseconds.

Insulation Coordination

It would be desirable, of course, to make a transmission line proof against all flashovers, if that were possible. It is doubtful whether it is possible so to insulate a line as to achieve this result. Lightning voltages are so great that any amount of insulation that is economically feasible would probably fail to eliminate flashovers. Increasing the insulation

of the transmission line itself only transfers the flashovers to the terminal equipment where they cause greater damage than they would on the line. Therefore, the insulation of terminal equipment should be greater and better protected than the line insulation. Lightning arresters, shielding, and other methods are feasible and economical at substations but not on the line. For line insulation, protection arcing and grading rings, ground wires

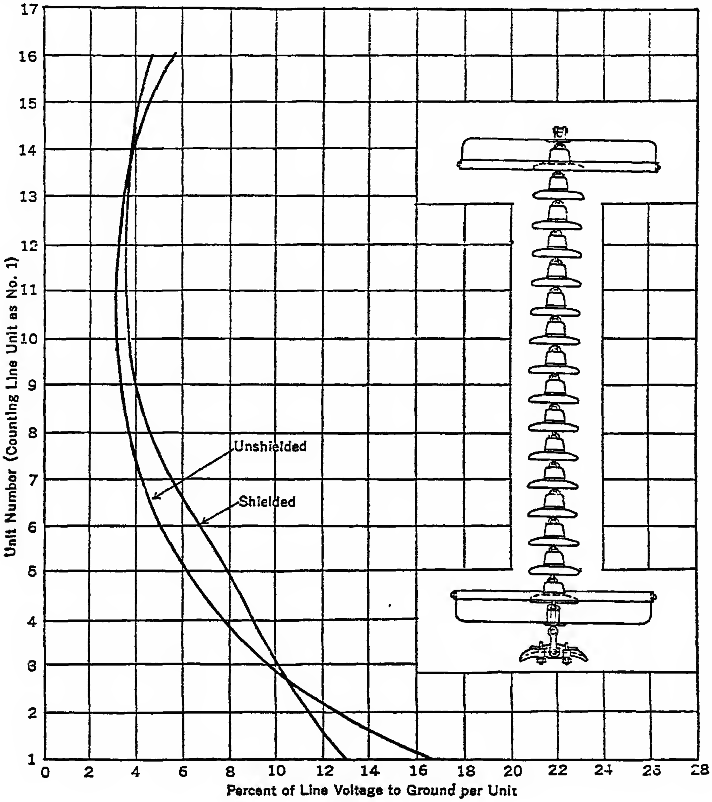

Fig. 38

and counterpoises, and expulsion protective gaps have been used with fairly good results. (See Bibliography.)

Strain Insulators

Insulators of this type are used primarily for insertion in guy wires. Most porcelain strain insulators are designed to place the porcelain under compression and are of cylindrical form with grooves for the loops of the guy wires. The design is such as to allow the loops of the two sections of guy wire to be interlaced with porcelain between them. Insulators for the higher voltages are provided with deep fluting or fins to provide additional leakage surface. This design prevents mechanical failure of the guy even if breakage of the porcelain takes place. Insulators of this type are also used for dead-ending circuits up to about 2500 volts. Several styles of metal clevises are available for attaching them to poles or cross-arms when used for dead-ending purposes. Fig. 39 shows a typical strain insulator.

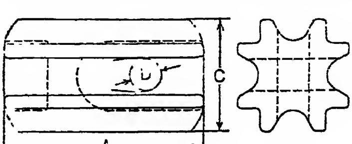

Fig. 39

Wood strain insulators are coming into greater use for guys on wood transmission line structures in order to increase the overall insulating value of the wood structure against

lightning voltages. The flashover value of insulation under lightning voltages depends primarily on its length between live parts and grounded parts. Wood provides a long flashover path at lower cost than any other material of the same mechanical strength. The wood is used in tension and is connected to the guy wires by metal end pieces specially designed to develop the full mechanical strength of the wood. Arcing horns are provided to make sure that any possible arc across the insulator will be kept away from the wood.

Tests of Insulators

Tests of insulators may be classified as: (1) design tests, made on a few insulators to determine the electrical and mechanical characteristics of each different design; and (2) routine tests, made on all or a certain percentage of each lot of insulators purchased, to detect defects of material or workmanship. Standard 41 of the A.I.E.E. gives complete test specifications for porcelain insulators, which are generally accepted. These specifications, slightly modified, may also be used for glass insulators. Space does not permit the incorporation of these specifications here. They are reprinted in several manufacturers' catalogs.

DESIGN TESTS include: wet and dry flashover, corona, and puncture tests. For suspension insulators a combined mechanical and electrical test is required.

ROUTINE TESTS include: (1) Continuous dry flashover for 3 minutes for pin-type insulators, both on individual parts and on the assembled insulator. For suspension insulators the flashover is continued for 5 minutes. (2) Thermal change tests on the assembled suspension insulators, and on individual parts as well as assembled pin-type insulators. (3) Porosity tests. (4) Puncture tests. (5) High-frequency and combined mechanical and electrical tests on suspension insulators only.

TESTING ON THE LINE. Periodic testing of insulators on the line should be carried out to eliminate deteriorating units before they fail and interrupt service. Various patented methods are available for line testing of insulators. Generally these use the voltage gradient method. Measurement of the dielectric power factor is an excellent method for testing off the line.

Selection of Insulators

The first step in the selection of insulators is to determine the insulation level of the line relative to that of the terminal equipment. Both normal-frequency flashover voltages and lightning voltages should be considered. The type of supporting structures (wood or steel) has a considerable effect on the insulation efficiency of the line. The possibility of overvoltages due to switching surges and to arcing grounds on isolated neutral systems should be taken into account. The mechanical strength requirements should also be evaluated. Manufacturers' catalogs now give very complete and reliable data to aid in insulator selection.

Installation of Insulators

See the article on Line Erection.

29. INSULATOR PINS AND HARDWARE

Insulator pins are made of wood or steel. Insulator hardware is made of steel or malleable iron.

Wood Pins

All-wood pins are now used only for lines up to 7500 volts. They are unsatisfactory on higher voltages for they deteriorate rapidly owing to the combination of moisture and leakage current. Black and yellow locust are the most satisfactory woods for the purpose, although eucalyptus, oak, birch, osage orange, gum, and other woods have been used successfully. Fig. 40 shows the N.E.L.A. standard wood pin. It should be noted that the 4 1/4-in. shank will not develop the full strength of cross-arms 4 1/2 in. or more in depth.

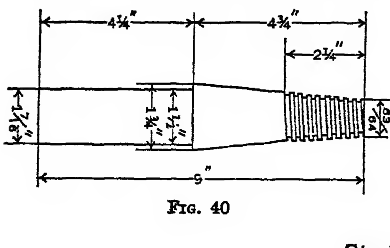

FIG. 40

Steel Pins

Steel pins are available in many designs for any voltage class of insulators. They are made of forged steel and are usually galvanized. Steel pins for low-voltage insulators

are made with long and short bolt shanks and also with lag screw shanks. The threaded head is made of cast lead, pressed steel, or a helix of spring steel wire. The lead head has the advantage of giving a tight fit without tending to burst the insulator under temperature changes. Fig. 41 shows a typical pin of this type.

Steel pins for higher-voltage insulators are similar to those for low voltages, except in size and strength. In addition to being made with lead heads they are also made with special threaded tops for screwing into thimbles which are cemented into the insulators before placing on the pins. Fig. 42 shows a pin of this general type.

Since steel pins may be designed to have greater strength than obtainable with wood pins they are more desirable for heavy conductors and long spans. Owing to the smaller size of shank, steel pins require less wood to be removed from the cross-arms and therefore do not reduce the cross-arm strength as much as wood pins do. Cross-arms rot around the pinholes rather quickly when wood pins are used, which is not the case when steel pins are used.

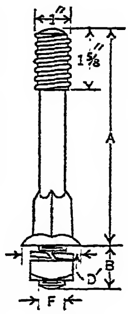

Fig. 41

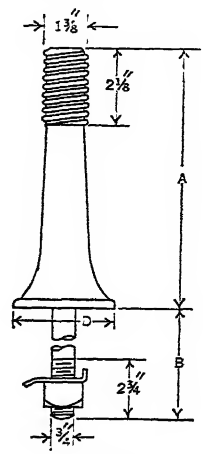

Fig. 42

Attachment of Pins to Arms

Pinholes in cross-arms for wood pins are bored $1\,^{17}/_{32}$ in. and the diameter at the top of the shank of pins is required to be $1\,^{1}/_{2}$ in. On account of swelling or shrinking of the wood the pins may fit tightly or loosely. In order to make sure that the pins will not pull out of the arm a 4 or 6 penny galvanized nail is driven through the arm and the pin. Pinholes for steel pins are bored $^{1}/_{16}$ in. larger than the shank diameter. The end of the pin shank is threaded, and in addition to the nut the pin is furnished with a locking washer. Drawing the nut tight insures a rigid connection and seals the pinhole against moisture, thus retarding decay.

Suspension Insulator Hardware

The suspension insulator type of construction requires a wide variety of hardware fittings. For attaching insulators to structures, use is made of eyes, shackles, clevises, or hooks. These are usually designed to permit free swing of the insulator string in every direction. For any particular situation fittings should be selected which will provide the simplest form of connection. Eyes of some form are usually provided on tower cross-arms for attaching insulators. Eye bolts or specially designed fittings are used to attach insulator strings to wood cross-arms. With parallel strings of insulators for extra strength, yokes are used to provide a common connection for the strings and to space them apart properly. These yokes are, of course, used at each end of the multiple string.

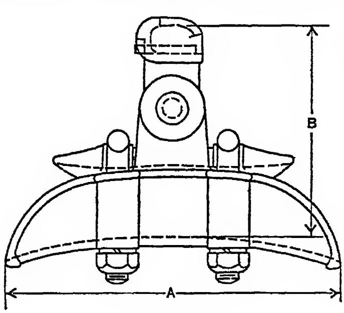

Fig. 43

Suspension clamps or strain clamps serve to attach conductors to insulator strings. Suspension clamps are used on insulator strings at all structures at which the conductor is not dead-ended. This includes tangent sections, slight angles, and angles which are turned with flying corner construction. Such clamps are generally made of galvanized malleable iron with steel U-bolts or J-bolts, although cast or forged steel has been employed in some cases. These clamps should have smooth, well-rounded seats and corners. They should be quite light in weight and therefore of small inertia to reduce conductor vibration. They are provided with aluminum liners, if desired, for use with aluminum cables. Fig. 43 shows a typical suspension clamp. Recent developments at Boulder Dam and elsewhere have led to the development of clamps for extra-long spans, with special features to prevent concentration of vibration on the cable near the clamp.

Strain clamps, used for dead-ending conductors, are made in a wide variety of styles and sizes to meet the tension requirements imposed by the line conductors. They are commonly made in such form as to put a bend in the conductor to provide some snubbing action and are also provided with steel U-bolts or J-bolts, and pressure pieces to hold the conductor by friction. Slight waves are often made in the surfaces of pressure pieces and conductor seats to increase the snubbing effect. Strain clamps are designed to hold without slippage the largest conductor they will fit. They are also provided with liners for aluminum cable, if desired. Fig. 44 shows a typical strain clamp. Clamp bodies are usually made of galvanized malleable iron or steel. Another form of a strain clamp which is coming into general use provides nearly 360 deg snubbing effect combined with clamping. This type is used with the smaller conductors on distribution lines.

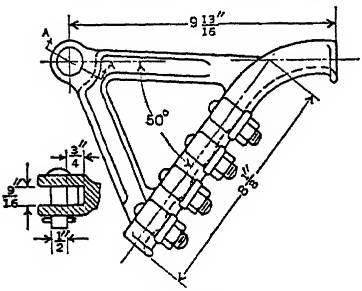

FIG. 44

30. CONDUCTOR INSTALLATION

Stringing conductors is the final major job in erecting overhead lines. For city distribution lines the process is somewhat different from that for transmission lines, although the underlying principle is the same.

On Distribution Lines

In stringing conductors on lines on city streets the conductors are either coiled on pay-off reels or left on shipping reels which are supported on reel jacks. The reels may either be mounted on the line truck or set up on the ground underneath the line. On account of the short spans the conductors may be pulled in by manila hand-lines running over the cross-arms. Power winches mounted on the line trucks are often employed for pulling heavy conductors. Secondary conductors to be run on vertical racks are often laid out along the line and lifted by hand-lines to the rack points where they are held loosely in place behind the rack rods. Where conductors are being strung on poles already carrying energized circuits, every possible precaution should be taken to prevent injury to the linemen. Throughout the process of stringing, sagging, and tying-in, the live conductors should be covered at the poles with rubber blankets, line hose, or similar protective devices, and the linemen should be required to wear their rubber gloves and glove protectors. They should be required to have their safety belts fastened around the pole at all times when in working position. Hand-lines should be perfectly dry and clean.

After pulling the wires over the arms and securing them temporarily at the dead-end arms, they are tied in permanently at one dead-end arm. They are then pulled up at the other end to proper sag for each span. It is not feasible to measure tension in the conductors for such construction, and therefore the sag is measured by sighting across targets fixed in proper position on adjacent poles. After completion of the sagging operation the other dead-end is tied in permanently, and then the wires are lifted upon the insulators at each intermediate pole and tied to them by tie wires. Long-span rural line conductors on wood poles are strung in much the same manner.

On Transmission Lines

Conductors for long-span transmission lines are shipped in long lengths on wooden reels. These reels of wire are distributed along the line in accordance with the length of wire on each. The reels are set up on reel jacks which allow the reels to revolve when the wire is paid out. Stringing sheaves which have very free-running rollers are hung on the towers in place of the insulator strings. The conductors are placed in the sheaves on the first tower away from the reel set-up and pulled to a point beyond the next tower. The pulling is stopped until the conductors are lifted to the sheaves on the second tower, and the pulling process is then continued as before until the next reel set-up is reached. When the first set of reels are emptied the conductors are tied temporarily to the adjacent dead-end tower. The conductors of the second set of reels are then pulled into their section of the line, and when the reels are emptied the conductors of both sections are spliced together. Pulling of conductors is usually done with horses. Under favorable conditions tractors provide excellent power for the purpose. In mountainous country oxen have

been used successfully when horses or tractors were out of the question. Locomotives have sometimes been used for lines along railroads. Care should be exercised in pulling conductors not to injure them by sharp bends, faulty cable grips, or dragging them over sharp stones.

This general procedure is followed until the next dead-end tower is reached, when the entire section between dead-ends is pulled up to proper sag and tension for the prevailing temperatures. If prestretching is done in the field the conductors should be pulled up to nearly the elastic limit before adjusting to normal sag and tension. (See article on Mechanical Design.) Dynamometers are used to measure the tension in conductors for prestretching for approximate sagging. Since dynamometers are not sufficiently accurate the final determination of correct sag and tension should be made by the use of surveying instruments. Wind loads on conductors at the time of stringing should be taken into account if they are of sufficient magnitude to affect the tension more than a negligible amount.

Where ground wires or telephone wires are also on the tower, it is equally important that they be strung at the proper tension; otherwise they may cross and ground the conductors.

Suspension towers (i.e., those intermediate between the dead-end towers) are ordinarily not strong enough to stand the strain of dead-ended cables during high winds and heavy sleet storms; consequently care must be used if cables are temporarily dead-ended on them during construction.

After the conductors are properly sagged the insulators are put in position and connected to the conductors. Care should be taken to connect the insulators at proper points on the conductors so that the tension will not be changed and so that suspension insulators will hang in a vertical plane. At dead-end towers the jumpers must be clamped together and bent to shape and not left so that they may ground to the tower due to twist of cable or wind pressure.

Conductor Splicing

Conductors are spliced by the use of twisting sleeves or compression sleeves or by twisting the conductors together without sleeves. The latter method is used only with small conductors and is being superseded by the other methods to some extent. Twisting sleeves are used quite generally for splicing conductors which are not too large to be twisted and where the conductors strength will be less than the strength of the twisted splice, by a safe margin. Fig. 45 shows a typical twisting sleeve and a twisted joint.

FIG. 45

Twisting sleeves are made with a figure-eight cross-section as well as elliptical. It is claimed that greater strength is obtained with the figure-eight shape. Twisting sleeve joints are twisted by hand wrenches which are of various sizes depending on the conductor size. Ratchet wrenches are quite popular for this purpose.

Compression sleeves are made in various forms and for various methods of application. A common form used for steel-reinforced aluminum cable really consists of two sleeves, a steel sleeve for the steel core and an aluminum sleeve covering the whole splice. In making the splice a machine known as a compressor is used which applies the pressure hydraulically to properly shaped dies. The aluminum sleeve is placed on the cable and slid back some

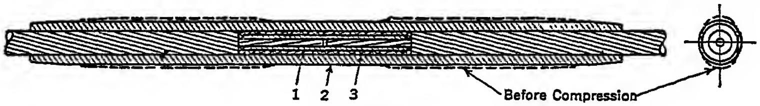

FIG. 46

distance from the joint. The aluminum strands are cut off to expose the steel core a little more than one-half the length of the steel sleeve. Both the aluminum and steel strands are wrapped near the ends with wire to prevent spreading. Both ends of cable are dipped into red lead paint to cover slightly more than the total length of the joint. The steel sleeve is slipped over the ends of the steel cores and centered and then compressed with the machine. The steel sleeve is wrapped with aluminum wire to the diameter of the outer aluminum strands, after which the aluminum sleeve is slipped over the joint, centered, and compressed. Fig. 46 shows such a joint. Item 1 of the figure is the steel sleeve, 2 is the aluminum sleeve, and 3 is the filler wire.

Compression joints for copper conductors are quite similar, and many patented methods have been devised to make them up in the field. One method for small conductors involves the use of a die which rolls from one end of the sleeve to the other. Another method employs stationary dies with the pressure applied by exploding gunpowder cartridges. Hydraulic compressors are also used. In general the strength of sleeve joints depends on the length of the sleeve to give adequate bond, and on its thickness to give adequate tensile strength.

Transposition of Conductors

Communication circuits closely paralleling a-c power circuits are susceptible to inductive interference. The interference is brought about by the linking together of the two separate systems by means of their own external electromagnetic or electrostatic fields. Interference may also arise due to the fact that the earth forms a common part of the circuits of both systems. Interference is proportional to the closeness of the two circuits, the length of parallelism or exposure, and the magnitude of the interfering voltage or current.

Certain frequencies produce more serious effects than others. In telephone circuits, frequencies from 800 to 1500 cycles per second are most troublesome from the standpoint of noise in the telephone circuit. Eleven hundred cycles is by far the worst noise frequency. Telegraph circuits are susceptible to frequencies of 60, 180, 300, and 420 cycles. The triple harmonics and their odd multiples together with the odd non-triple harmonics are those which may be present and which require attention from an interference standpoint.

Perfectly balanced symmetrical three-phase systems may have the fundamental frequency and all odd harmonics present, but there will be no single-phase or residual components of current or voltage. Unbalance either as to voltage between phases, currents, or capacity to ground produces a residual component. The third harmonic and its odd multiples may be produced by certain connections of transformers or by the generator wave shape. The third harmonic and its multiples are best taken care of by suppressing them. This may be accomplished by providing a delta-connected winding across the phases. (See Transformer Connections.) Residuals are best taken care of by repressing them either by delta-Y grounding banks or by wave traps. The inductive effects of balanced voltages and currents may be overcome by a coordinated system of transpositions of both power and communication lines.

Transposition of power lines is necessary only within the distance in which the lines are closely parallel. This distance is called the length of exposure. A transposition

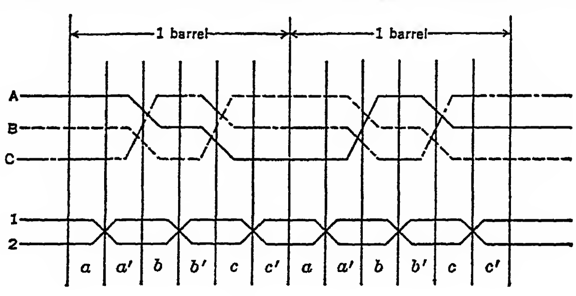

Fig. 47

scheme requires that each conductor of the power line shall have equal coupling to the communication line. This is accomplished in three-phase lines by successively bringing each of the three conductors into the position previously occupied by one of the other conductors. At each transposition structure such a replacement of conductors takes place in such a manner as to rotate the phases spirally along the line. The length of line in which each conductor occupies each position for an equal distance is called a "barrel." The number of barrels necessary in a given exposure depends upon the amount of inductive coupling between circuits.

In order to neutralize the voltages induced between the two sides of the communication circuit it is also necessary to transpose the communication circuit. The two transposition schemes must, therefore, be coordinated. Fig. 47 shows a typical coordinated scheme.

The conditions required for balance for such a coordinated scheme are shown in the following table.

Component of Voltage or Current in Power Line	Voltage Induced in Communication Circuit	Condition Required for Balance
Balanced	Between wires	$a - a' = b - b' = c - c'$
Balanced	To ground	$a + a' = b - b' = c + c'$
Residual	Between wires	$a + b' + c = a' + b + c'$

Specially designed structures are usually necessary at transposition points to provide proper clearances between conductors and between conductors and the structures. For more complete discussion of inductive interference and remedial measures see Pender & McIlwain's Handbook of Communication Engineering.

31. INSPECTION OF POLE AND TOWER LINES

All parts entering into a line are usually tested and inspected before shipment; therefore, in general, the final inspection of a line before putting it into service consists in inspecting the final assembly. This is ordinarily done by an inspection crew under the supervision of an engineer. This crew gives a visual inspection of all parts to detect breakage or other defects which can be seen. Tightness of bolts, position of insulators, alignment of structures, clearances of jumpers, conductor connections, rust spots on steel, and condition of right-of-way are among the items checked. Phasing out of the lines is usually done by the crews working on the electrical construction of the substations. After the line is in operation periodic inspections are made similar to the final construction inspection, and the inspection crews usually carry out the necessary maintenance work.

The proper maintenance of a line includes resetting foundations that have settled; covering of foundations with earth to proper depth after heavy rains; repainting of towers before they are affected by rust; renewal of rusted ground cables; replacement of cracked or partially defective insulators that have not failed; correcting sag of any cable where sag has changed as a result of stretch of cable, change of length during emergency repairs, etc.

In addition to the periodic inspections which are made one or two years apart, the lines should be patrolled every few days by regular patrolmen who should note and report the condition of foundations, poles, towers, insulators, and conductors. They should keep weeds, brush, and inflammable material away from the structures and also make minor and emergency repairs. Arrangements should be made for means of communication for patrolmen. If a private telephone line is constructed along the power line right-of-way, telephones for patrolmen should be located at convenient intervals. Emergency repair storerooms are also located at strategic points along the line.

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UNDERGROUND CONDUITS

By W. A. Del Mar

33. TERMINOLOGY

There is no accepted standard terminology, but N.E.L.A. Reports have, in general, been consistent with the following definitions.

Duct, a pipe designed or used for accommodating electric cable.

Service Duct, a duct entering a consumer's premises.

Duct-bank, a group of ducts.

Manhole, or Splicing Chamber, a chamber giving access to a duct-bank.

Distributor Box, a small manhole for connecting distributing mains to consumer's service ducts.

Conduit, usually a duct-bank with its manholes, but sometimes used to designate a duct. The word will be used herein to designate a duct-bank with manholes.

Manhole Frame, a casting enclosing the entrance to a manhole.

Manhole Cover, a casting which closes the opening in a frame.

34. USE OF CONDUITS

Conduits are used for housing underground electric cables. They permit the installations and removal of cable at manholes without disturbing the surface of the street. They are used for transmission, distribution, and service cables.

Transmission conduits are generally laid with manhole spacings of 400 to 600 ft, and they are made as straight as conditions permit.

The length of sections between manholes is largely determined by local conditions. Large cables have been successfully pulled up to 1200-ft lengths, but difficulties encountered in replacing cables in such long sections indicate that 600 ft is the maximum length desirable. Pulling stresses vary from about 1000 lb in the shorter pulls to 7000 lb in the longer pulls of a 1000-ft cable, with cable weighing about 12 lb per ft.

The cost of replacement of street pavement constitutes a large part of the total. Duct lines are often run through parkways, side streets, and alleys, thus avoiding the expense of repaving. Manholes located at alley intersections also have the advantage of being more readily accessible than those at street intersections, where surface traffic is heavier.

Distribution conduits are usually shorter than those of a transmission system, and usually consist of one or two ducts, 3 1/2- to 4-in. diameter, placed over the transmission ducts, if these are in the same street. The routes follow main streets or avenues, and the cables are subject to frequent disturbances owing to changes in service connections. All classes of service which include feeders, secondaries, street lighting, and control cables are found in distribution duct-banks.

In the main avenues, secondary cable ducts usually are placed in the streets. Frequent distribution boxes are installed to provide for consumer service connections. For side streets in residential districts, it may be more desirable to install the ducts inside the curb, in order to avoid the cost of repaving the street.

Service pipes, run from distribution box to consumer's premises, usually consist of 2-in. steel or fiber pipe.

In localities where the soil is free from large stones, steel pipe may be pushed from the cellar wall by means of a pneumatic gun. Ordinarily it is laid by digging a trench, which, however, often necessitates breaking up some of the sidewalk.

Services are usually grouped by fours or less, so as to make each one not more than 80 ft long. Service boxes are located at least 50 ft apart in a heavily loaded district.

35. TYPES OF DUCTS

The most frequently used materials are tile, fiber, and monolithic concrete. Iron pipe and sometimes creosoted wood duct are often used for service connections.

Tile duct is less commonly used for power cable purposes than fiber duct, owing largely to the quicker installation of fiber duct with its longer lengths, lighter weight, and less trouble from breakage in transit.

Tile ducts are usually laid without any separation, which does not always isolate trouble to the duct in which it occurs. Tile ducts also fuse badly, sometimes making it difficult to remove cable after a burnout. They are likely to be rougher at the joints than fiber ducts because of the easier entrance of concrete at these points.

Multiple Tile duct is seldom used for power cables because of the danger from short circuits being communicated to other cables at the joints.

"Soapstone" duct is now being used, which consists of a mixture of cement and ground soapstone. The chief claim for it is low frictional losses. It is mechanically strong, and it possesses good arc-resisting properties, as it does not melt into sharp needles, but rather flakes away, leaving a smooth surface for the cable. The individual units are 3 ft long and weigh about 6 2/3 lb per ft.

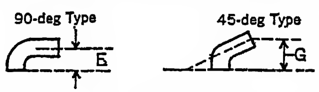
Fiber Conduit consists of tubes of wood pulp fiber formed on a mandrel, dried, and impregnated with a bituminous compound. The ends are tooled to the desired form for joint construction.

The standard sizes are as shown in Table I.

Table I. Standard Fiber Ducts

Inside Diameter, in.	Wall Thickness, in.
2	1/4
2 1/2	1/4
3	1/4
3 1/2	1/4
4	1/4
4 1/2	5/16
5	3/8

Table II. Standard Elbows for Fiber Conduit



Internal Diameter, in.	Radius of Center Line, in.	*E*	*G*
2	2	6	8 1/2
2 1/2	2 1/2	6 1/2	9
3	3	6 1/2	9
3 1/2	3 1/2	7	9 1/2
4	4	7 1/2	10 1/2
4 1/2	4 1/2	8	12
5	5	11	16

There are three types of joints, as follows:

Socket type, with mortise and tenon 3/8 in. deep, slightly tapered.

Sleeve (or Harrington) type, having a sleeve machined internally tapering from both ends to center, the bore being such that conduits do not butt.

Standard elbows are shown in Table II.

Tile ducts are made with either round or square bore and octagonal outside. They are made by mixing surface clay or fireclay to the proper consistency and pressing the plastic mass through a die. They are dried and burned, and covered inside and out with salt glaze. A well-made duct will give a clear ringing sound when struck with a piece of steel. A dull dead sound indicates softness or porosity. Standard sizes are shown in Table III.

Table III. Standard Single Tile Ducts

Bore of Duct Hole, in.		Length, in.	Approximate Outside Width, in.
Nominal	Actual		
3 1/4 Round	3 3/8	18	4 1/2
3 1/2 Round	3 5/8	18	4 3/4
4 1/4 Round	4 3/8	18	5 1/2
3 1/2 Square	3 5/8	18	5
4 1/4 Square	4 1/4	18	5 3/4

Pump Log, which comes in lengths 6 to 7 ft and may be installed without concrete, is used largely where traffic conditions compel rapid installation and immediate back-filling.

It is made of southern yellow pine or Norway spruce steamed and impregnated with creosote oil. It is joined by means of mortise and tenon joints about 1 1/8 to 1 1/4 in. long. Lengths are laid with joints staggered horizontally and vertically.

Monolithic Concrete is made by pouring concrete over long rubber forms and pulling out the rubber when the concrete sets.

Care must be exercised, in all concrete duct work, to use a grade of concrete which will not cause chemical corrosion of the cable sheaths.

The depth of top of duct structure below street surface is usually not less than 2 ft 6 in., but is often fixed by municipal ordinance.

Proper selection of manhole location will permit ducts to drain into manholes, using a minimum slope 3 in. in 100 ft. On very steep streets, it is best to build in 3 or 4 steps, alternating level and sloping, to avoid cable creepage.

36. SPLICING CHAMBERS

Splicing chambers are designed to permit the highest-voltage cables which they are to contain, to be trained around the walls and spliced without excessive bending. It is necessary to leave room for cable expansion and contraction. The dimensions should

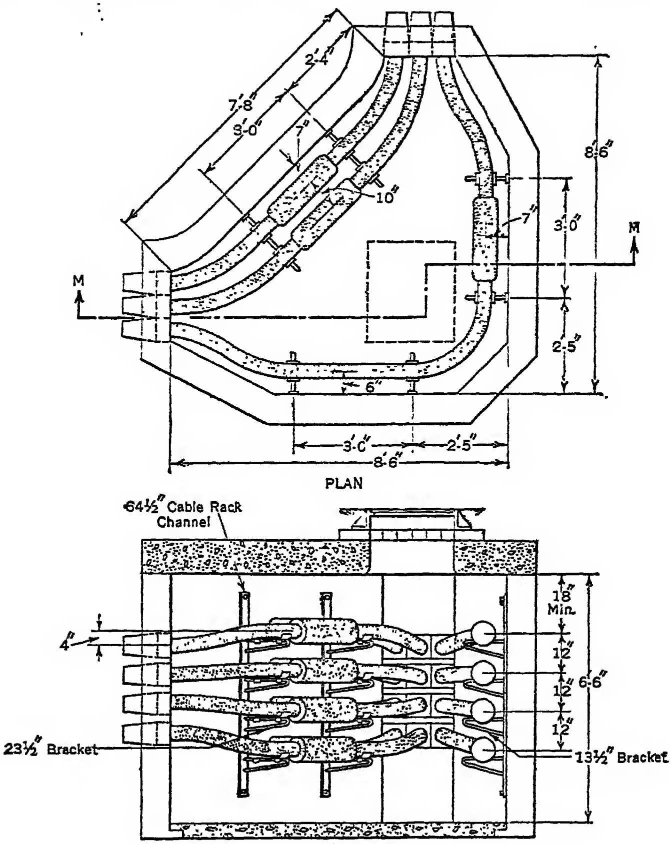


Fig. 1

also be ample to permit splicers to work efficiently. The height is usually 6 ft or more.

Roofs should be designed to withstand static wheel loads of 15,000 lb plus an allowance for impact, usually 50 per cent, making 22,500 lb. Dry earth is usually estimated to weigh 100 lb per cu ft. Horizontal pressures are usually taken as one-third the vertical.

The total vertical load varies from 2000 lb per sq ft at a depth of 1 ft, to 1000 lb per sq ft at 2 ft to 6 ft, then again rising to 2000 lb at 15 ft.

Typical Splicing Chambers are shown in Figs. 1 to 6, the dimensions being suitable for 12,000-volt cables. (From N.E.L.A. reports.) For higher voltages, the dimensions will have to be greater. In oil-filled cable systems, there must be space for reservoirs and other accessories.

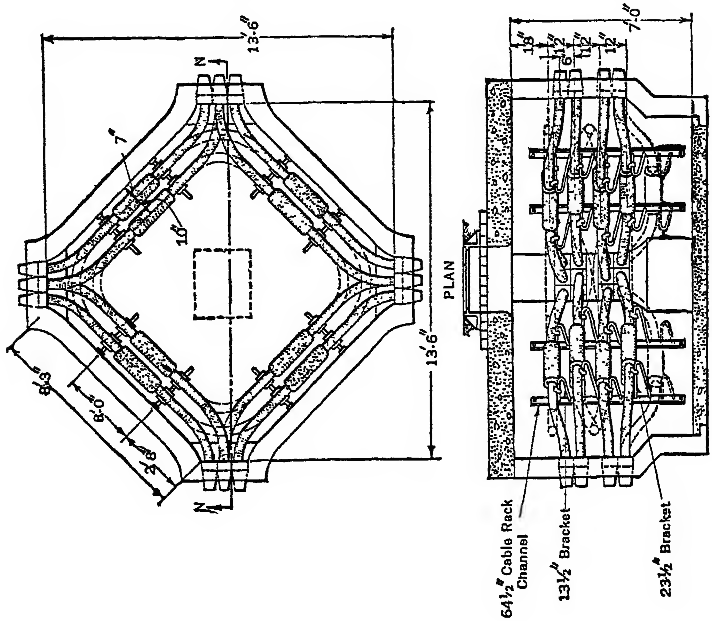

PLAN
ELEVATION SEC. N-N
13′-6″
7′-0″
8′-3″
6′-0″
2′-8″
64½″ Cable Rack Channel
13½″ Bracket
23½″ Bracket

FIG. 3

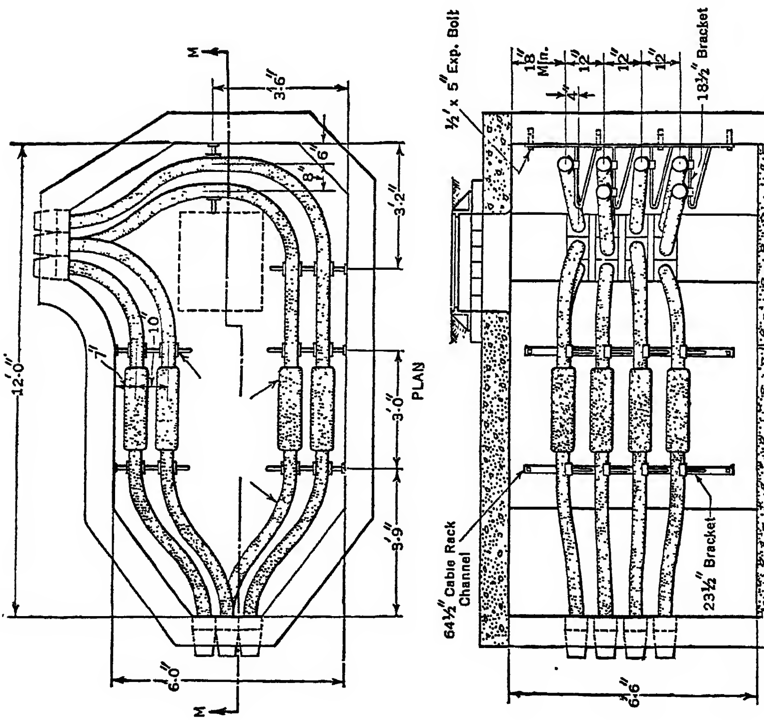

PLAN
ELEVATION SEC. M-M
12′-0″
3′-6″
3′-2″
3′-0″
3′-9″
6′-0″
6′-6″
½″ x 5″ Exp. Bolt
18½″ Bracket
64½″ Cable Rack Channel
23½″ Bracket

FIG. 2

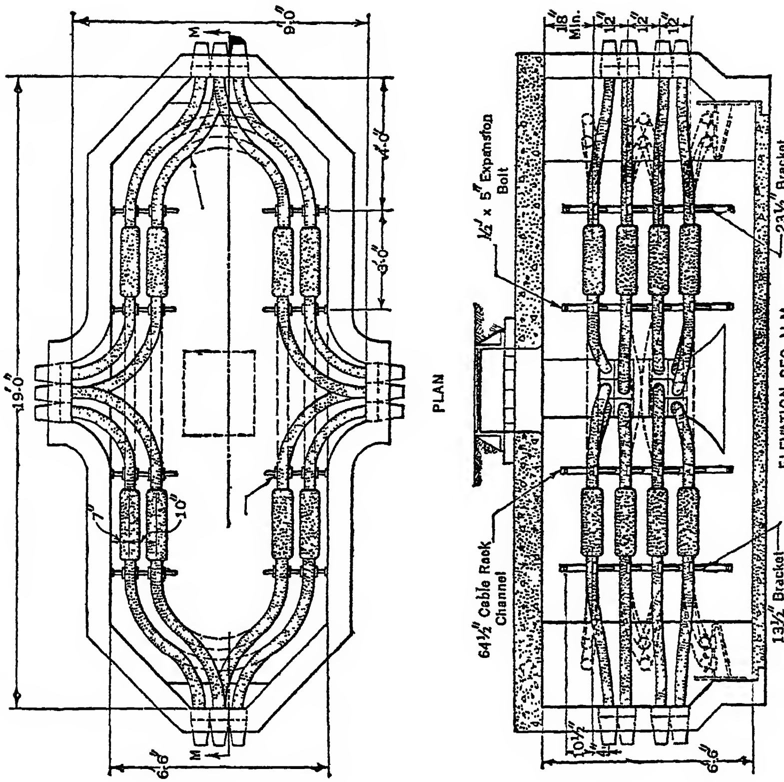

9'-0"
M
19'-0"
4'-0"
3'-0"
7"
10"
6'-6"
PLAN
18" Min.
12"
12"
12"
½" x 5" Expansion Bolt
23½" Bracket
64½" Cable Rack Channel
13½" Bracket
10½"
4"
6'-6"
ELEVATION SEC. M-M

FIG. 5

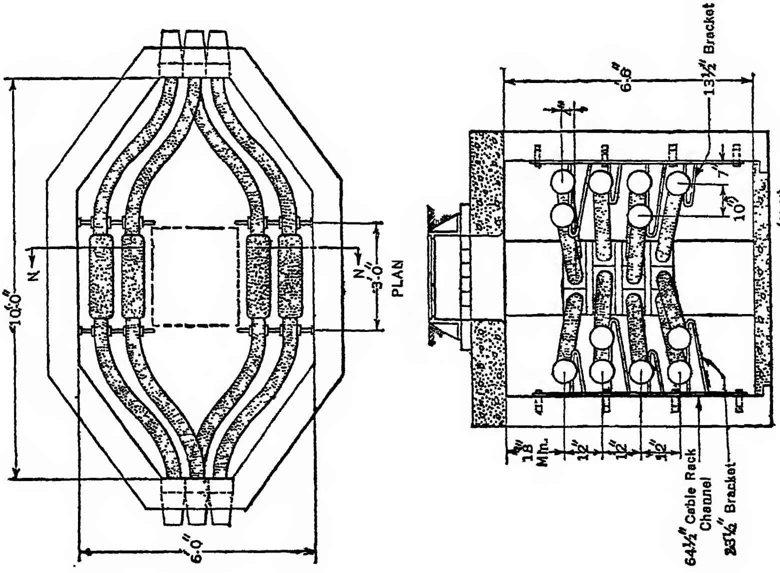

10'-0"
N
3'-0"
6'-0"
PLAN
6'-6"
13½" Bracket
4"
7"
10"
18" Min.
12"
12"
12"
64½" Cable Rack Channel
23½" Bracket
ELEVATION SEC. (N-N)

FIG. 4

DUCT ENTRANCES. It is customary to construct the duct entrance into the manhole with a flare. This provides easier cable bending conditions and lessens the danger of damage to cable at the duct edge, at the same time increasing the effective length of the manhole and permitting the cable fireproofing being carried back into the entrance of the duct. The flare is sometimes constructed with a tapered wooden plug, made to fit the duct at one end and covered with paper so that it can easily be removed after the duct window has become firm. Often it is fashioned by hand with an appropriately shaped wooden tool, before the cement has become too hard. Another method is to use a form, which simplifies the procedure and can be used repeatedly.

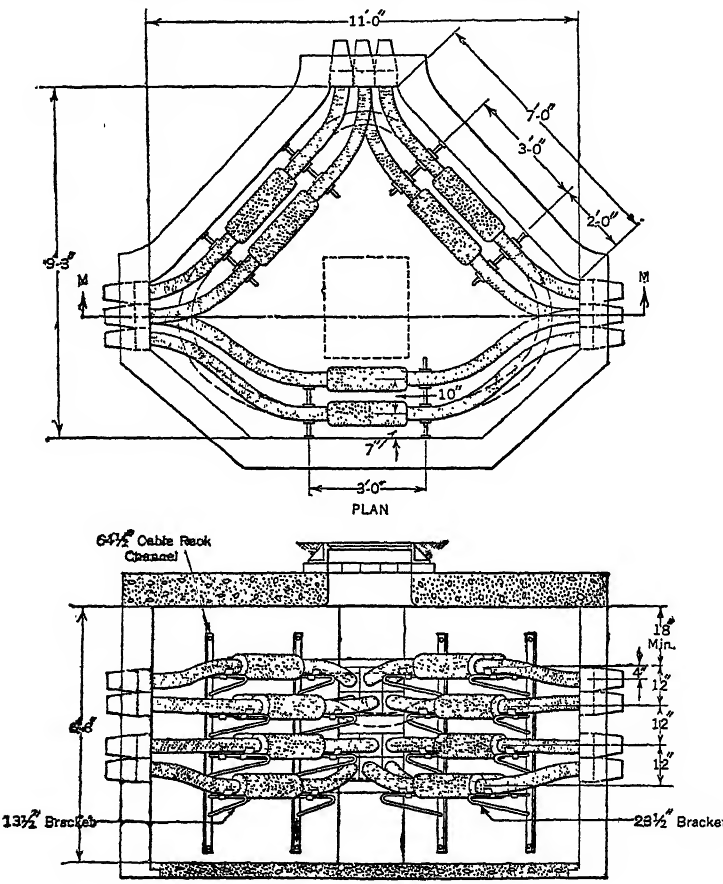


Fig. 6

MANHOLE FRAMES AND COVERS. Manhole frames are usually made with two covers, the outer of which should be strong enough to stand the weight of the heaviest vehicle, and the inner should be as light as consistent with the rough usage the frames receive in handling. Both covers should be provided with means to grip them with a hook-bar. Where power cables are used, it is essential to provide ventilating holes. Some companies lock the inner cover with a locking bar and substantial brass padlock with protected keyhole.

Round manhole covers are preferred to square, because they cannot slip through the hole. The hole in the frame should not be over any cable and should be well located to

give a clean sweep of cable in installation. A ratio of 1 sq in. of holes to 30 sq in. of cover is a good balance between strength and ventilation. A typical design is shown in Fig. 7.

The (clear) diameter of the opening is usually either 22, 24, 27, or 30 in., except for transformer vaults, where sizes 33, 36, 42, and 48 in. are used.

Frames with inner covers are 11 in. high; those without, 7 in.

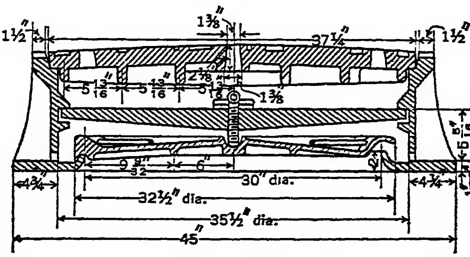


FIG. 7

WATERPROOFING AND DRAINAGE. There is some diversity of opinion with regard to the value of waterproofing splicing chambers, although the modern practice is to omit waterproofing and provide effective drainage.

Every chamber should be provided with a sump into which the water can drain, Fig. 8. It is desirable to connect the sump to the sewer through a syphon and backwater valve. If this is not practicable, the sump may be drained through the manhole opening by a hand pump.

Where natural drainage cannot be secured, as, for example, where the chamber is below the sewer level, it is good practice to provide a special drain pipe to which all the chambers

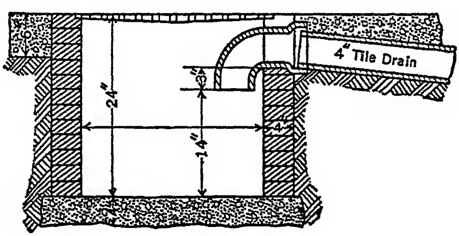


FIG. 8

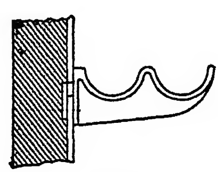


FIG. 9

are connected, the drainage being towards a general sump pit, which is kept dry by an automatic sump pump.

The sump of a splicing chamber should be covered with an easily removable grating; wooden ones are often used on account of their property of floating when the chamber is flooded, thereby leaving the sump open for the pipe of a hand pump.

CABLE SUPPORTS. Cables in splicing chambers are usually supported on iron brackets which engage steel uprights imbedded in the chamber walls, as shown in Fig. 9.

37. GENERAL DESIGN OF CONDUIT LINES

The duct line design should take into account the facility of heat dissipation from the cables to the earth. Assuming all ducts to be equally loaded, the ratio of the temperature rise of the hottest inside duct air to that of the coolest outer duct air will be 1.6 for a 2 wide by 6 deep bank and 2.5 for a 3 wide by 4 deep bank. The 2 by 6 construction, having all ducts outside, is the ideal type; and where the 3 by 4 type is used, the center ducts are commonly used for control, telephone, and pressure wires which are not heat-producing.

Sufficient spare ducts to take care of probable load increases and extensions should be provided.

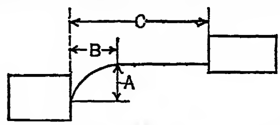
Curves should be as gradual as possible. Tables IV and V give the recommended minimum lengths of curve to take care of offsets in the conduit line.

Manhole lengths are determined from the following details. The parts occupied by cable curves are usually based on radii of curvature of 7 1/2 times the cable diameter. The

straight part is the sum of the splice length, something slightly over the sleeve length, to allow for disposal of the sleeve in splicing. There should also be allowance for curves to change levels, horizontal bend space in L manholes, and expansion clearance to duct edge.

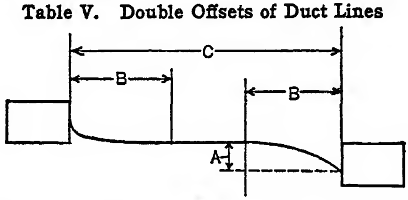
Network manholes must be specially designed to provide space for the network switches and apparatus.

Table IV. Single Offsets of Duct Lines



Offset of Curve *A*	Minimum Distance, ft *B*	Maximum Length Section, ft *C*: 2 and 4 Ducts	Maximum Length Section, ft *C*: 6, 9, 12 and 16 Ducts	Offset of Curve *A*	Minimum Distance, ft *B*	Maximum Length Section, ft *C*: 2 and 4 Ducts	Maximum Length Section, ft *C*: 6, 9, 12 and 16 Ducts
0–1′	5	800	700	3′ 0″– 3′ 9″	45	485	425
1– 3″	10	740	650	3′ 9″– 4′ 7″	50	470	410
3– 6″	15	685	600	4′ 7″– 5′ 6″	55	445	390
6–10″	20	640	560	5′ 6″– 6′ 6″	60	425	370
10–15″	25	600	525	6′ 6″– 7′ 6″	65	400	350
15–21″	30	570	500	7′ 6″– 8′ 9″	70	370	325
21–28″	35	545	475	8′ 9″–10′ 0″	75	345	300
2′ 4″–3′ 0″	40	515	450				

Table V. Double Offsets of Duct Lines



Offset of Curve *A*	Minimum Distance, ft *B*	Maximum Length Section, ft *C*: 2 and 4 Ducts	Maximum Length Section, ft *C*: 6, 9, 12 and 16 Ducts	Offset of Curve *A*	Minimum Distance, ft *B*	Maximum Length Section, ft *C*: 2 and 4 Ducts	Maximum Length Section, ft *C*: 6, 9, 12 and 16 Ducts
0 –1″	5	800	700	3′ 0″–3′ 6″	60	420	365
1 –3″	15	740	650	3′ 6″–4′ 0″	65	400	350
3 –6″	25	655	575	4′ 0″–4′ 6″	70	385	335
6″–1′ 0″	35	570	500	4′ 6″–5′ 0″	75	365	320
1′ 0″–1′ 6″	40	515	450	5′ 0″–6′ 0″	80	345	300
1′ 6″–2′ 0″	45	485	425	6′ 0″–7′ 0″	85	320	280
2′ 0″–2′ 6″	50	455	400	7′ 0″–7′ 6″	90	310	270
2′ 6″–3′ 0″	55	435	380				

38. RODDING AND WIRING OF CONDUITS

The only test which it is usual to apply to conduit lines is rodding with a mandrel in order to ascertain whether the ducts are continuous and unobstructed.

The rods used for this purpose are of hickory about 1 in. in diameter and 3 or 4 ft long and are fitted at the ends with steel couplings such as shown in Fig. 10. The first rod is attached to a mandrel and pushed into the duct. Another rod is coupled to the first and the pair pushed further into the duct. By successively coupling other rods and pushing them into the duct, the mandrel is made to travel from one chamber to another. As soon as the mandrel emerges into the receiving chamber, the rods are pulled through, dragging a cleaner, and uncoupled. If an obstruction stops the

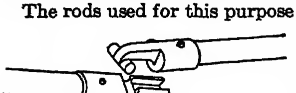


Fig. 10

mandrel, an attempt is made to force it through by repeated blows with a cutting tool attached to the rods, failing which, it becomes necessary to cut into the conduit line from the side.

MANDRELS FOR RODDING. Various types of mandrels are used for testing and clearing ducts, some hollow and smooth with sharp cutting edges (Fig. 11), and others

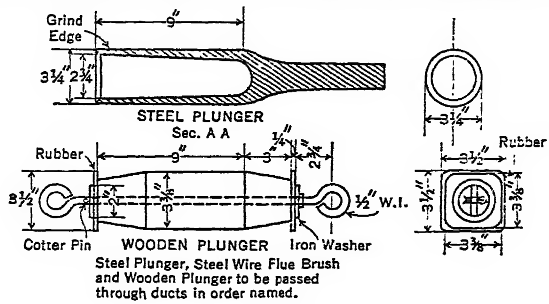


FIG. 11

fitted with numerous sharp projections and known as hedgehogs (Fig. 12).

WIRING THE DUCTS. It is usual to attach a galvanized steel wire (No. 8, 10, or 12) to the last rod and leave the wire in the duct after the removal of the rods. The ducts may also be "wired" by the use of a conduit machine. This consists of a reel of steel tape and means for winding and unwinding the tape into the duct at the rate of about 10 ft per sec. This tape or "snake," having been pushed through the duct, is pulled out again with the wire attached to its end. In recent years a piston pushed through the duct by compressed air has found favor.

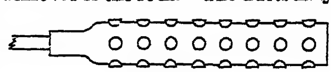


FIG. 12

39. SPECIFICATIONS FOR CONDUIT LINES

Many cable failures occur from injury to the lead sheaths during installation. Sharp projections in the ducts should therefore be carefully guarded against, as a cable pulled over such a projection will have a groove cut along much of its length and the effective thickness of the sheath will be thereby materially reduced. It is a matter of practical importance to give each duct length a rigid inspection before installation, rejecting any which have roughness or irregularity. Scarcely less important is rigid inspection during installation, in order to assure cleanliness and good alignment.

DETAILS OF CONSTRUCTION. The following items should be covered:

Tile Ducts:

Single-way, or four-way.

Holes circular or square with rounded corners.

Inside diameter and tolerances.

Outside diameter and tolerances.

Freedom from blisters, cracks, and other imperfections which, in the opinion of the engineer, will tend to injure the cables to be accommodated therein.

Good-quality tile, thoroughly glazed inside and outside.

Shall be straight and true.

Shall be provided with holes for dowel pins if ducts are four-way. Dowel pins may also be called for. The sides of single duct conduits shall be combed with two (2) sets of three (3) longitudinal combings each, each combing to have a width of one-quarter ($^1/_4$) inch and a depth of one-sixteenth ($^1/_{16}$) inch. Multiple-duct conduits shall be scored transversely near the ends.

Nominal length and tolerances. It is usual to call for a certain percentage of shorter lengths (generally 1 per cent), in order to finish runs or stagger joints.

Fiber Ducts:

Nominal inside diameter and tolerances.

Minimum wall thickness.

Nominal lengths and tolerance.

Shall be straight.

Type of joint.

Ability to resist warm water, oil, acid, alkali, and temperatures up to 80 deg cent. Suitable tests are suggested in Underground Systems Reference Book of N.E.L.A.

Cement Mortar:

Stated proportions of cement and sand (usually 2 $^1/_2$ to 1 of cement).

Concrete mortar shall not be laid in freezing weather, and shall not be used after initial set has taken place.

Excavation:

Shall always be of such depth as to leave a stated minimum distance between the top of the concrete over the conduits and the surface of the ground.

Ground on which conduits are laid shall be rammed solid before any concrete is laid.

Refilling Excavations:

The best part of the material excavated shall be used.

Surplus material shall be carted away by the contractor (or will be carted away by the company).

Filling shall be thoroughly tamped and rolled, or flushed, as seems necessary to the engineer, and shall be done in a manner to prevent, as far as possible, a settling of the earth after completion.

Obstructions:

Obstructions encountered in the course of the work shall be overcome in a manner to be approved by the engineer.

Laying Conduits:

Shall be laid with ends square so as to leave a tight, well-fitting joint.

Joints shall be staggered horizontally and vertically.

Ducts shall be laid in a bed of cement mortar of about $^1/_4$ in. thickness.

Each joint of tile duct shall be wrapped with two strips of burlap, muslin, or cheesecloth, 6 in. wide and coated with either asphaltic cement or neat cement mortar, the ends of the wrap to lap 4 in. (It will insure more careful work if it be specified that the contractor shall supply rubber gloves to the men who lay the burlap.) Where conduits are laid on curves, the wraps shall be doubled if required to protect the openings between the ends of the ducts on the outside of the curve, and to exclude mortar from said openings. (Wrapping is not universal. If another method is to be used, corresponding details should be given.)

Tile ducts shall be laid with a mandrel of specified length and width and provided at one end with a rubber washer for wiping the joints. Tile ducts shall be laid on a bed of concrete of stated depth, usually 3 in., shall be covered at the top with a stated depth of concrete, usually 3 in., and shall have a stated thickness of concrete on each side. Where the conduit line goes under railroad tracks, the concrete shall be suitably thickened and reinforced.

If tile ducts are four-way, they shall be laid with dowel pins at joints.

The alignment horizontally and vertically shall be satisfactory to the engineer.

Drainage of Conduit Lines. The grade of all conduit lines shall be such that water cannot stand in the ducts but shall drain into one or both splicing chambers.

Brick:

Shall be of good commercial hard-burned sewer brick, or other stated variety.

Concrete:

Stated proportions of cement, sand and stone (usually 1 part of cement, 2 parts of sand, and 4 parts of stone).

Maximum size of aggregate (usually $^3/_4$ in.)

If wraps are not used at the joints of tile ducts, the sand and cement shall be mixed dry and wetted with only sufficient water to make a stiff paste. If wrapped tile or fiber duct is used, the concrete may be of normal consistency. The stone having been previously wetted shall be added while wet and thoroughly mixed until all the stones are covered with mortar. It shall then be deposited as rapidly as possible. Machine-mixed concrete will be accepted if made in a manner approved by the engineer. (See Art. 40 under Laying Conduits.)

Terminals of Conduit Lines. Whether they go into power stations, etc.

Splicing Chambers. Shall be built according to plans supplied, unless local conditions interfere, in which case the suggested modifications shall be approved by the engineer.

State any details pertaining to the design of the chamber, duct windows, and cable supports which may not be clear from the plans.

Details of sewer connections or other drainage system should be clearly stated.

RODDING, CLEANING, AND WIRING.—After the conduits are laid and the cement is sufficiently set, they shall be rodded, and the contractor shall draw after such rods, wire brushes and a mandrel of specified dimensions. All mortar and other foreign matter shall be removed. If obstructions are found which cannot be removed by cleaners so as to pass the specified mandrel, the ducts shall be removed and relaid. Any expense incurred by such work shall be borne by the contractor. A galvanized wire of stated size shall be left in each conduit from splicing chamber to splicing chamber, and sufficient length shall be left at each end to permit it to be bent in order to prevent it from slipping into the duct.

40. INSTALLATION OF CONDUIT LINES

The problems connected with the installation of conduits vary greatly with the local conditions. They involve the choice of methods of trench excavation, drainage methods, removing or avoiding obstructions, concrete mixing, and so on. The following details are gleaned from first-class examples of the various kinds of work described but should in no sense be regarded as standard.

CROSS-COUNTRY CONDUIT LINES. Excavation for conduit lines in the country and suburban districts may often be made partly by a trenching machine and partly by a trench plow. A typical trenching machine of the " caterpillar " type will dig a trench 18 in. wide and 3 ft deep at the rate of 3 ft of clean trench for each minute of actual working time. An operator and two assistants are required to operate the machine, replacing, it is estimated, 50 laborers.

A wheel-type trenching machine has also been successfully used. This machine has a cutting wheel 7 1/2 ft in diameter, digging a trench 15, 18, or 21 in. wide and with a maximum depth of 5 1/2 ft. The machine weighs 9 tons and is operated, at a speed of 3 1/2 ft per min, by an operator and one helper.

CONDUIT LINES FOR RAILROADS are usually difficult to construct and operate for the following reasons:

1. Owing to the right-of-way being usually on made ground, excessive quantities of concrete and reinforcement are required to make a reasonably strong duct construction.

2. Owing to the width of the right-of-way being usually very restricted it is necessary to shore-up tracks in order to excavate close to them.

3. Owing to the vibration caused by heavy trains, it is necessary to bury the conduits at a greater depth, first to avoid undue stress on the conduit, and second to avoid intercrystalline fracture of the cable sheaths.

In order to have the conduits below the frost level, the depth of ballast must be neglected, as it has been found that, with stone ballast on top of the ground, the frost penetrates the ground about as far as if the ballast were not there, unless the ballast is very dirty.

4. There is considerable difficulty in obtaining best results from labor where there are continual interruptions from trains. On a busy section a duct construction gang engaged for 10 hours can possibly work 2 full hours.

5. Owing to the right-of-way being often quite low, in many cases alongside of rivers, duct construction is likely to be seriously impeded by the flooding of trenches.

6. Where the right-of-way shows signs of settlement, as, for example, on marshy ground, continuous piling is necessary to support the ducts. This involves the use of the track for construction purposes for long periods, and thereby not only impedes but also endangers traffic.

7. Duct-line construction generally involves interference with signal and interlocking apparatus, thereby introducing danger and expense.

8. Bridge abutments, bridges, culverts, and in fact all special right-of-way construction, present complicated problems which can be solved only at great expense.

CONDUIT LINES IN CITIES. The obstructions due to sewer, water, and gas pipes, car tracks, and foreign conduit lines render conduit construction in city streets a complicated problem. Plans made in the office can seldom be followed, without change, in the field, as municipal pipe plans are seldom exact, and the supervision of an experienced civil engineer is needed to solve the numerous problems which constantly arise. Excavation is almost invariably performed by hand labor, and when the conduits have been laid in covered with concrete, it is common practice to lay a plank over them in order to warn future excavators. Obstructions are often avoided by changing the grouping of the conduits.

LAYING CONDUITS. Conduits must be laid so that joints are mechanically strong and the ducts unobstructed. With this in view, joints should be staggered horizontally

and vertically. Some engineers require a muslin or cheesecloth wrap, 6 to 8 in. wide, at each joint, to keep out concrete, especially at curves. This is applied either with asphaltum cement or thin cement mortar. When wraps are used, the ducts may be laid in a thin concrete or mortar. As this may enter through the flutings, there is a general tendency to omit wraps and use a stiff concrete or mortar. The ducts are spaced with only sufficient cement mortar between, to bed them properly and fill the voids.

The ducts are laid on a concrete layer about 3 in. thick and covered all around with 3 in. of concrete.

When laying tile ducts it is necessary to have a long mandrel to remove loose cement. This mandrel is usually provided with a rubber washer at the rear, and a hook-eye at the front end. The conduit layer is provided with a hook rod by means of which he draws the mandrel after him as he lays the conduit.

It is general practice to lay fiber ducts with at least 1 in. of concrete separation, in order to lessen the danger of electrical faults spreading. Three inches of concrete are placed all around the duct bank.

FOUR-DUCT CONDUITS are usually provided with holes for dowel pins by means of which the ducts are aligned. The joints are wrapped in burlap soaked in asphalt and afterwards painted with asphalt. No mandrel is used in this type of construction.

CONCRETE COVERING. However the conduits may be laid, the complete group is always inclosed in concrete to secure rigidity and protection, and the finished ducts are cleared of rubbish and obstructions by pushing a steel plunger through them by means of the rods described above. A steel plunger for this purpose is shown in Fig. 11, in which is also shown a wooden plunger with rubber washers, which should be drawn through the ducts to collect the loose particles left behind the steel plunger.

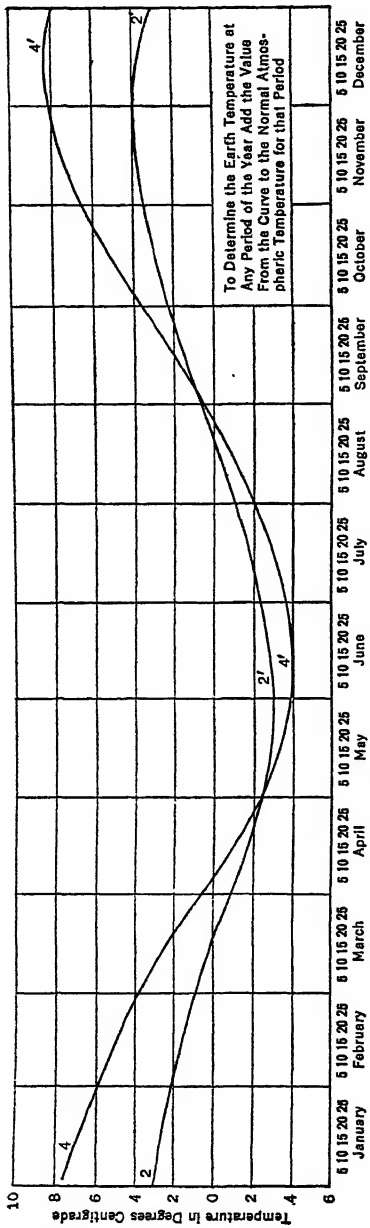


FIG. 13. Earth Temperature Curve (N.E.L.A. Report)

41. MAINTENANCE OF CONDUIT LINES

The principal items of conduit-line maintenance are those relating to keeping the line clean and safe, namely, pumping out water, blowing out gas, removing mud, opening and closing manholes for the benefit of cable workers, and inspecting the line to guard against theft and injury. Generally, large systems have one or more wagons equipped with apparatus required for these purposes, and men ready to go out with it upon emergency calls.

Removal of Water is usually the most important of maintenance items, especially where no drainage system is installed. When cable accidents occur, it is important to have the chambers accessible without delay, and a portable pump is required. For this purpose a small gasoline or electric pump is useful, having a capacity of about 50 gal per min.

Ventilation does not occur naturally in conduit lines, because the cold air contained in them has no tendency to rise. Noxious gases therefore tend to accumulate in splicing chambers, endangering workers and making explosions possible. No permanent system of ventilation has proved successful for general use, as it is found that pressure is maintained only at or near the blowing points. When it is necessary to blow out the chambers, it is therefore usual to employ a portable blower in conjunction with an air-tight false manhole cover.

TEMPERATURES. Cooling duct lines is sometimes desirable in order to increase the carrying capacity of the cables in them. L. E. Imlay accomplished this at Niagara by keeping the surrounding soil moist. The Duquesne Light Co. of Pittsburgh installed several blowers each of 600 cu ft per min capacity and forced air into alternate splicing chambers. A reduction of temperature from 125 to 96 deg fahr was obtained.

An N.E.L.A. Report gives the following data on earth temperatures.

1. The earth temperature varies with the depth below the surface, the maximum temperature decreasing and the minimum temperature increasing with the depth below the surface.
2. The temperature at the same depth varies, owing to the influence of the soil conditions, adjacent sources of heat, and the heat received from solar radiation.
3. The earth temperature at one particular locality seems to repeat itself with reasonable accuracy each year, following a cyclic almost sinusoidal curve (Fig. 13).
4. Paving, and a high percentage of moisture in the soil, ordinarily increase the maximum summer and decrease the minimum winter temperatures.
5. Below some depth, which may be somewhere between 25 and 80 ft below the surface, the earth temperature has no seasonal variations.
6. The maximum and minimum earth temperatures lag the corresponding air temperatures by a period which increases progressively with the depth, the lag at the surface probably being practically zero.

Fig. 14 gives data on the amount of air required for transformer manhole ventilation.

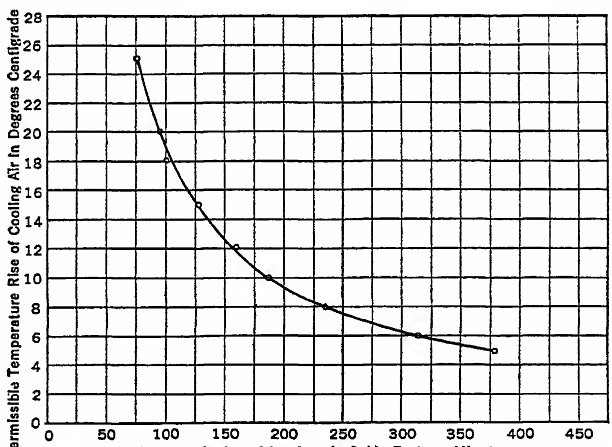


FIG. 14. Air for Manhole Ventilation (N.E.L.A. 1929)

Electrolysis is more fully treated in the article on Electrolysis. It should be noted that the prevention of electrolytic corrosion of cable sheaths depends more upon efficient drainage than anything else.

Corrosion of Lead Sheaths by lime is likely to occur if ducts are used before the concrete has been leached out by ground or rain water.

REPAIRS. The principal repairs to conduit lines are those due to settlement and to damage done by adjacent building operations, such as the construction of sewers or railway tracks. It is sometimes desirable to replace the conduit lines without disturbing the cables they contain. In such cases, the conduits are broken, the utmost care being taken to avoid injuring the cables. New conduits are then relaid on a firm foundation, after having been split longitudinally so as to fit over the cables. The whole construction is then rendered rigid by being inclosed in concrete.

42. BIBLIOGRAPHY

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Ruhling, T. C. Underground Systems for Electric Light and Power. New York, 1927. More than 50 per cent of this is devoted to conduit construction.
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DISTRIBUTION

By W. A. Del Mar

This section covers the distribution of energy for light and power from substations to customers' premises.

A distribution system consists of two parts: the **primary distribution**, which extends from substation to distribution transformers; and the **secondary distribution**, which extends from these transformers to the customers' premises, the latter being at the utilization voltage.

Two types of construction are employed, overhead and underground. Overhead construction has the following advantages (1) lower first cost, (2) easier to repair, (3) easier to change; underground construction has the advantages (1) less unsightly, (2) less dangerous to the public, (3) less subject to damage by external agencies. The majority of circuits in use are overhead. Underground circuits are principally used in the central portions of the larger cities. When underground circuits are used a large part of the construction consists of a composite of underground and overhead construction.

43. DISTRIBUTION CIRCUITS

Primary Systems

Primary distribution systems are usually 2300/4000 volt three-phase circuits with grounded neutrals. Various other systems are in quite extensive use, however, to meet special conditions, such as

2300	volts	delta
13,200	"	delta
6600/11,000	"	Y
4600	"	delta
2300	"	2-phase

RADIAL DISTRIBUTION. Radial distribution is a system of supplying power from a substation to a district by means of one or more feeders to which mains are connected. These mains run to the distribution transformers.

1. Radial Main. A feeder from a substation runs to a point near the load center of a district and mains branch therefrom to the distribution transformers, Fig. 1. This type of feeder is most suitable for areas of medium low load density. It permits the use of three-phase equipment over a large territory and load may be balanced between phases.

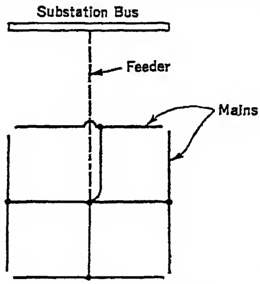

FIG. 1. Radial Main

The **tree** or **herringbone** type of feeder is where the feeder supplies the mains at a point near the substation instead of near the center of the load. Its application is principally in small towns and rural districts as the regulation and losses are greater.

2. Feeder and Main. A sectionalized main (4 conductors) runs through a district, and laterals run from each section to the distribution transformers, Fig. 2. Each section of the main is fed by a separate feeder from a substation, not necessarily the same substation.

This is the best type of circuit for narrow districts of medium load density with laterals reaching out to adjacent lower density load areas. The phases may be readily balanced and extensions made with little new construction. During low load periods, the feeder may be taken out of

service and the main tied to an adjacent main section. This permits maintenance of feeders and switches without interruption of service.

3. Segregated Phase. This is similar to the radial system except that each phase is run in a different direction and handles all the load in a definite phase area, instead of carrying the three phases into each district.

This type circuit is most economical for districts of low load density covering a considerable territory. It is difficult to balance phases, and three-phase equipment cannot be generally used. Extensive rearrangement is necessary to take care of added load.

4. Duplicate Service. This is a modification of the radial system, where each customer transformer is fed by two mains, one operating, and the other for emergency, Fig. 3. This system is used for important load areas where continuity of service is very important. It permits taking feeders out of service without serious interruption of service.

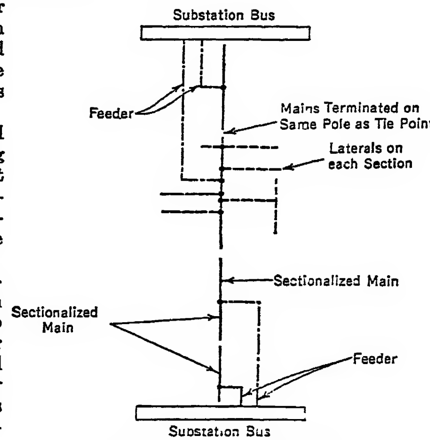

FIG. 2. Feeder and Main

Two oil switches or sets of fuses are required at each installation. It is the general opinion that, for supplying business areas, this system is seldom preferable to a multiple primary feed secondary network.

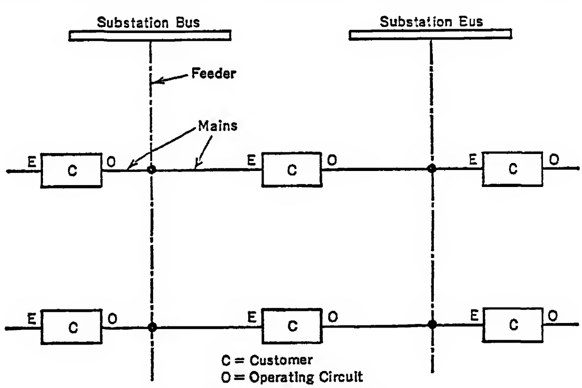

FIG. 3. Duplicate Service

LOOP DISTRIBUTION. Loop distribution is a system of supplying power from one or two substations by means of a sectionalized feeder which is fed from both ends.

1. Customer's Loop. A feeder extends from a substation to a group of customers connected in series and either back to the same substation or to one adjacent, Fig. 4. Protection equipment is provided on both sides of each customer's service.

2. Distribution Loop. A feeder extends from a substation to a number of automatic sectionalizing switches in series, Fig. 5. Lateral mains run from each section of the feeder. The loop is relayed with time delay relays or pilot wire protection so that the two switches nearest the trouble will open, leaving only that section out of service.

Either loop system is very reliable for combined power and light loads. It necessitates two oil switches at the substation for each circuit.

NETWORK. A number of radial transmission circuits extend from a substation each to a step-down transformer bank. The secondary of such bank, presumably at 4000 volts, would be connected to a bus through an automatic oil switch, from which lines extend in four directions, each tying with a similar bus some distance away. These tie lines, from which laterals would be installed, are protected by an automatic oil switch at

each end. Distribution transformers would be connected to these ties and laterals Fig. 6.

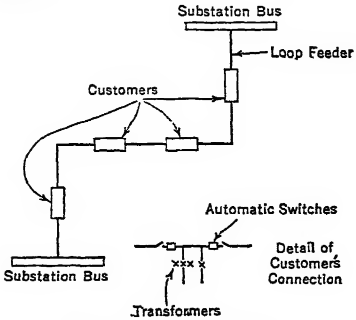

FIG. 4. Customers' Loop

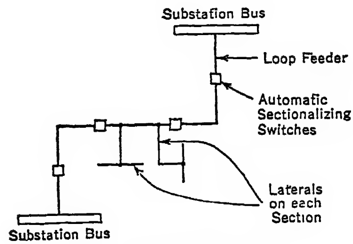

FIG. 5. Distribution Loop

This system permits a very reliable source of supply for moderately loaded areas where the service rendered by a secondary network is desirable but the load does not warrant its adoption.

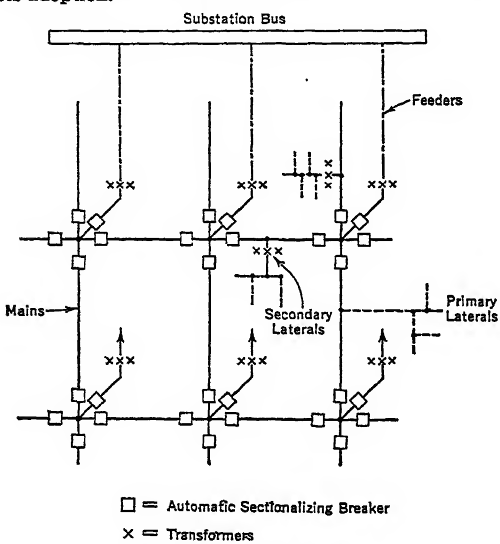

FIG. 6. Primary Network

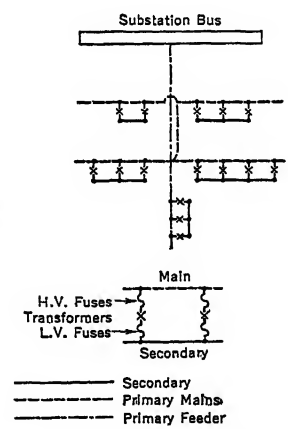

FIG. 7. Secondary Network with Radial Primary Feed

Secondary Systems

Secondary distribution systems are usually at 110 or 120 volts.

RADIAL. Radial distribution is a system of supplying power from distribution transformers to customers where a number of isolated secondaries are each supplied by one transformer or transformer bank.

NETWORK. Network distribution is a system of supplying power from distribution transformers to customers where several transformer banks feed into an electrically solid network of secondaries. Networks are used both for overhead and underground distribution, although those for the latter purpose are more common. The 3-phase, 4-wire system is usual, with small customers supplied by single-phase, 2-wire, 120-volt circuit. The secondary network may be associated with the following primary feeding systems.

Radial Primary Feed. Several sets of transformer banks common to the same primary and secondary lines are banked with protection on both the primary and secondary sides, Fig. 7. This ensures service on the secondary in the event of a transformer having failed. This system is especially suitable for overhead territories.

Multiple Primary Feed. All distribution transformers are tied solidly to a primary feeder of the radial type and in turn tied to a common secondary bus through automatic low-voltage network protectors (Fig. 8).

Multiple primary feed is especially adaptable to concentrated business areas, where continuity of service is of major importance. Any primary feeder may be out of service without materially affecting the transformer loadings or secondary voltage. It also permits the combination of light and power on the same secondary.

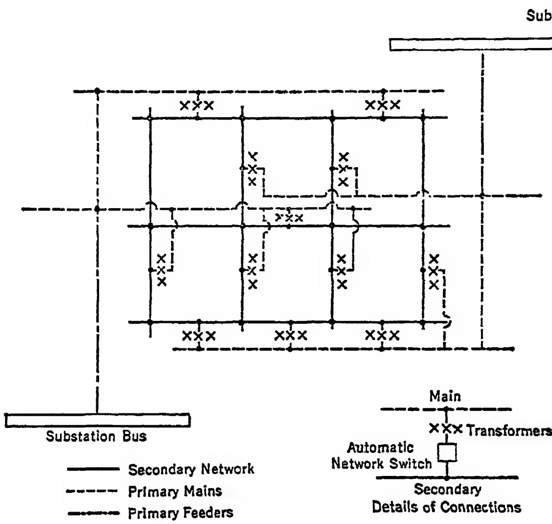

FIG. 8. Secondary Network with Multiple Primary Feed

Loop Primary Feed. All distribution transformers are tied to protected sections of a sectionalized loop feeder (Fig. 9). Such an installation does not necessitate so much spare transformer capacity, as only a small section of a feeder may be out in the event of a primary feeder failure.

Network Protectors and Cables. Network protectors are automatic circuit breakers between the distribution transformers and the secondaries. They are made in the submersible and non-submersible types to suit manhole conditions and are relay-controlled circuit breakers of either the solenoid or motor type, the former being generally used for ratings of 250 to 800 amp, and the latter for larger ratings. The relays are of the reverse-power type with low settings and function to open the breaker when there is a reverse power flow from the network to the transformer, thus isolating the primary circuit on which a fault has occurred without an interruption to the network. The relays also automatically reclose the circuit breaker when the voltage and phase relation are such as to feed power into the network.

Secondary network cables are usually 250,000- to 500,000-circular-mil single-conductor cables. These are tied solidly together at the intersections and connected to the transformers by 1,000,000-circular-mil cables.

Network voltage regulation methods include bus regulation and feeder regulation, the former for high-voltage primary feeders and the latter for 2.3/4.0 kv lines.

The permissible regulation at the customer's services varies from 1 to 8 per cent, the usual value, however, being 3 per cent.

The 120/208 volt system of secondary voltages is the most favored, but the following are also in use: 115/199, 115/230, 117/202, 120/208, 120/240 volts.

Any short circuit which is likely to occur will be burned off and cleared from the system. A current of 1740 amp will fuse the conductor of a 250,000-circular-mil cable in about 20 minutes. A current of 5000 amp will clear a 500,000-circular-mil cable. Fuses are used at remote parts of the network, where the current capacity of the system is insufficient to melt off a secondary cable.

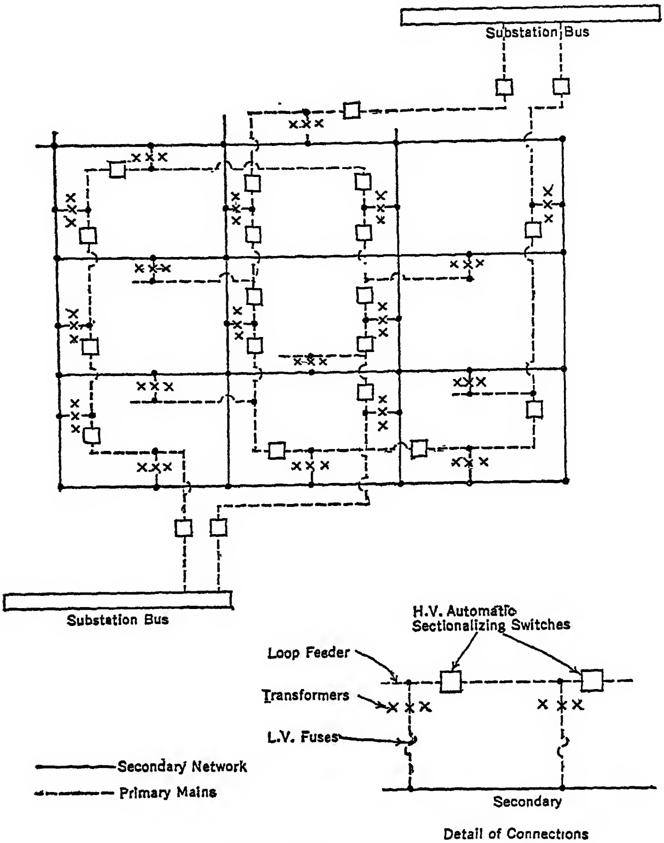

FIG. 9. Secondary Network with Loop Primary Feed

44. OVERHEAD DISTRIBUTION

Line Materials

CONDUCTORS. Overhead distribution is usually effected by means of weatherproof wire, i.e., copper wire covered with saturated braids. The usual sizes are Nos. 4 and 6 A.W.G., but larger sizes, up to 500,000 circular mils, are often used. Medium hard-drawn copper is preferred by most companies, but annealed copper is used to a large extent for spans up to 150 ft.

Special rubber-insulated service cables are often used for secondary distribution, and special rubber-insulated wire, known as tree wire, is used where the distribution lines have to run through dense foliage.

COVERING ON OVERHEAD WIRES. The covering on overhead conductors is solely for the purpose of limiting the short-circuit current due to an accidental cross or grounding. The normal insulation of the line is maintained by the insulators alone; any reinforcement obtained from the insulation on the conductors is neglected in practice. While weather-proof braid is an imperfect insulator, it serves to eliminate the greater proportion of the short-circuits and arcs which would occur, due to momentary contact, were bare wires used.

Bare wire is generally used on circuits operating at 10,000 volts and above to avoid giving a false sense of security. For voltages between 2500 and 10,000 weatherproof wire is often used, though the protection afforded against short circuits is doubtful.

INSULATORS FOR OVERHEAD LINES. City distribution circuits are ordinarily carried on double-petticoat deep-groove glass (D.P.D.G.) insulators; see Art. 28.

Tie Wire. The conductor is attached to the insulator by a tie wire of the same material as the conductor, though soft wire is usually employed even for hard-drawn conductors; the tie wire is either bare or insulated to correspond to the conductor. The size of tie wire is often the same as that of the conductor; for small wires it is merely a piece of conductor. With large conductors it may be as much as three sizes smaller. See Art. 30.

Design of City Overhead Distribution Lines

City distributing systems should be designed so that service may be given to any building in the city and ultimately to every building present and future. On certain streets pole lines may be omitted without defeating this object. The arrangement of lines which will serve scattered customers with the least number of poles will usually contain many poles which should not be used if the ultimate arrangements were immediately constructed. Preliminary studies and designs should be made, first, of arrangements suitable for servicing the initial expected customers; second, of arrangements for ultimately servicing customers on every lot in the city; third, of a plan of extension by which the initial arrangement can be extended to the ultimate with the minimum expense in changing lines and services.

THE POLE LINE. The pole lines perform two functions: (1) of carrying feeders from the station to the mains, and (2) of carrying the mains supplying services to buildings immediately adjacent. In old cities where the streets are crooked, general rules for systematic line work cannot be followed far. In the newer cities the streets are laid out at regular intervals and at right angles, dividing the city into rectangular blocks of equal size. In such cases the following rules should be followed:

1. A pole line should continue on the same side of the street throughout its entire length and disconnected lines built in the same street should be on the same side, so that they may be connected when desired without crossing the street.

2. The spacing between poles should be an exact divisor of the length of a block (including cross street), giving a uniform number of poles per block.

3. Whenever the line crosses a street where there is, or may be, an intersecting line there should be a corner pole on the proper side of the intersecting street for making a junction.

Trunk Lines. If the location of power house or substation is fixed, trunk lines must be laid out from such point, but often a study of possible trunk-line arrangements made before the location of the power house is fixed will show that other locations are more advantageous. If distributing station (power house or substation) is centrally located, there should be at least four main trunk lines (of poles) from it, say north, east, south, and west. A short distance from the station they should be divided into branches, then subdivided into smaller branches, and finally merged into the service lines. The trunk lines should be laid out: (1) on back streets where the large poles, numerous and heavy wires, and heavy guying will not be conspicuous; (2) on side streets or streets little built up so that interruption to service due to fire in adjacent buildings will be infrequent; (3) on streets where there are few trees; (4) on streets where there are no jogs or offsets to weaken the line and require heavy guys; (5) on streets which lead directly to the section supplied, penetrate its center, and intersect the maximum number of service lines. Even when the station location is excellently chosen these desirable conditions will have to be compromised to a considerable degree.

Street and Alley Service Lines. With the symmetrical arrangement of lots described above there is often an alley through each block parallel with the principal streets. Under these conditions there is therefore a choice of two methods of laying out the lines: (1) run the lines on the principal streets servicing the houses from the front, and (2) run the lines in the alleys servicing the houses from the rear. The disadvantages of the first are unsightliness of the poles and wires, difficulty of avoiding or of trimming shade trees; the disadvantages of the second are discontinuity of alleys, proximity of buildings (inflammable barns and outbuildings in residence districts and of windows and fire escapes in business districts), and lack of established grade.

COMPOSITE DISTRIBUTION. Where underground distribution is used it is sometimes desirable to have overhead services. In such cases a pipe runs from the conduit line into the interior of each block, where the pipe comes to the surface at the foot of a terminal pole up which the cables run. At the top the cables connect to overhead wires. The interior of each block contains a complete overhead distribution system of poles, transformers, secondary mains, and services. Sometimes the servicing is done from a single centrally located pole; at other times there may be a pole line of several spans length run-

ning longitudinally through the block. The most serious disadvantage of this method is that the poles and wires frequently have to be on private property, where no permanent rights may be obtained.

The composite system has the advantage of the overhead system in less unsightliness, less accessibility of public to high-voltage wires, and less trouble from trees. In thickly built-up blocks its use results in a tangle of wires over roofs and along walls which may be as dangerous in case of fires or high winds as an overhead system. The composite system is used (1) in small cities where the load density is small, and (2) in large cities in an annular district between overhead and underground construction; in either case it is usually an intermediate step from overhead to underground construction.

CROSSING OF WATERWAYS; SUBMARINE CABLES. Where a distributing system is divided by a navigable waterway the connection is usually made by submarine cables laid on the surface of the bottom, or better, below the surface in trenches dredged for the purpose. Cables may be single or multiple conductors, the latter being usually used for alternating currents to avoid reactance due to wire armor of cable. In laying cables it is desirable to keep them approximately parallel. Where one crosses under another it may be impossible to remove it. Each cable should be in a single length without joints, and the length should be made as short as possible, as repairs are very difficult and often impossible. At ends unimportant cables may be brought up a pole and connected to overhead wires; important cables should land in a splicing chamber with suitable provision for disconnecting the cable or for transferring the overhead circuit to a spare cable in case of trouble. Submarine cables are weak links in a distribution system and may sometimes be avoided by aerial crossings at sufficient height to clear the masts of ships.

CALCULATION OF SIZE OF WIRES. The size of wire to be used depends upon the voltage drop which should be permitted, considering the probable growth of the load. The following table of per cent voltage drop in the various lines is representative of ordinary practice.

	Per Cent		Per Cent
House wiring	2	Transformers	2
Service wires	2	Primary mains	5
Secondary mains	5	Primary feeders	10

The drop in the feeders is usually compensated for by raising the voltage at the substation or power station or by using voltage regulators.

Formulas for calculating the size of wire for a given length of line, given load, and given distribution of load are given in the chapters on Electrical Design and Wiring of Buildings. On account of the uncertainty regarding the probable increase of load, a close calculation of the size of wire is seldom made, the engineer relaying largely on his experience and judgment, making only a rough calculation as a check.

EFFECT OF DIVERSITY OF FACTOR. It should be noted that in a distribution circuit the maximum load on a feeder is less than the sum of the maximum loads on the mains which it feeds, these in turn are less than the sum of the maximum loads on the transformers connected to these mains, and so on. Therefore, whenever a circuit divides or subdivides, the aggregate sectional areas should ordinarily be greater after division than before. The total drop in voltage from power house or feeding point to a customer's lamp or motor is also usually less than the sum of the maximum drops in the parts of the circuit which are in series (such as house wiring, services, secondary mains, etc.) because these component drops do not have their maximum value simultaneously.

Poles

ARRANGEMENT OF WIRES ON POLES.—The arrangement of wires is governed by mechanical, electrical, and practical considerations. For mechanical reasons it is desirable that:

1. The largest wires be on the lowest cross-arm, in order to reduce the bending stress on the pole to a minimum.

2. The largest wires be on the pins nearest the pole, in order to reduce the bending stress on the cross-arm to a minimum.

3. The wires be arranged symmetrically on the two sides of the pole, especially those which end at the pole, in order to reduce the twisting stress on the pole to a minimum.

For electrical reasons (which, however, are of minor importance) it is desirable that:

4. The wires of any one circuit be as close together as practicable (on adjacent pins), in order to reduce the self-inductance of the circuit.

5. The wires of a three-phase circuit be arranged to form the edges of an equilateral prism and the wires of a two-phase circuit be arranged to form the edges of a square prism, in order to render the inductances and capacities of the wires respectively equal.

6. The wires of different circuits be placed as far apart as practicable, in order to reduce their mutual inductance.

For practical reasons it is desirable that:

7. The highest voltage wires be on the top cross-arms and on the pins farthest from the pole, in order to reduce the danger of accident to linemen.

8. The mains, which have the greatest number of taps, be on the lowest cross-arms, in order to avoid danger of accidental crossing (with contact) of the wires.

9. The arrangement be systematic throughout; this is absolutely essential for safe and economical operation.

As it is impossible to meet all of these conditions the actual arrangements used are compromises and are governed by the relative importance attached to the several desirable conditions. In some cases the electrical requirements (4) to (6) have been considered of most importance, resulting, for example, in an arrangement subordinated to the idea that the three wires of a three-phase circuit must be arranged exactly in an equilateral triangle. It appears, however, that these electrical requirements are really the least important of the considerations and that in most practical cases can be entirely neglected.

TRANSPOSITIONS. Due to the effect of mutual electromagnetic induction an alternating or varying current flowing in one circuit will induce a voltage, and therefore a current, in any parallel circuit; and due to the effect of mutual electrostatic induction (see Capacitance) an alternating or varying voltage in one circuit will induce currents in a parallel circuit, even though there is no metallic connection between the two circuits. It is possible, however, by properly transposing equal alternate lengths of the wires forming the two sides of each of the parallel circuits, to neutralize these effects. Fig. 10 shows diagrammatically a scheme of transposition whereby a two-wire circuit can be protected from both electromagnetic and electrostatic induction from a parallel circuit and vice versa, provided neither circuit is grounded. The more frequent the transpositions the more thoroughly are the effects due to inequality in the spacing of wires and poles eliminated. Transpositions cannot be made effective in eliminating inductive effects when either circuit is grounded, otherwise than at the neutral point. See also Art. 30.

Fig. 10

Methods of calculating the induced voltage and induced currents in one line due to currents and voltage in a neighboring line are indicated in Art. 22.

TREES. Trees constitute a serious obstacle to the proper construction and operation of overhead lines. The principal methods of meeting this difficulty are (1) avoiding them, (2) going over them, (3) going under them, (4) going through them. In most cases a combination of these methods is used. Where trees are a serious factor it is necessary to examine the route of every line in detail, and the size and location of the trees may become the determining feature of the whole design. In such cases nearly all rules of good construction and systematic arrangements are violated in the interests of expediency.

The methods by which trees may be avoided are: using alleys instead of streets, or vice versa; choosing streets without trees for important lines; taking side of street with fewer trees; and finally the very bad arrangement of crossing the street back and forth to avoid the trees either individually or in groups.

The plan of going over the trees is a proper one in the case of all small trees and is perfectly satisfactory until the trees grow up and touch the wires. It is therefore only a temporary method, especially where the trees are of tall, quick-growing varieties. In going over small trees it is well to have poles tall enough to allow for wires clearing after several years' growth. It is usually impracticable to go over large full-grown trees because of cost of poles, unsightliness of very tall poles, and the difficulty of properly guying them to resist wind and the unbalanced pull of wires, which is magnified by the great leverage. It is also difficult or impracticable to take off service wires over the tops of tall trees.

With trees of moderate size it is usually necessary to take the wires through the trees among the leaves and small branches. Special **tree wire** should be used for this purpose. It is rubber insulated and covered with abrasion-resistant braid or armor. It is carried on insulators. The branches and leaves should be trimmed from around the wires as much as possible, including not only those in contact with the wires but such as will be brought into contact by wind or which will grow into contact during the season. Another plan is to protect the wires by split tubes of wood or rolls of baekelized fabric.

Pole Transformers

POLE TRANSFORMERS. The primary (2300–4600 volts) mains are usually run to transformers mounted on poles and the voltage there stepped down to the lamp or motor voltage (110 or 220 volts), and secondary mains run from the transformer to the buildings

in the immediate vicinity. These transformers range in size from 1 1/2 to 100 kv-a, but transformers larger than 100 kv-a are sometimes used in factory districts where the unsightliness of the supporting structure (several poles framed together) does not have to be considered.

Use of Single- and Three-phase Transformers. The main advantage of three single-phase transformers over one three-phase transformer is the greater flexibility in connections, voltage combinations, and spares. For example, if one unit of a delta-delta bank fails, the other two can be operated in open delta at 58 per cent of the capacity of the bank. Initial expenditure at customers' substations may often be kept down by installing two single-phase transformers in open delta, a third one being added when the load has increased sufficiently, thus increasing the bank capacity 73 percent.

The main advantages of the three-phase transformers are higher efficiency, lower cost, saving in space, and simplicity of connections.

Voltage Ratios of Pole Transformers. The usual voltages are 2300 and 4000 volts on the high-tension side and 115, 120 and 125 volts on the low-tension side.

Services

SERVICE WIRES. The service drop wires are those which connect the house wiring with the main on the street. Usually these wires extend in a single span from the nearest pole to the house. At the house they are fastened to insulators similar to those used on the line and mounted on brackets attached to the house. These brackets are often the ordinary wooden bracket used in line work, though the special iron brackets made for the purpose are neater and more secure. The bracket should take the strain of the span, so that, where the service passes through the wall it will not be under strain.

At the pole the service wires are sometimes attached directly to the mains that supply them; although this is the easiest method it has the disadvantage that the strain in the service wire will come on the mains, which may also be injured by the attaching and detaching of numerous service wires. When the service wires do not slope upward or downward at a considerable angle they are likely to become crossed with the main of opposite polarity. Since the general direction of the service wire is at right angles to the main, the service wire should naturally originate on a cross-arm at right angles to the arm carrying the mains. In good service work consequently a cross-arm is generally attached to the pole below the main, and all the service wires to both sides of the street run from this. One tap to each of the wires constituting the main can then be used for a number of service wires.

Present-day tendencies are toward the substitution of two-, three-, or four-conductor cables for the separate wires. A construction which is much favored consists of a group of rubber-insulated wires, with asphalt-saturated braids on each, twisted together without any overall covering.

Where the service entrance and distribution point are in the attic, the service wires pass through the wall in individual porcelain bushings and should have a drip loop between the bracket and bushing so that water will not follow the wire into the building. Where service wires do not conveniently reach the house at point of entrance, they are carried along the wall to such point, being supported at intervals by insulators on brackets. (See also Wiring of Buildings.)

More commonly, however, the service entrance and distribution point are in the basement, and in that case the service drop wires or cables terminate outdoors at the supporting insulators, and there is a separate cable, called a **service entrance cable**, which runs from a service head to the meter. This cable may be in a conduit, in which case it is braid covered; or it may be simply clamped to the wall, in which case it is armored or partially armored in steel wire or steel tape. In some cases, the neutral is bare and is placed over the other conductor or conductors. (See Art. 78.)

The **service head** is a metal hood with its opening covered with a porcelain bushing plate, the opening and plate facing downward so that rain drip will not enter. The single conductors of the service entrance cable run through separate holes of the bushing and, after forming a rain drip loop, curve up to the service drop cable, to which they are connected.

FRONT AND REAR SERVICING. When houses are serviced from a pole line on the street the service wires usually enter the front of the house; if serviced from alley, they enter the rear. Service wires can be run around detached houses from front to rear on brackets on the wall, but this is unsightly and expensive, and it increases the service wire drop. Service entrances can be changed by changing the inside wiring, but the expense is usually heavy and the damage to decoration of rooms sometimes makes it prohibitive.

Sectionalizing of Distribution Circuits, Fuses and Cut-Outs

An overhead a-c system supplied from a single bus is sectionalized: (1) at the switchboard in power station or substation into circuits having no external interconnection by knife switches or oil switches with fuses or automatic trip; (2) at the poles where long branches leave, by transformer cut-outs, either fused or solid, or by pole-type oil switches; (3) at transformer primaries by fused cut-outs. Switches and fuses are used but sparingly in the primary mains, and in a small compact circuit none are necessary. No switches or fuses are used on the secondary side of transformers or on secondary mains except as described in Art. 43.

When several transformers feed the same secondary main, the opening of a primary cut-out, either by blowing of fuse or by hand, does not make the transformer primary dead, as it is still alive from the secondary side. If the fuse of one transformer on a secondary network blows, the additional load thrown on an adjacent transformer may cause the fuse of that to blow, and all transformers to go out in succession. Under these conditions the fuses cannot be replaced in one transformer at a time. To avoid the interruption to service involved in leaving the fuses out until a time of day when one transformer can carry the whole load temporarily, or in having the whole primary circuit out while the fuses are replaced, **secondary network protectors** are essential as described in Art. 43.

Lightning Protection

(See also Section 12, Art. 22.) Practically all disturbances from lightning enter a system through the overhead distributing circuits. The most serious effects are not to the distributing circuit itself but to the switchboard and machinery in the station. The effects on the circuit consist of splitting of poles, puncturing of insulators, puncturing of transformers, blowing of transformer fuses. There may also be damage to meters or appliances on the premises of consumers. The protection principally used includes (1) lightning arresters and choke coils in the station (principally to protect station apparatus); (2) lightning arresters at intervals on the lines; (3) ground wires over the lines; (4) lightning rods on poles; and (5) grounding of the circuit.

USE OF LIGHTNING ARRESTERS. On alternating circuits lightning arresters are used at intervals on the primary but not on the secondary circuits. The amount of line which a lightning arrester will protect is less the more severe the lightning discharge. It is impracticable, and probably impossible, to space the arresters close enough together to protect a circuit absolutely. Theoretically the number of arresters used should be such that the sum of the loss due to the damage by lightning and the expense of providing and maintaining arresters to avoid damage is a minimum. The effects of lightning are too variable and the money loss due to interruption of service is too indeterminate to admit of correct distribution of arresters being determined by calculation. In practice it has been found that when no arresters are used many lightning discharges of considerable intensity do no great amount of damage; also that where the arresters are used much of the damage done is to the arresters themselves, and that their failure is likely to be a cause of interruptions to service. Ordinarily it is well to begin by using arresters sparingly on the lines and putting additional arresters on if found necessary. The maximum number of arresters would be reached when one was provided for each transformer bank.

The importance of **lightning protection** is greatest in a composite system, and in most cases **underground or submarine cables** under 2000 ft in length should be protected by lightning arresters wherever they connect to exposed overhead circuits. Longer ones are considered to be self-protecting.

Ground wires are principally used over transmission lines but may be used to advantage over city distribution wires in exposed places. Where adjoining buildings and trees are higher than the pole line, these foreign objects answer the same purpose and little additional screening effect will be obtained from a special ground wire.

Records of Circuits

Overhead line construction is constantly changing because of erection, moving, and removal of poles; extension of mains; connection and disconnection of service; erection and changing of location and size of transformers. The ease with which changes may be made in the lines necessitates a system of records in such form that any details can be changed at frequent intervals without making a completely new record. The great amount of complicated detail subject to frequent change makes it impracticable to record every feature.

Maps of circuits have usually become useless, soon after they have been prepared, either because obsolete from lack of correction or unintelligible because of successive

erasures and interlineations. The following method of keeping map records has been found to give good results: (1) a map of the circuits is prepared on tracing cloth, giving the circuits on the date of preparation; (2) a blueprint is taken from this tracing, and all changes marked in pencil or crayon on this print, the print and not the tracing being used for correcting and reference; the tracing is not subject to wear or tear; (3) before the blue print becomes illegible from correction or wear the accumulated corrections are made on the tracing and a new up-to-date blueprint substituted, and the process repeated. The corrections on the blueprint can be made by line foremen or other unskillful persons, at the time that the line changes are made; those on the tracing should be made by a draftsman who will do them neatly and with minimum damage to the cloth. If no colored inks are used on parts subject to change, such a tracing will last a long time and will have to be redrawn only when changes have become so numerous or radical as to amount to a rebuilding of the circuits.

45. UNDERGROUND DISTRIBUTION

CONDUCTORS. Underground primary distribution is effected equally by means of three-conductor and single-conductor cables, either of the paper-lead or rubber-lead type, but four-conductor cables are used to a considerable extent, and the two-conductor type in a few places. The usual sizes are No. 4/0 A.W.G. to 350,000 circular mils, although the range extends from No. 6 A.W.G. to 1,500,000 circular mils.

In underground networks it is desirable to have secondary distribution cables which will withstand grounding without creating such disturbance as to necessitate the opening of circuit breakers or fuses. With this in view, cables without lead sheaths are favored in some quarters, and insulation which has the least tendency to burn or to form explosive mixtures with air, when volatilized, are desired. At the present time (1936) there is no thoroughly waterproof insulation which has the desired explosion-proof qualities.

Secondary distribution of the radial type usually employs paper-lead cables, but rubber-lead is also used in many cities. Paper has the advantage of greater carrying capacity, whereas rubber presents less of a problem at its terminations.

SECTIONALIZING DEVICES are almost invariably of the oil cut-out or oil switch type, without remote control.

It is general practice to have fuses on the primary side of transformers, and therefore, fused cut-outs may be used for disconnecting purposes. These are, of course, totally enclosed and provided with wiped outlets and means for venting the gases liberated when fuses operate. Such fuses are usually rated on the basis of faults in the transformer, rather than overload.

Figs. 6 to 9, inclusive, show the use of fuses and sectionalizing switches in networks.

For details of apparatus, see Section 10.

CONDUIT LINES. See Underground Lines, Arts. 33–39.

TRANSFORMERS. Transformers are placed in manholes or vaults under the street or sidewalk. Sometimes, however, they are placed on the upper floors of large buildings. Subway-type transformers differ from the line type in being equipped with a waterproof case, the cover being gasketed and tinned brass wiping sleeves being provided for connecting to cable sheaths.

As for line transformers, the single-phase type is generally preferred, but the three-phase type is coming into favor for networks, as the loss in transformer connection flexibility is compensated by the more flexible circuit. The average size of single-phase transformers is 100 kva, but sizes range from 50 to 150 kva. Three-phase transformers run to three times these capacities.

Primary voltage ratings of 2300, 4000, 11,000, 13,200, and 27,000 volts are in very general use, with delta-connection. The usual secondary voltages are 115, 120, and 125 volts. With three-phase transformers, the secondary voltages are usually 199, 208, and 216 volts, so as to give 115, 120, or 125 volts between line and neutral with star connection.

Network transformers usually have high impedance, often as high as 10 per cent, as compared with about 2.5 per cent for 2300 and 4000 volts, and 4 1/2 to 5 per cent for 11,000 volts and higher, in transformers of the ordinary type.

46. BIBLIOGRAPHY

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BARE WIRES AND CABLES

By W. A. Del Mar

47. DIMENSIONS, WEIGHTS, AND RESISTANCES

A wire may be either solid or stranded, i.e., made up of a number of smaller wires twisted or braided together. A large bare stranded wire is usually called a bare cable. Data on the insulation and protection of wires and cables will be found in the next chapter. Data on **resistance wires** will be found in Section 2, Art. 6.

Resistance of Wires and Cables

CONDUCTIVITY. Per cent conductivity refers to the "**International Annealed Copper Standard.**" On the assumption of a resistivity temperature coefficient of 0.00393 at 20 deg cent this per cent conductivity is 0.283 per cent higher than the conductivity referred to Matthiessen's Standard. If the length of a given wire is L centimeters, its cross-section A square centimeters, and its resistance at 20 deg cent is R_{20} ohms, then the conductivity of this wire is

$$C = \frac{15.328\,L}{88{,}900\,AR_{20}} \text{ per cent}$$

Annealed copper usually has a conductivity of about 100 per cent; hard-drawn copper, a conductivity of about 97 per cent. Ordinarily hard-drawn aluminum has a conductivity of 61 per cent. The conductivity of iron or steel wire ranges from 8 to 16 per cent.

RESISTANCE AT ANY TEMPERATURE. The tables give resistances at 20 deg cent. These may be converted to resistances at other temperatures by the following formulas.

Let C = per cent conductivity.

R_{20} = resistance of 100 per cent conductivity wire at 20 deg cent (from table).

R_t = resistance of wire of conductivity C at temperature t deg cent.

For copper, $$R_t = \frac{100}{C} R_{20}[1 + 0.00393(t - 20)]$$

aluminum, $$R_t = \frac{61}{C} R_{20}[1 + 0.004(t - 20)]$$

steel, see Table VIII.

Solid Wires

TABLES. Tables I–VII for copper and aluminum are compiled from tables in Circular 31 of the Bureau of Standards. Table VIII is compiled from data published by the American Steel and Wire Co.

TROLLEY WIRE. Trolley wires of two different sections are in use in the United States. The sections shown in Fig. 1 are known as the "American Standard," and

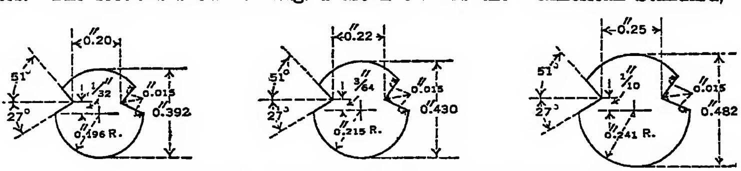

Fig. 1. Grooved Trolley Wire Sections (American Standard)

those in Fig. 2 as the "figure 8" sections. The American Society for Testing Materials recommend that the sizes be specified in circular mils, and not as gage numbers; the

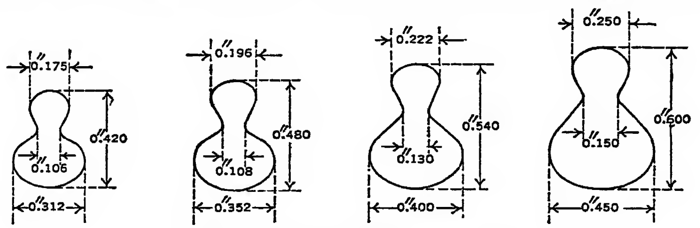

Fig. 2. Figure 8 Trolley Wire Sections

sizes shown in the figure differ in the area of the cross-section from the gage numbers given in parentheses by less than 5 parts in 1000.

Trolley wires are usually of hard-drawn copper; the electrical and mechanical properties are the same as for round hard-drawn wire of the same cross-section. Copper alloys are sometimes used for trolley wires.

,Table I. Solid Copper Wire

A. W. G. or B. & S. Gage; English Units

100 per cent conductivity; density 8.89 at 20 deg cent

Gage No.	Diameter in Mils	Cross-section		Resistance at 20° C or 68° F		Weight in Pounds		Feet per Pound
		Circular Mils	Square Inches	Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile	
0000	460.0	211,600	0.1662	0.04901	0.259	640.5	3380	1.561
000	409.6	167,800	0.1318	0.06180	0.326	507.9	2680	1.968
00	364.8	133,100	0.1045	0.07793	0.411	402.8	2130	2.482
0	324.9	105,500	0.08289	0.90827	0.519	319.5	1680	3.130
1	289.3	83,690	0.06573	0.1239	0.654	253.3	1340	3.947
2	257.6	66,370	0.05213	0.1563	0.825	200.9	1060	4.977
3	229.4	52,640	0.04134	0.1970	1.04	159.3	841	6.276
4	204.3	41,740	0.03278	0.2485	1.31	126.4	667	7.914
5	181.9	33,100	0.02600	0.3133	1.65	100.2	529	9.980
6	162.0	26,250	0.02062	0.3951	2.09	79.46	420	12.58
7	144.3	20,820	0.01635	0.4982	2.63	63.02	333	15.87
8	128.5	16,510	0.01297	0.6282	3.32	49.98	264	20.01
10	101.9	10,380	0.008155	0.9989	5.28	31.43	166	31.82
12	80.81	6.530	0.005129	1.588	8.38	19.77	104	50.59
14	64.08	4,107	0.003225	2.525	13.3	12.43	63.3	80.44
15	57.07	3,257	0.002558	3.184	16.8	9.858	52.0	101.4
16	50.82	2,583	0.002028	4.015	21.2	7.818	41.3	127.9
17	45.26	2,048	0.001609	5.064	26.7	6.200	32.7	161.3
18	40.30	1,624	0.001276	6.385	33.7	4.917	26.0	203.4
19	35.89	1,288	0.001012	8.051	42.5	3.899	20.6	256.5
20	31.96	1,022	0.0008023	10.15	53.6	3.092	16.3	323.4
21	28.46	810.1	0.0006363	12.80	67.6	2.452	12.9	407.8
22	25.35	642.4	0.0005046	16.14	85.2	1.945	10.3	514.2
23	22.57	509.5	0.0004002	20.36	108	1.542	8.14	648.4
24	20.10	404.0	0.0003173	25.67	135	1.223	6.46	817.7
25	17.90	320.4	0.0002517	32.37	171	0.9699	5.12	1,031
26	15.94	254.1	0.0001996	40.82	216	0.7692	4.06	1,300
27	14.20	201.5	0.0001583	51.46	272	0.6100	3.22	1,639
28	12.64	159.8	0.0001255	64.90	343	0.4837	2.55	2,067
29	11.26	126.7	0.00009953	81.84	432	0.3836	2.03	2,607
30	10.03	100.5	0.00007894	103.2	545	0.3042	1.61	3,287
31	8.928	79.70	0.00006260	130.1	687	0.2413	1.27	4,145
32	7.950	63.21	0.00004964	164.1	866	0.1913	1.01	5,227
33	7.080	50.13	0.00003937	206.9	1,090	0.1517	0.814	6,591
34	6.305	39.75	0.00003122	260.9	1,380	0.1203	0.635	8,310
35	5.615	31.52	0.00002476	329.0	1,740	0.09542	0.504	10,480
36	5.000	25.00	0.00001964	414.8	2,190	0.07568	0.400	13,210
37	4.453	19.83	0.00001557	523.1	2,762	0.06001	0.317	16,660
38	3.965	15.72	0.00001235	659.6	3,480	0.04759	0.251	21,010
39	3.531	12.47	0.000009793	831.8	4,392	0.03774	0.199	26,500
40	3.145	9.888	0.000007766	1049	5,540	0.02993	0.158	33,410
41	2.800	7.842	0.000006159	1323	6,983	0.02374	0.125	42,130
42	2.494	6.219	0.000004884	1668	8,806	0.01882	0.0994	53,120
43	2.221	4.932	0.000003873	2103	11,100	0.01493	0.0788	66,990
44	1.978	3.911	0.000003072	2652	14,000	0.01184	0.0625	84,470

Table II. Solid Copper Wire

A. W. G. or B. & S. Gage in Metric Units

100 per cent conductivity; density 8.89 at 20 deg cent

Gage No.	Diameter, mm	Cross-section, sq mm	Ohms per Kilometer 20° C	Kilograms per Kilometer
0000	11.68	107.2	0.1608	953.2
000	10.40	85.03	0.2028	755.9
00	9.266	67.43	0.2557	599.5
0	8.252	53.48	0.3224	475.4
1	7.348	42 41	0.4066	377.0
2	6.544	33.63	0.5126	299.0
3	5.827	26.67	0.6464	237.1
4	5.189	21.15	0.8152	188.0
5	4.621	16.77	1.028	149.1
6	4.115	13.30	1.296	118.2
7	3 665	10.55	1.634	93.78
8	3.264	8.366	2.061	74.37
10	2.588	5.261	3.277	46.77
12	2.053	3.309	5.211	29.42
14	1.628	2.081	8.285	18.50
15	1.450	1.650	10.45	14.67
16	1.291	1.309	13.18	11.63
17	1.150	1.038	16.61	9.226
18	1.024	0.8231	20.95	7.317
19	0.9116	0.6527	26.42	5.803
20	0.8118	0.5176	33.31	4.602
21	0.7230	0.4105	42.00	3.649
22	0.6438	0.3255	52.96	2.894
23	0.5733	0.2582	66.79	2.295
24	0.5106	0.2047	84.22	1.820
25	0.4547	0.1624	106.2	1.443
26	0.4049	0.1288	133.9	1.145
27	0.3606	0.1021	168.8	0.9078
28	0.3211	0.08098	212.9	0.7199
29	0.2859	0.06422	268.5	0.5709
30	0.2546	0.05093	338.6	0.4527
31	0.2268	0.04039	426.9	0.3590
32	0.2019	0.03203	538.3	0.2847
33	0.1798	0.02540	678.8	0.2258
34	0.1601	0.02014	856.0	0.1791
35	0.1426	0.01597	1079	0.1420
36	0.1270	0.01267	1361	0.1126
37	0.1131	0.01005	1716	0.08931
38	0.1007	0.007967	2164	0.07083
39	0.08969	0.006318	2729	0.05617
40	0.07987	0.005010	3441	0.04454
41	0.07113	0.003973	4339	0.03532
42	0.06334	0.003151	5472	0.02801
43	0.05641	0.002499	6900	0.02222
44	0.05023	0.001982	8700	0.01762

Table III. Solid Copper Wire

British Standard Wire Gage; English Units

100 per cent conductivity; density 8.89 at 20 deg cent

Gage No.	Diameter, mils	Cross-section		Ohms per 1000 ft, 15.6° C or 60° F *	Pounds per 1000 ft
		Circular Mils	Square Inches		
7-0	500	250,000	0.1964	0.04077	756.8
6-0	464	215,300	0.1691	0.04734	651.7
5-0	432	186,600	0.1466	0.05461	564.9
4-0	400	160,000	0.1257	0.06370	484.3
3-0	372	138,400	0.1087	0.07365	418.9
2-0	348	121,100	0.09512	0.08416	366.6
0	324	105,000	0.08245	0.09709	317.8
1	300	90,000	0.07069	0.1132	272.4
2	276	76,180	0.05983	0.1338	230.6
3	252	63,500	0.04988	0.1605	192.2
4	232	53,820	0.04227	0.1894	162.9
5	212	44,940	0.03530	0.2268	136.0
6	192	36,860	0.02895	0.2765	111.6
7	176	30,980	0.02433	0.3290	93.76
8	160	25,600	0.02011	0.3981	77.49
9	144	20,740	0.01629	0.4915	62.77
10	128	16,380	0.01287	0.6221	49.59
11	116	13,460	0.01057	0.7574	40.73
12	104	10,820	0.008495	0.9423	32.74
13	92	8,464	0.006648	1.204	25.62
14	80	6,400	0.005027	1.592	19.37
15	72	5,184	0.004072	1.966	15.69
16	64	4,096	0.003217	2.488	12.40
17	56	3,136	0.002463	3.250	9.493
18	48	2,304	0.001810	4.424	6.974
19	40	1,600	0.001257	6.370	4.843
20	36	1,296	0.001018	7.864	3.923
22	28	784.0	0.0006158	13.00	2.373
24	22	484.0	0.0003801	21.06	1.465
26	18	324.0	0.0002545	31.46	0.9807
28	14.8	219.0	0.0001720	46.54	0.6630
30	12.4	153.8	0.0001208	66.28	0.4654
32	10.8	116.6	0.00009161	87.38	0.3531
34	9.2	84.64	0.00006648	120.4	0.2562
36	7.6	57.76	0.00004536	176.5	0.1748
38	6.0	36.00	0.00002827	283.1	0.1090
40	4.8	23.04	0.00001810	442.4	0.06974
42	4.0	16.00	0.00001257	637.0	0.04843
44	3.2	10.24	0.000008042	995.3	0.03100
46	2.4	5.760	0.000004524	1,769	0.01744
48	1.6	2.560	0.000002011	3,981	0.007749
50	1 0	1 000	0.0000007854	10,190	0.003027

* Let C = per cent conductivity, R_{60} = resistance of 100 per cent conductivity wire at 60 deg fahr (from table), R_t = resistance of wire of conductivity C at any temperature t deg fahr; then

$$R_t = \frac{100}{C} R_{60} [1 + 0.00223(t - 60)]$$

Table IV (*a*). Solid Copper Wire

"Millimeter Gage"; Metric Units and Circular Mils

100 per cent conductivity; density 8.89 at 20 deg cent

Diameter, mm	Cross-section, sq mm	Ohms per Kilometer, 20° C	Kilograms per Kilometer	Cross-section, cir mils*
10 0	78.54	0.2195	698.2	155,000
9.0	63.62	0.2710	565.6	125,550
8.0	50.27	0.3430	446.9	99,200
7.0	38.48	0.4480	342.1	75,950
6.0	28.27	0.6098	251.4	55,800
5.0	19.64	0.8781	174.6	38,750
4.5	15.90	1.084	141.4	31,380
4.0	12.57	1.372	111.7	24,800
3.5	9.621	1.792	85.53	18,990
3.0	7.069	2.439	62.84	13,950
2.5	4.909	3.512	43.64	9,690
2.0	3.142	5.488	27.93	6,200
1.8	2.545	6.775	22.62	5,010
1.6	2.011	8.575	17.87	3,970
1.4	1.539	11.20	13.69	3,040
1.2	1.131	15.24	10.05	2,230
1.0	0.7854	21.95	6.982	1,550
0.90	0.6362	27.10	5.656	....
0.80	0.5027	34.30	4.469	....
0.70	0.3848	44.80	3.421	....
0.60	0.2827	60.98	2.514	....
0.50	0.1964	87.81	1.746	....
0.45	0.1590	108.4	1.414	....
0.40	0.1257	137.2	1.117	....
0.35	0.09621	179.2	0.8553	....
0.30	0.07069	243.9	0.6284	....
0.25	0.04909	351.2	0.4364	....
0.20	0.03142	548.8	0.2793	....
0.15	0.01767	975.6	0.1571	....
0.10	0.007854	2195	0.06982	....
0.05	0.001964	8781	0.01746	.....

* One square millimeter equals 1973.52 circular mils.

Table IV (*b*). Large Metric Conductor

Square-Millimeter Gage; Metric Units, Circular Mils and Square Inches

Cross-Section Sq. mm.	Cross-Section		Cross-Section Sq. mm.	Cross-Section	
	Circ. Mils	Sq. in.		Circ. Mils	Sq. in.
1.5	2,960	.002325	150	296,000	.2325
2.5	4,934	.003875	185	365,100	.2867
4.0	7,894	.006200	240	473,600	.3720
6.0	11,840	.009300	300	592,100	.4650
10.0	19,740	.01550	400	789,400	.6200
16.0	31,580	.02480	500	986,800	.7750
25	49,340	.03875	625	1,233,000	.9687
35	69,070	.05425	800	1,579,000	1.240
50	98,680	.07750	1000	1,974,000	1.550
70	138,100	.1085			
95	187,500	.1472			
120	236,800	.1860			

Table V. Solid Copper Wire; Ohms per Unit Weight

A. W. G. or B. & S. Gage; English and Metric Units

100 per cent conductivity; density 8.89 at 20 deg cent

Gage No.	Ohms per Pound			Ohms per Kilogram		
	0° C 32° F	20° C 68° F	50° C 122° F	0° C	20° C	50° C
0000	0.00007051	0.00007652	0.00008554	0.0001554	0.0001687	0.0001886
000	0.0001121	0.0001217	0.0001360	0.0002472	0.0002682	0.0002999
00	0.0001783	0.0001935	0.0002163	0.0003930	0.0004265	0.0004768
0	0.0002835	0.0003076	0.0003439	0.0006249	0.0006782	0.0007582
1	0.0004507	0.0004891	0.0005468	0.0009936	0.001078	0.001206
2	0.0007166	0.0007778	0.0008695	0.001580	0.001715	0.001917
3	0.001140	0.001237	0 001383	0.002512	0.002726	0.003048
4	0.001812	0.001966	0.002198	0.003995	0.004335	0.004846
5	0.002881	0.003127	0.003495	0.006352	0.006893	0.007706
6	0.004581	0.004972	0.005558	0.01010	0.01096	0.01225
7	0.007284	0.007906	0.008838	0.01606	0 01743	0.01948
8	0.01158	0.01257	0.01405	0.02553	0.02771	0.03098
9	0.01842	0.01999	0.02234	0.04060	0.04407	0.04926
10	0.02928	0.03178	0.03553	0.06456	0.07006	0.07833
11	0.04656	0.05053	0.05649	0.1026	0.1114	0.1245
12	0.07404	0.08035	0.08983	0.1632	0.1771	0.1980
13	0.1177	0.1278	0.1428	0.2595	0.2817	0.3149
14	0.1872	0.2032	0.2271	0.4127	0.4479	0.5007
15	0.2976	0.3230	0.3611	0.6562	0.7121	0.7961
16	0.4733	0.5136	0.5742	1.043	1.132	1.266
17	0.7525	0.8167	0.9130	1.659	1.800	2.013
18	1.197	1.299	1.452	2.638	2.863	3.201
19	1.903	2.065	2.308	4.194	4.552	5.089
20	3.025	3.283	3.670	6.670	7.238	8.092
21	4.810	5.221	5.836	10.60	11.51	12.87
22	7.649	8.302	9.280	16.86	18.30	20.46
23	12.16	13.20	14.76	26.81	29.10	32.53
24	19.34	20.99	23.46	42.63	46.27	51.73
25	30.75	33.37	37.31	67.79	73.57	82.25
26	48.89	53.06	59.32	107.8	117.0	131.8
27	77.74	84.37	94.32	171.4	186.0	207.9
28	123.6	134.2	150 0	272.5	295.8	330.6
29	196.6	213.3	238.5	433.3	470.3	525.7
30	312.5	339.2	379.2	689.0	747.8	836.0
31	497.0	539.3	602.9	1,096	1,189	1,329
32	790.2	857.6	958.7	1,742	1,891	2,114
33	1,256	1,364	1,524	2,770	3,006	3,361
34	1,998	2,168	2,424	4,404	4,780	5,344
35	3,177	3,448	3,854	7,003	7,601	8,497
36	5,051	5,482	6,128	11,140	12,080	13,510
37	8,032	8,717	9,744	17,710	19,220	21,480
38	12,770	13,860	15,490	28,150	30,560	34,160
39	20,310	22,040	24,640	44,770	48,590	54,310
40	32,290	35,040	39,170	71,180	77,260	86,360
41	51,340	55,720	62,290	113,200	122,800	137,300
42	81,640	88,600	99,050	180,000	195,300	218,400
43	129,800	140,900	157,500	286,200	310,600	347,200
44	206,400	224,000	250,400	455,000	493,900	552.100

Table VI. Solid Aluminum Wire

A. W. G. or B. & S. Gage; English Units

61 per cent conductivity; density 2.70

Gage No.	Diameter, mils	Cross-section: Circular Mils	Cross-section: Square Inches	Resistance at 20° C or 68° F *: Ohms per 1000 ft	Resistance at 20° C or 68° F *: Ohms per Mile	Weight in Pounds: per 1000 ft	Weight in Pounds: per Mile	Feet per Pound
0000	460.0	211,600	0.1662	0.0804	0.424	195	1027	5.14
000	409.6	167,800	0.1318	0.101	0.535	154	815	6.48
00	364.8	133,100	0.1045	0.128	0.675	122	646	8.17
0	324.9	105,500	0.08289	0.161	0.851	97.0	512	10.31
1	289.3	83,690	0.06573	0.203	1.073	76.9	406	13.00
2	257.6	66,370	0.05213	0.256	1.353	61.0	322	16.39
3	229.4	52,630	0.04134	0.323	1.706	48.4	255	20.7
4	204.3	41,740	0.03278	0.408	2.15	38.4	203	26.1
5	181.9	33,100	0.02600	0.514	2.71	30.4	160.7	32.9
6	162.0	26,250	0.02062	0.648	3.42	24.1	127.4	41.4
7	144.3	20,820	0.01635	0.817	4.31	19.1	101.0	52.3
8	128.5	16,510	0.01297	1.03	5.44	15.2	80.2	65.9
10	101.9	10,380	0.008155	1.64	8.65	9.55	50.4	104.8
12	80.81	6,530	0.005129	2.61	13.76	6.00	31.7	166.6
14	64.08	4,107	0.003225	4.14	21.9	3.78	19.93	265
15	57.07	3,257	0.002558	5.22	27.6	2.99	15.81	334
16	50.82	2,583	0.002029	6.59	34.8	2.37	12.54	421
17	45.26	2,048	0.001609	8.31	43.8	1.88	9.94	531
18	40.30	1,624	0.001276	10.5	55.3	1.49	7.89	670
19	35.89	1,288	0.001012	13.2	69.7	1.18	6.25	844
20	31.96	1,022	0.0008023	16.7	87.9	0.939	4.96	1,065
21	28.46	810.1	0.0006363	21.0	110.9	0.745	3.93	1,343
22	25.35	642.4	0.0005046	26.5	139.8	0.591	3.12	1,693
23	22.57	509.5	0.0004002	33.4	176.3	0.468	2.47	2,130
24	20.10	404.0	0.0003173	42.1	222	0.371	1.961	2,690
25	17.90	320.4	0.0002517	53.1	280	0.295	1.556	3,390
26	15.94	254.1	0.0001996	67.0	353	0.234	1.233	4,280
27	14.20	201.5	0.0001583	84.4	446	0.185	0.978	5,400
28	12.64	159.8	0.0001255	106	562	0.147	0.776	6,810
29	11.26	126.7	0.00009953	134	709	0.117	0.615	8,580
30	10.03	100.5	0.00007894	169	894	0.0924	0.488	10,820
31	8.928	79.70	0.00006260	213	1127	0.0733	0.387	13,650
32	7.950	63.21	0.00004964	269	1421	0.0581	0.307	17,210
33	7.080	50.13	0.00003937	339	1792	0.0461	0.243	21,700
34	6.305	39.75	0.00003122	428	2260	0.0365	0.1929	27,400
35	5.615	31.52	0.00002476	540	2850	0.0290	0.1530	34,510

* Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20 deg cent (from table), R_t = resistance of wire of conductivity C at any temperature t deg cent; then

$$R_t = \frac{61\,R_{20}}{C}[1 + 0.004(t - 20)]$$

Table VII. Solid Aluminum Wire

A. W. G. or B. & S. Gage in Metric Units

61 per cent conductivity; density 2.70; temperature 20 deg cent or 68 deg fahr *

Gage No.	Diameter, mm	Cross-section, sq mm	Ohms per Kilometer	Kilograms per Kilometer
0000	11.68	107.2	0.264	289
000	10.40	85.03	0.333	230
00	9.266	67.43	0.419	182
0	8.252	53.48	0.529	144
1	7.348	42.41	0.667	114
2	6.544	33.63	0.841	90.8
3	5.827	26.67	1.06	72.0
4	5.189	21.15	1.34	57.1
5	4.621	16.77	1.69	45.3
6	4.115	13.30	2.13	35.9
7	3.665	10.55	2.68	28.5
8	3.264	8.366	3.38	22.6
10	2.588	5.261	5.38	14.2
12	2.053	3.309	8.55	8.93
14	1.628	2.081	13.6	5.62
15	1.450	1.650	17.1	4.46
16	1.291	1.309	21.6	3.53
17	1.150	1.038	27.3	2.80
18	1.024	0.8231	34.4	2.22
19	0.9116	0.6527	43.3	1.76
20	0.8118	0.5176	54.6	1.40
21	0.7230	0.4105	68.9	1.11
22	0.6438	0.3255	86.9	0.879
23	0.5733	0.2582	110	0.697
24	0.5106	0.2047	138	0.553
25	0.4547	0.1624	174	0.438
26	0.4049	0.1288	220	0.348
27	0.3606	0.1021	277	0.276
28	0.3211	0.08098	349	0.219
29	0.2859	0.06422	440	0.173
30	0.2546	0.05093	555	0.138
31	0.2268	0.04039	700	0.109
32	0.2019	0.03203	883	0.0865
33	0.1798	0.02540	1110	0.0686
34	0.1601	0.02014	1400	0.0544
35	0.1426	0.01597	1770	0.0431

* Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20 deg cent (from table), R_t = resistance of wire of conductivity C at any temperature t deg cent; then

$$R_t = \frac{61\,R_{20}}{C}[1 + 0.004(t - 20)]$$

The temperature coefficient is approximate only.

Table VIII. Solid Steel Wire

American Steel Wire Gage; English Units

12.5 per cent conductivity; density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C or 68° F *		Weight in Pounds		Feet per Pound
	In.	Mils	Circular Mils	Square Inches	Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile	
	1/2	500.0	250,000	0.1964	0.332	1.752	662.5	3499	1.51
7–0		490.0	240,100	0.1886	0.346	1.825	636.3	3360	1.57
	15/32	468.8	219,800	0.1726	0.378	1.993	582.4	3075	1.72
6–0		460.0	211,600	0.1662	0.392	2.07	560.8	2961	1.78
	7/16	437.5	191,400	0.1503	0.433	2.29	507.2	2678	1.97
5–0		430.0	184,900	0.1452	0.449	2.37	490.0	2587	2.04
	13/32	406.3	165,000	0.1296	0.503	2.65	436.8	2306	2.28
4–0		393.8	155,100	0.1218	0.535	2.82	411.9	2175	2.42
	3/8	375.0	140,600	0.1104	0.590	3.12	372.6	1967	2.68
3–0		362.5	131,400	0.1032	0.631	3.33	348.2	1839	2.87
	11/32	343.8	118,200	0.09280	0.702	3.71	313.1	1653	3.19
2–0		331.0	109,600	0.08605	0.757	4.00	290.3	1533	3.44
	5/16	312.5	97,660	0.07670	0.850	4.49	258.8	1366	3.86
0		306.5	93,940	0.07378	0.883	4.66	249.0	1315	4.02
1		283.0	80,090	0.06290	1.036	5.47	212.2	1121	4.71
	9/32	281.3	79,100	0.06213	1.049	5.54	209.6	1107	4.77
2		262.5	68,910	0.05412	1.204	6.36	182.6	964.1	5.48
	1/4	250.0	62,500	0.04909	1.328	7.01	165.6	874.5	6.04
3		243.7	59,490	0.04665	1.397	7.38	157.4	831.0	6.35
4		225.3	50,760	0.03987	1.635	8.63	134.5	710.2	7.43
	7/32	218.8	47,850	0.03758	1.734	9.15	126.8	669.5	7.89
5		207.0	42,850	0.03365	1.936	10.22	113.6	599.5	8.81
6		192.0	36,860	0.02895	2.25	11.88	97.7	515.8	10.23
	3/16	187.5	35,160	0.02761	2.36	12.46	93.2	491.9	10.73
7		177.0	31,330	0.02461	2.65	13.98	83.0	438.4	12.04
8		162.0	26,240	0.02061	3.16	16.69	69.6	367.2	14.38
	5/32	156.3	24,410	0.01917	3.40	17.95	64.7	341.6	15.46
9		148.3	21,990	0.01727	3.77	19.92	58.3	307.8	17.16
10		135.0	18,200	0.01431	4.55	24.0	48.3	255.0	20.70
	1/8	125.0	15,630	0.01227	5.31	28.0	41.4	218.6	24.15
11		120.5	14,520	0.01140	5.71	30.2	38.5	203.2	25.98
12		105.5	11,130	0.00874	7.45	39.4	29.5	155.7	33.90
	3/32	93.8	8,789	0.00690	9.44	49.8	23.3	123.0	42.94
13		91.5	8,372	0.00658	9.91	52.3	22.1	117.2	45.16
14		80.0	6,400	0.00503	12.96	68.5	17.0	89.55	58.97
15		72.0	5,184	0.00407	16.01	84.5	13.7	72.53	72.80
16		62.5	3,906	0.00307	21.2	112.1	10.4	54.66	96.60
	1/16	62.5	3,906	0.00307	21.2	112.1	10.4	54.66	96.60
17		54.0	2,916	0.00229	28.5	150.2	7.73	40.80	129.5
18		47.5	2,256	0.00177	36.8	194.2	5.98	31.57	167.2
19		41.0	1,681	0.00132	49.4	261	4.45	23.52	224.4
20		34.8	1,211	0.00095	68.5	362	3.21	16.95	311.5
21		31.8	1,008	0.00079	82.3	435	2.67	14.11	374.4
	1/32	31.3	977	0.00076	85.0	449	2.59	13.66	386.5
22		28.6	818	0.00064	101.4	536	2.17	11.45	461.1
23		25.8	666	0.00052	124.6	658	1.76	9.31	567.0
24		23.0	529	0.00042	156.8	828	1.40	7.40	713.5
25		20.4	416	0.00033	199.4	1053	1.10	5.82	907.0

* Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20 deg cent (from table),
R_t = resistance of wire of conductivity C at any temperature t deg cent; then

$$R_t = \frac{12.5\,R_{20}}{C}[1 + 0.006(t - 20)]$$

The temperature coefficient is approximate only.

Table VIII. Solid Steel Wire.—(*Continued*)

American Steel Wire Gage; English Units

12.5 per cent conductivity; density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C or 68° F *		Weight in Pounds		Feet per Pound
	In.	Mils	Circular Mils	Square Inches	Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile	
26		18.1	328	0.00026	253	1337	0.87	4.58	1152
27		17.3	299	0.00024	277	1464	0.79	4.19	1261
28		16.2	262	0.00021	316	1669	0.70	3.67	1438
29		15.0	225	0.00018	469	1947	0.60	3.15	1677
30		14.0	196	0.00015	424	2240	0.52	2.74	1925
31		13.2	174	0.00014	476	2510	0.46	2.44	2166
32		12.8	164	0.00013	506	2670	0.43	2.30	2303
33		11.8	139	0.00011	596	3150	0.37	1.95	2710
34		10.4	108	0.00008	767	4050	0.29	1.51	3489
35		9.5	90	0.00007	919	4850	0.24	1.26	4193
36		9.0	81	0.00006	1023	5410	0 21	1.13	4659

* Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20 deg cent (from table),
R_t = resistance of wire of conductivity C at any temperature t deg cent; then

$$R_t = \frac{12.5\,R_{20}}{C}[1 + 0.006(t - 20)]$$

The temperature coefficient is approximate only.

COPPER-CLAD STEEL WIRE. This wire consists of a steel core and a concentric coat of copper permanently welded thereto. It is used chiefly for long-span transmission and telephone wire. It is made in several grades, which differ in the relative amounts of steel and copper. The grades are designated by the corresponding conductivity expressed as percentages of the Annealed Copper Standard: e.g., 40 per cent grade has a conductivity of 40 per cent.

Table IX. Copper-clad Steel Wire

A. W. G. or B. & S. Gage; English Units

40 per cent conductivity; density 8.26

Gage No.	Diameter, mils	Cross-section		Resistance at 23.9° C or 75° F *		Weight in Pounds		Feet per Pound
		Circular Mils	Square Inches	Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile	
0000	460.0	211,600	0.1662	0.123	0.649	595	3140	1.68
000	409.6	167,800	0.1318	0.154	0.813	471	2490	2.12
00	364.8	133,100	0.1045	0.195	1.03	374	1970	2.67
0	324.9	105,500	0.08289	0.246	1.30	297	1570	3.37
1	289.3	83,690	0.06573	0.310	1.64	235	1240	4.26
2	257.6	66,370	0.05213	0.390	2.06	186	982	5.38
3	229.4	52,630	0.04134	0.492	2.60	148	781	6.76
4	204.3	41,740	0.03278	0.622	3.28	117	618	8.55
5	181.9	33,100	0.02600	0.782	4.13	92.9	491	10.76
6	162.0	26,250	0.02062	0.987	5.21	73.7	389	13.57
7	144.3	20,820	0.01635	1.25	6.60	58.5	309	17.09
8	128.5	16,510	0.01297	1.57	8.29	46.4	245	21.6
9	114.4	13,090	0.01028	1.98	10.5	36.8	194	27.2
10	101.9	10,380	0.008155	2.50	13.2	29.2	154	34.2
11	90.74	8,234	0.006467	3.15	16.6	23.1	122	43.3
12	80.81	6,530	0.005129	3.97	21.0	18.3	96.6	54.6
13	71.96	5,178	0.004067	5.00	26.4	14.6	77.1	68.5
14	64.08	4,107	0.003225	6.31	33.3	11.5	60.7	87.0

* Let C = per cent conductivity,
$R_{23.9}$ = resistance of 40 per cent conductivity wire at 23.9 deg cent (from table),
R_t = resistance of wire of conductivity C at temperature t deg cent; then

$$R_t = \frac{40\,R_{23.9}}{C}[1 + 0.00432(t - 23.9)]$$

The temperature coefficient is approximate only.

ALLOY WIRES OF HIGH TENSILE STRENGTH. Copper alloys having a low conductivity but a tensile strength from 50 per cent to 100 per cent greater than that of copper are sometimes used where strength or hardness is a primary requisite, as in long spans of small wires or for trolley wires. See Art. 48.

Stranded Conductors

FACTORS AFFECTING DIMENSIONS, WEIGHT AND RESISTANCE OF STRANDED WIRES. Individual stranded wires or cables are of four different types, namely: (*a*) bunched wire; (*b*) wire braids; (*c*) concentric-lay cables; and (*d*) rope-lay cables.

Bunched Wires. Bunched wires are used especially for those extra-flexible cables known as cords, wherein the individual wires are so small that concentric stranding is not necessary to keep them together. The wires are assembled parallel and then generally given a slight twist. Sometimes they are kept together by being wound with soft cotton thread which also serves to prevent adhesion between the insulation and wires.

Wire Braids. In the flat form, wire braids are used for potential leads, etc., in lighting cables, where a flexible flat conductor is necessary. Tubular wire braids, known as **basket weave,** are also frequently formed over the insulation of cables in order to afford mechanical protection. Cables for naval or marine purposes and for automobile work are frequently thus protected.

Concentric-lay Cables. A concentric-lay cable is a stranded conductor composed of a central core surrounded by one or more layers of helically laid wires. A rope-lay cable is a stranded wire made up in the same manner by using stranded wires instead of individual solid wires for the core and layers. The cores of concentric-lay cables may be composed of one, two, three, or four wires of equal diameter. A five- or six-wire core would not be symmetrical, and seven wires would themselves constitute a core and a layer.

Number of Wires in Concentric-lay Cables. Table X gives all the possible concentric-lay cables with eight or less layers of equal size wires and formulas for calculating the number of wires with any number of layers. (See Table XIII.)

Table X. Number of Wires in Concentric-lay Cables

(All wires of same diameter)

Number of Layers over Core	Number of Wires in Core			
	1	2	3	4
0	1	2	3	4
1	7	10	12	14
2	19	24	27	30
3	37	44	48	52
4	61	70	75	80
5	91	102	108	114
6	127	140	147	154
7	169	184	192	200
8	217	234	243	252
n	$3n^2 + 3n + 1$	$3n^2 + 5n + 2$	$3n^2 + 6n + 3$	$3n^2 + 7n + 4$

The number of wires per layer increases by six for each successive layer when the core has one wire, the first layer over the core having six. With cores having more than one wire, the increment per layer is not constant.

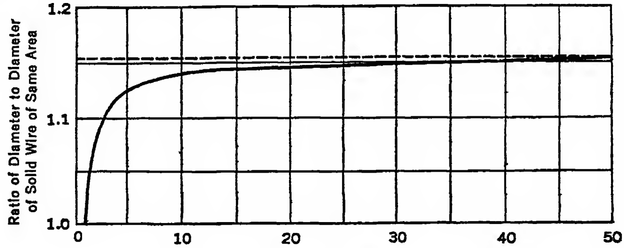

Fig. 3

Diameter of Concentric-lay Cables. The diameter of the circumscribing circle of any of the above cables is equal to $(2n + b)$ times the diameter of each wire, where n is the

number of layers over the core and b has the following values: 1 wire in core, $b = 1$; 2 wires in core, $b = 2$; 3 wires in core, $b = 2.155$; 4 wires in core, $b = 2.414$.

The relation between the number of component wires and the diameter of the cable is shown in Fig. 3.

Rope-lay Cables. As already noted, a rope-lay cable is made up in the same way as a concentric-lay cable except that stranded wires are used for the core and layers instead of individual solid wires. Rope strands are used for large conductors which would be too stiff if stranded concentrically. The formulas for regular concentric-lay cables may be readily modified to apply to rope-lay cables, as each stranded wire bears the same relation to the rope as each individual solid wire does to the concentric-lay cable. Table XI gives the principal forms of rope-lay cables.

Table XI. Wires in Rope-lay Cables

Number of Layers over Core	Number of Strands *	Total Number of Wires				
		Wires per Strand *				
		7	19	37	61	91
0	1	7	19	37	61	91
1	7	49	133	259	427	637
2	19	133	361	703	1159	1,729
3	37	259	703	1369	2257	3,367
4	61	427	1159	2257	3721	5,551
5	91	637	1729	3367	5551	8,281
6	127	889	2413	4699	7747	11,557
n	$3(n^2+n)+1$	$21(n^2+n)+7$	$57(n^2+n)+19$	$111(n^2+n)+37$	$183(n^2+n)+61$	$273(n^2+n)+91$

* By "strand" is here meant the group of stranded wires of which the rope is built up.

The number of wires in a rope-lay cable is frequently designated by a product; thus, 7 × 19 indicates a conductor made up of 7 strands, each strand containing 19 wires.

Table XII. Properties of Concentric-lay Cables

		Regular; $d_c = d$	Special; $d_c \neq d$
I.	Number of wires in terms of number of layers (and core diameter)	$N = 3(n^2+n)+1$ (including core)	$N = 3\left(n\frac{d_c}{d}+n^2\right)$ (excluding core)
II.	Diameter of cable in terms of diameter of wires and number of layers	$D = d(1+2n)$	$D = d_c + 2nd$
III.	Diameter of cable in terms of total area and number of wires	$D = 10^{-3}\sqrt{\frac{1}{3}\left(4-\frac{1}{N}\right)}\cdot\sqrt{A}$	
IV.	Ratio of wire area to area of circle circumscribing the outside of cable	$R = \frac{3(n^2+n)+1}{(2n+1)^2}$	
V.	Weight of cable in terms of weight of wire, number of layers and pitch factors	$W = w(1 + 6p_6 + 12p_{12} + \text{etc.})$	$W = w_c + w(6p_6 + 12p_{12} + \text{etc.})$
VI.	Strength of cable in terms of strength of the component wires and the pitch factors	$T = \frac{\pi}{4}d^2\left[s + t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$	$T = \frac{\pi}{4}\left[sd_c^2 + d^2 t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$
VII.	Minimum pitch in terms of wire diameter and core diameter	$\frac{3\pi d_p}{\sqrt{(\pi+3)(\pi-3)}} = 10.1$ times pitch diameter	$\frac{\pi d_p l d}{\sqrt{(\pi d_p)^2-(ld)^2}}$
VIII.	Diameter of wires in terms of total conductor area and number of wires	$d = \frac{1}{1000}\sqrt{\frac{A}{N}}$	

Diameters of Component Wires in Commercial Stranded Conductors. Table XVII gives the diameters of the component wires in the types of stranded conductors ordinarily used.

Effect of Lay on Resistance and Weight. In Tables XIV, XV, and XVI, for stranded cables, the values given for "ohms per unit length" and "weight per unit length" are 2 per cent greater than for a solid rod of cross-section equal to the total cross-section of the wires of the cable. This increment of 2 per cent means that the values are correct for cables having a lay of 1 in 15.7. For any other lay, equal to 1 in n, resistance or mass may be calculated by increasing the above tabulated values by

$$\left(\frac{484}{n^2} - 2\right) \text{ per cent}$$

Table XIII. Number of Wires for Concentric Standing

(I.P.C.E.A.)

Class A for bare, weatherproof, and slow-burning cables.
Class B for rubber, varnished-cambric, and paper cables.
Class C, Class D } for use where greater degrees of flexibility are desired.

M.C.M. or A.W.G.	Class A	Class B	Class C	Class D	M.C.M. or A.W.G.	Class A	Class B	Class C	Class D
5000	169	217	271	271	500	37	37	61	91
4500	169	217	271	271	450	37	37	61	91
4000	169	217	271	271	400	19	37	61	91
3500	127	169	217	271	350	19	37	61	91
3000	127	169	217	271	300	19	37	61	91
2500	91	127	169	217	250	19	37	61	91
2000	91	127	169	217	4/0	7–19	19	37	61
1900	91	127	169	217	3/0	7–19	19	37	61
1800	91	127	169	217	2/0	7	19	37	61
1750	91	127	169	217	1/0	7	19	37	61
1700	91	127	169	217	1	7	19	37	61
1600	91	127	169	217	2	7	7	19	37
1500	61	91	127	169	3	7	7	19	37
1400	61	91	127	169	4	7	7	19	37
1300	61	91	127	169	5	7	7	19	37
1250	61	91	127	169	6	1	1	19	37
1200	61	91	127	169	7	1	1	19	37
1100	61	91	127	169	8	1	1	19	37
1000	61	61	91	127	9	1	1	7	19
900	61	61	91	127	10	1	1	7	19
800	61	61	91	127	12	1	1	7	19
750	61	61	91	127	14	1	1	7	19
700	61	61	91	127	16	1	1	7	7
650	61	61	91	127	18	1	1	7	7
600	37	61 *	91	127	20	1	1	7	7
550	37	61 *	91	127					

* Thirty-seven wires permissible for rubber-insulated cables.

General Formulas for Properties of Cables in Terms of the Properties of the Constituent Wires. Table XII gives the principal formulas for concentric-lay cable having a core of one wire.

A = total area in circular mils of the component wires measured at right angles to their axes, when laid out straight.
D = diameter of cable overall, in inches.
d = diameter of each of the component wires, in inches.
d_c = diameter of core, in inches.
d_p = pitch diameter, in inches, of any layer (= mean diameter of the helix made by any layer).
e = elongation, per cent, at which the wires (other than the core) break.
l = number of wires in any layer having pitch diameter d_p.
N = total number of wires except where the core is of special size, in which case N is the number of wires exclusive of the core.
n = number of layers of wire over the core.

P = pitch of any layer of wires = distance in inches measured along the axis of the cable for one complete turn of the helix formed by any wire of this layer.
p = pitch-factor of any layer of wires = ratio of the actual length of a wire to the corresponding axial length of the cable.
R = ratio of wire area to the total area of the circle circumscribing the outside of the conductor.
s = stress in pounds per square inch in the core when the elongation is e.
t = tensile strength of each outer wire, in pounds per square inch.
T = tensile strength of conductor, in pounds.
W = weight of conductor, in pounds per foot.
w = weight of each wire of the cable, in pounds per foot.
w_c = weight of the core of the cable, in pounds per foot.

Table XIV. Copper Cables, Concentric-lay

Circular Mils and A. W. G. or B. & S. Gage; English Unite

100 per cent conductivity; density 8.89 at 20 deg cent

Circular Mils and A.W.G.	Resistance at 25° C or 77° F		Weight in Pounds, Bare		Class B Stranding			Class C Stranding		
	Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile	Number of Wires	Diameter of Wires, mils	Outside Diameter, mils	Number of Wires	Diameter of Wires, mils	Outside Diameter, mils
2,000,000	0.00539	0.0285	6180	32600	127	125.5	1631	169	108.8	1632
1,900,000	0.00568	0.0300	5870	31000	127	122.3	1590	169	106.0	1590
1,800,000	0.00599	0.0316	5560	29300	127	119.1	1548	169	103.2	1548
1,700,000	0.00634	0.0335	5250	27700	127	115.7	1504	169	100.3	1504
1,600,000	0.00674	0.0356	4940	26100	127	112.2	1459	169	97.3	1460
1,500,000	0.00719	0.0380	4630	24500	91	128.4	1412	127	108.7	1413
1,400,000	0.00770	0.0407	4320	22800	91	124.0	1364	127	105.0	1365
1,300,000	0.00830	0.0438	4010	21200	91	119.5	1315	127	101.2	1315
1,200,000	0.00899	0.0475	3710	19600	91	114.8	1263	127	97.2	1264
1,100,000	0.00981	0.0518	3400	17900	91	109.9	1209	127	93.1	1210
1,000,000	0.0108	0.0570	3090	16300	61	128.0	1152	91	104.8	1153
950,000	0.0114	0.0600	2930	15490	61	124.8	1123	91	102.2	1124
900,000	0.0120	0.0633	2780	14670	61	121.5	1093	91	99.4	1094
850,000	0.0127	0.0670	2620	13860	61	118.0	1062	91	96.6	1063
800,000	0.0135	0.0712	2470	13040	61	114.5	1031	91	93.8	1031
750,000	0.0144	0.0759	2320	12230	61	110.9	998	91	90.8	999
700,000	0.0154	0.0814	2160	11410	61	107.1	964	91	87.7	965
650,000	0.0166	0.0876	2010	10600	61	103.2	929	91	84.5	930
600,000	0.0180	0.0949	1850	9780	61	99.2	893	91	81.2	893
550,000	0.0196	0.1036	1700	8970	61	95.0	855	91	77.7	855
500,000	0.0216	0.1139	1540	8150	37	116.2	814	61	90.5	815
450,000	0.0240	0.1266	1390	7340	37	110.3	772	61	85.9	773
400,000	0.0270	0.1424	1240	6520	37	104.0	728	61	81.0	729
350,000	0.0308	0.1627	1080	5710	37	97.3	681	61	75.7	682
300,000	0.0360	0.1899	926	4890	37	90.0	630	61	70.1	631
250,000	0.0431	0.228	772	4080	37	82.2	575	61	64.0	576
0000	0.0509	0.269	653	3450	19	105.5	528	37	75.6	533
000	0.0642	0.339	518	2735	19	94.0	470	37	67.3	471
00	0.0811	0.428	411	2170	19	83.7	418	37	60.0	420
0	0.102	0.540	326	1720	19	74.5	373	37	53.4	374
1	0.129	0.681	258	1364	19	66.4	332	37	47.6	333
2	0.162	0.858	205	1082	7	97.4	292	19	59.1	296
3	0.205	1.082	163	858	7	86.7	260	19	52.6	263
4	0.259	1.365	129	680	7	77.2	232	19	46.9	234
5	0.326	1.721	102	540	7	68.8	206	19	41.7	209
6	0.410	2.170	81.0	428	7	61.2	184	19	37.2	186
7	0.519	2.74	64.3	339	7	54.5	164	19	33.1	166
8	0.654	3.45	51.0	269	7	48.6	146	19	29.5	147

Table XV. Copper Cables, Concentric-lay

Circular Mils and A. W. G. or B. & S. Gage in Metric Units

100 per cent conductivity; density 8.89 at 20 deg cent

Circular Mils and A.W.G.	Total Cross-section, sq mm	Ohms per Kilometer at 25° C	Kilograms per Kilometer, Bare	Class B Stranding: Number of Wires	Class B Stranding: Diameter of Wires, mm	Class B Stranding: Outside Diameter, mm	Class C Stranding: Number of Wires	Class C Stranding: Diameter of Wires, mm	Class C Stranding: Outside Diameter, mm
2,000,000	1013	0.0177	9190	127	3.19	41.4	169	2.76	41.4
1,900,000	963	0.0186	8730	127	3.11	40.4	169	2.69	40.4
1,800,000	912	0.0197	8270	127	3.02	39.3	169	2.62	39.3
1,700,000	861	0.0208	7810	127	2.94	38.2	169	2.55	38.2
1,600,000	811	0.0221	7350	127	2.85	37.1	169	2.47	37.1
1,500,000	760	0.0236	6890	91	3.26	35.9	127	2.76	35.9
1,400,000	709	0.0253	6430	91	3.15	34.7	127	2.67	34.7
1,300,000	659	0.0272	5970	91	3.04	33.4	127	2.57	33.4
1,200,000	608	0.0295	5510	91	2.92	32.1	127	2.47	32.1
1,100,000	557	0.0322	5050	91	2.79	30.7	127	2.36	30.7
1,000,000	507	0.0354	4590	61	3.25	29.3	91	2.66	29.3
950,000	481	0.0373	4370	61	3.17	28.5	91	2.60	28.5
900,000	456	0.0393	4140	61	3.09	27 8	91	2.53	27.8
850,000	431	0.0416	3910	61	3.00	27.0	91	2.45	27.0
800,000	405	0.0442	3680	61	2.91	26.2	91	2.38	26.2
750,000	380	0.0472	3450	61	2.82	25.3	91	2.31	25.4
700,000	355	0.0506	3220	61	2.72	24.5	91	2.23	24.5
650,000	329	0.0544	2990	61	2.62	23.6	91	2.15	23.6
600,000	304	0.0590	2760	61	2.52	22.7	91	2.06	22.7
550,000	279	0.0643	2530	61	2.41	21.7	91	1.97	21.7
500,000	253	0.0708	2300	37	2.95	20.7	61	2.30	20.7
450,000	228	0.0786	2070	37	2.80	19.6	61	2.18	19.6
400,000	203	0.0885	1840	37	2.64	18.5	61	2.06	18.5
350,000	177	0.101	1610	37	2.47	17.3	61	1.92	17.3
300,000	152	0.118	1380	37	2.29	16.0	61	1.78	16.0
250,000	127	0.142	1150	37	2.09	14.6	61	1.63	14.6
0000	107	0.167	972	19	2.68	13.4	37	1.93	13.5
000	85	0.211	771	19	2.39	11.9	37	1.71	12.0
00	67.4	0.266	611	19	2.13	10.6	37	1.52	10.7
0	53.5	0.334	485	19	1.89	9.46	37	1.36	9.50
1	42.4	0.423	385	19	1.69	8.43	37	1.21	8.46
2	33.6	0.533	305	7	2.47	7.42	19	1.50	7.51
3	26.7	0.673	242	7	2.20	6.61	19	1.34	6.68
4	21.2	0.849	192	7	1.96	5.88	19	1.19	5.95
5	16.8	1.07	152	7	1.75	5.24	19	1.06	5.30
6	13.3	1.35	121	7	1.56	4.67	19	0.944	4.72
7	10.5	1.70	95.7	7	1.39	4.16	19	0.841	4.20
8	8.37	2.14	75.9	7	1.23	3.70	19	0.749	3.74

Table XVI. Aluminum Cables

A. W. G. and Circular Mils; English Units

A. W. G. Gage or Circular Mils		Usual Number of Strands	Diameter of Bare Cable, in.	Resistance at 25° C or 77° F		Weight in Pounds	
Copper (97 per cent) Equivalent	Aluminum 61 per cent			Ohms per 1000 ft	Ohms per Mile	per 1000 ft	per Mile
1,000,000	1,590,000	61	1.454	0.0111	0.0587	1462	7872
950,000	1,515,500	61	1.416	0.0118	0.0618	1393	7482
900,000	1,431,000	61	1.380	0.0124	0.0652	1317	7086
850,000	1,351,500	61	1.341	0.0131	0.0691	1243	6695
800,000	1,272,000	61	1.301	0.0139	0.0734	1171	6299
750,000	1,192,500	61	1.257	0.0148	0.0783	1098	5908
700,000	1,113,000	61	1.215	0.0159	0.0839	1025	5512
650,000	1,033,500	37	1.170	0.0171	0.0903	950	5116
600,000	954,000	37	1.124	0.0186	0.0979	877	4726
550,000	874,500	37	1.077	0.0202	0.107	805	4330
500,000	795,000	37	1.026	0.0223	0.117	732	3939
450,000	715,500	37	0.974	0.0247	0.131	658	3543
400,000	636,000	37	0.918	0.0278	0.147	585	3152
350,000	556,500	19	0.856	0.0318	0.168	512	2756
300,000	477,000	19	0.793	0.0371	0.196	439	2362
250,000	397,500	19	0.724	0.0444	0.235	365	1969
0000	336,400	19	0.657	0.0525	0.278	310.2	1666
000	266,800	7	0.586	0.0662	0.350	245.7	1322
00	211,950	7	0.522	0.0836	0.441	195	1048
0	167,800	7	0.464	0.105	0.556	155	831
1	133,077	7	0.414	0.133	0.702	122.6	659
2	105,535	7	0.368	0.167	0.885	97.2	523
3	83,693	7	0.328	0.211	1.12	77	414
4	66,371	7	0.293	0.267	1.41	61.2	329
5	52,635	7	0.258	0.336	1.77	48.5	261
6	41,741	7	0.232	0.423	2.24	38.5	207

Table XVII. Commercial Stranded Conductors

Area of Conductor, cir mils	Number of Wires in the Stranded Conductor								
	7	19	37	7×7=49	61	91	127	169	217
	Diameter, in Inches, of Each Wire in the Cable								
2,000,000	0.5345	0.3244	0.2325	0.202	0.181	0.1482	0.1255	0.1086	0.096
1,750,000	0.5000	0.3035	0.2175	0.189	0.169	0.1387	0.1174	0.1020	0.090
1,500,000	0.4629	0.2810	0.2013	0.175	0.157	0.1285	0.1087	0.0940	0.083
1,250,000	0.4226	0.2565	0.1838	0.1507	0.143	0.1174	0.0992	0.0860	0.076
1,000,000	0.3779	0.2294	0.1644	0.1429	0.1285	0.1048	0.0887	0.0769	0.0678
950,000	0.3684	0.2236	0.1602	0.1392	0.1247	0.1021	0.0864	0.0749	0.0661
900,000	0.3585	0.2176	0.1559	0.1355	0.1214	0.0995	0.0841	0.0729	0.0644
850,000	0.3484	0.2115	0.1515	0.1317	0.1180	0.0966	0.0818	0.0709	0.0625
800,000	0.3380	0.2050	0.1470	0.1278	0.1145	0.0937	0.0793	0.0687	0.0607
750,000	0.3273	0.1986	0.1423	0.1237	0.1108	0.0907	0.0769	0.0666	0.0588
700,000	0.3163	0.1919	0.1375	0.1195	0.1071	0.0887	0.0742	0.0643	0.0567
650,000	0.3047	0.1849	0.1325	0.1152	0.1032	0.0845	0.0715	0.0620	0.0547
600,000	0.2927	0.1776	0.1273	0.1107	0.0991	0.0812	0.0687	0.0595	0.0525
550,000	0.2803	0.1701	0.1219	0.1060	0.0949	0.0777	0.0658	0.0571	0.0503
500,000	0.2673	0.1622	0.1162	0.0110	0.0905	0.0741	0.0628	0.0543	0.0480
450,000	0.2535	0.1538	0.1103	0.0958	0.0858	0.0703	0.0595	0.0516	0.0455
400,000	0.2390	0.1457	0.1039	0.0904	0.0809	0.0663	0.0561	0.0486	0.0429
350,000	0.2236	0.1357	0.0972	0.0845	0.0757	0.0620	0.0526	0.0455	0.0401
300,000	0.2070	0.1256	0.0903	0.0783	0.0701	0.0574	0.0486	0.0421	0.0371
250,000	0.1889	0.1147	0.0824	0.0714	0.0640	0.0524	0.0443	0.0384	0.0339
Size A.W.G.									
0000	0.1739	0.1055	0.0756	0.0657	0.0589	....	....	....	....
000	0.1548	0.0940	0.0674	0.0586	0.0525	....	....	....	....
00	0.1379	0.0837	0.0600	0.0521	0.0467	....	....	....	....
0	0.1228	0.0745	0.0534	0.0464	0.0416	....	....	....	....
1	0.1094	0.0664	0.0475	0.0413	....	....	....	....	....
2	0.0974	0.0591	0.0424	0.0369	....	....	....	....	....
3	0.0867	0.0525	0.0377	0.0327	....	....	....	....	....
4	0.0772	0.0468	0.0335	0.0291	....	....	....	....	....
5	0.0688	0.0418	0.0299	0.0260	....	....	....	....	....
6	0.0612	0.0372	0.0266	0.0231	....	....	....	....	....
7	0.0545	0.0331	0.0237	0.0206	....	....	....	....	....
8	0.0484	0.0294	0.0211	0.0184	....	....	....	....	....
9	0.0432	0.0263	0.0188	0.0164	....	....	....	....	....
10	0.0386	0.0233	0.0168	....	....	....	....	....	....
12	0.0306	0.0185	0.0133	....	....	....	....	....	....
14	0.0242	0.0148	0.0105	....	....	....	....	....	....

Weights in pounds per 1000 ft of all bare copper cables are computed by multiplying the circular mils by 0.00309.

Table XVIII. Resistance of Wires and Cables at Various Temperatures

(Based on the Standards of the A.I.E.E.)
Resistance, Ohms of Wire or Cable which is 1000 ft Long at 20 deg C
(Stranded except for sizes smaller than No. 6 A.W.G.)

Size A.W.G. or Cir. In.	20° C 68° F	25° C 77° F	30° C 86° F	35° C 95° F	40° C 104° F	45° C 113° F	50° C 122° F
14	2.525	2.574	2.624	2.674	2.723	2.773	2.822
12	1.588	1.619	1.650	1.682	1.713	1.744	1.775
10	0.9989	1.018	1.038	1.058	1.077	1.097	1.116
8	0.6282	0.6404	0.6527	0.6651	0.6774	0.6898	0.7021
6	0.403	0.410	0.419	0.427	0.435	0.442	0.450
4	0.253	0.259	0.263	0.268	0.273	0.278	0.283
2	0.159	0.162	0.166	0.169	0.172	0.175	0.178
1	0.126	0.129	0.131	0.134	0.136	0.139	0.141
0	0.100	0.102	0.104	0.106	0.108	0.110	0.112
00	0.0795	0.0811	0.0826	0.0842	0.0857	0.0873	0.0888
000	0.0630	0.0642	0.0655	0.0667	0.0680	0.0692	0.0705
0000	0.0500	0.0509	0.0519	0.0529	0.0539	0.0549	0.0559
Cir. In.							
0.25	0.0423	0.0431	0.0440	0.0448	0.0456	0.0465	0.0473
0.35	0.0302	0.0308	0.0314	0.0320	0.0326	0.0332	0.0338
0.50	0.0211	0.0216	0.0220	0.0224	0.0228	0.0232	0.0236
0.75	0.0141	0.0144	0.0147	0.0149	0.0152	0.0155	0.0158
1.00	0.0106	0.0108	0.0110	0.0122	0.0114	0.0116	0.0118
1.25	0.00846	0.00863	0.00879	0.00896	0.00913	0.00929	0.00946
1.50	0.00705	0.00719	0.00733	0.00747	0.00760	0.00774	0.00788
1.75	0.00604	0.00616	0.00628	0.00640	0.00652	0.00664	0.00676
2.00	0.00529	0.00539	0.00550	0.00560	0.00570	0.00580	0.00591

Size A.W.G. or Cir. In.	55° C 131° F	60° C 140° F	65° C 149° F	70° C 158° F	75° C 167° F	80° C 176° F	85° C 185° F
14	2.872	2.922	2.971	3.021	3.071	3.120	3.170
12	1.806	1.838	1.869	1.900	1.931	1.962	1.994
10	1.136	1.156	1.175	1.195	1.215	1.234	1.254
8	0.7144	0.7268	0.7391	0.7515	0.7638	0.7762	0.7885
6	0.458	0.466	0.474	0.482	0.490	0.498	0.506
4	0.288	0.293	0.298	0.303	0.308	0.313	0.318
2	0.181	0.184	0.188	0.191	0.194	0.197	0.200
1	0.144	0.146	0.149	0.151	0.154	0.156	0.158
0	0.114	0.116	0.118	0.120	0.122	0.124	0.126
00	0.0904	0.0920	0.0935	0.0951	0.0967	0.0982	0.0998
000	0.0717	0.0729	0.0742	0.0754	0.0767	0.0779	0.0791
0000	0.0569	0.0578	0.0588	0.0598	0.0608	0.0618	0.0628
Cir. In.							
0.25	0.0481	0.0490	0.0498	0.0505	0.0514	0.0523	0.0531
0.35	0.0344	0.0350	0.0356	0.0362	0.0377	0.0378	0.0379
0.50	0.0241	0.0245	0.0249	0.0253	0.0257	0.0261	0.0266
0.75	0.0160	0.0163	0.0166	0.0169	0.0171	0.0174	0.0177
1.00	0.0120	0.0122	0.0125	0.0127	0.0129	0.0131	0.0133
1.25	0.00962	0.00979	0.00996	0.0101	0.0103	0.0105	0.0106
1.50	0.00802	0.00816	0.00830	0.00844	0.00857	0.00871	0.00885
1.75	0.00687	0.00699	0.00711	0.00723	0.00735	0.00747	0.00759
2.00	0.00602	0.00612	0.00622	0.00633	0.00643	0.00654	0.00664

48. STRENGTH, ELASTICITY, AND EXPANSION COEFFICIENTS OF WIRES

The strength and elasticity of a wire of any material depend to a considerable extent upon the method of manufacture, heat treatment, etc. The tensile strength of soft copper is between 25,000 and 35,000 lb per sq in., as against 60,000 lb per sq in. for hard-drawn copper. Again, because of the greater relative thickness of the hard " skin " and comparatively soft " core " of small hard-drawn copper wires as compared with large wires, the tensile strength, in pounds per square inch, of a small hard-drawn copper wire is greater than that of a large hard-drawn wire. For example, a No. 0000 A.W.G. hard-drawn copper wire has a tensile strength of about 50,000 lb per sq in. as against approximately 65,000 lb per sq in. for a No. 18. A similar but smaller variation holds for soft annealed wires. The tensile strength of steel wire depends to a very great extent upon the composition of the steel.

Metals Available for Conductors

The following table gives, for a No. 0 A. W. G. wire, representative values of the various quantities stated for representative metals and alloys used for wires and cables. These values do not hold, except to a rough approximation, for other sizes of wire.

Strength, Elasticity, and Coefficient of Expansion
Of a No. 0 A. W. G. Wire

Kind of Wire	Tensile Strength, lb per sq in.	Elastic Limit, lb per sq in.*	Modulus of Elasticity, lb-in. units	Coefficient of Linear Expansion	
				per °F	per °C
Copper, soft-annealed......	36,000	....	....	9.6×10^{-6}	17×10^{-6}
Copper, hard-drawn.......	57,330	30,000	16×10^6	9.6×10^{-6}	17×10^{-6}
Aluminum, soft-annealed...	16,000	....	....	12.8×10^{-6}	23×10^{-6}
Aluminum, hard-drawn....	25,000	25,000	9×10^6	12.8×10^{-6}	23×10^{-6}
Copper-clad steel, 40% grade	60,000	51,000	22×10^6	6.7×10^{-6}	12×10^{-6}
P. M. G. (Phelps-Dodge) bronzes...............	60,000 to 140,000	27,000 to 63,000	15×10^6	9.6×10^{-6}	17×10^{-6}
Anaconda copper alloys....	69,000 to 135,000	....	13×10^6 to 15×10^6	....	....
Phonoelectric bronze.......	75,800	55,000	18×10^6	8.3×10^{-6}	14.9×10^{-6}
Steel, ordinary............	68,000	40,000	24×10^6 to 30×10^6	7.0×10^{-6}	12.6×10^{-6}
Steel, Siemens-Martin......	90,000	45,000		7.0×10^{-6}	12.6×10^{-6}
Steel, high-strength........	150,000	82,000		7.0×10^{-6}	12.6×10^{-6}
Steel, extra high-strength...	225,000	135,000		7.0×10^{-6}	12.6×10^{-6}

* There is no elastic limit for soft annealed copper, and the elastic limits of hard-drawn copper and aluminum are not precise quantities.

TENSILE BREAKING LOAD. The tensile strength in pounds for solid wires from $^1/_{16}$ to $^1/_2$ in. in diameter are given in Table XIX.

Table XIX. Breaking Load for Solid Wires in Pounds per Wire

Gage No. A.W.G. or B. & S.	Diameter In.	Diameter Mils	Hard-drawn Copper (A.S.T.M.) *	Hard-drawn Aluminum (23,000 to 33,300 lb per sq in.)	Copper-clad Steel, 40 per cent Grade	Steel (100,000 lb per sq in.) †
	1/2	500	9310	4520	11,400	19,640
0000		460	8140	3820	10,000	16,620
	7/16	437	7500	3460	9,250	15,030
000		410	6720	3030	8,300	13,180
	3/8	375	5800	2540	7,150	11,040
00		365	5540	2400	6,850	10,450
0		325	4520	1910	5,700	8,289
	5/16	312	4220	1770	5,400	7,670
1		289	3680	1530	4,800	6,573
2		258	3000	1240	4,000	5,213
	1/4	250	2830	1170	3,780	4,909
3		229	2420	1000	3,200	4,134
4		204	1950	810	2,600	3,278
	3/16	187	1680	693	2,300	2,761
5		182	1570	655	2,200	2,600
6		162	1270	532	1,800	2,062
7		144	1020	432	1,450	1,635
8		129	822	351	1,200	1,297
	1/8	125	780	335	1,150	1,227
9		114	660	287	975	1,028
10		102	528	234	800	816
11		91	423	191	650	647
12		81	337	155	510	513
13		72	268	126	410	407
14		64	213	103	330	323
	1/16	62	203	98	310	307

* Tensile strength in pounds per square inch ranging from 49,000 for No. 0000 to 66,200 for No. 14; see below.

† For wires having a tensile strength of S pounds per square inch, multiply by $S/100{,}000$. The tensile strength of steel varies from 60,000 to 225,000 lb per sq in.

Table XX. Physical Properties of Hard-drawn Copper Transmission Cables

Size A.W.G. or B. & S.	Area, cir mils	Number of Wires	Diameter of Wires, nominal mils	Diameter of Cable, nominal inch	Elastic Limit		Breaking Strength Nominal lb	Tensile Strength, Minimum, lb per sq in	Weight, Nominal, per 1000 ft, lb
					Cable lb	Lb per sq in.			
4	41,740	3	118.0	0.254	1128	34,400	1879	57,330	128.9
3	52,640	3	132.5	0.285	1415	34,240	2359	57,060	162.5
2	66,370	3	148.7	0.320	1748	33,530	2913	55,890	204.9
2	66,370	7	97.4	0.292	1827	35,050	3045	58,410	204.9
1	83,690	3	167.0	0.360	2172	33,050	3620	55,080	258.4
1	83,690	7	109.3	0.328	2282	34,720	3804	57,870	258.4
1/0	105,500	7	122.8	0.368	2851	34,400	4752	57,330	325.8
2/0	133,100	7	137.9	0.414	3556	34,020	5926	56,700	410.9
3/0	167,800	7	154.8	0.464	4420	33,530	7366	55,890	518.1
4/0	211,600	7	173.9	0.522	5492	33,050	9154	55,080	653.3
4/0	211,600	12	132.8	0.552	5690	34,240	9483	57,060	653.3
4/0	211,600	19	105.5	0.528	5770	34,720	9617	57,870	653.3
Circular Mils									
250,000		12	144.3	0.600	6680	34,020	11130	56,700	771.9
250,000		19	114.7	0.574	6754	34,400	11260	57,330	771.9
300,000		12	158.1	0.657	7900	33,530	13170	55,890	926.3
300,000		19	125.7	0.629	8105	34,400	13510	57,330	926.3
350,000		19	135.7	0.679	9352	34,020	15590	56,700	1081
400,000		19	145.1	0.726	10540	33,530	17560	55,890	1235
450,000		19	153.9	0.770	11850	33,530	19750	55,890	1389
500,000		19	162.2	0.811	13170	33,530	21950	55,890	1544
500,000		37	116.2	0.813	13510	34,400	22510	57,330	1544

Hard-drawn Copper Cables

Table XX gives the mechanical properties of hard-drawn copper transmission cables.

MODULUS. Tests by B. Welbourne (*J. I.E.E.*, 1917, Vol. 56, p. 53) indicate that the elastic modulus of copper cables varies with the number of strands, as shown in Table XXI.

Table XXI

Strands	Modulus, lb-in. units
7	20.0×10^6
19	17.5×10^6
37	15.5×10^6
61	14.0×10^6
91	12.5×10^6

49. CABLES FOR HIGH VOLTAGES AND LONG SPANS

Hollow Copper Cables

LARGE-DIAMETER CABLES. Aerial cables for high voltages should be of sufficiently great diameter to keep the corona loss down to an economic minimum. The diameter required may be calculated from the formulas in Art. 19. As the cross-section is generally determined by the current, conductivity and span, high-voltage lines often require conductors whose cross-section and diameter must be independent of each other, if minimum weight and cost are to be attained. Such conductors are made in a variety of forms, as follows:

P.D.C.P. Hollow Core Cable. This consists of a thin-walled copper core tube which is concentrically stranded with one or more layers of copper wires. These wires may be either solid or hollow, or a combination of solid and hollow wires, as shown in Fig. 4.

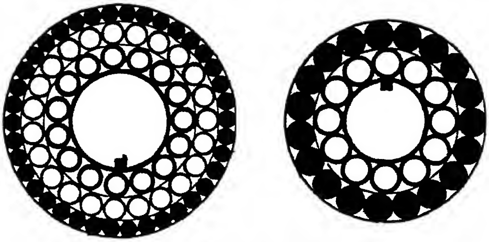

Fig. 4. Type P.D.C.P. Conductors

Anaconda I-Beam Cable. A copper section like a thin I-beam is given a helical twist and used as the core on which one or more layers of wire are stranded, as shown in Fig 5.

Type HH (or Heddernheim) Cable. Interlocking segments of copper are stranded together as shown in Fig. 6, forming a single flexible tube.

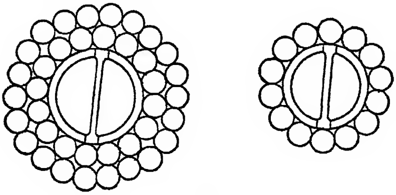

Fig. 5. I-Beam Type Conductors

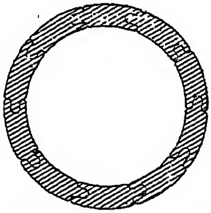

Fig. 6. Type HH Conductor

A.S. & W. Type Cable. Similar to Type HH except that the segments are grooved at both ends and held together by flat elliptical key wires which fit into opposing grooves.

These various types differ somewhat in diameter, corona-suppression, a-c resistance, ease of installation and splicing, ability to resist vibration, and general mechanical stability in tension with and without lubrication, all of which properties should be given due weight. On long spans, consideration must be given, especially during installation, to proper balance of turning moments of the helically laid wires. Where members have substantially different lays, the distribution of load between members must also be considered.

Aluminum-steel Cables

Cables made of a central core of steel, solid or stranded, surrounded by one or more layers of aluminum wire are in extensive use for aerial lines. Their great advantages are that they may be erected not only with less sag than aluminum but with from 60 to 70

Table XXII. Aluminum-steel (A.C.S.R.) Cable *

Size of Conductor: Circular Mils (Aluminum)	Size of Conductor: A.W.G. or B.&S.	Cross-section, sq in.: Aluminum	Cross-section, sq in.: Total	Copper Equivalent Based upon Copper 97% Aluminum 61 %	Stranding: Aluminum	Stranding: Steel	Outside Diameter, in.	Per Cent Aluminum: By Weight	Per Cent Aluminum: By Area	Ultimate Strength lb	Weight per 1000 ft, lb.	D-c Resistance in Ohms per Mile at 0 Amp and 25° C†
1,590,000	....	1.249	1.4071	1,000,000	54x.1716	19x.1030	1.545	73.4	88.5	56,000	2033	0.0587
1,510,500	....	1.186	1.3367	950,000	54x.1673	19x.1004	1.506	73.4	88.5	53,200	1932	0.0618
1,431,000	....	1.124	1.2664	900,000	54x 1628	19x 0977	1 465	73.4	88.5	50,400	1830	0.0652
1,351,500	....	1.062	1.1959	850,000	54x.1582	19x.0949	1 424	73.4	88.5	47,600	1728	0.0691
1,272,000	....	0.9990	1.1256	800,000	54x.1535	19x.0921	1.382	73.4	88.5	44,800	1627	0 0734
1,192,500	....	0.9366	1.0553	750,000	54x.1486	19x 0892	1.338	73.4	88.5	43,100	1526	0.0783
1,113,000	....	0.8741	0 9850	700,000	54x 1436	19x 0862	1.293	73.4	88.5	40,200	1424	0.0839
1,033,500	....	0.8117	0.9170	650,000	54x.1384	7x.1384	1.246	72.9	88.5	37,100	1330	0.0903
954,000	....	0.7493	0.8464	600,000	54x.1329	7x.1329	1.196	72.9	88 5	34,200	1227	0.0979
900,000	....	0.7069	0.7985	566,000	54x.1291	7x.1291	1.162	72.9	88.5	32,300	1158	0.104
874,000	....	0.6868	0.7759	550,000	54x.1273	7x.1273	1.146	72.9	88 5	31,400	1125	0.107
795,000	....	0.6244	0.7054	500,000	54x.1214	7x.1214	1.093	72.9	88.5	28,500	1024	0.117
795,000	....	0.6244	0.7261	500,000	26x.1749	7x.1360	1.108	68.2	86.0	31,200	1094	0 117
795,000	....	0.6244	0.7668	500,000	30x.1628	19x.0977	1.140	60.5	81.5	38,400	1234	0.117
715,500	....	0.5620	0.6348	450,000	54x.1151	7x.1151	1.036	72.9	88.5	26,300	920	0.131
715,500	....	0.5620	0.6535	450,000	26x.1659	7x.1290	1.051	68.2	86.0	28,100	984	0.131
715,500	....	0.5620	0.6900	450,000	30x.1544	19x.0926	1.081	60.5	81.5	34,600	1109	0.131
666,600	....	0.5235	0.5914	419,000	54x.1111	7x.1111	1.000	72.9	88.5	24,500	857	0.140
636,000	....	0.4995	0.5642	400,000	54x.1085	7x.1085	0.977	72.9	88.5	23,600	819	0.147
636,000	....	0.4995	0.5808	400,000	26x.1564	7x.1216	0.990	68.2	86.0	25,000	875	0.147
636,000	....	0.4995	0.6135	400,000	30x.1456	19x.0874	1.019	60.5	81.5	31,500	987	0.147
605,000	....	0.4752	0.5369	380,500	54x.1059	7x.1059	0.953	72.9	88.5	22,500	779	0.154
605,000	....	0.4752	0.5525	380,500	26x.1525	7x.1186	0.966	68.2	86.0	24,100	833	0.154
556,500	....	0.4371	0.5083	350,000	26x.1463	7x.1138	0.927	68.2	86.0	22,400	766	0.168
556,500	....	0.4371	0.5391	350,000	30x.1362	7x.1362	0.953	59.9	81.1	27,200	871	0.168
500,000	....	0.3927	0.4843	314,500	30x.1291	7x.1291	0.904	59.9	81.1	24,400	782.8	0.187
477,000	....	0.3746	0.4357	300,000	26x.1355	7x.1054	0.858	68.2	86.0	19,430	656.6	0 196
477,000	....	0.3746	0.4620	300,000	30x.1261	7x.1261	0.883	59.9	81.1	23,300	746.8	0.196
397,500	....	0.3122	0.3630	250,000	26x.1236	7x.0961	0.783	68.2	86.0	16,190	546.8	0.235
397,500	....	0.3122	0.3850	250,000	30x.1151	7x.1151	0.806	59.9	81.1	19,980	622.4	0.235
336,400	....	0.2642	0.3073	0 000	26x.1138	7x.0885	0.721	68.2	86.0	14,050	643.0	0.278
336,400	....	0.2642	0.3259	0 000	30x.1059	7x.1059	0.741	59.9	81.1	17,040	526.7	0.278
300,000	....	0.2356	0.2739	188,700	26x.1074	7x 0835	0.680	68.2	86.0	12,650	412.7	0.211
300,000	....	0.2356	0.2906	188,700	30x.1000	7x.1000	0.700	59.9	81.1	15,430	469.7	0.211
266,800	....	0.2095	0.2367	000	6x 2109	7x.0703	0.633	72.9	88.5	9,645	343.3	0.350
266,800	....	0.2095	0.2436	000	26x.1013	7x.0788	0.642	68.2	86.0	11,250	367.2	0.350
211,600	0000	0.1662	0.1939	00	6x.1878	1x.1878	0.563	67.6	85.7	8,420	293.4	0.441
167,806	000	0.1318	0.1537	0	6x.1672	1x.1672	0.502	67.6	85.7	6,675	232.4	0.556
133,077	00	0.1045	0.1219	1	6x.1490	1x.1490	0.447	67.6	85.7	5,345	184 5	0.702
105,535	0	0.0829	0.0967	2	6x.1327	1x.1327	0.398	67.6	85.7	4,280	146.4	0.885
83,693	1	0.0657	0.0767	3	6x.1182	1x.1182	0.355	67.6	85.7	3,480	116.1	1.12
66,371	2	0.0521	0.0608	4	6x.1052	1x.1052	0.316	67.6	85.7	2,790	92.1	1.41
52,635	3	0.0413	0.0482	5	6x.0937	1x.0937	0.281	67.6	85.7	2,250	73.0	1.78
41,741	4	0.0328	0.0383	6	6x.0834	1x.0834	0.250	67.6	85.7	1,830	57.9	2.24
33,102	5	0.0260	0.0308	7	6x.0743	1x.0743	0.223	67.6	85.7	1,460	45.8	2.82
26,251	6	0.0206	0.0240	8	6x.0661	1x 0661	0.198	67.6	85.7	1,170	36.4	3.56

* From Electrical Characteristics of A.C.S.R. Cable, issued, 1934, by Aluminum Co. of America.

† A-c resistances are higher, owing to skin effect, especially at high current densities. Values for various current densities are given in Electrical Characteristics of A.C.S.R. Cable, which also gives tables of inductance, reactance, and capacitance and a chart for calculating carrying capacity.

per cent of that of hard-drawn copper and that their diameter is often sufficiently great, except at the highest voltages, to prevent corona formation.

The modulus of elasticity and temperature coefficient of expansion depend upon the relative cross-section of aluminum and steel. The anchor clamps hold both the aluminum and steel, so that at low temperatures the load is divided between the two metals. As the temperature rises, the aluminum expands faster than the steel and the load gradually shifts over to the steel which in turn stretches more to keep pace with the expanded length of aluminum, tending to eliminate relative motion between the two metals. Eventually the steel carries the total load, and the expansion follows the coefficient for steel only.

Let H_a = fraction of area covered by aluminum.
H_s = fraction of area covered by steel.
M_a = modulus of elasticity of aluminum (9,000,000).
M_s = modulus of elasticity of steel (30,000,000).
M = modulus of elasticity of aluminum-steel cable.
G_a = coefficient of expansion of aluminum (0.0000128).
G_s = coefficient of expansion of steel (0.0000064064).
G = coefficient of expansion of cable.

$$M = M_a H_a + M_s H_s = (9 \times H_a + 30\,H_s)10^6,$$

$$G = \left\{\frac{1}{1 + \dfrac{M_a H_a}{M_s H_s}}\right\} G_s - \left\{\frac{1}{1 + \dfrac{M_a H_a}{M_s H_s}} - 1\right\} G_a$$

$$= 0.0000064 \left\{\frac{1}{1 + 0.3\dfrac{H_a}{H_s}}\right\} - 0.0000128 \left\{\frac{1}{1 + 0.3\dfrac{H_a}{H_s}} - 1\right\}$$

Table XXII gives the principal characteristics of aluminum-steel cables.

Steel Cable for Catenary Construction

A modulus of 22×10^6 lb-in. units is representative of ordinary steel messenger cable. Table XXIII compiled from tables published by the General Electric Co. (*Bulletin 4538*). " High " and " Extra-high " strength steel should be used only where absolutely necessary, as, on account of its stiffness, it requires special mechanical fastenings.

Table XXIII. Steel Cable for Catenary Suspension

Extra-galvanized Siemens-Martin Steel Strand, 90,000 lb per sq in.

Diameter, in.	Tensile Strength, lb	Elastic Limit, lb	Elongation, per cent	Lay, in.
1/4	3,060	1,830	6 to 9	3
5/16	4,860	2,910	6 to 9	3 1/2
3/8	6,800	4,080	5 to 8	4
7/16	9,000	5,300	5 to 8	4 1/2
1/2	11,000	6,600	5 to 8	4 1/2
5/8	19,000	11,400	4 to 6	5
Extra-galvanized High-strength Crucible Steel Strand				
1/4	5,100	3,315	3 to 5	3 1/2
5/16	8,100	5,265	3 to 5	4
3/8	11,500	7,475	3 to 5	4 1/2
7/16	15,000	9,500	3 to 5	5
1/2	18,000	11,700	3 to 5	5
5/8	25,000	16,250	2 to 4	5 1/2
Extra-galvanized Extra-high-strength Plow Steel Strand				
1/4	7,600	5,700	2 1/2 to 4	4
5/16	12,100	9,075	2 1/2 to 4	4 1/2
3/8	17,250	12,930	2 1/2 to 4	5
7/16	22,500	16,800	2 1/2 to 4	5 1/2
1/2	27,000	20,250	2 1/2 to 4	5 1/2
5/8	42,000	31,500	1 1/2 to 3	6

50. SKIN-EFFECT AND A-C RESISTANCE

The a-c resistance of conductors is greater than the d-c resistance given in the preceding tables, owing to skin effect. (See Section 3, Art. 24.) The **skin-effect ratio** is the ratio

of the a-c resistance to the d-c resistance. Table XXIV gives these ratios for concentric-lay conductors, and Table XXV for annular conductor cables, such as are used in insulated cables. The skin-effect ratios for annular and similar cables for aerial transmission should be obtained from the manufacturers.

Table XXIV. Skin Effect in Large Concentric-lay Conductors

Class B Stranding (see Table XIII)

Size, cir mils	Skin Effect Ratio at 65° C		Size, cir mils	Skin Effect Ratio at 65° C	
	25 Cycles	60 Cycles		25 Cycles	60 Cycles
5,000,000	1.237	1.765	2,250,000	1.058	1.276
4,500,000	1.200	1.685	2,000,000	1.048	1.233
4,000,000	1.165	1.615	1,750,000	1.036	1.185
3,500,000	1.130	1.513	1,500,000	1.027	1.142
3,000,000	1.100	1.425	1,250,000	1.020	1.103
2,500,000	1.071	1.326	1,000,000	1.011	1.067

Table XXV. Specifications for Annular Conductor Cable

(I.P.C.E.A.)

Nominal Size, cir mils	Approximate Rope Size, in.	Diameter Wires, in.	Number of Strands in Each Layer				Actual Cir-mil Area	Maximum O.D., in.	Approximate Weight Copper, lb per 1000 ft	Approximate D-c Resistance ohms per 1000 ft		Skin Effect Ratio at 65° C	
			1st	2nd	3rd	Total				25° C	65° C	25 Cycles	60 Cycles
750,000	0.375	0.1172	12	18	24	54	741,735	1.108	2,412	0.0146	0.0171	1.004	1.021
800,000	0.468	0.1110	16	21	28	65	800,865	1.164	2,523	0.0135	0.0158	1.004	1.020
900,000	0.500	0.1172	16	22	28	66	906,565	1.234	2,833	0.0119	0.0139	1.005	1.025
1,000,000	0.563	0.1255	16	21	28	65	1,023,766	1.346	3,348	0.0106	0.0124	1.006	1.031
1,250,000	0.750	0.1255	21	26	33	80	1,260,020	1.533	4,022	0.00864	0.0101	1.007	1.034
1,500,000	1.000	0.1255	26	32	38	96	1,512,024	1.783	4,924	0.00719	0.00840	1.007	1.037
1,750,000	1.125	0.1280	30	35	42	107	1,753,088	1.923	5,509	0.00628	0.00734	1.008	1.043
2,000,000	1.3125	0.1284	34	40	46	120	1,978,387	2.114	6,223	0.00556	0.00649	1.009	1.045
2,500,000	1.500	0.1440	34	40	46	120	2,488,320	2.394	7,942	0.00442	0.00516	1.012	1.066
3,000,000	1.625	0.1620	33	38	45	116	3,044,304	2.627	9,762	0.00361	0.00417	1.019	1.105
3,500,000	2.000	0.1620	40	45	52	137	3,595,428	3.007	11,507	0.00306	0.00353	1.020	1.110
4,000,000	2.250	0.1620	45	51	57	153	4,015,332	3.262	12,672	0.00274	0.00316	1.021	1.116
4,500,000	2.500	0.1620	50	56	62	168	4,408,992	3.517	13,836	0.00249	0.00288	1.022	1.118
5,000,000	2.875	0.1620	57	63	69	189	4,960,116	3.897	15,582	0.00222	0.00256	1.023	1.121

51. CURRENT-CARRYING CAPACITY OF BARE WIRES AND CABLES

Values of Thermal Constant K
Deg cent per watt per sq in.

Diameter, in.	Tarnished	Bright
0.1	850	605
0.2	670	490
0.3	490	410
0.4	410	340
0.5	380	302
0.6	340	264
0.7	320	245
0.8	302	226
0.9	282	208
1.0	282	208
1.1 and larger	264	188

Let d = diameter of conductor in inches, T = permissible temperature rise in degrees centigrade above surrounding medium, r = resistance of conductor in ohms per 1000 ft at final temperature, I = current per conductor in amperes.

$$I = \sqrt{\frac{1000T}{rR}}$$

where R is the thermal resistance from conductor surface to ambient air per foot and equals $\dfrac{0.0041\,K}{d}$, where K depends upon the condition of the surface of the wire, and upon the amount of heat convection due to air currents. Values of the constant K for still air are given in the accompanying table.

From data by George E. Luke, based on tests for a temperature rise of 40 deg cent.

52. TESTS OF BARE WIRES AND CABLES

The usual tests on bare wires are gaging diameter, measuring tensile strength, elongation, modulus, elastic limit, and electrical conductivity.

GAGING DIAMETER. (See also Art. 53.) The best type of gage for measuring wire diameters is that shown in Fig. 7. The wire is placed between the measuring surfaces and the screw adjusted until a click occurs. The number of large divisions exposed on the axis is multiplied by 100; the number of small divisions exposed on the axis is multiplied by 25; and the sum of these two items added to the number indicated on the revolving scale. The sum will be the diameter of the wire in mils.

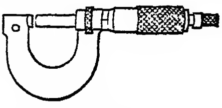

FIG. 7

TENSILE STRENGTH, ELONGATION, MODULUS, AND ELASTIC LIMIT. The essential features of a wire-testing machine are a means of applying a measurable pulling force to the wire, and a means of taking up the elongation. Accordingly, the usual testing machine consists of two pairs of jaws for gripping the wire, one pair being connected to a balance lever and the other to a power-driven mechanism which draws it in the direction of the axis of the wire. A typical machine is shown diagrammatically in Fig. 8, where *A* and *B* are the two pairs of jaws between which the wire is stretched. The machine is operated by setting in motion the mechanism which makes the jaw *A* move steadily in the direction indicated. The operator then moves the counterpoise *C* by hand, in the direction indicated, so as to keep the beam balanced. This operation is continued until the wire breaks, when the elongation of the sample is measured by the travel of the jaw *A* and its breaking strength by the weight indicated on the balance beam at the counterpoise *C*.

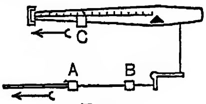

FIG. 8

Measurement of Strain. The amount by which the wire is stretched is measured by means of an extensometer which consists of a pair of clamps to grip the wire at points a definite distance apart, and a magnifying scale for measuring the increase of distance between these clamps as the wire stretches. The stress-strain curve obtained by plotting the elongations thus measured against the stresses measured by the machine described above is not a true one, as there is initially an abnormal elongation due to the straightening of the wire, as shown by curve *OA* in Fig. 6. The standard method of overcoming this, is described in Section 8 of the A.S.T.M. Specification for Hard-drawn Copper Wire.

Modulus of Elasticity. The modulus of elasticity is obtained from the slope of the straight part of the corrected curve. In Fig. 9 the modulus of elasticity is *OD–CD* pound-inch units.

Elastic Limit. The elastic limit can be obtained only by applying a series of increasing loads, releasing the load (leaving, however, a sufficient load to keep the wire straight), and measuring the elongation between successive loads. The load at which a permanent elongation begins is the elastic limit.

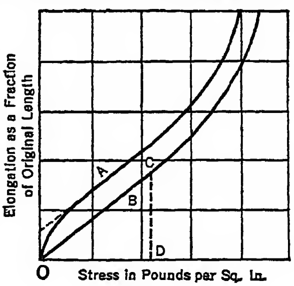

FIG. 9

CONDUCTIVITY. In order to maintain a wire at a uniform and known temperature, it must be short. Unless the wire is very small the test sample will therefore have a very low resistance, and an ordinary Wheatstone bridge will not be sufficiently accurate to measure it. This difficulty is avoided by using a bridge of either the Kelvin, Hoop, Willyoung or Reeves type (see Section 5, Art. 7).

53. SPECIFICATIONS FOR COPPER WIRE AND CABLE

The specifications almost universally used for bare copper wire and cable are those prepared by the American Society for Testing Materials, which are abstracted below.

Specification for Soft or Annealed Copper Wire

GENERAL. This specification covers untinned, drawn, and annealed round wire.

SHIPMENT; COILS, SPOOLS AND REELS. Wire may be shipped in coils or on reels as agreed upon by the purchaser and manufacturer. In Table XXVI (below) there are stated the maximum and minimum weights of wire of the stated sizes which may be

shipped in any one package, whether coil, reel, or spool; for wire larger than 0.010 in. in diameter, the maximum and minimum package weights are net, and for wire 0.010 in. and less in diameter, the maximum package weights are gross, and the minimum package weights are net. The table also states the limiting dimensions of the coils, reels, and spools on which wire may be shipped.

Table XXVI

Diameters, in.	Package Weights, lb		Diameter of Draw-block for Final Drawing, in.	Dimensions of Reels and Spools, in.		
	Maximum	Minimum		Maximum Diameter	Maximum Length	Diameter of Hole for Rod
0.460 to 0.360	520	290	24	32	21	1 1/2 to 2 1/2
0.359 to 0.258	430	290	24	32	21	1 1/2 to 2 1/2
0.257 to 0.129	290	140	22	24	12	1 1/2 to 2 1/2
0.128 to 0.102	230	95	22	24	12	5/8 to 1 1/2
0.101 to 0.083	230	75	22	24	12	5/8 to 1 1/2
0.082 to 0.081	200	75	16	24	12	5/8 to 1 1/2
0.080 to 0.064	200	50	16	24	12	5/8 to 1 1/2
0.063 to 0.051	120	50	16	24	10	5/8 to 1 1/2
0.050 to 0.041	100	50	16	24	10	5/8 to 1 1/2
0.040 to 0.032	50	20	8	24	8	5/8 to 1 1/8
0.031 to 0.020	25	15	8	10	6 1/2	5/8 to 7/8
0.019 to 0.011	10	5	8	5 1/2	4	3/8 to 11/16
0.010 to 0.008	5	2 1/2	8	4	4	3/8 to 11/16
0.007 to 0.0056	2 1/2	1	6	2 1/2	4	3/8 to 11/16
0.005	1 1/2	5/8	6	2 1/2	4	3/8 to 11/16
0.004	1 1/2	3/8	6	2 1/2	4	3/8 to 11/16
0.003	1	1/4	6	2 1/2	4	3/8 to 11/16

SPECIFIC GRAVITY. For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be take as 8.89.

SIZE AND GAGING. (*a*) Size shall be expressed as the diameter of the wire in decimal fractions of an inch.

(*b*) Wire shall be accurate in diameter; permissible variations from nominal diameter shall be:

For wire 0.010 in. in diameter and larger, 1 per cent over or under.

For wire less than 0.010 in. in diameter, 0.1 mil (0.0001 in.) over or under.

(*c*) Each coil shall be gaged at three places, one near each end and one approximately at the middle; from spools, approximately 12 ft shall be reeled off, and the wire shall be gaged in six places between the second and twelfth foot from the end. The coils or spools will be rejected if the average of the measurements obtained is not within the limits specified in (*b*).

Table XXVII. Annealed Copper Wire

Diameter, in.	Tensile Strength, lb per sq in.	Elongation in 10 in., per cent
0.460–0.290	36,000	35
0.289–0.103	37,000	30
0.102–0.021	38,500	25
0.020–0.003	40,000	20

TENSILE STRENGTH AND ELONGATION. Wire shall be so drawn and annealed that its tensile strength shall not be greater and its elongation not less than the values stated in Table XXVII. For wire whose nominal diameter is between listed sizes, the requirements shall be those of the next larger size included in the table.

RESISTIVITY. Electric resistivity shall be determined upon fair samples by resistance measurements at a temperature of 20 deg cent (68 deg fahr) and it shall not exceed 891.58 lb per mile-ohm (0.15614 ohm per meter-gram).

Specification for Hard-drawn Copper Wire

GENERAL. These specifications cover hard-drawn round wire, grooved trolley wire, and figure-eight trolley wire, as hereinafter described.

A section of wire containing a braze must show at least 95 per cent of the tensile strength of the unbrazed wire. Elongation tests are not to be made upon test sections including brazes.

SPECIFIC GRAVITY. For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.89 at 20 deg cent.

HARD-DRAWN ROUND WIRE. (*a*) *Size* shall be expressed as the diameter of the wire in decimal fractions of an inch, using not more than three places of decimals; i.e., in mils.

(*b*) Permissible *variations from nominal diameter* shall be:

For wire 0.100 in. in diameter and larger, 1 per cent over or under;

For wire less than 0.100 in. in diameter, 1 mil over or under.

(*c*) Each coil is to be *gaged* at three places, one near each end, and one approximately at the middle; the coil may be rejected if, two points being within the accepted limits, the third point is off gage more than 2 per cent in wire 0.064 in. in diameter and larger, or more than 3 per cent in the wire less than 0.064 in. in diameter.

Wire shall be so drawn that its *tensile strength* and *elongation* shall be at least equal to the value stated in Table XXVIII.

Electric Resistivity shall be determined upon fair samples by resistance measurements at a temperature of 20 deg cent (68 deg fahr).

The wire shall not exceed the following limits:

For diameters 0.460 in. to 0.325 in., 900.77 lb per mile-ohm at 20 deg cent.

For diameters 0.324 in. to 0.040 in., 910.15 lb per mile-ohm at 20 deg cent.

GROOVED TROLLEY WIRE. *Standard sections* shall be known as the " American Standard " grooved trolley-wire sections, the shape and dimensions of which are as shown in Fig. 1, above.

Table XXVIII. Hard-drawn Copper Wire

Approximate Gage No., B. & S.	Diameter, in.	Area, cir mils	Tensile Strength, lb per sq in.	Elongation, per cent
				in 10 in.
0000	0.460	211,600	49,000	3.75
000	0.410	168,100	51,000	3.25
00	0.365	133,225	52,800	2.80
0	0.325	105,625	54,500	2.40
1	0.289	83,520	56,100	2.17
2	0.258	66,565	57,600	1.98
3	0.229	52,440	59,000	1.79
				in 60 in.
4	0.204	41,615	60,100	1.24
5	0.182	33,125	61,200	1.18
	0.165	27,225	62,000	1.14
6	0.162	26,245	62,100	1.14
7	0.144	20,735	63,000	1.09
	0.134	17,956	63,400	1.07
8	0.128	16,385	63,700	1.06
9	0.114	12,995	64,300	1.02
	0.104	10,815	64,800	1.00
10	0.102	10,404	64,900	1.00
	0.092	8,464	65,400	0.97
11	0.091	8,281	65,400	0.97
12	0.081	6,561	65,700	0.95
	0.080	6,400	65,700	0.94
13	0.072	5,184	65,900	0.92
	0.065	4,225	66,200	0.91
14	0.064	4,096	66,200	0.90
15	0.057	3,249	66,400	0.89
	0.051	2,601	66,600	0.87
16	0.045	2,025	66,800	0.86
18	0.040	1,600	67,000	0.85

(*a*) Size shall be expressed as the area of cross-section in circular mils, the standard sizes being as follows:

211,600 cir mils, weighing 3386 lb per mile.
168,100 cir mils, weighing 2690 lb per mile.
133,200 cir mils, weighing 2132 lb per mile.

(*b*) Grooved trolley wire may vary 4 per cent over or under in weight per unit length from standard, as determined from the nominal cross-section.

The **physical tests** shall be made in the same manner as those upon round wire. The tensile strength of grooved wire shall be at least 95 per cent of that required for round

wire of the same sectional area; the elongation shall be the same as that required for round wire of the same sectional area.

The requirements for electric resistivity shall be the same as those for round wire of the same sectional area.

FIGURE-EIGHT TROLLEY WIRE. Standard sections of figure-eight trolley wire shall be as shown in Fig. 2, above.

The requirements for weight, physical properties, and electric resistivity of figure-eight trolley wire shall be the same as for the same sizes of grooved trolley wire.

Specification for Bare Concentric-lay Copper Cable: Hard, Medium-hard, or Soft

PRODUCTS COVERED. These specifications cover bare concentric-lay cables made from round copper wires laid helically around a central core in one or more layers. The central core shall be made of wire having the same quality and temper as the concentric layers, unless otherwise especially provided for in separate specifications governing the individual case.

Table XXIX

Area of Cross-Section, cir mils	Approximate A. W. G. or B. & S. Gage Sizes	Class A		Class B		Class C	
		Number of Wires	Diameter of Wires, mils	Number of Wires	Diameter of Wires, mils	Number of Wires	Diameter of Wires, mils
2,000,000	......	91	148.2	127	125.5	169	108.8
1,900,000	......	91	144.5	127	122.3	169	106.0
1,800,000	......	91	140.6	127	119.1	169	103.2
1,700,000	......	91	136.6	127	115.7	169	100.3
1,600,000	......	91	132.6	127	112.2	169	97.3
1,500,000	......	61	156.8	91	128.4	127	108.7
1,400,000	......	61	151.5	91	124.0	127	105.0
1,300,000	......	61	146.0	91	119.5	127	101.2
1,250,000	......	61	143.2	91	117.2	127	99.2
1,200,000	......	61	140.3	91	114.8	127	97.2
1,100,000	......	61	134.3	91	109.9	127	93.1
1,000,000	......	61	128.0	61	128.0	91	104.8
950,000	......	61	124.8	61	124.8	91	102.2
900,000	......	61	121.5	61	121.5	91	99.4
850,000	......	61	118.0	61	118.0	91	96.6
800,000	......	61	114.5	61	114.5	91	93.8
750,000	......	61	110.9	61	110.9	91	90.8
700,000	......	61	107.1	61	107.1	91	87.7
650,000	......	61	103.2	61	103.2	91	84.5
600,000	......	37	127.3	61	99.2	91	81.2
550,000	......	37	121.9	61	95.0	91	77.7
500,000	......	37	116.2	37	116.2	61	90.5
450,000	......	37	110.3	37	110.3	61	85.9
400,000	......	19	145.1	37	104.0	61	81.0
350,000	......	19	135.7	37	97.3	61	75.7
300,000	......	19	135.7	37	90.0	61	70.1
250,000	......	19	114.7	37	82.2	61	64.0
212,000	4/0	7–19	173.9–105.5	19	105.5	37–61	75.6–58.9
168,000	3/0	7–19	155.0– 94.0	19	94.0	37–61	67.3–52.5
133,000	2/0	7	138.0	19	83.7	37	60.0
106,000	1/0	7	122.8	19	74.5	37	53.4
83,700	1	7	109.3	19	66.4	37	47.6
66,400	2	7	97.4	7	97.4	19	59.1
52,600	3	7	86.7	7	86.7	19	52.6
41,700	4	7	77.2	7	77.2	19	46.9
33,100	5	7	68.8	7	68.8	19	41.7
26,300	6	7	61.2	7	61.2	19	37.2
20,800	7	7	54.5	7	54.5	19	33.1
16,500	8	7	48.6	7	48.6	19	29.5

NOTE: Class A cable, sizes 4/0 and 3/0, is usually 7-strand when bare and 19-strand when weatherproof, etc.

CLASSES. The purpose for which the several classes of concentric-lay cables are generally used are as follows:

Class *A*, for bare, weatherproof, slow-burning, and slow-burning weatherproof cable for aerial use;

Class *B*, for various insulated cable, such as rubber, paper, varnished cloth, etc.

Class *C*, for cable where greater flexibility is required than in Class *B*.

REQUIREMENTS OF WIRES. The copper wires entering into the construction of standard concentric-lay cable shall, before stranding, meet all the requirements of that one of the Standard Specifications of the American Society for Testing Materials for Hard-drawn, Medium Hard-drawn, or Soft or Annealed Copper Wire (Serial Designations: *B* 1, *B* 2, or *B* 3), which applies.

BRAZES. Brazes may be made in the wire when finished and ready for cabling. No brazes in cable made from hard, or medium hard-drawn copper wire may be closer together than 50 ft.

PITCH AND LAY. The pitch of standard cable shall not be less than 12 nor more than 16 diameters of the cable, and the lay may be right or left-handed, unless one direction of lay is specified by the purchaser.

TESTING. Tests for the physical and electrical properties of the wire composing the cables may be made before, but not after, stranding. Experience indicates that the tensile strength of concentric-lay copper cable of standard pitch is at least 90 per cent of the total strength required of the wires forming the cable.

WEIGHTS AND AREA. For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.89 at 20 deg cent. Two per cent shall be taken as the standard increment of resistance and of mass. If the lay is definitely known, the increment shall be calculated and not assumed.

VARIATION IN AREA. The area of cross-section of the completed cable shall not be more than 2 per cent below the area specified, as determined by weight.

CONSTRUCTION. The area of cross-section, number, and diameter of wires, in standard cable Classes *A*, *B*, and *C*, shall be as specified in Table XXIX.

INSPECTION. All testing and inspection, both of individual wires entering into the construction of the cable, and of the completed cable, shall be made at the place of manufacture. Tests on individual wires shall be made on samples taken before cabling, and not on wires removed from the completed cable.

54. BIBLIOGRAPHY

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See also Art. 71.

INSULATED WIRES AND CABLES

By W. A. Del Mar

55. TERMINOLOGY

WIRE. A wire is a slender rod or filament of drawn metal. The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition, the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire; though primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated, the term "wire" will be understood to include the insulation.

CONDUCTOR. A conductor is a wire or combination of wires not insulated from one another, suitable for carrying an electric current.

STRANDED CONDUCTOR. A stranded conductor is a conductor composed of a group of wires, or of any combination of groups of wires. The wires in a stranded conductor are usually twisted or braided together.

STRAND. A strand is one of the wires, or groups of wires, of any stranded conductor.

CABLE. A cable is either a stranded conductor (single-conductor cable), or a combination of conductors insulated from one another (multiple-conductor cable).

The first kind of cable is a single conductor; the second kind is a group of several conductors. The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering.

The term " cable " is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of " cable." The term " cable " is a general one, and in practice, it is usually applied only to the larger sizes. A small cable is called a " stranded wire " or a " cord," both of which are defined below. Cables may be bare or insulated, and the latter may be sheathed with lead, or armored with wires, or bands.

CORD. A cord is a small very flexible insulated cable. There is no sharp dividing line in respect to size between a " cord " and a " cable."

CONCENTRIC-LAY CONDUCTOR. A concentric-lay conductor is a conductor composed of a central core surrounded by one or more layers of helically laid wires. In the most common type of concentric-lay conductor, all wires are of the same size and the central core is a single wire.

ROPE-LAY CONDUCTOR OR CABLE. A rope-lay cable is a cable composed of a central core surrounded by one or more layers of helically laid groups of wires. This kind of cable differs from the preceding in that the main strands are themselves stranded. In the most common type of rope-lay conductor or cable, all wires are of the same size and the central core is a concentric-lay conductor.

CONCENTRIC-LAY CABLE. A concentric-lay cable is either: (*a*) A concentric-lay conductor as defined. (*b*) A multiple-conductor cable composed of a central core surrounded by one or more layers of helically laid insulated conductors.

***N*-CONDUCTOR CABLE.** An N-conductor cable is a combination of N conductors insulated from one another. It is not intended that the name as here given be actually used. One would instead speak of a "three-conductor cable," a "twelve-conductor cable," etc. In referring to the general case, one may speak of a "multiple-conductor cable."

***N*-CONDUCTOR CONCENTRIC CABLE.** An N-conductor concentric cable is a cable composed of an insulated central conductor with $(N - 1)$ tubular stranded conductors laid over it concentrically and separated by layers of insulation. This kind of cable usually has only two or three conductors. Such cables are used particularly for alternating currents. The remark on the expression " N-conductor " given for the preceding definition also applies here.

DUPLEX CABLE. A duplex cable is a cable composed of two insulated stranded conductors twisted together. They may or may not have a common insulating covering.

TWIN CABLE. A twin cable is a cable composed of two insulated stranded conductors laid parallel, having a common covering.

TWIN WIRE. A twin wire is a cable composed of two small insulated conductors laid parallel, having a common covering.

TWISTED PAIR. A twisted pair is a cable composed of two small insulated conductors, twisted together, without a common covering. The two conductors of a " twisted pair " are usually substantially insulated, so that the combination is a special case of a " cord."

TRIPLEX CABLE. A triplex cable is a cable composed of three insulated single-conductor cables twisted together. They may or may not have a common insulating covering.

SECTOR CABLE. A sector cable is a multiple-conductor cable in which the cross-section of each conductor is substantially a sector, an ellipse, or a figure intermediate between them. Sector cables are used in order to obtain decreased overall diameter and thus permit the use of larger conductors in a cable of given diameter.

ROUND CONDUCTOR. A round conductor is either a solid or stranded conductor of which the cross-section is substantially circular.

SPLIT CONDUCTOR CABLE. A split conductor cable is one in which each conductor is composed of two or more insulated conductors normally connected in parallel.

SHIELDED CONDUCTOR CABLE. A shielded conductor cable is a cable in which the insulated conductor or conductors is/are enclosed in a conducting envelope or envelopes.

FACTOR OF ASSURANCE. The factor of assurance of wire or cable insulation is the ratio of the voltage at which it is tested to that at which it is used.

INSULATION RESISTANCE. The insulation resistance of an insulated conductor is the resistance offered by its insulation, to an impressed direct voltage tending to produce a leakage of current through the same.

MIL. A mil is the one-thousandth part of an inch. There are 1974 circular mils in a square millimeter.

CIRCULAR MIL. A circular mil is a unit of area equal to $\pi/4$ ($= 0.7854 \ldots$) of a square mil. The cross-sectional area of a circle in circular mils is therefore equal to the square of its diameter in mils. A circular inch is equal to one million circular mils.

LAY. The lay or pitch of any helical element of a cable is the axial length of a turn of the helix of that element.

Among the helical elements of a cable may be each strand in a concentric-lay cable, or each insulated conductor in a multiple-conductor cable.

DIRECTION OF LAY. The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

56. DESIGN AND CONSTRUCTION

Materials

CONDUCTORS. Round copper wire, solid or stranded, is almost invariably used for insulated wires and cables. Aluminum, requiring a larger cross-section for the same conductance per unit length, requires more insulating material for the same thickness of insulation.

Stranding. Conductors are stranded in order to make them more flexible. The flexibility is approximately proportional to the square root of the number of strands. In large cables, flexibility is required first in order to permit them to be put on reels; second, to permit easy installation; and third, to permit them to be bent around the walls of splicing chambers. Cords, elevator cables, mining machine cables, etc., must have considerable flexibility in order to permit them to be readily shifted from place to place in service.

For conduit work it is found unnecessary to strand conductors smaller than No. 6 A.W.G., although some contractors use stranded conductors as small as No. 12.

Sector-shaped Conductors. Two-, three-, and four-conductor cables, with sector-shaped conductors are in general use, the three-conductor type being most common. The principal advantage of this type of cable is the greater carrying capacity for a given outside diameter.

PREPARATION OF CONDUCTORS FOR INSULATION. Where rubber insulation is used it is necessary to cover the conductor with a thin film of tin or lead alloy, or with a separator, as described below.

Tinning. Copper and rubber when brought into contact react upon one another chemically. It was formerly thought that this action was due entirely to the sulfur in the rubber compound combining with the copper. Pure rubber, however, also reacts with the copper, the rubber breaking down into a gluey, sticky mass. Tinning the copper affords the necessary protection as tin does not react with either dry rubber or sulfur.

Separators. Small stranded conductors are usually covered with a winding of soft cotton threads for the following purposes:

1. To protect the copper and rubber from mutual chemical action where a coating of tin on the copper cannot be used.
2. To hold together a group of fine wires so that individual wires will not stand up and penetrate the insulation during the manufacturing process.
3. To prevent adhesion between the rubber and copper in order that the copper may be easily bared for making connections.

A separator composed of a wind of soft cotton is used on cords of various types for the first and second reasons. A similar cotton wind on small conductors, or a paper or dry muslin tape on large conductors, is used on cables for car wiring for the third reason given above.

INSULATION. The materials used for insulation are vulcanized rubber, gutta-percha, varnished cloth, impregnated paper, asbestos, cotton and silk thread, enamel, etc.

RUBBER INSULATION. There are two processes by which rubber compound is ordinarily put on the wire, the strip and the tube or seamless processes. By the former, the compound is first made into long narrow strips and then pressed around the wire; by the latter, the wire is run through a die through which the compound is pressed onto the wire. Insulation made by the former process shows a seam or ridge where the sides of the strip have united, which is often partially obliterated by a tape applied before vulcanization; that made by the tube process is seamless.

Rubber insulating compounds are made by mixing new rubber with some or all of the following ingredients:

Vulcanizing agent—either sulfur or sulfur-bearing organic compound capable of liberating sulfur at vulcanizing temperatures.

Accelerator—either litharge or an organic compound, usually of the basic nitrogenous type.

Activator—usually zinc oxide.
Filler—inert mineral matter, such as talc, clay, etc.
Hardener—usually carbon black or clay.
Softener—paraffin wax, ozokerite, resins, reclaimed rubber.
Anti-oxidant—an organic compound usually of nitrogenous type.
Ozone-resistant—vulcanized oil, ceresin, etc.
Vulcanizing is effected in dry steam at a temperature of about 300 deg fahr.
Several processes for direct deposition of **rubber latex** on wire are also in use.
Percentages of rubber vary from about 20 per cent in Code compounds to 60 per cent in low-capacity communication cable compounds.
The principal grades of rubber compound are as follows:

Quality Controlled by Composition	Code grade, for ordinary house wiring (N.E. Code Specification). Intermediate grade, for house wiring where a better quality is desired (N.E.M.A. Specification). Thirty per cent, Type A, formerly the highest grade for wiring of buildings (A.S.T.M. & N.E.M.A. Specifications).
Quality Controlled both by Composition and Accelerated Life Tests	Thirty per cent, Type AO, an improvement on Type A in regard to aging qualities (A.S.T.M. Specification).
Quality Controlled by Accelerated Life Tests	Performance Test Type, the quality of which is gaged entirely by accelerated life tests (A.S.T.M., N.E.M.A. and "Performite" Specifications). High Tension Type, an ozone-resistant or corona-deterrent type of high dielectric strength.
Special Compounds Requiring Special Tests to Control Quality	Submarine Communication Cable Grade—low S.I.C. low water absorption compound with 50–60 per cent of rubber. Cord Sheath Grade—abrasion-resistant compound of high carbon-black content with 40–60 per cent rubber.

VARNISHED CAMBRIC. Cotton fabric coated with multiple layers of varnish is applied to the conductor in the form of tape, the successive turns being staggered. A thin layer of oil or petroleum is applied between layers. Where the cable is to be used for vertical risers, only a small quantity of this "slipper compound" should be used.

IMPREGNATED PAPER. Tapes of special wood pulp paper, 5 to 8 mils thick, are applied to the conductor and then dried and thoroughly impregnated with heavy oily compound, sometimes containing about 15–20 per cent of rosin. The utmost care is required in the drying, evacuating, and impregnating to keep out every trace of air and moisture, especially for high-voltage cables. The lead is applied after impregnation. Oil-filled cable is similar except that a thin oil is used and impregnation is effected after application of the sheath.

ASBESTOS. Asbestos may be applied in felted form, as a tape, or as a braid. Sometimes it is impregnated with asphaltic material, and it is often associated with varnished cambric, as its own electrical properties are very poor.

COTTON AND SILK. Cotton or silk insulation consists of one to three layers of thread spun on to the wire. It is usually paraffined or varnished.

ENAMEL. Wire is passed through successive baths of quick-drying enamel until the requisite thickness is attained.

FILLERS. Fillers, used to round out multiple-conductor cables, are usually made of crinkled paper for paper-insulated cables and of jute for rubber or varnished-cambric insulated cables. The jute may be paraffined, tarred, or vacuum-impregnated with asphaltic compound, depending on the degree of waterproofness required.

PROTECTIVE COVERINGS. Coverings of lead, steel wire, steel tape, treated cotton, asphalted jute, rubberized cotton tape, reinforced rubber, hardened rubber, vulcanized oil, treated paper, etc., are used to protect cable insulation from mechanical injury or water.

Cotton Braid. The most usual covering for rubber-insulated conductors is a cotton braid saturated with asphaltic material and filled with stearine pitch. The larger sizes of cables have a rubberized cotton tape under the braid.

Weatherproofing. Saturated cotton braid is also put on uninsulated hard-drawn copper wire for overhead service, in order to protect the wire from destructive arcing due to accidental contact with tree branches and other foreign bodies.

Lead Sheath. Lead sheath is put over the insulation by passing the cable through a die, while hot lead is pressed hydraulically around it through an annular die, forming a continuous and close-fitting pipe. Were it not for electrolytic corrosion, it would be practically permanent. Unfortunately, lead is subject to electrolytic and even chemical corrosion. It is also rendered brittle and eventually breaks into pieces when exposed to vibration, this effect being due to separation along intercrystalline surfaces. Pure lead is suitable for cables which are hard and compact. Other cables require a small quantity of tin, antimony, or calcium to harden the lead. For a given thickness pure lead is cheaper, but for a given tensile strength, alloy is cheaper. The usual proportions for alloy sheath are either 2 per cent of tin or $^3/_4$ per cent of antimony. Ternary alloys of lead, tin, and cadmium are also used. Such alloyed lead is more resistant to intercrystalline cracking, at atmospheric temperatures, than pure lead.

Non-metallic Sheaths. Sheaths of vulcanized oil, thiokol, asphalt-saturated asbestos, and other materials water-resistant in various degrees, are being largely used for rubber-insulated cables which are to be buried underground.

Armor Wire and Tape. Submarine cables are usually covered with steel wire armor, and cables to be buried direct in the ground are usually covered with galvanized steel tape armor. In either case the armor is usually covered with asphalted jute.

57. APPLICATIONS OF VARIOUS TYPES

The type of insulation and protection to employ in any instance depends upon the purpose for which the conductor is to be used and the place in which it is to be installed.

Power Wires and Cables, that is, those for the transmission or distribution of electric energy, may be installed in buildings, in cars, in underground conduits, on pole lines, buried in the ground, or under water.

Wires and cables for **buildings** are usually rubber-insulated and covered with a saturated braid. Varnished cambric with saturated braid is often used for the larger sizes.

Wires and cables for **power houses** are usually varnished-cambric insulated with either a saturated or a flameproof braid.

Wires and cables for **underground conduit lines** are generally paper-insulated, but rubber or varnished cambric is sometimes used.

Wires for **mining** machinery and for **railway signals** are rubber-insulated.

Submarine Power Cables are usually insulated with rubber, sometimes sheathed in lead, and armored with steel wire. Paper-insulated lead-sheathed and armored submarines are in extensive use for high voltages.

Insulated Conductors for Instrument and Machine Windings: Enameled wire or wire insulated with cotton or silk is used for the former, while varnished cambric, mica, and asbestos compounds are used for the latter.

58. DESIGN

Insulation Thickness

The thickness of insulation required for low-voltage cables is merely that necessitated by inevitable irregularities of manufacture and roughness in handling. At voltages above 1000, the dielectric stress begins to have an influence and, when 2000 volts are reached, becomes the dominant factor. If certain assumptions are made, the thickness of insulation corresponding to a given dielectric stress may be calculated, but these assumptions are only partially realized in practice. These assumptions are:

(*a*) That the radial depth of the insulation or dielectric is the same at all points, i.e., that the cross-section of the cable is bounded by a perfect circle with the conductor section (also a perfect circle) exactly in the center of the insulation. Owing, however, to the crinkling of tape, the pressure of the braid, or other accidents of manufacture, this is never the case. The eccentricity of the conductor with respect to the insulation may, however, be allowed for by adding to the theoretical thickness of insulation an additional thickness, known as the " error thickness " or excess thickness.

(*b*) That the dielectric is perfectly homogeneous throughout. This is never realized in practice. It is probable that even in the most carefully made insulation there is a minute amount of air and moisture, but sufficient to modify considerably any conclusions based on absolute homogeneity of the dielectric.

(*c*) That when the electric stress, or potential gradient, at any part of the dielectric exceeds a certain value F_s known as its dielectric strength, that part of the dielectric becomes a partial conductor even though there be no actual rupture or puncturing.

THEORETICAL THICKNESS OF INSULATION. Let

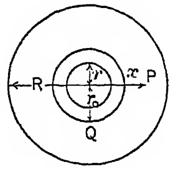

FIG. 1

F = potential gradient, in volts per mil, at any point P in the dielectric at a distance x mils from the center of the wire, Fig. 1.
E = volts between wire and sheath (or outside surface of insulation).
r = radius of wire, in mils.
R = outside radius of insulation, in mils.
F_s = dielectric strength of the insulation, in volts per mil.

Then, on the assumption of a perfectly homogeneous dielectric and perfect symmetry between conductor and insulation,

$$F = \frac{E}{x \log_\epsilon \left(\frac{R}{r}\right)}$$

That is, the potential gradient is the greatest at the surface of the conductor and decreases toward the outer surface of the insulation as shown in Fig. 2. The potential gradient at the surface of the conductor is

$$F_{\max} = \frac{E}{r \log_\epsilon \left(\frac{R}{r}\right)}$$

For the same outside diameter of the insulation this stress at the surface varies with the radius of the conductor as shown in Fig. 3, and has a *minimum* value theoretically when $r = R/2.72$, but actually when R/r is somewhat greater than 2.72.

The volt-ampere characteristics of insulation are generally of the form shown in Fig. 4. Owing to the form of the curve at high stresses, it is evident that, when the critical stress F_c is reached, the current will rise indefinitely, i.e., the insulation will fail, even if the stress is lowered. If, however, in the case of cylindrical insulation, the inner layers are stressed to the value F_c, the remainder may act as a ballast resistor to keep the current down. Then

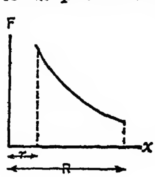

FIG. 2

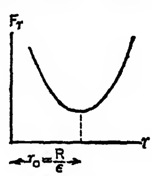

FIG. 3

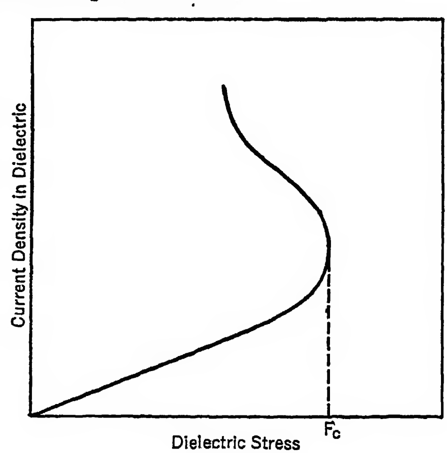

FIG. 4

the inner insulation will be overstressed but uninjured. The above formulas for stress, therefore, are not applicable at or near the breakdown point.

In impregnated-paper insulation, a potential gradient of about 50 volts per mil at the conductor surface will start ionization of occluded gas, but ionization is likely to die out unless the stress is about 60 volts per mil. Oil between paper tapes ionizes at an overall stress of about 230 volts per mil, at atmospheric pressure. (Inge & Walther.)

Rubber insulation is more likely to be injured by excessive minimum than maximum stress as air at the outer surface of the insulation may be ionized and converted into ozone, which rapidly destroys the rubber.

Tables I to VI inclusive give the standard thicknesses of insulation in American practice.

RUBBER INSULATION. The thicknesses of insulation in Table I for use with grounded neutral voltages are practically in agreement with A.I.E.E. Standards. Tabulated

thicknesses for use with ungrounded neutral voltages are based on the same standard but assume that the effective voltage across the insulation is intermediate between the circuit voltage and the voltage to ground. From 0 to 5000 volts the thicknesses are in accord with N.E. Code of 1935.

The thicknesses in Table I apply to single-conductor cable and the individual conductors of multiple-conductor cables, leaded or braided, except special applications such as aerial, non-leaded submarine, vertical riser, and series street lighting. For series street lighting cables, find the *rated voltage* from Table III and the corresponding thickness from Table IV.

Code, intermediate, or other compounds inferior in quality to 30 per cent grades are not acceptable for circuit voltages 5001 volts and above.

The thickness of insulation for the various systems shall be determined as follows,

(*a*) For three-phase systems with grounded or ungrounded neutral, use thickness values in accordance with the respective columns in Table I.

(*b*) For single- or two-phase systems up to and including 5000 volts, use thickness values in accordance with the grounded neutral column of Table I. Where a single-phase line is a tap from a three-phase line, experience indicates that rule (*a*) should be followed.

(*c*) For single- or two-phase systems operating at over 5000 volts with one side grounded, multiply the circuit voltage (phase to phase) by 1.73 and use the resulting voltage value to select the corresponding insulation thickness in the grounded neutral column of Table I.

(*d*) For single- or two-phase systems operating at over 5000 volts with the center grounded, multiply the circuit voltage (phase to phase) by 0.866 ($= 1/2\sqrt{3}$) and use the resulting voltage value to select the corresponding insulation thickness in the grounded neutral column of Table I.

(*e*) For single- or two-phase ungrounded systems operating at over 5000 volts, multiply the circuit voltage (phase to phase) by 0.866 ($= 1/2\sqrt{3}$) and use the resulting voltage value to select the corresponding insulation thickness in the ungrounded neutral column of Table I.

For non-leaded submarine cables, 1/32 in. should be added to the specified respective walls of insulation in Table I for all sizes and voltages.

See below for Shielding standards.

Table I. I.P.C.E.A. Insulation Thicknesses for All Grades of Rubber Insulation

(For series lighting cables, see Tables II and III.)

Rated Circuit Voltage	Size of Conductor A. W. G. or M. C. M.	Insulation Thickness on Each Conductor 64ths inch		Rated Circuit Voltage	Size of Conductor A. W. G. or M. C. M.	Insulation Thickness on Each Conductor 64ths inch	
		Grounded	Ungrounded			Grounded	Ungrounded
0–600	14–9	3	3	2,001–3,000	14–8	7	7
Code grade	8–2	4	4		7–2	8	8
	1–4/0	5	5		1–4/0	8	8
	225–500	6	6		225–500	9	9
	525–1000	7	7		525–1000	9	9
	Over 1000	8	8		Over 1000	10	10
0–600	14–9	3	3	3,001–4,000	14–8	9	9
Intermediate,	8–2	4	4		7–2	9	9
30%, and	1–4/0	5	5		1–4/0	9	9
higher grades	225–500	6	6		225–500	10	10
	525–1000	7	7		525–1000	10	10
	Over 1000	8	8		Over 1000	11	11
601–1,000	14–8	4	4	4,001–5,000	14–8	10	10
	7–2	5	5	(Highest	7–2	10	10
	1–4/0	6	6	N. E. Code	1–4/0	10	10
	225–500	7	7	voltage)	225–500	11	11
	525–1000	8	8		525–1000	11	11
	Over 1000	9	9		Over 1000	12	12
1,001–2,000	14–8	5	5	5,001–6,000	14–8	10	12
	7–2	6	6	(30% hevea	7–2	10	12
	1–4/0	7	7	and higher	1–4/0	10	12
	225–500	8	8	grades)	225–500	11	12
	525–1000	9	9		525–1000	11	12
	Over 1000	9	9		Over 1000	12	13

Table I. I.P.C.E.A. Insulation Thicknesses for All Grades of Rubber Insulation (*Continued*)

Rated Circuit Voltage	Size of Conductor A. W. G. or M. C. M.	Insulation Thickness on Each Conductor 64ths inch	
		Grounded	Un-grounded
6,001–7,000	7–2	11	14
	1–4/0	11	14
	225–500	11	14
	525–1000	11	14
	Over 1000	12	15
7,001–8,000	7–2	12	16
	1–4/0	12	16
	225–500	12	16
	525–1000	12	16
	Over 1000	13	17
8,001–9,000	7–2	13	17
	1–4/0	13	17
	225–500	13	17
	525–1000	13	17
	Over 1000	14	18
9,001–10,000	7–2	14	18
	1–4/0	14	18
	225–500	14	18
	525–1000	14	18
	Over 1000	15	19
10,001–11,000	7–2	15	20
	1–4/0	15	20
	225–500	15	20
	525–1000	15	20
	Over 1000	16	21
11,001–12,000	4–4/0	16	22
	225–500	16	22
	525–1000	16	22
	Over 1000	17	23
12,001–13,000	4–4/0	17	23
	225–500	17	23
	525–1000	17	23
	Over 1000	18	24
13,001–14,000	4–4/0	18	25
	225–500	18	25
	525–1000	18	25
	Over 1000	19	26
14,001–15,000	4–4/0	19	27
	225–500	19	27
	525–1000	19	27
	Over 1000	20	28
15,001–16,000	2–4/0	20	
	225–500	20	
	525–1000	20	
	Over 1000	21	
16,001–17,000	2–4/0	21	
	225–500	21	
	525–1000	21	
	Over 1000	22	
17,001–18,000	2–4/0	22	
	225–500	22	
	525–1000	22	
	Over 1000	23	
18,001–19,000	2–4/0	23	
	225–500	23	
	525–1000	23	
	Over 1000	24	
19,001–20,000	1–4/0	24	
	225–500	24	
	525–1000	24	
	Over 1000	25	
20,001–21,000	1–4/0	25	
	225–500	25	
	525–1000	25	
	Over 1000	26	
21,001–22,000	1–4/0	26	
	225–500	26	
	525–1000	26	
	Over 1000	27	
22,001–23,000	1/0–4/0	27	
	225–500	27	
	525–1000	27	
	Over 1000	28	
23,001–24,000	2/0–4/0	28	
	225–500	28	
	525–1000	28	
	Over 1000	29	
24,001–25,000	2/0–4/0	29	
	225–500	29	
	525–1000	29	
	Over 1000	30	
25,001–26,000	2/0–4/0	30	
	225–500	30	
	525–1000	30	
	Over 1000	31	

[M.C.M. signifies thousands of circular mils.]

VARNISHED-CAMBRIC INSULATION. Table V gives the recommended thickness of insulation for lead-covered cables, for braided cables up to 7500 volts between phases, and for braided cables for all voltages when strung on insulators. See Shielding.

The thicknesses of insulation for various systems shall be determined as follows:

For three-phase systems with grounded or ungrounded neutral, use thickness values in accordance with the respective columns in Table V.

For single- or two-phase systems up to and including 5000 volts, use thickness values in accordance with the ungrounded neutral column of Table V.

For single- or two-phase systems operating at over 5000 volts with one side grounded, multiply the circuit voltage (phase to phase) by 1.73 and use the resulting voltage value to select the corresponding insulation thickness in the grounded neutral column of Table V.

Table II. Thickness of Rubber Insulation for Railway Signal Cables (A.A.R.)

666 Volts or Less			
Single Conductor		Multi-conductor	
Size of Wire, A. W. G. or B. & S.	Insulation Thickness, 64ths inch	Size of Wire, A. W. G. or B. & S.	Insulation Thickness, 64ths inch
18–16	4	16	3
14–9	5	14–10	4
8–4	6	9–6	5
2–0	8	4	6

Table III. Rating Data for Series Lighting Cable

Rated voltage in terms of transformer rating.

(Use thickness from Table IV.)

Transformer Rating, kw	Maximum Operating Voltage		Recommended Voltage Rating		Transformer Rating, kw	Maximum Operating Voltage		Recommended Voltage Rating	
	Open Circuit	Closed Circuit	Without Protectors	With* Protectors		Open Circuit	Closed Circuit	Without Protectors	With* Protectors
1	225	152	600	600	20	4,115	3,030	5,000	4,000
2	430	303	600	600	25	5,110	3,787	6,000	4,000
3	660	454	600	1,000	30	6,130	4,545	7,000	5,000
5	1,080	757	2,000	1,000	40	8,260	6,060	9,000	7,000
7 1/2	1,600	1,137	2,000	2,000	50	10,350	7,580	11,000	8,000
10	2,090	1,515	3,000	2,000	60	12,400	9,100	13,000	10,000
15	3,090	2,272	4,000	3,000	70	14,500	10,600	15,000	11,000

* Where protectors are used for economy rather than for added security.

Table IV. Thickness of Rubber Insulation for Series Lighting Cable (I.P.C.E.A., 1934)

(See Table III for rated voltage in terms of power rating and operating voltages.)

Rated Voltage	Insulation Thickness 64ths inch		Rated Voltage	Insulation Thickness 64ths inch	
	No. 8 A. W. G.	No. 7–No. 4 A. W. G.		No. 8 A. W. G.	No. 7–No. 4 A. W. G.
600	4	4	8,000	12	12
1,000	4	5	9,000	13	13
2,000	5	6	10,000	14	14
3,000	7	8	11,000	15	15
4,000	9	9	12,000	16	16
5,000	10	10	13,000	17	17
6,000	10	10	14,000	18	18
7,000	11	11	15,000	19	19

For single- or two-phase systems, operating at over 5000 volts with the center grounded, multiply the circuit voltage (phase to phase) by 0.866 ($= \frac{1}{2}\sqrt{3}$) and use the resulting voltage value to select the corresponding insulation thickness in the grounded neutral column of Table V.

For single- or two-phase, ungrounded systems operating at over 5000 volts, multiply the circuit voltage (phase to phase) by 0.866 ($= \frac{1}{2}\sqrt{3}$), and use the resulting voltage value to select the corresponding insulation thickness in the ungrounded neutral column of Table V.

For d-c systems up to and including 2000 volts, use thickness values in accordance with the grounded neutral column of Table V.

For d-c systems over 2000 volts, consult the manufacturers.

For series lighting cables, use Tables III and VII.

See next article for Shielding standards.

Table V. Recommended Thickness of Varnished-Cambric Insulation (I.P.C.E.A.)

Single-conductor Cable and Multiple-conductor Shielded Cable

Rated Voltage Volts Phase to Phase	Size A. W. G. or 1000 cir mil	Ungrounded Neutral Wall of V. C. in 64ths inch (and mils)	Grounded Neutral Wall of V. C. in 64ths inch (and mils)	N.E. Code 1935
0–600	14–8	3 (47)	3 (47)	3 (47)
	7–2	4 (63)	4 (63)	4 (63)
	1–4/0	5 (78)	5 (78)	5 (78)
	213–500	6 (94)	6 (94)	6 (94)
	501–1000	7 (109)	7 (109)	7 (109)
	1001 and larger	8 (125)	8 (125)	8 (125)
601–1,000*	14–2	4 (63)	4 (63)	4 (63)
	1–4/0	5 (78)	5 (78)	5 (78)
	213–500	6 (94)	6 (94)	6 (94)
	501–1000	7 (109)	7 (109)	7 (109)
	1001 and larger	8 (125)	8 (125)	8 (125)
1,001–2,000	12–2†	5 (78)	5 (78)	5 (78)
	1–4/0	6 (94)	6 (94)	6 (94)
	213–500	6 (94)	6 (94)	6 (94)
	501–1000	7 (109)	7 (109)	7 (109)
	1001 and larger	8 (125)	8 (125)	8 (125)
2,001–3,000 (incl. 2,500*)	10–2†	6 (94)	6 (94)	6 (94)
	1–4/0	6 (94)	6 (94)	6 (94)
	213–500	7 (109)	7 (109)	7 (109)
	501–1000	7 (109)	7 (109)	7 (109)
	1001 and larger	8 (125)	8 (125)	8 (125)
3,001–4,000	8–4/0	7 (109)	7 (109)	7 (109)
	213–500	8 (125)	8 (125)	8 (125)
	501–1000	8 (125)	8 (125)	8 (125)
	1001 and larger	9 (141)	9 (141)	9 (141)
4,001–5,000 (incl. 4,500*)	8–4/0	9 (141)	8 (125)	9 (141)
	213–1000	10 (156)	9 (141)	10 (156)
	1001 and larger	10 (156)	9 (141)	10 (156)
5,001–6,000	8–4/0	10 (156)	9 (141)	
	213–1000	11 (172)	9 (141)	
	1001 and larger	11 (172)	9 (141)	
6,001–7,000	8 and larger	11 (172)	10 (156)	
7,001–8,000 (incl. 7,500*)	6 and larger	12 (188)	11 (172)	
8,001–9,000	6 and larger	13 (203)	11 (172)	
9,001–10,000	6 and larger	15 (234)	12 (188)	
10,001–11,000	6 and larger	16 (250)	13 (203)	
11,001–12,000	6 and larger	16 (250)	14 (219)	
12,001–13,000	6 and larger	18 (281)	15 (234)	
13,001–14,000	6 and larger	19 (296)	15 (234)	
14,001–15,000*	6 and larger	21 (328)	16 (250)	
15,001–16,000	4 and larger	22 (344)	17 (266)	
16,001–17,000	4 and larger	23 (359)	18 (281)	
17,001–18,000	4 and larger		19 (296)	
18,001–19,000	4 and larger		20 (313)	
19,001–20,000	2 and larger		21 (328)	
20,001–21,000	2 and larger		22 (344)	
21,001–22,000	2 and larger		23 (359)	
22,001–23,000*	2 and larger		24 (375)	
23,001–24,000	2 and larger		25 (391)	
24,001–25,000	2 and larger		26 (406)	
25,001–26,000	2 and larger		27 (422)	
26,001–27,000	2 and larger		28 (438)	
27,001–28,000	1 and larger		29 (453)	

* The ratings marked * are those recommended by the N.E.M.A.-N.E.L.A. Joint Committee on Voltage Standardization as "Preferred Voltage Ratings" for "general apparatus" except 1000 which has been added because of the large amount of very low-voltage cable used.

† N.E. Code specifies No. 8 minimum size and does not mention shielded cable.

All cables have an operating tolerance of 5 per cent above the rated voltage except those rated at 15,000 volts and below which have no operating tolerance. All cables for three-phase circuits are rated on the conductor-to-conductor basis.

Unless otherwise specified, two-conductor cable will be of the round type; when laid parallel, the thickness of the insulation shall be the same as that for single-conductor cable for the same voltage.

The thicknesses given for 7500 volts and over should be used for single-conductor braided cables only if supported on insulators, or suitably shielded.

Table V. Recommended Thickness of Varnished-Cambric Insulation—*Continued*

Multiple-conductor Belted Cable

Rated Voltage Volts Phase to Phase	Size A. W. G. or 1000 cir mil	Ungrounded Neutral Wall of V. C. in 64ths inch (and mils)		Grounded Neutral Wall of V. C. in 64ths inch (and mils)	
		On Condrs. (C)	On Belt (B)	On Condrs. (C)	On Belt (B)
0–600	14–8	3 (47)	0 (0)	3 (47)	0 (0)
	7–2	4 (63)	0 (0)	4 (63)	0 (0)
	1–4/0	5 (78)	0 (0)	5 (78)	0 (0)
	213–500	6 (94)	0 (0)	6 (94)	0 (0)
	501–1000	6 (94)	2 (31)	6 (94)	2 (31)
	1001 and larger	7 (109)	2 (31)	7 (109)	2 (31)
601–1,000*	14–2	4 (63)	0 (0)	4 (63)	0 (0)
	1–4/0	5 (78)	0 (0)	5 (78)	0 (0)
	213–500	6 (94)	0 (0)	6 (94)	0 (0)
	501–1000	6 (94)	2 (31)	6 (94)	2 (31)
	1001 and larger	7 (109)	2 (31)	7 (109)	2 (31)
1,001–2,000	12–2†	5 (78)	0 (0)	5 (78)	0 (0)
	1–4/0	6 (94)	0 (0)	6 (94)	0 (0)
	213–500	6 (94)	0 (0)	6 (94)	0 (0)
	501–1000	6 (94)	2 (31)	6 (94)	2 (31)
	1001 and larger	7 (109)	2 (31)	7 (109)	2 (31)
2,001–3,000 (incl. 2,500*)	10–2†	5 (78)	2 (31)	5 (78)	2 (31)
	1–4/0	6 (94)	2 (31)	6 (94)	2 (31)
	213–500	6 (94)	2 (31)	6 (94)	2 (31)
	501–1000	6 (94)	3 (47)	6 (94)	3 (47)
	1001 and larger	7 (109)	3 (47)	7 (109)	3 (47)
3,001–4,000	8–4/0	6 (94)	3 (47)	6 (94)	3 (47)
	213–500	6 (94)	3 (47)	6 (94)	3 (47)
	501–1000	6 (94)	4 (63)	6 (94)	4 (63)
	1001 and larger	7 (109)	4 (63)	7 (109)	4 (63)
4,001–5,000 (incl. 4,500*)	8–4/0	6 (94)	4 (63)	6 (94)	4 (63)
	213–1000	7 (109)	4 (63)	7 (109)	4 (63)
	1001 and larger	7 (109)	5 (78)	7 (109)	5 (78)
5,001–6,000	8–4/0	6 (94)	5 (78)	6 (94)	5 (78)
	213–1000	7 (109)	5 (78)	7 (109)	5 (78)
	1001 and larger	7 (109)	5 (78)	7 (109)	5 (78)
6,001–7,000	8 and larger	7 (109)	6 (94)	7 (109)	5 (78)
7,001–8,000 (incl. 7,500*)	6 and larger	7 (109)	7 (109)	7 (109)	6 (94)
8,001–9,000	6 and larger	8 (125)	8 (125)	8 (125)	6 (94)
9,001–10,000	6 and larger	9 (141)	9 (141)	9 (141)	6 (94)
10,001–11,000	6 and larger	10 (156)	10 (156)	10 (156)	6 (94)
11,001–12,000	6 and larger	10 (156)	10 (156)	10 (156)	7 (109)
12,001–13,000	6 and larger	11 (172)	11 (172)	11 (172)	7 (109)
13,001–14,000	6 and larger	12 (188)	12 (188)	12 (188)	7 (109)
14,001–15,000*	6 and larger	13 (203)	13 (203)	13 (203)	7 (109)
15,001–16,000	4 and larger	14 (219)	14 (219)	14 (219)	7 (109)
16,001–17,000	4 and larger	14 (219)	14 (219)	14 (219)	7 (109)

* The ratings marked * are those recommended by the N.E.M.A.-N.E.L.A. Joint Committee on Voltage Standardization as "Preferred Voltage Ratings" for "general apparatus" except 1000 which has been added because of the large amount of very low-voltage cable used.

† N.E. Code specifies No. 8 minimum size.

All cables have an operating tolerance of 5 per cent above the rated voltage except those rated at 15,000 volts and below which have no operating tolerance. All cables for three-phase circuits are rated on the conductor to conductor basis.

Unless otherwise specified, two-conductor cable will be of the round type; when laid parallel, the thickness of the insulation shall be the same as that for single-conductor cable for the same voltage.

IMPREGNATED-PAPER INSULATION. The thicknesses given in Table VI are considered to be representative of good practice for normal conditions, i.e., where impregnation can be maintained, voltage transients are not unusual, etc. See below for Shielding standards.

Oil-filled Cable uses thinner walls, as shown in Table VIII, owing to the absence of gaseous ionization. These walls apply to hollow core conductors of 0.5 or 0.69 in. internal diameters.

Pressure Cables use insulation walls about the same as oil-filled cable, but no standards have been published.

For Circuits Other Than Three-phase, the thickness should be that for grounded neutral with the rated three-phase voltage which gives the same voltage across the insulation. See above under Varnished-cambric Insulation.

Table VI. Recommended Minimum Thickness of Insulation and Minimum Size Conductors

Impregnated-paper Cable

(I.P.C.E.A., 1934)

(The thicknesses for grounded neutral are also "recommended" by A.E.I.C. specifications.)
(G = Grounded Neutral and U = Ungrounded Neutral)

Rated Three-phase Voltage Phase to Phase	Size* Conductor A. W. G. or Circular mils	Minimum Average Insulation Thickness, 64ths in.					
		Single-conductor		Multiple-conductor Shielded		Multiple-conductor Belted	
		G	U	G	U	G	U
0– 1,000	8–4/0	4	4	—	—	4 × 2	4 × 2
	225,000–500,000	4	4	—	—	5 × 2	5 × 2
	525,000–1,000,000	5	5	—	—	5 × 2	5 × 2
	Over 1,000,000	6	6	—	—	—	—
1,001– 2,000	8–4/0	5	5	—	—	5 × 2	5 × 2
	225,000–500,000	5	5	—	—	5 × 3	5 × 3
	525,000–1,000,000	6	6	—	—	5 × 3	5 × 3
	Over 1,000,000	7	7	—	—	—	—
2,001– 3,000	8–500,000	5	5	—	—	5 × 3	5 × 3
	525,000–1,000,000	6	6	—	—	5 × 3	5 × 3
	Over 1,000,000	7	7	—	—	—	—
3,001– 4,000	8–1,000,000	6	6	—	—	6 × 3	6 × 3
	Over 1,000,000	7	7	—	—	—	—
4,001– 5,000	8–1,000,000	6	6	—	—	6 × 4	6 × 4
	Over 1,000,000	7	7	—	—	—	—
5,001– 6,000	6–1,000,000	7	7	—	—	6 × 4	6 × 4
	Over 1,000,000	8	8	—	—	—	—
6,001– 7,000	6 and larger	8	9	—	—	7 × 4	7 × 6
7,001– 8,000	6 and larger	9	10	—	—	7 × 4	7 × 7
8,001– 9,000	6 and larger	9	11	—	—	8 × 4	8 × 8
9,001–10,000	6 and larger	10	12	10	12	8 × 4	8 × 8
10,001–11,000	6 and larger	10	12	10	12	8 × 5	8 × 8
11,001–12,000	6 and larger	11	13	11	13	9 × 5	9 × 9
12,001–13,000	6 and larger	11	14	11	14	9 × 5	9 × 9
13,001–14,000	6 and larger	12	15	12	15	10 × 5	10 × 10
14,001–15,000	6 and larger	13	16	13	16	10 × 5	10 × 10
15,001–16,000	4 and larger	13	17	13	17	For these voltages shielded type is recommended.†	
16,001–17,000	4 and larger	14	18	14	18		
17,001–18,000	4 and larger	15	18	15	18		
18,001–19,000	4 and larger	15	19	15	19		
19,001–20,000	2 and larger	16	19	16	19		
20,001–21,000	2 and larger	16	20	16	20		
21,001–22,000	2 and larger	17	21	17	21		
22,001–23,000	2 and larger	17	22	17	23		
23,001–24,000	2 and larger	18	22	18	22		
24,001–25,000	2 and larger	18	23	18	23		
25,001–26,000	2 and larger	19	24	19	24		
26,001–27,000	2 and larger	19	25	19	25		
27,001–28,000	1 and larger	20	26	20	26		
28,001–29,000	1 and larger	21	27	21	27		
29,001–30,000	1 and larger	21	27	21	27		
30,001–31,000	0 and larger	22	28	22	28		
31,001–32,000	0 and larger	22	28	22	28		
32,001–33,000	0 and larger	23	29	23	29		
33,001–34,000	0 and larger	24	30	24	30		
34,001–35,000	0 and larger	24	31	24	31		

* The minimum conductor sizes given are for round conductors. For sector conductors the following are the minimum sizes:

Circuit Voltage	Size	Circuit Voltage	Size
0–20,000	1/0	25,001–30,000	3/0
20,001–25,000	2/0	30,001–35,000	4/0

† Belted cables are, however, in common use up to 27,000 volts and, rarely, 33,000 volts.

Table VI. Recommended Minimum Thickness of Insulation and Minimum Size Conductors—*Continued*

Impregnated Paper Insulation

Single-conductor Cables for Grounded Neutral Systems of over 5500 Volts

Rated Voltage Phase to Phase	Minimum Size Conductor	64ths inch	Rated Voltage Phase to Phase	Minimum Size Conductor	64ths inch
35,001–36,000	2/0	25	52,001–53,000	250,000	35
36,001–37,000	2/0	26	53,001–54,000	250,000	36
37,001–38,000	2/0	26	54,001–55,000	250,000	36
38,001–39,000	2/0	27	55,001–56,000	250,000	37
39,001–40,000	2/0	27	56,001–57,000	250,000	37
40,001–41,000	3/0	28	57,001–58,000	300,000	38
41,001–42,000	3/0	28	58,001–59,000	300,000	38
42,001–43,000	3/0	29	59,001–60,000	300,000	39
43,001–44,000	3/0	29	60,001–61,000	300,000	39
44,001–45,000	3/0	30	61,001–62,000	300,000	40
45,001–46,000	3/0	30	62,001–63,000	300,000	40
46,001–47,000	4/0	31	63,001–64,000	300,000	41
47,001–48,000	4/0	32	64,001–65,000	350,000	42
48,001–49,000	4/0	32	65,001–66,000	350,000	42
49,001–50,000	4/0	33	66,001–67,000	350,000	43
50,001–51,000	4/0	34	67,001–68,000	350,000	44
51,001–52,000	4/0	34	68,001–69,000	350,000	44

Table VII. Thickness of Varnished-Cambric Insulation for Series Lighting Cable

(I.P.C.E.A. 1934)

(See Table III for rated voltage in terms of operating voltage.)

Rated Voltage	Insulation Thickness, 64th in.	
	8 A. W. G.	7 to 4 A. W. G.
600	3	4
1,000	4	4
2,000	5	5
3,000	6	6
4,000	7	7
5,000	8	8
6,000	9	9
7,000	10	10
		6 to 4 A. W. G.
8,000	—	11
9,000	—	11
10,000	—	12
11,000	—	13
13,000	—	15
15,000	—	16

Shielding

It is customary to shield the outer layers of insulation, where danger from gaseous ionization exists, with a metallic screen in close contact with the insulation. The standard practice for shielding is given in Table IX.

Such shielding, however, is useful for other reasons. It eliminates stresses along the surface of the insulation, in three-phase cables, i.e., tangential stresses, which are likely to exist because the insulation has to carry the circuit voltage along the axes between conductors and $1/\sqrt{3}$ of the circuit voltage along the radii between conductors and ground. Shielding of unleaded cables also affords a protection against shock, as the shield is always grounded and may be connected to give a good return path for the short-circuit current, in the event of a failure of the insulation, thereby making the operation of protective devices more certain. Shields should always be terminated in cones of insulating tape (usually of varnished cambric) to eliminate edge stresses.

Certain rubber compounds have been developed which are substantially ozone-proof and under some conditions may be used without shielding. Examples are the wires used for 7500–15,000 volt neon gas tubes and for ignition systems of internal-combustion engines.

Table VIII. Recommended Insulation Thicknesses for Oil-filled Cable
(1935)

Rated Circuit Voltage, kw	Range of Conductor Sizes, A. W. G. and thousands of circular mils: Single-conductor*	3-conductor, Round†	3-conductor, Sector†	Insulation Thickness, mils
20	1/0 to 2500	1 to 500	0 to 500	130
23	"	"	"	145
25	"	"	"	155
30	"	"	"	170
34.5	"	0 to 600	00 to 600	190
35	"	"	"	190
40	2/0 to 2500	"	"	210
45	"	"	"	225
46	"	0 to 750	00 to 750	225
50	"	"	"	245
55	"	"	"	260
60	"	00 to 750	000 to 750	280
65	"	"	"	295
69	"	"	"	315
70	"			320
75	3/0 to 2500			340
80	"			355
90	"			390
100	"			430
110	"			465
115	4/0 to 2500			480
120	"			500
130	"			535
138	"			560
140	"			570
150	"			610
160	250 to 2500			645
230	750 to 2500			925

* For sizes over 2,500,000 cir mils, add 15 mils.
† For sizes up to and including 750,000 cir mils, add 15 mils.

Table IX. Shielding of Insulation
(a) Shielding of Rubber Insulation
Circuit Voltages at Which Shielding is Required*

Type	Grounded Neutral: Single-conductor	Grounded Neutral: Multiple-conductor	Ungrounded Neutral: Single-conductor	Ungrounded Neutral: Multiple-conductor
Braided, not supported on insulators	6001 and over	6001 and over	3501 and over	3501 and over
Lead covered	....	10,000 and over	....	6001 and over
Leadless submarine (Shore ends)	6001 and over	6001 and over	3501 and over	3501 and over

* Corona-restraining or ozone-resisting insulations or coverings are sometimes used above these voltages without shielding.

(b) Shielding of Paper or Varnished-cambric Insulation
Grounded or Ungrounded Neutral

Type	Varnished Cambric: Single-conductor	Varnished Cambric: Multiple-conductor	Impregnated Paper: Single-conductor	Impregnated Paper: Multiple-conductor
Braided, and in contact with a conducting surface	7500 and over	7500 and over*	....	....
Lead Covered	....	....	40,000 and over	Over 15,000*

* Shield may be over belt, in a belted cable.

Coverings

LEAD SHEATHS. Standard lead sheath thicknessess are given in Table X. These apply to both pure and alloyed lead.

ARMOR. Table XI gives the standard dimensions of armor and jute for armored cables. The I.P.C.E.A. " Metallic Coverings " Specifications also give standards for interlocked armor, duck tape over sheaths, and braid over sheaths.

BRAIDS. Three grades of braid are in use, as follows:

(a) **Class A (N.E.M.A.) Braid Specifications,** which are to be used only for code and intermediate grade wires and cables. The thickness shall be as specified in Table XII.

(b) **Class B (A.S.T.M. "Standard") Braid Specifications,** which are designed for indoor service, installation in conduits, or as the inner braids for multiple-conductor cables, both for rubber and varnished cambric. The thickness is given in Table XIII.

(c) **Class C (A.S.T.M. "Heavy") Braid Specifications,** which are designed for outdoor and rough service. The thickness is given in Table XIV.

(d) Unless otherwise specified, Class B (A.S.T.M. "Standard") Braid Specifications are used for 30 per cent grade.

Table X. Lead Sheath Thickness

64ths Inch

(I.P.C.E.A., 1934)		(I.P.C.E.A. & A.E.I.C., 1934)		(A.E.I.C., 1934)	
Core Diameter, in.	For Rubber or Varnished Cambric	Core Diameter, in.	For Paper "Solid Type" Cable	Core Diameter, in.	For Paper Oil-filled Type
0–0.425	3	0–0.400	5	up to 1.800	8
0.426–0.700	4	0.401–1.000	6	1.801 to 2.800	9
0.701–1.050	5	1.001–1.800	7	2.801 and over	10
1.051–1.500	6	1.801–2.900	8		
1.501–2.000	7	2.901–3.200	9		
2.001–3.000	8	3.201 and over	10		
3.001 and over	9				

For twin flat construction use major core diameter.
Submarine cable sheath should never be less than 5/64 in.
Pressure cable of the Hochstadter type has thinner lead than ordinary solid or oil-filled cable.

Table XI. Armor Dimensions and Thickness of Jute Bedding for Armored Cable

(I.P.C.E.A., 1934)

64ths Inch

(a) Jute Bedding				(b) Width and Thickness of Steel Tapes			(c). Size of Galvanized Steel Armor Wire	
	Minimum Thickness of Jute							
Diameter of Cable under Jute, in.	Steel Tape Parkway Cable	Round Wire Armor Cables: Lead Sheathed	Round Wire Armor Cables: Non-leaded	Diameter of Cable under Jute Bedding, in.	Minimum Steel Tape Thickness, mils	Maximum Width of Steel Tape, in.	Diameter of Cable under Jute Bedding, in.	Minimum Size of Armor Wire, B. W. G.
0 to 0.450	2	3	5	0 to 0.450	20	0.75	0 to 0.750	12
0.451–0.750	3	3	5	0.451–0.750	20	1.00	0.751–1.000	10
0.751–1.000	3	4	6	0.751–1.000	20	1.00	1.001–1.700	8
1.001–2.500	4	5	7	1.001–1.400	30	1.25	1.701–2.500	6
2.501 and larger	4	6	8	1.401–2.000	30	1.50	2.501 and larger	4
				2.001 and larger	30	2.00		

Table XII

N.E.M.A. Class A Braids

Diameter under Braid, in.	Minimum Thickness of Braid, in.	Corresponding Minimum Size and Ply of Cotton Yarn
0–0.200	0.015	30/2 or 14/1
0.201–0.350	0.017	26/2 or 12/1
0.351–0.800	0.020	20/2 or 10/1
0.801–1.500	0.025	12/2
1.501–3.000	0.031	8/2

RUBBERIZED TAPE. Rubberized tape has a thickness of not less than ten (10) mils, and is applied with a lap of not less than 10 per cent of its width. The base sheeting of the tape has not less than 56 × 60 picks per inch.

Table XIII

A.S.T.M. Standard or Class B Braids

Diameter under Braid, in.	Minimum Thickness of Braid, in.	Corresponding Minimum Size and Ply of Cotton Yarn
0–0.200	0.016	30/2
0.201–0.350	0.017	26/2
0.351–0.800	0.020	20/2
0.801–1.500	0.025	12/2
1.501–3.000	0.031	8/2

Table XIV

A.S.T.M. Heavy Braids

Diameter under Braid, in.	Minimum Thickness of Braid, in.	Corresponding Minimum Size and Ply of Cotton Yarn
0–0.200	0.020	20/2
0.201–0.300	0.022	16/2
0.301–0.600	0.025	12/2
0.601–1.000	0.031	8/2
1.001–1.500	0.037	6/2
1.501–2.000	0.044	4/2
2.001–3.000	0.053	4/3

59. TESTS

GENERAL. Rubber-insulated Cables are tested for imperfections by means of a high-voltage test, Table XV, and for dryness by a megohm test. General quality is tested by tensile strength, elongation, and permanent set measurements. Life expectancy is estimated by accelerated aging tests in which oxidation is accelerated by heat.

High-voltage rubber is tested for corona suppression by noting the voltage at which visual corona appears when viewed in the dark, and for ozone resistance by exposure to ozone of concentration 0.02 per cent.

Water resistance is tested by noting the rise of S.I.C. when immersed in water either at room temperature or 70 deg cent.

Varnished-cambric Cables are given high-voltage and megohm tests for the reasons given above for rubber (Table XVI). They are occasionally tested for power factor and ionization. See I.P.C.E.A. specifications.

Impregnated Paper Cables are given high-voltage and megohm tests for the reasons given above (Table XVII).

Cables for circuit voltages of 7500 and above are tested for power factor and dielectric loss both as an economic measure and to guard against accumulative heating which might result from high dielectric loss.

The limiting power factors of A.E.I.C. (1936) are as follows:

Temperature of Cable, ° C	Power Factor, %	
	7.5 to 20 kv	20.1 kv and over
Room to 60	1.1	0.90
70	1.6	1.30
75	1.9	1.50
80	2.3	1.80
85	2.8	2.20
90	3.4	—

Cables for circuit voltages of 7500 and above are tested for ionization, i.e., rise of power factor from 20 to 100 volts per mil, average stress, to detect entrained gas. The usual ionization limits are as follows:

No. of Conductors	Rated Kilovolts	Ionization Factor Rise of Power Factor
Single	7.5–20.0	0.3
	20.1–35.0	0.2
	35.1 and over	0.1
Multiple, shielded	7.5–20.0	0.3
	20.1 and over	0.2
Multiple, belted	7.5 and over	0.5

TEST VOLTAGES. Factory test voltages on full reel lengths are given in Tables XV, XVI, and XVII.

In all cases, if a test voltage is to be applied after armoring it should be 80 per cent of that applied before armoring.

Tests after installation and proof tests at intervals are made at the following values, the percentages being taken on the full reel final factory test voltage unless otherwise stated.

	Rubber	V. C.	Paper
Test after installation	80% for same period	70% for 5 min	60% for 4 hr 80% for 1/4 hr
Proof test	60% for same period	60% for 5 min or 150% of rated voltage	60% for 5 min. or 175% of rated volts
Ratio of d-c to a-c test	2.2	2*	2.4*

* The d-c test may be made in place of any of the a-c tests listed. If the cable temperature exceeds 25 deg cent the tabulated ratio shall be reduced by 0.013 for each degree over 25 deg cent.

BENDING TESTS. These tests are made to ensure flexibility of paper and varnished-cambric cables. In paper cables they are made at diameter of 12 times the diameter of the cable, and in varnished cambric, at a diameter ratio of 8.

Table XV. Test Voltages for Rubber Insulation

Kilovolts for { 5 min, 30% grade; 3 min, Intermediate; 1 min, Code (see Note 1, below) }

(I.P.C.E.A., 1934)

64ths Inch	Size Conductor, A.W.G. or Circular Mils					
	14 to8	7 to 2	1 to 4/0	225,000 to 500,000	525,000 to 1,000,000	Over 1,000,000
3	3					
4	4*	3.5				
5	5	5	4			
6	6	6	6	5		
7	7	7	7	7	6	
8	8	8	8	8	8	7
9	9	9	9	9	9	9
10	10	10	10	10	10	10
	All Sizes					
Over 10	Test voltage kv = $10 + 1\,1/2\,(T - 10)$ where T = thickness in 64ths inch; e.g., if insulation thickness is 20/64 in., $T = 20$ and the test voltage equals $10 + 1\,1/2\,(20 - 10) = 25$ kv					

* 3.5 on No. 8 A.W.G.

1. The word "Code" is here used in the sense of Code Grade Insulation. Where National Electrical Code labels are to be applied, the tests should be in accordance with National Electrical Code Specifications.

The "Code" requirements for 0–600 volt wire and cable are as follows:

Size A.W.G. or 1000 in mils	Test Voltage 1 min	Size A.W.G. or 1000 in mils	Test Voltage 1 min
14–9	1.5	225–500	3.0
8–2	2.0	525–1000	3.5
1–4/0	2.5	Over 1000	3.5

2. These tests are based on A.I.E.E. Standards but certain departures therefrom have been made, as dictated by experience obtained since those Standards were formulated. These departures are all confined to thicknesses below 9/64ths inch.

Table XVI. Voltage Tests for Varnished-cambric Insulation

Kilovolts for 5 min

(From Standards of I.P.C.E.A., 1934)

Insulation Thickness, 64ths in.	Size A.W.G. or Cir Mils						
	14 to 8	7	6	4 to 2	1 to 4/0	250,000 to 500,000	Larger than 500,000
3	2.5	—	—	—	—	—	—
4	3.5	3.5	4.5	5.0	—	—	—
5	4.5	4.5	5.0	5.5	6.0	6.5	—
6	—	7.0	7.5	8.0	8.5	9.0	9.0
7	—	9.5	10.0	10.5	11.0	11.5	11.5
8	—	11.5	12.0	12.5	13.5	14.0	14.0
9	—	14.0	14.5	15.0	15.5	16.0	16.0
10	—	16.0	16.5	17.0	18.0	18.5	18.5
11	—	18.0	18.5	19.0	19.5	20.0	20.0
12		19.5	20.0	20.5	21.0	22.0	22.0
13		21.5	22.0	22.5	23.0	23.5	24.0
14		23.0	23.5	24.0	24.5	25.5	26.0
15			25.5	26.0	26.5	27.0	27.5
16			27.0	27.5	28.0	29.0	29.5
17			29.0	29.5	30.0	30.5	31.0
18			30.5	31.0	31.5	32.5	33.0
19			32.5	33.0	33.5	34.5	35.0
20			35.0	35.5	36.0	37.0	37.5
21				38.0	38.5	39.0	39.5
22				40.5	41.0	41.5	42.0
23				43.0	43.5	44.0	44.5
24				45.0	45.5	46.0	46.5
25				47.5	48.0	48.5	49.0
26				49.5	50.0	51.0	51.5
27				50.5	51.0	52.0	52.5
28				52.0	52.5	53.5	54.0
29				53.5	54.0	55.0	55.5
30				55.0	55.5	56.5	57.0

The above table is for single-conductor and multiple-conductor shielded cables.

For multiple-conductor non-shielded cables, the test between conductors shall be twice that given above for a single-conductor cable having the same insulation thickness as each conductor. The test pressure to sheath or ground shall be 58 per cent of this value except for multiple-conductor cables for ungrounded circuits over 6000 volts working pressure, where the test pressure shall be 80 per cent of the pressure between conductors.

Table XVII. Voltage Tests for Impregnated-paper Cables

(From Specifications of A.E.I.C., 1934)

(a) Test Voltages (Kilovolts) for Stranded Conductor Cables Rated at 7.5 kv or Less, Phase to Phase

Time of application, 5 min

Thickness of Insulation, 64ths in.	Single-conductor cable Test volts, kv (cond.-sheath)				Multiple-conductor cable, belted Test volts, kv (cond.-cond.)	
	No. 1 to No. 4/0	213,000 to 500,000	501,000 to 1,000,000	Over 1,000,000	No. 1 to No. 4/0	225,000 and larger
4	5.0	5.0	—	—	6.0	—
5	8.0	8.0	6.0	—	14.0	14.0
6	12.5	12.5	11.0	7.5	25.0	25.0
7	18.0	18.0	17.0	13.0	93.0	39.0
8	24.0	24.0	23.0	22.0	—	—
9	31.0	31.0	31.0	31.0	—	—

Note 1. Three conductor belted cables shall be tested from conductor to sheath at $\frac{1}{\sqrt{3}}$ times the tabulated (c-c) value, except that, whenever a separate c-s test is required, it shall be made at an average stress (c-s) equal to 80 per cent of the average stress (c-c) determined from the above table.

Note 2. Cables having stranded conductors smaller than No. 1 A.W.G. shall be tested at 85 per cent of the voltage specified for No. 1 A.W.G. and larger. If such cables have solid conductors, they shall be tested at 70 per cent of the voltage specified for stranded cables with conductors No. 1 A.W.G. and larger.

(b) Test Voltages for Cables Rated at Over 7.5 kv, Phase to Phase

Time of application, 15 min

Type of cable	Test volts, volts per mil of specified insulation thickness
Single-conductor	
up to 50 kv	220
over 50 kv	230
Multiple-conductor	
Shielded	200
Belted, cond.-cond.	180
Belted, cond.-sheath	145*

* The conductor-sheath test voltage for three-conductor cable may be $\frac{1}{\sqrt{3}} \times$ conductor-conductor test voltage.

Concentric conductor cable to be tested as single-conductor cable.

(c) Oil-filled Cable. Single-conductor cable is tested at 300 volts per mil and three-conductor cable at 270 volts per mil of specified insulation thickness for 15 minutes.

INSULATION RESISTANCE. The insulation resistance of a cable is usually expressed in megohm-miles, sometimes erroneously called megohms per mile. The total insulation resistance of a cable varies inversely as its length; e.g., a cable 2 miles long has half the resistance between conductor and sheath that 1 mile of this cable has. The formulas for insulation resistance of various types of cables are given below.

The insulation resistance of a single-conductor cable of length l centimeters is

$$R' = \frac{\rho}{2\pi l} \log_\epsilon \frac{D}{d}$$

where d = diameter of conductor, D = outside diameter of insulation, R' = insulation resistance, l = axial length centimeter, and ρ = specific resistance.

From the above formula the megohm-miles are

$$R = K \log_{10} \frac{D}{d}$$

where R = the insulation resistance in megohms for a specified unit length.

K = megohms constant—when the insulation resistance is to be determined in megohm-mile units, K equals 0.00000228 times the resistivity of the insulation expressed in megohm-centimeter units.

D = outside diameter of insulation.

d = diameter of conductor.

Table XVIII. Values of Log D/d

Equivalent Insulation, Thickness

Cond. Size A.W.G. or M. Cir. Mils.	3	4	5	6	7	8	9	10	12	14	16	18	20	22
	64th Inches													
	Decimals													
	(0.047)	(0.063)	(0.078)	(0.094)	(0.109)	(0.125)	(0.141)	(0.156)	(0.188)	(0.219)	(0.250)	(0.281)	(0.313)	(0.344)
14*	0.392	0.470	0.537	0.594	0.645	0.691	0.732	0.770	0.836	....	....	....	....	....
12*	.334	.405	.467	.520	.568	.611	.651	.686	.751	0.806	0.856	....	....	....
10*	.283	.348	.404	.453	.498	.538	.575	.609	.670	.723	.771	0.814	0.853	0.889
8*	.239	.296	.347	.392	.432	.470	.505	.537	.594	.645	.691	.731	.770	.804
6	....	.225	.267	.305	.340	.373	.403	.431	.483	.529	.570	.608	.643	.676
5	....	.206	.245	.281	.314	.346	.373	.401	.450	.495	.535	.572	.606	.637
4	....	.187	.224	.257	.289	.318	.345	.371	.418	.460	.500	.535	.568	.598
3	....	.171	.204	.236	.265	.293	.318	.343	.388	.429	.466	.500	.532	.562
2	....	.155	.186	.215	.243	.269	.293	.316	.359	.398	.433	.466	.497	.526
1	....	....	.168	.195	.220	.244	.267	.288	.328	.365	.399	.431	.461	.487
1/0	....	....	.152	.177	.201	.223	.244	.264	.302	.337	.369	.399	.428	.454
2/0	....	....	.138	.161	.183	.204	.223	.242	.278	.311	.342	.370	.397	.422
3/0	....	....	.125	.146	.166	.185	.204	.221	.255	.286	.315	.342	.367	.392
4/0	....	....	.113	.132	.151	.168	.187	.202	.233	.262	.289	.315	.339	.362
250	....	....	....	.123	.140	.157	.173	.189	.218	.246	.272	.296	.320	.342
300	....	....	....	.113	.130	.145	.160	.175	.203	.229	.254	.278	.300	.321
350	....	....	....	.106	.121	.136	.150	.164	.190	.215	.239	.262	.283	.303
400	....	....	....	.0995	.114	.128	.142	.155	.181	.204	.227	.249	.269	.289
450	....	....	....	.0944	.108	.122	.136	.148	.172	.195	.217	.238	.258	.277
500	....	....	....	.0901	.103	.116	.129	.141	.165	.187	.208	.228	.248	.266
600	....	....	....	....	.0952	.107	.119	.130	.152	.173	.193	.212	.230	.248
650	....	....	....	....	.0919	.104	.115	.126	.147	.168	.187	.206	.223	.241
700	....	....	....	....	.0888	.100	.111	.122	.143	.163	.181	.199	.217	.234
750	....	....	....	....	.0861	.0971	.108	.118	.139	.157	.176	.194	.211	.228
800	....	....	....	....	.0836	.0943	.105	.115	.135	.154	.172	.189	.206	.222
900	....	....	....	....	.0793	.0895	.0994	.108	.128	.146	.164	.180	.196	.212
1000	....	....	....	....	.0755	.0851	.0948	.104	.122	.140	.157	.173	.189	.203
1250	....	....	....	....	....	.0770	.0856	.0943	.111	.127	.142	.157	.172	.186
1500	....	....	....	....	....	.0708	.0789	.0870	.102	.116	.132	.146	.159	.172
1750	....	....	....	....	....	.0658	.0734	.0810	.0954	.110	.123	.136	.149	.162
2000	....	....	....	....	....	.0619	.0691	.0761	.0898	.103	.116	.128	.141	.153
2500	....	....	....	....	....	.0558	.0623	.0687	.0812	.0934	.105	.117	.128	.139

* Solid conductor. Conductors larger than No. 8 are assumed to be stranded. For intermediate thickness interpolate.

Table XVIII. Values of Log D/d—*Continued*

Equivalent Insulation, Thickness

Cond. Size A.W.G. or M. Cir. Mils.	64th Inches													
	24	26	28	30	32	34	36	38	40	44	48	52	56	60
	Decimals													
	(0.375)	(0.406)	(0.438)	(0.469)	(0.500)	(0.531)	(0.562)	(0.594)	(0.625)	(0.687)	(0.750)	(0.812)	(0.875)	(0.937)
10*	0.922	....	....	....	....	....	....	....	....	....	....	....	....	....
8*	.836	0.866	0.894	....	....	....	....	....	....	....	....	....	....	....
6	.706	.734	.760	0.786	....	....	....	....	....	....	....	....	....	....
5	.667	.694	.720	.744	....	....	....	....	....	....	....	....	....	....
4	.625	.653	.678	.703	0.725	....	....	....	....	....	....	....	....	....
3	.589	.615	.640	.663	.685	0.706	....	....	....	....	....	....	....	....
2	.553	.578	.602	.625	.646	.666	0.686	....	....	....	....	....	....	....
1	.513	.538	.561	.583	.603	.623	.642	0.661	....	....	....	....	....	....
1/0	.479	.502	.525	.546	.566	.585	.604	.622	0.639	0.671	....	....	....	....
2/0	.446	.469	.490	.511	.531	.549	.567	.584	.601	.632	0.662	....	....	....
3/0	.414	.436	.457	.476	.495	.513	.531	.547	.563	.594	.622	0.649	....	....
4/0	.384	.405	.425	.443	.462	.480	.491	.512	.527	.557	.585	.610	0.635	0.658
250	.363	.383	.402	.420	.438	.455	.471	.487	.502	.530	.557	.583	.607	.630
300	.341	.360	.379	.396	.413	.430	.445	.460	.475	.503	.529	.554	.578	.600
350	.323	.341	.359	.376	.393	.408	.424	.438	.453	.480	.506	.530	.553	.575
400	.308	.326	.343	.360	.376	.391	.406	.420	.434	.461	.486	.510	.532	.553
450	.295	.312	.329	.345	.361	.376	.391	.405	.418	.444	.469	.492	.514	.535
500	.284	.301	.317	.333	.348	.363	.377	.391	.404	.430	.454	.477	.498	.519
600	.265	.281	.297	.312	.326	.340	.354	.367	.380	.405	.428	.450	.471	.491
650	.257	.273	.288	.303	.317	.331	.345	.357	.370	.395	.417	.439	.460	.480
700	.250	.266	.281	.295	.309	.323	.336	.349	.361	.385	.408	.429	.450	.469
750	.243	.259	.273	.288	.302	.315	.328	.341	.353	.376	.399	.420	.440	.459
800	.237	.252	.267	.281	.294	.308	.320	.333	.345	.368	.390	.411	.431	....
900	.227	.242	.255	.269	.282	.295	.307	.319	.331	.354	.375	.396	....	....
1000	.218	.232	.245	.259	.271	.284	.296	.308	.319	.341	.362	....	....	....
1250	.199	.212	.225	.237	.249	.261	.272	.284	.294	....	....	....	....	....
1500	.185	.197	.210	.221	.233	.244	.255	.265	....	....	....	....	....	....
1750	.174	.185	.197	.208	.219	.229	.240	....	....	....	....	....	....	....
2000	.164	.176	.187	.197	.207	.218	.228	....	....	....	....	....	....	....
2500	.150	.160	.170	.180	.190	.199	....	....	....	....	....	....	....	....

* Solid conductor. Conductors larger than No. 8 are assumed to be stranded. For intermediate thickness interpolate.

The value of K for various types of insulation at 60 deg fahr, after an electrification of approximately 1 min under a constant d-c voltage, is given in the accompanying table. K varies with the time of electrification, the temperature, and the humidity.

Insulation	K at 60° F (15.5° C)	
	Limits	Usual Values
Vulcanized rubber	780 to 10,000	2,000 to 6,000
Gutta-percha	500 to 4,000	2,500
Varnished cloth	400 to 2,000	1,000
Impregnated paper	500 to 3,000	1,000
Commercial Grades of Rubber		
Code		3,800 for 1,000 ft
Intermediate		8,000 for 1,000 ft
30% and performance type		21,120 for 1,000 ft

Table XVIII gives the value of $\log_{10}\frac{D}{d}$ for various sizes of wire and thicknesses of insulation.

If the insulation resistance be measured with alternating current, it will appear to be much less than with direct current because of the dielectric loss.

Multiple-conductor Cables, Concentric. The insulation resistance between the inner conductor and the adjacent conductor of a concentric cable is calculated as for a single-conductor cable. The insulation resistance between any other conductor and its adjacent conductors, or conductor and sheath (if the sheath is adjacent), of a concentric cable, is the product divided by the sum of the resistances of the layers of insulation adjacent to such conductor, each being calculated separately as for a single-conductor cable.

Multiple-conductor Cables, Non-concentric, Round or Sector Type. The insulation resistance of each conductor to all other conductors connected to the sheath or water shall be calculated assuming the multiple conductor cable to be replaced by a single-conductor cable having a round conductor of the same cross-sectional area and an outside diameter over insulation of

$$D = \frac{(d + 3c + 2b)}{d}$$

where d is the diameter of a round conductor of the same cross-sectional area, c is the thickness of conductor insulation, and b is the thickness of belt insulation.

Where the braid or outer surface of the insulation acts as a conducting surface (as is sometimes the case in multiple-conductor, rubber-insulated cables), the values of D are as in single-conductor cables (A.I.E.E. Standard No. 30).

60. CURRENT-CARRYING CAPACITY OF WIRES AND CABLES

The maximum permissible temperature rise of insulated conductors is limited by the effect of heat upon the insulation. The highest temperatures which may be safely attained continuously are given in Table XIX. The voltage E is taken as line voltage for three-conductor belted cable and voltage to ground for three-conductor shielded and single-conductor cables. Minimum temperature for impregnated paper 60 deg cent, maximum temperature 85 deg cent. The first three items are A.I.E.E. Standards.

Table XIX.

Insulation	Maximum Permissible Temperature Centigrade	
	Low voltage	At E kv between conductors
Impregnated paper	85	$90-E$
Varnished cambric	75	$75-E$
Rubber insulation (30% hevea)	60	$60-0.25E$
Rubber insulation (Code)	50	......
Oil-filled cable 75° C for 69 kv and lower.		
" " " 70° C for over 69 kv.		

When rubber is heated, the usual order of events is as follows: Oxidation is accelerated if air is present, "devulcanization" commences; i.e., the rubber molecule breaks up without, however, liberating any sulfur. In varnished cambric, both the insulation resistance and dielectric strength fall quite rapidly after the limiting temperature is attained. The same applies to impregnated paper in a less degree. Furthermore, at high voltages, the dielectric loss is often considerable, and if neglected in calculating carrying capacity, is likely to make one overlook possibilities of temperatures, far in excess of those to be expected on the basis of I^2R losses alone.

Continuous carrying capacity is calculated by equating for each conductor the sum of the I^2R loss and equivalent dielectric loss at the conductor surface, with the temperature rise per unit of heat resistance from each conductor to the ambient earth or air. With current in amperes, resistance in ohms per foot, temperature rise in degrees centigrade, the thermal resistance will be in watts per foot per degree centigrade. The following tables are calculated on this basis. Recent work by Schering has shown that the temperature rise between conductor and sheath for a given loss in the dielectric is half as much as if the loss were at the conductor.

Current-carrying Capacity of Impregnated-paper Insulated Cable

(Publication A14 of E.E.I., 1933)

The tables for ordinary, "solid type," paper-insulated cable are based on standard conditions adopted by the Edison Electrical Institute and are representative of conservatively average field conditions in this country. These are:

(*a*) Thermal resistivity of insulation 700 (watt-cm units).

(*b*) Emissivity of sheath 1200 (watt-cm units).

(*c*) Daily load factor 50, 75, and 100 per cent and corresponding loss factors of 33, 62.5, and 100 per cent in underground ducts, and any load factor from 50 to 100 per cent in air.

(*d*) Duct constants, H, for

1 cable = 1.45 thermal-ohms
3 cables = 1.04 "
6 cables = 0.82 "
9 cables = 0.75 "
12 cables = 0.72 "

(*e*) All cables in group of similar size, equally loaded and in outside ducts only.

(*f*) Average soil conditions (not dry or "hot spot" conditions).

(*g*) Groups of cable in ducts are referred to ambient earth temperature without external source of heat.

(*h*) Groups of cable in air are referred to surrounding air temperature at full-load (corresponding to duct temperature underground).

(*i*) A.I.E.E. Standard Temperature Rule for paper cable, as given above.

(*j*) A.E.I.C. Standard thicknesses of insulation, adopted in 1930, were used in calculating these tables. See Table VII. Deviation, within reasonable limits, from these standard thicknesses in any one voltage class does not introduce serious error in current rating.

(*k*) Standard logarithmic formula was used in calculating the thermal resistance of single-conductor cable. The thermal resistance of three-conductor cable, both round and sector, was determined by methods given in *Elec. J.*, July, 1932, p. 336.

DIELECTRIC LOSS. The reduction in carrying capacity due to dielectric loss, as included in the tables, is based on the following 1934 A.E.I.C. *maximum* dielectric power-factor values:

Temperature, °C	Per Cent Power Factor—60 Cycles		Temperature, °C	Per Cent Power Factor—60 Cycles	
	7.5 to 20 kv	20 kv and over		7.5 to 20 kv	20 kv and over
Room to 60	2.0	1.5	80	4.0	3.5
70	2.9	2.4	85	4.7	4.2
75	3.4	2.9	90	5.5	5.0

The S.I.C. of the paper insulation is assumed to have its maximum value of 4.0 for all temperatures, and the calculated dielectric loss is based on uniform temperature for the cable cross-section; also, this loss is added directly to the copper loss. This, of course, results in slightly lesser carrying capacity, but is on the safe side and is practically negligible.

ADDITIONAL A-C LOSSES. In addition to dielectric loss, both single- and three-conductor cables have other a-c losses produced by magnetic induction, such as skin-

effect and **proximity-effect** losses in the conductors, losses in the lead sheath, and relatively small losses in the metal shielding and metal binding tape, where used.

The correction factors used for three-conductor cable of both the belted and shielded types, exclusive of dielectric loss, are as follows:

Conductor Size, 1000 cir mils	Reduction Factor for Current Rating	Conductor Size, 1000 cir mils	Reduction Factor for Current Rating
0 A.W.G.	0.99	500	0.94
0000 A.W.G.	.98	600	.93
300	.97	750	.91
350	.96		

For single-conductor cable, the tables include only dielectric and skin-effect losses (proximity-effect losses are negligible). No attempt is made to include induced sheath current loss for " short-circuited sheath operation " since this varies with spacing between cables, etc. See below. The tables, therefore, represent " open-circuited sheath operation " of single-conductor cable, i.e., cases where there are practically no sheath losses.

D-C CARRYING CAPACITY. The following table gives the factor by which the a-c rating of single-conductor standard strand cables at 5000 volts should be multiplied to obtain the d-c rating, for d-c voltages up to 1500 volts.

Conductor Size, cir mils	Correction Factor	Conductor Size, cir mils	Correction Factor
300,000	1.005	800,000	1.025
350,000	1.006	1,000,000	1.036
400,000	1.009	1,250,000	1.054
500,000	1.011	1,500,000	1.072
600,000	1.016	1,750,000	1.092
700,000	1.020	2,000,000	1.115
750,000	1.023		

FORMULAS USED

T_c = copper temperature.
T_a = ambient air temperature.
T_e = ambient earth temperature.
W = watts loss per foot of single cable.
R_{th} = thermal ohms per foot of cable.
$R_{th} = R_i + R_s$, where:
R_i = thermal ohms from conductor to sheath.
R_s = thermal ohms from sheath to ambient air.

Table XX. Single-conductor Standard Strand Paper Cables in Ducts

Based on 60-cycle alternating current, and an earth temperature of 20 deg cent

Conductor Size A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded: 7500			15,000			23,000			34,500			46,000			69,000		
Maximum Copper Temperatures, Degrees Centigrade	85			81.5			76.5			70.0			63 5			60.0		
Per Cent Load Factor	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100
	(a) Amperes Per Conductor—Three Loaded Cables in Duct Bank																	
0	240	230	218	238	225	214	228	217	206	214	204	194	...	...	...	...	...	...
00	282	266	250	266	258	236	260	248	236	244	233	222	...	...	...	...	...	...
000	325	306	288	310	296	272	300	284	266	282	268	250	263	250	234	...	...	...
0000	378	353	330	362	342	314	346	328	304	324	308	286	305	288	270	...	...	...
250	416	390	364	400	378	348	384	362	336	358	339	316	336	315	294	...	...	...
300	465	438	406	450	421	390	432	405	376	400	378	350	372	351	326	...	...	...
350	510	478	445	496	461	428	470	442	410	438	413	380	408	384	354	378	353	328
400	555	520	482	538	501	464	514	478	442	478	446	412	442	413	382	408	378	352
500	640	596	552	622	575	532	592	548	506	548	510	470	504	471	436	468	434	400
600	718	666	614	698	641	590	664	612	566	612	568	520	560	524	480	520	482	443
700	794	728	670	766	701	644	728	668	616	670	619	568	610	572	524	568	525	482
750	827	758	696	798	731	668	758	695	640	698	644	590	634	594	544	590	545	500
800	862	789	722	828	757	692	786	721	662	726	669	610	658	618	562	612	565	518
1000	986	896	816	940	860	784	914	818	746	830	759	692	752	699	640	694	638	586
1250	1120	1011	922	1070	970	880	1018	922	838	940	853	780	850	784	706	786	720	658
1500	1235	1101	1006	1180	1069	963	1122	1015	916	1032	939	850	936	862	784	860	788	716
1750	1338	1202	1082	1270	1154	1022	1208	1086	982	1114	1018	910	1010	928	838	928	845	762
2000	1440	1283	1150	1360	1228	1100	1288	1161	1046	1190	1065	968	1080	989	888	990	897	808

Table XX. Single-conductor Standard Strand Paper Cables in Ducts—*Continued*

Based on 60-cycle alternating current, and an earth temperature of 20 deg cent

Conductor Size A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded																		
	7500			15,000			23,000			34,500			46,000			69,000		
	Maximum Copper Temperatures, Degrees Centigrade																	
	85			81.5			76 5			70.0			63.5			60.0		
	Per Cent Load Factor																	
	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100
	(*b*) Amperes per Conductor—Six Loaded Cables in Duct Bank																	
0	235	218	200	228	212	195	220	204	188	207	192	177	...	...	...	...		...
00	270	250	234	252	244	214	248	234	212	233	220	199	...	...	...	...		...
000	313	290	267	294	280	248	285	268	240	272	250	228	254	234	212			...
0000	362	331	306	342	321	283	330	308	276	312	288	262	292	268	244	...		...
250	398	365	334	382	353	318	366	338	304	344	316	288	322	294	266	...		...
300	446	407	370	430	392	356	412	376	338	384	350	318	358	326	296	...	...	...
350	490	446	404	474	431	392	470	414	368	422	386	346	390	358	322	360	329	296
400	530	482	434	514	465	422	492	445	398	458	414	376	422	385	346	390	354	318
500	610	553	494	592	534	477	564	507	454	524	470	424	480	436	390	446	401	360
600	686	612	552	654	590	526	624	563	502	586	523	468	532	485	430	496	445	394
700	752	668	614	714	644	572	680	612	548	640	569	508	580	529	470	538	482	428
750	784	695	670	744	669	594	706	636	568	666	590	526	604	546	488	560	502	442
800	814	720	694	774	692	614	733	660	588	690	613	544	628	567	506	580	518	458
1000	924	815	722	885	784	694	840	746	660	784	692	612	716	640	566	656	584	512
1250	1038	917	815	998	880	774	955	836	738	904	774	686	810	712	628	744	654	576
1500	1142	1005	882	1102	964	844	1048	916	802	970	850	744	906	783	686	812	715	624
1750	1238	1094	942	1192	1039	904	1128	989	858	1040	914	794	940	840	735	870	765	662
2000	1334	1151	998	1272	1100	956	1202	1046	903	1104	967	838	1018	868	772	926	808	696
	(*c*) Amperes per Conductor—Nine Loaded Cables in Duct Bank																	
0	230	207	187	221	202	182	212	194	174	199	182	164	...	...	...			...
00	262	237	214	242	230	200	240	220	198	226	207	186	...	...	...	...		...
000	298	272	244	282	264	230	276	252	222	262	236	212	246	220	192	...		...
0000	350	314	280	330	303	266	320	289	276	302	270	232	282	252	220	...	...	...
250	382	344	307	366	332	296	354	319	280	334	298	262	310	277	242	...		...
300	426	382	338	414	369	330	397	353	312	370	330	292	344	307	268	...	...	...
350	470	419	370	442	404	360	432	387	340	406	360	314	376	334	296	344	303	270
400	508	450	398	490	435	386	470	415	366	438	387	338	406	359	322	370	331	290
500	586	512	455	563	494	434	539	471	416	500	439	384	460	407	352	426	374	327
600	654	570	498	622	549	477	600	523	462	554	486	424	510	450	390	468	414	360
700	717	618	540	680	596	519	654	568	502	606	529	458	556	490	424	510	448	388
750	745	643	560	710	619	538	680	590	518	630	547	474	580	506	438	530	464	402
800	772	667	580	736	641	557	704	612	536	654	568	490	600	525	452	548	481	414
1000	875	753	652	840	724	628	798	688	594	742	639	552	680	594	502	620	538	462
1250	984	842	734	948	807	700	902	768	662	838	703	620	770	658	566	702	601	513
1500	1076	922	794	1042	884	760	990	840	718	918	767	670	840	717	608	768	654	554
1750	1162	991	847	1122	949	810	1062	901	768	982	823	714	904	767	652	822	699	580
2000	1246	1050	893	1194	1006	854	1128	956	810	1038	871	752	962	811	684	872	736	619
Deg Cent	Correction Factors for Various Earth Temperatures																	
10	1.07			1 08			1.09			1.10			1.11			1.12		
20	1.00			1.00			1.00			1.00			1.00			1 00		
30	0.92			0.92			0.90			0.90			0.88			0.87		
40	0.83			0.82			0.80			0.78			0.74			0 71		
50	0.73			0.72			0.69			0 63			0.56			0 50		
	(*d*) Amperes per Conductor—Twelve Loaded Cables in Duct Bank																	
0	226	200	176	214	193	170	205	184	164	193	173	154	...	...	...			...
00	254	227	200	242	220	188	236	210	186	222	197	175	...	...	...	...		...
000	290	260	228	278	251	217	268	240	206	252	224	200	235	209	180	...		...
0000	338	297	260	322	286	230	308	275	238	290	256	226	270	233	204	...		...
250	370	326	286	352	315	276	342	301	262	320	281	248	298	263	224	...	...	...
300	412	362	316	396	350	306	380	334	290	356	312	274	332	290	248	...	...	...
350	450	395	343	436	381	332	414	364	314	388	338	296	360	314	268	330	289	250
400	488	426	370	470	409	358	450	392	340	418	366	318	388	340	284	358	311	268
500	558	484	420	538	467	402	514	445	384	478	413	358	440	382	324	406	351	300
600	624	534	462	598	515	444	554	491	424	528	456	392	490	421	354	450	387	327
700	680	580	502	653	560	482	626	532	458	578	495	426	532	458	382	490	418	352
750	706	604	518	678	581	498	650	552	476	600	513	440	554	474	396	508	433	367
800	732	624	536	702	600	516	672	571	534	622	530	454	572	490	406	526	448	378
1000	828	703	602	797	675	578	760	642	544	704	595	508	648	550	452	592	500	422
1250	934	784	672	896	754	642	850	715	608	796	662	565	732	610	502	668	558	468
1500	1002	857	730	982	820	694	932	779	660	864	722	610	798	665	542	728	605	502
1750	1098	918	777	1040	879	740	1020	835	700	926	774	648	854	712	574	778	645	530
2000	1170	968	820	1120	929	780	1065	880	738	980	815	680	906	749	600	824	677	558
Deg Cent	Correction Factors for Various Earth Temperatures																	
10	1.07			1.08			1.09			1.10			1.11			1.12		
20	1.00			1.00			1.00			1.00			1.00			1.00		
30	0.92			0.92			0.90			0.90			0.88			0.87		
40	0.83			0.82			0.80			0.78			0.74			0 71		
50	0.73			0.72			0 69			0 63			0.56			0 50		

For single conductor cable,

$$R_i = 8.40 \log_{10} \frac{d_2}{d_1}$$

$$R_s = \frac{4.92}{d_3}$$

where d_1 is diameter of conductor.
d_2 is outside diameter of insulation.
d_3 is outside diameter of sheath.
H = duct constant or thermal ohms per foot of total duct bank.
D = duct correction factor for a single cable of conduit group.
N = number of cables in duct bank.
L = daily loss factor, per cent.

For a single, isolated cable in still air:

$$T_c - T_a = W \cdot R_{th} \tag{1}$$

For one of a group of equally loaded cables in a duct bank:

$$T_c - T_e = W(R_{th} + D) \tag{2}$$

where:

$$D = H.L.N/100 \tag{3}$$

Table XXI. Single-conductor Annular Paper Cables in Ducts

Based on 60-cycle alternating current, and an earth temperature of 20 deg cent

Conductor Size M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded																	
	7500			15,000			23,000			34,500			46,000			69,000		
	Maximum Copper Temperatures, Degrees Centigrade																	
	85			81.5			76.5			70 0			63.5			60.0		
	Per Cent Load Factor																	
	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100	50	75	100
	(*a*) Amperes Per Conductor—Three Loaded Cables in Duct Bank																	
700	820	747	684	786	718	658	742	682	624	690	633	580	628	586	536	580	538	492
800	892	813	742	856	781	714	814	742	680	752	689	630	686	636	582	632	584	532
1000	1030	938	850	990	900	816	940	855	776	868	794	720	790	732	664	730	666	608
1250	1200	1079	976	1150	1033	930	1086	982	888	1002	910	822	912	838	758	840	760	690
1500	1358	1214	1086	1318	1162	1040	1226	1105	990	1130	1023	916	1026	940	842	944	856	770
1750	1510	1335	1184	1422	1277	1136	1362	1211	1080	1256	1119	998	1134	1028	918	1042	940	840
2000	1642	1445	1276	1550	1380	1224	1492	1312	1162	1378	1215	1074	1238	1111	988	1138	1020	904
2500	1882	1645	1446	1800	1575	1384	1690	1493	1316	1556	1379	1218	1404	1264	1114	1288	1145	1010
	Six Loaded Cables in Duct Bank																	
700	768	683	608	738	658	584	702	625	556	650	581	516	600	536	476	552	493	438
800	838	741	658	804	713	632	764	679	602	710	632	558	652	584	516	600	534	470
1000	963	853	750	926	816	720	880	777	682	816	720	634	748	664	585	688	607	530
1250	1120	973	856	1066	931	818	1010	886	776	938	822	718	858	758	662	788	687	598
1500	1258	1087	950	1200	1042	908	1140	990	860	1054	918	796	966	845	732	884	770	664
1750	1384	1185	1232	1318	1137	986	1258	1078	933	1162	999	864	1058	920	794	974	841	722
2000	1500	1277	1104	1426	1223	1057	1364	1162	1000	1264	1075	926	1144	989	850	1052	908	774
2500	1716	1447	1240	1640	1384	1186	1550	1316	1124	1434	1217	1036	1304	1117	950	1188	1011	856
	(*b*) Amperes per Conductor—Nine Loaded Cables in Duct Bank																	
700	728	633	550	700	608	530	666	578	504	618	537	470	572	496	432	526	456	398
800	792	685	596	762	659	574	726	627	544	674	583	506	620	539	464	570	493	424
1000	912	784	674	878	752	648	832	715	616	774	663	572	710	611	522	650	556	474
1250	1056	892	766	1004	856	736	952	813	696	888	753	644	814	693	586	742	630	533
1500	1175	994	842	1126	953	806	1066	903	768	938	835	712	910	768	646	830	700	590
1750	1288	1078	916	1236	1035	870	1170	982	830	1086	909	772	993	835	696	914	761	636
2000	1390	1159	984	1336	1111	930	1272	1054	888	1174	975	826	1070	896	744	986	818	676
2500	1588	1305	1092	1520	1250	1050	1440	1185	988	1338	1095	916	1214	1005	824	1104	908	744
	Twelve Loaded Cables in Duct Bank																	
700	696	593	512	668	570	490	636	542	464	590	504	430	544	465	390	500	425	360
800	754	641	550	726	617	526	690	587	500	640	545	465	592	504	414	544	458	385
1000	864	729	622	828	699	594	790	665	565	732	617	524	676	568	464	620	516	432
1250	994	827	708	948	792	670	904	753	634	838	699	586	772	643	520	705	581	480
1500	1106	918	780	1060	880	736	1010	835	696	938	772	644	860	707	570	786	644	525
1750	1210	994	850	1160	953	790	1102	905	754	1020	837	696	938	768	612	860	697	566
2000	1340	1066	908	1250	1021	842	1188	968	810	1096	896	744	1010	822	650	926	746	600
2500	1480	1198	1000	1414	1147	950	1348	1086	890	1244	1001	820	1140	916	712	1036	825	654
Deg Cent	Correction Factors for Various Earth Temperatures																	
10		1.07			1.08			1.09			1.10			1.11			1.12	
20		1.00			1.00			1.00			1.00			1.00			1.00	
30		0.92			0.92			0.90			0.90			0.88			0.87	
40		0.83			0.82			0.80			0.78			0.74			0.71	
50		0.73			0.72			0.69			0.63			0.56			0.50	

Table XXII. Three-conductor Sector Shielded Type H Paper Cables in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	15,000			23,000			34,500		
	Maximum Copper Temperature, deg cent								
	81.5			76.5			70.0		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	Amperes per Conductor Three Loaded Cables in Duct Bank								
0	186	170	153	....	....	....	....	....	....
00	209	193	171	204	187	166	....	....	....
000	239	218	196	234	212	188	....	....	....
0000	274	249	224	268	242	216	255	230	202
250	300	270	242	292	263	233	276	246	218
300	334	300	268	327	290	258	306	274	240
350	364	326	288	356	316	277	333	296	259
400	390	347	308	382	337	295	358	314	276
500	442	390	345	432	381	331	401	354	310
600	487	430	378	473	415	362	442	388	338
700	530	464	408	512	446	390	478	418	363
750	546	476	420	528	458	400	492	430	373
	Amperes per Conductor Six Loaded Cables in Duct Bank								
0	175	153	135	....	....	....	....	....	....
00	197	174	151	190	167	146	....	....	....
000	224	197	171	205	189	166	....	....	....
0000	256	224	196	250	216	188	235	203	175
250	278	242	210	272	234	203	256	219	188
300	311	268	233	302	258	222	284	240	206
350	336	291	250	326	280	239	307	261	222
400	360	308	266	350	297	254	328	277	236
500	405	346	296	397	331	284	370	311	263
600	445	379	322	434	364	308	404	340	286
700	482	408	348	467	390	331	436	364	307
750	498	419	358	480	400	340	448	373	315
	Amperes per Conductor Nine Loaded Cables in Duct Bank								
0	165	141	121	....	....	....	....	....	....
00	185	159	134	183	154	132	....	....	....
000	210	179	153	208	174	149	....	....	....
0000	241	204	173	235	197	168	218	185	156
250	263	221	186	254	212	180	238	197	167
300	291	244	206	281	232	198	264	216	183
350	314	263	221	302	252	211	287	235	196
400	335	279	234	324	267	224	305	249	207
500	377	313	261	366	300	252	342	278	230
600	414	342	284	400	326	272	374	304	250
700	446	368	304	428	350	291	403	326	267
750	461	376	310	438	357	296	414	333	274
	Amperes per Conductor Twelve Loaded Cables in Duct Bank								
0	156	132	112	....	....	....	....	....	....
00	175	148	125	170	142	120	....	....	....
000	196	167	142	194	161	135	....	....	....
0000	227	189	161	221	182	153	208	168	139
250	245	204	172	240	194	164	224	181	149
300	274	225	189	264	214	180	246	200	165
350	295	243	202	284	231	192	266	215	175
400	314	256	214	303	246	203	284	227	185
500	354	287	237	340	274	225	319	254	203
600	388	313	256	371	298	242	348	276	218
700	420	337	273	400	319	259	374	294	232
750	430	344	288	409	326	266	384	302	236

Deg Cent	Correction Factors for Various Earth Temperatures		
10	1.08	1.09	1.10
20	1.00	1.00	1.00
30	0.92	0.91	0.90
40	0.82	0.80	0.78
50	0.72	0.69	0.63

(a) Correction faction for round conductor cable is 0.990.

Table XXIII. Three-conductor, Round or Sector, Belted-type Paper Cable in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded											
	4,500			7,500			15,000			23,000		
	Maximum Copper Temperature, deg cent											
	85.0			82.5			75.0			67.0		
	Per Cent Load Factor											
	50	75	100	50	75	100	50	75	100	50	75	100
	Amperes per Conductor—Three Loaded Cables in Duct Bank											
8	57	55	52	......	......	......	......	......	......	......	......	......
6	75	71	67	74	70	66	70	67	63	......	......	......
4	99	93	86	97	92	86	92	87	82	......	......	......
2	130	122	113	128	121	113	122	115	107	114	107	100
1	150	140	130	145	138	127	138	131	121	132	124	112
0	173	160	147	170	158	144	160	148	137	151	140	128
00	197	181	165	191	179	163	181	169	154	169	157	143
000	226	210	191	220	206	186	208	192	173	195	180	163
0000	262	241	218	256	237	214	241	221	201	222	206	187
250	287	261	238	280	257	233	263	242	218	244	223	204
300	320	291	265	314	286	260	295	268	242	272	247	227
350	348	315	287	341	310	281	322	292	263	294	270	243
400	374	338	307	366	333	300	344	312	281	314	285	258
500	424	384	345	413	376	338	388	350	316	352	322	290
600	466	423	378	455	415	371	427	388	338	388	354	317
700	506	459	409	495	450	402	462	420	376	421	382	344
750	522	474	420	510	465	414	475	432	388	434	395	354
	Amperes per Conductor—Six Loaded Cables in Duct Bank											
8	56	51	49	......	......	......	......	......	......	......	......	......
6	73	67	65	72	66	64	69	63	61	......	......	......
4	96	87	80	95	86	79	91	82	75	......	......	......
2	126	113	105	125	111	99	117	106	97	109	99	90
1	144	130	119	141	128	113	133	121	108	125	113	101
0	165	147	136	162	144	128	151	137	122	143	128	114
00	189	166	150	183	164	143	172	156	136	154	144	128
000	216	190	170	211	187	166	197	176	156	175	165	146
0000	248	218	194	243	214	190	227	201	178	213	187	167
250	271	238	210	265	233	206	246	219	194	230	204	180
300	300	265	234	294	260	229	275	242	215	254	227	198
350	329	286	252	320	280	246	298	264	232	276	244	213
400	348	306	271	339	299	263	320	280	247	295	260	227
500	395	345	302	388	338	295	362	316	276	332	290	253
600	436	380	330	427	372	323	399	348	302	364	320	277
700	475	410	355	463	402	349	432	376	326	394	344	298
750	490	423	364	478	415	358	446	388	337	406	354	308
	Amperes per Conductor—Nine Loaded Cables in Duct Bank											
8	55	49	46	......	......	......	......	......	......	......	......	......
6	71	63	61	70	62	60	67	59	57	......	......	......
4	93	82	75	92	81	72	88	77	69	......	......	......
2	123	106	95	120	105	93	109	99	87	101	92	81
1	140	122	108	135	119	103	126	113	99	118	107	93
0	157	137	121	154	135	119	143	127	112	136	119	105
00	178	156	136	175	153	135	164	143	125	154	133	117
000	203	176	154	201	172	152	186	163	152	175	152	132
0000	234	202	176	230	198	172	216	186	162	201	172	151
250	254	219	191	250	215	187	234	202	174	218	188	163
300	284	244	211	277	240	207	260	224	193	242	206	178
350	306	263	229	302	258	224	283	243	207	261	223	192
400	330	281	243	324	276	238	303	258	220	279	236	204
500	372	316	272	366	309	265	341	290	246	313	265	226
600	411	348	295	402	340	289	358	317	269	344	292	246
700	446	373	317	436	366	312	408	343	289	374	314	263
750	460	386	326	450	377	322	422	350	297	384	322	270

Table XXIII. Three-conductor, Round or Sector, Belted-type Paper Cable in Ducts—*Continued*

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded											
	4,500			7,500			15,000			23,000		
	Maximum Copper Temperature, deg cent											
	85.0			82.5			75.0			67.0		
	Per Cent Load Factor											
	50	75	100	50	75	100	50	75	100	50	75	100
	Amperes per Conductor—Twelve Loaded Cables in Duct Bank											
8	54	46	43	......	......	......	......	......	......	......	......	......
6	69	60	57	69	59	56	66	56	54	......	......	......
4	88	78	69	88	77	67	84	73	64	......	......	......
2	114	100	88	114	98	87	108	94	80	101	88	75
1	132	114	99	128	112	97	120	106	91	115	100	85
0	150	128	112	146	127	110	137	120	103	131	113	96
00	170	145	125	165	143	123	157	135	116	146	126	106
000	194	166	142	188	163	140	177	153	131	166	142	120
0000	224	189	163	218	185	159	204	175	149	190	162	136
250	243	204	175	238	201	172	221	188	161	206	175	147
300	272	227	195	264	223	190	246	204	177	231	193	162
350	294	245	210	287	240	204	267	226	189	248	207	174
400	314	261	222	306	256	217	287	240	201	263	222	133
500	354	293	246	344	288	241	322	267	223	295	245	203
600	388	321	268	378	314	261	352	293	241	323	269	220
700	418	344	286	410	338	280	384	316	258	350	288	236
750	432	356	293	423	348	286	396	322	264	360	296	241
Deg Cent	Correction Factors for Various Earth Temperatures											
10	1.07			1.08			1.09			1.10		
20	1.00			1.00			1.00			1.00		
30	0.92			0.92			0.91			0.89		
40	0.83			0.83			0.80			0.76		
50	0.73			0.72			0.68			0.60		

NOTE: This table can be used for either sector or round conductor cable but the minimum sizes are for round conductors.

For multiple-conductor cable,

$$R_i = \frac{3.65G_1}{n}$$

where n is the number of conductors in the cable and G_1 is the geometric factor, values of which are given by D. M. Simmons, *Trans. A.I.E.E.* 1923, vol. XLII, p. 600, and *Elec. Jour.* 1932, vol. 29, p. 339.

Table XXIV. Single-conductor Paper Cables in Air

Based on 60-cycle alternating current, and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded					
	7,500	15,000	23,000	34,500	46,000	69,000
	Maximum Copper Temperature, deg cent					
	85.0	81.0	76.5	70.0	63.5	60.0
	Standard Stranding—Amperes per Conductor					
6	89	86	....	....	....	....
4	118	116	....	....	....	....
2	157	154	148	....	....	....
1	181	178	171	....	....	....
1/0	214	206	194	175	....	....
2/0	249	238	223	201	....	....
3/0	289	275	256	230	204	....
4/0	337	320	298	266	235	....
250	373	354	330	296	260	....
300	419	398	370	331	292	....
350	466	441	410	365	321	278
400	508	479	447	398	350	300
500	588	558	516	458	399	344
600	660	625	579	514	447	384
700	730	689	639	566	493	422
750	762	721	666	590	510	439
800	796	749	694	616	534	457
1000	917	864	799	705	610	520
1250	1052	986	910	804	693	594
1500	1174	1099	1013	896	772	655
1750	1273	1191	1102	973	844	710
2000	1377	1293	1192	1047	901	758
	Annular Cables—Amperes per Conductor					
700	757	712	659	583	505	435
800	835	783	725	641	556	476
1000	978	918	848	748	649	551
1250	1154	1082	998	879	761	636
1500	1313	1231	1134	998	863	728
1750	1473	1379	1269	1113	958	813
2000	1616	1513	1390	1219	1046	895
2500	1890	1767	1620	1415	1212	1020
Deg Cent	Correction Factor for Various Ambient Air Temperatures					
20	1.20	1.22	1.25	1.29	1.36	1.42
30	1.11	1.12	1.13	1.15	1.20	1.23
40	1.00	1.00	1.00	1.00	1.00	1.00
50	0.88	0.87	0.85	0.82	0.76	0.71

(*a*) Core Diameters Assumed for Single Conductor Annular Cables

M.C.M.	Core Diameter	Outside Diameter	M.C.M.	Core Diameter	Outside Diameter
700	0.40	1.04	1500	0.95	1.71
800	0.45	1.13	1750	1.16	1.92
1000	0.56	1.31	2000	1.25	2.11
1250	0.72	1.49	2500	1.38	2.36

(*a*) Any load factor from 50 to 100 per cent.

NOTE: These tables are relatively accurate within 2 per cent for other core sizes, as follows: up to a conductor size of 1,250,000 cir mil core diameters can be increased or decreased 20 per cent; above 1,250,000 cir mil core diameters can be increased 9 per cent or decreased 15 per cent.

Table XXV. Three-conductor, Sector, Shielded, Type H Cables in Air

Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded		
	15,000	23,000	34,500
	Maximum Copper Temperature, deg cent		
	81.5	76.5	70.0
	Amperes per Conductor		
0	171	...	...
00	196	188	...
000	226	216	...
0000	262	248	234
250	287	274	253
300	324	306	285
350	355	334	310
400	380	356	332
500	434	408	377
600	481	455	417
700	526	494	453
750	546	512	476
Deg Cent	Correction Factors for Various Ambient Air Temperatures		
20	1.22	1.25	1.29
30	1.12	1.13	1.15
40	1.00	1.00	1.00
50	0.87	0.85	0.82

(a) Any load factor from 50 to 100 per cent.
(b) Correction factor for round-conductor cables is 0.990.

Table XXVI. Three-conductor, Round or Sector, Belted-type Cables in Air

Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded			
	4,500	7,500	15,000	23,000
	Maximum Copper Temperature, deg cent			
	85	82.5	75.0	67.0
	Amperes per Conductor			
8	51	...	...	...
6	67	66	62	...
4	89	88	83	...
2	118	116	109	101
1	137	135	124	111
0	158	154	142	128
00	181	177	163	143
000	210	205	187	166
0000	246	237	216	190
250	268	262	238	208
300	300	292	266	232
350	330	322	292	254
400	355	348	314	272
500	412	400	356	308
600	459	445	400	342
700	500	488	437	373
750	522	506	450	386
Deg Cent	Correction Factors for Various Ambient Air Temperatures			
20	1.20	1.21	1.25	1.32
30	1.11	1.11	1.13	1.17
40	1.00	1.00	1.00	1.00
50	0.88	0.88	0.85	0.80

(a) Any load factor from 50 to 100 per cent.
(b) This table can be used for either sector or round conductor cable.
(c) The above minimum sizes are for round conductors.

Current-carrying Capacity of Varnished-cambric Insulated Cable

(I.P.C.E.A.)

The tables for varnished-cambric insulated cable are based on standard conditions adopted by I.P.C.E.A. and are representative of conservatively average field conditions in this country. These are:

(*a*) Thermal resistivity of varnished-cambric insulation 600 (watt-cm units).

(*b*) Emissivity of lead sheath or braid 1200 (watt-cm units).

(*c*) Daily load factor 50, 75, and 100 per cent in underground ducts, and any load factor from 50 to 100 per cent in air.

(*d*) Duct correction factor, D:

Cables in Duct Bank	Load Factor...50%	75%	100%
	Loss Factor....33%	62.5%	100%
3	1.0	1.9	3.1
6	1.6	3.0	4.9
9	2.2	4.2	6.8
12	2.8	5.4	8.6

(*e*) All cables in group of similar size, equally loaded and in outside ducts only.

(*f*) Average soil conditions (not dry or "hot spot" conditions).

(*g*) Groups of cable in ducts are referred to ambient earth temperature without external source of heat.

(*h*) Groups of cable in air are referred to surrounding air temperature at full load (corresponding to duct temperature underground).

(*i*) A.I.E.E. Standard Temperature Rule for varnished cambric cable, Table XIX.

(*j*) I.P.C.E.A. standard thicknesses of insulation were used in calculating these tables. See Table V. Deviation, within reasonable limits, from these standard thicknesses in any one voltage class, does not introduce serious error in current rating.

(*k*) Standard logarithmic formula was used in calculating the thermal resistance of single-conductor cable. The thermal resistance of three-conductor cable was determined by methods cited on p. 199.

(*l*) The tables include correction for standard 60-cycle I.P.C.E.A. dielectric loss and all other forms of extra a-c losses as outlined in the following.

DIELECTRIC LOSS. The reduction in carrying capacity due to dielectric loss, as included in the tables, is based on the following I.P.C.E.A. maximum dielectric power-factor values:

Temperature in deg cent	Single- or Multiple-Conductor Shielded Cable	Multiple-Conductor Non-shielded Cable
Room to 40	7.0	10.0
60	10.0	15.0
80	16.0	25.0

The S.I.C. (specific inductive capacity) of the varnished-cambric insulation is assumed as 5.5 for all temperatures, and the calculated dielectric loss is based on uniform temperature for the cable cross-section; also, this loss is added directly to the copper loss. This, of course, results in slightly lesser carrying capacity, but is on the safe side. It is the most practical method for general use and introduces discrepancies only in extreme cases of large groups of extra high-voltage cable.

ADDITIONAL A-C LOSSES. For single-conductor cable, the tables include only dielectric and skin-effect losses (proximity-effect losses are negligible). No attempt is made to include induced sheath current loss for "short-circuited sheath operation," since this varies with spacing between cables, etc. See below. The tables, therefore, represent "open-circuited sheath operation" of single-conductor cable.

For three-conductor cable, the tables include corrections for dielectric loss, and, also, all other forms of 60-cycle extra a-c losses, such as skin-effect, proximity, and sheath losses. The correction factors used for three-conductor cable of both the belted and shielded types, exclusive of dielectric loss, are as given above for paper cables.

These factors hold, within acceptable limits, for all types of cable and thicknesses of insulation.

If current-carrying capacity for other values of extra a-c loss is desired, it can easily be obtained by first dividing the current value in the table by the above correction factor. This gives a value of current without correction for extra a-c loss. This value of current should then be divided by the square root of the ratio of watts loss including extra a-c losses, and watts loss without including extra a-c losses.

The above correction factors for extra a-c loss assume this as part of the copper loss. This is, of course, approximate but is always on the safe side and introduces no appreciable discrepancy except, possibly, for very large conductors. For belted and non-magnetic binder, shielded-type cable the greater part of the total extra a-c loss occurs in the sheath.

For magnetic binder, shielded type cable, on the other hand, the greater part of the extra loss occurs in the conductor. Where the exact proportions of such loss are known, the carrying capacity value in the table can be corrected by an obvious variation of the methods described.

Table XXVII. Three-conductor Shielded Type H Cable in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded					
	7500			15,000		
	Maximum Copper Temperature, deg cent					
	70.5			66.5		
	Per Cent Load Factor					
	50	75	100	50	75	100
	(*a*) Amperes per Conductor—Three Loaded Cables in Duct Bank					
8	59	55	51	57	53	48
6	77	71	66	74	68	62
4	100	92	84	96	87	79
2	132	120	109	125	113	102
1	150	136	123	143	128	116
1/0	171	155	140	163	146	131
2/0	196	178	159	187	166	148
3/0	223	201	160	213	188	167
4/0	257	231	199	243	214	188
250	283	253	222	264	233	203
300	311	277	244	290	255	222
350	337	300	264	315	275	238
400	360	321	281	338	293	252
500	411	362	315	380	329	281
600	451	398	345	414	359	305
700	489	428	370	448	384	325
750	506	441	380	463	396	334
Deg Cent	Correction Factors for Various Earth Temperatures					
10	1.10	1.10	1.10	1.12	1.12	1.12
20	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.89	0.87	0.86	0.86
40	0.77	0.77	0.76	0.71	0.70	0.69
50	0.62	0.61	0.60	0.51	0.49	0.45
Cond. Size	(*b*) Amperes per Conductor—Six Loaded Cables per Duct Bank					
8	56	51	46	54	49	43
6	73	66	58	70	63	54
4	95	85	74	90	80	69
2	124	110	96	116	103	88
1	141	124	108	133	117	100
1/0	160	141	122	151	132	112
2/0	184	161	139	172	149	126
3/0	208	181	156	195	168	142
4/0	240	207	178	222	189	159
250	262	226	194	242	205	171
300	287	246	212	265	225	185
350	312	265	227	287	242	198
400	334	283	242	317	257	210
500	377	318	269	344	286	231
600	414	348	293	377	309	249
700	445	375	314	403	330	264
750	460	386	323	415	339	271
Deg Cent	Correction Factors for Various Earth Temperatures					
10	1.10	1.10	1.10	1.12	1.12	1.13
20	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.89	0.87	0.86	0.85
40	0.76	0.76	0.75	0.71	0.69	0.66
50	0.61	0.61	0.60	0.50	0.45	0.38

Table. XXVII. Three-conductor Shielded Type H Cable in Ducts—*Continued*

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded					
	7500			15,000		
	Maximum Copper Temperature, deg cent					
	70.5			66.5		
	Per Cent Load Factor					
	50	75	100	50	75	100
	(c) Amperes per Conductor—Nine Loaded Cables per Duct Bank					
8	54	47	41	52	45	40
6	70	61	53	64	57	49
4	90	78	68	85	72	61
2	117	102	88	108	93	77
1	133	114	99	124	105	87
1/0	151	129	111	142	118	98
2/0	173	147	127	161	133	110
3/0	195	165	142	182	150	124
4/0	224	188	160	207	169	138
250	245	204	172	225	182	149
300	268	221	187	246	197	159
350	290	238	200	266	212	170
400	310	255	213	283	224	179
500	348	285	236	315	249	197
600	382	312	256	344	268	210
700	410	333	273	368	285	219
750	422	343	281	379	293	223
Deg Cent	Correction Factors for Various Earth Temperatures					
10	1.10	1.10	1.10	1.12	1.13	1.14
20	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.88	0.86	0.85	0.84
40	0.76	0.76	0.75	0.70	0.67	0.63
50	0.61	0.60	0.58	0.48	0.41	0.32

The figures for extra a-c loss were based on data taken on lead-sheathed cables without armor. They will apply approximately to rubber-hose-jacket cable without lead sheath, but with magnetic armor, or for lead-sheathed cable, with magnetic armor and binder. They should not be used for lead-sheathed cable, with magnetic armor, and non-magnetic binder.

BRAIDED CABLE. It would appear at first sight that a cable with black weatherproof braid should have a greater current-carrying capacity than a leaded cable, since the braid has better radiating qualities than a lead sheath. However, it must be remembered that the braid is not so stable chemically as a lead sheath. If run at too high a temperature over an extended period of time, it will tend to become dry and eventually to rot off. Moreover, braided cables are commonly used around stations where practice should always be very conservative. For these reasons it is recommended that the same ratings be used for braided cables as for leaded cables.

GENERAL USE OF TABLES. In using these loading tables, the preceding conditions and limitations should be kept in mind. Where the actual conditions depart from these, proper allowance should be made.

It should be emphasized that these loading tables are based on standard *maximum* allowable copper temperature, and therefore, represent *maximum* allowable loading. Conservative loading (oversize conductors) is always a wise investment. It is recommended that conservative current loading practice be followed wherever possible. *When in doubt select the next largest size conductor.* Reduction in line losses and surplus capacity for emergency use justify this practice.

The column headings of the tables refer to N.E.L.A.–N.E.M.A. " Standard Preferred Voltage Ratings " for General Apparatus. If current-carrying capacity at other voltages and temperatures is desired, it can be obtained by interpolation.

FORMULAS USED. The formulas used in calculating the current tables are those given above for impregnated-paper cables.

Table XXVIII. Three-conductor Belted Type Cables in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	4500			7500			15,000		
	Maximum Copper Temperature, deg cent								
	70.5			67.5			60.0		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	(*a*) Amperes per Conductor—Three Loaded Cables per Duct Bank								
8	53	50	47	51	48	45	48	45	41
6	68	64	60	67	63	59	61	57	52
4	91	85	78	87	81	76	79	73	66
2	120	111	102	114	106	98	104	95	86
1	137	127	116	131	121	112	118	108	97
1/0	156	144	132	149	138	126	135	123	110
2/0	181	167	152	173	159	145	154	140	125
3/0	208	190	172	197	183	165	177	160	141
4/0	239	218	197	226	205	186	202	182	159
250	261	238	215	249	226	203	224	200	174
300	287	261	234	272	247	222	246	219	192
350	313	283	253	296	267	240	267	237	207
400	337	305	272	319	288	258	286	252	221
500	365	345	306	363	326	290	320	281	244
600	426	382	337	403	360	317	351	307	264
700	464	413	365	440	390	341	380	330	281
750	481	428	377	456	404	353	394	341	290
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.10	1.10	1.10	1.11	1.11	1.11	1.14	1.15	1.15
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.89	0.88	0.88	0.88	0.84	0.83	0.82
40	0.77	0.77	0.77	0.74	0.74	0.74	0.63	0.61	0.58
50	0.62	0.62	0.62	0.57	0.57	0.56	0.32	0.26	0.19
Cond. Size	(*b*) Amperes per Conductor—Six Loaded Cables per Duct Bank								
8	51	47	43	49	45	41	46	41	37
6	65	60	54	64	59	53	59	52	46
4	87	79	70	83	76	68	75	66	58
2	114	103	91	108	98	88	98	87	74
1	130	117	103	124	112	99	111	98	84
1/0	148	133	118	141	127	112	127	111	94
2/0	171	153	135	163	146	128	143	126	106
3/0	195	174	153	186	166	145	164	143	120
4/0	225	199	175	212	188	165	187	161	134
250	245	216	190	233	205	180	207	174	146
300	269	235	207	255	222	195	228	193	160
350	292	256	224	277	242	209	245	209	173
400	314	275	240	298	259	224	263	222	184
500	357	310	268	338	291	252	292	245	202
600	395	340	293	373	320	275	320	266	216
700	429	368	315	405	345	297	345	285	228
750	444	380	325	419	357	305	355	294	233
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.10	1.10	1.10	1.11	1.11	1.11	1.14	1.15	1.15
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.89	0.88	0.88	0.87	0.83	0.82	0.80
40	0.77	0.77	0.77	0.74	0.74	0.73	0.62	0.58	0.51
50	0.62	0.62	0.62	0.56	0.56	0.54	0.27	0.08	....

Table XXVIII. Three-conductor Belted Type Cables in Duct—*Continued*

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	4500			7500			15,000		
	Maximum Copper Temperature, deg cent								
	70.5			67.5			60.0		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	(*c*) Amperes per Conductor—Nine Loaded Cables per Duct Bank								
8	49	44	40	47	42	35	44	38	33
6	63	56	51	61	54	46	56	48	41
4	83	74	65	79	70	60	71	61	51
2	109	96	85	103	91	79	93	79	65
1	124	110	95	117	103	90	105	89	73
1/0	141	124	108	134	117	102	120	101	83
2/0	163	142	124	154	134	116	136	114	93
3/0	185	160	139	175	151	131	156	128	104
4/0	212	182	157	199	170	146	177	144	116
250	231	198	170	219	186	159	195	156	125
300	253	216	185	238	203	172	213	172	137
350	275	238	200	258	220	186	229	185	147
400	297	250	213	278	235	198	243	197	156
500	335	281	239	315	263	220	270	215	169
600	369	318	252	348	278	239	293	233	179
700	400	333	280	377	310	256	315	247	187
750	419	344	288	390	320	264	325	253	189
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.10	1.10	1.10	1.11	1.11	1.11	1.14	1.16	1.19
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.89	0.89	0.89	0.88	0.88	0.87	0.83	0.80	0.77
40	0.77	0.77	0.76	0.74	0.73	0.72	0.62	0.54	0.42
50	0.62	0.62	0.61	0.56	0.55	0.53	0.26	....	....

Table XXIX. Single-conductor Standard Strand Cables in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	4500			7500			15,000		
	Maximum Copper Temperature, deg cent								
	72.5			70.5			66.5		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	(*a*) Amperes per Conductor—Three Loaded Cables per Duct Bank								
8	72	69	66	72	69	66	71	68	65
6	95	91	87	95	91	87	93	89	85
4	123	119	113	123	119	113	121	116	110
2	163	156	148	163	156	148	160	153	143
1	189	180	171	189	180	171	184	175	164
1/0	219	209	197	219	209	197	212	200	187
2/0	251	239	224	251	239	224	243	230	214
3/0	293	278	261	290	274	256	282	265	246
4/0	342	322	299	337	317	295	325	303	282
250	378	356	330	373	350	325	360	335	311
300	424	398	368	418	390	361	401	372	344
350	465	436	404	463	432	398	441	410	375
400	506	473	437	503	468	430	476	440	405
500	588	546	501	576	534	491	547	503	460
600	658	610	558	646	596	544	612	558	508
700	724	668	610	710	652	594	674	622	566
750	754	694	632	741	680	617	700	638	580
800	784	722	658	769	705	640	726	660	597
1000	894	816	742	882	804	724	830	748	674
1250	1022	928	838	1000	906	814	935	838	751
1500	1125	1025	916	1090	984	884	1035	926	832
1750	1230	1100	985	1200	1075	960	1120	993	884
2000	1310	1175	1047	1286	1145	1016	1195	1066	945
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.09	1.09	1.09	1.10	1.10	1.10	1.11	1.11	1.11
20	1.00	1.00	1.00	1.00	1.00	1 00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.88	0.88	0.88
40	0.79	0.79	0.79	0.77	0.77	0.77	0.73	0.73	0.72
50	0.65	0.65	0.65	0.63	0 63	0.62	0.55	0 55	0.54
Cond. Size	(*b*) Amperes per Conductor—Six Loaded Cables per Duct Bank								
8	70	67	62	70	67	62	69	66	61
6	92	87	82	92	87	82	90	85	79
4	120	113	105	120	113	105	118	111	102
2	158	148	137	158	148	137	155	144	132
1	183	171	158	183	171	158	178	165	151
1/0	212	198	182	212	198	182	203	188	173
2/0	243	225	207	243	225	207	234	216	196
3/0	283	262	240	279	257	236	270	[illegible]8	224
4/0	328	301	274	323	297	270	310	[illegible]4	256
250	363	333	303	356	327	297	343	313	282
300	406	372	336	398	364	330	381	346	310
350	445	405	367	441	400	361	418	378	337
400	483	440	396	478	433	390	451	408	360
500	558	506	452	546	492	441	516	464	411
600	625	563	500	610	548	489	578	514	454
700	684	616	546	669	598	533	642	570	494
750	712	639	566	697	622	552	659	584	510
800	740	663	587	724	645	572	680	605	528
1000	840	748	660	826	730	644	774	680	591
1250	956	845	740	934	820	722	867	760	658
1500	1053	927	808	1013	890	780	954	840	716
1750	1137	995	866	1110	966	842	1032	891	762
2000	1220	1060	918	1185	1025	890	1110	952	803
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.09	1.09	1.09	1.10	1.10	1.10	1.11	1.11	1.10
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.88	0.87	0.87
40	0.79	0.79	0.79	0.77	0.77	0.77	0.73	0.73	0.72
50	0.65	0.65	0.65	0.63	0.63	0.62	0.55	0.54	0.52

Table XXIX. Single-conductor Standard Strand Cables in Ducts—*Continued*

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	4500			7500			15,000		
	Maximum Copper Temperature, deg cent								
	72.5			70.5			66.5		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	(c) Amperes per Conductor—Nine Loaded Cables per Duct Bank								
8	69	64	59	69	64	59	68	63	58
6	90	84	77	90	84	77	88	81	74
4	117	108	99	117	108	99	115	105	95
2	154	141	129	154	141	129	150	136	123
1	178	163	148	178	163	148	172	155	140
1/0	205	187	169	205	187	169	197	178	159
2/0	235	213	191	235	213	191	226	202	180
3/0	273	247	223	270	243	218	260	232	206
4/0	315	283	254	311	279	249	298	265	234
259	349	313	279	343	307	273	329	292	257
300	390	347	309	383	341	302	365	322	284
350	427	380	336	423	374	330	400	351	307
400	463	411	363	458	404	355	431	378	330
500	534	469	412	522	458	401	492	429	371
600	596	520	456	582	508	443	546	474	408
700	652	568	495	637	554	473	608	516	443
750	678	589	514	663	575	496	623	534	458
800	704	612	532	688	596	516	644	554	471
1000	797	688	596	783	672	579	729	621	529
1250	903	774	668	882	754	644	816	692	584
1500	993	845	724	956	816	696	900	754	634
1750	1070	907	774	1045	882	750	965	806	673
2000	1140	962	820	1110	934	790	1035	850	706
Deg Cent	Correction Factors for Various Earth Temperatures								
10	1.09	1.09	1.09	1.10	1.10	1.10	1.11	1.11	1.12
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.87	0.87	0.86
40	0.79	0.79	0.79	0.77	0.77	0.77	0.73	0.72	0.70
50	0.65	0.65	0.65	0.62	0.62	0.61	0.54	0.52	0.48

Table XXX. Single-conductor Annular Cables in Ducts

Based on 60-cycle alternating current and an earth temperature of 20 deg cent

Conductor Size in A.W.G. or M.C.M.	Rated Three-phase Line Voltage, Neutral Grounded								
	4500			7500			15,000		
	Maximum Copper Temperature, deg cent								
	72.5			70.5			66.5		
	Per Cent Load Factor								
	50	75	100	50	75	100	50	75	100
	Amperes per Conductor—Three Loaded Cables in Duct Bank								
750	775	713	647	755	694	627	705	643	580
1000	960	880	791	937	851	763	863	780	700
1500	1275	1140	1014	1235	1095	975	1135	1005	880
2000	1532	1350	1190	1480	1310	1140	1345	1180	1025
2500	1740	1530	1340	1680	1475	1285	1530	1335	1160
3000	1940	1693	1470	1865	1620	1410	1695	1465	1260
4000	2310	2030	1740	2270	1940	1660	2050	1750	1470
5000	2740	2320	1985	2625	2205	1870	2360	1975	1642
Deg Cent	Correction Factors for Various Ambient Earth Temperatures								
10	1.09	1.09	1.09	1.10	1.10	1.10	1.10	1.10	1.10
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.87	0.87	0.87
40	0.78	0.78	0.78	0.77	0.77	0.77	0.73	0.69	0.65
50	0.65	0.65	0.65	0.62	0.61	0.60	0.54	0.51	0.49
Cond. Size	Amperes per Conductor—Six Loaded Cables in Duct Bank								
750	732	652	576	711	631	555	660	585	510
1000	908	800	700	875	766	670	805	703	605
1500	1180	1022	882	1140	985	840	1040	890	744
2000	1410	1200	1023	1355	1145	980	1225	1035	860
2500	1590	1350	1150	1530	1295	1095	1395	1165	960
3000	1765	1485	1260	1700	1422	1195	1535	1275	1050
4000	2120	1760	1470	2035	1675	1390	1835	1500	1200
5000	2445	2000	1665	2340	1900	1550	2085	1670	1330
Deg Cent	Correction Factors for Various Ambient Earth Temperatures								
10	1.10	1.10	1.10	1.10	1.10	1.10	1.11	1.12	1.13
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.87	0.86	0.85
40	0.78	0.78	0.78	0.77	0.77	0.77	0.72	0.70	0.67
50	0.65	0.65	0.64	0.61	0.61	0.60	0.52	0.48	0.42
Cond. Size	Amperes per Conductor—Nine Loaded Cables in Duct Bank								
750	695	602	527	673	580	500	625	535	454
1000	857	732	629	821	702	600	760	638	535
1500	1105	927	786	1065	890	765	970	800	656
2000	1307	1080	910	1250	1035	860	1130	920	745
2500	1475	1210	1015	1415	1155	960	1285	1025	820
3000	1630	1330	1100	1560	1270	1045	1405	1125	892
4000	1942	1560	1290	1855	1480	1200	1665	1300	1010
5000	2220	1755	1450	2120	1660	1335	1885	1430	1080
Deg Cent	Correction Factors for Various Ambient Earth Temperatures								
10	1.09	1.09	1.09	1.10	1.10	1.10	1.12	1.12	1.14
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.90	0.90	0.90	0.89	0.89	0.89	0.87	0.86	0.83
40	0.78	0.78	0.78	0.76	0.76	0.75	0.71	0.68	0.63
50	0.65	0.65	0.64	0.61	0.60	0.58	0.50	0.43	0.31

Table XXXI. Varnished-cambric Insulated Cable

Three-conductor Round Shielded Type H Cables in Air

Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Maximum Three-phase Line Voltage, Neutral Grounded		
	7500	15,000	23,000
	Maximum Copper Temperature, deg cent		
	70.5	66.5	61.5
	Amperes per Conductor		
6	68	...	...
4	89	81	...
2	112	103	91
1	129	117	104
0	150	136	119
00	171	155	135
000	197	177	153
0000	238	205	176
250	255	228	193
300	286	254	214
350	314	277	233
400	338	298	250
500	383	338	280
600	422	372	307
700	460	404	331
750	479	419	343
Deg Cent	Correction Factor for Ambient Temperature		
10	1.43	1.58	1.74
20	1.30	1.39	1.53
30	1.15	1.21	1.30
40	1.00	1.00	1.00
45	0.91	0.88	0.84
50	0 81	0 74	0 51

Round conductors only recommended for varnished-cambric cable.

Table XXXII. Varnished-cambric Insulated Cable
Three-conductor Round Belted Type Cables in Air

Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Maximum Three-phase Line Voltage, Neutral Grounded		
	4500	7500	15,000
	Maximum Copper Temperature, deg cent		
	70.5	67.5	60.0
	Amperes per Conductor		
8	44	...	...
6	52	48	...
4	70	64	59
2	94	84	75
1	110	97	86
0	134	117	100
00	154	139	116
000	179	162	134
0000	208	190	155
250	229	212	171
300	255	235	190
350	280	259	206
400	303	280	222
500	345	318	250
600	385	354	276
700	425	390	300
750	442	407	312
Deg Cent	Correction Factor for Ambient Temperature		
10	1.41	1.48	1.74
20	1.29	1.34	1.53
30	1.15	1.18	1.29
40	1.00	1.00	1.00
45	0.91	0.90	0.83
50	0.81	0.78	0.58

(*a*) Any load factor from 50 to 100 per cent.
(*b*) Round conductors only recommended for varnished-cambric cable.

Table XXXIII. Varnished-cambric Insulated Cable

Single-Conductor Standard Strand Cables in Air

Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size in A.W.G. or M.C.M.	Maximum Three-phase Line Voltage, Neutral Grounded			
	4.5	7.5	15.0	23.0
	Maximum Copper Temperature, deg cent			
	72.5	70.5	66.5	61.5
	Amperes per Conductor			
8	56	....	....	....
6	73	72	....	....
4	97	96	91	....
2	130	128	121	106
1	153	151	142	124
0	180	179	165	143
00	208	206	189	163
000	244	239	218	188
0000	286	278	253	219
250	320	310	282	244
300	360	348	315	272
350	402	384	347	298
400	440	422	378	324
500	512	492	438	375
600	576	553	490	420
700	636	612	540	462
750	666	638	566	481
800	692	664	589	500
1000	795	760	676	568
1250	909	871	768	644
1500	1020	976	853	717
1750	1135	1082	942	776
2000	1200	1142	1001	835
Deg Cent	Correction Factor for Ambient Temperature			
10	1.39	1.42	1.52	1.68
20	1.27	1.29	1.36	1.49
30	1.14	1.16	1.20	1.26
40	1.00	1.00	1.00	1.00
45	0.92	0.91	0.88	0.83
50	0.83	0.82	0.75	0.62

Any load factor from 50 to 100 per cent.

Table XXXIV. Varnished-cambric Insulated Cable
Single-conductor Annular Cables in Air
Based on 60-cycle alternating current and an ambient air temperature of 40 deg cent

Conductor Size M.C.M.	Maximum Three-phase Line Voltage, Neutral Grounded			
	4.5	7.5	15.0	23.0
	Maximum Copper Temperature, deg cent			
	72.5	70.5	66.5	61.5
	Amperes per Conductor			
750	678	650	577	488
800	716	687	608	512
1000	858	820	723	607
1250	1019	971	850	708
1500	1170	1115	970	806
1750	1305	1232	1070	886
2000	1420	1343	1166	964
2500	1630	1550	1350	1116
3000	1880	1790	1560	1250
4000	2380	2240	1940	1530
5000	2840	2690	2310	1785
Deg Cent	Correction Factor for Ambient Temperature			
10	1.40	1.42	1.52	1.68
20	1.27	1.30	1.37	1.49
30	1.15	1.16	1.20	1.27
40	1.00	1.00	1.00	1.00
45	0.92	0.92	0.88	0.83
50	0.83	0.81	0.75	0.62

(*a*) Any load factor from 50 to 100 per cent.
(*b*) For core diameters on which these ratings are based see Table XXIV, note (*a*).

Current-carrying Capacity of Rubber-insulated Wires and Cables

At the date of publication there are no tables of carrying capacity of rubber-insulated cables which take into account different types of cables and installations, but I.P.C.E.A. has such tables in preparation.

The values given in Table XXXV are those now approved by the National Board of Fire Underwriters for indoor installations.

The formulas given under Paper and Varnished Cambric may be employed, however, using the following constants.

Thermal resistivity....................................500 (watt-cm units)
Thermal emissivity of braid............................950 (watt-cm units)

Braid emissivity resistance values with age, color, diameter and conditions of installation, over a range of 600 to 1200.

Maximum permissible continuous operating temperature.

Heat-resisting rubber.................................. 70 or 75 deg cent *
30 per cent hevea...................................... 60 deg cent
Code... 50 deg cent

There are special rubber compounds that may be operated at higher temperatures. No allowance need be made for dielectric loss, as it is low and changes little with temperature.

Formula for Carrying Capacity in Earth

The carrying capacity of a cable buried in the earth may be derived from that in air, by the following relation:

Let I_a = continuous carrying capacity in air.
I_e = continuous carrying capacity buried in the earth.

* As recommended by manufacturer.

Table XXXV. Carrying Capacities, in Amperes, for Interior Copper Conductors, Rubber Insulated, National Board of Fire Underwriters

(For aluminum 84 per cent of these currents is allowed)

Single-conductor cables or each conductor of multiple-conductor cable

Size		Amperes	Size		Amperes	Size		Amperes
A.W.G.	Area, cir mils		A.W.G.	Area, cir mils		A.W.G.	Area cir mils	
18	1,624	3	0	105,500	125	...	900,000	600
16	2,583	6	00	133,100	150	...	1,000,000	650
14	4,107	15	000	167,800	175	...	1,100,000	690
12	6,530	20	...	200,000	200	...	1,200,000	730
10	10,380	25	0000	211,600	225	...	1,300,000	770
8	16,510	35	...	250,000	250	...	1,400,000	810
6	26,250	50	...	300,000	275	...	1,500,000	850
5	33,100	55	...	400,000	325	...	1,600,000	890
4	41,740	70	...	500,000	400	...	1,700,000	930
3	52,630	80	...	600,000	450	...	1,800,000	970
2	66,370	90	...	700,000	500	...	1,900,000	1010
1	83,690	100	...	800,000	550	...	2,000,000	1050

Then, assuming the same ambient temperature in both cases,

$$I_e = I_a\sqrt{\frac{R_i + R_s}{R_i + R_e}}$$

where R_s = thermal resistance from the surface of the cable to the air, in degrees centigrade per watt-foot.

R_i = thermal resistance of insulation, etc., from conductor to the surface of the cable, in degrees centigrade per watt-foot.

R_e = thermal resistance of earth from the surface of the cable to the surface of the ground in degrees centigrade per watt-foot.

$$= 0.008\, g \log_{10} \frac{48\, L}{D}$$

where L = depth of conductor, feet

g = thermal resistivity of soil in watt-centimeter centigrade units, usually taken as 180.

	H_2O%		
g	Sandy Loam	Heavy Clay	Chalky
340	0	1	2
180	5	17	10
120	10	..	16
90	15	..	20

D = overall diameter of cable, in.

Table XXXVI. Approximate Carrying Capacity of Cables Buried in the Earth

600 volt cables 18 in. deep; 15° to 65° = 50° C temperature rise

11,000 volt cables 36 in. deep; G = 180 watt-cm-C°

A.W.G. or Cir Mils	Amperes			A.W.G. or Cir Mils	Amperes		
	One-conductor	Three-conductor			One-conductor	Three-conductor	
	600 Volts	600 Volts	11,000 Volts		600 Volts	600 Volts	11,000 Volts
4	180	120	100	350,000	590	380	350
2	230	150	130	400,000	640	400	370
1	270	170	150	500,000	730	450	420
0	300	190	160	600,000	820	500	460
00	330	210	190	750,000	930	570	520
000	390	250	220	1,000,000	1100	...	...
0000	440	280	250	1,250,000	1270	...	...
250,000	490	300	280	1,500,000	1420	...	...
300,000	540	340	310				

Sheath Losses and Carrying Capacity

The effect of sheath currents on the carrying capacity may be calculated by substituting for the thermal resistance from conductor to ground, $R_1 + R_2$, the larger quantity

$$R_1 + R_2\left(1 + \frac{W_s}{W_c}\right)$$

where R_1 = thermal resistance from conductor to sheath.
R_2 = thermal resistance from sheath to ambient earth or air.
W_c = watts loss in conductor plus dielectric loss.
W_s = watts loss in sheath.

Temperature Rise of Cable with Variable Load

$$T = T_m(1 - \epsilon^{-t/t_1}).$$

where T = temperature rise at time t hr deg cent.
T_m = ultimate temperature rise, deg cent.
ϵ = base of Naperian logarithm.
t_1 = time constant or time (hr) required to reach 63 per cent of final temperature rise with a constant power loss.
= SH
where H = heating factor or thermal resistance in degrees centigrade per watt per foot.
S = thermal storage factor in watthours to raise the copper in 1 ft of cable 1 deg cent.
This may be calculated from the following factors:

Material	Watthours stored per lb per deg cent
Copper	0.048
Paper	0.137
Oil	0.237
Lead	0.016

The ratio $\frac{T}{T_m}$ or $(1 - \epsilon^{-t/t_1})$ for a period $t = 1$ hr is called the hourly attainment factor, A. Its usual value is between 0.4 and 0.8.

This factor may be used to calculate the copper temperature curve for any load curve which may be divided for convenience into hourly steps. If the copper temperature rise at the beginning of any hour is represented by T_0 and the ultimate temperature rise by T_m, corresponding to the average watts lost during that hour, the change in temperature rise during the hour will be $A(T_m - T_0)$. The copper temperature rise at the end of the hour (T_t) is then equal to $T_0 + A(T_m - T_0)$, and the copper temperature is equal to $T_t + T_d$ where T_d is the temperature of the air in an adjacent empty duct, which may be assumed constant. By evaluating $T_0 + A(T_m - T_0)$ for each hourly period, the entire temperature curve may be plotted step by step.

61. A-C RESISTANCE AND REACTANCE

Ratio of A-c Resistance to D-c Resistance, Three-conductor Cables

The following are maximum values of this ratio, at 60 cycles, which may be expected in cables (I.P.C.E.A.) having concentric stranding with alternately reversed layers. Lower values for the larger sizes are obtained with compactly crushed conductors with all layers in the same direction.

Size of Each Conductor, cir mils	Ratio A-c/D-c Resistance
0	1.00
200,000	1.05
300,000	1.07
400,000	1.10
500,000	1.13
600,000	1.16
700,000	1.19
800,000	1.23

For single-conductor cables, see chapter on Bare Wires and Cables, Tables XXIV and XXV.

REACTANCE OF TRIPLEX CABLES. The reactance of three-conductor cables without magnetic binder tapes or armor, may be calculated by the ordinary formulas for reactance of parallel conductors. See Arts. 16 and 22. It is of the order of 0.027 to 0.035 ohm per 1000 ft at 60 cycles. The effect of steel binder tape is to increase the reactance about 25 per cent.

62. FLEXIBILITY

Table XXXVII

Thickness of Conductor Insulation	Minimum Bending Radius as a Multiple of Overall Diameter	
	Up to 500,000 Cir Mils	500,000 Cir Mils and over
Up to 8/64 in.	6	8
9/64 to 12/64 in.	8	10
13/64 to 20/64 in.	10	12
21/64 in. and over	12	12

RUBBER-INSULATED CABLES. The minimum permissible radius to which rubber-insulated cables may be bent is as given in Table XXXVII.

VARNISHED-CAMBRIC CABLES. A minimum bending radius of 5 1/2 times the cable diameter is recommended for voltages up to 10,000. Above 10,000 volts, the minimum ratio should be 7.

PAPER CABLES. A minimum bending radius of 8 times the cable diameter is recommended for 0 to 20,000 volts, and a minimum ratio of 10 for over 20,000 volts.

63. SINGLE-CONDUCTOR CABLES FOR THREE-PHASE SYSTEMS

It is common practice, for transmission lines operating at 40,000 volts or over, to use three single-conductor, paper-insulated, lead-sheathed cables, for three-phase systems. Voltages are induced along the sheaths, and therefore considerations of safety make it necessary to ground them. If any sheath is grounded, or connected to another sheath at more than one point, current will flow, which will lower the carrying capacity of the cable and cause expensive energy loss. Means for keeping down the sheath voltages and currents have been devised, such as by transposition of sheatns. (See Halperin and Miller, *Trans. A.I.E.E.*, 1929, Vol. 48, p. 399.)

The following formulas are approximate, the exact ones being unwieldy for practical purposes:

Let b = radius of conductors, inches, carrying I_c amperes each.
e = inner radius of lead sheath, inches.
f = outer radius of lead sheath, inches.
D = distance between centers of conductors, inches, assuming them to be set in an equilateral triangle.
M = mutual inductance of conductor and sheath, henrys per 1000 ft.
W_s = power loss in sheath, watts per 1000 ft of each cable.
E_s = emf, volts, induced per 1000 ft of sheath. The potential difference between sheaths at 1000 ft from any bond will be $1.73E_s$.
$\omega = 2\pi f$, where f = frequency.
L_0 = inductance of each cable, with sheath open-circuited, henrys per 1000 ft.
L_c = inductance of each cable with sheath short-circuited, henrys per 1000 ft.
L_s = inductance of the sheath of each cable, henrys per 1000 ft.
I_s = sheath current, amperes when bonds of negligible resistance are used.
R_s = resistance of sheath, ohms per 1000 ft.

$$M = 1.405 \times 10^{-4} \log_{10} \frac{2D}{e+f}$$

$$E_s = \omega M I_c.$$

$$L_0 = \left[0.152 + 1.4 \log_{10} \frac{D}{b}\right] \times 10^{-4}$$

$$L_c = L_0 - M.$$

$$L_s = \frac{1.4}{10^4} \log_{10} \frac{D}{f}$$

$$R_s = \frac{0.036}{f^2 - e^2} \text{ at 40 deg cent.}$$

$$I_s = \frac{E_s}{\sqrt{R_s^2 + (\omega L_s)^2}}$$

$$W_s = I_s^2 R_s$$

An important group of papers on this subject will be found in *J.I.E.E.* (London), 1928 and 1929. Halperin and Miller, *Trans. A.I.E.E.*, April, 1929, p. 399, give formulas for groupings other than equilateral.

EFFECT OF MAGNETIC ARMOR.

Let μ = permeability of armor.

$$a = \frac{1-\mu}{\mu}.$$

k = ratio of total flux in magnetic armor to flux that would exist if armor were replaced by air, usually from 20 to 40.

Then
$$k = \frac{1}{a}\left\{\frac{\pi}{2} - \frac{2}{\sqrt{1-a^2}}\tan^{-1}\frac{1-a}{1+a}\right\}$$

Let L = inductance, millihenrys per mile.
d_1 = diameter over armor.
d_2 = diameter under armor.
D = distance between wires.
d = radius of conductor.

$$L = 0.0805 + 0.741\left[\log_{10}\frac{D}{d} + k\log_{10}\frac{d_1}{d_2}\right]$$

This assumes perfect magnetic contact between armor wires. In practice, owing to loose contact, the inductance will be about 0.7 of the above value. This formula requires some assumption to be made regarding the permeability of the wire, a factor which depends both on the material of the armor and the current in the cable. Numerical calculations require the permeability to be deduced from the magnetic flux density in the armor, a quantity which may be estimated from k and the conductor current.

64. EXPANSION AND CONTRACTION IN IMPREGNATED-PAPER CABLES

The ordinary impregnated-paper cable is characterized by high resistance to fluid flow both radially and longitudinally, due to the density of the paper, the compactness of construction, and the viscosity of the oil. As the oil has a large coefficient of thermal expansion, compared to the other materials of the cable, the effect of thermal cycles is to create, alternately, local high pressures and local vacua. The former may cause stretching of the sheath and thus accentuate the vacua in subsequent cycles. The local vacua are weak spots because they ionize at low voltage, causing hot spots. Therefore, either the insulation thickness must be great enough to overcome this weakness, or means must be adopted to prevent the formation of voids.

In ordinary cables, the insulation is made thick enough to reduce the stresses to a point at which destructive ionization, in minute voids, does not occur. The elimination of voids may be accomplished in either of two ways, to be described below.

65. IONIZATION-FREE CABLES FOR HIGH VOLTAGES

PRESSURE CABLE. If the entire lead-sheathed cable be enclosed in a steel pipe and subjected to an external gas pressure of 12–15 atmospheres, the pressure will reach the interior of the cable and prevent the formation of voids. The insulation may then be of about one-half the ordinary thickness. (Hochstadter, *Royal Soc. Arts*, London, 1931.) Another type of pressure cable, the Oilostatic, uses a leadless paper cable in a pipe containing cable oil under similar pressure.

OIL-FILLED CABLE. The formation of voids may be prevented by the use of oil of low viscosity, not over 20 minutes Saybolt, and preferably much less. This permits the oil to flow radially through the insulation, and longitudinally through oil ducts furnished to carry the oil along the cable from reservoir-equipped joints. As with the *pressure cable*, the insulation thickness is about one-half the ordinary value, permitting the use of cables of practicable diameter for voltages as high as 250,000, three-phase.

Oil-filled Cable Approximate Formulas

The frictional resistance of a hollow core, to the flow of oil, may be calculated by the following formula, the factor b being that to be used in the formula for pressure drop given below.

Let Z = viscosity of oil, centipoises.
d = effective channel diameter.

$$b = \frac{0.0000283\ Z}{d^4}.$$

If the resistance is to be measured by forcing external oil through the hollow core, let

p = pressure, pounds per square inch.
q = flow of oil, cubic inches per second.
l = length of cable, feet.

$$b = 0.4\frac{p}{q^l}.$$

The usual order of value of b is about 0.05.

The maximum rate of oil demand of a cable may be approximated on the assumption that, at the beginning of heating or cooling, the entire heating or cooling depends on the thermal capacity of the copper and oil contained therein.

Let a = oil demand, cubic inches per feet per second.
α = coefficient of expansion of oil per degree centigrade, usually about 0.00074.
v = volume of oil in conductor (including hollow core), cubic inches per foot of cable.
W = watts per foot lost in cable.
$h = 1899\ (0.511\ W_0 + 0.101\ W_c)$.
W_0 = weight of oil in conductor, pounds per foot.
W_c = weight of copper, pounds per foot.

$$a = \frac{\alpha v W}{h}.$$

The usual order of value of a is about 50×10^{-6}.

The pressure drop in the hollow core, due to oil demand, may be calculated as follows:

Let p = pressure drop, pounds per square inch to distance x from oil-feeding point.
a = oil demand, defined above.
b = resistance factor, defined above.
l = total length of section, feet.

$$p = 2\,ab\left(lx - \frac{x^2}{2}\right)$$

At the end of the line, where $x = l$,

$$p = abl^2$$

It is usual to design the line so that p will not exceed 15 lb per sq in. at any point, nor fall below atmospheric pressure.

66. WIRE AND CABLE SPECIFICATIONS

Wires and cables constitute such a large proportion of electric systems that their quality is a matter of great importance, from the standpoint of safety, economy, and continuity of service. Practically all wires and cables are therefore purchased under standard specifications. The principal of these are as follows:

Subject	Standardizing Organization	Standard or Specification Number
Definitions, Methods of Test, etc.	A.I.E.E.	30
	A.S.A.	C–8–a
Bare Soft Annealed Copper Wire	A.S.T.M.	33
	A.I.E.E.	61
	A.S.A.	C–8–b–2
Tinned Soft Annealed Copper Wire	A.S.T.M.	B–33
	A.I.E.E.	60
	A.S.A.	C–8–b–1
Rubber-insulated House Wire and Cord	N.B.F.U.	Nat. Elec. Code
	A.S.A.	C–8–11
	N.E.M.A.	......
Rubber-insulated House Wire and Cord Intermediate Grade	N.E.M.A.	......

Rubber-insulated Wire, 30 per cent Hevea, Class A..	A.S.T.M. A.I.E.E. N.E.M.A.	393 63 ......
Rubber-insulated Wire, 30 per cent Hevea, Class AO	A.S.T.M. A.S.A.	D–27 C–8–d–1
Rubber-insulated Wire, Performance Type.........	A.S.T.M.	D–253
Rubber-insulated House Wire and Cord Performance Type..	N.E.M.A. A.S.T.M.	...... D–353
Impregnated-paper Cables (Including Oil-filled).....	A.E.I.C. I.P.C.E.A. A.S.A.	....
Varnished-cambric Wires and Cables..............	I.P.C.E.A.	......
Magnet Wire..................................	A.I.E.E. A.S.A.	69, 70, 71 C–8–J–1, 2, 3.
Cable Shipping Reels...........................	I.P.C.E.A. E.E.I.	....
Braid Colors for Control Cable....................	E.E.I. I.P.C.E.A.	....
Marine Wires and Cables.......................	A.I.E.E.	45

(a) The braces indicate that the specifications included by them are identical in substance.
(b) A.I.E.E. = American Institute of Electrical Engineers.
A.S.A. = American Standards Association.
A.S.T.M. = American Society for Testing Materials.
E.E.I. = Edison Electrical Institute.
I.P.C.E.A. = Insulated Power Cable Engineers Association.
N.B.F.U. = National Board of Fire Underwriters.
N.E.M.A. = National Electrical Manufacturers Association.
(c) A.S.T.M. and N.E.M.A. Specifications for Perfcrmance Type Rubber differ principally in that the former contains a low water absorption requirement which is absent from the latter.

67. INSTALLATION

For the installation of wires and cables in buildings see Wiring of Buildings. For the construction of the pole lines carrying the messenger wire see Arts. 25 and 26. For the construction of conduit systems see chapter on Underground Lines. Below is given a brief description of modern practice in installing insulated wires and cables on messenger wires and in underground conduit systems. See also the articles on Distribution Lines and Transmission Lines.

MESSENGER CONSTRUCTION. The messenger wire is erected in the same manner as an ordinary line wire; see Art. 30. A " leading-up " wire is stretched from the bottom of one pole to the messenger wire on the starting pole, forming an incline on which to pull up the cable. A pulling rope is then fastened to the end of the cable by means of a cable grip and carried alongside of the messenger to the point where the cable is to reach, thence through a snatch-block down to the terminal pole to a second block at the bottom and thence to a capstan, winch, locomotive. or whatever is to be used for pulling. Either temporary rollers should be provided on the poles over which the rope runs, or the rope should be suspended from the messenger by wire hooks. The cable is then slowly drawn up the inclined wire and along the messenger, attaching temporary carriers to the cable as it is paid out and hooking them over the messenger to carry the weight of the cable. Linemen must be stationed on each pole to pass these carriers around the messenger clamp or insulator. The final suspension of the cable may be accomplished in either of the following ways: (1) When the end of the cable arrives at the beginning of the last span, the lineman on each pole replaces the temporary carriers by permanent hangers, spacing them regularly along the cable, so that when the last span is pulled, all the hangers will be in place. (2) When the cable has been pulled all the way, a lineman rides along the messenger wire in a carriage replacing each temporary carrier by a hanger. This plan is preferable as the hangers may be attached more tightly to the messenger wire, and are less likely to slip on the cable.

INSTALLATION OF CABLES IN DUCTS. The conduit system having been constructed, it must be prepared for the reception of the cable by being cleaned out or rodded as described in the chapter on Underground Lines.

If several ducts are available, the choice of the particular duct to be used should be governed by the following considerations: (1) avoiding unnecessary crossings of cable in

the splicing chambers, substations, etc.; (2) avoiding the obstructing of empty ducts; (3) keeping the cables cool; and (4) keeping d-c cables away from others. The coolest and best heat-radiating ducts are those located at the lower corners of the system; next are those nearest to the outside of the system, and lastly the middle and top ducts which not only take up heat from the lower cables, but must also dissipate heat through adjoining ducts.

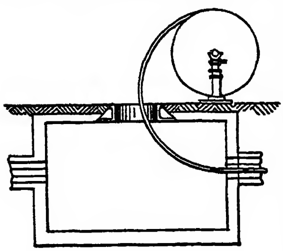

Fig. 5

The next step is to set the cable reel on the shaft of a pair of wheels of slightly greater diameter than the reel itself or on jacks and raise it slightly above the ground, taking care to locate it as shown in Fig. 5, so that the cable will unreel into the manhole without making a reverse bend.

The pulling rope having been left in the duct after rodding, the cable is unreeled sufficiently to bring its end close to the mouth of the latter and a wire pulling grip (Fig. 6) is drawn over its end. Several companies prefer to use pulling eyes soldered to the conductors and sheath of the cable. The end of the grip or pulling eye is hooked to the rope and the rope pulled from the other end. The pulling may be done by capstan, winch, motor truck, horse, or by hand, depending upon the size and amount of cable to be pulled and upon local conditions. The cable should be carefully guided into the duct so as to avoid sharp bends and abrasions, and a small quantity of grease, about 100 lb. per mile, may with advantage be spread over the cable as it enters the duct. It is also necessary to cover the edges of the duct with pieces of lead or other suitable material to prevent abrasion of the cable.

Fig. 6

PROTECTION OF CABLES IN SPLICING CHAMBERS. Wherever there are several large cables in a splicing chamber, there is always danger that a burn-out of one cable will involve some or all of the remainder. Hence it is usual to protect such cables by means of one or more of the following methods:

1. Cement coating with $^1/_4$-in. rope bond.
2. Asbestos tape saturated with silicate of soda.
3. Asbestos tape covered with soft steel-tape armor.
4. Asbestos rope.
5. Special trade-marked products designed for this purpose.

68. CABLE JOINTS

The problem of cable splicing may be resolved in the following elements:

(*a*) Joining the conductors to retain their full carrying capacity and requisite tensile strength.

(*b*) Insulating the joint to resist both the normal radial stresses and the tangential stresses caused by the termination of the sheaths.

(*c*) Reducing tangential stresses as far as practicable in high-tension joints.

(*d*) Keeping out air and moisture.

(*e*) In oil-filled cables, to provide (1) an oil barrier, (2) a continuous channel for the oil, (3) suitable oil passages for connecting the cable oil channels to an external reservoir.

Conductors

The joining of conductors is a standard procedure, involving the cleaning and "tinning" of the conductors with stearine flux and 50/50 solder. The conductor ends thus prepared are inserted into standard N.E.L.A. connectors of the dimensions shown in Table XXXVIII. Solder, 50/50, is then ladled over the connector until all interstices are filled, and then all surplus solder is wiped or filed off, so as to leave the connector and abutting conductors free of roughness or projections. In the case of sector conductors, it is usual to hammer the ends into cylindrical form and use standard connectors, of the next larger size in the table.

Insulation

The next step is to prepare the cable insulation surface to receive the joint insulation. Except in certain special designs, the insulation is penciled down from its full diameter to approximately the diameter of the connector, and the short spaces ($^1/_4$ to $^1/_2$ in.) between the penciled ends and the connector filled with insulating tape of the same material, but much narrower, as is used for the joint insulation. The length of the bared insulation

Tables XXXVIII. Nominal Dimensions Straight Tinned Copper Connectors

Conductor Size	Inside Diameter, mils	Outside Diameter, mils	Wall Thickness, mils	Slot Width, mils	Length, inches
8	151	201	25	30	1 1/2
7	169	225	28	30	1 1/2
6	189	251	31	30	1 1/2
5	211	281	35	30	1 1/2
4	237	315	39	30	2
3	265	353	44	30	2
2	297	395	49	30	2
1	337	449	56	70	2
0	378	504	63	70	2
00	423	565	71	70	2
000	475	635	80	70	2
0000	533	713	90	70	2 1/2
250 M	581	777	98	120	2 1/2
300 M	635	849	107	120	2 1/2
350 M	690	920	115	120	2 1/2
400 M	740	986	123	120	3
450 M	784	1046	131	120	3
500 M	826	1102	138	120	3
550 M	868	1154	143	175	3
600 M	906	1206	150	175	3 1/2
650 M	948	1260	156	175	3 1/2
700 M	983	1307	162	175	3 1/2
750 M	1018	1356	169	175	3 1/2
800 M	1052	1400	174	175	4
850 M	1083	1441	179	220	4
900 M	1115	1483	184	220	4
950 M	1145	1525	190	220	4
1000 M	1175	1565	195	220	4 1/2
1250 M	1320	1754	217	220	4 1/2
1500 M	1440	1912	236	280	5
1750 M	1560	2074	257	280	5 1/2
2000 M	1664	2214	275	280	6
2500 M	1855	2455	300	280	6 1/2

commonly used is shown in Fig. 7. The penciling is usually about 0.2 in. in length per 64ths inch of thickness. Some engineers prefer to have the insulation stepped, rather

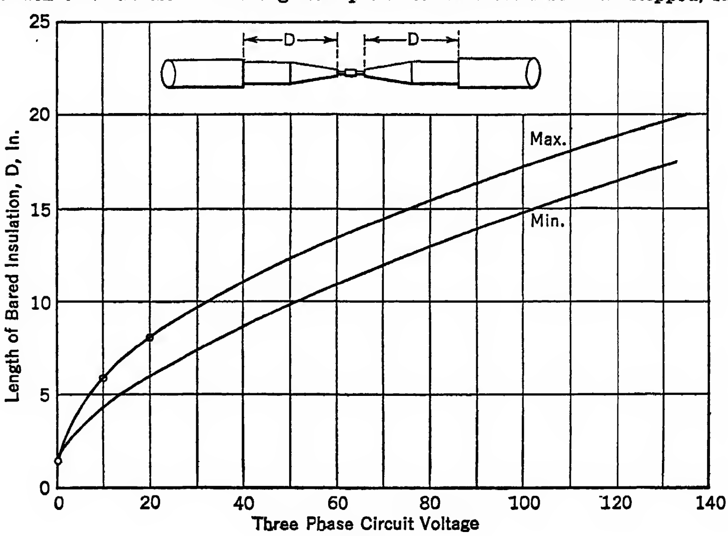

FIG. 7

than penciled gradually, in which case the slope may be somewhat steeper, as the unequal stretching of the tape does not have to be considered.

The main insulation of a joint consists of tape applied lapped, in cigar form, from beyond the ends of the penciling on one side, to a corresponding point on the other, with a maximum thickness over the connector. This maximum is usually about 40 per cent more than the thickness of cable insulation.

For rubber-insulated cables, rubber tape is used. Both for varnished cambric and impregnated paper, it is common to use bias-cut varnished-cambric tape. For high-voltage cables, say 11,000 volts and over, the varnished-cambric tape is generally of the low dielectric loss type. The usual thicknesses of varnished-cambric tape are 7, 10, or 12 mils. Rubber tape up to 25 mils thick is used. Care must be taken not to stretch the tape unduly, lest its dielectric strength be impaired.

Some power companies use impregnated-paper tape, usually 5 or 6 mils thick, and in a few instances, wide rolls of paper with converging sides are used, the purpose of the convergence being to make the paper roll up with conical ends.

The higher-voltage cables generally begin the slope of the tape at the end of the lead and the slope is covered with a flared metallic shield, usually of copper gauze tape.

Where the cable insulation is shielded, it is usual to carry the shield over the entire joint insulation. The proper slope of shield has much to do with the reduction of tangential stresses.

The design of joints from the standpoint of stress is based on the formula for the capacity between concentric cylinders and on the fact that the total voltage divides inversely as the capacities. By means of these relations the voltages at the various layers may be calculated and the axial potential drops calculated therefrom.

It is necessary to keep air and moisture out of a joint, and to this end, it is customary, with either varnished cambric or paper tape, to apply a liberal amount of oil or compound as each layer of tape is being applied. The insulation is finally " boiled out " by pouring hot compound over it.

Compound

When the joint is equipped with its sleeve, it is filled with compound of one or other of the following types in accordance with Table XXXIX.

A, insulating oils, that remain liquid over the operating temperature range encountered and which migrate into cable.

B, insulating greases and viscous compounds (not an asphaltic compound) having melting points within the operating temperature range and which migrate (more or less) into cable.

C, plastic and hard compounds having melting points above the maximum operating temperature encountered and which do not migrate into cable.

Table XXXIX. Recommended Types of Joint-filling Compounds Standard Walls of Insulation and Lead

	0–18 Kv	19–23 Kv	24–34.5 Kv (Note 3)	Above 34.5 Kv (Note 3)
Belted 3-conductor cable...	*C*	......	....	....
Shielded 3-conductor cable.	*C*	*C*	*A*, for underground *B*, for underground (Note 1) *C*, for submarine or aerial	*A*
Single conductor cable....	*C*	*C*	*A*, for underground *B*, for underground (Note 1) *C*, for submarine or aerial	*A*, for underground *C*, for submarine (Note 2)

Notes: For heavier walls of insulation than standard the limits of Class *C* compound may be raised 10 percent.

(1) With proper servicing.

(2) For submarine use, a full length of cable without joints is obviously first choice. Where piers for bringing ends out of water are feasible, oil-filled joints are recommended. Where this is not possible, solid filled joints (fully shielded) can be used.

(3) There has been limited experience in this country with joints filled with Class *C* compounds as follows:

Shielded, 3-conductor cable up to 34.5 kv.

Single conductor cable, up to 46 kv.

When further experience is available, a modification of this rule may be possible.

Coverings

Lead-sheathed cable joints are usually enclosed in lead sheaths, but very long joints sometimes have brass or copper sleeves. The sleeve diameter is generally based on a clearance of about $^1/_4$ to $^1/_2$ in. over the joint insulation.

The lead sleeve is slipped over the cable before the conductors are joined and is brought into position when the structure of insulation and shielding has been completed. The ends are beaten down to meet the cable sheath and joined thereto by wiping with 45 tin/55 lead solder.

Rubber cables with braid covering do not have lead sleeves on their splices but are finished either with "anhydrous tape" or friction tape, over which asphaltic compound is poured. Low-voltage rubber joints are often made without other compound, but high-voltage rubber-lead cables always have their sleeves filled with plastic compound.

Reservoirs

Experience has shown that insulating greases and viscous compounds *B* cannot be used satisfactorily as a filling medium without reservoirs or other means of maintaining pressure and supplying surplus compound. This type of filling material does not act as a definite seal and allows the entrance of moisture and air when accidental leakage occurs in the outer metallic casing. Also, these compounds have a tendency to migrate into the cable, causing vacuum and collapse of the joint casing. For these reasons, this filling material is not recommended unless accompanied by reservoir, or servicing, to maintain surplus supply and positive pressure. These reservoirs usually have about 1 to 1 $^3/_4$ gallons capacity of working oil.

Oil-filled Cables present special problems of their own, which are treated in *Trans. A.I.E.E.*, 1928, p. 200, and *Elec. World*, 1929, p. 97.

The success of joint making for high voltages depends on careful design with respect to potential distribution, mechanical features, and care in workmanship to avoid leaving dirt, air pockets, sheath leaks, and rough edges on conducting parts.

69. FAULT LOCATION

Faults may be classified as follows:

(*a*) Conductors not burned apart.

1. Solid ground on one or more conductors with one clear conductor.
2. Solid ground on all conductors.
3. Short or cross between two or more conductors with one clear conductor.
4. Short or cross between two conductors with one conductor grounded.
5. Short or cross between all conductors and ungrounded.

(*b*) One or more conductors burned apart.

1–5. Conditions 1–5 inclusive (see above) from either end of cable.

6. All conductors burned off and clear of grounds.

The general procedure followed in locating a fault varies somewhat with local conditions, but is typified by the following:

Failure
|
Sectionalization as far as possible
|
Test Analysis by above classification
|
Fault Reduction, as required

1	2	3	4	5
Inspection	Cut and Test until Localized	Tone Test	Loop Test	Capacity Test
		Final Cut and Test	Tone Test	Cut and Test for Approximate Location
			Final Cut and Test	Final Cut and Test

In short cables, the fault is usually located by inspection, i.e., looking for smoking manholes or listening for crackling sounds when the kenotron is applied to the faulty cable.

The final step in every fault hunt is a cut and try, although ordinarily the fault is found at the first cut.

The two major methods of locating circuit faults are the **tracing current** or **tone test method** and the **bridge method.** The former is more generally useful and certainly more accurate, although the latter is quite indispensable for long transmission lines.

TONE TEST METHOD. The tracing current method depends on the Lundin, or some similar, fault locator, which includes three main elements, the interrupter, the analyzer, and the pick-up or listening equipment.

The interrupter uses a 110-volt alternating current source of 50-amp capacity and applies current of suitable voltage to the cable and analyzer in characteristic periodic impulses, which may be picked up along the cable by the listening equipment.

The analyzer, with its reactive loading coils, indicating lamp, and step-up transformers, enables an experienced operator to obtain a description of the fault.

The procedure of analysis is as follows:

The inductance coils of the analyzer are connected in series with the conductor to be studied. If this conductor is open and clear of ground, the current, in passing through the inductive loading coils, will produce a voltage drop across the capacitative conductor, which is greater than the applied voltage. If, however, there is a high-resistance fault, the voltage may be equal to or less than the applied voltage. Where a solid ground exists, the circuit voltage is reduced nearly to zero. These voltage conditions are tested by means of a lamp which is bright, dim, or extinguished according to which of the conditions exists.

A partial ground is usually reduced by the application of sufficient voltage, in order to facilitate the locating process.

The locating depends upon an abrupt decrease or change in the signal at a certain point along the cable, to indicate that the fault has been passed. A pick-up coil with telephone receiver is used for detecting "grounds."

In the case of triplex cables, if the detector is moved circumferentially around the cable, it will sometimes be near and sometimes away from the conductor carrying the tracing current. The resultant variation in loudness, called the "hump effect," always indicates that the fault is farther on. Sheath current produces no hump effect. The change from humped to uniform sound around the cable gives the precise location of the trouble. If the trouble is in a duct, it is usual to check the location by "ringing" the lead at the near joint and checking again with the earphone and by trying for a spark across the cut.

Another device is a bar listening coil, which is used on the surface of the ground, and which, up to the fault, gives a louder sound with the bar vertical than when horizontal, and beyond the fault gives a louder sound with the bar horizontal than when vertical.

In some cases, a large listening coil is used on an automobile which is driven along the route of the cable until the sound changes.

In single-conductor cables, where there is no "hump effect," the stray sheath currents beyond the fault will make locations uncertain. In such cases a heavy copper strap is used to divert current from the sheath locally; the exploring coil being applied between the sheath and conductor current when the fault is beyond, the signal will be strengthened, and when it is behind, it will be weakened, as the shunt is applied.

The usual cable fault is at its lowest resistance when a current of 1 to 5 amp is flowing through it. Less current will allow the fault to cool, and more, to burn up, either tending to increase the resistance. If the pilot lamp flickers when starting to reduce the fault at 1100 or 2200 volts, it is probable that there is moisture at the break. In such cases it is usual to resort to the bridge method.

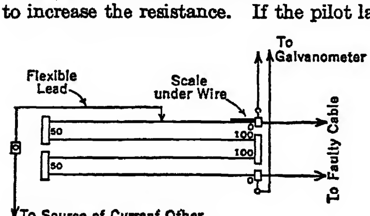

FIG. 8. Murray Loop

BRIDGE METHOD. The bridge method, moreover, is much used for transmission lines because the tracing current method is necessarily slow on long lines and because high-voltage cable faults are often hard to reduce to low enough resistance to be located by a tracing current.

The bridge uses the Murray loop as shown in Fig. 8. It consists essentially of a resistance wire graduated into two equal scales each reading from 0 to 100, the zero points being at the ends of the wire. A galvanometer is connected across the zero points, and a flexible lead connected to the positive terminal of a source of direct current is put

in sliding contact with the wire. The negative terminal of the d-c source is grounded. A protective gap is often placed across the galvanometer terminals to protect against surges which occur when a sudden change in fault resistance occurs. The usual source of d-c supply for a bridge test is a motor-generator set giving about 1500 volts.

Capacity tests for open-circuited conductors without ground are usually made by measuring the charging current with an a-c milliameter.

Trouble in buried series lighting cable is usually located by tests at the lamps. The section having been found in this way, the location in the section is found by a bridge test, using a return conductor above ground.

The usual time required to locate a service fault in a cable in a duct line is about 2 to 3 hours. Test failures may take about an hour longer.

LOCATION OF FAULTS IN NON-METALLIC SHEATH TYPE CABLES FOR SERIES LIGHTING. Such cables are usually buried back of the curb or under pavements, making inspection impossible.

Grounds can be located in a section between adjacent lights by noticing which lamps do not burn properly. After the ends of the faulty section of cable are exposed at the lamp posts, a loop test with portable battery energy can be used for approximate localization. It is necessary to lay a temporary return wire along the ground between the exposed cable ends. Then the cut and test method must be used for final localization. Faults can generally be found within 10 ft of loop test locations.

In the case of an open circuit, the cables being shielded by the ground, the electrostatic field will not extend outside the sheath. However, the lamp filaments, being unshielded, may still be used as sources of sound for a Lundin detector. By this means opens may be located between lamps. The exploring coil may occasionally be used to trace opens, but here, the entire current from the station being charging current, the signal dies out uniformly to the point where the open exists. Usually, the trouble hunter can get to within 200 ft of the open before the signal becomes inaudible.

In some of the recent types of series circuits with buried cables and buried insulating transformers at the base of the lighting standards, the electrostatic field does not extend to the secondaries and lamp filaments. In this case, however, the charging current of the cable up to the fault is transformed and circulated through the lamp filaments. For this condition, a special coil, vertically mounted, is attached to the end of a fish pole, and connected to the earphones or amplifier, which picks up the sound at the lamp. This satisfactorily locates the fault between two lamps. More precise location may sometimes be made with triangle and amplifier from the surface of the ground, if the fault is solid; otherwise digging will be necessary.

If the circuits do not enter the stations, it is necessary to move a portable set to the location to analyze and apply tracing current to the faulty circuit.

70. WEIGHTS OF INSULATED WIRES AND CABLES

So many variables enter into the weight of an insulated wire or cable, and so many forms are in use, that it is impossible to give comprehensive tables here. The weight of a cable may be calculated from its dimensions by finding the sum of the weights of the conductors and the weights of the insulation, braid, tape, sheath, etc. The weight of the conductor may be found in the wire tables under Bare Wires and Cables. The weight of the insulation, braid, tape, or sheath may be found by calculating the cross-section of each of these materials and multiplying by the length of the cable and a factor proportional to the density of the material; see Table XL.

The cross-section of a tube having an internal diameter d and thickness t is $\pi t(d + t)$. When the diameter and thickness are in inches and the specific gravity of the material forming the tube is δ, then the weight in pounds per 1000 feet of tube is

$$W = 1362\ \delta t(d + t)$$

The values of the specific gravity δ, the product 1362 δ, and the weight per cubic inch for the common materials used in the construction of cables are given in Table XL. From this relation the weight per 1000 ft of any tubular (circular cross-section) layer of insulation, braid, tape, or sheath may be readily calculated. The formula is, therefore, directly applicable to single-conductor tables.

The weight of duplex or triplex cables is calculated as for a group of single-conductor cables, except with regard to the fillers. The cross-section of the filling material is most readily calculated by subtracting from the cross-section of the entire cable the cross-sections of the individual conductors and the tubular insulation, sheath, etc. The diameter of the circle circumscribing three round insulated conductors of diameter d is 2.15 d.

The weight of separators in pounds per 1000 ft is usually between 5 and 10 times the mean diameter, expressed in inches, of the conductor and separator.

The weight of saturated braids per 1000 ft is from 12 to 30 times the diameter over the insulation, expressed in inches. The corresponding multiplier for 12-mil rubber-filled cotton tape with 1/4 lap is 24.

Table XL. Weights of Cable Insulation

Material	Specific Gravity, δ	Pounds per Cubic Inch, $0.03613\,\delta$	$1362\,\delta$
Rubber compound, 30 per cent para:			
Organic base	1.2	0.0434	1635
Mineral base	1.5 to 2.0	0.054 to 0.072	2043 to 2724
Varnished cloth	1.11	0.401	1515
Paper, impregnated	1.17	0.0423	1594
Lead	11.37	0.411	15,530
Braid, untreated	1.11	0.0402	1515
Braid, saturated	1.33	0.0480	1809
Tarred jute, in cable	0.63	0.0228	858
Dry jute, in cable	0.267	0.00965	364
Dry hemp, in cable	0.267	0.00965	364
Gutta-percha	1.0	0.0361	1362
Rubber-filled tape	1.0	0.0361	1362
Copper	8 89	0.3212	12,108

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WIRING OF BUILDINGS

By George W. Zink

72. GENERAL REQUIREMENTS

Wires and fittings designed to conduct electricity in a building should be selected as to size and insulation and installed in such a manner that: (1) the attending fire risk and the possibility of an electric shock to the inhabitants shall be a minimum; (2) the electric power efficiency of the system shall be reasonable; (3) the voltage at the receiver shall approximate the rated voltage of the receivers and shall remain sensibly constant; (4) the mechanical arrangement of the system shall be simple and convenient for inspection and use; (5) the conductors shall be mechanically protected from external injury; (6) the service shall not be interrupted under normal load; (7) the cost of the materials, labor of installation, and replacement due to depreciation shall not be excessive, and (8) the entire wiring system shall conform to the rules and regulations of any authority having jurisdiction over the building in question.

73. SYSTEMS OF WIRING

For d-c and single-phase distribution the two-wire system and the three-wire system are employed. For two-phase distribution either three or four wires are used. For three-phase distribution three wires are usually employed, although a fourth or neutral wire is sometimes installed.

D-C AND SINGLE-PHASE DISTRIBUTION. In d-c or single-phase a-c circuits, the two-wire or the three-wire system is used. The two-wire system is used for the greater part of interior wiring, the three-wire system being used chiefly for feeders and mains. (See Arts. 1 and 11.) The neutral wire of a three-wire system is usually of the same size as either outside wire, the saving in copper over the two-wire system then being $^5/_8$ or 62.5 per cent. In some cases, buildings are wired with the three-wire system in which the neutral wire is made twice the size of either outside wire so that, if necessary, the system may be operated either as a two-wire or a three-wire system; in the former case the two outside wires are connected in parallel. Power may then be supplied to the building from a local two-wire source of supply (isolated plant in the building) or from an emergency three-wire street service.

In this case wires of such size would be used as would give normal efficiency and regulation when used as a two-wire system; the loss when used as a three-wire system would then be only half as great.

The first cost of a three-wire system may not be less than that of a two-wire system because of the increased cost of the fittings, insulation, and the labor of installation. The

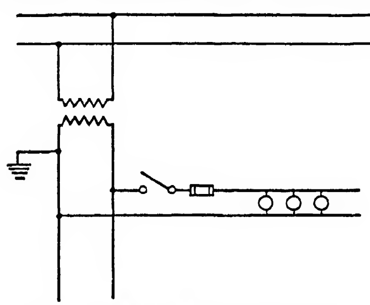

Fig. 1. Single-phase Two-wire System

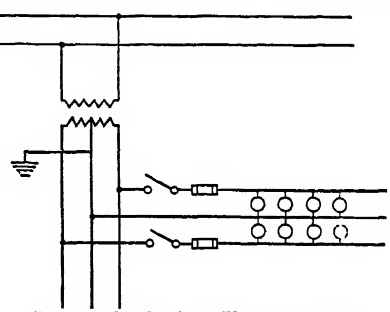

Fig. 2. Single-phase Three-wire System

three-wire system is not as simple as the two-wire system, and is subject to more disturbances unless the load is kept balanced. Some electric power companies limit the power which they will supply to a two-wire system, and in such cases in new installations buildings taking power above the two-wire limit must be wired with the three-wire system. The two-wire system is used for either lighting or power loads; the three-wire system, for either lighting or mixed lighting and power loads, motors in the latter case being connected between the outside wires. (See Figs. 1 and 2.)

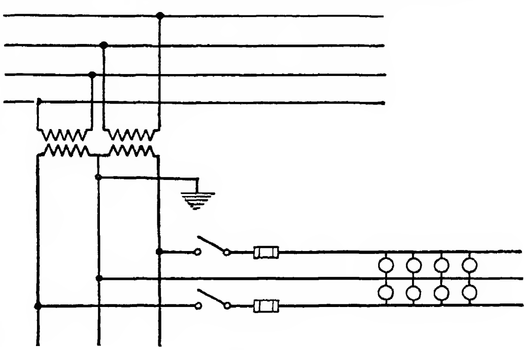

Fig. 3

TWO-PHASE DISTRIBUTION. For distributing two-phase alternating currents, either three or four wires are employed. A four-wire two-phase system may be treated as two separate two-wire systems, which cannot in any case be connected in parallel. A single wire 41 per cent larger than either of the wires it displaces may be substituted for any two of the wires of the four-wire two-phase system thus making a three-wire two-phase system. Either three or four wires may be used for two-phase lighting or power loads.

All the three or four wires are used for power loads, the motors being connected to

both phases. Lighting loads are connected to each phase, in the three-wire system, one wire being common to both phases. It is important that the phases are kept in balance by maintaining equal loads on each side of the circuit.

THREE-PHASE DISTRIBUTION. This system is the most usual one for power distribution and for mixed power and small lighting loads. Where power is the only load, a

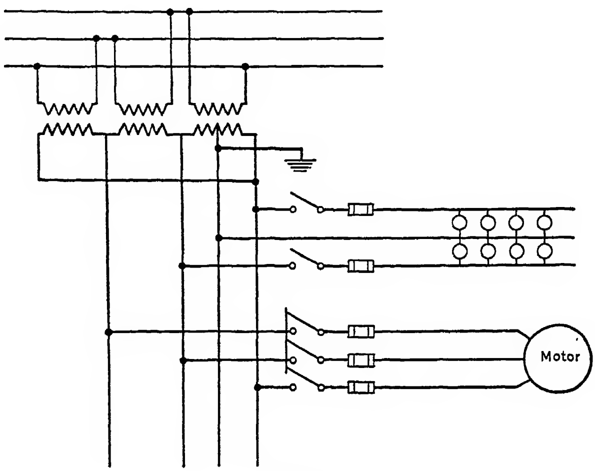

Fig. 4

three-wire system is used. If a small lighting load is required on a three-phase, 230-volt system, one phase may have a neutral tap and a three-wire lighting circuit obtained. This, however, will tend to throw the system out of balance and can be used only for small loads. (See Fig. 4.)

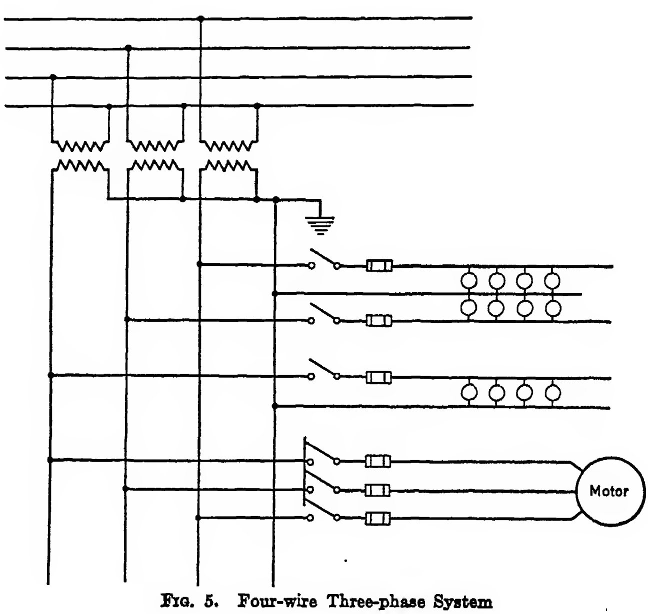

Fig. 5. Four-wire Three-phase System

Four-wire Three-phase System. This system is in general use where the power load is subordinate to the lighting load. In this case, the fourth wire is the neutral and usually grounded, and there should be no fuse in this wire in any part of the system. Branch lighting circuits may be run either two-wire, using one outside wire and the neutral; or three-wire, using two outside wires and the neutral. Care must be taken to distribute the lighting load so as to keep all phases of the system in balance. Power loads are connected to the three outside wires. This system is usually 120–208 volts, that is, 120 volts from any outside wire to neutral and 208 volts between the outside wires. (See Fig. 5.)

74. STANDARD VOLTAGES FOR LIGHTING AND POWER

The standard voltages used in connection with the wiring of buildings are usually 115–230–460–550 or 2300 volts. The National Electrical Code does not permit voltages above 7500 for general wiring of buildings. Higher voltages, however, are sometimes used for service connections from the power company's distributing lines to a transformer vault within the building. These cases are considered extensions of the street mains and not general wiring.

The usual lighting circuits are normally 115 volts, although in industrial plants 230 volts are sometimes used. Series lighting or constant-current systems use higher voltages, but this type of lighting is not permitted by the National Electrical Code for installations indoors except in a few restricted locations.

The voltage of power circuits is determined by the character of the load, distance from the center of distribution, and other local conditions. For plants with small and medium-sized motors, 230 volts are most frequent, and where large motors or other units of large capacity are required, as in steel mills, voltages of 2300 or higher are commonly used.

75. STANDARD FREQUENCIES

Throughout the United States, alternating current for light and power is usually supplied at a standard frequency of 60 cycles. Some isolated plants use other frequencies such as 25 or 40 cycles, and 50 cycles is a standard frequency abroad. Transformers, motors, and other equipment are regularly furnished for three-phase 60-cycle current in all the standard voltages; equipment for other frequencies is not readily available. For lighting circuits, frequencies less than 60 cycles are not satisfactory, as the lamps have a noticeable flicker.

76. AUTHORITIES GOVERNING THE INSTALLATION OF WIRING

Before proceeding with a wiring layout, it is necessary to become familiar with the laws and rules governing the installation of wiring and electrical equipment in the locality where the work is being done. In general, the authorities having jurisdiction are (1) the National Board of Fire Underwriters, (2) the local municipal authorities, and (3) the utility company supplying the current.

The National Electrical Code was originally issued in 1897 by a group of interested organizations in an effort to establish rules, governing the construction and installation of electrical equipment, which would reduce fire hazard to a reasonable minimum. The Code is now sponsored by the National Fire Protection Association, under the rules of procedure of the American Standards Association, and is published and distributed by the National Board of Fire Underwriters.

The Code is revised every two years by the Electrical Committee of the National Fire Protection Association and issued so as to become effective, usually, on November 1, of the odd years. If progress in the art or other considerations warrant it, interim revisions or additions may be made at any time.

The National Electrical Code rules are drawn up primarily in connection with fire insurance risk, but recently they have been giving consideration to accident risk as well. Accident risks, however, are more specifically covered by municipal regulations and various safety codes.

UNDERWRITERS' LABORATORIES, INC. It is to be noted that the Code does not specify test requirements for equipment, but refers to "approved" devices or apparatus. This means approved by the Underwriters' Laboratories, Inc., an organization established to examine and test materials and equipment to determine their fitness for use under the Code. Detailed specifications for the construction and performance under test

and in service of approved electrical fittings and material will be found in the Standards of the Underwriters' Laboratories.

The "List of Inspected Electrical Appliances" is published semiannually by the Underwriters' Laboratories, and all "listed" materials bear some evidence of their approval. Permission must be obtained to use unlisted material.

Copies of the National Electrical Code and the List of Inspected Electrical Appliances may be obtained from most local inspection bureaus or from either the National Board of Fire Underwriters, 85 John Street, New York, N. Y., or the Underwriters' Laboratories, 109 Leonard Street, New York, N. Y. (Chicago address: 207 East Ohio Street).

MUNICIPAL REGULATION. Most states and municipalities have laws concerning the installation of electric wiring, and these laws usually include the National Electrical Code with such modifications as considered advisable to suit local conditions. These modified rules are often more rigid than the Code in that they prohibit certain practices recognized by the Code. For example, concealed knob and tube wiring is one method of wiring recognized by the Code, but prohibited by most municipalities. Local regulations are often broader than the Code in that they cover accident risk, inspection, licensing of electricians, and penalties for violations. Municipal authorities should be consulted with regard to obtaining copies of local regulations.

REGULATIONS OF THE UTILITY COMPANIES. Most lighting and power companies have rules which must be observed by consumers obtaining service from their lines. These rules are made necessary by conditions pertaining to the operation of their particular systems and usually concern the location of meters, the type and location of service entrances, and, in the case of power loads, the type and size of motors, etc.

ENFORCEMENT AND INSPECTION. The National Electrical Code, as issued by the National Board of Fire Underwriters, has in itself no legal status. However, inasmuch as the Code rules form part of ordinances of most cities throughout the country, it has almost universal enforcement by the police power of such municipalities. Where not so enforced, insurance companies require the Code to be observed if the buildings in question are to be insured. In order to ensure the observance of the Code, the Fire Underwriters and most municipalities have inspectors to check the electrical work in buildings. These two inspection agencies may act independently, but often they cooperate and have one office handle both inspections. Before a new building is occupied, and when the occupancy of an old building is changed, it is advisable to be sure that proper inspection has been made.

77. METHODS OF WIRING

The National Electrical Code recognizes a number of methods of wiring, one or more of which must be selected for any wiring layout. Inasmuch as certain types of wiring recognized by the Code are not permitted in some municipalities, this should be checked before deciding upon any particular method. The methods listed by the 1935 Edition of the Code are as follows:

(1) Installed on insulators.
- (*a*) Open wiring.
- (*b*) Concealed knob and tube.

(2) The "pull-in and pull-out" systems.
- (*a*) Conduit work—rigid or flexible metallic conduit.
- (*b*) Electrical metallic tubing.
- (*c*) Wireways and busways.
- (*d*) Underfloor raceways.
- (*e*) Cast in place raceways.

(3) Cable assemblies.
- (*a*) Armored cable.
- (*b*) Non-metallic sheath cable.
- (*c*) "Non-metallic" wiring systems for use in wet places.

(4) Extension materials.
- (*a*) Surface metal raceways.
- (*b*) Surface wooden raceways.
- (*c*) Under plaster extensions.
- (*d*) Non-metallic surface extensions.

(5) Special feeders.
- (*a*) Bare bus-bars and risers.
- (*b*) Service entrance conductors.

OPEN WIRING. This method is used extensively where the appearance of exposed wiring is not objectionable, as in industrial buildings, and where not exposed to mechanical injury. It is a satisfactory method in buildings or rooms subject to moisture, corrosive vapors, or heat, such as paper mills, dye houses, or dry kilns. Its use is permitted for voltages up to 600 volts.

In installing, single-conductor wires are supported on knobs or cleats. Wires shall never be fastened with staples. For voltages not exceeding 300 volts, wires shall be separated 2 1/2 in. from each other and 1/2 in. from the surface wired over. For voltages from 301 to 600, the wires shall be separated 4 in. and at least 1 in. from the surface wired over. In damp places, 1-in. separation from the surface wired over must be maintained for all voltages. Wires must be supported at least every 4 1/2 ft, except that in buildings of mill construction wires No. 8 A.W.G. and larger may be separated 6 in. and run from timber to timber with supports on each timber.

Where exposed to mechanical injury, the wires must be protected by suitable guard strips—boxing or other means approved by the National Electrical Code. Wires passing through floors, walls, timbers, or partitions shall be protected by insulating tubes or bushings. Insulating tubes must be used over the wire where within 2 in. of piping or other conducting material. Open wiring must not be located in hoistways.

Wire suitable for dry locations are approved rubber-covered (Type R), slow-burning (Type SB), slow-burning weatherproof (Type SBW), varnished-cambric (Type VC), or asbestos-covered (Type A).

In damp places or buildings especially subject to moisture, rubber-covered wire must be used.

Wires subject to corrosive vapors must be rubber-covered, varnished cambric, or weatherproof.

Knobs must conform to the following minimum dimensions and have a body designed to maintain the wire at least 1 in. from the surface wired over.

Size Wire A.W.G.	Size of Base			Solid Knobs Groove, Inches		Thickness of Caps and Split Knobs inches from top of groove
	Diameter of Circular Knobs	Single Wire Cleats and Square Knobs		Depth	Diameter	
		Width	Length			
14–10	1 1/8	3/4	1 3/4	3/16	1/4	3/8
8–4	1 1/2	7/8	2	5/16	5/16	5/8
2–2/0	2	1	2 1/4	7/16	5/8	5/8
3/0–300,000 cir mils	2 1/2	1 1/8	2 3/4	7/16	25/32	7/8
400,000–1,000,000 cir mils	3	1 3/8	3 3/4	5/8	1 1/4	1

Multiple wire cleats for voltages up to 300 may be used if designed to maintain the separation of wires previously mentioned.

Approved tubes and bushings conform to the following minimum dimensions:

Diameter of Hole	External Diameter	Thickness of Wall	External Diameter of Head	Length of Head
5/16	9/16	1/8	13/16	1/2
3/8	11/16	5/32	15/16	1/2
1/2	13/16	5/32	1 3/16	1/2
5/8	15/16	5/32	1 5/16	1/2
3/4	1 3/16	7/32	1 11/16	5/8
1	1 7/16	7/32	1 15/16	5/8
1 1/4	1 13/16	9/32	2 5/16	5/8
1 1/2	2 3/16	11/32	2 11/16	3/4
1 3/4	2 9/16	13/32	3 1/16	3/4
2	2 15/16	15/32	3 7/16	3/4
2 1/4	3 5/16	17/32	3 13/16	1
2 1/2	3 11/16	19/32	4 3/16	1

CONCEALED KNOB AND TUBE. This type of wiring is used in buildings of frame construction where economical concealed wiring is desired. Most cities and towns prohibit this method of wiring, and its use therefore is restricted largely to isolated rural dwellings. It cannot be used for circuits of more than 300 volts between wires or 150 volts to ground.

The method of installing consists of running single insulated wires in the hollow spaces of walls and between floors. Where run parallel to timbers or floor, the wires are supported on single wire insulators or knobs and where passing through timbers or floors by insulating tubes. Wires must be separated at least 5 in., and where this spacing cannot be maintained, as in entering outlet boxes, it shall be encased in flexible tubing which shall extend from the last knob into and be secured to such boxes. The requirements as to supporting, clearances, etc., which are specified for open wiring, apply also to knob and tube.

Approved rubber-covered wire (Type R) is required.

CONDUIT WORK. The best and most extensively used type of wiring consists of wires enclosed in metallic conduit, either rigid or flexible. Rigid conduit is standard for general work; flexible conduit is frequently used for short runs such as from the end of a rigid conduit to a motor mounted on a machine frame. Conduit is employed in all types of buildings and may be run exposed or concealed and may be used for all voltages.

Rigid conduit is standard steel pipe, protected against corrosion by galvanizing, sherardizing, enameling, singly or combined—or pipe of non-corrosive metal may be used. It is regularly furnished in 10-ft lengths and cannot be used in sizes smaller than 1/2 in. The system must be installed complete from outlet to outlet and be securely fastened in place. The wires shall not be pulled in until all mechanical work on the building is completed.

Numerous designs of fittings are made for joining or terminating the conduit. These may be of the threaded or threadless type, and in either case must be installed to give adequate electrical continuity to the system, which must be grounded, as provided for by the Code.

A run of conduit between outlets or fittings must not contain more than four quarter bends, the radius of the curve of any field bend being not less than six times the internal diameter of the conduit.

Wires of different systems must not occupy the same conduit, outlet box, fitting, or cabinet. In a-c systems, all wires of the same circuit must be placed in one conduit; this practice should also be followed for d-c systems to provide for possible future change-over. For voltages above 600, wire must be installed as cable.

Wires in vertical conduits must be supported by means of suitable clamps or other approved means, such as deflecting the wires at 90 deg over insulating supports in junction boxes. The intervals of supports are as follows:

No. 14 A.W.G. to No. 1/0	not over 100 ft
No. 2/0 A.W.G. to No. 4/0	not over 80 ft
250,000 cir mils to 350,000 cir mils	not over 60 ft
350,001 cir mils to 500,000 cir mils	not over 50 ft
500,001 cir mils to 750,000 cir mils	not over 40 ft
Over 750,000 cir mils	not over 35 ft

Special vertical riser cables designed to be self-supporting may be used for feeders.

Wires in conduit should be rubber-covered (Type R) except as follows: Where the ambient temperature exceeds 120 deg fahr (49 deg cent), asbestos-covered (Type A) or slow-burning (Type SB) shall be used. Where permanently dry, and where the temperature of the insulation will not exceed 167 deg. fahr. (75 deg. cent) varnished-cambric (Type VC) may be used. With flexible conduit in damp locations, lead-covered wire (Type RL) must be used. With rigid conduit in permanently wet locations, underground, or in concrete slabs in contact with moist earth, lead-covered wire (Type RL) must be used.

The Tables I, II, III, and IV show the number and size wire permitted in various-sized conduits. Where other combinations are used, the combined cross-sectional areas of the several wires should not be more than the per cent of the interior cross-sectional area of the conduit shown in the following table.

	Per Cent of Internal Area of Conduit				
	Number of Conductors				
	1	2	3	4	over 4
Conductors not lead covered...	53	31	43	40	40
Conductors lead covered.......	55	30	40	38	35

Electrical Metallic Tubing. A form of rigid conduit having the same internal diameter as standard conduit, but with a much thinner wall and referred to as "electrical metallic tubing," may be used in the same manner as rigid conduit with certain restrictions imposed because of the lighter material. The tubing, unless of noncorrodible metal, must be protected with a rust-resisting coating. It is installed under the same rules as rigid conduit, except as noted below.

Table I. Standard Rigid Conduit
Made in 10-ft lengths

Standard Size of Pipe, in.	Internal Diameter, in.	External Diameter, in.	Nominal Weight per 100 ft, lb	Number of Threads per In. of Screw	Approximate Internal Cross-sectional Area, sq in.
1/2	0.62	0.84	85	14	0.30
3/4	0.82	1.05	112	14	0.53
1	1.04	1.31	167	11 1/2	0.86
1 1/4	1.38	1.66	224	11 1/2	1.50
1 1/2	1.61	1.90	268	11 1/2	2.04
2	2.06	2.37	361	11 1/2	3.36
2 1/2	2.46	2.87	574	8	4.79
3	3.06	3.50	754	8	7.38
3 1/2	3.54	4.00	900	8	9.90
4	4.02	4.50	1066	8	12.72
4 1/2	4.50	5.00	1254	8	15.95
5	5.04	5.56	1462	8	20.00
6	6.06	4.62	1897	8	28.89

Table II. Two-wire and Three-wire Systems

Size of Wire	Number of Wires in One Conduit								
	1	2	3	4	5	6	7	8	9
	Minimum Size of Conduit in Inches								
No. 14	1/2	1/2	1/2	1/2	3/4	3/4	3/4	1	1
12	1/2	1/2	1/2	3/4	3/4	1	1	1	1 1/4
10	1/2	3/4	3/4	3/4	1	1	1 1/4	1 1/4	1 1/4
8	1/2	3/4	1	1	1 1/4	1 1/4	1 1/4	1 1/4	1 1/2
6	1/2	1	1 1/4	1 1/4	1 1/2	1 1/2	2	2	2
5	3/4	1 1/4	1 1/4	1 1/4	1 1/2	2	2	2	2
4	3/4	1 1/4	1 1/4	1 1/2	2	2	2	2	2 1/2
3	3/4	1 1/4	1 1/4	1 1/2	2	2	2	2 1/2	2 1/2
2	3/4	1 1/4	1 1/2	1 1/2	2	2	2 1/2	2 1/2	2 1/2
1	3/4	1 1/2	1 1/2	2	2	2 1/2	2 1/2	3	3
0	1	1 1/2	2	2	2	2 1/2	3	3	3
00	1	2	2	2 1/2	2 1/2	3	3	3	3 1/2
000	1	2	2	2 1/2	3	3	3	3 1/2	3 1/2
0,000	1 1/4	2	2 1/2	2 1/2	3	3	3 1/2	3 1/2	4
200,000 cir mils	1 1/4	2	2 1/2	2 1/2	3	3	3 1/2	3 1/2	4
225,000	1 1/4	2 1/2	2 1/2	3	3	3 1/2	......	......	......
250,000	1 1/4	2 1/2	2 1/2	3	3	3 1/2	......	......	......
300,000	1 1/4	2 1/2	3	3	3 1/2	3 1/2	......	......	......
350,000	1 1/4	2 1/2	3	3 1/2	3 1/2	4	......	......	......
400,000	1 1/4	3	3	3 1/2	4	4	......	......	......
450,000	1 1/2	3	3	3 1/2	4	4 1/2	......	......	......
500,000	1 1/2	3	3	3 1/2	4	4 1/2	......	......	......
550,000	1 1/2	3	3 1/2	4	4 1/2	5	......	......	......
600,000	2	3	3 1/2	4	4 1/2	5	......	......	......
650,000	2	3 1/2	3 1/2	4	......	......	......	......	......
700,000	2	3 1/2	3 1/2	4 1/2	......	......	......	......	......
750,000	2	3 1/2	3 1/2	4 1/2	......	......	......	......	......
800,000	2	3 1/2	4	4 1/2	......	......	......	......	......
850,000	2	3 1/2	4	4 1/2	......	......	......	......	......
900,000	2	3 1/2	4	4 1/2	......	......	......	......	......
950,000	2	4	4	5	......	......	......	......	......
1,000,000	2	4	4	5	......	......	......	......	......
1,100,000	2 1/2	4	4 1/2	6	......	......	......	......	......
1,200,000	2 1/2	4 1/2	4 1/2	6	......	......	......	......	......
1,250,000	2 1/2	4 1/2	4 1/2	6	......	......	......	......	......
1,300,000	2 1/2	4 1/2	5	6	......	......	......	......	......
1,400,000	2 1/2	4 1/2	5	6	......	......	......	......	......
1,500,000	2 1/2	4 1/2	5	6	......	......	......	......	......
1,600,000	2 1/2	5	5	6	......	......	......	......	......
1,700,000	3	5	5	6	......	......	......	......	......
1,750,000	3	5	5	6	......	......	......	......	......
1,800,000	3	5	6	6	......	......	......	......	......
1,900,000	3	5	6	......	......	......	......	......	......
2,000,000	3	5	6	......	......	......	......	......	......

It shall not be used for circuits over 600 volts or for conductors larger than No. 4 A.W.G. Threadless fittings must be used throughout. It shall not be used in sizes larger than 2 in. or less than 1/2 in. electrical trade size, except in under-plaster extensions where 3/8 in. may be used. It cannot be laid in cinder concrete or fill, or where subject to severe mechanical injury.

Bushings, Bends, etc., in Rigid Conduit. All parts of the conduit system including outlet or junction boxes must be mechanically secured in place and must be electrically connected and grounded. At outlets or junction boxes rigid conduits must be provided with bushings or nipples to protect the wires from abrasion.

Table III. Three Conductors

Two Numbers and One Number		Size Conduit, in.	Two Numbers and One Number		Size Conduit, in.
14	10	3/4	000	400,000	2 1/2
12	8	3/4	0000	550,000	3
10	6	1	250,000 cir mils	600,000	3
8	4	1	300,000	800,000	3
6	2	1 1/4	400,000	1,000,000	3 1/2
5	1	1 1/4	500,000	1,250,000	4
4	0	1 1/2	600,000	1,500,000	4
3	00	1 1/2	700,000	1,750,000	4 1/2
2	000	1 1/2	800,000	2,000,000	4 1/2
1	0000	2			
0	250,000 cir mils	2			
00	350,000	2 1/2			

Table IV. Lead-covered Wires and Cables—600 Volts

Size of Wire	Size of Conduit to Contain Not More than Four Cables											
	Single-conductor Cable				Two-conductor Cable				Three-conductor Cable			
	1	2	3	4	1	2	3	4	1	2	3	4
	Cables in One Conduit				Cables in One Conduit				Cables in One Conduit			
14	1/2	3/4	3/4	1	3/4	1	1	1 1/4	3/4	1 1/4	1 1/2	1 1/2
12	1/2	3/4	3/4	1	3/4	1	1 1/4	1 1/4	1	1 1/4	1 1/2	2
10	1/2	3/4	1	1	3/4	1 1/4	1 1/4	1 1/2	1	1 1/2	2	2
8	1/2	1	1 1/4	1 1/2	1	1 1/4	1 1/2	2	1	2	2	2 1/2
6	3/4	1 1/4	1 1/2	1 1/2	1 1/4	1 1/2	2	2 1/2	1 1/4	2 1/2	3	3
4	3/4	1 1/4	1 1/2	1 1/2	1 1/4	2	2 1/2	2 1/2	1 1/2	3	3	3 1/2
3	3/4	1 1/4	1 1/2	2	1 1/4	2	2 1/2	3	1 1/2	3	3	3 1/2
2	1	1 1/4	1 1/2	2	1 1/4	2	2 1/2	3	1 1/2	3	3 1/2	4
1	1	1 1/2	2	2	1 1/2	2 1/2	3	3 1/2	2	3 1/2	4	4 1/2
0	1	2	2	2 1/2	2	2 1/2	3	3 1/2	2	4	4 1/2	5
00	1	2	2	2 1/2	2	3	3 1/2	4	2 1/2	4	4 1/2	5
000	1 1/4	2	2 1/2	2 1/2	2	3	3 1/2	4	2 1/2	4 1/2	4 1/2	6
0,000	1 1/4	2 1/2	2 1/2	3	2 1/2	3	3 1/2	4 1/2	3	5	6	6
250,000	1 1/4	2 1/2	3	3	......	......	......	......	3	6	6	.....
300,000	1 1/2	3	3	3 1/2	......	......	......	......	3 1/2	6	6	.....
350,000	1 1/2	3	3	3 1/2	......	......	......	......	3 1/2	6	6	.....
400,000	1 1/2	3	3	3 1/2	......	......	......	......	3 1/2	6	6	.....
450,000	1 1/2	3	3	4	......	......	......	......	4	6	6	.....
500,000	1 1/2	3	3 1/2	4	......	......	......	......	4	6	......	.....
600,000	2	3 1/2	4	4 1/2	......	......	......	......	......	......	......	.....
700,000	2	4	4	5	......	......	......	......	......	......	......	.....
750,000	2	4	4	5	......	......	......	......	......	......	......	.....
800,000	2	4	4 1/2	5	......	......	......	......	......	......	......	.....
900,000	2 1/2	4	4 1/2	5	......	......	......	......	......	......	......	.....
1,000,000	2 1/2	4 1/2	4 1/2	6	......	......	......	......	......	......	......	.....
1,250,000	3	5	5	6	......	......	......	......	......	......	......	.....
1,500,000	3	5	6	6	......	......	......	......	......	......	......	.....
1,750,000	3	6	6	......	......	......	......	......	......	......	......	.....
2,000,000	3 1/2	6	6	......	......	......	......	......	......	......	......	.....

The above sizes apply to straight runs or with nominal offsets equivalent to not more than two quarter-bends.

WIREWAYS AND BUSWAYS. In industrial plants where exposed work is not objectionable and a readily accessible low-voltage wiring system is desired, branch circuits are often run part of their length in wireways. These consist of sheet-metal troughs with hinged covers and made in convenient length so designed as to be securely fastened together. Convenient knockouts are provided in the sides and bottom so that extensions of conduit, tubing, armored cable, etc., may be run from any point. Rigid supports must be provided at least every 5 ft. Wireways may be used only in dry locations and for voltages not exceeding 600 volts.

The wires are placed, after the system is installed, by raising the cover and laying the wires in place. Not more than 30 wires may be placed in one wireway unless the excess are signal or control wires. No conductor larger than 500,000 cir mils shall be used, and the cross-sectional area of all cables shall not exceed 20 per cent of the area of the wireway.

Rubber-covered wire (Type R) and varnished-cambric (Type VC) are used, unless the ambient temperature exceeds 120 deg fahr (49 deg cent), in which case slow-burning or asbestos-covered wire (Types SB or A, respectively) shall be used.

Busways are similar in construction to wireways and governed by the same rules as to location and use. Instead of insulated wire, bare copper buses mounted on suitable insulators are used. Their chief application is for large feeders.

UNDERFLOOR RACEWAYS. In buildings of fire-resisting construction, metal or fiber raceways laid in concrete floors are extensively used for running branch circuits. This method is particularly adaptable to office buildings, where the location of partitions is frequently changed with a change of occupancy.

The use of this type of raceway is restricted to dry locations and to circuits where the voltage between wires or voltage to ground does not exceed 300 volts.

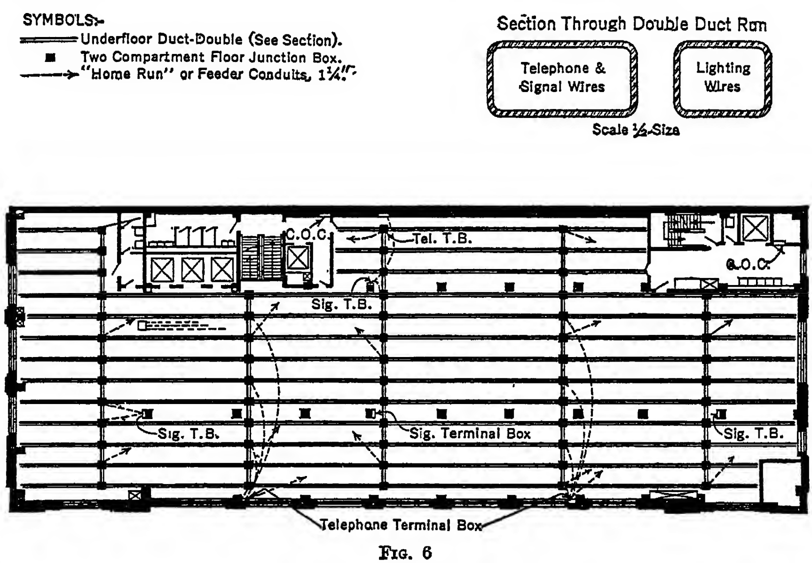

FIG. 6

Raceways are made in several styles, the metal ones usually being of rectangular section and the fiber ones of semicircular or oval section, and have conveniently spaced outlets formed in the duct. They are usually installed in the form of a grid covering an entire floor in such a manner that, no matter what the office layout may be, there will be an outlet convenient to any desk or other equipment.

The ducts are often run parallel in pairs, one for light and power wire and one for telephone or signal systems. Fig. 6 shows a typical floor layout of this kind.

Raceways may be placed in the concrete fill between the rough and finished floor, or in the slab, provided raceways of half-round section or of flat-top section not over 4 in. in width, shall have at least $^3/_4$ in. of concrete above the raceway, except that in office occupancies, metal flat-top raceways not over 2 in. in width may be laid flush with the concrete if covered with substantial linoleum or equivalent floor covering.

It shall not be used for circuits over 600 volts or for conductors larger than No. 4 A.W.G. Threadless fittings must be used throughout. It shall not be used in sizes larger than 2 in. or less than 1/2 in. electrical trade size, except in under-plaster extensions where 3/8 in. may be used. It cannot be laid in cinder concrete or fill, or where subject to severe mechanical injury.

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5	1	1 1/4	500,000	1,250,000	4
4	0	1 1/2	600,000	1,500,000	4
3	00	1 1/2	700,000	1,750,000	4 1/2
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00	350,000	2 1/2			

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	Cables in One Conduit				Cables in One Conduit				Cables in One Conduit			
14	1/2	3/4	3/4	1	3/4	1	1	1 1/4	3/4	1 1/4	1 1/2	1 1/2
12	1/2	3/4	3/4	1	3/4	1	1 1/4	1 1/4	1	1 1/4	1 1/2	2
10	1/2	3/4	1	1	3/4	1 1/4	1 1/4	1 1/2	1	1 1/2	2	2
8	1/2	1	1 1/4	1 1/2	1	1 1/4	1 1/2	2	1	2	2	2 1/2
6	3/4	1 1/4	1 1/2	1 1/2	1 1/4	1 1/2	2	2 1/2	1 1/4	2 1/2	3	3
4	3/4	1 1/4	1 1/2	1 1/2	1 1/4	2	2 1/2	2 1/2	1 1/2	3	3	3 1/2
3	3/4	1 1/4	1 1/2	2	1 1/4	2	2 1/2	3	1 1/2	3	3	3 1/2
2	1	1 1/4	1 1/2	2	1 1/4	2	2 1/2	3	1 1/2	3	3 1/2	4
1	1	1 1/2	2	2	1 1/2	2 1/2	3	3 1/2	2	3 1/2	4	4 1/2
0	1	2	2	2 1/2	2	2 1/2	3	3 1/2	2	4	4 1/2	5
00	1	2	2	2 1/2	2	3	3 1/2	4	2 1/2	4	4 1/2	5
000	1 1/4	2	2 1/2	2 1/2	2	3	3 1/2	4	2 1/2	4 1/2	4 1/2	6
0,000	1 1/4	2 1/2	2 1/2	3	2 1/2	3	3 1/2	4 1/2	3	5	6	6
250,000	1 1/4	2 1/2	3	3	......	......	......	......	3	6	6	......
300,000	1 1/2	3	3	3 1/2	......	......	......	......	3 1/2	6	6	......
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450,000	1 1/2	3	3	4	......	......	......	......	4	6	6	......
500,000	1 1/2	3	3 1/2	4	......	......	......	......	4	6	......	......
600,000	2	3 1/2	4	4 1/2	......	......	......	......	......	......	......	......
700,000	2	4	4	5	......	......	......	......	......	......	......	......
750,000	2	4	4	5	......	......	......	......	......	......	......	......
800,000	2	4	4 1/2	5	......	......	......	......	......	......	......	......
900,000	2 1/2	4	4 1/2	5	......	......	......	......	......	......	......	......
1,000,000	2 1/2	4 1/2	4 1/2	6	......	......	......	......	......	......	......	......
1,250,000	3	5	5	6	......	......	......	......	......	......	......	......
1,500,000	3	5	6	6	......	......	......	......	......	......	......	......
1,750,000	3	6	6	......	......	......	......	......	......	......	......	......
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The wires are placed, after the system is installed, by raising the cover and laying the wires in place. Not more than 30 wires may be placed in one wireway unless the excess are signal or control wires. No conductor larger than 500,000 cir mils shall be used, and the cross-sectional area of all cables shall not exceed 20 per cent of the area of the wireway.

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Busways are similar in construction to wireways and governed by the same rules as to location and use. Instead of insulated wire, bare copper buses mounted on suitable insulators are used. Their chief application is for large feeders.

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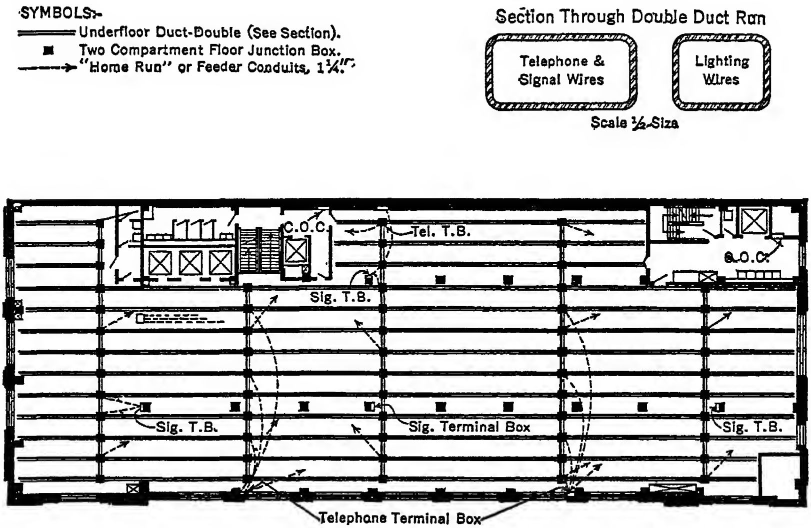

Fig. 6

Raceways are made in several styles, the metal ones usually being of rectangular section and the fiber ones of semicircular or oval section, and have conveniently spaced outlets formed in the duct. They are usually installed in the form of a grid covering an entire floor in such a manner that, no matter what the office layout may be, there will be an outlet convenient to any desk or other equipment.

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Raceways may be placed in the concrete fill between the rough and finished floor, or in the slab, provided raceways of half-round section or of flat-top section not over 4 in. in width, shall have at least $^3/_4$ in. of concrete above the raceway, except that in office occupancies, metal flat-top raceways not over 2 in. in width may be laid flush with the concrete if covered with substantial linoleum or equivalent floor covering.

Open-bottom raceways must be laid on a smooth concrete pad at least 1 in. thick, extending 1 in. on either side of the raceway, and the joint must be filled with waterproof cement.

Where metal raceways are used, they must be continuous, both electrically and mechanically, be properly grounded, and conform in general to the rules for rigid conduit.

Not more than 10 wires may be used in any one raceway, and no wire larger than No. 8 shall be used. Individual circuits shall not be fused for more than 30 amp, nor shall the combined cross-sectional area of all conductors exceed 40 per cent of the cross-sectional area of the raceway.

" Duplex " or " flat twin " rubber-covered wire (Type RD), armored cable or non-metallic sheath cable must be used in open-bottom raceways. In closed-bottom raceways, rubber-covered wire (Type R) may be used.

CAST-IN-PLACE RACEWAYS. In fire-resisting buildings, raceways may be cast in place in concrete floor slabs, walls, or ceilings, provided they do not interfere with the strength or fire-resistive value of the unit. Such raceways are in effect non-metallic rigid conduit. Fiber or tile ducts imbedded in concrete would also come under this classification, although, at present, they are not referred to in the National Electrical Code.

Cast-in-place raceways are formed by pouring concrete around rubber cores, similar to a heavy rubber hose, which is pulled out as soon as the concrete is sufficiently hard. Such raceways are terminated in special fittings designed for the purpose. The practice regarding the number of wires, their support in vertical runs, the type of wires and other features, is the same as for rigid metallic conduit.

ARMORED CABLE. Armored cable, sometimes known as BX or ABC cable, is employed almost universally in residence wiring and is also a satisfactory method of installing concealed wiring in old buildings. It is frequently used for branch circuits in installations where the feeders are run in conduit. It is permitted for all voltage up to 600 volts and where the ambient temperature does not exceed 120 deg fahr (49 deg cent).

The cable consists of one or more rubber-insulated conductors, protected by an interlocking metal strip, usually galvanized steel, wound helically to form a tube similar to flexible conduit. Where of more than one conductor, the insulated conductors are cabled together and enclosed in a braided or impregnated fiber or paper covering before the armor is applied.

Armored cable may be installed exposed or concealed, may be run in the hollow spaces of walls or between floors, or may be " fished " through such spaces in completed buildings.

Special fittings or outlet boxes are available to fit cables of various sizes. All cable must be continuous from outlet to outlet and be connected both electrically and mechanically thereto, and the armor shall be properly grounded.

Where exposed to weather, continuous moisture, or imbedded in masonry or otherwise exposed to materials likely to affect the rubber insulation, a type of cable having a lead covering under the armor shall be used.

NON-METALLIC SHEATHED CABLE. This material is used in a manner similar to armored cable but requires more protection where exposed to mechanical injury. It is prohibited by the Code for certain types of buildings. It may be used for voltages up to 600 volts.

The cable consists of rubber-insulated conductors in two- and three-wire assemblies, protected by paper wrappings and cotton braids thoroughly impregnated with moisture-resisting compounds and the outer covering treated to make it fire-resisting. It is furnished with or without a bare ground wire in the assembly.

It may be installed exposed or concealed in dry locations where the ambient temperature does not exceed 120 deg fahr (49 deg cent). It is not permitted in sizes larger than No. 4 B.W.G.

NON-METALLIC WIRING FOR USE IN WET PLACES. In ice plants, breweries and other wet locations, rubber sheathed multiple-conductor cable may be used as the wiring material. When so used the individual conductors shall not be smaller than No. 12 A.W.G. and the cable shall be supported on insulators spaced at intervals not exceeding 3 ft, except when passing through walls, where it shall be enclosed in conduit or insulating tubing.

Special fittings which provide a moisture-proof seal by means of a gasket must be used with this system of wiring.

SURFACE METAL RACEWAYS. Metal Raceways, sometimes called metal molding, is used for exposed surface wiring, usually in completed buildings for the extension of existing systems where a neat and inconspicuous metallic wire enclosure is desired and the voltages do not exceed 300 volts between wires or 150 volts to ground.

Raceways are made in various designs and sizes of substantially rectangular cross-section, with removable covers or backs. They are made in various sizes, together with the

necessary fittings, to accommodate from two to ten wires. The strips are fastened in place by screws or bolts so placed as not to interfere with the wire space.

The same rules regarding electrical and mechanical continuity between outlets or fittings and the same grounding rules apply as for rigid conduit.

Surface raceways are not permitted for wire sizes larger than No. 8 or circuits fused for more than 30 amp. They must not be used in damp locations or where exposed to corrosive fumes or where subject to severe mechanical injury.

Rubber-covered wire (Type R) is required except where the ambient temperature exceeds 120 deg fahr (49 deg cent), in which case asbestos wire (Type A) or slow-burning wire (Type SB) shall be used.

SURFACE WOODEN RACEWAYS. This method of wiring, although mentioned in the National Electrical Code, is practically obsolete and is prohibited in most localities. It is used in the same general manner as surface metal raceways. The raceways are made of waterproofed hard wood with a half-inch barrier between the wire grooves and a suitable wooden cap.

UNDER-PLASTER EXTENSIONS. Where the absence of hollow spaces in the walls and floors of finished buildings does not permit fishing between outlets, extensions may be made to existing systems by means of under-plaster extensions. This method is permitted in fire-resisting buildings only and used mostly in short runs. Such extensions must not extend beyond the limits of the floor where they originate.

The installation is made by channeling the plaster and fastening the raceway, cable, or conduit to the wall, then plastering over flush with the original finish.

Special small sizes of conduit, tubing, and raceways are permitted for these extensions, which are not permitted for other wiring, either open or concealed. "Ovalflex," a flat armored cable (National Electric Products Co.), is also made for this purpose. Standard "all metal" wiring systems are permitted for under-plaster extensions, but usually they are too large to be properly covered.

NON-METALLIC SURFACE EXTENSIONS. For short extensions from existing convenience outlets, a non-metallic surface extension material may be used for short runs not extending beyond the room in which they originate. The inclusion in the National Electrical Code of properly supervised material of this nature is intended to discourage the dangerous practice of using lamp cord for this purpose.

The material consists of two insulated conductors enclosed in a suitable fabric or other covering so designed that it may be tacked or otherwise fastened to walls or trim. It can be used only for 15-amp circuits up to 150 volts in residences or offices and is of value where the original wiring layout is inadequate.

BARE BUS-BARS AND RISERS. Conductors serving as main feeders in buildings of fire-resisting construction may be run without insulating coverings when suitable provisions are made in the design of the building. Such installations are treated as special cases and subject to review by the authority enforcing the Code in the locality where the installation is made.

Such installations may not be used for circuits in excess of 600 volts, and the conductors are supported on insulators in properly guarded shafts or channels, provided with fire cut-offs at each floor.

The conductors used may be bars of round or rectangular section or they may be of tubing. Copper tubing made in standard iron pipe sizes is frequently used. This gives a tubular section and has the advantage in the larger sizes on a-c systems of reducing the skin-effect losses which may be quite appreciable on conductors having an area above 500,000 cir mils.

In connection with the use of water pipe grounds, it should be noted that the American Waterworks Association at their May 1935 meeting passed a resolution stating that this Association does not approve of the practice of grounding electric light and power circuits to water pipes.

CONCENTRIC WIRING. This method of wiring at the present time (1936) is approved by the National Electrical Code only for service entrance cable and may not be used for interior wiring except that for range circuits only, an approved entrance cable with an uninsulated grounded conductor may be used provided the cable has a final non-metallic outer covering.

Several experimental installations of concentric wiring in interior wiring systems have been made, however, and the Electrical Committee of the National Fire Protection Association has approved certain provisions governing further trial installations. Also the Underwriters' Laboratories will "list" approved wire assemblies for such trial installations. When and if experience proves this type of wiring safe and otherwise satisfactory, it will be included in the Code.

A summary of the more important rules for trial installations and those which would

presumably be included later in the Code, are as follows: Limit to alternating current supply where voltage to ground does not exceed 150 volts. Must be grounded to an electrically continuous metallic underground water system. No fuses or switches in neutral conductor. Must be used for fixed wiring only and in new installations. May not be used for extensions to any other existing system.

The material consists of one insulated conductor and one bare conductor, in an assembly; the bare wire being the neutral conductor and, presumably, being at all times at ground potential, should require no insulation. Adequate grounding is essential to the safe use of such a system.

The usual construction consists of the neutral conductor being wound helically about the insulated conductor, either in the form of a conducting strip or a number of fine wires. This assembly usually has some overall protection, such as a weatherproof braid.

78. SERVICE ENTRANCE CABLE

This is a wire assembly or cable designed to be used as a feeder running from the service company's lines outside of the building to the service equipment within the building. It is used in place of conduit and is fastened to the side of the building in a similar manner. The most frequent application is for residence services, where it is less conspicuous than conduit. There are no restrictions, however, regarding its use for any type of service up to 600 volts.

Entrance cable is made in protected and unprotected types and may have a bare neutral wire. The protected type has a heavy armor over the assembly, and the unprotected type usually has the bare neutral, composed of several small wires, wound helically about the insulated conductors over which may be placed a metal or fiber tape and protecting braids. The unprotected type is approved only for use where not subject to mechanical injury.

Underground service entrance cables are also made in several types, with both metallic and non-metallic coverings. A typical non-metallic construction is shown in Fig. 7.

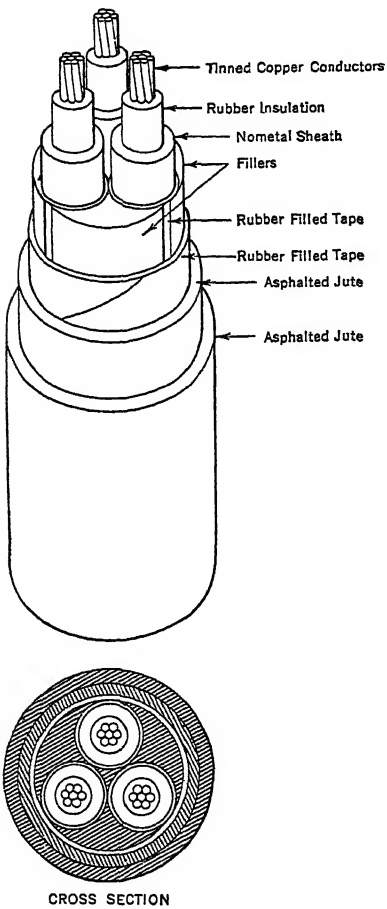


Fig. 7. Underground Non-metallic Sheath Service Entrance Cable

79. VERTICAL RISER CABLES

In tall buildings, it is common practice to locate the service transformers on a floor about half way up the building and distribute low-voltage current from this point. The service is then carried from the basement, as an extension of the street mains, up through the building to the transformer vault by means of a vertical riser cable. The voltage most frequently used is 13,200 volts.

Vertical riser cables are armored with steel wires wound helically about the insulated conductors and so designed that the entire cable may be suspended from its upper end by means of the armor wire. Inasmuch as long cables suspended from one end tend to untwist and cause damage to the insulation, the design of the complete cable must be such as to prevent untwisting.

The insulation may be rubber, varnished cambric, or paper, but is usually rubber or varnished cambric, so made as to prevent " bleeding " of the compound used between the tapes as a slipper.

80. WIRE AND WIRING ACCESSORIES

PANEL BOXES. Distributing panels mounted in metal boxes are available for any number of branch circuits and contain the switches, circuit breakers, or fuses necessary for controlling these circuits. For lighting branch circuits the switches are readily accessible for convenient operation, and the fuses and live parts are enclosed in such a manner as to be accessible only to authorized persons. Panels having automatic cut-outs or circuit breakers, in combination with the control switches, are frequently used, in which case the conventional type of fuse is not required for individual branches.

KNIFE SWITCHES. For the control of small motors, service entrances, and the like, enclosed knife switches may be used. Such switches are mounted, together with fuses, in a metal box so designed that the switch may be operated by means of a handle outside the box. The box can be opened for access to the fuses only when the switch is in the " off " position. For service entrances, the Code requires that enclosed switches be used, and it is common practice to provide meter connections and such facilities as the power company requires for meter testing in the same cabinet as the service switch.

SNAP SWITCHES. For the control of branch circuits up to 60 amp, 250 volts, either from panel boards or from isolated wall switches, single-pole snap switches are used. These devices are so designed as to give a quick break and reduce arcing when opened; they may be obtained for any type of mounting. The familiar push-button, toggle, and pull switches are of this type. For controlling circuits from more than one location, three- and four-way switches are made. (See Fig. 8.)

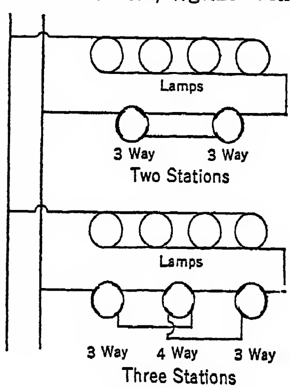


Fig. 8

Electrolier switches are used for controlling different groups of lights in the same fixture.

It should be noted that these are all classed as single-pole switches and must not be used in the neutral wire of any circuit. If for any reason a switch is placed in the neutral wire of a circuit, it must be so designed as to open all other wires of the circuit simultaneously.

OVERCURRENT PROTECTION DEVICES. To protect circuits from overloads, fuses, circuit breakers, or thermal cutouts are used, and all are designed to open the circuit when the current exceeds some predetermined value. Circuit breakers are magnetically operated, thermal cut-outs operate by means of a heating unit melting a fusible link, and fuses by the melting of short section of wire or strip.

Thermal cut-outs, not being designed to operate on short circuits but on small overloads of long duration, must be used in connection with fuses.

Fuses are the most common protective devices, and the enclosed types are completely standardized by the National Electrical Code as to rating and dimensions. The open link type of fuse is no longer permitted, except for a few special applications.

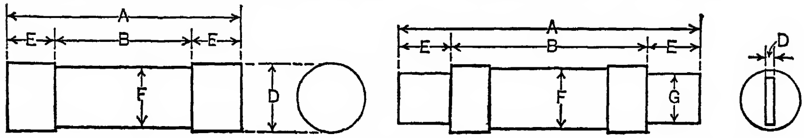
Plug fuses are designed with a screw base and are used in connection with a cut-out base having a screw socket similar to a lamp socket. They are made for the following ratings:

Non-tamperable type	Not over 250 volts	0–15 amp
Non-tamperable type	Not over 250 volts	16–25 amp
Standard screw shell type	Not over 250 volts	0–30 amp

Cartridge fuses have the fusible element enclosed in an insulating tube and are made in two styles, one having a ferrule contact and one a knife-blade contact. Renewable fuses are made to the same dimensions as cartridge fuses but differ from them in that the fusible element can be replaced in the case of a blown fuse and the fuse casing used over again.

Standard ratings and dimensions of fuses are shown in Table V.

Table V. Dimensions of National Electrical Code Standard Cartridge Fuses



Voltage	Capacity, Amp	A Length Overall	B Distance between Contacts	C Width of Contact	D Diameter of Ferrules or Thickness of Blade	E Minimum Length of Ferrule or Blade	F Diameter of Tube	G Width of Blade	
Not over 250	0– 30	2.0	1.0	0.5	0.5625	0.50	0.50	....	Ferrule type
	31– 60	3.0	1.75	0.625	0.8125	0.625	0.75	....	
	61–100	5.875	4.0	0.875	0.125	1.00	1.00	0.75	Knife-blade type
	101–200	7.125	4.5	1.25	0.1875	1.375	1.50	1.125	
	201–400	8.625	5.0	1.75	0.250	1.875	2.00	1.625	
	401–600	10.375	6.0	2.125	0.250	2.25	2.50	2.000	
Not over 600	0– 30	5.0	4.0	0.5	0.8125	0.50	0.75	....	Ferrule type
	31– 60	5.5	4.5	0.625	1.0625	0.625	1.00	....	
	61–100	7.875	6.0	0.875	0.125	1.00	1.25	0.75	Knife-blade type
	101–200	9.625	7.0	1.25	0.1875	1.375	1.75	1.125	
	201–400	11.625	8.0	1.75	0.250	1.875	2.50	1.625	
	401–600	13.375	9.0	2.125	0.250	2.250	3.00	2.000	

81. WIRES AND CABLES

The following is a brief description of wires and cables commonly used in connection with the various wiring systems. For details of construction, weights, dimensions, and carrying capacities, see chapter on Wires and Cables.

RUBBER-INSULATED WIRE (TYPE R) consists of rubber-insulated conductors protected by braids or tapes and braid. Sizes Nos. 6 A.W.G. and larger must have two braids or one tape and one braid; smaller sizes may have only one braid. Used in dry or moist locations.

RUBBER-INSULATED TWIN WIRE (TYPE RD) consists of two Type R wires laid parallel and protected with a braid or tape on each conductor and a braid overall. For use in dry or moist locations. Cannot be used for open wiring.

RUBBER-INSULATED AND LEAD-COVERED (TYPE RL) consists of Type R wires, single or multiple conductor, enclosed in a lead sheath. Used in permanently wet locations.

VARNISHED-CAMBRIC INSULATED WIRE (TYPE VC) consists of conductors insulated with layers of varnished-cloth tapes and protected with at least one braid. Can be used only in dry locations.

ASBESTOS-COVERED WIRE (TYPE A) consists of an asbestos-covered conductor saturated with some moisture-resisting compound. For use in dry, hot locations where other types of coverings would perish. Frequently used in switchboard wiring. A special type (Type AF) is designed for fixture wiring.

SLOW-BURNING (TYPE SB) consists of three braids impregnated with a flame-retarding compound and is used in a similar manner to Type A. A special type (Type CF) is used for fixing wiring.

SLOW-BURNING WEATHERPROOF (TYPE SBW) consists of one weatherproof and one flame-retarding braid. For use in open wiring in dry locations.

WEATHERPROOF WIRE (TYPE WP) consists of three braids, or the equivalent, saturated with a weatherproof compound. Used on insulators outdoors or where moisture is certain to be present.

FIXTURE WIRES. In addition to the flame-retarding types noted above, several types of rubber-insulated fixture wires are made for use in fixtures in moist locations. They are designated as Type RF for solid or stranded and Type FF for flexible. The

suffix 64 or 32 indicates the thickness of insulation. Fixture wires are permitted in sizes Nos. 18 and 16, whereas the smallest size permitted in other types is No. 14 A.W.G.

82. LAYOUT OF A WIRING SYSTEM

DEFINITIONS. The following definitions apply to the terminology commonly used in connection with Building Wire Circuits:

Appliance. Appliances are current-consuming equipment, fixed or portable, such as small heaters or motor-operated devices.

Approved. Acceptable to the authority enforcing the National Electrical Code.

Branch Circuit. That portion of a wiring system extending beyond the final automatic overcurrent protective device. **Lighting branch circuits** supply lighting outlets only. **Appliance branch circuits** supply either permanently wired appliances or attachment plug receptacles or a combination of the same with no permanently connected lighting fixtures.

Cabinet or Cut-out Box. An enclosure designed for surface mounting, having swinging doors or covers.

Demand Factor. Ratio of maximum demand to the total connected load.

Feeder. The conductors extending from the source of energy to a distribution center. This includes service conductors, mains, and subfeeders.

Service. A service includes the "service entrance" conductor and all "service equipment" required for bringing energy to the wiring system of a building.

Panelboard. A single or multiple unit, including buses and with or without switches and protective devices, used for the control of any type of circuit; designed to be placed in a cabinet and accessible only from the front. (This is distinct from a switchboard, which is generally accessible from the rear as well as the front.)

Adequate Wiring

In laying out a wiring system, in order to ensure its being adequate, safe, and efficient, consideration should be given certain general principles and accepted practices. The more important of these follow.

In designing new buildings, provision should be made for channeling and pocketing of buildings to accommodate the various wiring systems, keeping in mind that telephone, fire alarm, or other systems should be provided for but must be kept separate from light and power wiring.

Distribution centers should be centrally located in easily accessible places where switches and fuses for branch circuits can be grouped for convenient operation.

Branch circuit loads should be evenly distributed and complicated wiring avoided, so that the mechanical execution of the work will be as simple as possible.

With the increasing use of electrical devices, consideration should be given to possible future loads and provision made for ample capacity in branch circuits and mains. This applies particularly to residences and apartments. In addition to the desired lighting outlets, it is considered good practice to provide receptacle outlets so that no point on the walls shall be more than 10 ft from such outlet. Separate appliance branch circuits should be run to supply receptacle outlets in kitchen, dining room, and laundry. Such appliance branches are required to be of larger size wire than for lighting branch circuits, regardless of the immediate contemplated load.

Branch Circuits

The Code recognizes three types of branch circuits: a 15-amp branch circuit, an appliance branch, and a motor branch circuit.

A 15-amp branch circuit must not be smaller than No. 14 wire and must have overcurrent protection not exceeding 15 amp at 125 volts or 10 amp at 250 volts. Not more than 12 outlets shall be connected to such a circuit. Such circuits may supply lighting or receptacle outlets or permanently connected appliances except that no appliance so connected shall exceed a rating of 1320 watts.

It should be noted that 12 outlets does not imply 12 sockets, as a multiple light fixture may be connected to one outlet.

APPLIANCE BRANCH CIRCUITS. Wires smaller than No. 12 are not permitted for appliance branches, and overcurrent protection shall not exceed 25 amp. Such circuits shall supply only receptacle outlets or permanently connected appliances. No appliance connected to such a circuit shall be rated at more than 15 amp at 1650 watts. It should be noted that, if the 25-amp limit for fuses is adhered to, the wire size must be No. 10 as No.

12 cannot be fused for more than 20 amp, which is the maximum current-carrying capacity of No. 12 wire. (See Current-carrying Capacity of Wires and Cables.)

One side of all appliance branch circuits must be grounded, and the voltage to ground shall not exceed 150 volts.

INDIVIDUAL APPLIANCE BRANCH CIRCUITS must be used for all appliances rated over 15 amp or 1650 watts. That is, there shall be no receptacle outlets on such circuits.

MOTOR BRANCH CIRCUITS. Except for small motors used on appliance branch circuits, motor branch circuits are used. The conductors for supplying individual motors shall have a current-carrying capacity of 125 per cent of the full-load current of the motor. The overload protection requirements for such circuits are too involved to be given here but are fully covered by the National Electrical Code. In general, the motor is protected from continuous overload by means of some form of time delay device, and both the motor and conductor are protected from short circuit by means of fuses or circuit breakers, or both.

OVERCURRENT PROTECTION. Fuses or other overcurrent protection shall be located at the point where the size of a conductor changes and are so placed to protect the smaller conductor. Therefore, if fuses are used, they should not be rated higher than the current-carrying capacity of the conductor they protect. This does not apply when connection is to a grounded neutral, as no overcurrent device is permitted in conductors which are permanently grounded, unless it simultaneously opens all conductors of the circuit. This means that fuses can never be used in grounded conductors, as obviously they cannot open more than one conductor.

The overload protection of motors, and the use and location of circuit breakers in various types of circuits, are extensively covered in the National Electrical Code, Art. 8.

83. GROUNDING PRACTICE FOR INTERIOR WIRING SYSTEMS

Grounding in interior wiring systems is done primarily as a safety measure, and two classes of grounds are concerned:

In a **system ground,** one of the current-carrying conductors of a system is connected to ground.

An **equipment ground** is one where non-current-carrying parts are connected to ground. This covers such things as metallic conduit, cable armor, motor frames, switch boxes, and the like.

The purpose of equipment grounds is to keep all exposed metal parts at as near ground potential as possible by providing a low-resistance path to ground so that in case of a breakdown in the insulation, the non-current-carrying parts of the equipment cannot be raised above ground potential sufficiently to be dangerous to a person coming in contact therewith. In ungrounded equipment, it is also possible to have a high-resistance path to ground through some part of the building structure, such as metal lath, which would introduce a fire hazard as well as the accident risk, due to shock.

Equipment Grounds are required on exposed metal parts of practically all fixed equipment, including all metal duct and raceway systems. There are a few exceptions to this rule, the most important one being that grounding of conduit or armored cable runs under 25 ft is not required if free from metallic contact with the ground or grounded metal parts.

The exposed metal parts of portable equipment operating at more than 150 volts to ground must be grounded, and when possible, other portable equipment should be grounded. The attachment cords for grounded portable equipment are made with an extra conductor for this purpose.

The sizes of grounding conductors for equipment grounds are determined by class of equipment being grounded and by the size of the overcurrent device limiting the current.

System Grounds on interior wiring systems protect the low-voltage wiring in case of accidental contact with higher-voltage lines. Where the neutrals of both systems are grounded, a low-resistance path to ground is provided through the neutral of the low-voltage system, and in the event of a cross between the two systems a fuse should blow and isolate that part of the system where the trouble occurred.

If such a cross occurs with an ungrounded system, the low-voltage system will be brought up to the potential above ground of the high-voltage line. Under these conditions, there is danger of shock to persons coming in contact with the low-voltage system, as well as probable damage to the insulation of the low-voltage system and the equipment connected to it.

In order to be effective, a ground connection must be of low resistance. Therefore,

the grounding electrode must make good contact with moist earth. Continuous water piping systems afford the most convenient and best grounds. Where such a ground is not available, the metal frame of a building, a gas piping system, or a properly constructed "artificial ground" consisting of a buried plate or drain pipe may be used. Artificial grounds must not have a resistance of over 25 ohms and should be less. Water pipe grounds are of the order of 3 ohms or less.

Interior wiring systems require grounding as described in Art. 94.

The size of a grounding conductor for a system ground for a d-c system shall not be less than the largest conductor supplied by the system and in no case less than No. 8 A.W.G. copper.

The grounding conductor for an a-c system shall have a current-carrying capacity not less than one-fifth that of the conductor to which it is attached and in no case less than No. 8 A.W.G. copper.

84. RESIDENCE WIRING

SERVICE ENTRANCE AND EQUIPMENT. The service entrance must first be located with reference to the type of dwelling and position of the power company's lines. In general, the power company will supervise this part of the work and quite frequently install the complete service through the meter and service switch, up to the point where the branch circuits are connected. Service conductors are brought in overhead or underground, usually through conduit or by means of service entrance cables, as previously described. Two types of overhead entrances are shown in Fig. 9.

The service equipment located within the building should be easy of access, and provision should be made on the panel for extra branch circuits in order to take care of circuits that may be added in the future.

A master service switch, when installed, must open all wires simultaneously unless a ground terminal block within the cabinet provides means for disconnecting the grounded wire, in which case the switch may open only the current-carrying conductors.

Under certain conditions (specified in Art. 405 of the National Electrical Code) the master switch may be omitted in residence wiring.

In residence wiring, the limitation of 12 outlets on 15-amp branch circuits, as mentioned above, may be varied, provided at least one branch circuit is installed for every 500 sq ft of floor area in the building. These circuits are to be in addition to any appliance branch circuits which may be installed.

The branch circuits should be so distributed that the blowing of a fuse in one circuit will not leave an entire floor without light.

Basement, stair, and hall lights should be so wired as to operate from more than one location, and it is desirable to have several conveniently located lights operated from a switch at the main entrance.

Care should be taken in locating wall switches, to have them near the most-used entrance to the room, on the latch side of the door.

Electric ranges, water heaters, or other appliances having a rating of more than 1650 watts must be served by a separate branch circuit and, unless equipped with a plug-type connection, must have a disconnecting switch of the indicating type.

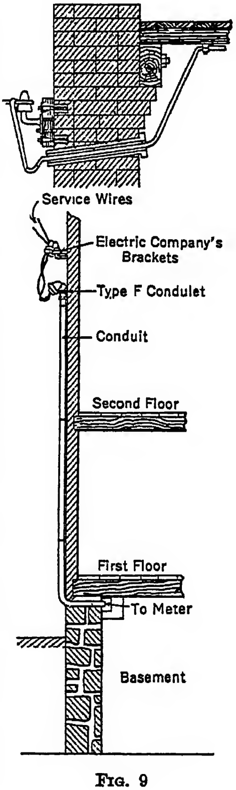


FIG. 9

In calculating wire sizes for ranges or heaters, care should be taken not only to see that the conductors are large enough to carry the rated current, but also, unless the feeders are short, to be sure that the voltage drop is not great enough to impair the efficiency of the appliance.

85. WIRING OF LARGE BUILDINGS

The first step in laying out a wiring system for buildings other than residences is to locate on a floor plan the lighting and appliance outlets and, if possible, the probable location and capacity of power equipment. Panelboard locations should then be determined in such a manner that the branch circuits serviced from them shall be of as nearly the same length as possible and preferably no longer than 100 ft.

In multistory buildings, the distributing boxes, from which mains run to the various panelboards, should be placed in the same locations on each floor. It is customary to provide pockets or closets in the building structure for such boxes and channels for the feeders to them.

The load for each panel should be calculated for use in determining the capacity of feeders. (See Wiring Calculations in this section.)

In laying out appliance branch circuits, a certain amount of judgment is required to determine the proper number of circuits. The probable load which may be applied is estimated by assuming the capacity of the devices most likely to be used.

Table VI gives the current drawn by some commonly used appliances:

Table VI. Watts Taken by Appliances

Device	Watts	Device	Watts
Flatirons	350–1200	Office machines	150
Immersion heater	300– 500	Curling iron	25
Water heaters	600–5000	Vibrators	50
Chafing dishes	420	Vacuum cleaner	150
Grill	660	Stove	5000–7500
Coffee percolators	420	Bake oven	5000–80,000
Toasters	500	Dish washer	200
Heating pads	60	Washing machine	600
Cigar lighters	70	Ironing machine	2500–6000
Glue pots	70–880	Hot plates (2)	2300
Luminous radiators	750–1500	Hot plates (3)	3100
Reflection heater	500–1000	Laboratory plate	860–4300
Air heater	500–5000	Industrial disc heater	1000–1800
Sewing machine	50	Soldering iron	100–500
Sun lamps	100–1500	Office fans	30–60
Refrigerators	100–350	Household stoves	4000–7500

The Number of Feeders depends upon the occupancy of the building and also upon (1) the power taken by the receiving devices, (2) the degree of control required at the main distributing center or switchboard, (3) the character of the receiving devices, and (4) the uniformity of voltage required.

In multiple-family dwellings one feeder is required for each apartment, the main distributing center being a metering panelboard where the meters for the entire building, or a large section of it, are grouped.

In other buildings if it is not required to control each floor separately; one set of feeders may be used for the entire building or a large section thereof.

It is advisable to sectionalize public buildings, or buildings occupied by a large number of people, in such a manner that not all the lights will fail in the event of an open circuit on any one set of feeders. It is not good practice to supply lighting loads and large fluctuating loads from the same feeders, as the voltage will vary with the load and cause undesirable flickering of the lamps. This means that power and lighting loads should not be supplied from the same feeders.

The maximum size of feeders should not exceed 1,000,000 cir mils, as it is not practicable to run larger conductors. Where larger capacity is required, several conductors having the necessary combined carrying capacity should be used. Large conductors on a-c systems are also undesirable because of loss due to skin effect.

86. DESIGNS AND PLANS

WIRING DIAGRAM. When the location of outlets, panelboards, and distributing centers, and the number of feeders, have been determined, a wiring diagram should be made showing the location and length of all feeders and branch circuits. The size of conductors for all feeders and mains should be indicated. A typical wiring diagram for an office building is shown in Figs. 10 and 11, and a list of standard symbols for electrical equipment in Fig. 12.

SPECIFICATIONS. A complete set of specifications should be prepared, giving the details of the installation. Typical specifications prepared by various committees and associations are readily obtainable.

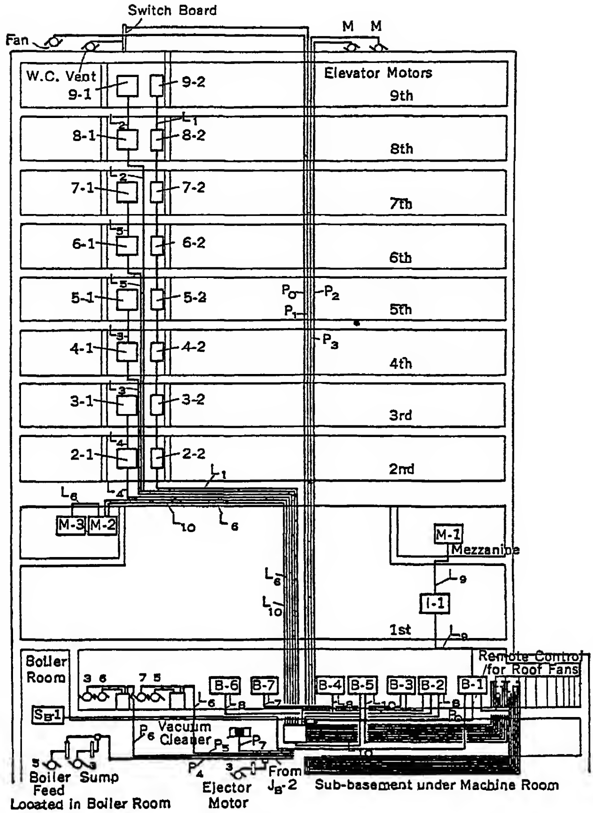

Switch Board
Fan
M M
W.C. Vent
9-1
9-2
Elevator Motors
9th
8-1
8-2
8th
7-1
7-2
7th
6-1
6-2
6th
5-1
5-2
5th
4-1
4-2
4th
3-1
3-2
3rd
2-1
2-2
2nd
M-3
M-2
M-1
Mezzanine
I-1
1st
Boiler Room
B-6
B-7
B-4
B-5
B-3
B-2
B-1
Remote Control for Roof Fans
Vacuum Cleaner
Boiler Sump Feed
Located in Boiler Room
Ejector Motor
From J_B-2
Sub-basement under Machine Room

FIG. 10

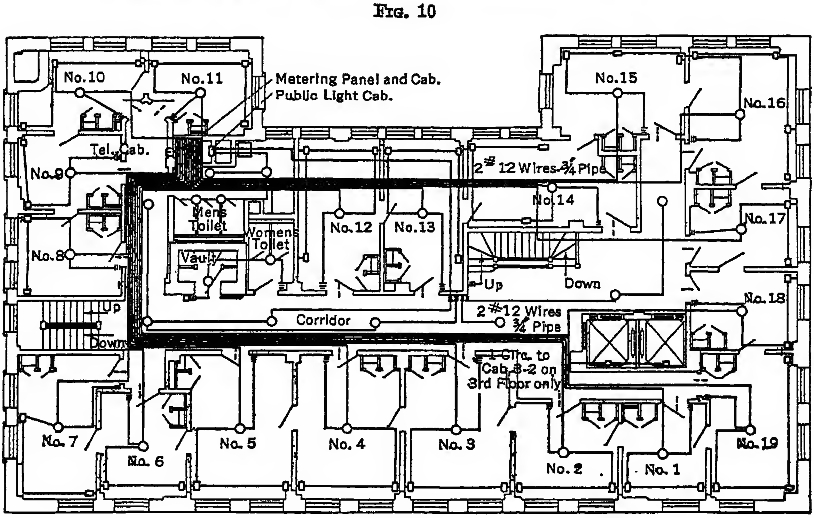
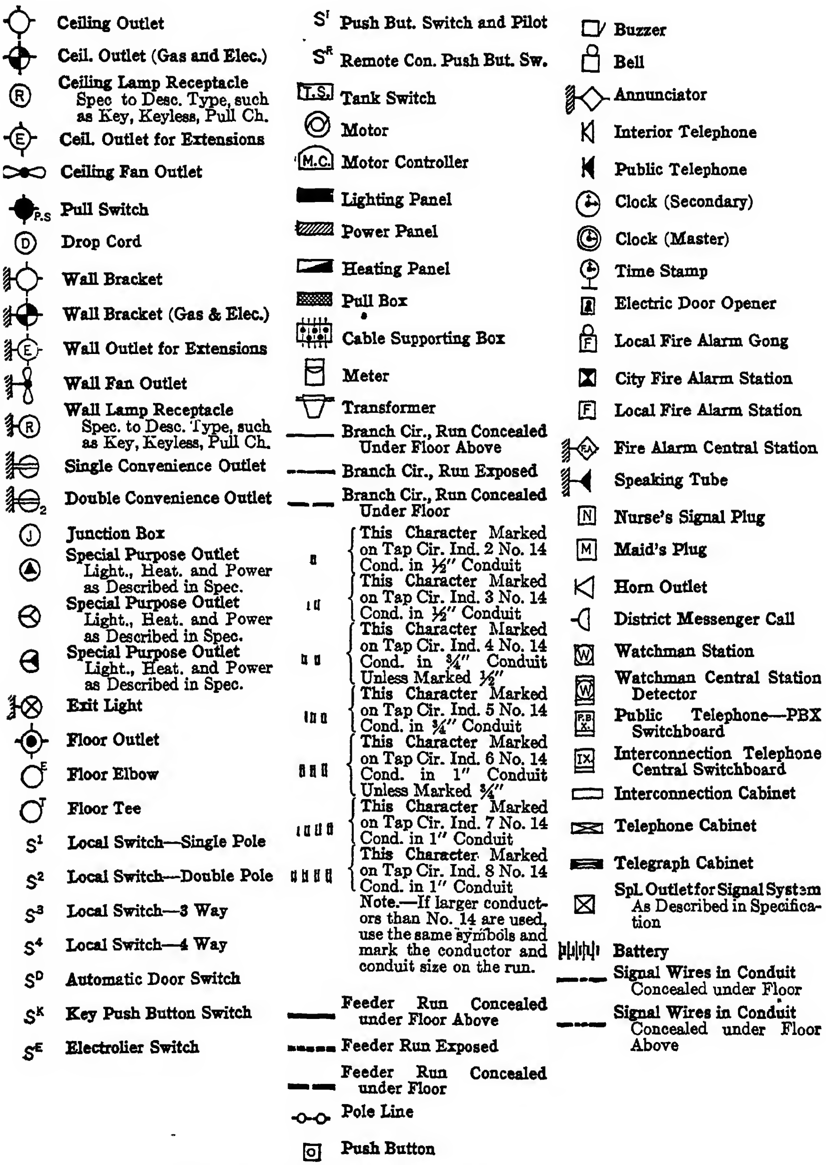

No. 10
No. 11
Metering Panel and Cab.
Public Light Cab.
No. 15
No. 16
Tel. Cab.
No. 9
2 #12 Wires-¾ Pipe
No. 14
Mens Toilet
Womens Toilet
No. 12
No. 13
No. 17
No. 8
Vault
Up
Down
No. 18
Corridor
2 #12 Wires ¾ Pipe
1 Circ. to Cab. 3-2 on 3rd Floor only
No. 7
No. 6
No. 5
No. 4
No. 3
No. 2
No. 1
No. 19

FIG. 11



Height of Switches (unless otherwise specified), 4′–0″
Heights of Center of Wall Outlets (unless otherwise specified):

Living Rooms......	5′–6″	Offices............	6′–0″
Chambers.........	5′–0″	Corridors..........	6′–3″

Fig. 12. Standard Symbols for Wiring Plans Recommended by Am. Assn. of Electragists, Am. Inst. of Architects, Am. Inst. Elect. Eng. and approved by A.S.A.

WIRING CALCULATIONS. The size of conductor required for any circuit is determined by the current-carrying capacity and the allowable potential drop. The current-carrying capacity for building wires is fixed by the National Electrical Code. (See table of current-carrying capacities in section on Wires and Cables.) The allowable potential drop depends upon the character of the device being served. In lighting circuits, 3 per cent is considered satisfactory, and in power circuits 5 to 10 per cent.

CURRENT TO BE CARRIED BY FEEDERS. The amount of current carried by any feeder is related to the connected load but is seldom equal to the total connected load, inasmuch as not all devices provided for will be in use at the same time. The maximum demand on any feeder, therefore, will be some percentage of the connected load, and it varies with the type of occupancy of the building.

The probable maximum loads for lighting and appliance circuits in the more usual occupancies has been determined by experience and demand factors have been established for standard loads. The National Electrical Code gives such demand factors as a means of determining feeder capacities. These factors give the minimum capacity required and do not include heavy current-consuming loads such as ranges and motors, which are usually run on separate feeders distinct from the lighting and appliance load and treated separately.

For convenience, the factors as given in Section 2011 of the 1933 Code are condensed and tabulated below:

Class of Occupancy	Watts per Sq Ft Standard Load	Area Limits for Standard Loads *	Demand Factor, per cent
Single-family dwellings	1.0	First 2000 sq ft or less Excess over 2000 sq ft (Add 1000 watts for appliances)	100 60
Multi-family dwellings	1.0	First 2000 sq ft or less	100
Apartments and apartment hotels (See Note below)	1.0	Excess over 2000 sq ft 1–10 apartments 11–40 apartments 41 or over (Add 1000 watts for each apartment for appliances)	 70 60 50
Hotels with no provision for electric cooking in rooms, exclusive of ballrooms	1.0	First 10,000 sq ft 10,000–50,000 sq ft Excess over 50,000 sq ft	100 80 70
Stores, exclusive of show windows	2.0	Total area (Add 25 watts per lin ft for counter cases and 50 watts per lin ft for standing cases)	100
Show windows	...	200 watts per lin ft	100
Office buildings	2.0	First 10,000 sq ft or less Excess over 10,000 sq ft	100 70
Commercial or loft buildings occupied by more than one tenant	1.0	Total area	100
Garages	0.5	Total area	100
Hospitals, exclusive of operating and x-ray rooms	0.75	First 25,000 sq ft or less Excess over 25,000 sq ft	100 60
Schools	1.5	First 10,000 sq ft or less Excess over 10,000 sq ft	100 50
Storage warehouses	0.25	First 50,000 sq ft or less Excess over 50,000 sq ft	100 50

NOTE: Demand factors must not be applied to appliance loads in computing subfeeders to individual apartments, but may be applied in computing combined lighting and appliance loads for other feeders.

* Information in parentheses refers to special loads.

Where a common neutral is used for several feeders additional demand factors may be applied to neutral conductors, as follows:

Type of System	Current Load in Outside Conductors after Applying Demand	Further Demand for Neutral Conductors, per cent
3-wire, d-c or 1-phase, and 4-wire, 3-phase	0 to 200 amperes	100
3-wire, d-c or 1-phase, and 4-wire, 3-phase	above 200 amperes	70
5-wire, 2-phase	0 to 200 amperes	140
5-wire, 2-phase	above 200 amperes	100

For factories, theaters, libraries, and other buildings for special purposes, the feeders shall be designed for the specific load which they are to serve.

Electric-heated cooking appliances rated above 1650 watts shall have feeders computed on the following basis:

No. of Ranges	Demand Factor	No. of Ranges	Demand Factor	No. of Ranges	Demand Factor	No. of Ranges	Demand Factor
1	80	7	56	13	43	20	35
2	75	8	53	14	41	21	34
3	70	9	51	15	40	22	33
4	66	10	49	16	39	23	32
5	62	11	47	17	38	24	31
6	59	12	45	18	37	25	30
				19	36	Over 25	30

Example. A single-family residence 40 by 50 ft having two floors and finished basement, unfinished attic.

Area occupied $40 \times 50 \times 3 = 6000$ sq ft.

Watts load on feeders:

2000 sq ft at 1 watt per sq ft at 100% demand	= 2000 watts
4000 sq ft at 1 watt per sq ft at 60% demand	= 2400 watts
Allowance for appliances	1000 watts
Total	5400 watts

Current for 110 volt, 2-wire system

$5400 \div 110 = 49$ amp

Size conductor neglecting voltage drop = No. 6

Current for 110–220 volt, 3-wire system

$5400 \div 2 \times 110 = 24.5$ amp per wire

Size conductor neglecting voltage drop = No. 10

Example. An apartment house, with total area of 48,000 sq ft, having 60 apartments divided into three groups of 20 each, with meter banks in the basement.

Area of each apartment = 800 sq ft.

(1) Feeder for each apartment: 800 sq ft $\times$ 1 watt per sq ft = 800 watts

Appliance load 1000 watts

Total 1800 watts

Being less than 2000 sq ft, there is 100% demand.

Current for 110 volt, 2-wire = $1800 \div 110 = 16.4$ amp

Minimum wire size = No. 12

Current for 110–220 volt, three-wire system = $1800 \div 220 = 8.2$ amp

Minimum wire size = No. 14

(2) Feeder to each meter bank:

Total area supplied by each bank $20 \times 800 = 16{,}000$ sq ft. In this case the demand factor for groups between 11 and 40 apartments is 60 per cent after the deduction of the first 2000 sq ft at 1 watt per sq ft, or 2000 watts. We then have

16,000 sq ft at 1 watt per sq ft	= 16,000 watts
Allowed for appliances 20 × 100	= 20,000 watts
Computed load	= 36,000 watts
of which we have 2000 watts at 100%	= 2,000 watts
and 34,000 watts at 60%	= 20,400 watts
Maximum demand	= 22,400 watts

For 2-wire, 110 volts, the current = 22,400/110 = 203.6 amp
Minimum wire size = No. 1/0 (for rubber-insulated wire)

(3) Service feeders for the entire building, there being over 40 apartments, the demand after the first 2000 volts will be 50%. We then have

48,000 sq ft × 1 watt per sq ft	= 48,000 watts
Allowed for appliances 60 × 100	= 60,000 watts
Computed load	= 108,000 watts
2000 watts at 100%	= 2,000 watts
106,000 watts at 50%	= 53,000 watts
	55,000 watts

For 2-wire, 110 volts, the current will be 55,000/110 = 250 amp
Minimum size of feeder = 250,000 cir mils.

Feeders for Motor Loads

Feeders for individual motors must have a capacity of 125 per cent of the full load current of the motor.

Feeders supplying a group of two or more motors shall have a current-carrying capacity not less than 125 per cent of the full-load current of the largest motor plus the sum of full-load ratings of all the motors in the group.

Where two or more motors of the same horsepower are on the same feeder, it is assumed that one of these motors is larger for the purpose of calculating.

Certain exceptions are made to the above where the operation of motors is not continuous.

The current required by the various types of motors is given in Tables VII, VIII, IX, and X and the wire and fuse size for branch circuits in Table XI.

For industrial applications of motors, see Section 16.

Table VII. Full-load Motor Currents

Direct-current Motors

Amperes

Horsepower	115 Volts	230 Volts	550 Volts	Horsepower	115 Volts	230 Volts	550 Volts
1/2	4.5	2.3	....	30	220	110	45
3/4	6.5	3.3	1.4	40	294	146	61
1	8.4	4.2	1.7	50	364	180	75
1 1/2	12.5	6.3	2.6	60	436	215	90
2	16.1	8.3	3.4	75	540	268	111
3	23.0	12.3	5.0	100	....	357	146
5	40	19.8	8.2	125	....	443	184
7 1/2	58	28.7	12.0	150	....	....	220
10	75	38	16.0	200	....	....	295
15	112	56	23.0				
20	140	74	30				
25	185	92	38				

Table VIII. Full-load Motor Currents

Single-phase A-c Motors

Amperes

Horsepower	110 Volts	220 Volts	440 Volts	Horsepower	110 Volts	220 Volts	440 Volts
1/6	3.34	1.67	....	1 1/2	15.2	7.6	....
1/4	4.8	2.4	....	2	20	10	....
1/2	7	3.5	....	3	28	14	....
3/4	9.4	4.7	....				
1	11	5.5	....	5	46	23	....
				7 1/2	68	34	17
				10	86	43	21.5

For full-load currents of 208- and 200-volt motors, increase corresponding 220-volt motor full-load current by 6 and 10 per cent, respectively.

Table IX. Full-load Motor Currents

Two-phase A-c Motors (4-wire)*

Horse-power	Induction-type Squirrel-cage and Wound Rotor Amperes					Synchronous-type Unity Power Factor † Amperes			
	110 volts	220 volts	440 volts	550 volts	2200 volts	220 volts	440 volts	550 volts	2200 volts
1/2	4.3	2.2	1.1	.9	....	....	....	....	....
3/4	4.7	2.4	1.2	1.0	....	....	....	....	....
1	5.7	2.9	1.4	1.2	....	....	....	....	....
1 1/2	7.7	4.0	2	1.6	....	....	....	....	....
2	10.4	5	3	2.0	....	....	....	....	....
3	....	8	4	3.0	....	....	....	....	....
5	....	13	7	6	....	....	....	....	....
7 1/2	....	19	9	7	....	....	....	....	....
10	....	24	12	10	....	....	....	....	....
15	....	33	16	13	....	....	....	....	....
20	....	45	23	19	....	....	....	....	....
25	....	55	28	22	6	47	24	19	4.7
30	....	67	34	27	7	56	29	23	5.7
40	....	88	44	35	9	75	37	31	7.5
50	....	108	54	43	11	94	47	38	9.4
60	....	129	65	52	13	111	56	44	11.3
75	....	156	78	62	16	140	70	57	14
100	....	212	106	85	22	182	93	74	18
125	....	268	134	108	27	228	114	93	23
150	....	311	155	124	31	....	137	110	28
200	....	415	208	166	43	....	182	145	37

* Values of current in common wire of two-phase three-wire system will be 1.41 times value given.

† For 90 and 80 per cent power factor the above figures should be multiplied by 1.1 and 1.25, respectively.

POTENTIAL DROP. In most installations the potential drop *per conductor* between service entrance or switchboard and the farthest receiver is taken as approximately 3 per cent of the *voltage to neutral* at the service entrance or switchboard. This is equivalent to a *total drop* (both wires) of 3 per cent of the *voltage between* wires in a two-wire d-c or single-phase system. In three-wire systems the two outside wires are to be regarded as the feeders.

In d-c or single-phase a-c systems the maximum potential drop *per conductor* will then be $0.015 \times 110 = 1.65$ volts for a 110-volt system, 3.3 volts for a 220-volt system, etc. In three-phase systems the potential drop *per conductor* will be $0.03 \times \frac{110}{\sqrt{3}} = 1.91$ volts for a 110-volt system, 3.82 volts for a 220-volt system, etc. In feeders and branch circuits the drop in the feeders is usually made two-thirds and the drop in the branch circuits one-third of the total drop.

Table X. Full-load Motor Currents
Three-Phase A-c Motors

Horse-power	Induction-type Squirrel-cage and Wound Rotor Amperes					Synchronous-type Unity Power Factor * Amperes			
	110 volts	220 volts	440 volts	550 volts	2200 volts	220 volts	440 volts	550 volts	2200 volts
1/2	5	2.5	1.3	1	....	....	....	....	....
3/4	5.4	2.8	1.4	1.1	....	....	....	....	....
1	6.6	3.3	1.7	1.3	....	....	....	....	....
1 1/2	9.4	4.7	2.4	2.0	....	....	....	....	....
2	12	6	3	2.4	....	....	....	....	....
3	....	9	4.5	4	....	....	....	....	....
5	....	15	7.5	6	....	....	....	....	....
7 1/2	....	22	11	9	....	....	....	....	....
10	....	27	14	11	....	....	....	....	....
15	....	38	19	15	....	....	....	....	....
20	....	52	26	21	....	....	....	....	....
25	....	64	32	26	7	54	27	22	5.4
30	....	77	39	31	8	65	33	26	6.5
40	....	101	51	40	10	86	43	35	8.6
50	....	125	63	50	13	108	54	44	10.8
60	....	149	75	60	15	128	64	51	13
75	....	180	90	72	19	161	81	65	16
100	....	246	123	98	25	211	106	85	21
125	....	310	155	124	32	264	132	106	26
150	....	360	180	144	36	....	158	127	32
200	....	480	240	195	49	....	210	168	42

For full-load currents of 208- and 200-volt motors, increase the corresponding 220-volt motor full-load current by 6 and 10 per cent, respectively.

* For 90 and 80 per cent power factor the above figures should be multiplied by 1.1 and 1.25, respectively.

Note that in the case of a lamp load the allowable drop of 3 per cent is based on the current corresponding to the *total connected load*, i.e., this is the maximum drop when *all* lamps are burning. In ordinary buildings the actual maximum load is seldom more than one-third the connected load; consequently when the wiring is designed on this basis the voltage at the lamps will seldom be more than 1 per cent lower than the voltage at the service connection.

CALCULATION OF SIZE OF WIRE. Let

I = current per conductor in amperes.
v = allowable potential drop *per conductor* in volts.
l = length of the conductor in feet.†

Then the required cross-section of a copper wire in circular mils is

$$A = \frac{KlI}{v} \tag{1}$$

where K is a factor depending upon the specific resistance of the wire, the size and spacing of the wires, the frequency and the power factor of the receiver. The factor K for a-c circuits is therefore not a constant, but for *preliminary* calculations its values for the sizes of wires and spacings (1 to 6 in.) ordinarily used for interior wiring is approximately as given in Table XII. The a-c values apply to single-phase, 2-phase 4-wire, and 3-phase 3-wire systems at any frequency from 25 to 60 cycles per second. For a d-c or single-phase 3-wire system proceed as for a 2-wire system, neglecting the presence of the neutral wire; the neutral wire should then be made the same size as each outside wire as thus calculated.

† Note that l is the length of *each conductor*; the total length of wire for a two-wire line is $2l$, for a three-wire line $3l$, etc.

Table XI. For Selecting Wire and Fuse Sizes for Motor Branch-circuits

Full-load Current Rating of Motor, Amperes	Minimum Allowable Size of Copper Wire, Am. Gauge or Cir. Mils.			For Running Protection of Motors		Maximum Allowable Rating of Branch Circuit Fuses		
	Rubber	Varnished Cambric	Slow Burning	Max. Rating of N.E.C. Fuses, Amperes	Max. Setting of Time-limit Protective Device, Amperes	Single-phase and Squirrel-cage and Synchronous (Full voltage, Reactor and Resistor Starting), Amperes	Squirrel-cage and Synchronous (Auto-transformer Starting). High-reactance Squirrel-cage, Amperes	D-c and Wound Rotor A-c, Amperes
Col. 1	2	3	4	5	6	7	8	9
1	14	14	14	2	1.25	15	15	15
2	14	14	14	3	2.50	15	15	15
3	14	14	14	4	3.75	15	15	15
4	14	14	14	6	5.0	15	15	15
5	14	14	14	8	6.25	15	15	15
6	14	14	14	8	7.50	20	15	15
7	14	14	14	10	8.75	25	20	15
8	14	14	14	10	10.0	25	20	15
9	14	14	14	12	11.25	30	25	15
10	14	14	14	15	12.50	30	25	15
11	14	14	14	15	13.75	35	30	20
12	14	14	14	15	15.00	40	30	20
13	12	14	14	20	16.25	40	35	20
14	12	14	14	20	17.50	45	35	25
15	12	12	14	20	18.75	45	40	25
16	12	12	14	20	20.00	50	40	25
17	10	12	12	25	21.25	60	45	30
18	10	12	12	25	22.50	60	45	30
19	10	12	12	25	23.75	60	50	30
20	10	12	12	25	25.0	60	50	30
22	8	10	10	30	27.50	70	60	35
24	8	10	10	30	30.00	80	60	40
26	8	8	8	35	32.50	80	70	40
28	8	8	8	35	35.00	90	70	45
30	6	8	8	40	37.50	90	70	45
32	6	8	8	40	40.00	100	70	50
34	6	6	8	45	42.50	110	70	60
36	6	6	8	45	45.00	110	80	60
38	6	6	8	50	47.50	125	80	60
40	6	6	8	50	50.00	125	80	60
42	5	6	6	50	52.50	125	90	70
44	5	6	6	60	55.0	125	90	70
46	4	6	6	60	57.50	150	100	70
48	4	6	6	60	60.00	150	100	80
50	4	5	6	60	62.50	150	100	80
52	4	5	6	70	65.0	175	110	80
54	4	4	6	70	67.50	175	110	90
56	4	4	6	70	70.00	175	120	90
58	3	4	5	70	72.50	175	120	90
60	3	4	5	80	75.00	200	120	90
62	3	4	5	80	77.50	200	125	100
64	3	4	5	80	80.00	200	150	100
66	2	4	4	80	82.50	200	150	100
68	2	4	4	90	85.00	225	150	110
70	2	3	4	90	87.50	225	150	110
72	2	3	4	90	90.00	225	150	110
74	1	3	3	90	92.50	225	150	125
76	1	3	3	100	95.00	250	175	125
78	1	2	3	100	97.50	250	175	125
80	1	2	3	100	100.00	250	175	125
82	0	2	2	110	102.50	250	175	125
84	0	2	2	110	105.00	250	175	150
86	0	2	2	110	107.50	300	175	150

Table XI. For Selecting Wire and Fuse Sizes for Motor Branch-circuits—*Continued*

Full-load Current Rating of Motor, Amperes	Minimum Allowable Size of Copper Wire, Am. Gauge or Cir. Mils.			For Running Protection of Motors		Maximum Allowable Rating of Branch Circuit Fuses		
	Rubber	Varnished Cambric	Slow Burning	Max. Rating of N.E.C. Fuses, Amperes	Max. Setting of Time-limit Protective Device, Amperes	Single-phase and Squirrel-cage and Synchronous (Full voltage, Reactor and Resistor Starting), Amperes	Squirrel-cage and Synchronous (Auto-transformer Starting). High-reactance Squirrel-cage, Amperes	D-c and Wound Rotor A-c, Amperes
Col. 1	2	3	4	5	6	7	8	9
88	0	2	2	110	110.00	300	200	150
90	0	1	2	110	112.50	300	200	150
92	0	1	2	125	115.00	300	200	150
94	0	1	2	125	117.50	300	200	150
96	0	1	2	125	120.00	300	200	150
98	0	0	2	125	122.50	300	200	150
100	0	0	2	125	125.00	300	200	150
105	00	0	1	150	131.5	350	225	175
110	00	0	1	150	137.5	350	225	175
115	00	0	1	150	144.0	350	250	175
120	00	0	1	150	150.0	400	250	200
125	000	00	0	175	156.5	400	250	200
130	000	00	0	175	162.5	400	300	200
135	000	00	0	175	169.0	450	300	225
140	000	00	0	175	175.0	450	300	225
145	200,000	000	0	200	181.5	450	300	225
150	200,000	000	0	200	187.5	450	300	225
155	200,000	000	0	200	194.0	500	350	250
160	200,000	000	0	200	200.0	500	350	250
165	0000	000	00	225	206.	500	350	250
170	0000	200,000	00	225	213.	500	350	300
175	0000	200,000	00	225	219.	600	350	300
180	0000	200,000	00	225	225.	600	400	300
185	250,000	200,000	000	250	231.	600	400	300
190	250,000	200,000	000	250	238.	600	400	300
195	250,000	0000	000	250	244.	600	400	300
200	250,000	0000	000	250	250.	600	400	300
210	300,000	0000	000	250	263.	....	450	350
220	300,000	250,000	000	300	275.	....	450	350
230	350,000	250,000	200,000	300	288.	....	500	350
240	350,000	250,000	200,000	300	300.	....	500	400
250	400,000	300,000	0000	300	313.	....	500	400
260	400,000	300,000	0000	350	325.	....	600	400
270	500,000	350,000	250,000	350	338.	....	600	450
280	500,000	350,000	250,000	350	350.	....	600	450
290	500,000	350,000	300,000	350	363.	....	600	450
300	500,000	400,000	300,000	400	375.	....	600	450
320	500,000	500,000	300,000	400	400.	....	....	500
340	600,000	500,000	350,000	450	425.	....	....	600
360	600,000	500,000	350,000	450	450.	....	....	600
380	700,000	500,000	400,000	500	475.	....	....	600
400	700,000	600,000	400,000	500	500.	....	....	600
420	800,000	600,000	500,000	600	525.	....	....	....
440	800,000	700,000	500,000	600	550.	....	....	....
460	900,000	700,000	500,000	600	575.	....	....	....
480	900,000	700,000	500,000	600	600.	....	....	....
500	1,000,000	800,000	600,000	....	625.	....	....	....
520	1,000,000	800,000	600,000	....	650.	....	....	....
540	1,000,000	900,000	600,000	....	675.	....	....	....
560	1,200,000	900,000	700,000	....	700.	....	....	....
580	1,200,000	1,000,000	700,000	....	725.	....	....	....
600	1,300,000	1,000,000	700,000	....	750.	....	....	....
625	1,400,000	1,000,000	800,000	....	782.	....	....	....

Table XII

System	Power Factor	Values of K for Copper
Direct current..............	....	11 *
Alternating current:		
Lighting load only.........	1.00	11
Lighting and power loads...	0.95	12
Lighting and power loads...	0.90	13.5
Power loads..............	0.85	15
Power loads..............	0.80	17

* This value is practically exact, the others are approximate only.

Commercial sizes of wire differ successively by about 25 per cent in the larger sizes and 60 per cent in the smaller sizes; the odd-numbered sizes smaller than No. 1 are not generally manufactured, although Nos. 3, 5, and 9 are sometimes made. From Table XIII* select the size of wire corresponding to the calculated area. Unless the calculated area in circular mils agrees very closely with the area of one of the conductors given in the table, the next larger size of wire should be selected.

Table XIII. 600-volt Rubber-insulated Copper Wire *

(Single braid, N.E.C. standard; conductivity of copper, 98 per cent)

Size, A.W.G.	Cross-section of Copper, cir mils	Overall Diameter, in.	Ohms per 1000 Ft at 77° F	Pounds per 1000 Ft of Insulated Wire	Reactance per 1000 Ft of Each Wire			
					Wires in Contact †		6 In. between Wires ‡	
					25 cycles	60 cycles	25 cycles	60 cycles
		Stranded	Stranded	Stranded				
	1,000,000	1.50	0.0110	3556	0.0130	0.0313	0.0283	0.0679
	900,000	1.44	0.0122	3223	0.0132	0.0317	0.0288	0.0692
	800,000	1.37	0.0138	2888	0.0134	0.0320	0.0292	0.0702
	700,000	1.31	0.0157	2554	0.0136	0.0328	0.0298	0.0717
	600,000	1.24	0.0184	2215	0.0138	0.0332	0.0306	0.0732
	500,000	1.09	0.0220	1805	0.0135	0.0324	0.0312	0.0749
	400,000	1.00	0.0275	1477	0.0141	0.0340	0.0322	0.0773
	300,000	0.90	0.0367	1133	0.0144	0.0347	0.0333	0.0800
	250,000	0.84	0.0441	962	0.0148	0.0354	0.0341	0.0820
0000	212,000	0.77	0.0520	807	0.0144	0.0347	0.0348	0.0835
000	168,000	0.71	0.0656	656	0.0149	0.0358	0.0358	0.0860
00	133,000	0.66	0.0827	535	0.0151	0.0362	0.0368	0.0883
0	106,000	0.61	0.104	438	0.0159	0.0381	0.0378	0.0908
1	83,700	0.57	0.133	350	0.0165	0.0396	0.0390	0.0936
		Solid	Solid	Solid				
1	83,700	0.53	0.126	340	0.0159	0.0381	0.0388	0.0933
2	66,400	0.47	0.159	268	0.0160	0.0385	0.0398	0.0958
3	52,600	0.44	0.201	221	0.0166	0.0400	0.0410	0.0986
4	41,700	0.41	0.253	184	0.0168	0.0403	0.0421	0.1011
5	33,100	0.39	0.320	153	0.0176	0.0422	0.0432	0.1037
6	26,300	0.37	0.403	129	0.0179	0.0430	0.0442	0.1061
8	16,500	0.27	0.641	76	0.0181	0.0434	0.0464	0.1112
10	10,400	0.23	1.02	54	0.0193	0.0464	0.0495	0.1165
12	6,530	0.21	1.62	40	0.0210	0.0505	0.0507	0.1218
14	4,110	0.20	2.58	30	0.0221	0.0532	0.0529	0.1270
16	2,580	0.16	4.10	20	0.0231	0.0554	0.0550	0.1320
18	1,620	0.14	6.51	13	0.0245	0.0588	0.0572	0.1372

* More extended wire tables will be found in the chapter on Bare Wires and Cables.
† Two insulated wires side by side, the insulation of the two wires in contact.
‡ Measured from insulation to insulation, equals distance between centers minus twice the thickness of insulation on either wire.

CHECK ON CALCULATION OF SIZE OF WIRE. CALCULATION OF ACTUAL POTENTIAL DROP. From the table, corresponding to the size of wire selected, take

r = the resistance per 1000 ft of conductor.
x = the reactance per 1000 ft of conductor.

and put

l = length of each conductor in feet.
I = amperes per conductor.
V = volts to neutral at receiver.*
$\cos\phi$ = power factor of receiver.

$$R = \frac{rl}{1000} = \text{total resistance per conductor.}$$

$$X = \frac{xl}{1000} = \text{total reactance per conductor.}$$

Then the actual volts drop *per conductor* is

$$v = \sqrt{(V\cos\phi + RI)^2 + (V\sin\phi + XI)^2} - V \qquad (2)$$

To a very close approximation in all ordinary cases this is equal to

$$v = RI\cos\phi\left(1 + \frac{X}{R}\tan\phi\right) \qquad (3)$$

If the drop as calculated does not check within a reasonable value with the drop assumed in calculating the size of the wire, select another size of wire and recalculate, etc. This refinement will be found necessary, only with low power factors and when the size of wire as calculated is either exceptionally large or exceptionally small, in which case the values given for the factor K in the formula for A, eq (1), may be in error by a large amount (in limiting cases 50 per cent or more).

In addition to making sure that the drop will be within a reasonable amount, one should also note whether the conductor selected is large enough mechanically and has the proper current-carrying capacity (see above), remembering also that the National Electrical Code requires that conductors through which power is supplied to a motor shall have a current-carrying capacity equal to 125 per cent of the full-load current taken by the motor.

Example of Calculation for a D-c or Single-phase System. Incandescent lamps taking 0.5 amp each are supplied with power from a 115-volt 60-cycle service through a set of feeders 200 ft in length which terminate at a cut-out cabinet as shown in Fig. 13. The lamps are grouped in sets of 10 lamps each (only one set is shown) and each set is connected to the cut-out cabinet by branch circuit conductors averaging 25 ft in length. Assuming a total potential drop of 3 volts, the drop per conductor will be 1.5 volts. The drop per conductor in the branch circuit should then be one-third of 1.5 or 0.5 volt, and the drop per conductor in the feeder circuit should be two-thirds of 1.5 or 1 volt. The respective calculations of the areas of the conductors in the feeder and branch circuits may then be carried on independently. In the branch circuit, referring to eq (1), $I = 10 \times 0.5 = 5$, $l = 25$ and $v = 0.5$, whence for copper wire

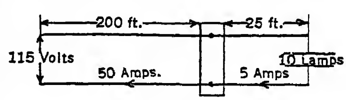


FIG. 13

$$A = \frac{11 \times 5 \times 25}{0.5} = 2750 \text{ cir mils}$$

From the wire table it is found that the next larger commercial size is No. 14 A.W.G., which has a resistance of 2.58 ohms per 1000 ft, and a reactance (assuming a 2.5-in. spacing) of 0.106 ohms per 1000 ft.

Referring to eq. (3), $V = 57.5$, $\cos\phi = 1$, $\tan\phi = 0$, $R = 2.58 \times 25/1000 = 0.0645$, $X = 0.106 \times 25/1000 = 0.00265$, whence

$$v = 0.0645 \times 5 = 0.323$$

Eq (2) gives the same value. This voltage is 35.4 per cent less than the assumed voltage of 0.5 volt, but since No. 13 wire is not a commercial size a closer realization of the assumed voltage is impracticable.

In the same manner, the required area of the conductors used in the feeder circuit is

* For d-c or a-c single-phase or a-c 2-phase, 2-wire systems $V = 1/2 \times$ (volts between wires); for a 3-phase system $V = \frac{1}{\sqrt{3}} \times$ (volts between wires); for a 2-phase 3-wire system $V = 1/2 \times$ (volts between either outside wire and middle wire). In calculating the normal drop in a single-phase 3-wire system pay no attention to the middle wire, i.e., assume a balanced load.

determined by substituting the following values in eq (1): $I = 100 \times 0.5 = 50$ amperes, $l = 200$ ft, and $v = 1$ volt. Hence

$$A = \frac{11 \times 50 \times 200}{1} = 110{,}000 \text{ cir mils}$$

From the wire table it is found that a No. 00 A.W.G. wire must be used.

The above calculations although made for a lighting load apply equally well to a motor load, except that in the final selection a wire must be chosen which will carry 125 per cent of the full-load current of the motor.

Example of Calculation for a Three-phase System. Power is to be supplied by three feeders each 400 ft in length to a three-phase, 60-cycle, a-c motor (power factor 80 per cent), the voltage between any two line wires at the service entrance being 550 volts. At full load the current taken by the motor is 50 amp. The voltage to neutral at the service entrance is $\frac{550}{\sqrt{3}}$ or 318 volts. The wires are to be spaced 6 in. apart.

Allowing a drop in each conductor of 3 per cent of the voltage to neutral, the drop will be 3 per cent of 318 or 10 volts approximately. The following values should then be substituted in eq. (1): $K = 17$, $I = 50$ amp, $l = 400$ ft and $v = 10$ volts, giving

$$A = \frac{17 \times 50 \times 400}{10} = 34{,}000 \text{ cir mils}$$

From the wire table it is found that a No. 5 A.W.G. wire might be used considering the potential drop alone but since the carrying capacity of the conductor must be 1.25×50 or 62.5 amp, it would be necessary to use a No. 4 wire.

Adopting a No. 4 wire and referring to eq. (3), $V = 318$, $\cos\phi = 0.8$, $\tan\phi = 0.75$, $R = 0.253 \times 400/1000 = 0.101$, $X = 0.101 \times 400/1000 = 0.040$, whence

$$v = 0.101 \times 50 \times 0.8 \left(1 + \frac{0.04 \times 0.75}{0.101}\right) = 5.3 \text{ volts}$$

87. WIRING OF BUILDINGS FOR MISCELLANEOUS DEVICES

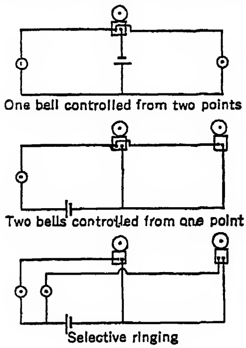
In addition to the wires designed to conduct electrical energy to lamps, motors and heating devices in a building, wires may be installed in connection with telephone, telegraph, district messenger and call-bell circuits, fire and burglar alarms, door-opening devices, watchman's clocks, and electric clocks. Since all these devices are operated at low voltage, it is unnecessary to use the same care in selecting and installing the wires as in the higher-voltage systems, except that in all low-voltage systems care must be exercised that the conductors shall not become crossed with light and power circuits.

PROTECTION OF LOW-VOLTAGE WIRING. When the conductors of any low-voltage system are brought into a building from the outside, an approved protective device must be located as near as possible to the entrance of the wires to the building. With the exception of instrument circuits of telegraph systems, where cut-outs only are required, protective devices must contain a lightning arrester with a ground connection and a cut-out or heat-coil (see Pender & McIlwain). The conductors beyond the protective device in low-voltage systems need be insulated only sufficiently to prevent short circuits and the consequent interruption of service. When bunched together in vertical runs the wires must be inclosed in a fire-resisting covering to prevent the wires carrying fire from floor to floor. Low-voltage circuits may be run in the same shaft with light and power circuits, provided the two classes of wires are separated by at least 2 in. or one of the classes is run in a non-combustible tubing. Low-voltage wires may not be run in any case in the same conduit or raceway with lighting or power wires.

BELL WIRING. In its simplest form a bell-wiring system consists of a battery (or a bell ringing transformer connected to the lighting circuit), a bell-push, a bell or buzzer, and the connecting wire. Paraffin-impregnated double-cotton covered wires, or rubber-covered fixture wire in damp locations, ranging in size from No. 16 to No. 22 A.W.G. gage, are usually employed, either singly or in the form of twin wires. The wires may be fished, concealed in molding, or run exposed along the finish of the room. When a large number of bell-wires must be run together through a building, as in a hotel annunciator system, the wires are frequently inclosed in a cotton braid or lead sheath. In many buildings supplied with an a-c source of power a small step-down transformer is used in place of the batteries, thus effecting a saving in the cost of the installation and eliminating the possibility of run-down batteries. In Fig. 14 are shown several systems of bell wiring; these figures are self-explanatory.

BURGLAR ALARMS. In place of the bell-push used in bell-wiring systems, the bell circuit may be closed by some circuit-closing device attached to windows, doors, etc.,

so that the opening of any window or door in the building will be made known by the ringing of a bell. As such a system has the objection that it will not operate if the battery is run down or if the wires are cut, a closed-circuit system is preferable, in which the opening of a window or door opens the circuit and a relay in turn closes the bell circuit. In Fig. 15 is shown a typical open-circuit and in Fig. 16 a typical closed-circuit system. In Fig. 17 are shown a door spring A and a window spring B. Photoelectric cells are also used for actuating alarms.

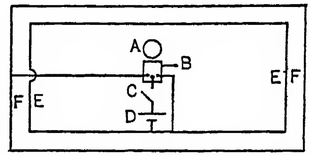


One bell controlled from two points

Two bells controlled from one point

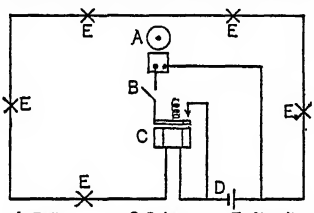
Selective ringing

FIG. 14



A Bell B Constant Ringing Drop
C Day Switch D Battery
E and F Wires to which closing springs are connected

FIG. 15



A Bell C Relay E Circuit-opening Springs
B Day Switch D Battery

FIG. 16

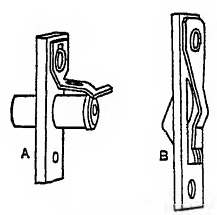


FIG. 17

FIRE ALARMS. In the manually operated system glass disks are placed at convenient points throughout the building, and with each disk is provided a hammer, with which the disk may be broken in case of fire. The breaking of the disk opens or closes an electric circuit by means of which electric gongs are rung throughout the building. In the automatic system the expansion or fusion by heat of elements in circuit-closing or -opening devices, usually placed on ceilings, causes gongs to ring throughout the building or in the watchman's office.

DOOR-OPENING DEVICES AND ELECTRIC CLOCKS. Doors may be opened by pressing a button at some distant point and thereby closing a circuit, through which current flows, energizing a magnet, which releases the nosing in the door frame. In the same manner, a moving disk on a master clock may periodically close a circuit, through which a magnet is energized which operates a pawl attached to the hands of another clock.

WATCHMAN'S CLOCK. Circuit-closing devices are placed at different points throughout a building so that a watchman on his rounds may close the circuit by means of a handle or special key. The closing of the circuit at any station makes a record on a dial in the watchman's clock indicating the time at which the circuit was closed at each particular station, the stations being designated by numbers. To obviate the possibility of run-down batteries, magneto systems are often installed, a small magneto being placed at each station.

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GROUNDING OF ELECTRIC CIRCUITS AND ACCESSORIES

By R. F. Abele

89. PURPOSES OF GROUNDING

Some electric circuits are operated insulated, that is, with no intentional electrical connection from any point of the circuit to the ground; others are grounded by one or more conductors provided for the purpose.

Among the considerations influencing the decision of this point are: (1) danger of shock to persons touching the circuit; (2) danger of fire from escaping current, spark, or arc; (3) elimination of electrical oscillations set up in a circuit by the sudden changes in current resulting from the making and breaking of an arc, usually to ground, usually referred to as an " arcing ground "; such grounds may give abnormal voltages particularly on a-c circuits where the arc forms and is extinguished twice every cycle; (4) decreased strain on circuit insulation in case of accidental ground; (5) improvement of relaying, i.e., circuit protection by operation of relays; (6) the utilization of the ground and conductors on or in the ground for carrying return currents; (7) the danger of electrolysis and disturbance of telegraph, telephone, and radio systems by currents escaping to the ground.

In general, high-tension systems are grounded to protect apparatus and service and to aid relaying, whereas low-tension systems are grounded to protect life and guard against fire hazards.

Enclosures, frames, supports, etc., which may become energized owing to breakdown of electrical insulation are usually grounded, especially where others than trained employes are exposed.

90. ELECTRIC SHOCK FROM GROUNDED AND UNGROUNDED CIRCUITS

Of the various considerations affecting the desirability of grounding a circuit, that of shock is perhaps the most important. A person touching a circuit at any two points between which there is a difference of potential will receive a shock. The danger or severity of shock from touching simultaneously two line wires of a circuit is not affected by grounding the circuit, but the danger or severity of shock from touching simultaneously either wire and the ground does depend upon whether the circuit is grounded or not.

It should be noted that the sensitiveness of different people to shock varies a great deal and that the severity of a shock depends as much on the surface resistance of the skin (whether dry or wet), and on the parts and organs of the body through which the current passes, as on the voltage. It is therefore impossible to say that any voltage used in practice is so low that under no condition can it give a dangerous shock. However, considering ordinary conditions and the result of a majority of the shocks, circuits of voltages up to and including 150 volts from conductor to ground may be considered as not likely to cause a serious shock, whether grounded or ungrounded.

VOLTAGE TO GROUND OF GROUNDED CIRCUIT. When a circuit is intentionally grounded, the voltage of the grounded point is made permanently that of the earth, and the potential of every other point of the circuit becomes fixed with respect to the ground and may be readily calculated. Each conductor then carries a definite, predetermined risk, instead of an indefinite risk, which may be nothing or may be very great.

CURRENT THROUGH GROUND CONNECTION. In case of a ground, either intentional or accidental, the grounded point of the circuit is brought to ground potential, and the resultant charge on the earth is no longer zero. This charge must come by conduction from the circuit and must therefore enter the earth through the ground connection. As long as there is a change in voltage of the wires of the circuit due to the ground connection a current will flow through the ground connection. Such a current is only momentary in the case of a d-c circuit, but flows as long as the ground continues with alternating currents.

The number of amperes which flows through such a ground connection is roughly proportional to the voltage of the system and its electrostatic capacity to ground. In addition to current due to the capacity to ground there may be a leakage current due to imperfect insulation.

SHOCK FROM SINGLE CONTACT WITH HIGH-VOLTAGE CIRCUITS. Circuits having a normal line voltage of 11,000 volts or above are generally of sufficient extent (and therefore have sufficient electrostatic capacity) so that a fatal shock may be received from wire to ground, due to the current passing to ground through the body, even though the circuit be otherwise perfectly insulated from ground.

SHOCK FROM INTERMEDIATE VOLTAGE CIRCUITS. In general the power available for shock in the current escaping from an insulated circuit to ground is small compared to that in case of contact with the two wires of the circuit. Consequently, there is a class of circuits whose voltages are high enough to give serious or fatal shocks where contact is made with two points of the circuit, but not high enough to give serious (or sometimes even appreciable) shocks from wire to ground when the circuit is ungrounded. These circuits are intermediate between the low-voltage (220-volt) and high-voltage (11,000-volt) circuits above mentioned. This class includes 440- and 550-volt power circuits and *sometimes* 1100- and 2200-volt primary circuits, if leakage is small.

91. FIRE HAZARD

The ground current should be limited in amount or duration so that an arc cannot form and the ground conductor cannot burn off.

The National Electrical Code is the authority on the subject of grounding from the standpoint of fire hazard. These rules, given in the National Electrical Code (1935 edition), are abstracted in Table I.

92. ARCING GROUNDS

In normally ungrounded high-voltage circuits the current to ground is sometimes sufficient to maintain an arc between one of the wires and a grounded conductor near it but not quite in contact with it, producing a so-called " arcing " ground. Such grounds have been found to produce destructive voltages on the circuit, sometimes as great as six times the normal voltage. By metallically grounding the neutral or one wire of the circuit the rise of voltage is reduced.

LIMITING SHORT-CIRCUIT CURRENT BY RESISTANCE. In a grounded circuit every accidental ground becomes a short circuit. As such short circuits may be destructive to generating machinery, the current is sometimes limited by a resistance or reactance in the ground connection (usually made at the neutral). The amount of resistance required depends on the percentage of short-circuit current which is permissible. The resistance has, however, the disadvantage that it increases the strain on the insulation, so that, although the short-circuit current is reduced, the benefit to the insulation is also reduced. Where the circuit is grounded as a remedy for arcing grounds and no increased factor of safety on the insulator for normal condition of operation is desired, the use of resistance in the ground connection is allowable.

A system may be regarded as effectively grounded when its neutral is grounded in such a way that any arcing grounds which may occur cannot build up dangerous voltages.

Such a system may be either (*a*) directly or solidly grounded, (*b*) resistance grounded, (*c*) reactance grounded.

(*a*) A system is said to be **directly or solidly grounded** when its neutral is grounded in such a way that only the impedance in the ground leads and that of the ground itself are included and are of relatively small value. Under such conditions there can be no overvoltages due to arcing grounds.

(*b*) A system is said to be **resistance grounded** if resistance of an appreciable value is introduced in the neutral ground or ground leads. Such a system is effectively grounded if the resistance does not exceed the critical value above which maximum voltage to ground exceeds normal voltage between phases at normal frequency.

(*c*) A system is said to be **reactance grounded** if reactance of appreciable value is introduced into the neutral ground or ground leads, either in the form of reactance coils or of grounding transformers. Such a system is effectively grounded if the neutral shift under short-circuit conditions does not exceed 40 per cent of the line to neutral voltage, as under such conditions dangerous overvoltages cannot occur from arcing grounds.

LIMITING INSULATION STRAIN. In high-voltage circuits the cost of insulators is an important element. In an ungrounded circuit each insulator must be large enough to stand full line voltage (between wires), for any phase may become grounded accidentally. Where the neutral of the circuit is grounded the maximum voltage on any insulator can never exceed the voltage to ground. The voltages to neutral of a single- or two-phase system is 50 per cent of the voltage between lines, and the voltage to neu-

tral of a three-phase system is 58 per cent of the voltage between lines. A circuit with grounded neutral therefore requires insulators of from 50 to 58 per cent of those for same circuit with insulated neutral, or for the same size insulator the line voltage can be from 73 to 100 per cent higher with grounded neutral than without. These figures neglect the fact that the maximum strain is constant for grounded neutral, and occurs only for a short time at irregular intervals with insulated neutral.

93. RELAYING

Grounding is of assistance in facilitating operation of relays, especially on transmission lines. There are circuits, however, where it is better practice to operate without a system ground, in order to prevent operation of relays. For example, it is desired to keep the system alive, even if one side of the circuit should become grounded, in some types of primary distribution networks.

See Section 12, Art. 12.

94. SYSTEM GROUNDING

The practice of different companies is not consistent with respect to insulating or grounding of various classes of circuits, but the following rules cover the usual practice.

1. **TRANSMISSION SYSTEMS** do not involve questions of hazard, as far as circuit grounding is concerned, as they necessarily are dangerous. Neutrals are usually grounded to reduce insulation stresses and arcing grounds, and facilitate automatic tripping of the line. Where this last feature is not desired, grounding is, of course, a disadvantage.

2. **PRIMARY DISTRIBUTION SYSTEMS** usually have grounded neutral, not necessarily because of inherent advantages, but often because many such systems are of the 4000 volt 4-wire Y-type, converted from an older 2300-volt delta system. Ungrounded delta systems are likely to show better continuity of service.

3. **SECONDARY DISTRIBUTION SYSTEMS** are grounded as a protection to life and against fire, particularly in view of the possibility of crosses with the primary lines.

4. **D-C SYSTEMS, EXCEPT THREE-WIRE SYSTEMS,** are generally considered safer ungrounded. Neutrals of three-wire circuits are grounded if grounding will limit the voltage of the outside wires, to ground, to 150 volts or less. Where d-c circuits can come in contact with high-tension a-c lines, it is better to ground. Railway circuits are invariably grounded, in order to enable track rails to be used for returns.

TRANSMISSION LINES. The various methods used in grounding transmission circuits are as follows:

A. Solid ground—a low-resistance connection.

B. Resistance ground—connection through a resistor.

C. Reactance ground—air core or iron core reactor.

The Report of the A.I.E.E. Committee on Grounding, 1931, shows that systems of 110 kv and above are nearly always classified as solidly grounded. However, there appears to be a tendency in high-voltage systems to limit the ground current by some kind of impedance, resistance, reactance, or partial grounding of the total transformer capacity. On systems of 66 kv and below, the solid grounding practice is less pronounced. In this range, answers to a questionnaire sent out by A.I.E.E. show 51 per cent solidly grounded, 9 per cent grounded through resistance, 8 per cent grounded through reactors or grounding transformers, and 32 per cent with free neutrals.

There are three conditions to be considered in relation to grounding of neutrals:

A. Neutral anchored to ground potential and system provided with prompt disconnection in event of one leg being grounded.

B. Neutral potential not anchored, but system provided with prompt disconnection in event of one leg being grounded.

C. Neutral potential not anchored and system capable of operating for long period with one leg grounded, i.e., while repairs on grounded leg are being made.

PRIMARY DISTRIBUTION LINES. Not all grounds on a grounded system necessarily trip the circuit because such grounds are likely to have high resistance. The actual difference between grounded and ungrounded systems, as regards service protection, is not as great as might be expected. With regard to life hazard from fallen wires, grounding, for the same reason, is not a perfect protection.

The **Hood system,** or common neutral ground, in which the same ground wire is used for primary and secondary circuits, tends to give a low impedance for line-to-ground faults.

A-C SECONDARY SYSTEMS. Table I shows the National Electrical Code requirements. In addition to the service grounds, some companies ground the neutrals at the supply transformers. See also Art. 83.

Table I

Type of Current	Type of Circuit	Point of Attachment of Ground
D-c	Two wire—not to exceed 300 volts	At the supply station only. Recommended that ground be eliminated if voltage to ground of either conductor is over 300 volts after grounding.
	Three wire	Neutral, at one or more supply stations, but not at individual services.
A-c	Two wire or three wire	At every service end, provided voltage to ground does not exceed 150 volts. Recommended for over 150 volts but not over 300 volts. Additional grounds optional at transformer or one or more points of system.

95. STATION GROUNDING

GROUND MASS. The ground mass of a station consists of the ground electrodes buried in the earth to provide earth connection for the station. It is usual to provide a low-impedance bus network, with numerous connections to the station ground mass, to which equipment to be grounded is connected. The building steel is usually tied to this network, as is also the water piping if big enough to carry the current that may flow in it. See Arts. 83 and 98.

NEUTRALS. It is usual to connect the neutral of generators and transformers to the protective ground system. Lightning arresters may also be tied to the station ground mass.

FRAMES AND CASES. Machine frames, meter cases, cable sheaths, etc., are connected to the ground mass if they contain conductors which are or may be at 150 volts or more above ground. Instrument transformer grounds should be made in a way to avoid errors due to " sneak currents " in parallel paths. Suitable board frames are usually not grounded if they support bare conductors operating at 150 volts or more.

OUTDOOR SUBSTATIONS. In the case of outdoor substations, it is usual to provide fences with special grounds and not connect to the ground mass.

96. POLE LINE GROUNDING

Steel towers should be grounded so that power arcs will not create dangerous potential gradients at and near their bases.

OVERHEAD GROUND WIRE. Where an overhead ground wire is used, it should be designed so as not to flash over to the line insulators, i.e., its surge impedance, together with that of the tower or ground lead, should be greater than that of the system earth connection itself.

COUNTERPOISE GROUND. If the soil is of high resistivity, where steel towers are used, a counterpoise ground may be used, consisting of a wire or network of wires laid in the earth at a considerable distance from the tower and connected to it.

WOODEN POLES. The value of a ground wire on a wooden pole line is questionable, as without it the pole adds so much to the insulation. The pole has a flashover strength of about 160 kv per foot of length.

The grounding of transformer cases, on poles, is not universal practice, and there are advantages in not grounding.

97. CABLE SHEATH GROUNDING

Cable sheaths are grounded to prevent their attaining voltages dangerous to workers, when faults occur. Single-conductor high-tension a-c cable sheaths are likely to have induced high voltages under normal conditions, unless they are grounded. Sheaths should be grounded at station ends, at connections to overhead lines, and at manholes, where practicable.

98. GROUND CONNECTIONS

TYPES. Good ground connections require contact with permanent earth moisture and should extend below the frost line. Most of the resistance of a ground connection is in the shell of earth immediately surrounding it.

Water pipes make an ideal ground connection for alternating currents but cannot be used for direct current because of electrolysis.

The next best ground is a pipe, or group of pipes, driven into the ground. Plate grounds are now virtually obsolete.

GROUNDING TO CONDUCTORS USED FOR OTHER PURPOSES. Where an extensive system of water pipes buried in the earth is available, a satisfactory ground connection can be made directly to the pipes. This practice, however, is now opposed by the American Waterworks Assn., 1935. Gas pipes are not suitable for ground connections, as in case of broken joints the resulting arc might ignite the escaping gas. Steam heat pipes are heat insulated (and therefore partially electrically insulated) from the earth. Where there are two ground connections, one to an electric railway track and the other to a water pipe, there is danger that railway current may thereby be introduced into the water piping system and produce electrolysis. Sheaths of cables are not good ground as the cables might be injured by the currents in their sheaths, either from electrolysis or unforeseen heating. In fact, cable sheaths, like other buried conductors, have a tendency to collect stray currents from electric railways, and special ground connections sometimes have to be made to permit these currents to escape without producing electrolysis. Steel frames of buildings present only a limited surface to ground at the foundations and are therefore good only when the current to be discharged will be proportionally small.

CONNECTIONS TO PIPING SYSTEM. The National Electrical Code (1935) contains the following requirements:

Size of Ground Wires. The grounding conductor for a d-c supply system shall have a current-carrying capacity not less than that of the largest conductor supplied by the system and in no case smaller than No. 8 A.W.G. The grounding conductor for an a-c supply system shall have a current-carrying capacity not less than one-fifth that of the conductor to which it is attached except that in no case shall it be smaller than No. 8.

Method of Making Ground Connections. The grounding conductor shall be attached to circuits, cabinets, equipment and the like, which are to be grounded, by means of suitable lugs, clamps, blocks, or other approved means.

The grounding conductor shall be attached to the pipe or rod (1) by means of an approved bolted clamp to which the conductor is soldered or otherwise connected in an approved manner or (2) by means of a pipe fitting, a plug, or approved device screwed into the pipe or into the fitting, or (3) by other approved means.

Where a non-conductive protecting coating such as paint or enamel is used to protect the equipment, conduit, couplings, and fittings, such coating shall be completely removed from threads and other surfaces in order to ensure a good contact between ground clamp and equipment. Pipes and rods used as ground electrodes shall have clean metal surfaces, and shall not be covered with paint, enamel, or other poorly conducting materials.

GROUNDING TO SPECIAL PIPES, PLATES, ETC. When suitable buried conductors are not already available, direct connection to the ground may be made (1) by driving a pipe or rod vertically into the earth, (2) by burying a wire or ribbon in the earth, or (3) by burying a plate in the earth. The pipe or rod form of ground connection is now generally accepted as the best.

FACTORS AFFECTING THE IMPEDANCE OF GROUND CONNECTIONS. The impedance offered to a flow of current into the ground depends almost entirely on the nature of the soil. To obtain low ground impedance, it is necessary to have electrolytic moisture in contact with the earth connection, or to have a very large area of earth cross-section for the current. Table II shows a typical case of the influence of moisture on soil resistivity. Other factors affecting the impedance are (1) the extent of metal surface in contact with the earth, (2) the thoroughness of this contact, (3) the wetness of the earth, (4) the distance between this ground connection and the ground connections (if any) at which the current leaves the earth.

The effect of the distance upon the impedance between a pair of ground electrodes, such as ground pipe and a test electrode, is frequently overlooked. If the electrodes are close together, the impedance will be low on account of the shortness of the current path. If the distance between electrodes be increased, the impedance will be increased until a certain distance, usually about 25 ft, is reached, after which it will steadily decrease on account of the increase in the cross-sectional area of the earth path (Fig. 1).

The potential drop in the soil near a ground connection is surprisingly high, as shown by a typical case in Table IX.

DEFINITION OF THE RESISTANCE AND IMPEDANCE OF A SINGLE GROUND CONNECTION. If the distance between a pair of ground connections be fixed and the drop of potential from one of them G_1 and a point P between them be plotted against the distance of P from G_1 a curve such as $G_1\,DA$, Fig. 2, will be obtained, if the two ground connections G_1 and G_2 are similar. If, on the other hand, the ground connection G_1 has very much more perfect and extensive contact with the earth, the curve of

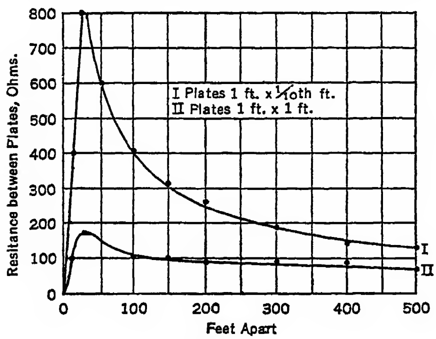

FIG. 1

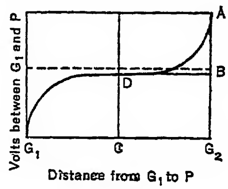

FIG. 2

potential drop will have the shape $G_1\,DB$. Both curves have the peculiarity of being steep near the restricted electrodes and of becoming relatively flat between them, indicating that the greater part of the resistance occurs near the electrodes, where the current is restricted to a limit area, practically no drop occurring where the current spreads out.

Table II. Effect of Moisture Content of Red Clay Soil on Resistance

Per Cent Moisture in Terms of Dry Earth	Resistance, ohms per cc	Per Cent Moisture in Terms of Dry Earth	Resistance, ohms per cc
12	170,000	20	9000
14	65,000	30	5000
16	17,000	60	5000
18	10,000		

The impedance of the ground connection G_1 is then defined as the ratio of the potential drop CD, to the current entering the ground at G_1, and similarly for the ground connection G_2. The resistance may be approximated by measuring the impedance at different frequencies, and by extrapolation, finding the impedance at zero frequency. A more accurate way, however, is to use a special bridge which will give a phase balance as well as a resistance balance. See Bureau of Standards Technological Paper 108; also Section 5, Art. 9 and Section 6, Art. 8.

RESISTANCE OF SOME TYPICAL GROUND CONNECTIONS. The resistance of ground connections varies so greatly with the condition and nature of the soil that data on resistance have only local application. The data in Table III are presented merely to give a general idea of the results which are obtained in practice.

Table III

Authority	Time of Year	Description of Soil	Type of Ground Connection	Dimensions	Depth, feet	Ohms
Hoxie.......	Feb.-June	Surface loam	Plate	1 ft × 1 ft	..	1940
	Feb.-June	Moist black loam	Plate	6 ft × 2 ft 3 in.	6	113
	Feb.-June	River bottom	Plate	1 ft × 1 ft	..	132
	Feb.-June	Gravelly soil	Pipe	1 1/4 in.	5	630
	Feb.-June	Gravelly soil	30 Pipes	1 1/4 in each	5	13
	Feb.-June	Swampy soil	7 Pipes	1 1/4 in each	5	15
Del Mar....	Summer	Rock and loam fill	Plate	15 in. × 15 in.	6	155
Hayden.....	Aug.-Sept.	Clay loam	Pipe	2 1/2 in.	3 3/4	26.1
Hayden.....	Feb.-March	Clay loam	Pipe	2 1/2 in.	3 3/4	120
Hayden.....	June-July	Clay loam	Pipe	2 1/2 in.	3.1	35.4
	March-April	Clay loam	Pipe	2 1/2 in.	3.1	240

The Bureau of Standards (T–108) gives the following comparative table of ground pipe resistances in various kinds of soil:

Number of Grounds Tested	Soil	Resistance Ohms		
		Average	Minimum	Maximum
24	Fills and ground containing more or less refuse as ashes, cinders, and brine waste.	14	3.5	41
205	Clay, shale, adobe, gumbo, loam, and slightly loam with no stones or gravel.	24	2.0	98
237	Clay, adobe, gumbo, and loam mixed with varying proportions of sand, gravel, and stone.	93	6.0	800
72	Gravel, sand, or stones with little or no clay or loam.	554	35	2700

The Bureau of Standards has made measurements of resistance of a ground connection using a driven pipe. The pipe was $^3/_4$ in. galvanized and driven in rather stony clay ($\rho = 7420$). The figures in Table IV give the results obtained with accuracy within 2 to 3 per cent.

Table IV

Number of Specimens	Depth, feet	Average Measured Resistance, ohms	Number of Specimens	Depth, feet	Average Measured Resistance, ohms
8	1	167	1	6	42.5
8	2	76.3	3	7	31.0
8	3	47.2	1	8	22.5
8	4	37.5	1	9	23.5
8	5	49.5	8	10	36.5

The irregularity of the soil produces the variance in figures. The lower depths were found to be of boulders, 2 to 5 in. in diameter, which have high resistance. These resistances were obtained by the ammeter-voltmeter method, using 60-cycle current.

Design of Ground Connections

It is as a rule impracticable to install a ground connection of sufficiently low conductivity to allow the passage of the maximum current it may have to carry without producing a considerable rise of voltage. The conductivity which a ground connection should have is therefore largely a matter of experience and judgment rather than of exact calculation. Cheapness and durability generally determine the material, usually iron or copper, and the thickness.

COPPER PLATES. Formerly the most common form of ground connection consisted of a copper plate imbedded in charcoal or coke in moist earth. Coke, so often recommended for earth connections, is not a good conductor in itself. It attracts and holds moisture, but since that moisture does not contain an electrolyte in solution, it may leave the earth connection with high resistance. Furthermore the sulfur in the coke is likely to corrode the copper plate and thus increase its resistance. Some engineers are of the opinion that neither charcoal nor coke is of any particular use in connection with a ground plate. The corrosion of copper ground plates is greatly reduced by coating with tin. As the conductance of a ground connection is not proportional to the area of the metal plate, the use of a single large plate is an uneconomical method of obtaining a low resistance.

COPPER WIRE OR RIBBON. Wire or ribbon has the advantage of a lower resistance than a plate for the same area and is useful where bedrock is near the surface. The wire has the further advantage that no special material or connection is required. The wire may be laid out in a straight line or coiled with turns wide apart into a plane spiral. The latter form is sometimes made by wrapping a number of turns of wire around the butt of a pole before setting it.

PIPES DRIVEN INTO THE GROUND. On the basis of the first cost, and of inspection, resistance measurement, etc., the driven pipe or rod is probably the best of all

forms of direct ground connections. Iron is the cheapest available metal and has proved its serviceability even when imbedded in salty marshes.

The pipe is often galvanized, sherardized, or copper covered, from $^3/_4$ in. to 2 in. in nominal size, driven 6 ft or more into the moistest ground available. The effect of depth on resistance is shown in Table V. The diameter of a pipe has little effect upon its earth resistance, as shown by Table VI. To facilitate driving, the pipe is sometimes provided with a pointed casting at the lower end and cap at the upper end. The pipe has the advantage that it is cheap to install, as no hole is dug, and being driven vertically, it can tap any conducting stratum of the earth which is practically accessible. It is highly efficient as regards conductivity per square foot of surface exposed to the earth, and while the resistance of a single pipe will exceed that of a large plate, a higher and more permanent conductivity can be obtained from several pipes driven at some distance apart and connected in multiple, than from an equal expenditure in money on a single large ground plate. See Tables VII and VIII. Ground pipes or plates in multiple should be not less than 6 ft, and preferably 10 ft, apart, in order to avoid superimposing two zones of high current density.

Table V. Effect of Depth upon Resistance of Pipe in Fairly Wet Soil

Depth in the Fairly Wet Soil, ft	Resistance, ohms	Depth in the Fairly Wet Soil, ft	Resistance, ohms
1	152	6	37
2	90	7	32
3	66	8	29
4	52	9	26
5	42	10	24

Table VI. Effect of Pipe Diameter upon Resistance (Pipe buried 10 ft)

Overall Diameter of Pipe, cm	Resistance, ohms	Overall Diameter of Pipe, cm	Resistance, ohms
1	37	5	28 1/2
2	33	6	27 1/2
3	31	7	26 1/2
4	29 1/2	8	26

Table VII. Effect of Number on Parallel Ground Pipes on Resistance, Pipes Being 10 Ft Deep and 10 Ft Apart

Number of Pipes	Per Cent of Resistance of One Pipe	Number of Pipes	Per Cent of Resistance of One Pipe
1	100	5	19
2	57	6	12
3	38	7	9
4	27	8	8

Table VIII. Effect on Resistance of Separation of Pipes, 5 Ft Deep

Distance Apart, ft	Per Cent of Resistance with No Separation	Distance Apart, ft	Per Cent of Resistance with No Separation
0	100	3	61
1	72	4	58
2	65		

Table IX. Voltage Drop at Various Distances from a Pipe Ground

Distance from Ground Pipe, ft	Per Cent of Total Voltage Drop	Distance from Ground Pipe, ft	Per Cent of Total Voltage Drop
0	0	6	76
1	34	10	88
2	48	Infinite	100
4	65		

If the ground available is naturally dry the conductivity may be improved as much as 50 per cent by cupping out the soil around the pipe at the surface and adding salt and

water. A concentration of 1/2 per cent calculated on the soil moisture accomplishes nearly 90 per cent of the highest concentration and may be used as a basis for calculations of amount of salt required. A neat form of earth unit is shown in Fig. 3. A cylinder of metal or earthenware of any available diameter is set around the pipe at the surface of the ground and covered by a lid. This receptacle will hold the salt. Its advantages lie in the easy construction of the connection and protection of surrounding vegetation from the saline water.

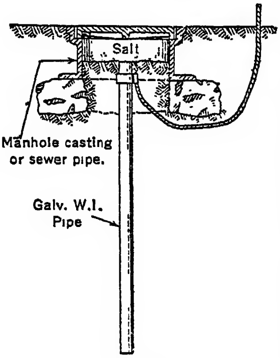

FIG. 3

PIPE GROUNDS FOR LIGHTNING ARRESTERS. A number of grounds is desirable at each installation. These numerous ground connections are joined together by a copper connection. It sometimes happens that a good ground cannot be conveniently made near the arrester, or that a better one can be made at a more distant point. In this case it is recommended that the principal ground be made at the more distant point, but that a ground of some sort, the best possible under the conditions, be made directly underneath the arrester.

CONNECTIONS TO GROUND PLATES AND PIPES. The wire connecting the apparatus to be grounded to the buried conductor should lead directly to the latter with as few bends as possible; this is particularly important in grounding lightning arresters, in which case a flat strip is also better than a round wire. All exposed joints and those buried in the ground should be made in such a way that they will not rust off. The buried connection from a ground plate to the surface should be so protected that it will not rust off or be cut off when excavations are made for other purposes. It is also necessary that the connections be protected from mechanical injury and theft, from the surface of the ground to above the level at which it can be reached. To insure reliability, connections should as far as possible be made so that their continuity can be determined by inspection.

W. W. Lewis' formula gives the circular mils A to carry the maximum current of I amperes for S seconds:

$$A = 10.6\,IS$$

This is based on clamped, bolted, or soldered connections and 250 deg cent final temperature. For brazed joints and a final temperature of 450 deg cent the numerical factor is 8.7. The carrying capacity of ground electrodes is treated theoretically and experimentally by H. G. Taylor, *Jour. I.E.E.* (London), 1935, vol. 77, pp. 542–560.

Tests of Ground Connections

Ground connections should be frequently tested to determine their continuity and occasionally to determine their impedance. Alternating current should be used, in order to avoid the effects of polarization. In order to pass a current through a ground connection it is necessary to have a second connection in order to complete the circuit. Where a large water-piping system is available for the return connection, an approximate measure of the resistance can be obtained by passing a current from the ground connection to an accessible part of the piping, say a hydrant, and computing the resistance by dividing the volts by the amperes. The resistance thus obtained includes both the resistance to be measured and that of the connection through the piping system. As the latter is probably much the smaller and may be insignificant, the result gives an approximate idea of the resistance and locates a discontinuity as an infinite resistance, that is, no current flowing. Where there are two similar grounds, current may be passed from one to another and the sum of the two resistances, or the average resistance, obtained.

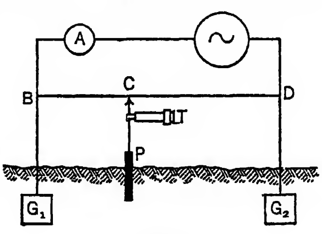

FIG. 4

When a more accurate determination of the individual resistances of two or more ground connections is desired, the connections shown in Fig. 4 may be employed and a curve obtained by test, similar to that shown in Fig. 2. The apparatus required comprises a source of alternating current, a rod P which may be driven into the earth, a high-resistance wire BD stretched between the two ground connections G_1 and G_2, an ammeter A, and a telephone receiver T. The

contact C is slid along the resistance wire until the telephone receiver is silent. The drop of potential between G_1 and the rod is then the same as between G_1 and the contact C. The drop between G_1 and P is therefore equal to

$$\frac{(\text{Length of wire } BC)}{(\text{Length of wire } BD)} \times (\text{Total drop from } G_1 \text{ to } G_2)$$

By taking a series of observations with P driven into the ground at various points the curve of potential drop is readily plotted and the resistance of each ground connection, as defined above, obtained. Bureau of Standards Technological Paper No. 108 describes an improved Kohlrausch bridge, for the same purpose.

On grounded services, a very simple way of determining whether or not a satisfactory ground has been obtained is as follows: (1) On a 220–110 volt secondary system connect the underground leg through a 5-amp fuse to an available point on the water-piping system. If sufficient current flows to blow the fuse, the ground connection may be considered satisfactory. (2) On a 440–220 volt system, the current should be sufficient to blow a 10-amp fuse.

Under some conditions it is of advantage to have a disconnecting clamp in the ground circuit. It decreases the hazard to men working on the circuit and also affords a means of disconnecting the ground for test purposes.

Care of Ground Plates and Pipes

The greatest trouble with ground plates and pipes arises from electrolysis, and periodic resistance measurements should be made as described above. When a ground connection shows an abnormally high resistance, it is usual to supplement it with a new ground connection, as it seldom pays to remove the old ones. Ground connections deteriorate rapidly if equipped with salt boxes; but even under the worst conditions they are likely to last many years. J. L. R. Hayden cites the case of some pipe grounds which maintained the same average resistance for three years.

SPECIFICATIONS FOR GROUND PIPES AND PLATES. In the case of ground plates, the following data should be specified: material of plate; size of plate, area, and thickness; depth at which the plate shall be buried; amount of charcoal below and above it; method of connecting the plate to its cable or wire (usually soldered and bolted); whether the plate shall also be connected to the track rails and if so, how.

In the case of pipe ground connections, the following data should be specified: diameter of pipe; whether galvanized, sherardized, etc.; depth to which the pipe shall be driven; method of finishing the top of the pipe; height the pipe shall project above the ground.

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ELECTROLYSIS OF UNDERGROUND STRUCTURES

By W. A. Del Mar

100. STRAY CURRENTS

The following discussion will be limited to the electrolysis of underground structures, that is, to the corrosion of metal work in the earth, due to the action of stray currents from the grounded rails of electric railroads.

The track rails of electric railways practically always form part of the electric circuit feeding the cars. As these rails are laid either in the ground or on ballast on the ground, and as the ground is a partial conductor, some of the current will leave the rails and flow in the ground. If pipes, lead-sheathed cables, or structural foundations happen to be in the ground near the railway, part of the stray current from the rails will enter these under-

ground structures and flow along them, leaving them at some other point to return to the station bus. If the current is direct, the underground structure from which current leaves to enter the earth becomes the anode of an electrolytic cell, the earth containing the electrolyte. This anode corrodes as the result of the liberation of acids from the salts contained in the earth. Occasionally the cathode, or underground structure into which current enters from the earth, is liable to corrosion by secondary causes. In the case of railways using alternating current for distribution, the stray currents are usually a large proportion of the total current, but the electrolytic effects are negligible. (See J. L. R. Hayden, *Trans. A.I.E.E.*, Vol. 26, p. 221.)

In d-c railways, it is therefore desirable to confine the current to the track rails as far as practicable. This may be done by maintaining the rail bonds in a high degree of efficiency and by keeping down the voltage drop in the rails by means of insulated negative feeders, attached thereto at suitable intervals.

While such means will assist materially in mitigating electrolysis troubles, it is often necessary to take special steps to protect underground structures without relying upon the railway companies to confine their currents to the rails.

MINIMUM VOLTAGE TO PRODUCE ELECTROLYSIS. The conduction of current through moist ground is almost entirely electrolytic, ordinary conduction being almost negligible. Electrolytic corrosion occurs whenever current flows from metal into the ground, regardless of whatever difference of potential may exist between different parts of the circuit. In order that current may flow, however, it is in general necessary that the difference of potential between anode and cathode exceeds the algebraic sum of the emf's of combination and separation of the compounds involved in the electrolytic process. An iron anode in a soil containing iron salts in solution will be attacked when the emf is infinitesimal.

101. CORROSION EFFICIENCY AND PASSIVITY

When an electric current is passed through an electrolyte, the mass of the anode dissolved may be calculated theoretically by Faraday's law (see Sec. 6). In practice this amount may be exceeded, or the reverse. The term " coefficient of corrosion " or " corrosion efficiency " is used to designate the ratio of the actual to the theoretical mass of the anode dissolved. The corrosion efficiency is usually less than 100 per cent, owing to the occurrence of secondary reactions. Thus the corrosion of iron, which primarily occurs by the formation of ferrous salts, should theoretically occur at the rate of 1.045 grams per ampere-hour, but in practice the rate may be anywhere from zero to 40 per cent more than the theoretical value. In the case of iron in soil, without protection, the corrosion efficiency is practically independent of the temperature over a range from 0 to 40 deg cent. It is also independent of the kind of iron, whether wrought iron, steel, or machined cast iron. The efficiency is greater the lower the current density, varying from 20 to 140 per cent for a range of current densities between 5.0 and 0.05 milliamperes per square centimeter. It is greater with increasing moisture content up to saturation of the soil. Data on steel encased in concrete are given below, under Protection of Steel by Concrete.

102. DISTRIBUTION OF EARTH CURRENTS

The distribution of stray currents in the earth depends not only upon the resistance of the earth but also upon the resistance and distribution of the conducting structures in the earth.

EARTH RESISTANCE. As the earth conducts electrolytically, its specific resistance can be calculated from the concentration of the salt solutions it contains. Such calculations are, generally useless, however, as, even with moderate specific conductance, the extent of the current path in the earth is so great that its resistance is very low, except near the electrodes. The effect of the electrodes is to concentrate the current streams into limited areas, thereby increasing the effective resistance of the earth. It is obvious, therefore, that the resistance of the earth depends more upon the electrode area, and the dampness of the soil around the electrodes, than anything else.

The resistance of the earth is low when the electrodes consist of considerable lengths of track rails. This is particularly noticeable in single-phase lines, owing to the skin effect and inductance of the rails. Thus J. Dalziel and J. Sayers (Inst. Civil Engineers, London, 1909) found that, on the Midland Railway, most of the current did not continue along the rails for any considerable length, but within a few hundred yards of the car sank gradually into the earth to re-enter the ballast at a very short distance from the power station. H. F. Parshall found that on a line eight miles long, the earth-return current

was 60 per cent of the total. Probably the most exact data of this kind are those due to A. W. Copley (*Trans. A.I.E.E.*, Vol. 27, p. 1171, 1908) who stated that on the New York, New Haven & Hartford Railroad, before the adoption of the three-wire system, the percentage of current in the earth was 25 per cent on the four-track sections and 60 per cent on the single-track sections.

ELECTROLYTIC ZONE. When a drop of potential occurs in a track rail, the current spreads out from it into the soil in much the same way that the leakage lines of magnetic force from a long electromagnet spread out into the air; that is, almost the entire leakage current is confined to a limited zone of the earth, the extent of this zone depending upon the potential gradient and the length of the line. The stream lines of the current will tend to crowd into a pipe or cable sheath (on account of its low resistance) if such conductor is within this zone, but this tendency rapidly diminishes as the distance between the conductor and the rails increases. Conductors at a comparatively short distance from the rails are, therefore, not liable to electrolysis unless the difference of potential between them and the rails is large or the leakage path of the current from the rails to the conductors is of low resistance. Thus in England, where the permissible drop in the grounded return is limited to a total of 7 volts, Messrs. Cunliffe found experimentally that the electrolytic zone is confined to within 3 ft of the track. The voltage drop under this rule is taken as the mean between the average and the momentary maximum values during a 15 to 30 min period of a schedule run, at the time of maximum traffic, exclusive of exceptional occasions.

103. STRUCTURES AFFECTED BY CORROSION

Stray currents can set up corrosion of the rails, rail spikes, cross-bonds, steel columns, etc., through which they escape into the earth, and they can set up corrosion of gas and water pipes, cable sheaths, building structures, etc., which they may enter and leave in their course through the earth. The protection of the former class of materials is entirely the concern of the railroad company which originates the earth currents; not so with the latter class, which is often of great concern to the municipalities or public-service corporations which own them. Electrolytic surveys should, therefore, be made with a clear understanding of which class of property is under suspicion of danger. As a rule, electrolysis is most destructive to the grounded metal of the railroads; foreign pipes, cable sheaths, etc., being affected in comparatively rare instances. Of all properties foreign to the railroads, the thin sheaths of telephone cables are most susceptible to electrolytic corrosion.

DESTRUCTION OF CONCRETE BY ELECTROLYSIS. Where iron imbedded in concrete becomes an anode, not only is the iron corroded but the concrete also is cracked. This action is due to the internal stress set up by the increase of volume which the iron suffers when it changes to an oxide or salt. The current has no direct effect upon the concrete at the anode even when sufficiently strong to liberate chlorine from the brine used to impregnate it. (C. E. Magnusson, *Trans. A.I.E.F.*, June, 1911.) At the cathode, however, the concrete becomes softened and loses its bond with the electrode. See papers by E. B. Rosa, B. McCallum, and A. S. Peters, Nat. Assn. of Cement Users, 1912, and abstracted in *Eng. News*, Vol. 68, p. 1162, 1912. No action, however, occurs on concrete through which current flows, except at the electrodes.

104. ELECTROLYTIC SURVEYS

An electrolytic survey is an investigation made to determine the condition of grounded metallic structures and the soil in which they are imbedded and of their electrical conditions with the view of ascertaining what conditions tending to produce damage may exist.

POTENTIAL READINGS. Random readings between rails and pipes give no quantitative information about electrolytic conditions. As a matter of fact, a difference of potential between a pipe and rail at any point is often greater the less the stray currents at that point. To illustrate this, we may assume a pipe to be outside the electrolytic zone and, therefore, safe from electrolytic corrosion. If, at any point, the rail is at the same potential as the pipe, the potential of the rail at every other point will differ from the pipe. Also, if the part of the rail which is at the potential of the pipe happens to be at some distance from the power station, the pipe will be positive to the rail at all points between the equipotential point and the power station. Hence the fact that the pipe is positive to the rail does not indicate that it is subject to electrolysis. A more important illustration is a pipe carrying little current itself but connected to some distant pipe which is carrying considerable current. In such a case, a pipe may be highly electropositive to the rails and yet be quite innocent of electrolytic tendencies. These illustrations are merely specific

instances of the general principle that potential readings are useless unless taken with reference to the resistance of the earth between rails and conductors, which in turn depends upon the direction and distribution of the current stream lines. The potential difference between the structure tested and earth affords more complete information than can be secured from any other practicable class of observations. For this purpose, it is necessary to use a non-polarizable electrode such as that shown in Fig. 1, which is known as the **Haber electrode.** The zinc sulfate paste shown therein is made by adding saturated zinc sulfate solution to fine zinc sulfate crystals until the mixture has attained a semi-fluid condition.

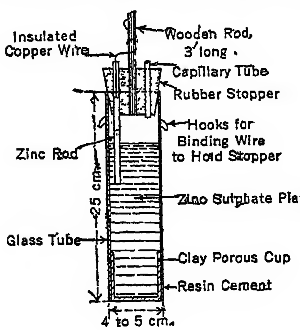


Fig. 1

MEASUREMENT OF LEAKAGE CURRENT IN GROUNDED RETURN SYSTEM. The columns and foundations of elevated structures and subways, rail spikes, bare negative cables, and bonds are the parts most likely to be attacked electrolytically.

In the elevated railways of New York City, after about nine years' operation with the steel structure a part of the return system, it was found necessary to remove all metallic connections in the Borough of Manhattan between the track rails and structure and to install negative feeder cables to compensate for the conductivity thereby sacrificed.

The corrosion of rail spikes, though happily rare, is a matter of such grave concern to the railroads and public that it should be carefully watched for. It occurs when ties are old and water-logged or when improperly treated with preservative. Timber is ordinarily classed with the non-conductors, because, when dry and well-seasoned, it has considerable dielectric strength and resistance. When green or moist, however, it becomes an electric conductor of comparatively low resistance. The resistance along the grain is much less than across it, and porous woods, such as oak, are better conductors than the non-porous woods such as pine. The conductivity of wood is due primarily to the presence in its pores of electrolytes formed from the salts found in natural timber, from preservatives and from salts originating from coal fumes or ashes. The flow of current, from the rails and spikes into the ties, fills the pores of the wood with iron salts, which add to the electrolytic conductivity and permit the leakage of more current. The effect is, therefore, cumulative, the leakage current increasing until the pores of the wood are completely saturated with electrolyte. Cases have been known where spikes were pitted more than half way through and where the rail flanges were badly corroded. Creosote has but little effect upon conductivity. A mixture of creosote and gas-oil, however, is a preservative which greatly reduces conduction. Red oak treated with zinc chloride is a bad tie from the electrolytic point of view.

Columns of elevated railroads, subways, passenger stations, etc., are best tested for electrolytic trouble by means of a sensitive galvanometer used in the following way. Iron clamps with pointed tips are fastened to the column at points 4 or 5 ft apart, care being taken that the points penetrate the paint and make metallic contact with the steel. Wires are run from these clamps to a galvanometer and the deflection noted. The galvanometer having been calibrated, this gives the drop of potential in the column. The cross-sectional area of the column being known, its resistance may be calculated from the known resistivity of structural steel (usually 11 times that of copper). The potential drop, divided by the resistance, gives the flow of current in the column. The direction of flow, its amount, and the efficiency of corrosion being known, the actual damage being done by electrolysis may be calculated as a definite weight of metal per annum. This method has been used in the Grand Central Terminal, New York, the galvanometer employed being a Queen & Co.'s E-8010 with tube E-8011, a calibration resistance, and a tripod. A deflection of one scale division is equivalent to about 0.000003 volt. Considering an average column with a resistance of 0.000,004 ohm for a 4-ft length, a deflection of one scale division corresponds to three-fourths of an ampere. Where the columns are to be encased in concrete, permanent testing terminals should be provided, preferably in the form of small iron pipes screwed into the steel and ending flush with the concrete.

MEASUREMENT OF LEAKAGE CURRENT IN PIPES, CABLE SHEATHS, ETC. It is not uncommon to find potential readings taken between different systems of pipes, without regard to the location of the connections, the results being recorded as differences of potential between those systems of pipes. The potential of a pipe system, however, may vary from point to point, and consequently such readings have no significance unless the points between which the potential difference is measured are specified. The signifi-

cance of potential readings between specific points is that they afford an indication of the potential gradient normal to the tracks and thereby help to determine the electrolytic zone.

Making use of all possible connections to the pipe, the potential difference between these points and the anode end of the grounded system should be determined as described below and the limits of the " electrolytic zone " ascertained by noting where the potential gradient in the pipe becomes negligible. For this purpose, water hydrants and water pipes constitute the best connection points.

Potential readings between points on the same pipe line or cable sheath are sometimes made the basis of potential curves showing the drop in the pipe or sheath. Such potential curves may be very significant when taken from cable sheaths, but are of little use for pipes on account of the variable joint resistance of the latter. Thus in a cable sheath of uniform resistance along the entire length tested, where the potential curve is flat, the sheath carries no current; when it is a straight sloping line, the sheath carries current without giving current to or taking current from the earth; where it changes its slope, current is either entering or leaving the sheath.

Hering's Method. The current in a pipe may also be measured by the following method. (C. Hering, *Trans. A.I.E.E.*, June, 1912.) The fundamental principle is as follows. Let *P*, Fig. 2, be a part of an underground pipe which has been uncovered and through which an unknown current *I* is flowing. Let *D* be a sensitive galvanometer, millivoltmeter, or any other suitable form of detector; there should preferably be no variable resistance like an unbonded pipe joint between the two contact points. Let *A* be an ammeter, *B* a few battery cells, and *R* an adjustable resistance; the shunt circuit containing them is connected as shown, anywhere outside of the points of application of the voltage detector, the farther away the better; they may even be on the other side of a joint.

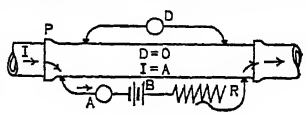


FIG. 2

To find the current flowing in the pipe adjust the resistance *R* until *D* reads zero; then the current due to the battery *B* will be exactly equal and opposite to the current in the pipe. Hence the reading of the ammeter *A* gives the current in the pipe.

If *D* is a galvanometer with proportionate deflections, instead of a mere detector, then by taking a deflection immediately after the shunt circuit has been opened a reading proportionate to the drop of voltage for that current will be obtained. The instrument *D* is thereby calibrated to read the pipe currents directly and can be used for this purpose thereafter; the test with the battery current is therefore merely of the nature of a preliminary calibration, and need be carried out only once for each station.

Instead of attempting to adjust the current in the shunt to bring the voltage *D* to zero, it is often more convenient to use a regular measuring instrument for *D* instead of a mere zero detector, and to pass a definite current through the shunt, say 10, 50, or 100 amp. reading the two deflections of *D* when this current is on and when it is off; this had best be repeated several times. The difference between these two readings then corresponds to the current in the pipe. The best current to use is that which will reduce the original deflection as much as possible. By thus using the difference between a large and a small deflection the errors due to a loose zero, which are so common with highly sensitive instruments, are reduced.

Methods for overcoming fluctuating currents are given by Hering in the paper cited above.

Having thus calibrated the voltmeter *V* at each of two neighboring stations, the currents which enter or leave the pipe between them may be determined, with the fluctuating currents, by taking the readings of the two instruments simultaneously by means of visual or telephonic signals, preferably at times when the currents are momentarily steady.

Measurement of Current Leaving Pipe at Any Point. The strength of the current at the point where it is leaving the sheath may be determined by the following test, which involves the use of a temporary bond in the form of a stout copper cable, electrically connected through an ammeter with both sheath and rail. Let

V = total drop of potential in the rails from *their* anodic center to the point where the temporary bond is connected.

By " anodic center of the rails " is meant the center of gravity of the leakage current *leaving* the rails.

v = difference of potential between the sheath and rail at the bonding point prior to the insertion of the bond.

v_1 = same, after the insertion of a bond which carries I amperes. The current in the bond is read from the ammeter.

Then the earth current, i, after the insertion of the bond will be

$$i = I \cdot \frac{k}{1 - k}$$

where
$$k = \frac{v_1}{v} \cdot \frac{V - v}{V - v_1}$$

The earth current from the pipe, without bonding, would be

$$\frac{V - v}{V - v_1}(I + i)$$

For example, suppose the drop in the rails from the anodic center to the bond to be 100 volts, and the differences of potential between the pipe and the rails at the bonding point to be 5 volts before the bond is inserted and 1 volt after. Suppose the current in the bond is found to be 50 amp.

Then $\quad V = 100, \quad v = 5, \quad v_1 = 1, \quad I = 50$

$$k = \frac{1}{5} \cdot \frac{100 - 5}{100 - 1} = 0.19$$

$$i = 50 \cdot \frac{0.19}{1 - 0.19} = 12 \text{ amp, approximately}$$

Without bonding, the current would be

$$= \frac{100 - 5}{100 - 1}(50 + 12) = 59 \text{ amp, approximately}$$

The above formula is based upon the following assumptions:

(1) That the drop in the rails V is not affected by the insertion of the bond.

(2) That the area over which the current from the pipe or sheath enters the earth is short. For proof of formula, see *Elec. World*, 1910, Vol. 35, p. 407.

105. MITIGATION OF ELECTROLYSIS

Where stray currents are unavoidable, electrolysis may be mitigated by the use of drainage bonds, insulation, electric shielding, boosters, or cathode maintaining generators.

PROTECTION FROM ELECTROLYSIS BY DRAINAGE BONDS. Bonds are not only useful for testing purposes; they may be applied permanently to reduce the earth currents. Certain effects, however, may render their use inadvisable.

1. If one piping system is connected to the rails or bus, a difference of potential will be established between it and all other underground metallic structures and it will, therefore, attract current from the latter and expose them to electrolytic danger. A bonded piping system thus becomes a part of the trolley return circuit, and the owner may become a party to whatever damage may occur in the other structures.

2. A considerable current in a gas pipe is a serious fire hazard and in a lead cable sheath is a menace to continuity of service.

3. Electrolysis is promoted at all imperfect joints and connections.

4. In power cables, the sheath currents may be great enough to add unduly to the heating of the cables.

In spite of these objections bonds are very largely used to protect cable sheaths from corrosion. To be most effective, the sheath should be connected to the negative bus by an insulated cable and the bus itself should be connected to the track rails by insulated cables only.

PROTECTION OF STEEL BY CONCRETE. The conductivity of concrete depends upon its porosity and wetness. Tests have shown that when concrete is wet its specific resistance may be as low as 20 or 30 ohms per yard cube, and when dry, as great as 2000 ohms. When the potential gradient in the concrete around an anode exceeds about 2 volts per centimeter, the temperature rises and the corrosion of iron is usually so rapid as to cause the concrete to crack open. At ordinary temperatures, the efficiency of corrosion of iron in concrete is low, but if the temperature is high, owing either to the high current density or external causes, the efficiency of corrosion may become quite high, as shown by Table I compiled from Tech. Paper 18 of the Bureau of Standards. For any fixed temperature the amount of corrosion for a given number of ampere-hours is independent of the current strength.

Passivity at ordinary temperatures is due principally to the drying of the film of anode rust by endosmose (see Section VI, Art. 15) and to a less extent to the concentration of calcium hydrate near the cathode, where it becomes converted into chalk, which fills the pores

of the concrete. Both these actions have the effect of increasing the resistance from anode to cathode, but the effect of the former action, which is the principal one while the current flows, does not last long after the stoppage of current (W. A. Del Mar and D. C. Woodbury, *Elec. World*, 1917, Vol. 70, p. 916).

Table I. Corrosion Efficiency and Temperature

Temperature, degrees, centigrade	Corrosion Efficiency, per cent	Temperature, degrees, centigrade	Corrosion Efficiency, per cent
10	2.0	70	22.5
20	2.5	80	37.5
30	3.0	90	44.5
40	3.5	100	47.0
50	5.0		
60	11.5		

Briefly stated, concrete affords good protection to steel work if the current density is not high enough to heat the concrete excessively.

Salt should never be used in concrete if there is the slightest probability of action by electric currents, since the addition of even a fraction of a per cent of chlorine is sufficient to increase the rate of corrosion a hundred fold.

PROTECTION OF STEEL BY PAINT. Unpainted steel imbedded in concrete can be electrolytically corroded at the anode, but a good insulating paint applied to the steel prevents such corrosion. Acid-proof paints with tar or asphalt base such as are commonly used to protect steel imbedded in concrete are usually effective. A typical paint of this kind has the following composition: 16 parts coal-tar paint, 4 parts Portland cement, 3 parts kerosene. When the voltage between cathode and ground does not exceed 5 volts the corrosive efficiency of an anode, so protected, is usually less than 1 per cent.

PROTECTION OF CABLE SHEATHS. No entirely satisfactory protection is available which is both inexpensive, flexible and able to resist heat and electrolytes. Asphalted hessian or duck tapes have been fairly successful but tend to loosen from the lead in time. Special rubber compounds are now being tried with some promise of success.

PROTECTION BY INSULATED JOINTS. In a number of installations, flow of stray current on metallic pipe lines has been prevented by the use of a sufficient number of insulating joints. It is found that, where a pipe line is laid with every joint insulated, the line has such a high electrical resistance that no measurable current flows, although considerable potential gradient may exist in the earth parallel to the pipe line. Insulating joints in cable sheaths are not found to afford an effective means of preventing electrolysis.

PROTECTION BY SHIELDING. Underground structures have been protected from electrolysis by connecting to the structure an auxiliary metallic conductor located so as to cause the current to flow to earth from the auxiliary conductor. The shielding conductor must be so placed as to prevent the current from leaving the structure to be protected, or at least, to cause its magnitude to be greatly reduced.

PROTECTION BY BOOSTER. It is often proposed to render grounded metal work electronegative to the rail return by means of a booster, but such proposals are seldom carried into execution on account of the expense they involve.

PROTECTION BY CATHODE (MAINTAINING) GENERATOR. The following system is used for protection of oil pipes against electrolysis and natural corrosion. At places where there is danger, electrodes are placed in the earth in the neighborhood of the pipes and these electrodes are connected to the positive pole of a low-voltage generator while the pipe to be protected is itself connected with the negative pole. In this way electric current is forced to enter the pipe from the earth so that anodic destruction of the pipe is impossible.

The same system is used to prevent the pitting of condenser tubes, especially where salt water is used for cooling.

106. BIBLIOGRAPHY

A very complete bibliography of the subject up to 1908 is given in a paper by W. H. Gee, Electrolytic corrosion, *J.I.E.E.*, 1908, Vol. 41, p. 425. In addition to these references and those given in the text, the following papers should be consulted:

Report of the American Committee on Electrolysis, 1921. (This contains a great deal of valuable information on the subject, including a description of European practice and protective legislation. It also contains a bibliography of contributions which are regarded by the Committee as having permanent value.)

Bureau of Standards Technologic Papers, 15, 18, 25, 26, 27, 28, 32, 52, 54, 55, 62, 63, 72, and 75. (These papers cover almost every phase of the subject.)

SECTION 15

LIGHTING AND HEATING

LIGHTING

By F. C. Caldwell

ART.		PAGE
1.	Illumination and Light	02
2.	Light Sources	07
3.	Vision	21
4.	Radiation, Colored Light, and Artificial Daylight	23
5.	Light-modifying Devices	27
6.	Light Projection	29
7.	Specification, Design, and Maintenance of Illumination	33
13.	Ranges	45
14.	Bibliography	46

WATER HEATING

By C. J. Fay and J. H. Reifenberg

15.	Principles	46
16.	Design of Hot Water Heaters	47
17.	Testing	48
18.	Performance	49
19.	Installation	49
20	Operation	49
	ibliography	50

:ATING AND AIR CONDITIONING OF BUILDINGS

By Otto W. Walter

	eat Losses and Calculations	50
	ystems	55
	ir Conditioning	60
	ibliography	67

LIGHTING

By F. C. Caldwell

1. ILLUMINATION AND LIGHT

Units, Definitions, and Laws

I.E.S. Illuminating Engineering Nomenclature and Photometric Standards

For the purpose of illuminating engineering, light is radiant energy, evaluated according to its capacity to produce visual sensation. (See also Sec. 9 of *Fundamentals of Engineering* for Theory of Radiation, and Sec. 5, Art. 21, for Photometry.) The units associated with the applications of light are based upon the "international candle."

LUMINOUS FLUX. (Also called light flux.) The rate of flow of light, as defined above, from a luminous body. Generally only a small portion of the emitted radiation is thus effective, and the effectiveness of that portion varies as indicated by a visibility factor, shown in Fig. 14. The luminous flux is the integral of luminous intensity, I, that is, $F = \int_0^\omega I\,d\omega$, where F is the flux and ω is a solid angle.

LUMEN. The unit of luminous flux. It is the flux emitted from a uniform point source of 1 candle through 1 unit of solid angle, or 1 steradian. On a sphere of 1-ft radius this unit angle subtends 1 sq ft. There are, therefore, a total of 4π or 12.6 (12.5664) lumens emitted by a source of 1 mean spherical candle, or, if I_s = mean spherical candles, $F = 12.6\,I_s$; and for any zonal solid angle, ω, $F_\omega = \omega I_{s\omega}$.

LUMINOUS INTENSITY. The uminous intensity of a point source, in any direction, is the luminous flux per unit solid angle, emitted by that source, in that direction. If the distance at which the intensity is measured is large relative to the dimensions of the source (about 10 times) this source may be taken as a point. The light-giving body, though often a prime light source, may also be a surface which reflects light. Luminous intensity is also called candlepower. It may be applied to certain average or mean values, thus:

Mean spherical intensity $-I_s$; mean lower hemispherical intensity $-I_{mlh}$; mean upper hemispherical intensity $-I_{muh}$; mean horizontal intensity $-I_h$.

INTERNATIONAL CANDLE. The arbitrary unit of luminous intensity established in 1909 for the United States, Great Britain, and France. It is usually called a "candle"; its value is maintained by groups of standardized lamps at the U. S. Bureau of Standards and other national laboratories. The Hefner unit, German standard = 0.9 candle. (See also Sec. 5, Photometry.)

ILLUMINATION. The effect produced at any surface by flux falling upon or passing through it. The term is used both qualitatively and quantitatively. Quantitatively, illumination is measured by the density of the luminous flux at the point in question. Illumination is, therefore, independent of the character or color of the surface at which it is measured. Illumination, as the most usual application of light, is the most important phenomenon connected with it, and the unit of illumination is the most important unit.

FOOT-CANDLE. The unit of illumination used by English-speaking peoples. It is the illumination produced by luminous flux having a density of 1 lumen per square foot. As explained under *lumen*, 1 lumen per square foot is received at a distance of 1 ft from a light source of 1 candle. Thus, a foot-candle illumination is produced by 1 candle falling on a surface at 1 ft normal distance, from 1 candle.

LUX, PHOT. (Metric units.) These are given by 1 lumen per square meter and per square centimeter, respectively. They are also the illumination values produced by a standard candle on a surface at 1 meter and 1 cm normal distance, respectively. One foot-candle = 10.76 lux (10.76 sq ft in a square meter). One lux = 0.093 ft-c. The phot = 10,000 lux.

LUMEN-HOUR. This is the unit of radiant energy, referred to its effective value for vision. It is, for instance, used in measuring the total output of an incandescent lamp during its life.

BRIGHTNESS. This is the luminous intensity of a surface, per unit area, as projected on a plane normal to the line of sight. Using s = area and θ = angle of surface with normal, brightness = $dI/ds \cos\theta$. The brightness of a surface depends upon the relative directions of the incident light and the line of sight, to the plane of the surface. Brightness is proportional to the product: illumination times reflection factor.

THE CANDLES PER SQUARE INCH (projected area) is the most-used unit of brightness, based on the English system of units. The *candle per square foot* is also used for low brightness-values of reflecting surfaces.

LAMBERT (metric unit) is the brightness of any perfectly diffusing surface, emitting or reflecting 1 lumen per square centimeter. It is also the average brightness of any surface which emits or reflects 1 lumen per square centimeter. **Millilambert,** 0.001 lambert, is more generally used; because of its smaller value it is better suited to most cases. To reduce candles per square centimeter to lamberts, multiply by π. To reduce candles per square inch to lamberts, multiply by 0.487 ($\pi/6.45$, approximately $^1/_2$).

The foot-lambert is the average brightness of any surface emitting or reflecting 1 lumen per square foot. One foot-lambert = 1.076 millilamberts.

POINT-SOURCE, OR EXTENDED SURFACE. From a point-source the light rays spread out in the form of a cone. Thus the areas intercepted by planes at distances d_1 and d_2 are proportional to $(d_1/d_2)^2$, and the illumination, being inversely proportional to the areas over which the flux is spread, will be inversely proportional to the squares of the distance from the source. This is the inverse-square law; it is true only if the source is a point, but it is essentially true if the diameter of the source is not greater than $^1/_{10}$ the distance to the illuminated surface. On the other hand, if the light rays are effectively parallel, owing either to large area of light source or to great distance (sunlight), the illumination is independent of the distance. Illumination of interiors, due to reflection from light colored ceilings, generally lies between these extremes. The inverse-square law may be applied for artificial illumination of interiors with dark ceilings and walls and for many cases of exterior lighting, but effectively parallel illumination is never met with, even in searchlight beams. Sunlight is an example of parallel radiation.

THE COSINE LAW. If the surface upon which a given beam of light is falling normally be tilted through an angle θ, the area of the illuminated surface is increased in the ratio $1/\cos\theta$; the illumination is thus decreased in the ratio $\cos\theta/1$. The angle θ is also the angle made by the light rays with a line normal to the surface, which is the "angle of incidence of the light," as in Fig. 1. Thus illumination varies as the cosine of the angle of incidence.

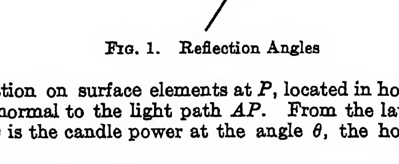


FIG. 1. Reflection Angles

CALCULATION OF ILLUMINATION, POINT-BY-POINT METHOD. In Fig. 1 let A be the location of an illuminant and P a point at which the illumination from A is to be determined. The three most important cases of this problem refer respectively to the illumination on surface elements at P, located in horizontal and vertical planes, and in a plane normal to the light path AP. From the laws of inverse squares and of cosines, and if I_θ is the candle power at the angle θ, the horizontal illumination at P is

$$E_h = \frac{I_\theta \cos\theta}{d^2} = \frac{I_\theta \cos^3\theta}{v^2}.$$

The vertical illumination at P is $E_v = \dfrac{I_\theta \sin\theta}{d^2} = \dfrac{I_\theta \sin^3\theta}{h^2} = \dfrac{I_\theta \cos^2\theta \sin\theta}{v^2}$.

The normal illumination at P is $E_n = \dfrac{I_\theta}{d^2} = \dfrac{I_\theta \cos^2\theta}{v^2} = \dfrac{I_\theta \sin^2\theta}{h^2}$.

Also, $E_h = \dfrac{I_\theta \cos\theta}{h^2 + v^2} = \dfrac{I_\theta v}{(h^2 + v^2)^{3/2}}$.

If v is small compared to h, then approximately, $E_h = \dfrac{I_\theta v}{h^3}$.

Also, under these conditions, approximately, $\sin^2\theta = \sin^3\theta = 1$, hence $E_v = E_n = I_\theta/h^2$ (1 per cent error for $h = 7v$).

For other methods of point-by-point calculation and useful tables see Benford, *Trans. I.E.S.*, 1912, Vol. 7, p. 695.

REFLECTION, TRANSMISSION, AND ABSORPTION. When light falls upon the surface of an object, these three phenomena occur. Depending upon which predominates, the object (*a*) has a high reflection factor, is a good reflector; (*b*) has high transmission, is transparent or translucent; (*c*) has high absorption, is opaque.

Reflection Factor. Is defined as the ratio of luminous flux reflected to that incident upon a surface. It is thus a percentage of the incident flux. Some typical values are given in Table I. Reflection is specular, as from a mirror, or diffuse, as from blotting

Table I. Reflection Factors for Typical Materials

(Poller and Meaker, *Trans. I.E.S.*, 1931, Vol. 26, p. 1029)

Material	Reflection per cent
Paint:	
Flat white	75–85
Gloss white	75–80
Vitreous porcelain enamel on steel:	
Matte white	60–83
Glazed white	65–77
Plaster, white	90–95
Terra-cotta:	
White and cream—smooth and matte	60–80
Sandstone, one sample	41
Limestone	35–58
Marble ($CaCO_3$):	
One side polished	30–71
Impregnated	27–54
Alabaster ($CaSO_4$):	
Veined	49–67
Colored	27–29
Matte-finished metal:	
Oxidized or etched aluminum	70–89
Aluminum paint	60–65
Polished metal:	
Silver	90–92
Chromium	63–66
Aluminum *	62
Monel metal	49–55
Chromium-nickel (stainless) steel	55

* After electrolytic treatment 83 per cent. Dickerson, *Trans. I.E.S.*, 1934, p. 358.

paper, or a combination of both. In specular reflection the angle of reflection is equal to the angle of incidence as in Fig. 2(*a*). Fig. 2(*b*) shows perfect diffusion with equal

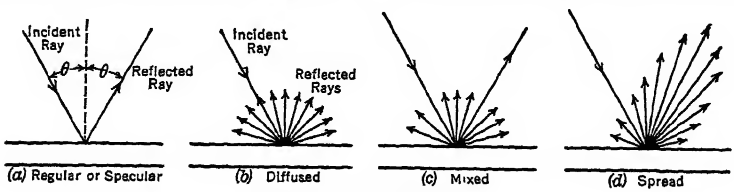


Fig. 2. Types of Reflection

candlepower radiated in all directions. Fig. 2(*c*) shows mixed reflection, as from an enameled surface. Fig. 2(*d*) shows imperfect diffusion or spread reflection. Reflection is generally selective as to wavelength, thus resulting in the appearance of color. This is treated in Art. 4 under Color. **Transmission factor,** or **transmission,** is also given as a percentage of the incident luminous flux. It varies from nearly 100 per cent, as in clear glass, to almost zero. Some light is transmitted through or into even the most dense substances, as witness a greenish glow through thin gold leaf. Transmission depends upon the thickness of the object, being roughly inversely proportional to the thickness. Some values are given in Table II. In considering the transmission through thin glass sheets, the reflection from the two surfaces, which will amount to about 8 per cent or more, must be taken into account, that is, only about 90 per cent of the incident light will come through the clearest glass, with normal incidence. If this angle of incidence is large the reflection is much greater. **Absorption factor,** or **absorption,** is equal

to 100 per cent less the sum of the reflection and transmission factors. It represents that part of the light radiation which is converted into invisible heat energy in the object or material under test. It is approximately proportional to thickness.

REFRACTION. The phenomenon which accounts for the bending of a light ray when it passes from one substance to another of different density is important because of its use in redirecting light rays in prismatic globes. (See Art. 5.)

Table II. Transmission, Reflection, and Absorption Factors for Typical Translucent Materials

(Poller and Meaker, *Trans. I.E.S.*, 1934, Vol. 26, p. 1031)

Type	Thickness, in.	Transmission, per cent	Reflection, per cent
Configurated,* obscure clear glass	0.12–0.23	57–90	7–24
Clear glass, satin finish:	0.075	85–89	6–8
Alabaster glass	0.125–0.19	60–70	20–30
Flashed opal glass	0.08 –0.13	27–66	31–67
Composition, white diffusing	0.010–0.025	0–41	32–75
Solid opal glass	0.06 –0.14	12–51	40–78

CANDLEPOWER DISTRIBUTION CURVES. The performance of any light source as to candles emitted in different directions is shown in a candle power "distribution curve." On this curve each point is determined by the length of the radius in the appropriate direction, as in Fig. 3. Most light sources, though varying greatly as to distribution in a vertical plane, through the axis, are practically constant as to distribution in horizontal planes at right angles to the axis. The principal exceptions are asymmetric refractors, designed for street lighting (Art. 9). In such cases the distribution may be more or less completely defined by the curves for several vertical planes. Another method of representation useful for asymmetric light sources (Benford, *Gen. Elec. Rev.*, 1925, p. 271; *Trans. I.E.S.*, 1926, p. 129) uses the curve sheet shown in Fig. 4. Isocandle curves for the light source are drawn upon this sheet, and thus the performance of the source can be easily seen. In this curve sheet the vertical space per degree is constant and is equal to the horizontal space at the equator. The horizontal spaces in any zone are proportional to sin θ, where θ is the midzone vertical angle; sin θ measures the relative area of this zone on a sphere. This set of isocandle curves, besides giving the graphical presentation of the candlepower distribution of the light source, is useful for the determination of light flux.

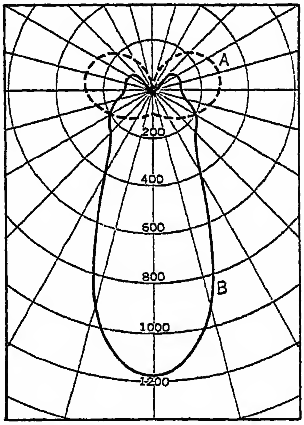


FIG. 3. Distribution Curve

CALCULATION OF LIGHT FLUX AND MEAN INTENSITIES FROM POINT SOURCES. Because of the variation of solid angle (and subtended area) of zones, which have equal vertical angles, the area of the vertical distribution curve is not a measure of the total flux. This is shown in Fig. 3, which represents the distribution curve for a bare incandescent lamp, A, and for the same lamp with a concentrating prismatic reflector, B. Curve A, of course, shows a greater flux than curve B. Various methods of calculation of flux from the vertical distribution curve have been used, of which two will be given. The fact that light flux is usually measured by the spherical photometer (see Sec. 5, Art. 21), rather than calculated, decreases the importances of such methods.

Let I_θ be the mean intensity of a light source at an angle θ from the equatorial plane at right angles to the axis. The flux within an elementary zone at this angle of inclination is then

$$dF = I_\theta \, d\omega = 2\pi I_\theta \cos\theta \, d\theta; \quad F = \sum_{-\pi/2}^{+\pi/2} 2\pi \cos\theta \times I_\theta \, \Delta\theta$$

If the 180 deg from $-\pi/2$ to $+\pi/2$ be divided into n equal angles, then $\Delta\theta = \pi/n$, and F' for one zone $= \left(\dfrac{2\pi^2 \cos\theta}{n}\right) I_\theta$, and the total flux is the sum of these zone-fluxes

corresponding to the n midzone values of I_θ. These values of I_θ are read from the distribution curve for the light source. For 18 zones of 10 deg each the constant by which each value of I_θ must be multiplied is given in Table III.

Table III. Constants for Flux Calculation

Midzone Angle from Equator—θ	$\frac{2\pi^2}{n}\cos\theta$ K_θ	Midzone Angle Measured from the Perpendicular	
5	1.091	85	95
15	1.058	75	105
25	0.992	65	115
35	0.897	55	125
45	0.774	45	135
55	0.628	35	145
65	0.463	25	155
75	0.283	15	165
85	0.095	5	175

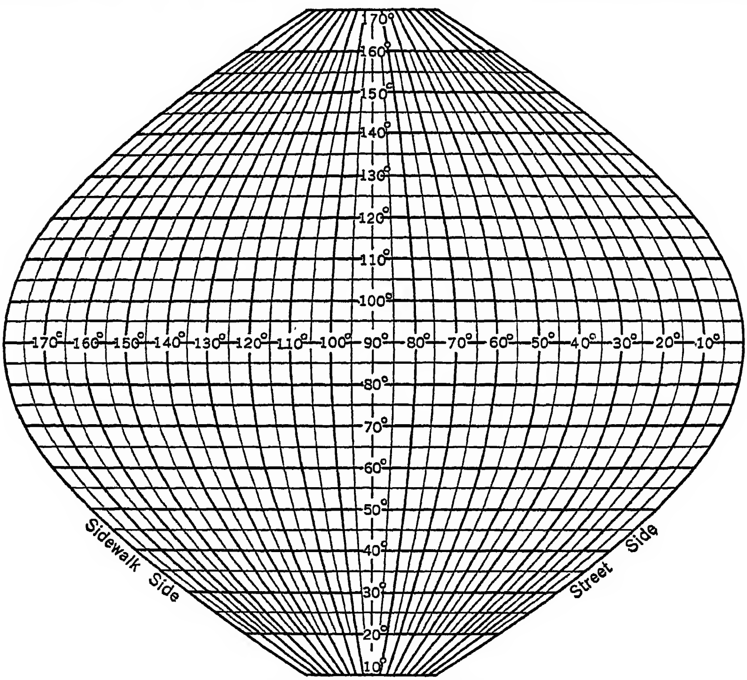


FIG. 4. Benford Isocandle Web

The total flux is the sum of the products of $K_\theta \times I_\theta$ for the eighteen 10-deg zones. A larger number of zones would give more exact values of flux. The flux for the lower hemisphere, or for any zone, as for instance for 20 deg below the horizontal, may be easily determined by this method. (Kennelly, *Elec. World,* 1908, p. 645; Caldwell, Modern Lighting, p. 92.)

PROTRACTOR METHOD. This method is adapted to quick approximations of flux output. The method assumes a sphere of light distribution divided into zones of equal solid angular content, i.e., of equal altitude. A direct average of the mean candlepower of the several zones gives the general mean for all the zones included, since each component zone bears equal weight. For convenience it may be assumed generally that the mean candlepower in any zone agrees closely with that at the angle which bisects the area of the zone. Fig. 5 shows a convenient subdivision of the sphere based on this assump-

tion. To obtain the mean spherical candlepower, the intensities at all the angles indicated by radial lines are averaged. This result multiplied by 12.6 gives the total flux from the source. For convenience in practice a transparent protractor may be prepared with the reference angles marked by lines. This protractor may be placed over the polar distribution curve and the values for averaging read off directly.

For asymmetrical light sources, as street lighting refractors, the Benford isocandle curves (Fig. 4) are useful, since in these curves the areas of isocandle bands are actually proportional to the flux. If a uniform source of I candles were placed within the sphere represented by the Benford web, I candles would be recorded at all points and the flux would be $12.6\,I$. An area of 10 deg × 10 deg at the equator is 0.00243 of the total area of the sphere, and I candles will give, for this equatorial 10-deg square area, $0.00243 \times 12.6\,I$, or $0.0305\,I$ lumens. This figure, 0.0305, then becomes the constant by which the candle value for each 10-deg square at the equator must be multiplied to convert candles into lumens. At any other vertical angle $-\theta$, measured from the nadir, the area of a 10-deg square is proportional to $\sin\theta$, and the lumens become $0.0305 \sin\theta\, I_\theta$.

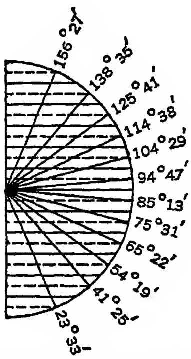


FIG. 5. Equal Area Zones for Flux Calculation

To obtain the total flux, therefore, the number of 10-deg squares covered by an isocandle band in the zone with mid-angle θ will be multiplied by $0.0305 \sin\theta$; the same operation will be performed for each other candle value in the θ-deg zone, the boundaries of the bands being lines midway between the isocandle lines. This operation will be repeated for all other isocandle bands in each of the other zones. The sum of all such products will be the total flux. For any angular area h degrees wide by v degrees high, with a mean angle of elevation θ, the lumens equal $0.000305\, hv\, I_\theta \sin\theta$. To obtain the flux falling upon any rectangular area, great circles which cut the boundaries of the area being studied will be drawn on the web, and the flux within the area on the web so delineated will be determined. (Benford, *Trans. I.E.S.*, 1926, Vol. 21, p. 129.)

MEASUREMENT OF ILLUMINATION. From the point of view of the user of illumination, the measurement of existing installations is of great importance. Instruments and methods are described in Sec. 5, Art. 21, under Photometry. The Weston photronic illumination meter (Goodwin, *Trans. I.E.S.*, 1932, Vol. 27, p. 828) and the foot-candle meter are calibrated directly in foot-candles. The former requires only the reading of a pointer on a calibrated scale and calls for little expert skill on the part of the observer. This instrument is made in a variety of forms, suited to the use of non-technical persons, and is largely superseding the foot-candle meter which does call for some skill and experience in order to obtain satisfactory results. These instruments in the forms known as *sightmeters* and *lightmeters*, are inexpensive and should be in the hands of persons who are responsible for the design or maintenance of lighting. The Macbeth and similar instruments, which depend upon the inverse-square law, require considerable photometric skill.

2. LIGHT SOURCES

Although flames were the only lamps for many thousand years, and are even now perhaps the most numerous, space permits the consideration here of only present-day electric light.

Incandescent Lamps

SOLID FILAMENT TYPE. The temperature to which an electrically heated conductor, as a lamp filament, rises, is such that the heat dissipated is equal to I^2R. When the vaporization temperature of a solid substance is approached, disintegration becomes rapid, which limits the permissible operating temperature of a lamp. Based on these facts, and the laws of the electric circuit (see Sec. 3), the following general statements with regard to incandescent lamps may be made.

(*a*) The higher the lamp voltage, the less the current and the thinner the filament (watts constant); the resistance must vary as the square of the voltage. Hence the limit of practicable voltage, in the United States generally to about 115 (250 volts is common in Europe), also higher permissible temperatures, and hence higher efficiencies for low-voltage lamps. (*b*) To obtain high efficiency a lamp is operated near to its vaporization temperature; thus a small increase in voltage greatly shortens the lamp life through disintegration

of the filament. (See Table IV.) (c) With increase in voltage, and thus of temperature, the watts-input increases, but not so rapidly as the lumen-output, and hence the efficiency increases. The effect of voltage variation on life, watts, lumens, and lumens-per-watt for tungsten lamps is shown in Fig. 8. These data are all based upon rated voltage as 100 per cent and hold for only a limited range of voltage.

SELECTIVE LUMINOUS RADIATION. Bodies which radiate owing to temperature are either selective or non-selective (the latter also called **black bodies**; see Sec. 2, Art. 21). As the result of a selectivity, which causes a relative increase in the visible radiation, the efficiency of some filament substances is considerably increased. This raises the efficiency of a tungsten lamp about 25 to 30 per cent. (E. P. Hyde, Lectures on Illuminating Engineering, p. 57.) Carbon, on the other hand, is practically non-selective. The ratio of visible to invisible radiation, and hence the luminous efficiency, increases rapidly with temperature as seen in Figs. 15 and 16.

FILAMENT MATERIAL. A material for lamp filaments should have, in large measure, the following characteristics: (a) Highest practicable vaporization temperature. (b) Melting temperature high enough to prevent softening and deformation of filament. Example: though carbon has a higher melting temperature than tungsten, its vaporization temperature is lower. (c) Favorable selectivity. (d) High mechanical strength, which is not seriously affected by temperatures experienced during the operation of the lamp. (e) High resistivity to give thicker filaments. (f) A high positive temperature coefficient to counteract voltage variation. (g) Ductility, to permit formation into a suitable filament. (h) Moderate cost. Carbon and tungsten alone, up to the present time, have been extensively used for filaments. Tantalum was the first metal used, but was soon superseded.

BLACKENING OF THE BULBS. The particles of filament material which are thrown off from the filament by the disintegration at high temperature are largely deposited on the bulb. This results in a progressively increasing absorption of light, and is a considerable factor in the loss of lumen-output of the lamp during its life. In large lamps, where blackening is particularly serious, granulated metal is enclosed in the bulb. By sliding this around the blackening is largely removed.

CARBON-FILAMENT LAMP. This was the first and for over 20 years the only incandescent lamp. Carbon melts at about 4300 deg K, but vaporizes at about 3800 deg K. These lamps are made only in small sizes and give about 3.3 lumens per watt. This efficiency is so low that their use, except for special purposes, cannot be economically justified.

TUNGSTEN, ITS PROPERTIES. This metal is obtained from a tungstate of iron and manganese called wolframite, a tungstate of calcium called scheelite, and certain other ores. Its properties are much affected by previous treatment. Melting point 3380 deg cent; boiling point 5560 deg cent. A very heavy metal with an atomic weight of 184 and a specific gravity of 19 (gold 19, lead 11). Tensile strength very high, $^1/_2$ lb per sq mil, about 10 per cent higher for very fine wire (high-grade steel, $^1/_4$ lb). Resistivity, ohms per mil-foot at 25 deg cent, hard-drawn 37, soft-drawn 31. Temperature coefficient 0.0045 per degree at 20 deg cent, 0.0057 at 500 deg, 0.0089 at 1000 deg (Somerville). Ratio of resistance at operating temperature to cold resistance for vacuum-type lamps, 12 to 14; for gas-filled lamps, 15 to 17. Ductile tungsten suitable for drawing into a wire is made by sintering powdered metal into a bar in an electrically heated and hydrogen-filled furnace. Because of its great hardness the tungsten wire is drawn through dies made from diamonds.

TUNGSTEN FILAMENT LAMPS. These are either of the vacuum type or are filled with a mixture of approximately 86 per cent argon and 14 per cent nitrogen. The nitrogen is added to increase dielectric strength. Argon is characterized by low heat conductivity and a high degree of inertness. It is a rare gas found in the earth's atmosphere. The gas is used at about atmospheric pressure. By reason of this gas pressure of about 15 lb per sq in. on the filament, vaporization of the tungsten is decreased, so that the operating temperature may be higher and the efficiency increased. See Table IV.

Another important function of the gas is the carrying, by convection currents, of the vaporized tungsten to the top of the bulb, where it is least effective in reducing the useful light. The larger sizes of gas-filled lamps have an elongated bulb, the upper portion of which serves as a cooling chamber. The walls of this chamber receive the black deposit from the filament. The larger sizes of gas-filled lamps are standard for operation in a pendent position (within 45 deg) only.

The presence of gas in the lamp bulb also promotes heat convection. This effect decreases with increased diameter of filament. This leads to higher efficiency in three classes of lamps: large lamps of the 115-volt type, low-voltage constant-potential lamps, and constant-current street lamps, which also operate at low voltage. See Table V. If a

filament is coiled into a spiral whose diameter is several times that of the wire, it acts, from the point of view of convection, like a wire of the diameter of the spiral. Most lamps now use spiral filaments. Concentration of the light-giving area, which is essential for projection lamps, and for other applications where definite modification of the light distribution

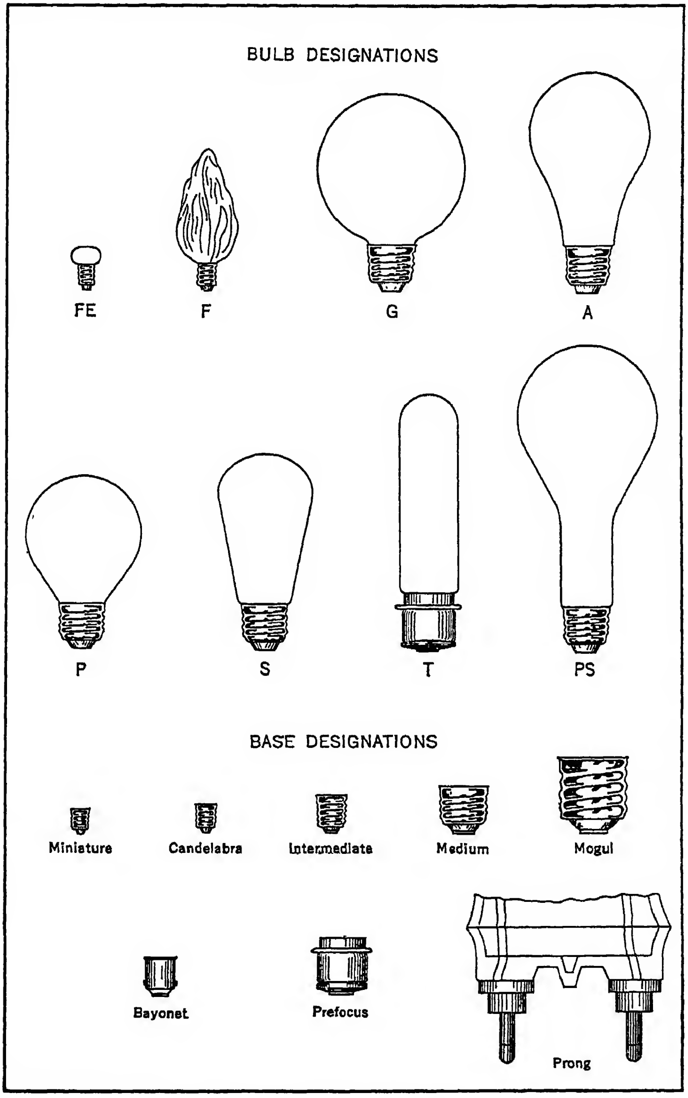


Fig. 6. Standard Types of Bulbs and Bases

by prisms or lenses is involved, is promoted by coiling the filament. Also, breakage due to blows is decreased by the spiral spring effect produced. The tensile strength of tungsten is high before use, but it is greatly reduced after long operation, so that the filament is easily broken.

BULBS, BASES, AND FILAMENTS. The more common forms of bulbs and bases are illustrated in Fig. 6. In the bulb designation, such as S–19, the letter indicates the type of bulb, and the number the diameter of the bulb in eighths of an inch. " S " indicates a " straight-side " or conical-sided bulb; " G," a round or globular bulb; " T," a tubular bulb; and "PS," a pear-shaped bulb. In the " A " bulb, the " inside frost " is also implied. The inside-frosted bulb is standard for all general-service lamps from 15 to 100 watts, and for many special lamps. It greatly reduces the brightness of the lamp, while absorbing less than 2 per cent of the light. The smooth outside surface is a great advantage over outside frosting, for which the loss of light is much greater.

The PS-shape is the one most commonly used for general-service lighting. This covers wattages from 150 to 1500. The G-bulb is much used for special service lamps such as floodlighting, spotlighting, etc. The T-bulb is made in lengths from 5 1/2 to 34 in. for standard bases and 17 7/8 in. for the 30- and 60-watt lamps in T8 bulbs based at both ends. Filament dimensions are shown in Fig. 7.

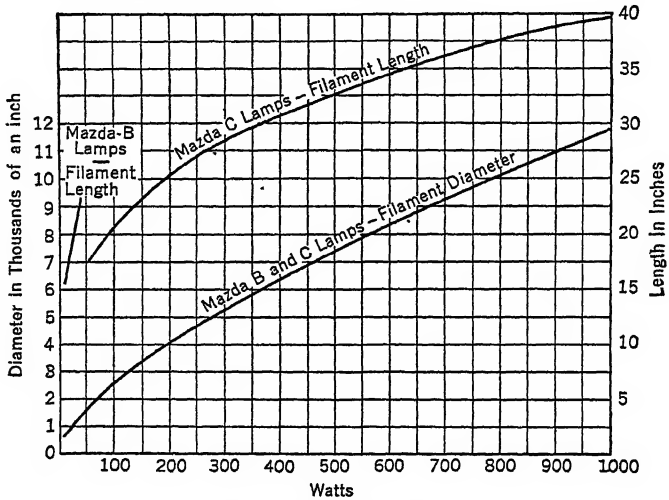


FIG. 7. Filament Dimensions

PREFOCUSED AND BIPOST BASES. When, for projection, it is important to have the space relation of the filament to the lamp base especially exact, the bulb, with its base and filament mounting, may be finally adjusted and soldered to the screw base with the filament lighted and its image projected upon a screen. When the image of the filament is on the screen cross-lines, the attachment to the screw base is made. For large lamps a loose threaded sleeve is used. For automobile headlamps a collar is used, which surrounds the base and has three small bosses. These are designed to set on the reflector-surface, when the bulb is inserted in the headlamp, and thus bring the filament to focus.

In the larger sizes of lamps, also used for projection, the same end is accomplished by using the bipost base. This is a prong base in which the prongs and filament-supports (see Fig. 6) form one rigid structure with, therefore, a definite space relation between prongs and filament. A glass cup is molded around the bipost structure and the bulb is sealed to this cup.

LUMEN OUTPUT, EFFICIENCY, AND LIFE WITH VOLTAGE VARIATION. The high rate of change of light intensity and of filament-life, with temperature, causes incandescent lamps to be extremely sensitive to variations in voltage. The positive temperature coefficient of resistance of the tungsten filament affords a partial compensation to voltage variations and increases the stability of the lamp's performance. The relations of voltage to lumen output, watts, lumens per watt, and life are shown, for modern types of lamps, in Fig. 8. High voltage increases the lumen output and efficiency, but greatly reduces the life of the lamp. Operating conditions are always a compromise between life and efficiency. Under ordinary conditions, circuit voltage and rated lamp voltage should agree closely. The lowering of the luminous flux with reduced voltage is

marked. Satisfactory service requires that the voltage be maintained within 3 per cent, at most, of the rated value for the lamp.

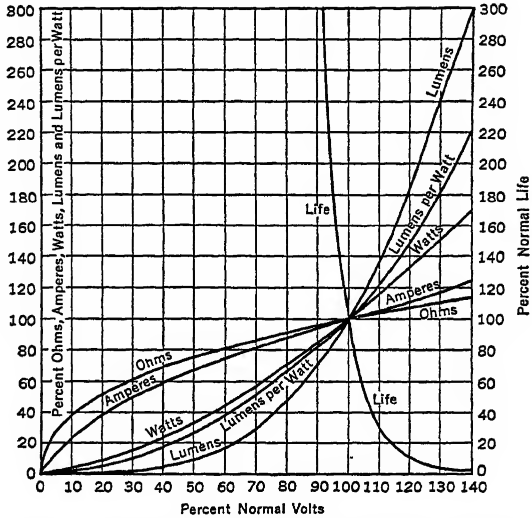


FIG. 8. Properties of Mazda Lamps as Functions of Voltage

The relations between lamp performance and voltage may be completely expressed by a series of simple proportions and exponents given in Table IV. These exponents hold accurately only for a small deviation from normal voltage. The variations of these values, together with those of amperes and ohms, are shown in Fig. 6.

The changes in efficiencies with lamp size, for 115-volt Mazda B and Mazda C lamps, are shown as curves in Fig. 9.

The data for the curves of Fig. 8 may also be expressed as follows: 1 per cent change in volts produces approximately the following changes: ohms, 0.4 per cent; amperes, 0.5 per cent; watts, 1.5 per cent; lumens, 3.5 per cent; efficiency, 1.8 per cent; life, 13 per cent; these figures apply only to the straight portion of the curves, near 100 per cent normal volts. The life characteristics of vacuum and gas-filled lamps, Mazda B and C, are shown in Fig. 10.

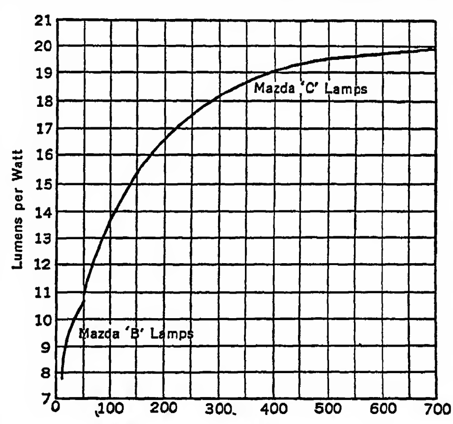


FIG. 9. Efficiency and Lamp Size

Table IV.* Effects of Voltage Increase

Effects of increase in voltage, $V_1/V_2 = R$, upon increase in lumen-output, L; watts, P; lumens per watt, K; and decrease in hours of life, H.

Filament Material	Effects Due to Voltage Change in Ratio $R = V_1/V_2$			
	L_1/L_2	P_1/P_2	K_1/K_2	H_1/H_2
Carbon	$R^{5.62}$	$R^{1.96}$	$R^{3.66}$	$1/R^{20.5}$
Mazda B	$R^{3.51}$	$R^{1.58}$	$R^{1.93}$	$1/R^{13.5}$
Mazda C	$R^{3.38}$	$R^{1.54}$	$R^{1.84}$	$1/R^{13.1}$

* General Electric Co. Bul. LD 4.

LAMP RATING AND PERFORMANCE. As all but a small fraction of the lamps made in this country are manufactured under license from the General Electric Co., their published data are given, with acknowledgment, as representative of American practice.

The lamp manufacturers' schedules of standard large lamps show approximately 140 different types of lamp for use on constant-potential circuits. These differ in rated watts and volts, also in shape, color, and other characteristics of the bulbs and bases. In addi-

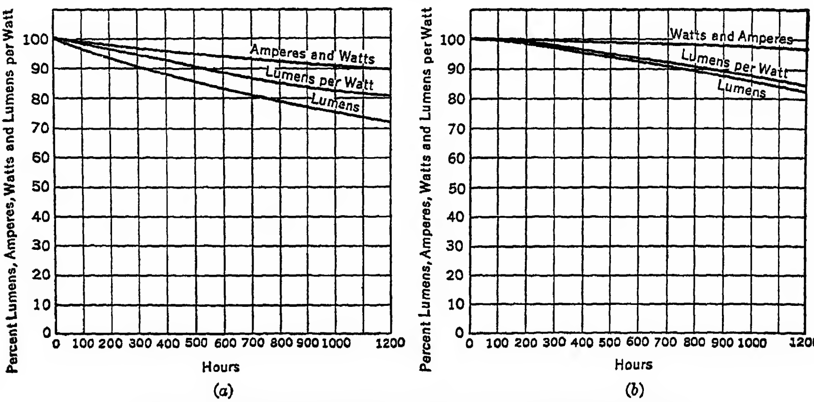


FIG. 10. Life Characteristics of Vacuum (*a*) and Gas-filled Lamps (*b*)

tion to general-purpose lamps, these include lamps for many specialized uses—airway beacons, locomotive and other headlights, floodlights, indicators, locomotive cabs, projection, traffic signals, street-railway cars, spotlights, etc. The differences between these do not include small differences in voltage (110, 115, 120 volts). There are also approximately 40 miniature lamps (Table IX) and 9 constant-current street-lighting lamps. Many lamps are covered in federal specifications WL–101.

Constant-potential lamps are rated in watts, and for the voltage at which they are designed to give a life of 1000 hours and presumably operate most economically. The

Table V. Standard Large 115-volt Mazda Lamps for General Service

1935 data

Watts	Mazda B or Mazda C	Rated Initial Lumens	Rated Initial Lumens per Watt	Mean Lumens Per Cent of Average Initial Lumens
3	B	14	4.7	..
6	B	37	6.1	..
10	B	74	7.4	..
15	B	138	9.2	88
25	B	252	10.1	86
40	C	428	10.7	91
60	C	750	12.5	93
100	C	1,510	15.1	91
150	C	2,400	16.0	91
200	C	3,400	17.0	89
300	C	5,490	18.3	88
500	C	9,800	19.6	88
750	C	14,550	19.4	88
1000	C	20,700	20.5	87

Where, from 150 to 500 watts, both inside-frosted and clear PS bulbs are available, the light data are the same. All the above data are based on 1000-hour life except for the 6- and 10-watts lamps (for decoration, indicators, and sign use) which have 1500-hour life, and the 3-watt "all nite" lamps for which the life is not established. This lamp has the standard prongs of an attachment plug. Three-light, 115-volt lamps have two filaments and three terminals. Either filament or both may be used at once, thus giving a choice of three degrees of illuminaton. The life of these lamps is 1000 hours, and the efficiency of each filament is the same as in the corresponding single-filament lamps. This lamp is available in three sizes: for general lighting, base-up operation, 50–100-watt, 150–200-watt, and 200–300-watt; and for indirect lighting, base down, 100–200-watt.

standard voltages are 110, 115, and 120 volts, and lamps of these three voltages are rated as in the 115-volt class. Similarly the 230-volt class includes lamps of 220, 230, 240, and 250. The lumen-output, given by each size of lamp, is specified in the lamp schedules and determines the performance of the lamp. Efficiency in lumens per watt is also given. A nominal candlepower rating for street series or constant-current lamps was formerly used on the basis of 1 candlepower per 10 lumens output but is now discarded. The standard manufacturing tolerance is ± 5 per cent on watts and ± 7 per cent on lumens per watt. (Lieb, *Trans. I.E.S.*, Vol. 25, 1930, p. 639.) Data on several most-used types of Mazda lamps are given in Tables V to IX.

Table VI. Standard Large Daylight Lamps

1000-hour life

Watts	Mazda B or Mazda C	Rated Initial Lumens *	Rated Initial Lumens per Watt *	Watts	Mazda B or Mazda C	Rated Initial Lumens *	Rated Initial Lumens per Watt *
25 †	B	163 †	6.5 †	150	C	1560	10.4
50	B	325 †	6.5 †	200	C	2210	11.0
60	C	490	8.2	300	C	3570	11.9
100	C	980	9.8	500	C	6370	12.6

* Approximate values.
† Designed for sign use.

Table VII. Standard Constant-current Street Lamps

Mazda C, 1350 hours' life. Mogul base.

Rated Initial Lumens	Amperes	Average Volts	Average Watts *	Rated Initial Lumens per Watt	Mean Lumens, Per Cent of Average Initial Lumens *
1,000	6.6	9.7	63.7	15.7	100
2,500	6.6	22.2	146.2	17.1	100
4,000	6.6	33.9	223.5	17.9	100
4,000	15	14.3	215.1	18.6	98
6,000	6.6	50.5	333.3	18.0	98
6,000	20	15.3	306.1	19.6	95
10,000	20	25.4	507.6	19.7	92
15,000	20	37.9	757.6	19.8	88
25,000	20	60.7	1213.6	20.6	84

The good maintenance of lumen-output in series lamps is due to the fact that, with constant current, the heat, I^2R, increases as the resistance increases, with the decrease of filament area. During the first half of the rated life, the lumen-output increases about 2 per cent, after which it again falls to about the initial value at its rated life. Characteristic curves for these lamps are shown in Fig. 12.

Type D lamps are not given the name "Mazda." Their rated life is 500 hours, and the cost is about two-thirds that of the Mazda lamps of the same size. Data for the four sizes now (1936) being sold are given in Table VIII.

Table VIII. Type D Large Lamps

Watts	Bulb	Rated Initial Lumens (Approx.)	Rated Initial Lumens per Watt (Approx.)	Mean Lumens, Per Cent of Initial (Approx.)
7 1/2	G 11	51.3	6.85	..
15	A 15 inside frosted	142	9.5	88
30	G 19 *	330	11.0	86
60	A 19	822	13.7	93

* Inside frosted or outside colored.

LAMPS FOR PROJECTION. These are made in sizes from 50 to 1000 watts. They are all in cylindrical bulbs (T8 to T20). The life ranges from 25 to 50 hours. The efficiencies range from 16 to 28 rated initial lumens per watt. The filament is specially designed to give a concentrated light source. The sections of the filament are arranged in one plane (monoplane), or two planes (biplane), so that the combined effects is that of a luminous square. In the biplane grouping there is a staggered arrangement so that the sections of filament in the rear plane are seen through the spaces between those in the front plane. Special glass of high melting point is used with the larger lamps, and forced ventilation is essential when they are enclosed, as for instance, in projection lanterns.

PHOTOFLOOD LAMPS. These are lamps so overrated as to voltage that, when operated on a 115-volt circuit, they give a very white light, equivalent in its actinic effect to lamps of much higher wattage operated at normal voltage. In consequence the rated life is much shortened. As the periods of operation need be measured in seconds only, this short life is not a serious defect. Two sizes of the photoflood lamp are available: a 250-watt lamp in an A21 bulb giving 9000 lumens and having a life of 2 hours, and a 1000-watt lamp in a PS35 bulb giving 35,000 lumens and with a 10-hour life. Both lamps operate with an efficiency of about 35 lumens per watt. The 120-watt lamp has the actinic value of a standard 750-watt lamp at normal efficiency. While arranging the picture a standard lamp of large size may be substituted for the photoflood lamp.

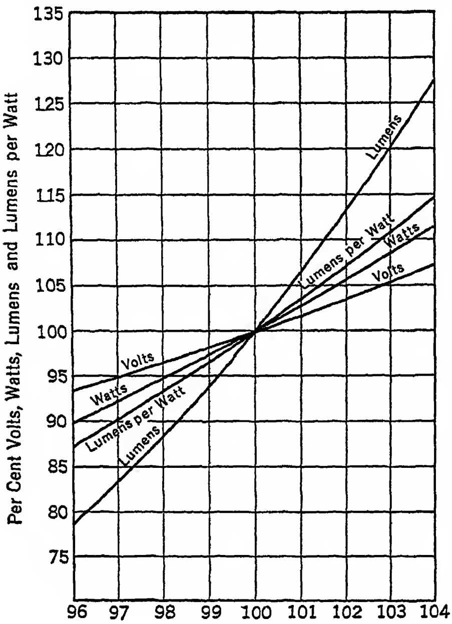


Fig. 11. Characteristics of Series Lamps

MINIATURE LAMPS. In 1929 more than 278 million miniature lamps were sold. Of these 49.8 per cent were used for automobiles, 18.4 per cent for flashlights, 25.4 per cent for Christmas-tree decorations, and the remaining 6.4 per cent for miscellaneous uses, including physical and dental service, signals, indicators, miners' caps, and many others. Except for Christmas-tree decorations, which are usually operated in series groups on 115-volt circuits, the current for these lamps is usually supplied from storage or dry-cell batteries, so that the voltage is very variable, especially with dry cells. Since in the smallest flash-

Table IX. Miniature Mazda Lamps

Mazda Lamp Number	Service	Mazda B or Mazda C	Design Volts	Design Amperes	Bulb Diameter, in.	Rated Average Life, hrs.
Mazda 1	Flashlight	B	2.25	0.25	15/32	3
Mazda 10	Flashlight	B	2.33	0.27	7/16	5
Mazda 13	Flashlight	B	3.70	0.30	7/16	14
Mazda 14	Flashlight	B	2.47	0.30	7/16	14
Mazda 19	Toy	B	1.25	0.60	9/16	75
Mazda 41	Radio panel	B	2.50	0.50	13/32	3000
Mazda 63 3 cp	Auto rear, inst., step, side, aux. lp.	C	6.85	0.54	3/4	250
Mazda 81 6 cp	Auto, dome and panel	C	6.90	0.88	3/4	250
Mazda 87 15 cp	Auto, signal and backing lp.	C	6.75	1.71	1 1/8	150
Mazda 1000 32 cp	Auto, head lp. / depressible beam	C	6.20 / 6.20	3.78 / 3.78	1 1/4	125
Mazda 1110 32 cp	Auto, head lp. / depressible beam	C	6.50 / 6.50	2.45 / 2.45	1 1/4	125
Mazda 1116 32–21 cp.	Auto, head lp. / depressible beam	C	6.20 / 6.50	3.78 / 2.52	1 1/4	125
Mazda 1129 21 cp	Auto, head and spot lp.	C	6.50	2.39	1 1/4	125
Mazda 1133 32 cp	Auto, head and spot lp.	C	6.20	3.61	1 1/4	125
Mazda 1158	Auto, combination rear and signal lp. and head lp, for older Ford cars	C	6.50 / 6.85	2.49 / 0.53	1 1/4	125 / 250

light unit the periods of use are very short, and for compactness high efficiency is needed, the lamps are designed for only about 3 hours of life. At the other extreme are some radio panel signal lights, where efficiency is of no importance, and where a life of 3000 hours is provided. The schedule of lamps given in Table IX covers more than 90 per cent of the consumption of miniature lamps.

Photoflash Lamps

In an oxygen-filled bulb (A19 or A23) is a small low-voltage filament adjacent to a large crumpled piece of thin aluminum foil. When voltage is applied to the lamp terminals, the filament is momentarily raised to a high temperature and burns out, but in doing so it ignites the aluminum foil which is consumed in about $1/_{50}$ second, with a flash of light equal in intensity, for the largest size, to that of five hundred 1000-watt lamps. This lamps replaces magnesium flashpowder. It will operate on any emf from 2.5 volts up. It is made in 3 sizes, Nos. 10, 20, and 75.

Lamps for the Production of Infra-red and Ultra-violet Radiation

Lamps commonly used in equipment for infra-red medical treatment are large lamps, usually with a carbon filament, underrated as to voltage. These lamps thus have a relatively large proportion of infra-red radiation. They probably offer no advantages, however, over the use of ordinary standard large lamps. (Porter and Ditchman, *Trans. I.E.S.*, 1933, Vol. 28, p. 465.)

Among the light sources designed to give ulta-violet radiation, only one type is distinctly an incandescent lamp. (See also Gaseous Conductor Light Sources.) The Mazda CX lamps are standard 60-, 250-, and 500-watt lamps, except that the bulb is made of a special ultra-violet-transmitting glass, which passes a high percentage of the small volume of ultra-violet produced by filaments operated at a temperature giving a 500-hour life. (*Trans. I.E.S.*, 1932, Vol. 27, p. 28.)

Temperature of Filament and Lamp

The lamp filament, like any other body, will operate at such a temperature that the heat dissipated is equal to the heat produced. The heat produced is definitely determined by the watts consumed. In the small vacuum-type lamp, the heat is nearly all radiated from the filament, though the absorption of the heat by the glass bulb results in convection currents in the air, which carry off the heat from the surface of the lamp. In the gas-filled lamps, convection currents within the lamp itself play an important part in dissipating the heat from the filament. Also, in the larger Mazda C lamps, and in the low-voltage lamps, the heat conducted away by the lead-in wire is a considerable factor.

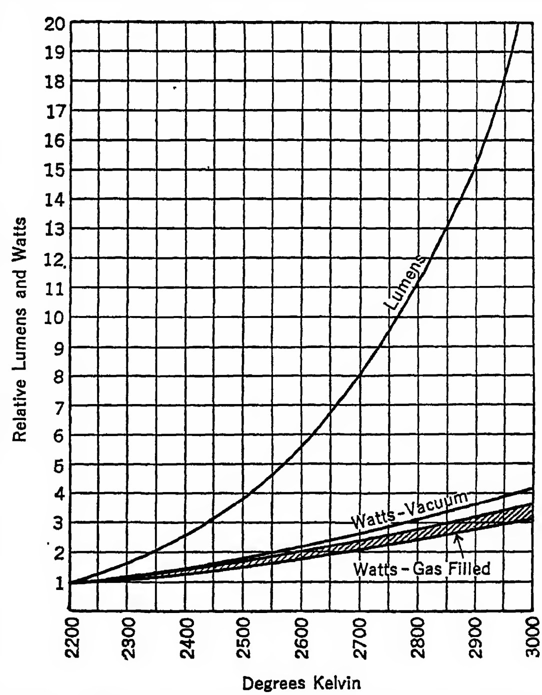


FIG. 12. Relation of Lumens and Watts to Filament Temperature

Filament temperatures range, for Mazda B lamps, between approximately 2400 and 2570 deg K. The temperatures (K) corresponding to any given efficiency for a B lamp (coiled filament) may be obtained from the following equation: $T = 2075 + 47\ l$, where l

is the lumens per watt. The temperature of Mazda C lamps varies from approximately 2680 to 3250 deg K, the latter temperature corresponding to an efficiency of 28 lumens per watt, in the 3- to 10-kw lamps with a 100-hour life. The photoflood lamp has a temperature of 3490 deg at an efficiency of 35 lumens per watt. The temperature for Mazda C lamps is given by the following equation: $T = 2320 + 33.3\,l$. The relation of lumens and watts to filament temperature is shown in Fig. 12.

The temperature of the surface of small lamps is so low as not to be a source of trouble, unless the lamp is in contact with inflammable material; but the temperature of the outside of the bulb of the larger lamps rises to a considerable figure. Examples of maximum temperatures found on the bulbs of the larger lamps when operated in the open air are given in Table X. This maximum temperature is generally found slightly above the level of the filament.

Optimum Lamp Life and Cost of Light

By changing resistance and thus designing the lamp to operate at higher or lower temperature, the maker may select any reciprocal relation of life and efficiency desired. If a low temperature is selected, the life will be long, but the efficiency will be so low that the large amount of energy taken will make the cost per lumen-hour excessive. On the other hand, if a high temperature is used, high efficiency will keep down the cost of energy, but the resulting short life of the lamp will again raise the unit cost of light.

Although varying conditions of operating would suggest a large variety of lamp ratings, it is fortunate that over a considerable range of temperature the combined expense of energy and lamp renewal is so nearly constant as to make two standard temperatures giving standard lives of 1000 and 500 hours respectively satisfactory for all ordinary service. It is only for specialized uses that other temperatures are used.

Let L = the average lumen-output during the given life; C = the average cost (cents) per hour for the power used; c = the cost of the lamp; t = the number of hours operated. Then the total cost per lumen-hour, $K, = (Ct + c)/Lt$. By calculating this cost for several assumed life-periods for a given lamp, the life to give minimum cost per lumen-hour may be determined. The same formula may be used to determine the optimum efficiency to be used, or to compare the cost per lumen-hour of different types of light source. Calculations made in this way indicate, for normal conditions, life of less than 1000 hours, but the matter of inconveneince due to replacement and the psychological objection to short life must be considered.

Table X. Bulb Temperature of Mazda Lamps

Size of Mazda C Lamps, watts	Maximum Bulb Temperature, deg fahr	Size of Mazda C Lamps, watts	Maximum Bulb Temperature, deg fahr
50	257	300	297
100	275	500	379
150	338	750	310
200	330	1000	371

The rated life given as 1000 hours, 500 hours, etc., must be taken as the average life of a considerable number of lamps when operated at the rated voltages. Thus Fig. 13 is given by the makers as a typical burn-out curve for Mazda lamps. It will be noticed that although a few lamps burned out in less than 500 hours, and a few lasted nearly 2000 hours, the great majority held rather closely to the rated life of 1000 hours.

Arc Light Sources

(See Sec. 3, Art. 25, for the theory of the electric arc.) Here will be considered only those light sources in which the current passes through the atmosphere between two solid electrodes. (See also Gaseous Conductor Light Sources.) The positive and negative electrodes are termed respectively anode and cathode. The active conducting medium of the arc is supplied by the electrovaporization of the cathode. With the exception of the carbon arc, it takes a higher, and generally a much higher, voltage to maintain an arc with alternating than with direct current.

VOLTAGE DROP ACROSS AN ARC. The voltage drop across an arc comprises three elements: (1) a sensibly constant drop at the anode, (2) a similar but much smaller drop at the cathode, and (3) a variable drop in the arc stream. The electrode drops in air depend only on the nature of the electrodes. The drop in the arc stream varies with the current according to Ohm's law, but the relation is greatly complicated by the changes in the resistance of the arc stream. Steinmetz (Radiation, Light and Illumination, p.

139) proposes the following general expression for plain carbon arc conduction in air: $E = \{36 + 130\,(l + 0.33)\}/\sqrt{I}$, where E is the total voltage, 36 the electrode drop, l the arc length in inches, and I the current.

STEADYING RESISTANCE. An arc on a constant potential circuit must be compensated by a considerable ballast of stable series resistance or reactance. (See Pender's Handbook, 1922 Ed., p. 50.)

POWER FACTOR. Owing to the distortion of the current wave by the varying resistance of the arc, the a-c solid carbon arc has a power factor of about 85 per cent, although the current and voltage pass through their zero values simultaneously; with some flame arcs the power factor may be nearly 90. (See Sec. 3, Art. 4.) Reactive ballast coils when used in constant-potential solid electrode lamps reduce the overall power factor to values between 60 and 75 per cent.

SOURCES OF LUMINOSITY. There are three distinct modes of light production by electric arcs, viz.: (1) by incandescence of the electrodes, due to their high temperature; (2) by the luminescence, in the arc, of salts derived from mineralized carbon electrodes; and (3) by the luminescence, in the arc, of the conducting vapors from the cathode. The first is exemplified by the plain carbon arc. This arc was the first form of electric light, but is no longer used for general lighting. It is, however, still used to some extent for small projectors. With mineralized electrodes, arc lamps are used for large projection equipment, photography, and the supplying of ultra-violet radiation. The color of the light and its efficiency depend on the nature of the luminescent salts and the temperature attained by the arc. The third type of light production is exemplified by the magnetite or luminous arc.

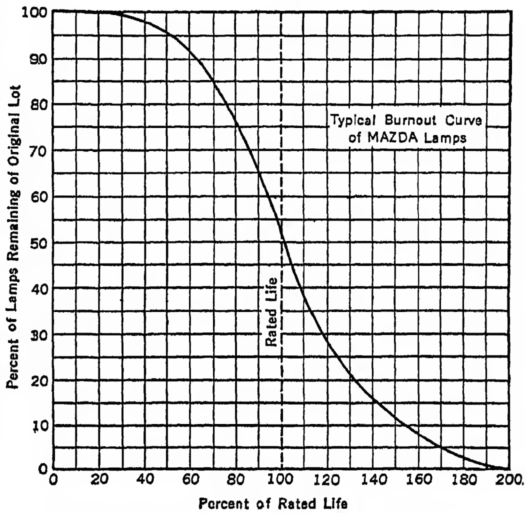


FIG. 13. Typical Burnout Curve of Mazda Lamp

THE CARBON ARC. (See, also, National Projector Carbons, the National Carbon Co., Cleveland, Ohio.) This arc was originally used with two solid carbon electrodes. When on direct current, the upper electrode is used as the positive, or anode; owing to the peculiar action of the arc, its tip becomes hollowed out into a crater and attains a very high temperature. The lower, negative electrode becomes pointed and, while white hot, is still far below the temperature of the positive. To reduce the tendency of the arc to move from point to point on the positive electrode, a core of soft carbon, impregnated with the arc-supporting salts of the alkali metals, is provided in the positive electrode. Also, the diameter of the negative electrode is reduced, thus further steadying the arc. To make up for the reduced carrying capacity of the smaller electrode, a copper coating is provided. A core is also sometimes used in the negative electrode. For currents from 25 to 140 amp, National positive carbon diameters run from $^5/_8$ to $1^1/_8$ in. and negative from $^5/_{16}$ to $^1/_2$ in. Their lengths are 12 and 6 in., respectively. When used on alternating current, the plain carbon arc with unimpregnated electrodes loses the advantage of the high temperature in the positive crater and gives only about half as much light from it as on direct current. To rectify this, for light-projection work, where alternating current must be used, electrodes with cores impregnated with rare earth salts are used. When focused on the positive electrode, and with the white light of the arc which results from the impregnating salts, a good projection light is obtained. For a-c arcs the National carbons range from $^1/_2$- to $^7/_8$-in. diameter for both upper and lower, corresponding to currents of 25 to 100 amp. Reflector lamps use 9- to 14-mm positive carbons and 6.4 to 10 mm negatives. The corresponding currents range from 10 to 35 amp. An improved type of carbon electrode, National SRA, made in 12- and

13-mm sizes, increases the practicable positive carbon current density from between 120 and 165 amp per sq in. to 205. Two types of low-voltage high current intensity a-c arcs, using 6-, 7-, and 8-mm carbons, for 40 to 45, 60 to 65, and 75 to 80 amp, respectively, have recently been developed. These electrodes have a current density of 800 to 1090 amp per sq in., and are metal coated. (Joy and Downes, *Soc. Motion Picture Engrs.*, Vol. 21, 1933, p. 116.) The operation of this a-c lamp with a specially designed constant-current transformer results in high efficiency. High-intensity arcs for direct current, which use cerium fluoride in the positive cores, were first made for searchlights. This type of positive electrode is continually rotated by a motor. These are designed for from 75 to 160 amp. They are also used for condenser-type projection equipment in large theaters; also, taking from 60 to 85 amp, in reflector-type projectors. In a recent further development the positive carbon is non-rotating, and with 6- and 7-mm positive carbons 32 to 65 amp are used. In the 9- to 16-mm rotating carbon type as used for large searchlights and projectors, the candlepower per square millimeter is 400 to 800. The action of these high-intensity arcs depends upon the formation in the positive crater of cerium carbon particles which are heated to a temperature of about 5000 deg cent. (Joy and Downes, *Soc. Motion Picture Engrs.*, 1934, Vol. 22, No. 1, p. 42; T. R. Bassett, *Trans. I.E.S.*, 1932, Vol. 27, p. 623.)

ARC LIGHT SOURCES FOR THE PRODUCTION OF ULTRA-VIOLET RADIATION. (Coblentz, *Trans. I.E.S.*, 1928, Vol. 23, p. 223; Greider and Downes, *ibid.*, p. 637.) See Pender and McIlwain for ultra-violet radiation and its application. The neutral or unimpregnated arc has so little ultra-violet radiation at moderate current values as to be of much less value for therapeutic purposes and similar applications than the later types of arcs. The use of different impregnating materials in the cores of the carbons offers unlimited possibilities as to the type of radiation produced. Cerium and other rare earth salts give a white flame similar to that used for the high-intensity projection arc, but for ultra-violet radiation, are not operated at such high intensity. A core impregnated with metallic salts, including iron, gives about the same radiation just below the visible spectrum with several times as much below about 0.3 micron. Differing types of lamps are built for industrial radiation, individual therapeutic, and solarium therapeutic uses. For the last purpose, lamps taking 60 amp and 50 volts at the arc, or 20.5 amp on a 230-volt circuit, and 3100 line watts input, may be grouped either 2 or 4 in a single housing. In these lamps, the carbons are moved up and down automatically by a small electric motor to adjust the length of the arc, which is also maintained at an approximately fixed location in the lamp. With increase in current, the radiation increases more rapidly than the increase in current, and this is especially true of the ultra-violet portion of the spectrum. This increase, though irregular, reaches, in some cases, several times the increase in current.

Gaseous Conductor Lamps

Here are considered only those lamps in which the gaseous conduction takes place inside of a sealed tube, as distinguished from the atmospheric conduction of the lamps classed above as arcs. (Cady and Dates, *Illuminating Engineering*, p. 131.) At atmospheric pressures, all gases offer very high resistance to the passage of electric current. When, however, a gas is highly rarefied the resistance is greatly decreased; and with sufficient potential, a luminous discharge takes place within the enclosing tube. The phenomena in such tubes are quite complicated and dependent upon the emission of electrons and the ionization of the gas. Where a cold cathode is used, several hundred volts are consumed in the ionization of the gas at that electrode. For a given gas and a given electrode, and up to a critical current density, this is a constant potential gradient through which the ions must drop in order to produce the necessary emission of electrons by the bombardment of the cathode, and since no corresponding light production results this drop tends seriously to reduce the efficiency of this type of light source, and necessitates the use of long tubes and correspondingly high voltages. Step-up transformers with 2000 to 15,000 volts on the secondary, adapted for 9-mm tubes from 2 to 35 ft in length, and for 15-mm tubes from 2 1/2 to 65 ft are used. Besides loss of efficiency and the necessity for high voltage, these cathode phenomena cause two other difficulties. The bombardment of the cathode is so violent that particles are actually torn off and projected to the walls of the tube, thus causing blackening. The bombardment is also associated with a gradual absorption of the gas, with consequent objectionable decrease of pressure. These phenomena affect the satisfactory life of the tube, which, however, even so, is estimated as several thousand hours. The use of reactance as ballast to stabilize the arc results in a low power factor. In the case of neon and other signs it is practicable to compensate with a condenser on the primary side which may well be combined in one unit with the transformer.

HIGH-VOLTAGE, GASEOUS CONDUCTOR LIGHT SOURCES. The use of cold-cathode, gaseous conductor lamps is largely confined to electric signs and decorations. Neon has been the gas most extensively used for this purpose. The diameter of the tubing for these signs varies from about $^3/_8$ in. to about 2 in., though usually not over 1 in. Actual data derived from a survey by H. P. Cook (*Trans. I.E.S.*, 1929, Vol. 24, p. 133) gave the following: watts per foot, 5.5 to 11.6, average 8; power factor 34 to 56 per cent, average 40 per cent; 15-mm tubing, watts per foot, 3.9 to 8.6, average 6.1; power factor 36 to 73 per cent, average 50 per cent. Average period of burning about 14 hours. In addition to the well-known orange color of the neon sign, blue is obtained by combining neon and mercury, and green by using a tube of fluorescent uranium glass. Other colors are being developed.

The neon glow lamp is a cold-cathode lamp in which the electrodes are separated only $^1/_{16}$ in. On a 135-volt circuit, a glow forms, covering both electrodes for alternating, and only one for direct, current. A resistance of several thousand ohms in the base holds the current down so that the watt-consumption ranges from $^1/_4$ in the smallest lamps to 2 in the largest now made. The light flux produced is very low, but enough to be of value for signal purposes and for low illumination for dark-adapted eyes.

THE HOT CATHODE. By substituting for the emission of electrons by bombardment, thermal emission, at the cathode, the drop at this electrode may be reduced to a few volts, thus permitting the use of shorter tubes and higher current density; this likewise eliminates sputtering and the injurious lowering of the gas pressure. Here, also, coating the electrode with an oxide, or with the metal thorium, greatly improves the electron emission. (See Pender and McIlwain, also, Hull, *Trans. A.I.E.E.*, 1928, Vol. 47, p. 743; Found and Forney, *ibid.*, p. 747.)

MERCURY-VAPOR LAMPS. The cathode is not separately heated in mercury-vapor lamps, but the thermal electron emission phenomenon is due to the characteristics of the mercury surface, which results in a very high-temperature spot that serves as a cathode, wandering over the surface of the mercury pool, which connects to the negative terminal of the lamp. The high temperature of this spot results both in the emission of the electrons and in the evaporation of the mercury, thus producing the mercury vapor. The low-pressure mercury lamp, sometimes called the Cooper Hewitt lamp, employs, as generally used, a 1-in. tube, 50 in. long. Tubes 35 in. long are also used, and larger and longer tubes for special purposes. In the d-c lamp, the watts per lamp are nominally 385 for direct, and 450 for alternating, current. Rated lumens are 5200 and 5625, and lumens per watt, with reflector, 13.4 and 12.5. The light output falls 20 per cent in 1800 hours, but only 9 per cent in the next 2500 hours. The a-c power factor is about 85 per cent. The discharge is started by a small mercury switch, which, being tilted by the application of the line voltage, breaks the current shunted through a reactance. As in all other arc and gaseous discharge lamps, the use of a ballast is necessary; this reduces the efficiency and, with alternating current, the power factor of the lamps to the values given above. The color of the light is characteristic of the mercury-vapor spectrum at low pressure. (Buttolph, *Gen. Elec. Rev.*, 1920, p. 741.) The fact that the mercury at the pressure used has a prominent line in the yellow and a lesser one in the yellow-green has led to the claim for monochromatic quality, which has been seemingly substantiated by many successful applications where observation of fine detail is essential. The presence of strong lines at the blue end of the spectrum casts some doubt on the monochromatic claim. The character of the light also increases the effectiveness of the light at low intensities in accordance with the Purkinje effect. (See Art. 3.) It is also useful for certain types of inspection and for photographic work. Combined with incandescent lamps, mercury vapor can be made to produce a fairly satisfactory white light. (See Art. 4.) This low-pressure lamp is also made with a special ultra-violet-transmitting glass, for the purpose of giving sufficient ultra-violet radiation to promote the health of workers who stay for long hours under its illumination.

HIGH-INTENSITY MERCURY LAMP. This a-c lamp, announced in June, 1934, gives 16,000 lumens at 40 lumens per watt, initial net efficiency, thus consuming 400 watts. Mean lumens, 85 per cent of initial; estimated life, 2000 hours. The lamp itself now requires 240 volts to start, but operates with 155 volts. For operation on a 115-volt circuit each lamp requires a 2.8-amp transformer with high reactance, with which is combined a 20-μf condenser, thus raising a 60 per cent power factor to about 92 per cent. The lamp bulb is 1 $^3/_8$ in. in diameter, and is made of a heat-resisting glass, which permits of operation at a high temperature corresponding to about atmospheric pressure. The luminous discharge takes place between tungsten spiral electrodes near the ends of the 7 $^1/_2$-in. bulb; coats of barium compounds on the tungsten spirals provide electrons for starting. Argon also assists in this. A 2-in. by 12 $^3/_4$-in. outside tube conserves heat and protects the circuit run on the outside of the inner bulb to the tip electrode. The

lamp takes about 12 min to come to normal operation, but it reaches 60 per cent of its luminous output in about 4 min. A combination of about equal lumens of high-intensity mercury and of tungsten incandescent (500-watt) is claimed to give a fair approach to daylight quality. Similar lamps giving 4000, 6000 and 10,000 lumens are now made (1936) and smaller lamps are being developed.

When enclosed in a tube molded from fused silica or quartz, the mercury-vapor discharge becomes an important source of ultra-violet radiation. (See Pender and McIlwain.) In this form, mercury-vapor lamps have the trade name Uviarc. (Uviarc Lamps and the Ultra-violet, by L. J. Buttolph, Gen. Elec. Vapor Lamp Co.) These lamps take about 4 amp on a 220-volt d-c circuit and are provided with a transformer and rectifier for use on alternating current. They operate at about atmospheric pressure, and have about 25 volts drop per inch in the arc as compared to 1.5 volts in the low-pressure arc. The operating temperature approaches 1400 deg cent, the softening temperature of fused silica. The spectrum radiated extends from 1.014 μ in the infra-red to 0.185 μ in the ultra-violet.

By the use of a suitable filter, any narrow portion of the spectrum, including a characteristic mercury line or band, can be isolated, thus giving light of a highly monochromatic character. A smaller equipment is also made, especially for laboratory use.

The Sunlight Lamp (General Electric trade name) combines an incandescent filament and a mercury arc between tungsten terminals. This is made in two sizes, the S1 and S2 lamps. A special transformer which accompanies the S1 lamp has a starting voltage of 33, which lights the filament. The heat of the filament vaporizes the mercury and permits the arc to form between the tungsten electrodes. The voltage then falls to 11, and the current rises to 30 amp. The tungsten electrodes are incandescent and give about 68 per cent of the light, 35 per cent coming from the arc and 7 per cent from the filament. The glass of the bulb is especially designed to pass ultra-violet radiation, but to cut it off at 0.28 μ. With the transformer, the S1 lamp takes 400 watts. The smaller S2 lamp takes 130 watts, with 8.5 amp at 15 volts. The approximate lumens are 6000 and 1600 with 18.3 and 12.5 lumens per watt. The power factor for each lamp is about 50 per cent, and the lamps are designed for a life of 300 hours. These lamps are equipped with aluminum reflectors because of the high reflection factor of that material for ultra-violet radiation. (Porter, Egler, and Sturrock, *Trans. I.E.S.*, 1932, Vol. 27, p. 23.) The Type G lamp has a small bulb with two oxide-coated tubular electrodes heated from within by tungsten filaments. It gives but 150 lumens at 3.0 lumens per watt, but a relatively large amount of ultra-violet radiation.

THE HOT-CATHODE NEON LAMP is used on either direct or alternating current. The 2.2-cm diameter lamp, 55 cm long, operates satisfactorily on 125 volts direct current. The a-c tube, either by the use of a transformer with central connection to the cathode and end connections to two anodes, as in rectifiers, or by the use of electrodes in each end of the tube, which will operate either as anodes or hot cathodes, uses both halves of the cycle, and gives a practically non-flickering light. The 220-volt a-c tube, 2.2 cm in diameter and 125 cm long, takes 3 amp and has a luminous output of 10,000 lumens. These tubes are estimated to have a life of more than 5000 hours. In addition to their advertising and display applications, the tubes are used for beacons, and combined with mercury tubes, to obtain an approximately white light satisfactory for photography and for other purposes where an approximation to white is satisfactory.

SODIUM LAMPS. A gaseous conductor lamp using sodium vapor as its light-producing element was introduced in 1934. This lamp combines with sodium, as gaseous conductor, some other gas, such as argon, to facilitate conduction and to make starting possible. At atmospheric temperatures the sodium is solid, but is vaporized by the heat of the discharge through the auxiliary gas. The sodium vapor then takes part in the conduction of the current and gives to the light produced the characteristic yellow sodium color. To maintain the necessary temperature in the lamp it is enclosed in a double-walled tube, or globe, from which the air is exhausted, as in the Dewar flask. Sodium lamps are used only on alternating current and have an anode and a hot cathode in each end of the tube. The following are approximate 1935 data on these lamps.

Lumens	Total	Tube	Net	Tube	Candles per sq in.	Arc Current	Arc Volts	Arc Length, in.	Over-all Length, in.
6,000	165	135	36	40	35	5.0	24	7	14 1/2
10,000	220	200	45.5	50	37	6.6	28	9	16 1/2

The above efficiencies are with an insulating transformer. For series operation, 10,000 lumen lamps may be operated in series, without individual transformers, with an effi-

ciency of 51.3 lumens per watt. The normal life of both lamps is 2000 hours and the arc power factor, 90 per cent. (Buttolph, *Trans. I.E.S.*, 1935, Vol. 30, p. 147; Cleaver, *ibid.*, p. 703.)

Sodium lamps promise to be especially suited to highway and street lighting. The monochromatic character of their light will also adapt them to special uses. While the lamps starts quickly, stable conditions are reached only after some 30 minutes.

3. VISION

The Eye

The pupil, the biconvex lens, and the retina give to the eye the performance of a camera. The lens possesses the image-forming function, and accommodation to distance is provided by its varying curvature. The contraction and dilation of the pupil give automatic accommodation to the intensity and quantity of light entering from the field of view. The retina also adapts itself automatically in sensibility to the flux density of the light falling upon it. The combined effect of these two phenomena is to make the range of sensitiveness of the eye as great as 1 to 8000. The extreme range of brightness of recognizable objects is over 50 billions. (Nutting, *Trans. I.E.S.*, 1925, Vol. 20, p. 529.)

The retina comprises an elaborate structure of microscopic nerve terminals, known as rods and cones. In its periphery, rods predominate; but the ratio of cones to rods increases steadily toward the center or fovea, where the cones greatly predominate. The sharpness of perception diminishes markedly in passing from the fovea to the periphery. At low intensities, rod vision predominates. At moderate and high intensities cone vision predominates.

The adaptation of the eye from darker to brighter surroundings is much more rapid than that for the reverse operation. Thus, to perform 77 per cent of the complete opening of the pupil requires 10 sec, as against 1.6 sec for 77 per cent of the complete closing; that is, the adaptation to brightness, so measured, is more than 6 times as rapid as adjustment to darkness.

Reactions of the Eye

THE SENSIBILITY, OR RELATIVE VISIBILITY CURVE, OF THE HUMAN EYE. The sensibility of the eye to radiation of varying wavelengths differs greatly. Below 0.4 micron and above 0.7 micron radiation does not affect the eye. The maximum sensitivity is for yellow-green light of 0.556 micron. A light source converting all applied energy into radiation of that wavelength would produce approximately 620 lumens per watt. The sensibility curve established as an American standard, as a result of the work of several investigators (*Trans. I.E.S.*, 1924, Vol. 19, p. 176; 1925, Vol. 20, p. 632), is shown in Fig. 14. Lumens per watt for any wavelength may be obtained by multiplying 620 by the proper relative visibility factor from Fig. 14. This curve explains the low visibility found under deep red or deep blue light.

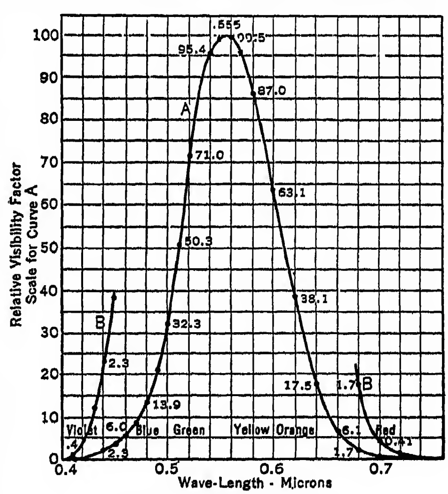


FIG. 14. Sensibility Curve

Purkinje Effect. Transition from vision at high to low intensity involves relative loss of sensibility to red and gain of sensibility to blue, so that two lights of reddish and bluish hue respectively show a ratio of intensities which varies with the brightness of the field of view. This is known as the Purkinje effect. In fields above a brightness of 1 lumen emitted per square foot (1 ft-lambert), the shift is slight. Photometric comparisons involving color differences should be made only in at least moderately bright fields.

VISUAL PHENOMENA. The performance of the eye under varying conditions of illumination is covered in detail in Analysis of the literature concerning the dependency of

visual function upon illumination intensities, by L. P. Troland, *Trans. I.E.S.*, 1931, Vol. 26, pp. 107 to 196, from which the following items are abstracted.

Brightness Discrimination. Fechner's law states that the lowest difference in brightness between adjacent surfaces that can be discerned is a constant fraction of the illumination. From about 7 to 70 ft-c illumination on an 80 per cent reflecting surface, this fraction is in the neighborhood of 1 1/2 per cent. This figure varies considerably, depending upon the conditions under which the test is made.

Effect on Visual Acuity of Increased Illumination. The increase in visual acuity follows an approximately logarithmic law, at least to the neighborhood of 30 ft-c. This makes the increase small above 7 ft-c. These data are based on conditions of high contrast, black on white.

Effect on Speed of Vision of Increased Illumination. The work of many investigators shows a roughly logarithmic relation between speed of vision and illumination. Quantitative results are much affected by such conditions as contrast, size of object viewed, etc. According to this law, speed of vision would show as much improvement between 1/2 and 2 ft-c as between 2 and 8. The logarithmic relation, however, does not hold for very high intensities, probably not above 100-ft-c (100 millilamberts). Luckeish found more than three times as much increase in speed for a certain increase in illumination when reading black type on gray paper of 22 per cent reflection factor, as for black type on white paper. Ferree and Rand found almost twice as much increase in speed of vision when the test object was reduced in diameter from approximately 2 in. to 1 in.

Pupilary Change. Pupilary contraction occurs continuously from 0.0001 ft-c to about 1000 ft-c on white paper. The range in pupil-diameter is from about 8 to about 2 mm. An average diameter of 5 mm occurs for 1 ft-c on white paper.

Effect of Intensity of Illumination upon the Eye. The highest practicable artificial illumination intensities, where properly applied without glare and without excessive contrast, are much lower than natural illumination intensities near north windows, such illuminations as are considered entirely satisfactory. On the other hand, glare from bright or large lights in the field of view, and especially ultra-violet radiation coming directly to the eyes, is likely to be seriously injurious.

Influence of Illumination Intensity upon Practical Operations

(See also Art. 7.) Many practical tests carried on in industrial establishments have indicated considerable increases in production with increases in illumination intensities up to the neighborhood of 10 ft-c for work of medium fineness. Further increase in production with still higher intensities is probable, but except for very fine work such increases are small. Averages taken from 19 sets of figures show an increase of production amounting to 13.8 per cent resulting from an increase in illumination from an original intensity of 2.7 to an approved level of 10.9 ft-c. Some authorities, however, claim that much higher intensities would justify themselves through increased production.

Luckeish and Moss (*Trans. I.E.S.*, 1932, Vol. 27, p. 699) present a new point of view with regard to the advantage of high intensity of illumination. They claim that although very high values cannot be justified from the point of view of increased production, they can on account of decreased drain of nervous energy, as measured by decrease in nervous muscular tension of the subject under test under increased illumination. Over 110,000 observations were taken on 14 subjects. By their methods of measurement, the nervous muscular tension under 1 ft-c illumination averaged 63.2; under 10 ft-c, 54.1; and under 100 ft-c, 43.0. Their probable error was calculated to be less than 1 per cent. Plotted to the logarithms of these three foot-candle values, namely, 0, 1, and 2, these values of nervous muscular tension lie upon a straight line. It is claimed that this saving in nervous strain would justify much higher intensities than are now in general use.

GLARE. Glare may be defined as light shining into the eyes in such a way, or of such quantity, as to cause discomfort, annoyance, interference with vision, or injury to the eyes. "Glare is produced in one or more of three ways: (*a*) excessive flux of light entering the eye, as from a large source of low brightness; (*b*) excessive brightness of any light-source within the field of view, as in the case of unshaded incandescent lamps; (*c*) excessive contrast in brightness between the light-source and surrounding surfaces. Of these three causes of glare, excessive brightness is the cause of more cases of glare than either excessive flux, or contrast." (From Caldwell, Modern Lighting, by permission of the Macmillan Co., publishers.)

One or more of the foregoing conditions must exist in order that glare shall be present, The seriousness of such glare, however, depends upon the following three conditions, not connected with the eye of the observer: (*a*) the duration of exposure to the glare; (*b*) the angle between the source of glare and the line of vision; (*c*) the distance from the

source of glare. (Caldwell, Modern Lighting, p. 18.) If the angle from the line of sight is greater than 30 deg the resulting glare will not interfere with vision, though it may be annoying if the light is too bright. Close to the line of sight, a very small light source may be annoying; but if it is 90 deg or more removed, glare will not result, no matter how bright or large the light source. Light coming from below the line of sight is particularly troublesome. Snow blindness is an example.

In addition to these objective conditions, the health and sensitiveness of the eye itself are important factors in determining the effect of glare. Sensitiveness to glare is a protection to the eyes, and should be cultivated. "Three effects of glare should be differentiated: (*a*) temporary blinding or impairment of vision; (*b*) discomfort, as when looking at the sun, or at a bright artificial light close by. This may be instantaneous or it may be due to continued exposure to a source of light which would not be at once felt as glare; (*c*) injury due to prolonged exposure to glare, often unrecognized. Of these three the last is evidently much the most serious case, though enforced exposure of the second class may easily evolve into it." (From Caldwell, Modern Lighting, by permission of the Macmillan Co., publishers.)

Blinding due to glare is approximately independent of either brightness or area of light sources, but varies as the total light flux transmitted from a source of glare, which enters the eye. That is, the blinding effect of a lamp is not reduced by surrounding it with a globe, except as the total flux is decreased. The comfort and appearance, however, are improved. Brightness values of more than 2 or 3 candles per square inch are glaring if near the line of sight, and $^1/_2$ candle per square inch is to be preferred. Large units of considerable height may be given 5 candles per square inch. In the matter of contrast, a ratio of 200 : 1 is to be regarded as the extreme permissible limit, with 100 : 1 preferred.

Reflected Glare from polished surfaces, as glass desk tops, is particularly bad, because of coming from below. So-called veiling glare reflected from glazed paper, may seriously interfere with the legibility of the type.

Glare is to be avoided by (*a*) keeping down the brightness and light flux of light sources which must come into the field of view; (*b*) avoiding the placing of light sources within the field of view as far as practicable; (*c*) avoiding contrast, by the use of light ceilings and walls. (See also *I.E.S.* Codes of lighting for Factories and Schools.)

SHADE, SHADOWS AND DIFFUSION. Except for the effect of color differences, the forms of objects are recognized only by shades and shadows upon them, and by shadows cast by them upon other objects. While too great uniformity of illumination is therefore undesirable, practical limitations, even with indirect lighting, will hardly ever permit the elimination of shadows to a serious extent. Shades and shadows should not be too black and should not be sharp at the edges. This condition will be accomplished by a suitable number and diffusion of light sources. Diffusion has two different but related meanings. Illumination is diffused when sharp black shades and shadows, and strong contrasts, are avoided. A luminaire is diffusing when it is of sufficient size to give satisfactory diffused illumination. This is in contrast to a virtual point light source such as a bare incandescent lamp. Multiple shadows from a number of sources without sufficient diffused light are annoying.

4. RADIATION, COLORED LIGHT, AND ARTIFICIAL DAYLIGHT

Radiation

The discussion of radiation in the present section is confined to that portion of the general radiation spectrum which affects the eye, namely from 0.4 to 0.7 micron (μ = 0.001 mm). For the fundamentals of radiation, see Sec. 9, Eshbach, *Fund. of Eng.* For vision in connection with radiation, see Art. 3.

In light sources we have to deal with two forms of radiation, and with combinations of these. These are continuous spectra and line spectra. Incandescent solids, as exemplified by lamp filaments, and arc electrodes, give continuous spectra. Starting with a heated body, not yet visible to the eye by its own radiation, as represented by curve a in Fig. 15, and raising the temperature stepwise, a series of radiation curves, b, c, d, e, f, and g, are shown for the temperatures indicated. To obtain relative values the ordinates of these curves must be multiplied by the following factors: a — 1, b — 1.1, c — 8, d — 33, e — 103, f — 265, g — 1100. In so far as these curves cross the region of the visible spectrum between the lines $v - v'$ at 0.4 and 0.7 μ, light is produced, and the efficiency of the body as a source of visible radiation is equal to the ratio of the area cut off between the lines $v - v'$ to the total area under the curve. Such radiation efficiencies are shown for different temperatures in Fig. 16, curve A. It is evident, from this curve, why the

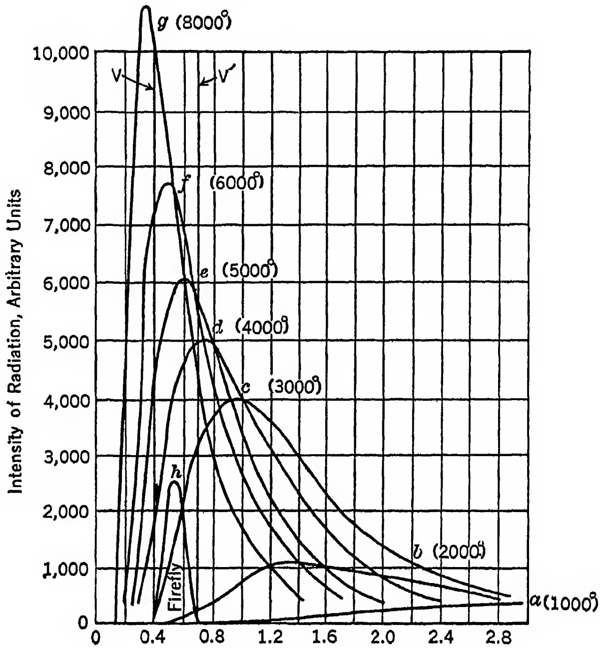


FIG. 15. Radiation Curves for Hot Bodies

efficiency of incandescent lamps increases with temperature, and also why the efficiency of such light sources can never be high. The curve h represents, to its own scale, the curve of radiation from a firefly, with nearly 100 per cent radiation efficiency. All radiation which falls upon and is absorbed by an opaque body heats that body, though the temperature rise may be very small. Cold light cannot exist.

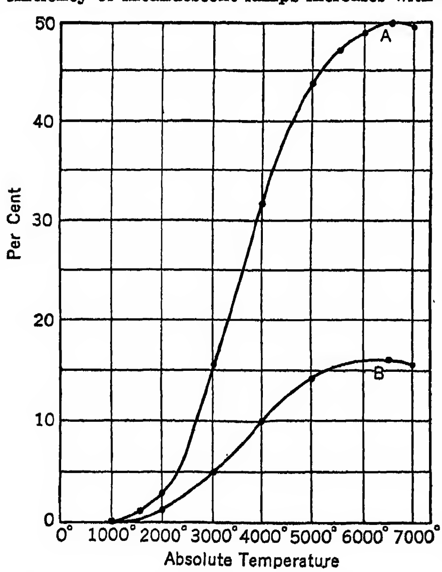


FIG. 16. Radiation Efficiency and Temperature

The curves of Fig. 15 show the total radiation produced at each wavelength, regardless of its effect on the eye or its luminosity. To obtain the luminous effect on the eye, the radiation at each wavelength must be multiplied by the corresponding ordinate of the sensibility curve of Fig. 14. Performing this operation, and then obtaining the efficiencies as for curve A, gives curve B in Fig. 15 (Hyde, I.E.S. Lectures on Illuminating Engineering, 1911), the net luminous efficiency of a black body at different absolute temperatures. Any material whose radiation curve at a given temperature differs in form from the curves of Fig. 15 has selective radiation. If the selectivity is such as to raise the part of the curve between 0.4 and 0.7 μ above normal, there will obviously result an increase in the efficiency of that material. This is the case with

tungsten. (See Art. 2.) The color corresponding to different temperatures, from a just perceptible red to the white light of the sun, is also explained by the relative proportions of the different wavelengths from 0.4 to 0.7 μ, corresponding to the different temperatures. Although sunlight represents the highest black-body temperature of experience, since a definite law between color and temperature exists up to that point it is convenient to extrapolate according to this law for colors, such as skylight, having a still larger percentage of blue than sunlight. (I. G. Priest, *J. Opt. Soc. Am.*, 1923, p. 1179.) Table XI gives color temperatures, abstracted from a table by Norman Macbeth, *Trans. I.E.S.*, 1928, Vol. 23, p. 308. (See also A. H. Taylor, *Trans. I.E.S.*, 1930, Vol. 25, p. 154.) Fig. 17 shows the relative amounts of the various wavelengths relative to white light in different light sources. For each curve the radiation at different wavelengths is compared with that at 0.556 μ as 100 per cent and all curves are scaled to have the same radiation at that wavelength.

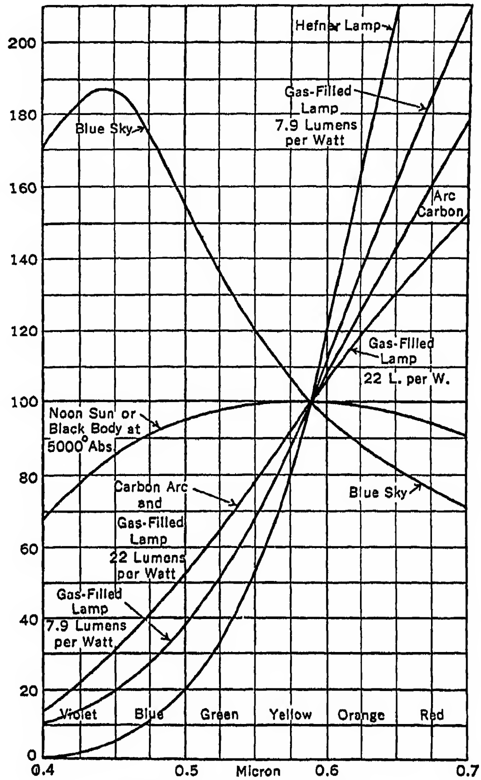


FIG. 17. Relative Radiation

Selective Reflection and Transmission

Any surface exhibits a given color according as it absorbs certain wavelengths of the light which fall upon it, and reflects the remainder. If the incident light is white, then the color, as determined by the combination of colors reflected, is the complement of the color absorbed. White light, or some form of daylight, is the standard for color discrimination. A surface can reflect light only of wavelengths that exist in the incident light. Consequently, if such light is lacking in any wavelength which the material normally reflects, the color of the object will be modified accordingly. If none of the

Table XI. Color Temperatures for Daylight and Artificial Light

Sun, 17 min before sunset	1,900	Sun, 63 min before sunset	3,500
Candle flame	1,930	Mazda daylight, 150-watt	3,600
Sun, 25 min before sunset	2,100	Crater solid carbon arc	3,700
Treated carbon filament, 3.4 lpw	2,170	Tungsten electrode arc	3,800
Tungsten filament in vacuum 5.4 lpw	2,250	Sun, 95 min before sunset	4,000
Acetylene flame	2,380	Noon sun, Washington, average	5,300
Sun, 33 min before sunset	2,500	Uniform overcast sky	6,000 to 7,000
40-watt tungsten filament in vacuum, 10 lpw	2,510		
100-watt gas-filled lamp, 12.9 lpw	2,740	Moore carbon dioxide tube lamp	7,800
Sun, 40 min before sunset	2,900	Blue sky	10,800 to 24,500
500-watt gas-filled lamp, 18.1 lpw	2,910		
1000-watt gas-filled lamp, 20 lpw	2,980		
Projection type, 900-watt, 27.3 lpw	3,220		

Opal glasses generally introduce some long-wave radiation and show color temperatures of 2700 to 2800. Some "white" diffusing glasses go as low as 2600. Blue glasses designed to produce artificial daylight may be had up to 30,000 deg. at 8.7 mm thick. The color temperature for any blue glass rises with its thickness.

wavelengths which the surface normally reflects are present in the incident light, no color will appear; the surface will be black or gray.

In the same way, a transparent or translucent colored material transmits selectively the colors which are not absorbed. If a pure blue glass is placed in front of a pure red, the combination ceases to be transparent, or passes only a little light, if the colors are not entirely pure. For the purpose of obtaining light of special spectral qualities, screens which cut off definite portions of the spectrum, as indicated by transmission or absorption curves, are available. These curves give as ordinates the transmission corresponding to each wavelength of the visible spectrum. Such screens, or filters, are available either in glass (H. B. Gage, *Trans. I.E.S.*, 1916, Vol. 11, p. 1050; also, *Trans. Soc. Motion Picture Engrs.*, May, 1924; see also *Bur. Standards Scientific Paper* 325, Vol. 14, p. 653), or in stained gelatin (Wratten Light filters, Eastman Kodak Co.). A greater variety of colors is available in gelatin, but the spectral characteristics are not as permanent as with glass.

Artificial Daylight

Because of the standard character of daylight-colored light, it is desirable for various purposes to produce it artificially. The principal use of artificial daylight is where color discrimination or color matching is important. The imitation of daylight can be so satisfactorily accomplished that it is customary to shut out the daylight, which is highly variable, both in color and intensity, and perform color-matching operations entirely by a standardized artificial daylight. Some persons who have difficulty in working under the usual artificial lighting claim to find artificial daylight much more comfortable for their eyes. The production of artificial daylight is generally accomplished by the use of a carefully designed blue glass filter, which takes out the excess of radiation of the long wavelengths, and thus passes the light having the proper proportions of the long and short waves. For accurate color matching, the light so produced should have a color temperature closely approximate to that of clear noon sunlight and the accompanying sky light. According to Taylor (*Trans. I.E.S.*, 1930, Vol. 25, p. 154), any color temperature between 5500 and 6500 deg K will give a satisfactory equivalent of daylight. Taylor gives the color temperatures of 100-, 300-, and 500-watt daylight lamps as 3600, 4250, and 4500 deg K. These pass from 62 1/2 to 64 per cent of the lumens produced. He also gives data on other "daylight" units running up to one of 15,000 deg, which passes only 8.7 per cent of the produced lumens. For noon sunlight color, 5800 deg K is required, the efficiency for which is 14.7 to 17.2 per cent of the lumens generated. If the comparison is made for both bare lamp and daylight units in aluminum reflectors, the above percentage becomes for noon sunlight 20.5 to 24 per cent and for north skylight 11.3 to 16 per cent. For many purposes the blue "daylight" lamps, although passing the yellow color of late afternoon sunlight, give entire satisfaction as an imitation of sunlight.

Light which is apparently white may be obtained by mixing different groups or pairs of complementary colors which, nevertheless, are far from giving, by their combination, a continuous color spectrum, and therefore are imperfect for color-matching purposes. Also, a blue filter may give what seems to the eye to be white light, and which yet seriously distorts some colors viewed by it. The true whiteness of a light cannot be determined by its appearance to the eye.

Gaseous Conductor Light Sources

(See also Art. 2.) Hitherto, only the continuous spectrum, and the effects of absorbing filters upon such light, have been considered. The gaseous conductor light sources, as distinguished from the continuous spectrum light sources, give line spectra. That is, their light flux is produced at a number of definite wavelengths. Usually certain of these wavelengths predominate, thus giving the light an especially monochromatic color. Spectra for a number of gases and metal vapors are given by Rentschler, *Trans. I.E.S.*, 1934, Vol. 29, p. 439. Spectra and other data are given by Buttolph, *Trans. I.E.S.*, 1933, Vol. 28, p. 153. He also gives the following data on the efficiency, in lumens per watt, of various gaseous conductor lamps, including accessories. Sodium arc, on a constant potential, 32; mercury arc, 21; low-pressure arc, 18; neon, with inductive ballast, 17; mercury, with tungsten electrode, 12; helium with resistance ballast, 4. For series operation, sodium lamps have an efficiency in the neighborhood of 50 lumens per watt. The strong colors produced at high efficiencies by these lamps are of value for advertising and spectacular effects. The monochromatic character of their lights limit them in their availability for lighting purposes, though the sodium lamp and the high-pressure mercury lamp bid fair to be useful for highway and perhaps for street lighting. By a combination of about 2 watts of high-efficiency incandescent light, with 1 watt of mercury-arc light, or 2 1/2 lumens of incandescent to 1 lumen of mercury arc, a fairly satisfactory

imitation of daylight can be produced with an efficiency far higher than is possible by means of glass filtering equipment. (Grandy, *Trans. I.E.S.*, 1933, Vol. 28, p. 762.)

Colored Incandescent Lamps. Apart from their use for the production of artificial daylight, colored incandescent lamps are in extensive use for decorative purposes. The effect of color is obtained at a considerable expense because of absorption of light. (Taylor, *Trans. I.E.S.*, 1924, Vol. 19, p. 136.) Luckeish and Taylor give as practicable efficiencies for colored lamps as compared to unmodified light: amber, 40 to 60 per cent; red, 15 to 20 per cent; green, 5 to 10 per cent; blue, 3 to 5 per cent. An important application of colored bulbs is the "flame tint" color used in small lamps to imitate flames. Luckeish (Color and Its Applications, p. 254) states that the flame-tint glass used for this purpose passes 80 per cent of the light produced. He also compares the energy required to produce the same color by means of a carbon lamp at 4 watts per mean horizontal candle and finds it to be about 2 1/2 times that required for the tungsten lamp in the flame-tint bulb. The use of colored lamps also has an extensive place upon the stage, as well as for signal lights, such as automobile tail lights, port and starboard lights on shipboard, etc., traffic signals, and railway signals.

5. LIGHT-MODIFYING DEVICES

PURPOSES OF LIGHT-MODIFYING DEVICES. The two principal purposes of such equipment are: first, to prevent glare and promote diffusion, by cutting off the direct vision of the light source, or increasing its size; second, to change the natural distribution of the light source by reflectors, diffusing globes, or refractors, in order to adapt it better to the purpose in hand. Such equipment also improves the appearance of the light source and, out-of-doors, protects it from rain and snow. A classification of modifying equipment may be made as follows:

Reflectors, direct or inverted: Opaque—enameled steel, aluminum, stainless steel, metal plated with silver or chromium, silvered glass. Translucent—white glass, prismatic glass, synthetic materials.

Globes: lanterns, urns, etc., white glass, prismatic glass.

Shades and Screens: art glass, parchment, fabric.

REFLECTORS. As compared with globes, reflectors are less expensive and more efficient. They have the disadvantage, however, of showing the exposed lamp from below the luminaire. Globes and similar devices lend themselves better to artistic effects. For the use of reflectors in light projection and floodlighting, see Art. 6.

Opaque Reflectors cut off all upward-directed light and should therefore be used only with dark ceilings, or where the illumination of the ceiling is not desired. With the diffusing surface of enameled steel, the dome-shaped reflector of large diameter absorbs only about half as much light as the deep bowl-shaped reflector. The General Electric Co. has copyrighted the trade name RLM, and permits its use only on reflectors that meet certain specifications, among which are: a minimum diameter of 12 to 20 in., a maximum cut-off angle of 17 1/2 deg from the horizontal, and a minimum reflection factor for the enamel of 70 per cent. Such reflectors may be provided with a glass neck to throw some light upon the ceiling. Aluminum is much used for small, close-fitting reflectors but hitherto has suffered from considerable tarnishing. Recent developments promise notable improvement, especially in permanence. (Edwards, *Trans. I.E.S.*, 1934, Vol. 29, p. 351; Dickerson, *ibid.*, p. 358.) Because of high reflection factor for ultra-violet radiation, about 80 per cent, aluminum is regularly used for this purpose. (See Sec. 9, Eshbach, *Fund. of Eng.*) Silver, which gives the highest reflection factor when clean, suffers greatly from tarnishing. Chromium and stainless steel have lower reflection factors, but maintain them much better than silver. Silvered glass has high reflection factor and greatest permanency. Its principal use is for small reflectors for indirect lighting luminaires, also for industrial lighting, the lighting of show windows and show cases, and for floodlighting. These glass reflectors are also especially adapted to concealed lighting, either built into the walls and ceilings or mounted above a ceiling skylight. Their specular reflection surface makes the exact design of distribution curve practicable, but is sometimes objectionable from the point of view of glare, where it is in a position to be visible to the eye.

Translucent Reflectors, white glass and synthetic compositions, are like enameled metal, in that the form of the reflecting surface does not determine the distribution curve. They have wide application, however, because of their good appearance, low cost, low brightness, and provision of light for the ceiling. Prismatic glass reflectors are smooth on the inside, and covered with 45-deg prisms on the outside. Light from the lamp passes through the smooth interior, is reflected 90 deg by one of the 45-deg surfaces of a

prism, strikes the other side of the prism, also at 45 deg, and thus is reflected back at an angle with its original path, determined by its angle of incidence to the inside surface. As referred to the prism this phenomenon is called "total reflection." Rounded prism edges and fillets between them allow some diffused light to pass through. A high reflection factor is thus obtained, together with complete control of the distribution curve. Thus Fig. 3 shows a concentrated distribution from such a prismatic glass reflector. The reflection factor is considerably decreased by any dirt that sticks to the reflector. For large industrial reflectors, therefore, a close-fitting aluminum cover is provided.

GLOBES. The best white glass, as used in globes, combines efficient diffusion, designed to give uniform brightness, with low absorption. Standards for the characteristics of white glass and other diffusing media are now being developed by committees of the Illuminating Engineering Society (1936). Globes are usually either flattened or elongated. The distribution curve for lights with globes is considerably affected by the shape of the globe. Thus Harrison gives the data for two globes, having ratios of vertical to horizontal candlepower of $^3/_4$ and 2, respectively. The flattened globe also gives considerably more light flux to the upper 45 deg. Owing to internal reflection, followed by transmission, the overall efficiency of a globe may be much higher than the transmission factor of the glass itself. Satisfactory diffusion requirements limit the transmission of white glass to about 60 per cent, but a globe of such glass may have an overall efficiency as high as 85 per cent.

The prismatic glass globe has horizontal prisms surrounding it, which bend the light by refraction. (See Sec. 9 of *Fundamentals of Engineering*.) These prisms may be designed to give any practicable vertical distribution desired. On the inside of the globe, as used for interior lighting, are vertical flutes for the purpose of spreading and diffusing the light. Such globes are called "refractors." As used for street lighting, these globes are made in two pieces that fit inside of each other, with an air space between. The inner bowl is smooth on the inside and has the horizontal prisms on the outside. The outer bowl has the vertical fluting on the inside and is smooth on the outside. The two are joined together with an airtight seal. The fluting may be replaced by prisms of designed form, by which the horizontal distribution can also be adjusted to throw the light upon the street as desired. Such refractors are called asymmetric. (See Art. 9, also Fig. 21.) Table XII gives the brightness for different sizes of enclosing globes fitted with a 100-watt lamp. Table XIII gives recommended globe sizes for different size lamps, as taken from the I.E.S. Standards of School Lighting, p. 33. In all refractors, as well as in all reflectors with specular reflecting surfaces, the position of the lamp filament vitally affects the form of the distribution curve.

SYSTEMS OF LIGHTING. Lighting systems are classified as general and local "For rooms of considerable size, general lighting has the following advantages over local lighting: (*a*) the whole floor is illuminated so that work can be done at any point; (*b*) troublesome shadows are eliminated; (c) danger of glare is reduced; (*d*) the luminaires are up out of the way of breakage and theft; (*e*) the large lamps are more efficient, and the light is whiter; (*f*) there are fewer lamps to clean and maintain, and the cost of replacement is less than where many local lamps are used; (*g*) the appearance of the room is better." (From Caldwell, Modern Lighting, by permission of the Macmillan Co., publishers.) General lighting is direct, semi-indirect, or indirect. Direct lighting includes all downward-directed reflectors and all globes which throw at least as much light downward as upward. Semi-indirect luminaires emit from 65 to 90 per cent of the light upward, the remainder passing through the bowl, or if closed, through the lower part of the globe. Indirect lighting reflects all the light upward to the ceiling, from which it is

Table XII. Brightness and Size of Enclosing Globes of Good Diffusing Glass *

Light source—100-watt Mazda lamp

Diameter of globe, in.	5	6	7	8	9	10	12	14
Candles per sq in.	4.5	3.1	2.3	1.8	1.4	1.1	0.8	0.6
Lamberts	2.2	1.5	1.1	0.9	0.7	0.5	0.4	0.3

* The globes are assumed to be fairly uniform in brightness and to have an overall efficiency of 85 per cent.

Table XIII. Recommended Minimum Diameters of Enclosing Globes

Lamp watts	75	100	150	200	300
Diameter, in.	10	12	14	16	18

re-reflected downward. Direct lighting includes opaque reflectors, which have 100 per cent downward distribution, translucent reflectors, and most globes and other enclosing units. Semi-indirect and indirect lighting, though desirable from the point of view of

diffusion, necessarily give low efficiency and, especially when the units are open toward the ceiling, depreciate greatly from dust. Their efficiency is also dependent upon the maintenance of a high reflection factor of the ceiling. Indirect lighting by lamps in specular reflectors concealed in coves around the ceiling, ornamental wall brackets, and floor columns is to be preferred, from an esthetic viewpoint, to that from luminaires suspended from the ceiling.

BUILT-IN LIGHTING. In this type of lighting, recesses are built in walls, ceilings, and columns, in which the lamps, with suitable reflecting equipment, are placed. By careful design very pleasing effects with good illumination and fair efficiency can be obtained. For design data, see Beggs and Woodside, *Trans. I.E.S.*, 1931, Vol. 26, p. 1007; Potter and Meaker, *ibid.*, p. 1025; Higbie and Bychinsky, *ibid.*, 1934, Vol. 29, p. 225.

6. LIGHT PROJECTION

METHODS. Light projection is accomplished by: (1) **lenses,** (*a*) simple, (*b*) compound; (2) **reflectors,** (*a*) parabolic, (*b*) spherical, (*c*) elliptical, (*d*) non-symmetrical forms.

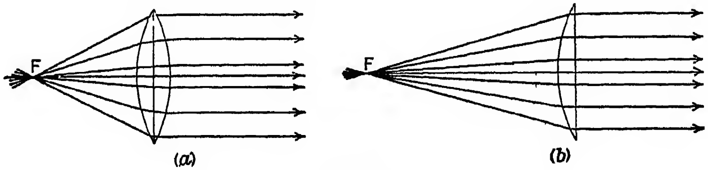


FIG. 18

LENSES. A lens bends the light rays by refractive action of its convergent surfaces, as in a prism. If the light is placed at the proper point, *F*, in Fig. 18, the rays will leave the lens as an approximately parallel beam, as in a spotlight. If it is desired to concentrate the light at a point, as in lighting the stage of a microscope or in the condenser of a lantern, moving the light nearer to the lens will cause more bending and hence a convergence of the rays.

A spherical lens will not bring the rays to an exact focus. It is less costly to correct this error by making a compound lens than by making a non-spherical lens. The compound lens is a combination of two lenses made from glass of different densities and cemented together.

CHROMATIC ABERRATION. The diffraction of the light into different wavelengths, as in the spectrum, also occurs in lenses. This defect also may be corrected by the compound lens. Both defects are lessened by the use of the center of the lens only, with the help of a diaphragm as in a camera. **The Fresnel lens,** shown in Fig. 19, is spherical as to its component rings, but these are offset to keep down the thickness and weight of the lens as a whole. It is extensively used for railway signal lights.

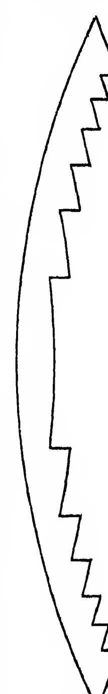


FIG. 19.

Reflectors

THE PARABOLIC REFLECTOR. (Benford, *Trans. I.E.S.*, 1915, Vol. 10, p. 905.) This reflector depends upon the characteristics of the parabola. If a line be placed tangent to the surface of a parabola at any point, and then a line drawn from the point of tangency to the focus, and also one parallel to the axis, these two lines will make the same angle with the tangent. Thus, if a light is placed at the focus, *F*, Fig. 20, a ray striking an element, *M*, of the reflector is reflected parallel to the axis. (See Art. 1.) If the light is moved along the axis toward the reflector, the rays will be caused to diverge; if away from the reflector, the rays will converge, cross, and then diverge. This phenomenon is a means of adjusting the angle of spread of the beam, which, owing to the fact that the light source is not a point, and to imperfections of the paraboloid, is never cylindrical. If the light source is moved upward, that portion of the reflector back of the focal plane will reflect the light downward, and the portion forward of the focal plane will reflect it upward. For a light source of uniform distribution, one-half of the flux is reflected back of the focal plane. The part of the flux reflected forward of the focal plane will depend upon the depth of the reflector, or the angle of the aperture.

The focal length is the distance from the focus to the vertex of the reflector. On its value depends the form of the reflector. A short focal length gives a narrow, deep reflector; a long focal length, a shallow, wide reflector. The 8- to 10-in. automobile headlamps of the recent past have focal lengths from 1 $^9/_{16}$ to 1 $^3/_4$ in. and intercept 60 to 70 per cent of the light from the lamps. (Brown and Roper, *Trans. I.E.S.*, 1934, Vol. 29, p. 128.) A 5-in. reflector intercepting the same amount of light would have a focal length of $^7/_8$ in. Where the focal length is increased much above these values, as in searchlights, the diameter must be much increased, or the proportion of the light flux intercepted is decreased, or both. (See Searchlights.) Because of the long distance of the light source from the reflector surface (lever arm), deviations of the surface from the true paraboloid produce much less effect on the beam in a long-focus projector than in a short—hence the advantage of the long focus for high-grade searchlight projectors. When an actual parabolic mirror is observed from a point in a beam near the reflector, light is received from only a small unit of the mirror surface. As the eye is withdrawn further from the reflector, the area producing illumination at a given point increases, owing to the finite area of the light source. After a certain point has been reached, the whole surface of the reflector appears luminous and gives light to every point in the beam. From this point on, the illumination varies inversely as the square of the distance. Benford gives this distance as:

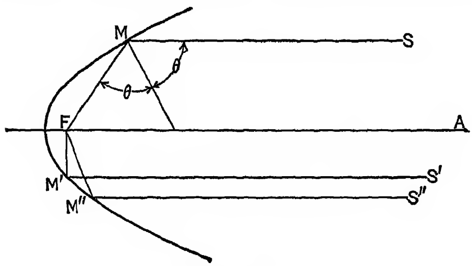


Fig. 20.

$$L = (F + R^2/4F)(R/12\,r),$$

where R is the radius of the mirror in inches, F the focal length, and r the radius of the source, in inches. (*Trans. I.E.S.*, 1915, Vol. 10, p. 905.) All tests on candlepower of reflectors should, therefore, be made at a greater distance than this. The I.E.S. code specifies 60 ft for tests on automobile headlamps. With a finite source the spread of a beam is directly proportional to the diameter of the source, and inversely proportional to the focal length. Because of the longer distance to the reflector surface, the outer portion of the surface of the parabolic reflector is more accurate in its light projection than the portion near the vertex.

THE SPHERICAL REFLECTOR reflects back the light along the radial line of incidence, if the light source is at the center, because the radius is perpendicular to the tangent. Then so much of this light as can pass the body of the light source joins with the emergent flux, and increases the effective lumens. If the light source is placed at one-half radial distance from the spherical reflector, the flux emerging from it is nearly parallel. This fact was taken advantage of in the Mangin mirror, which is a spherical reflector the two surfaces of which have different centers, the glass becoming thicker towards the edges of the reflector. The back is silver plated, and the prismatic or refracting effect of the glass is used to bring the rays reflected from this silvered surface into parallelism.

THE ELLIPSOIDAL REFLECTOR. This has the characteristic that, if a point source is placed at one focus, all reflected rays will pass through the other focus. Such a reflector can therefore be used if it is desired to concentrate the light upon a small object, as on a moving-picture film, by means of a reflector. This reflector is used with an arc-light in picture projection to concentrate the light flux on the film-picture area. The effect of the angular direction from which the reflected rays come to the conjugate focus is to give a highly concentrated form of distribution curve after the flux leaves this focus. (Benford, *Gen. Elec. Rev.*, 1923, Vol. 26, p. 163.)

Comparison of the Use of Lenses and Reflectors

The greatest difference in the use of these two projection devices is that in the lens the light source can be completely enclosed, so that the only emergent light is that coming from the lens. With the reflector, the open front necessarily permits a considerable part of the flux from the light source to pass out along conical lines, thus illuminating the surroundings. Reflectors direct from 30 to 70 per cent of the flux from the light source into the beam. The small angle subtended by a lens inevitably reduces this figure, for lenses,

to 20 per cent. Usually a reflector will be less expensive than the lenses which will perform a similar function.

Applications of Projection

PICTURE PROJECTION. The most important application of lenses, for picture projection, is so extensive and technical that it cannot be treated here. The reader is referred to Cady and Dates, Illuminating Engineering, pp. 481–497; Richardson, Handbook of Projection, Chalmers Publishing Co.; Farnham, *Trans. I.E.S.*, 1928, Vol. 23, p. 1241, for further information.

SEARCHLIGHTS. In the smaller sizes, up to 30 in., 1000-watt incandescent lamps are used, with parabolic or Mangin mirrors, and with the generation of several million candlepower at the center of the beam. Such searchlights have a wide application on shipboard and for other commercial uses. For the larger projectors, beginning with 30-in. and running up to the largest 60-in. reflectors, open arcs with special carbons are used. (See Art. 2.) The focal length of the mirror is about 40 per cent of its diameter. The positive electrode is directed along the axis of the reflector, and thus, in spite of the long focal length, nearly all the flux reaches the mirror, which subtends an angle of about 125 deg. Beams of as little as 1 1/2 deg spread are used. Searchlights, especially those for military purposes, may have louver or iris shutters for cutting off the light, without extinguishing the arc. All searchlights are arranged for easy orientation in any direction, which their application demands, and the large ones are operated with remote-controlled motors, so that the operator may not experience the blinding effect of the beam. The success of the searchlights of the present day is largely dependent upon the high-intensity arc. (See Art. 2.)

FLOODLIGHTING PROJECTORS. These are projectors having a fairly wide angle of spread, usually from 10 to 45 deg; they range in diameter from 11 to 24 in., being usually about 18 in. They use incandescent lamps ranging from 200 to 1500 watts. The maximum candlepower of floodlights of the narrowest spread ranges up to nearly 40,000 candlepower.

Floodlights are, as a rule, permanently located for the lighting of a given area of wall, ground, waterfall, or other object. Thus, though they are designed for convenient permanent orientation, facility in adjustment is not usually included. If this is needed, a small searchlight is preferred. The mirrors are of silver- or chromium-plated metal, or silvered glass. The last has high efficiency and permanence, but suffers from fragility. Fluted cover glasses may be used to spread the beam, either horizontally or vertically, where needed. Straight or circular louvers may be provided to reduce spilled light.

In the design of floodlighting, the problem is to obtain efficient illumination on the surface to be lighted, and sufficient uniformity so that the variations will not be noticed, even when the surface is observed from such a distance that the whole area enters into a single field of view. Madgick gives recommended illumination values for different types of surface, as in Table XIV. To obtain satisfactory uniformity, every point on the area should

Table XIV. Desirable Illumination Values for Floodlighting

Building Surfaces	Foot-candles *	Miscellaneous Applications	Foot-candles
White or cream terra cotta	4– 8	Flags and signs	5–25
Select gray limestone	5–10	Beaches and swimming pools	3– 6
Indiana or Bedford stone	5–10	Golf greens	2– 6
Buff limestone	6–12	Stadia	4– 8
Buff artificial stone	6–12	Pageants	10–20
Standard gray limestone	6–12	Playgrounds	1– 5
Smooth buff face brick	6–12		
Brief Hill sandstone	8–16		
Smooth gray brick	8–16		
Gray limestone	8–16		
Common tan brick	10–22		
Dark field gray brick (rough finish)	14–28		

* Depreciation in service may usually be taken as of the order of one-third.

NOTE.—Buildings composed of dark red brick or brownstone should not be floodlighted unless there is a large amount of light trim. Often, when color is employed, lower foot-candle values (but greater wattages) will be effective.

receive considerable illumination from at least two projectors. In producing an esthetic effect, where, usually, the light must come from below, the loss of the shadows produced by horizontal projecting architectural elements, as normally lighted by sunlight, should be

considered, also the shadows produced by architectural elements, when it is necessary to place the floodlights close to the surface to be lighted. (Cady and Dates, Illuminating Engineering, p. 419; Hallman, *Trans. I.E.S.*, 1934, Vol. 29, p. 287.)

Although the average illumination over a given area may be determined by dividing the total lumens by the area lighted, as in interior illumination (see Art. 7), the need for a fair degree of uniformity calls for a point-by-point study based on the distribution curve of the projectors used. For this purpose, Benford proposes the use of a spherical web, similar to the one used for isocandle curves (Art. 1). For this purpose he uses a special chart. (*Gen. Elec. Rev.*, 1923, Vol. 26, p. 579.) The artistic and spectacular possibilities of floodlighting are greatly increased by the use of color, though the necessary power to produce effective lighting is also much greater.

FLOODLIGHTS FOR INDUSTRIAL LIGHTING, TEMPORARY AND PERMANENT. Banks of floodlights are mounted on tall structural columns and on buildings, and directed down toward the ground for illumination of railway switching yards and other outdoor working places. (Mahan and Swackhamer, *Trans. I.E.S.*, 1928, Vol. 23, p. 189.) For temporary use on construction work, and when mounted at a sufficient height above the workers, floodlamps are very useful, but the danger of blinding glare must always be kept in mind.

AUTOMOBILE HEADLIGHTING. This is, probably, the most extensive and important form of light projection by means of reflectors. The difficult problem, with present-day road speeds and braking equipment, is to provide light enough along the highway so as to prevent colliding with any obstacle on the road, even when such an obstacle is to the rear and right of an oncoming car provided with similar headlight equipment. Also while having such adequate road illumination, to avoid causing troublesome glare for the oncoming driver. The headlighting should likewise facilitate vision of pavement defects, of persons and other objects close to the side of the road, and of road signs. In addition, there should be enough light above the horizontal to make visible any overhanging object. Incidentally the headlight also serves as a warning to cars approaching on cross roads. (See also, Sec. 17, Art. 20.)

At present the most generally accepted solution of this problem is the use of two beams, the upper for driving on a clear road, and the lower for meeting and for city driving. Model specifications for these two beams were drawn up by committees of the Illuminating Engineering Society and the Society of Automotive Engineers. With increasing speeds, the use of these two-beam lamps has become less and less satisfactory. The upper beam is not adequate when driving at the rate of, say, 50 miles an hour. This has resulted often in the tilting up of the headlamps so as to get more light at a distance. Again, drivers have been unwilling to sacrifice their road visibility, at such a speed, by depressing when meeting other cars.

Specifications for a new type of headlamp, known as asymmetric, using three instead of two beams, were recommended in 1934 by the Illuminating Engineering Society and the Society of Automotive Engineers, which, it was hoped, would eventually better the situation. In addition to upper and lower beams, as in the earlier type, a third beam, the meeting beam, is provided. In this beam, the right side remains high, while the left side is lowered to a position corresponding to the old depressed beam. With this third beam made available for meeting, it was proposed to set no upper limit on the beam for clear road use.

Obviously, the success of this system depends upon the possibility of training drivers to make use of the meeting beam at all proper times. State automobile authorities have questioned the practicability of this, and the unlimited clear road beam has not yet received general approval.

Except that the earlier limitations on the candle-power of the upper beam have been retained, this three-beam system is now being used on several important makes of cars.

The headlighting situation is in a condition of rapid development at the present time, so that only general statements may advantageously be made with regard to candle-power distribution. The latest authoritative data, at any time, may be obtained from the Illuminating Engineering Society, 29 W. 39th St., New York.

To obtain adequate illumination of objects on the road at 300 to 400 feet, an intensity of several thousand candles is needed straight ahead and on the horizontal 3000 candles with a 32-cp lamp has been specified, but this is an unsatisfactory compromise. In this direction and just below it, the desirable candles are limited only by danger of glare due to the occasional tilting up of the beam.

Such tilting may be due either to bad adjustment, to loading of the back seat, especially in smaller cars, or to slight convex curvature of the road surface. An intensity of 7500 candles at 1 deg down has been specified, which is also an unsatisfactory compromise.

Most of the light-flux from the lamps should be emitted within the upper two deg of

the light beam, but below this, the intensity should fall off rapidly, but uniformly, to prevent excessive brightness of the road-surface near the car.

To make objects ahead of the car, say at 100 feet, which are at or near the side of the road, visible, spreads of the beams of about 6 deg to each side at 1 deg down, and 12 deg at 2 deg down are needed. Thus the top of the bright portion of the beam cross-section should be curved, falling off toward each side of the road. This also makes possible the limitation of the intensity at 1 deg up and 4 deg left to 1000 candles or less, thus keeping down the glare for a car which is being met. For the same reason there should not be over about 2500 candles 1 deg up, along the axis of the car. A plan for using polarized light for headlighting is being developed, which may revolutionize this art.

Contrary to the usual understanding, several surveys have shown that more traffic accidents are due to inadequate road illumination than to glare. A large part of present-day headlighting trouble is due to failure of the automobile owner to keep his headlamps in proper adjustment, particularly as to vertical direction of the headlighting beam. It is a simple matter, at night, to determine whether the top of the beam is approximately horizontal, and if not, to adjust the lamps to make it so.

The recently developed **prefocused automobile bulb** (see Art. 2) is making it possible to reduce the diameter of headlamps to 4 or 5 in., and still obtain satisfactory distribution. Also building the lamps into the front of the car, to obtain streamlining effects, should improve the permanence of their mounting. (Brown and Roper, *Trans. I.E.S.*, 1934, Vol. 29, p. 175.)

7. SPECIFICATION, DESIGN, AND MAINTENANCE OF ILLUMINATION

USES OF ILLUMINATION. Illumination may be classified under three heads, as follows: (1) To facilitate activity; this includes all kinds of work or play. (2) To facilitate the observation of objects; this may be called exhibitive lighting. (3) To promote general comfort and convenience; this is the case where no definite activity or observation is to be provided for. Of the requirements for good lighting, the absence of glare, proper diffusion, shades and shadows, and color have been discussed in Arts. 3 and 4; adequacy remains to be considered.

ADEQUACY. The amount of illumination required for a particular case depends upon the following: (*a*) reflection factor of object, or objects, to be observed; (*b*) fineness of detail; (*c*) length of period during which attention must be fixed upon fine detail; and (*d*) need for the maintenance of bright and cheerful surroundings.

In line with the above principles, much consideration has been given to illumination intensities needed for different activities, and various tables of data have been published; the most authoritative of these are given in the I.E.S. code for the lighting of factories, and for school lighting. An abridgement of these data is given in Table XV. The latest edition of either code, and of various other authoritative publications on the subject of illumination, may be obtained from the Illuminating Engineering Society, 29 W. 39th St., New York City. Mention was made in Art. 3 of the work done by Luckeish and Moss on the relation of the illumination to the nervous strain involved in work. In connection with their claim that, from this point of view, much higher intensities can be justified than are now in use, and recommended in the codes, they call attention to the fact that, since vision follows a logarithmic law, illumination must be increased 10 times in order to double its effectiveness for vision. Hence, they say, a range of illumination such as from 8 to 12 ft-c is not a reasonable one. (Luckeish and Moss, *Trans. I.E.S.*, 1931, Vol. 26, p. 1061.) It should also be said that, from the beginning of modern lighting, illumination intensities to be considered satisfactory have been steadily increasing. Also, that since the best artificial illumination values now in general use are not more than $^1/_5$ to $^1/_{10}$ of that received on a desk by a north window, there seems to be no practical limitation to the process of increase which has been going on. It thus appears logical to use higher, rather than lower, values than those now generally accepted. Incidentally, electric wiring should certainly be designed with the possibility of increased demand in view.

LUMENS REQUIRED FROM THE LIGHTING SOURCES. Two methods are in use for the determination of the required number and size of electric lamps. The point-by-point method is applicable to places where reflection from walls and ceilings is negligible, and where the lights may be regarded as essentially point sources. This is based on the inverse-square and cosine laws, explained in Art. 1. In most cases of interior lighting, however, the contribution of flux from the ceiling and walls is so great as to vitiate the method entirely. When this method is used, it is assumed that the distribution curve of the complete luminaire is known. For determining the size of lamp to be used, the

Table XV. Recommended Illumination Values

Abstracted from I. E. S. Industrial and School Lighting Codes, and Nela Park Bulletin

	Foot-candles Recommended
Aisles, stairways, passageways	3– 2
Assembling:	
Rough	8– 5
Medium	12– 8
Fine	20–12
Extra fine	100–25
Bakeries	12– 8
Chemical works:	
Tanks for cooking, extractors, etc.	10– 6
Cloth products:	
Cutting, inspecting, sewing	
Light goods	15–10
Dark goods	100–25
Coal breaking and washing	5– 3
Construction—indoor, general	5– 3
Dairy products	12– 8
Electric manufacturing:	
Storage battery, molding of grids, charging room	10– 6
Coil and armature winding, mica working, insulation processes	20–12
Elevator—freight and passenger	8– 5
Engraving	100–25
Forge shops and welding	10– 6
Foundries:	
Fine molding and core making	15–10
Garage—automobiles:	
Repair department and washing	15–10
Inspecting:	
Medium	15–10
Extra fine	100–25
Jewelry and watch manufacturing	100–25
Locker rooms	6– 4
Machine shops:	
Rough bench and machine work	10– 6
Medium bench work, etc.	15–10
Fine bench and machine work, etc.	100–25
Offices:	
Close work	15–10
No close work	10– 8
Drafting room	25–15
Paint shops:	
Fine hand painting and finishing	15–10
Extra fine hand painting and finishing (automobile bodies, piano cases, etc.)	100–25
Power plants:	
Engine rooms, boilers, coal and ash handling	5– 3
Auxiliary equipment, oil switches and transformers	8– 5
Power plants:	
Switchboard, engines, generators, blowers, compressors	10– 6
Printing industries:	
Matrixing and casting, miscellaneous machines, presses	12– 8
Linotype, monotype, typesetting, imposing stone, engraving	100–25
Receiving and shipping	6– 4
Steel and iron mills, bar, sheet and wire products:	
Charging and casting floors	6– 4
Muck and heavy rolling, shearing (rough by gage), pickling and cleaning	8– 5
Plate inspection, chipping	25–15
Store and stock rooms:	
Rough bulky material	3– 2
Medium or fine material requiring care	8– 5
Structural steel fabrication	10– 6
Toilet and washrooms	6– 4
Warehouse	3– 2
Woodworking:	
Rough sawing and bench work	8– 5
Fine bench and machine working, fine sanding and finish	15–10
Recommended Values of Illumination for School Interiors *	
School classrooms	12–8
Auditoriums, assembly rooms, cafeterias, etc.	5–3
Recommended Values of Illumination for Commercial Interiors †	
Lunch rooms	12– 8
Market	12– 8
Moving picture theater:	
During intermission	5– 3
During pictures	– 0.1
Restaurants	8– 5
Show windows:	
Large cities—brightly lighted business district	150–30
Medium cities—brightly lighted district	75–30
Small cities and towns	50–30
Stores, medium size	15– 8

* From Standards of School Lighting.
† From General Electric Co., Nela Park Bul. LD-6.

illumination values with an approximately correct size of lamp have to be determined, then, correcting for any differences in the distribution curve, the size of lamp to give the desired values of illumination can be determined by proportionality.

DEPRECIATION FACTOR. In both the point-by-point and the light flux methods, the lumen data used are generally based on new and clean lamps and accessories. During the life of a lamp, there is always a considerable falling off in lumen output, owing both to the depreciation of the lamp itself, and to the accumulation of dust and dirt on the lamp and accessories. A larger lamp, therefore, must always be used than the illumination required would indicate. This may be allowed for by the use of depreciation percentages, which depend upon the type of equipment, the surrounding conditions, and the frequency of cleaning. The depreciated light flux will usually lie between 60 and 80 per cent. It

should probably never be lower than 50 per cent, which indicates a serious loss of light. See also the data of Table XVII.

LIGHT FLUX METHOD. In designing the lighting for large rooms, it is usually assumed that the lamps are close enough together so that the average instead of the minimum illumination may be used. The lumens to be provided from the lamps are then determined from the following equation: $F = \frac{e_h \times a}{u \times d}$, where F = flux to be supplied, e_h = horizontal illumination required, a = area to be lighted, d = depreciated percentage, u = utilization coefficient.

UTILIZATION COEFFICIENT. This is the ratio of lumens available on the working plane or illuminated surface to the total lumens produced by the lamps. It increases with increase of reflection factor of ceilings and walls, and with the ratio of horizontal to vertical dimensions of the room to be lighted. It also depends upon the type of accessories, as indicated in Table XVII.

The success of the light flux method depends largely upon the knowledge of the proper utilization coefficient to use. For this purpose, a large amount of data was gathered together by Harrison and Anderson, and given in *Trans. I.E.S.*, 1920, Vol. 15, p. 97. These data have been revised and extended in *Bul. LD-6*, Nela Park Engineering Department, General Electric Co., in which will also be found a useful chart of reflection factors. A similar reflection factor chart is given in Cady and Dates, p. 298, which also quotes the utilization coefficients table. An abstract of the data of *Bul. LD-6* is given in Tables XVI and XVII. By interpolation, quite satisfactory values of utilization coefficients can be obtained from the figures of Table XVII, not only for the lighting units given, but also for others similar in form and characteristics. The RLM type of reflector with glass neck, known as the glass steel diffuser, runs from 3 to 4 per cent below the RLM with a white bowl lamp, and is given 5 per cent more depreciation. The oxidized aluminum reflector with a clear lamp runs 1 to 2 per cent below the mirror glass reflector. The enclosed semi-indirect unit, for which data are given, is the most efficient of four shown, values on the others running from 1 to as much as 7 per cent lower, the latter figure for high values of the room index.

Table XVI. Room Indexes.

For use on Table XVII. Abstract of Table 6, Bul. LD-6, Incandescent Lamp Department, General Electric Co.

Room Width, ft W	Mounting Height of Luminaires, ft								
	7 to 7 1/2			10 to 11 1/2			17 to 20		
	Square	*$L = 2W$	$L = 4W$	Square	$L = 2W$	$L = 4W$	Square	$L = 2W$	$L = 4W$
14	1 1/2	2	2 1/2	1	1.2	1 1/2	0.6	0.6	0.6
17	2	2 1/2	2 1/2	1	1.2	1 1/2	0.6	0.6	0.8
20	2	2 1/2	3	1	2	2	0.6	0.8	1
25	2 1/2	3	3	1 1/2	2	2	0.8	1	1
30	3	4	4	2	2 1/2	2 1/2	1	1.2	1 1/2
35	4	5	5	2	3	3	1	1 1/2	1 1/2
40	5	5	5	3	3	3	1.2	1.2	2
50	5	5	5	3	3	3	1 1/2	2	2
60	5	5	5	4	4	4	2	2 1/2	2 1/2
75	5	5	5	5	5	5	2 1/2	3	3
90	5	5	5	5	5	5	2 1/2	3	3

* L = length, W = width.

For indirect lighting, use ceiling height and add 2, 4, and 8 ft, respectively, to the mounting heights given.

LUMINAIRE MAINTENANCE. The bad effects of luminaire depreciation have already been considered. To the causes of depreciation should be added the loss due to blackening of the ceiling. Where this is supposed to be light colored, it should be regarded as a part of the lighting system, and the maintenance of its reflection factor considered from that point of view. The elements of depreciation, then, are: (*a*) reduced output of the lamp; (*b*) soiling of the lamp; (*c*) soiling of the reflector or globe; (*d*) soiling of the ceiling. Assuming that the illumination is not allowed to fall below a satisfactory minimum, the cost of lost lumen-hours should be balanced against the cost of cleaning, and an economical minimum illumination or an economical cleaning period should be established. This period is likely to differ at different times in the year. Two abnormal causes of loss of light are: reduced voltage due to inadequate wiring (see Fig. 6) and the failure to replace burned-out lamps.

Table XVII. Coefficient of Utilization and Depreciated Percentage

Abstract of Table 7, Bul. LD 6, Nela Park Engineering Department, General Electric Co.

Lighting Unit	Probable Average Illumination—as fraction of initial illumination			Ceiling	Very Light (70%)		Fairly Light (50%)		Fairly Dark (30%)	
				Walls	Fairly Light (50%)	Very Dark (10%)	Fairly Light (50%)	Very Dark (10%)	Fairly Dark (30%)	Very Dark (10%)
	Clean Conditions	Average Conditions	Dirty Conditions	Room Index*	Coefficients of Utilization					
Wide dome				0.6	0.32	0.25	0.32	0.25	0.27	0.25
White bowl lamp				1.0	.43	.37	.42	.37	.39	.37
RLM	0.80	0.75	0.65	1.5	.48	.43	.47	.43	.45	.43
90°–180° 0%				2.0	.52	.48	.51	.47	.49	.47
0°– 90° 66%				3.0	.57	.53	.56	.52	.54	.52
				5.0	.61	.57	.60	.57	.58	.56
RLM Dome				0.6	.34	.24	.34	.24	.28	.24
Clear Lamp				1.0	.46	.39	.45	.39	.42	.39
90°–180° 0%	.80	.75	.70	1.5	.53	.46	.52	.46	.48	.45
0°– 90° 76%				2.0	.58	.51	.57	.51	.53	.51
				3.0	.64	.58	.63	.58	.60	.58
				5.0	.69	.65	.67	.64	.65	.63
Concentrated prismatic				0.6	.42	.38	.41	.38	.40	.37
reflector				1.0	.54	.52	.53	.51	.51	.50
Aluminum cover	.80	.75	.65	1.5	.61	.57	.58	.56	.57	.55
Clear lamp				2.0	.63	.60	.62	.59	.59	.58
90°–180° 5%				3.0	.68	.64	.66	.63	.63	.62
0°– 90° 72%				5.0	.72	.68	.68	.66	.65	.64
Mirrored glass reflector				0.6	.42	.39	.42	.39	.41	.38
Clear lamp				1.0	.55	.53	.54	.52	.52	.51
90°–180° 3%	.80	.70	.60	1.5	.61	.58	.59	.57	.58	.56
0°– 90° 73%				2.0	.64	.61	.62	.60	.61	.59
				3.0	.69	.65	.67	.64	.64	.63
				5.0	.72	.69	.69	.66	.66	.65
Flattened white glass				0.6	.22	.14	.20	.13	.14	.12
enclosing globe				1.0	.31	.23	.28	.21	.22	.19
90°–180° 35%	.80	.75	.65	1.5	.38	.29	.34	.27	.27	.24
0°– 90° 45%				2.0	.42	.33	.38	.31	.31	.28
				3.0	.49	.40	.43	.36	.36	.33
				5.0	.55	.47	.49	.42	.40	.38
Prismatic glass enclosing unit				0.6	.28	.18	.26	.17	.19	.16
				1.0	.38	.29	.36	.28	.30	.27
90°–180° 27%	.80	.70	.60	1.5	.46	.36	.43	.34	.35	.33
0°– 90° 53%				2.0	.51	.42	.47	.40	.40	.38
				3.0	.58	.50	.54	.47	.46	.44
				5.0	.65	.57	.60	.53	.52	.50
Enclosed semi-indirect				0.6	.19	.11	.16	.10	.10	.08
enameled bottom,				1.0	.27	.19	.23	.17	.16	.14
etched top	.75	.70	..	1.5	.34	.25	.29	.22	.20	.18
90°–180° 48%				2.0	.38	.29	.32	.25	.23	.21
0°– 90° 32%				3.0	.44	.36	.38	.31	.28	.26
				5.0	.51	.43	.43	.37	.33	.31
Open indirect	.70	.60	..	0.6	.15	.10	.11	.07	.05	.04
				1.0	.22	.16	.15	.11	.08	.07
90°–180° 80%				1.5	.27	.21	.20	.16	.10	.09
0°– 90° 0%				2.0	.30	.25	.22	.17	.11	.10
				3.0	.36	.30	.26	.22	.14	.13
				5.0	.42	.37	.30	.26	.17	.15

* See Table XVI.

SPACING OF LAMPS. This is likely to be largely affected by the dimensions of the room or the beams on the ceiling. A single lamp should be used only in very small rooms; two lamps, only in long, narrow rooms. In general, the smallest number of lamps used

should be four. As far as practicable, the distance between lamps should be the same in both directions, both across and along the room. Data with regard to mounting height and spacing of units, given by Ward Harrison, are reproduced in Table XVIII. (Cady and Dates, p. 288.) In determining mounting heights, it should be remembered that, with the light ceilings, illumination does not vary inversely as the square of the distance, but is not greatly affected thereby; also that greater mounting height promotes uniformity of illumination. Where very high mounting has to be used, on account of cranes, or for some other reason, a concentrating type of lighting unit should be used.

ILLUMINATION ON VERTICAL SURFACES. So far, only illumination on a horizontal working plane has been considered. Frequently illumination on a vertical plane or some inclined plane is important. In such cases, lighting units must be selected which have a relatively large horizontal component of candlepower in their distribution curve. In some places, where it is desired to provide special illumination for a fixed vertical surface, angle reflectors are recommended. Frequently, in a system of general lighting, special local lighting is provided where extra high intensities are needed. The general illumination should then be sufficient to prevent strong contrasts with the local illumination. It will be noted from Table XVIII that, for moderate mounting height, spacing equal to the mounting height gives satisfactory uniformity.

Table XVIII. Mounting Height of Lighting Units

Direct Lighting Units				Semi-indirect and Indirect Lighting	
Actual Spacing between Units (*D*), ft	Distance of Units from Floor Not Less Than (*H*), ft	Desirable Mounting Height in Industrial Interiors (*R*)	Desirable Mounting Height in Commercial Interiors (*R*)	Actual Spacing between Units (*D*), ft	Recommended Suspension Length (Top of Bowl to Ceiling) (*S*), ft
7	8	12 ft above floor if possible—to avoid glare, and still be within reach from stepladder for cleaning	The actual hanging height should be governed largely by general appearance, but particularly in offices and drafting rooms the minimum values shown in Column *H* should not be violated	7	1–3
8	8 1/2			8	1–3
9	9			9	1–3
10	10			10	1 1/2–3
11	10 1/2			11	2–3
12	11			12	2–3
14	12 1/2			14	2 1/2–4
16	14			16	3–4
18	15			18	3–4
20	12 1/2			20	4–5
22	14			22	4–5
24	15			24	4–6
26	21			26	4–6
28	22			28	5–7
30	24			30	5–7

G. E. Co. Incandescent Lamp Dept. Bul. LD-6.

8. INTERIOR ILLUMINATION. SPECIAL CASES

LIGHTING FOR ACTIVITY. INDUSTRIAL LIGHTING. (See also Kotch, Sturrock, and Staley, *Trans. I.E.S.*, 1933, Vol. 23, p. 57.) Good factory lighting improves operation in the following particulars: (*a*) increases production, by promoting more rapid perception; (*b*) increases accuracy of work; (*c*) promotes healthful conditions for the eyesight of the workers; (*d*) promotes cheerfulness of surroundings; (*e*) tends to prevent accidents; (*f*) facilitates careful oversight by the supervisors; and (*g*) promotes order and neatness. For good industrial lighting, in addition to the features already specified (Art. 7), emphasis is to be placed upon absence of reflected glare, where work is being done on polished materials, high illumination on vertical surfaces, absence of dense shadows, gradual transition from high-grade interior lighting through lower values in hallways to low exterior illumination. Inadequate illumination and blinding glare play parts of considerable importance in industrial accidents. Simpson (*Trans. I.E.S.*, 1920, Vol. 15, p. 576) estimated the cost of industrial accidents where poor lighting was involved as $300,000,000, which exceeded the whole factory lighting bill for the country. Prevention of accidents is the basis upon which legal lighting codes are enacted. Such

codes are usually based upon the I.E.S. Model Code of Lighting Factories, Mills and Other Work Places. *Trans. I.E.S.*, 1930, Vol. 25, p. 607.

Office Lighting. Large clerical offices should be provided with general lighting, usually with large enclosing globes of low brightness. (See Art. 5.) Where typewriters are used, a large component of illumination on vertical surfaces is needed.

Drafting Rooms. Lighting of these calls for special care in the prevention of glare. To provide the high illumination needed, a general system, giving about 10 ft-c, combined with local luminaires with opaque reflectors, may be advisable.

EXHIBITIVE LIGHTING. In exhibitive lighting three general principles apply: (*a*) the light sources must not be visible to the observer—they must either be at his rear, or be screened by reflectors; (*b*) the light sources must be placed to give the most effective distribution over the object to be observed—for flat objects, uniform distribution is generally required; (*c*) an illumination high, as compared with the surroundings, must be provided, if emphasis is desired; (*d*) the direction of the light rays must prevent reflected glare from the surface of the object; (*e*) the color of the light must be suitable. Generally white light is appropriate.

Illuminated Signs, as distinguished from those where the lights themselves form the letters, are usually lighted by one of three methods. (*a*) Floodlights or spotlights (see Art. 6); (*b*) small lamps in frames surrounding the sign; (*c*) the most usual method, lamps with specular reflectors on brackets, generally above the sign, which must be from 1 to 2 ft above the sign to minimize the effects of glare, and for uniformity of illumination must be in front of the sign, by a distance of 60 to 70 per cent of its height. The general range of sign illumination is 2 to 15 ft-c. (Atherton, *Trans. I.E.S.*, 1925, Vol. 20, p. 148.)

Picture Lighting is similar to sign lighting, except that it calls for much more careful design, especially in the matter of color. For pictures painted out-of-doors, daylight color should be supplied. Wall decorations, which will be seen principally by artificial light, as in theaters, should be painted under the light by which they will be seen. Ten to twenty foot-candles is recommended for pictures. The lighting of important pictures by projection lanterns placed at a distance should be considered.

For **show window and show case lighting,** special angle reflectors of mirrored glass or prismatic glass with narrow beams are available. A careful consideration of the general principles of exhibitive lighting is needed. Use of spotlight, possibly colored, to emphasize certain objects in a window, may be effective.

Stage Lighting is, perhaps, the highest form of exhibitive lighting. For lectures and concerts, care must be taken to avoid having the background so bright that the speaker or singer appears as a silhouette. Spotlights directed downward from the sides of the halls to illuminate the speaker or singer are effective. In the footlights and borders, lamps of alternating colors are used. Red, blue, white, and green are desirable. Vacuum tubes are now used to control dimming.

The following additional forms of spectacular lighting may be mentioned: fountains, waterfalls, trees, shrubbery, and garden planting. (Powell, *Trans. I.E.S.*, 1929, Vol. 24, p. 325; La Wall and Cutler, *ibid.*, 1933, Vol. 28, p. 239; Caldwell, Modern Lighting, p. 174; see also Art. 6, Floodlights.)

LIGHTING FOR PHOTOGRAPHY. Mercury-vapor tubes and carbon-arc lamps with special impregnated electrodes are often used. The special photoflood and photoflash lamps (see Art. 2) are inexpensive and convenient. (See also Arts. 2 and 7.)

LIGHTING FOR COMFORT AND CONVENIENCE takes a wide variety of forms. Simple hallways and passageways call for only a moderate illumination—1 to 2 ft-c recommended. (I.E.S. Lighting Codes.) Stairways, on account of the danger of falling, call for special care in lighting. Three to five foot-candles are recommended. Lights must not be so placed that they will shine in the eyes of a person descending the stairs. The use of contrasting color, or reflection factor, on the upper surface at the edges of stair-treads is recommended.

Auditoriums and Churches constitute a high type of lighting for comfort and convenience. Where little or no use of the eyes for reading will be made, 2 or 3 ft-c suffices, unless a higher intensity is desired to produce a bright and cheerful effect. Where reading, as of hymn books, is expected, at least 5 ft-c should be provided. In lecture halls and churches, the lighting on the speaker's manuscript calls for particular care in order to avoid glare in the eyes of the audience. A lens spotlight, mounted in the ceiling, has been used successfully for this purpose. If there is a gallery, care must be taken that its occupants do not suffer glare from low-hung luminaires. Auditoriums, and all rooms, such as hotel dining rooms, which are likely to be used as audience rooms, should be wired for connecting a projection lantern, and also for the control of at least a low, and preferably for both low and high, values of illumination from the lantern location. In the lighting of Gothic churches, special consideration to the esthetic effects is needed.

Residence Lighting. (Caldwell, Modern Lighting, p. 200; Powell and Harrington, *Trans. I.E.S.*, 1919, Vol. 14, p. 394; Artificial light and its application in the home, I.E.S. Committee on Residence Lighting.) This type of lighting, so important to everyone, does not generally receive the careful design which it should have. At least one light should always be placed beside the front entrance, preferably on the wall at the side of the doorway. A low illumination in the hallway is permissible, both for the adaptation of the eyes in coming from out-of-doors, and for economy's sake. Provision, however, should be made for additional illumination for special occasions. For the living room, the taste of the owner should be given special consideration. The choice lies between a general, more or less shadowless illumination from a central ceiling luminaire, or the effects obtained by several portable luminaires and wall brackets. These give a moderate general lighting, with higher local lighting for points where this is needed. More artistic effects can be obtained with this type of lighting. Lamps for reading and writing, especially by students, call for particular attention. A portable lamp, specially designed for this purpose by a committee of the Illuminating Engineering Society, is known as the **I.E.S. study lamp,** and is made by over 40 manufacturers. It carries a special I.E.S. certificate tag. (*Trans. I.E.S.*, 1934, Vol. 29, p. 327.) The exhibitive aspect should always be an important consideration in residence lighting. The **dining room** calls for a central luminaire, throwing the light down upon the table, which should be the point most emphasized in this room. The **kitchen** and other service rooms call for application of the principles of industrial lighting. In the kitchen, two or three wall luminaires may be provided, to light the three important points, the table, the stove, and the sink. **Bedrooms and bathrooms** call for low-placed light sources of moderate brightness at one or both sides of the dressing table. In the bedroom these may well be provided by luminaires attached to the dressing table, and supplied from an attachment outlet in the baseboard. There should also be an attachment receptacle by the bed for a light mounted on the bed, or a portable on a bedside stand. A ceiling luminaire is of doubtful value in a bedroom. A wall bracket or other provision for a 1- to 3-watt night lamp is desirable.

SCHOOL LIGHTING. This is especially important because children's eyes are immature and easily deformed. The I.E.S. Standards for School Lighting covers the subject adequately. In the schoolroom where the children sit and study, particular care must be taken. A minimum illumination of 10 ft-c, obtained generally from 4 luminaires, should be provided. Enclosing globes of low brightness are essential. In special cases, the principles of sign lighting may be applied to lighting a blackboard.

9. STREET ILLUMINATION

PURPOSES. Street lighting is intended to promote public safety from traffic accidents and crime, to promote the comfort and convenience of the users of the street, and to increase its attractiveness. Experience shows that good street lighting attracts traffic and stimulates retail trade. The most important factors affecting the degree of street illumination required are the density of traffic, prevalence of retail business, degree of police supervision needed, and the architectural character of the thoroughfare. In dim light, vision depends primarily on differences of brightness and is but slightly assisted by color distinctions.

TRAFFIC ACCIDENTS AND CRIME. Extensive studies have been made of the effect of lack of visibility, due to inadequate lighting, upon the occurrence of traffic accidents. Through a comparison of the number and character of such accidents during those evening hours which are light in summer and dark in winter, it has been clearly demonstrated that a large part of the accidents which occur after dark could be prevented if streets and highways were more adequately lighted, and that, from a broad point of view, the economic savings resulting from the prevention of accidents would be much more than enough to pay for the needed additional illumination. Conclusions formed in this way have been confirmed by other methods. Rolph, *Trans. I.E.S.*, 1931, Vol. 26, p. 787; Caldwell, *ibid.*, 1932, Vol. 27, p. 828; Simpson, *ibid.*, 1933, Vol. 28, p. 780.) One study of crime made in Cleveland indicated a reduction of 40 per cent due to improved lighting in the retail district; and another study indicated 25 per cent less crime on those thoroughfares, and 50 per cent less on those residence streets, where there was adequate illumination.

STREET CLASSIFICATION. The Committee on Street Lighting of the Illuminating Engineering Society, in its 1927 report (*Trans. I.E.S.*, 1927, Vol. 22, p. 107), recommended the following classification of streets from the point of view of lighting: primary business streets; secondary business streets; thoroughfares, heavy, medium, and light traffic;

residence streets; alleys; highways. Primary business streets generally include those where retail stores, hotels, and theaters are located. Secondary business streets include wholesale districts, factory districts, and sometimes minor retail districts. In primary business streets, extra lighting for the purpose of greater attractiveness is often provided by special assessments. Thoroughfares, sometimes called arterial, are those streets which are the principal routes from the suburbs to the center of the city. Their major lighting problem is due to the heavy automobile traffic between five and six in the winter evenings. Residence streets have usually been quite inadequately lighted. The large

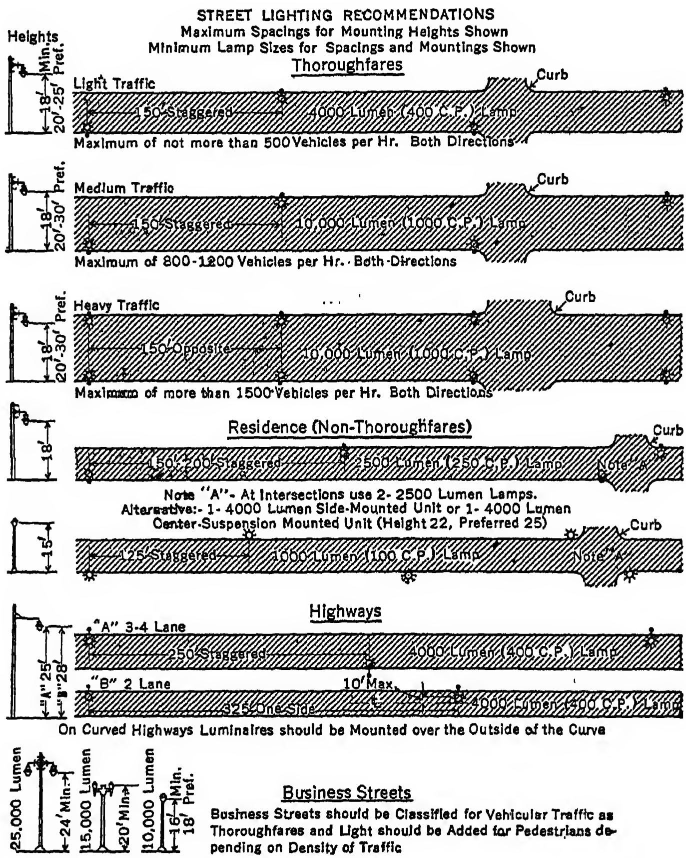


Fig. 21. I.E.S. Street Lighting Recommendations. (Based on the most effective equipment used in the most effective manner.)

mileage of this type of street makes the cost of sufficient lighting a serious problem. Extra lighting of sections of such streets by special assessment is sometimes provided. Lighting data for these different types of streets are shown graphically in the chart of Fig. 21, which was drawn up from the recommendations of the I.E.S. Committee by A. F. Dickerson.

Lighting Conditions on Streets

DIFFERENCE BETWEEN INDOOR AND OUTDOOR LIGHTING. The conditions which differentiate street lighting from interior lighting may be listed as follows: large areas per lamp and hence low illumination; long and narrow areas to be lighted, with special emphasis on illumination of the ground surface; little or no useful light by reflection; vision principally of large objects, generally with limited time for observation.

METHODS OF VISION ON STREETS. Except on unusually well-lighted streets, objects are generally perceived as dark spots against a lighter background. This is known as **silhouette vision**, and largely takes the place of the direct vision of interior lighting. Direct vision, however, plays a greater or lesser part near all street lamps, and on especially well-illuminated streets. Discernment may take place occasionally by the specular reflection of the street lights from a wet or polished surface, and by the shadow caused by some object on the street pavement, or by the sides of a hole in the pavement.

CHARACTER OF PAVEMENT SURFACE. Pavement surfaces may be light, dark, or medium in color, and may be mat or specular, as to their reflecting characteristics. Obviously, so far as the silhouette effect is concerned, the reflection factor of the pavement plays an important rôle. If a dark paving material is used, twice as much light will be needed to produce a given quality of vision as would be needed with a material of double the reflection factor. Every effort should therefore be made to develop and maintain light-colored pavements. When a pavement is wet, the specular reflection from the glossy surface determines the silhouette production, and the color becomes of minor importance.

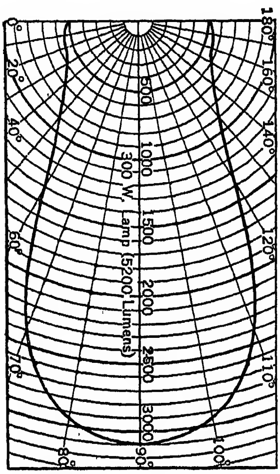
GLARE IN STREET LIGHTING. The code of street lighting (*Trans. I.E.S.*, 1931, Vol. 26, p. 15) gives the following as the effects of glare in street lighting: (1) reduction in ability to see; (2) ocular discomfort; (3) detraction from good appearance of streets. The following facts with regard to glare are well established: all effects of glare are diminished as the glaring light sources are removed further from the center of the field of vision, as by the use of higher mounting. Glare decreases as the candlepower of the light source is decreased, but, for a light source of a given candlepower, glare, as reducing ability to see, is not materially affected by change in brightness, that is, by the presence or size of an enclosing globe. On the other hand, comfort and street appearance are improved by reduction in brightness in the manner indicated above.

Arthur J. Sweet performed an extended series of tests in 1914. (Caldwell, Modern Lighting, p. 223.) Mr. Sweet found that the blinding effects of glare varied as the square root of the candlepower of the light source. He also determined the following relations between height and glare: at 15 ft the glare was $1/2$ of its value at a 12 $1/2$ ft mounting. At 22 ft it was about $1/4$ and at 32 ft $1/8$ of the value at 12 $1/2$ ft. In a test of three systems of 10,000-lumen lamps 300 ft apart with mounting heights of 16, 21, and 26 ft (Harrison, Haas, and Reid, Street Lighting Practice, p. 114), made by some 2000 observers, about 80 per cent picked the 21-ft mounting height, and the other 20 per cent picked the 26-ft mounting height. None chose the 16-ft as being desirable. Increasing mounting height decreases glare, but it also decreases illumination, especially that on the pavement below the lamp, which produces silhouette vision. Many tests made by shielding off the street lighting from the eyes of the observer have indicated that the effect of glare was to reduce the effectiveness of the street lighting from 25 to 50 per cent. Globes of square horizontal cross-section which can be equipped with a rectangular screen to keep glare off of residence porches and windows are made. The trunks of trees planted between the sidewalk and the curb frequently cause serious shadows on the sidewalk. It is very desirable, for this and other reasons, that street trees should be planted on the lot-line instead of near the curb line.

LIGHT DISTRIBUTION OF STREET-LIGHTING LUMINAIRES. Peculiar conditions found in street lighting put special importance on the light distribution of the luminaires. As already indicated, the relative importance to be given to direct vision and to silhouette effects is largely determined by the grade of illumination to be found upon the streets. In the best of business street lighting, direct vision is important, but as the grade of illumination is lowered, on second-grade business streets and thoroughfares, the use of direct vision decreases, and there is a considerable range where the relative importance of the two types of vision is open to argument. To promote direct vision, it is evidently important that there should be a high candlepower, somewhat below the horizontal. On the other hand, for silhouette vision on moderately light mat pavements, the greatest effect is usually attained by silhouette against the bright spots below and near the lamps. With highly specular pavements, no such spots exist, and the silhouette effect also will be due to the light directed slightly below the horizontal.

To meet these different conditions and points of view, a variety of modifying accessories, diffusing and prismatic globes, urns, bowls, and lanterns, are available. A plain white or diffusing glass globe of sufficient density gives a fairly uniform candlepower distribution, resulting in a great difference in illumination at different points on the street, but with comparatively low brightness of the luminaire surface. In diffusing bowls and in lanterns with metal frames, reflectors are often used over the lamps to bend down the upward-directed light. By making the reflector an inverted cone with properly curved elements,

some of the upward-directed light may be sent along the street. Where such opaque reflectors are used on business streets, an unpleasant effect is often produced by the sharply cut-off shadows on the fronts of buildings, above the level of the lamps. Desirable architectural features may also be concealed. Similar conical reflectors may be used below the lamps, but here the question arises as to whether the light so reflected would not be more useful in producing the bright spots for silhouette effects on the pavement. This flux, because of the nearness of the pavement surface, is quite effective in producing a high illumination near the lamp, but the same flux spread out over 360 deg near the horizontal will add very little to the illumination in this direction.



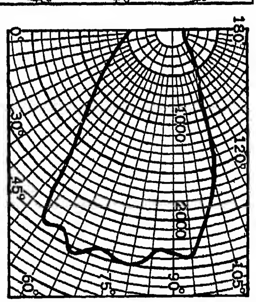


Fig. 22. Horizontal or Lateral Distribution Curves for Holophane Refractors. *A*. Refractor No. 4376, 300-watt lamp. *B*. Refractor No. 4377, 300-watt lamp.

The horizontal prisms on refractors (see Art. 5) may be designed to give almost any vertical distribution of the light which is desired for the lighting of the street, and it is upon these that principal dependence has been placed for obtaining high candlepower directed about 15 deg below the horizontal, at the expense of upward- or downward-directed light or both. Where it is desired to use only the upward-directed light in this way, a hemispherical refractor is used above the light source and extending slightly below it. Such a lighting unit is often enclosed in a stippled or rippled glass bowl with a metal cap. Deep bowl-shaped refractors which surround the lamp and fit into a metal top may be designed to take as much of the downward-directed light as can be spared from the pavement beneath the lamp, and to redirect it nearer to the horizontal. Asymmetric refractors (see Art. 5) are used to direct the light in two beams 180 deg apart, or in four beams 90 deg apart, or in two beams somewhat less than 180 deg for lamps mounted at the curb. Fig. 22 shows the general character of distribution curves for two such refractors.

LAMPS FOR STREET LIGHTING. The only lamps now in extensive use for street lighting are gas-filled incandescent lamps, either constant current or constant potential. (See Art. 2 for data.) The use of constant-current series systems has come over from the days of arc-lamps, and it seems not unlikely that the future will see a gradual change to constant-potential lamps, which are, indeed, already in extensive use for street lighting.

"There are no conditions of street lighting prevailing in the United States which justify the use of smaller than 1000-lumen lamps. The 2500-lumen lamp is the smallest size which may be used with good economy." (Street Lighting Code, *Trans. I.E.S.*, 1927, Vol. 22, p. 107.)

Luminous or magnetite arc lamps were installed extensively at one time, and many are still in use, but they are no longer being put in as new equipment. Two other light sources are seriously advocated for street and highway lighting. These are the sodium-vapor and the high-pressure mercury-arc lamps. (See Art. 1 for data.) One or both of these types of lamps will probably replace incandescent lamps in some cases, especially for highway lighting, on account of their high efficiency. How much their peculiar color characteristics will affect their usefulness remains to be seen. Carefully designed aluminum reflectors are claimed to increase the effectiveness of incandescent lamps, about four times.

LOCATION AND SPACING OF LIGHTS. Two principal methods of mounting street lamps are: (*a*) upon brackets, or yardarms, carried on ornamental poles, trolley poles, or distribution line poles; (*b*) on the tops of ornamental columns or posts. The first of these methods is the one most commonly used on residence streets and thoroughfares. Except where the illumination is high, this method is much to be preferred to the mounting on the tops of columns, both because it permits of higher mounting and also because it facilitates having the lamp over the street pavement, thus greatly improving silhouette vision, especially when the pavements are wet or polished.

Spacing, mounting heights, and lamp sizes are given in the lighting code chart of Fig. 21. It is to be noted that the figures given therein are not claimed to be optimum, but to repre-

sent good current practice. As indicated above under *Traffic Accidents and Crime*, it is probable that much higher illumination than that obtained with these recommendations can be justified from a broad economic point of view. As shown in Fig. 21, one-side mounting should be used only on low-traffic highways. Staggered mounting is suitable for all except heaviest traffic thoroughfares and business streets, and here the opposite mounting recommended is principally on account of appearance. The overhang, beyond the curb, of lamps upon brackets or yardarms, is determined principally by appearance and cost. From the point of view of illumination, the center of the traffic lane is the ideal location, but conditions of cost and appearance and the cleaning of lamps often necessitate shorter arms.

There should always be at least one lamp at every intersection. This is especially important on residence streets and minor thoroughfares, where driving speeds are likely to be great and illumination values low. On curves, it is best to have all lights on the outside of the curve; also on dead-end streets, to have a lamp opposite the center line.

APPEARANCE OF THE STREETS. The increasing importance that is being given to esthetic values indicates the careful selection of all street-lighting equipment from this point of view. In such selection of equipment, it is desirable to have involved some one with artistic and architectural training, or better still, a committee of several such persons.

SPECIFICATION AND JUDGING OF GOOD STREET LIGHTING. The design of lighting should, but seldom does, begin with the reflection factor of the pavement. The selection of type of equipment, size of lamp, mounting height, and spacing should be made in line with the principles and practice set forth in the foregoing statements. In the construction specifications, these specific data should be definitely set forth. In comparing different systems as to their effectiveness in street lighting, it is advisable to analyze them into a number of elements, and then, by using weightings for these various elements, as judged on a percentage basis, the net or total percentage value for each of the two or more equipments may be obtained. Such an analysis with recommended weightings was undertaken by the Street Lighting Committee of the Illuminating Engineering Society. These weightings were never published as an authoritative report, but doubtless they constitute the best information at present available. These are given in Table XIX. (Charles J. Stahl, Electric Street Lighting, p. 187.)

Because of surrounding conditions, illumination tests made upon streets are very unsatisfactory. It is much better to make or obtain distribution curves or isocandle curves for the units to be used, and then to calculate the illumination values on the street from these data. To facilitate this work, useful curve sheets and nomograms have been developed by Benford, Goodall, Putnam, and others.

Table XIX. Suggested Weightings for Judging Street Lighting

	Maximum Points Allotted	
	Residence Side Street	Traffic Thoroughfare
A. To Motorist:		
1. Average pavement brightness	23	25
2. Distribution of brightness	11	14
3. Vertical illumination	11	17
Negative—Glare		
B. To Pedestrian between Curbs Including All Cross Walks		
1. Conspicuity	11	11
2. Visibility	8	8
Negative—Glare		
C. To Pedestrian on Sidewalk:		
1. Horizontal illumination	6	4
2. Vertical illumination	8	5
3. Distribution of illumination	9	6
Negative { Glare, Shadows		
D. To Occupants of Property:		
1. Protection afforded	9	7
2. Visibility	4	3
Negative—Glare		
Total	100	100

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HOUSEHOLD HEATING APPLIANCES AND COOKING

By Frank Thornton, Jr.

11. POWER RATING OF APPLIANCES

The following table shows the range of rated inputs for the principal household applications of electricity and also the probable average yearly consumption of power, which is, of course, subject to wide variation depending on habit, size of family, climate, etc.

Application	Power Consumption		Application	Power Consumption	
	Input, watts	Annual Consumption, kwhr		Input, watts	Annual Consumption, kwhr
Hair curler	18	1.0	Portable heater	600–1250	96.0
Waffle iron	600–900	30.0	Range	7000–10000	1,600.0
Toaster	500–850	50.0	Water heater	1500–6000	3,900.0
Percolator	450	50.0	Ironing machine	1320	100
Pressing iron	550–1000	72.0			

See Sec. 2, Art. 6 and Sec. 18, Art. 21 for resistor materials. Also Sec. 2, Art. 10 and Sec. 18, Art. 22 for insulating materials.

12. SMALL APPLIANCES

The most widely used appliance is the pressing iron. These irons are usually 6 lb in weight, highly polished, and either nickel or chromium plated. Non-automatic irons are 550 watts input. The automatic iron contains a built-in thermostat to regulate the temperature, thereby eliminating the necessity of watching the temperature to avoid scorching.

Adjustment of the temperature is possible by means of a button on the top of the iron. The maximum is set at some point below 600 deg fahr surface temperature, because it has been found that this is above the usual ironing temperature and still not high enough to ignite readily the padding on an ironing board and start a fire if inadvertently left standing too long.

Automatic irons with inputs up to 1000 watts are now available. These irons are fast enough to avoid all delays due to wet cloth or fast work by the operator.

Waffle irons are also available with automatic temperature regulation.

Toasters are on the market with time control features.

Percolators have protective thermostats or fuses.

There is a multitude of small appliances of many different types of construction. Nickel chromium wire or ribbon is used in all of them. For electrical insulation a number of different materials are used, the principal one being mica. The usual practice is to design a mica form to fit the space available for the heater and wind the wire or ribbon on it. This is insulated on both sides with other sheets of mica and clamped solidly between two metal parts. Another type consists of a tube containing a coil of wire embedded in powdered magnesia. The tube is contracted upon the insulation to make good thermal contact, and the whole assembly formed by bending into a shape to fit the available space in the appliance.

Radiant heaters are made by assembling a helical coil of resistance wire on a porcelain insulator of spool or other shape. Reflector-type portable radiators are examples of this construction. Mica is often used as the support for radiant heaters as in toasters.

In view of the fact that the heater must be designed as a part of the completed appliance there are no standardized heating elements for use in these devices.

13. RANGES

HOUSEHOLD RANGES. Household ranges are built in a wide varaiety of sizes and shapes. They all consist of an oven and two or more surface burners. Some include warming compartments and storage spaces.

Ovens are most commonly available in 14-, 16-, and 18-in. widths. They are usually well insulated with mineral wool to reduce loss of heat.

Surface burners are usually approximately 6 or 8 in. in diameter. Different makes vary slightly from these dimensions because of differences in design details. The following table gives approximately the range of watts input found in ovens and surface burners.

Table of Heaters in Electric Ranges

Heater	Wattage
Bottom heater—14-in. oven	1320 to 1500
" " —16-in. oven	1500
" " —18-in. oven	1500 to 1800
Surface burner—6 in.	1200
" " —8 in.	1500 to 2250
Warming compartment	300

Oven heaters consist of open coils supported in heat-resisting porcelain insulation arranged to radiate heat directly into the oven. Top heaters are generally duplicates of the bottom heaters and are used only for quick heating of the oven, broiling, browning, or searing.

Hand control is provided for each heater. In some cases three heats are provided; others provide only single heat. Ovens are usually equipped with automatic temperature control and often with a clock so that the cooking can be automatically started at a predetermined time.

Surface burners are of two types—open and enclosed. Open burners are most commonly used and consist of a special heat-resisting porcelain molded into a circular disk with grooves in the surface in which the bare wire coils are assembled. Transfer of heat to the bottom of the cooking utensil is largely by radiation from the red-hot wires. Enclosed burners are made in several ways, and in all designs the heating coils are enclosed in a metal casing of some kind. In some makes this takes the form of flat castings with grooves in the bottom in which the coils are embedded. Another has a sheet-metal casing containing heater coils embedded in a cement processed in a special manner. In a third make the casing is a tube in which the heater coil is assembled with a compacted insulation of magnesia powder.

Surface burners are each controlled by a three-heat series-parallel switch.

The total capacity of an electric range may be from 7000 to 10,000 watts depending on size and number of burners, but the average is about 8000 watts. All burners are almost never used simultaneously at full heat so that the maximum demand is usually much less than this value.

THE COST OF COOKING depends on the number of persons in the family, the kind of meals served, the skill of the user, and the rate for current. The curve in Fig. 1 gives an idea of the power requirements for electric ranges under average conditions. This does not include the power required for lights, extra appliances, water heating, etc.

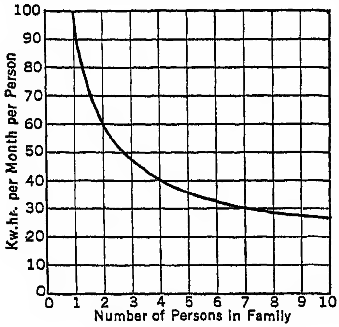

Fig. 1.

COMMERCIAL COOKING. Hotel and restaurant kitchen equipment is available for all the operations of cooking that are needed.

RANGES consist of a flat cooking top with oven beneath. Automatic controls are available for ovens. A typical range is as follows:

Cooking surface 36 in. wide by 24 in. deep with four cooking plates, two front plates at 5000 watts each and two rear plates at 4000 watts each.

Oven 22 in. wide by 28 in. deep by 14 in. high, heaters in top and bottom each 3000 watts.

Total maximum load, 24,000 watts.

Bread ovens are usually of the sectional type capable of baking 20, 40, 60, and 100 loaves of bread (1 lb per loaf) on each shelf. Sometimes two and occasionally three shelves may be assembled one above the other on the same base. Each shelf is a complete oven in itself with heaters and automatic temperature control.

The average energy consumption will be between 100 and 200 watthours per loaf, depending on the character of the dough, size of loaf, and skill of the baker.

Other appliances are coffee urns, steam tables, waffle irons, sandwich toasters, hot plates, griddles, fry kettles, etc.

The cost of operation varies in different kinds of restaurants. Institutions in which a definite menu is prepared and served at a definite time can be operated at the lowest cost, whereas special-order restaurants which must be ready to serve at all hours must of necessity keep their kitchen equipment hot continuously. However, the following table will give some idea as to the approximate energy requirments of the completely electrified kitchens of various types of restaurants.

Energy Requirements

Type of Restaurant	Watthours per Meal Served
Cafeterias	300– 600
Clubs	600–1000
Hotels	350–1000
Hospitals	200– 350
Institutions	150– 350
Restaurants	600–1000

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WATER HEATING

By C. J. Fay and J. H. Reifenberg

15. PRINCIPLES

Water is usually heated by electricity by immersing an insulated resistance type of heater in the water to be heated. Occasionally resistance heaters are clamped to the outside of the tank or vessel in which the water is to be heated, and very rarely induction-type heaters are used. The size of heater for any particular application is dependent upon the amount of hot water required, its temperature, the capacity of storage tank, and the length of time the heater may be connected to the power circuit.

One Btu is required to heat 1 lb of water 1 deg fahr.
One gallon of water weighs 8.336 lb at 62 deg fahr.
One kwhr is equivalent to 3412 Btu.
2.443 watts will raise 1 gal of water 1 deg fahr in 1 hr.

$$\text{Size of heater in kilowatts} = \frac{\text{Gallons of water} \times \text{deg fahr rise desired} \times 2.443}{1000 \times \text{Time in hours}}$$

One kilowatthour will heat 4.1 gal of water 100 deg fahr.

For the various household uses the following table gives the approximate quantities of hot water required and the temperatures at which it is to be used.

	Quantity, gal	Temperatures, deg fahr
Bathtub	10 to 15	95 to 100
Shower	12 to 20	95 to 100
Dishes	2 to 5	115 to 145
Face washing	1/2 to 1	95 to 100
Hands	1/2 to 1	100 to 105
Washing machine	10 to 20	145 to 160
Rinsing—first	5 to 10	115 to 120
Rinsing—second	5 to 10	90 to 95

16. DESIGN OF HOT WATER HEATERS

Immersion heaters permit constructions of complete water heaters giving the most economical operation. Such complete heaters consist of storage tank completely surrounded by heat insulation, with the heating element mounted in the storage tank, and the whole encased in a sheet-metal casing. Thermostats for controlling the temperature of water are either mounted in the headers of the heating elements on the storage tank, or suspended in the tank, usually in tubular casings.

STORAGE TANKS. Storage tanks are usually constructed of sheet steel with electrically welded seams and joints and are completely galvanized inside and out. Where foreign matter in the water attacks and corrodes galvanized steel, copper, copper-alloy or monel tanks are used. On copper tanks, the seams are usually riveted and soldered while the copper-alloy or monel tanks have welded seams. Copper and copper-alloy tanks usually are lined with block tin, as some water will react with the copper to produce a bluish-green tinge in the water. Tank sizes vary from 2 to 120 gallon capacities with special ones even larger.

IMMERSION HEATERS. Immersion heaters are of two kinds: (1) helical coils of nickel-chromium wire surrounded by magnesium oxide, or one of the various insulating cements, within a tubing or casing, usually copper; and (2) helical coils of nickel-chromium wire mounted in or on blocks of porcelain or other refractory material. Heaters of class 1 are generally mounted in a header either singly or in multiple, and the header is threaded with standard pipe thread or arranged for bolting to adaptors welded to the tank. The casings of heaters of class 2 are usually water-tight tubular wells welded to the tank. Heaters may be inserted horizontally through the side wall of the tank or vertically through either top or bottom heads. In those designs using vertical mounting the heaters are usually surrounded by a circulation tube extending practically the full length of the tank. The cold water drawn in at the bottom end of the tube is heated by the heater and discharged from the top of the tube. In this manner it is possible to heat a small quantity of water rapidly and deposit it at the top of the tank. This is also accomplished in those tanks having heaters inserted in the side walls by using two heaters, one at the bottom and one near the top of the tank; the top heater will heat only the water above it.

EFFICIENCY. The efficiency of operation of an electric water heater depends primarily on the effectiveness of the heat insulation in reducing the radiation losses from the tank and its fittings. Balsam wool, mineral wool, paleo bark, hair felt, and granulated cork are the heat insulations generally used. Thickness of insulations vary from 2 1/2 to 5 in. For equal thicknesses the insulating values of these materials differ but slightly. Practically all outside casings are of sheet steel with japanned or enameled finishes.

THERMOSTATS. Thermostats are usually adjustable, permitting the maintenance of water temperatures most suitable for the particular user. The heat-responsive member may be bimetal in the form of a formed disk with snap action or a strip operating a toggle mechanism, gas-filled bellows, or rods of dissimilar metals. When non-adjustable thermostats are used, temperatures of water are usually maintained from 145 to 160 deg fahr.

When economy of operation is not an important factor immersion heaters may be assembled in the 1-in. openings in the side walls of the ordinary range boilers or in a pipe casing and used as a circulation heater outside of the tank, or strap-on heaters may be clamped around the tank. Since bare tanks will radiate heat very rapidly, the tanks should at least be covered with a commercial tank cover. With water 70 deg fahr above the temperature of the surrounding air, the ordinary 30-gal bare range boiler will radiate approximately 725 watthours per hour and a 40-gal tank approximately 850 watthours.

Commercial tank covers are built up of layers of air-cell-asbestos, magnesia, or similar materials formed to fit the ordinary tanks and covered with canvas or duck. Such covers will reduce the radiation losses approximately 75 per cent.

17. TESTING

Electric water heaters are tested for (1) rate of heating, (2) heat loss, and (3) draw off. Thermocouples and potentiometer are used for measuring temperatures of the water, the thermocouples being soldered to the storage tank, one on each tank head and four equally spaced from top to bottom on the side wall. For draw-offs additional thermocouples at inlet and outlet are used to measure temperatures of water drawn into and from tank.

RATE OF HEATING. In this test the tank is filled with cold water, the heater turned on and allowed to heat until the thermostat cuts the heater off. The efficiency of heating is determined by the following formula:

$$\text{Per cent efficiency} = \frac{C \times (T_2 - T_1) \times 0.00243 \times 100}{\text{Kwhr}}$$

where C = actual capacity to tank in gallons.
T_1 = average water temperature at start.
T_2 = average water temperature at end.
Kwhr = kilowatthours consumed by heater.

HEAT LOSS is determined by permitting the heater to operate under control of its thermostat, without any water being drawn from the tank, and measuring the wattage consumed. The water must be permitted to reach constant cut-off and cut-on temperatures, and then the test must be started just after the thermostat cuts off, be permitted to run for not less than 72 hours, and be terminated just after the thermostat cuts off.

$$\text{Heat loss in watthours per hour at } (T - t) = \frac{\text{Watthours input}}{\text{Time in hours}}$$

where T = average water temperature.
t = average room temperature.

All temperatures must be read at least every hour and at every cut-on and cut-off operation of the thermostat in order to get a correct average. For all practical purposes the heat loss varies directly as temperature rise $(T - t)$ for the well-insulated tanks.

DRAW-OFF TESTS. Extensive studies have shown that the average family of 4 to 5 persons in the northern part of the United States uses approximately 1285 gal of 145 deg fahr water per month. Different quantities of water are used each day, but the days may be divided into three classes, as minimum, normal, and maximum days, with average usages of 20, 50, and 75 gal, respectively. In a month there are 12 minimum, 13.4 normal, and 5 maximum days.

Table I

Time	Minimum Day	Normal Day	Maximum Day
6:30 A.M.	..	8	8
7:30 A.M.	3	4	4
8:00 A.M.	2	2	13
9:00 A.M.	2	3	10
10:00 A.M.	..	3	10
11:00 A.M.	2	3	3
12:00 N.	3	3	3
1:00 P.M.	1	5	5
5:00 P.M.	..	2	2
6:00 P.M.	1	3	3
7:00 P.M.	2	4	4
9:00 P.M.	4	..	..
10:30 P.M.	..	10	10
Totals. . .	20	50	75

On the basis of the above, standard draw-off schedules given in Table I are used for comparison of water heaters.

The heater should be preheated and idling on its temperature control, so that a tank full of hot water will be available at the start of the test. A test extends over a period of 24 hours. Water meter in the cold water line is used to measure quantity of water drawn. Curve-drawing temperature recorders should be used for measuring tank temperatures; however, when they are not available readings should be taken of each thermocouple 15 minutes before first draw off and at 30-minute intervals throughout the 24-hour test. Temperature must be read of each gallon of water withdrawn.

Records are made of the following: (1) average temperature of water in tank at beginning of test; (2) average temperature of water in tank at end of test; (3) average temperature of incoming water; (4) average temperature of water withdrawn; (5) average

room temperature; (6) kilowatthour input; (7) average temperature of water in tank during 24-hour period.

These data are corrected to a common basis of 75 deg fahr room temperature, 50 deg fahr incoming water, and 150 deg fahr water withdrawn, and the following calculations are made:

(1) Tank losses = Input − Output

(2) $\text{Tank efficiency} = \dfrac{\text{Output}}{\text{Input}}$

(3) $\text{Gallons of water of 100 deg fahr rise per kilowatthour} = \dfrac{\text{Input}}{\text{Gallons drawn}}$

(4) $\text{Daily load factor} = \dfrac{\text{Input}}{24 \times \text{Connected load}}$

By obtaining above results for each of the three daily demands, i.e., minimum, normal and maximum, and combining in the ratio of 12, 13.4, and 5, similar results are obtained for monthly operation.

18. PERFORMANCE

Since 1 kwhr will raise the temperature of approximately 4 gal of water 100 deg fahr the amount of water that any water heater will heat depends upon the rating of the heater and the length of time the heater may be connected to the power line. Radiation losses reduce the amount of effective heat supplied. In the factory-built complete tanks, radiation losses vary from approximately 35 watts for some of the smaller tanks to as much as 135 watts for some of the 120-gal sizes with water in tank 70 deg fahr above room temperature. For 30-gal range boilers equipped with immersion heaters or strap-on heaters and commercial tank covers, radiation losses vary from 105 to 185 watts under similar conditions, and 30-gal factory-built heaters from 60 to 90 watts.

The quantity of hot water that can be drawn from a tank at any one time depends on the storage capacity of the tank, the rating of the heating element, and the length of time the heating element has had to preheat the water.

19. INSTALLATION

Electric water heaters should be installed as closely as possible to the point at which the most hot water is used so as to require a minimum of hot water piping. All hot water piping should be covered with heat insulation to minimize radiation losses therefrom. Heat losses from bare pipe are shown in Table II.

Table II. Heat Loss from Bare Pipe

Size of Pipe, in.	Watthours Lost per Foot Length per Hour per Degree Fahrenheit Difference in Temperature	Size of Pipe, in.	Watthours Lost per Foot Length per Hour per Degree Fahrenheit Difference in Temperature
1/2	0.133	1 1/2	0.298
3/4	0.165	2	0.373
1	0.200	2 1/2	0.451
1 1/4	0.260		

Return circulation systems should be avoided since hot water is circulating continuously in such systems and, therefore, heat being lost by radiation continuously.

20. OPERATION

Water heaters are operated either continuously or intermittently. Continuous service may be on peak, off peak, or semi-off peak.

For intermittent service the heater is equipped with switch for manual operation and is turned on only when hot water is desired and turned off as soon as the immediate requirement is satisfied. This is the most economical method of operation, as radiation losses are very low. The user, however, must anticipate his needs far enough in advance to allow the heater time to heat the water to the temperature desired and must also remember to turn the heater off or equip it with a time limit switch. A thermostat prevents the water from heating above the maximum desired temperature. A small or medium-

sized tank (10 to 18 gal) with high-wattage heater (3 to 5 kw) is commonly used for this type of service.

For **on-peak service** the heater is connected to the power service permanently and its thermostat maintains the water between definite limits. Any combination of tank and heater can be used, depending upon the demands of the consumer and the capacity of the utilities transmission system and generating equipment. Hot water is always available. Operating costs are slightly higher since more hot water is used where it is constantly available. Standby losses are also slightly higher.

For **off-peak service** the heater is connected to the power service through a time switch or a relay, actuated from the power plant. Power is supplied to the heater only at such times as the demand for power for other service plus that for hot water service does not exceed the peak load of the system. Generally utilities sell power for off-peak service at lower rates than for on peak. Thermostats prevent the water from attaining temperatures higher than desired. Tanks must be of sufficient capacity to supply the maximum demand for hot water between heating periods, and heaters must be of high enough rating to reheat the full tank of water during the heating period. Proper selection of tank size and heater rating will give the lowest cost continuous hot water service.

Tanks for **semi-off-peak service** are equipped with two heaters: the bottom heater on off-peak service and the top heater on on-peak service. When abnormal demand exhausts the hot water before it is time for the bottom heater to reheat the tank again, the top heater will supply normal demands. Power for this top heater is usually paid for at the on-peak rate and for the bottom heater at the off-peak rate. Two meters are required.

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HEATING AND AIR CONDITIONING OF BUILDINGS

By Otto W. Walter

22. HEAT LOSSES AND CALCULATIONS

In order to maintain a constant room or building temperature, heat must be supplied at the same rate as that at which it is lost from the inside. The heat losses are of two kinds:

(*a*) Transmission losses by direct conduction through walls, windows, doors, ceilings, and floors.

(*b*) Infiltration losses due to air leakage around windows and doors, through cracks, and to some extent through porous walls, ceilings, and floors.

The evaluation of these losses in Btu per hour constitutes the correct basis for the selection of the proper size of heating equipment for any building.

HEAT LOSSES BY TRANSMISSION. Heat loss by conduction directly through walls, ceilings, and other areas occurs when the temperature on the outside is lower than the temperature on the inside.

Let H_t = heat lost by transmission in Btu per hour.
U = heat transmission coefficient in Btu per hour per square foot of surface (wall, window, ceiling, etc.) per degree Fahrenheit temperature difference.
S = area of wall, glass, ceiling, etc., in square feet.
t = temperature of air inside room in degrees Fahrenheit.
t_0 = temperature of outside air in degrees Fahrenheit.

$$H_t = US(t - t_0) \tag{1}$$

The value of U may be taken from Table I, or from more complete tables of these coefficients given in Mechanical Equipment of Buildings by Harding and Willard, the Guide of the American Society of Heating and Ventilating Engineers, and other handbooks. (See Bibliography.) The heat transmission coefficients may also be calculated from the thermal conductivities and the thickness of the wall or ceiling materials. For a wall of a single homogeneous material

$$U = 1\Big/\left(\frac{1}{f_i} + \frac{1}{f_0} + \frac{x}{k}\right) \tag{2}$$

Where f_i and f_0 are inside and outside air surface conductances for the material in contact

with air expressed in Btu per square foot per hour per degree Fahrenheit temperature difference; k is the thermal conductivity of the homogeneous building material expressed in Btu per hour per square foot per inch of thickness per degree Fahrenheit temperature difference; and x is the thickness of the building material.

For a compound wall having several materials and air spaces

$$U = 1 \Big/ \left(\frac{1}{f_i} + \frac{1}{f_0} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \ldots + \frac{1}{a_1} + \frac{1}{a_2} + \ldots \right) \tag{3}$$

where the x's and the k's are respectively the thicknesses and the thermal conductivities of the different homogeneous building materials, and the a's are the conductances of the air spaces. Thermal conductivities may be taken from Table II, and air space conductance from Table III. Thermal conductivities are given in Table II, that is Btu per square foot per inch thickness per degree Fahrenheit temperature difference, except in cases indicated where a standard thickness applies, when the conductance C is given in Btu per square foot per degree Fahrenheit per thickness specified. The value of f_i may be taken as 1.65, which is an average value for the air surface conductance for a smooth surface with still air expressed in Btu per hour per square foot per degree Fahrenheit temperature difference. The value of f_0 may be taken as 6.0, which is an average value of air surface conductance for a 15-mile-per-hour wind velocity expressed in Btu per hour per square foot per degree Fahrenheit temperature difference.

In some cases, in calculating the heat-loss coefficients for walls or ceilings which are insulated by filling between the studs or joists, it is necessary to take into account the fact that the space occupied by the studs or joists will not be filled with insulation. This can be done by considering the path through the studs and the path through the insulation as parallel paths analogous to parallel resistances in the electric circuit. In frame construction where 2-in. studs are 16 in. on centers about 15 per cent of the wall area is not filled with insulation if a small allowance is made for headers and fire stops. A close value of the average coefficient is obtained by calculating the coefficient for the section through the studding and the coefficient for the section through the insulation and combining these two values according to the law of parallel conductances by the formula

$$U = 0.85U_i + 0.15U_s \tag{4}$$

where U_i and U_s are the heat-transmission coefficients for the sections through the insulation and the studs, respectively. In ceilings where headers and obstructions are less likely to occur the space occupied by the joists may be taken as 10 per cent instead of 15 per cent, and formula (4) can be modified accordingly. This method of calculating the average coefficient assumes that the direction of the heat-flow lines are always perpendicular to the surface of the walls. Another method assumes infinite conductance to lateral flow of heat perpendicular to the wall surface. The true value lies between the values obtained by these two methods. In calculating the coefficients for complicated compound wall structures, a great deal of assistance will be obtained by considering the analogous electrical circuit.

INFILTRATION LOSSES. The calculation of the heat loss by infiltration is based on the volume of outside air leaking into the building

$$H_i = 0.018Q(t - t_0) \tag{5}$$

where H_i equals Btu per hour required to raise the temperature of the air leaking into the building, Q equals cubic feet of air entering per hour, t equals inside temperature in degrees Fahrenheit, t_0 equals outside air temperature in degrees Fahrenheit, and 0.018 equals the product of the density and specific heat of air (0.075 × 0.24), or the Btu to raise 1 cu ft of air 1 deg fahr.

An approximate method of getting the volume of air leakage is to assume a certain number of air changes per hour in accordance with the average values given in Table IV or Table VI. However, the more accurate method is to base the leakage estimate on crackage. The crackage around doors and windows and the area of any porous or poorly constructed walls are multiplied by leakage coefficients giving the cubic feet of air leakage per hour. A brief list of leakage coefficients summarized from research studies is given in Tables V and VII. Since the leakage is caused chiefly by pressure built up by the wind velocity, the cold air leaks in on the windward side of the building and the warm air leaks out on the opposite side. Therefore in calculating the volume of infiltration air only half of the total crackage of the building is used. In getting the feet of crackage, add the perimeters around all outside doors and window sash, including the perimeter along the meeting-rails of the sash.

In some cases it is necessary to provide air for ventilation in addition to that entering by natural infiltration. The amount of air required per person for various classes of

service for good ventilating practice is given in Table VI. The heat required to heat this air must be included in the total heat loss estimated.

HEAT-LOSS CALCULATIONS. A formal detailed calculation of the heat losses of the building is the most accurate and the proper basis for selecting heating equipment. However, a preliminary or rough estimate may be made by using any one of several rule-of-thumb methods, but these short rules should not be relied upon except as first approximations. In Carpenter's short rule the loss in Btu per hour is given by

$$H = (0.02\,NC + G + 0.25\,W)(t - t_0) \tag{6}$$

where N equals the number of air changes (see Table IV).
C equals the volume of the building in cubic feet.
G equals the glass and door area in square feet.
W equals net wall and ceiling area in square feet.
$(t - t_0)$ equals temperature difference between inside and outside air in degrees Fahrenheit.

Harding and Willard's rule is similar to Carpenter's rule except that the infiltration loss is based on crackage.

$$H = (1.2\,P + G + 0.25\,W)(t - t_0) \tag{7}$$

where the notation is the same as above except that P equals not less than one-half of the total window and door crackage in feet.

The following procedure is given for making a detailed formal calculation of the total heat loss for a building:

1. Ascertain the inside temperature which is to be maintained at the breathing line (5 ft from the floor) during the coldest weather.
2. Determine the minimum outside temperature to be used in the estimate, basing this temperature on Weather Bureau records.
3. Calculate and tabulate the net outside wall, ceiling, window, door and floor areas, as well as areas adjacent to unheated spaces.
4. Select the proper heat-transmission coefficient for each area from Table I, or calculate the coefficients by the use of formulas (2), (3), and (4).
5. Compute the heat loss for each area by multiplying the areas in square feet by the heat-transmission coefficients times the temperature difference. Increase the losses computed in this manner for the walls and glass on the sides of the building next to the prevailing wind by adding 15 per cent to allow for exposure and higher than 15 miles per hour wind.
6. Compute the cubic feet of cold air entering the building per hour, preferably by the crackage method. Calculate the Btu per hour required to warm this air by formula (5). The heat to warm any air admitted by artificial ventilation must also be included.
7. Add the heat losses obtained in 5 and 6. The result represents the total Btu per hour which must be supplied by the heating system on a cold day after the building is up to temperature and operating under steady state conditions.

INSIDE TEMPERATURE. The inside room temperature schedule should be stated in the heating specifications. Inside air temperature is understood to mean the temperature at the breathing line, 5 ft above the floor and not less than 3 ft from the side walls. Inside temperatures recommended for different types of buildings are given in Table VIII.

Owing to the tendency of warmer air to rise, the temperature is usually higher at the ceiling and lower at the floor than at the breathing line. The temperature at the floor may ordinarily be taken 5 deg fahr lower than at the breathing line provided the latter is not less than 55 deg fahr. For ceilings and other areas it is common practice to allow a 2 per cent increase in temperature per foot above the breathing line in estimating the probable air temperature. With unit heaters or other types of equipment with forced air circulation the temperature difference between ceiling and floor may be much less. However, the difference should not be assumed to be less than 1 per cent per foot unless the equipment is guaranteed by the manufacturer.

The comfort in a heated room is found to depend not only on the temperature, but also upon the humidity and the velocity of air circulation. Where the humidity is raised by artificial means, it is found that the same comfort is obtained with an inside temperature 1 deg fahr lower for each increase of 10 in the per cent relative humidity. Thus if the relative humidity is raised from 20 to 40 per cent, the same comfort will be felt at 68 deg fahr as was felt at 70 deg fahr before raising the humidity. Higher than 40 to 50 per cent relative humidity usually should not be used in freezing weather because of danger of condensation of moisture on walls and windows. Also humidities above 60 per cent may feel uncomfortable. Very low humidities are to be avoided not only because of the higher room temperatures required, but also because low humidities are apt to be unhealth-

ful and are injurious to furniture. The consensus of opinion is that 40 to 60 per cent relative humidity is the best from the standpoint of health.

OUTSIDE TEMPERATURES. The outside temperature to be used for estimating the capacity of heating equipment should be based on Weather Bureau records taken over a period of years. The temperature taken can ordinarily be slightly higher than the lowest temperature recorded for the locality, usually from 5 to 10 deg higher.

HEAT AVAILABLE FROM SOURCES OTHER THAN THE HEATING PLANT. Sometimes the heat supplied by the heating plant is supplemented by heat supplied by persons, lights, motors, and machinery. The amount supplied from these sources is usually so small that it may be neglected, but in certain instances it must be evaluated and taken into account, as in assembly halls, theaters, and some industrial plants. In special cases waste heat may be used to supply all or most of the heat required.

A man at rest gives off about 400 Btu per hr, and at work about 500 Btu per hr. This quantity of heat is not of sufficient importance to be taken into account except in assembly halls and theaters.

Lamps. The heat (Btu) supplied by the electric lamps in a room is equal to

$$3.415 \times \text{(Watts per lamp)} \times \text{(Number of lamps)}$$

The heat supplied by gas lamps may be calculated from the following data:

1 cu ft producer gas	150 Btu
1 cu ft illuminating gas	530 Btu
1 cu ft natural gas	1000 Btu

A Welsbach burner averages 3 cu ft of gas per hr, and a fishtail burner 5 cu ft per hr.

Motors and Machinery. Motors and the machinery which they drive, if both are located in the room, convert all the electrical energy supplied to them into heat which is retained in the room if the product being manufactured is not removed until its temperature is the same as the room temperature. In this case

$$\text{Btu supplied per hour} = 2546 \left(\frac{\text{Hp output of motor}}{\text{Efficiency of motor}}\right)$$

If power is transmitted to the machinery from the outside, then only the heat equivalent of the brake horsepower supplied is used, in which case

$$\text{Btu supplied per hour} = 2546 \times \text{(Brake horsepower)}$$

In high-powered mills the heat given out by the machinery is the chief source of heating and is frequently sufficient to overheat the building even in zero weather, thus requiring cooling by ventilation the year round.

In a transformer substation or other room from which electrical energy is transmitted, the heat supplied by the electrical apparatus is of course only the losses in the apparatus. The loss is equivalent to 3415 Btu per kw hr of loss.

SEASONAL HEATING LOAD. The number of degree-hours (or degree-days) per heating season is usually accepted as a fair index of the seasonal heating load. (One degree-day equals 24 degree-hours.) The number of degree-hours for a given heating period for a building in which a given inside temperature is maintained is found by a summation over the period of the hourly differences in temperature between the inside and outside air. Tables giving the seasonal degree-hours for residence heating may be found in several heating and ventilating handbooks and journals (see Bibliography), or the value for any given locality may be calculated from hourly weather data which can be secured from the Weather Bureau.

An estimate of the fuel consumption of a building for a heating season may be made by the following formula:

$$H_s = (H_c/Dc)D_s \tag{8}$$

where H_s = the Btu which must be delivered by the heating system for the season.
H_c = Btu required by the building for 24 hrs on the cold day.
D_c = degree-hours (or degree-days) for the cold day.
D_s = degree-hours (or degree-days) for the season.

The heat requirements for a given season will depend upon the degree hours of the actual season, the room temperature schedule actually followed, habits of the occupants of the building, average wind velocities over the season, and numerous other factors. Therefore, the actual consumption may vary considerably from that estimated for normal conditions. The variation of the degree-hours for the severe and mild season is different for different localities, but the average variation is of the order of 20 per cent above or below normal.

The Btu requirement for the season divided by the product of the average overall seasonal efficiency of the heating system and the Btu content per unit of fuel gives the required fuel consumption for the season.

BUILDING INSULATION. There are several important advantages in building insulation. Fuel savings as high as 50 per cent may be obtained with complete insulation where 2 or 3 in. of insulation are applied to the walls and ceilings, and weather strips and double glass are used on windows and doors. Aside from fuel savings, insulation gives a more uniform inside temperature in winter and makes the building more comfortable in summer.

Insulation is often installed on the basis of the most economical thickness. This thickness can be estimated from the formula

$$t = \left(\frac{U\,D_s\,F}{n\,B\,M\,r} \times 10^6\right) - \frac{k}{U}. \tag{9}$$

where t = economical thickness of insulation in inches.
U = heat-transmission coefficient of wall or ceiling in Btu per square foot per hour per degree temperature difference before insulation is applied.
D_s = degree-hours per season.
F = cost of fuel in dollars per unit quantity.
n = overall efficiency of heating system for the season in per cent.
B = Btu value per unit of fuel.
M = cost of insulation installed in cents per square foot per inch of thickness used.
r = return required on insulation investment in per cent.
k = conductivity of insulation in Btu per square foot per hour per inch of thickness per degree temperature difference.

This formula assumes that M, the cost of insulation per square foot per inch of thickness installed, is proportional to the thickness used. Where this does not hold, it will be necessary to assume values of M and solve the equation by trial and error.

For electric heating, eq. (9) can be expressed in a more convenient form, as follows:

$$t = \frac{U\,D_s\,e}{0.3415\,n\,M\,r} - \frac{k}{U} \tag{10}$$

where t, U, D_s, n, M, r, and k are defined the same as in eq (9), and e is the cost of electrical energy in cents per kilowatthour.

It should be noted that the economic thickness of insulation increases directly with the cost of fuel. Therefore with the use of electricity, gas, or any higher-priced fuel, a greater thickness of insulation will be justified.

In installing insulation greater efficiency is obtained with a given amount of insulation where it is applied over the whole heat-loss area rather than where the insulation is concentrated in a thick layer over a small part of the heat-loss area. Where sufficient insulation is used, it is possible to make a reduction in the size and cost of the heating system. If this saving is to be taken into account in determining the amount of insulation justified, it will be necessary to make a detailed heating estimate both with and without insulation.

HEATING APPARATUS. For dwellings, small stores, shops, etc., steam, hot water, warm air, and electric heating systems are used. For large buildings less than three or four stories high, either hot water or steam may be used, but in tall buildings only steam or vapor may be used because of the high static head encountered with hot water.

In steam and hot water systems the heat is delivered by standard metallic radiators of various types. Some of the enclosed and recessed types are more properly called convectors. It is now the general practice of the manufacturers to rate all radiators in Btu per hour, in 1000 Btu per hour designated as Mb per hour, or in equivalent square feet of heating surface. One square foot of equivalent heating surface emits 240 Btu per hr when located in a room of 70 deg fahr and supplied with steam at 215 deg fahr. For other than standard conditions, the transmission may be taken as equal to

$$H_r = H_s \frac{(t_s - t_r)^{1.3}}{(215 - 70)} \tag{11}$$

where H_r = the rate of heat emission in Btu per hour per equivalent square foot of surface at temperature t_r.
H_s = the rate of heat emission in Btu per hour per equivalent square foot of surface under standard conditions.
t_s = the temperature of radiator (assumed to be temperature of steam or hot water).
t_r = the temperature of the room in which the heater is located.

When hot water is used in the radiators, the emission is assumed to be the same as for steam of the same average temperature as the water. Where the average hot water temperature is 170 deg fahr and the radiator is located in a room at 70 deg fahr the emis-

sion is usually taken as 150 Btu per hr per sq ft of equivalent heating surface. Conversion factors based on eq. (11) are given in Table IX for convenience in calculating the emission where the conditions are not standard.

There are three principal classifications of solid fuel boilers in general use: (1) cast-iron sectional boilers, (2) steel boilers of the fire-tube or water-tube type, (3) magazine-fed boilers.

Working pressures in most cases are limited to 15 lb per sq in. for steam, and 30 lb per sq in. for hot water. Boiler ratings are given in Btu per hour, in equivalent square feet of radiation, or in boiler horsepower which is equivalent to 33,471.9 Btu per hr. The boiler should have sufficient capacity to supply (1) the estimated emission for the connected radiators, plus (2) the estimated emission of the piping connecting the radiation and boiler, plus (3) an allowance for starting up cold radiators. The piping loss may vary considerably, but it is common practice to add a flat percentage, 25 per cent for steam systems and 35 per cent for hot water systems, to the connected radiation to take care of this loss. The allowance for heating up varies with conditions and the size of the system. The minimum requirements specified by the American Society of Heating and Ventilating Engineers' code are as follows:

Up to 100,000 Btu per hr add 65 per cent.
100,000 to 200,000 Btu per hr add 60 per cent.
200,000 to 600,000 Btu per hr add 55 per cent.

These percentages are percentages of the radiation plus the piping loss.

Other important factors affecting the operation of the boiler which should be carefully considered are: (1) chimney and draft capacity, (2) fuel contemplated, (3) quality and frequency of attention, (4) design of piping and heat-emitting surfaces, (5) character of water for use in boiler. Proper selection of equipment with reference to these points can usually be assured by careful study of the recommendations and data supplied by manufacturers.

23. SYSTEMS

ELECTRIC HEATING. The many advantages of the use of electricity for building heating, together with the attractiveness of this load for the power companies, have led to lower rates and new developments in electrical heating equipment. Three different classes of equipment are now in use: (1) direct radiators and space heaters which take electrical energy coincidental with the heating demand, (2) thermal storage equipment which takes electrical energy only at off-peak hours when the power system is operating sufficiently below rated load to allow it to carry the added heating load without additional plant or distribution capacity, and (3) equipment using the reversed refrigeration cycle.

From the standpoint of the heating customer, advantages offered by electrical heating are: (1) elimination of dirt, soot, and fumes; (2) elimination of firing and other furnace labor; (3) elimination of fuel and combustion products from the premises; (4) accurate and reliable room temperature regulation; (5) minimum radiation losses in the basement and other parts of the building where heat is not required; (6) minimum maintenance and reconditioning; (7) efficient and low-cost servicing and supervision by competent public utility staff in territories where heating rates are established. From the standpoint of the power company, the heating load offers an opportunity of improving the system load factor and power factor, and at the same time greatly increasing the kilowatthour output. The off-peak load especially is very desirable since it fills in the valleys in the load curve and does not increase the generating or distribution capacity required. In some territories, usually where hydroelectric power is available, rates of 2 cents per kwhr or less are available for on-peak electric space heating. Where the power use is controlled by a time switch and limited to the off-peak hours, off-peak rates, as low as 1 $^1/_2$ to $^3/_4$ cents per kwhr, have been made available. In heating their own buildings with off-peak energy utilities often figure the energy as low as $^1/_2$ cent per kwhr.

A comparison of the cost of heating by electricity with that of heating with coal, oil, or gas is readily obtained by assuming the efficiencies that may be expected and calculating the equivalent prices for these fuels to give the same cost for a given amount of heat delivered to the radiators. On this basis the equivalent rate for coal in dollars per ton to give the same cost of heating as electricity at any given rate per kilowatthour is given by the formula

$$R_c = (R_e/E_e)\ (5.85\ B_c\ E_c) \times 10^{-3} \tag{12}$$

where R_c = cost of coal in dollars per ton (2000 lb).
R_e = cost of electricity in cents per kilowatthour.

E_e = overall efficiency for electric heating in per cent.
B_c = Btu per pound of coal.
E_c = overall efficiency for coal heating in per cent.

The equivalent rate for oil is given by

$$R_o = (R_e/E_e)\ (2.92\ B_o\ E_o) \times 10^{-4} \tag{13}$$

where R_o = cost of oil in cents per gallon.
R_e = cost of electricity in cents per kilowatthour.
E_e = overall efficiency for electric heating, in per cent.
B_o = Btu per gallon of oil.
E_o = overall efficiency for oil heating in per cent.

The equivalent rate for gas is given by

$$R_g = (R_e/E_e)\ (0.292\ B_g\ E_g) \tag{14}$$

where R_g = cost of gas in cents per thousand cubic feet.
R_e = cost of electricity in cents per kilowatthour.
E_e = overall efficiency for electric heating in per cent.
B_g = Btu per cubic foot of gas.
E_g = overall efficiency for gas heating in per cent.

If electric thermal storage heating at 1 cent per kwhr is compared with heating with manufactured gas by substitution in formula (14), using an overall efficiency of 90 per cent for electricity, 530 Btu per cu ft for gas, and an overall efficiency of 60 per cent for gas heating, the equivalent price of gas is found to be $1.03 per 1000 cu ft. The 60 per cent overall efficiency for gas heating assumes a boiler efficiency of 70 per cent and a basement piping loss of approximately 15 per cent, giving an overall efficiency of 60 per cent. The overall efficiency for electrical heating assumes 10 per cent tank and basement piping loss for the storage system.

If electric thermal storage heating at 1 cent per kwhr is compared with coal heating by substitution in formula (12), assuming $B_c = 13{,}000$, $E_c = 36$ (45 per cent heater efficiency and 20 per cent piping loss), and $E_e = 90$, the equivalent price of coal is $30.80 per ton.

The overall efficiency for electric space heaters and direct radiators is 100 per cent. The efficiencies used in the examples above for electric thermal storage, gas, and coal are representative, but these values vary with conditions and the design of the equipment. A representative overall efficiency for oil heating may be taken as 50 per cent, which assumes a seasonal heater efficiency of 60 per cent and a piping loss of about 17 per cent. Fuel oil used for heating runs around 143,000 Btu per gal.

DIRECT RADIATORS AND SPACE HEATERS. This class of equipment includes radiant heaters, convection heaters, unit heaters, and electric hot water and electric steam radiators using immersion-type heating units. Each has certain characteristics and certain advantages.

The first cost of this equipment is usually low. Another advantage is that it may be made portable so that it can be moved from place to place where needed. Small heaters may be connected directly to appliance outlets, but larger ones involve the installation and cost of electric wiring of the proper capacity. Care must be exercised to comply with the National Electrical Code of the National Board of Fire Underwriters which specifies (1) that portable electric heaters rated at 6 amp or 660 watts or less may be used on lighting branch circuits or on combination lighting and appliance branch circuits, (2) fixed or portable electric heaters of not over 1320 watts may be supplied by an ordinary appliance branch circuit, (3) up to 1650-watt equipment may be supplied by a medium-duty or heavy-duty appliance branch circuit. Equipment of higher rating be supplied by power circuits of proper capacity.

The radiant electric heaters deliver most of their heat by radiation. Reflectors direct the heat rays in the direction desired. This type of heat gives an immediate and pleasant sensation of warmth when the heat rays strike the body. These heaters have a high efficiency of utilization, but are not satisfactory for general heating, as radiant rays do not warm the air through which they pass. The location of the heaters is important, since they must be directed toward the objects to be heated. They must not face a window, because any radiation toward the window would pass through and be lost. Radiant heaters are built in portable types, and in wall types for both flush and recessed mounting.

Convection types of electric heaters are built in a number of different forms. They are designed to deliver heat by convection air currents set up by the tendency of the heated air to rise. The heating units are made of coils of resistance wire, metal ribbons,

grids, and other forms and shapes of resistance materials. The heating elements should have comparatively large area and operate at moderately low temperatures (temperatures below that which makes the element visibly red). The enclosures should give proper stack effect in order to draw cold air from the floor and direct it into the room.

Electric unit heaters are similar to steam or hot water unit heaters in that they combine the heater with a fan or blower. They are made in portable, wall mounting, and built-in types in sizes ranging from 1 to 50 kw or larger if required. This type is the best form of electric heater for many applications. The warm air can be directed toward the floor or in any direction desired. The positive air circulation insured by the fan reduces stratification of the air and high ceiling temperatures. Because of the forced air circulation the area of the heating elements may be made smaller and the weight and cost of the resistors reduced to a minimum. None of the electrical energy taken by the fan is lost, since it is all delivered to the air as heat and kinetic energy. Portable units of artistic design, free from radio interference, are available for home use. Larger units may be used for industrial plants, power plants, substations, and for temporary use during construction work, repairs, etc.

Electric hot water and electric steam radiators have been built by one or two manufacturers. In this design immersion heating elements are used to heat water in the lower part of the radiator. The temperature of the water or the pressure is controlled by a thermostat or by a thermostat and a pressure switch which automatically cuts the heating element off and on as required. The cost of this type of equipment is high because of the relatively expensive construction required. The only advantage is a larger area of low-temperature direct radiating surface. The water and steam do not increase the efficiency of the heating elements.

ON-PEAK CENTRAL HEATING. Central warm air electric heating systems which take electrical energy coincidental with the heating requirements have the heating installed in a plenum chamber connected to the warm air duct system. A fan or blower circulates the air past the heating units and through the ducts. In this system it is important to provide the proper value of air in order to give the desired air temperature. Since the electric heating units supply a constant amount of heat, the air temperature equals the kilowatts times 3415 divided by the product of the cubic feet of air times the density times the specific heat of the air at the warm air temperature.

Central electric hot water or steam systems differ only in the use of electric boilers to supply the hot water or steam. For hot water, small, heavily insulated boilers with immersion-type heating units are used. The same type of boiler can also be used for steam. In Canada and Europe for larger installations an electrode boiler is used for generating steam. In this type the resistance of the water between the electrodes is used to generate the heat. The amount of heat is controlled automatically by raising and lowering the movable electrodes. To cut off the electric current the upper electrodes are raised above the water into the steam space. There is no danger of injury to the boiler due to low water as the current is automatically interrupted when the water becomes low. Only alternating current may be used in this type of boiler because of electrolysis. Some trouble has been experienced with electrode boilers on account of the formation of gases which have caused explosions in the boiler. Manufacturers have been able to overcome this trouble by proper design.

On-peak central electric heating is usually expensive with the rates available for this purpose. However, in certain localities rates of 2 cents per kw and below are available. In mild climates this method of heating may be permissible with these rates. It may be found economical to shut down the main boilers and use electrical boilers as an auxiliary source of supply at the beginning and end of the heating season.

OFF-PEAK THERMAL STORAGE SYSTEMS. A number of different types of thermal storage electric heaters have been developed in Europe and America. One arrangement is a modification of the old stone warming block. An appropriate size metal container, insulated on the outside, is filled with solid material in which electric heating elements are imbedded. The storage material is heated during the off-peak hours. The stored heat is utilized as needed by passing air through ducts in the heated material. These units are placed inside the room where heat is desired; they are semi-portable, although somewhat heavy to move.

Another type of thermal storage system uses a well-insulated central storage tank filled with water. The tank is heated to a temperature of 250 to 300 deg fahr during hours designated by the power company. Charging periods ranging from 12 to 20 hr are used, depending upon the characteristics of the utility's load curve. During the charging period the storage tank absorbs enough heat to meet the demands of the 24-hr period. Heat is delivered under thermostatic control as required to maintain the desired room temperature. Several different arrangements of the equipment have been developed,

depending on the type of radiation used to distribute the heat to the building. These may be listed as follows:

(a) Hot water with thermal circulation through standard hot water radiation.

(b) Hot water with pump circulation through standard hot water radiation.

(c) Warm air utilizing a central blast heater.

(d) Warm air utilizing unit heaters.

(e) Warm air utilizing stored heat in connection with waste heat from synchronous condensers or other equipment.

The essential parts of all the different arrangements of this equipment are:

(a) Heavily insulated storage tank.

(b) Immersion-type heating elements.

(c) Automatic charging control.

(d) Radiation system.

(e) Automatic room temperature control system.

The heating element capacity required is found by dividing the Btu requirement of the building for the coldest day by the product of 3415 times the number of hours that charging is permitted. This should be increased to allow for a small tank loss, irregularity in heating element rating, and low voltage, remembering that the heating element capacity varies as the square of the voltage. Where the voltage remains practically constant 10 to 15 per cent is sufficient allowance to cover these factors.

The size of the storage tank depends upon the following factors:

(a) The number and allocation of the hours during which heat must be supplied from storage.

(b) The temperature range over which the storage water is worked.

(c) The Btu requirements of the building during the time that heat is supplied from storage.

Various charging schedules have been used, depending on the times that peak loads occur on the electric system. A number of schedules have permitted charging of the tank at any time except for 4 to 7 hr during the evening lighting peak. In other places, charging has been prohibited for an additional 2 hr in the morning to care for a morning peak. Charging schedules as short as 12 hr have been used successfully by power companies having a large industrial load and a high load factor. Where the hours of heating from storage are broken up the tank will recover or partly recover its charge between discharge periods so that the size of the tank is materially reduced.

It has been found that with the use of standard hot water radiation a storage temperature of 250 deg fahr giving a pressure of 15 lb per sq in. may be used, and that the water may be returned from the radiation at a temperature of about 150 deg fahr, giving a working range of 100 deg fahr. The temperature of the water to the radiators is varied by means of a mixing valve controlled automatically by the heat requirements of the building. The return water stratifies at the bottom of the storage tank so that the hot water above remains at almost a constant temperature until the whole tank is nearly discharged. For warm air systems the storage tank is used to supply steam and vapor to unit heaters or blast heaters, in which case it is found that a working range of 300 to 200 deg fahr can be obtained. The working temperature range can be increased by increasing the amount of radiation used, which lowers the return water temperature, or by raising the storage tank temperature to the limit that the pressure will permit. Where the heating from storage is confined to a single period, the capacity of the storage tank in gallons equals the maximum Btu storage required divided by the product obtained by multiplying the working temperature range of the storage water by the weight of a gallon of water at the fully charged temperature by the storage tank efficiency allowing for a small tank radiation loss. Where the hours of heating from storage are broken or short, and the heat required is low, the storage tank acts like a storage battery floating on the line. The effect is to smooth out the charging curve and reduce the connected load to an average value over the whole charging period. Since in this case the heat stored in the tank fluctuates up and down, it is necessary to calculate the input and output over the whole 24-hr cycle in order to determine the maximum Btu storage at any one time and the size of the tank.

The space required for the tank and the auxiliary equipment is usually less than that allowed for the coal bin with a coal heater. In new buildings where openings to the basement permit, factory-fabricated tanks are used; otherwise, the tanks are electrically welded in the basement. As yet there has been no definite specifications of storage tanks with reference to the boiler code, but this has been under consideration by the Hartford Boiler Code Committee. The present tendency is to limit electrical welding to tanks of not over 4 $^1/_2$-ft diameter and not over 15 lb per sq in. gage pressure (250 deg fahr), and use a single V butt weld on both circumferential and longitudinal seams, all edges being bevel planed. Tanks of larger diameter and higher pressures are usually riveted in accord-

ance with the American Society of Mechanical Engineers' code for unfired pressure vessels. All tanks should be provided with manholes for inspection.

Standard immersion-type heating elements, usually 5 or 10 kw in size, are inserted directly into the tank in a group assembly in a nozzle welded into the tank, or they are installed singly through smaller nozzles. The tank construction, piping, and the connections to the tank are made quite simple, so that the whole unit lends itself to efficient insulation. The automatic charging control is obtained by the use of a synchronous time switch and a suitable tank thermostatic switch which opens and closes the main magnetic contractors supplying current to the heating elements.

A great deal of improvement in storage heating equipment would result if a better and lower-cost storage medium could be found. An ideal storage liquid would be one that is dense, with a high specific heat, that could be raised to a high temperature without developing pressure. It should be non-inflammable and should not break down with continued heating. A chlorinated diphenyl product which has been developed by the Swan Research Corporation for heat-transfer purposes shows promise for heat-storage applications.

Over a period of a number of years a few off-heat thermal storage installations have been made in Europe. Between 1928 and 1932 approximately 75 installations were made in the United States. These were installed in residences, office buildings, substations, and other buildings. The connected load for these installations varied from 30 to about 300 kw. The first cost of this equipment varies greatly, but in general it is only slightly higher than that of modern gas and oil heating equipment with fully automatic control. If put on a regular production basis, the cost could be made as low as that of other types of equipment.

HEATING BY THE USE OF A REVERSED REFRIGERATION CYCLE. Considerable discussion and experimental investigation have been made of the application of Lord Kelvin's "warming engine" or "heat pump" as a practical means of heating. The method utilizes a reversed refrigeration cycle whereby heat is absorbed from a cold body and rejected to a warm body. Electrical energy for heating is first used to operate a refrigeration unit. The evaporator absorbs heat from a water supply or from the outside air. The condenser is arranged so that the heat given off is delivered to the building for heating purposes. Thus the total heat obtained is equal to the amount expended in driving the refrigeration apparatus plus that extracted by the evaporator. The maximum heat obtainable is that given by the ideal case where the system operates on the reversed Carnot cycle. In refrigeration this ideal cycle of operation gives a coefficient of performance (ratio of heat extracted to heat expended) equal to $T_2/(T_1 - T_2)$, where T_1 equals the absolute temperature of the hot body and T_2 equals the absolute temperature of the cold body. In applying reversed refrigeration to heating, the ratio of the heat delivered to that expended, which may be called the coefficient of heating, is equal to $[T_2/(T_1 - T_2)] + 1$, since the heat rejected equals the heat extracted plus the heat expended. When this expression is simplified it gives a heating coefficient equal to $T_1/(T_1 - T_2)$, which is the maximum theoretical value.

In commercial refrigeration practice about 1 hp-hr of work (2546 Btu) is required per ton of refrigeration (12,000 Btu), and 14,546 Btu are rejected through the condenser. If the same operating temperatures could be used, this would mean a coefficient of heating of approximately 5.7, that is 5.7 Btu of heating would be obtained for each Btu expended to drive the refrigerator. However, the working temperatures would usually be less favorable and the actual performance would fall a great deal short of that indicated by the ideal coefficient of heating.

A number of installations have been made over the past few years, mainly in warmer climates. In most of them outside air has been used for the evaporator. A disadvantage in this case is that the lowest heating capacity occurs in the coldest weather when maximum capacity is needed. Recently an installation has been made in a two-story office building of the American Gas and Electric Co. of Salem, N. J., which uses water at about 57 deg fahr pumped from a well instead of outside air for the evaporator. The reversed refrigeration is obtained by four four-cylinder compressors each driven by a 5-hp three-phase motor. The refrigerant is raised to a temperature of 135 deg fahr in the condenser. Air, which is circulated over the condenser, is heated to 115 deg fahr, humidified, filtered, and carried to various parts of the building by the duct distribution system. In severe weather it has been found that an electrical input of approximately 20 kw for the driving motors produces a heat delivery to the building equal to about 70 kw or about 240,000 Btu per hr. A novel feature of the equipment is that by opening and closing several valves the evaporator is changed into a condenser and the condenser into an evaporator, so that the equipment cools rather than heats. Two units of the four have a capacity of approximately 100,000 Btu per hr when operating on the cooling cycle.

Heating equipment based on the heat pump principle is theoretically and practically sound. The principal drawbacks are (*a*) the high cost of the equipment; (*b*) the objection to installation of rotating machinery, which decreases safety and simplicity of operation; (*c*) increased cost of supervision and maintenance. These objections have been greatly decreased by the development of improved equipment for cooling and air conditioning. Where a part of the cost of equipment can be charged to cooling and air conditioning it will be found that in many cases the use of the heat pump for heating can be justified. New developments in this type of equipment seem sure to widen the field of application.

PANEL HEATING AND FLOOR WARMING. A new type of heating has been developed in Europe, principally in England, which is known as panel heating and floor warming. Only a few installations have been made in America, one of which is in the British Embassy in Washington.

Low-temperature hot water (130 deg fahr or below) is circulated through pipes imbedded in the ceilings, walls, or the floors of the building; or electricity is utilized by imbedding heating elements instead of pipes. The heating is accomplished mainly by a low-temperature radiant heat. The piping or heating elements are distributed over a large area so that radiant heat impinges on the body from all directions to give the warmth required. That the heating is principally by radiant heat will be clear in the case where the heating elements are confined to the ceiling area. Here a thin blanket of heated air lies against the ceiling, and the only heating below is by radiation. It is claimed that comfort is obtained with room temperatures from 10 to 15 deg fahr below that required with other systems, and that healthfulness is increased. With lower average room temperature the heat losses from the building are proportionately reduced, resulting in economy in operating costs. It should be possible to apply this method of heating in connection with the heat pump, using the equipment for cooling in summer as well as heating in winter.

24. AIR CONDITIONING

AIR CONDITIONING. In the last few years great advances have been made in new methods and new equipment for increasing comfort and healthfulness in homes and public buildings. The engineer, who was once called upon for the design and installation of equipment for heating and the supply of a sufficient quantity of fresh air, must now concern himself also with the quality of the air, and in many cases he must provide cooling in summer as well as heating in winter. Many new factors affecting comfort are now better understood. The temperature, humidity, purity, odor, and movement of the air are all controlled in the modern air-conditioning system.

The temperature sensation of the human body depends not only upon the dry-bulb temperature of the surrounding air, but also upon the humidity (or wet-bulb temperature) and upon air movement. This is because of the effect of humidity and air movement on the dissipation of body heat. The best representation of the combined effects of these factors on comfort is given by comfort charts which have been prepared by the American Society of Heating and Ventilating Engineers and published in the Guide and *Transactions* of this society.

In addition to heating, winter air conditioning consists in humidification as well as filtering and sometimes deodorizing. As stated under the section on heating, relative humidities between 40 per cent and 60 per cent are desirable in winter, which usually requires the addition of moisture to the air. In warm air systems this is accomplished by passing the air through sprays or over surfaces wetted by a continuous flow of water. With steam or hot water heating systems the moisture is added by unit humidifiers, or, where more adequate equipment cannot be supplied, some improvement in the humidity may be obtained by the use of radiator pans.

Complete summer air conditioning consists of lowering the effective temperature by cooling, and by reducing the relative humidity, filtering, deodorizing, and circulating the conditioned air. However, the temperature must not be lowered too far below that of the outside, and the relative humidity must be kept low enough (50 per cent or less) to induce a rate of evaporation which will keep the clothing and skin dry. The desirable indoor conditions corresponding to given outdoor temperatures are given in Table X.

There are four methods of lowering the effective temperature as follows:

1. Lowering the dry-bulb temperature by the removal of sensible heat without change of the dew-point temperature through the use of some form of refrigeration.
2. Dehumidifying and lowering of the dew-point temperature without change of the dry-bulb temperature.
3. Evaporative cooling, which lowers the dry-bulb temperature by evaporation of moisture without addition or subtraction of heat.
4. Increasing the air movement, which results in higher evaporation from the skin.

The best method to employ depends on climatic and other conditions. The first two methods are as a general rule the most satisfactory. Satisfactory cooling by evaporation and air motion alone requires a low relative humidity, and thus depends upon climatic conditions beyond the engineer's control. With the use of refrigeration both the removal of sensible heat and dehumidification are usually obtained.

Dehumidification may be obtained in three ways:

1. Cooling the air below the dew-point by refrigeration.
2. Extraction by moisture absorption.
3. Extraction by adsorption.

Dehumidification by the first method is based on the fact that as the temperature of air is lowered the amount of moisture that can be retained is decreased. For a given moisture content (grains per cubic foot) there is a definite temperature called the dew-point temperature for which the air will be saturated (100 per cent humidity). If the temperature is dropped below the dew-point temperature, a definite amount of moisture condenses out until the amount left corresponds to that required to give 100 per cent relative humidity for the new temperature. Thus by lowering the temperature to the proper point, reducing the moisture, and reheating, air at any desired temperature and relative humidity can be obtained.

In extraction by absorption, moisture is taken up by a substance which undergoes a change in chemical or physical structure; in extraction by adsorption a substance is used which does not undergo a change in chemical or physical structure. Absorbers include sulfuric acid, calcium chloride, etc.; adsorbers include such substances as silica gel and lamisilite. Silica gel (SiO_2) will absorb about 25 per cent of its own weight of moisture from air passed over it. After the substance has become saturated, the moisture is driven off by heat without change in structure.

COOLING LOAD. The cooling load for any building or part of a building is made up of four parts:

1. Heat transmitted through walls, roofs, and glass with allowances for sun-exposed surfaces and heat capacities.
2. Solar radiation transmitted through windows.
3. Heat and moisture from infiltration of outside air.
4. Heat and moisture from occupants, lights, and machinery.

The transmission for surfaces not exposed to the sun is calculated by using the heat-transmission constants and temperature difference in the same manner as for a heat loss. For surfaces exposed to the sun, the heat transmission will be greater. This may be taken into account by raising the outside air temperature by a certain amount depending on the color and character of the surface and the angle of incidence of the sun as indicated in Table XI.

One hundred per cent of the sun's solar energy is transmitted into a building through an opening which is without glass or other obstructions, and which is normal to the sun's rays. The solar radiation for the different hours of the day and for different angles of incidence of the sun's rays is given in Table XII. The note given below indicates the percentage of the solar heat which should be taken if the opening is obstructed by glass, shades, etc.

AIR-CONDITIONING EQUIPMENT. Air-conditioning systems may be classified as central air-conditioning systems and unit air conditioners. The first is used principally in theaters, restaurants, office buildings, and manufacturing plants. The second is used in homes, separate offices, and for other applications where the requirements are not too great. In the central system the fans, dehumidifiers, filters, etc., as well as the refrigeration units are located in a suitable apparatus room from which the conditioned air is supplied through distribution and return ducts. In the unit conditioner the fans, dehumidifier, cooling unit, controls, and other equipment form a complete unit which is assembled at the factory. The various parts of the central air-conditioning system are usually purchased and assembled by the heating and air-conditioning contractor, who guarantees the performance of the complete equipment. The manufacturer is responsible for performance of the unit conditioner.

Unit air conditioners usually have capacities below 30,000 Btu per hr for cooling and below 60,000 Btu per hr for heating. The different types may be classified as all-year units, summer units, and winter units. The all-year units perform all the functions of air conditioning: namely cooling, dehumidification, air circulation, air cleaning, heating, and humidification, with suitable controls. Summer units provide cooling, dehumidification, air cleaning, and air circulation. The winter units provide heating, humidification, air cleaning, and air circulation. Units which do not provide simultaneous control of at least four of the recognized functions of an air conditioner are usually more properly classified as unit heaters, unit ventilators, window-type ventilators, humidifiers, coolers, or dehumidifiers.

Cooling is usually accomplished by the use of mechanical refrigerators using low-temperature evaporation of a liquid to absorb the heat. Air or water is cooled by passing it over the evaporator. The evaporated liquid is compressed and then cooled by passing air or water over the condensing coils. The apparatus exclusive of the evaporator or cooling coils is called the condensing unit and is usually located in the basement or some other convenient place. There are two methods of applying the refrigeration. In the direct-expansion system the evaporator is located in the assembly in the room where cooling is required. The expansion valve is next to the evaporator so that the refrigerant lines need not be insulated. In the indirect-expansion system the evaporator is located near the condensing unit and water surrounding the evaporator is chilled and circulated through coils in the room unit. For the direct system, odorless, non-toxic refrigerant fluids, such as methyl chloride or freon, are used. Indirect systems may use ammonia, sulfur dioxide, and carbon dioxide, since the piping is not located within the conditioned areas. Compressors of multicylinder or rotary designs driven by electric motors are used. These units have very carefully designed shaft seals which minimize leaks. The whole assembly is carefully mounted to give quiet operation. Much of the success in building cooling is due to the improvements in the design, efficiency, and dependability of this part of the equipment.

Filters or air cleaners are of three general types: (1) dry air filters, (2) air washers, (3) viscous filters.

Dry air filters are made up in the form of a screen from felt, cloth, wool, and other materials. The filter material may be designed for vacuum cleaning or for replacement as it becomes filled with dust and dirt. The air-washer type cleans the air by passing it through sprays. In the viscous type of filter the dust and dirt are trapped by allowing them to impinge on viscous sheets placed in the air stream from the fan. A great many different arrangements and substances are used. The viscous screens are washed and the viscous coating renewed either manually or automatically.

The automatic controls for heating and air conditioning constitute a very important part of the equipment. This apparatus is made up of devices sensitive to temperature, devices sensitive to relative humidity, and devices sensitive to pressure, together with relays, contactors, valves, dampers, time switches, and other auxiliary equipment. Compressed air and electricity are both used for operating the auxiliary equipment. The temperature-sensitive devices are used to actuate electric or pneumatic switches and are called thermostats. Depending on their intended use, thermostats may be classified as room, duct, pipe, or tank thermostats, etc. They may also be classified according to the type of temperature-sensitive element, as straight, curved, or spiral bimetal strip thermostats, diaphragm type filled with either volatile liquid or a gas, and liquid bulb type. Devices which are sensitive to changes in humidity are used to cause the humidifying apparatus to increase or decrease the supply of moisture. These devices, which are called humidistats, or hydrostats, operate on several different principles. One type which is directly sensitive to humidity uses wooden blocks, hair, fiber, paper, and membranes. A second type uses a wet-bulb and a dry-bulb thermostat; still another type uses a photoelectric cell which is so arranged that it cuts off the moisture supply whenever moisture condenses on the windows.

Table I. Coefficients of Heat Transmission (*U*)* for Walls, Ceilings, Floors, Roofs and Glass

No.	Type	Description of Construction	Btu per Sq Ft per Hr per 1° F
1	Solid brick wall....	8-in., no interior finish	0.50
2	" " "	8-in., 1/2-in. plaster on brick	0.46
3	" " "	8-in., 3/4-in. plaster on metal lath—furred	0.30
4	" " "	8-in., 1/2-in. plaster on 1-in rigid insulation—furred	0.16
5	" " "	12-in., no interior finish	0.36
6	" " "	12-in., 1/2-in. plaster on brick	0.34
7	" " "	12-in., 3/4-in. plaster on metal lath—furred	0.25
8	" " "	12-in., 1/2-in. plaster on 1-in. rigid insulation—furred	0.14
9	Brick and tile wall	4-in. brick, 6-in. hollow tile, no interior finish	0.36
10	" " " "	4-in. brick, 6-in. hollow tile, 1/2-in plaster on tile	0.34
11	" " " "	4-in. brick, 6-in. hollow tile, 3/4-in. plaster on metal lath—furred	0.26
12	" " " "	4-in. brick, 6-in. hollow tile, 1/2-in. plaster on 1-in. rigid insulation—furred	0.14
13	" " "	4-in. brick, 8-in. hollow tile, no interior finish	0.35
14	" " " "	4-in. brick, 8-in. hollow tile, 1/2-in. plaster on tile	0.33
15	" " "	4-in. brick, 8-in. hollow tile, 3/4-in. plaster on metal lath—furred	0.25
16	" " " "	4-in. brick, 8-in. hollow tile, 1/2-in. plaster on 1-in. rigid insulation—furred	0.14
17	Frame wall.......	Wood siding, 1-in. sheathing, 2 × 4-studs, wood lath and plaster	0.24
18	" "	Wood siding, 1-in. sheathing, 2 × 4-studs, metal lath and plaster	0.25
19	" "	Wood siding, 1-in. sheathing, 2 × 4-studs, rock wool fill, metal lath and plaster	0.075
20	Stucco frame wall..	1-in. stucco, 2 × 4-studs, wood lath and plaster	0.46
21	" " " ..	1-in. stucco, 1-in. sheathing, 2 × 4-studs, wood lath and plaster	0.30
22	" " " ..	1-in. stucco, 2 × 4-studs, rock wool fill, metal lath and plaster	0.090
23	Brick veneer wall..	4-in. brick, 1-in. sheathing, 2 × 4-studs, wood lath and plaster	0.26
24	" " " ..	4-in. brick, 1-in. sheathing, 2 × 4-studs, rock wool fill, wood lath and plaster	0.076
25	Floor............	1-in. (25/32-in.) yellow pine, no ceiling	0.46
26	"	1-in. yellow pine, metal lath and plaster	0.31
27	"	13/16-in. maple on 1-in. yellow pine, no ceiling	0.35
28	"	Floor No. 27 with metal lath and plaster ceiling	0.23
29	"	Floor 27 with 1 1/2-in. corkboard and 1/2-in. plaster ceiling	0.11
30	"	4-in. bare concrete on ground	1.07
31	Roof and ceiling †..	Wood shingles on wood strip, no attic flooring, wood lath and plaster	0.40
32	" " " ..	Slate, tile, or asphalt shingles on wood strips, no attic flooring, wood lath and plaster	0.50
33	" " " ..	No. 31 with attic floored	0.28
34	" " " ..	No. 32 with attic floored	0.34
35	" " " ..	No. 31 with 4-in. rock wool fill in ceiling	0.061
36	" " " ..	No. 32 with 4-in. rock wool fill in ceiling	0.063
37	Glass (doors) ‡....	Windows and skylights, single glazing	1.13
38	" "	" " " double "	0.45

* Calculated from conductivities and conductances recommended by Am. Soc. Heat. & Vent. Eng., 1935 Guide. See Harding and Willard or the Guide for more complete lists.

† The coefficients given for roof and ceiling are the combined coefficients of the roof and ceiling expressed in Btu per hr per sq ft of ceiling area and apply only where the attic is unheated. The combined value is obtained by calculating the separate constants for the roof (U_r) and ceiling (U_c), and then combining these two values by the law of series thermal resistances by the formula

$$U = \frac{N\,U_r \times U_c}{N\,U_r + U_c}$$

where N equals the square feet of roof area per square foot of ceiling area. The values given in the table were calculated using a 1/3 roof pitch (making $N = 1.2$), but the coefficients are sufficiently accurate for pitches ranging from one-fourth to full pitch.

The combined coefficients for roof and ceiling should not be used except where the attic is tight and the infiltration is low. If there is much infiltration of cold air the attic temperature should be taken as equal to that of the outside and only the resistance of the ceiling should be considered.

‡ It is sufficiently accurate to use 1.13 as the coefficient for doors which have wood panels, making it possible to combine glass and door areas for convenience.

Table II. Conductivities (k) and Conductances (C) of Building Materials and Insulations *

Material	Description	Density, lb per cu ft	Mean Temperature, deg fahr	Conductivity (k) or Conductance (c)	Authority
Air spaces	Over 3/4 in., ordinary material	...	40	1.10 †	(a)
Ordinary surface	Still air (f_i)	...	...	1.65 †	(a)
" "	15 mph air (f_0)	...	...	6.00 †	(a)
Brick	Common	...	...	5.00	...
"	Face	...	...	9 20	...
Brickwork	Damp or wet	...	...	5.00	(b)
Concrete	Typical	...	...	12.00	...
Concrete blocks	Typical 8 in	...	...	1.00 †	...
" "	Typical 12 in	...	...	.80 †	...
Stone	Typical	...	...	12.50	...
Stucco	Typical	...	...	12.00	...
Tile	4 in. hollow clay	...	...	1.00 †	...
"	6 in. " "	...	...	0.64 †	...
"	8 in. " "	...	...	0.60 †	...
"	10 in. " "	...	...	0.58 †	...
"	12 in. " "	...	...	0.40 †	...
"	16 in. " "	...	...	0.31 †	...
Tile or tarrazzo	Typical flooring	...	...	12.00 †	...
Wood (across grain)	Maple or oak	...	...	1.15	...
" " "	Yellow pine or fir	...	...	.80	...
Building boards	Compressed asbestos and cement sheets	...	...	2.70	(c)
" "	Plaster board, 3/8 in. thick	...	...	3.73 †	...
" "	" " 1/2 " "	...	...	2.82 †	...
Building constructions	1-in. fir sheathing and building paper	...	30	0.71 †	(a)
" "	1-in. fir sheathing, building paper, and yellow pine lap siding	...	20	0.50 †	(a)
" "	1-in. fir sheathing, building paper, and stucco	...	...	0.82 †	(a)
" "	Maple flooring across grain	...	...	1.20	(d)
" "	Battleship linoleum (1/4 in.)	...	...	1.36 †	...
Plaster	Cement	...	...	8.00	...
"	Gypsum	...	...	3.30	...
"	Metal lath and plaster	...	...	4.40 †	...
"	Wood lath and plaster	...	...	2.50 †	(a)
Roofing	Asphalt composition or prepared	...	75	6.50 †	(d)
"	Built up—3/8 in. thick	...	...	3.53 †	...
"	Built up, bitumen and felt gravel or slag surfaced	...	...	1.33 †	(b)
	Plaster board, gypsum fiber, concrete and 3-ply roof covering	...	76	0.58 †	(a)
Shingles	Asbestos	65.00	75	6.00 †	(d)
"	Asphalt	70.00	75	6.50 †	(d)
"	Slate	201.00	...	10.37	(e)
"	Wood	...	...	1.28 †	...
Insulation, loose	Glass wool	1.50	75	0.27	(d)
" "	Granular silicate of lime and alumina	4.20	72	0.24	(d)
" "	Dry celllular gypsum	30.00	90	1.00	(c)
" "	" " "	18.00	90	0.59	(c)
" "	" " "	12.00	90	0.44	(c)
" "	Gypsum flaked dry and fluffy	24.00	75	0.48	(d)
Rock wool	Made from limestone	21.00	90	0.30	(c)
" "	" " "	10.00	90	0.27	(c)
" "	With binding agent	14.50	77	0.33	(c)
Sawdust	Ordinary	...	86	1.04	(c)
Shavings	"	...	86	0.71	(c)
Rigid insulation	Corkboard	...	...	0.30	...
" "	Fiber	...	...	0.33	...

* For additional data on conductivities see the Guide of the American Society of Heating and Ventilating Engineers or Data Book of the American Society of Refrigerating Engineers.

† These values are conductances for the actual thickness stated, and not for 1 in. of thickness.

(a) F. B. Rowley; (b) A. C. Willard, Lichty and, L. A. Harding; (c) U. S. Bureau of Standards; (d) J. C. Peebles; (e) Lees and Charlton.

Table III. Conductances of Air Spaces (*a*) at Various Mean Temperatures in Btu per Square Foot *

Mean Temperature, deg fahr	Conductances of Air Spaces for Various Widths in Inches				
	0.128	0.250	0.493	1.00	1.500
20	2.30	1.37	1.10	1.03	1.02
40	2.47	1.48	1.19	1.11	1.11
60	2.65	1.69	1.29	1.19	1.19
80	2.83	1.70	1.39	1.28	1.27
100	2.99	1.81	1.49	1.36	1.35

* Abstracted from thermal resistance of air spaces, by F. B. Rowley and H. B. Algren. *A.S.H.V.E. Trans.*, Vol. 35, 1929.

Table IV. Air Changes Taking Place under Average Conditions Exclusive of Air Provided for Ventilation

Kind of Room or Building	Number of Air Changes per Hour	Kind of Room or Building	Number of Air Changes per Hour
Room, 1 side exposed..............	1	Reception halls............	2
Room, 2 sides exposed..............	1 1/2	Living rooms...............	1 to 2
Room, 3 sides exposed..............	2	Dining rooms..............	1 to 2
Room, 4 sides exposed..............	2	Bathrooms................	2
Room with no windows or outside doors	1/2 to 3/4	Drug stores................	2 to 3
Entrance halls.....................	2 to 3	Churches, factories.........	1/2 to 3

Table V. Infiltration or Air Leakage Coefficients for Windows in Cubic Feet per Hour per Foot of Crackage

Part or Type of Window	Description	Cubic Feet of Leakage per Foot for Various Miles per Hour Wind Velocities					
		5 mph	10 mph	15 mph	20 mph	25 mph	30 mph
Around window frame	Uncalked...........	1.4	11.3	22.6	31.1	....	53.6
Double-hung wood sash windows with 1/16-in. cracks and 7/64-in. clearance *	No weather strips, unlocked............	39.3	84.9	124	161	....	233
	Weather stripped.....	3	11.7	22.9	34.9	....	59.6
Double-hung metal windows	No weather strips, locked............	20	45	70	96	125	154
	No weather strips, unlocked............	20	47	74	104	137	170
	Weather stripped, unlocked............	6	19	32	46	60	76
Rolled section steel sash windows	Industrial pivoted, 1/16-in. crack.......	52	108	176	244	304	372
	Architectural projected, 3/64-in. crack.......	20	52	88	116	152	208
	Residential casement, 1/32-in. crack.......	14	32	52	76	100	128
	Heavy casement, projected, 1/32-in. crack	8	24	38	54	72	96
Hollow metal, vertically pivoted.......	....	30	88	145	186	221	242

* Crack is the clearance of the window in the frame in the plane of the wall. Clearance is the amount the window may be pushed in or out in the direction perpendicular to the plane of the window. Most new sash are fitted with cracks of at least 1/16 in. which become greater as the sash dries out and shrinks. The values given in the table for double-hung wood sash apply for windows with cracks up to 1/4 in.

Table VI. Air Required for Ventilation *

Type of Building	Cu Ft of Air per Hour per Person	Type of Building	Cu Ft of Air per Hour per Person
Hospitals, ordinary	2500	Schools, theaters, prisons	1800
Surgical cases	3000	Factories and shops	2000
Contagious diseases	6000	Factories (unhealthy trades)	2500

* Especial care should be taken to comply with state and city laws in the case of schools and public buildings.

Table VII. Infiltration of Air Leakage Coefficients through Walls in Cubic Feet per Hour per Square Feet of Wall Area

Type	Description	Cubic Feet of Leakage per Square Foot for Various Miles per Hour Wind Velocities					
		5 mph	10 mph	15 mph	20 mph	25 mph	30 mph
13-in. brick wall	Plain	1.4	3.9	7.5	11.6	16.3	21.2
	Plastered	0.005	0.013	0.025	0.043	0.067	0.097
Frame wall	Lath and plaster	.03	.07	0.13	0.18	0.23	0.26

Table VIII. Inside Air Temperature Usually Specified

Type of Building	Temperature, deg. fahr	Type of Building	Temperature, deg fahr
Warm air baths	120	Schools	70
Steam baths	110	Public buildings	68–70
Hospitals, private rooms	70	Factories and machine shops	60–65
Hospitals, operating rooms	85	Stores	65–68
Hospital, bath rooms	70–80	Hotels	70
Paint shops	80	Theaters, seating space	68–72
Residences	70		

Table IX. Conversion Factors for Direct Cast-iron Radiation *

Steam Pressure		Temperature of Steam or Water, deg fahr	Temperature of Room					
Vacuum, in. Hg	Lb Absolute		80° F	75° F	70° F	65° F	60° F	50° F
22.4	3.71	150	0.388	0.424	0.462	0.499	0.538	0.617
20.3	4.75	160	0.462	0.499	0.538	0.577	0.617	0.697
17.8	5.99	170	0.538	0.577	0.617	0.657	0.697	0.782
14.7	7.51	180	0.617	0.657	0.697	0.740	0.782	0.868
10.9	9.33	190	0.697	0.740	0.782	0.825	0.868	0.955
6.5	11.52	200	0.782	0.825	0.868	0.911	0.955	1.045
Lb. Gage								
1	15.7	215	0.911	0.955	1.000	1.045	1.091	1.183
2	16.7	219	0.947	0.991	1.036	1.081	1.128	1.220
3	17.7	222	0.973	1.018	1.063	1.109	1.155	1.249
4	18.7	225	1.000	1.045	1.091	1.137	1.183	1.277
5	19.7	227	1.018	1.063	1.109	1.155	1.202	1.296

* These conversion factors multiplied by the heat emission of a radiator operating at 215 deg fahr in a room at 70 deg fahr give the heat emitted by the radiator operating under the conditions indicated.

Table X. Desirable Inside Air Conditions in Summer Corresponding to Outside Temperatures

Outside Temperature, deg fahr Dry Bulb	Inside Air with Dew-point Constant at 57° F		Effective Temperature *
	Dry Bulb	Wet Bulb	
95	80.0	65.0	73
90	78.0	64.5	72
85	76.5	64.0	71
80	75.0	63.5	70
75	73.5	63.0	69
70	72.0	62.5	68

* The effective temperature of air at any relative humidity for any given air motion is the temperature of saturated air which will induce the same sensation of warmth or cold with an air velocity of 15 to 25 ft per min.

Table XI. Degrees Fahrenheit to be Added to Outside Walls and Roofs to Allow for Solar Heat

Type of Surface	Black	Red Brick or Tile	Aluminum Paint
Roof horizontal	45	30	15
East or west wall	30	20	10
South wall	15	10	5

Table XII. Solar Intensity on Surfaces for Different Angles of Incidence of the Sun's Rays at Different Hours of the Day *

Time	Btu per Hour per Square Foot				
	Surface Normal to Sun	East Wall	South Wall	West Wall	Horizontal Surface
5 A.M.	0	0	0	0	0
6 "	60	55	0	0	20
7 "	175	160	0	0	70
8 "	250	210	25	0	150
9 "	285	185	65	0	210
10 "	300	135	90	0	250
11 "	310	70	110	0	280
12 N.	312	0	120	0	285
1 P.M.	310	0	110	70	280
2 "	300	0	90	135	250
3 "	285	0	65	180	210
4 "	250	0	25	210	150
5 "	175	0	0	165	70
6 "	60	0	0	55	20
7 "	0	0	0	0	0

* The heat transmission for angles of incidence which differ from those represented in the table can be calculated by multiplying the values given for a surface normal to the sun by the cosine of the actual angles of incidence. The amount of the heat transmitted through bare and shaded windows may be taken as 97 per cent for bare window glass, 28 per cent where shaded by canvas awning, 45 per cent where inside shade is fully drawn, and 68 per cent for shade one-half drawn.

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SECTION 16

INDUSTRIAL APPLICATIONS OF MOTORS

BY

FRANCIS A. WESTBROOK

ART.		PAGE
1.	Advantages of Electric Drive	02
2.	Group versus Individual Drive	02
3.	Selection of Motors	03
4.	Specific Applications of Motors	19
5.	Bibliography	34

INDUSTRIAL APPLICATIONS OF MOTORS

By Francis A. Westbrook

1. ADVANTAGES OF ELECTRIC DRIVE

The more important advantages of electric drive are the following:

The various machines can be placed in almost any desired position, and the use of portable tools is readily made possible.

Clear headroom is obtained by the elimination of belting. This gives better illumination and ventilation and permits overhead cranes to be used freely.

Power can readily be distributed from a central supply station to the different buildings, and changes or additions to the system can always be made without difficulty.

The electric system offers greater reliability than belt drive as a breakdown is usually confined to a single machine or group.

Meters of either the recording or indicating type can be installed easily where desired and the performance of every individual machine ascertained. It is then possible to maintain all the machinery in the best operating condition. Any excess power taken is at once readily detected and the defect can be promptly corrected. An accurate record can also be kept of the cost of power for the different operations.

2. GROUP VERSUS INDIVIDUAL DRIVE

There are two general systems of drive, namely, group and individual. When line shafting is to be changed to an electric drive, the most obvious and simple way of making the change is to split the shafting up into such sections as would be most convenient, and drive each of these by means of a comparatively large motor. On the other hand, it may frequently happen that it is necessary to operate only one machine of the entire group for a considerable time, as in overtime work, and to do this it would then be necessary to keep the motor and the line shafting of the whole group running. Since the efficiency of the motor at this light load would be small, and the friction losses of the entire drive would have to be supplied, it is evident that such a method of operation would result in a waste of power and be most inefficient.

On the other hand, if a large number of similar machines is involved which run continuously if at all, as with much textile machinery, they are often driven in comparatively small groups. If the plant is not on full production certain groups are shut down while others will be operated without any individual being idle in the working group. In this way the investment in motors is reduced but the advantages of simplified shafting, and protection against complete paralysis of operations due to the breakdown of one motor, are realized.

Modern installations, therefore, indicate a tendency toward the use of both the group and individual drive.

INFLUENCE OF CHARACTER OF LOAD. It is generally agreed that all large tools or other machinery should be equipped with individual motors, especially if their service is of an intermittent nature. With the group drive there are two distinct loads, the variable load of the machines and the friction of the line shafting and belting.

INFLUENCE OF SPEED. Wide ranges of control and the possible variations of speed are reasons which in many cases are sufficient in themselves for the selection of an individual drive. With group drive the methods of speed control for the individual machines are obviously more limited. It is then generally accomplished by shifting of belts on cone pulleys or by change of gears. Both of these methods, however, take a considerably longer time than the simple manipulation of the controller with the individual electric drive.

INFLUENCE OF RELATIVE COST. The question of whether group or individual drive is to be installed is a financial one, and each case must be properly analyzed. Individual drive necessarily means a larger investment, but in nearly all cases a much greater percentage income will be realized than if line-shaft drive were employed. With individual

drive the total horsepower rating of the motors installed in the plant will be considerably greater than with group drive, but the maximum power demand of the plant is approximately the same in either case. If power is purchased the price should be based on the actual maximum power demand and not, as sometimes is required, on the total connected horsepower capacity of the motors.

3. SELECTION OF MOTORS

GENERAL CONSIDERATIONS. The installation of a motor having too large a capacity should be avoided, unless an increase in the load is to be expected in the near future, because the efficiency of a motor is usually a maximum at its normal rated output. With a-c motors the effect of the power factor must furthermore be considered. This decreases rapidly below normal load, and on account of the penalty exacted by many public service companies for low power factor, and because of its bad effect on the regulation of the system, it should be kept as high as possible, which can only be done by operating the motors as nearly fully loaded as possible. Ordinarily, however, it is possible so to group the machinery that the motors may be operated near their rated output at all times. Too small a motor is naturally also very undesirable, as it would then in all probability be subject to overloads, which might result in overheating and a burn-out of the motor, causing a shutdown of the machinery which it drives. The operating conditions of the plant may furthermore be such, as for example in steel mills, that the failure of a single motor may necessitate the shutting down of the entire mill.

USE OF STANDARD MOTORS. The American Institute of Electrical Engineers has devised and promulgated standards on which the ratings of motors used by the most competent manufacturers are based. These are the best to use because of the standardized basis of comparison. In addition to this, many of the better motor manufacturers have been gradually changing the motor frames so as to conform with standardized motor mounting dimensions adopted under the guidance of the National Electrical Manufacturers Association (N.E.M.A). The interchangeability thus realized is obviously a great advantage because, among other things, the user is not committed to one make of motor. It is to be noted, further, that N.E.M.A. standards cover ratings, performance, and manufacture of motors in general.

TORQUE AND SPEED. Some machines will require motors with very heavy starting torque, although running under light load when up to speed; for others the requirements may be just the opposite. With a varying-speed motor the torque-speed characteristics should agree as nearly as possible with the load which the motor is to drive. For example, to start and accelerate the bridge of a crane requires a motor capable of developing a high starting torque, but after the bridge is accelerated comparatively little power is required to keep it in motion.

The condition of maximum torque must also be given due consideration. A motor driving a heavy punch may, in spite of the flywheel, develop insufficient torque to keep up the speed. As a rule, however, where the motor is large enough for starting and normal operation, but not large enough for the maximum overload required for perhaps only a second or two, the addition of a suitable flywheel will sometimes cut down the maximum torque required.

AVERAGE LOAD. The average load usually should be the determining factor, although under many conditions found in practice the starting load controls the size of the motor. The frequency of starting and the duration of the starting period, in addition to unusual starting current conditions, are necessarily of great importance in selecting the right motor. The required amount of power to drive a given machine can be ascertained from experience, but in any event it should be known as accurately as possible. See Section 14, Art 86, for current and voltage calculations, and tables of motor currents.

HIGH-SPEED MOTORS. It may be said that, in general, high-speed motors cost less and are more efficient from the operating standpoint than low-speed motors. It is, therefore, best to employ motors having a high normal speed unless it means the use of expensive mechanical transmission equipment where slow-moving machines are to be driven. Close speed regulation is required for machine tools and textile machinery, and under other conditions where the work must be kept running at substantially constant speed. On the other hand, where there are peak loads likely to damage the motor if it does not slow down automatically, or where the machine speed is partially governed by means of a flywheel, as with punch presses, wide speed regulation is necessary. For still other kinds of work calling for variable speed control by an operator it is necessary to use wound-rotor motors or d-c motors. In other words, the character of the drive must be fully taken into account in making the selection of a motor for any particular service.

GENERAL-PURPOSE MOTORS. Motors which are not designed for some special application are generally referred to as general-purpose motors. They are constant-speed motors but are so designed as to be able to carry reasonably varying loads above or below normal. They are made by most of the manufacturers in sizes up to 200 hp and with speeds of 450 rpm and higher. They meet most requirements, and their use is consequently so wide that the National Electrical Manufacturers Association has adopted a **service factor** for all general-purpose motors. This is applied to the normal horsepower rating and gives the overload at which a motor may be run with safety. It appears on the name plates of new motors and reads "Service Factor 1.15 at rated volts and cycles." In other words, if the rating of a motor is 50 hp the permissible loading will be $1.15 \times 50 = 57.5$ hp, provided the line voltage and frequency do not change. This means that if the load requirement should be 55 hp a 50-hp motor will be large enough.

INFLUENCE OF ENVIRONMENT. The physical environment in which the motor has to function should be given careful consideration. Many manufacturers make an effort to design motors for operation under almost every imaginable working condition such as heat, cold, moisture, dust, inflammable or corrosive gases, etc. The users should get the benefit of this by consulting with reliable makers, which costs nothing. The right motor may cost a little more, but it is poor economy to save in this way as it is almost sure to be at the sacrifice of quality and suitability and consequently at the expense of production.

There is no reason for installing d-c motors except where an adjustable-speed service is required, and in small plants where such a service is predominating the d-c system would naturally be the one to install. If a varying-speed feature is required only intermittently, the phase-wound induction motor may be used to advantage; but if the motor must run at reduced speed a considerable portion of the time, the varying-speed, brush-shifting motor should be given careful consideration. As a rule, a considerable part of industrial machinery will require a constant-speed service, for which the a-c motor is admirably adapted. The constant speed induction motor is adaptable to such a wide range of applications that this type covers the great majority of uses. It is, moreover, the least expensive, the most simple and rugged in design and the most economical from the standpoint of maintenance. Should direct current be required it can be obtained by installing a motor-generator set.

As a rule it may be said that the a-c system should be selected if possible, even for the isolated industrial plant, if for no other reason than that it permits of throwing over to a central-station service, in case it should be found that power could be more economically purchased from the central station than be generated on the premises, or in case emergency central-station current were needed.

Motors are of two classes: d-c and a-c according to the system from which they are to be operated. Their subclasses are given below.

Points to Consider in Selecting a D-c Motor

The d-c motors are subdivided into three types, namely, series, shunt, and compound motors.

SERIES MOTOR. This motor is used when a powerful starting torque and rapid acceleration are required, without an excessive instantaneous demand of energy. The torque is practically independent of the voltage and at low flux densities varies directly as the square of the current, but as the magnetization approaches saturation it becomes more nearly proportional to the first power of the current. The maximum torque exists at low speed, this being the most valuable feature of the series motor. Dangerously high speeds may be attained by the armature with very light loads, and for this reason series motors should be either geared or direct-connected to the load.

The Speed of a Series Motor on constant potential varies automatically with the load, increasing as the load decreases. The speed may be adjusted, however, if some means of varying the impressed voltage is provided. As the work required of a series motor is very often intermittent in character, the insertion of resistance in the armature circuit to reduce the speed is permissible from an economic standpoint in such cases. In others, where two or four motors are used, reduced voltage is most readily and economically obtained by connecting the motors in series or in series-parallel.

SHUNT MOTOR. This motor has good starting characteristics and a practically constant speed, varying only slightly with load changes. The speed can be adjusted, however, either by changing the emf impressed on the armature, or by changing the field flux.

Speed Adjustment by Armature-voltage Control, i.e., by changing the emf impressed on the armature, does not change the full-load torque which the motor is capable of

exerting, since the rated torque depends only upon field flux and rated armature current. These methods are therefore constant-torque methods and are properly adapted to loads in which the torque remains constant regardless of speed. The method most generally used for varying the impressed emf with a single-voltage system is by inserting resistance in series with the armature. The efficiency with this method is, of course, very low at slow speeds. The speed regulation with varying loads may also be very poor.

There are several systems of controlling motor speeds by applying different voltages, such as by the use of three-wire generators or two-wire generators with balancer sets or by the Ward-Leonard system. This latter system, which is the most practicable, consists of a constant-speed motor driving a generator which supplies current to the motor whose speed is to be adjusted. This arrangement is very satisfactory, but on account of the expense of providing three full-sized machines instead of one to perform the work, the cost may be prohibitive except with large individual motors, such as for hoists, etc.

Speed Adjustment by Shunt-field Control, i.e., by inserting resistance in the shunt-field circuit, is the simplest of all methods of speed variation, but with ordinary shunt motors, the range of speed variation by this means is small. Where a variation of more than from 20 to 30 per cent is desired, a motor of modified design and of a certain increased size is generally required because the field must be more powerful with respect to the armature than in standard single-speed motors. Varying or adjustable speed motors of the field-weakening type are not constant-torque, but constant-output motors, i.e., the torque falls proportionally as the speed increases.

As a rule, a speed variation up to 3 to 1 meets all requirements, and such motors can readily be obtained in commercial sizes. Should a greater speed variation be desired, say 4 to 1 or 5 to 1, it is possible to accomplish this by the commutating-pole shunt motor with field control only. A combined field and armature control, however, would be a better method.

COMPOUND MOTOR. This motor is provided with both a series and a shunt field. The two fields are usually connected so that they act in the same direction, in which case the motor is called a "cumulative" compound motor. "Differential" compound motors, with the two fields opposing, are sometimes employed for special services. The cumulative, or ordinary, compound motor combines the characteristics of the shunt and and series motors, having a speed not extremely variable under load changes, but developing a powerful starting torque and an increasing torque with decreasing load. Motors having a comparatively weak series field are employed extensively in shop practice where the motor may be required to start under heavy load but must maintain an approximately constant speed after starting, or when the load is removed. The heavily compounded motor is used where powerful starting torque and rapid acceleration are necessary, with a speed not varying too widely under load changes, such as for rolling mills, etc.

The speed control employed with compound motors may be any of the various methods explained in connection with the shunt motor. For certain service the control may be entirely rheostatic, the series winding being cut out after the motor has come up to speed.

Points to Consider in Selecting an A-c Motor

As a general thing, single-phase a-c motors are preferable in sizes of $^1/_2$ hp or less. They should also be used up to 5 hp in preference to polyphase motors when installed in places which are inaccessible or otherwise out of the way. This is because if a fuse blows on one phase of a polyphase motor, it has the effect of a heavy overload which causes excessive heating, and serious damage to the motor may result. If a fuse blows on a single-phase motor, the motor simply stops.

Except under these conditions two- or three-phase motors are preferable because of their simplicity, ruggedness, and fool-proof qualities.

SINGLE-PHASE INDUCTION MOTORS. These have their rotors wound for repulsion starting and provide for a starting torque of 2 to 4 times full-load torque. As the motor comes up to the speed at which it runs as an induction motor, a governor automatically short-circuits the rotor windings. The starting characteristics may be varied by shifting the brushes so that there will be an extra heavy torque when starting if the machine has a marked flywheel effect, or so that the greatest torque will come when the machine reaches full speed, as is required for such drives as blowers, centrifugal pumps, etc.

POLYPHASE MOTORS, SQUIRREL-CAGE TYPE. The following analysis of seven different types of squirrel-cage motors now on the market will form a basis of comparison for industrial applications. The type numbers refer to Table IV.

General-purpose Squirrel-cage Motors—with Normal Torque (Type 1) Standard squirrel-cage motors represent the simplest possible machines for converting electrical energy into mechanical work.

With rated voltage applied at the terminals, four-pole standard squirrel-cage motors develop a minimum starting torque of 150 per cent of full-load torque. (See Fig. 1 and Table 1.) The torque increases gradually until it reaches a maximum of from 250 to 300 per cent at about 75 per cent full-load speed. At no load, the speed is practically synchronous; as load is applied, speed falls off uniformly up to full load, where it is approximately 95 per cent of synchronous speed. Since these motors operate at nearly constant speed from no load to full load, they are often referred to as "constant-speed motors." These electrical characteristics qualify standard squirrel-cage motors for an almost unlimited variety of applications, as listed in Table III.

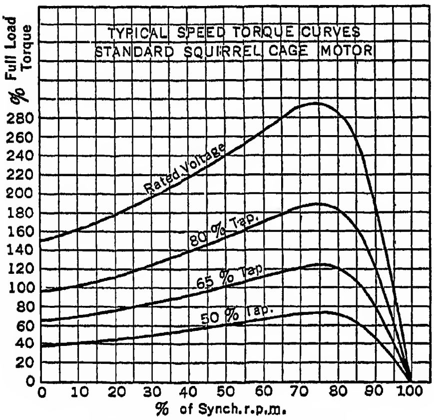

FIG. 1. Speed-torque Curves

Many power companies have established limitations on the amount of current motors may draw at starting, limitations usually based on N.E.L.A. 1923 (now E.E.I.) starting current recommendations, but in some cases higher or lower than those figures. The purpose is to reduce power fluctuations and prevent flickering of lights. The regulations apply to installations of a single polyphase motor of 1 hp and over; the larger the motor, the stricter the regulations. (See Table II.)

Table I. National Electric Manufacturers Association Minimum Starting Torques in Percentage of Full-load Torque, of Standard Squirrel-cage Motors, 25 to 60 Cycles, with Rated Voltage Applied, at the Instant of Starting from Rest

Poles	Torque	Poles	Torque	Poles	Torque	Poles	Torque
2	150	6	135	10	120	14	110
4	150	8	125	12	115	16	105

Where E.E.I. starting current recommendations have been accepted, or where even lower limits are the rule, standard squirrel-cage motors of 7 $^1/_2$ hp and larger cannot be thrown directly across the line. Their starting currents are higher than the E.E.I. recommendations shown in the table. Hence, current-reducing devices must be used to start these larger ratings when it is necessary to meet starting current limitations.

In localities where starting currents are limited, the application of standard squirrel-cage motors is limited to machines which do not require a heavy starting torque, such as fans and blowers, compressors, centrifugal pumps, line shafting, machine tools, etc. If the starting torque under reduced voltage is below that required, an across-the-line type motor should be used, as described below.

The data of Table II are from Appendix I, Revision of Motor Rules, N.E.L.A., April 16, 1923. The currents are the free rotor values, i.e., current values indicated by a well-damped ammeter located in the motor-circuit on the line side of the starter, if any, with the motor connected to its load and the starter in the position necessary to start the motor. These currents are three-fourths of the blocked rotor values and therefore will have to be increased one-third where the blocked rotor values govern.

Experience since 1923 has led to the conclusion that motor-starting current rules based not only on the capacity of the motor but also on that of the system, at a given point, would result in much more economical installations for consumers in the large majority of cases and would also prevent uneconomical demands for excessive starting currents at remote points. Where an installation consists of several motors, the diversity factor further extends the permissible starting current per motor. These considerations led the Electrical Apparatus Committee of N.E.L.A. in 1929 to withdraw their support from the 1923 Rules. Nevertheless, the Rules, although not followed rigorously, continue to be used as a general guide.

The initials E.E.I. will be used herein to designate the Edison Electric Institute or its predecessor, the N.E.L.A., and their respective committees. (See Edison Electric Institute Bulletin, July, 1934, page 251; July, 1935, page 271 and August, 1935, page 333.)

Tables of current at rated load data on overload protection and formulas for calculating wire sizes will be found in Section 14, Art. 86.

Low-torque across the Line Squirrel-cage Motors, 30 Hp and Smaller (Type 2). These motors are designed to develop starting torques approximately as given in Table I, and can be started directly across the line without exceeding E.E.I. starting-current recommendations shown below. No current-reducing starting devices are required; a simple hand-operated (manual) starting switch—or in case of remote-control, an across-the-line magnetic contactor operated by push-button or other pilot-circuit device—is all the starting equipment needed.

These motors are intended for installation where starting-current limitations require the use of compensators with type 1 motors, and where the reduction of voltage at starting lowers the starting torque of type 1 motors below that required.

These motors with across-the-line type starters cost considerably less than type 1 motors with compensators, as they require no compensators, are easier to install, and involve less maintenance. The cost of a 7 1/2-hp, 220-volt, 1800-rpm type 2 motor with push-button starter, for example, is 66 per cent of that of a type 1 motor of the same rating with compensator. That is not the only saving, however. The starting torque of a type 1 motor is greatly reduced by the compensator; the type 2 motor, on the other hand, starting at rated voltage, makes available the *full* inherent starting torque for " break-away " and acceleration. This feature makes it possible in many applications to use a type 2 motor of lower rating than would be necessary if a type 1 motor and compensator were used—reducing the initial investment still further.

Table II. E.E.I. (1923) Starting-current Recommendations—60 Cycles Only

Hp	Amperes per Phase (75 per cent of blocked rotor current)							
	220 volts		440 volts		550 volts		2200 volts	
	3 Ph.	2 Ph.	3 Ph.	2 Ph.	3 Ph.	2 Ph.	3 Ph.	2 Ph.
7 1/2	86	74	43	37	34	30	7 1/2	7 1/2
10	106	92	53	46	42	37	10	10
15	148	128	74	64	59	52	15	15
20	188	166	94	83	76	66	20	20
25	228	203	114	102	92	81	25	25
30	270	240	135	120	108	96	30	30
40	285	250	142	125	114	100	30	30
50	300	260	150	130	120	104	30	30
60	360	312	180	156	144	126	36	30
75	450	390	225	195	180	157.5	45	37.5
100	600	520	300	260	240	210	60	50

Low-torque across the Line Squirrel-cage Motors, 40 Hp, and Larger (Type 3). These large motors can be started directly across the line without exceeding E.E.I. starting-current recommendations. No current-reducing starting devices are required; a simple hand-operated (manual) starting switch—or in case of remote control, an across-the-line type magnetic motor switch operated by push button or other device—is all the starting equipment needed.

The starting torque of type 3 motors, is, however, somewhat lower than that of type 1 motors of 30 hp and below, the starting amperes per horsepower recommended by E.E.I. are high enough to permit E.E.I. starting torques. But above 30 hp the starting current allowed is so low that this starting torque cannot be developed. Designed to operate within E.E.I. limitations, type 3 motors develop a starting torque of about 80 to 100 per cent of full-load torque, and are therefore known as "low-torque " motors.

These low-torque motors are suitable for such applications as listed in Table III.

High-torque across the Line Double Squirrel-cage Motors, 30 Hp and Smaller (Type 4). Double squirrel-cage motors develop a starting torque of from 200 to 250 per cent of full-load torque, a starting torque from 1/3 to 2/3 higher than that of the standard squirrel-cage motors. Notwithstanding the high starting torque, the starting current is within E.E.I. recommendations, and the motors can therefore be thrown directly across the line without the use of current-reducing starting devices.

The absence of current-reducing starting devices and the high starting torque of these motors make them the preferred type for a wide variety of industrial applications. They are particularly suitable for such applications as listed in Table III, where heavy starting resistance must be overcome and starting current must be held down. As these motors can be started at rated voltage, they permit applying motors rated very closely to the actual horsepower requirements. Thus it is possible in applications requiring a high starting torque to install type 4 motors of considerably lower horsepower rating than

would be necessary if type 1 motors with compensators were installed, eliminating expensive overmotoring and assuring better power factor. The double squirrel-cage motor has what its name denotes: two separate squirrel cages, one of smaller dimensions inside the other, and both embedded in the same laminated iron field.

High Torque Double Squirrel-cage Motors, 40 Hp and Larger (Type 5). These motors are the same as type 4 motors, electrically and mechanically. They should, however, be treated as a separate group, as their starting currents exceed E.E.I. recommendations shown below—for which reason they involve the use of current-reducing starting devices wherever starting currents are regulated.

With full voltage applied, these motors develop the same high starting torques as the type 4 motors just described. When started on reduced voltage to meet E.E.I. starting-current limitations, their starting torques vary as the square of the voltage applied to the motor terminals. At 80 per cent rated voltage, the starting torque of a 4-pole type 5 motor is reduced to approximately 160 per cent (250×0.80^2—taking 250 per cent as the starting torque at rated voltage), which is naturally considerably higher than that of a type 1 motor started at the same reduced voltage. At 65 per cent rated voltage, the starting torque developed is approximately the same as that of the type 3 motors started on full voltage. See Figs. 2 and 3.

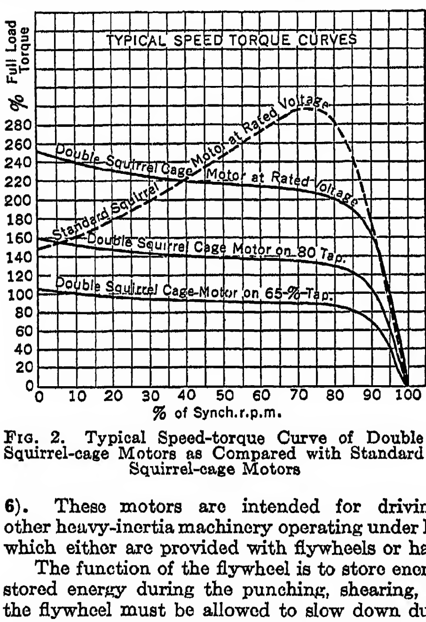

FIG. 2. Typical Speed-torque Curve of Double Squirrel-cage Motors as Compared with Standard Squirrel-cage Motors

The use of these motors is dependent upon starting-current regulations. Where full voltage can be applied, these motors are the logical selection for such applications as crushers, plunger-pumps, belt conveyors starting under load, large air compressors, and other machinery involving heavy inertia at starting. Where compensators are required and the starting torque obtained is insufficient to start a given load, the only alternative is the use of wound-rotor motors, such as the slip-ring and Fynn-Weichsel motors.

Ten Per Cent Slip Squirrel-cage Motors for Flywheel Applications (Type 6). These motors are intended for driving punch presses, shears, bulldozers, and other heavy-inertia machinery operating under heavy, fluctuating load conditions, machines which either are provided with flywheels or have flywheel effect.

The function of the flywheel is to store energy as the machine idles, and to deliver the stored energy during the punching, shearing, or other operation. In order to do this, the flywheel must be allowed to slow down during each operation, and must be quickly accelerated to full speed immediately after to prepare it for the next operation. The proper motor for such a service is one which has (1) high slip to allow the flywheel to release its stored energy, (2) high starting torque to overcome inertia of the heavy flywheel, (3) quick acceleration to bring the flywheel up to full speed at the start and after each operation.

These punch press motors have a slip high enough so that they slow down while the punch press releases stored-up energy. They develop a starting torque very much greater than that of standard squirrel-cage motors, accelerate quickly to full-load speed, and are designed for across-the-line starting. They are not suitable, however, for machinery not flywheel operated which requires motors of the standard, across-the-line, and double squirrel-cage types. On the other hand, these types are not suitable for flywheel machinery.

Standard squirrel-cage, double squirrel-cage and across-the-line motors are low-slip, constant-speed motors, inherently unable to slow down to let the flywheel do the work; they would attempt to keep the flywheel revolving at a *constant* speed, and would consequently create considerable line disturbance by drawing excessive current each time the machine is doing work. Furthermore, standard squirrel-cage and across-the-line motors have considerably lower starting torque than punch press motors, for which reason machines would have to be heavily overmotored to start the heavy flywheels.

High-torque Squirrel-cage Motors for Elevators, Small Hoists, etc. (Type 7). Elevator motors are intended for elevator service, or such similar applications as cranes and hoists requiring: (1) very high starting torque; (2) frequent starting, stopping, and reversing;

and (3) low starting current; and which do *not* require: (1) constant speed; (2) adjustable varying speed; or (3) long periods of operation.

A comparison between the type 7 elevator and standard squirrel-cage motors shows important differences in electrical characteristics, briefly stated as follows:

(*a*) Maximum torque of elevator motors occurs at the start rather than at $2/3$ full-load speed or thereabouts, as in standard squirrel-cage motors. This is necessary to overcome the inertia of machinery for which elevator motors are intended.

(*b*) Elevator motors have a slip of about 20 per cent as compared with 5 per cent or less for standard squirrel-cage motors. The high slip minimizes heavy jerking of the machinery, which would occur if low-slip motors were applied.

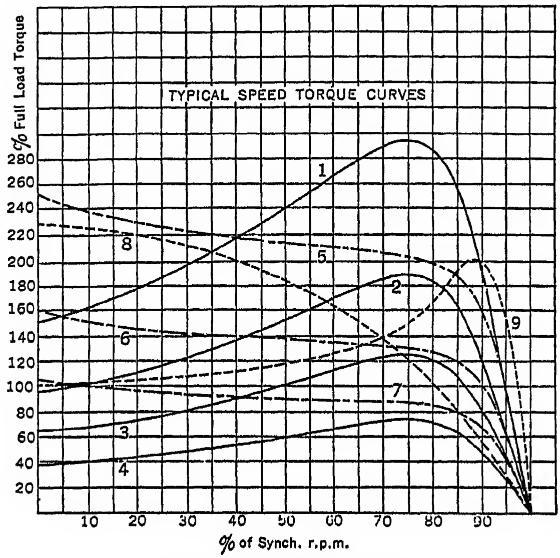

FIG. 3. Squirrel-cage Motors

Curve 1—Standard;	Rated Voltage	Curve 5—Double;	Rated Voltage
2— " ;	80% "	6— " ;	80% "
3— " ;	65% "	7— " ;	65% "
4— " ;	50% "	8—Elevator;	Rated "
		9—Low Torque;	" "

(*c*) Starting current drawn by elevator motors is much lower than that drawn by standard squirrel-cage motors, eliminating the need of current-reducing starting devices.

(*d*) Because of the high rotor resistance necessary to effect high starting torque, elevator motors are designed for intermittent loads only, and overheat when used for continuous service. They are intended for a maximum service of 30 minutes, over which period the temperature rise will not exceed 50 deg cent.

Multi-speed Squirrel-cage Motors. Both two- and four-speed squirrel-cage motors are now being used to a considerable extent and are available in the constant-torque, variable-torque, and constant-horsepower types. Multispeed, squirrel-cage motors are taking the place of the slip-ring variable-speed motor for many applications where only certain definite fixed speeds are required.

SLIP-RING INDUCTION MOTORS. The following discussion of the characteristics of slip-ring motors has been taken largely from bulletins of the Wagner Electric Corp. and tells why motors of this type are necessary for satisfactory service for certain industrial applications.

Mechanical and Electrical Characteristics. The outstanding difference between squirrel-cage and slip-ring motors is in the rotor winding. Since the rotor bars and end rings of squirrel-cage motors have fixed resistances, such characteristics as starting and pull-out torques, rate of acceleration, and full-load operating speed cannot be altered for a given squirrel-cage motor installation. In slip-ring motors, the ends of the windings are brought out to three collector rings, and the currents induced in the rotor are carried through the slip rings to external control equipment which inserts varying values of

Table III. Comparison between Seven Different Types of Squirrel-cage Motors with Average Characteristics and Typical Applications

Type	Standard Ratings (Single Speed)	Starting Current	Starting Torque	Approximate Full-load Slip	Mechanical Variations	Typical Applications
1	1/10 to 400 hp 3 or 2-phase 25 to 60 cycles 110 to 2200 volts	Above that permitted by most power companies for 10 hp and larger; hence, current-reducing starting devices ordinarily used.	Varies with the square of the voltage applied to the motor terminals: approximately 150% of full-load torque at rated voltage; 96% at 80% voltage; as low as 38% at 50% rated voltage.	4 to 5%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	Group or individual drives, in machine shops, on machine tools, fans and blowers, compressors, centrifugal pumps—on any application where normal torque motors are satisfactory.
2	7 1/2 to 30 hp 3 or 2-phase 60 cycles 110 to 2200 volts	Within E. E. I. starting-current recommendations, hence within most power companies' limitations, and therefore suitable for starting directly across the line.	Approximately same as type 1 motors. As these motors can be started across the line at *rated* voltage, they can start considerably more load than type 1 motors started under *reduced* voltage.	4 to 5%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	Same as standard squirrel-cage motors, type 1.
3	40 to 100 hp 3 or 2-phase 60 cycles 220 to 2200 volts	Within E. E. I. starting-current recommendations, hence within most power companies' limitations, and therefore suitable for starting directly across the line.	Centrifugal pump service does not require heavy starting torque. Therefore these motors have starting torque of 80 to 100% of full-load torque.	4 to 5%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	Centrifugal pumps, agitators, fans and blowers, generator sets, and other applications requiring low starting torque.
4	3 to 30 hp 3 or 2-phase 60 cycles 110 to 2200 volts	Within E. E. I. starting-current recommendations, hence within most power companies' limitations, and therefore suitable for starting directly across the line.	About 200 to 250% of full-load torque.	4 to 5%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	Crushers, plunger pumps, belt conveyors starting under load, large air compressors, large refrigerating machinery, mixers, and other applications requiring high starting torque.
5	40 to 100 hp 3 or 2-phase 60 cycles 220 to 2200 volts	These motors have starting currents above most power companies' limitations; hence, current-reducing starting devices ordinarily used.	Varies with the square of the voltage applied to the motor terminals: approximately 200 to 250% of full-load torque at rated voltage; 130 to 160% at 80% voltage; 85 to 106% at 65% rated voltage.	4 to 5%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	The logical motor to use for applications where the torque produced by type 1 motors is not sufficient. If the starting torque of type 5 motors under reduced voltage is too low for any application, slip-ring or Fynn-Weichsel motors should be used.
6	1 to 150 hp 3 or 2-phase 25 to 60 cycles 110 to 2200 volts	Low enough to start across the line.	About 350% of full-load torque.	10 to 13%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical	Punch presses, shears, bulldozers, metal-drawing operations, balers, and other machinery equipped with flywheels or having flywheel effect.
7	1 to 30 hp 3 or 2-phase 25 to 60 cycles 110 to 550 volts	Lower than any other type of squirrel-cage motor. No current-reducing starting devices used.	Approximately 250% of full-load torque.	15 to 17%	Sleeve or rolling bearings open or totally enclosed horizontal or vertical.	Elevators, cranes, hoists, dumbwaiters.

resistance in the rotor circuit, thus effecting (1) variation in starting torque and current, (2) smooth acceleration, and (3) variation in operating speed—depending, of course, entirely upon the resistors incorporated in the control equipment.

It also follows that, because the resistance of the rotor circuit is controlled externally and thus is subject to change at will, (1) a relatively large starting torque can be obtained with relatively low starting current, without affecting operating characteristics, and (2) the greater part of the heat developed during starting or reduced speed operation is dissipated in the external resistors rather than in the rotor.

These desirable characteristics of slip-ring motors not only assure more satisfactory service, but there are other advantages such as (*a*) prolonging the life of machinery and belts through smoother starting, (*b*) preventing overmotoring, and (*c*) reducing voltage fluctuations on distribution circuits.

Starting Heavy Loads. Slip-ring motors have the ability to start extremely heavy loads; hence they are suitable for (1) driving various types of machinery which require development of considerable starting torque to overcome friction, (2) accelerating extremely heavy loads which have flywheel effect or inertia, and (3) overcoming back-pressures set up by fluids and gases in the case of reciprocating pumps and compressors.

Double squirrel-cage motors are also applicable on many of the heavier machines involving the problems mentioned above. But if a considerable length of time is required to accelerate the load to full speed, double squirrel-cage rotors may burn out before full speed is attained—and for that reason slip-ring motors should be used instead. Frequent starting has the same effect of overheating double squirrel-cage motors—for which reason slip-ring motors should be used on machinery started frequently.

Variation in Operating Speed. Variations in operating speed are essential on many applications. It is often desirable to vary the operating speed of conveyors, compressors, pulverizers, stokers, and the like in order to meet varying production requirements. Slip-ring motors, because of their adjustable varying speeds, are ideally suited for such applications. However, if the torque required does not remain constant, the speed of the slip-ring motor will vary over wide limits—a characteristic constituting one of the serious objections to the use of slip-ring motors for obtaining reduced speeds. Another factor which must be taken into consideration when selecting slip-ring motors for reduced-speed operations is that of lowered motor efficiency.

When slip-ring motors are used for cranes, hoists, and elevators, machinery operated intermittently and not over long periods, poor speed regulation and loss in efficiency are of little consequence. However, if lowered speeds are required over longer periods, poor speed regulation and loss in efficiency may become prohibitive.

Smooth Starting. Through the use of external resistors in the slip-ring rotor windings, a wide variation in rotor resistance can be obtained with a resultant variation in acceleration characteristics. Thus a heavy load can be started as slowly as desired, without a jerk, and can be accelerated smoothly and uniformly to full speed. It is merely a matter of supplying the necessary auxiliary control equipment to insert sufficiently high resistance at start, and to reduce this resistance gradually as the motor picks up speed.

Low Starting Current. Many power companies have established limitations on the amount of current motors may draw at starting—limitations usually based on Edison Electric Institute starting-current recommendations, but in some cases higher or lower than those figures. The purpose is to reduce voltage fluctuations and prevent flickering of lights. Because of such limitations, the question of starting current is often the deciding factor in the choice of slip-ring motors instead of squirrel-cage motors.

Slip-ring motors with proper starting equipment develop a starting torque equal to 150 per cent of full-load torque with a starting current of approximately 150 per cent of full-load current—which compares very favorably with squirrel-cage motors, one type of which requires a starting current of as much as 600 per cent of full-load current to develop the same starting torque of 150 per cent.

Such a high starting current causes all but the smallest of squirrel-cage motors to exceed the E.E.I. recommendations, and such squirrel-cage motors must therefore be started with current-reducing devices to meet starting-current limitations established by the power companies. Any reduction, so obtained, in starting current of squirrel-cage motors involves a sacrifice in starting torque which frequently results in overmotoring in order to provide sufficient starting torque.

Types of Slip-ring Motors. Two types of slip-ring motors may be obtained, namely, type 1—continuous duty; type 2—intermittent duty. The two types are alike in practically every detail; the difference is in frame size for a given horsepower and rpm rating, the type 2 motors being built in smaller frames than the type 1. Type 2 is intended for elevator, crane, hoist, and like services requiring but short periods of motor operation, making a reduction in frame size permissible.

Type 1 can be subdivided into two classes: (1) constant speed; (2) adjustable varying speed.

In either case, the electrical differences are entirely in the control equipment, and principally the external resistor units. The motors themselves are the same. The starting devices used for both types of service are simple, but the control devices for adjustable varying speed are much larger in order to dissipate the greater amount of heat developed when the rotor resistances (determining operating speeds for a given torque) remain in the rotor circuit for a considerable period of time.

CONTROL EQUIPMENT FOR SLIP-RING MOTORS. Slip-ring motors have certain inherent speed-torque characteristics, which can be altered considerably by the secondary control equipment which introduces into the rotor circuit varying values of resistance. Since the control equipment has this ability to change the slip-ring motor characteristics, the following brief discussion should assist motor users to understand the various types of control apparatus needed with slip-ring motors.

Motor Starting Equipment. Standard manual starting equipment for slip-ring motors up to and including 20 hp consists of an enclosed primary magnetic contactor providing thermal overload and undervoltage protection, electrically interlocked with a secondary face plate starter with self-contained N.E.M.A.-15 starting duty resistors. For motors 25 hp and over, standard manual starting equipment consists of a primary magnetic contactor providing thermal overload and undervoltage protection, electrically interlocked with a non-reversing drum and externally mounted N.E.M.A.-35 starting duty resistors. For 2200-volt motors, an enclosed manually operated double-break, 3-pole, oil circuit breaker replaces the primary magnetic contactor employed on low-voltage circuits.

The electrical interlocking of the primary and secondary control circuits makes it always necessary to start the motor from the off position of the secondary controller, and assures that the motor is always started with the full resistor in the rotor circuit.

In the event that the starting duty requirements are quite severe, resistors built to N.E.M.A.-15 and N.E.M.A.-35 service classifications may not prove ample. In this case the proper service classification resistors listed in Table IV may be selected to correspond to the amount of starting torque required and the length of time the resistors will remain in the secondary circuit during the starting period. Cases may also arise where the starting service for motors 25 hp and under may require that a drum-type secondary starter be substituted for the standard face plate secondary with N.E.M.A.-15 resistors.

Should reverse rotation of the motor be required, this can be effected by means of a reversing drum controller in the secondary circuit similar to the non-reversing drum controller except that it is provided with a set of primary contacts which serve to reverse one phase of the primary or stator circuit.

Oftentimes it is desirable to start slip-ring motors entirely automatically. Standard non-reversing full magnetic remote-control slip-ring motor starters consist of a panel upon which is mounted a three-pole primary magnetic contactor with thermal overload relays, and secondary accelerating contactors governed by timing relays. The resistors (N.E.M.A.-36 for standard automatic starters) are mounted in the rear of the panel. A start and stop three-wire control push-button station completes the standard equipment.

Full magnetic slip-ring motor starters may also be entirely automatically operated from accessories such as float switches, thermostats, and pressure regulators.

Type 1 Motor Speed Regulating Equipment. Standard manual speed-regulating control equipment for slip-ring motors 15 hp and under consists of an enclosed primary magnetic contactor providing overload and undervoltage protection, electrically interlocked with a secondary face plate speed regulator with self-contained resistors designed for either fan or machine duty.

The interlocking of the primary and secondary control circuits makes it always necessary to start the unit from the off position of the controller, and assures that the motor is always started with full resistor in the rotor circuit of the motor.

Standard manual speed-regulating control equipment for slip-ring motors 20 hp and upwards consists of a primary magnetic contactor providing overload and undervoltage protection for low-voltage circuits. For 2200-volt circuits, an enclosed manually operated, double-break, 3-pole oil circuit breaker is used, electrically interlocked with a non-reversing drum controller and separately mounted fan-duty (N.E.M.A.-93), or machine-duty (N.E.M.A.-95) speed-regulating resistors.

A reversing drum controller can be supplied if reverse operation of the motor is required. Such a reversing drum has a set of primary contacts mounted in the drum which serves to reverse one phase of the primary (stator) circuit, thereby reversing the motor.

Upon the choice of the proper secondary resistors depends the success of any reduced or variable speed slip-ring motor installation since the speed and the torque are mutually

related. Great care should, therefore, be taken in determining the nature of the load as far as its torque requirements are concerned at reduced speeds.

Machine duty resistors (N.E.M.A.-95) are designed for constant-torque loads—that is, irrespective of speed, the *torque* required remains essentially the *same*. Machine-duty resistors are designed to reduce the motor speed 50 per cent with 80 per cent of normal current flowing. They are used with reciprocating pumps, positive pressure blowers, etc.

Fan-duty resistors (N.E.M.A.-93) are designed for loads where the torque decreases with the speed, and are designed to reduce the speed 50 per cent with approximately 40 per cent normal current flowing. They are used with ventilating fans, centrifugal pumps, etc. The characteristics of this type of load are such that the horsepower required to drive the load varies approximately as the cube of the speed.

Fan-duty and machine-duty resistors will perform satisfactorily only when employed with the type of load for which they are specifically designed.

MOTOR CONTROL EQUIPMENT. Since type 2 motors are intermittently rated motors used for crane, hoist, and elevator work (where the motor is under control of the operator, most, if not all of the time it is in operation), the standard manual control consists of a reversing drum controller with the proper separately mounted secondary resistors. A set of primary contacts in the drum serves to reverse one phase in the primary (stator) winding, thereby securing two directions of rotor rotation, or up and down travel of the driven load. Primary magnetic contactors are also employed with type 2 motor control in order to supply "limit protection" so as to insure that the load will not overtravel in the up and down, or back and forth directions.

Limit protection is provided by interlocking the primary magnetic contactor with limit or hatchway switches which open the pilot circuit of this primary magnetic contactor, thereby disconnecting the motor from the line. Overload relays may be included with this primary magnetic contactor to afford overload protection, but accurate overload protection is made difficult by the intermittent nature of the load.

In ordering control for type 2 motors, the service for which the apparatus is intended should be specified in all cases. If the resistors are to be designed for crane service, this should be stated; if for hoist service, full information should be given as to whether the resistors are to be used with a vertical or a slope hoist.

Brake Equipment. Type 2 motors for gear drive are generally furnished with a double extended shaft. One end of the shaft drives the load; the other is so arranged that a brake wheel can be mounted on it. A solenoid brake shoe of the spring-set or gravity-set type acts on the brake wheel. The solenoid coil of the brake is connected across the power supply, and as long as the motor is connected to the line the solenoid coil overcomes the tendency of the spring or gravity to set and seize the brake wheel.

Table IV. N.E.M.A. Resistor Service Classifications

Periodic Ratings

Approximate Percentage of Full-load Current on the First Point	15 Sec out of 4 Min	30 Sec out of 4 Min	45 Sec out of 4 Min	1 Min out of 4 Min	1 1/2 Min out of 4 Min	2 Min out of 4 Min	Continuous Duty
	Numbers						
25	11	31	41	51	61	71	91
50	12	32	42	52	62	72	92
75	13	33	43	53	63	73	93
100	14	34	44	54	64	74	94
150	15	35	45	55	65	75	95
200 or Over	16	36	46	56	66	76	96

NOTE: 15 sec out of 4 min means that the resistor will operate at its specified duty not more than a total of 15 sec during any 4 min period. Unless otherwise specified, Wagner furnishes a N.E.M.A.-35 starting duty resistor, which provides for 150 per cent torque on the first point of drum controllers, good for 30 sec starting duty. If the load is extremely hard to start and requires more that 30 sec to accelerate to full-load speed, a N.E.M.A.-55 (1 min duty out of every 4 min) or a N.E.M.A.-75 (2 min duty out of every 4 min) may be required. If the load is of such a nature that a 100 per cent starting torque is required, and if for the purpose of protecting the machinery from the shock of too rapid starting it is found necessary to consume 1 min in bringing the motor to full-load speed, a N.E.M.A.-54 starting duty resistor should be used as indicated in the above table. These same remarks apply to speed-regulating resistors as far as torque is concerned, as all speed-regulating resistors, whether for machine or fan duty, have a continuous duty rating. (See Continuous Duty column.)

The moment the motor is disconnected from the line the solenoid brake coil is de-energized and the brake sets, owing to the action of the spring, gravity, or both.

Type 2 motors for direct-connected drive are generally furnished with single shaft extension, the metal coupling between the motor and the driven load serving as the brake wheel, the rest of the solenoid brake mechanism being similar to that previously described.

SPLIT-PHASE MOTORS. This type of motor is going out of use because the excessive starting current causes flickering of lights fed from the same transformer. As a general thing, power companies are opposed to the use of these motors except in very small sizes ($^1/_4$ hp or less). Their application is practically entirely for domestic appliances and only very rarely industrial.

CONDENSER MOTORS. These are single-phase squirrel-cage motors with a condenser across one phase to correct power factor and provide starting torque for small sizes up to 5 hp, for appliances such as fans, unit heaters, domestic refrigerators, etc. The industrial uses, other than for unit heaters, have so far not been developed to any extent.

TOTALLY ENCLOSED AIR-JACKETED MOTORS. Several of the leading manufacturers are building motors which are totally enclosed and ventilated and fan cooled. They are designed for locations where dust, fumes, and moisture are present in sufficient quantity to clog, corrode, short-circuit, or wear out the open type or where inflammable dust or vapors are present in sufficient quantity to cause explosions or fires. In some cases enclosed motors should be used throughout the plant, and in others only in locations where conditions make them necessary. Obviously the use of such motors is more satisfactory in the long run than providing improvised unventilated boxes for their protection or placing them in a separate room or building.

Where Used. The following brief analysis gives an idea of the services to which motors of this type may be applied. It is worth bearing in mind that insurance rates in hazardous locations are lower where enclosed air-jacketed motors are used and that they are furthermore of advantage in that they act as a safeguard against loss of production, not covered by insurance, in atmospheres where there would be danger of fire or explosion with open-type motors, even when boxed or partitioned.

(*a*) *Abrasive Dust.* Any kind of dust of sufficient density to act as an abrasive—such as iron, glass, sand, rock, coke and coal—exerting wear on moving parts and windings, and shortening life of motor.	Steel mills and all other metal-working industries. Glass plants—especially plate glass works. Rock crushers. Foundries.
(*b*) *Conducting Dust.* Current-carrying substances, such as certain metals and their salts, which will short-circuit the motor windings.	Steel mills. Metal-working plants.
(*c*) *Explosive Dust.* Inflammable material present in sufficient quantity to ignite and explode—such as grain, feed, flour, coal, coke, lint, wood, fiber and starch.	Grain elevators. Flour and feed mills. Pulverized coal plants. Planing mills.
(*d*) *Corrosive Gases or Fumes.* Any alkaline or acid substance in gaseous state which will attack windings and corrode the motor—such as found in process industries, dipping and plating departments.	Gas plants. Chemical plants. Paint shops. Dipping and plating departments. Dye houses.
(*e*) *Steam and Moisture.* Free steam or any kind of vapor, splashes of water and other liquids, which will short-circuit or corrode the motor.	Laundries. Meat-packing plants. Paper and pulp industries. Glass factories. Any outdoor service.

DRIP-PROOF MOTORS. This type of motor has the upper halves of the heads closed so that materials, including liquids, cannot fall into it. They are used in locations where danger of this exists.

GEAR MOTORS. For many applications requiring reduction of speed, a type of motor with the reducing gears forming a part of it, has the advantage of compactness, quietness and low maintenance cost.

SYNCHRONOUS MOTOR. The speed of a synchronous motor is constant, being fixed by the number of poles and the frequency of the applied voltage. The single-phase

type is not self-starting, and the polyphase type has in itself a very low starting torque. They may, however, be made self-starting in the same manner as squirrel-cage induction motors, by the use of an amortisseur or cage winding, similar in construction to that used for induction motors. This is now the usual practice.

The speed-torque curve of a synchronous motor is similar to that of an induction motor except that the torque values are lower for a given resistance of rotor winding on account of the construction of the machine. The starting winding must be designed with both the load at start and the load at synchronous speed in mind, because too great a slip may cause the motor to shut down when the field is put on. It is seldom, however, that the same motor will be called upon to start a heavy load and at the same time synchronize a heavy load, as the load usually consists principally of either static friction, as in motor-generator sets, line shafting, etc., or it comes up with the speed as in a fan blower or centrifugal pump. The former case would be met by a high-resistance squirrel-cage winding, and the latter would require a low resistance.

FYNN-WEICHSEL MOTOR. The Fynn-Weichsel motor is designed to supply its own magnetizing current and supply it to the line for other motors so as to correct overall power factor. It starts as a slip-ring induction motor, and when reaching synchronous speed operates as a self-excited synchronous motor up to about 150 per cent of full-load torque, when it again resumes operation as an induction motor until the excessive overload has passed, when it automatically returns to synchronous operation. These characteristics make it widely applicable to industrial loads of considerable variety and irregularity.

Points to Consider in Selecting a Motor for a Given Application

1. Make allowance for manufacturing tolerances, difference in materials, etc., in the motor and the driven machine, bearing in mind that such differences exist in machines that are manufactured as exact duplicates.

2. Make certain that the normal rating of the motor is sufficient to handle the normal load properly. If the correct rating is selected for the normal load the reserve power will take care of all emergencies such as low voltage, tight bearings, congealed oil, line surges, etc.

3. Where a careful application test is desired on small machines it is preferable for the machine manufacturer to ship a complete machine to the motor manufacturer's plant where facilities for making tests of this nature are usually available.

4. Where a contract for a number of motors is involved and where it is impossible to send the machine, it is generally possible to have the motor manufacturer send an engineering representative to study conditions and make recommendations.

PULLEYS AND PINIONS. In connection with belted installations, select a pulley that is at least $1/4$ in. ($1/2$ in. preferable) wider than the belt. Belt speeds above 5000 ft per min should be avoided. Belt speeds of 3500 to 4500 ft per min are preferable. The following formula may be used for obtaining belt speeds where the diameter of the motor pulley and its speed in revolutions per minute are known.

$$\text{Belt speed in feet per minute} = .2618\, ds$$

where d equals the diameter of the pulley in inches and s the rated load speed of the motor in revolutions per minute.

Paper pulleys have a transmitting capacity approximately 65 per cent greater than that of iron pulleys and 30 per cent more than wood. For motors below 2 hp, and in case the belt contacts for both driving and driven pulleys is as much as approximately 170 deg of the pulley circumferences, the belt slippage is negligible, but where quantity is involved and it is important that the driven machine should run extremely close to a certain speed, it is well to put on a pulley of the calculated size and try it before deciding definitely as to correctness of size.

In order to calculate the proper pulley size use the following formula:

$$ds = DS$$

where ds represents diameter in inches and revolutions per minute respectively of motor pulley, and DS the diameter in inches and revolutions per minute of the driven pulley. For example, it is required to determine the diameter of a pulley for a 1725-rpm motor to drive an appliance having a 10-in. diameter pulley, and running 345 rpm. Substituting in the above formula:

$$1725\, d = 10 \times 345$$

$$d = 2 \text{ in.}$$

The direction of rotation should be such as to cause a pull on the bottom side of the belt.

POWER TRANSMITTED BY BELTS. The power a belt will transmit depends upon its width, its thickness, and the speed at which it travels, and the unit for measuring the capacity of belts is ordinarily taken as the speed in feet per minute required to transmit 1 hp with a belt 1 in. in width.

For single or light double belts, the value is 700 ft per min; for heavy double belts, 450 ft per min.

The formulas given below are intended to indicate the safe running load that can be transmitted. In selecting a belt, however, the buyer should be guided by the load required to start and the width of the pulley specified.

$$Hp = \frac{\text{Ft per min} \times W}{700} \text{ for single and light double belts}$$

$$Hp = \frac{\text{Ft per min} \times W}{450} \text{ for heavy double belts}$$

Hp represents horsepower and W represents belt width in inches.

GEAR DRIVE. In case gear drive is used, much care should be exercised when lining up the motor shaft pinion with the gear, so that no end thrust will occur on the motor bearings. Improper aligning causes excessive noise and undue wear. Where T and t represent the number of gear and pinion teeth respectively, and S and s speeds in revolutions per minute, of gear and pinion, respectively, one of these quantities can be determined if the other three are available by the use of the formula:

$$ts = TS$$

For example, it is required to determine the number of pinion teeth for a 1725-rpm motor necessary to drive a machine running 149 rpm and having 232 teeth in its gear. Substituting in the above formula:

$$t \times 1725 = 232 \times 149$$
$$t = 20$$

Tests

APPLICATION TEST DATA. It is impossible in the limited space available to outline the proper procedure in testing various kinds of appliances or machines. Each machine requires somewhat different treatment. However, certain general data can be supplied that will be of great assistance in determining the proper motor for the machine.

These data consist of the following:

1. Kind of machine to which the motor is to be applied.
2. Name of the manufacturer of the machine.
3. Serial number of the machine.
4. Type of drive to be used (whether belted or direct driven).
5. Size of driving and driven pulley, or gears (if the latter give the number of teeth in each).
6. Some idea as to the general location of the equipment after it is installed.
7. A blueprint or sketch showing just how the motor is to be mounted, and in what position.
8. The estimated horsepower required to drive the machine at normal load.
9. A general idea of the load characteristics. This includes information as to whether the motor will be called upon to carry normal operating load continuously or whether the service will be intermittent, and if the latter some idea of the frequency and duration of the load and rest periods.
10. The direction of rotation desired for the motor (whether clockwise or counter-clockwise when facing pulley end).
11. In case the machine is belt-driven, supply the distance between pulley centers.
12. Supply circular or catalog of the machine, if possible.

SPECIAL DATA. In some types or makes of machines there is some certain point in the cycle of operation where the starting conditions are more severe; therefore the starting period should be watched carefully to see that the motor will bring the machine up to speed under the very worst possible starting condition.

TESTS TO DETERMINE SIZE OF MOTOR. The test data given below are for the guidance of those users who desire to make tests or to obtain data that will enable the maker's engineers to give a definite motor recommendation.

One of the important phases of making the proper motor application is to secure wattage input tests under all conditions of load, especially during the starting and other periods of severe load.

There are many ways of making these tests, and the same results can be obtained by

various methods. The wiring diagrams illustrated here represent methods which have given good results.

It is of first importance to make sure that the various instruments used, which consist of a voltmeter, an ammeter, and one or more wattmeters, be properly connected into the supply line. Fig. 4 shows the correct way of connecting a voltmeter. Most voltmeters may be used on either a-c or d-c circuits, although there are some instruments made for either d-c or a-c use, but not for both. All modern voltmeters have separate binding posts marked 150 and 300 for use upon 110- or 220-volt circuits, respectively. The binding post marked ± is common to both voltages.

Fig. 4 represents the connection to be used for 110 volts, and by shifting the one voltmeter lead from the 150- to the 300-volt binding post it can be used to register on a 220-volt line. The main thing to remember in connecting a voltmeter is that the leads should be connected to opposite sides of the power supply line and as close to the motor terminals as is practicable. The voltmeter leads are generally clipped on the motor switch-blades by means of small clips made for that purpose.

Fig. 5 illustrates the proper ammeter connection. The ammeter is used to register the amount of current taken by the motor. Instead of being connected to opposite

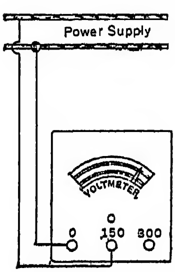

Fig. 4
Voltmeter

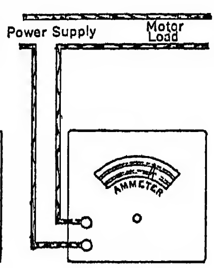

Fig. 5
Ammeter

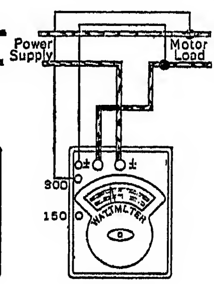

Fig. 6
Wattmeter

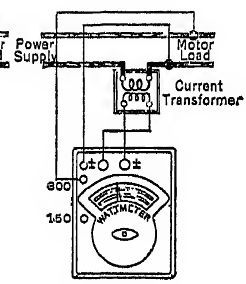

Fig. 7
Wattmeter

Wiring Diagrams for Test Instruments

sides of the supply line the ammeter leads are always connected in series with one side of the line, and when so connected the instrument registers the amount of current flowing through the line in which it is connected. On a two-wire circuit such as is used in direct current or single-phase alternating current it does not matter to which wire the ammeter is connected.

In connecting the ammeter for test, it is sometimes necessary to shift the ammeter leads at the ammeter binding posts until the current is flowing through the instrument in the proper direction. The ammeter coils inside the box are wound in such a way that when the current goes through the instrument in the wrong direction the needle will tend to go off the scale at the zero end and will not register. In case this happens when the ammeter is connected in the circuit just reverse the leads at the binding posts.

Fig. 6 represents a standard wattmeter connection. The wattmeter is a combination of voltmeter and ammeter, and this explains why the leads of the wattmeter are connected in exactly the same way as those of a standard voltmeter and ammeter. In connecting a wattmeter into the circuit it is well to remember that the voltage connection or the voltage leads should always be connected between the instrument and the motor load so that the true voltage across the terminals of the motor will be registered and the voltage drop across the wattmeter coils eliminated.

On motors of small size the average portable wattmeter can be used without any external equipment, but if the wattage input required by the motor is larger than that shown on the scale of the wattmeter it is then necessary to use a current transformer. The wattmeter connection with current transformer is shown in Fig. 7. Current transformers are made up in various ratios between the primary and secondary coils, and the wattmeter reading must always be multiplied by the correct ratio to obtain the true wattage input. For example, if the ratio of the primary to the secondary coil of the current transformer is 2 to 1, and the reading on the wattmeter scale is 100, the true reading would be 100 × 2, or 200 watts. If the current transformer ratio is 4 to 1 the true wattmeter reading would be 100 × 4 or 400 watts.

It will be noted that the wattmeter shown in Figs. 6 and 7 have three binding posts for the voltage connections, the same as a standard voltmeter, the maximum voltage

that can be used being 300 volts. In using a wattmeter of this type upon supply lines having voltages higher than 300 volts it will be necessary to use a potential transformer which would be connected into the voltage leads in exactly the same manner as the current transformer is connected into the current leads in Fig. 7. For example, let us suppose that a test is being made upon a 440-volt motor. This voltage is beyond the voltage range of the wattmeter shown. By the use of a potential transformer having a ratio of 2 to 1 the primary voltage of 440 is reduced to 220 volts, and the secondary leads of the potential transformer are then connected to the binding posts on the instrument marked respectively ± and 300.

When both potential and current transformers are used in the circuit the scale reading of the wattmeter must be multiplied by both the ratio of the current transformer and the ratio of the potential transformer. For example, assume the use of a potential and current transformer having a ratio of 2 to 1 and a wattmeter scale reading of 100. The actual number of watts consumed would then be the scale reading of the wattmeter multiplied by 2 (the ratio of the potential transformer) and further multiplied by 2 (the ratio of the primary and secondary clause of the current transformer) or $100 \times 2 \times 2 = 400$ watts.

The wiring diagrams shown in Figs. 8, 9, and 10 indicate a simple method of determining the wattage input to a motor driving an appliance or machine.

The heavy lines indicate that portion of the circuit represented by the permanent wiring; the light lines indicate the wire that has to be connected in to make the test.

The wiring diagram for d-c motors is shown in Fig. 8. It is not necessary to use a

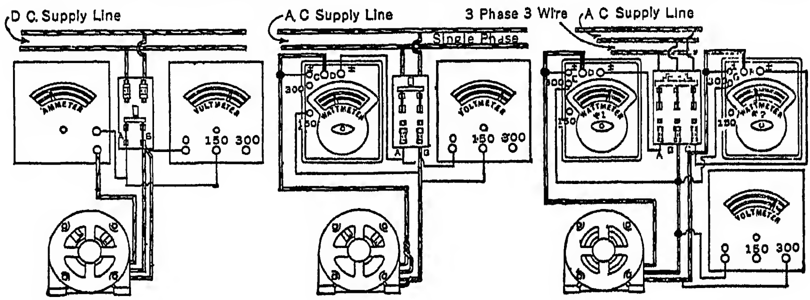

Fig. 8 Direct Current | Fig. 9 Single-phase | Fig. 10 Three-phase

Wiring Diagrams to Determine Power Input to Motors

wattmeter when measuring d-c power as the true wattage input to the motor will be the volts indicated by the voltmeter multiplied by the amperes indicated by the ammeter.

The wiring diagram shown in Fig. 9 is arranged for the test of a single-phase a-c motor operating on a 110-volt circuit. A 220-volt test can be made by changing the one voltage lead on voltmeter and wattmeter to the binding post marked 300.

Although the scale reading of the wattmeter is sufficient to give the wattage input to the motor, the voltmeter is also installed in a-c test circuits. Owing to the low-voltage conditions which exist in various localities and the effect which voltages below normal have upon the wattage input it is necessary to know the exact voltage condition prevailing at the time the wattmeter reading is taken. Note that the voltmeter is used also in the test of a three-phase a-c motor.

Fig. 10 indicates the correct wiring diagram to be used upon a three-phase squirrel-cage induction motor (three-wire supply circuit). It is possible in making a test of this kind to use a polyphase wattmeter in place of the two single-phase wattmeters shown.

The input to the motor in the circuit shown in Fig. 10 will be the arithmetical sum of the two wattmeter readings—providing the power factor of the motor is greater than 50 per cent. If the power factor is below 50 per cent the input delivered to the motor will be the arithmetical difference between the two wattmeter readings. For example, assume a scale reading on wattmeter 1 of 150, and on wattmeter 2 of 125. With the motor power factor above 50 per cent the total input to the motor would be 150 plus 125 or 275 watts. If the motor power factor is below 50 per cent the total input to the motor would be 150 less 125 or 25 watts.

Whether to add or subtract the readings can be determined as follows: If both of the scale pointers deflect toward the top of the scale add the reading; if one pointer deflects

toward the low part of the scale, reverse the current leads on the instrument and subtract the readings.

The diagram as shown is arranged for testing a 220-volt, three-phase motor. A 110-volt motor may be tested in the same manner by connecting the one voltage terminal of the two wattmeters and the voltmeter to the binding post marked 150.

Rating of Motors

See N.E.M.A. Motor and Generator Standards. These rules recommend that, with the exception of railway motors, all motor ratings shall be expressed in horsepower output.

It is also highly desirable that the motor ratings should closely conform to the actual service requirements, and for this reason the rules also recommend the following two kinds of ratings:

CONTINUOUS DUTY is a requirement of service which demands operation at substantially constant load for an unlimited period.

INTERMITTENT DUTY is a requirement of operation or service consisting of alternate periods of load and rest so apportioned and regulated that the temperature rise at no time exceeds that specified for the particular class of apparatus under consideration.

Speed Classification of Motors

Motors may, for industrial application, be classified with reference to their speed characteristics. The N.E.M.A. classification is as follows:

Constant-speed Motors, in which the speed is either constant or does not materially vary such as synchronous motors, induction motors with small slip, and ordinary d-c shunt-wound constant-voltage motors.

Multispeed Motor (change-speed motors), which can be operated at any one of several definite speeds, each being practically independent of the load; for example, a d-c motor with two armature windings, or an induction motor with a primary winding capable of various pole grouping.

Adjustable-speed Motors, in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load, such as shunt motors with resistance control, designed for a considerable range of speed adjustment.

Varying Speed Motors, in which the speed varies with the load, ordinarily decreasing when the load increases, such as series motors, compound-wound motors, and series-shunt motors.

Adjustable Varying Speed Motors, in which the speed can be varied over a considerable range but when once adjusted to a given load will vary in considerable degree with change in load.

4. SPECIFIC APPLICATIONS OF MOTORS

The preceding pages have covered motors in a general way in order that the reasons for the specific recommendations may be the more readily understood. The balance of this article is devoted to the discussion of specific applications of motors to a variety of those drives most commonly encountered in actual industrial practice. It is impossible within the limits of such a presentation as this to cover every conceivable case, but it is hoped that enough ground has been covered to indicate the type of motor which is applicable under ordinary circumstances.

Whenever there is any doubt as to what should be done the only reasonable procedure is to consult with one or more reliable motor manufacturers. In many instances certain builders of motors have specialized on particular types and their knowledge and experience should be utilized.

MACHINE TOOLS. The equipping of a machine tool with motor drive should be undertaken with the closest cooperation between the maker of the machine and the maker of the motor. If the user of the machine tool is changing from belt to individual drive he will in all probability ask some reliable motor manufacturer how his purpose can best be accomplished, or he may turn to the maker of his machine and ask the same question. Unless some past experience is being duplicated this is the most reasonable and safest course to pursue. If a machine is being purchased which is to be equipped for individual drive the buyer will naturally expect to be told what sort of motor he should provide; or if a special motor is required, one of proper design will be supplied or specified by the tool manufacturer.

There are some other equally important reasons for close cooperation between machine

tool makers and motor manufacturers. Among these is the desirability of so designing the machine tool that, wherever practicable, a standard motor and control apparatus can be used. Motor frames and control equipment are being standardized by most of the best-known makers, and it is generally possible to make provision in the design of the tool for mounting such standard equipment on or in it.

On the other hand, certain types of tools require special motors. Some tools are run at specified constant speeds; others require variable speeds; planers call for reversing motors; grinders must avoid vibrations; and the size of the machine, nature of the work, depth of cut, speed of feed, kind of metal, etc., are all factors on which the horsepower and design of the motor depend and whether intermittently rated or continuously rated motors must be supplied. Finally where machine tools are to be used for general manufacturing, rather than for one particular operation, the size of the motor will depend on the range of work which is likely to be performed. This applies especially to standard tools such as shapers, lathes, planers, etc., on which a wide variety of work may be done.

Shunt Motors are used in the following cases: when the work is of a fairly steady nature; when considerable range of adjustment of speed is required, as on lathes and boring mills, and on group and line-shaft drives, etc.

Compound-wound Motors are used where there are sudden calls for excessive power of short duration, as on belt-driven planers, punch pressers, etc. When a 50 per cent shunt, 50 per cent series, field is used, a motor is obtained which will develop nearly as much torque per ampere as a series motor with the advantage that light-load speed will not be greater than 150 per cent full-load speed.

Series Motors should be used where speed regulation is not essential and where excessive starting torque and slow starting speeds are required, as, for instance, in moving carriages of large lathes, in raising and lowering the cross rails of planers and boring mills, and for operating cranes.

When in doubt as to the choice of compound or series motors of small horsepower, the choice might be determined by the simplicity of control in favor of the series motor. Series motors, however, should never be used when the motor can run without load, as the speed would accelerate beyond the point of safety.

Induction Motor. The a-c motor of the squirrel-cage rotor type corresponds to the constant-speed, shunt, d-c motor, but with a high-resistance rotor it approaches more closely the characteristics of a compound motor. It is understood that the variable-speed machines, checked in Table V under the a-c squirrel-cage rotor column, have the necessary mechanical speed changes.

The slip-ring induction motor with external rotor resistance would be used for variable speed, but this must not be construed to mean that it corresponds to a d-c adjustable-speed motor, as it has the characteristics of a d-c shunt motor with armature control.

The self-contained, rotor-resistance type would be used for line-shaft drives, and for groups when of sufficient size.

Multi-speed A-c Motors are those giving a number of definite speeds, usually 600 and 1200 or 600, 900, 1200, and 1800 rpm, and are made for both constant horsepower and constant torque. These motors would be used where alternating current only was available, or direct current limited; and the speed range of the motor, together with one or two change gears, would give the required speeds.

Shaft Couplings. In connection with the selection of motors, standard shafts and shaft extensions should be chosen so that spare parts and interchanges may be made with the least cost and time.

Table of Motor Ratings. Table V will aid in the choice of the proper motor for machine tool application.

It must be kept in mind that various circumstances, such as size or roughness of work, flywheel capacity, etc., may call for radical departures in choice of motors, this list being compiled to meet average conditions.

Control Equipment. The choice of control, whether it be for old or new tools, in the majority of cases is fully as important as that of the motor. In selecting the control it is necessary to consider the nature of the work, the accessibility of the controller to the operator, the method of attaching it to the tool and in some cases its relative position to other tools; for instance, an open-type starting rheostat should not be exposed to danger of short circuit from flying chips.

When installing controllers, accessibility in case of accident should be kept in mind, even though of little importance as far as starting up is concerned. The starting apparatus should be placed where the motor or some of the moving parts can be seen by the operator. On individual motor-driven tools, where the motor is started and stopped many times a day or where the starting conditions are of a severe nature, or where tools are edged along, drum-type controllers with extra heavy starting resistance should be used. For

Table V. Motors for Machine Tools

Tool	D-c			A-c (See Footnotes)		
	Shunt	Comp., %	Series	*	†	‡
Bolt cutter	√			*		
Bolt and rivet header		20		*	†	
Bulldozers		20		*	†	
Boring machines	√			*		
Boring mills	√			*		
Raising cross rails on boring mills and planers		20-50	√		†	
Boring bars	√			*		
Bending machines		20-50		*	†	
Bending rolls		50				‡
Corrugating rolls		20–50		*	†	
Centering machines	√			*		
Chucking machines	√			*		
Boring, milling and drilling machines	√			*		
Drill, radial	√			*		
Drill press	√			*		
Grinder—tool, etc	√			*		
Grinder—castings	√			*		
Gear cutters	√			*		
Hammers—drop		20			†	
Keyseater—milling—broach	√			*		
Keyseater—reciprocating	√	20		*		
Lathes	√			*		§
Lathe carriages			√		†	
Milling machines	√			*		
Heavy slab milling	√			*		
Pipe cutters	√			*		
Punch presses ¶		20		*	†	
Planers—belt driven		20		*		
Planers—reversing motor	√			*		
Planers—rotary	√			*		
Saw—small circular	√			*		
Saw—cold bar and I beam		20		*		
Saw—hot		20		*		
Screw machine	√			*		
Shapers	√	20		*		
Shears		20		*	†	
Shears—rotary type	√			*		
Swaging	√	20		*	†	
Tappers	√			*		
Tumbling barrels or mills—individual		20		*		
Tumbling barrels or mills—group	√			*		

* Squirrel-cage rotor.
† Squirrel-cage rotor—high starting torque.
‡ Slip-ring induction motor with external rotor resistance.
§ Might be used for tire lathes as it allows slowing down when cutting hard spots.
¶ Small punch presses running at high speed can be driven by shunt motors.

adjustable speed motors, using the drum-type control, the field control should be through fingers making contact on segments of the controller drum and not by sliding contacts on a dial. Motors above 40 or 50 hp under these severe conditions are best operated by a master controller which operates contactors for cutting out steps of starting resistance, and if adjustable speed, the field control should be taken care of by fingers making contact on segments of the drum. This class of starting apparatus will stand any quantity of abuse and, by the addition of a simple current limit relay device, becomes practically a fool-proof protection for the motor. There are cases where it might be advantageous to use master controllers and contactors even with smaller motors. The controlling apparatus as well as the motor in the case of individual drives should be attached directly to the tool when possible. This arrangement allows moving the tool by simply disconnecting the leads and connecting them in the new position. In portable tools this, of course, is an absolute necessity.

Upon the convenient arrangement of the control depends, to a considerable degree, the output of the tool. The importance of the arrangement from the standpoint of the operator cannot be ignored, since the output of a tool will be materially increased when an operator can start and stop the tool and obtain at all times maximum cutting speeds by simply turning a handle. The controller must be placed in a safe position and should

be accessible for repairs, which very often means that some arrangement is necessary to bring the operating handle within easy access of the operator.

The convenience of control, which bears directly on production is ignored in the majority of tools where the control is of the greatest importance. A familiar illustration of the convenience of control is the arrangement so commonly seen on lathes, whereby the operating handle travels with the tool carriage and allows the operator at all times a complete control of his tool.

Application of Reversing Motors. One of the most interesting motor applications is the use of reversing motors for machine tools. The large increase in production due to this form of drive on planers is generally appreciated, but the application of the reversing motor drive in its various forms (which is almost unlimited) is not so well understood. This reversing drive is applicable not only to planers, new and old, but to screw, worm, and rack-driven slotters, keyseaters, turret lathes, wire and tube drawing machinery, grinder tables, lapping machines and boring mills, when machining projections which are short in comparison with the total travel of the mill or when machining surfaces where projections prevent a complete revolution.

The motors recommended for this service are of the commutating-pole type with a speed range usually of from 300 to 1200 rpm for the small sizes and 250 to 1000 rpm for the larger sizes. Other speeds can be obtained when required. The speeds given allow the motor, in the majority of cases, to be coupled direct to the driving shaft of the machine.

The requirements for reversing planer motors are ruggedness to withstand the strain of heavy service due to the reversing action and long armatures of small diameter to reduce the flywheel effect and facilitate reversal. Carefully made and insulated armature coils to withstand the mechanical and electrical strains of reversing, the windings designed to provide satisfactory commutation under quick reversing and to carry heavy currents at the time of reversal under heavy overload when extra deep cuts are being made, are further requirements for these motors.

Steel motor frames of extra-heavy mechanical strength to take care of the heavy stresses, and substantial feet, accurately machined for the direct connection of the motors, are also necessary. Because of the length and small diameter of the armature the frame is longer and lower than with standard general-purpose motors. Armature shafts are made of high-carbon steel of exceptional strength on account of the strains of reversing. Commutators should be liberal in size with deep wearing surfaces and have extra-thick mica insulation. Field coils of bobbin construction are used by many of the best manufacturers in order that they may be securely held in place under the vibration of the machine.

The control must automatically and very accurately control the operation of the planer at a number of different speeds. The cutting and return speeds must be separately adjustable—the former from 25 to 60 ft per min and the latter 50 to 100 ft per min, and in some cases up to 150 ft per min. The control equipment is the most vital part of this type of reversing motor drive. It is usually placed in a steel, dust-proof panel which may be mounted either on the machine or on the floor as is more convenient. It has two rheostat handles, one for controlling the cutting speed and the other for the return speed. The reversing contactors within the panel are generally carbon to copper to avoid the fusing of metal to metal. Some controllers are designed for reversing by "plugging" because this acts more quickly than dynamic braking. On the other hand, plugging takes more current than dynamic braking. The user will have to decide whether economy of time or economy of power is the more profitable. There are usually pendent push-button control stations with push buttons for "inching" the planer table while work is being set up, starting the machine and stopping it quickly by means of having the motor act as a self-excited generator and thus accomplishing dynamic braking.

The advantages of the reversing motor are (1) increased production due to the availability of the most economical cutting and return speeds, (2) decrease in cost of maintaining belts which wear quickly under the conditions of this service, (3) efficient setting up of jobs because of the easy control, or "inching," possible with direct motor drive and elimination of slipping of belts with their resultant losses.

Cross-rail and Tool Traverse Motors. Electrically driven cross-rail and tool traverse equipment is supplied by some of the motor manufacturers for mounting on boring mills, planers, and other machine tools where a rapid tool reverse is economical. They are the means, in many instances, of saving much time. The equipment consists of intermittently rated standard motors, either squirrel-cage or d-c motors, with heavily compounded windings to give high starting torque. Small enclosed drum controllers are used whereby the operator can easily start, stop, or reverse the motor.

Grinding Machine Motors. Grinding machine motors, where internal or external surface grinders for producing work of a high degree of accuracy are concerned, must meet special requirements to avoid vibrations. This is because vibrations set up by the

motor are communicated to the mirror finishes of the surfaces. For this reason a uniform air gap between rotor and stator is essential, for the variations in magnetic pull, where the gap is even only extremely slightly irregular, results in vibrations. Consequently it is preferable to have the bores of the motor stator ground. Ball bearings are necessary in order to keep the rotor accurately centered at all times, so as to reduce friction. For these reasons it probably will generally be best to let the machine tool builder supply the motor which he can have built according to the specific requirements of his machine.

PRINTING PRESSES. Owing to the great variety of work performed, printing machinery as a rule requires a certain degree of speed variation in order that, with a given equipment and operating force, the maximum of high-grade production may be turned out.

Individual drive is particularly applicable to printing establishments, as in this class of service the ratio is small between the power required to drive the machines running idle and when performing actual productive work. With group drive the ratio between average and connected load is high, owing to the large and constant character of the friction losses. The economy in power consumption is, therefore, in favor of separately driven units, which may be shut down when not operating productively.

Many motor manufacturers have specialized on printing press motors, and it is strongly recommended that prospective buyers consult with such manufacturers in order that they may secure motors which are adapted to their particular requirements. It is frequently practicable to get motors which may be mounted on a given machine with minimum waste of floor space or other inconvenience.

Job Presses. Job presses are the smallest type and require motors of $1/4$ to $1\ 1/2$ hp in either a-c or d-c types. The following table taken from one of the bulletins of the Cline Electric Manufacturing Co. gives an idea of the different sizes of motors applicable to different-sized presses.

Table VI

Hp	Maximum Rpm	Suitable for Press Size		
		Hand Feed	With Peerless or Miller Feeder	With Brandtjen and Kluge Feeder
1/4	1800	8 × 12	......	.....
1/4	1200	8 × 12	......	.....
1/3	1800	10 × 15	......	.....
1/3	1200	10 × 15	......	.....
1/2	1800	12 × 18	8 × 12	.....
1/2	1200	12 × 18	8 × 12	.....
3/4	1800	Craftsman	10 × 15 & 12 × 18	10 × 15
3/4	1200	Craftsman	10 × 15 & 12 × 18	10 × 15
1	1800	14 1/2 × 22	Craftsman	12 × 18
1	1200	14 1/2 × 22	Craftsman	12 × 18
1 1/2	1800	.....	......	Craftsman
1 1/2	1200	.....	......	Craftsman

High-speed Presses. For most of the high-speed automatic presses single-phase, polyphase, or d-c motors may be used. With d-c motors speed control is secured by shunt field or armature control. Dynamic braking stops the machine quickly and is also used with the a-c motors. With single-phase motors the speeds are obtained by varying the voltage. Motors of this type giving 12 speeds are on the market, and two- or three-phase slip-ring motors with 18 speeds.

Offset presses, small rotary presses, and cylinder presses where the service is alternating current require motors providing a strong starting torque and low starting current because of the heavy inertia and friction they must overcome in starting up the presses. They must do this with minimum loss of time and without causing line disturbances. The motors must be of the slip-ring type on account of the speed-control features. When d-c service is available shunt-wound motors are used with shunt field control. Special attention should be given to securing sparkless commutation in order to avoid unnecessary fire hazard.

Unit-type Presses. On unit-type presses it is generally desirable to have an individual motor for each printing unit and folder because of the comparative simplicity and lower cost of the drives and the more satisfactory speed control of each motor. When the speed has been set for each unit the whole group of motors driving the press is controlled by means of a single controller so that all start simultaneously and with equal pull on each web.

ELEVATORS. Elevators may be divided in two general classes, freight and passenger. The former may be divided into (1) slow-speed elevators (75 ft per min or less), (2) infrequent-duty elevators, and (3) high-speed (250 ft per min or less) material-handling elevators provided with accurate means for landing so as to accommodate wheel vehicles. The latter may be divided into local service (450 ft per min and less), which requires rapid rates of acceleration and accurate landing; and high-speed express (600 ft per min and more) which requires smooth acceleration and efficient landing. These express elevators sometimes must be operated as local elevators for a few floors at the top of the building and are often of the two-speed type.

Power Required. The horsepower of an elevator motor depends on the load to be raised, the speed of travel, and the elevator efficiency, that is, the combined efficiency of the motor, the gear, and the drive. It is customary to counterbalance the weight of the cage, and if it is under or overbalanced this amount must be added to or deducted from the live load to get the actual load.

$$\text{Horsepower} = \frac{(\text{Unbalanced load in lb}) \times (\text{Speed of elevator in ft per min})}{33{,}000 \times (\text{Efficiency})}$$

An efficiency of 0.50 is generally used in problems of this kind.

As the load is intermittent the motors are generally rated on the basis of 55 deg cent rise, with full load for one-half hour. The starting torque should be from 2 to 2 1/2 times full-load torque.

D-c and A-c Systems are both in general use for elevator service. On a-c systems the induction motor, either of the squirrel-cage or wound-rotor type, is used. These motors may be either single-speed or multispeed, depending upon the kind of service.

Single-speed, Slip-ring Elevator Motors are applicable to passenger and freight elevators where the speeds, not exceeding 200 ft per min, are within the limits of satisfactory mechanical braking. By inserting resistance in series with the rotor windings it is possible to obtain a starting torque of 2 1/2 times full-load torque with low starting current. With this type of motor, also, the heat generated is dissipated from the external resistance and the regulation is good. This type is particularly well adapted to the operation of heavy elevators calling for motors larger than 25 hp.

Two-speed Squirrel-cage Motors have given good results in elevator operation because the elevator can be started on the slow-speed winding. As the motor accelerates the resistance in series with this winding automatically cuts out until it is running at full voltage. It is then switched on to the high-speed winding with its set of resistors in series which are switched out as the elevator reaches full speed. All this switching takes place automatically after the operator moves the switch handle from "off" to "full speed." There is an intermediate position for the switch handle used when the car is to travel only short distances and which permits use of only the low-speed winding. When stopping the elevator the slow-speed winding has a braking action. For speeds over 250 ft per min reactor coils placed in the line circuit smooth out sudden changes of current and provide for correspondingly smooth changes in the speed of the elevator.

D-c Motors are, of course, the ideal for elevator service so far as accurate speed control and low-speed operation without gears is concerned. For passenger service exceeding 250 ft per min shunt-wound motors are used. Compound motors are used for low and moderate single-speed elevators. When the motor comes up to speed the series field is short-circuited and the motor runs as a shunt motor.

Variable-voltage Control Motors for High Buildings. The following summary of a paper by Edgar Bouton, presented at the Midwinter Convention of the A.I.E.E. at Philadelphia, Feb. 4–8, 1924, will give an excellent idea of the application and advantages of variable-voltage control for elevators used in high buildings.

Low-speed electric elevators, using d-c motors, came into use about 1890. Later, a-c motors were employed, but on account of the difficulty of speed control could not be used for the high speeds necessary in tall buildings. Since the height of buildings is dependent upon the elevator system, and in many districts only alternating current is available, the need for high-speed equipments that can be operated from alternating current is evident. A solution to the problem is found in the variable-voltage system of control.

In this system each elevator motor is supplied by an individual generator driven by a motor operating from the a-c or d-c supply voltage. The generator's voltage, and hence the elevator motor's speed, are controlled by varying the field of the generator.

The apparatus consists of:

1. An ordinary shunt-wound d-c elevator motor.
2. A d-c generator of special design for the elevator motor.
3. Control panel for the generator and elevator motor.

4. An a-c or d-c motor to drive the generator.
5. A starting device for the driving motor.
And if the supply is alternating current:
6. A direct-connected exciter for the field, brake, and control circuits.

The control panel (1) makes the proper connections for up and down motion of the car; (2) releases, or sets, the brake; (3) controls the speed; (4) discharges the generator field during retardation, and on stopping; (5) demagnetizes the generator; and (6) opens the circuit between the armature of the generator and the armature of the elevator motor.

From the tests the following conclusions in favor of the variable-voltage over the rheostatic control system are drawn:

1. High-speed installations are now possible for any commercial a-c voltage and frequency.

2. The rate of acceleration increases gradually to about half speed, then decreases uniformly until full speed is reached. The time and power required for acceleration are less than with rheostatic control. The time remains practically the same for all loads. The higher rate of acceleration and retardation permits higher speeds, and the smoothness, besides reducing wear and tear on the machinery, makes riding entirely comfortable to passengers. The impression of falling which is often given, under rheostatic control, by a heavily loaded car when descending, is inherently avoided, since the car follows the generator voltage which at no time changes suddenly.

3. The speed regulation remains flat at as low as one-tenth full speed. Consequently it is easier to make an accurate landing, fewer false stops are made, and the car may be "inched" easily and quickly. In stopping, regenerative braking is set up which brings the car quickly but smoothly to a low speed before the friction brake is applied. Positive speed control enables the limit stops to be made in less time and shorter distances, and without overtravel.

4. Less power is required in acceleration and retardation. This saving of power over the rheostatic control is greatest when the number of starts and stops is large. Power is returned to the line while making the limit stops. Power consumption is not increased in making small movements of the car or in running at low speed. During idle periods standby losses may be eliminated by shutting down the motor-generator set.

5. Since the switching of major currents is eliminated and they are controlled indirectly, the number of arc rupturing contacts is reduced to a minimum, the control as a whole is simpler, less adjustments are necessary, and maintenance costs are lowered.

6. Inherent safety features make higher speeds possible. Limit stops are made accurately and positively. A second independent means is provided for stopping the car in emergencies. On failure of power a dynamic braking circuit closes and a field is maintained, on the elevator motor, making certain the stopping of the car. An overspeed contact on the motor-generator set opens the safety circuit independently of the speed governor.

Energy Consumption. The use of power increases in proportion to the number of starts and stops. This is a natural result of the use of the electric motor, in which the largest part of the energy is required for the process of acceleration, the next largest for electrical and mechanical retardation, and a lesser amount for the actual running, with a small proportion devoted to the continuous excitation of the field. It therefore follows that the main part of the consumption of power is directly related to the number of stops and starts which result from the number of persons carried.

POWER SHOVELS. The operating cycle of an electrically operated shovel is about 20 sec, the component times being: hoist 8 to 10 sec, thrust 10 sec, and swing 10 to 12 sec, the thrust being in operation at the same time as the hoist and swing are operating. The motors to meet these requirements must have a sufficiently low armature inertia to permit of rapid acceleration under small power, and are therefore generally of the crane or mill-type construction.

Hoist Motors. In the case of the hoist, considerable advantage may be gained in this respect by using two motors of one-half the capacity each instead of one motor of the full capacity, as the power required for accelerating is much less. For example: Assume a shovel that requires a 225-hp, 514-rpm motor for the hoisting operation. Such a motor requires 144-hp-sec for bringing up to full speed, whereas if two 115-hp, 600-rpm motors are substituted in its place, each motor requiring 63.5 hp-sec or both 127 hp-sec to bring up to speed, there would be a saving of about 12 per cent in the power required to accelerate the motor alone. On the other hand, such an arrangement may require the use of bevel gears and additional shafting.

Swing Motor. This motor, although not subject to the severe overloads and shocks encountered on the hoist motors, is subject to frequent reversals, and, as rapid acceleration is required, a motor of similar design as for the hoist should be used.

Thrust Motor. This motor differs somewhat in its operation in that it is practically stalled during the digging operation, although it may revolve or overhaul according to conditions, and is operated at full speed only after the hoist operation is completed. Its duty is to keep the dipper against the bank, and it must therefore stand still and exert torque most of the time. For this reason its design should be very rugged, and the motor should be able to develop a heavy torque for short intervals of time while standing still, or rotating very slowly.

Location of Motors. The hoist and swinging motors are as a rule located on the car and are geared to the drums through suitable reducing gears, while the thrust motor as a rule is mounted directly on the boom and communicates its motion to the bucket staff through reducing gears connected to a pinion engaging a rack on the staff.

Type and Size of Motors. It is possible to obtain successful operation with a-c induction motors but it can be done only at the expense of larger capacities and increased power consumption, owing to the characteristic of the motor, which radically differs from that of the steam engine ordinarily used on shovels and from the d-c series motor.

A careful analysis of the typical steam-shovel engine reveals a characteristic which is not unlike that of the series d-c motor in that it speeds up under light loads, slows down under heavy loads, and possesses a certain elasticity in operation which minimizes mechanical strains on the shovels. Because of these facts it is not only desirable but advisable to adhere to the d-c series motor on the electric shovel. Practically all large power supplies are alternating current, but the smaller capacities in d-c series motors permit the use of a synchronous motor-generator set mounted on the shovel at practically the same cost as the a-c induction motors with transformers, with the advantages in operating characteristics and power consumption all favorable to the d-c equipment.

Variable-voltage System. The variable-voltage system is the latest development and makes possible simple and flexible control. Each d-c motor is supplied by its separate d-c generator, the particular motion in question being controlled by varying the voltage of the generator. These generators are the differentially compound-wound type. There is a separately excited shunt winding, a self-excited shunt winding, and a differential series winding. The main driving motor which drives the d-c generators is either a squirrel-cage induction motor, on the smaller shovels, or a synchronous motor for the large shovels. Mill-type motors are used for driving the different motions, ventilated for the hoist and swing motions and totally enclosed for the thrust. Different manufacturers use different field windings, including shunt, compound, and series windings, depending in part on the service for which the shovel is intended and in part on its design.

Control Equipment. With the variable-voltage shovels, the practice is to use small drum controllers with series resistors in circuit with the separately excited shunt fields.

Brakes for the different shovel motions are generally of the air-brake type, a small air compressor being provided on the shovel for this purpose.

WOODWORKING INDUSTRY. The cutting heads of woodworking machinery generally operate at high speeds, and in order to eliminate high-speed belts, save floor space, decrease maintenance and fire hazard, and secure a safe compact machine there is a distinct advantage in using high-speed direct-connected motors. When there are several cutting heads, each having its individual motor, the group of motors is interlocked electrically so that the stopping of one stops all.

Motor Requirements. The motors recommended for this service by one of the important manufacturers are of squirrel-cage construction made in sizes up to 35 hp and with speeds from 3600 to 10,000 rpm and even as high as 20,000 rpm when used in conjunction with a frequency changer. Double-end ventilation is preferable, but single-end will suffice if mounting conditions make this necessary. Multispeed, generally two- or four-speed depending on the nature of the work, motors are applied to the feed mechanism. The controls of all motors are located conveniently so that the operator can control both feed and cutting.

TEXTILE MACHINERY. Individual drive is largely used for pickers, spinning, roving frames, twisting frames, cotton cards, looms, and other machines, especially on new installations. In mills where a change is being made from mechanical to electrical drive two or four frames are frequently driven from one motor to reduce first cost.

Motors have been specially designed for these services, or certain types are particularly well adapted, as outlined below:

Roving, Spinning and Twisting Frames—Two and Four Frame Drive.

(*a*) Motor has shaft extending on each end and two frames can be belt driven from each end, motor mounted on ceiling.

(*b*) Motor has long extension on one end of shaft only for use where aisle space between driving ends of frames is limited.

(*c*) There are squirrel-cage motors treated to resist moisture from the humidity of

the mill. There are no screens, and the openings in the brackets are large to permit of easy cleaning with compressed air. Air deflectors are omitted to reduce the amount of lint and dirt drawn in.

Spinning, Roving and Twisting Frames, Pickers, Cone Winders, etc.—Individual Drive. (*a*) Squirrel-cage motors with screens to prevent entrance of lint into interior because the motor is mounted on the machine.

(*b*) Specially treated motors to resist moisture of mill.

(*c*) Dust-proof waste-packed bearings to exclude foreign material.

Carding Machines—Individual Drive. High starting torque motor required to take care of inertia of heavy cylinder. Starter with means for reversing the cylinder for grinding.

Looms. Individual Drive. Totally enclosed induction motor.

PAPER INDUSTRY. The following analysis will give an idea of the motors applicable to the various machines used in the making of paper:

BEATERS. Wound rotor induction motors are applicable where the starting conditions are severe and where the starting current must be kept low. Squirrel-cage motors, because of their simplicity, should be used when starting conditions permit. Synchronous motors, where desirable to improve power factor, may be used provided starting torque requirements may be met by overmotoring or by the use of a slip belt or a magnetic clutch.

Paper Machine.—Single Motor Drive. D-c variable speed.

Paper Machines.—Sectional Drive. Many advantages in economy of operation are claimed for driving each section of a paper machine by its individual motor.

Super-calenders. There are three principal methods of driving these, namely, single motor, two motors, and single motor dual frequency.

(*a*) With single motor drive the wound rotor type of motor is used and the various speeds are obtained by means of gears and clutches.

(*b*) With two motor drive a large wound rotor motor is used for driving the super-calender at "production" speed, and for "threading-in speed" a small squirrel-cage motor, both operating through gears.

(*c*) The dual frequency motor has a wound rotor designed to operate at two frequencies, one frequency being that of the power system and the other obtained from a frequency changer. The low frequency is used for the threading in and the high frequency for production speed. This system has several advantages over the others such as increased flexibility, less noise, less fire hazard, greater safety and cleanliness, and less floor space.

(*d*) D-c motors are in many ways preferable for super-calender drives on account of the greater facility of speed control. The most usual methods are the single motor, two motor, and the Ward-Leonard system. Either shunt or compound wound motors may be used in either of the first two systems. With the Ward-Leonard system only one motor with shunt wound characteristics is necessary and presents substantially the same advantages as the dual frequency a-c arrangement.

Super-calender Reel Stand. Squirrel-cage motors are generally used for this drive.

Sheet Calenders. Wound rotor motors for a-c service or shunt wound motors for direct current are the usual practice.

Platers. High torque and low flywheel characteristics are called for. With a-c operation wound rotor motors with a permanent resistance across the rotor circuit give satisfactory service because with this arrangement maximum starting torque is available at any time and excessive heating takes place in the resistance outside the motor. With d-c service compound-wound motors with small flywheel effect are used.

Rewinders. D-c or a-c motors or Ward-Leonard system.

Cutters. Wound rotor motors, with variable resistance in the rotor circuit to secure speed changes, are used. With d-c service the speed is regulated by shunt field control.

Trimmers. When d-c drive is employed the shunt-wound motor is required, and for alternating current a motor with high slip characteristics is desirable so that the shock of the cutting stroke will be taken by the flywheel.

SUGAR MILLS. Cane Crushers and Rolls. These call for close speed control, and as alternating current is commonly used motor secondaries are required.

Pumps and Auxiliaries. Squirrel-cage motors are applicable where constant speed is called for and where there is no objection to high starting current, or wound rotor motors where variable speed is required for hoists, car dumps, etc., and where it is desired to keep the starting current not much in excess of full-load current when starting at full-load torque.

Centrifugals. Squirrel-cage motors designed to be thrown directly on the line by a drum controller are generally used.

OIL FIELDS. Well Drilling, Pumping, etc. Two-speed wound rotor induction motors have a wide application in the oil fields. They are made in open and air-jacketed enclosed types, the latter for use in outdoor locations. The low speed is used for pumping. Other operations, such as pulling, cleaning, swabbing, bailing, and redrilling, are done at the high speed. Enclosed standard controls are used. The same motors are applicable to cable tool drilling, in which case twin motors are used. Motors are of proper size for pumping the well after it is drilled, provided that two motors of such size are large enough to do the drilling. When the well has been brought in, one of the motors is left on the job for pumping, etc. Single motor drilling is practicable with this type of motor for depths of about 4000 ft. The size of motor for drilling equipment for various depths is given in Table VII.

Table VII

Equipment	Maximum Band Wheel Speeds, rpm	Maximum Recommended Depth, ft
15–35 hp twin-motor equipment	70–80	2500
20–50 hp twin-motor equipment	70–80	4000
25–65 hp twin-motor equipment	60–90	7000
35–75 hp twin-motor equipment	60–90	9000
25–65 hp single motor with auxiliary control	55–80	4000
35–75 hp single motor with auxiliary control	55–80	4500
75 hp single motor equipment	55–80	4000

Pipe Line Pumping. Squirrel-cage motors have the widest application because of their low first cost and simplicity, and the fact that constant speed is generally all that is called for. Of course, where variable speeds are required the slip-ring motor is the next best thing. Synchronous motors are entirely applicable to this service and should be used where low power factor is penalized. The size of motor required may be figured by means of the following formula:

$$Hp = \frac{1.7 \times Q \times P}{\text{Eff.}}$$

where P = pressure in hundreds of pounds per square inch or static head plus friction head on pipe line.
Q = thousands of barrels of output per day.
Eff. = 65–75 per cent for centrifugal pumps and 85–95 per cent for plunger pumps.

Rotary Drilling. Motors for this service have been specially designed to provide for continuous carrying of rated load and several hundred per cent overload for short periods due to the meeting of sudden unexpected obstructions. Wide speed range is also required because at light load for making and breaking pipe, circulating, fishing, etc.

Band Wheel Drive. For this a standard two-speed oil well motor is usually the most satisfactory drive. A reduction gear is mounted on the same bed plate as the motor.

Slush Pumps are generally driven by single-speed, wound rotor motors.

COAL MINES. Most of the operations in coal mining have been electrified in one way or another. Some of the electrified equipment, such as cutters and locomotives, have motors which form an integral part of each unit, and there is seldom, if ever, any occasion for the mine operator to consider what type of motor is needed. On the other hand, there are a number of drives, such as for tipples and breakers, ventilation, pumping, and dumping, and hoisting, where data concerning the type of motor which should be used are frequently needed. A further point of importance is that, in gaseous mines, all motors and controls must be fully enclosed so that there will be no danger of sparks setting off an explosion. Motors specially designed for this service are on the market.

Tipples and Breakers. For tipple and breaker drives either d-c or a-c motors may be used. For reasons of economy and simplicity of operation squirrel-cage and wound rotor motors are used to the greatest extent. Synchronous motors are also installed where power-factor correction is an item of consideration.

Ventilation of coal mines is of course of particular importance. Squirrel-cage, wound rotor and synchronous motors are applicable.

Pumping. In mine pumping the motor must withstand severe service conditions. The locations are usually very damp, there is likely to be a good deal of sweating, and sulfuric acid in the water causes corrosion. Consequently, enclosed air-jacketed motors and tightly sealed bearings are very necessary. The squirrel-cage motor or, if it is desired to run at half speed part of the time, a two-speed squirrel-cage motor is generally the most economical and satisfactory. In some instances wider speed ranges are called for,

and then the wound rotor type is needed. Synchronous motors are coming into use to an increasing extent and are applicable to either centrifugal or plunger pumps.

Hoisting. For hoisting either a-c or d-c motors may be used. Variable-voltage d-c hoisting motors, supplied from a flywheel motor-generator set, have advantages in that they give fast acceleration and high speed economically and safely.

CRANES. Cranes may be divided into six general types, namely, overhead traveling cranes, gantry cranes, locomotive cranes, jib cranes, shipbuilding cranes, and wharf cranes. As a general thing the crane builder takes care of the selection of the proper motors for their use, and it is assumed that a clear and unmistakable picture has been given of the service which is to be required. The electrical equipment is about the same for all of them. For instance, the overhead traveling cranes and gantry cranes usually have three motors. One is for the hoist, the second for the travel of the trolley, and the third for the bridge travel. Sometimes a crane has two trolleys, and not infrequently there are two hoists for a single trolley to handle loads of different size.

Motor Data. It is essential, of course, that the motors for crane service be strong and rugged, have high starting torque, large short-time overload capacity, large shafts and bearings, armatures of small diameter to reduce the power for quick acceleration and braking, and other consistent characteristics. Control may be magnetic or manual, with drum-controllers. The Westinghouse Electric and Manufacturing Co. has devised the following table covering the types of motors to be used in crane service:

Motors		Control		Brakes	
D-c	A-c	D-c	A-c	D-c	A-c
Type HK	Type CI	Type S Drum	Type FA Drum	Type HB	Type A
Type K	....	Type C Magnetic	Type F Magnetic	Type KT	....
Type MC	....	....	....	Type B	....

Coal and Ore Bridges. These are actually special kinds of gantry cranes with exceptionally long bridges. The selection of the proper motors for the drives is highly specialized, and as a general thing the purchaser of the handling equipment buys it complete with the necessary motors. This is in fact the safest and most satisfactory procedure to follow ordinarily. Either a-c or d-c motors can be used for the bridge travel, trolley, and hoist.

Coal Towers. These are another variation of crane service as they consist of a boom with a trolley, equipped with a hoist. Ordinarily wound rotor motors are used, although for very high-speed operation d-c motors with variable-voltage control are generally more economical.

CONVEYORS. Conveyors for the mechanical handling of materials are very widely used in industry. There are many types which may be divided into two general classes, namely, conveyors for handling packages and conveyors for handling loose, bulk material.

Package Conveyors. All package conveyors consist essentially of one or two parallel endless chains or cables or of endless belts. The ordinary chain conveyor may be used for horizontal travel or steep inclines. Sometimes these consist of single chains, sometimes of a pair of chains with cross-bars; they are used mainly for handling heavy materials, and provision may be made for suspending slings between the cross-bars. The well-known automobile assembly line is a form of chain conveyor. Belt conveyors are so well known as not to call for any special description. In addition to this there is the continuous overhead conveyor made of a chain or cable supported by rollers. A single exception to the endless chain or belt system is the roller conveyor which is made up of a series of horizontal rollers all driven simultaneously in the same direction. These are used for moving steel ingots in rolling mills and elsewhere, logs in sawmills, etc. In some cases the rotation of the rolls must be reversed to return the ingot or log for further operations.

Bulk Conveyors. Bulk material conveyors embrace bucket conveyors, belt conveyors, including portable car loading conveyors and the like, pan conveyors, and drag conveyors, all dependent on the endless-line principle. In addition there are screw conveyors, shaking conveyors, and pneumatic conveyors.

Application of Motors. The application of motors is fairly simple. Generally the conveyors are run at constant speed so that induction motors may be used. Where starting is heavy and reduced speed operation is desirable the wound rotor type should be selected as it may be brought up to full speed by short-circuiting the external resistor through the controller. Where the starting torque is low the squirrel-cage type of motor is

permissible and preferable because of its simplicity. It is also advantageous under operating conditions involving dust and dirt and danger of explosions.

D-c motors, either constant or variable speed, are suitable for these conveyor services. Enclosed types should be used where operating conditions may make this necessary or desirable.

Where exposed to the weather the appropriate type of motor should be used.

Aerial Tramways. Aerial tramways are used for the handling of materials of many kinds, sometimes for distances of several miles. It is in general an economical method of transportation, especially over rough country, for the output of quarries and mines, etc. Aerial tramways are also used for short hauls for the disposal of waste from industrial plants. Motors are wound rotor with magnetic control.

DREDGES. Dredges have many of the characteristics of shovels. They may be supplied with electrical power either by means of a submarine cable or more often from their own power plants. Diesel engines direct-connected with the generator or steam turbines direct-connected are used. With Diesel engine drive there are usually several generating units which make possible variable-voltage d-c control. With such arrangements some of the generators are constant voltage for constant-speed drives, and others have variable voltage for the variable-speed drives. A-c motors are also used with success. In any event, each motor performs one distinct duty so that several are used per dredge. One of the very large harbor dredges has four Diesel engine d-c generator units, a 2700-hp motor for the pump, plus twenty-five other motors between 5 and 250 hp.

CAR DUMPERS. Car dumpers are designed for the dumping of open top railroad cars, from small mine cars to full-size gondola cars. D-c series motors are preferred, but in some cases wound-rotor a-c motors are applicable. In selecting motors for this it is safest to draw on the experience of the manufacturer of the dumping equipment as to just what type of motor is likely to give satisfactory results.

TRUCKS, TRACTORS, ROAD TRUCKS, LOCOMOTIVES, AND LORRY CARS. These pieces of handling equipment are supplied by their manufacturers complete with motors installed. For details concerning the selection of the proper unit for particular service see discussion under the respective headings.

LAUNDRY MACHINERY. The machines found in laundries which are individually motorized in up-to-date installations consist of washers, drying tumblers, centrifugal extractors, and ironers.

Washing Machines and Drying Tumblers call for reversing motors. Reversal is automatic, by means of the reversing switch. With a-c motors this is accomplished by throwing it across the line. A low-torque motor is used so as to cause it to start gradually and smoothly on reversal. These motors must also be so designed as not to overheat even when reversed as many as six times per minute. When the service is direct current compound-wound motors are used. The automatic control does not permit the starting voltage to be applied for reversal until the machine has been stopped by means of dynamic braking. Bearings which are proof against the entrance of moisture, lint, etc., and the leakage of oil are essential for these applications.

Centrifugal Extractors. For centrifugal extractors the motor bearings must also be closely sealed for the same reasons, and splash-proof covers for the ends are necessary to keep out water and clothes which might become entangled in the motor. The motors should also be well impregnated to protect the winding against moisture. Low starting current motors, both alternating and direct current, which bring the basket up to speed smoothly and quickly, have been especially designed for this application.

Ironers. For the ironers either of two types of standard a-c motors may be applied. One of these is the squirrel-cage induction motor for constant-speed machines, and the other, for large flatwork ironers, a four-speed motor with push-button starter and drum controller. Standard shunt-wound motors with either constant or adjustable speed are used where the service is direct current.

RUBBER MILLS AND CALENDERS. Rubber mills are used for the various steps in producing commercial rubber compounds from the raw rubber. They may be driven either by wound rotor or synchronous motors. In those applications where the motors operate at the speed of the mill line, synchronous motors are generally preferable because they cost less and perform more satisfactorily than induction motors. With geared drive the principal advantage of synchronous motors is due to their corrective effect on the plant power factor. So far as actual running of the mills is concerned, experience has shown that either type of motor will give satisfactory performance. When the synchronous motor is run at the comparatively low speed of the mill it is brought up to speed with the clutch thrown out so that the stored energy in rotor will make up for the lower starting torque as compared to the induction motor. For geared drives where the motor runs at something like 600 rpm it has been found that the starting torque is sufficient.

Quick Stopping. In order to obtain quick stoppage as a safety measure the wound rotor motor drives and synchronous motor direct drives should be equipped with clutch brakes. With synchronous motor geared drives where the clutch is eliminated, quick stopping is secured by disconnecting the motor armature from the a-c source and connecting it to a proper resistance, leaving the field energized at full-load value.*

Rubber Calenders are used for the final treatments of the previously prepared rubber compound. A great deal of speed adjustment is called for, and because of this the best results from electrical operation can usually be obtained by means of adjustable-speed d-c motors. Either single- or double-voltage motors may be used; magnetic-type controllers are supplied for either single- or double-voltage equipments. Speed variation with the former is by means of a field rheostat, and with the latter, with a drum-type master switch. Wound rotor induction motors can be used for rubber calender drives, but as the speed regulation is very limited, the different kinds of rubber compound that can be calendered and the rate of production are correspondingly limited.

Ratings for Rubber Calender Motors. The ratings in Table VIII, taken from the bulletin Electric Drive for Rubber Calenders, of the Westinghouse Electric and Manufacturing Co., apply to d-c rubber calender motors for operating on single- and double-voltage circuits:

Table VIII

Horsepower at Maximum Speed	Single Voltage		Horsepower at Maximum Speed	Double Voltage	
	Volts	RPM		Volts	RPM
25	230	300-1200	50	115-230	287-575-1150
35	230	300-1200	60	115-230	237-475- 950
50	230	300-1200	75	115-230	237-475- 950
60	230	300-1200	90	115-230	237-475- 950
75	230	300- 900	100	115-230	225-450- 900
75	230	300-1200	100	115-230	175-350- 700
75	230	225- 900	125	115-230	225-450- 900
100	230	300- 900	150	115-230	225-450- 900
100	230	300-1200	200	115-230	200-400- 800
125	230	300- 900	250	115-230	175-350- 700

All motors are rated at 50 deg on a constant-torque basis. The single voltage motors are good for 80 per cent rating at 600 rpm. The double-voltage motors are good for 80 per cent of full-load rating at high voltage and half speed.

LOGGING AND SAWMILLS. Electrical drive has been very completely applied in this industry; instead of discussing each machine separately, primary details of the motors required are given in Table IX taken from the Westinghouse bulletin entitled "From Log to Lumber by Electric Power."

CHEMICAL INDUSTRIES. Most of the applications in the chemical industries can be made with a-c squirrel-cage motors. They are particularly desirable because many of the operations are continuous and very frequently the service conditions are severe in other ways. That is, there are very likely to be fumes, dust, and moisture. The simple construction of squirrel-cage motors with no small parts and no sliding contacts makes them peculiarly well adapted for this service. Motors must be strongly constructed, and they should be of the air-jacketed enclosed type to keep out oil, fumes, and moisture, and as a protection against deterioration from dusty chemicals. Motors should be selected with tight, dust-proof bearings, automatic oiling, and of a design which will not permit accumulated dust to interfere with proper ventilation.

Dusty Locations. For very dusty locations, as in cement mills and potash, fertilizer, and other factories where there is a great deal of dust, the motors should be enclosed.

Variable Speeds. Where variable speeds or especially strong starting torque is called for, wound rotor motors must be used for a-c service.

D-c Applications. About the only usual application of d-c motors in the chemical and electrochemical industries is for the operation of cranes, hoists, and other material-handling equipment requiring a wide range of speed adjustment.

HEATING AND VENTILATION OF BUILDINGS. In the heating and ventilating of buildings electric drive for the fans circulating the air are of extreme importance. Two types of fans are generally used, the propeller or disk fan and the centrifugal fan. The former are best adapted for exhausting air from rooms and for use with unit heaters. Centrifugal fans should be used with duct systems of ventilation and with certain kinds

* For further details see Synchronous motor drive for rubber mills, by C. W. Drake, a paper presented at Spring Convention of the A.I.E.E., St. Louis, April 13-17, 1925.

of unit heaters. Great care must be exercised in selecting the proper fan for the work to be done and in choosing the right motor to go with it. Consultation with reliable fan manufacturers is strongly recommended.

Table IX

Electric Drive for Logging and Sawmills

	Motor			Method of Drive *
	Type	Hp	RPM	
Log haul	WR–SC	25– 75	900	Belted or geared
Log lift	WR †	35– 75	900	Geared to drum
Carriage set works	SC	5– 10	900	Geared, belted, or chain driven
Rock saw	SC	10– 15	1200	Belted
Head saw (band)	WR	100–300	600– 710	Belted or direct connected
Live rolls and transfer chains	SC ‡	3– 15	600– 900	Geared and chain driven
"Jump" or "swing" saws	SC	7 1/2– 25	600–1200	Belted
Slasher	SC	35– 75	900–1200	Direct connected
Edger	SC	35–400	1200–1800	Direct connected
Trimmer	SC	25– 75	900	Direct connected
Gang saw	WR	75–250	720	Belted
Sorting table	SC	5– 10	900–1200	Direct connected
Resaw	WR–SC	50– 75	900	Belted or direct connected
Stacker	SC–WR	5–7 1/2	1200	Belted
Unstacker	SC	3	1200	Belted
Timber sizer	SC	50– 75	900	Direct connected
Hog	SC–WR	15–450	750–1200	Direct connected or belted
Rolls, various kinds	SC ‡	3– 15	900–1200	Chain driven or geared
Saw grinders, stretcher, retoother and scarfing machines	SC	2– 5	1200	Belted
Exhaust fans and blowers	SC–Syn	20–450	600–1200	Belted or direct connected
Compressors	SC–Syn	5–300	1200–1800	Direct connected
Conveyors	SC ‡	10– 25	900–1200	Geared
				LATH MACHINES
Lath bolter	SC	40– 50	1800	Direct connected
Lath mill	SC	25– 40	3600	Direct connected
Lath trimmer	SC	5– 10	1200	Direct connected
				SHINGLE MILLS
Shingle mill: Main saw	SC	20	1800	Direct connected
Shingle mill: Trimmer saw	SC	3	1800	Direct connected

SC—Squirrel Cage SYN—Synchronous WR—Wound-rotor

* Where motors are direct connected, flexible couplings are recommended.
† Solenoid brake generally supplied. ‡ These motors are of the high starting torque type.

Selection of Motor. The selection of the motor will depend on several service conditions, which include the speed variation desired, available voltage and space, and the horsepower necessary. The following equation for determining the required horsepower for estimating purposes is taken from an electrical manufacturer's bulletin, but accurate figures should be obtained from the manufacturer's curves:

$$Bhp = \frac{QI}{6356 \times \text{Eff.}}$$

Q = cubic feet of air required per minute; I = water gage pressure in inches; Eff. = blower efficiency in decimals (usually from 50 to 60 per cent); pressure in pounds per square foot = 5.2 × water gage in inches; pressure in ounces per square inch = water gage in inches ÷ 1.73. The pressure I should be measured at the fan. The amount of air required (Q) varies under different circumstances and should be substantially as given in Table X.

D-c Motor Applications. D-c motors of the shunt-wound type furnish satisfactory drives for fans and blowers where variable speed is required. The greatest flexibility is realizable with variable voltage control as explained under elevator and hoist drives. Where the fan or blower is to be operated with low-speed direct drive, d-c motors are particularly satisfactory because they operate at high efficiency at all speeds. The benefits of this higher efficiency are so marked that where slow-speed direct drives are required in localities where direct current is not available it may be worth while to install a motor-generator set. This holds particularly if the fan is to run more or less continuously.

Table X. Air Required

Cubic feet of new air per person per minute

	Without Humidification or Recirculation	With Humidification but without Recirculation	With Humidification and Recirculation	Number Air Changes per Hour
Classrooms	30	20	5 to 10	......
Assembly rooms	15 to 30	10 to 20	5 to 15	......
Gymnasiums	30	25	15 to 20	......
Theaters	30 to 50	20 to 30	10 to 15	......
Hospital wards	30 to 40	20 to 30	......	......
Toilets	......	......	......	10 to 20
Locker rooms	......	......	......	5 to 10
Kitchens	......	......	......	20 to 60
Dining rooms	......	......	......	10 to 20
Ballrooms	......	......	......	5 to 10
Work space	......	......	......	5 to 10

Squirrel-cage Motors. Squirrel-cage motors should be used for constant-speed drives, or where, by having two- or four-pole combinations providing wide steps in speed, regulation will be satisfactory.

Slip-ring Motors. Wound rotor or slip-ring motors should be used where speed regulation is required in small steps. Synchronous motors are well adapted for slow-speed operation, but unless large units are involved the power is too small for this type.

STEEL MILLS. The application of electric motors to steel mill service may be divided in two general classes: (1) that dealing with the application of motor drive to the main rolls, that is, to those rolls in which the ingot or billet is reduced in section; and (2) that dealing with the problems of electrifying the numerous other auxiliary machines and devices, such as tables, screw-downs, charging machines, etc.

Main-roll Application. The term "mill" is sometimes used to designate a single stand or group of stands, and sometimes to include the main rolls and all auxiliaries involved in the production of a given class of materials. The stands are generally classified according to the arrangement of rolls and method of operation, that is, two-high or three-high, the two-high being either reversing or non-reversing.

Continuous Mills. Continuous mills should also be included in which a number of stands are placed in series, the metal from the rolls of one stand passing directly to the rolls of the next. The stands are operated at such speed as to take care of the increased length. The speed of successive stands is accurately controlled to prevent the formation of loops or the stretching of the metal between passes.

Reversing and Non-reversing Mills. The great majority of rolling mills in this country are of the non-reversing type where the rolls run continuously in one direction. The large flywheel effect of the motors used for this class of service, together with the great capacity of the electrical supply systems at the present time, renders the use of separate flywheels unnecessary in most cases, in spite of the heavy peak loads encountered.

For reversing mill duty d-c motors are used in connection with flywheel motor-generator sets. The motors of such motor-generator sets are of the slip-ring type with either a constant resistance in series with the rotor, to increase the motor slip and enable the flywheel to give up its stored energy during peaks, or with a liquid slip regulator to perform the same function with a more definite limit on the motor input. To reduce the flywheel effect of the d-c reversing motors they are usually built with two armatures mounted on the same shaft.

Non-reversing Mills. For non-reversing mills rolling to definite sizes where no variation in speed is required, slip-ring induction motors are used, and more recently synchronous motors. Although the latter do not possess as good starting characteristics their effect on the power factor of the system in many cases more than offsets the disadvantages as regards starting torque.

Variable Conditions. Where mills are called upon to roll a variety of sections, and also in continuous mills where it is essential to adjust the speeds of successive stands to take up the increased length due to reduction of cross-section, the use of d-c motors having field control is now standard practice. They are used in connection with large motor-generator sets.

Speed Control. The use of the Kraemer and Scherlius systems of speed control for large slip-ring a-c motors has largely been given up in favor of d-c adjustable-speed motors. These are now built in capacities up to 7000 hp with a speed range from 50 to 120 rpm.

Power Required. Rolling mill loads are irregular in the extreme owing to the inter-

mittent character of the process. For any particular ingot the motor load consists of periods of heavy duty increasing in length with the length of the metal in the pass, interspersed with periods of friction load only. With a given mill the load varies widely with the difference in the section rolled, differences in temperatures of metal, personal equation of the operator, etc. The practical determination of the power required to roll steel is a matter of elaborate and extensive tests under widely varying conditions. The information for predetermining the sizes of motor and flywheel for an installation must cover the following points:

Type of mill; rail, plate, etc.
Diameter and speed of rolls.
Weight of ingot.
Initial and final section.
Number of passes.
Time between passes.
Elongation in each pass and total.
Initial length.
Average and maximum rate of rolling.
Temperature of metal.
Character of metal.

Auxiliary Motor Applications. Electric motors have been used for a long time for auxiliary steel-mill machinery. The nature of this service is unusually severe and has led to the development of the mill-type motor for both d-c and a-c service. These motors are designed to withstand heavy overloads and abnormally rapid acceleration. Their insulation consists of mica and asbestos (class B insulation) to withstand the high temperatures encountered.

All A-c versus Mixed System. It is to be assumed that power will primarily be alternating current, as the transmission distances ordinarily preclude the use of d-c generators. It might therefore seem to be simplest and most efficient to step down to a suitable voltage through static transformers and use a-c motors. But, with many of the auxiliaries, such as screw-downs, live-roll tables, etc., the advantages in favor of d-c motors are so great that they are almost universally adopted. This system involves additional expense for motor-generator sets and entails considerable power loss due to the low efficiency of conversion with intermittent loads. On the other hand, d-c motors of this type are lower in first cost than corresponding induction motors, and a higher power factor is maintained on the entire system where they are used. An increase in power factor is effected by eliminating the lagging current of the induction motors, and, in addition, the motor-generator sets can be equipped with synchronous motors which will take a leading current from the line and offset part of the lagging current on the rest of the system. The increase in power factor enables a reduction to be made in the size and cost of transformers and generators, and also increases their efficiency owing to the lower currents which they are required to handle and to the decreased excitation required by the generators.

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See also, bibliographies of Sections 8 and 9.

SECTION 17

TRANSPORTATION

ELECTRIC TRACTION

By H. A. Currie and Robt. S. Rhodes

Art.		Page
1.	Traction Systems	02
2.	Energy Requirements and Motor Equipment (W. A. Del Mar)	06
3.	Electric Locomotives	24
4.	Electric Cars and Trolley Buses	27
5.	Control Systems for Railway Motors	29
6.	Current Collectors	38
7.	Brakes and Braking Systems	39
8.	Lighting of Trains by Electricity	41
9.	Third-rail or Contact-rail Systems	43
10.	Overhead Contact Systems	51
11.	Underground Trolley Systems	61
12.	Track and Third Rails	62
13.	Railway Track Bonds	64
14.	Dispatching of Trains by Telephone	69
15.	Car Barns, Inspection Sheds, and Repair Shops	69
16.	Bibliography	71

ELECTRICAL EQUIPMENT OF INTERNAL-COMBUSTION AUTOMOBILES

By H. Chase

Art.		Page
17.	Electric Ignition	72
18.	Starting and Lighting Systems	84
19.	Battery	88
20.	Lamp Equipment	90
21.	Instruments, Fuses, etc	92
22.	Starting Motor	93
23.	Wiring	97
24.	Bibliography	101

ELECTRIC AUTOMOBILES

By W. P. Kennedy

Art.		Page
25.	Development	101
26.	Applications of Electric Trucks	102
27.	Transmission Systems	104
28.	Batteries	105
29.	Controllers	105
30.	Performance; Energy Consumption	106
31.	Operation	106
32.	Bibliography	107

ELECTRIC PROPULSION OF SHIPS

By W. N. Zippler

Art.		Page
33.	Fundamental Propulsion Requirements	108
34.	Steam Turbine Drive	111
35.	Turbine Electric Drive	113
36.	Advantages of Electric Propulsion	115
37.	Internal-combustion Engine Drive	119
38.	Typical Electrically Propelled Ships	123
39.	Bibliography	123

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Article 14 is a revision of an article in the previous edition by **S. G. McMeen.**

B. P. Coulson was author of chapter on "Electric Propulsion of Ships" in previous edition and the present chapter is a revision thereof.

Signalling is treated in **Pender & McIlwain, Communication Engineering Handbook.**

ELECTRIC RAILWAYS

By H. A. Currie and Robt. S. Rhodes

1. TRACTION SYSTEMS

APPLICATIONS OF ELECTRIC TRACTION. There are certain well-defined fields in which electric traction is superior to other methods, the most important of which are the following:

1. Where the frequency of stops is so great as to require a high rate of acceleration in order to make a good schedule speed. Motor buses are replacing trolley lines to an increasing extent for such service. If the service requires trains of several cars, any desired number of these cars can be made motor cars and thus a sufficient weight on the driving wheels can be economically secured to give the required adhesion and tractive effort.

2. Where local conditions prohibit the nuisance of the smoke, exhaust gases, and noise of steam locomotives, as in cities, tunnels, and mines.

3. In heavy trunk-line service where the density of traffic is so great that a high load factor can be obtained with respect to both the power house and distribution system. The *operating* cost of an electric train is always less than the operating cost of a steam train but the *fixed* charges (interest on investment and depreciation) of the electric system are high, for the first cost for the electric equipment, viz., locomotives, motor-car equipment, distribution system, and power house, is much greater than the first cost of steam locomotives, and their accessory equipment. If there are sufficient trains in a system so that the pro rata share of the fixed charges for each train is less than its share of the difference between the operating cost of an equivalent steam equipment and the operating cost of the total electric equipment then electric traction is advantageous even in the absence of the secondary and intangible advantages which usually exist.

4. Where cheap electric power is available and fuel is expensive.

SYSTEMS. Three different types of motors are in use for electric traction, the d-c motor, the single-phase commutator motor, and the three-phase induction motor; see chapter on Motors. The operating voltages (i.e., volts between trolley and track rails or between third rail and track rails or, in double-trolley roads, between the two contact wires) and the motor voltages employed are as follows:

D-c Systems. Trolley or third-rail voltages of 600, 750, 1200, 1500, 2400, and 3000 are in use. The motors for the 600- and 750-volt systems are designed for operation at full trolley voltage. For all the other systems, two motors are usually connected permanently in series electrically, each designed for half the trolley voltage, but insulated for the full voltage.

Single-phase Systems. Trolley voltages (third rails are not used) of from 3000 to 11,000 volts and frequencies of 15 and 25 cycles per second are in use. In the United States 11,000-volt single-phase 25 cycles is standard, and in Europe 15,000-volt 15 or 16 2/3 cycles. The trolley voltage is stepped down to values suitable for direct application to the motors, from 200 to 1150 volts.

Three-phase Systems. Two trolley wires for each track are required, the two wires and the track forming the necessary three conductors for the three-phase distribution. This system is extensively used in Italy.

The voltages used between the two contact wires and between each contact wire and track range from 3700 to 10,000, and the frequency is 15, 16 2/3, and 45 cycles. A few installations have also been made in Switzerland, Spain, and Austria using 25 and 40 cycles, and as low as 750 volts. Voltage is stepped down for the motors as in the single-phase system.

Comparison of the Various Systems. For ordinary street railway service the 600-volt d-c system is almost universally employed, but for interurban and trunk-line service there has been much difference of opinion as to which of the various systems is the most economical when all the factors are taken into account. The factors which must be considered in comparing the three systems in any particular case are the following:

1. For a given weight and length of trolley or third rail the per cent power loss for a

given amount of power transmitted varies inversely as the square of the trolley or third-rail voltage.

2. The higher the trolley or third-rail voltage the smaller is the number of substations required for the same efficiency of distribution and weight of conductor.

3. The higher the trolley or third-rail voltage the more costly is the insulation and supporting structure, and also the greater is the cost of maintenance of the distribution system.

4. Both the first cost and the annual expense of the substations are less for the a-c systems than for the d-c systems, since for the former static transformers only are required whereas for the latter mercury-arc rectifiers or rotating machinery must be used for conversion.

5. The less-than-unity power factor of a-c motors (90 to 95 per cent) as well as of the line (due to the reactance of the trolley wire and track return) gives rise to a greater power loss in the a-c distribution system for the same power delivered than in the d-c system, and makes necessary generating apparatus of greater kv-a capacity.

6. The high-voltage d-c motor, for the same horsepower rating and speed, costs more, weighs more, and occupies more space than the 600-volt d-c type or the modern a-c motor.

7. With the a-c motors transformers are required on the locomotive or car, which adds to the cost and weight of the equipment.

8. The 600-volt d-c motor costs less to maintain and is liable to fewer operating troubles than any of the other motors.

9. With the commutating type of a-c motor the power lost in the control equipment is practically negligible, since the "potential" type of control can be used. For both the d-c motor and the induction motor a resistance control is necessary, with consequent loss in power (see Art. 5, Control Systems for Railway Motors).

10. The induction motor is inherently a constant-speed machine, and consequently the power input varies directly as the opposing resistance. The d-c motor and the a-c commutator motor are inherently variable-speed machines, and the power input varies approximately as the square root of the opposing resistance, the speed at the same time falling off.

11. The three-phase induction motor, when kept connected electrically to the source of power, automatically operates as a generator when the train is going down grade at a speed greater than the synchronous speed of the motor, the motor thus returning power to the line and at the same time acting as a brake preventing any considerable increase in speed. "Regeneration," as this action is called, can also be obtained with the other types of motors but only at increased expense for the additional control equipment required.

Examples of the Use of the Various Systems. Tables I and II list the most notable examples of electrified steam railroads.

ANALYSIS OF AN ELECTRIC RAILWAY PROJECT. In the analysis of a particular problem the general line of procedure indicated below is followed, referring to Art. 2 for methods of calculation.

1. Determine the number and capacity of cars to supply the service desired.

2. Determine the power and energy required to propel these cars at the schedule speed desired.

3. Select the motors to correspond to the power determined in 2.

4. Lay out distribution of cars, by train diagrams if necessary.

5. Calculate the capacity of the low-potential distribution system and of the substations.

6. Determine the capacity of the generating stations and transmission system.

7. Estimate the first cost of the system.

8. Estimate the cost of operation.

9. Estimate the earning power of the system.

A. In the case of a new road the earning power must exceed the sum of the operating cost and fixed charges by an amount sufficient to pay dividends.

B. In the case of the electrification of a steam road it must be possible to show either: (1) that the result of electrification has been to reduce the operating charges by an amount more than sufficient to pay the fixed charges on the electrical apparatus, or, (2) that the result of electrification will increase the capacity of the road or attract sufficient new business so that the increased earning power will more than balance the increased fixed charges.

C. The study should include other traction systems, such as internal-combustion engine systems.

Table I. The Most Notable Examples of Electrified Steam Roads in the United States and Canada (1935)

Name of Road	Date Installed	Miles of Single Track	Main Line, Tunnel, or Terminal	Trolley or Third Rail	Trolley Voltage	System	Locomotive (L.) or Motor Cars (M.C.)
Baltimore & Ohio	1895	9.1	Tun.	3rd R.	650	D-c	L.
B. & O. Staten Island R. T.	1925	44.7	M. L.	3rd R.	650	D-c	M. C.
Boston & Maine, Hoosac Tun.	1911	21.4	Tun.	T.	11,000	S-p	L.
Boston, Revere Beach & Lynn	1928	34.5	M. L.	T.	600	D-c	M. C.
Butte, Anaconda & Pacific	1913	144.7	M. L.	T.	2,400	D-c	L.
Canadian National R. R.	1908	15.0	M. L.	T.	3,300	A-c	L.
C., M., St. P. & Pacific	1915	892.1	M.L.	T.	3,000	D-c	L.
Cleveland Union Terminals Co.	1930	56.0	Term.	T.	3,000	D-c	L.
Delaware, Lackawanna & Western R. R.	1930	160.0	Term.	T.	3,000	D-c	Both
Great Northern Railway	1927	87.5	M. L.	T.	11,000	S-p	L.
Illinois Central R. R.	1926	156.6	Term.	T.	1,500	D-c	Both
Long Island R. R.	1905	448.1	Term.	3rd R.	600	D-c	Both
Long Island R. R., Bay Ridge	1927	84.4	M. L.	T.	11,000	S-p	L.
Michigan Central R. R.	1910	28.5	Term.	3rd R.	600	D-c	L.
New York Central R. R.	1906	381.6	Term.	3rd R.	650	D-c	Both
N. Y., N. H. & H. R. R.—A-c	1907	597.5	M. L.	T.	11,000	S-p	Both
N. Y., N. H. & H. R. R.—D-c	1895	65.1	Term.	T. & 3rd R.	600	D-c	M. C.
New York, Westchester & Boston Ry.	1912	82.4	M. L.	T.	11,000	S-p	M. C.
Norfolk & Western Ry.	1915	208.4	M. L.	T.	11,000	S-p	L.
Norfolk & Southern Ry.	1910	54.3	M. L.	T.	600	D-c	Both
Pennsylvania Railroad Company	1930-1935	1353.6	M. L.	T.	11,000	S-p	Both
New York Terminal	1910	110.1	Term.	3rd R.	600	D-c	Both
Pennsylvania-Reading Seashore Line	1906	82.6	M. L.	3rd R.	600	D-c	Both
Reading Company (Philadelphia)	1931	197.8	Term.	T.	11,000	S-p	M. C.
Southern Pacific (O. A. & B. Div.)	1911	118.0	Term.	T.	1,200	D-c	M. C.
Virginian Railway	1925	231.0	M. L.	T.	11,000	S-p	L.

Table II. Selected Examples of Foreign Railroad Electrifications (1933)

Name of Road	Date Installed	Miles of Single Track	Main Line, Tunnel, or Terminal	Trolley or Third Rail	Trolley Voltage	System	Locomotives (L) or Motor Cars (M. C.)
Americas							
Canada							
Canadian National Ry.	1918	32.2	Tun.	T.	2,400	D-c	Both
Montreal Harbor	1922	63.0	Term.	T.	2,400	D-c	L.
London & Port Stanley	1915	44.9	Term.	T.	1,500	D-c	Both
Canadian Nat'l—Grand Trunk	1908	15.2	Tun.	T.	3,300	1-ph	L.
Cuba							
Hershey Cuban	1920	110	M. L.	T.	1,200	D-c	Both
Mexico							
Mexican Ry. Co., Ltd.	1924	70.23	M. L.	T.	3,000	D-c	L.
South America							
Bethlehem Chile Iron Mines Co.	1916	24.0	M. L.	T.	2,400	D-c	L.
Anglo-Chilean Consol. Nitrate Corporation	1927	33.47	M. L.	T.	1,500	D-c	L.
Chilean State Railways	1924	244.6	M. L.	T.	3,000	D-c	L.
Paulista Railway	1921	252.5	M. L.	T.	3,000	D-c	L.
Transandine Railway	1927	23.7	M. L.	T.	3,000	D-c	L.
West of Minas	1930	46.0	M. L.	T.	1,500	D-c	Both
Australasia							
Victorian Railways	1919	439.0	Term.	T.	1,500	D-c	M. C.
New South Wales Railways	1926	278.4	Term.	T.	1,500	D-c	M. C.
New Zealand Govt. Railways	1924	13.4	Tun.	T.	1,500	D-c	L.
New Zealand Govt. Railways	1929	18.6	Tun.	T.	1,500	D-c	L.
Austria							
Austrian Federal	1912	820.0	M. L.	T.	15,000	1-ph	L.

Table II. Selected Examples of Foreign Railroad Electrifications (1933)—*Continued*

Name of Road	Date Installed	Miles of Single Track	Main Line, Tunnel, or Terminal	Trolley or Third Rail	Trolley Voltage	System	Locomotives (L) or Motor Cars (M. C.)
Belgium							
Belgian State Railways........	1935	54	Term.	T.	3,000	D-c	M. C.
China							
South Manchurian Railway...	1914	157.8	M. L.	T.	1,200	D-c	Both
France							
Midi Railway................	1929	1502.5	M. L.	T.	1,500	D-c	L.
P. L. M. Railway............	1925	165.1	M. L.	3rd R.	1,500	D-c	L.
Paris-Orleans................	1924	1931.2	M. L.	T.	1,500	D-c	Both
State........................	1924	154.5	Term.	3rd R.	650	D-c	M. C.
Germany							
German State Rys.—16 2/3 cy..	1914–28	627.6	M. L.	T.	15,000	1-ph	Both
German State Rys.—15 cy.....	1913	63.0	M. L.	T.	15,000	1-ph	L.
German State Railways—25 cy.	1908–24	68.5	Term.	T.	3,000–6,000	1-ph	M. C.
German State Railways.......	1924	365.6	Term.	3rd R.	800	D-c	M. C.
Italy							
Italian State Railways........	1901–27	127	Term.	3rd R.	650	D-c	M. C.
Italian State—3-ph. 15 cy.....	1902	77	M. L.	T.	3,300	3-ph	L.
Italian State—3-ph. 16 2/3 cy...	1911–29	1366.8	M. L.	T.	3,700	3-ph	L.
Italian State—3-ph. 45 cy......	1928	150	M. L.	T.	10,000	3-ph	L.
Italian State—3000 V. D-c....	1927	87	M. L.	T.	3,000	D-c	Both
North of Milan..............	1929	62	M. L.	T.	3,000	D-c	Both
Valli Di Lanzo...............	1921	26.1	M. L.	T.	4,000	D-c	Both
India							
Bombay, Baroda & Central India	1928	75.1	Term.	T.	1,500	D-c	Both
Great Indian Peninsula Ry....	1926	571.4	M. L.	T.	1,500	D-c	Both
So. Indian Railway...........	1931	43.1	Term.	T.	1,500	D-c	Both
Japan							
Imperial Government Ry......	1908–28	602	M. L.	T.	1,500	D-c	Both
Morocco							
Moroccan Railway...........	1927	237.4	M. L.	T.	3,000	D-c	Both
Netherlands							
Netherlands Ry..............	1927	292.1	M. L.	T.	1,500	D-c	M. C.
Netherland East Indies							
Java State Railways..........	1925	165.0	Term.	T.	1,500	D-c	Both
Norway							
Norwegian State—15 cy......	1923	40.4	M. L.	T.	16,000	1-ph	L.
Norwegian State—16 2/3 cy....	1922–7	159.0	M. L.	T.	15,000	1-ph	L.
Norwegian State—16 2/3 cy....	1911	22.4	M. L.	T.	10,000	1-ph	L.
Poland							
Polish State Railways.........	1935	134	Term.	T.	3,000	D-c	Both
Spain							
Santander-Bilbao............	1928	30.6	M. L.	T.	1,650	D-c	L.
Spanish Northern............	1924	49.7	M. L.	T.	3,000	D-c	L.
Vascongados Railway.........	1929	153.6	M. L.	T.	1,650	D-c	L.
Spanish Northern............	1929	367.5	M. L.	T.	1,500	D-c	Both
South Africa							
South African Rys.—Natal....	1926	352.2	M. L.	T.	3,000	D-c	L.
South African Rys.—Cape Town	1927	61.9	Term.	T.	1,500	D-c	M. C.
Sweden							
Nordmark Klaralvens Ry.–25 cy.	1921	123.3	M. L.	T.	16,000	1-ph	L.
Swedish State Rys.—15 cy.....	1915	365.0	M. L.	T.	16,000	1-ph	L.
Swedish State Rys.—16 2/3 cy..	1925	488.0	M. L.	T.	16,000	1-ph	L.
Switzerland							
Swiss Federal—16 2/3 & 25 cy..	1906–28	2408.5	M. L.	T.	3,300 5,500 15,000	1-ph	Both
Bernese Alps—16 2/3 cy........	1910–27	110.0	M. L.	T.	15,000	1-ph	L.
Montreux-Oberland...........	1901	50.5	M. L.	T.	1,000	D-c	L.
Rhaetian Rys.—16 2/3 cy......	1913–22	198.5	M. L.	T.	11,000	1-ph	L.
United Kingdom							
London, Midland & Scottish...	1904	195.7	Term.	3rd R.	600	D-c	M. C.
London & Northeastern.......	1914	48.25	M. L.	T.	1,500	D-c	L.
Southern Railway............	1909–29	799.0	Term.	3rd R.	650	D-c	M. C.
London & Northeastern.......	1904	80.5	M. L.	3rd R.	600	D-c	Both

2. ENERGY REQUIREMENTS AND MOTOR EQUIPMENT

By W. A. Del Mar

From a consideration of the forces acting on a moving train it is possible to determine the motor capacity and energy required to operate it when the profile and contour of the road, the time table, and the characteristics of the available motors are known.

UNITS AND ABBREVIATIONS. Throughout this article the various quantities employed will be expressed in the following units unless specifically stated otherwise: distances in feet, weights in tons of 2000 lb, forces in pounds, speeds in miles per hour (abbreviated mph), accelerations in miles per hour per second (abbreviated mphps), mechanical power in horsepower, energy in watthours.

FORCES ACTING ON A TRAIN. The forces tending to accelerate a train are the tractive effort developed by the motors and the component of the weight along the track on down grades. The forces which retard the motion of the train are the various frictional forces and the component of the weight along the track on up-grades; also in braking, the frictional force due to the brakes. All the various frictional forces, except the braking resistance, such as track friction, journal friction, air friction, etc., which oppose the motion of a train on a straight track are usually considered together and are referred to as the "train resistance." The extra friction due to track curvature is usually considered as an equivalent up-grade.

Tractive Effort and Draw-bar Pull. The tractive effort of a motor is the force exerted by the motor at the rim of the driving wheel to which the motor is geared. The tractive effort of a locomotive is the force exerted by the locomotive at the rim of the drivers. The draw-bar pull of a locomotive is the force transmitted through the draw-bar of the locomotive,* and is less than its tractive effort by an amount equal to the resistance due to the rolling friction of the locomotive wheels on the track and the air resistance of the locomotive.

D-c motors deliver a uniform tractive effort for a given current; the tractive effort of a-c motors pulsates to some extent with the alternations of the current; the tractive effort of a steam locomotive varies from 28 to 50 per cent above and below its average value during each revolution of the drivers.

Train Resistance. The total train resistance may be expressed as the sum of four terms, described as follows by W. J. Davis, Jr., in the *General Electric Review*, 1926. It is now usual to add 10 per cent to the values obtained from these equations.

The first two terms of the equations represent journal friction almost entirely. They have been derived from dynamometer and coasting tests on standard freight and passenger cars and electric locomotives and are based on oil lubrication with average temperature conditions. Journal friction may be increased 20 to 40 per cent at temperatures below freezing.

The third term comprises resistances due to flange friction, concussion, swaying, and miscellaneous frictions proportional to the speed. The factor for this element is decreased by increase in length of truck wheel base and increased by poor roadbed conditions and inferior riding qualities of motor cars.

The last term gives air resistance for average weight of car or locomotive in pounds per ton for standard types of equipment. No allowance is made for head winds or strong side winds.

Locomotive resistance represents tractive effort delivered to driving axles and does not include friction losses in gears, motor bearings, or other parts of the driving equipment, as these are usually covered in the motive power efficiency.

The formulas in Table III are based on tests taken under mild weather conditions. Values obtained from them may be used as modified above in calculations relating to electric distributing systems, substations, energy consumption, and power demand. In the determination of electric motor characteristics and gear reductions to meet particular speed requirements, however, it may be desirable to add a small percentage to the required speed as a protection against unusual conditions.

The accuracy of these formulas at very high speeds is unconfirmed.

Train Resistance at Starting. The formulas given above are not applicable to speeds below about 10 mph. The New York Central tests on electric trains show that the train resistance decreases with decrease in speed to 10 mph, but that as the speed still further

* In the case of a steam locomotive, the draw-bar pull usually refers to the force transmitted through the coupling between the tender and train; i.e., the tender is considered as a part of the locomotive.

Table III

Symbols

R = Tractive resistance in pounds per ton (2000 lb) on tangent level track.
A = Area in square feet of cross-section of locomotive or car body and trucks.
V = Speed in miles per hour.
n = Number of axles per car.
w = Average weight per axle in tons.
wn = Average weight of locomotive or car.

Values of A

Locomotives: 50 tons	105	sq ft
" 70 tons	110	"
" 100 tons and over	120	"
Freight cars	85–90	"
Passenger cars	120	"
Multiple-unit cars	100–110	"
Motor cars: 2 trucks	80–100	"
" " 1 truck	70–75	"

Where Used		Usual Formulas Recommended for convenience in calculation. Approved for axle weights in excess of 5 tons.	General Formulas Applicable to all axle weights. To be used when axle weights are less than 5 tons
Locomotives		$R=1.3+\frac{29}{w}+0.03\,V+\frac{0.0024\,AV^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.03V+\frac{0.0024AV^2}{wn}$
Freight cars		$R=1.3+\frac{29}{w}+0.045\,V+\frac{0.0005\,AV^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.0005AV^2}{wn}$
Passenger cars (vestibuled)		$R=1.3+\frac{29}{w}+0.03\,V+\frac{0.00034A\,V^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.03V+\frac{0.00034\,AV^2}{wn}$
Multiple-unit trains	Leading car. (vestibuled)	$R=1.3+\frac{29}{w}+0.045\,V+\frac{0.0024\,AV^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.0024AV^2}{wn}$
	trailing cars	$R=1.3+\frac{29}{w}+0.045\,V+\frac{0.00034\,AV^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.00034\,AV^2}{wn}$
Motor cars		$R=1.3+\frac{29}{w}+0.09\,V+\frac{0.0024\,AV^2}{wn}$	$R=\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.09V+\frac{0.0024AV^2}{wn}$

decreases the resistance per ton increases. The resistance at starting may be from 6 to 18 lb per ton, depending upon the condition of the bearings, track, etc., and upon the duration of the stop preceding the starting. These figures also apply to freight trains. Tests on the Rock Island system showed that in a train which had stood overnight in cold weather (i.e., which had become " frozen up "), the starting resistance was 30 lb per ton. The slack in the car couplings, however, renders it unnecessary for a locomotive to exert sufficient effort to start all the cars at once.

Grades and Curvature. An actual up-grade of G per cent produces a retarding force of 20 G pounds per ton, and a down-grade of G per cent produces an accelerating force of 20 G pounds per ton. A curve gives rise to a retarding force which may be represented by the formula:

$$r = 0.058\,SC \tag{1}$$

where r = (excess) curve resistance in pounds per ton.
S = speed, miles per hour.
C = degree of curvature.

(E. C. Schmidt and H. H. Dunn, Univ. of Illinois, Bulletin 92.)

Note that for angles of curvature up to 12 deg the angle in degrees may be taken equal to 5730 ÷ R where R is the radius of curvature in feet.

ACCELERATION AND BRAKING. The permissible rate of acceleration depends upon a number of factors.

1. The rating of the motors; the larger the motors the higher the tractive effort they can develop and therefore the greater the acceleration.
2. The weight on the driving wheels of the car or locomotive; the maximum tractive effort that a motor car can exert without slipping the wheels is from 15 to 20 per cent of the weight on the drivers (see below under Adhesion Coefficient).
3. The comfort of the passengers. This also depends to some extent upon the uniformity of the acceleration.
4. To make a given schedule speed with the least amount of energy the acceleration rate should be as high as possible. Very high rates of acceleration, however, are not in general justified on this score, as the increase in the size of the motors required may more than offset the saving of energy.

The rates of acceleration in Table IV represent common practice:

Table IV. Acceleration Rates

Service	Miles per hour per second
Steam locomotive, freight service...........	0.1 to 0.2
Steam locomotive, passenger service.........	0.2 to 0.5
Electric locomotive, passenger service........	0.3 to 0.6
Electric motor cars, interurban service.......	1.0 to 1.5
Electric motor cars, city service.............	2.0 to 4.75
Electric motor cars, rapid transit service.....	1.5 to 2.0
Highest practical rate, service conditions.....	2.5 to 4.75
Highest practical rate, emergency...........	7 to 8

Braking. The maximum retardation in braking is limited by the comfort of the passengers, and injury to equipment, a retardation of 1.5 mphps being the usual practical limit for electric or steam passenger trains, although 2.5 mphps is sometimes attained. For freight trains the braking retardation is from 0.7 to 8.0 mphps. The higher the rate of braking the less the energy consumption for a given schedule speed.

The tractive effort required to give to 1 ton (2000 lb) a linear acceleration of 1 mphps is 91.2 lb. To accelerate a train of W tons requires a tractive effort of 91.2 aW lb to produce a linear acceleration of a miles per hour per second, but on account of the accompanying angular acceleration of the rotating parts an additional force is required. This additional force is proportional to the linear acceleration a and also depends upon the radius of gyration (see Vol. I, Fund. of Eng.) of all rotating parts, and upon the gear ratio of the motors (i.e., ratio of number of teeth in gear to number of teeth in pinion). The effect of the moment of inertia may be looked upon either as increasing the effective weight W or as increasing the acceleration constant, the acceleration constant being defined as the quotient of total accelerating force divided by the product of weight and linear acceleration.

The increase in effective weight (in tons) due to any rotating axle or wheel is $\dfrac{M}{2000}\left(\dfrac{K}{r}\right)^2$ where M is the weight in pounds of the part in question, K its radius of gyration, and r its actual radius, both in feet. Each motor armature adds to the effective weight $\dfrac{M}{2000}\left(\dfrac{\rho K}{r}\right)^2$ tons, where M is the weight of the armature in pounds, K the radius of gyration of the armature in feet, r the radius in feet of the wheel to which it is geared, and ρ is the gear ratio. The total additional weight W_r is the sum of the above items for all the rotating parts. The total force in pounds required to produce the acceleration of a miles per hour per second is then CaW, where W is the actual weight of the train in tons and $C = 91.2\left(1 + \dfrac{W_r}{W}\right)$. This quantity C is the **corrected acceleration constant** and this corrected value should be used in all calculations.

The acceleration constant is raised by the flywheel effect discussed above by about 5 per cent (i.e., $W_r/W = 0.05$) for heavy cars and locomotives, and between 5 per cent and 10 per cent for light low-speed cars, 8 per cent being an average figure. However, C is usually taken as 100, correspondence to an increase in effective weight of about 10 per cent. A given linear acceleration of a miles per hour per second then requires an accelerating force of 100 a pounds per ton.

TRACTIVE EFFORT REQUIRED.

Let F = tractive effort, in pounds per ton, exerted by motors.
G = per cent actual grade (+for up-grade).
g = degrees of curvature.
r = train resistance, in pounds per ton.
a = acceleration in miles per hour per second (−for retardation).

Then the tractive effort required per ton of total train weight is

$$F = 100\,a + r + 20\,G + g \tag{2}$$

Example. Given a train of three 45-ton cars moving with a speed of 20 mph and accelerating at a rate of 1.5 mphps up a 1 per cent grade on a straight track; what is the total tractive effort required?

Answer: $(100 \times 1.5 + 7.8 + 20 \times 1)3 \times 45 = 24{,}000$ lb

GEAR RATIO AND SPEED. By gear ratio is meant the ratio of the number of teeth in the gear on the wheel axle to the number of teeth in the pinion on the motor shaft. A gear ratio greater than 6 : 1 is seldom used for railway motors. For a given torque developed by the driving motor, the tractive effort at the wheel rim and the linear speed for a given current depend upon the gear ratio and wheel diameter. Let D = the diameter of the wheel in inches, K = the gear ratio, F = the tractive effort for a given current

input; then the tractive effort F_1 for this same current input but for a wheel diameter D_1 and gear ratio K_1 is

$$F_1 = \frac{DK_1}{D_1 K} F$$

If V is the speed corresponding to the tractive effort F for a given motor voltage, then the speed V_1 corresponding to the tractive effort F_1 for the same motor voltage and current is

$$V_1 = \frac{D_1 K}{DK_1} V$$

If the gear ratio is low, the maximum speed will be high and the rate of acceleration low; if the gear ratio is high, the maximum speed will be low and the rate of acceleration high.

For a given motor equipment, train weight, schedule, and profile the energy consumption and temperature rise of the motors depend upon the gear ratio selected, since this in turn determines the amount of coasting; see below on Importance of Coasting. The proper gear ratio can be found only by trial calculation, plotting speed-time and distance-time curves from motor curves based upon different gear ratios, and calculating the energy consumption and temperature rise in the motors as described below.

MAXIMUM POSSIBLE TRACTIVE EFFORT—ADHESION COEFFICIENT. The adhesion or "tractive" coefficient is the quotient (expressed usually as per cent) of the tractive effort in pounds which will slip the drivers, divided by the weight in pounds on the drivers. Burch gives the values in Table V. The maximum possible tractive effort is the product of the adhesion coefficient (as a decimal fraction) by the weight (in pounds) on the drivers.

Table V. Adhesion Coefficients

Condition of Track	Without Sand	With Sand
Most favorable condition	35	40
Clean, dry rail	28	30
Thoroughly wet rail	18	24
Greasy moist rail	15	25
Sleet-covered rail	15	20
Dry-snow-covered rail	11	15

Maximum Grade Train Can Ascend.

Let W = total weight of train in tons.
W_d = total weight on all drivers in tons.
p = adhesion coefficient in per cent
r = train resistance in pounds per ton of total weight.
G = per cent grade.
g = degree of curvature.
a = acceleration in miles per hour per second.

Then the maximum tractive effort which the drivers can exert is 20 pW_d pounds, and therefore $(r + 20\,G + g + 100\,a)\,W$ must be less than 20 pW_d, or the maximum per cent grade which the train can ascend is *

$$G = \frac{pW_d}{W} - \frac{(r + g + 100\,a)}{20} \tag{3}$$

This grade is greater the less the acceleration, or the greater the retardation. The greater the speed before the train strikes the grade, the greater may the retardation be without bringing the train to rest on the grade, and therefore the steeper the grade it may ascend.

Example. Assume no acceleration or retardation and no curvature, a train resistance of 8 lb per ton, an adhesive coefficient of 15 per cent, and 25 per cent of total weight of train on drivers. Then the maximum grade the train can ascend is $G = 15 \times 0.25 - 8/20 = 3.35$ per cent.

The highest permissible grade is when all the weight is on the drivers, e.g., single cars or trains of motor cars with all axles equipped with motors. On steam freight roads the maximum grade seldom exceeds 2 per cent, and is usually considerably less, except in very mountainous country.

Weight of Locomotive. The weight of locomotive required to accelerate a train weighing W tons at the rate of a miles per hour per second up a grade of G per cent on a g degree curve against a frictional resistance of r pounds per ton, when the q per cent of the

* To be exact W_d should be multiplied by $\sqrt{1 - (G/100)^2}$, but except for very heavy grades this correction is negligible.

weight is on the drivers and the coefficient of adhesion is p per cent, is given by the following formula:

$$\text{Weight of locomotive} = \frac{5\,W}{pq}(100\,a + r + 20\,G + g)$$

Example. What weight of locomotive is required to accelerate a 400-ton train at the rate of 0.5 mphps up a 0.1 per cent grade against a frictional resistance of 8 lb per ton, when 80 per cent of the weight is on the drivers and the coefficient of adhesion is 20 per cent.

$$\text{Weight of locomotive} = \frac{5 \times 400}{20 \times 80}(50 + 8 + 2) = 75 \text{ tons.}$$

POWER REQUIRED AT GIVEN SPEED.

Let r = train resistance in pounds per ton of total train weight.
G = per cent grade.
g = degree of curvature.
a = acceleration in miles per hour per second.
v = speed in miles per hour.
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99\,v(r + 20\,G + g + 100\,a)$ watts per ton, or $p_0 = 2.67 \times 10^{-3}\,vW(r + 20\,G + g + 100\,a)$ horsepower, total. (4)

The *power input* p_i to the car or locomotive is equal to the power at the rims of the drivers divided by the over-all efficiency ϵ of the controller, motors and gears, i.e.,

$$p_i = \frac{1.99\,Wv(r + 20\,G + g + 100\,a)}{1000\,\epsilon} \quad \text{kilowatts} \tag{5}$$

Efficiency of Motors. The overall efficiency of the motors and gears when the motor is operating at full line voltage does not vary considerably for loads ranging from 50 to 150 per cent of rated load. The maximum efficiency is usually at about rated load, and has the values given in Table VI. At 50 and 150 per cent load, the efficiency may be from 3 to 10 per cent less, depending upon the design and the type of motor. The variation in efficiency with load is usually greater with a-c series than with d-c motors.

Table VI. Maximum Overall Efficiency of Motors and Gears at Rated Voltage

Horsepower, 1-hr rating	Kind of Motor	Maximum Efficiency per cent
30–100	D-c single-reduction spur geared...	83–88
30–100	D-c single reduction worm geared..	79–83
30–100	D-c double reduction spur geared..	82–86
100–250	D-c geared.....................	88–89
250–500	D-c gearless....................	91–93
50–200	A-c series geared................	70–80*
200–500	3-phase induction geared.........	85–89

* Including step-down transformers.

Average Over-all Efficiency During Controller Period. The overall efficiency of the motors and controller depends upon the type of control employed and upon the resistance inserted or the connections made by the controller. Most modern controllers for interurban cars are provided with a current-limiting device which limits the current to a given value, usually the value of the current corresponding to the 1-hr rating of the motor. Under these conditions the *average* overall efficiency of the motors and controller over the whole of the *controller period* (i.e., from time of starting until full line voltage is established across each motor or until all resistance is cut out) may be expressed as follows, on the assumption (which is very nearly exact) that the speed of the motor varies directly as the voltage impressed across its terminals:

Let ϵ = the overall efficiency of the motors and gears at end of controller period. Then for straight resistance control with d-c or induction motors the average efficiency during the controller period is $\epsilon/2$; for series-parallel control with 2 d-c motors the average efficiency during the controller period is $2\,\epsilon/3$; with 4 d-c motors first all in series, then 2 series sets in parallel, and then all 4 motors in parallel the average efficiency during the controller period is $8\,\epsilon/11$; for a-c series motors with voltage control the average efficiency during the controller period (including losses in step-down transformers) is about 0.90ϵ. Hence for a motor having an 85 per cent efficiency at the 1-hr rating and the starting current limited to the corresponding current rating, the average overall efficiency of the controller, motors, and gear during the controller period is

For straight resistance control................. 43 per cent
For series-parallel control, d-c motors.......... 57 per cent
For series-series-parallel control, d-c motors..... 62 per cent

Example. Given a train of three 45-ton cars each car equipped with two 200-hp d-c motors. What is the input to each motor if the speed is 30 mph and the train is accelerating at 1.0 mphps up a 2-per-cent grade, the track being straight, and full line voltage being across each motor?

Answer: From Table III the train resistance is $r = 10.5$, and from eq. (4) the total power output is

$$p_0 = 2.67 \times 10^{-3} \times 30 \times 135(10.5 + 20 \times 2 + 100 \times 1)$$
$$= 1627 \text{ hp}$$

or $1627/6 = 271$ hp per motor. Assuming an efficiency of 87 per cent, the input to each motor is $271 \times 0.746/0.87 = 232$ kw, and the total input to the train $6 \times 232 = 1392$ kw.

If the train accelerates at this same rate from rest to a speed of 30 mph at the end of the controller period (series multiple control) the grade being 2 per cent throughout, the average output of the motor would be the output corresponding to half speed or 15 mph, or 136 hp per motor, requiring an average input of $136 \times 0.746/0.57 = 178$ kw, and the average input to the train during the controller period would be $6 \times 178 = 1068$ kw.

SPEED-TIME AND DISTANCE-TIME CURVES. To determine the energy required to propel a car or train a given distance over a given track in a given time requires the consideration of a number of factors which can best be taken into account by the construction of various kinds of time curves. Such curves may be constructed with practically

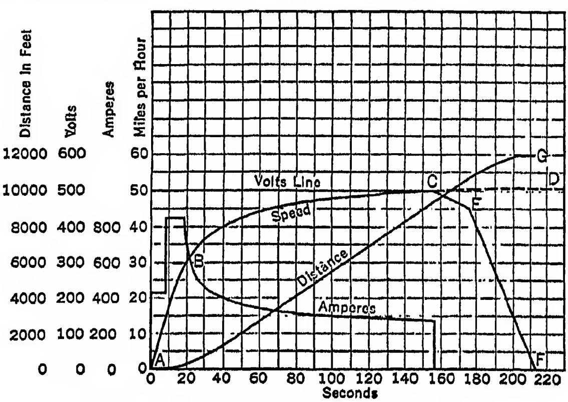

Fig. 1. Speed, Distance, and Current Curves

any degree of accuracy desired when the profile of the track, the weight of the train, the various resistances, schedule speed, time of stops, etc., are accurately known. Such data are, however, seldom known with any great precision, and consequently elaborate methods of plotting and calculation are seldom justified. Below will be given (1) some results of actual tests, (2) a rough but simple method of approximating the energy requirements, (3) a step-by-step method, which, though tedious in application, is capable of any degree of accuracy desired, provided the given data are accurately known, and (4) an outline of an analytical method by which the effect of changes in the time of coasting, rates of acceleration and braking, etc., may be predetermined.

The following terminology will be employed:

Speed-time Curve. A curve showing the speed (in miles per hour) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve *ABCEF* in Fig. 1. A speed-time curve may be conveniently divided into four parts, namely:

Controller Period. The period from starting until full line voltage is established across each motor; i.e., the portion *AB* of the curve in Fig. 1.

Motor-curve Period. The period during which the motor is operating on full line voltage; i.e., the portion *BC* in Fig. 1. The relation between speed and tractive effort during this portion of the run is fixed by the motor characteristics, specifically by the speed-torque curve of the motor and the gear ratio.

Coasting Period. The period during which the car or train is coasting; i.e., the portion *CE* in Fig. 1.

Braking Period. The period during which the brakes are applied; i.e., the portion *EF* in Fig. 1.

Distance-time Curve. A curve showing the distance covered (in feet) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve AG in Fig. 1.

Current-time Curve. A curve showing the line current (in amperes) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve marked "Amperes" in Fig. 1.

Voltage-time and Power-time Curves. Curves showing respectively the voltage per motor (or the line voltage) and the power input to the motor (or train) plotted as ordinates against elapsed time (in seconds) as abscissas.

Average Speed and Schedule Speed. The average speed V is the total distance run L' (in miles) divided by the time (in hours) the train is actually running. The schedule speed S is the total distance run (in miles) divided by the total time (in hours) of the run from one end of the road to the other, including time of all stops at intermediate stations. If the *total* time of all the *intermediate* stops is T_s' seconds, then

$$V = \frac{S}{1 - \dfrac{T_s' S}{3600 L'}} \tag{6}$$

where V and S are in miles per hour and L' is the *total* length of route in *miles*.

Duration and Frequency of Station Stops. The duration of each stop for surface cars ranges from 5 to 10 sec. for elevated and subway trains from 10 to 30 sec. for interurban trains from 10 to 40 sec. The stops per mile are the reciprocal of the average distance in miles between stops. For the six most important elevated and subway lines in Europe and America, the stops per mile average 2.5.

Average Equivalent Grade (G). Grades may be taken into consideration by calculating the sum H_1 of all the rises on up-grades and the sum H_2 of all the drops on down-grades, and taking for the average "equivalent" up-grade in percentage

$$G = \frac{100(H_1 - 0.5\,H_2)}{L} \tag{7}$$

where H_1 and H_2 are in feet and L is the total length of the route in feet. On a round trip $H_1 = H_2$, and the "equivalent" grade in per cent is

$$G = \frac{50\,H_1}{L}$$

This method of dealing with grades is equivalent to assuming that half the kinetic energy stored in the train on down-grades is utilized in taking the train up the following up-grade. The amount of energy thus rendered available of course will depend upon the amount of braking necessary on down-grades to prevent excessive speeds and also upon the location of the stops with respect to the grades. The figure $^1/_2$ is taken as an approximate average; it may be varied as seems reasonable in view of the actual profile.

Average Angle of Curvature (g). Curves may be taken into account by finding the average curvature, i.e., finding for each curve the product of the degree of curvature by the length of the curve, adding all these products, and dividing by the length of the route.

ENERGY CONSUMPTION FROM TESTS. Table VII gives the energy consumption, as found by tests, for a number of typical services. This table will be found useful as a

Table VII. Electric Trains: Energy at Collector, from Actual Tests

	New York Central R. R.	New York, New Haven & Hartford R. R.					Interborough Subway, N. Y.		Urban Trolley Lines
							Express	Local	
Service	P.	F.	F.	P.	P.	P.	P.	P.	P.
A-c or D-c	D-c	A-c	A-c	A-c	A-c	A-c	D-c	D-c	D-c
Locomotive or cars	L.	L.	L.	L.	L.	C.	C.	C.	C.
Train weight, including locomotive, tons	1226	3060	4049	909	519	529	355	226	4–12
Length of run, miles	32.7	69	87	60	60	21	10.2	15.2	....
Average grade, per cent	0	0.033	−0.055	−0.02	−0.02	−0.033	0	0	....
Average speed, including stops	37.4	21.7	16.1	47.4	38.7	23	24.4	16.5	....
Stops per mile, including one terminal	0.092	0.043	0.034	0.017	0.18	0.67	0.59	2.6	2–10
Energy consumption, watt hours per ton-mile	28.3	17.0	22.2	28.3	48.8	72.0	43.6	104.2	100 to 160

rough check on any calculations made for a specific service. Methods of making such calculations are given below.

APPROXIMATE METHOD OF CALCULATING ENERGY CONSUMPTION. The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the step-by-step method.

When the method is applied to checking purposes, the column of Table VIII headed "Actual Energy Output" should be used, and the input calculated from the known efficiencies. When applied to rough calculations, the column headed "Approximate Electrical Energy Input" should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required.

Table VIII. Output at Wheel Rim and Input to Car in Watthours per Ton-mile

Energy for	Actual Energy Output at Wheel Rims of Cars	Approximate Electrical Energy Input to Cars
Acceleration	$\dfrac{V^2_m}{36.2L}$	$\dfrac{K^2 n V^2}{25}$
Train resistance	$\dfrac{1.99\, rL_p}{L}$	$\dfrac{2.9\, r}{Q}$
Grades	$\dfrac{39.8\, GL_p}{L}$	$\dfrac{57\, G}{Q}$
Curves	$\dfrac{1.99\, gL_p}{L}$	$\dfrac{2.9\, g}{Q}$
Total	Sum	Sum

Let V = average running speed in miles per hour.
V_m = maximum speed in miles per hour.
L = length of run in miles.
L_p = distance traveled, with power on, in miles.
n = $1/L$ = number of stops per mile including one terminus.
r = average train resistance, in pounds per ton. (Say that corresponding to a speed from 10 to 20 per cent greater than the average speed.)
G = average equivalent grade, in per cent.
g = average curvature in degrees.

$$K = \frac{V_m}{V} = \text{ratio of maximum to average speed; see Table IX.}$$

$$Q = \frac{L}{L_p} = \text{ratio of length of run to distance traveled with power on; see Table VIII.}$$

NOTE: $25 = 36.2\,\epsilon$, $57 = 39.8 \div \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of 1 lb per ton, which is probably high.

Table IX. Values of K and Q

(D. C. Woodbury and W. A. Del Mar)

Stops per Mile n	K: Locomotive Passenger Trains	K: Single Cars, Multiple-unit Trains and Freight Trains	Q: All Trains	Stops per Mile n	K: Locomotive Passenger Trains	K: Single Cars, Multiple-unit Trains and Freight Trains	Q: All Trains
0	1.00	1.00	1.00	1.4	1.93	1.59	2.34
0.1	1.18	1.10	1.11	1.6	1.94	1.62	2.44
0.2	1.35	1.18	1.24	1.8	1.94	1.65	2.52
0.3	1.48	1.25	1.38	2.0	1.95	1.68	2.58
0.4	1.60	1.31	1.52	2.5	1.95	1.75	2.71
0.5	1.68	1.36	1.67	3.0	1.96	1.80	2.81
0.6	1.75	1.40	1.78	3.5	1.96	1.85	2.87
0.7	1.82	1.44	1.89	4.0	1.97	1.90	2.91
0.8	1.86	1.47	1.99	4.5	1.97	1.94	2.95
0.9	1.90	1.50	2.07	5.0	1.98	1.97	3.00
1.0	1.93	1.52	2.15	over 5.0	2.00	2.00	3.00
1.2	1.93	1.56	2.24				

Example. A multiple-unit train has a speed (excluding stops) of 25 mph and makes 0.8 stop per mile. It ascends an average grade of 0.143 per cent. What will be its energy consumption in watthours per ton-mile?

From Table IX we find for $n = 0.8$, that $K = 1.47$ and $Q = 1.99$. Then using the formulas in Table VIII, the results in Table X are obtained, assuming 6.5 lb per ton for friction.

Table X

Energy for	Watthours per Ton-mile	
Acceleration	$\frac{1.47^2 \times 0.9 \times 25^2}{25} =$	48.5
Train resistance	$\frac{2.9 \times 6.5}{1.99} =$	9.5
Grades	$\frac{57 \times 0.143}{1.99} =$	4.1
Total		62.1

Efficiency of Run. The formulas given above enable one to judge the effect, upon the energy consumption, of altering any of the principal physical elements upon which the run is based.

From the formulas for kinetic energy it is obvious that a low value of K means a low energy consumption. A low value of K, however, means a "square" speed-time curve; i.e., for low energy consumption the controller, acceleration, and braking periods should be as short as possible, or in other words, the rate of acceleration and braking should be as great as practicable.

The quantitative effect of changing any of these variables may be estimated by the analytical method outlined on p. 22.

This example worked out by the step-by-step method gave 60.5 whr per ton-mile.

STEP-BY-STEP METHOD OF PLOTTING SPEED-TIME CURVES. There is no way of exactly predetermining a speed-time curve except by a number of successive trials. That is to say, the time the current is kept on, the time of coasting, and the time of braking must each be guessed, and it is usually necessary to make a number of trials, by varying the proportion of motor run, coasting, and braking, before the given distance is traversed in the desired time.

If the characteristics of the train and its equipment are expressed numerically, the principles of mechanics enable such trial runs to be plotted on paper and the proper proportion of motor run, coasting, and braking selected to make the train travel the desired distance in the given time. For a given motor equipment on a given route it is possible to plot by a step-by-step method these speed-time curves, and then from these curves and the characteristic curves of the motors the various characteristics, such as energy consumption, root-mean-square current, etc., may be determined. The accuracy of this method depends solely upon the accuracy with which the assumed data are known. The necessary data are:

Profile and alignment of road.
Characteristic curves of the motors.

W = total weight of train in tons.
T = the time of run in seconds between successive stops.
I_0 = the permissible starting current, or
a = the acceleration in miles per hour per second during the controller period.
b = the braking rate in miles per hour per second.
r = the train resistance in pounds per ton at any speed.

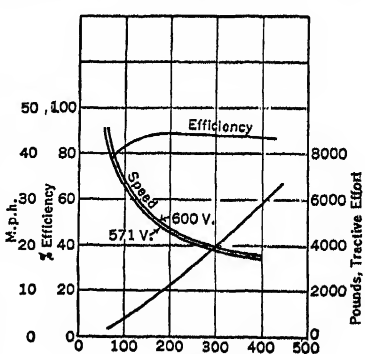

Fig. 2

Determination of Acceleration and Retardation Rates. From the motor characteristics, which are usually given in the form indicated in Fig. 2, determine the tractive effort and speed corresponding to the permissible starting current. This is usually taken as the current corresponding to the 1-hr rating; in the case of the motor whose characteristics are shown in Fig. 2, this current is 315 amp, the speed 19.6 mph, and the tractive effort 4300 lb. If each car is equipped with two motors, the weight corresponding to each motor is half the weight of the car. If the cars weigh 44 tons each, then the tractive effort per ton developed by the motors at this speed is 4300/22 = 195 lb per ton. At the point where the motors are changed from series to parallel the speed will be approximately one-half the speed at rated voltage or 9.8 mph. If the average line voltage is less than the

rated voltage, these speeds should be reduced in proportion to the ratio of the actual to the rated voltage. In the case selected the average line voltage is 571 volts; hence the speeds corresponding to the parallel and series points are 18.6 and 9.3 mph respectively. These speeds and the corresponding tractive efforts are entered into Table XI.

Table XI. Acceleration Rates

Speed, mph		Motor Tractive Effort, lb per ton	Train Resistance, lb per ton	Acceleration rates, mphps, = a						
				Per Cent Equivalent Grade Given in First Line						
	v	f	r	+3	+2	+1	0	−1	−2	−3
Accelerating, power on	9.3	195	6	1.29	1.49	1.69	1.89	2.09	2.29	2.49
	18.6	195	8	1.27	1.47	1.67	1.87	2.07	2.27	2.47
	20	159	9	0.90	1.10	1.30	1.50	1.70	1.90	2.10
	22	112	10	0.42	0.62	0.82	1.02	1.22	1.42	1.62
	25	74	11	0.03	0.23	0.43	0.63	0.82	1.03	1.23
	30	45	13	......	......	0.12	0.32	0.52	0.72	0.92
	35	27	15	......	......	......	0.12	0.32	0.52	0.72
	40	18	18	......	......	......	0.00	0.20	0.40	0.60
Retarding, power on	40	18	18	−0.60	−0.40	−0.20	......	......	......	......
	35	27	15	−0.48	−0.28	−0.08	......	......	......	......
	30	45	13	−0.28	−0.08	......	......	......	......	......
Coasting, no power	50	0	34	−0.94	−0.74	−0.54	−0.34	−0.14	+0.06	+0.26
	45	0	31	−0.91	−0.71	−0.51	−0.31	−0.11	+0.09	+0.29
	40	0	28	−0.88	−0.68	−0.48	−0.28	−0.08	+0.12	+0.32
	35	0	25	−0.85	−0.65	−0.45	−0.25	−0.05	+0.15	+0.35
	30	0	23	−0.83	−0.63	−0.43	−0.23	−0.03	+0.17	+0.37
	25	0	21	−0.81	−0.61	−0.41	−0.21	−0.01	+0.19	+0.39
	20	0	19	−0.79	−0.59	−0.39	−0.19	+0.01	+0.21	+0.41

NOTE. The train resistance is assumed to be 6 lb per ton from zero speed to 9.3 mph.

From the motor characteristics determine the tractive effort (f) in pounds per ton and the corresponding speeds (v), corrected for line voltage, up to the maximum permissible speed, and enter these in Table XI. Also enter in this table the train resistance corresponding to the various speeds. In the table given, the train resistance is calculated from Burch's formula for three 44-ton cars, the cross-section of each car being 120 sq ft, and the constant K is taken equal to 0.0050 (subway service). The formula is then

$$r = 4.0 + 0.13\,v + 0.0055\,v^2$$

where v is the speed in miles per hour.

The available tractive effort in pounds per ton for acceleration on any grade is then $f - r - 20\,G'$, and the acceleration rate is (assuming an acceleration constant of 100)

$$a = \frac{f - r - 20\,G'}{100}$$

where G' is the "equivalent" grade (including curvature, see above), in per cent. The value of a is calculated for grades of 0, 1, 2, 3, etc., per cent, both up and down, including the largest up- and down-grade. These values are entered in Table XI. The maximum speeds are the speeds for which the acceleration becomes zero. These can be obtained by plotting the accelerations against speed, and finding the speed at which the curves cross the speed axis.

Similar calculations should also be made for the retardation on up-grades when the train strikes such a grade, with power on, at a higher speed than the free running speed on the grade, and also for the coasting period when no power is on. When power is on, the friction of the gears and motors is allowed for in the motor tractive effort curve, the tractive effort curve being the gross tractive effort less these losses. The friction of the gears and motors may be taken as approximately 5 per cent of the rated tractive effort, which, in the special case under consideration, is $0.05 \times 195 = 10$ lb per ton. The total train resistance in coasting is then the normal train resistance plus the resistance of gears and motors.

In the tables a + acceleration signifies an actual increase in velocity, a − acceleration a retardation, or decrease in velocity.

The braking rate is assumed constant, usually $b = 1.5$ mphps.

Construction of Acceleration and Retardation Time Curves. The next step is to con-

struct a set of acceleration speed-time and distance-time curves, a set of coasting speed-time and distance-time curves, and a braking speed-time and distance-time curve. The construction of these curves is based on the following relations:

Let v_1 = the speed in miles per hour at time t_1.
v_2 = the speed in miles per hour at time t_2.
$v = \dfrac{v_1 + v_2}{2}$ = average speed during the interval $t_2 - t_1$.
a_1 = the acceleration in miles per hour per second at time t_1.
a_2 = the acceleration in miles per hour per second at time t_2.
$a = \dfrac{a_1 + a_2}{2}$ = average acceleration during the interval $t_2 - t_1$ (for the first step take a speed corresponding to half the speed at end of controller period).
x_1 = the distance in feet from the starting point at time t_1.
x_2 = the distance in feet from the starting point at time t_2.

Then, for a small change in speed,

$$t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \text{seconds} \tag{8}$$

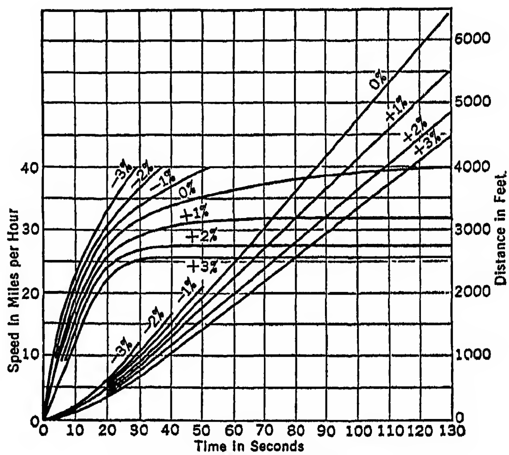

FIG. 3. Accelerating Curves

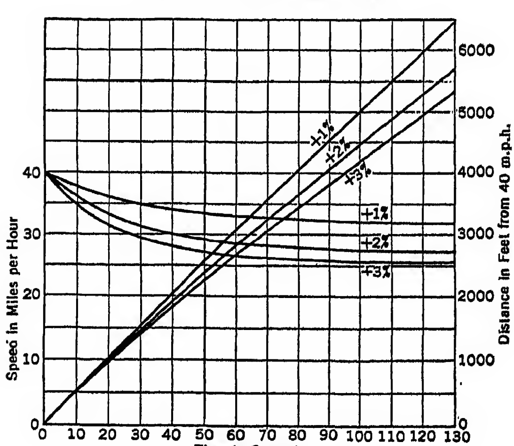

FIG. 4. Retarding. Power On

and the distance covered in this interval is

$$x_2 - x_1 = 1.466\, v(t_2 - t_1) \qquad \text{feet} \tag{9}$$

or, when the speed is plotted against time,

$$x_2 - x_1 = 1.466 \times (\text{Area of speed-time curve between } t_2 \text{ and } t\,) \tag{10}$$

From eq. (8), using the values of a given in Table XI, the time at which any speed is reached may be calculated. The results of such calculations for the special case under consideration are given in Table XII, and are plotted in Figs. 3, 4, and 5. The distance-time curves are found by planimetering the speed-time curves and multiplying by 1.466 (see eq. 9), and are also plotted in Figs. 3, 4, and 5.

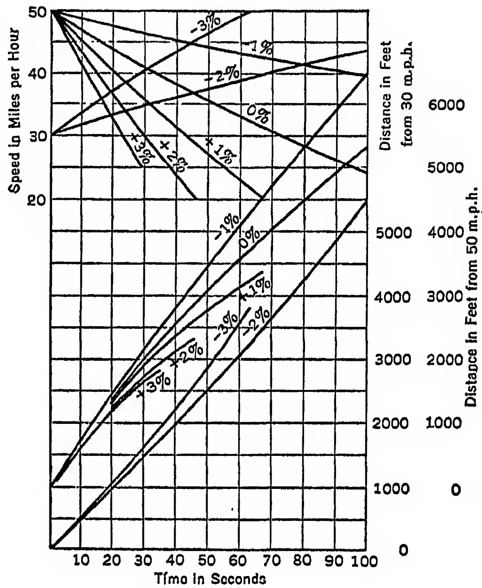

FIG. 5. Retarding, Power Off

The speed during the braking period t seconds *before* the train stops is $v = bt$, and the distance to travel to come to rest is $x = {}^1/_2\, bt^2$.

In the special case of a braking rate of 1.5 mphps,

$$v = 1.5\, t \text{ miles per hour}$$

and the distance to travel to come to rest is

$$x = \frac{1.466 \times 1.5\, t^2}{2}$$

The corresponding curves are given in Fig. 6.

Time Curves for Given Profile and Alignment. With these four sets of curves the speed-time curve for any given run with this particular equipment may be rapidly constructed. For intermediate grades interpolation may be readily made. In Fig. 7 is given a profile and alignment between two stops. The first step is to make up a table like Table XIII, dividing the route into sections such that the " equivalent " grade (= actual grade plus, say, 0.05 per cent for each degree of curvature, assuming a resistance of 1 lb per ton per degree of curvature) is the same throughout each section.

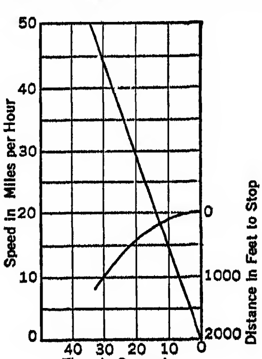

Fig. 6. Braking

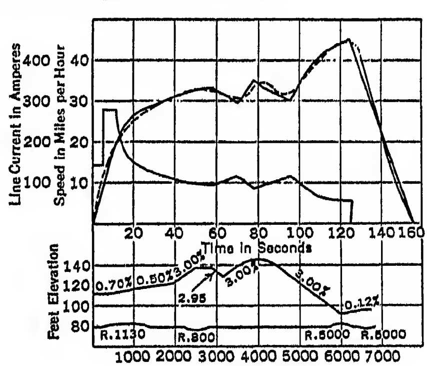

FIG. 7

For a complete round trip over the entire route the time curves must be plotted for the entire route in both directions. The run in one direction between two stations only will be considered in the numerical calculations given below.

Next lay off on a piece of tracing cloth, see upper part of Fig. 7, a distance equal to

Table XII. Data for Accelerating, Coasting, and Retarding Speed-time Curves

	Speed v	Total time in seconds to accelerate from rest						
		Per cent equivalent grade						
		+3	+2	+1	0	−1	−2	−3
Accelerating, power on	0.0	0	0	0	0	0	0	0
	9.3	7	6	6	5	4	4	4
	18.6	14	12	11	10	9	8	7
	20	17	13	12	11	10	9	8
	22	20	16	14	12	11	10	9
	25	33	23	19	16	14	12	11
	30	Max. speed = 25.3	Max. speed = 27.5	37	26	21	18	16
	35	......	......	Max. speed = 32	49	33	26	22
	40	......	......	......	132	52	37	29
		Total time in seconds to retard from 40 mph						
Retarding, power on	40	0	0	0	......	......	......	......
	35	9	15	36	......	......	......	......
	30	22	43	Min. speed = 37	......	......	......	......
	....	Min. speed = 25.3	Min. speed = 27.5	......	......	......	......	......
		Time in seconds required for speed to decrease 5 mph to speed given in first column.					To increase 5 mph from given speed	
Coasting, power off	45	5	7	10	15	40	67	18
	40	6	7	10	17	53	48	16
	35	6	8	11	19	77	37	15
	30	6	8	11	21	125	31	14
	25	6	8	12	23	250	28	13
	20	6	8	13	25	......	25	13

Table XIII. "Equivalent" Grades

Stop	Distance between Stops, ft	Length of Section, ft	Per Cent Grade = G	Radius of Curvative, ft	Degree of Curvature = g	"Equivalent" Grade $G' = G + 0.05\,g$
A	......	505	......	......	......	......
	......	145	......	1130	5.0	+0.25
	......	908	+0.70	1130	5.0	+0.95
	......	77	+0.50	1130	5.0	+0.75
	......	633	+0.50	......	......	+0.50
	......	273	+3.00	......	......	+3.00
	......	273	+3.00	800	7.2	+3.36
	......	247	......	800	7.2	+0.36
	......	94	......	......	......	......
	......	308	−2.95	......	......	−2.95
	......	800	+3.00	......	......	+3.00
	......	1862	−3.00	......	......	−3.00
	......	192	−3.00	5000	1.2	−2.94
	......	234	+0.12	5000	1.2	+0.18
	......	178	+0.12	......	......	+0.12
	......	69	+0.12	5000	1.2	+0.18
	......	358	......	5000	1.2	+0.06
B	7321	165	......	......	......	......

the time of run between the two stations (156 sec in the example), to the same scale as Figs. 3 to 6. The braking speed-time curve can be laid off directly at the far end of the run by placing Fig. 6 under the tracing cloth and tracing the curve. Similarly, by placing Figs. 3 and 4 under the tracing cloth and tracing for the proper distance the curve corresponding to the proper grade, an acceleration curve can be built up until this curve intersects the braking curve. If the total distance, as read off from the corresponding distance-time curve, is greater than the actual distance, it will be necessary to introduce a coasting period of proper duration to make the total distance as read off from the distance-time

curves equal to the actual distance. This can be done by placing Fig. 5 under the tracing cloth, and by cut and try finding the proper amount of coasting.

If there are curves or crossovers, requiring a reduction in speed at certain points, these reductions should be allowed for in plotting the speed-time curve. A rough rule for the permissible speeds on properly constructed curves is that

$$\text{Speed on curve} = \sqrt{\text{Radius of curve}},$$

where the radius is in feet and the speed in miles per hour.

The dotted curve in Fig. 7 was obtained from test.

Current-time Curve. The motor current for any speed may be taken directly from the speed-time curve and motor characteristics. From the motor currents the line current is readily found by multiplying by *half* the number of motors during the *series* portion of the controller period and by the total number of motors during the rest of the time power is on. Current-time curves (line current) for the various grades may also be drawn once for all in the same manner as the speed-time curves in Fig. 3. The curve with the square shoulder in Fig. 7 is the current-time curve for the example considered.

Watthours per Ton-mile. During the first half of the controller period the input per motor is equal to the product of the current per motor by approximately one-half the line voltage (series-parallel control assumed); during the rest of the time that power is on, the input per motor is equal to the product of the motor current by the line voltage. Hence, calling A the area of the current-time curve from the start until power is shut off, and I_0 the starting current, E the average line voltage, T_1 the seconds' duration of the controller period, L the length of the run in miles, M the number of motors, and W the weight of the train in tons, then to a fair approximation,

$$\text{Watthours per ton-mile} = \frac{EM(A - 0.25\, I_0\, T_1)}{3600\, WL}$$

A more accurate, but tedious, method is to plot a power-time curve by multiplying the total current per train at successive intervals of time by the average line voltage during this interval, and then integrating this curve to find the total watt-seconds. This area, in watt-seconds, divided by (3600 WL) will give the watthours per ton-mile. In making such a calculation note that during the series part of the controller period the current per train is equal to the current per motor multiplied by *half* the number of motors.

Root-mean-square Current per Motor. This may be found by squaring the ordinates of the current-time curve (current per *motor*), plotted as described above, and dividing by the time of run including stops and taking the square root of the quotient. Or the current-time curve may be plotted in polar coordinates, taking time in seconds as the angle in degrees and current in amperes as the radius vector. See Fig. 8. Calling B the area of this curve, the unit of area being the square whose side has a length corresponding to 1 amp, then the root-mean-square current is

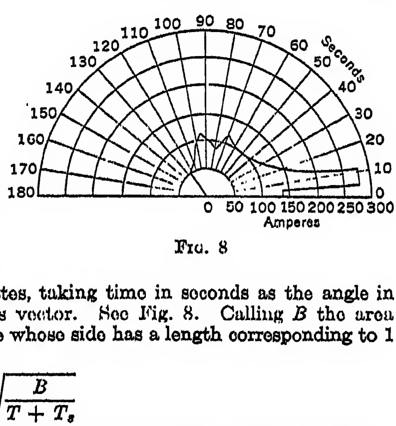

FIG. 8

$$I_e = 10.7\sqrt{\frac{B}{T + T_s}}$$

where T is the time that the train is moving and T_s the standing time, both in seconds.

Average Motor Voltage. For a simple run, such as shown in Fig. 1, average motor voltage during the controller period is equal to approximately 55 per cent of the line voltage, assuming 10 per cent of line voltage across the motors at the instant of starting.

Let E = average line voltage.
T_1 = time of controller period, from speed-time curve.
T_2 = time motor is running on full line voltage.
T = total time that the train is moving.
T_s = total standing time.

Then average motor voltage for the entire run is

$$E_m = \frac{0.55\, ET_1 + ET_2}{T + T_s}$$

For a complex run, where the controller is shut off and put on again during the run, a voltage-time curve may be plotted and the average ordinate obtained by integration.

MOTOR CAPACITY. A railway motor is usually rated in terms of the output in kilowatts or horsepower which it will give when run for 1 hr at rated voltage with a temperature rise above the surrounding air not exceeding 75 deg cent in any part of the motor, other than the commutator. (See Standards of the A.I.E.E.)

The size of motor required for any particular service (i.e., for a given route, schedule, weight of car, line voltage, and per cent coasting) depends upon two factors: (1) the motor must be of such a size that the maximum current required will not produce harmful sparking at the brushes or dangerous mechanical stresses in any part of the motor, and (2) the temperature must not rise to a value which will cause the insulation to deteriorate.

Size of Motor Limited by Commutation and Mechanical Stresses. The maximum current is usually that required at starting, and since the starting current remains practically constant up to the point where full line voltage is impressed across the motor, the corresponding maximum horsepower output of the motor can be calculated directly from eq. (5), when v is taken as the speed at the end of the controller period and W as the weight of the train per motor, i.e., W is taken equal to the total weight of train divided by the number of motors. A safe rule for non-interpole motors in single-car or multiple-unit service is to limit the starting current to a value equal to the rated current. For interpole motors in like service the starting current may safely be 25 to 50 per cent in excess of the rated current.

In locomotive work a heavier starting current is sometimes demanded, and due to the low acceleration rate during the starting period the motor must carry this current for a longer interval than in single-car or multiple-unit service. In selecting a motor for such service, information should be obtained from the manufacturer as to the maximum current which the motor can safely carry during a limited period, say for 5 min. This maximum current may be limited by sparking at the commutator, by mechanical stresses, or by local heating of the windings. See also Size of Motor Limited by Short-time Heating.

Size of Motor Limited by Heating. The heat developed in a railway motor is carried partly by conduction through the several parts and partly by convection through the air to the motor frame, whence it is distributed to the outside air. As the temperature of the several parts is thus dependent not only upon their own internal losses, but also upon the temperature of the neighboring parts, it becomes necessary to determine the actual value and distribution of losses in a railway motor for a given service in order to determine with precision what the temperature rise will be; or, vice versa, to determine what size of motor will be required to avoid too great a temperature rise.

For ordinary electric railway calculations, however, in view of the other uncertain elements which enter, it is usually sufficiently accurate to assume that the *relative* temperatures in the different parts of the motor are independent of the *relative* values of the copper and core losses. The copper loss, i.e., the rate at which heat is developed in the windings, is proportioned to the square of the current, and the core loss, i.e., the rate at which heat is developed in the core, due to hysteresis and eddy currents, may be taken as proportional to the first power of the voltage across the motor terminals (this latter relation being approximate only). When the motor reaches a constant temperature under a constant load, the temperature of any part will then be proportional (approximately) to the total power (kilowatts) lost in the windings and core. Similarly, under a fluctuating load continuing over a long period during which there are no excessively long breaks or excessive overloads, the temperature becomes fairly constant and the rise is proportional to the average power (kilowatts) lost during this period. There will be times at which the temperature rise will exceed this average and times at which it will be less, but on account of the heat-storage capacity (or thermal capacity) of the materials of which the motor is made the fluctuations in temperature will be very much less than the fluctuations in the load.

Size of Motor Limited by Average Temperature Rise. The manufacturers supply information as to the current which any motor will carry continuously (on stand test) without overheating, at various voltages from one-half to full voltage; see Table XIV. From this information, making the assumptions noted in the preceding paragraph, it is possible to determine the approximate temperature of the motor for any given run or series of runs. The process is to calculate the root-mean-square current per motor and the average motor voltage for the particular service contemplated, using the methods given above in the paragraph headed Root-Mean-Square Current per Motor and Average Motor Voltage. Call these values of the rms service current and average motor voltage, I_s and E_m respectively. Let I_c be the continuous-current capacity at a given voltage E_c, as given by the manufacturer (see Table XIV), and let T_c be temperature rise corresponding to this continuous rating. (Motors having ordinary fibrous insulation are rated on the basis of a 75 deg cent temperature rise on stand test, which corresponds to about 65 deg cent rise in actual service, due to better ventilation; see Standards of the A.I.E.E.; hence for

Table XIV (a). Rated and Continuous Capacity of Westinghouse Railway Motors

Type Westinghouse	Rated Hp	Rated Voltage	Rated Amperes	Continuous Current Capacity		Weight in Pounds
				At 300 Volts	At 450 Volts	
Surface Cars—Subway—Elevated						
508–A	25	600	37	32	35	1035
510–E	35	600	51	37	39	1475
514	40	600	60	35	36	1650
516–A	50	600	72	47	51	1950
535–A	60	600	85	52	54	2400
306–CV4	65	600	94	58	60	2750
548–C8	100	600	138	90	93	3175
333–V8	125	600	180	110	115	3850
557–A8	140	600	202	130	135	4050
570–D5	190	600	255	160	165	5300
Locomotives						
543–C7	55	600	79	48	50	3175
552–A7	75	600	106	62	63	4050
562–A5	100	600	140	35	87	4900
582–D5	140	600	195	115	120	6000
583–D5	240	600	326	185	190	6000
585–A2	310	600	415	290	300	6300

Table XIV (b). Rated and Continuous Capacity of General Electric Co. Railway Motors, 600-volt Rating

Type	Rated Hp, 1 hr	Rated Amperes, 1 hr	Continuous Current Capacity		Weight in Pounds
			At 300 Volts	At 450 Volts	
GE–264	25	37	30.0	33.0	1045
GE–702	35	50	41.0	43.0	1500
GE–247	40	60	34.0	37.0	1840
GE–301	50	70	51.0	55.0	2100
GE–1198	55	156	135.0	....	700†
GE–1196	60	172	155.0	....	1050†
GE–154*	65	90	68.0	73.0	785†
GE–706	100	138	114.0	118.0	2670
GE–259	120	171	120.0	127.0	4000
GE–254	150	212	124.0	131.0	4700
GE–714	190	255	160.0	165.0	5200
GE–700‡	255	137	93.0	96.0	6840

* Light-weight high-speed motors for trolley buses or street cars.
† Not axle suspended; hence weights not comparable with those of other motors.
‡ 1500/3000-volt motor—rating at 1500 volts rated amperes and at 750 and 1125 volts continuous.

ordinary motors T_c is 65 deg cent). Let J_c be the corresponding core loss and K_c the corresponding copper loss and $L_c = J_c + K_c$ be the corresponding total electrical losses. Then the total electrical losses corresponding to the average load are

$$L_a = \frac{E_m}{E_c} J_c + \left(\frac{I_c}{I_c}\right)^2 K_c$$

and the average temperature attained by the motor in service will be approximately

$$T_a = \frac{L_a}{L_c} \cdot T_c$$

For safe operation the average temperature rise T_a should never exceed the value T_c, which for motors with ordinary fibrous insulation is 65 deg cent.

Approximate Values of J_c and K_c. When the core loss and copper loss are not given separately, a rough estimate of J_c and K_c may be made by assuming that at rated load (1-hr rating and line voltage) the core loss is, say, one-fourth of the total electrical losses.

The total electrical losses L_r in kilowatts at rated load may be found from the characteristic curves of the motor by using the formula

$$L_r = P\left(\frac{0.97}{\varepsilon} - 1\right)$$

where P is the 1-hr rating in kilowatts, ε the efficiency of the motor with gears, and the 0.97 takes into account the frictional losses in the motor and the gears. Let I_r be the rated current and E_r the rated voltage; then

$$J_c = \frac{L_r}{4}\frac{E_c}{E_r} \qquad \text{and} \qquad K_c = \frac{3\,L_r}{4}\left(\frac{I_c}{I_r}\right)^2$$

Size of Motor Limited by Short-time Temperature Rise. When the service is such that the motor must take a heavy current for a comparatively long interval (e.g., a long starting period or a heavy grade for a considerable distance) followed by a like period of light load or no load, the average temperature for the run, as calculated above, may be within the required limits, but the short-time temperature rise may be excessive. This short-time temperature rise depends upon the heat-storage capacity of the motor, i.e., upon the energy loss (number of kilowatthours of heat developed in it) required to raise its temperature 1 deg, say, assuming no radiation of heat from its surface. The 1-hr rating of a motor is an indirect measure of this heat-storage or thermal capacity.

The temperature-time curve during the first hour's application of a load is practically a straight line whose slope is proportional to the load. The rise in temperature of the motor due to a short-time load may then be assumed to be proportional to the *energy* (kilowatthour) input during this time, and the factor of proportionality may be obtained from the 1-hr rating as follows: Let T_r = the temperature rise at the end of 1 hr due to a load equal to the 1-hr rating of the motor (rated current and rated voltage), L_r = the total electrical losses in kilowatts corresponding to the rated load, L_a = the total electrical losses in kilowatts corresponding to the average load in service (L_r and L_a may be estimated by the method given in the preceding section), L_p = the total electrical losses in kilowatts corresponding to any given short-time or peak load, t_p = the number of minutes' duration of this peak load. Then during this interval t_p the rise of temperature above the average value T_a, is

$$T_p - T_a = \frac{t_p\,T_r}{60\,L_r}\cdot(L_p - L_a)$$

T_p as calculated from this formula gives approximately the maximum temperature rise during the run. For safe operation this maximum temperature rise T_p should not exceed the safe limit stated by the manufacturer. For motors with ordinary fibrous insulation T_p should not exceed 75 deg cent.

Final Choice of Motor. No motor should be employed for a given service which does not meet the above requirements regarding the maximum current and heating limits. A larger motor than that fixed by these requirements may prove the cheaper in the long run, if by using such a motor the energy consumption can be materially reduced by increasing the amount of coasting during the run. In any event the motor should be of sufficient capacity to permit of a reasonable amount of coasting under normal conditions, so that there will be a sufficient margin in which to make up for lost time, due to unexpected slow-downs or extra stops.

ANALYTICAL METHOD OF PREDETERMINING ENERGY AND MOTOR EQUIPMENT. A useful analytical method was developed by Cary T. Hutchinson, in two papers in the *A.I.E.E. Trans.*, Vol. 19, p. 129, 1902, and Vol. 22, p. 657, 1903. In the method a speed-time curve similar in shape to the curve *ABCEF* in Fig. 1 is assumed. That is, the acceleration during the controller period, the train resistance, and the braking retardation are all assumed constant, but a "motor-curve" period (*BC* in Fig. 1) is also taken into account, this last constituting the essential difference between this method and and the "straight-line" speed-time curve method frequently employed for approximate calculations. The introduction of this motor-curve period in the calculations enables one to approximate much more closely actual working conditions, and the results are much more accurate. In addition this method enables one to predetermine, without choosing any particular equipment, the effect of rate of acceleration, rate of braking, per cent of coasting, etc. Where extensive calculations are to be carried out, it would be well worth while to master the above references and apply this method.

Importance of Coasting and Selection of Gear Ratio. A study of Fig. 9 shows that a much less energy consumption would be required for a coasting period of 50 per cent instead of the actual coasting period of 10 per cent, namely 72 watt-hr instead of 88 watt-hr. The starting current required in order to obtain this higher coasting time is 406 instead of 345 amp, or an increase in the starting current of 18 per cent. If no change in the size of

motor were made, this would require approximately the same percentage increase in the gear ratio. The root-mean-square current would increase from 131 to 150, but the average motor voltage would drop from 350 to 170. The motor could therefore probably operate at this higher gear ratio without seriously overheating, but it would be safer to use a larger motor, particularly as the starting current of 406 amperes is also close to the safe commutating limit.

TRAIN AND LOAD DIAGRAMS. The current-time curve for a train making a number of stops may be represented as shown by (*a*) in Fig. 9. On a railway line where there are several trains, the total current may be obtained by placing the current curve for each train at its proper place in the time scale, and adding the ordinates of the curves. Such a process is very tedious and unnecessary where there is a large number of trains. In such cases the high and low parts of the curves become staggered with respect to one another more or less according to the laws of chance, so that each current curve may be replaced by a rectangle of the same area but with a base extending over the entire running

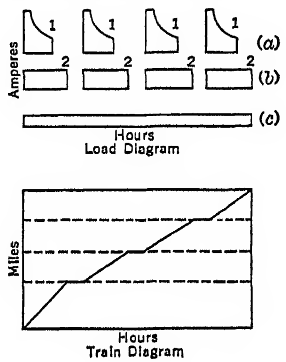

Fig. 9

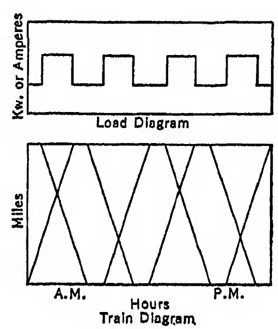

Fig. 10

time as shown by (*b*) in Fig. 9. When this is done the kilowatts and amperes per train are derived from the watthours per ton-mile by the following formulas:

$$\text{Kilowatts} = \frac{WV}{1000} \times \text{(Watthours per ton-mile)}$$

$$\text{Amperes} = \frac{WV}{E} \times \text{(Watthours per ton-mile)}$$

where W = weight of train in tons.
V = average running speed (excluding stops) in miles per hour.
E = line voltage.

The time when the current is cut off is indicated by 1, and that when the train stops, by 2, in Fig. 9.

Another approximation, which is even more often used, is to replace the series of rectangles shown at *b*, by a single rectangle as shown at *c* in Fig. 9. The area of this rectangle will be equal to the sum of the areas of the smaller rectangles or current curves. Using this approximation, kilowatts or amperes may be obtained from the above formulas, taking, however, V to be the schedule speed (i.e., speed including stops). The procedure is to plot a train diagram showing when each train comes on and off the line, neglecting intermediate stops, as shown for a simple case in Fig. 10. Each time a train comes on or off, the corresponding kilowatts or amperes are added to, or subtracted from, the load diagram.

Power Required for Car Heating and Lighting. In addition to the energy required for propelling the

Table XV. Heating and Lighting of Cars

Length of Car, ft	Average* Kilowatts for Lighting	Average† Kilowatts for Heating	
		Average Conditions	Severe Conditions
14–20	0.25	3.5	4
20–28	0.35	4.5	5.5
28–34	0.55	5.5	7.5
34–40	0.70	7.5	10.5

* During the hours lights are on, using tungsten lamps.
† During the time car is in service.

cars, a very appreciable amount is also required in the winter, for heating them, and a small amount at night for lighting. In making up a load diagram this energy should be included.

The average power for car heating varies, of course, with the climate and time of year. The figures in Table XV represent usual requirements in the northern parts of the United States.

Substation and Power-station Loads. The load diagram obtained as described above gives the total load at the trains. To obtain the load at a substation, the kilowatts or amperes must be increased by a suitable amount to allow for the losses in the distribution system. The load diagram of the power station should allow for all transmission and distribution losses between the power house and substation and also for all auxiliary power, such as that required for station lighting, shop machinery, etc.

3. ELECTRIC LOCOMOTIVES

By H. A. Currie and Robt. S. Rhodes

For many years a new type of electric locomotive was designed for nearly every new proposition. This was due in part to the wide diversity in the conditions to be fulfilled. However, with the very numerous applications of electric locomotives of from 30 to 80 tons weight for slow freight and switching service on interurban roads and in terminals, the double-truck bogie type finally demonstrated its superior fitness and came to be adopted almost universally in America for all work involving speeds less than 45 mph. For higher speeds special provision must be made for guiding the locomotive around curves by the addition of guiding trucks or axles, for placing as much of the weight of the motors as possible on springs and for raising the center of gravity to a reasonable height (5 ft or over). With a low center of gravity every sidewise movement of the mass of the locomotive strikes a blow sidewise on the track, but with a high center of gravity a side swaying is transformed into a downward thrust on the track. As the track is not usually designed to withstand great side thrusts it is better to avoid a low center of gravity in high-speed locomotives. For higher capacities, the cab underframe must be relieved of the draft forces by articulating the trucks with a hinge joint, and mounting the draft gear on the truck frame.

The coefficient of adhesion which can be relied upon in electric locomotives at running speeds is from 18 to 22 per cent, tending to decrease at higher speeds. It is higher for electric than for steam locomotives on account of the uniform torque of the electric motor, while the steam locomotive has the advantage of coupled drivers which minimizes the effect of local slippery spots on the rail. The coefficient at starting is usually taken at 25 per cent, and with clean dry rails may at low speeds be as high as from 30 to 40 per cent.

CLASSIFICATION. Locomotives are usually classified by the arrangement of their wheels and the subdivision of the wheels into driving wheels and guiding wheels. The Whyte classification for steam locomotives has been applied to electric locomotives to some extent, but a distinct system, which distinguishes more clearly between driving and idle axles, has come into use both in the United States and abroad. Numerals are used to represent the number of idle axles, either in a guiding truck or simple weight-bearing axles in a driving truck. The number of adjacent drivers in one truck or driving wheel-base is indicated by the letters A, B, C, D, etc., for one, two, three, and four axles respectively. For example, the symbol 2 – D – 2 represents an electric locomotive which, in the Whyte classification, would be designated as 4 – 8 – 4. If two driving wheel-bases are used, two letters are required separated by a minus sign in case of a non-articulated unit (transmitting the tractive force through the truck center pins and the cab underframe) or a plus sign in case of an articulated arrangement. For example, 2 – C + C – 2 would represent two "ten wheeler" or 4 – 6 – 0 wheel arrangements coupled back to back with an articulated joint or hinge.

TYPES OF MOTORS. Locomotives are built with various types of motors and operate from various systems of electrical distribution, e.g.,

(*a*) D-c System. D-c series motors operating from 600,- 750,- 1200,- 1500,- 2400-, or 3000-volt trolleys. In all cases each motor must be insulated for full trolley voltage, but on the higher voltages, two or more motors are operated in series.

(*b*) A-c System.

1. A-c series motors fed from transformer secondaries at approximately 250 volts at 25 cycles. The contact line may be 3300, 6600, 11,000, or 22,000 volts, though 11,000 is by far the most common. Freedom from the restriction of inconvenient high trolley potential on the motor commutator is a strong point in favor of the a-c system.

2. D-c series motors fed from a motor-generator carried in the locomotive cab. This

makes available the advantages of the high-voltage, a-c distribution and the ruggedness of the d-c series motor. Economical use of this type of unit is confined to heavy-drag, slow-speed service.

3. Three-phase induction motors fed from a single-phase trolley through a Scott-connected transformer and phase converter. Acceleration is secured by inserting resistance, usually by means of a liquid rheostat, in the rotor circuits.

4. Three-phase induction motors fed through a transformer, from a three-phase trolley system. This system is widely used in Italy, but the only installation of its sort in the United States has been superseded. The complexity of the three-phase distribution is a great handicap to this system.

CONTROL SYSTEMS. The control of all modern electric locomotives is by some form of remotely operated power control, as the currents required are too large or the voltage too high for a hand-operated control. Many locomotives are equipped with multiple unit control, one engineman operating and controlling two or three locomotive units at the same time.

TYPES OF ELECTRIC LOCOMOTIVES. The simplest form of locomotive is a two-truck car ballasted to provide greater adhesion. Freight and baggage cars are frequently so used on interurban roads. In general, the ordinary car motors are not suited to locomotive service, being wound for too high speed.

The type of locomotive used for light haulage and terminal switching consists of a cab containing the control equipment and supported by two four-wheeled trucks with two axle-hung motors each. The tractive forces are transmitted through the cab underframe. Such units range in weight up to 100 tons and a horsepower of 1850. They are well adapted to switching purposes in terminal freight yards, and in smaller units to hauling freight trains on interurban electric railways.

A further development of this type of unit is exemplified in the Detroit River Tunnel locomotives of the Michigan Central Railroad. The heaviest of these units weigh 126 tons. They are similar to the type described above except that the two trucks are fastened together by an articulated coupling. The draft gear is carried at the ends of the truck frames, thereby relieving the cab underframe of all tractive and buffing forces.

A still further step is illustrated by the West Side freight locomotives of the New York Central Railroad which have two three-axle articulated trucks to obtain greater motor capacity and to subdivide the load sufficiently to keep the axle loading within limits.

The Cleveland Union Terminal passenger locomotives have a two-axle truck for guiding purposes at each end of a wheel-base similar to the New York Central units just described.

While the above types have been evolving by steps from the street-car prototype, another type has been developing, designed especially for high-speed railway service. Several earlier attempts have contributed features. The New York, New Haven and Hartford Railroad, in 1906, purchased 41 passenger locomotives having a number of new features. In addition to being the first a-c installation of considerable size, these locomotives introduced the quill drive. To relieve the dead weight on the axle, attendant upon an axle-hung motor, the motor armature was mounted on a quill which was frame-supported while surrounding the axle and driving through flexible spring elements. In 1912 the New Haven again advanced the art by introducing the twin motor. A gear was mounted on the quill which was supported in bearings below a frame-mounted twin motor, both pinions meshing with the gear.

Meanwhile the Pennsylvania was using locomotives patterned closely after the steam locomotive, driving from one or two large motors by means of side rods. Their most recent development, in connection with the most comprehensive electrification yet attempted, abandons the side rod in favor of the individually powered quill-drive axle mounted in a frame with integral cab similar to a steam locomotive. Their latest passenger unit has a $2 - C + C - 2$ wheel arrangement, each of the six driving axles having a twin motor and quill drive.

Table XVI gives data on a number of electric locomotives.

SPECIFICATION FOR ELECTRIC LOCOMOTIVE. The following memoranda are intended to assist in writing specifications. See also article on Specifications.

On important high-speed systems it is usual for the design of the locomotive to be worked out by the purchaser and manufacturer working in collaboration, and the design is usually specified in detail. In other cases, it is more usual to specify the operating characteristics and leave the design to the manufacturer. The following memoranda are to assist in the preparation of a specification of the latter type.

General Description of Service. Whether for direct or alternating current, single-phase or three-phase, freight or passenger hauling, overhead trolley or third rail. Line voltage, etc.

Specific Details of Work To Be Performed by Locomotive. Weight of cars loaded

Table XVI. Electric Locomotives

	Cleveland Union Terminals—Passenger	New York, New Haven & Hartford R. R.—Passenger	New York Central Railroad—Passenger	Chicago, Milwaukee, St. Paul & Pacific Railroad—Gearless Passenger	Chicago, Milwaukee, St. Paul & Pacific Railroad—Geared Quill Pass.	Great Northern Ry. Passenger and Freight M. G. Type	Great Northern Ry. Freight M. G. Type
Date installed	1929	1931	1926	1919	1920	1930	1926
System	D-c	Single-phase	D-c	D-c	D-c	Single-phase	Single-phase
Trolley voltage	3,000	11,000	600	3,000	3,000	11,000	11,000
Service	Passenger	Passenger	Passenger	Passenger	Passenger	Freight & pass.	Freight
Weight in working order (lb) { Total	419,000	403,000	285,000	528,000	620,000	539,000	368,600
Weight in working order (lb) { drivers	312,000	272,000	285,000	464,000	420,000	426,200	282,700
Weight of equipment	144,220	173,000	110,800	234,000	270,000	278,500	176,800
Number of motors	6	6 (twin)	8	12	6 (twin)	6	4
Diameter, drivers	48 in.	56 in.	36 in.	44 in.	68 in.	55 in.	56 in.
Total wheel-base	69 ft 0 in.	66 ft 0 in.	46 ft 5 in	67 ft 0 in.	79 ft 10 in.	58 ft 8 in.	62 ft 10 in.
Rigid wheel-base	15 ft 0 in.	13 ft 8 in.	6 ft 6 in.	13 ft 9 in.	16 ft 9 in.	15 ft 4 in.	16 ft 9 in.
Total horsepower—1-hr rating	2,900	3,440	2,488	4,020	4,680	3,260	2,165
Tractive effort—1-hr rating	29,200	25,200	18,440	59,500	66,000	67,200	56,250
Speed, miles per hour—1-hr rating	37.3	51.2	50.6	25.3	26.7	18.2	14.0
Classification	2–C+C–2	2–C+C–2	B–B+B–B	1B+D+D+B1	2–C–1+1–C–2	1C+C1	1–D–1

	New York Central Railroad—Freight	Mexican Railway Co.—Freight and Passenger	Virginian Railway—Freight	New York Central Railroad—Switcher (3-power)	Illinois Central Railroad—Switcher	Pennsylvania Railroad—Freight	Pennsylvania Railroad—Passenger
Date installed	1930–1	1925	1925	1930	1930	1932	1935
System	D-c	D-c	Single-phase	D-c	D-c	Single-phase	Single-phase
Trolley voltage	600	3,000	11,000	600	1,500	11,000	11,000
Service	Freight	Freight	Freight	Switching	Switching	Freight	Passenger
Weight in working order (lb) { Total	266,000	308,000	429,530	257,000	200,000	392,000	460,000
Weight in working order (lb) { drivers	266,000	308,000	315,100	257,000	200,000	220,000	300,000
Weight of equipment*	106,200	135,000	157,400	132,600	73,000	160,000	164,000
Number of motors	4	6	2	4	4	3 (twin)	6 (twin)
Diameter, drivers	44 in.	46 in.	62 in.	44 in.	45 in.	72 in.	57 in.
Total wheel-base	41 ft 0 in.	40 ft 6 in.	37 ft 6 in.	34 ft 1 in.	27 ft 3 in.	49 ft 10 in.	69 ft 0 in.
Rigid wheel-base	14 ft 6 in.	9 ft 2 in.	16 ft 6 in.	8 ft 3 in.	8 ft 7 in.	20 ft 0 in.	13 ft 8 in.
Total horsepower—continuous	2,490	3,030	2,030	1,665	1,550	3,750	4,620
Tractive effort—continuous	41,800	58,000	54,000	34,100	20,400	22,300	19,140
Speed, miles per hour—continuous	22.3	19.6	14.1	18.3	28.5	63.0	90
Classification	C+C	B+B+B	1–D–1†	B–B	B–B	2–C–2	2–C+C–2

* Includes all electrical equipment, and air brake parts except foundation brake rigging and air brake cylinders.
† Three cabs normally operated as a single locomotive.

and empty. Maximum train weight. Average train weight. Time to make typical run of stated length. Number and duration of stops in typical run. Ton miles per day per locomotive. Maximum speed on level with average load. Maximum speed on maximum grade with maximum load. Acceleration (miles per hour per second), with maximum load. Hours per day in regular service. Amount of time in shifting and yard service.

Profile and Plan of Line. Grades and curves.

Clearances and Limiting Dimensions. Gage of track, clearance diagram of right-of-way. Maximum and minimum height of trolley wire or third-rail location. Height of coupler. Wheels, tread and flange (A.A.R., or special). Weight of rail. Minimum radius of curve. Wheel diameter. Maximum permissible weight per running foot of right-of-way.

Operating Characteristics. Absence of nosing or lateral swing. Absence of rail pounding. Temperature-rise limitations. Efficiency.

Control. See Control Systems for Railway Motors.

Motors. See articles on Motors.

Air Brake. Straight, automatic, or combined. General characteristics.

4. ELECTRIC CARS AND TROLLEY BUSES

The cars used on electric railways are of both single- and double-truck types, closed bodies with fully enclosed platforms or vestibules being now almost universally used. The cross-bench continuous step open car, and the open platform car, long the standard, have been largely made obsolete in order to eliminate the accident hazard and to permit the same car to be used the year around, relieving railway companies of the burden of maintaining distinct summer and winter equipments.

SINGLE-TRUCK CARS. Single-truck cars were used extensively in the early days of electric railways, but were very generally replaced by the larger-capacity double-truck car; about 1914 the single-truck one-man safety car was developed, and many hundreds were placed in service. The characteristics of a typical single-truck safety car are given in Table XVII.

Table XVII

Item	Dimension
Length overall	27 ft 10 in.
Truck wheel-base	8 ft 0 in.
Seating capacity	32
Maximum capacity	65
Weight complete unloaded	16,000 lb
Weight complete with maximum load	25,100 lb
Maximum speed	25 mph
Schedule speed	10 mph
Number of motors	2
Motor capacity, total	70 hp

These cars are of light weight and economical construction and are equipped with motors, control, and air brake of such characteristics as permit them to meet the growing competition of buses and automobiles. The safety car makes possible faster and more frequent service at lower labor cost than the buses. Safety devices interlocking with the air-operated doors and air brakes are also included in the equipment to permit operation of cars by one man, who combines the duties of motorman and conductor.

TROLLEY BUSES. The trackless trolley or trolley bus has been in limited use for twenty years in many parts of the world. Large installations have been made recently, notably in Chicago. The modern buses have street-car-type bodies mounted on automotive chassis, with conventional bus wheels and tires, steering gears, air and hand brakes and axles, except that the rear axle is adapted for drive by two motors each driving one wheel only, without the usual differential. Electrical equipment comprises two 50-hp motors with automatic acceleration controller carried in a compartment at rear of bus, and a master controller actuated by pedal. A similar pedal mechanism operates the air brake valve thus duplicating driving conditions on gasoline buses including acceleration, steering, and braking. The characteristics of a typical trolley bus are given in Table XVIII.

Table XVIII. Trolley Bus

Item	Dimension
Length overall	32 ft 0 in.
Wheel-base	16 ft 0 in.
Seating capacity	40
Maximum capacity	80
Weight complete (unloaded)	17,500 lb
Weight complete with maximum load	29,500 lb
Maximum speed	35 mph
Schedule speed	12 mph
Number of motors	2
Motor capacity, total	100 hp

DOUBLE-TRUCK CARS. Electric railway cars are now almost entirely of double-truck type and range in size from 42 ft 0 in. overall length, with seating capacity of 42 passengers and maximum speed of 35 mph for city service, to 80 ft 0 in. overall length, seating capacity of 121 passengers, and maximum speed of 75 mph for interurban and electrified steam railroad suburban service.

The body of these cars is supported by two bolsters, each having a lubricated pivot bearing, and two bearing plates located about 2 ft 6 in. each side of longitudinal center line of car. These bearings rest upon similar bearings on the truck, body and trucks being held together by a large king pin. This arrangement permits the truck to swivel through a large angle in running on curves as sharp as 30-ft radius.

Construction of Trucks. The truck consists of a rigid frame of either structural steel, pressed steel, or cast steel, or a combination of all three, having vertical guides at all four corners in which slide journal boxes equipped with either plain babbitt-lined bronze bearings or anti-friction bearings. The journal boxes rest upon journals on the axles, which are of carbon or alloy steel, in some cases heat treated. The wheels and motor gears of wrought steel are pressed on and generally no keys are used, the torque of the motors being transmitted through the press fit. The car body is supported on truck bolsters which are carried on springs, either full elliptic, semi-elliptic, helical, or some combination of these, which rest on seats hung by swing hangers from the main cross-members of truck frame known as transoms, the seats being connected together across the truck, and the swing hangers permitting the bolster, springs, and seats to swing transversely when the car enters a curve. The truck frames are supported on springs resting on the journal boxes, or on equalizer beams spanning both journal boxes. Special wheel and axle arrangements such as radial axles and "maximum traction" trucks (having large wheels on one axle and smaller wheels on the other, and one motor per truck) are now seldom built. Trucks may be of motor or trailer types, arranged for either one or two motors. Wheels are from 22-in. diameter for city service to 42-in. maximum diameter for electrified suburban service, and wheel-bases are from 5 ft 2 in. to 8 ft 4 in. Truck weights for motor trucks range from 13,500 lb per pair to 56,250 lb per pair complete with motors.

Suspension of Motors. Motors, until recently, have been almost universally mounted by the nose suspension method with motors located between truck transoms and axles. Motor frames have two split bronze bushed bearings which bear on the axle, and a nose which bears on the transom, with wear plates and springs between nose and transom. Power is transmitted by single reduction spur gears, sometimes with helical teeth. Some experimental special drives with completely spring-borne high-speed motors have been applied, one embodying an application of automotive type axle and worm drive with armature shafts longitudinal, others using double reduction spur gears.

CAR BODIES. Most cars now have fully enclosed bodies, convertible, semi-convertible, and open bodies being now practically obsolete. The arrangement and dimensions of car bodies have become fairly well standardized according to the service for which they are designed, and of four general types: city, interurban, rapid transit (subway and elevated) and suburban. The following describes typical cars of each type.

City car bodies are of two different sizes, the smaller usually arranged for double-end operation and having exit and entrance doors at both ends, with an average seating capacity of 46, about two-thirds of the seats being of the transverse type. The larger type is arranged for single-end operation, has front entrance and center exit doors, seating capacity about 54, two-thirds of which is also transverse and located in rear of car to provide a large standing space in front portion of car and to facilitate free circulation of passengers entering and leaving. This type is used largely in the larger cities. Both types of city car bodies are equipped with deep spring-cushioned seats upholstered generally in leather.

Interurban car bodies are usually provided with front entrance and rear exit doors on both sides, but some have been built with a single entrance door at front for single-end operation by one man. Seating capacity is from 45 to 50, practically all transverse seats of the reclining type upholstered in various fabrics.

Rapid-transit car bodies are built with two, three, or four doors per side to facilitate loading and unloading from station platforms level with the car floor. Seats are provided for 50 to 65 passengers and are usually all longitudinal and upholstered in rattan.

Suburban car bodies for electrified steam railroad service are generally duplicates of steam road practice, modified by restrictions of weight and clearance. Enclosed vestibules are provided, with covered connecting doorways in some cases, for passing between cars. Seating capacity is from 82 to 120. Nearly all seats are transverse, upholstered in rattan or imitation leather.

Construction of Car Bodies. Car bodies are built entirely of metal except floors of city cars, which are usually of wood, and roofs, which are of wood, covered with canvas. Some of the recent designs of bodies have had very low weight per seated passenger,

achieved in some cases by the use of strong aluminum alloys and in others by careful design and arrangement.

DIMENSIONS AND WEIGHTS OF TYPICAL EQUIPMENTS. Table XIX gives the principal data for electric cars operated in various typical services.

Table XIX. Data on Electric Cars and Trolley Buses

	(1) City	(2) City Trolley Bus	(3) City	(4) City	(5) Interurban	(6) Subway N. Y. City	(7) Subway Philadelphia	(8) Suburban Lackawanna	(9) Suburban N. Y., N. H., & H. R. R.
Seating capacity	32	40	44	52	38	60	75	84	120
Number of trucks	1	1	2	2	2	2	2	2	2
Length overall, ft	28	32	41.5	46	43.75	60.5	67.5	70.3	79.6
Length of body, ft	18	30	40.5	45	43	59.2	66.2	60.0	69.0
No. of motors	2	2	4	4	4	2	2	4	4
Horsepower of each motor	35	50	35	50	100	190	210	230	260
Weight of body, lb, less electrical equipment	9,650	10,150	15,350	17,140	23,100	45,400	64,000	69,800	84,400
Weight of trucks, less motors, lb	4,000	5,000	8,350	11,440	12,300	25,400	31,800	35,700	43,200
Weight of electrical equipment, less motors	750	700	1,200	1,480	1,740	2,500	2,800	19,800	23,720
Total weight of motors, lb	2,100	1,650	6,100	8,740	10,660	10,700	11,400	21,900	26,680
Weight total, lb	16,500	17,500	31,000	38,800	47,800	84,000	110,000	147,200	178,000
Weight of seated passenger load, lb, at 140 each	4,480	5,600	6,160	7,280	5,320	8,400	10,500	11,750	16,800
Weight per seated passenger, lb	516	475	706	746	1,260	1,400	1,467	1,752	1,483

5. CONTROL SYSTEMS FOR RAILWAY MOTORS

The function of the control equipment is to regulate the speed and direction of the motors by certain definite systematic changes in connections. The speed of d-c railway motors is controlled in two ways: (1) by connecting suitable resistances in series with the motors, which will reduce the voltage across the motors and thereby the current which they will take; (2) by changing the connection of the motors so that they will be connected at first in series, thereby applying half of the line voltage to each motor, and then in parallel across full line voltage. In most control equipments a combination of both methods is used. For the speed control of a-c railway motors an auto-transformer, or compensator, is used instead of the resistances.

The direction of rotation of the motors, d-c or a-c, is changed by changing the direction of the current in either the fields or the armatures; it is customary to connect the terminals of each field coil to a reversing switch in order to accomplish this effect.

TERMINOLOGY. The following terms are in general use.

Cylinder or Drum Control or Direct Control are names commonly applied to an equipment in which all the connections are made by contacts on a cylinder or drum which is manually operated by the motorman and located on the platform. This may, therefore, be called direct control.

Multiple-unit, Indirect, Remote Control or Train Control are names applied to an equipment in which the changes in connection of the main power circuit are made by switches called "contactors," usually located underneath the floor of the car, and controlled by electric circuits coming from a small master controller on the platform. Two systems of multiple-unit control are in use in this country, viz.:

Magnetic Contactor System. In this system the contactors are closed by electromagnets which force a plunger against a spring, which normally holds the switch open.

Pneumatic Contactor System. In this system the contactors are closed by compressed air, from the air-brake system, the air valves at the switches being controlled electrically from the master controller.

Non-Automatic Control is a term applied to that method of control in which the motorman has it in his power to regulate the current to any value he pleases by moving the controller handle, the change in connections depending only upon the motion of the handle.

Automatic Control, as distinguished from non-automatic, is a type of control in which certain automatic devices prevent the motorman from causing the motors to take a cur-

rent greater than a predetermined value. With this method the motors start with a definite current and as soon as the current has decreased to a specified value a change in the connections is automatically made. Thus the rate of acceleration and the current are kept practically uniform throughout the period of control. It is nearly always used in connection with multiple-unit control.

Rheostatic Control consists in connecting a resistance in series with the motor and short-circuiting consecutively parts of this resistance. It is seldom used at present, except on mining locomotives and for single-motor operation.

Series-parallel Control, which is used on practically all railway equipments, includes the feature of connecting two motors and their resistances in series on the first step, then short-circuiting portions of the resistance consecutively until all resistance is cut out, under which condition the motors will operate efficiently at approximately half speed. On the next step of the controller the two motors with resistance in series are connected in parallel and subjected to full line voltage. There are three methods of accomplishing the change from series to parallel.

Transition with Power Off. In the so-called type L controller power is entirely cut off from both motors while the change in connection is being made. This was formerly used for large motors and locomotives but is not at present much used.

Transition with Series Resistance. During the transition from series to parallel a resistance is placed in series with one motor and the other motor is first short-circuited, then disconnected from the main circuit, and finally, placed in parallel with the other motor. This method is in general use in equipments of small motors with the so-called type K controller.

Bridge Transition. The so-called " bridge " method consists in grouping the motors and their resistances like the arms of a Wheatstone bridge, so that after the two motors are in full-series position the resistances may be placed in circuit again in parallel with the motors, without opening the circuit; the two motors are then connected in parallel with each other and each in series with its own resistance.

Series-parallel Control with Four-motor Equipment. Whereas the three methods just described apply particularly to two-motor equipments, they are equally applicable to four-motor equipments by connecting two motors permanently in parallel and treating them as a unit.

DRUM CONTROLLERS. The type of control as well as the construction of the controllers for ordinary single-car equipments have been practically standardized in this country, the chief manufacturers supplying control equipments which are practically identical. Railway drum controllers are designated by the type letters K, B, R, and L with appropriate subnumbers and subletters to distinguish smaller variations. The most common form is the K controller. On the first notch this controller connects the motors in series with each other and with the starting resistance. The resistance is then cut out by steps until the motors alone are in series. Transition is then effected from series to parallel connection of motors with resistance in the circuit. The resistance is then again cut out in steps until the motors are connected directly across the line. B controllers in addition to the starting steps include a certain number of braking steps. They find little use in the United States where air brakes are nearly universal. L controllers have connections similar to those of K controllers except that transition from series to parallel is made by completely opening the series circuit and then applying the parallel circuit; they are practically obsolete. R controllers provide resistance steps only, without series-parallel connection of motors. Applications of R controllers are largely confined to single-motor equipments and to very small locomotives where simplicity is desirable.

Railway drum controllers generally include two drums held in a cast frame. A cover of insulating material encloses the front and sides and when removed gives access to all working parts. The main drum, which makes the resistance and transition connections, consists of an insulated shaft carrying a set of contacts. Fingers press on these contacts to make the proper connections. Between fingers are barriers of arc-resisting material. A magnetic blowout operates between these barriers to assist in extinguishing arcs. The second drum is used to reverse the direction of motion of the motors. It consists of a wooden drum with copper contacts with fingers pressing on the contacts. It is mechanically interlocked with the main drum so that it cannot be moved with power applied.

Railway drum controllers find their widest application on cars used on city streets. Except in special cases they are not applied to equipments of an aggregate motor capacity of more than about 260 hp.

Where type K controllers are used for motors aggregating more than about 100 hp, they are sometimes adapted with a modification of the remote control by the addition of an electrically operated line breaker, placed underneath the floor of the car, the function of which is to open the main power circuit every time it is necessary that it should be

opened and thus remove all flashing and arcing from the controller. This expedient makes it possible to use a smaller controller for a given capacity of motors and obviates all danger to the passengers from fire and fright. The scheme is accomplished by substituting for the main power circuit on the controller an auxiliary circuit carrying only a fraction of an ampere, and every time this auxiliary circuit is opened in the main controller the line breaker underneath the car opens the power circuit. When the auxiliary circuit is closed the line breaker closes the main circuit. By means of an overload trip operated by a coil in the main circuit this breaker is also used as a circuit breaker, and if the current taken by the car exceeds a certain value a relay opens the auxiliary circuit which in turn causes the line breaker to open.

Capacity and Weight of Type K Controllers. The more usual forms of type K controllers and the capacity in motors for which they are adapted are as shown in Table XX.

Table XX. Type K Controllers

Type	No. of Motors	Maximum Allowable Capacity, each Motor, Hourly Rating Hp at 600 Volts	Number of Points		Approximate Weight, lb
			Series	Parallel	
K-35-JJ	4	65	5	3	290
K-35-KK	4	65	5	3	228
K-35-PP	4	65	5	3	228
K-35-QQ	4	65	5	3	290
K-68-A	2	70	4	4	225
K-39-C*	4	70	4	4	230
K-40-B*	4	65	5	3	280
K-51-D†	2	70	5	4	250
K-63-G	2	40	4	3	135
K-64-D	4	110	6	4	450
K-75-A	4	50	5	3	148
K-80-B	2	50	5	3	148

* For metallic-return circuit. † For tapped-field motors.

Method of Operation of the Type K Control. The principle of the type K control for small and moderate-size motors is shown in Fig. 11. The controller has an operating handle which moves the main cylinder and thereby changes the connections, and also a reversing handle which moves the reversing cylinder. The latter merely changes the direction of the current through the fields of all the motors with respect to the armature. These two handles or cylinders are interlocked so that the reversing handle can be moved only when the operating handle is in the off-position, thus preventing reversal with voltage on the motors.

One terminal of one of the motors is grounded throughout. The first three points are known as accelerating steps. As resistance is in circuit for each of these points the controller should not be left on any of them for a considerable length of time, for there is a considerable power loss in the rheostats and they are not designed for continuous operation. The fourth step, full series, is an efficient "running point," giving about half normal speed. The next two steps are transition steps and are not marked as points on the controller as they must be passed over rapidly. During this period one terminal of the second motor is grounded, thus short-circuiting the motor, which has one terminal grounded initially; the connection between the two motors is then opened; finally the

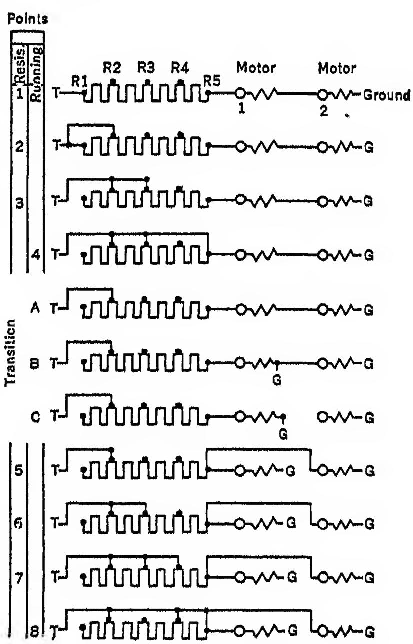

Fig. 11. Type K Control

two motors are connected in parallel but in series with a part of the rheostat. Points 5, 6, and 7 are accelerating steps with motors in parallel and resistance in series and are therefore not to be used continuously. At the last point the motors are in parallel, and all resistance is cut out; it is therefore an efficient high-speed running point. The two terminals of the armatures of each motor are led to fingers on the reversing cylinder as are also one terminal of each field and the two points of the main circuit to which the motors are connected. Thus, in reversing, the current is reversed in the armatures, but not in the fields.

In all controllers a magnetic blowout and an " arc chute " are employed to interrupt the current quickly and direct it away from the contacts, in order to prevent short-circuiting other contacts. In the older types of controllers this was obtained from one large magnet coil and a large iron pole piece covering all the contacts. In the later forms each contact has an independent blowout coil and small pole pieces. This gives a more powerful effect and more accurately directs the arc in the proper direction.

REASONS FOR MULTIPLE-UNIT CONTROL. When the total capacity of the motors on a car or locomotive exceeds 300 hp it is advisable, and when the capacity exceeds about 450 hp it is necessary, to use the indirect or multiple-unit control, for the cylinder type of controllers required to handle the large currents becomes too bulky and dangerous to place on the platforms of passenger cars. The cylinder control is also inadequate when it is desired to control simultaneously the motors on the several cars in a train, which is necessary in order to obtain the high tractive effort necessary in high-speed service on elevated and underground railways.

For these two reasons the system of multiple-unit or train control was developed. As originally proposed by Sprague this consisted of a large cylinder controller on each car and each controller was actuated by a small motor instead of by hand. These small motors were controlled synchronously from a single point by means of auxiliary control circuits. With the growth in the capacity of the motors this system became inadequate and was replaced by the systems now in use.

GENERAL ELECTRIC MULTIPLE UNIT CONTROL. Two types are in use, the type M and the type MA. The chief difference is that the type M is non-automatic

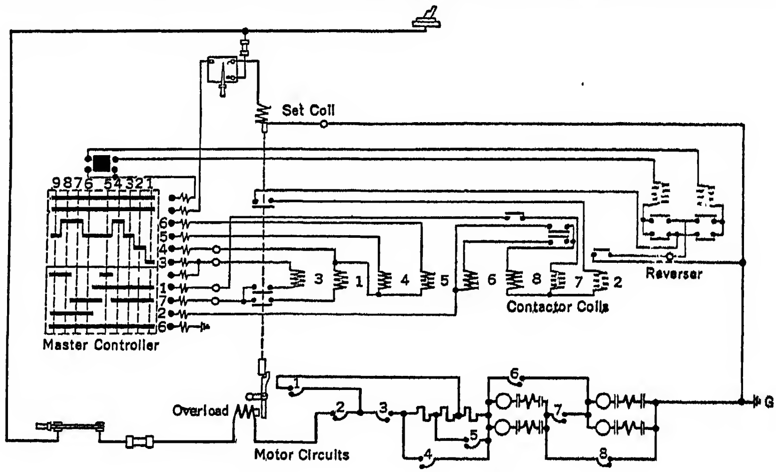

Fig. 12. General Electric Type M Control

whereas the type MA is provided with a current-limiting relay. Fig. 12 is a diagram of the type M, showing the control circuits in light lines and the motor circuits in heavy lines.

The material included in the control equipment of a motor car consists of:

2 master controllers.
1 motor controller containing 8 or 10 contactors and a reverser.
3 master control switches.
1 main switch.
1 main fuse box.
1 set of rheostats.
Cables, train couplers.

Master Controller. This is very similar to an ordinary railway drum-type controller but is much smaller, as the current carried by it is small. Each master controller is equipped with an operating handle, reversing handle, individual magnetic blowouts and (optionally) a "deadman's handle," which automatically interrupts the current and applies the brakes when the motorman's hand is removed from the button located in the top of the handle.

Motor Controller. This consists of an iron box lined with asbestos in which the several contactors and the reverser are placed and is mounted under the car.

Contactors. Each contactor consists of a powerful magnet operating an arm by means of a toggle joint against a spring pressure. This arm closes and opens the circuit in a strong magnetic field which acts as a blowout. As one contactor can carry and break currents of several thousand amperes no extra circuit breakers are required. Each contactor is provided with interlocks in the form of relays so that contactor 2 cannot be operated until after contactor 1 is closed, thus providing the proper sequence of operation under all conditions.

Reverser. This is a switch with several circuits and contacts and is comparable to the reverser cylinder in an ordinary controller. These contacts are mounted on a rocker-arm actuated by two electromagnets, one for moving the switch to the forward and the other to the reverse position. The electromagnets receive their current from the master controller and are interlocked so that only one can be operated at a time.

Switches and Fuses in Control Circuit. Motor cut-out switches are located on the reverser to permit cutting out a disabled motor. In the control circuit there is one main control switch and fuse to protect the control circuits, and near each master controller is located a "control and reset" switch, which in one position closes the circuit to the resetting coil of the overload relay in the main controller and in the other position closes the supply circuit for the master controller.

Main Switch. A knife-blade switch is placed in the power circuit and is intended to disconnect the motor circuits from the trolley when it is desired to test the motor controller.

Train Line Couplers are provided where cars are to be operated in trains. These couplers or jumpers provide a means of connecting together similar control circuits of the different cars. They contain from 8 to 12 wires and are so designed that it is impossible to couple the cars together improperly.

Current-limiting Relay. For automatic control, or acceleration at a predetermined current, a current-limit relay is provided on each car, and this prevents each successive contactor from operating until the current in the motors has decreased to a predetermined value. On roads having a fairly level profile and operating with frequent stops this refinement is desirable, as it makes it possible for the motorman to accelerate the train every time at the maximum allowable rate and yet never exceed that rate except on a down grade. When this relay is provided the control equipment is known commercially as the type MA control.

PC Control. This type was first introduced in 1914 and is a simplification of the older multiple unit systems. It uses air from the air-brake system to operate the switches in groups, thus requiring only two or three air cylinders and valves instead of many, which is the cause of its simplicity and improved reliability. On account of its cheapness, it is quite generally used for single cars as well as trains.

The "PC" control (manufactured by the General Electric Co.) uses a combination of electrically controlled pneumatic cylinders to rotate, notch by notch, a main camshaft, and the cams on this shaft open and close spring actuated switches or "contactors." These cam-operated switches cut out the starting resistance and change the motor connections from series to parallel. A separate air cylinder operates the line breaker, while other cylinders actuate the reversing drum. The line breaker has suitable magnetic blowout coils and serves as a circuit breaker by means of a series trip coil.

The advantages claimed for both forms of this type of control are:

1. All power circuits and the weight of the main control apparatus are removed from the car platform, i.e., the system is operated by remote control.
2. The main line current is always broken by large, suitably designed breakers.
3. Multiple unit operation may be secured with less apparatus and therefore adapted to smaller equipments.
4. The operation of the contactors being mechanical, it is more positive and therefore the time element is definite.
5. Automatic or selective acceleration may be provided with less complexity.
6. A greater number of motor cars may be operated in multiple in one train (16 motor cars in one train is claimed).
7. The current from the control circuits is so small that storage batteries may be used, thus making this control easily adaptable for operation on a-c or high-voltage d-c systems.

WESTINGHOUSE "UNIT SWITCH" SYSTEM. The Westinghouse "unit switch control" is designed for either multiple-unit operation of several motor cars in a train or operation of single cars or locomotives using either large currents or high voltage. Unit switch control equipments are classified according to whether they are arranged for hand or automatic acceleration, whether they employ energy from the line or from a battery, and whether they have certain other features such as field control. The standard designations are as follows: HB, ABF, HL, HLF, AL, VA, etc. Thus it will be noted that "H" signifies hand acceleration, "A" automatic, "B" battery control, "L" line control, "F" field control of motors, "VA" variable automatic, etc.

RAPID TRANSIT CONTROL. ABF control is automatic in its progression, being under control of a limit relay which permits each additional step to be taken when motor current has dropped to a predetermined value. It provides series-parallel operation of motors using bridging transition which insures smooth, uniform acceleration. The distinctive features of ABF control are the three-wire system which makes it necessary to complete three circuits at the controller before power can be applied to the motor, inherent rail gap protection, and the use of low battery voltage.

The apparatus included in ABF control includes a master controller, line switch, a group of unit switches, and the necessary protective and accelerating relays. The unit switches are electropneumatically operated and are used to make the motor and resistance connections.

ABF control is suitable for all classes of rapid transit service, such as subway, elevated, and interurban lines.

Operation. The interlocking shown in Fig. 13 provides a simple and effective means for automatic progression. The unit switches are provided with interlocks which are electrically connected with the valve magnet in such a manner that the closing of one switch energizes the magnet of the next, thus producing automatic progression of the unit switches under the control of a limit relay. This relay controls the rate at which the resistance is cut out of the circuit so as to give uniform accelerating current. The relay is of the solenoid type, being operated by one motor current. When this current exceeds a predetermined value for which the relay is adjusted, the control feed circuit is opened, thus halting the switch progression. When the current falls below the predetermined value, the control feed circuit is closed and allows the unit switches to continue their progression.

The air required for operating the unit switches is taken from the air-brake system through a reducing valve at a constant pressure of 70 lb per sq in.

HL CONTROL. HL control provides series-parallel operation of motors with resistance steps in both series and parallel like the K controller. The changes in motor and resistance connections are made by a number of independent pneumatic switches, each provided with a strong magnetic blowout and held normally open by a powerful spring. The switches are controlled by a magnet valve which admits or exhausts air from the switch cylinder. A pneumatically operated reverser controls the direction of motion of the car. This reverser is operated by two pistons similar to those used in the unit switches.

Energy to operate the magnet valves is taken from the trolley and reduced to a low voltage by taking taps across a resistor which is connected from trolley to ground. This low-voltage energy is distributed by a master controller to the various magnet valves of the unit switches and reverser. These same wires supplying energy to the magnet valves also run to each end of the car from where they may be run to other cars by means of jumpers. In this way multiple operation of a number of cars may be obtained from a single master controller. The number of master controller notches is the same as the number of steps of operation, each step being under direct control of the operator.

HL control is generally used on equipments with aggregate motor capacity of more than 200 or 250 hp where stops are relatively infrequent. It is also used on smaller equipments where train operation is required.

VA CONTROL. VA control provides series-parallel operation of motors but includes a larger number of resistance steps than is found in HL or K control. VA control is automatic in its progression, being under control of a limit relay which permits each additional step to be taken when motor current has dropped to a predetermined value. One of the most distinctive features of VA control is that the limit relay setting may be changed by changing the controller position. This gives the operator direct control of the accelerating rate of the car within certain limits.

The apparatus included in VA control includes a master controller, a group of unit switches either pneumatically or magnetically operated, a sequence switch, and a limit relay. The unit switches make the motor and resistance connections. The sequence switch is a pneumatically operated drum which has as many positions as there are steps

of operation. The drum carries a set of contacts which open and close the unit switches in the proper order. The advance of the sequence switch is controlled by a current limit relay which permits the next step to be taken when the motor current has dropped to a predetermined value. This predetermined value of current may be changed within limits by the master controller.

VA control is applied largely to street cars in city service where stops are frequent and

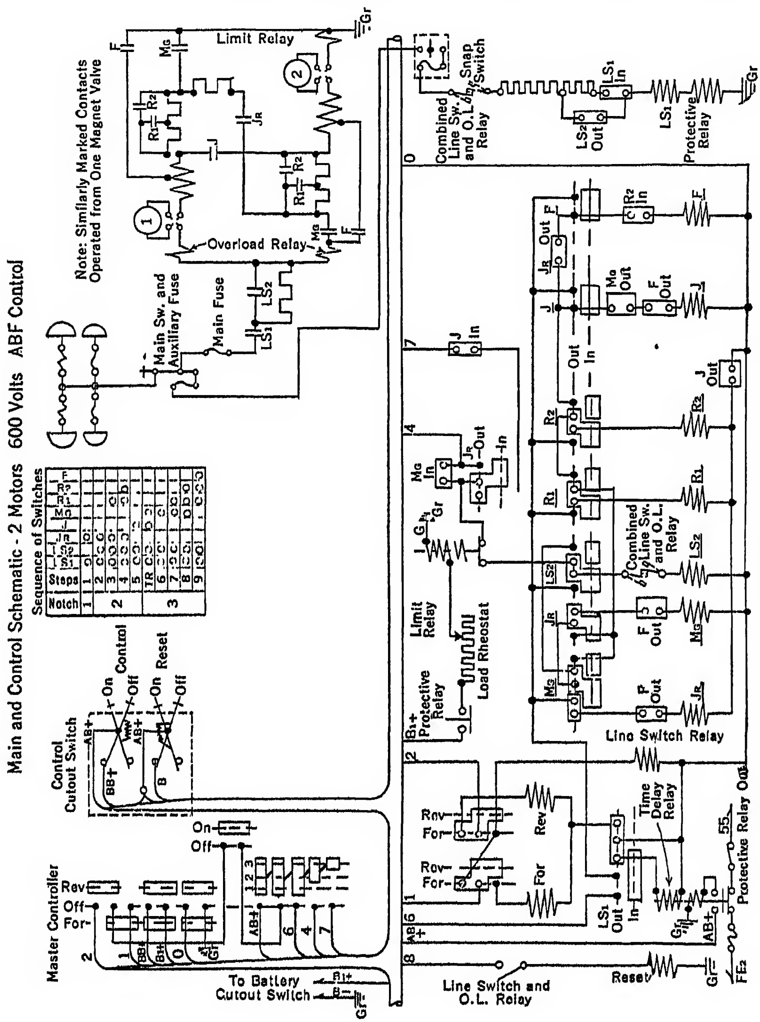

FIG. 13. Westinghouse Unit Switch Control

acceleration rates high. It also finds some use on foot-operated equipments where automatic acceleration is desirable.

CAPACITY AND WEIGHT OF CONTROL EQUIPMENTS. Table XXI gives the capacity and weight of some typical control equipments.

CABLES FOR CAR EQUIPMENT. Typical cables for car equipment have seven tinned copper strands for sizes smaller than No. 1 and nineteen for No. 1 and larger. Cotton or paper, depending upon the size of cable, is used as a separator between the bare cable and rubber compound, the compound conforming to National Electric Code standards. As

Table XXI

Number of Motors	Horsepower of Each Motor	Type of Control	Weight of Control Equipment, lb	Weight of Each Motor, lb	Total Weight of Equipment, lb
4	25	K–75	1,200	1,050	5,400
4	50	K–75	1,400	2,100	9,800
4	60	K–35	1,800	2,550	12,000
4	100	Multi-unit	2,600	2,740	13,560
2	200	Multi-unit	2,750	6,240	15,230

a final protection, the rubber is covered with a single layer of rubber-treated tape and one of cotton braid, and the cable is then subjected to a weather-proofing process.

In the Table XXII, 1-motor, 2-motor, etc., indicate that the cable carries the current

Table XXII. Sizes of Cable for Car Equipments

Horsepower of Motor at 600 Volts	Trolley and Ground Cable		Motor Cable		Resistor Cable		
	2-motor	4-motor	1-motor	2-motor	1-motor	2-motor	4-motor
25	5	2	6	5	6	6	5
40	4	1	6	4	6	6	4
50	4	0	6	3	6	6	3
65	3	2/0	5	1	6	5	1
75	1	3/0	4	0	6	4	0
100	0	4/0	3	2/0	6	3	2/0
125	2/0	300,000 cir mil	2	3/0	5	2	3/0
140	3/0	350,000 cir mil	1	4/0	5	1	4/0
165	3/0		1		4	1	
190	4/0		0		3	0	

of a single motor in the former and of two motors in the latter case, etc. The numbers are American Wire gage sizes.

CONTROL FOR MINING AND INDUSTRIAL LOCOMOTIVES. On small mining and industrial locomotives where space limitations will not permit of mounting full magnetic or pneumatic control, a semi-magnetic or semi-pneumatic system is often used.

With this type of control all accelerating connections of the motors and main resistors are made with switches, magnetically or electropneumatically operated. The controller consists of a master drum which controls the sequence of operation of the switches, and a reverse drum through which the motor connections are made for the desired direction of operation.

The term " permissible " as applied to control equipment implies that, when this equipment is mounted and wired with the apparatus necessary to the operation of a storage-battery locomotive or other mining machinery, the complete locomotive or machine is approved by the U. S. Bureau of Mines for operation in gaseous mines. This control may be of the drum or semi-magnetic type. All apparatus is enclosed in metal enclosures with metal-to-metal joints between cover and container, the joints being of such a width or construction that no sparks or flame will pass from the interior of the enclosure to the outside in case of an explosion on the inside.

The term " explosion-tested " or " explosion-proof " as applied to control equipment implies that the enclosures for the same have passed the explosion tests prescribed by the U. S. Bureau of Mines for permissible control but, because of some method of operation or condition of installation, the complete locomotive or mining machine does not have the approval of the Bureau of Mines.

CONTROL OF HIGH-VOLTAGE D-C MOTORS. The motors operating on systems of from 1200 to 1500 volts are usually designed to operate two in series, thus each receives normally 600 or 750 volts. However, the insulation of each motor must be designed to withstand the whole line potential, and each motor must be able to withstand momentarily the line voltage across its commutator, for if one motor slips the voltage will be unevenly divided between them. For operation on high voltage two motors in series are normally treated as a unit and the series and parallel connection made with these double units. The multiple-unit control is preferable with these high voltages and contactors or unit switches similar to those for 600 volts are used. To operate the control it is customary to supply either a self-starting dynamotor which provides 600 volts or a motor-generator supplying 32 volts for this purpose as well as for the lights and other auxiliary apparatus.

Provision for 600-volt Operation. Where these equipments operate also over 600-volt sections of road, provision has to be made to change the connection of the dynamotor or motor-generator when the transfer is made. If the cars are to operate at reduced speed on the lower voltages, as is usually the case on entering the city districts, no change need be made in the motor connections. But if the cars must operate at 600 volts at high speed over an interurban section, then provision must be made to separate the pairs of motors so that all motors will be in parallel for full-speed operation on 600 volts. This requires a commutating switch with automatic protection in order to provide that it is always changed when the car passes from one section to another.

3000-volt Systems. For locomotive work 1500-volt motors are constructed to operate two in series on 3000 volts. The control for such a locomotive is similar to the control for a 1500-volt equipment.

CONTROL OF A-C COMMUTATOR MOTORS. To transform the line voltage (3000, 6000, or 11,000 volts) to a voltage suitable for the motors, which is usually from 200 to 300 volts per armature, early equipments used a compensator or auto-transformer. Most of the later equipments use a two-circuit transformer which, though it is slightly heavier for a given capacity, has the advantage of operating with the secondary ungrounded. Taps on the low-voltage side of the compensator or transformer provide the various voltages necessary to start and control the motors, and thus there is no need of series-parallel control or rheostats, and the energy loss in the rheostats is obviated. The transformer is usually suspended from the bottom of the car body. Early transformers for cars were oil-insulated. The modern transformer for both cars and locomotives is the dry type, air blast. The use of oil, especially in passenger service, is objectionable as a fire hazard. Fewer steps (5 to 7 in all) are required for the control of a-c motors on account of the reactance of the circuits. To avoid open-circuiting the connection to the motors in changing from tap to tap or short-circuiting the portion of the transformer between the taps a " preventive " resistance or reactance is connected in the circuit momentarily during the transition. A reverser is provided to reverse the connection of the series fields or exciting windings.

The control may be either of the cylinder or multiple-unit type. In the former type a standard controller may be adapted for the work. In a-c multiple-unit equipments a storage battery is used to supply current for the control circuit.

Provision for D-c Operation. For operation on direct current as well as alternating current provision must be made to perform the following operations: (1) cut the transformer out of circuit, (2) connect the motors for series-parallel control, (3) connect rheostats in circuit, (4) change the field connection of the motors, (5) change the connections of the compressor motors, (6) change the connections of the lighting circuits. All this is done by a " commutating switch " which is thrown over at the instant the change is made. This is so arranged that it can only be moved when the controller is at the off-position. The commutating switch is usually operated by the motorman when the car reaches a dead section of the trolley between the a-c and d-c sections.

CONTROL OF THREE-PHASE INDUCTION MOTORS. Three-phase induction motors for railway work may be controlled by three methods, viz.: (1) changeable pole windings, (2) concatenation of two motors, (3) variable resistance in secondary.

The first two methods were given a considerable trial by German manufacturers some years ago, and have been practically abandoned on account of their complications. In addition to their complications the variable resistance method must also be used with them to provide the smaller gradations of speed.

Variable-resistance Methods. The secondaries of the motors have a definite winding and the terminals are brought to collector rings by means of which a three-phase starting resistance is connected into the circuit. The speed of the motors is controlled by varying this resistance. Under these conditions the operating characteristics of the motors are similar to those of a d-c shunt motor. At all fractional speeds a considerable amount of energy is wasted in the rheostats and there is only one efficient running speed. For prolonged running at fractional speed the rheostats must have considerable heat-dissipating capacity. To reverse the direction of the motors a reverser is employed which reverses the connections of two of the three primary leads on each motor. Either the cylinder or multiple-unit control may be used. With induction motors it is desirable to provide a separate set of resistances for each motor to avoid the tendency of the motors to exchange current and " buck," which would occur if the driving wheels were not of exactly the same diameter and one set of resistances were used for all motors. With several induction motors on one car it is desirable to accurately maintain the same diameter of driving wheels on all axles in order to divide the load equally between the motors.

6. CURRENT COLLECTORS

Current collectors for electric cars or locomotives are divided into three classes in accordance with the form of the working conductor from which they collect current, as follows:

(*a*) Overhead collector, which may be of the pole type with wheel or sliding shoe contact, the pantograph type with sliding or roller contact, or the bow type.

(*b*) Third-rail shoes, which may be of the over-running or under-running type.

(*c*) Underground conduit plow.

WHEEL TROLLEY. The wheel trolley consists of a grooved brass or copper wheel held in bearings in a prong called a " harp " at the end of a steel pole which is pressed upward by a system of springs and levers carried by a spring-equipped base, with provision for vertical movement and horizontal rotation, together with adjustment for pressure against the contact wire.

The contact wire is generally from 17 to 18 ft above rails on urban and interurban roads and 22 ft on electrified steam roads with this type of collector. The pressure between wheel and contact wire varies between 20 lb and 35 lb, depending upon speed and current to be collected. The current-collecting capacity of the wheel collector, though governed by speed and by the condition of the overhead system, is generally given as from 400 to 450 amp for average urban and interurban operating conditions. As much as 1200 amp can be collected during the brief period of acceleration.

The wheel collector is generally used on street railways, and is subject to objections for heavier work on account of limited capacity, wear on rotating parts, liability of dewirements, and necessity for reversal in case of reverse movements, unless they are limited in extent.

Sliding Shoes. The slider shoe is a grooved block of bronze or steel which is used in the " harp " in place of the wheel. It gives sliding contact between the collector and the contact wire. The sliding shoe is being used to replace the wheel on many interurban and some city lines and very generally on trolley buses. With lubricated contact wires, longer shoe and wire life is obtained and radio interference is reduced.

PANTOGRAPH COLLECTOR. The pantograph collector, as generally used, consists of one or two flat sliding shoes, mounted on a collapsible frame of pantograph form. Adjustable pressure is provided by springs, compressed air, or both. A roller has been used to a limited extent instead of the shoe or shoes, but, except in one case, has been replaced by the shoe on account of undesirable weight, limited capacity, and rotating parts.

With relatively small current values, 150 to 200 amp, a plain steel shoe is used, without lubrication. For higher values, steel shoes are fitted with renewable wearing strips of copper, copper and steel, or copper alloy and are lubricated. It is practicable to collect from 1000 to 2000 amp, with one collector, with shoes so fitted and lubricated and with suitable overhead construction. Pantographs have been manufactured with operating ranges as high as 129 in. Pantograph pressures vary from 10 to 20 lb per shoe.

BOW COLLECTOR. The bow collector consists of a contact member of bow shape, generally having a renewable part of aluminum or copper alloy, mounted on one or two flexible poles carried by a spring-equipped base. For high-speed, heavy traction work, the bow is sometimes mounted on a pantograph frame. It is suitable for only small currents. The bow is little used in the United States but is used extensively in Europe.

THIRD-RAIL COLLECTORS. A third-rail collector consists of a shoe or slipper, made of cast iron or steel, which makes contact with the third rail, together with the necessary frame and attachments. The gravity type, for over-running rail, suspended by links, has been replaced largely by a type employing springs or compressed air for pressure. Collectors are of the over-running, under-running, or side contact type, to suit the type of third rail involved.

Third-rail shoes have relatively high current collection capacity—about 2000 amp at speeds of from 30 to 35 mph and from 500 to 600 amp at 60 mph. (See article on Third Rails.) As they are self-adjusting two or three may be placed on a locomotive or car just as well as one, so that there is practically no limit to the current that can be collected in this way. In fact it is customary to put two on each side of each locomotive or car in order to prevent a cessation of current when passing over breaks in the third rail due to switches or crossings.

UNDERGROUND CONDUIT PLOW. The underground plow consists of an insulated steel plate hung from a movable structure on the car. On the two sides of this plate and thoroughly insulated from it are two shoes pressed outward from the plate by springs. These shoes press against the two working conductors which are usually steel tees separated from each other by about 6 in. and supported on some form of ceramic insulator. The current is led from the shoes to the car body by flexible insulated conductors.

7. BRAKES AND BRAKING SYSTEMS

(See also Electric Cars, Art. 4, Energy Requirements, Art. 2.) In order to stop a car or train a torque must be applied to the wheels in a direction opposite to the direction of motion of the car. This may be accomplished by applying a frictional retarding force to the wheel rims, by applying a retarding force to the axle by means of a magnetically operated friction clutch, by applying a reverse torque to the axles by operating the motors as generators, or by applying a frictional force to the rails directly by a "track brake." The method of applying a retarding frictional force to the wheel rims is the most general one and lends itself most readily to the system of manipulating the brakes by compressed air. This system has done much to improve the safety of travel on railroads. The other methods have all been tried and some have been put into practical operation, but only to meet special local conditions.

FRICTIONAL RESISTANCES IN BRAKE-SHOE SYSTEM. The application of the usual brake-shoe system makes use of the frictional adhesion between the wheels and the track and between the brake shoes and the wheels. Both these quantities vary throughout a considerable range and it is therefore necessary to adjust the pressure between the various members so that it is possible to rely on a definite minimum value.

Adhesion between Wheel Rim and Rails. The coefficient of adhesion between the wheel rims and rails varies from less than 15 per cent to over 30 per cent depending upon the condition of the track and the relative motion between the track and wheel rim. (See Energy Requirements, Art. 2.) An adhesion of 15 per cent can usually be depended upon with normal track, and this can be increased to 25 per cent by the use of sand. But these values only obtain while the wheels are rolling on the track. If they begin to slide, the coefficient decreases considerably. For this reason the braking effort must always be controlled so that the wheels do not slip.

Adhesion between Brake Shoe and Wheel Rim. When the brakes are applied, the retarding force is applied below the center of gravity of the car body. The latter is therefore subjected to a couple and tends to press downward at the forward end and upward at the rear end, thus changing the distribution of weight on the axles and decreasing the adhesion on some axles or trucks. It is therefore not possible to figure on using for braking purposes the same weight per axle as exists at standstill. For this reason the brake-shoe adhesion must be less than the track adhesion. The coefficient of adhesion between the customary cast-iron brake shoe and the steel tire of the wheel varies with the speed, and decreases as the time of application increases. As the speed increases the coefficient drops off, being a maximum of from 30 to 25 per cent at speeds from 0 to 5 mph, 20 per cent at 20 mph, 14 per cent at 40 mph, and 7.5 per cent at 60 mph. Thus at high speeds a heavy pressure may be applied without stopping the wheels, while as the speed of the car diminishes the pressure on the brake shoes must be decreased in order to prevent gripping the wheels and causing them to slide on the track.

Effect of Angular Momentum. In addition to overcoming the linear momentum of the cars the brakes must overcome the angular momentum of the gears and motor armatures. The effect of the latter is to introduce a tendency of the whole motor to rotate around the car axle and introduce additional strains on the gears and on the trucks. For this reason brake shoes hung between the wheels of a truck are better than those hung on the outside of the wheels.

HAND BRAKES are always provided on cars and electric locomotives whether power brakes are employed or not, as they are necessary to hold a car left out of service on a grade, because the air brakes will not hold a car standing idle for any length of time. In hand braking equipment the "foundation" brakes (see next paragraph) are actuated through a drum or lever system which is hand operated.

Power Brakes

The "foundation" brakes are that part of the brake equipment usually furnished separately from the power-braking equipment, and consist of the brake shoes, hangers, equalizers, levers, etc., back to the brake cylinder. To this is attached the desired form or make of power brake. Of the various forms of power brakes in use, viz., air, electric, regenerative, and electropneumatic, the air brake is the most generally used.

AIR BRAKES. In electric railway practice three systems of air brakes are in use, each of which is best suited to a definite type of service and has its particular field, as follows: (1) straight air-brake system for cars always operated singly; (2) semi-automatic air-brake system for cars operated in trains of two or three but never more than three cars; (3) automatic air-brake system for cars operated in trains of any number of cars; (4) combined automatic and straight air system for cars operated singly or in trains of any number of cars.

Straight Air-brake System. The equipment for the straight air-brake system consists essentially of a motor-driven compressor, a reservoir, a brake cylinder, a motorman's valve, a train pipe, and the foundation brakes. The brakes are applied by direct pressure, that is, by admitting air from the reservoir directly to the cylinder. The advantages of the system are that it is quick-acting and the braking effort is easily controlled to any value desired. Its disadvantage is that in trains a break in the train pipe renders the brakes ineffective so that there is no means of applying the brakes on the trail-cars if the train should break apart. Sometimes instead of a compressor the motor-car carries a large reservoir which is charged at stations.

Semi-automatic Air-brake System. This system involves the use of anemergency valve and an emergency pipe on each car so arranged that if the air pressure in the emergency pipe drops, due to cars breaking apart, this valve automatically operates to admit pressure into the brake cylinders and apply the brakes. The usual operation in service is like that of the straight air brake.

Automatic Air-brake System. This system involves the use of an auxiliary reservoir and a " triple valve " on each car. Whenever the air pressure in the train pipe is reduced, either intentionally or accidentally, this triple valve admits the air pressure of the auxiliary reservoir on each car into the brake cylinders. The brakes are applied by the motorman by opening the train pipe to the atmosphere by means of the engineer's valve. The engineer's valve has at least four positions; "release," "lap," "service," and "emergency." By turning the handle to the service position the train pipe is opened to the atmosphere and the air continuously escapes. This applies the brakes with a continuously increasing pressure. When the desired pressure has been reached the handle is turned to the lap position, and the pressure of air in the train pipe and the pressure of the brakes on the wheels remain constant. By turning the handle to the emergency position a very rapid escape of air from the train pipe causes a quick, heavy application. By turning the handle to the release position pressure is restored in the train pipe, the brakes are released, and the reservoirs recharged.

Combined Automatic and Straight Air-brake System. This system combines the advantages of the flexibility and promptness of straight air operation on a single car, and the safety features of the automatic air brake.

Electro-pneumatic Brakes. This system is now in extensive use in subway and suburban service. It involves controlling the application and release of the brakes on each car by means of electromagnets receiving current from the locomotive cab. By energizing these circuits the motorman can apply or release the air brakes on all cars practically simultaneously. Each car is equipped with the usual automatic brake mechanism, so that the brakes operate pneumatically in case of failure of the electric circuits.

ELECTRIC BRAKES. Electric brakes in most cases are used to supplement the standard air brake equipment. Three types are in common use: rheostatic brakes, electromagnetic track brakes, and regenerative braking.

Rheostatic Brakes. With this type of equipment the motors are arranged to operate as generators supplying current to rheostats which absorb the energy. This type of braking is effective, of course, only when the car is in motion. It is possible to operate rheostatic brakes without the refinements in control that are required with regenerative braking.

Electromagnetic Track Brakes. For this type of braking equipment electromagnetic brake shoes energized by trolley current are spring-suspended from the car trucks, normally clear of the rails. When a brake application is desired the shoes are moved down to the rails by means of small air cylinders. There is no mechanical connection between the electric brake shoes and the wheel brake, and the air control is also separate. By exciting the magnet coils in the brake shoe the friction between the shoe and the rail acts to retard the motion of the car. When the brake is applied no weight is taken from the car wheels, and full braking is therefore available with the air brakes. Since the excitation current for the shoe is taken from the trolley there is no regenerative load on the motors. Braking is not dependent on the motion of the car. At the end of 1931 there were in operation in the United States 151 cars using this type of braking, distributed among nine operating companies.

As an outgrowth of this type of brake, experiments have been made with similar magnetic shoes, mounted in similar position but kept out of contact with the rail and designed to exert a downward pull, increasing the effective weight on the wheels by as much as 25 to 30 per cent and making possible a corresponding increase in brake-shoe pressure and in rate of retardation.

REGENERATIVE BRAKING. The term "regenerative braking" is normally used to apply to equipments where the motors act as generators and return current to the trolley line, thus effecting a retardation of the locomotive by making use of the stored

energy in the moving vehicle. This system has been most extensively used for heavy locomotive duty. It was first used by the Chicago, Milwaukee, St. Paul and Pacific Railroad on large d-c freight locomotives and subsequently on similar locomotives for various American-equipped foreign d-c electrified roads. The principal feature of these equipments is the provision of an accurately controlled exciter which separately energizes the fields of the motor, thus giving a definite control of the voltage generated. On some locomotives a separate motor-generator set, and on others a mechanically driven exciter mounted on the idle trucks, supplies this current. A further variation is the use of one or more of the motor armatures as the source of excitation for the other motors.

There are three installations in this country using regenerative braking with single-phase a-c supply. On the Norfolk and Western and the Virginian Railway polyphase induction motors are used and three-phase regeneration occurs automatically when the locomotive exceeds synchronous speed. On the Great Northern Railway, with motor-generator type locomotives, converting the single-phase supply to direct current for driving the traction motors, an unusually wide range of regeneration is obtained by controlling the fields of the d-c generators.

A number of installations have been made of light-weight street-car equipment providing regenerative braking by use of field control through an exciter. On most systems, however, it is difficult to justify the additional expense required for this type of braking.

With d-c motors and with single-phase commutator motors it is possible to secure regenerative braking over quite a wide range of speeds. With the three-phase motor regenerative braking is limited to a range fairly close to the synchronous speed of the motor. Half speed on three-phase motor equipments is usually obtained by a change in the number of field poles permitting half speed regeneration as well as motoring.

The use of regenerative braking not only relieves the air-brake equipment but also effects a saving of an appreciable amount of energy and a reduction in wear on the wheel tires, brake shoes, and tracks. Savings in power have been reported for the Chicago, Milwaukee, St. Paul and Pacific as averaging 11 per cent of the power consumed by the motors. On other installations the savings have been even greater.

8. LIGHTING OF TRAINS BY ELECTRICITY

Electricity is now used almost exclusively by the railroads of the United States for the lighting of trains. It is estimated that, at the present time (1936) of the total of about 50,000 passenger cars, including Pullman cars on Class I roads, 40,000 are lighted by electricity. The remainder, lighted by oil or gas, include a considerable number of old or obsolete cars subject to early retirement.

SYSTEMS USED. The following systems are used for the purpose of electric lighting:

a. Axle generator system.
b. Head end system.
c. Straight storage system.

AXLE GENERATOR SYSTEM. This method of lighting is by far the most popular. The recommended practice as to design, application, and methods of rating equipment of this system as formulated jointly by the Mechanical Division of the Association of American Railroads and the Association of Railway Electrical Engineers, and published in their respective Manuals of Recommended Practice, is now followed almost universally by American railroads. The principal parts of the axle generator system are:

(1) *A d-c axle generator* mounted either on the truck or under the car body and in most cases driven by a belt from a pulley on the truck axle.

The generator is usually operated at about 38 volts and delivers current to 16 cells of lead storage battery, or a larger number of cells of nickel-iron storage battery, through suitable regulating devices. The capacity of the generators, when the load is principally lighting, varies usually from 1 kw to 5 kw, depending upon the service requirements. A 1-kw machine is found to be of sufficient capacity for a baggage car. An ordinary coach requires a 2-kw machine, while the additional load on a dining car often requires a generator of 5 kw capacity. When an air-conditioning load is added, the generator capacity is generally 20 kw for motor-driven systems and up to 10 kw for steam-jet and mechanically-driven systems.

(2) *A storage battery*, usually of 16 cells, of from 150 to 1000 amp-hr capacity, with some installations of two or even three sets in parallel for a maximum total of over 2000 amp-hr.

The battery is installed in a suitable ventilated box hung under the car body. The amount of charge is regulated by an automatic device mounted on the regulator panel inside the car. When the car reaches a speed resulting in a predetermined generator voltage, an automatic switch cuts in and connects the generator to the battery. This

switch automatically disconnects the generator from the battery when the generator voltage falls below a predetermined limit.

Until recent years the preferred type of lead battery was the Planté type except where the required capacity made the use of the lighter-weight paste type necessary in order to keep within reasonable weight limits. At the present time the paste type is gaining rapidly in popularity owing to its more advanced development, improvements in the regulating equipment which contribute largely to its more successful performance, and increasing load demands with consequent emphasized advantage of its lower weight and smaller physical size characteristics as compared with the Planté type.

(3) *A regulator panel* on which are mounted a regulator controlling the output of the generator, the automatic switch, and a lamp regulator which holds a constant voltage on the lamps.

(4) *A drive* which, for generators up to 10 kw, consists usually of a belt connecting pulleys on the generator shaft and on the car axle. Rubber or balata belts are most common, with limited use, particularly in winter, of chain drive with sprocket pulleys.

Generators of larger capacity make use of gear and shaft drives, of which several forms have been developed, with or without an intermediate V-belt.

(5) *A suspension.* The former general practice of suspending the axle generator from the truck is being rapidly superseded by the body type of suspension.

By mounting the generator under the car body a much lighter form of suspension can be used. The machine is also better protected from shock due to rough track, crossing switches, etc., than when mounted on the truck, and unbalancing of the truck is avoided.

HEAD END SYSTEMS. In so-called head end systems or locomotive train lighting systems, current is supplied to the cars from a generator on the locomotive or a baggage car. Earlier systems, with the generator in a baggage car, either direct connected to a steam engine receiving steam from the locomotive or driven by a belt through the car floor from a pulley on the axle, have been largely displaced by locomotive train lighting systems, in which a d-c turbo-generator set is mounted on the locomotive or tender.

The generator, generally of 5- to 20-kw capacity, with some 2 $^1/_2$-kw installations, supplies direct current through a train line, using the loop system of wiring to make the lamp voltage uniform throughout the train. Voltages are 32, 64, or 110, the tendency being toward the higher voltages when locomotive lighting is taken care of by a separate 32 volt turbo-generator. Some use is also made of dual-voltage turbo-generators supplying current at 32 volts for the locomotive and at 64 volts for the cars. In some cases a battery is provided on one or more cars to carry the load when the locomotive is not coupled or the set is inoperative. Without batteries, these equipments are suitable only for trains which run solid between terminals, but on account of low initial and maintenance cost are being used to advantage for suburban trains, branch line trains, etc. The principal disadvantages are limited lengths of train possible without excessive voltage drop, and loss of light when the locomotive is uncoupled or the set becomes inoperative.

An a-c system has been introduced, which employs a 25-kw turbo-generator supplying 60-cycle, single-phase, alternating current at 220 volts through a train line. Each car is equipped with a transformer to reduce the voltage to the standard of 32 volts. Considerable flexibility is provided, since batteries may be used on each car and a-c supply from local power lines may be utilized when cars are parked for extended periods or standing in yards.

STRAIGHT STORAGE SYSTEM. This is the simplest method of lighting by electricity but has very largely passed out of use on account of inflexibility and high cost of operation.

Each car is equipped with one or two 16-cell storage batteries suspended under the car body and arranged for convenient charging at terminal yards.

This system requires elaborate charging facilities at terminals where it is necessary to "place" cars and either hold them long enough to give the battery proper time to charge or to have spare batteries fully charged to replace the discharged ones.

At first sight this system appears inexpensive to operate. However, when the maintenance and operation, together with fixed charges of the charging plant are considered, it is perhaps the most expensive method of lighting cars.

ILLUMINATION. Considerable study has been given to car illumination for many years. When cars were first lighted electrically no shades were used to conceal the glare of the lamp filament from the eye. Now great care is taken to protect against objectionable glare, to give as far as possible uniform illumination on the reading plane, and to secure decorative effects from the units.

LAMPS. Depending upon the style of fixtures employed, and their location in the car, lamps range from 15 to 100 watts. The tungsten-filament, gas-filled inside-frosted lamp is used almost exclusively.

The standard voltages in use are:

Straight storage and old style head end systems.......	64 volts
Modern head end (locomotive train lighting) system...	32, 64, and 110 volts
Axle generator system..........................	32 and 64 volts

There have been great improvements in the manufacture of lamps since the introduction of electricity for car lighting. The original type carbon-filament lamps operating at from 3.5 to 4 watts per candlepower have, through progressive development, been replaced by tungsten-filament, gas-filled lamps operating at approximately 1 watt per candlepower or 9 to 16 lumens per watt.

FIXTURES. As in ordinary lighting two standard types of fixtures are used: viz.: direct and semi-indirect. Fixtures are sometimes suspended from the center deck and sometimes side deck mounting is preferred. The modern trend toward pleasing decorative lighting effects has resulted in a number of interesting installations in dining cars, business cars, etc.

The Post Office Department of the U. S. Government after many exhaustive tests issued a rigid specification for the lighting of railway mail cars. This specification covers the location of light units, limits for illuminating values, emergency lighting requirements, etc., and is revised from time to time to keep it fully abreast of latest developments in the art.

GENERAL. From an operating standpoint the axle generator system is more flexible than the others. A car so equipped may be operated on any division independent of charging facilities at the division terminal points or train line connections between cars; it may also be cut out of one train at a junction point for connection to another train without special arrangements being made to supply light from an outside source.

9. THIRD-RAIL OR CONTACT-RAIL SYSTEMS

The contact rail, or third rail, is a conductor supported on insulators near the ground and presenting a continuous contact surface to a collector or shoe attached to the rolling stock. Electrical continuity between adjacent lengths is obtained by copper bonds across the joints.

TERMINOLOGY. The following terms and definitions are used in connection with third-rail systems. See also Standards of the A.I.E.E.

Contact Shoe. A third-rail contact shoe is a conductor, fastened to the rolling stock, which is designed to make electrical contact with the third rail. This is hereinafter referred to as the "shoe."

Contact Surface. The contact surface of a third rail is the surface against which the shoe presses.

Gage of Track. The minimum clearance between the inside surface of the heads of the two track rails; see Fig. 18. The standard track gage in the United States is 4 ft 8 1/2 in.

Third-rail Gage. The distance measured parallel to the plane of the running rails, between the gage line of the nearer track rail and the inside gage line of the contact surface of the third rail; see Fig. 15.

Location. Third rail locations may be described in terms of the third-rail gage and the elevation or vertical distance (see Fig. 14) between the normal contact surface and the normal top of the track rail. See also Table XXIV below.

Top-contact Rail. A top-contact third rail is one on which the contact surface is on the upper side of the rail.

Under-contact Rail. An under-contact third rail is one on which the contact surface is on the under side of the rail.

Third-rail Insulator. A third-rail insulator is that portion of the third-rail support which forms the principal electrical insulation.

Insulator Base or Bracket. A third-rail insulator base or bracket is a device used to support the third-rail insulator.

Incline. An incline is a portion of third rail sloped to gradually bring the shoe from its free position into contact with the normal surface of the third rail.

End Incline. An end incline is an incline at the end of a run of third rail and is made to receive shoes moving in line with the third rail.

Offset-end Incline. An offset-end incline is an incline *at the end* of a run of third rail and is made to receive shoes moving laterally towards the third rail.

Cross Incline. A cross incline is a combination in one piece of two end inclines from adjacent third rails which meet at a turn-out.

Side Incline. A side incline is an incline at *the side of* a third rail and is made to receive shoes moving laterally towards the third rail.

Offset-side Incline. An offset-side incline is an incline offset from the standard third rail in order to gain clearance between the third rail and the maximum equipment line of the rolling stock.

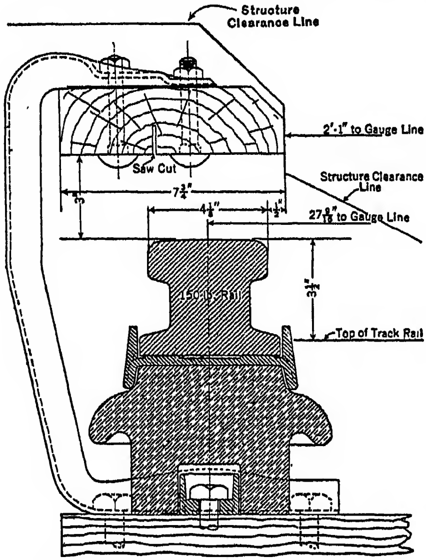

Fig. 14. Interborough Top-contact Type

Third-rail Protection. A third-rail protection is a partial covering of the third rail which affords some degree of protection from the weather or from accidental contact with foreign bodies.

Third-rail Anchorage. A third-rail anchorage is a device to prevent the third-rail creeping longitudinally.

TYPES OF CONSTRUCTION. Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

Interborough Top-contact Type (Fig. 14). One of the most commonly used types is illustrated in Fig. 14. It is often called the "Interborough Type" on account of its use in the subways of the Interborough Rapid Transit Co., of New York. The rail rests on porcelain insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

Pennsylvania Top-contact Type (Fig. 15). Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail, when repairs are being made, but it cannot be relied upon as well as the Interborough type to keep the rail and protection in perfect alignment.

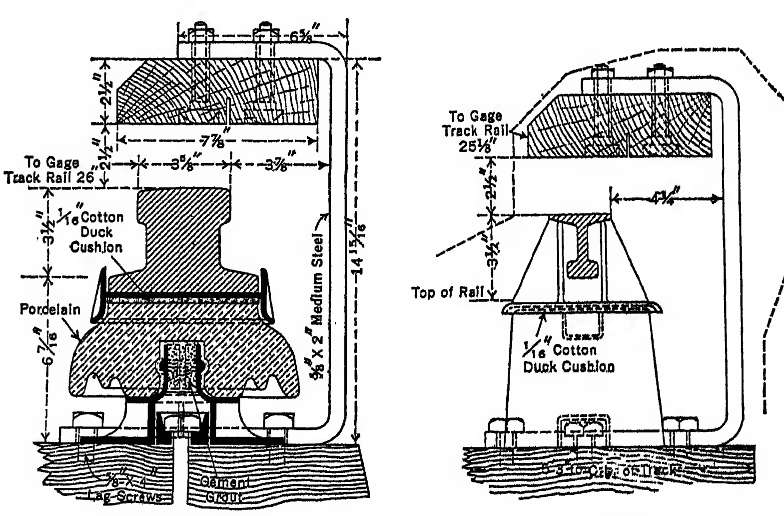

Fig. 15. Pennsylvania Top-contact Type

London Tube Type (Fig. 16). The mounting and location of the third rail and the negative contact rail of the London Tube Railways are illustrated in Fig. 16.

New York Central Under-contact Type (Fig. 17). Although the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed

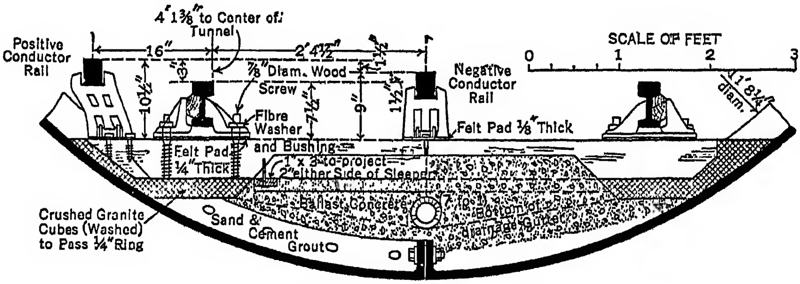

Fig. 16. London Tube Type

locations as they cannot be wholly protected from snow, ice, and sleet. The lower part is only a few inches from the ties, and holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow and ashes and from flooding. The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped in insulators by hook bolts hung from brackets, with the top and sides of the rail completely sheathed in an insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow, and spray. With this type of rail (Fig. 17) the protection is of such character that there is no packing of snow between the protection and the contact rail, as in some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but are easily broken off by the passing shoe.

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type.

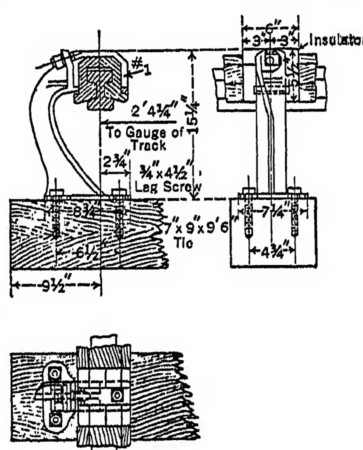

Fig. 17. New York Central Under-contact Type

Protection and Special Work. The protection between the insulator blocks, depending upon local conditions and the price of materials, as well as the potential used, is usually formed of three wooden strips, one grooved on the under side and inclosing the head of the rail, and the other two, attached to and dependent from it, reaching in toward the web of the rail. Soft rubber has sometimes been used in places where brine and manure drippings have tended to reduce the insulation resistance of wood sheathing.

Combined Top- and Under-contact Shoes. The employment of collecting shoes on rolling stock so constructed as to press upwards on the under-contact rail and downwards on the top-contact type solves the question of interchange between railroads not using the same type.

CONTACT RAILS VERSUS OVERHEAD TROLLEY. The contact rail is used as a part of the positive conductor system whenever the current to be collected by each collector exceeds the amount which can be taken safely from a trolley wire, or whenever the total current taken by a train exceeds the amount that can economically be carried by conductors of such expensive metal as copper or bronze.

Positive Contact Rails. Considered as a part of the positive conductor system, the contact rail and overhead trolley possess the relative qualifications given in Table XXIII.

Table XXIII. Relative Qualifications of Third-rail and Overhead Contact Systems
(By C. E. Eveleth)

I Protected Under-running Third Rail	II Overhead High-voltage Catenary Construction
Interference with track maintenance.	Entirely clear of roadbed.
Mechanical details can be maintained by section gangs.	Requires special crews with ladders or work trains.
Generally damaged by train derailments. Does not interfere with wrecking derricks. Quickly replaced by section gangs.	Offers obstruction to wrecking derrick. Partially overcome by insulating boom and help from linemen. Derailment liable to affect all tracks. Delays minimized by use of bridge supports and proper temporary measures.
Obstruction and hazard to brakemen and yardmen, particularly in freight yards.	Overhead bridge warnings confined to signal lights or similar devices.
Interferes with clearing snow between tracks.	No interference.
Use in heavy snow territory questionable.	Not affected.
Provides high current-collection capacity on account of large contact surface obtained with both rail and shoe.	Adequate current collection provided by suitable collectors and overhead construction.
Mechanical details can be inspected by track walker.	Some mechanical details require a lineman.
With proper precautions, may be worked on while alive.	With higher voltages, light repairs made from ladder. Heavy repairs require de-energizing catenary affected.
No interference with visual signals.	Modern light signals and methods of mounting have eliminated objections in this connection.
Absolute freedom from lightning disturbances.	Though more exposed to lightning, shielding by auxiliary circuits and proper insulation offer ample protection.
Freedom from inductive effects on nearby parallel circuits.	Some systems require adoption of measures to correct inductive effects on parallel and nearby communication circuits.
Operating voltage, to date, limited to 1500.	Maximum voltage used to date 22,000.
Contact rail must be interrupted at grade highway crossings.	Not affected.
Increases cost of track maintenance.	No effect.
Use limited by side clearances of permanent structures and equipment.	Use limited by overhead clearances of permanent structures.

ELECTRICAL DESIGN. The calculations of potential drop, network, resistance, etc., are treated in Art. 10. The composition, weight, dimensions, resistance, and reactance of rails will be found in Art. 12.

Selection of Suitable Rail. When the various types of rail are under consideration it is well to arrange a table with the following headings, in order to compare the relative economy of the different types: (1) circular mils of copper equivalent to rail; (2) additional circular mils of copper required to equal the rail of highest conductivity; (3) cost of rail for the entire railroad; (4) cost of additional copper for entire railroad; (5) total cost of rail and additional copper for entire railroad.

MECHANICAL DESIGN. The general design of the rail itself having been settled, the next step is to secure a set of track plans on which to lay out the special work. When an entirely new railway is being projected, the track designer and the third-rail designer can work together; but when an existing line is being converted, the general track plans cannot be used as they are seldom sufficiently accurate for the electrical engineer's purpose. In this case, the contact-rail engineer has to take measurements of the track work in order to make drawings of the cross-overs and other complications.

Location and Weight of Third Rails (Table XXIV). There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right-of-way and in the maximum equipment lines of various railroads. The following clearances are the standards of the Association of American Railroads, and the American Transit Association:

The clearance lines for third-rail and permanent-way structures and rolling equipment to be as shown in Fig. 18, thus reserving the space within lines *AT*, *BT*, *CT*, *DT*, *ET*, *FT*

and *AT*, *JT*, *KT*, *LT*, *MT* for third-rail structures; rolling equipment not to encroach upon the third-rail space under conditions of maximum wear and deflection beyond the line *AE*, *BE*, *CE*, *DE*, *EE*, *FE*, *GE*, and permanent-way structures not to encroach upon the third-rail space beyond the line *AS*, *JS*, *KS*, *LS*, *MS*; this leaves a clearance space or neutral zone of 1 in. both horizontally and vertically upon which neither the third-rail structures nor equipment shall encroach. On curves of less radius than 800 ft, the third rail must be moved back and the equipment may be allowed to swing outward as indicated by the lines and notes on the diagram.

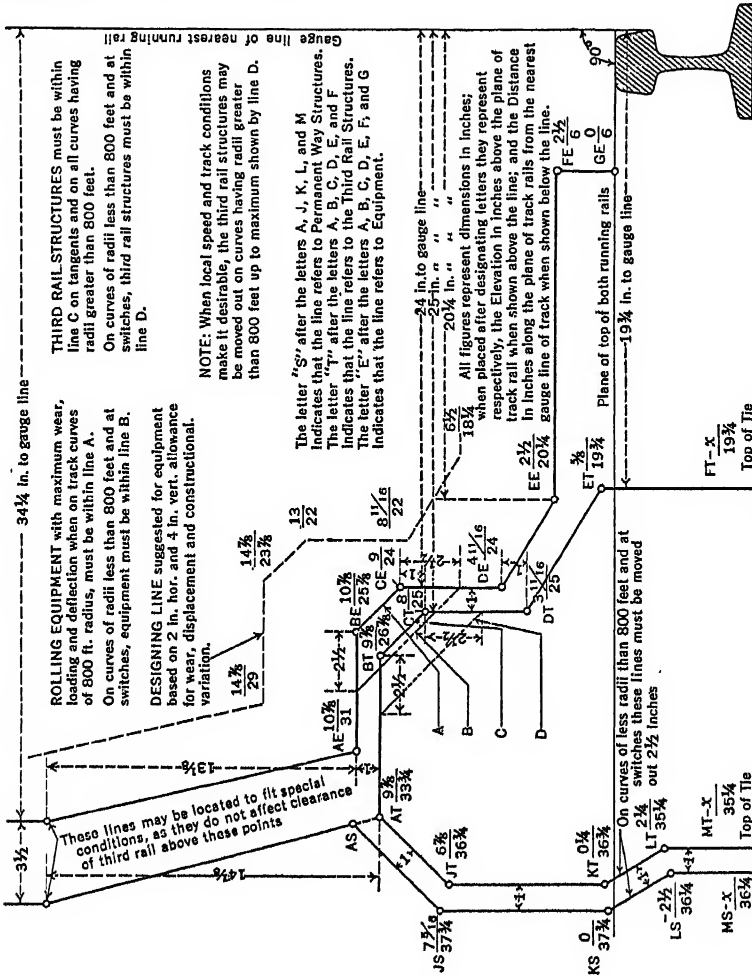

FIG. 18. Clearance Lines for Third Rail, Permanent Structures and Rolling Equipment

In the design and construction of new rolling stock to be used in interchange service allowance must be made for such horizontal and vertical variations as may in any reasonable probability occur in combination at one time as well as allowance for deflections on curves. Encroachments on normal clearance due to deflection of springs and wear vary with the type of construction used, and the practice of the respective roads as regard permissible wear and deflection before repairs and adjustments are made. For the general guidance of the roads, in designing equipment, but not as a standard, therefore, a dotted line is shown on the diagram located 2 in. distant horizontally and 4 in. distant vertically from the limiting clearance line for equipment.

Table XXIV. Location and Weight of Third Rail

Name of Railroad	Center of Third Rail to Near Gage Line, in.	Contact Face above Top of Running Rail, in.	Weight, lb per yd
American Railroads			
Baltimore & Ohio	30 1/2	3 1/2	150
Boston Elevated and Subway	20 13/32	5 15/16	85
Brooklyn Manhattan Transit			
Elevated	21 11/16	6	70
Subway	27 1/16	3 1/2	150
Central California Traction*	28 1/2	3	......
Chicago, Aurora & Elgin	20 1/2	6 5/16	100
Columbus, London & Springfield	27	6	......
Columbus & Newark	27	6	......
Chicago Rapid Transit	20 1/8	6 1/2	48
Hudson & Manhattan R.R.	26	4	75
Interborough Rapid Transit (N. Y.)			
Subway	26	4	150
Manhattan Elevated	20 3/4	7 1/2	100
Jointly operated	27	4	150
Lackawanna & Wyoming Valley R.R.	19	3 3/4	......
Long Island R.R.	26	3 1/2	35–100–150
Michigan Central R.R.*	28 1/4	2 3/4	70
New York Central R.R. (New York)*	28 1/4	2 3/4	70
New York City, Municipal subway	26	4	150
Northwestern Pacific R.R.	27 3/4	6	......
North Shore R.R. (Cal.)	27	6	50–60
Pennsylvania Railroad			
New York Division	27 13/16	3 1/2	150
New York Division Yard Tracks	27 3/8	3 1/2	25
West Jersey & Seashore	26	3 1/2	100
Philadelphia & Western Ry.	26	3 1/2	70
Philadelphia Rapid Transit			
Broad Street Subway	28 1/16	3 1/2	150
Frankfort Subway & El.*	27	6	70
Sacramento Northern Ry.	27 1/2	5 9/16	......
Sciota Valley Ry.	28	6	......
Seattle & Tacoma R.R.	20	7 1/2	......
Staten Island Rapid Transit Ry.	27 9/16	3 1/2	150
Wilkes-Barre & Hazleton Ry.	28	5 1/16	80
Foreign Railroads			
Berlin Elevated & Subway	14 3/8	7	......
Central London Railway	Center	1 1/2	......
Fayet-Chamounix	23	9	......
Great Northern Ry. (England)	19 1/4	......	80
Lancashire & Yorkshire Ry.	19 1/4	3	70
Liverpool Elevated	Center	1 1/2	......
Mersey Ry.	22	4 1/2	......
Metropolitan & District (London)	16	3	100
Milan Gallerate	26 5/8	7 1/2	......
Northeastern Ry. (England)	19 1/4	3 1/4	80
Paris Orleans Ry.	25 5/8	7 7/8	......
Paris Versailles	25 5/8	7 7/8	......
Wannseebahn (Berlin)	33 1/2	12 5/8	......

* Bottom contact surface. All others have top-contact surface.

Turnouts. A typical turnout is indicated diagrammatically in Fig. 19, which shows the lengths which should be measured; F is the distance from the point of the switch to the point of the frog, and is called the "frog distance"; D is the length between the adjacent frogs, and Z the distance apart of the track centers. In such a diagrammatic view the lines represent the gage lines of the track rails. When, however, a contact rail is represented by a line, it is the center line that is given. Having measured the four distances specified above, the radius of the turnout curve may be calculated from the following formulas:

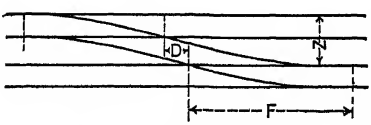

FIG. 19. Typical Turnout

$R = 0.127\ F^2$, when the turnout is from a straight track.

$R = 0.271\ F^2$, when the turnout is from a track of equal radius to the turnout.

Graphical Method of Locating Contact Rail. The track plans having been drawn from the field sketches, to a scale of, say, 1/4 in. to the foot, the contact rail may be drawn in, as outlined below. An aid in this work is a template of the car or locomotive to the same scale as the track drawings, as shown in Fig. 20, in which the points W represent the wheels, S the contact shoes, and K the king-pins. This had better be made of celluloid with a large pinhole at each point S. In order to lay out the contact rail the car template is drawn along the tracks with the points W on the rails and a pencil stuck in one of the pinholes. The line traced by the pencil represents the center line of the shoe path, and therefore that of the contact rail. Such a line should be made on each side of the car. In cases where there are sharp curves, it will not do to run the wheel points W along the tracks, as an error will arise due to the truck rotation not being represented. In such a case the king-pin points must be run along a track center line.

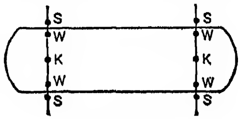

Fig. 20

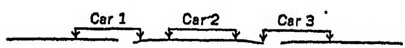

Fig. 21

Location of Inclines. The exact location of the contact-rail inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the equalizer bar and journal box project outward. If there is a train-bus line connecting all the cars, the end inclines may be situated many feet back from the switch point or frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.

Sectionalizing the Contact Rail. Where a train-bus line is not used, it is customary to break the third rail in front of substations and use an isolated section of third rail between the two main sections, as shown diagrammatically in Fig. 21. The length of this isolated section and of the gaps which bound it should be such that both gaps are never spanned by cars at the same time. If the isolated section is connected to one of the main sections by a switch, the cost of a separate feeder to the isolated section is saved.

Let L = car length, feet.
T = distance between shoe centers (usually the same as distance between truck centers).
S = shoe length.

Then
$$\text{Total space must be} > L + T + S$$
$$\text{Rail section must be} < 2L - T - S$$
$$\text{Length of each gap must be} < T - S$$

Cross and End Inclines. Inclines serve the purpose of assisting the contact shoes to rise (or fall) from their free position to the position of contact with the rail. They should, therefore, have sufficient slope to prevent any undue shock either to the rail or to the shoes. Low-speed lines and sidings may therefore have shorter inclines than high-speed lines. The usual length of straight slope (i.e., without end nose and flat surface at top) is from 60 to 70 in. for high-speed work and 30 or 40 in. for sidings. The slope ranges from 1 in 45 for high-speed work to 1 in 30 for urban railways. Inclines are almost invariably made of cast iron. A typical design is shown in Fig. 22.

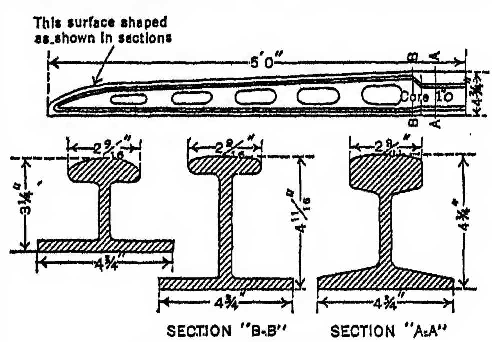

Fig. 22. Typical Inclines

Third-rail Insulators. Third-rail insulators should have the following qualifications. Strength to withstand weight and vibration; surface impervious to moisture; resistance wet shall not be less than 1 megohm; shall have a drip edge; shall allow free motion of rail laterally, longitudinally, and vertically to allow for expansion, contraction, and tie motion; and must be capable of easy and quick removal without disturbing the rail.

Spacing of Insulators. A spacing of 10 ft between insulators is recommended for 30-ft rails, 11 ft for 33-ft rails, and 9.75 ft for 39-ft rails. Location on joint ties should be avoided. Proper and systematic relation of insulators and joints is facilitated by having third rail and track rail of the same length.

INSTALLATION OF CONTACT RAIL. In electrifying an existing railway, the first step is to replace standard ties at proper intervals by long ones. A man then proceeds along the line with a template and marks on them the location of the screw holes of the insulator bracket. He is followed by an auger gang which drills the holes. Meanwhile the brackets (in the case of an under-contact rail) or the stool and insulators (in the case of a top-contact rail) are distributed along the line, and a gang following the auger gang screws the brackets or stools to the ties.

In the case of under-contact rails the brackets must be accurately checked for principal dimensions before being distributed.

The rails and fish-plates, etc., are next distributed along the line on that side of the track where they are to be installed, and a gang follows which installs the rail, and, in the case of an under-contact rail, attaches the insulators and hook bolts; in the case of a top-contact rail, another gang follows, attaching the clips or whatever is used to fasten the rail to the insulators. Another gang follows to install the fish plates, expansion joints, and anchorages, if any. If there are no anchorages or expansion joints, expansion and contraction are provided against by making an allowance at all joints, depending on the temperature at which the rails are installed. This is done by inserting an " expansion shim " while the rails are being bolted together. The thickness of shim for different temperatures on a 30-ft rail is given in Fig. 23.

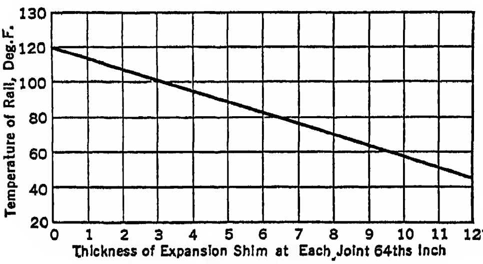

Fig. 23

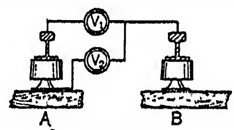

Fig. 24

OPERATION AND MAINTENANCE OF CONTACT RAIL. The wear of the third rails is negligible, and even without painting the deterioration due to rust is very slow. The life of a rail of any of the usual sections should, even under the worst combination of circumstances, seldom be less than 25 years, and present experience seems to indicate a life of upwards of 40 years. The principal items of maintenance are the insulators, protection, and bonds. Unless the supports are carefully designed, insulators are shattered by " tie motion," i.e., by the depression and rebound of the ties as the trains pass over them.

Removal of Sleet. The best protection against sleet is undoubtedly the under-contact third rail, and the next best is a top-contact third rail with a wide protecting board. Where these are impracticable the deposit of ice on the rail cannot be prevented and means for its removal have to be adopted. The two principal means are scrapers and hot water. Electrical heating of the rail has also been suggested but found to require too much energy. When the hot-water method is used, some salt such as calcium chloride, or some other substance, which lowers the freezing point of water, must be added to the water.

TESTING OF CONTACT RAILS. The Conductivity Test is best performed by passing a considerable current through a length, measuring the drop with a millivoltmeter, and calculating the resistance by Ohm's Law.

The Insulation Test is best performed by means of a pair of voltmeters, as shown in Fig. 24. The rail A to be tested is connected to a live rail B through a voltmeter, and the voltage V_1 between the two noted. At the same instant the voltage V_2 between the

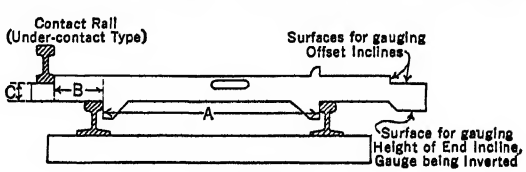

Fig. 25

rail B and ground is also read. Let L be the length in miles of the rail A. Then the insulation resistance of the rail A in megohms per mile is

$$\frac{rL}{10^6}\left(\frac{V_2}{V_1} - 1\right)$$

where r is the resistance in ohms of the voltmeter on which V_1 is read.

Testing of Gage. The gage line of a third rail is measured by means of a template which fits over the track rails as shown in Fig. 25.

10. OVERHEAD CONTACT SYSTEMS

General

CONTACT WIRE. The contact wire is usually made of hard-drawn copper. Where a harder material is required to withstand greater wear and give a larger life, a bronze alloy is used. The conductivity of alloy contact wire is usually between 55 and 80 per cent of that of copper, depending on the particular materials used in the alloy and on the degree of wear resistance desired.

SUSPENSION. The contact wire is usually suspended at heights varying from 15 to 25 ft above the top of the running rails. In any one particular installation, however, it is unusual to find a variation in the height of the contact wire of more than 6 ft. The contact wire presents a continuous contact surface (except for section insulating gaps in some types of construction) to the trolley wheel, bow, or pantograph mounted on the rolling stock.

There are two general classes of contact wire suspensions, direct and catenary.

Direct Suspension. In the simple direct suspension class the contact wire is suspended by insulators from span wires stretched across the tracks between poles or building walls, or from bracket arms attached to the poles. A combination of the two types is sometimes made by using a short-span wire connecting two bracket arms on the same pole. In this single bracket type of construction the poles are usually placed from 90 to 120 ft apart on tangent track, and the contact wire sags considerably between supports, allowing a vertical motion between supports with a very decided "hard spot" at bracket arm supports. This construction is unsuited for high-speed service, owing to the considerable collector motion with resultant arcing and decreased life of the collector and contact wire.

Catenary Suspension. The catenary suspension was designed to overcome the difficulties encountered with the previous type of construction. In general it consists, in its simplest form, of a messenger wire sagging between supporting structures with the contact wire supported from the messenger wire by hangers spaced at equal intervals. The hanger spacing varies from 15 to 30 ft. This method of support results in the contact wire being in a plane approximately parallel to the plane of the running rails by using hangers of varying length at the different points in the span. The bracket arm, span wire, or cross-bridge type of support may be used with the catenary system with supporting structures placed usually 150 to 300 ft apart on tangent track.

Applications of Various Types of Construction

The direct suspension type of contact wire construction is generally used on urban street railways, where it is not objected to on the grounds of unsightliness or danger due to the exposed construction. It is also used on some of the older interurban installations. Center pole construction with bracket arms is usually chosen for street railways for double-track work where it is not objectionable so to locate the poles. The alternate method for street railways is the use of cross-span wires with the poles located near the curb or by attaching the span wires to buildings. This latter method is used quite extensively in Europe.

The catenary-type suspension is generally used for all other overhead contact systems except where the current required by the rolling stock is too great to be economically carried by copper wires or to be satisfactorily collected without undue arcing. This is illustrated by the overhead third-rail construction for special conditions.

Electrical Design of Conductors

In designing the overhead contact system careful attention must be given to both the electrical and the mechanical features. In this division the electrical features will be treated and the mechanical features in a following division.

FEATURES TO BE CONSIDERED. The electrical design of railway distribution systems involves the consideration of potential drop, economy in use of copper, and the heating of the conductors. The heating of conductors is seldom an important feature in

railway distribution system design, as it is generally necessary to use a low current density in order to keep the potential within the limits required. A discussion of the heating of conductors will be found in Section XIV, Art. 51. The feeder system being sufficient to meet the conditions imposed by the allowable potential drop, it will be economical to install more copper, if the saving in the cost of energy which will result is greater than the increase in interest and other charges on the additional investment, plus any additional maintenance charges that may be encountered because of the new facility.

ALLOWABLE POTENTIAL DROP. The maximum allowable potential drop is limited by various features; such as

1. The necessity of running the trains or cars at certain speeds and meeting schedule requirements.
2. The necessity of keeping the illumination in street railway or multiple unit cars at an intensity which varies with the requirements of various localities.
3. The danger of electrolysis in the grounded portion of the return circuit. The maximum drop in grounded circuits, under maximum load conditions, is limited by law in certain localities. In Great Britain the limit is 7 volts drop between any two points in the negative system. In Germany it is generally 1 volt per kilometer (1.61 volts per mile). A usual limit for some cities in the United States is 1 volt per 1000 ft.
4. The necessity of maintaining voltage at the rolling stock to such a value that satisfactory operation of the control circuits will be obtained.

CALCULATION OF POTENTIAL DROP. The method of calculation of potential drop depends upon the following conditions: (1) whether the current is direct or alternating; (2) whether the load is concentrated at one point, sparsely distributed or evenly distributed; (3) the distribution of metal in the feeder circuits, and (4) whether the section is being fed by one or by two or more substations. Calculations for a-c lines differ from those for d-c lines only in taking into account the inductance, as described below. On city railways it is usual to assume the load to be evenly distributed, it being stated as a given number of amperes per foot. (See Art. 2.) If the load is actually concentrated at n equidistant points, the drop will exceed that calculated on the assumption of uniform distribution by about $\frac{100}{n}$ per cent. On interurban and trunk lines, the cars are usually concentrated at one or two points between substations, making the assumption of uniform distribution impracticable. In such cases the loads should be located so as to give the worst conditions, and calculations made as for any network.

Where electrolytic damage is to be guarded against, the drop of potential in the track rails themselves has to be calculated, as well as the total drop in the rails and feeders.

Resistance of Trolley and Track. Values of the resistance of trolley wires to direct current will be found in Section 14, Art 47, and values of the resistance of rails to direct current will be found in Art. 12. It should be noted that the resistances of the trolley and positive feeders are in parallel and that the track rails and negative feeders are in parallel. Also note that in high-voltage systems a considerable portion of the current returns through the earth and not through the rails, and consequently the drop in the rails is due only to that part of the current which returns through them. For preliminary calculations, however, the full current may be assumed as returning through the rails.

FORMULAS FOR D-C TROLLEY CIRCUITS. The following formulas apply to certain typical circuits which frequently occur in practice.

Let I = total current in amperes taken by all cars on section considered.
L = total length of section in 1000 ft.
V_p = total drop in volts, in positive conductors between substation bus and far end of line.
V_n = total drop in volts in negative conductors between substation bus and far end of line.
$V = V_p + V_n$ = total drop in volts in both positive and negative conductors.
r_p = resistance in ohms of all the positive conductors in multiple per 1000 ft of line.
r_n = resistance in ohms of all the negative conductors in multiple per 1000 ft of line.
$r = r_p + r_n$ = total resistance in ohms per 1000 ft of line.
l = distance in 1000 ft, from far end of line to any point P.
v = drop to the point P, subscripts used as for V.

Uniformly Distributed Load, Uniform Conductor, Fed from One Substation. Then

$$v = \frac{rIl^2}{2L}$$

$$V = \frac{rIL}{2}$$

These formulas are applicable to either the positive or negative conductors considered separately or to both in series.

Uniformly Distributed Load, Conductor Tapered to Give Minimum Weight of Metal, Fed from One Substation. For minimum weight the tapering must be such that at any point P

$$r = \frac{3V}{2I\sqrt{L}\sqrt{l}}$$

i.e., the cross-section, if all the conductors are of the same metal, must increase directly as the square root of l. The drop to the point P is

$$v = \frac{V\sqrt{l^3}}{L^3}$$

These formulas also apply to either the positive or negative conductors separately or to both in series.

Uniformly Distributed Load, Conductor Divided into Sections (Fig. 26); Each Section of Constant Resistance, Fed from One Substation. The drop from P_n to the far end of line is

$$\frac{I}{2L}\sum_{n=1}^{n=n} r_n[l^2{}_n - l^2{}_{(n-1)}]$$

This formula is applicable to either the positive or negative conductors. The subscript n is here the general form of the subscript corresponding to each section, represented by 1, 2, 3, and 4 in Fig. 26.

Concentrated Load, Section Fed from One End. Fig. 27 shows a 4-track road with 4 trolley wires and 3 feeders with the tracks cross-bonded at intervals. The solution given below is a general one, and may be applied to any case from 1 track and 1 trolley wire up. Let all distances be expressed in 1000 ft and let

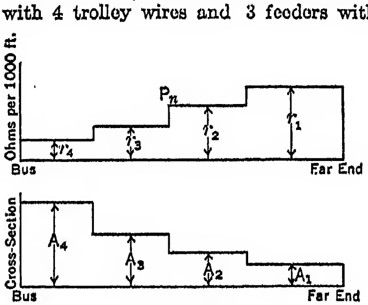

Fig. 26

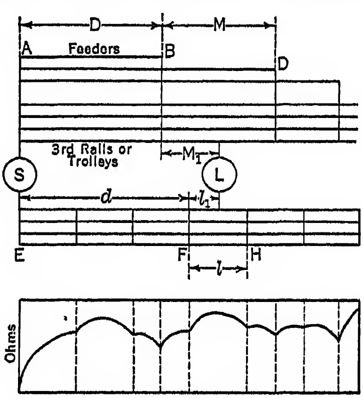

Fig. 27

N_f = number of feeders in section considered, e.g., for the section AB, $N_f = 3$, and for the section BD, $N_f = 2$.

N_c = number of contact conductors, trolley wires or third rails in section considered, e.g., four are shown in Fig. 27.

N_t = number of tracks in section considered.

R_f = resistance of each feeder per 1000 ft.

R_c = resistance of each contact conductor per 1000 ft.

R_t = resistance of each track per 1000 ft (1 rail or 2 rails in multiple depending upon whether 1 or 2 rails are used for return conductor).

$n = N_c + \frac{R_c}{R_f} N_f$ for the section considered.

Then for the section in which the load may be, the resistance of the positive conductors from the load to the end of that section in the direction of the substation, e.g., the resistance from L to B, is

$$R_c M_1\left[1 - \frac{(n-1)M_1}{nM}\right]$$

The resistance of the positive conductors in any section such as BA is

$$\frac{R_c D}{n}$$

(Note that the value of n for this section is not the same as for the section BD.)

The resistance of the negative conductors from the load to the first cross bond in the direction of the substation, e.g., the resistance from L to F, is

$$R_t l_1 \left[1 - \frac{(N_t - 1) l_1}{N_t l} \right]$$

The resistance of the remaining portion of the negative conductors, e.g., from F to E, is

$$\frac{R_t d}{N_t}$$

(Note that if negative feeders are used, each negative feeder having a resistance of $R_f{}'$ per 1000 ft, then for N_t in the last two formulas substitute $n' = N_t + \frac{R_t}{R_f{}'} N_f{}'$, where $N_f{}'$ is the number of negative feeders for that section.)

The total resistance from the load to the substation is the sum of the resistances as above calculated.

Concentrated Load, Section Fed from Both Ends, Substation Voltage at the Two Ends the Same. The most convenient method of treating such problems is to plot an "equivalent-resistance-distance" curve such as shown in Figs. 27 to 30. By "equivalent resistance" is here meant that resistance by which the total current taken by the load must be multiplied to give the total drop in voltage between the load and either substation. For example, if the substation voltage is 600 at each end, the voltage across the load is 550 and the current taken by the load is 200 amp, then the equivalent resistance is $R = \frac{600 - 550}{200} = 0.25$. This method avoids the determination of the distribution of the current in the various parts of the network, and the resistance when once determined can be applied to any load.

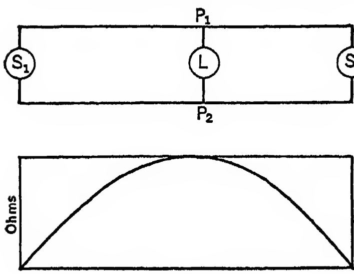

FIG. 28

1. In Fig. 28 is shown a single track and single trolley. S_1 and S_2 are substations; L is a load placed arbitrarily between corresponding points P_1 and P_2 on the positive and negative conductors respectively.

Let a = resistance of the conductors between the points $S_1 P_1 P_2 S_1$;
b = resistance of the conductors between the points $S_2 P_1 P_2 S_2$.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. Then the equivalent resistance is

$$R = \frac{ab}{a + b}$$

2. In Fig. 29 is shown a 4-track road with 4 trolleys, both track and trolley cross-bonded. Fig. 29a is a simplified diagram of Fig. 29, corresponding points being designated by identical letters.

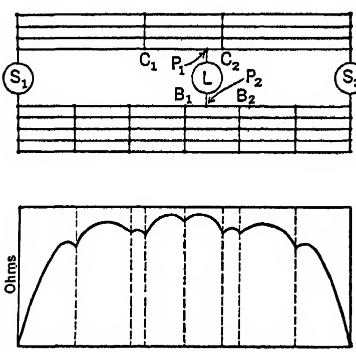

FIG. 29

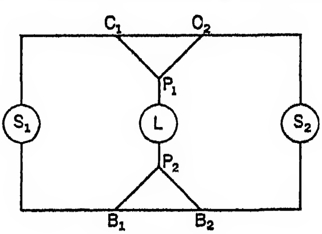

FIG. 29a

S_1 and S_2 are substations; L is a load placed arbitrarily between points P_1, on the positive system and P_2 on the negative system; C_1 and C_2 are ties between positive conductors and B_1 and B_2 ties between negative conductors.

Let

a = resistance of loop $S_1\,C_1\,P_1\,LP_2\,B_1\,S_1$.
b = resistance of loop $C_1\,P_1\,C_2\,C_1$.
c = resistance of contact conductor $C_1\,P_1$.
d = resistance of contact conductor $C_2\,P_1$.
e = resistance of loop $B_1\,P_2\,B_2\,B_1$.
f = resistance of track $B_1\,P_2$.
g = resistance of track $B_2\,P_2$.
h = resistance $\dfrac{d^2}{b} + \dfrac{g^2}{e}$.
i = resistance $\dfrac{c^2}{b} + \dfrac{f^2}{e}$.
j = resistance $S_2\,C_2\,P_1\,LP_2\,B_2\,S_2$.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. The equivalent resistance is

$$R = \frac{aj - ah - ji - 2\left(\dfrac{fg}{e} \times \dfrac{cd}{b}\right) + \dfrac{f^2\,d^2 + c^2\,g^2}{eb}}{a + j - h - i - 2\left(\dfrac{fg}{e} + \dfrac{cd}{b}\right)}$$

Concentrated Load, Section Fed by Feeders from Both Ends. Fig. 30 shows a simple case with feeders for the positive conductors only. No general formula is available for such a circuit. Any such network can, however, be calculated by Kirchhoff's Laws (q.v.), using the numerical values of the resistances for the various resistances. The equivalent resistance from the substations to the load is then the drop in voltage for a load of one ampere.

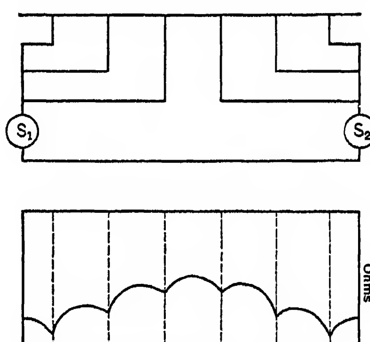

FIG. 30

METHODS OF REDUCING TOTAL DROP. Three methods are employed for reducing the total drop between the substation and the load. The one in most general use is the installation of additional copper as feeders connected to the contact wire, third rail, or track at frequent intervals. These feeders are usually bare and carried on insulators on the pole line. The resistance of the positive or contact wire side of the circuit is usually high compared to the negative or rail side of the circuit, and the feeders are therefore added usually on the positive side of the circuit only. Feeders on the negative side of the circuit are usually installed only as an electrolysis protective measure, but are sometimes required in order to permit track repairs on a single-track line to prevent the danger of a high voltage existing across the gap when track rails are removed at the time that a car or train may be using power. Figs. 27 and 30 indicate two methods of making feeder connections.

The second method, which is not in general use, is to connect a feeder to the contact wire or third rail at one point only, using a booster in the feeder circuit at the substation. The principle of this method is to make the feeder carry more current than it would on the basis of resistance ratios. The first cost and operating costs are usually prohibitive from an economical standpoint, when measured by the improvement obtained. This method is, however, successful and economical on the negative side of the circuit as a means of electrolysis protection, in which case, however, boosters are usually omitted.

The third method consists of the use of an additional substation either fixed or portable at or near the point of low voltage.

A-c Railway Circuits

When alternating current is used the current returning in the rail does not distribute itself uniformly throughout the rail because of the skin effect. On account of the irregu-

larity of the perimeter of a rail a simple mathematical formula cannot be given for the effective resistance or for the internal reactance. At the present time there are two general methods of attack in estimating the constants of an a-c railway system. One method is to use constants which have been measured on a road which has been electrified. This method is most useful when extending a system already in operation or in making calculations on a new system having the same-size rails and same overhead circuits as an electrification already in existence. The other method is to use certain approximations which have been found satisfactory.

In an a-c railway system the return current divides between the rails, the earth, and any other return conductors in parallel with the rails. This division is to a large extent influenced by the mutual effects between the outgoing and returning currents especially when the load and supply points are far apart. The current in the rail is out of phase with the current in the contact wire by a few degrees, but this may be neglected without serious error. For a railroad having the contact wire approximately 20 ft above the rails it may be assumed that the current divides between the rails and ground approximately as follows:

Single track—40 per cent in rails—60 per cent in ground
Double track—60 per cent in rails—40 per cent in ground
Four track—70 per cent in rails—30 per cent in ground

The a-c resistance of a single rail may be calculated by means of the following approximate formula:

$$R_{a-c} = \frac{0.91\sqrt{f}}{P}$$

where f = frequency in cycles per second.
P = perimeter of rail in inches.

The equivalent resistance of earth and rail in parallel may be taken as

$$R_{eq.} = \frac{a}{b} R_{a-c}$$

where a = the percentage of catenary current in the rail.
b = the number of rails in parallel.

The ground current may be assumed to return in the image of the overhead conductors at a depth of from 500 to 3000 ft, depending on the locality and the nature of the strata.

The reactance components are calculated by considering the division that has been assumed and applying the law that the voltage drops in parallel paths are equal. The current in each path sets up a flux which is assumed to extend to a finite distance. When all the linkages have been considered this factor cancels out. The assumed divisions of currents appear as exponents of the different separations.

In calculating the constants of three-wire systems two methods may be followed, depending on whether the equivalent circuit is to be set up with the three circuits intact, or in the calculations to have circuits replaced by single impedances. The fundamentals are the same in both cases. The vector sum of the currents in the feeder, catenary, and track is equal to zero, but the division of currents between the feeder, catenary, and rail depends on the circuit constants.

If an equivalent system is to be set up with the three circuits intact, the current in the catenary is assumed to be I and the current in the rail aI. The equivalent impedance is calculated by determining the voltage drop in each line and dividing by the current in the line. It will be found that the finite distance cancels out and that the assumed division of current appears as an exponent of the separations of catenary, feeder, and rail.

If an equivalent circuit is to be set up in which circuits are replaced by equivalent impedances, the impedance of each circuit and the mutual impedance between circuits are calculated separately. In a three-wire system there are three circuits—the catenary-rail circuit, the feeder-rail circuit, and the feeder-catenary circuit. The *self*-impedance of each of these circuits is calculated as if it existed alone. The *mutual* impedance between the catenary-rail and feeder-rail circuit is calculated by assuming a unit current in one circuit and calculating the voltage in the other when it is open-circuited. In the same way the mutual impedances between the catenary-rail and the feeder-catenary circuits and between the feeder-rail and catenary-rail circuits are calculated. The system may be represented by any two of the three circuits and the mutuals between the circuits selected.

The current division in a two-wire or three-wire system may be calculated most easily by using a calculating board. The a-c board involves the least amount of labor but it is possible to obtain sufficiently accurate results by using a d-c board.

The more laborious method is to work back to the source of supply, by starting from

the point where the load is being considered, and then expand the circuit as the current divisions and voltage drops are determined. In general a separate calculation must be made for each load point and for each source of power. A unit current load at zero angle is assumed at each substation, and the current division throughout the network is determined. The load between substations is assumed to be equivalent to a load at the substation on either side of the load. The equivalent load at a substation is assumed to be inversely proportional to the distance of the substation from the load. The principle of superposition is then applied to determine the current division for the entire system.

High-voltage transmission lines for a-c railway systems are usually single phase. In two-wire systems and in some three-wire systems the transmission line must act as a distribution line in addition to its normal function as a transmission line. The constants of the line may be calculated by the methods used for calculating the constants of three-phase lines. The problems of wave form, relaying, insulation coordination, stability, and other problems of high-voltage engineering require special methods of attack, but the same fundamental principles may be used as in considering three-phase systems.

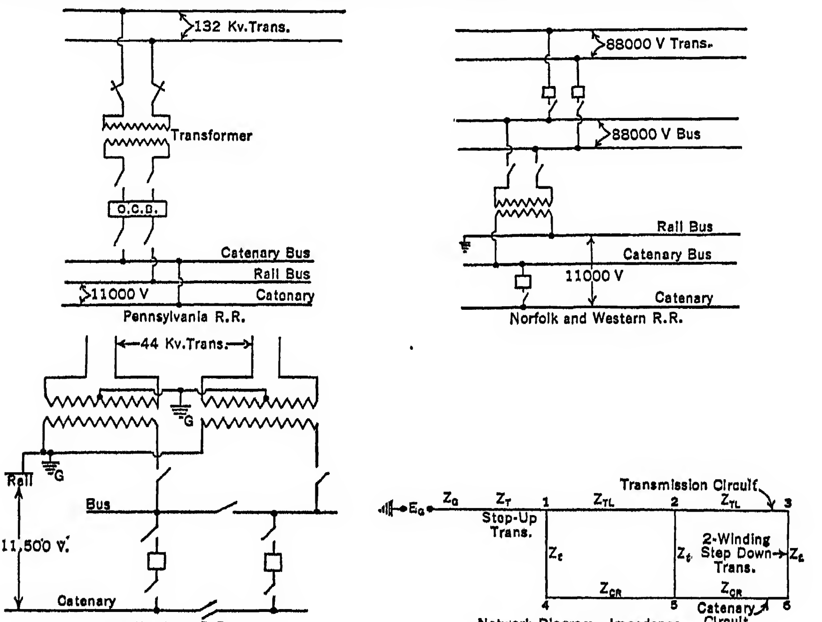

FIG. 31. Two-wire A-c Railway Distribution System

Inductive coordination must be considered in all railway systems whether d-c or a-c. In a-c systems acoustic shock is probably the most serious problem. On the part of the railroads the application of the a-c high-speed breaker has done much to reduce this trouble in addition to protecting overhead lines, current collectors, and apparatus. In laying out a power system for a railroad all interferences are considered and the system is so designed as to reduce such interferences to the minimum consistent with economical design.

The three distinct types of a-c railway distribution systems are illustrated in Figs. 31, 32, and 33.

THE TWO-WIRE SYSTEM (Fig. 31). In this system the power, after being transformed to a suitable transmission voltage, is carried by single-phase lines to step-down transformer stations at suitable locations, stepped down to the catenary voltage, and connected to the catenary and rail. Examples of this type of system are:

Railroad	Transmission Voltage	Catenary Voltage
Pennsylvania R.R.	132 and 44 kv	12 kv
Norfolk and Western R.R.	88 and 44 kv	11 kv
Great Northern R.R.	44 kv	11.5 kv

THE THREE-WIRE SYSTEM (Fig. 32). For systems of limited extent (25 to 30 miles either way from the power supply source), the transmission voltage may be fairly low and the catenary is used as one side of the transmission circuit. The other wire is called the feeder. Three-winding transformers are generally used at the power source. Power is transformed to a suitable transmission voltage, usually 24 or 36 kv, and fed over the transmission circuit consisting of the catenary and feeder to auto-transformer stations where it is transformed to catenary-rail voltage. The three-winding transformer at the power source also supplies power directly to the catenary-rail circuit. By varying

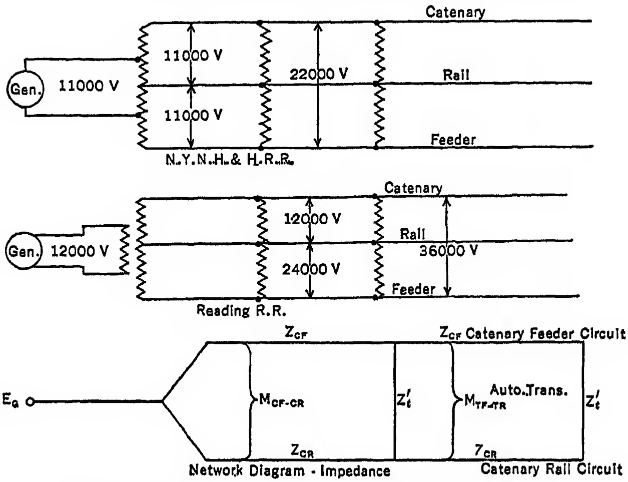

FIG. 32. Three-wire A-c Railway Distribution System

the reactance of the three-winding transformer the power flow may be proportioned between catenary-rail and catenary-feeder circuits to reduce interference in communication circuits. The following are examples of this system:

Railroad	Catenary-feeder Voltage	Catenary-rail Voltage
New York, New Haven & Hartford R.R.	22 kv	11 kv
Reading Railroad (initial installation)	36 kv	12 kv

THE THREE-WIRE SYSTEM WITH TRANSMISSION CIRCUIT (Fig. 33). Economies may sometimes be effected by the use of a combination of the two previously

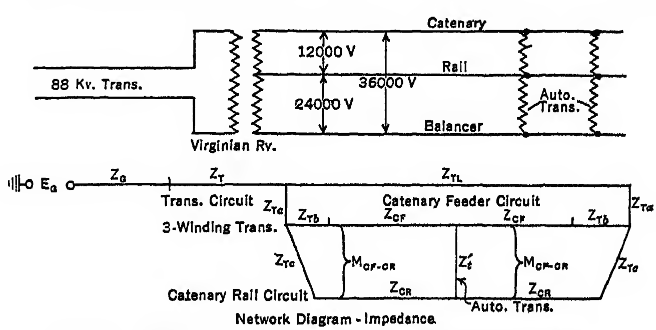

FIG. 33. Three-wire A-c Distribution System with Transmission

described systems. The transmission circuit may be run on a shorter route cross-country meeting the railroad right-of-way only at infrequent intervals with the three-wire system between these points. The three-wire system may be used for the initial electrification and the transmission system added in later years when increased power demand requires it. Examples of this system are:

Railroad	Transmission Voltage	Catenary-feeder Voltage	Catenary-rail Voltage
Virginian Ry.	88 kv	36 kv	12 kv
Reading R.R.	66 kv	36 kv	12 kv

IMPEDANCE DIAGRAMS. In the figure for each system there is shown a single line impedance diagram for use in calculating short-circuit currents, regulation, induction, load division, relaying problems, etc. The constants of the various branches having been determined, these networks can then be solved by the usual "network methods." Impedances for all apparatus and aerial circuits can be calculated the same as for power systems. Impedances of circuits having ground and rail return, such as a catenary-rail circuit, can be calculated the same as for ground return power circuits in which ground wires are accounted for. However, a larger proportion of the current generally returns in the rails than in the ground wires. Average values for 100-lb rails are given in Table XXV. Since a rail is made of steel its internal reactance is high so that a fictitiously low equivalent radius must be used to account for this. The radius for a 100-lb rail has been found experimentally to be 0.35 in. Based on this equivalent radius for 1 rail, geometric mean radii for 1, 2, 3, and 4 tracks are given in Table XXV.

Table XXV

No. of Tracks (Pairs of Rails)	Rail Current in Percentage of Catenary Current	Geometric Mean Radius, ft
1	40	0.37
2	60	2.16
3	65	4.54
4	70	7.17

Owing to the large skin effect the ratio of a-c to d-c resistance of rails is high, the 25-cycle resistance per pair of rails being 0.092 ohm per mile.

On the basis of the proportions of rail current given in Table XXV total impedances of typical catenary circuits are given in Table XXVI. Variations in earth conductivity or catenary height encountered in practice would not ordinarily cause variations of more than 10 per cent in the values given in this table.

Table XXVI *

Catenary-rail Circuit	Catenaries	Tracks	Catenary Circuit	Return Circuit	Total Impedance
3/0 55% 1/0 copper 100-lb rails	1	1	0.316 + j 0.306	0.037 + j 0.084	0.353 + j 0.390 = 0.526
	1	2	0.316 + j 0.310	0.028 + j 0.073	0.344 + j 0.383 = 0.514
	1	4	0.316 + j 0.322	0.016 + j 0.051	0.332 + j 0.373 = 0.498
	2	2	0.158 + j 0.170	0.028 + j 0.073	0.186 + j 0.243 = 0.305
	2	4	0.158 + j 0.180	0.016 + j 0.049	0.174 + j 0.229 = 0.289
	4	4	0.079 + j 0.094	0.016 + j 0.049	0.095 + j 0.143 = 0.171
3/0 55% 2/0 copper 100-lb rails	1	1	0.270 + j 0.309	0.037 + j 0.084	0.307 + j 0.393 = 0.498
	1	2	0.270 + j 0.313	0.028 + j 0.073	0.298 + j 0.386 = 0.488
	1	4	0.270 + j 0.325	0.016 + j 0.051	0.286 + j 0.376 = 0.472
	2	2	0.135 + j 0.172	0.028 + j 0.073	0.163 + j 0.245 = 0.294
	2	4	0.135 + j 0.182	0.016 + j 0.049	0.151 + j 0.231 = 0.275
	4	4	0.068 + j 0.095	0.016 + j 0.049	0.084 + j 0.144 = 0.166
3/0 55% 3/0 copper 100-lb rails	1	1	0.232 + j 0.312	0.037 + j 0.084	0.269 + j 0.396 = 0.478
	1	2	0.232 + j 0.316	0.028 + j 0.073	0.260 + j 0.389 = 0.467
	1	4	0.232 + j 0.327	0.016 + j 0.051	0.248 + j 0.378 = 0.450
	2	2	0.116 + j 0.173	0.028 + j 0.073	0.144 + j 0.246 = 0.282
	2	4	0.116 + j 0.183	0.016 + j 0.049	0.132 + j 0.232 = 0.266
	4	4	0.058 + j 0.096	0.016 + j 0.049	0.074 + j 0.145 = 0.162
4/0 55% 4/0 copper 100-lb rails	1	1	0.189 + j 0.309	0.037 + j 0.084	0.226 + j 0.393 = 0.452
	1	2	0.189 + j 0.313	0.028 + j 0.073	0.217 + j 0.386 = 0.442
	1	4	0.189 + j 0.325	0.016 + j 0.051	0.205 + j 0.376 = 0.428
	2	2	0.094 + j 0.172	0.028 + j 0.073	0.122 + j 0.245 = 0.273
	2	4	0.094 + j 0.182	0.016 + j 0.049	0.110 + j 0.231 = 0.256
	4	4	0.047 + j 0.095	0.016 + j 0.049	0.063 + j 0.144 = 0.157
4/0 copper 100-lb rails	1	1	0.260 + j 0.368	0.037 + j 0.084	0.297 + j 0.452 = 0.540
	1	2	0.260 + j 0.372	0.028 + j 0.073	0.288 + j 0.445 = 0.528
	1	4	0.260 + j 0.384	0.016 + j 0.051	0.276 + j 0.435 = 0.513
	2	2	0.130 + j 0.200	0.028 + j 0.073	0.158 + j 0.273 = 0.315
	2	4	0.130 + j 0.210	0.016 + j 0.049	0.146 + j 0.259 = 0.296
	4	4	0.065 + j 0.109	0.016 + j 0.049	0.081 + j 0.158 = 0.177

* From Regulation and inductive effects in single phase railway circuits, by A. W. Copley, in *Elec. J.*, August, 1920.

Mechanical Design of Overhead Systems

The American Electric Railway Engineering Association has adopted specifications for overhead line material and construction which cover most of the mechanical details pertaining to ordinary contact wire suspension systems.

DIRECT SUSPENSION CONSTRUCTION. The contact wire is secured to an " ear " by soldering, clamps, or other means; the ear is bolted to a " suspension " which may or may not be provided with an insulating portion. These suspensions may be fastened directly to the bracket arm, or carried by span wires which in turn are fastened to the poles or bracket arms. If the ear is not insulated, strain insulators are inserted in the span wire between the ear and the point of attachment to the poles or bracket arms. A slightly different construction called " pull-off " is used on curves. In a system designed for use with trolley wheels, " trolley-frogs " must be used at turnouts to guide the trolley wheels. Complete information and dimensions of the various parts referred to are contained in the A. E. R. E. A. specifications previously cited. A few of the more important items of construction are covered briefly in the following paragraphs.

LENGTH OF SPAN. In general this type of construction should not be used for spans greater than 150 ft, 90 to 110 ft being most generally used.

HEIGHT OF CONTACT WIRE. The height of contact wire above the top of the rail varies between 15 and 22 ft with a recommended height of 18 ft in streets and general locations, except where special conditions govern; the usual height over steam railroad crossings should be 22 ft. These values are for conditions of maximum sag.

RAKE OF POLES. Wood poles for bracket-arm construction should in general have a backward rake from the track of 6 in. in 24 ft. For cross-span construction wood poles should have a rake of 12 in. in 24 ft. Steel poles should have a rake half that of wood. Center poles (between tracks) should be set without rake.

ANCHORAGE. A permanent anchorage for the contact wire should be provided at intervals of $^1/_4$ to $^3/_4$ mile and at both ends of every grade and curve.

CURVES. On curves of less than 500-ft radius a pole spacing of 50 ft should be used; for 500 to 800-ft radius, 75-ft spacing; and over 800-ft radius the same spacing should be used as for tangent track but not over 100 ft.

At curves the contact wire is made to follow the curvature of the track by pull-offs (wires pulling the contact wire toward the outside of the curve). Pull-offs should be radial to the contact wire where possible and not too expensive.

The number of pull-offs should be such that the contact wire is kept within about 2 $^1/_2$ in. from the theoretical curve and is given by the formula

$$L = \sqrt{\frac{2aR}{3} - \left(\frac{a}{6}\right)^2}$$

where L = distance between pull-offs in feet.
R = radius of curve in feet.
a = offset of contact wire in inches from the theoretical curve, midway between pull-offs, the ears being assumed to be on the theoretical curve.

Or
$$L = 0.815\sqrt{aR}$$

with an error less than $^1/_4$ per cent for all radii greater than 40 ft. If a is to be 2 $^1/_2$ in. then

$$L = 1.29\sqrt{R}, \text{ approximately.}$$

CATENARY SUSPENSION CONSTRUCTION. In general, catenary construction may be divided into simple and compound. In the former the contact wire or wires are supported by hangers directly from the messenger wire. In the latter another wire, called the auxiliary messenger, is used to support the contact wires by clips or very short hangers, being itself supported by hangers from the main or primary messenger wire.

The compound catenary is more flexible than the simple catenary unless some form of flexible or loop hanger is used which might necessitate the use of shunts in each span in order to prevent arcing at the hangers. Flexibility in the overhead construction is very important especially where high-speed operation is encountered, as a " hard spot " in the overhead results in arcing with decreased life of contact wire and collector device. An aid to flexibility is the use of two contact wires in approximately the same horizontal plane, each wire being fastened to alternate hangers or clips. The choice between simple and compound construction also depends on the total conductivity required.

The same general requirements for pull-offs, anchorages, etc., are encountered with catenary construction with the direct suspension type.

At curves the catenary construction may be handled by either the " tangent chord " or the " inclined " construction. This latter type does not require pull-offs but does require more detailed attention to hanger design.

It is almost the universal custom to provide "steadies" in tangent track at each supporting structure to prevent side sway caused by wind or resulting from rolling stock motion. "Steadies" are very similar in construction to pull-offs. See Table XVII.

Table XXVII. Overhead Catenary Construction

Railroad	Trolley Voltage	Equivalent Conductivity, cir mils	Span Length Tangent, ft	Type of Curve Construction	Main Messenger Sag	Main Messenger Kind	Main Messenger Diameter, in.	Auxiliary Messenger Material	Auxiliary Messenger Diameter	Grooved Contact Wires	
										Material and Size	No. Used
Chi. M. St. P. & P....	3,000	443,200	150	Chord	20 in.	SM	0.500	......	....	Cu 4/0	2
Black River........	3,000	450,000	150	"	20 in.	(1)	0.375	......	....	Cu 4/0	2
Cleve. Union Term....	3,000	793,800	300	"	4 ft 4 in.	(3)	0.878	Cu	0.528 in.	Bz 4/0	2
Illinois Central........	1,500	838,900	300	"	4 ft 9 in.	(2)	0.810	Cu	0.512 in.	Bz 3/0	2
" "	1,500	898,700	300	"	4 ft 9 in.	(2)	0.810	Cu	0.375 in.	Cu 4/0	2
" "	1,500	840,000	300	"	4 ft 9 in.	(1)	0.810	Cu	0.512 in.	Bz 3/0	2
D. L. & W. R.R.......	3,000	792,000	300	"	5 ft 0 in.	(3)	0.821	Cu	0.414 in.	Bz 4/0	2
N. Y. N. H. & H.											
Woodln.-Stamford....	11,000	335,360	300	"	6 ft 5 in.	(4)	9/16	Cu*	0.482 in.	Bz 4/0	1
Har. Stam.-N. Haven	11,000	320,640	{300} {150}	Inclined	15 in. for 150-ft span	(5)	5/8	Cu*	0.482 in.	Bz 4/0	1
Danbury............	11,000	315,800	250	"	4ft9½in.	(5)	9/16	Cu*	0.482 in.	Bz 4/0	1
Norfolk & Western....	11,000	216,700	300	"	4 ft 6 in.	(6)	1/2	Cu*	0.392 in.	Bz 3/0	1
Penn. R. R. "Paoli"...	11,000	335,600	285	"	5 ft 5 in.	(6)	0.625	Cu	0.482 in.	Bz 4/0	1
" Wilmington.....	11,000	334,200	300	"	5 ft 6 in.	(6)	0.625	Cu	0.460 in.	Bz 4/0	1
" Trenton........	11,000	373,370	300	"	5 ft 6 in.	(6)	0.625	Cu	0.460 in.	Bz 4/0	1
N. Y., N. H. & H., Yds. Sidings............	11,000	61,760	300	{Mostly chord	5 ft 3 in.	(6)	3/8	Cu	....	Bz 2/0	1
Virginian Railway.....	11,000	714,600	320	Inclined	5 ft 0 in.	(3)	0.725	Cu	2-3/0	Bz 3/0	1
" "	11,000	645,000	320	"	in	(3)	0.725	Cu	2-2/0	Bz 3/0	1
" "	11,000	512,000	320	"	300 ft	(3)	0.555	Cu	2/0	Bz 3/0	1

SM—Siemens Martin hi-strength steel.
(1) Hi-strength rod Brass, 7 strand.
(2) Hi-strength copper-covered steel, 7 strand steel, 12 strand copper.
(3) Hi-strength bronze, 19 strand bronze, 16 strand copper.
(4) Hi-strength steel gal. ex. hi. 7-strand-2-cables.
(5) Hi-strength steel gal. ex. hi. 19-strand.
(6) Hi-strength steel gal. ex. hi. 7-strand.

* Grooved auxiliary messenger.

Chi. Mil. St. P. & Pacific. Modified simple catenary, 885.21 miles, wood pole, bracket construction with cross-span at sidings and passing tracks. Feeder tap every 1000 ft. Contact trolleys side by side.

Cleveland Union Terminal; Compound catenary cross-bridge construction, 56 miles. Two trolleys; on main, side by side; in yards, one above the other.

Ill. Central. Compound catenary cross-bridge and center pole construction, 154.5 miles. Cross-span construction in yards.

D. L. & W. R. R. Compound catenary cross-bridge construction on main line, 160 miles. Simple catenary on side tracks and yards. Bracket construction over single-track branch.

New York, New Haven & Hartford R. R.: 576.8 miles. Three point compound catenary, regular compound catenary, special double compound catenary, and simple catenary in yards. Cross bridge on main line, cross span in yards, and pole and bracket on the Danbury branch.

Norfolk & Western: 208.63 miles of cross-span compound catenary construction. Poles and brackets on single-track sections.

Penna. R. R.: 1400 miles. Cross span compound catenary construction. H poles and tubular poles are used which also carry feeders.

Virginian R. R.: 228.54 miles. Cross span compound catenary construction. Some sections have two auxiliary messengers in vertical plane with contact wire. Single track, pole and bracket construction.

11. UNDERGROUND TROLLEY SYSTEMS

This system is little used on account of its cost, the installations at Budapest, New York, Washington, and London being the only notable ones. The essential feature is the underground conductor which is reached from the car by a "plow" extending through a continuous slot parallel to the tracks.

NEW YORK SYSTEM. The New York type, in successful use for many years, is described briefly below.

General Description. The street is excavated and cleared of obstructions for a width of about 5 ft 6 in. and a depth of about 3 ft and yokes (Fig. 34) set in the excavation about 5 ft apart. The track and slot rails are supported on these yokes and the whole

system made solid with concrete which fills the excavation from the foundation to near the top of the rails leaving only a tunnel under the slot free from masonry. The conductors, of a special rolled-steel flanged tee section, are suspended in this tunnel, by special strain insulators, no part of the electrical system being grounded. The use of two insulated conductors avoids trouble in case of accidental grounding.

Yoke. The yoke shown in Fig. 34 is made in three parts: an I-beam which rests on the floor of the excavation; and two castings riveted to this, each supporting one track rail and one slot rail. The track rail is carried on a timber stringer which extends from yoke to yoke along the whole line. The stringers are held to the yokes by bolts which serve also to fasten the rails to the stringers. The rails are further secured by long bolts running to the center of the yoke. The slot rails are bolted direct to the yoke and are connected to the track rails by long bolts every 30 in.

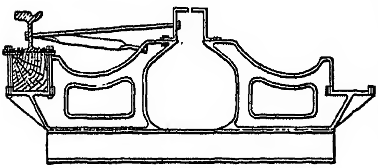

Fig. 34

Insulator Boxes. Cast-iron boxes are laid across from the slot rails to the track rails every 15 ft and bolted to them. These contain the insulators which support the contact conductors. In each box is a pair of shelves; resting on these shelves is a cast-iron bridge which holds the insulator. A cast-iron cover is flush with the street surface.

Concrete Work. The tunnel which is to protect the conductors and do the draining is formed with collapsible forms. Concrete is then poured into the excavation, completely filling every part but the tunnel and its offsets at the insulator boxes to within the height of a paving block of the rail tops. The depth of the completed tunnel is 18 in. from the base of the slot rails.

THE LONDON SYSTEM has alternate long and short yokes, the long ones fulfilling the same function as the New York ones, the short ones serving merely to give additional support to the slot rails. No stringers are used, the rails resting on hardwood blocks at the yokes only.

THE BUDAPEST CONSTRUCTION has the slot in the track rail, thus saving considerable iron, but necessitating an excessively wide slot to accommodate the wheel flanges.

12. TRACK AND THIRD RAILS

(See also Bonds, Railway Track; Traction Systems for Electric Railways; Third-rail Systems; Trolley Systems.)

The American Electric Railway Engineering Association has standardized the rail sections summarized in Table XXVIII, which are described fully in its Engineering Manual.

Table XXVIII. Rail Sections

Designation of Rail	Height, in.	Weight, lb per yd	Designation of Rail	Height, in.	Weight, lb per yd
Standard section.....	5	80	Girder grooved......	7	103, 122
" "	5 5/8	90	" "	9	134
" "	6	100	" guard........	7	140
Plain girder..........	7	82, 92, 102	" "	9	152

The standard (" tee ") and plain girder sections are listed for use on private right-of-way and in macadam or other shallow paving, and the girder grooved and guard sections for paved streets.

Standard lengths are as follows: plain girder and girder grooved rail, 60 and 62 ft; girder guard rail, 30 and 32 ft; standard section (" tee ") rail, 33 or 39 ft as ordered, 62 ft when so specified.

RESISTANCE AND CHEMICAL COMPOSITION. The chemical composition of steel rails with respect to the impurities or elements other than iron (chiefly carbon, manganese, phosphorus, sulfur, and silicon) varies over a considerable range, depending upon the process of manufacture. According to J. A. Capp, the specific resistance of an ordinary* steel rail may be taken as a rough indication of the total impurities present. Fig. 35 shows the results of tests on a number of samples of different makes, ranging in total impurities from 0.1 per cent to 1.9 per cent, the specific resistance referred to copper

* This does not apply to special forms of rails submitted in the process of rolling to extra heavy pressures.

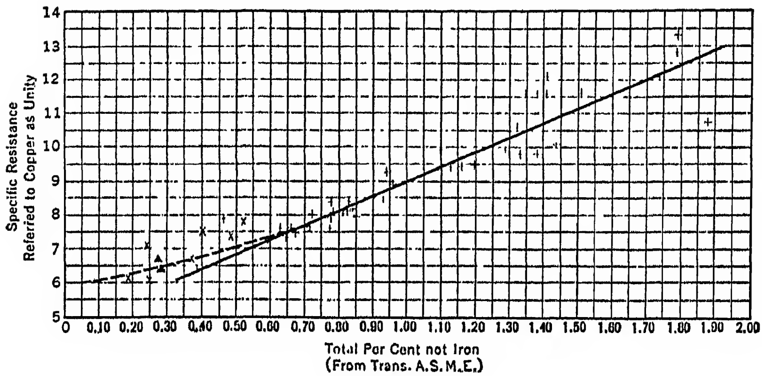

FIG. 35

Table XXIX. Chemical Composition, Track Rails

Designation of Rail	Weight, lb per yd	Carbon, %	Manganese, %	Phosphorus, %	Silicon, %
Standard Section Rails	50-69	0.50-0.63	0.60-0.90	< 0.04	> 0.15
	70-84	0.53-0.70			
	85-100	0.62-0.77			
	101-120	0.67-0.83	0.50-0.90		
	121-140	0.72-0.89			
Girder Rails	A *	0.60-0.75	0.60-0.90	< 0.04	0.15-0.40
	B *	0.70-0.85			
	C *	0.75-0.90			

* Girder guard rails are class A. Plain or grooved girder rails 135 lb or over are class C; under 135 lb may be specified A or B.

Table XXX. Chemical Compositions and Resistances, Third Rails

	New York Central R. R.	Pennsylvania R. R.	Long Island R. R.	Baltimore & Ohio R. R. (Staten Island)	Board of Transportation N.Y. City	Manhattan Elevated R. R.
Weight per yard, lb	70	150	150	....	150	100
Process	O.H.	O.H.	O.H.	O.H.	O.H.	Mfr's option
Length, ft	33	39	39	33	33	60
Specific resistance (ratio to copper) not to exceed:						
No. 1 rails	7.	7.	7.	7.	6.85	7.
No. 2 rails	7.35	7.35	7.35	7.35	7.	7.3
Bonus below	...	6.85	6.85	....	....	....
Chemical composition, %:						
Carbon (Not to exceed)	0.20	0.06,*0.10	0.06 *	0.06*	Note A	Note B
Sulfur (Not to exceed)	.06	.06	.06 *	.05*	Left to discretion of manufacturer	
Manganese (Not to exceed)	.30	.20	.15 *	.15*		
Phosphorus (Not to exceed)	.04	.015	.008*	.025*		
Silicon (Not to exceed)	.05	.05	.04 *	.04*		
Copper from	.20	.15	.15 *	.20		
to	.30	.30	.30 *	.30		
Total, other than iron Not to exceed	...	.70	.70	.70		

* "Desired" (not positive requirement.)
A "May be low carbon, but should have sufficient stiffness to retain shape during shipment and installation."
B Desired not less than 0.15; must not be less than 0.12.

as unity ranging from 6.1 to 13.3. The greater hardness caused by the presence of impurities is an advantage in the case of track rails which offsets the disadvantage of low conductivity, but it is usually economical to employ for the third or contact rail a rail of fairly high conductivity.

The compositions given in Table XXIX are recommended by the American Electric Railway Engineering Association. The specific resistance of track rails, referred to copper as unity, ranges between 10 and 12.5.

As shown, by specification requirements, in Table XXX, current practice favors the use of copper in third rails and low content of carbon and sulfur, to reduce both corrosion and electrical losses.

ALLOWANCE FOR WEAR IN RESISTANCE CALCULATIONS. Resistance calculations should be made for rails worn down to the weight at which they will be scrapped. It is usual to scrap rails when they have lost from 10 to 20 per cent of the original weight, depending upon the importance of the line. The resistances in Table XXXI should therefore be increased from 10 to 20 per cent.

Table XXXI. Resistance of Steel Rails

(Full Cross-section)

Weight, lb per yd	Cross-section, sq in.	Area, millions of circular mils	Specific Resistance 12.5 times that of copper		Specific Resistance 8 times that of copper *	
			Ohms per 1000 ft	Ohms per mile	Ohms per 1000 ft	Ohms per mile
40	3.90	4.95	0.0261	0.138	0.0167	0.0882
45	4.40	5.60	0.0231	0.122	0.0148	0.0782
50	4.90	6.23	0.0208	0.110	0.0133	0.0702
55	5.40	6.86	0.0189	0.0996	0.0121	0.0637
60	5.90	7.50	0.0173	0.0911	0.0110	0.0583
65	6.40	8.14	0.0159	0.0840	0.0102	0.0538
70	6.90	8.77	0.0148	0.0779	0.00944	0.0499
75	7.437	9.45	0.0138	0.0729	0.00884	0.0467
80	7.80	9.9	0.0131	0.0689	0.00835	0.0441
85	8.34	10.5	0.0122	0.0645	0.00781	0.0413
90	8.83	11.2	0.0115	0.0609	0.00738	0.0390
95	9.30	11.8	0.0109	0.0570	0.00701	0.0370
100	9.82	12.5	0.0104	0.0547	0.00664	0.0350
105	10.31	13.1	0.00991	0.0521	0.00632	0.0333
110	10.80	13.8	0.00945	0.0497	0.00604	0.0318
115	11.29	14.4	0.00904	0.0476	0.00577	0.0304
120	11.78	15.0	0.00867	0.0456	0.00553	0.0292
125	12.27	15.6	0.00832	0.0438	0.00531	0.0280
130	12.77	16.2	0.00800	0.0421	0.00511	0.0269
135	13.26	16.9	0.00770	0.0405	0.00492	0.0259
140	13.75	17.5	0.00743	0.0391	0.00474	0.0250
145	14.24	18.1	0.00717	0.0377	0.00458	0.0241
150	14.73	18.7	0.00693	0.0365	0.00443	0.0233
155	15.22	19.4	0.00671	0.0353	0.00428	0.0226
160	15.71	20.0	0.00650	0.0342	0.00415	0.0219

* To find the resistance of rails of any specific resistance x referred to copper as unity multiply these resistances by x and divide by 8.

A-c Resistance and Reactance. See articles on Overhead Trolley Systems; Railway Signaling.

13. RAILWAY TRACK BONDS

(See also Third Rail Systems; Track and Third Rails; Wires and Cables.) Rail bonds are electrical conductors for bridging the joints of rails. They consist of stranded or laminated copper conductors welded or pressed into copper or steel terminals. Sometimes the conductors are made up of copper and steel wires stranded together.

TYPES OF BONDS. Bonds may be classified, according to the method of fastening them to the rail, as welded bonds, soldered bonds, brazed bonds, and bonds applied by mechanical pressure.

Welded Bonds (Figs. 36 and 37) are designed for oxy-acetylene flame welding, steel-electrode arc welding, copper-electrode arc welding, and carbon-electrode arc welding.

They usually consist of all-copper or bimetallic copper and steel stranded conductors with mechanically attached steel or copper terminals or with steel terminals butt-welded to the conductors. The flame-welded bonds are usually installed with a flux-containing copper-alloy welding metal. The arc weld bonds are installed with steel electrode or flux-containing copper-alloy electrode, or with carbon electrode in conjunction with a carbon mold with flux-containing copper as the welding metal.

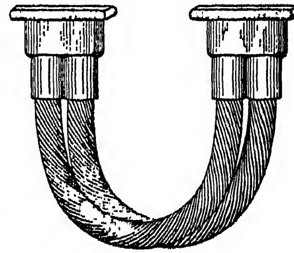

FIG. 36

The designs of welded bonds are many in number and are changing so rapidly that no details of construction are given.

Soldered Bonds (Fig. 38) usually consist of a series of thin strips of annealed copper with tinned terminals. They are soldered direct to the head of the rail. One or more bonds per joint may be used. They are suitable for use only on third rails.

Brazed Bonds resemble soldered bonds except that the terminals are enveloped in brass. They are brazed or welded to the rail by heat generated electrically in a carbon which constitutes the clamp holding the bond against the rail.

Expanded and Compressed Terminal Bonds. Bonds fastened to the rail by mechanical pressure may be divided into two general classes, expanded terminal and compressed terminal bonds. Both kinds are called "stud terminal" bonds.

Pin-expanded Terminal Bonds (Fig. 39) have their heads drilled with an axial hole,

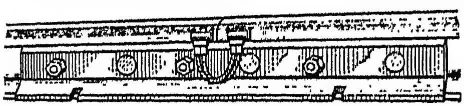

FIG. 37

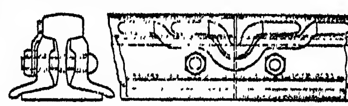

FIG. 38

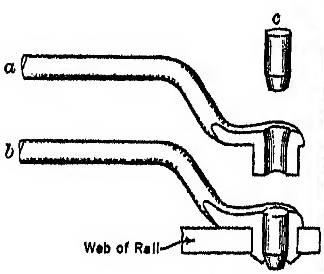

FIG. 39

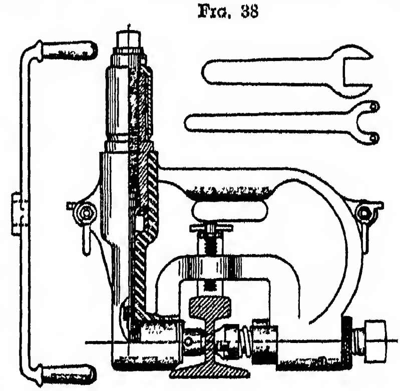

FIG. 40

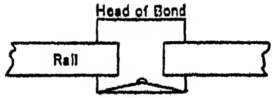

FIG. 41

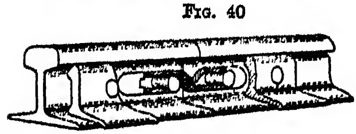

FIG. 42

through which a tapered steel pin *d* is driven, forcing the copper outward and against the steel. This type of bond is fastened to the web of the rail.

Compressed Terminal Bonds have solid copper terminals and are installed with pressure applied at both ends of the stud. The bond is applied to the web of the rail by means of a heavy screw or hydraulic press (Fig. 40) which engages the bond head and causes it to compress longitudinally and expand laterally as the pressure is applied, bringing the copper into firm contact with the steel and spreading the projecting end of the terminal into a button-shaped rivet-head, as shown in Fig. 41.

Exposed Versus Concealed Bonds. Bonds may be either exposed or concealed (Fig. 42) under the fish-plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made.

Welded type bonds are always of the exposed type and are applied either to the head or the top of the flange of the rail. Stud terminal bonds (pin or compressed) may be of the concealed or exposed type. They are always applied in holes drilled in the web of the rail. The choice of type is largely a matter of economics. While concealed bonds are necessarily applied to the web of the rail, exposed bonds may be applied to the foot or head. Head bonds have the advantage of being short. Web or base bonds, unless concealed, have to be long in order to span the splice bars.

USES OF VARIOUS TYPES OF BONDS. Bonds are used for track rails, third rails, and girders of elevated and subway lines. Soldered bonds find their best application in third-rail work, where good electrical contact is of greater importance than mechanical strength. Expanded or compressed terminal bonds, especially of the concealed type, are excellent for heavy traction work, where space underneath the splice bars is adequate. Welded bonds are nearly always used on the head of the rail, within the limits of the splice bars; they are easy to inspect, but are more subject to damage from vibration where traffic conditions are severe.

SUBSTITUTES FOR BONDING. Several efficient substitutes for bonding are now in use, such as electrical welding, cast welding, and Thermit welding. Welded joints are used almost exclusively in paved streets.

Electric Welding is almost exclusively of the arc welding type. Various methods of "sewing" the splice bar along its top and bottom edges to the rail head and rail base by different methods of arc welding are in general use. Resistance welding of the splice plate, because of its relatively high cost, has given way almost entirely to arc welding.

Cast Welding is accomplished by setting a mold around the rail joint and pouring molten iron around it. The Thermit process is a modification of this, the iron being liberated at a white heat from a mixture of iron oxide and aluminum, which is ignited in a crucible.

Flame-welding by oxyacetylene torch is also in successful use.

The **Coullié Bond,** used in France, is a short mechanical splice bar of special shape such that, when the joint is depressed by a wheel, four points make rubbing contact with the head and flange of the rail, thereby maintaining clean electrical contacts.

SELECTION OF TYPE AND SIZE OF BOND. Considerations determining the choice between concealed and exposed bonds are liability to theft, electrolytic corrosion, facility of inspection, and injury to bonds in service.

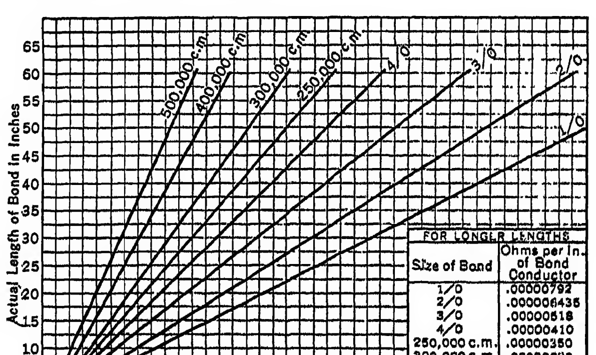

Fig. 43

The choice between mechanically attached and welded bonds depends largely upon the importance of rapid installation, the mechanical stresses to be withstood in service, the type of labor available, and the facilities for the use of drills, presses, etc.

Single vs. Double Bonding. Joints are sometimes bonded with one and sometimes with two bonds. Double bonding has the advantages of less chance of complete failure

and greater carrying capacity for a given cross-section of copper. It has the disadvantages of being more expensive and giving uncertain results in testing, however.

Selection of Cross-sectional Area of Bond. The cross-sectional area of a rail bond should, as a rule, be not greater than is necessary to keep its temperature at a safe working value, unless greater area is required for mechanical strength. The resistance of the bonded joint is of secondary importance unless very high, because the resistance of the joints is usually a mere fraction of the total track-rail resistance (see below).

Carrying Capacity of Bonds. The excellent heat conductivity of copper and the large heat-storage capacity of steel rails tend to make the carrying capacity of bonds in cold weather considerably greater than that of free wire of the same size, especially if the bonds are short. In hot weather, however, the rails and consequently the bonds are likely to become hot from exposure to the sun's rays, thereby reducing the effective carrying capacity of the bonds.

It is not safe to set up a formula for carrying capacity. A very conservative rule is to use 1 amp per 500 cir mil cross-sectional area. If 250 cir mils per amp is used there will be no injurious heating of the bond but the length of bond conductor is a very important factor. Very short bonds will carry considerably more current than long bonds.

Resistance of Bonds. The ohmic resistance of stud terminal rail bonds, at 68 deg fahr, including rail contact resistance, is given in Fig. 43.

Although the resistance values of welded bonds are in general higher, the stud terminal bond resistance values can be used with fair accuracy for welded bonds of all types. There is very little difference between the resistance of one form of welded bond and another when properly applied.

Resistance of Bonded Joint. The conductivity of the splice bars may be neglected. The resistance of a bonded joint can be computed as follows:

Let b = resistance of joint, ohms.
a, a_1, a_2 = resistance of bond A, A_1, A_2 (extended length) ohms, from Fig. 43.
d = resistance of length D of rail, ohms.

Then, for a single bonded joint, $b = a$. For a joint bonded with two equal bonds, lapped, Fig. 44*a*, $b = {}^1/_2\,(a + d)$, and for a joint bonded with two unequal bonds, Fig. 44*b*, $b = \dfrac{a_2(a_1 + 2d)}{a_2 + a_1 + 2d}$

Resistance of Bonded Rail. The resistance of bonded rail can be computed from the following formula:

$$x = nb + r\,\frac{(5280 - ns)}{5280}$$

where x = resistance of bonded rail per mile, ohms.
b = resistance of joint, ohms, from preceding paragraph.
s = length between outside bond terminals, feet (Fig. 44).
n = number of joints per mile.
r = resistance of rail per mile, ohms. (See Rails, Art. 12.)

Length of Bond. The length of concealed bonds is necessarily determined by the spacing of the bolt holes. The bonds for attachment to the head of the rail are usually 7 or 7 $^1/_2$ in. long for single bonding and about 8 $^1/_2$ in. long for double bonding where U-shaped bonds are used. Bonds to span the splice bar must be of sufficient length to have the terminals beyond the ends of the splice bar and far enough away to permit the bond conductor to clear the splice bar properly.

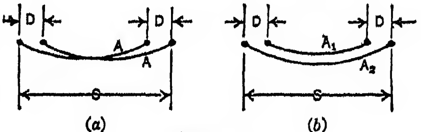

FIG. 44

SPECIFICATIONS. (See also article on Specifications and Contracts in Fund. of Eng.) Specifications for rail bonds should state the exact service conditions under which the bond is to be used, the style of attachment desired, the part of the rail they are to be applied to, the style of conductor (ribbon, solid wire, or cable), the cross-sectional area, the contact area of the stud, the formed length between centers of terminals, and the fish-plate and bolt layout, if the bonds are to be concealed.

INSTALLATION. An important consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Welded Bonds. For the correct installation of any type of welded bond it is essential that the proper welding apparatus and welding metal for applying the bonds be selected. Flame-welding requires the use of flux-containing copper alloy of the proper composition

Exposed Versus Concealed Bonds. Bonds may be either exposed or concealed (Fig. 42) under the fish-plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made.

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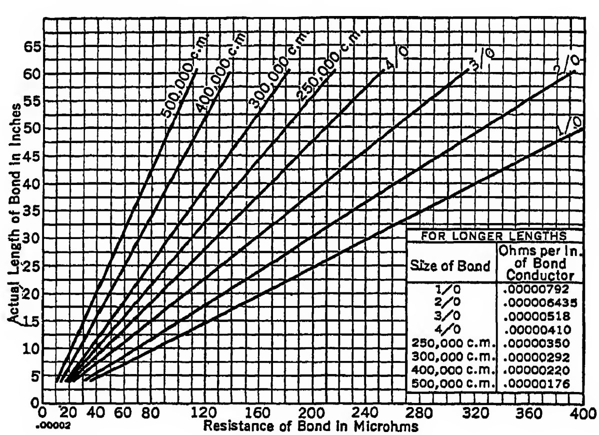
SELECTION OF TYPE AND SIZE OF BOND. Considerations determining the choice between concealed and exposed bonds are liability to theft, electrolytic corrosion, facility of inspection, and injury to bonds in service.



FOR LONGER LENGTHS	
Size of Bond	Ohms per In. of Bond Conductor
1/0	.00000792
2/0	.000006435
3/0	.00000518
4/0	.00000410
250,000 c.m.	.00000350
300,000 c.m.	.00000292
400,000 c.m.	.00000220
500,000 c.m.	.00000176

Fig. 43

The choice between mechanically attached and welded bonds depends largely upon the importance of rapid installation, the mechanical stresses to be withstood in service, the type of labor available, and the facilities for the use of drills, presses, etc.

Single vs. Double Bonding. Joints are sometimes bonded with one and sometimes with two bonds. Double bonding has the advantages of less chance of complete failure

and greater carrying capacity for a given cross-section of copper. It has the disadvantages of being more expensive and giving uncertain results in testing, however.

Selection of Cross-sectional Area of Bond. The cross-sectional area of a rail bond should, as a rule, be not greater than is necessary to keep its temperature at a safe working value, unless greater area is required for mechanical strength. The resistance of the bonded joint is of secondary importance unless very high, because the resistance of the joints is usually a mere fraction of the total track-rail resistance (see below).

Carrying Capacity of Bonds. The excellent heat conductivity of copper and the large heat-storage capacity of steel rails tend to make the carrying capacity of bonds in cold weather considerably greater than that of free wire of the same size, especially if the bonds are short. In hot weather, however, the rails and consequently the bonds are likely to become hot from exposure to the sun's rays, thereby reducing the effective carrying capacity of the bonds.

It is not safe to set up a formula for carrying capacity. A very conservative rule is to use 1 amp per 500 cir mil cross-sectional area. If 250 cir mils per amp is used there will be no injurious heating of the bond but the length of bond conductor is a very important factor. Very short bonds will carry considerably more current than long bonds.

Resistance of Bonds. The ohmic resistance of stud terminal rail bonds, at 68 deg fahr, including rail contact resistance, is given in Fig. 43.

Although the resistance values of welded bonds are in general higher, the stud terminal bond resistance values can be used with fair accuracy for welded bonds of all types. There is very little difference between the resistance of one form of welded bond and another when properly applied.

Resistance of Bonded Joint. The conductivity of the splice bars may be neglected. The resistance of a bonded joint can be computed as follows:

Let b = resistance of joint, ohms.
a, a_1, a_2 = resistance of bond A, A_1, A_2 (extended length) ohms, from Fig. 43.
d = resistance of length D of rail, ohms.

Then, for a single bonded joint, $b = a$. For a joint bonded with two equal bonds, lapped, Fig. 44a, $b = {}^1/_2\,(a + d)$, and for a joint bonded with two unequal bonds, Fig. 44b, $b = \dfrac{a_2(a_1 + 2d)}{a_2 + a_1 + 2d}$

Resistance of Bonded Rail. The resistance of bonded rail can be computed from the following formula:

$$x = nb + r\,\frac{(5280 - ns)}{5280}$$

where x = resistance of bonded rail per mile, ohms.
b = resistance of joint, ohms, from preceding paragraph.
s = length between outside bond terminals, feet (Fig. 44).
n = number of joints per mile.
r = resistance of rail per mile, ohms. (See Rails, Art. 12.)

Length of Bond. The length of concealed bonds is necessarily determined by the spacing of the bolt holes. The bonds for attachment to the head of the rail are usually 7 or 7 ${}^1/_2$ in. long for single bonding and about 8 ${}^1/_2$ in. long for double bonding where U-shaped bonds are used. Bonds to span the splice bar must be of sufficient length to have the terminals beyond the ends of the splice bar and far enough away to permit the bond conductor to clear the splice bar properly.

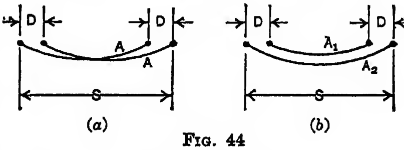


FIG. 44

SPECIFICATIONS. (See also article on Specifications and Contracts in Fund. of Eng.) Specifications for rail bonds should state the exact service conditions under which the bond is to be used, the style of attachment desired, the part of the rail they are to be applied to, the style of conductor (ribbon, solid wire, or cable), the cross-sectional area, the contact area of the stud, the formed length between centers of terminals, and the fish-plate and bolt layout, if the bonds are to be concealed.

INSTALLATION. An important consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Welded Bonds. For the correct installation of any type of welded bond it is essential that the proper welding apparatus and welding metal for applying the bonds be selected. Flame-welding requires the use of flux-containing copper alloy of the proper composition

which the leading manufacturers have developed. Steel electrode welding requires the use of low-carbon steel. Coated electrodes are very desirable. Copper-electrode welding requires the use of a flux-containing copper-alloy electrode and high amperage in order to " puddle " the metal as much as possible. Carbon-electrode welding requires the use of a carbon mold to surround the bond terminal. Flux-containing copper alloy (same as copper-alloy electrode) is melted in the carbon arc until the mold cavity is filled. Proper polarity of electrode is essential. Steel electrodes may be used either positive or negative. Copper electrodes should always be positive. Carbon electrode must be negative.

Brazed Bonds. The rail surface is first ground clean. The bond terminal, being covered with a brass plate, is then placed against the clean surface. A carbon block is pressed against the surface of the bond terminal opposite the rail. The carbon block is then heated to incandescence by means of an electric arc, which does not come in contact with the bond. Borax flux is introduced between the bond terminal and the rail. When the terminal reaches the proper temperature a braze is effected between the bond terminal and the rail.

Soldered Bonds. The rail surface is brightened by means of a silicon carbide (Carborundum) or emery wheel, then tinned, using an acid flux. The bond is then clamped in place and the rail and bond heated by means of a blow-torch, to a temperature at which the solder will melt and cause the bond to adhere firmly to the rail.

Pin-expanded Bonds. The rail is drilled through the web, with or without lubricant. Drilling without lubricant has the advantage of giving a perfectly clean hole. Holes are drilled to the same diameter as the bond studs, and rust or scale on the web of the rail is removed around the hole. The bond stud is then inserted into the hole, and a long taper punch lubricated with grease is driven entirely through the terminal. Then a short drift pin is driven home, as shown in Fig. 39. About 100 bonds may be installed with one taper punch.

This type of bond requires a smaller equipment in tools and materials than most other types and does not necessitate the use of any apparatus which obstructs the track and thereby endangers traffic.

Compressed-terminal Bonds. The drilling having been performed as for a pin-expanded bond, and the bond studs inserted into their respective holes, a screw or hydraulic compressor, as shown in Fig. 40, is applied at both ends of the bond head, the conical point of the press fitting into the conical depression of the bond. Pressure is applied, either until a collar on the ram touches the rail, or until the head of the bond acquires the proper shape. Where no collar is used the point of the press (if of the screw type) sometimes cuts into the bond head; this may be avoided by placing a small amount of flake graphite mixed with oil in the depression of the bond head.

For durable contact the bond terminal stud should be installed with a direct pressure of at least 25 tons per square inch of contact area. Actual contact resistances of stud terminals at 15 tons per square inch contact pressure are given in Table XXXII.

Table XXXII. Contact Resistance of Stud Terminals

Diameter of Terminal Stud, in.	Area of Contact, sq in.	Contact Resistance, ohm	Diameter of Terminal Stud, in.	Area of Contact, sq in.	Contact Resistance, ohm
1	1.77	0.00000040	5/8	1.10	0.00000064
7/8	1.55	0.00000045	1/2	0.88	0.00000080
3/4	1.33	0.00000053	2 twin terminal studs	2.00	0.00000035

The copper of a bond head is hardened by the pressure it is subjected to, and, like the steel, is distorted within its elastic limit, causing the surfaces to adhere even if the pressure is reduced to one-third its original value, say 10,000 lb per sq in. Between these two pressures, the electrical resistance does not vary. Expansion due to heat, therefore, has no effect upon the resistance of bonds.

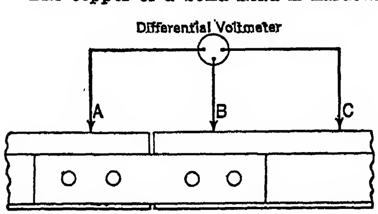


Fig. 45

TESTS AFTER INSTALLATION. The usual method of testing is to measure the drop of potential across the bonded joint and find simultaneously the length of continuous rail in which the same drop occurs, i.e., the " equivalent " length of the bonded joint. Several ingenious instruments have been devised for making this comparison with ease and accuracy.

Voltmeter Method. The simplest form of bond tester consists of a center zero millivoltmeter with three connections to the rail. (See Fig. 45.) With current flowing through the rail in the direction from A to C, the current flows through the millivoltmeter movable coil in the direction A to B to indicate the difference of potential between A and B. The drop between B and C is indicated by the current flowing through the same coil in the direction B to C, thus opposing the drop between A and B, and indicating the difference. By varying the distance BC the drop across the joint and the drop from the solid rail can be equalized. The distance between B and C is then measured and represents the length of rail equivalent to the resistance across the joint.

Many commercial bonding sets have rail contact devices with a fixed spacing of the contacts, which span the rail joints, of 3 ft. In order to obviate correction of the readings of the rail joints as tested, the resistance of 3 ft of joint may be calculated and used for comparison.

14. DISPATCHING OF TRAINS BY TELEPHONE

(See also Pender and McIlwain.) Up to 1907 train dispatching was done almost entirely by the telegraph. In that year the telephone began to replace the telegraph for this purpose. At the beginning of 1935 approximately 151,000 miles of railroad in the United States was being dispatched by telephone, as against approximately 94,000 miles dispatched by telegraph.

Telephone train dispatching has many advantages over the former method. The dispatcher's duties consist in the main of gathering information and issuing orders for train movements. This work is performed much more rapidly and no less accurately by the use of the telephone. Special training in the use of the apparatus is not necessary, as when telegraphy is employed. All the agencies are brought into closer personal relations, with the result that there are closer cooperation and better discipline. The dispatcher can call and speak at the same time, which is impossible by telegraph.

It now is possible to operate dispatching lines as long and with as many stations as railway conditions permit. The length of line and number of stations are not limited by the nature of the electrical equipment. Lines of 300 miles length are in operation, and as many as 115 selector stations, and numerous pole boxes without selectors, on one line are in successful use.

Dispatching circuits are multi-station lines, like the telegraph-dispatching lines they displace. Broadly speaking, they are selective party lines, but are equipped with systems of apparatus which allow the placing of many more stations on a line than is possible with any other party-line system so far developed.

Several train-dispatching systems are on the market, the principal ones being known as the Gill System and the Western Electric System. These are in general similar in that the dispatcher has means of calling any station selectively. In some systems he can call several stations at once by setting keys for them in advance and operating a common calling key.

Each way-station is equipped with a device called a selector. These selectors are operated only by the dispatcher. They work on the step-by-step principle, and the whole object of the device is to close a contact at the station called and thus to ring a bell. No provision is made for cutting the way-station telephone set on and off the line, as secrecy is not aimed at. Any way station may communicate with the dispatcher at will by lifting the telephone receiver. Loud-speaking receivers are generally provided for the dispatcher's use, with head sets for optional use, the dispatcher in either case listening on the line at all times.

Train orders are written as given and received and are read back to the dispatcher by each station, the dispatcher underscoring each word once for each correct repetition from a way station.

15. CAR BARNS, INSPECTION SHEDS, AND REPAIR SHOPS

In every electric railway system it is necessary to provide inspection sheds where the cars may be taken regularly for a systematic inspection. In some cases these buildings are used for the storage of cars and in special cases are equipped as repair shops. On street railroad systems they are generally combined with the necessary facilities for conducting transportation (crew rest rooms, sand facilities, ticket receivers' and superintendent's offices, etc.). Construction of indoor storage space for cars has practically ceased, since the increased deterioration of the cars left in the open is likely to be less than the fixed charges on a car barn. In the case of urban roads, the high value of land may justify a car barn of more than one story because of the smaller area occupied.

LOCATION. The location of a car barn and inspection shed may be determined by several considerations. They may be located in the city on account of the convenience of the employees, the possibility of placing the general offices in the same building, and on account of the fact that the city service is an important part of the whole system. They may be located in the suburbs where real estate is cheap and ample space available. They may be located at a point selected to make the dead mileage of the cars between the ends of their runs and the barns, and between the barns and inspection sheds (if not located together), a minimum. Finally, they may be located adjacent to the power station or other company facilities so that the activities of the two departments may be combined and thus be more efficient.

Repair shops are now generally located where land is inexpensive, so that all tracks may be laid on the ground, and all buildings containing tracks may be approached by ladder tracks, avoiding the expense of transfer tables, car elevators, and the heavier construction required by second-story tracks. For a large shop, location permitting connection with a steam railroad is economical. In remote locations, lacking convenient access to suitable residence districts for employees, the difficulty of securing labor must be considered.

CONSTRUCTION. Preparatory to designing a car house inspection shed, or repair shop, it is necessary to become acquainted with municipal and underwriters' regulations. Facilities required in a centralized building include space for car storage, inspection, administration offices, line department, road department, car employees' lobby, car sign storage, sand drying and storage, salt storage, oil storage, wash-room and toilets.

The standard construction consists of a steel frame on a concrete foundation, and brick walls, but many of the smaller railroads have mill frame buildings of slow-burning construction. The floors are of concrete with cement finish, and sloped for drainage. In most localities it is desirable to provide roller-shutter doors, which can be handled with less labor and greater safety than large hinged doors. Provision must be made for lifting at least one end of any unit of rolling stock handled; depending on the size of the shop this may be fixed hoists or a traveling crane, hand or power operated.

INSPECTION PITS. Inspection pits between the rails are usually 4 ft 6 in. deep below the top of rails. It is important to have these pits thoroughly drained and well illuminated, with recessed and guarded lights. Ample outlets for electric and pneumatic tools should be provided.

In locomotive shops and large car shops a drop pit, equipped with a jack arranged to lower a truck, a pair of wheels, or any heavy underbody apparatus into the pit and transfer it to an adjacent track, will repay its cost.

TRACKS. Tracks in a carhouse used for storage only need be spaced just far enough apart to leave walking space between rows of cars, or between cars and columns or walls; in inspection sheds and shops the track spacing should be several feet greater. Placing top of rail 12 to 15 in. above floor level facilitates work on trucks and underbody parts; the floor must be ramped up to rail level at transverse aisles. The tracks at the entrance to the car house should be designed to permit the rapid movement of cars with the least interruption to traffic.

ILLUMINATION. Illumination should be both general and local. The general illumination should consist of incandescent lights, suspended from overhead between the tracks and spaced about a car length apart. These may be run five in series on a 600-volt trolley circuit, although an ungrounded system is better. The intensity of illumination on the floor should be from 1 to 1.5 ft-c or lumens per square foot. Frequent sockets for drop lights must also be provided, for many portions of the car equipment are in dark corners.

POWER. On low-voltage trolley systems the trolley wire may be run into buildings, but on third rail and high-voltage trolley systems, the contact conductor should end at the doors and movements within the building be made by the use of portable contacts applied to the third-rail shoe or trolley and connected by flexible insulated cable either to a fixed outlet or a roller contact on a live rail overhead between tracks.

The traction power supply is in most cases unsuitable for machine tools, etc., because of voltage fluctuations.

FIRE RISKS. Attention should be given to minimizing fire losses by: (1) use of automatic sprinklers; (2) provision of two sources of water supply such as mains and tank; (3) division of building by fireproof walls; (4) avoiding proximity to inflammable buildings; (5) having ample provision of water pails, sand pails, chemical extinguishers and hoses; (6) locating near a fire station; (7) building tracks on a grade so that cars may be easily pushed out by hand; (8) having auxiliary fire alarms; (9) consulting underwriters when planning; and (10) care in storage of inflammables.

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ELECTRICAL EQUIPMENT OF INTERNAL-COMBUSTION AUTOMOBILES

By Herbert Chase

17. ELECTRIC IGNITION

FUNCTION. It is the function of the ignition system of an internal-combustion engine to initiate the burning of the charge of fuel within the engine cylinder at the proper instant in the cycle; i.e., when the explosive mixture of gas and air is at or near the point of maximum compression. The typical system, see Fig. 1, comprises (1) a source of electrical energy (battery or magneto generator); (2) a timer for correlating the production of the spark with the rotation of the engine (this may be either a " timer " or " commutator " which closes the primary circuit of a vibrator system at the proper instant, or a " breaker " which interrupts the primary circuit of an induction coil); (3) an induction coil (which may be incorporated as part of the magneto) for transforming the energy to a high voltage; (4) a distributor for leading the high voltage discharge to the successive cylinders in the proper sequence; and (5) a spark plug in each cylinder to provide a fixed insulated gap at which the igniting spark is produced.

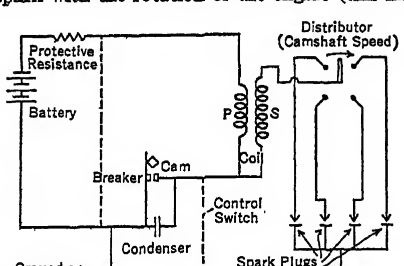


Fig. 1. Circuits of Typical Single-spark, Jump-spark Ignition System. Solid lines show battery system. If the battery and resistance are omitted, the dotted connections added, and the " coil " taken to represent the windings of a magneto, the circuits become those of a high-tension magneto ignition system.

IGNITION OF GASES. When combustible mixtures of air and gas or vapor, or even of air and finely powdered solid combustible fuel, are ignited, a wave of flame spreads through the mixture. Inflammation of the mixture is not instantaneous. In general, the rate of flame propagation is relatively slow, but under some conditions the velocity increases with great rapidity and the charge is said to **detonate.** Detonation is accompanied by exceedingly sudden and violent rises in pressure and so rapid a release of energy that overheating of parts exposed to the high temperature is likely to occur. If these parts reach incandescence or become hot enough to ignite the charge before the normal time, preignition starts and may result in serious injury to engine parts. It is desirable to avoid detonation not alone because of the high temperatures that accompany it but also because the rapid pressure rise may result in mechanical injury. The rate of flame propagation, prior to detonation, varies with different gases, with different proportions of air and fuel

in otherwise similar mixtures, with the density and the temperature of the charge, and probably with other factors that are less definitely understood.

When detonation occurs, the power developed decreases, but, if the compression pressure and consequent density of the gases compressed can be increased without detonation, power output can be materially increased. Knowledge of this fact and of the destructive effects of detonation have led to the extensive use of fuels less subject to detonation and to the use of certain so-called "dopes," small quantities of which have a marked tendency to prevent detonation. One of the most effective dopes is tetraethyl lead. Even a few cubic centimeters of this liquid per gallon of fuel will suppress detonation and permit marked increase in the allowable compression pressure and in power output. Changes in the molecular grouping of hydrocarbon fuels, especially of petroleum derivatives, such as can be effected by variations in the refining process or by hydrogenation, are capable of producing fuels that are not subject to detonation even at the highest compressions ordinarily encountered. The availability of these two types of fuels has led in recent years to the use of much higher compression pressures and to obtaining much higher power outputs from engines of a given displacement. This in turn has necessitated important change in ignition equipment some of which have been made necessary by the increase in voltage required at the spark plug to break down the gap between spark-plug electrodes resulting from the greater density of gases surrounding the gap.

Detonation is much more likely to occur if ignition takes place at a point in the cylinder remote from a highly heated area, such as that around the exhaust valve, so that the flame wave is propagated toward the hot area, than if the reverse condition is established. For this reason, it is now common practice to place the spark plug over the exhaust valve instead of over the inlet valve where it was formerly customary to place it. Regardless of the location of the ignition source, however, there is an appreciable time lag between the instant of combustion and that at which the entire charge is inflamed. It is necessary, therefore, in order that the charge may all be ignited at or very near the top of the stroke, to ignite the charge well in advance of top center, and usually to increase this advance by some means as the engine speed increases. To prevent back kicks or sudden reversal of the direction of rotation during the cranking period, it is usually necessary to provide means for retarding the spark during this period. Both hand-operated and automatic means of effecting the required advance and retard of spark timing are now in extensive use, and in many instances a combination of hand and automatic controls is employed. As timing of the spark has a marked effect both upon power output and upon specific fuel consumption, as well as upon the general responsiveness of the engine to demands for change in speed and power output, increasing attention has been paid in recent years to a variety of means for securing an automatic control by the use either of centrifugally operated mechanism or of devices actuated by the pressure difference between the inlet manifold and outside air. In some instances both centrifugal and vacuum controls are applied.

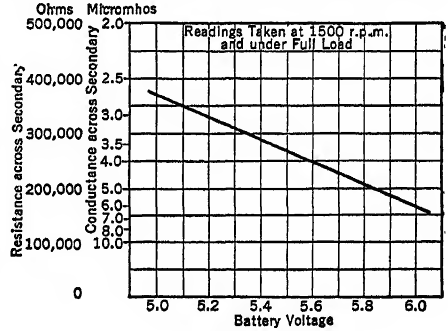


FIG. 2

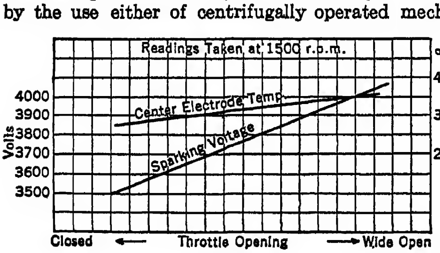


FIG. 3

According to Hector Rabezzana and Donald W. Randolph (*The Automobile Engineer*, England, June, 1930), the voltage required to cause a spark to jump the gap in a spark plug, when the gap measures less than 0.035 in., is nearly proportional to the width of the gap and to the density of the gases in the gap. Under apparently identical conditions in the engine, however, consecutive sparks may have as much as 50 per cent variation in the breakdown or sparking voltage at the plug. In addition to the size of the gap and the density of the gases in the

gap the sparking voltage is affected mainly by the shape, temperature, material, and deterioration of the electrode material. It is also affected materially by electrically conducting deposits on the insulator, especially when, as is often the case, these deposits are not composed entirely of carbon but contain iron oxide or other metallic materials some of which are known to come from certain dopes. The lower the resistance of the shunt which this deposit forms, the lower is the voltage which the spark coil is capable of delivering to the gap. The allowable secondary leakages, resulting from fouling of the spark-plug insulator, for different primary voltages, for an engine running at full load and 1500 rpm, are as shown in Fig. 2.

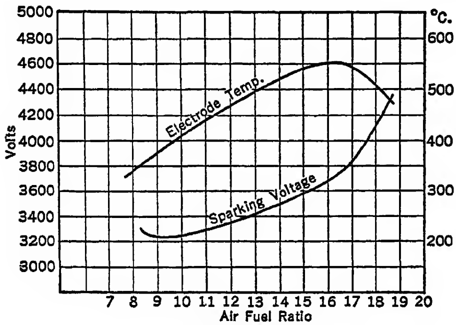


FIG. 4

Other conclusions of Rabezzana and Randolph may be noted as follows: For a given engine speed, the actual compression pressure and resulting density of the charge at the time of ignition increase as the throttle is opened. The secondary or sparking voltage rises accordingly, even though the temperature of the center electrode also rises. See Fig. 3. The sparking voltage and [the temperature of the electrode also vary with changes in the air to fuel ratio, as shown in Fig. 4. Atmospheric humidity, as well as barometric pressure and turbulence of the gases in the cylinder at the time of ignition, have an effect upon combustion temperature, and also upon sparking voltage and electrode temperature. The effect of humidity on these factors under full-load engine conditions is shown in Fig. 5.

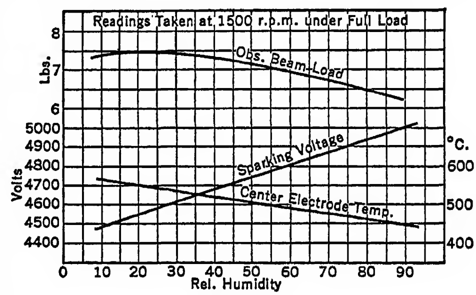


FIG. 5

Sparking voltage increases as the gap between electrodes increases, but not in a straight-line ratio, as shown in Fig. 6, which applies to an engine of rather low compression. A small increase in conductance (resulting from fouling of the insulator) or in the size of the spark-plug gap may result in missing. This is shown by reference to Fig. 7, which shows the maximum allowable fouling conductance on the insulator for a given gap size.

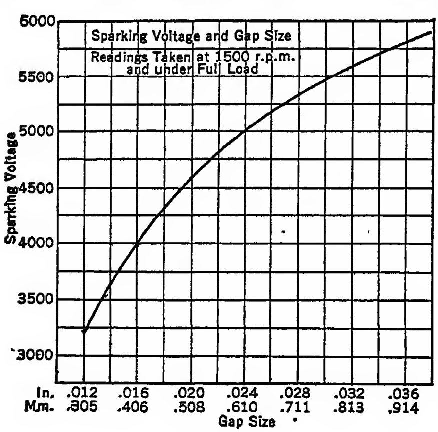


FIG. 6

Sharp edges on electrodes may slightly lower the sparking voltage, but such edges soon wear away after a few hours of engine operation. In other respects the shape of the electrodes has little effect. As hot electrodes emit a greater number of electrons than those that are relatively cool, the sparking voltage falls off quite rapidly as the temperature of the electrodes rises. See Fig. 8. This is an important effect frequently overlooked, and is one reason why a higher voltage is required in starting a cold engine than is needed to cause sparking when the engine and electrode become heated. Since a spark jumps a gap because the gases in

the gap become conductors as a result of the presence of ions and free electrons in the gap, and most of the electrons are furnished by the electrode wire itself, the belief that the material of the electrode has no effect upon sparking voltage is not admissible. The characteristics of the ordinary commercial electrode alloy are such that the emission of electrons decreases as the plug is used and requires a proportional increase in the sparking voltage necessary to break down the gap. The increased voltage required sometimes amounts to as much as 3500 volts under the same gap conditions. Continued use of the conventional electrode tends to exhaust the electron-emitting material in the surface of the wire and results in wide variations from spark to spark in the voltage required to break down the gap. There is also a physical disintegration of the wire resulting from the action of combustion gases at high temperature. Individual particles are loosened by intercrystalline corrosion and become detached with resultant widening of the gap. A new type of electrode material, called Isovolt, retains the same sparking voltage throughout its life and also requires a lower voltage for sparking than the usual commercial electrode wire. So long as the spark gap is not increased, corrosion of this new alloy does not increase the sparking voltage. Tests with it indicate easier starting in cold weather, and the greater resistance of the electrodes to corrosion enables the engine to run longer without regapping the plugs. In addition, the sparking voltage from spark to spark remains substantially constant.

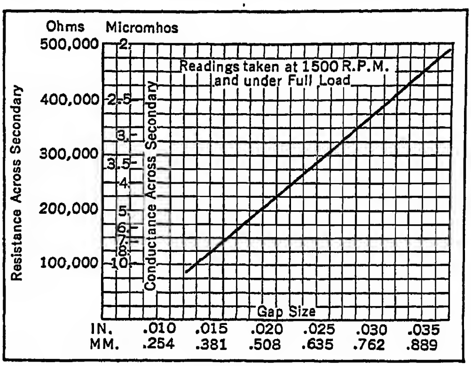


Fig. 7

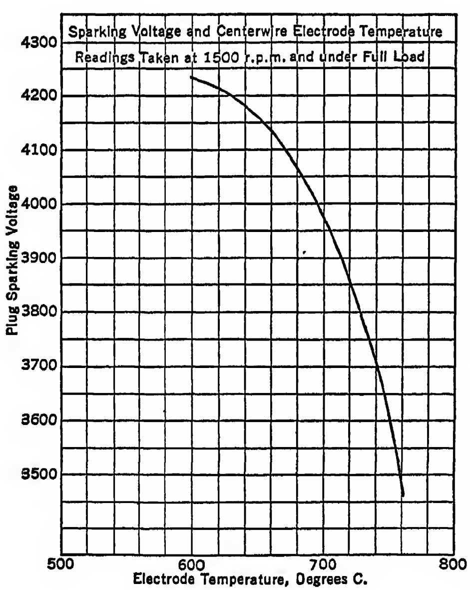


Fig. 8

As a result of the work of Rabezzana and Randolph the following recommendations are made to minimize spark-plug missing: (1) spark-plug gaps of 0.015 to 0.018 in. for high-compression engines and of 0.018 to 0.022 in. for low-compression engines; (2) electrode shape least affected by corrosion; no sharp edges or points at gap; (3) good electrode wire; (4) correct carburetion; (5) proper position of plug in combustion chamber; (6) correct breaker point gaps; (7) clean and square breaker points; (8) correct plugs for conditions under which engine is used; (9) avoidance of metal shielding on secondary leads and minimum length of said leads.

TYPES OF IGNITION SYSTEMS. Virtually all ignition systems in general use today are high-tension jump-spark types in which the initial or primary current is generated at a comparatively low voltage which is stepped up to a voltage sufficiently high to break down the gap at the spark plug and produce an ignition arc or spark at the gap.

Make-and-break Low-tension Systems were extensively used at one time but are now seldom found in practical use except on old engines. This type of system employs a

high inductance connected to a pair of contacts within the combustion chamber of the engine. These contacts are arranged so that mechanical separation occurs at a predetermined point in the cycle, resulting in an arc which ignites the charge. This type of system is not well suited for high-compression engines nor for those operating at high speed, and it also suffers the disadvantage of mechanical complexity.

Jump-spark or High-tension Systems are the most general source of electric ignition. The high voltage for jumping the spark gap is induced in the secondary winding of an induction coil, which in some cases forms a part of a magneto winding, when the primary circuit is broken. Systems of this type are sometimes classified as follows:

Vibrator Systems, in which the primary circuit is broken by vibrating contacts, as on Model T Ford cars, are now nearing obsolescence.

Single-spark Systems, in which a single spark is produced by one interruption of the primary circuit for each power stroke of the engine, are now practically universal on American passenger car engines and are extensively used on many other types of engines. In general, the source of electrical energy is a storage battery, usually floating on the line with a charging generator, but it may be a set of dry cells or a magneto. As a rule, the primary circuit remains closed during a considerable proportion of the engine cycle in order that the flux in the induction coil may build up to a maximum value. The field thus created collapses rapidly, of course, when the primary circuit is broken by separating the cam-actuated contacts, the lines of force cutting the secondary winding and building up a voltage high enough to jump the gap of the spark plug. Because of the long time that the primary circuit of this type of system remains closed as compared to so-called open-circuit types, the current flow is greater and the energy in the spark is correspondingly increased.

Electric sheets of high-silicon steel or annealed core wire are used for the core of the induction coil in order to break up eddy currents, to avoid residual magnetism, and to insure the most rapid collapse of the magnetic field when the primary circuit is broken. In general, the primary winding has from 150 to 225 turns and is wound on the core, the size of wire running from No. 19 to No. 22 A.W.G. From two to four layers are used, and enameled wire is generally employed except on the better grades of coil, in which cotton-covered wire is used. The latter is more resistant to overheating in the event that the user leaves the ignition switch closed when the engine is not operating. Under the latter conditions enamel insulation is likely to lose its insulating value. Many coils, however, are protected to some extent by a resistance unit with a high temperature coefficient so arranged that, if the current becomes excessive, the increasing resistance of the heated resistance unit is sufficient to prevent an excessive flow of current. Another benefit derived from the use of a resistance unit in series with the primary is that it permits of a higher current flow when the engine is being started and the unit is still cool. During operation at low speed, when the primary circuit is closed for comparatively longer intervals, the resistance unit heats and reduces the current flow, whereas at higher speed the reverse effect is secured. This tends to equalize the current and resulting sparks at all speeds.

Secondary windings of No. 38 plain enamel wire are commonly used. About 1 volt per turn is generated, hence if there are 250 turns per layer the difference in voltage per layer approximates 500. Insulating paper is therefore applied between layers, which in the better grades of coil are wound outside the primary. If, as often happens, the insulation between layers of the coil becomes punctured, missing of the engine is likely to occur, especially under heavy loads or at high speeds. In cheaper grades of coils the secondary is sometimes wound next to the core, with the primary outside. Although this is a less favorable arrangement from the standpoint of the magnetic circuit, it makes it possible to eliminate a resistance unit, since the primary is made to act in place of such a unit in addition to performing its normal function. This arrangement also makes possible the use of a less expensive case (stamped metal in place of molded phenolic material) and of a cheaper secondary winding.

COIL AND CONDENSER. In the ordinary ignition system using a coil connected as shown in Fig. 1, when the switch is closed, the current flows through the resistance unit, through the primary winding, through the closed contact points, and back to the battery. Opening of the contact points by the cam provided for this purpose, and driven, of course, in timed relation to the engine, causes a surge of current in the primary and a sudden increase in voltage which, except for a condenser in parallel with the breaker gap, would tend to continue the flow of current in arcing at the gap and result in burning the breaker points. The condenser temporarily absorbs the surge, reduces the arcing, and reverses the direction of current flow in the primary, thus aiding the rapidity of the collapse of the magnet field in and around the core and correspondingly increasing the voltage induced in the secondary winding. The condenser should be of sufficient capacity to prevent excessive arcing at the contact points, but according to some authorities, if it

has sufficient capacity to prevent any visible flash upon breaking of the contact a blue coating will form on the contacts, producing a high resistance and making starting more difficult.

Some observers have contended that so-called hot sparks (presumably those in which a relatively large amount of energy is dissipated) are of little advantage, so long as a spark occurs at all at the spark-plug gap. This belief appears to have been discredited, however, partly because coils of low inductance which build up rapidly and give adequate voltage to jump the plug gap have not proved conducive to easy starting and good operation, especially when the engine is cold. Coils of higher inductance, in combination with cam and breaker designs such as to increase the *dwell* or period during which the contacts remain closed and permit the magnetization of the coil to build up to high values, are said to improve running conditions and be more economical of fuel, especially in engines that are inclined to run too cold in service. In any case, some makers of ignition systems have found it advantageous to use coils of high inductance along with breaker and cam designs such as to increase the period during which the circuit is closed, especially in high-speed engines. The advantages gained are attributed to the larger current induced in the secondary and the resultant increase in spark heat.

MAGNETOS. Highly successful ignition from battery sources has resulted in practically eliminating magneto ignition on American passenger cars and trucks and on many other types of automotive equipment in which a battery is required anyway for lighting and/or starting purposes. This is in no sense an indication that magneto ignition is less successful than formerly, but rather that the higher cost of a magneto equipment is not justified when less expensive and satisfactory ignition means are available. For airplane engines, many forms of industrial engines, tractor engines, and some of the smaller marine engines, magnetos are extensively used with excellent results.

Magnetos are electric generators in which the field is produced by permanent steel magnets rather than by excitation through a field winding. In some cases there is a single primary winding on the rotating armature which produces a low-tension current that is afterward stepped up to sparking voltage in a separate induction coil. More frequently, however, the armature has a high-tension as well as a primary winding and then the magneto is termed a high-tension type. The latter is the most widely used type and is fitted with fixed U-shaped magnets between the ends of which the armature turns. An alternative arrangement, however, involves the use of a primary and secondary winding that are fixed while the magnet or magnets are rotated.

Magnetos are also classified as single-spark and two-spark types, the former being the more common, and having one end of the secondary grounded through the primary while the other end is connected to a distributor. So-called two-spark magnetos have both ends of the secondary insulated and connected through a double distributor to two spark plugs in the same cylinder. With this arrangement the charge is ignited at two points simultaneously. If the points are correctly located, the distance that the flame must travel as combustion proceeds may be materially decreased and the time for combustion correspondingly decreased, often with some increase in power output. The shorter average distance which the flame must travel may also result in avoiding detonation under conditions where otherwise it might occur. Magnetos usually are driven in timed relation to the engine and are fitted with a breaker mechanism by which the primary circuit is opened at the desired instant. High-tension types also incorporate a distributing device for connecting spark plugs successively to the terminal or terminals of the secondary winding.

A further classification of magnetos follows:

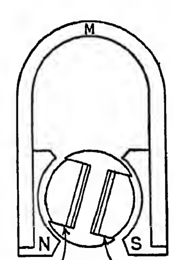


FIG. 9

High-tension Shuttle-type. As shown in Fig. 9, this type has an H-shaped shuttle or armature on which both primary and secondary windings are placed. The armature rotates in ball bearings with a minimum gap between the soft iron laminated pole pieces that are attached to the ends of the U-shaped magnets. The primary usually has about 150 turns of No. 22 wire and is next to the laminated shuttle. Secondary turns may approximate 10,000, the layers being insulated with oiled paper and/or oiled silk. One end of the secondary is usually grounded to the primary and the other is connected to a collector ring which in turn is connected to the central point of the distributor. The latter has a rotating brush which is driven by gearing from the armature shaft within a distributor head having one metal segment for each plug to which the high-tension current is to be led. The rotating brush contacts with or comes close to the metal segments so that the current may be led successively to various spark-plug wires, either by jumping the gap between the rotating arm and the segments or by actual contact of a carbon

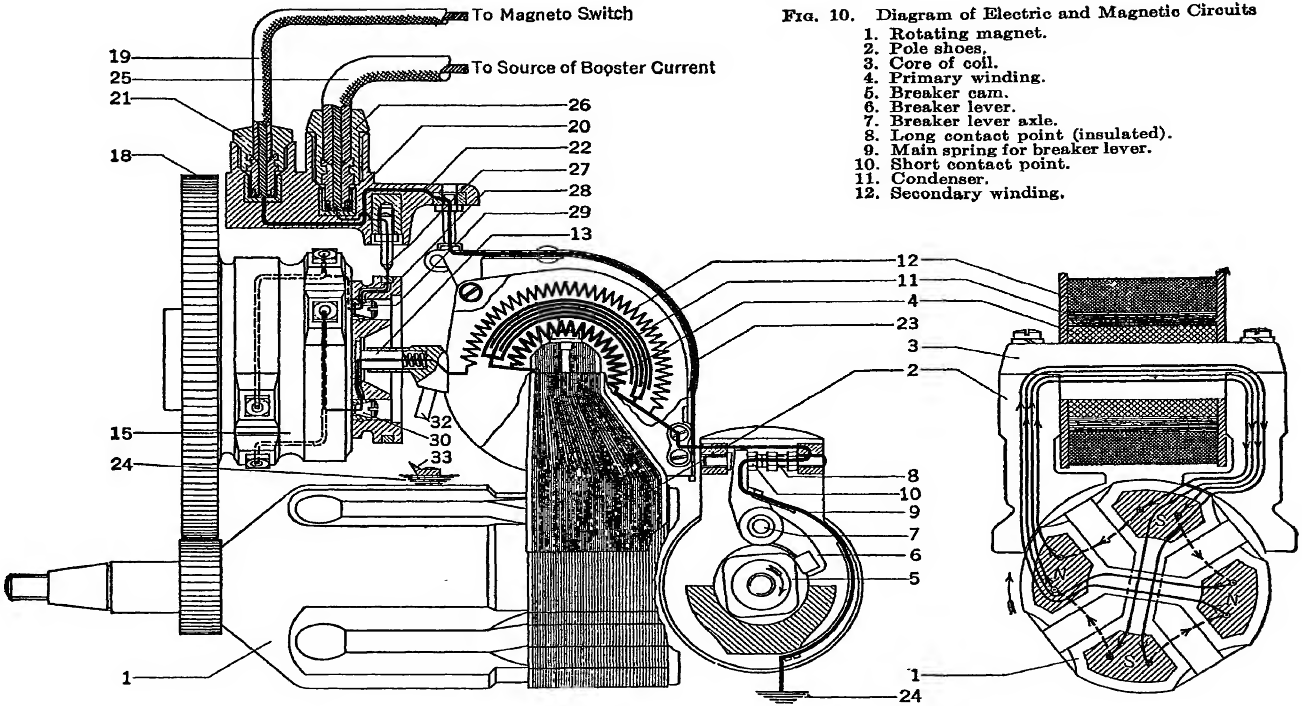


Fig. 10. Diagram of Electric and Magnetic Circuits

1. Rotating magnet.
2. Pole shoes.
3. Core of coil.
4. Primary winding.
5. Breaker cam.
6. Breaker lever.
7. Breaker lever axle.
8. Long contact point (insulated).
9. Main spring for breaker lever.
10. Short contact point.
11. Condenser.
12. Secondary winding.
13. High-tension carbon brush.
14. Distributor cylinder.
15. Large distributor gear.
16. Ground wire.
17. Booster and ground connection block.
18. Fastening screw for ground wire.
19. Stud for ground contact.
20. Primary bridge.
21. Ground through the magneto and engine.
22. Booster cable.
23. Fastening screw for booster wire.
24. Electrode for booster current.
25. Collector ring for booster current.
26. Fastening screw for collector ring (in booster circuit).
27. Fastening screw for collector ring (in secondary circuit).
28. Safety gap electrode.
29. Ground plate for safety gap.

brush on the arm with them. The rubbing surface between contact segments against which the brush bears is usually made from vulcanized rubber to minimize the effect of arcing, but the remainder of the distributor is usually molded from phenolic materials which have excellent insulating qualities and are not softened by heat. The electric circuit is closely analogous to that of the battery system, as shown by the dotted lines in Fig. 1. To operate the breaker mechanism two cam surfaces are provided on the breaker cover, which is coaxial with the armature, and a plate carrying the breaker contacts. One of the latter is on an L-shaped lever one end of which strikes the cams as the breaker plate rotates with the armature. This mechanically opens the breaker points. The breaker cover and the cams attached to it can be turned by a lever which is attached to an advance and retard linkage. With this type of magneto there are two sparks per revolution of the armature, hence for a four-cylinder four-cycle engine the magneto is driven at crankshaft speed and for a six-cylinder engine at one and one-half times crankshaft speed.

Inductor-types. Inductor-type magnetos differ from the shuttle-type in that the windings are stationary and the magnets themselves or their pole pieces rotate. A diagram of one inductor-type magneto that has seen extensive use, especially on aircraft engines, is shown in Fig. 10. In this make, the rotating magnets form a bell-shaped structure with either two or four poles. The rotor is made from a special chrome magnet steel, and the poles have laminated extremities which are held in place by an end plate on which a breaker cam is mounted. The rotating magnet, in the design diagrammed in Fig. 10, has four poles, but in other designs of the same make, two poles are used. The pole extremities of the magnets rotate in close proximity to the two pole shoes that are fastened to the core of the coil. The latter has an inner primary winding and an outer secondary winding with a condenser between the two. The paths of the magnetic flux are indicated by the arrows in Fig. 10, the flux being reversed each time that a pole piece passes through the gap between pole shoes. One breaker point is carried on a rocker lever the other end of which has a shoe riding on cam. The cage which carries the pivot for this rocker lever carries the second contact and is pivoted so as to turn about the armature axis in advancing and retarding the spark. Both points are made from an alloy containing 75 per cent platinum and 25 per cent iridium. In this, as in some other magnetos, especially those used in aircraft work, a tap is sometimes provided whereby an external source of high-tension current can be applied during the starting period, connections being as indicated in the diagram.

Other Inductor-type Magnetos of several designs are used to some extent, a schematic drawing of the polar type being shown in Fig. 11. Its coils are wound on a laminated core having extended pole pieces between which rotate a pair of soft iron blocks mounted on opposite sides of a non-magnetic shaft. The outer ends of these blocks form disks which rotate close to the poles of a permanent U-shaped magnet. Four or more pole pieces can be used, the number of sparks produced per revolution being equal to the number of poles. In the **sleeve-inductor-type** magneto there is a fixed shuttle-type armature. Between this and fixed permanent magnet poles rotates a sleeve of non-magnetic material having segments of soft iron. As the sleeve rotates there is a reversal of flux through the armature core. With two segments four sparks per revolution can be produced.

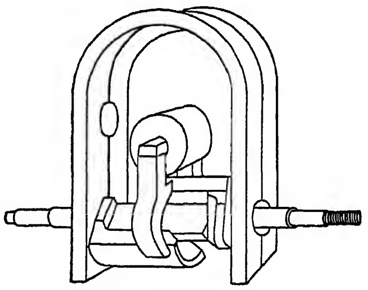


Fig. 11

Miscellaneous Magnetos: Low-tension Magnetos have only a primary winding the current from which must be stepped up in an external coil in order that it may jump a spark-plug gap. Such magnetos sometimes have a self-contained breaker for interrupting the primary circuit unless the coil employed is a vibrating type such as was used in the Model T Ford car. In this case a timer is required to close the circuit through the coil.

Other Types of Magnetos, of which there are many, include the **oscillating type** in which the armature is rotated through part of a revolution against springs and is then released so that it is snapped back across the position of flux reversal, thereby generating enough energy to produce a spark. Another type sometimes used has a single winding comprising many turns of fine wire. Translatory motion of a pair of soft-iron cores into and out of contact with the poles of a permanent magnet reverses the flux with sufficient rapidity to produce a high-tension current that will jump a spark-plug gap.

Impulse Couplings are often fitted between the magneto shaft and the engine shaft

which drives the magneto. Their purpose is to snap the rotating part of the magneto through a fraction of a revolution at a relatively high speed at the instant when a spark is required during the cranking or starting period of the engine, instead of allowing these parts to turn at the slow uniform speed which a positive coupling would impose. This sudden acceleration produces a relatively high voltage and a correspondingly hot spark, which is a decided advantage, especially when the engine and its charge are cold. The coupling includes a spring and latch device so arranged that the spring is wound up during a part of the revolution and is then suddenly released, imparting to the rotating part of the magneto a momentary angular velocity equivalent to that at about 800 rpm uniform speed. When the engine starts and the magneto attains a speed of 150 to 200 rpm a centrifugal device renders the latch inoperative and the coupling then functions at a normal uniform angular velocity.

Combination Ignition Systems, involving two or more systems that are more or less independent but to some extent interconnected, were widely employed, chiefly on automobile engines, at one time. The terms " dual," " duplex," " double," and " two-spark " have been used in different senses by different makers who have sought to increase dependability, provide more convenient starting, or boost the power output of the engine by this means. Today such combinations are not common, largely because of their added expense and because the degree of dependability of a single system is sufficient to meet most requirements. At least one American automobile manufacturer, however, fits so-called " twin " ignition, involving the use of two spark plugs per cylinder, on some models, and motor-coach manufacturers apply two more or less independent ignition systems to some bus engines. Combinations of magneto and battery ignition, once very popular, are seldom employed today.

CYCLE OF OPERATION. There are four periods in the cycle of operation of all jump-spark systems, each marked by an abrupt change in conditions: During the *first period*, the impressed voltage from the battery or magneto builds up the primary current and consequent storage of magnetic energy. With a battery system the build-up is exponential and gradually approaches the closed-circuit value. In the magneto, the emf wave is peaked, the short-circuit current wave at moderate and high speeds being flat-topped but of sufficient value to produce a spark if the break occurs over a 90-deg electrical half wave. At slower speeds the current wave is more peaked and the spark can be produced only over a narrower range of timing. The normal current at break ranges from 3 to 5 amp. The inductance of the magneto is usually greater, hence the heat energy in the spark is greater. Discharge of the battery on the closed circuit or the operation of the magneto as a short-circuited a-c generator is suddenly interrupted by the opening of the breaker contacts, introducing the second period.

The *second period* is of extremely short duration (of the order of 0.00005 sec) and extends from the opening of the breaker contacts to the initiation of the spark at the spark-plug gap. At the start of this period the primary current continues to flow unabated, charges the condenser, and introduces a back emf into the primary circuit. This decreases the primary current and the resultant flux in the core, and induces in the secondary a voltage which is higher than that of the primary in approximately the ratio of the number of turns, or from about 50 or 60 to 1. The secondary is thus charged as a condenser, one " plate " of which is the secondary winding, distributor, and spark-plug leads, and the other the engine and other grounded metal parts of the assembly. When the charge is sufficient to jump the gap at the spark plug, the discharge occurs, often when only a small fraction of the magnetic energy available has been transformed.

In the *third period*, immediately following attainment of the breakdown voltage of the gap in the second period, the gap becomes conducting and the secondary " condenser " discharges through it. After completion of this condenser discharge there follows the *fourth period* during which the magnetic energy still left in the coil continues to deliver an inductive spark for several thousandths of a second until the entire energy is dissipated. The spark delivered by a magneto is usually of longer duration than that delivered by a battery system, both because of the greater amount of stored magnetic energy and because the voltage generated in the secondary by continued rotation (if the spark is sufficiently advanced) may force an additional flow of current across the gap.

According to M. Mallory, a spark of low amperage and high voltage may jump the gap at the plug and still not ignite the mixture because the heat generated is insufficient. Under these conditions it is practically impossible to secure good engine performance. Again, a weak spark, even though it may ignite the charge, may result in burning so slowly as to cause back-spitting in the carbureter, indicating that the mixture is still burning when the intake valve opens to admit a fresh charge. This is true especially when the engine is cold. A lack of sufficient heat in the spark may result from too wide a gap at the spark plug, a gap in some other portion of the secondary circuit, an inadequate or imperfect

ignition coil, too short a dwell, leakage (or so-called " corona " effect) resulting from deterioration of the insulating qualities of secondary wiring, and several other factors. As engine speed increases, the length of time that the primary circuit remains closed decreases with a corresponding decrease in the time available for building up the flux in the core. The result is a weaker spark with more or less tendency toward missing at very high speed.

COIL PERFORMANCE. Fig. 12 shows the current output of three coils over wide ranges of speed. Curve *A* indicates good performance at engine speed up to about 1800 rpm, but above this speed the output is low and becomes zero at a little over 2200 rpm. At low speed and in starting, coil *A* is much better than coil *B*, but at high speeds, as shown by curve *B*, this coil continues to give a spark up to speeds approximating 4500 rpm. Its maximum output, however, never exceed 1.3 milliamperes. Coil *C*, as shown by curve *C*, has a higher output than either *A* or *B* through most of its range and a maximum more than twice as great as coil *B*. A coil such as *C*, if made with one secondary winding, would be costly, bulky, and difficult to insulate as there would be about 1000 volts pressure difference between layers. Outer layers would be far from the core and the length of winding so great as to increase the resistance to a point where it would affect performance. To avoid such drawbacks, some coils have been made with two secondary windings (on an elongated core) connected in series. In this case insulation problems are less difficult, the diameter is reduced, despite the use of larger wire of lower resistance, and a total of 30,000 turns, or 15,000 per coil, is feasible. Such coils give good starting

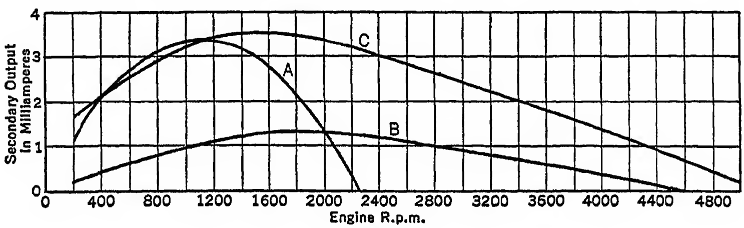


FIG. 12

sparks on only 2 volts and with a properly designed circuit breaker and distributor meet the requirements even of an eight-cylinder engine at speeds up to 5000 rpm.

During the starting period of engines that are cranked by a motor driven by energy from a storage battery, the heavy drain on the battery may drop its voltage as low as 2 volts, especially when the engine is cold. As ignition current is taken from the same battery during this period, only 2 volts may be available at the primary. This, together with the need for a hot spark under these conditions, imposes a difficult problem upon the designer of ignition coils. This has led to the development of so-called automatic coils, designed to give a hot spark at low voltage and automatically accommodating themselves to the higher voltage which obtains when the engine picks up speed and the generator raises the voltage to about 8 volts.

SPARK TIMING. To obtain maximum power output and greatest fuel economy from an engine depending upon electric ignition it is necessary to vary the timing of the spark with changes in speed and in throttle opening. This is partly because the time interval for completion of the engine cycle decreases in direct proportion to increase in speed and partly because the rate of combustion is influenced by changes in compression pressure which varies in turn (in the four-stroke throttling type of engine) with changes in throttle opening. For many years, hand setting of the spark timing device was depended upon to provide the optimum setting, and this method is still used extensively. Its chief disadvantage lies in dependence upon the human element, as its success, if best results are to be secured, depends upon the skill and willingness of the operator in varying the setting with changes in conditions. One alternative sometimes resorted to is to select the best fixed setting for average running conditions and permit the advance to remain at this fixed setting regardless of changing speed and load conditions, with the exception that a retard is provided to prevent a back kick in starting.

Automatic Spark Timing Controls. Two general types of automatic spark advance and retard mechanisms are now in quite general use either separately or in combination and with or without an added manual control. The first type is operated by a centrifugal governor designed to advance the spark with increase in speed but not necessarily in direct proportion thereto, although some designs have this objective. It should also be either capable of adjustment or especially designed for the particular engine, as engines

vary greatly in the degree of advance required, some needing only 10 deg or less and others requiring as much as 50 deg. Good centrifugal governors are generally considered a decided improvement over dependence upon hand advance mechanisms and are now extensively used. They have the disadvantage, however, that they are controlled only by variations in speed and not by changes in load.

Use of a vacuum-control, in addition to centrifugal control, has been made to overcome the disadvantage of a centrifugal device alone. The difference in pressure which actuates the device is that which exists between the interior of the inlet manifold of the engine and the atmosphere. Manifold depression decreases as the throttle is opened to meet increases in load, and increases again when the throttle is closed with decrease in load. Since spark advance should be greater when the throttle is closed, because the compression pressure is then lowered and flame propagation is then less rapid, the vacuum control is so arranged that the advance is increased as the load falls off, and vice versa. This type of control is superimposed upon that effected with a centrifugal governor, so that the combined effect is one of speed and load, as is desired.

SPARK PLUGS. (See also under Ignition of Gases.) The function of the spark plug is to provide a gap having a fixed breakdown voltage between two electrodes one of which is almost always grounded to the shell of the plug, which has a male thread and is screwed into a threaded hole in the wall of the combustion chamber, and the other (the central electrode) is insulated and connected to the secondary of the ignition coil. Among the requirements sometimes imposed by purchasers of spark plugs are the following: The voltage necessary to produce a spark shall not exceed 6000. Insulation resistance shall not be less than 100,000 ohms. The plug shall be substantially gas tight. In practice these conditions sometimes must be fulfilled under pressures as high as 1000 lb per sq in., and in the presence of gases that vary rapidly in temperature from atmospheric to 3000 deg fahr or above and frequently tend to deposit carbon on the surface of the insulator.

The electrode passes through the center of a vitreous ceramic insulator and is cemented in place in good electrical contact with a brass cap for receiving the secondary lead. Shells are of steel and are either crimped or spun over a shoulder on the insulator in such a way as to hold the latter permanently in gas-tight contact with a seat, or are provided with a threaded bushing which performs the same function but enables the insulator to be removed for cleaning and inspection. Gas-tight joints are essential, for if appreciable leakage occurs the flow of hot gases between the shell and the insulator is likely to cause overheating with consequent preignition of the charge. Most modern spark plugs have straight central electrodes. The grounded electrode is usually an L-shaped piece of wire one end of which is pressed into a hole in the shell and often welded to the shell to insure the good thermal contact which is necessary to prevent overheating. Electrodes are made from relatively non-corrosive wire usually containing a considerable percentage of nickel. Authorities differ as to the best setting for the gap. The optimum setting depends partly upon the compression of the engine and partly upon the voltage which the ignition system is capable of generating under the operating conditions that obtain. One authority recommends a gap of 0.015 to 0.018 in. for high-compression engines and of 0.018 to 0.022 in. for low-compression engines. Others recommend an average setting of 0.025 to 0.030 in., but advocate following the specific recommendations of the engine manufacturer or maker of the ignition equipment. It is probable that gaps in excess of 0.035 in. should not be used, as they are likely to result in leakage in the external secondary circuit. Gaps that are set too close are likely to result in excessive burning of electrodes and in rough engine performance. In setting the gap it is recommended that only the grounded electrode be bent, as bending of the center electrode is likely to result in breakage of the insulator at the time or later when it is heated in use.

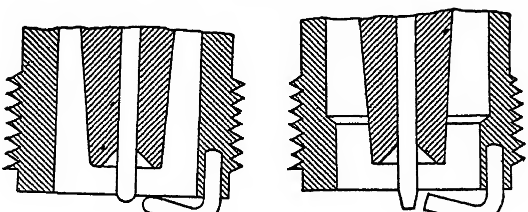


Fig. 13

Insulators in most modern spark plugs have what is termed a "semi-petticoat" tip such as is shown in Fig. 13. With this shape there is a heat gradient which prevents the lip or relatively thin edge from overheating but at the same time keeps it hot enough to burn off oil or carbon that may be deposited on it. Much care is usually exercised in locating the spark plug where the shell and the insulator in contact with it will be properly cooled, as otherwise overheating with consequent preignition is likely to result unless

conditions are such that the engine normally runs rather cool. On the other hand, excessive cooling of the plug is likely to result in fouling of the insulator with consequent formation of a high-resistance shunt across the gap. For these reasons, many plugs are specially designed to meet the particular cooling conditions that exist in engines of various types. Thus there are hot-running plugs for cool-running engines and cool-running plugs for hot-running engines, and it is important that the right plug for the particular conditions encountered be selected. Plugs designed to run hot in cold engines have an insulator that is relatively long from its firing end to the shoulder where it seats against the shell. Plugs designed to run cool in hot engines have an insulator which is relatively short between the points mentioned, so that the heat is conducted away from the hottest end more rapidly.

Spark-plug shells are also varied in length so as to fit into heads of differing thicknesses and in some cases to bring the spark gap well away from the inner wall of the combustion chamber so that the gases, which are rendered turbulent as a result of velocity imparted during the intake and compression stroke, may sweep across the gap during the ignition period. In some cases this velocity exceeds the rate of flame propagation, and the turbulence is depended upon to spread the flame quickly to parts well removed from the point of ignition. In some engines this turbulence is presumed to prevent or to minimize detonation by helping to bring about combustion before a wave of detonation velocity can be set up. Many engine manufacturers have made extensive tests to determine the best type, location, and size of spark plug to be used, and when new plugs are installed it is usually desirable to make sure that they duplicate the type selected as a result of such tests. Plugs used in **aircraft engines** that are air-cooled often run at much higher temperatures than are prevalent in conventional water-cooled engines, and in some instances special cooling provisions are required for the plugs of such engines, especially as high compressions and high specific power output are required in the average case. The combined cooler and radio shield used on some Wright aircraft engines is shown in Fig. 14. The finned shell is entirely of aluminum and is in intimate thermal contact with the cylinder. Its use resulted in a temperature drop of about 100 deg fahr in plugs fitted to the rear of the engine in question. With the cooler, erosion at the gap was less in 50 hours of service than in 10 hours on a plug not fitted with a cooler. The cooler also provides an effective shield against radio interference such as is required in planes fitted with radio equipment.

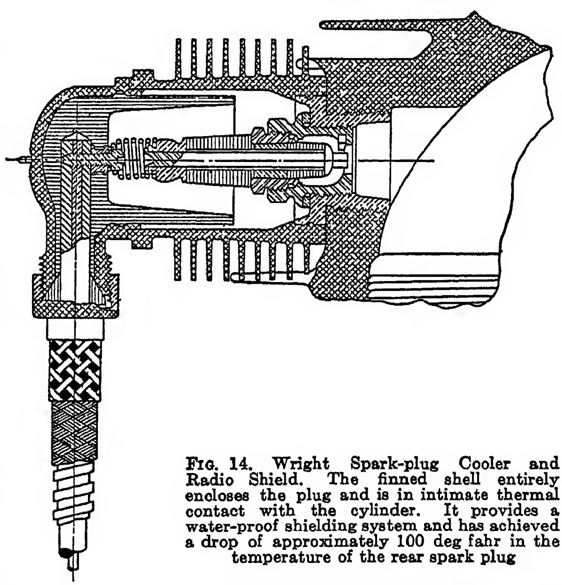
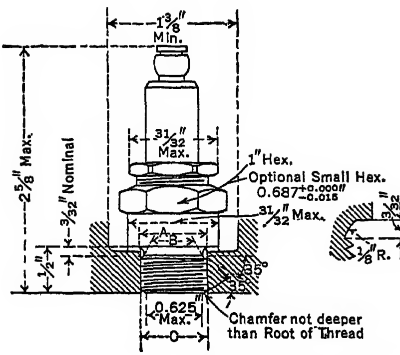


FIG. 14. Wright Spark-plug Cooler and Radio Shield. The finned shell entirely encloses the plug and is in intimate thermal contact with the cylinder. It provides a water-proof shielding system and has achieved a drop of approximately 100 deg fahr in the temperature of the rear spark plug

Spark-plug Standards. Three sizes of spark plugs are in more or less general use in this country, namely, the 1/2-in., the 7/8-in., and the 18-mm, the last-mentioned being generally used also in Europe. The 1/2-in. size has a standard 1/2-in. Briggs pipe thread. There are 14 threads per inch, and, as the thread is tapered, no gasket is required to make a tight joint. This size of plug is now rapidly going out of use. Shells having 7/8-in. and 18-mm threads have been standardized by the Society of Automotive Engineers, as shown by accompanying tables, and now are almost universally used on American automotive engines, the 18-mm size being especially recommended for aviation and other high-compression engines. Both the 7/8-in. size, which has 18 threads per inch, and the 18-mm size, which has 1 1/2 mm pitch (about 16 3/4 threads per inch), have straight threads and depend upon seating against a gasket to insure a gas-tight joint. Terminal posts for attachment of secondary leads to the upper ends of the plugs have also been standardized by the S.A.E. in accordance with dimensions given in Fig. 15.



Thread—18 mm diameter; pitch 1 1/2 mm.
Form of thread—International standard (same as American Standard except 1/2 as much truncation at root of thread).
Clearance—Minimum distance from terminal to nearest metallic object, 3/4 in.

SPARK-PLUG DIMENSIONS*

Dimensions	Maximum		Minimum	
	Mm	In.	Mm	In.
A	18.50	0.728	18.00	0.709
B	.....	0.633 †	.....	0.625
C	17.97	0.708	17.85	0.703

* These dimensions are the same as those of the regular S.A.E. metric spark-plug specifications.

† The thread and body dimensions, excepting the depth of chamfer at the skirt of the plug, are substantially in accordance with those recommended by the Inter-Allied Aircraft Standards Conference at London in April, 1918.

Fig. 15. Aeronautic Spark-plugs. S.A.E. Standard

Mountings for magnetos, as well as for timer-distributor units, the latter as used with battery ignition systems, have been standardized by the S.A.E.

18. STARTING AND LIGHTING SYSTEMS

The items comprising the electric starting and lighting equipment of a gasoline automobile are (1) the generator, (2) storage battery, (3) starting motor, (4) lamps, (5) wiring, (6) switches, cut-outs, and fuses, (7) ammeter. In the so-called single-unit systems the generator and starting motor are combined in a single unit, but this type of system is now almost obsolete. The battery is used not only to supply energy for starting and lighting but also for ignition and often for operating supplementary apparatus such as windshield wipers, cigar lighters, and radio receiving sets.

Generator

Practically all generators now employed on passenger cars are shunt wound. They are driven by the engine, sometimes through gears or silent chains, but more often by belts which as a rule are V-types. The primary function of the generator is to keep the storage battery, which floats on the line, charged, but in some installations, especially on buses, the generator is sometimes designed to carry the lighting and ignition load even though the battery becomes disconnected. Since with a shunt-wound generator the voltage normally varies in substantially direct proportion to the speed, and the speed constantly varies with changes in engine and vehicle speed, means for maintaining a nearly constant voltage have to be applied.

BATTERY CUT-OUT. To prevent the battery from discharging through the generator when the battery voltage exceeds that of the generator, it is usually necessary to provide an automatic switch, variously termed a cut-out, a circuit breaker, and a reverse-current relay, designed to open the circuit when the speed of the generator falls below that at which the battery is charged. A relay of this type is shown in Fig. 16; the wiring diagram, Fig. 17, shows the two windings of the relay and how it is connected in a normal automobile electric system. Essentially the relay consists of an iron core with a shunt and a series winding. The shunt winding is connected permanently across the main generator terminals, one of which is usually grounded. It serves to magnetize the core and attract a hinged armature as soon as the voltage is high enough and the magnetizing force great enough to overcome the tension of a spring which tends to hold the armature away from the core. When the armature is attracted, it closes a pair of contact points,

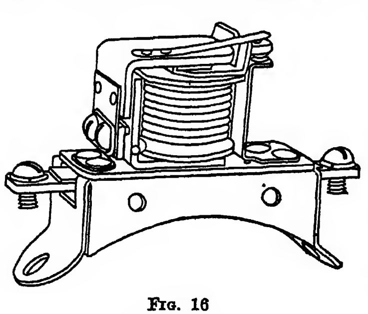


Fig. 16

thereby closing the battery circuit and also the series winding circuit of the cut-out. The additional flux added by this winding presses the contacts more firmly together. Both the spring and the stops that limit the travel of the armature of the relay are usually so set that the contacts close at 7 to 7.5 volts, equivalent to a charging rate of about 3 amp, with an ordinary automobile type relay for a 6-8-volt system. Contacts open again at a discharge current not greater than 2.5 amp, when the series coil bucks the shunt coil and decreases the pull on the armature enough for the spring to pull it away from the core. In general

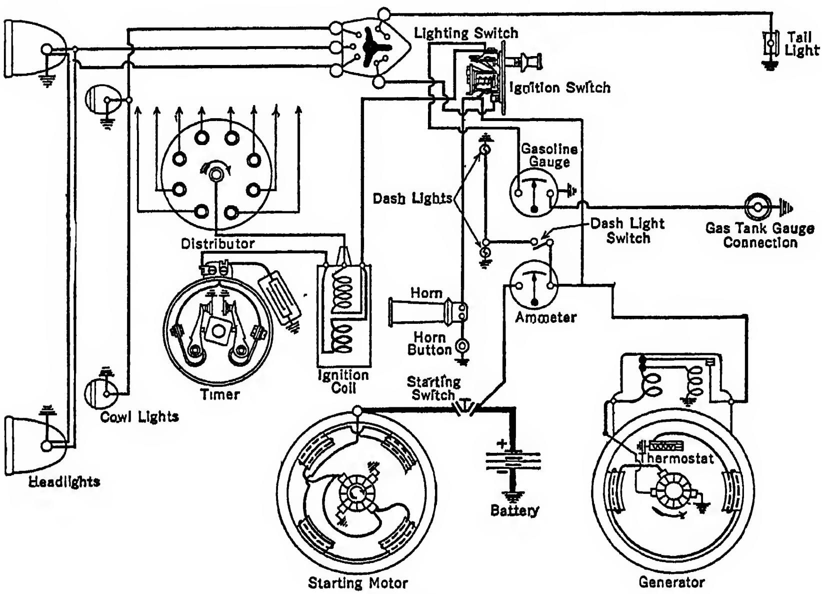


Fig. 17

this actuation of the relay takes place at an engine speed of about 600 rpm, equivalent to a car speed, in high gear, of around 10 mph.

COMMON TYPES OF VOLTAGE CONTROL. Only two types of voltage control are now in extensive use, and of these by far the most widely employed is the third-brush system. The other form is a vibrator regulator, which is most frequently employed on third-brush generators. A regulator operating upon a thermostatic principle has also seen some use in recent years, but it too is applied to third-brush generators. Like the vibrator regulator, its true function is to control the charging rate at the battery and to prevent overcharging of the battery with consequent gassing and loss of electrolyte.

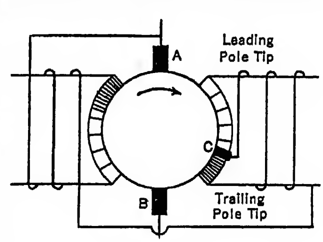


Fig 18

Third-brush Regulation. In this system, use is made of the distorting effect of armature reaction. When current flows through the armature, the resultant flux is skewed around in the direction of rotation (see Fig. 18), with the result that the number of lines of force entering the armature from the leading pole-tip is reduced and a corresponding increase in the number of lines of force takes place at the trailing pole-tip. Consequently, referring to Fig. 18, it is evident that the electromotive force generated between the brush A and the third brush C, located as shown, will decrease as the current through the armature increases. If the shunt field is connected between the brushes A and C, the current through the shunt field will therefore decrease as the current in the main circuit increases, which in turn will reduce the electromotive force between A and C still further. Consequently, if the battery is connected across the main brushes A and B, any tendency for the current to the battery to increase when the speed of the generator increases is counteracted by the

decrease in the field current. Such a generator when connected to a storage battery tends to act as a constant-current machine, irrespective of the speed. As a matter of fact, because of the effect of the short-circuit current in the coil passing under the brush *C*, which current is practically proportional to the speed and which produces a demagnetizing action, the current to the battery actually reaches a maximum value at a definite speed and then falls off.

The maximum current supplied to the generator is readily controlled merely by shifting the position of the intermediate brush. For a full treatment of the theory of third brush regulation, see Langsdorf's Principles of Direct-current Machinery.

Among the advantages of the third-brush control system are its simplicity and the fact that it requires, in most instances, no supplementary apparatus outside the generator. It also permits of ready variation of the current output by the simple expedient of shifting the brush. Shifting the brush in the direction of armature rotation increases the output, and shifting it in the reverse direction decreases the output. The output rises slightly as the battery becomes charged, and it is this characteristic that gives rise to overcharging of the battery when the output of the generator continues to progress much more rapidly than the battery is being discharged. It is partly to avoid this disadvantage that supplementary voltage regulators or battery charge regulators are sometimes used. Another reason for the use of voltage regulators is that, without them or some other protective device, an open circuit or high resistance in the charging circuit of a third-brush generator results in a rapid rise in voltage which is likely to burn out all the lights or cause a sufficiently heavy overload on the generator to damage its windings. A protective fuse in the field circuit of the generator is frequently provided.

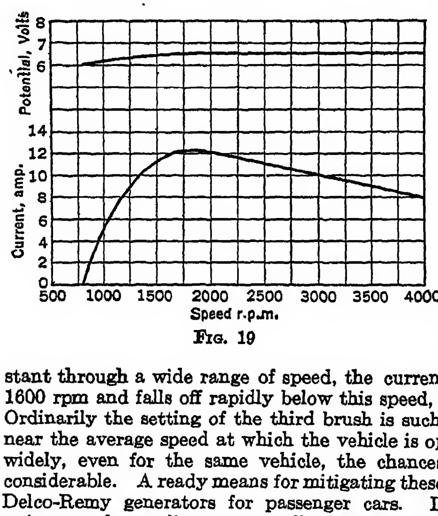


Fig. 19

In Fig. 19 are given the characteristic potential and current-speed curves of a third-brush 6-8-volt generator with a battery load. Although the voltage remains substantially constant through a wide range of speed, the current output reaches a maximum at about 1600 rpm and falls off rapidly below this speed, while above it the decrease is less rapid. Ordinarily the setting of the third brush is such that the peak of the curve comes at or near the average speed at which the vehicle is operated, but since this speed often varies widely, even for the same vehicle, the chances of overcharging or undercharging are considerable. A ready means for mitigating these disadvantages is that employed in many Delco-Remy generators for passenger cars. It consists of a simple thermostat and resistance shown diagrammatically in Fig. 20 and connected in the field circuit of the generator as shown in Fig. 17. This thermostat is placed inside the field frame of the

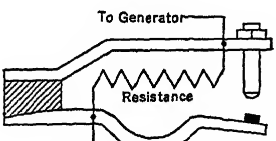


Fig. 20

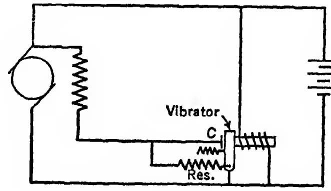


Fig. 21

generator, where the bimetallic element (the lower one in Fig. 20) is subjected to variations in the temperature of the generator itself. When the temperature rises to or about 165 deg fahr the contacts open and cut the resistance into the generator field circuit. The effect of this change is to decrease the output of the generator and thereby to lower its temperature. The resistance element, which is made easily replaceable, also acts as a fuse to protect the generator in case an open circuit occurs between the generator and the battery.

In starting, considerable current is drawn from the battery, so that, after starting, a

high charging rate is desirable. As the generator is then cold, the thermostat contacts are open and the field resistance is cut out, so that the high charging rate is secured. By or about the time that the energy taken from the battery in starting is replaced, the generator will have become warm and the contacts of the thermostat will open, cutting in the resistance and lowering the charging rate to a point where overcharging of the battery is unlikely to occur. If the car is used extensively for long trips with few stops, especially in warm weather, it sometimes becomes necessary to burn the headlights during the day or to resort to some other expedient to avoid overcharging of the battery. One of these is the voltage or battery-charge regulator designed to decrease the charging rate as the battery becomes charged. Such regulators are of special importance in buses, where the lighting load is likely to be heavy and a large generator with a high charging rate is required.

Vibrating Voltage Regulators. A simple form of voltage regulator is shown in diagram in Fig. 21. An external resistance is included in the field circuit, which resistance is short circuited when the vibrator is in contact with the contact *C*. When the vibrator is pulled away from the contact *C* by the electromagnet the resistance is inserted in the field circuit and the field current is thereby reduced. As long as the vibrator is operating, the field current varies between maximum and minimum values, and its average value depends upon the rate of vibration. The rate of vibration in turn depends upon the voltage across the winding of the electromagnet, and a very small increase in this voltage causes a relatively large increase in the rate of vibration, and therefore makes a substantial decrease in the average value of the field current. Consequently, as the speed of the engine varies, thereby tending to produce a varying voltage at the generator terminals, this tendency is counteracted by the reduction in the field current, so that the terminal voltage of the generator remains substantially constant for all values of the speed.

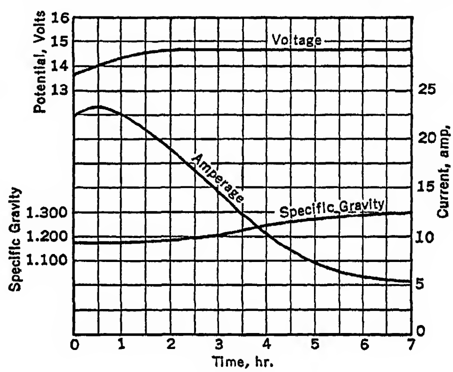


FIG. 22

Since the regulator holds the voltage substantially constant and that of the battery gradually increases as charging progresses, the voltage differential, and therefore the rate of charging, become steadily less as charging proceeds. In Fig. 22 is given a characteristic curve showing how the charging current of a voltage-controlled generator falls off as the battery becomes charged, until the current delivered assumes a low and almost constant value which is only slightly in excess of that normally consumed by the ignition system of the engine which drives the generator.

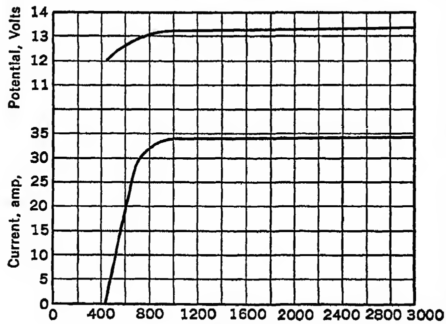


FIG. 23

Fig. 23 shows typical performance curves of a voltage-controlled generator operating with a battery load. It will be seen that both the voltage and current remain substantially constant over a three to one speed range from about 900 to 3000 rpm.

In a paper entitled "Generation of current for automotive apparatus" (*J. Soc. Automotive Engineers*, June, 1928, p. 658), from which Figs. 19, 22, and 23 are reproduced, B. M. Leece states the advantages of the voltage-regulated generator, which may be summarized as follows:

1. Connected batteries cannot be overcharged.
2. The rate of charge is proportionate to the need for charge, and gives a desirable trickling charge at the end.
3. There is less boiling away of battery electrolyte, hence less frequent filling is required.
4. Battery life is greatly increased.

5. Loose or corroded battery terminals do not cause excessive voltage with consequent burn-out of lamps.

6. Lamps can be operated directly from the generator when engine is running without a battery connected.

7. Generator output is proportional to load: no waste in overcharging battery.

8. In general, the generator can carry its load without assistance from the battery.

9. According to one large-scale operator of motor coaches, the cost of operating batteries from a voltage-regulated generator is as low as 0.0015 cent per coach-mile.

Constant voltage is beneficial in improving the performance and increasing the life of lamps as compared to systems in which the voltage constantly varies.

Thermostatic Battery-charge Regulator. Another form of regulator operated by a thermostat and shown diagrammatically in Fig. 24 has been produced by Owen-Dyneto and used on Packard cars. It has two arms of thermostatic bimetal, *k* and *l*, set firmly into an insulated block *n*, which can turn about the pivot *o*. The lower arm *l* is held by the spring *p* against the screw *q*, at the same time tending to rotate the arm *k* downward and close the contacts *r*, around which the resistance *s* is shunted. Thus *r* and *s* are in parallel and together are in series with the field of the generator, and the fuse *t*. Arm *l* is bare and consequently is affected thermostatically only by the atmosphere. Its purpose is to compensate for seasonal temperature changes in such a way as to give a higher charging rate in cold than in hot weather. Arm *k*, however, is wound with a heating coil of resistance wire which is connected directly across the generator and battery terminals. Its temperature varies, therefore, with the battery potential, being higher when the battery is fully charged and lower when it is discharged. When the battery is fully charged arm *k* becomes hot enough to open the contacts *r* and thus to throw the resistance *s* into series with the field, thereby reducing the generator output to such a value that the battery cannot be overcharged. Screw *q* is so adjusted that the contacts open when the battery is properly charged and just before the gassing point is reached. The generator is provided, of course, with the usual reverse current relay (not shown) for opening the circuit when the generator speed falls below the charging speed.

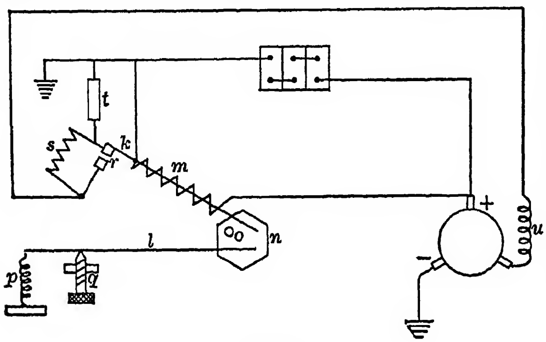


Fig. 24

GENERATOR MOUNTING STANDARDS. Generators for automotive lighting systems are usually mounted by a flange which bolts to the crankcase of the engine, but in some cases a base mounting is employed. Some of the larger generators such as are used in motor coach service employ a combination of flange and base mounting. Generators for some of the smaller cars are mounted on hinge pins in such a way that the generator can be swung about the pin to tighten the belt that drives the generator pulley. Three standard forms of mountings have been adopted by the Society of Automotive Engineers, each of which involves a standard shaft end such that generators are made readily interchangeable. There are also S.A.E. standards for generator brushes and cable terminals.

19. BATTERY

Storage batteries of the lead-acid type are universally employed. In general, jars are of hard rubber and usually are assembled in a one-piece molded case of acid-resisting material. All up-to-date American passenger cars are fitted with three-cell (nominally 6-volt) batteries, but motor coaches often use the six-cell type, and still higher voltages are required in some forms of automotive applications. Ampere-hour capacities for passenger cars vary from about 60 up to 140, but considerably larger batteries are used in some buses and for certain other automotive applications, notably in motor boats. Batteries for the smaller cars weigh from 55 lb upward, and the largest American cars have batteries weighing about 80 lb.

A number of standards covering dimensions and ratings for storage batteries have been adopted by the Society of Automotive Engineers. A summary of the more important follows:

Ratings. Batteries for combined starting and lighting service shall have three ratings, except as noted. The first rating shall indicate the lighting ability and shall be the capacity in ampere-hours of the battery when it is discharged continuously to an average final terminal voltage equivalent to 1.75 per cell, the temperature of the battery at the beginning of such discharge being 80 deg fahr, and an average temperature of 80 deg fahr being maintained during discharge, with a maximum variation of ±5 deg fahr from 80 deg fahr at the 20-hr rate for passenger-car and motor-truck service and at the 8-hr rate for motor-coach service. New batteries shall meet these ratings on or before the third discharge when discharged at a rate in amperes obtained by dividing the rated capacity in ampere-hours, as shown in the following tables, by the number of hours at which the rating is specified. The second rating shall be the minimum amperes when the battery is discharged continuously at the 20-min rate to an average final terminal voltage equivalent to 1.5 per cell, the temperature of the battery at the beginning of such discharge being 80 deg fahr. New batteries shall meet this rating on or before the third discharge, when discharged at the rate in amperes specified in the following tables. The third rating shall apply only to batteries used in passenger-car and motor-truck starting and lighting service. This rating shall indicate the cranking ability at low temperature, and shall be the minimum time in minutes when the battery is discharged continuously at 300 amp to an average final terminal voltage equivalent to 1 volt per cell, the temperature of the battery at the beginning of such discharge being 0 deg fahr. The batteries shall be prepared for this test by being placed in an atmosphere of 0 deg fahr ±1 deg fahr for not less than 24 consecutive hours and are to be tested in an atmospheric temperature of 0 deg fahr. New batteries shall meet this rating on the first discharge of a battery which has been fully charged following an initial discharge for 5 hr at a rate in amperes equal to one-seventh of the 20-min rate in amperes specified in the following tables, the temperature of the battery at the beginning of such preliminary discharge being 80 deg fahr.

Table I. Low-plate Type

S.A.E. Battery No.	No. of Cells	Minimum Capacity at 20-hr Rate, amp-hr	Minimum Current for 20 Min, amp	Minimum Time at 300 Amp and 0 Deg Fahr, min	Maximum Overall Dimensions, in.		
					Length * L	Width W	Height H
1	3	85	100	1.70	9 1/8	7 1/4	9 1/4
2	3	99	117	2.75	10 3/8	7 1/4	9 1/4
3	3	113	133	3.75	11 3/4	7 1/4	9 1/4
4	3	128	150	4.75	13 1/2	7 1/4	9 1/4
5	3	142	167	5.75	14 1/2	7 1/4	9 1/4

* Length dimensions do not include the ledge-type handles used on some types of rubber or composition case. This ledge shall not increase the case length more than 1 1/4 in. or extend downward more than 3/4 in.

Dimension Z shall not exceed 3 1/2 in.

Table II. High-plate Type

S.A.E. Battery No.	No. of Cells	Minimum Capacity at 20-hr Rate, amp-hr	Minimum Current for 20 Min, amp	Minimum Time at 300 Amp and 0 Deg Fahr, min	Maximum Overall Dimensions, in.		
					Length * L	Width W	Height H
6	3	90	105	1.80	9 1/8	7 1/8	9 3/4
7	3	108	126	3.00	10 3/8	7 1/8	9 3/4
8	3	126	147	4.25	11 3/4	7 1/8	9 3/4
9	3	144	168	5.50	13 1/2	7 1/8	9 3/4
10	3	162	189	6.75	14 1/2	7 1/8	9 3/4

* Length dimensions do not include the ledge-type handles used on some types of rubber or composition case. This ledge shall not increase the case length more than 1 1/4 in. or extend downward more than 3/4 in.

Table III. Motor-truck Battery Dimensions

Battery No.	No. of Cells	Minimum Capacity at 20-hr Rate, amp-hr	Minimum Current for 20 Min, amp	Minimum Time at 300 Amp and 0 Deg Fahr, min	Maximum Overall Dimensions, in.		
					Length * L	Width W	Height H
11	3	67	...	...	10 1/4	7 5/8	10 5/8
12	3	89	...	...	12 1/2	7 5/8	10 5/8
13	3	105	120	1.50	13 5/8	7 5/8	10 5/8
14	3	123	140	2.75	15 7/8	7 5/8	10 5/8
15	3	140	160	4.00	17 1/4	7 5/8	10 5/8

* Length dimensions do not include the ledge-type handles used on some types of rubber or composition case. This ledge shall not increase the case length more than 1 1/4 in. or extend downward more than 3/4 in.

Table IV. Motor-coach Battery Dimensions

Battery No.	No. of Cells	Minimum Capacity at 8-hr Rate, amp-hr	Minimum Current for 20 Min, amp	Maximum Overall Dimensions, in.		
				Length	Width	Height
22	3 *	129	157	19 1/2	7 5/8	10 7/8
23	3 *	178	225	26 1/4	7 5/8	10 7/8
25	6 †	88	112	21 5/16	9 3/4	10 7/8
26	6 †	104	135	21 5/16	11 7/16	10 7/8
27	6 †	118	160	21 5/16	11 7/16	10 7/8

* Side to side assembly of cells.
† Double row end-to-end assembly of cells.

Batteries 11 and 12 are for lighting service only; batteries 13, 14, and 15 are for combined starting and lighting service.

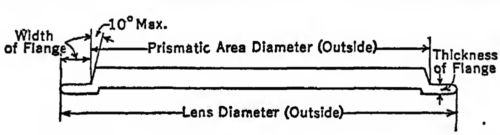
20. LAMP EQUIPMENT

Lighting equipment on most automotive vehicles includes a pair of headlamps, usually a separate pair of side (cowl or fender) lamps, a tail lamp (two often used) and various supplementary lights for illuminating instruments and body interiors, for stop signals, and for other purposes. Many states have stringent regulations governing the construction and adjustment of headlamps and in some cases of tail lamps, and often specifying the candlepower of the incandescent light source. The S.A.E. has standards covering all dimensions and tests of lamp equipment.

HEADLAMPS. The following is a summary of S.A.E. recommended practice pertaining to the construction of headlamps:

Means for holding the incandescent lamp base firmly should be provided. Both the focusing socket guide and the socket itself should be of non-ferrous metal. Not more than one focusing adjustment should be necessary, and the focusing screw should be readily accessible from outside the lamp. The focusing mechanism should permit of a 5/32-in. adjustment both forward and back of the focal point of the reflector. It should also maintain the relative position of lamp and reflector when the door and lens are removed. Paraboloidal reflectors should produce a beam, a normal section of which, at 100 ft from the lamp, should be entirely within a 105-in. circle. The mounting should be in the vertical center plane of the lamp, and fenders should not be tied together through the headlamps.

Table V. S. A. E. Standard Headlamp Lenses and Prismatic-area Diameters



Outside Diameter		Diameter of Prismatic Area, in.		Outside Diameter		Diameter of Prismatic Area, in.	
Wide Flange (Approx. 1/2 in.)	Narrow Flange (Approx. 1/4 in.)	Maximum	Minimum	Wide Flange (Approx. 1/2 in.)	Narrow Flange (Approx. 1/4 in.)	Maximum	Minimum
8.00	7.50	6.980	6.955	10.75	10.25	9.730	9.705
8.25	7.75	7.230	7.205	11.00	10.50	9.980	9.955
8.50	8.00	7.480	7.455	11.25	10.75	10.230	10.205
8.75	8.25	7.730	7.705	11.50	11.00	10.408	10.455
9.00	8.50	7.980	7.955	11.75	11.25	10.730	10.705
9.25	8.75	8.230	8.205	12.00	11.50	10.980	10.955
9.50	9.00	8.480	8.455	12.25	11.75	11.230	11.205
9.75	9.25	8.730	8.705	12.50	12.00	11.480	11.455
10.00	9.50	8.980	8.955	12.75	12.25	11.730	11.705
10.25	9.75	9.230	9.205	13.00	12.50	11.980	11.955
10.50	10.00	9.480	9.455				

HEADLAMP MOUNTINGS. Headlamp centers (according to S.A.E. standards) should be not less than 32 in. nor more than 42 in. from the ground, 36 in. being the preferred height.

HEADLAMP LENS DIMENSIONS. Standard dimensions for headlamp lenses adopted by the S.A.E. are given in Table V. The thickness of lens flange for the first 12 diameters listed in the table is $^5/_{32}$ in. at the edge of the prismatic area and not less than $^1/_8$ in. at the edge of the lens. For other sizes the corresponding dimensions are $^3/_{16}$ in. and $^1/_8$ in., minimum, respectively. The rear surface of the lens flange is plane. A lens notch, $^{17}/_{32}$ in. wide at the periphery, and $^3/_{32}$ in. deep, is located at the bottom of all lenses, except those having no prismatic structure, to insure correct location of the lens and prevent turning under vibration.

TAIL AND SIGNAL-LAMP LENS DIMENSIONS. S.A.E. standard for tail-light lenses provides that their diameter shall be 3 in. The minimum light opening shall be 2 $^1/_2$ in. Lens diameters for stop lights shall be 4 $^1/_2$ and 6 in., the minimum light opening in the lamp door being 3 $^1/_2$ and 5 in., respectively.

INCANDESCENT LAMPS. S.A.E. standards for incandescent lamps for automotive use provide that the nominal voltage shall be 6–8 or 12–16, for use respectively with three-cell and six-cell batteries. It is also provided that all such lamps for use in head lamps shall be of the gas-filled type and of 21 or 32 cp. Various filament forms in use are illustrated in Fig. 25. S.A.E. standard dimensions for incandescent lamps are given in Table

Table VI. Electric Incandescent Lamps

Dimension	G–6	S–8	S–10 *	Dimension	G–6	S–8	S–10 *
A—Aver...	3/4	1	1 1/4	C † Aver...	3/4	1 1/8	1 1/4
B—Aver...	1 7/16	2	2 3/8	D Max. ..	0.320	0.320	0.320

* The corrugations on electric incandescent lamps for headlight service shall be of sufficient depth to break up the filament image.

The spacing between the filaments of the depressed-beam or two-filament type of incandescent lamp shall be 0.140 in. ±0.016 in.

† Light-center length and axial alignment tolerances for headlight lamps are ± 3/64 in.

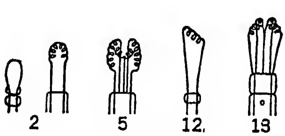


FIG. 25

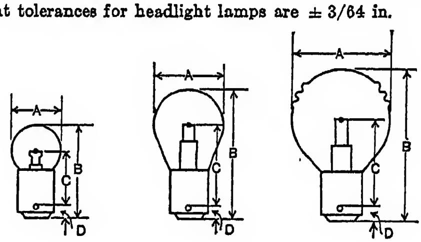


FIG. 26

VI, which refers to the dimensions shown in Fig. 26. Some lamps are provided with dual filaments one of which is so disposed in reference to the reflector as to give a depressed beam from a headlamp. As shown in Fig. 26, the S10 bulb is provided with corrugations which are intended to break up the filament image resulting from the use of a spherical surface. Bases are of the bayonet type, and some have single and some double contacts. Triple-contact bulbs are also used in connection with double-filament lamps. Standard bases, sockets, plugs, caps, and connectors are standardized by the S.A.E. Data on incandescent lamps published by the S.A.E., but not incorporated in its standards are given in Table VII. Bulb types refer to Fig. 26 and filament form to Fig. 25.

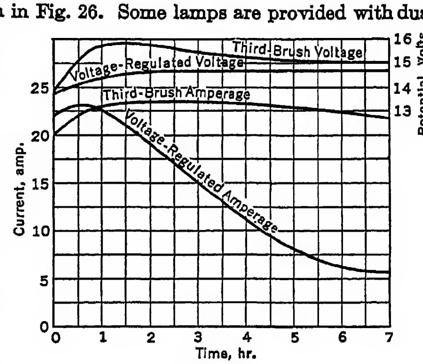


FIG. 27

Incandescent Lamps, Operating Characteristics. Although an increase of voltage from 6 to 7 increases the candlepower from 16.2 to 27, the life of the lamp is decreased to 45 per cent of its rated life if it is operated continuously at 7 volts. Extensive surveys

of automobile headlamp voltage indicate an average socket voltage of 6.35. The greater the lamp amperage, the lower is the line voltage, as the line drop is higher. Thirty-two-candlepower lamps give rated life and light output at 6.00 volts, and side and rear lamps at 6.85 volts. Because of the pronounced effect of voltage changes upon lamp life, close regulation of the voltage is desirable. That much closer regulation is obtained when the generator has a voltage regulator than when a third brush alone is depended upon for maintaining the desired output, is indicated by Fig. 27, which gives comparative curves of the two types in the case of 12-16-volt machine. The renewal rate is estimated at 3 to 4 lamps per car per year.

Table VII. Electric Incandescent Lamp Data

Candle-power	Voltage	Maximum Amper-age *	Bayonet-Base Contact †	Bulb Type	Bulb Diam-eter, in.	Maximum Overall Length, in.	Light Center Length, in.	Filament Form ‡
2	3–4	1.00	S or D	G–6	3/4	1 7/16	3/4	C–2
3	6–8	0.75	S or D	G–6	3/4	1 7/16	3/4	C–2
3	12–16	0.50	S or D	G–6	3/4	1 7/16	3/4	C–2
3	18–24	0.25	S or D	G–6	3/4	1 7/16	3/4	C–2
6	6–8	1.25	S or D	G–6	3/4	1 7/16	3/4	C–2
6	12–16	0.75	S or D	G–8	1	1 3/4	7/8	C–2
6	40–44	0.25	D	S–8	1	2	1 1/8	C–5
15	6–8	2.00	S or D	S–8	1	2	1 1/8	C–2
15	12–16	1.25	S or D	S–8	1	2	1 1/8	C–2
21	6–8	3.00	D	S–10	1 1/4	2 3/8	1 1/4	C–2
21		3.00					1 1/4	C–2
21	6–8	3.00	S or D	S–10	1 1/4	2 3/8	1 1/4	C–2
21	12–16	1.50	S or D	S–10	1 1/4	2 3/8	1 1/4	C–2
21	40–44	0.50	D	S–10	1 1/4	2 3/8	1 1/4	C–13
21	6–8	3.00	D	S–10	1 1/4	2 3/8	1 1/4	C–2
2		0.75		S–10	1 1/4	2 3/8		C–12
27	18–24	1.25	D	S–10	1 1/4	2 3/8	1 1/4	C–2
32	6–8	4.00	S or D	S–10	1 1/4	2 3/8	1 1/4	C–2
32	12–16	2.00	S or D	S–10	1 1/4	2 3/8	1 1/4	C–2

* Improvements in lamp design and manufacture from time to time make possible changes in ampere ratings. The figures given are therefore maximum and are for use in calculating wire sizes and battery capacities. For test purposes the exact amperage should be obtained from the lamp manufacturer.

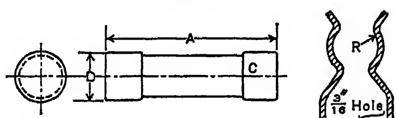
† S—Single-contact. D—Double-contact.

‡ C indicates a coiled-wire fiament.

21. INSTRUMENTS, FUSES, ETC.

FUSES AND FUSE CLIPS. S.A.E. standards for fuses and fuse clips provide, among other requirements, that fuses for circuits of 40 volts or under and 30 amp or less shall be of the closed type and so constructed that inspection will show whether or not they are burned out. The voltage (*V*) and the current capacity (*C*) shall be plainly marked on one ferrule. Fuses of the same dimensions as those of the National Electric Code shall be marked according to the requirements of that code. Fuses shall carry indefinitely, at a temperature of 75 deg fahr, a current 10 per cent above their rated capacity. A current

Table VIII



Capacity, amp	Length, in. (A)	Ferrule Diameter, in. (D)	Clip Radius (R)
5	1 1/4	1/4	3/32
10	1 1/4	1/4	3/32
15	1 1/4	1/4	3/32
20	1 1/4	1/4	3/32
30	1 1/2	13/32	5/32

50 per cent above rating shall cause the fuse to melt within 1 min. The temperature of the exterior of the fuse shall not rise more than 125 deg fahr above that of the surrounding air when the fuse is carrying its rated capacity. Fuse clips shall be so made that the fuse cannot slip out accidentally and shall be fastened to a base that cannot turn.

Standardized dimensions of fuses and clips are given in Table VIII.

INSTRUMENT DIMENSIONS. S.A.E. recommended practice relating to electrical instruments includes body diameters, drilling, and binding post sizes.

22. STARTING MOTOR

Starting motors for automotive engines are series-wound machines, designed to produce high torque at comparatively low speed and for use during only short time intervals. Consequently the motors are smaller and lighter than series motors for most other applications. Characteristic curves of a starting motor are given in Fig. 28, taken from a paper

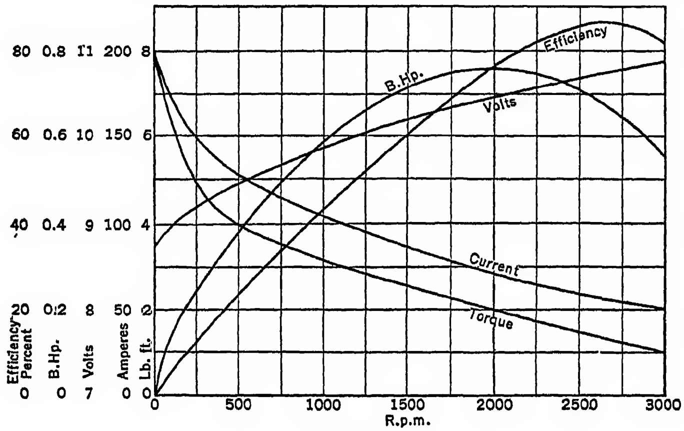


Fig. 28

entitled Electric starting and lighting equipment, by A. C. Burgoine (*The Automobile Engineer*, February, 1924). As this figure indicates, the torque is a maximum at zero speed, where, of course, the current drawn is a maximum. Both torque and current fall off rapidly with increase in speed. The power output increases with the speed, and in the motor in question reaches a maximum at about 2000 rpm, at which point the efficiency is approximately 76 per cent. Since motors of this class are rarely used for more than 1 min of continuous running, high efficiencies are not of special importance and ratings can be considerably higher than for series motors designed for continuous use. For the same reasons, the allowable rate of temperature rise may be much greater in the starting motor. High torque output is required, not alone to overcome the inertia of the parts put into motion, but also to break down the resistance or shear of the oil films in engine bearings and cylinders and to overcome the load imposed by compression of the charge in engine cylinders. Although a part of the energy used in compressing the charge is returned on the expansion stroke, even before ignition occurs, much of it is lost through wire-drawing of the gases and dissipation of the heat of compression to the cylinder jackets. Torque requirements vary during each revolution of the engine, the variation increasing as the number of cylinders in the engine decreases. Other factors also affect the torque required, one of the most important being the viscosity of the engine lubricant, which varies in turn with engine temperature, and is highest with a given oil when it is new and undiluted by fuel which passes the pistons. Chiefly because of higher oil viscosities in cold weather, low temperature starting imposes some of the most difficult limiting conditions in starter design. Comprehensive starting motor tests are reported in Low-Temperature Starting Development of Automobile Engines, by P. J. Kent (*S.A.E. Journal*, August, 1931, p. 141), from which some of the following particulars are abstracted.

A minimum cranking speed of 40 rpm is desirable. Fig. 29 illustrates the results of cranking tests of an eight-cylinder engine with two different starting motors and three combinations of starting motor and battery. Starting torque is a direct function of starting motor current, and the speed is a direct function of the terminal voltage minus the *IR* drop

in the motor. Above the starting motor voltage curve, Fig. 32, is a curve showing the required battery voltage for corresponding cranking speeds, calculated by adding the drop in the car-ground circuit and the starting motor cables. The resistance of this circuit in the average Chrysler car is 0.0012 ohm, which, when multiplied by the current, gives the drop in the line. Discharge curves of the batteries under consideration (based on a 5-sec discharge at 0 deg fahr) are also given in Fig. 29. The intersection of these curves with the curve of required battery voltage gives the cranking current for the particular battery, and projecting this point on the cranking speed curve gives the speed at which

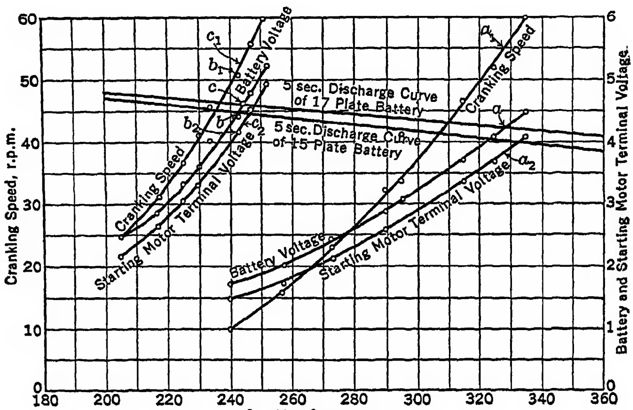


FIG. 29

the battery will crank the engine with the particular starting motor and gear reduction used. Fig. 29 applies to oil diluted with 15 per cent of kerosene. The various combinations tested are as follows:

Combination	*a*	*b*	*c*
Gear-reduction type	Direct	Back Geared	Back Geared
Ratio	9:115	11:115 and 14:29	11:115 and 14:29
Plates in battery	17	15	17
Cranking current, amp	328.0	242.5	244.0
Starting-motor voltage	3.85	4.15	4.25
Cranking speed, rpm	55.0	51.5	53.0

The intersection of the 5-sec battery-discharge curves with the battery-voltage curves at *a*, *b*, and *c* (Fig. 29) gives the battery voltage for the corresponding combinations tested. Projecting these points to intersect the cranking-speed curves at a_1, b_1, and c_1 and the starting-motor terminal-voltage curves at a_2, b_2, and c_2 gives the speed at which the engine would be cranked and the starting-motor terminal voltage for the corresponding battery, starting-motor and great-reduction combination; and on the base-line the cranking current is indicated.

Such data are sufficient to determine the correct equipment so far as initial cranking speed is concerned. Fig. 30 shows the results of three cranking capacity tests with the combinations indicated, the batteries first being tested to make sure that they are up to specifications. The engine is then cranked continuously until the potential across the starting motor terminals is reduced to 3 volts. Although the initial cranking speeds are not greatly different, the time that the engine can be cranked with the different combinations is shown to vary considerably. The current is less and the voltage at the starting motor (as well as that available for ignition) is greater in the back-geared motors. From the data in the cranking-speed tests, Fig. 29, the power required to crank the engine at various speeds can be determined, and if different starting motors and batteries are to be considered, and the performance curves of each motor with each battery are available, an ideal gear reduction can be selected and the cranking speed that each combination will give can be determined accurately. The horsepower required to crank the engine can be computed from the following formula:

$$\text{hp} = 2\pi NRT \div 33{,}000$$

where N is the cranking speed of the engine, R is the speed reduction ratio between the starting motor and flywheel gear, and T is the torque of the starting motor in pound-feet. If the actual engine cranking power is required, allowance must be made for the mechanical loss in the gearing. It is not necessary to know the loss in the gears, however, to determine the best gear reduction, if the torque has been measured with the gears in use. The power required to crank the engine, computed by the above formula, is plotted against cranking

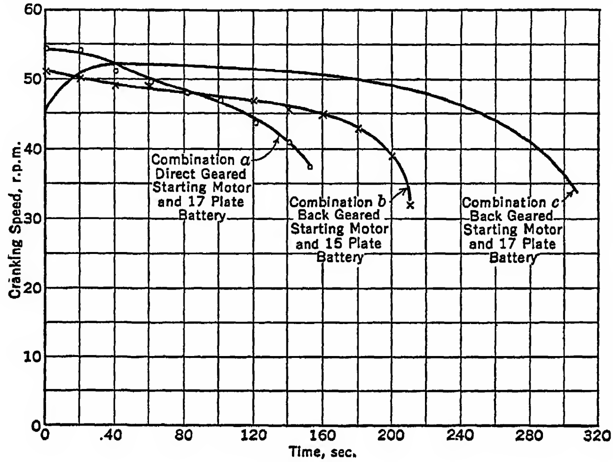


FIG. 30

speed as shown in Fig. 31. Tests of the torque *at the flywheel*, as developed by the starting motor, for different current values, are observed, using for each run a voltage at the starting motor terminals equal to the 5-sec discharge voltage of the battery minus the line loss. From these data the available power is computed and the complete performance curves of the motor are plotted. For accurate work, the starting motor should be tested at 0 deg fahr, or the normal temperature test results should be corrected to that tempera-

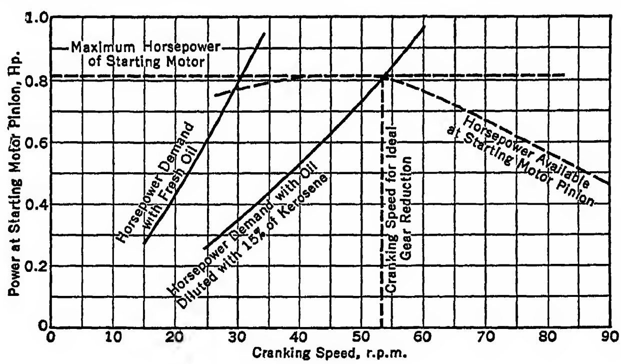


FIG. 31

ture, as was done in the curves plotted in Fig. 32. From the starting motor horsepower curve the maximum power available is determined. If this be transferred to horsepower-demand curve, Fig. 31, a line dropped to the base line, as indicated, gives the maximum cranking speed available from the particular motor and battery combination under consideration. This speed divided into the speed of the starting motor at the point of maximum power gives the ideal gear ratio for best cranking.

In Fig. 33 the curves show the actual relation between the piston displacement and the cranking speed for five different six-cylinder engines of similar design and approxi-

mately the same compression ratio, at 0 deg fahr, the same size of 13-plate battery being used in each case. Both motors used are of the direct-geared type, the gear ratio being 9 to 115. Starting motor *A* had four poles wound and motor *B* had field coils on only two poles. From the large volume of data accumulated, the following general conclusions are drawn: (1) The larger the engine, the more difficult is winter cranking. (2) For any given starting motor, increasing the size of the battery does not add greatly to cranking speed, but materially increases cranking capacity. (3) Cranking speeds of an engine may vary between wide limits, depending upon the type of starting motor and gear reduction employed.

STARTING MOTOR CONTROL MECHANISMS. Control mechanisms for starting motors are of three types: (1) Automatic engagement and disengagement of pinion (Pinion

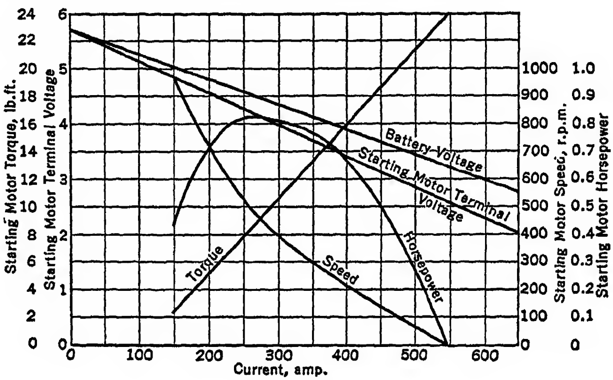


FIG. 32

type). (2) Pedal (or manual) engagement and automatic disengagement (**Bendix** type). (3) Pedal (or manual) engagement and disengagement in combination with an over-running clutch to release the armature at intervals between the time when the engine starts firing and that at which pedal disengagement occurs.

STARTING MOTOR CIRCUITS. S.A.E. standards provide that the starting motor circuit shall be so designed that the difference between the voltage of the storage battery terminals and that at the starting motor terminals shall not exceed 0.12 volt per 100 amp with the circuit at a normal temperature of 68 deg fahr.

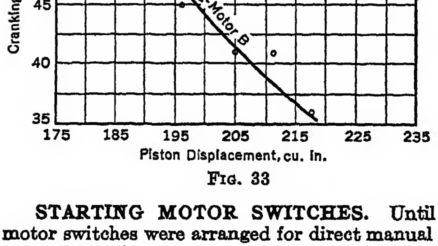


FIG. 33

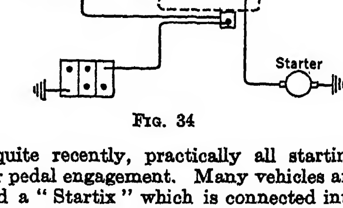


FIG. 34

STARTING MOTOR SWITCHES. Until quite recently, practically all starting motor switches were arranged for direct manual or pedal engagement. Many vehicles are now equipped with a relay starting switch termed a " Startix " which is connected into the starting system in the manner shown in Fig. 34. The Startix is so connected that the engine is started instantly and automatically when the ignition switch is closed and is automatically restarted in the event that it stalls; hence, to start the engine, the operator

need only turn the ignition key, without operating any supplementary switch or starting mechanism. The Startix is applicable, however, only to engines equipped with the Bendix drive, which automatically engages the starter pinion when the starting motor circuit is closed, and automatically disengages the starter pinion when the engine begins to fire. When the ignition key is turned on, current from the battery passes through the main switch solenoid of the Startix and closes this switch, whereupon the starter pinion is engaged and the engine is cranked. When the engine starts, the pinion is disengaged, a relay solenoid in the Startix is energized, and the inward travel of this plunger deflects a vibrator arm assembly which in turn interrupts the current through the main switch solenoid and thus breaks the starting motor circuit. When the engine starts, a second coil on the relay solenoid is energized, attracting a lever interconnected with the vibrator mechanism. If the engine stops, the lever is released along with the vibrating arm and the starting cycle is repeated. The vibrator introduces a time factor such that the engine starter will not be re-engaged until the engine comes to rest. A thermostatic safety device forming a part of the Startix is arranged to automatically open the starter circuit in the event that the starter pinion jams or fails to release. In this case the Startix produces a distinctive clicking sound until the ignition switch is turned off.

STARTING MOTOR PERFORMANCE DATA. Tests normally applied to starting motors in service and maintenance, as well as in factory inspection work, provide for measuring the current and speed at 5 or 6 volts (for 6–8 volt machines) and the current, voltage, and lock-torque when the armature is stalled and connected across a battery such as that with which it is normally employed, or some equivalent power source. The following table gives the performance data covering a number of Delco-Remy starting motors such as are widely used in automotive applications:

Model No.	No-load Test			Lock Test			Direction * of Rotation
	Amp	Volts	RPM	Amp	Volts	Torque, lb-ft	
710 C, AC	65	5	4500	450	3.70	10.0	CW
712 E, F	70	5	5000	450	3.60	11.0	CW
713 F, G, H, J	65	5	5000	475	3.63	12.0	CW
714 A, B, C, D, AX	65	5	5000	475	3.63	12.0	CW
716 A	70	5	3000	450	3.70	15.0	CW
717 A	70	5	5000	450	3.60	11.0	CW
720 J, K, L, M	65	5	6000	570	3.15	15.0	CW
720 N, P, Q, R, S, T, U, V, W, QX	65	5	6000	570	3.15	15.0	CW
723 A, B	70	5	1000	600	3.25	70.0	CW
724 C, D, E, F	70	5	3500	600	3.00	22.0	CW
725 A	65	5	6000	600	3.00	15.5	CC
732 B, C	65	5	2000	600	3.00	32.0	CW

* CW = clockwise, CC = counter-clockwise, when viewed from driving end.

23. WIRING

WIRING SYSTEMS. Although both double-wire and single-wire starting and lighting systems have seen extensive use, the single-wire type is by far the more general because of its lower cost and greater simplicity. In this system the frame and other metal parts of the car form a part of the circuit, either the positive or the negative terminal of the battery being grounded permanently to the frame or some other metal part of the car. For lighting circuits wires varing from No. 16 to No. 10 B. & S. gage are usually employed. Starting circuit cables usually range in size from No. 4 to No. 0 B. & S. gage, and their length is made as short as feasible to minimize voltage drop.

S.A.E. recommended practice for automobile grounded wiring systems is outlined in a code of which the following is an abstract:

INSULATED CABLE. Conductors should not be of smaller size than No. 16 B. & S. gage, and the potential loss at normal load should not exceed 3 per cent. Terminals on other than starting motor cable should be clamped to the insulation and soldered to the conductor.

CONDUIT. Insulated cables should be protected by metal armor or unpacked metallic or non-metallic covering unless otherwise protected or not in contact with metal. Ferrules having rounded edges should be used at open ends and at junction boxes. Wires not protected by conduit should be cleated at intervals not exceeding 28 in. When protected by conduit, metal clips secured by bolts or wood screws should be placed at intervals of not

less than 36 in. Staples should not be used. Cables not in conduit should be bushed with rubber where they pass through holes in metal.

GROUNDING. All ground connections should be accessible for repair. Ground return connections should be made to the chassis frame or to a substantial part firmly attached to the frame. The positive side of the battery should be grounded when a voltage control system is used. The surface on which grounding terminals make contact should be tinned and free from oxide or paint.

OVERLOAD PROTECTIVE DEVICES. Current to all low-tension circuits except starting motor and ignition circuits should pass through protective devices connected to the battery-feed side of switches. Circuits should be so arranged that the opening of a protective device will not extinguish all lights.

COLORING OF CABLES. To simplify the installation and connection of wiring and to afford a positive means of identification of the principal circuits, the following coloring is recommended:

Passenger car wiring, where cable is bought in coils: Red for unprotected live wires, and yellow for protected live wires.

Passenger car wiring where cables are bought in the form of a harness: Red for unprotected live wires; red with yellow tracer for low-tension or primary ignition; red with black tracer, ammeter to battery; yellow for protected live wire; brown with black tracer from lighting switch to junction block (parking lamp) and for all ground connections except battery ground; black, lighting switch to tail lamp; black with red tracer to bright headlamps (or upper beam); green to dim headlamps (or lower beam) and from switch to signal lamp.

Motor-coach and truck wiring (assumed to be bought in bulk coils): Red for unprotected live wires; yellow for protected live wires; brown with black tracer for generator cut-out or regulator to ground and all ground connections except that of battery; black for bright headlamps (or upper beam) and for bodylamp feed wires, switch to lamp; black with red tracer for dim headlamps (or lower beam); green from switch to signal-lamp and to signal-lamp indicator or pilot.

INSULATED CABLE. The following is an abstract of the S.A.E. standard specifications for insulated cable:

General Specifications. Conductors. Conductors shall be bunched or stranded as specified in each section below, and shall be of tinned annealed copper wire complying with specification No. B3-15 of the American Society for Testing Materials.

Rubber Insulation. Rubber insulation shall adhere closely but strip rapidly from the conductors, leaving them reasonably clean. It shall contain not less than 20 per cent by weight of good-grade new hevea rubber.

Varnished-cambric Tape. It shall not be less than 0.005 in. thick for wires of No. 10 A.W.G. and smaller, nor less than 0.007 in. thick for No. 8 A.W.G. and larger wires. In no case shall the thickness exceed 0.013 in. The instantaneous puncture voltage shall not be less than 750 volts per mil thickness.

Braids. Braids shall consist of closely woven cotton yarn not less than $^1/_{64}$ in. thick and shall be impregnated with at least two coats of insulating varnish or enamel resistant to heat, oil, and water, or with black water-proof wax compound that thoroughly saturates the braid and gives a smooth finish. When wound on reels, adjacent layers of cable shall not stick to each other at any temperature below 105 deg fahr.

Armor. Armor shall be solid D-shaped of either galvanized or sherardized dead-soft steel, soft brass, aluminum, or copper, applied in a close helix without overlapping turns, with dimensions as in Table IX.

Table IX. Armor Thickness and Width Dimensions

Armor	Thickness, in.			Width, in.		
	Minimum	Nominal	Maximum	Minimum	Nominal	Maximum
Small............	0.014	0.017	0.020	0.045	0.050	0.055
Large............	0.017	0.020	0.023	0.095	0.100	0.105

TESTS. Tinning Tests. A.S.T.M. standard two-cycle test.

Physical Tests. A 6-in. or longer sample of rubber insulation shall have marks placed on it 2 in. apart and shall be stretched until these marks are 6 in. apart at a stretching rate of 12 in. per min and immediately released. Thirty seconds after release the marks shall not be more than 2 $^1/_2$ in. apart. The specimen shall then be stretched until the marks are 7 in. apart without rupture. The tensile strength of rubber insulation shall not be less than 600 lb per sq in., based on original sectional area. Tests to be made at a

temperature between 50 and 90 deg fahr and sample to be cut with uniform cross-section. These physical tests apply to cables having a wall thickness of 0.045 in. or more. For thinner sections the initial and ultimate stretch shall be 5 and 6 in. respectively, and the tensile strength not less than 500 lb per sq in.

Oil Test for Braided Cables. This test shall be made on all cables, both high-tension and low-tension, having either a varnished or enameled braid covering. After submersion of all but the ends of a sample in a shallow dish containing equal parts of engine oil and gasoline for 24 hr it shall be shown that no oil has penetrated to the rubber and that the varnish or enamel does not show signs of softening or absorption.

HIGH-TENSION IGNITION CABLE. Conductors shall be of tinned copper wire and either plain rubber covered, rubber covered and varnished cambric taped and braided, rubber covered with single braid, or rubber covered with double braid, as ordered. Waxed braid shall not be used for this type of cable. When varnished-cambric strips are used, there shall be a 25 per cent overlap. Dimensions shall be as given in Table X, and when braided shall be varnished or enameled to withstand the oil test outlined above. The 7-mm size is recommended for all high-tension cables.

Table X. High-tension or Secondary Ignition Cable

Nominal Size		Number of Wires in Strand	Nominal Size of Wires in Strand, A.W.G.	Maximum Outside Diameter, in.	Minimum Outside Diameter, in.	Minimum Thickness of Rubber Wall, in.		
Mm	In.					Plain Rubber Covered	Single Braid	Double Braid
7	0.2756	12 19	26 (0.0159) 27 (0.0142)	0.285	0.265	0.097	0.081	0.066
9	0.354	19	27 (0.0142)	0.364	0.344	0.135	0.119	0.104

LOW-TENSION IGNITION CABLES AND STARTING-MOTOR AND LIGHTING CABLES. Conductors shall be bunched or stranded and insulated in one of the following seven ways: (1) plain rubber covered; (2) rubber covered and single-braided; (3) rubber covered and double-braided; (4) two layers of varnished cambric tape and single braid; (5) same as 4, but double-braided; (6) rubber covered with rubber-faced tape and single braid (for starting motor cables only); (7) rubber covered with rubber-faced tape and double-braided (for starting motor cables only). The braid shall be varnished, enameled, or weather-proofed, as provided above. The dimensions of low-tension ignition cable shall be as shown in Table XI.

Table XI. Low-tension Rubber-insulated Ignition Cable (Not Braided)

Nominal Size	Number of Wires in Strand	Nominal Size of Wires in Strand		Maximum Outside Diameter, in.	Minimum Outside Diameter, in.
		A.W.G.	In.		
5 mm (0.197 in.)	19	27	0.0142	0.207	0.187

The above cable is recommended as a grounding cable for short-circuiting magnetos.

Conductors of primary, lighting, and starting motor cables shall be of the sizes and construction given in Table XII. Conductors of cables composed of Nos. 16 to 10 A.W.G. inclusive shall be of tinned copper wire when rubber insulation is used and may be bunched or stranded as desired. For No. 8 A.W.G. and larger cables, conductors shall be stranded and may be of concentric or rope lay. In rubber-insulated cables, either tinned copper wires or bare copper wires with suitable separators may be used.

The total circular mil area of any strand shall not vary more than 2 1/2 per cent from the actual figures given.

Tolerances for the diameter of the rubber shall be as follows: No. 16 to No. 10 A.W.G. inclusive, plus or minus 0.005 in.; No. 8 to No. 00 A.W.G. inclusive, plus or minus 0.010 in.

The nominal diameter of the wires in the strands and the nominal circular mil area agree with figures issued by the Bureau of Standards in Circular 31, Table VIII.

When low-tension cables are insulated with varnished cambric and without rubber, there shall be two or more layers of overlapping cambric with alternate layers laid in opposite direction with 25 per cent overlap, and either tinned or bare copper wire may be used.

ARMORED LIGHTING CABLES. Cables of this class shall be insulated with: (1) two or more layers of varnished cambric with 25 per cent overlap and alternate

Table XII. Recommended Stranding and Dimensions of Lighting and Starting Cable

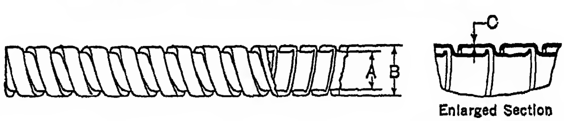
Nominal Size, A.W.G.	Number of Wires in Strand	Nominal Size of Wires in Strand		Circular Mils		Continuous Carrying Capacity, amp	Maximum Outside Diameter, in.	Thickness of Rubber Wall, in.
		A.W.G.	In.	Nominal	Actual			
	12	27	0.0142		2,420			
16	16	28	0.0126	2,580	2,556	6	0.200	0.0310
	19	29	0.0113		2,409			
	19	27	0.0142		3,831			
14	26	28	0.0126	4,110	4,154	15	0.223	0.0310
	19	25	0.0179		6,088			
12	26	26	0.0159	6,530	6,606	20	0.250	0.0310
	19	23	0.0226		9,679			
10	49	27	0.0142	10,400	9,880	25	0.275	0.0310
	19	21	0.0285		15,390			
8	49	25	0.0179	16,500	15,700	35	0.320	0.0370
	61	22	0.0253		39,200			
4	49	21	0.0285	41,700	39,689	70	0.420	0.0468
	127	22	0.0253		81,613			
1	133	22	0.0253	83,700	85,469	100	0.600	0.0625
	127	21	0.0285		102,866			
0	133	21	0.0285	106,000	107,726	125	0.635	0.0625
	127	20	0.0139		129,723			
00	133	20	0.0139	133,000	135,852	150	0.700	0.0625
	259	23	0.0226		131,936			

layers laid in opposite directions; (2) rubber; or (3) rubber and varnished-cambric tape. All cables shall have one or two treated braids, and armor over the braid.

FLEXIBLE STEEL CONDUIT AND TUBING. S.A.E. recommended practice in reference to flexible steel conduit and tubing is covered by a specification which provides that it shall be of the square locked type made by helical coils of formed steel strip. It is rated by nominal inside diameter. Standard dimensions are given in Table XIII.

Installation. To protect electric wires from abrasion and to prevent conduit from unwinding, exposed ends should be bussed with suitable ferrules soldered to the conduit. Where taps or joints are made, junction buses should be used and conduit ends carefully bushed.

Table XIII. Flexible Steel Conduit—Unpacked



Nominal Inside Diameter, in. (A)	Actual Inside Diameter, in.		Maximum Actual Outside Diameter, in. (B)	Minimum Thickness of Strip, in.* (C)	Approximate Minimum Weight, lb per 100 ft extended *	Minimum Tension Load, lb	Minimum Breaking Load, lb	Minimum Bending Radius, in.†
	Min	Max						
3/16	0.188	0.200	0.280	0.010	3.5	75	30	3/4
1/4	0.250	0.265	0.350	0.010	4.0	100	40	7/8
5/16	0.313	0.330	0.420	0.010	4.9	110	40	1
3/8	0.375	0.395	0.485	0.011	6.5	125	45	1 1/4
7/16	0.438	0.460	0.545	0.011	7.7	150	45	1 3/8
1/2	0.500	0.525	0.630	0.011	8.5	175	75	1 1/2
9/16	0.562	0.587	0.692	0.011	9.5	185	75	1 5/8
5/8	0.625	0.656	0.755	0.011	10.5	200	75	1 3/4
3/4	0.750	0.790	0.920	0.013	14.5	300	100	2

* These columns are for general information only.
† Inner radius of the bend without straining conduit.

NON-METALLIC CONDUIT. Non-metallic flexible conduit or loom, such as is recommended for use over insulated wire, metal tubing, or other parts requiring a covering that is proof against acid, oil, and water, and resistant to abrasion, is used also as a covering to prevent crystallization and rattles. See Table XIV.

Table XIV. Dimensions of Non-metallic Conduit

Nominal Size, in.	Inside Diameter, in.		Outside Diameter, in.		Nominal Size, in.	Inside Diameter, in.		Outside Diameter, in.	
	Min	Max	Min	Max		Min	Max	Min	Max
3/16 *	0.187	0.207	0.277	0.297	9/16	0.562	0.582	0.722	0.742
3/16	0.187	0.207	0.287	0.307	5/8	0.625	0.645	0.785	0.805
1/4 *	0.250	0.270	0.340	0.360	11/16	0.687	0.707	0.847	0.867
1/4	0.250	0.270	0.350	0.370	3/4	0.750	0.770	0.934	0.954
5/16	0.312	0.332	0.412	0.432	13/16	0.812	0.832	0.996	1.016
3/8 *	0.375	0.395	0.475	0.495	7/8	0.875	0.895	1.079	1.099
3/8	0.375	0.395	0.505	0.525	15/16	0.937	0.957	1.141	1.161
7/16	0.437	0.457	0.567	0.587	1	1.000	1.020	1.204	1.224
1/2	0.500	0.520	0.630	0.650					

* For use with standard ferrules.

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ELECTRIC AUTOMOBILES

By W. P. Kennedy

There are two types of electric automobiles, namely the battery vehicle and the "gasoline-electric" vehicle. The former type carries a storage battery from which energy is delivered to the driving motor; the latter type, in most instances, carries no battery, but electric energy is supplied to the motor by a generator which is itself driven by a gasoline engine. Formerly, storage batteries were used in combination with a motor-generator connected to the engine. Thus the machine was driven by energy from either battery or engine. The battery vehicle only is discussed in this article.

25. DEVELOPMENT

The first battery-driven automobile in this country was built by Fred M. Kimball in Boston in 1888. This machine had 6 cells of lead storage battery and could travel 10 miles on good roads at an average speed of 5 mph. In 1891 there was exhibited at the Mechanics'

Fair in Boston an electric surrey built by the Holzer-Cabot Electric Co. Little more was done in the development of electric automobiles until about 1897 when the passenger vehicle was developed in two directions: one for pleasure took the forms of victorias, runabouts, dos-à-dos, and surreys, and the other for business purposes in the form of public taxicabs or vehicles for hire. The latter development expanded during the following four years to the use of fleets of 100 to 400 cabs and broughams in New York, Boston, Philadelphia, Washington, Buffalo, and Chicago for public service on a hired basis; these disappeared from use about 1902 or shortly afterwards, largely for financial reasons, but also on account of the fact that large solid or pneumatic tires were not well developed at that date and storage batteries had not progressed beyond the heavy types used for stationary equipment in power stations.

From 1900 to 1903 the commercial type of electric truck began to be employed for merchandise delivery and progressed gradually but steadily until 1910, when the electric light companies recognized in it a desirable " off-peak " power consumer. The Electric Vehicle Association of America with considerable activity materially furthered the use of electric vehicles, both passenger and commercial, until 1914, when there were 37,000 in use; 25,000 passenger cars and 12,000 trucks (*Elec. World*, Jan. 3, 1914). The average price of the former at that date was $2200 and the latter $2800; this represented $88,000,000 invested by the public in this type of automobile. Following 1914 there was a rapid decline in the rate of production of electric vehicles owing largely to general business conditions, but principally to the competitive influence of mass production in the gasoline vehicle industry; although the electric still maintains its superiority in economic operation. In 1920 there were but three manufacturers of each type aggressively conducting production in their respective fields, but shortly afterwards passenger-car manufacture ceased. Another class of the electric vehicle, in the form of indoor industrial material and freight handling trucks, has been in progressive development since 1904. See Industrial Electric Trucks.

26. APPLICATIONS OF ELECTRIC TRUCKS

The three types of vehicles which are at present available for urban street haulage are the horse-drawn wagon, the gasoline motor truck, and the electric battery truck. In comparing these three types for use in any given service, the chief factors to be considered are relative speed, distance between stops, number and length of stops required by the service, expense per vehicle, and reliability (i.e., number of days per year upon which the vehicle can be depended for operation). The first three of these factors may be determined by trial or by observation.

The electric truck has found its widest commercial application in city services where the hauls are of moderate length, say from 2 to 10 miles, and where the daily service does not exceed 30 miles on the average, with occasional daily mileages of 40 to 50. For very short hauls where the standing time of the vehicle is a large proportion of the working time, so that the work per day of each delivery unit is not largely influenced by the speed of the unit, the horse-drawn wagon can perform the work cheaper than any motor vehicle. For the so-called " long hauls " the gasoline truck would be expected to work to better advantage than an electric truck because of higher running speed or greater distance capacity in a day, or both. The effect of hills or poor pavements would decrease both speed and mileage in electric trucks. Systems of operating in which discharged batteries are exchanged for charged batteries during the day in order to increase the mileage capacity beyond the rated value have been used with signal success in a few instances, but in general the battery equipment is usually of sufficient capacity to perform the daily service required, or is in some cases supplemented by high-rate boosting charge during the noon hour or other convenient period.

Design

The necessity of producing cars which would travel at moderate speeds with a low rate of energy consumption has forced the designers of electric automobiles to use equipment which would meet this requirement. Thus tires of low-energy consumption, batteries with large capacity per unit of weight, and " anti-friction " bearings in transmission and axles are used almost universally.

WEIGHT AND SPEED. In the design of an electric battery vehicle the two primary features are the total weight and the speed under normal conditions. In special instances a car may be built to meet unusual requirements of distance traveled per charge, in which case either the speed or weight must depart from the limitations of common practice. Any decrease in the total weight will result in an increase in either speed or

distance capacity or both; or if the weight is brought back to the original value by the use of additional battery capacity, the speed or distance may be still further increased. This has been one of the chief considerations leading to the adoption of lighter batteries, such as the Edison and " thin " pasted lead plate, in place of the heavier type lead plate batteries commonly used where their additional weight is not a critical factor. General practice as regards weight and speed for various types of electric automobiles is indicated in Table I.

Table I. Speeds and Weights of Electric Trucks

Rated Capacity of Truck	Approximate Weight in Pounds, Including Body and Load	Approximate Speed, miles per hour	Chassis Weight without Battery or Body, lb
1,500-lb truck	5,000	15 to 20	2800
2,000-lb truck	6,500	13 to 15	3500
4,000-lb truck	8,000	12 to 14	4000
6,000-lb truck	10,500	12 to 14	4800
8,000-lb truck	13,500	12 to 14	5300
10,000-lb truck	22,000	12 to 14	6400
12,000-lb truck	26,000	10 to 13	7600
15,000-lb truck	32,000	10 to 12	8500

Load Efficiency. By load efficiency of a motor truck is meant the ratio of the load on the machine to the gross weight of the machine, including chassis, body, and load. Thus, from the above table, a representative value for the rated load efficiency of a 5-ton capacity electric truck is $\frac{10{,}000}{10{,}000 + 22{,}000} = 0.31$. The rated load efficiencies for gasoline trucks of large capacity, 2 tons or more, are practically the same as the values for electric trucks, but small-capacity gasoline cars of good design may have a rated load efficiency as high as 30 per cent as compared to about 24 per cent for electric cars with the customary lead battery equipments.

MOTORS. Four-pole series motors are now used almost exclusively for electric automobiles. In the majority of cases but a single motor is used on each vehicle. The field coils are usually arranged in two groups which may be connected first in series and then in parallel. Motors will carry their normal rated load continuously with a maximum temperature rise of 65 deg cent above the surrounding air at 25 deg cent or will carry 2 1/2 times their rated load for 1 hr with a maximum temperature rise of 75 deg cent. Ratings are based on bench tests, so that when the motor is installed in a car the overload capacity is usually increased owing to the improved ventilation. Large overload capacity is necessary to meet severe street and grade conditions, and unusually good commutating characteristics are required.

An automobile motor must also operate satisfactorily over the varying range in the voltage furnished by the battery in the charged and discharged conditions. The motor speeds which are used with single reduction drives range from 750 to 1000 rpm; the speeds which are used with double reduction drives range from 1000 to 2000 rpm.

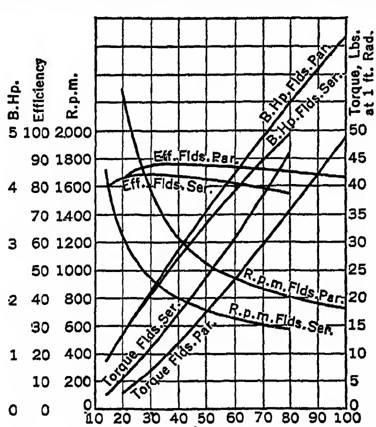

FIG. 1a

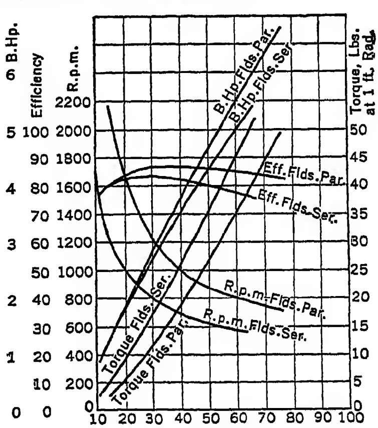

FIG. 1b

Number of Motors. Most electric trucks are now being equipped with a single motor. It was formerly common practice to use two motors on the larger trucks on account of flexibility in control, but a single motor is now being used on account of space limitations, saving in weight and better electrical efficiency.

The characteristic curves for a typical motor with the 60-volt and 80-volt windings are given in Figs. 1*a* and 1*b*. The relatively small change in efficiency for overload with the field coils in parallel is noteworthy. The "torque ratio" of this motor, i.e., the ratio of torque at 2 1/2 times rated current to torque at rated current, is approximately 5 with both windings.

Before recommending motor equipment for automobile service the motor manufacturers usually require that the purchaser submit data upon the weight, speed, battery, wheel dimension, etc., for the vehicle on which the motor is to be used.

SIZE OF MOTOR REQUIRED. In selecting the size of motor the voltage should be chosen on the basis of 1.9 volts per cell for lead batteries and 1.0 volt per cell for Edison batteries. The average discharge voltages of both types of batteries at their normal rates are higher than these values, but it has been found desirable to install motors on the basis of these values so that at high rates of discharge the terminal voltage may not fall too far below the rated motor voltage.

The current rating of the motor (and also of the battery) should be approximately the value of the current required to drive the car at the rated speed on hard level asphalt. The current input into a vehicle motor when driving a car at constant speed is given by the expression

$$I = \frac{2vWr}{E\epsilon}$$

where v = speed of car in miles per hour.
W = total weight of car and load in tons.
r = car resistance in pounds per ton of total car weight.
E = motor terminal voltage.
ϵ = overall efficiency of motor, controller, and transmission.

The value of ϵ at the rated speed is the product of the efficiencies of motor and transmission, usual values falling between 60 and 75 per cent. For value of r see next paragraph.

Car Resistance. The car resistance depends upon the following conditions: speed, grade, diameter of wheels, load per square inch of tire contact or per inch of tire width, construction and composition of tire, and road surface. The effect of speed is to change r in a manner dependent upon the type of tire, the value of r having a minimum usually at a speed between 6 and 10 mph; air resistance varies approximately as the square of the speed and need not be considered for speeds of less than 20 mph. The effect of up-grades is to increase r by 20 lb per ton for each per cent grade. The car resistance with pneumatic tires on hard, level asphalt is from 15 lb per ton for special tires to 35 lb per ton for the standard type of tire used on gasoline cars; the resistance is greatly affected by the air pressure, increasing rapidly as the air pressure is reduced.

Effect of Road Surface. Values of relative car resistance on various level road surfaces as compared to the value on hard, level asphalt are given in the Table II.

Table II. Relative Values of Car Resistance

Road Surface	Ratio	Road Surface	Ratio
Asphalt, hard	1.0	Snow, packed	1.3
Wood block	1.1	Snow, fairly hard, without chains	1.7
Macadam	1.15	Snow, fairly hard, with chains	1.9
Granite block	2.0	Snow, soft, about 3 in. deep	2.1
Dirt road	1.3	Sand	20.0
Brick	1.4		

27. TRANSMISSION SYSTEMS

The transmission systems between motor and wheel which are now in common use are gear drives. A single-speed reduction is used in a few makes of cars, but a double reduction is used on a majority of machines.

SHAFT OR GEAR DRIVE. With shaft drive the motor is usually mounted lengthwise of the car, and either a single- or double-speed reduction is used. For single reduction both bevel and worm gears are used as the connection between propeller shaft and differential in the rear axle. For double reduction the motor is usually connected to the pro-

peller shaft by spur gear, herringbone gear or silent chain, and the propeller shaft is connected to the differential by a bevel gear.

WORM DRIVE. There has recently been a decided tendency among designers of trucks to use the single-reduction worm gear instead of the double-reduction bevel or chain arrangements, the advantages claimed for the worm being better transmission efficiency and less trouble from adjustment.

WALKER BALANCED GEAR. In the trucks built by the Walker Vehicle Co. the motor is built into the rear axle, which is made hollow and of a sufficient diameter to contain the motor. The armature shaft is also hollow, so that the drive shafts may extend through from the sockets of the differential to the center of each rear wheel. The wheels are driven by a spur-gear reduction mounted inside the wheels. The wheels have steel-plate sides instead of spokes, so that the gears may run in oil and are thoroughly protected from dirt.

28. BATTERIES

There are three types of batteries in general use in electric vehicles, namely, the pasted lead, the "Iron-clad" lead, and the Edison. The performance characteristic (except life) of the first two are identical and are described in Section 7, Art. 7; the characteristics of the Edison battery are described in Section 7, Art. 13.

Number of Cells; Number of Plates. Practice among electric automobile manufacturers is now practically standardized upon the installation of 42 cells of lead battery and 60 to 72 cells of Edison battery. There are, however, in use old designs of cars in which from 24 to 38 lead cells are used, and the small-capacity delivery wagons in which 48 Edison cells are also employed. A 24-volt to 48-volt battery is used on industrial trucks.

Table III. Useful Data on the Edison Storage Battery

Cell Type	Normal Discharge Rate, amp		Rated Capacity	
	3 1/2 Hour	5 Hour	Ampere-hours	Watthours
A4	.....	30.0	150.0	180
A5	.....	37.5	187.5	225
A6	.....	45.0	225.0	270
A7	.....	52.5	262.5	315
A8	.....	60.0	300.0	360
A10	.....	75.0	375.0	450
A12	.....	90.0	450.0	540
A14	.....	105.0	525.0	630
A16	.....	120.0	600.0	720
C6	.....	67.5	337.5	405
C7	.....	74.7	393.7	473
C8	.....	90.0	450.0	540
C10	.....	112.5	562.5	675
C12	.....	135.0	765.0	810
G7	52.5	.....	175.0	210
G9	67.5	.....	225.0	270
G11	82.5	.....	275.0	330
G14	105.0	.....	350.0	420
G18	135.0	.....	450.0	540
Average voltage	1.2	1.2		

1. Type C, new in 1930, same area as A, but 5 in. higher. Capacity of C is therefore 50 per cent greater than A. Thus: A6—225 amp-hr, C6—337 1/2. Or A12—450 amp-hr, C12—675.

2. Trucks for city streets usually have 60, 64, 66, 70 or 72 cells. For high-speed trucks 80 cells are employed. Types A and C are used for ordinary service; type G is applied in special cases where greater capacity is required.

29. CONTROLLERS

Controllers are usually of the drum type with two running positions, known as the "field-series" and the "field-parallel" positions. The motor field coils are arranged in two groups for connection either in series or in parallel. The cells of the battery are connected permanently in series. In starting, either two or three resistance notches are used before the "field-series" notch is reached. A shunt on the fields is frequently used between the "field-series" and the "field-parallel" notches. For normal full-speed running the two groups of field coils are in parallel. An emergency speed is obtained by shunt-

Table IV. Useful Data on the Exide-Ironclad Battery

Number of plates	7	9	11	13	15	17	19	21	23	25	27
Ampere-hours' service capacity (over 6 hr)	102	136	170	204	238	272	306	340	374	408	442
Kilowatthours' service capacity (over 6 hr) at 1.98 volts per cell	0.202	0.269	0.337	0.404	0.471	0.539	0.606	0.674	0.741	0.808	0.876
Kilowatthours' service capacity (over 6 hr) for 42 cells at 1.98 volts per cell	8 48	11.31	14.14	16.96	19.79	22.62	25.45	28.27	31.10	33.93	36.76
Discharge in amperes for 6 hr at average voltage of 1.98 per cell	17	22.5	28.5	34	39.5	45.5	51	56 5	62.5	68	73.5
Charging rates — Start	19	24	30	35	40	45	51	56	61	66	72
Charging rates — Finish	8	10	12	14	16	18	20	22	24	26	28
Outside dimensions of cells, not including trays, in inches — Length	2 7/8	3 9/16	4 5/16	5 1/16	5 7/8	6 5/8	7 3/8	8 1/8	8 7/8	9 5/8	10 3/8
Outside dimensions of cells, not including trays, in inches — Width	6 3/16	6 3/16	6 3/16	6 3/16	6 1/4	6 1/4	6 1/4	6 1/4	6 1/4	6 1/4	6 1/4
Outside dimensions of cells, not including trays, in inches — Height, 2 3/4-in. ribs	15	15	15	15	15 1/16	15 1/16	15 1/16	15 1/16	15 1/16	15 1/16	15 1/16
Weight of electrolyte	4.6	5.9	7.3	8.6	9.9	11.3	12.6	13.9	15 3	16.6	17.9
Weight of complete cell	24	30 1/2	36 3/4	43 1/4	51 3/4	58	64 1/2	71	77 1/2	83 3/4	90 3/4
Approximate weight per cell, including tray	28	35	42	50	59	67	74	82	89	97	104

	Volts per Cell			Volts 42 Cells		
	Initial	Average	Final	Initial	Average	Final
At 6-hr rate	2.06+	1.98+	1.76+	86.89	83.21	74.08
At 3 times 6-hr rate	1.99+	1.90+	1.69+	83.87	80.18	71.06
At 5 times 6-hr rate	1.92+	1.83+	1.61+	80.84	77.16	68.03

Height given is from bottom of jar to top of intercell connector, except where vertical diagonal connector is used, when height should be increased 7/8 in.

ing a resistance around the fields when connected in parallel. It was formerly customary to split the battery into two groups which could be operated in series-parallel in order to obtain four economical running speeds, but trouble was experienced from the unequal discharge of the two halves of the battery. In the magnetic type of control which is used to some extent the changes in connection are affected by relays operated by an auxiliary electric circuit. Mechanical cam and spring type contactor controllers are also in successful use, avoiding the use of solenoid circuits.

TIRES. Tires for use on electric automobiles are generally more resilient than the standard types of tires commonly used on gasoline automobiles. "High efficiency" or special electric tires, both solid and pneumatic, have been developed by most of the tire manufacturers in response to a demand by the operators of electric vehicles for tires with low consumption of energy. Standard types of tires may consume as much as 100 per cent more energy than the special electric tires. Experience has shown that the smooth starting characteristic of an electric motor produces much less abrasion than the uneven acceleration with a gasoline motor, so that the rubber compounds in tires for electric cars are usually softer than the compounds in tires for gasoline cars and yet the lives of both types, expressed as distances traveled, are approximately the same.

30. PERFORMANCE; ENERGY CONSUMPTION

The energy consumed by an electric automobile will depend very largely upon the care of the driver in coasting up to stops instead of braking directly from full speed; other important factors in determining the energy consumption are efficiencies of battery and driving mechanism, tire equipment, nature of roads and grades, number of stops and miles per day, etc. However, for a car which is operated under similar conditions day after day, the energy consumption per mile should be reasonably uniform, so that any considerable increase in energy consumption may indicate either careless driving or poorly adjusted mechanism, as for instance a dragging brake.

31. OPERATION

In the operation of electric automobiles particular attention should be paid to the proper care of the storage batteries. The battery manufacturing companies issue complete instructions for the proper handling of their cells. The manufacturers of the battery, as well as the manufacturers of the vehicles, are usually ready to confer with the operator to

the end that he may obtain satisfactory service from his machines. They should be consulted on the first indication of trouble in their respective portions of the equipment, should the cause of the trouble not be understood.

The following points upon the care and operation of electric vehicles should be observed:

1. A battery must always be charged with direct current and in the right direction.
2. Never bring an exposed flame near a battery while charging or immediately afterwards.
3. Do not allow the battery temperature to exceed 110 deg fahr.
4. Keep the cells filled to the proper level by adding distilled water only. Never, under any circumstances, put acid in an Edison battery.
5. Keep the outside of the cells free from foreign substances, both solid and liquid.
6. For boosting a lead battery during a specified short period, the maximum current rate I which may be used without reaching the gassing point is the quotient of the ampere-hours Q previously discharged (read from ampere-hour meter), divided by 1 plus the hours H available for boosting, viz., $I = \frac{Q}{1 + H}$.
7. The mechanism of a car should be inspected carefully at least once in two weeks.
8. A car should be entirely overhauled at least once each year, in order that worn parts may be located and replaced.

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ELECTRIC PROPULSION OF SHIPS

By W. N. Zippler

33. FUNDAMENTAL PROPULSION REQUIREMENTS

Practically all vessels are now propelled by " screw " type propellers, the side wheel and stern wheel having generally passed out of use except for vessels operating in very shallow waters, as for instance in some parts of the Mississippi River. With the existing propeller designs, the best efficiencies are obtained at relatively low rpm. Propellers seldom exceed a speed of 400 rpm and are generally operated between 100 and 200 rpm. The propellers are usually driven by either reciprocating steam engines, steam turbines, or internal-combustion engines.

A ship may be equipped with one, two, three, or four propellers. The depth and speed of the vessel are, in general, the principal factors in determining the number of propellers. As the speed is increased, the power requirement increases very rapidly, and in order not to obtain a poor propeller efficiency, the area of the propeller must be increased with increased power. The limit of area per propeller is determined by the diameter that can be suitably accommodated with the external dimensions of the vessel. It is advantageous to have the propeller submerged at all conditions of draft and yet not project below the keel of the vessel to interfere with " dry-docking " or shallow-water operation—therefore, the necessity for more than one propeller. Maneuverability of a vessel is improved by the use of two propellers instead of one, owing to the " turning " force that can be obtained by operating one propeller in the ahead direction and the other astern. No accurate data have been obtained indicating there is a gain in four propellers over two in this respect.

PROPELLER CHARACTERISTICS. Fig. 1 shows the relation between the speed and torque of a propeller when the ship is being driven at constant speeds in smooth water and also when the ship is held stationary in the water. It will be noted that the running propeller torque increases approximately as the square of the speed. The power to drive a ship, therefore, increases approximately as the cube of the speed. The exact values, however, vary with each type of propeller design.

TORQUE REQUIRED TO STOP AND REVERSE. When the driving power is removed from a propeller with the ship traveling at a high speed, the propeller will continue to revolve in the same direction, being driven by the water. In order to stop the ship in the shortest time, or to maneuver quickly, it is necessary not only to stop the propeller against the action of the water but also to drive it in the reverse direction.

An experimental determination of the relation between the speed and torque of the propeller while being stopped and reversed with the ship traveling at constant speeds was made in the Model Tank at Washington, and later these data were found to agree approximately with tests taken on the U.S.S. *Jupiter* and the U.S.S. *New Mexico*. Fig. 2 shows such a propeller characteristic as obtained in the Model Tank.

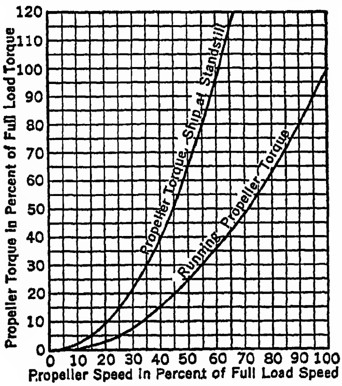

Fig. 1

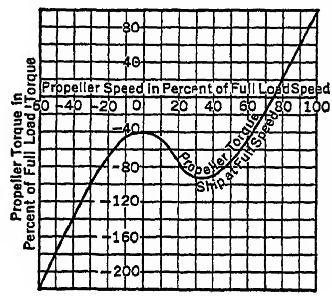

Fig. 2

Directly the power is removed, the propeller begins to slow down to approximately 73 per cent of its original speed, and, if power is now applied to the shaft, tending to stop the propeller, the torque of the propeller mounts up to approximately 100 per cent of the forward driving torque. This maximum torque is attained at between 35 and 40 per cent of normal propeller speed. From this point on, the torque decreases as the propeller speed decreases, until it comes to rest, when approximately 40 per cent torque is required to hold it stationary. If the propeller is now reversed, with the ship proceeding ahead at full speed, 100 per cent torque is attained at a propeller speed of about 30 per cent of the full-ahead revolutions. From this curve, therefore, it will be noted that it takes approximately 100 per cent torque to break the propeller away from the water, but only about 40 per cent torque to hold the propeller stationary after being stopped.

From experience obtained to date, it would seem that machinery for high-speed ships, having two or more propellers, should give 100 per cent reversing torque to bring the propellers quickly to rest when reversing from full speed. On lower-speed ships, indications are that the maximum stopping torque is considerably less than 100 per cent. On single-screw cargo boats, the stopping torque is probably quite low.

VARIATION OF PROPELLER TORQUE WITH PITCHING OF SHIP. The General Electric Co. has made a number of elaborate tests to determine the variation of the propeller torque when the ship is in a sea-way. An illustration of the variation which may be expected in moderately rough sea is shown on Fig. 3. Although the ship was only pitching 4 deg, the increase in torque varied from no load to 175 per cent of normal load.

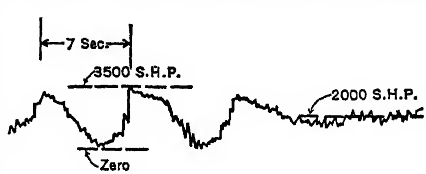

Fig. 3

These results were obtained on a cargo ship equipped with double reduction gearing. This turbine equipment did not have a governor, and the variation in torque may, in part, be due to the variation in speed of the propelling machinery.

Under worse conditions of sea, it may be expected that this variation in torque would be considerably increased.

It was observed that, when the propeller approached the surface of the water, the load was quickly released and was again gradually increased as the propeller went deeper

into the water. The record was taken with the ship in ballast, and it was observed that the propeller did not break water. This variable condition of torque has been confirmed on electrically driven ships by observing the variation of load by electrical instruments.

The electric motor is an ideal piece of machinery for absorbing these shocks without deterioration. As there is no mechanical connection between the turbine and the propeller, this shock is absorbed in the air gap of the motor, it being necessary only to give the electrical machinery a sufficiently strong magnetic bond to hold the generator and motor together. This bond is increased by simply increasing the degree of excitation. If the motors drop out of step with the generators, then, to bring them in step again, it is necessary only to reduce the turbine speed and increase the excitation. Then the turbine speed can again be increased to normal. No harm is done if these motors are left out of step for short periods as the generator is so proportioned that the increase in current cannot greatly exceed the normal.

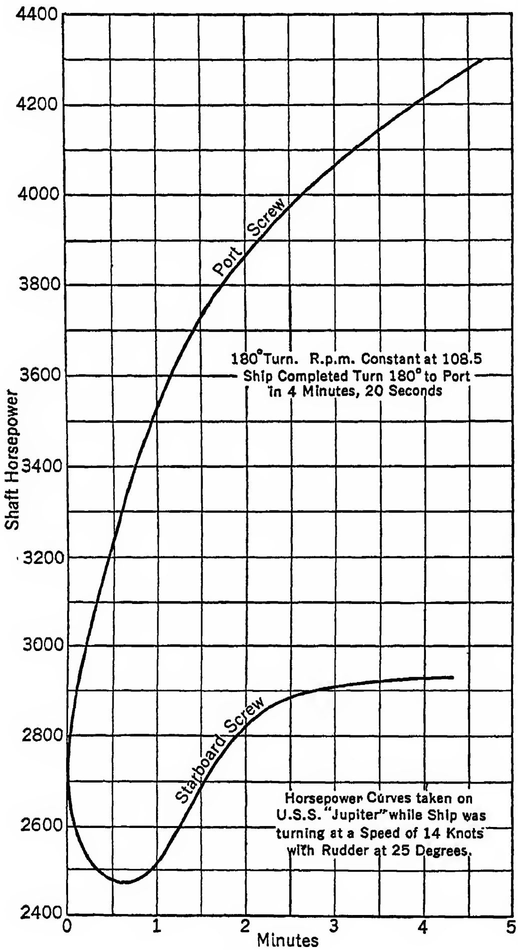

Fig. 4

Turning. Another condition of maneuvering that affects the design of electric propelling machinery is "turning." Most marine engines are not fitted with speed governors, the control of speed being entirely dependent upon the load and position of the throttle. With constant throttle opening, the reciprocating engine or turbine will develop practically constant torque regardless of the load on the propellers. Thus without a governor, the power varies directly with the revolutions. In the electric drive, the turbine is fitted with a speed governor which will hold the speed fairly constant, regardless of the load.

Commander S. M. Robinson, in his book, Electric Ship Propulsion, states:

"When turning a vessel, which is driven by a turbine, equipped with a constant speed governor, the power in the screw on the outboard side of the turning circle drops, and that on the screw on the inboard side of the turning circle rises. After a short time, the power on the outboard screw also rises. Fig. 4 shows how, in the case of the electric ship *Jupiter*, the power first drops on the outboard screw, reaching a minimum when the ship has turned through about 20 degrees; it then begins to rise, reaching its original value when the ship has turned through about 40 degrees, and reaching a constant value when the ship has turned through about 140 degrees. The power rises on the inboard screw from the beginning to the end of the turn. Thus in the case of the *Jupiter*, turning at 14 knots with the rudder at 25 degrees, at the completion of the turn, the increase of power on the outboard screw was 4.6 per cent and on the inboard screw 53 per cent, making a total increase of power of 29 per cent."

Although these figures vary with vessels of different size and speed, the data at least give a rough indication of what may be expected in all prime movers equipped with constant-speed governors.

It is for this reason that means are provided on governed turbines to limit the steam flow, thus producing a decrease in speed when the power exceeds a predetermined point.

34. STEAM TURBINE DRIVE

The relative light weight, small size, and high steam economy of high-speed steam turbines, compared with reciprocating engines, naturally suggest the use of such turbines for the propulsion of ships. Since the steam turbine has its best efficiency at a high speed and the propeller the best efficiency at a low speed, some means of speed reduction must be provided to obtain the best overall efficiency. At present, two means of reduction are available, namely: mechanical reduction gears and "electric gearing." Mechanical gearing, as used on ships, generally consists of a single or double set of spur, herringbone, or spiral-toothed gears, The "electric gearing" consists of a high-speed generator directly connected to the turbine and electrically connected to a low-speed motor, which is direct-connected to the propeller. By these two methods, speed reductions as high as approximately 40 to 1 may be obtained. Ships equipped with "electric gearing" are commonly referred to as being electrically propelled. There are many advantages and disadvantages for each type of reduction gearing.

The turbine has also permitted the use of much higher steam pressures and steam temperatures than had ever been considered for reciprocating engines. Some of the recently contemplated high-speed light-weight vessels are designed for steam temperatures as high as 850 deg fahr and 600 lb per sq in. gage pressure. Most merchant vessels of the present date operate with pressures of 275 to 300 lb per sq. in. gage and superheat temperatures of approximately 150 to 200 deg fahr. As is the case with the land steam plant, in recent years much more consideration has been given to increased pressures and temperatures; the increase, however, has been much slower in the marine plant than in the stationary plant. The German merchant marine boasts of an experimental cargo vessel that operates with a pressure of 1200 lb per sq in. gage, the boilers being of the "Benson flash type.".

Slow-speed turbines, which in comparison with high-speed machines are relatively inefficient, have been used for a number of years for driving ships. These turbines are directly connected to their propellers. In some large ships, a combination of reciprocating steam engines and a low-pressure turbine have been installed.

When the turbine is connected to the propeller by mechanical reduction gears, the drive is commonly called "geared turbine drive." Although as previously stated it is possible to obtain a turbine to propeller gear ratio as high as 40 to 1 by the use of a double set of gears, the ratio commonly used for passenger and cargo vessels seldom exceeds 25.

As is well known, superheat greatly improves the efficiency of steam turbines by reducing the friction and windage loss of turbine blades and disks. There is also a considerable gain due to increase in the available energy of the steam incident to increase in temperature. Steam consumption is reduced at least 1 per cent for every 12.5 deg fahr of superheat. With some installations, there is also a gain in boiler efficiency caused by increased heating surface exposed to the flue gases. Superheated steam also increases the life of the turbine blades and nozzles.

In America, superheat has, for a number of years, been extensively used in central stations, but only during the last few years has it come into extensive use on American merchant steamships.

TURBINES VERSUS RECIPROCATING STEAM ENGINES. Aside from questions of weight and cost, the higher steam economy of high-speed turbines is of itself ample in most cases to justify the adoption of the turbine for ship propulsion. The steam engine of the triple or quadruple expansion type, owing to the limitations in size of the low-pressure cylinder, cannot be made having a steam expansion ratio greater than about 16 to 1 or 20 to 1.

The reciprocating engine also cannot be used with highly superheated steam, on account of the difficulties in properly lubricating the cylinder walls and in obtaining a type of design and materials that can withstand the large stresses produced in the cylinder walls by the high pressures and temperatures.

The steam turbine has no such limitation and is capable of using, efficiently, steam expanded to any practicable extent. Where turbines are used, a vacuum of 28.5 in. is quite common, and 29.5 in. is being recorded on some vessels during winter months. The economy due to high pressure, high superheat, and large expansions is indicated by a comparison of the available energy of a pound of steam when expanded from boiler pressure to various degrees of vacuum, viz.:

200 lb pressure to 24-in. vacuum = 220,000 ft-lb
200 " " " 26-in. vacuum = 238,000 " "
200 " " " 28-in. vacuum = 265,000 " "
200 " " " 29-in. vacuum = 289,000 " "
600 " " 486 deg fahr (saturated) to 29-in. vaccum = 334,000 ft-lb
600 " " 850 deg fahr to 29-in. vacuum = 428,000 ft-lb

These figures give an indication of the saving that can be expected by the turbine which can use, efficiently, steam expanded to a low degree. Little advantage, however, is gained with a reciprocating engine when the expansion is carried below 24-in. vacuum. This greater available energy per pound means a saving not only of fuel, but also in the size and weight of boilers, auxiliaries, etc. Because of the above facts, the reciprocating engine has gradually passed into obsolesence.

LOSS DUE TO INCORRECT TURBINE AND PROPELLER SPEEDS. In many turbine-geared equipments, it has been considered advisable to depart from the most desirable propeller and turbine speeds so as to reduce the speed ratio of turbine and propeller and thus obtain a simple gearing. This sacrifice of turbine and propeller efficiency in some cases has caused an increase of fuel consumption amounting to 25 to 30 per cent. With electric propulsion a speed ratio as high as 40 to 1 can be used, the exact ratio being determined by the diameter of the motor that can be placed in the vessel, the output of the turbine, and the minimum number of poles that can be provided in the generator.

REVERSING. The propelling machinery of a ship must, of course, be capable of backing the ship as well as driving it forward. With reciprocating steam engines, there is no difficulty in this respect. A steam turbine, however, cannot be reversed. Consequently, when a ship is equipped with turbines directly connected to the propeller shafts, or connected thereto through mechanical gearing, each shaft must be equipped not only with a forward-driving turbine, but also with a turbine whose normal direction of rotation is opposite to that of the forward-driving turbine. Such turbines are called "reversing turbines" or "astern turbines." When the ship is driven forward the reversing turbine is driven backward, no steam being admitted to it. To reverse the direction of motion of the ship, the steam supply is cut off from the main turbine and admitted to the reversing turbine, the main turbine then being driven backward.

With electric gearing, reversing turbines are, of course, unnecessary, since the direction of rotation of the motors connected to the propeller shafts can be reversed merely by operating suitable switches, and the torque astern is equal to the torque ahead. This is a decided advantage of electric propulsion over direct-connected or mechanically geared turbines.

Losses in Reversing Turbines. Experiment has shown that a turbine forced in a direction opposite to its normal direction of rotation has about 10 times as much friction loss as when driven in its normal direction. This friction loss in a reversing turbine, which is always present when the ship is in motion, may amount to 1 per cent or more of the rating of the ahead turbine. This value varies with the rating and number of stages in the astern turbines.

TWO EFFICIENT SPEEDS. The propelling machinery of a war vessel should be capable of driving the ship efficiently not only at full speed but also at cruising speeds, which are usually from 40 to 70 per cent of full speed. The same is also desirable in certain passenger ships. As the power required to propel a ship varies approximately as the cube of the speed, this means that, at cruising speeds, the propelling machinery will be running at less than one-third of its maximum load. Since a steam turbine is most efficient at or near maximum load and at full speed, it follows that, when directly connected or mechanically geared turbines are employed, there is a sacrifice in efficiency at cruising speed, which is the speed at which a warship is driven by far the greater part of the time.

Cruising Turbines. To avoid this difficulty, it is possible to provide, in addition to the main turbines, auxiliary high-speed turbines geared to the propeller shafts, the speed reduction being such as to give the desired cruising speed. Provision is generally made for disconnecting the auxiliary turbines when the ship is driven at full speed by the main turbines. These auxiliary turbines are usually referred to as "cruising turbines." In all modern naval and merchant vessels, where long periods of operation at reduced power are contemplated, cruising turbines are generally provided.

With electric propulsion two efficient speeds are readily obtained without any such complication. All that is necessary is to change the ratio of the number of poles of the motors to the number of poles of the generators, which is readily accomplished by a simple switching operation. Since the propeller torque required at half speed is only about one-quarter that required at full speed, the voltage applied to the motors at cruising speed can also be reduced, thus keeping the motors operating at relatively high efficiency at reduced speed. U.S.S. *New Mexico,* which was the first electrically propelled battleship, being put

into commission in 1918, and all subsequent electrically propelled battleships and airplane carriers are arranged for pole changing. With reference to the *New Mexico*, Commander S. M. Robinson has stated: " In conclusion, it may be said that the performance of the *New Mexico* since commissioning has been entirely satisfactory in every way and that the expectations of those who were responsible for its installation have been more than realized."

GEAR LUBRICATION. In order to lubricate and carry away the heat generated in the reduction gears, large volumes of lubricating oil must be continually circulated through the gear case. This requires storage tanks, piping, oil pumps, and oil coolers. Sea water is used for cooling the oil. A storage tank is usually located above the machinery so as to insure an oil flow for several minutes upon failure of pump, thus allowing sufficient time to stop the gears before damage.

With turbine electric drive, a much smaller volume of lubricating oil is required because it is used only for lubricating the three or four turbine generator bearings and the two motor bearings.

35. TURBINE ELECTRIC DRIVE

Two Chicago fireboats, *Graeme Stewart* and *Joseph Medill* were equipped with electric propulsion in 1908. The first large ship equipped with electric propulsion for the U. S. Navy was the cutter *Jupiter*, which was placed in service in 1913. The latest example is the French Liner *Normandie*, which was completed in May, 1935, and is equipped with four turbine generating sets and four synchronous motors having a total output of 160,000 shaft horsepower.

The essential elements required for electric transmission between the turbines and the propellers of a ship are the generators, the motors, the cables connecting these two, and the switching and control devices.

ALTERNATING VERSUS DIRECT CURRENT. Where generators are driven by turbines, a-c apparatus is usually adopted for ship propulsion in preference to direct current. There are, however, a few cases where direct current is used when small powers are involved, and also when Diesel engines are used as prime movers.

Alternating current has many advantages over direct current, the chief of which may be stated as follows:

1. The combined transmission losses of the generator and motor, when a-c apparatus is used, is from 5 to 8 per cent less than with d-c apparatus.

2. High-speed prime movers and generators can be used with alternating current, whereas with direct current, when a high-speed steam turbine is used, it is necessary to have a reduction gear between this and the low-speed d-c generator. High-speed apparatus is cheaper and lighter.

3. A-c apparatus is, generally speaking, more reliable than d-c. As the distance between the generator and motor on shipboard is short, there is no advantage in using high-voltage transmission, as is the case on land. Lower voltages, therefore, are usually adopted. The range of potential is from 1100 to 6000 volts. The General Electric Co.'s records, over long periods of years and covering such electrical apparatus as is used on ships, show that only about $^1/_{10}$ of 1 per cent of machines of such voltage give any electrical trouble in a period of ten years. This electrical apparatus includes all motors and generators installed in all kinds of service.

TYPES OF MOTORS SUITABLE FOR ELECTRIC DRIVE. Five types of a-c motors have been used for ship propulsion:

1. Induction motors equipped with slip rings and external resistors.

2. Induction motors without slip rings, but with two squirrel-cage rotor windings. A high-resistance rotor winding is placed in the slot near the periphery of the rotor and beneath this a low-resistance winding. During normal running, when the current alternates slowly in the rotor, the low-resistance winding carries the current. When, however, there is a high-frequency current flowing in the rotor, as during reversals of the propeller, the self-induction of the low-resistance winding forces current into the high-resistance winding, and this produces a high torque.

3. Induction motors without slip rings but with one squirrel-cage rotor winding, consisting of deep, narrow bars. The action of this motor during reversal is similar to that of the double squirrel-cage motor.

4. Induction motors equipped with slip rings but no external resistors. The rotor is wound with a low-resistance winding near the periphery of the rotor, and beneath this a high-resistance squirrel-cage winding. The low-resistance winding is connected to the slip rings. To make the high-resistance winding carry the current during reversal of propeller, the low-resistance rotor winding is opened by means of contactor switches connected across

the slip rings. If this motor is wound to give two different speeds the slip rings can be made neutral for high-speed running and thus need not carry heavy current. All reversals are accomplished with low-speed connection.

5. Synchronous motors, the periphery of the poles being equipped with a low resistance bar winding. With this type of motor the propeller is brought to rest during reversal by reversing the phases with the circuit " dead " and then applying a strong field on the motor, leaving the field off the generator. The propeller thus drives the synchronous motor as a generator, being loaded on the turbine generator. Sufficient torque is developed to bring the propeller approximately to rest. Field is now applied to the main generator and removed from the motor. If the turbine governor has previously been set for a slow speed, the motor will now reverse and quickly reach approximate synchronous speed, at which time the field is again supplied. A small and simple air-cooled resistor can be switched in multiple with the generator during the stopping period. If this is done a higher torque can be developed with less excitation and with lower current flowing between the machines.

The following advantages are claimed for the synchronous motor over the induction motor:

1. A higher transmission efficiency due to operating at unity power factor.
2. Larger air gaps can be provided.
3. The generator and motor are lighter and cheaper.

The induction motor is better if two different numbers of poles are desired for running at different speeds, as in warships.

TYPES OF TURBINE GENERATORS SUITABLE FOR ELECTRIC DRIVE. The a-c propulsion turbine generator is of the conventional rotating-field central-station type, except as follows:

(*a*) It is designed for continuous operation at practically any speed from approximately 25 to 100 per cent of rating. In order to accomplish this requirement satisfactorily, it is necessary to design the turbine generator rotating parts so that there will be no serious critical speeds within the operating range. In the central-station unit, owing to operation at only one speed, it is only necessary to eliminate " criticals " in a narrow speed band.

(*b*) The field windings must withstand the large current required to prevent collapse of the generator voltage when starting the propulsion motor with the turbine operating at approximately 25 per cent speed. As soon as the motor locks into synchronism with the generator, the field current is reduced to normal. Approximately double normal excitation is used during this starting period.

(*c*) The governor must be designed to maintain approximate constant speed at any setting from 26 to 100 per cent. Since the a-c generators are not operated in parallel, it is not essential to have the turbine govern within the limits required for central-station equipment.

(*d*) It is also generally necessary, in a twin or quadruple propeller ship, to provide a steam limit device on the turbine to prevent overloading the electric equipment to the " pull-out point " when the ship is turned at full power. The increased power requirements are discussed in the paragraph on Turning. The steam limit simply limits the steam flow to the unit so that, when the overload occurs, the propeller and the turbine generator connected thereto will slow down, thus maintaining approximately constant torque.

The following are considered desirable turbine speeds (for the impulse type of turbine) for the corresponding generator output:

Horsepower	Rpm	Poles of Generator	Cycles per Second
7,000	3600	2 poles	60
10,000	3000	2 poles	50
16,000	2400	2 poles	40
29,000	1800	4 poles	60
65,000	1200	4 poles	40

CONTROL EQUIPMENT. The control equipment is usually placed in a wire mesh enclosed cell with a steel or aluminum panel front. All the operating equipment and meters are located on the front of the panel. The switches, rheostats, current transformers, potential transformers, and other high-voltage parts are enclosed in the control cell. The entrance door to this cell is equipped with an interlock which prevents application of excitation to the generators when the door is open.

All switches are of the air-break type, thus eliminating the fire hazard encountered with the oil-immersed type.

The connections are arranged so that the a-c generators cannot be operated in series or parallel, but either one, two, or four motors, depending on the number installed, can be

operated from one generator. When two or four motors are operated from one generator, all motors must operate at the same speed, in the same or opposite directions, due to the speed being determined by the setting of the turbine. The equipment in a four-propeller vessel is connected so that the two port motors are operated at the same speed and direction from a single control lever, and similarly for the starboard units. In all four-propeller vessels, it is customary to operate the port propellers as one unit and the starboard propellers as one unit, with provisions for operating the port and starboard units singly or from a single control lever.

When ready for operation, the turbine generator is started up by opening the hand throttle. All electrical circuits are open, and the turbine speed lever on the control board is at the minimum setting. The minimum speed setting is usually about 25 per cent of rated speed. When the throttle is wide open the governor takes charge and maintains approximately constant speed regardless of load.

After the " set-up " has been made, i.e., the selection of the motors and generators that are to be operated, the motor is started by operating the field control switch. The first step of this control closes the generator field circuit, applying approximately double normal current to the generator field windings. This condition is maintained until the motor or motors reach approximately constant speed; the field control lever is then moved to the next position, which applies current to the field of the synchronous motor. (This step is eliminated when induction motors are used.) As soon as the motor locks into synchronism, the load current decreases and the field control lever is moved to the next position, which reduces the generator field excitation to normal.

From this point on, the speed adjustment is made by operating the speed control lever which adjusts the governor setting of the turbine, the motor speed following the turbine speed.

The speed control lever and the field control lever are interlocked, so that the turbine speed cannot be increased until the field control lever is in the run position and the field cannot be removed until the speed control lever is in the minimum speed condition.

The motor reversing switches are also interlocked, so that they cannot be operated until turbine speed lever is in the minimum speed position and the field control lever is in the off position.

The operation of the overload device opens the generator field circuit, and, therefore, under no condition of operation is it possible to open or close the main circuit switch without first removing excitation from the generator fields.

VENTILATION OF ELECTRICAL APPARATUS. It is very important to ventilate the electrical apparatus in such a way that it will not collect moisture when shut down or running. To prevent this, the temperature of the apparatus must not be lower than that of the surrounding air. In order to prevent spray and water from being carried to the equipment, the ventilating air should be taken from the same room in which the apparatus is placed, except in Diesel engine rooms where other conditions affect the situation. Suitable dampers can be provided to prevent the engine room from becoming uncomfortably cold during the winter weather by recirculating some of the air.

In most recently constructed electric drive vessels, the closed system of ventilation is used, in which cases the air after leaving the generators and motors is passed around finned cooling tubes through which sea water is circulated. After passing the cooler, the air is returned to the generators or motors. Motor-driven fans are usually required for circulating the air in the motor cooling system, owing to the large size and low speed of the motor. Thus the same air is used continually. This method eliminates the large ducts leading from the upper decks of the ship and also insures the use of clean dry air at all times. These coolers can be made quite small for this purpose.

In order to keep the temperature of the internal parts of the motors and generators above the surrounding air during shutdown, steam or electric heaters are generally provided adjacent to the windings.

36. ADVANTAGES OF ELECTRIC PROPULSION

Some of the advantages of electric propulsion have already been noted. This subject has been very fully treated by Mr. W. L. R. Emmet in numerous papers (see Bibliography), and the following parts in quotation have been taken from his papers.

RELATIVE WEIGHTS OF ELECTRIC DRIVE AND GEAR DRIVE. Under some conditions, when properly designed, the weight of the machinery required for electric drive is but little in excess of the weight required for single reduction gear drive. Generally, the full theoretical advantage of the electric drive cannot be realized, because the dimensions of most vessels at the stern are insufficient to accommodate the large motor diameter required by a large speed ratio without excessive slope of the propeller shaft.

EFFICIENCY OF ELECTRIC TRANSMISSION. The greatest advantage for the electric transmission, from an efficiency standpoint, is in the higher powers, because the efficiency of motors and generators increases with increased ratings at a greater rate than with mechanical gears.

In the case of the 180,000 shaft horsepower airplane carriers, the overall motor and generator efficiency at full power, including excitation, power input to vent fans, control losses, etc., is approximately 92.7 per cent. A reputable manufacturer of large-powered reduction gears guarantees an efficiency of approximately 97.5 per cent for a set of double reduction gears having approximately the same ratio as provided for the electric transmission. From test data taken on turbines operating in the reverse direction, we may expect a loss of approximately 1.2 per cent in the reversing turbine when steaming ahead. This leaves a net difference in efficiency of 3.6 per cent in favor of the geared drive at full power for this particular installation.

When operating at reduced power and speed the reduction in efficiency of the electric drive shows a smaller decrease than the gear drive, owing to inherent characteristics of electric machinery in having a nearly uniform efficiency within certain limits of voltage reduction. In central-station generating equipment, the efficiency at reduced load shows a greater reduction because the voltage and speed are maintained at a constant value. In addition to this advantage, the electric drive permits the operation of a lesser number of turbines at the reduced power by virtue of its ability to operate more than one propeller from the same turbine. In the equipment specifically mentioned above, the overall efficiency at approximately 30 per cent power is only $^1/_2$ per cent less than at full power. Since in the turbo-electric drive the turbine speed is reduced in the same proportions as in a geared drive, the efficiency of the turbine would be the same in both.

In the example cited above, the motors are of the induction type; if synchronous motors had been used, the difference in overall efficiency at full load would have been reduced to approximately 2.5 per cent in favor of the geared drive.

GAIN IN EFFICIENCY AT LOW SPEEDS BY OPERATING AT REDUCED VOLTAGE. Electric motors for shore purposes usually operate at approximately constant voltage regardless of the load carried. At light loads, therefore, they are relatively inefficient owing to the iron losses remaining approximately constant. Motors, however, used for propelling ships usually obtain their power from a generator whose voltage can be varied to suit the motor's load. At reduced speeds, therefore, in an electrically driven ship, the voltage can be reduced; this reduction has the effect of materially reducing the losses in the iron laminations of both motor and generator. Thus, at reduced loads, the percentage of power lost is nearly the same as at full load, whereas with gears, the transmission loss remains quite high at reduced speeds. Also the percentage loss of power in the gears increases when the gears have become worn, but the efficiency of electric machinery is not affected by use.

LOCATION OF PROPELLING MACHINERY. When vessels are equipped with turbine electric propulsion, the driving motors can be placed as far aft as convenient, thus shortening the propeller shafting. The turbine-generator units can be placed near the boilers at any desired level, reducing to a minimum the length of steam piping and other piping systems. The controlling mechanism, as in an electric locomotive, can be placed at any convenient position.

EASE OF DISCONNECTION AND REPAIR OF PROPELLING MACHINERY. The following is quoted from Mr. Emmet: "In a geared equipment, each shaft has a system of turbines, gears, bearings, thrust-balancing devices and lubricating systems all mechanically locked together. With high-speed machinery any kind of trouble with any of these parts will almost certainly necessitate the immediate stoppage of the whole system. To keep a high-speed turbine running out of balance or with bearing trouble is impossible, and the gearing part would present almost equal difficulty. In the event of mechanical trouble of such character, a ship would have to be stopped until the damage could be cleared. The work necessary to uncouple and disconnect any part of such very heavy apparatus would be a serious matter involving much time, including that required to stop the ship.

"If it was found impracticable to make this disconnection, and the damage was such that the shaft could not be allowed to revolve, it would be necessary to lock the shaft to hold the propeller. This locked propeller acts as a very serious resistance to a vessel traveling at even moderate speeds. In fact, such a dragging screw may add 20 per cent to the horsepower and would in addition, materially reduce the maneuvering qualities of the vessel.

"In the electrically driven ship there is no mechanical connection of the shaft to anything but the rotors of motors. These are self-contained, iron-clad structure and cannot by any possibility be subject to mechanical interference. The shafts are subject

to the same possibilities of bearing or thrust trouble as shafts in other ships, but the presence of the motors does not increase this danger and the speed being low it is remote in any case. With this equipment any motor, generator or turbine, if in any kind of trouble, can be instantly disconnected without stopping the ship and with only a small loss from the highest speed capacity. Such a disconnection is made by simple switches."

Referring specifically to 180,000-hp, 33-5-knot airplane carriers now in the service of the U. S. Navy, Mr. Emmet continues: "Such interchangeability as this power of disconnection gives, constitutes one of the most important advantages of electric drive in such a ship. With one motor out of eight in trouble, only one-eighth of the maximum capacity is lost and the ship's maximum speed is impaired by only about 1 knot. If a generator or turbine is in trouble, the maximum speed is reduced only about 2 knots. With two generating units and four motors out, the ship can make 25 knots, and with three generators and four motors out, she can make 19 knots. If parts give trouble, they are simply cut out and repaired at leisure or as opportunity offers. The value of this interchangeability in a warship can hardly be over-estimated. It largely overcomes the military danger incident to accidents to the driving power, a danger which has always been of primary importance."

MUD IN CONDENSERS. In addition to the foregoing, there is a great advantage when maneuvering an electrically propelled vessel in shallow or muddy water in and around harbors, provided the ship is equipped with more than one turbine and condenser. Under these conditions, only one turbine generator is needed to drive the ship, and in case the condenser of this unit is plugged with mud, it can be immediately switched off and another turbine switched on to the driving motors.

In cases where turbines are mechanically connected to their respective propellers, under similar conditions of operation, it would be necessary to use all the turbines together with their condensers. If such a ship ran into mud, all condensers might become plugged at once but, if only one became plugged, the maneuvering qualities of the ship would be dangerously impaired. Commander Robinson states: "As an actual experience the *New Mexico* while entering New York harbor had to shift main generators twice owing to the plugging of her condensers with mud, and these shifts were made so quickly that they did not affect the operation of the ship at all."

BACKING POWER. Again quoting Mr. Emmet: "In geared turbine-driven ships, it is necessary to provide backing turbines which must run idle in the reverse direction when the ship is going ahead. These backing turbines involve complications which are very objectionable, and if these are reduced to a desirable minimum, the backing power will be greatly reduced to that easily provided with electric drive. Experiment has shown that a turbine forced in an opposite direction involves about ten times as much friction loss as when driven in its normal direction. This loss, therefore, is very appreciable in the backing turbines of ships. There are also serious difficulties and dangers in high-speed apparatus incident to the abrupt and wide changes of temperature where steam is suddenly admitted to a cold reversing or ahead turbine. With electric drive, the turbine never need be stopped when the ship is underway."

CRUISING ECONOMY. With the electric design, the number of motors and turbines used can, to a certain extent, be adapted to the demand for power, while with the other types all parts must be kept running. This gives very important gain in economy at all speeds below the maximum. At 18 knots (in the 180,000 hp airplane carriers for the U. S. Navy), only one turbine is required to drive the ship, and electrical arrangements are made by which the turbine can be run at full speed instead of running at half speed, as it would if the ratio was fixed as by gearing. Thus, the steam efficiency of the turbine at 18 knots, a desirable cruising speed, is equal to the best attainable at any speed, and the overall efficiency from turbine input to power developed at propeller shaft is only slightly less than at full power. The use of a cruising turbine will increase the efficiency over that developed by the main turbine at reduced speed, but in order to equal the electric drive, the efficiency of the cruising turbine must be sufficiently better than the main turbine to overcome the reduced efficiency of the reduction gear and the losses in the idling main turbine. Cruising economy gives increased cruising radius without renewal of fuel supply. This has always been considered a matter of the greatest importance in warships.

FLEXIBILITY OF INSTALLATION. A military advantage of great importance in connection with electric propulsion is the "flexibility of installation." The following is quoted from Commander Robinson (*Gen. Elec. Rev.*, April, 1919):

"The tendency in building modern capital ships is to provide for more and more torpedo protection and it becomes necessary to crowd the machinery away from the sides of the ship as much as possible. This arrangement is also desirable from the point of protection against gunfire for a similar reason. In this respect, electric drive has an enormous advantage over any other type of machinery in which the prime mover is mechanically con-

nected to the propelling shaft. The main turbine-generators may be placed in any part of the ship that is most desirable; they may be placed in compartments forward of each other and they may be raised up enough to place the main condenser underneath them—in fact, there is practically no limit, other than the head-room, as to the position of the main turbine-generators in the ship. This gives an enormous advantage to electric drive over all other types of machinery and enables the Naval Constructor to give far more adequate protection to the ship and machinery against damage by torpedo and gunfire. Those parts of the machinery—the main motors—which it is necessary to connect mechanically to the shafts, are comparatively small and take up only a small space so that they can be placed in small isolated compartments which will not menace the ship in case of flooding; since no main auxiliaries are required for the motors, the flooding of a motor room will not entail any loss in that respect. Also, the motors may be placed very much farther aft than can steam driven turbines and, therefore, the length of the main shafting may be very materially reduced. This constitutes a big advantage; both on account of less liability to derangement of the shafting itself, due to injury to the ship, and also of less danger to the ship itself because of the shafting not having to pierce a number of watertight bulkheads. These advantages of installation constitute the real and main reason for the adoption of electric drive for capital ships and any other advantages are minor compared with them. Utilizing these advantages to the fullest extent makes it possible to build capital ships which are far superior to any others fitted with any other form of machinery. In addition to advantages from the point of view of protection, there are also the advantages from an engineering standpoint. The shorter lengths of shafting make it easier to keep the shafts in line; the grouping of boilers around the machinery makes short and direct steam pipes with a consequent reduction in weight and complication and a smaller drop in steam pressure. The same may be said of practically all the other piping systems of the ship, such as feed lines, oil lines, exhaust lines, etc."

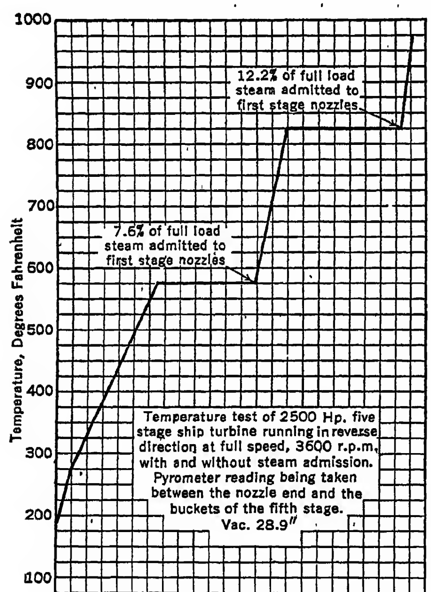

FIG. 5

SUPERHEAT AND REVERSING TURBINES. Whereas high superheat is beneficial to the operation and life of turbines, it has caused serious trouble in reversing turbines owing to expansion strains set up by rapid changes in temperature, and reversing turbines are necessary when mechanical connections between turbine and propeller are adopted. When steam expands without doing work, its temperature remains nearly constant, and if work is done upon it by a reversed turbine, this temperature is considerably increased. With superheated steam, these temperatures may be extreme and might be injurious to the main turbine as well as the reverse turbine, since it also must be used for reversal of direction and since the two often occupy the same casing.

Fig. 5 illustrates the possible steam temperatures which may be reached when maneuvering a ship equipped with reversing turbines. The turbine in this test was first run at full speed in the extremely high vacuum given on the record until a constant temperature was shown by the pyrometer. A little steam was then admitted and a constant temperature was again reached at 825 deg fahr, when more steam was admitted and the temperature then rose so quickly that the steam had to be shut off. In the General Electric Co.'s shops, it has been discovered that the reversing wheels of marine turbines turn blue with heat when operated at normal speed in a vacuum of 20 in.

When high superheat is used in combination with a reversing turbine, precautions can be adopted to preserve the turbine from damage. Various arrangements have been

proposed. One is to reduce the steam temperature before it enters the reversing turbine by injecting a spray of water, or by passing the steam through a cooler. Another method is to install a steam pipe connected between the boiler drum and the superheater—this also requires special maneuvering valves. With this latter method, greater care must be taken to prevent the superheaters from overheating during maneuvering operations. These complications are eliminated by the adoption of electric drive, which abolishes the astern turbine.

ELECTRIC DRIVE FOR AUXILIARIES. A source of considerable loss in the operation of many ships is the use of steam-driven auxiliaries. With electric drive, it is necessary to maintain a d-c electrical supply independent of the main generators for purposes of excitation. This same source can be used for lighting and driving electrical auxiliaries. If turbine units are chosen for this auxiliary power, the steam from these units can be exhausted into a suitable stage of the main turbine, and, by automatic means, exhausted into the main condenser when the load is removed from the main turbine. A more simple method is to design the auxiliary turbines as condensing units and connect their exhaust directly into the main condenser. This is the usual method of design. An auxiliary condenser is provided for use in port. In certain cases, it may be desirable to use a few steam-driven auxiliaries, such as boiler-feed pumps, and the exhaust from these can be utilized for feed heating. If necessary, this steam can be supplemented by either "bleeding" the main turbine or using part of the exhaust from the auxiliary units. In some installations, practically all feed-heating steam is bled from main turbine.

Some few ships are equipped with auxiliaries driven by a-c motors which receive their power from the main generating unit. In some of these installations, reduced speeds of the ship are obtained by running the turbine generator at approximately full speed and reducing the motor speed by means of an external resistance. The fuel consumption during these reduced speeds is greater with this method than when the turbine speed can be reduced, as there is a large loss of power in the external resistance.

In other installations, where the turbine speed varies with the propeller, the alternating current is taken from the main propulsion generator when the propulsion generator is operating between 66 and 100 per cent of rated speed. Since direct current is used for most auxiliaries, the alternating current taken from the propulsion generator operates an a-c motor which is directly connected to a d-c generator. The d-c generator is designed to give 100 per cent voltage at approximately 66 per cent speed. Since the speed of the a-c motor follows the propulsion generator, an automatic voltage regulator is provided to vary the generator field excitation automatically and thus maintain constant voltage at all speeds within the range from 66 to 100 per cent. When the propulsion generator is not in operation or the speed is below 66 per cent, the motor-generator set is driven by a steam turbine which is connected to it through reduction gears. The change-over from motor drive to turbine drive is done automatically when the propulsion turbine speed is reduced below 66 per cent. The change from turbine drive to motor drive is manual. When the a-c motor is driving the motor-generator set, the turbine is operated under vacuum with the steam supply shut off. This latter arrangement was adopted in the five U. S. Coast Guard vessels built in 1929.

In a few of the Coast Guard vessels, some of the auxiliaries, where the load varies with the speed of the vessel, are driven by a-c motors. These motors are supplied direct from the propulsion generator through step-down transformers or from the synchronous motor of the motor-generator set which may be operated as a generator when the motor-generator set is being driven by the steam turbine.

37. INTERNAL-COMBUSTION ENGINE DRIVE

DIRECT-DRIVE DIESEL ENGINE. For very small pleasure boats on inland waters and bays, the internal-combustion gasoline engine is used almost exclusively. Gasoline fuel is very expensive and presents a great fire hazard on a vessel. In recent years, the development of the heavy oil Diesel internal-combustion engine has progressed to such a state that it now finds many applications in overseas cargo and passenger vessels of the smaller sizes. The number of such applications is much greater on vessels of European registry than for the United States. The largest single Diesel engine in operation at the present day is of the double-acting four-cycle, type and has a rating of 6700 hp at 125 rpm.

As in all other types of prime movers, the weight, cost, and space required per horsepower for an internal-combustion engine decrease as the speed is increased. The use of reciprocating parts, however, limits the speeds far below those obtainable with the steam turbine, but speeds are obtained which are above the best propeller performance Therefore, in

this case, it is also possible to use the electric reduction gearing, the speed ratio for the larger size engines generally not exceeding approximately 5 to 1. Although some Diesel-engine-driven vessels have been equipped with reduction gears, the standard application is direct drive or through the electric reduction gear.

With the direct-drive Diesel, the engine must be stopped, started, and reversed many times during a vessel's entry into port. The general method of starting and reversing a Diesel is by operating it on compressed air until sufficient compression pressure is obtained in the cylinder to ignite the injected fuel oil. Since sometimes, during the docking of a vessel, orders may be received for as many as 30 starts, including reversals, it is very essential to have a large supply of compressed air, for without this, the engine is incapable of being started. This is one of the major disadvantages for the direct-drive internal-combustion-engine type of equipment. The fuel consumption per shaft horsepower-hour for the Diesel engine is slightly better than in the most modern high-pressure, high-temperature steam plant, but the present cost of Diesel fuel oil is much greater than for the grade of oil required in the steam plant. Because of this, the fuel cost per shaft horsepower-hour at present prices is about equal to or at least within debatable limits of equality with the modern steam plant.

In estimating the operating cost for a Diesel engine, some consideration must be given to the lubricating oil which is consumed; in the turbine plant the loss is practically negligible. Owing to the greater contamination and larger quantities of lubricating oil, the Diesel plant must be equipped with more purifying apparatus than is required by the turbine plant.

A review of the operating data of several direct-drive Diesel-engine-driven cargo vessels fails to indicate that the operating cost for a Diesel ship of American registry is materially less than for a modern turbine plant. The data also indicate that the maintenance cost increases very rapidly after the cylinder sizes exceed a certain dimension, owing to the increased difficulty in handling the parts in the limited space available on a vessel. For the large engines, it is generally necessary to have an outside contractor provide additional men and equipment in order to make a general overhaul of the machinery. This, of course, greatly increases the operating cost of the vessel and, in many cases, requires an arrangement of ship schedules which will allow a greater time in port.

The cost of these large Diesel engines (4000 hp or over) in the United States is also much greater than for some other types of machinery, because the quantity constructed is small, and the applications to date are so varied that each job is one of special design and construction. Stationary or industrial engines, except in special cases, are not applicable to marine installations on account of their higher speeds, constant direction of rotation, etc.

DIESEL-ELECTRIC DRIVE. The internal-combustion engine, when electrically connected to the propeller, is commonly known as the "Diesel-electric drive," and this type of installation overcomes several of the objections of the direct-drive Diesel.

The Diesel electric drive, like the turbine electric drive, consists of a generator directly connected to the Diesel engine and the propeller directly connected to an electric motor. In this case, the electric machinery is usually of the d-c type. This electric reduction gear removes the requirement for a very slow-speed engine, yet permitting a relatively slow-speed propeller. By the use of the d-c system, the speed ratio between the engine and propeller is readily changed from maximum design to zero and the propeller motor reversed with very little difficulty. The procedure is so easily accomplished that, in most Diesel electric installations, the speed of the propeller is adjusted and controlled directly on the bridge instead of the usual procedure of the bridge passing an order to the engine room where the actual speed adjustments are made.

In the Diesel electric installations, more than one engine can be used for each propeller with no operating difficulties. This permits the installation of more and smaller Diesel engines. The greater quantity reduces the cost per engine, and the smaller engine reduces the size of the parts to be handled, facilitating maintenance and repairs. The multiple engines also facilitate routine overhauls, in that, for multiple-engine ships, one engine can be switched out of the circuit and the ship operated on the remaining engines with only a slight reduction in speed.

The electric equipment for Diesel electric installations usually is arranged as follows:

A shunt-wound, separately excited, d-c generator is directly connected to each Diesel engine and a shunt motor directly connected to the propeller shaft. The propeller motor is frequently of the double type, i.e., it consists of two independent armatures on a single shaft with two bearings and two electrically independent field structures, to give increased reliability. If one motor is damaged for any reason, the vessel can still operate on the other motor at a reduced speed.

In addition to the propulsion generators, one or more exciters are usually installed. These exciters are either in tandem with and driven by the propulsion generator engines

or a separate unit directly connected to a separate Diesel engine. These exciters operate at constant voltage in most cases and supply excitation to the propulsion generators and propulsion motors and supply direct current to the other ship's auxiliaries, including lighting.

In order to keep the size of the motors and generators to a minimum and to insure uniform heat distribution in the equipment, both the motors and generators are usually force-ventilated. In some cases, the supply of air is taken from the engine room and discharged out the top of the vessel. This, however, is not the best practice, for invariably in a Diesel engine room there is considerable oil vapor in the air and, when it is forced through the motor and/or generator, the ventilation passages act as oil condensers, with the result that the windings are soon saturated with oil. The supply of this cooling air from outside the engine room is, therefore, essential, and except for the disadvantage to operating personnel, because of the high temperature that is produced, could be discharged to the engine room. The best installation, therefore, requires both supply and discharge ducts with careful design at the air intakes at the top of the vessel, to prevent water and spray from being carried into the electric equipment.

In one installation that the writer has been associated with, air coolers have been provided for the generators and motors. These coolers consist of finned tubes which are cooled with sea water, and the air is continuously circulated through motor and cooler, and generator and cooler, by motor-driven fans. No air is taken from the outside. This arrangement has the objection that, even with the most carefully designed ventilation systems, dust from the carbon brushes is circulated and eventually closes the ventilation passages through the windings unless frequently cleaned. This, of course, can be overcome by filters, but this adds equipment which requires space and increases the weight of the installation. Space is at a premium, because every cubic foot taken by machinery means a corresponding reduction of space available for revenue-producing passengers or cargo. The air ducts of the air cooler installation usually have an enlarged cross-section at one point, so that the air velocity will be reduced to such an extent that most of the carbon particles will drop to the bottom of the duct, where they can readily be cleaned out.

Since the d-c equipments permit a variable speed ratio between generator and motor, the Diesel engines for this type of equipment are automatically operated at practically constant speed by a governor. The engine is, therefore, never reversed and never started under load. This procedure reduces the starting air requirements, as the engine is not stopped until the vessel is securely fastened to the dock.

Since the armature circuits of the generators and motors are all connected in series, each generator is provided with a special switch which has two positions. The off position opens the armature circuit of the associated generator and at the same time keeps the propulsion armature circuit complete for the remaining equipment. This will permit operation with one, two, three, or four generators in series, depending on the number available. All switching equipment is interlocked, so that it is impossible to open or close any of the main power circuits without first opening the generator field circuits, thus reducing the terminal voltage of the generator to residual and preventing heavy arcs by breaking large currents. The main power circuit switches are provided with arcing tips to facilitate renewal from any burning that may be encountered.

In order to control the speed and direction of rotation of the propelling motors, a potentiometer-type reversing generator field rheostat is provided in the wheel house or adjacent thereto, with mechanical connection to a handle, wheel, or lever adjacent to the steering wheel. This field rheostat controls the amount and direction of the field current to the generators that have been "switched" into the propulsion circuit by the engineer in the engine room. This arrangement will permit operating the vessel at reduced speed with practically full-speed fuel consumption per horsepower-hour, provided the speed is selected so that the connected generators will be operated at rated load. For instance, in a vessel having four generators in series for full power operation, the fuel consumption per shaft horsepower hour for $^1/_4$, $^2/_4$, and $^3/_4$ full power will be approximately the same as for full power. This result is obtained because the generator always operates at the same speed, and with $^1/_4$ power and one generator operating, the generator voltage is the same as full-load. Therefore, the efficiency of the Diesel engine and generator is the same as for full load, the only additional loss being the difference in motor efficiency. The variation of motor efficiency from $^1/_4$ load to full load is considerably less than with the usual type of d-c motor because the reduced power is obtained at reduced speed which is procured by a reduction of the motor applied voltage and motor field excitation. If we assume that each generator has a rated voltage of 300 volts, then for $^1/_4$ power, one generator gives motor voltage of 300, $^1/_2$ power requires two generators with a motor voltage 600, etc.

The shunt fields of the generators are operated in parallel and each generator switch has sufficient contacts to close the circuit to the generator field at the same time the generator armature is connected to the power circuit.

Unfortunately for the electrical system, the power required to drive a propeller does not vary directly as the revolutions, but is generally rather close to the cube power. For example:

100% power—100% speed—100% voltage—100% current
75% power— 90.9% speed— 75% voltage—100% current
50% power— 79.4% speed— 50% voltage—100% current
25% power— 63.0% speed— 25% voltage—100% current

Since the generator is designed for the voltage required at full load, any increase in generator voltage to obtain a higher motor speed when operating with reduced number of generators would mean an increase in the cost and weight of the generators. Therefore, in order to absorb full power at a voltage corresponding to the sum of the ratings of the operating generators, the motor field excitation is reduced for each generator removed from the circuit.

In a four-generator installation, the characteristics of the motor may not permit a sufficient reduction of the field to give the speed required for 1/4 power without the motor becoming unstable. This is overcome in a double-motor installation by using only one motor armature for the one generator condition.

Since the Diesel engines are operated at rated rpm for all propeller speeds and electric machinery has the inherent characteristics of being able to withstand very great current overloads for short intervals of time, torques as great as 300 per cent may be developed by the propelling motor during the reversing period without overloading the Diesel engines—that is, with reduced motor speed and voltage, the current can be increased so that the product of the two approaches the rated horsepower of the engine. This high torque is not possible with any other type of drive.

In order to keep the voltage to ground and between wires to a minimum in a double-motor installation, each half of the double motor is connected between half the generators; i.e., in a four-generator installation with a double motor, the circuit is through two generators, one half the motor, the other two generators, then the other half of the motor. This arrangement is of no consequence where the applied voltage is relatively low.

The Diesel electric drive introduces an additional power loss between the prime mover and propeller, but this loss may be partially or even wholly made up in some particular cases by the better fuel consumption of the higher-speed engine and the increased efficiency of the lower-speed propeller. As the fuel consumption is approximately the same per horsepower-hour at full, 3/4 and 1/2 power, it requires only a very short period of operation at reduced power to effect a saving of fuel over that required for direct-drive Diesel equipment. In the direct-drive plant the fuel consumption increases quite rapidly per shaft horsepower-hour as the power and speed are reduced.

At the present time, there are many very successful Diesel electric installations, the greatest number of applications being in tugs, small harbor craft, etc., where maneuverability and pilot-house control are of great importance. One of the largest installations on a commercial cargo vessel consists of:

One 4000-hp separately ventilated 60-rpm 1500-volt, double-unit propulsion motor.

Four 800-kw, 375-volt, 250-rpm generators, each direct connected to an 8-cylinder 1200-bhp, Diesel engine.

The U. S. Army engineers have used Diesel electric equipment in some of their hopper dredges. In this type of vessel, the propeller and the dredge pump are each directly connected to a motor. When dredging, the propulsion generator is connected to the dredge pump motor and a smaller generating unit is connected to the propulsion motor. The speed of the vessel is very low when dredging; therefore, the power required to operate the propelling motor is very small. After the hoppers are filled, the pump motor is shut down and the propulsion generator connected to the propelling motor. This permits a reasonable speed of propulsion when proceeding to the dumping grounds with the minimum of idle equipment. This type of equipment makes it very easy to place the entire control of the dredging and maneuvering operations under one man on the bridge. It is understood that, with this type of dredging equipment, the cost per cubic yard has been considerably reduced over that obtained with other types of equipment.

This double use of the propulsion generator has also been adopted in a fire boat, the propulsion generator being connected to operate the fire pumps when at a fire and to the propulsion motor when on the way to the fire.

Summary of Advantages. The advantages of the electric reduction gear for Diesel engine installation are as follows:

(a) Direct control of vessel on bridge.
(b) Quicker reversal of propeller.
(c) Greater torque during reversing.
(d) Higher-speed Diesel engines.
(e) Less air tanks and smaller compressors.
(f) Small engines with small reciprocating parts.
(g) Less chances of sea or port detention, since failure of one engine will not prevent operation of vessel and repairs may be made at sea without stopping vessel.
(h) In some cases, fuel consumption at full load is same as for direct-drive Diesel.
(i) Practically same fuel consumption per shaft horsepower-hour at certain reduced speeds as at full speed.

Summary of Disadvantages.

(a) More equipment required than for direct-drive Diesel.
(b) In some cases, initial cost, and weight may be slightly more.

At the present time, the output per shaft with this type of drive is limited to about 8000 hp per propeller. This is due to the present limits of motor design and the inability to get the large-diameter motor in the vessel without producing too much slope in the propeller shaft.

38. TYPICAL ELECTRICALLY PROPELLED SHIPS

The following is a partial list of the more important turbine-electric-propelled ships:

Name	Type	No. of Propellers	Total Shaft Horsepower, Full Speed	Type of Motors	No. of Ships This Class	Reference
U.S.S. *Tennessee*	Battleship	4	28,000	Induction	5	1
Rodman Wanamaker	Ferry	2	2,200	Induction	2	2
U.S.S. *Lexington*	Airplane carrier	4	180,000	Induction	2	3
California	Pass.-cargo	2	17,000	Synchronous	3	4
Chelan	Coast guard	1	3,000	Synchronous	5	5
City of Saginaw	Car ferry	2	7,200	Induction	2	6
Morro Castle *	Pass.-cargo	2	16,000	Synchronous	2 *	7
President Coolidge	Pass.-cargo	2	26,500	Synchronous	2	8
Talamanca	Pass.-cargo	2	10,500	Synchronous	6	9
Corsair	Yacht	2	6,000	Synchronous	1	10
Normandie	Pass.-cargo	4	160,000	Synchronous	1	11

A partial list of representative Diesel-electric propelled ships is as follows:

Golden Gate	Ferry		750	D-c shunt	2	12
A. Mackenzie	Hopper dredge		1,600	Special	4	13
J. W. Van Dyke	Tanker		2,300	D-c shunt	1	14
N. Y. C. No. 34	Tug		650	D-c shunt	2	15
Fresno	Ferry		1,250	D-c shunt	6	16
Courageous	Cargo		4,000	D-c shunt	3	17

* The *Morro Castle* was burned in 1934.

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SECTION 18

ELECTROCHEMICAL AND ELECTROTHERMAL INDUSTRIES

ELECTROCHEMICAL PROCESSES

By M. de K. Thompson

ART.		PAGE
1.	Electroplating	01
2.	Galvanoplasty	06
3.	Electrolytic Refining	06
4.	Electrolytic Extraction of Metals	09
5.	Electrolytic Oxidation and Reduction	10
6.	Electrolysis of Sodium or Potassium Chloride	10
7.	Electrolysis of Water	11
8.	Ozone	11
9.	Bibliography	12

ELECTRIC FURNACES AND THEIR PRODUCTS

By M. de K. Thompson

ART.		PAGE
10.	Classification	12
11.	Temperature and Distribution of Heat	12
12.	Maximum Size of Electric Furnaces	13
13.	Design of Furnace Walls	13
14.	Design of Electrodes	14
15.	Small Laboratory Furnaces	14
16.	Electric-Furnace Products and Industrial Furnaces	16
17.	Electric Furnaces in Metallurgy	18
18.	Fixation of Atmospheric Nitrogen	21
19.	Bibliography	21

ELECTRIC RESISTANCE HEATING

By Frank Thornton, Jr.

ART.		PAGE
20.	General Principles	22
21.	Resistance Materials	22
22.	Electrical Insulation	23
23.	Industrial Heating Appliances	24
24.	Industrial Application Units	24
25.	Industrial Ovens	25
26.	Industrial Resistance Furnaces	26
27.	Bibliography	30

ELECTRICAL PRECIPITATION OF SUSPENDED PARTICLES

By N. W. Sultzer

ART.		PAGE
28.	Principles	31
29.	Equipment	31
30.	Operating Characteristics	33
31.	Typical Installations	33
32.	Bibliography	34

ELECTRIC WELDING

By H. M. Hobart

ART.		PAGE
33.	General Considerations and Definitions	34
34.	Resistance Welding	35
35.	Electric Arc Welding	41
36.	Manual vs. Automatic Welding	49
37.	Applications of Electric Welding	50
38.	Bibliography	51

ELECTROCHEMICAL PROCESSES

By M. de K. Thompson

(See also Electrochemistry.) The principal industrial electrochemical processes are the following:

Electroplating.
Galvanoplasty, including electrotyping.
Electrolytic refining of metals.
Electrolytic extraction of metals.
Electrolytic oxidation and reduction.
Electrolysis of sodium and potassium chloride solutions.
Electrolysis of water.
Electric-furnace processes (see Electric Furnaces).
Production of ozone.
The Fixation of Nitrogen.

Some of the more important of these various processes are briefly described below.

1. ELECTROPLATING

Electroplating consists in covering a conducting surface (usually metallic) with a thin, smooth, compact, well-adhering layer of metal, by depositing this metal electrolytically from an aqueous solution of one of its salts. Usually the anode of the electroplating vat consists of rods or plates of the same metal as that of the salt in solution and is connected to the positive terminal of the source of electricity. For some metals, for example, chromium, unattackable anodes are used. The object to be plated forms the cathode, and is usually connected to the negative terminal. The anode dissolves approximately to the same extent that the cathode gains, so that the amount of the metal ions in the bath remains nearly constant. There are cases in which this does not hold; for example, in brass plating. This results in a continuous change in the composition of the bath.

SUSPENSION OF OBJECTS TO BE PLATED. The cathodes are always suspended in the bath between two rows of anodes, so that they will be plated uniformly on both sides. When the cathode is of irregular shape, or very large, it must be turned frequently during the plating in order to get a uniform deposit. The cathodes and anodes are suspended by copper wires from horizontal metallic tubes, the ends of which rest on the edge of the plating vat. The metallic tubes are permanently connected to the source of the electricity, so that, as soon as the cathodes are suspended in the bath, electrolysis begins. Small objects, such as tacks, pins, and screws, are suspended in the vat in a wire basket, which is, of course, plated simultaneously. To get a uniform plated surface the objects should be well shaken during electrolysis. The anodes are removed from the bath only when they are nearly used up and have to be replaced.

CONSTRUCTION OF VATS. Large plating vats are made of wood or concrete lined with some specially prepared substance resembling pitch, with lead, or with rubber. Small tanks for silver or gold plating are usually porcelain lined.

VOLTAGE AND CURRENT DENSITY. Electroplating tanks are always connected in parallel, so that they will be electrically independent of each other.

Low-voltage generators of from 5 to 6 volts are, therefore, used in plating, and each tank must be connected directly to the generator through a rheostat in order to regulate the voltage.

The proper current density in any plating process is that density at which a good deposit is formed. This may vary within certain limits for a given solution and temperature; it is a function of the temperature and the nature of the solution.

WASHING AND PICKLING. In order to make the metal adhere well to the surface to be plated, the surface must be perfectly clean. Grease is removed by dipping it into a hot alkaline bath containing 10 per cent, by weight, of sodium carbonate or sodium hydrate. After washing off the alkali, the object is dipped into a bath called a " pickle," the purpose of which is to remove any oxide that may have been produced by the alkali, and to give a

bright surface. The pickle is then washed off with water and the object is suspended immediately in the electroplating vat. Degreasing by Trichlorethylene is a more recent method. (W. Burden, Metal Cleaning and Finishing, 1935, Vol. 7, p. 504.)

Pickling Solutions. The pickle varies with the nature of the metal treated. Cast iron and wrought iron are pickled in a solution consisting of 15 parts, by weight, of water to 1 part of concentrated sulfuric acid. A suitable pickle for zinc is dilute sulfuric or hydrochloric acid. Copper, brass, bronze, and German silver are pickled first in a bath consisting of 200 parts, by weight, of nitric acid of specific gravity 1.33, 1 part of common salt, and 1 part of lampblack. The lampblack is intended to form some nitrous acid from the nitric acid. The object is then washed in boiling water and is immersed in a "bright dipping bath," to give a bright surface. This bath consists of 75 parts, by weight, of nitric acid of specific gravity 1.38, 100 parts of concentrated sulfuric acid, and 1 part of common salt.

After the plating is finished, the object is removed from the plating bath, washed in hot water, and placed in warm sawdust or in a drying closet.

ELECTROLYTIC CLEANING. In electrolytic cleaning the object is made the cathode in a hot alkaline solution. A current density of 10 amp per sq ft (1 amp per sq dm) is used, which requires 6 to 10 volts. This emulsifies the grease. The time required is 3 to 5 minutes. The current may then be reversed for 10 to 15 seconds to dissolve any metallic deposit of zinc, lead, or tin that may have formed. This also tends to make the deposit more adherent. (De la Rive used electrolytic cleaning in sulfuric acid in 1842.) A suitable solution is 8 oz per gal (60 grams per liter) of sodium carbonate, and 2 oz per gal (15 grams per liter) of sodium hydroxide.

SIMULTANEOUS CLEANING AND PLATING. The object is electrolyzed as cathode in alkaline copper cyanide. The alkali removes the grease, and a thin film of copper is deposited where the surface is clean. The advantage is that the deposited copper shows when the surface is clean. It is chiefly used in the large-scale production of cheap articles, or for the preliminary coating of iron and steel with copper. The following solution may be used:

	per liter
1/2 lb H. V. Winkle's XXX lye	60 g
2 oz copper carbonate	15 g
4 oz ammonium carbonate	30 g
4 oz potassium cyanide	30 g
1 gal water	1000 g

(*Trans. Am. Electrochem. Soc.*, Vol. 27, p. 142, 1915.) If deposit of brass is desired, add 2 oz of zinc carbonate to this solution.

PLATING BY DIPPING. A thin film of metal may be deposited on a metal by dipping it into a solution of a salt of a metal which is electropositive (see Principles of Electrochemistry) with respect to the metal to be plated, e.g., by dipping iron into a copper-sulfate solution. A small amount of iron dissolves and an equivalent amount of copper is deposited on the remaining iron. Of course, only a thin film can be produced in this way, for, as soon as the iron is covered with copper, the action ceases. No external electric current is needed in this process.

PLATING BY CONTACT. When the metal to be plated is electropositive with respect to the metal to be deposited on it, the electrodeposition can be obtained by connecting the former to a zinc rod. In this case, the solution must be of a complex salt, in order to reduce the deposition of the metal in solution on the zinc itself. For example, silver may be deposited on copper by connecting a zinc rod or plate to the copper object by a wire and dipping both into a potassium-silver-cyanide solution; the zinc dissolves and silver is deposited on the copper, while some silver is also deposited on the zinc. No external current is needed in this process.

NICKEL PLATING. Numerous formulas for nickel-plating solutions are in use. The following is an example:

	Approximate Normality	Grams per Liter	Ounces per Gallon
$NiSO_4 \cdot 7H_2O$	0.75	105	14
NH_4Cl	0.25	15	2
$NiCl_2 \cdot 6H_2O$	0.13	15	2
H_3BO_3	0.25 molal	15	2

Hydrogen-ion concentration: $pH = 5.3$.
(Blum and Hogaboom, Electroplating and Electroforming, p. 259, 1930.)
Current density 0.5 to 2 amp per sq dm (5 to 19 amp per sq ft).

The following fluoride solution gives a fine-grained deposit:

		Grams per Liter	Ounces per Gallon
$NiSO_4 \cdot 7H_2O$	2 normal	281	37.5
NaF	0.2 normal	8.4	1.1
H_3BO_3	0.5 molal	31	4.2

Nickel anodes of at least 99 per cent purity should be used. The salts should be free from zinc and copper.

In recent years high-speed nickel plating with low *pH* (below 3) and high-current density (up to 18 amp. per sq. dm.) has been used (O. P. Watts. *Trans. Am. Electrochem. Soc.*, 1931, Vol. 59, p. 379; M. Ballay, ibid., 1932 vol. 62, p. 91.)

The Madsenell process of plating ductile and well-adhering layers of nickel and other metals consists in " degasification " of the base metal, such as iron or steel by treating as anode in 96 per cent sulfuric acid at 50 amp per sq ft. (F. M. Dorsey, *Ind. Eng. Chem.*, 1928, Vol. 20, p. 1094.)

COPPER PLATING. The metals on which copper is usually plated, such as zinc, iron, and tin, are more electronegative than copper. On dipping any of these metals into an acid copper-sulfate bath, they would become covered with a layer of copper, which in some cases is spongy and does not adhere well. In order to reduce the velocity with which this reaction takes place a solution of the double cyanide of copper and potassium, $KCu(CN)_2$, is used. This can be made by dissolving cuprous cyanide in potassium cyanide to form a 3 to 8 per cent solution,* with an excess of 0.2 per cent of potassium cyanide. This bath is generally heated to 50 to 60 deg cent. The proper current density at the cathode is about 0.5 amp per sq dm (4.6 amp per sq ft) which requires about 3 volts at room temperature. For further details, see Circular of the Bureau of Standards, No. 52.

Surfaces that have already received a thin coating of copper in a cyanide bath are sometimes thickened in an acid copper-sulfate bath. The cyanide must be washed off on transferring to the sulfate bath. A sulfate bath may be made by dissolving 150 grams of copper sulfate $CuSO_4 \cdot 5\,H_2O$ and 50 grams of concentrated sulfuric acid in 1 liter of water. The proper current density is about 0.7 amp per sq dm (6.5 amp per sq ft) which requires less than 1 volt.

Small springs are very much weakened by copper plating in a cyanide bath, and are very likely to break while suspended, slightly stretched, in the bath during plating. This may be due to absorbed hydrogen. See *Met. and Chem. Eng.*, Vol. 16, p. 83, 1917; *Trans. Am. Electrochem. Soc.*, 1917, Vol. 32, p. 247, and 1918, Vol. 33, p. 169. A copper sulfate-oxalate bath has been recommended as a substitute for copper cyanide (C. G. Fiuk and C. Y. Wong, *Trans. Am. Electrochem. Soc.*, 1933, Vol. 63, p. 65).

ZINC PLATING. Electrolytically deposited zinc is of a dull color and is not as pleasing in appearance as layers obtained by dipping in melted zinc, but electrolytic zinc has been shown to protect iron better for a given thickness of deposit than a coating made from melted zinc. (Burgess, *Electrochem. and Met. Ind.*, Vol. 7, p. 17, 1905.) A suitable solution consists of 240 grams of zinc sulfate, $ZnSO_4 \cdot 7\,H_2O$, 15 grams of ammonium chloride, 30 grams of aluminum sulfate $[Al_2(SO_4)_3 \cdot 18\,H_2O]$, and 1 gram of licorice per liter of solution. Enough sulfuric acid is added from time to time to keep the *p*H value from 3.5 to 4.5. The current density in the cathode is 5 amp per sq dm (47 amp per sq ft), which requires from about 1 to 2.5 volts. Zinc anodes are used. A little more zinc is dissolved than is deposited, owing to the free acid. The resistance may be reduced by warming the bath to 40 or 45 deg cent. A finer-grained deposit, and therefore better protection, are furnished by deposits from alkaline solutions. The following formula has been found satisfactory at the Bureau of Standards:

	Normality	Grams per Liter	Ounces per Gallon
ZnO	1.0	45	6
NaCN	1.5	75	10
NaOH	0.3	15	2

The current density is from 1 to 2 amp per sq dm (9.3 to 19 amp per sq ft), and the temperature is 40 to 50 deg cent (104 to 122 deg fahr).

Zinc anodes containing 0.5 per cent aluminum and 0.3 per cent mercury have been found superior to pure zinc anodes. (*Metal Ind.* 1933, Vol. 31, p. 195.)

* An *n* per cent solution of a substance contains *n* parts, by weight, of that substance in 100 parts, by weight, of the solution.

BRASS PLATING. If an acid solution of zinc and of copper sulfates were electrolyzed, only copper would be deposited. In a cyanide solution of zinc and copper, however, these metals are deposited simultaneously in the form of an alloy. (Spitzer, *Zeit. f. Electrochemie*, Vol. 11, p. 367, 1905.) The copper is deposited more easily than the zinc, so that at a low current density, 0.1 amp per sq dm (0.93 amp per sq ft), only a small amount of zinc is deposited, but at 0.3 amp per sq dm (2.8 amp per sq ft), the deposit contains only 80 per cent of copper. Increasing the current density changes the composition of the deposit only slightly.

A suitable bath for brass plating is made by substituting zinc cyanide for half of the copper cyanide in the solution given above for plating. Brass anodes are used. Other brass baths that have been found to give good deposits are the following:

	Normality	Grams per Liter	Ounces per Gallon
$CuCN$	0.30	27	3.6
$Zn(CN)_2$	0.15	9	1.2
$NaCN$	1.10	54	7.5
Na_2CO_3	0.60	30	4.0

(Blum and Hogabroom, Electroplating and Electroforming, p. 385, 1930.)

The temperature is 32 to 45 deg cent (90 to 113 deg fahr), and the current density 0.2 to 0.3 amp per sq dm (2 to 3 amp per sq ft). An important property of brass containing 65 per cent copper is that rubber can be made to stick to it with great tenacity. For this reason, brass plating is used extensively in the rubber industry.

SILVER PLATING. The double cyanide of potassium and silver is generally used for silver plating on account of the smooth deposit obtained with this solution. The deposit from a nitrate comes down in the form of isolated crystals, which do not cover the surface completely. The solution contains from 1 to 5 per cent silver as potassium silver cyanide, $KAg(CN)_2$, with 0.5 per cent of free potassium cyanide. Too much or too little free cyanide gives a bad color to the deposit. A good silver plating bath may be made up as follows:

20 grams potassium silver cyanide.
10–12 grams potassium cyanide, 99 per cent.
1 liter water.

The current density is 0.3 amp per sq dm, at about 1 volt. (Schlötter, Galvanostegie, 1. Teil, p. 149, 1910.) The anodes are silver.

In order to make thick deposits of silver adhere well on metals which alloy with mercury, a thin deposit of mercury is produced by dipping these metals in a "quicking bath," consisting of a solution of 30 grams of potassium mercury cyanide, $KHg(CN)_2$, and 30 grams of potassium cyanide in 1 liter of water. On removal from the quicking bath, articles are washed and placed immediately in the silvering bath.

At least two different silvering solutions are used. The first is called a "striking" solution, or "strike." It contains more cyanide and less silver than the regular silvering bath, for the purpose of reducing the silver-ion concentration in order to prevent deposition by immersion. In some cases a second striking solution is used.

The silver deposit from the solution given is not bright, but the addition of a small amount of the solution saturated with carbon bisulfide makes it so.

Silvering solutions of silver iodide dissolved in sodium iodide have recently been put into practice (C. W. Fleetwood and L. F. Yntema, *Ind. Eng. Chem.*, 1935, Vol. 27, p. 340.)

GOLD PLATING. The solution used in gold plating contains from 0.35 to 1 per cent of gold as the double cyanide of gold and potassium, $KAu(CN)_2$, with twice as much free potassium cyanide. The current density on the cathode is about 0.2 amp per sq dm (1.9 amp per sq ft), which requires about 1.5 volts. The anodes are pure gold. The solution may be used hot or cold. The deposit from a hot solution is more dense, more uniform, and of a richer color. The color of the gold deposit may be influenced by simultaneously depositing some other metal. Green gilding may be obtained by adding a little silver cyanide to the bath, until the desired tint is obtained. The solution should be cold. To give the deposit a red tint, a little copper cyanide is added to the solution.

CHROMIUM PLATING. In recent years chromium plating has become common. The following two formulas are in use:

	Grams per Liter	Ounces per Gallon
1. Chromic acid, CrO_3	250	33
Concentrated H_2SO_4	2.6	0.34
2. Chromic acid, CrO_3	400	54
Concentrated H_2SO_4	4	0.55

A good current density is 11 amp per sq dm (100 amp per sq ft), at 40 deg cent (104 deg fahr). Lead anodes are used. The deposit is bright.

PLATING ON ALUMINUM frequently does not wear well, on account of the difficulty in getting a perfectly clean aluminum surface on which to plate. This is due to the rapidity with which a thin invisible film of oxide or hydroxide forms on aluminum when exposed to the air or to any solution. One method of overcoming this difficulty is to immerse the aluminum in a solution of potassium hydrate until hydrogen is evolved, and then dip without previous washing in a potassium silver cyanide solution. The aluminum is immediately covered with a layer of silver. It is still better to amalgamate by dipping into a 0.5 per cent solution of mercuric chloride immediately after treating with hydrate. The chloride is rinsed off, and the object again treated with potassium hydrate and then immediately suspended in the silvering bath. (Langbein, Electrodeposition of Metals, 4th ed., p. 409, 1902.)

Burgess (*Electrochem. Ind.*, Vol. 2, p. 85, 1904) recommends cleaning first with dilute hydrofluoric acid, then with a mixture of 100 parts of sulfuric acid and 75 parts of nitric acid, both concentrated. After rinsing with water the surface is immediately plated with zinc, as this metal is found to adhere better than many others. Starting with the zinc surface, other metals are readily deposited.

OTHER PLATING PROCESSES. Plating with the following metals is also used: cobalt, cadmium, platinum, tin, lead, rhodium and iron. It has recently been found that tungsten can be deposited from aqueous solutions in a smooth, hard, coherent deposit, with a high luster. (Fink and Jones, *Trans. Electrochem. Soc.*, 1931, Vol. 59, p. 461).

2. GALVANOPLASTY

Galvanoplasty is the art of reproducing by electrolysis articles of various kinds, or of making finished products, such as set-up type, copper tubes, etc. "Electroforming" has been used as a substitute name for galvanoplasty.

ELECTROTYPING. The object is to make a copper plate which shall be an exact duplicate of type which has been set up ready for printing. First an impression of the type is made in wax, which is then covered with a thin layer of graphite, by dusting the fine powder over the wax surface with soft brushes. A thin layer of copper is then formed by sprinkling the surface with iron filings and pouring over the surface a solution of copper sulfate. The iron goes into solution, depositing copper on itself and on the graphite. The wax plate is then washed in water and suspended in an acid bath of copper sulfate, where copper is deposited electrolytically until a thin sheet that can be stripped from the wax has been formed. After removing the copper sheet from the wax plate, a melted lead-antimony alloy is poured on its reverse side, making a plate approximately 0.5 in. thick. This is then used for printing in place of the original type. Phonograph records are made by the same method.

The current density ranges from 1 to 2 amp per sq dm (0.9 to 1.8 amp per sq ft) and the volts per cell from 0.75 to 1.5.

Nickel is also used for electrotyping. The thickness of the deposit is 0.05 mm (0.002 in).

TUBES, ETC. Copper Tubes are made by the Elmore process by depositing copper on a conducting cylinder which rotates in an acid copper sulfate bath. The surface must be conducting, but the copper must not stick so firmly that the cylinder cannot be slipped out of the tube when finished. In order to keep the outer surface of the tube smooth, it is frequently polished during the deposition of copper. Copper sheets may be made by making tubes of large diameter and cutting them open. Copper sheets 1120 ft long, 31 in wide and 1.35 mils thick are now made by depositing on a rotating lead-covered copper cylinder, partly immersed in the solution, and stripping as the deposit emerges from the bath. The rate of deposition is such that 1120 ft are produced by one tank in a day. If greater thickness is desired, the sheet is passed through a second tank. (W. M. Shakespeare, *Trans. Am. Inst. Min. & Met. Eng.*, 1934, Vol. 106, p. 441.)

Spongy copper is being made electrolytically. This is mixed with graphite and is compressed to form self-lubricating bearing metal.

Electrolytic Iron Tubes were made for a number of years at Grenoble, but this has been given up.

Metallic Foil may be made by the electrolytic deposition of a thin metallic layer on a surface from which it can be removed.

3. ELECTROLYTIC REFINING

The method of refining metals electrolytically is as follows. The impure metal is made the anode of an electrolytic bath, the electrolyte of which is, at the start, a solution

of a pure salt of the metal to be refined, and the cathode is a sheet of the refined metal or of a metal from which the refined metal can be stripped. On passing the current, metal dissolves, along with certain of the impurities. The impurities which are electronegative with respect to the principal metal dissolve; those electropositive with respect to the principal metal remain adhering to the anode. When the latter finally drop from the anode they may be dissolved by the free acid, in which case they would be precipitated again on coming in contact with the anode or the cathode. Some metals are precipitated as an insoluble salt as soon as they dissolve, and are thus removed from the further action of the current.

When it goes into solution, the principal metal is, therefore, separated from those metals which are electropositive with respect to it. When it is deposited on the cathode, it is separated from those metals which are electronegative with respect to it. The bath thus becomes contaminated with certain of the impurities in the anode, and these would eventually be deposited on the cathode if the bath were not purified from time to time.

The metals which do not dissolve drop to the bottom of the tank forming the **anode mud.** It is from this mud that the platinum, gold, and silver are recovered in copper refining. For methods of working up anode mud, see Kern, *Met. and Chem. Eng.*, Vol. 9, p. 417, 1911.

COPPER REFINING. The object in refining copper electrolytically is to obtain as pure copper as possible for electric conductors and to obtain the precious metals contained in the crude copper. A representative composition of crude copper anodes for American refineries is the following:

Copper	98–99.5 per cent.
Silver	0–300 oz per ton.
Gold	0–40 oz per ton.
Arsenic	0–2 per cent.

The refined copper is about 99.95 per cent copper. The electrolyte is a solution of $CuSO_4$ and H_2SO_4 containing about 16 per cent of free acid and about 3 per cent of copper. The current density ranges from 0.43 to 4.8 amp per sq dm (4 to 45 amp per sq ft) of cathode surface, and the volts per cell range from 0.1 to 0.3. The electrolyte circulates slowly from tank to tank. The cathodes, called *starting sheets*, are thin sheets of refined copper.

Multiple and Series Systems of Arranging the Electrodes. The tanks for holding the solution and electrodes are made of wood or concrete and are lined with lead or asphalt. There are two methods of arranging the electrodes. In the " multiple system " all the cathodes of one tank are connected together, and likewise the anodes, the cathodes having an anode on each side, as shown in Fig. 1.

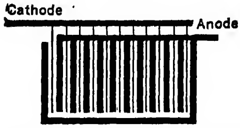

FIG. 1. Multiple System of Electrodes

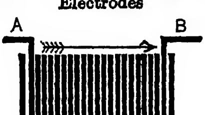

FIG. 3. Series System of Electrodes

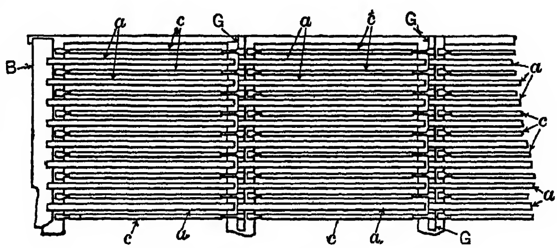

FIG. 2. Walker Multiple System

The most economical method as regards the use of copper of connecting tanks in the multiple system with each other is that devised by Walker (U. S. Pat. 687,800, 1901). In this system the current flows from tank to tank without being collected in a single bus-bar in this passage. The *cathodes* of the first tank rest on a conducting bar *G* (Fig. 2) on which the *anodes* of the second tank also rest. The figure shows three tanks so connected. *B* is the leading-in bus-bar; *a* are anodes, *c*, cathodes, and *G* is a copper bar connecting cathodes to anodes. A more recent improvement is to have the cathodes of a tank make direct contact with the anodes of the adjacent tank.

In the " series system," the electrodes are arranged as shown in Fig. 3. The impure copper plates are suspended in the solution at equal distances apart, only the end

ones being connected to the dynamo. The current enters at the electrode *A*, which dissolves, and on flowing through the tank passes through the intermediate plates. Pure copper is deposited on the sides of the plates facing *A*, and dissolved from the opposite surfaces facing *B*. Some conducting preparation is painted over the sides of the intermediate electrodes facing *A*, so that the copper deposited can be separated from the impure copper. When a certain amount of the impure copper has been dissolved, the electrodes are removed, the pure copper is separated from the impure, and the latter is melted and cast into new electrodes. This method is in use at the large plant of the Nichols Copper Co., Laurel Hill, N. Y., and at the Baltimore Copper Smelting & Rolling Co., Canton, Md. All others use the multiple system.

Comparison of the Two Systems. The energy required in the series system is about 70 per cent of that required in the multiple system. Contacts give more trouble in the multiple than in the series system. The electrodes in the series system cost more to prepare than those in the multiple system. The investment in a series plant of given capacity is less than in a multiple plant. Since lead-lined tanks cannot be used in the series system, the maintenance expense of a series plant is greater than that of a multiple plant. For recent improvements in the series system, see *Trans. Electrochem. Soc.*, 1930, Vol. 57, p. 231.

NICKEL REFINING. The International Nickel Co., Inc., extracts nickel from nickel-copper ore by flotation, top and bottom smelting, grinding, roasting, and smelting. The nickel thus made is 95 per cent pure, and this is cast into anodes and refined by the Hybinette process. Originally, the Hybinette process used anodes containing only 65 per cent nickel, when it would be considered an extraction process. Nickel starting sheets are used, and these are separated from the anodes by a canvas diaphragm.

Nickel sulfate solution circulates to the cathode and passes through the diaphragm, and copper is dissolved from the anode with the nickel. The copper is removed from the solution leaving the anode by cementation with nickel. The product is 99.9 per cent nickel and cobalt. The cobalt content is 0.4 per cent.

SILVER REFINING. Two cases arise: (1) the separation of silver from copper in an alloy consisting mainly of these two metals, and (2) the separation of silver from relatively small amounts of gold and platinum.

Dietzel Process. One method of separating silver and copper is the Dietzel process. This consists in dissolving both of the metals as anode in a weakly acid solution of copper nitrate. The solution is then transferred to another vessel and the silver is precipitated by metallic copper, following which the copper is deposited electrolytically.

The current density is about 1.5 amp per sq dm (14 amp per sq ft) of cathode surface, and the voltage per cell from 2.5 to 3.5 volts.

Moebius Process. Silver is separated from small amounts of other metals by the Moebius process. In this process, the anodes consist of the impure silver, the cathodes of thin sheet silver, and the electrolyte is a slightly acid dilute silver nitrate solution. In the earlier type of apparatus the cathodes were stationary; in the later type the cathode is a rotating sheet of silver. In both the pure silver is scraped off as crystals.

The current density is about 3 amp per sq dm (28 amp per sq ft) of cathode surface, and the voltage per cell from 1.4 to 1.5 volts. Another type is the Balbach-Thum cell (*Met. and Chem. Eng.*, Vol. 9, p. 444, 1911).

GOLD REFINING is carried out in a slightly acid solution of gold chloride. Gold anodes do not dissolve in a solution of gold chloride, $AuCl_3$; or of chloroauric acid, $HAuCl_4$, but the chlorine liberated comes off in the gaseous form. If some free alkali chloride or hydrochloric acid is present, the gold dissolves. The resulting gold is never less than 999.8 fine, and frequently is 1000 fine.

The baths contain about 3 per cent of free hydrochloric acid with 30 to 40 grams of gold as chloroauric acid per liter. The bath is heated to 60 or 70 deg cent, and the current density is about 10 amp per sq dm (93 amp per sq ft) at about 1 volt.

LEAD REFINING is carried out by the Betts process. The electrolyte is a solution of lead fluosilicate ($PbSiF_6$) containing 60 to 70 grams of lead and 120 to 130 grams of SiF_6 per liter, and 0.1 per cent of glue. The object in refining lead is to recover the copper, antimony, bismuth, gold, and silver. Tin cannot be separated from the lead electrolytically, on account of its proximity to lead in the electrolytic series.

The current density ranges from 1.3 to 1.7 amp per sq dm (12 to 16 amp per sq ft) of cathode surface, and the voltage per cell from 0.30 to 0.38 volt.

IRON REFINING has been carried out by different companies, as at Hawthorne, Ill., by the Western Electric Co., and at Grenoble, but has been given up at both places. (*J. Ind. & Eng. Chem.*, 1912, p. 752; *J. Iron & Steel Inst.*, Vol. 90, pp. 66–80, 1914; *Trans. Am. Electrochem. Soc.*, Vol. 29, pp. 357–367, 1916.)

TIN REFINING was carried out by the American Smelting and Refining Co. at Perth Amboy, N. J., but was given up several years ago. The solution contained 6 to 8 per cent sulfuric acid, 3 per cent of stannous tin, and 4 to 8 per cent of cresol-phenol sulfonic acid.

4. ELECTROLYTIC EXTRACTION OF METALS

The attempts that have been to extract metals directly from their ores or from mattes are based on the same principle as that underlying metal refining. These attempts have not been successful until recently, however, on account of the large amount of impurity that gets into the bath and on account of mechanical difficulties. The best known of such attempts are the following.

MARCHESE PROCESS. In the Marchese process it was attempted to obtain pure copper from a matte containing principally copper, lead, iron, and sulfur, by electrolyzing this matte as an anode in a sulfate bath. The process seemed a success at first, but finally failed.

SIEMENS AND HALSKE PROCESS. The Siemens and Halske process for extracting copper consists in electrolyzing a solution of iron sulfate and copper sulfate in a cell in which the anode and cathode are separated by a linen diaphragm. The anode was carbon; the cathode pure copper. In the cathode compartment the solution loses copper by deposition on the cathode. The solution then circulates to the anode compartment, where the ferrous sulfate is oxidized to ferric sulfate. See below, under Electrolytic Oxidation. From here the solution circulates to another vat where fresh ore, consisting of copper pyrites ($Cu_2SFe_2S_3$) that has been roasted so as to change the iron sulfide to oxide, is treated. The ferric sulfate dissolves the cuprous sulfide as follows:

$$Cu_2S + 2Fe_2(SO_4)_3 = 2CuSO_4 + 4FeSO_4 + S$$

This solution, enriched in copper, is then conveyed to the cathode compartment, and the cycle is completed. This process also failed when tried on a commercial scale.

HOEPFNER PROCESS. In the Hoepfner process the copper ore, consisting of copper sulfide, is dissolved in a cupric chloride solution containing a relatively large amount of sodium chloride. The reaction is as follows:

$$Cu_2S + 2CuCl_2 = 4CuCl + S$$

The cuprous chloride, insoluble in water, is held in solution by the sodium chloride. The anode and cathode of the electrolytic cell are separated by a diaphragm. The solution first circulates to the cathode compartment, where part of the copper is deposited, and then to the anode, where the cuprous chloride is oxidized to cupric chloride. The solution from the anode then circulates to fresh ore.

COPPER HYDROMETALLURGY AT CHUQUICAMATA. A process is being used by the Chile Exploration Co. for extracting copper directly from the ore of the Chuquicamata copper mine, in Chile. The ore is dissolved in dilute sulfuric acid in leaching tanks. The solution is then conducted to the electrolyzing tanks where the copper is deposited, using insoluble anodes and copper cathode starting sheets. Originally fused magnetite anodes were used, but these were replaced by *duriron* (ferrosilicon containing 12 per cent silicon), and duriron has been replaced by an alloy of copper, iron, and silicon.

COPPER HYDROMETALLURGY AT KATANGA AND MIAMI, ARIZONA. A similar process is used at Katanga in Central Africa. (F. Kroll, *Metall und Erz*, Vol. 25, p. 154, 1928.) A plant of the Consolidated Copper Co. at Miami, Arizona, leached mixed sulfide and oxide ore with acid ferric sulfate, and electrolyzed with lead anodes. (Van Arsdale, *Trans. A.I.M.E.*, Vol. 73, p. 58, 1926.)

SALOM PROCESS. Lead was at one time extracted from galena (PbS) by the Salom process, consisting in electrolyzing the powdered galena as cathode in sulfuric acid. The hydrogen deposited on the galena combines with the sulfur, forming hydrogen sulfide and leaving the lead as lead sponge. This process was given up, partly, at least, on account of the poisonous action of the hydrogen sulfide. (*Trans. Am. Electrochem. Soc.*, Vol. 1, p. 87, 1902; Vol. 4, p. 101, 1903.)

ELECTROLYTIC ZINC. Within recent years a large quantity of high-grade electrolytic zinc has been produced in the United States and Canada by a process similar to that used for copper at Chuquicamata. Sulfate solutions are used, with lead anodes and aluminum cathodes, from which the zinc is stripped in sheets $^1/_{16}$ in. thick. The solution must be free from impurities in order to get good deposits. See Hansen, *Bull. Am. Inst. Mining Eng.*, Vol. 135, p. 615, 1919; T. French, *Trans. Am. Electrochem. Soc*, Vol. 32, pp. 321–328, 1917; *Met. and Chem. Eng.*, Vol. 18, pp. 549–550, 1918.

DETINNING TIN SCRAP AND OLD CANS. The two principal methods of recovering tin from tin scrap and old tin cans are due to Goldschmidt. The first consists in

electrolytically dissolving the tin and depositing it in the metallic state; the second method, which, however, is not electrolytic, converts the tin into tin tetrachloride by treating with dry chlorine.

In the electrolytic method the solution consists of a sodium hydrate solution, containing a certain amount of carbonate, absorbed from the air, and sodium stannate. The best concentrations are: 10 to 12 per cent alkalinity, not over 7 per cent free alkali; carbonate not over 3 per cent; stannate not over 5 per cent. Temperature, 60 to 70 deg cent. About one-tenth of the solution must be replaced every week.

The tin scrap is suspended in the solution in baskets (39 by 35 by 18 in.) of perforated sheet iron, containing 100 to 130 lb of scrap. Old cans are cleaned, compressed, and then cut up. The cathodes are sheet iron 39 by 35 inches. The tin dissolves from the anode and is deposited as spongy tin on the cathode. The scrap is left in the bath for 3 hours. The spongy tin is compressed by a hydraulic press into cylinders weighing 6 lb; these are subsequently melted in a furnace in sealed tubes. This sponge contains 50 per cent tin and 50 per cent ash. The ash is subsequently reduced with carbon in an open-hearth furnace, yielding 70 per cent of the tin.

Very careful working is necessary to remove the tin so completely that the iron can be sold to open-hearth plants. Even then the iron contains 0.1 to 0.5 per cent tin.

The cathode current density is never over 0.75 amp per sq dm (7 amp per sq ft), at 2 to 3 volts per cell. (*Electrochem. and Met. Ind.*, Vol. 7, p. 79, 1909; *Met. and Chem. Eng.*, Vol. 10, p. 202, 1912.)

5. ELECTROLYTIC OXIDATION AND REDUCTION

Electrolytic oxidation and reduction are frequently carried out on substances in solution. One of the advantages of the electrolytic method is that no other substance has to be added to the solution. In carrying out an electrolytic oxidation or reduction a porous diaphragm is used to separate the anode and cathode compartments. The substance to be reduced is placed in the cathode compartment, where the whole or a part of the hydrogen that would be evolved by the current while in the nascent state acts on the substance in question. The intensity of the reduction may be varied by varying the potential difference between the solution and the cathode. This is accomplished either by increasing the current density on the cathode, or by making the cathode of different metals on which the overvoltage (see Principles of Electrochemistry) is different. It frequently happens that the metal composing the cathode has a marked accelerating effect on the velocity of the reduction. Thus the reduction may be much more complete on one metal than on another, though the potential difference between the metals and the solution is the same in both cases. The same considerations apply to oxidation on the anode.

TYPICAL REDUCTION AND OXIDATION PROCESSES. Examples where electrolytic reduction has been found useful are: in the manufacture of white lead by corroding a lead anode in a suitable solvent, e.g., the Sperry process; in the preparation of the lower salts of vanadium, molybdenum, and titanium. Examples where electrolytic oxidation has been employed are: in the oxidation of the lower cerium sulfate, $Ce(SO_4)$, to $Ce(SO_4)_2$, the latter being useful as an oxidizing agent; in oxydizing potassium manganate, K_2MnO_4, to permanganate, $KMnO_4$; in the oxidation of potassium ferrocyanide, $K_4Fe(CN)_6$, to the ferricyanide, $K_3Fe(CN)_6$; and of chromium sulfate to a chromate.

Sodium permanganate has been made on a semi-commercial scale by electrolyzing ferro-manganese anodes in sodium carbonate or hydrate solutions (*Trans. Am. Electrochem. Soc.*, Vol. 35, pp. 371–384, 1919). Potassium permanganate has been made in the same way (*Chem. and Met. Eng.*, Vol. 21, pp. 680–681, 1919).

6. ELECTROLYSIS OF SODIUM OR POTASSIUM CHLORIDE

The electrolysis of sodium or potassium chloride may yield several different products, depending on the kind of cell employed. In the following discussion the reactions for sodium chloride are given; these reactions also apply to potassium chloride when K is substituted for Na.

PRODUCTION OF ELECTROLYTIC BLEACH. If there is no diaphragm and the solution is kept cool, the chlorine and hydrate react on each other to form principally sodium hypochlorite or "electrolytic bleach," according to the equation

$$2NaOH + Cl_2 = NaOCl + NaCl + H_2O$$

As soon as hypochlorite is formed, it begins to change to chlorate, but at first not as rapidly as it is formed. As the concentration of the hypochlorite increases a larger and

larger proportion changes to chlorate, until finally the concentration of the hypochlorite reaches a limit, which depends on the temperature, original salt concentration, current density, and other factors. The hypochlorite then changes to chlorate as rapidly as it is formed.

The reactions by which hypochlorite changes to chlorate are two: (1) the action of the discharged ClO ion on water:

$$6ClO + 3H_2O = 2HClO_3 + 4HCl + 1^1/_2\, O_2$$

and (2) in a slightly acid solution:

$$2HOCl + NaOCl = NaClO_3 + 2HCl$$

If the solution is warmed to about 50 deg cent the conditions are so changed that the limiting concentration of hypochlorite is much lower than when cold. Sodium and potassium hypochlorites are used only in solution, but the chlorates are crystallized out.

By further electrolysis of a cooled sodium (or potassium) chlorate solution, perchlorate, $NaClO_4$, is produced by the action of the liberated ClO_3 ion on water:

$$2ClO_3 + H_2O = HClO_4 + HClO_3$$

and

$$HClO_4 + NaOH = NaClO_4 + H_2O$$

In most of the cells used for producing hypochlorite, a vessel is divided into a number of narrow compartments by bipolar or intermediate electrodes, consisting of carbon plates (Hass and Oettel) or platinum-iridium wire wound on glass plates (the Kellner cell). The solution is cooled by circulation. There is no essential difference between a hypochlorite and a chlorate cell, except that the latter is used at a higher temperature. (See numerous volumes in the Engelhardt Monographs.)

PRODUCTION OF CHLORINE AND CAUSTIC. If the anode and cathode are separated by a diaphragm of some kind, sodium hydrate is produced at the cathode by the removal of hydrogen ions, leaving an excess of hydroxyl ions, while at the anode chlorine is set free, which may be used to make bleaching powder.

Various devices are used to separate the hydrate and chlorine in cells intended for these products. In the McDonald cell (*Electrochem. Ind.*, Vol. 1, p. 387, 1903) and others a porous diaphragm is used. In the Hargreaves-Bird cell (U. S. Pat. 655,343, 1900; 596,157, 1897), the porous diaphragm is supported by a heavy copper wire screen cathode, which is wetted only by the solution percolating through the diaphragm. In the "bell jar process" the anode is inside an inverted nonconducting, nonporous bell jar, and the cathode is a conducting ring outside. Two important diaphragm cells are the Allen-Moore cell and the Nelson cell (*Trans. Am. Electrochem. Soc.*, Vol. 35, pp. 239–249, 1919.)

In the Castner cell the compartments are separated by a mercury diaphragm, acting as an intermediate electrode; and in the Solvay and the Whiting cells a mercury cathode is used, which, when charged with sodium in the electrolytic cell, is decomposed by water in a different compartment. The sodium amalgam reacts with water like metallic sodium. (See the Engelhardt Monographs.)

7. ELECTROLYSIS OF WATER

The electrolysis of water is carried out on a commercial scale for the production of hydrogen and oxygen. The electrolyte is usually a 15-per cent sodium hydrate solution. The cells generally use asbestos cloth diaphragms to prevent the mixing of hydrogen and oxygen.

In the cells of the Electrolabs Co. of Pittsburgh the anodes and cathodes are cobalt plated. The International Oxygen Co., of Newark, N. J., uses cast-iron nickel-plated anodes and cast-iron cathode.

Recently attempts have been made to produce hydrogen and oxygen at high pressure by electrolysis. (Noeggerath, *Zeitschr. d. V. deutscher Ing.*, Vol. 72, p. 373, 1928.)

8. OZONE

Ozone (O_3) is made by the silent discharge of electricity through dry air. The electrodes are water cooled, and protected to prevent the passage of sparks. From 8000 to 50,000 volts alternating are applied, this voltage being obtained from a small step-up transformer. For the Siemens and Halske ozonizer the concentration of the ozone is about 2 or 3 grams per cubic meter of the air which passes through it, and the yield is from 18 to 37 grams per kilowatthour. (*Electrochem. Ind.*, Vol. 2, p. 67, 1904.) Milk has been sterilized by high-tension discharges; one process being the Shelmerdine process

(*Electrical Times*, Vol. 45, pp. 502–505, 1914). Ozone is used for sterilizing the air in cold-storage warehouses and in public buildings.

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ELECTRIC FURNACES AND THEIR PRODUCTS

By M. de K. Thompson

10. CLASSIFICATION

Electric furnaces may be classified as follows:

Arc furnaces.
- *a.* High-tension.
- *b.* Low-tension.

Induction furnaces.
- *a.* High-frequency.
- *b.* Low-frequency.

Resistance furnaces.
- *a.* Current conducted by the materials heated:
  1. With electrolysis.
  2. Without electrolysis.
- *b.* Current conducted by a special resistor.

In the arc furnace the heat is produced by an electric arc (see Electric Arc) usually between carbon electrodes. The low-frequency induction furnace is essentially a static transformer (see Transformers) with the low-tension "winding" formed by the material to be heated. In the high-frequency furnace, the heating is due to eddy currents. In the resistance furnace the current is supplied to the material to be heated (i.e., to the furnace "charge"), or to the special resistor, by connecting the charge or resistor directly to the source of the current supply. The heat developed in both the induction and resistance furnaces arises from the passage of the current through the resistance offered by the charge or special resistor.

The arc furnace may be considered a resistance furnace in which the resistor is a gas. Since the resistance of a gas at atmospheric pressure is greater than that of any solid resistor of the same dimensions as the arc, the amount of heat that can be produced in a small space will be greatest with an arc furnace.

11. TEMPERATURE AND DISTRIBUTION OF HEAT

The advantage in electric heating is that a higher temperature can be produced than by using fuel, that the heat is produced inside the furnace where it is needed, and that the heat can be easily and accurately regulated. In the arc the hottest part of the positive carbon is estimated to be between 3900 and 4000 deg cent absolute. (Waidner and

Burgess, *Bull. Bur. Standards*, Vol. 1, p. 123, 1905.) The temperature of the arc itself increases with the current. (Kayser, Handbuch der Spektroscopie, Vol. 1, pp. 154–160, 1900. Chaney, Hamister and Glass, *Trans. Am. Electrochem. Soc.*, 1935, Vol. 1, p. 67, preprint 18.)

The electric energy delivered to the furnace as heat is used as follows: (1) to heat the charge to the desired temperature, which involves heating up the furnace walls, if cold at the start. This energy is equal to the mass of the charge times the temperature rise times the specific heat. If the charge is melted or vaporized in the furnace, additional heat must also be supplied.. This item may be reduced by delivering the charge to the furnace already hot as in steel refining. (2) To supply the energy needed for the reaction; the energy so required cannot be reduced. (3) To supply the loss due to conduction and radiation through the walls and electrodes, and the heat carried off by hot gases. A part of the heat carried off by hot gases may be recovered in heating water in boilers, so that it is not a complete loss.

12. MAXIMUM SIZE OF ELECTRIC FURNACES

The largest workable capacity of an open-arc single hearth with a compact bundle of electrodes has been found in practice to be from 2500 to 3000 kw at 30,000 to 40,000 amp and 75 to 90 volts. These large sizes are always used with the three-phase system, so that the maximum total power absorption of a furnace is from 7500 to 9000 kw. In carbide furnaces double three-phase furnaces are used with six electrodes, in place of three, in the same shaft, and the power absorbed is from 15,000 to 18,000 kw. Great progress has been made in electric-furnace construction, by closing the furnace at the top and by having special means of feeding in the charge, thereby avoiding the dust nuisance and protecting the workmen from the heat, as well as distributing the charge more uniformly. (Taussig, *VII Int. Cong. App. Chem.*, Sec. 10, p. 24, 1910; *Trans. Faraday Soc.*, Vol. 5, p. 254, 1909; *VIII Int. Cong. App. Chem.*, Vol. 21, p. 105, 1912.)

13. DESIGN OF FURNACE WALLS

To reduce the conduction of heat through the walls and electrodes to a minimum amount, these must be properly designed. The heat flow through the walls of three different shapes may be computed by the formulas below. In all cases

k = thermal conductivity of the walls, at the mean temperature $(t_1 + t_2)/2$. This coefficient k may be expressed in any convenient unit, e.g., gram calories per centimeter cube per second per degree centigrade, or watts per inch cube per degree centigrade. See Heat and Thermal Properties for values.

H = total heat conducted per second through the walls.

t_2 = temperature of the inside surface of the wall.

t_1 = temperature of the outside surface of the wall.

HOLLOW RECTANGULAR PARALLELEPIPED.

$$H = \left(\frac{A}{\vartheta} + 0.54\,\Sigma l + 0.15 n\vartheta\right) k(t_2 - t_1)$$

where A = the area of the six inner surfaces, ϑ = thickness of wall, Σl = the total length of all the inner edges, n = the number of corners. This applies where all three inner dimensions are greater than $^1/_5\,\vartheta$. (Langmuir, Adams, and Meikle, *Trans. Am. Electrochem. Soc.*, Vol. 14, p. 53, 1914.)

HOLLOW SPHERE.

$$H = \frac{\pi k D d\,(t_2 - t_1)}{l}$$

where D = outside diameter of sphere, d = inside diameter, l = thickness of wall, all in the same unit.

HOLLOW CYLINDER.

$$H = \frac{2\pi k L\,(t_2 - t_1)}{2.3 \log_{10} \dfrac{D}{d}} + \frac{\pi k D d\,(t_2 - t_1)}{2\,l}$$

where L = mean height of inner and outer walls, D = outside diameter, d = inside diameter, and l = thickness of top and bottom walls, all in the same unit. The first term gives the flow of heat through the cylindrical walls, the second the flow of heat through the top and bottom. (Hering, *Trans. Am. Electrochem. Soc.*, Vol. 14, p. 215, 1908.)

LININGS AND COMPOSITE WALLS. The most refractory substances do not have the lowest thermal conductivities. Consequently, it is advantageous to use a highly refractory substance only for the inner part or lining of the walls, using only such a thickness that the drop in temperature will be sufficient to permit of a less refractory substance of a lower conductivity being used for the next layer. "Graded" walls of several layers may be employed. See Ray and Kreisinger, Flow of Heat through Furnace Walls, Bull. 8, Dept. of the Interior, 1912.

REFRACTORIES FOR FURNACE WALLS. The most refractory substance is carbon, which, however, is a good heat conductor. Some of the products of the electric furnace, as silicon carbide and firesand (a substance containing varying amounts of carbon, silicon, and oxygen) stand next to carbon as refractories, and do not have such high heat conductivities. (See Fitzgerald, *Electrochem. Ind.*, Vol. 2, p. 439, 1904.) The numerical values of heat conductivities of refractories at high temperatures are known for only a few substances and then only approximately. See article on Heat and Thermal Properties; also article by Hering in *Met. and Chem. Eng.*, Vol. 9, p. 625, 1911, and the table below for graphite and carbon.

14. DESIGN OF ELECTRODES

The electrodes of an electric furnace should be so designed that the energy will be carried into the furnace with a minimum energy loss. The loss due to the electrical resistance is directly proportional to the electrode's length; this should therefore be made as short as convenient. The loss due to electrical resistance will be smaller the greater the cross-section of the electrode, but the heat loss from the furnace through the electrode will be directly proportional to the cross-section. It is therefore possible to find a cross-section of a given material which will give a minimum total loss for a given length. The cross-section that would give the minimum loss on certain assumptions, only approximately true, is found from the equation

$$S = 0.346\,LI\sqrt{\frac{r}{k(t_2 - t_1)}}$$

and the loss itself, watts, is

$$h = 2.89\,I\sqrt{kr\,(t_2 - t_1)}$$

where S = cross-section of electrode; L = its length; I = the current in amperes carried by the electrode; t_2 and t_1 the temperatures, in degrees centigrade, of the hot and cold ends of the electrode respectively; r = its mean electrical resistivity in ohms per unit cube; and k = its mean heat conductivity for the mean temperature $(t_1 + t_2)/2$. If S and L are in centimeters, r and k must be per cubic centimeter; if S and L are in inches, r and k must be per cubic inch.

The values of k and r for carbon and graphite are not known accurately at high temperatures. The following values have been computed from measurements of Hering (*Trans. Am. Electrochem. Soc.*, Vol. 17, p. 166, 1910).

Material	Temperature, ° C		Thermal Cond. g-cal per cm³ per ° C per sec	Electrical Resistivity, ohms per cm³
	Hot End	Cold End		
Carbon	300	40	0.0891	0.00422
	701	50	0.124	0.00381
	902	60	0.130	0.00377
Graphite	355	66	0.399	0.000837
	516	70	0.325	0.000827
	707	87	0.309	0.000802

NOTE: For other data see the International Critical Tables.

15. SMALL LABORATORY FURNACES

A great variety of electric furnaces have been devised. A few typical laboratory furnaces will be described in this article. Some industrial furnaces are described in the next article.

Moissan's Furnace (Fig. 1). Moissan's work was carried out in a furnace consisting of two horizontal electrodes, mounted so that the distance between the two ends could be adjusted longitudinally by a screw thread. An arc was formed between these electrodes in a cavity formed by some refractory material, such as lime. The substance to

be heated was placed in a crucible under the arc as shown in Fig. 1. When the substance was not to be exposed to the gases of the arc, a furnace was made with a carbon tube passing through it at right angles to the electrodes and immediately below the arc. The substance to be heated was placed inside this tube.

BORCHERS' FURNACE (Fig. 2). A type of furnace due to Borchers consists in a carbon rod placed between two larger electrodes. The charge is either packed around

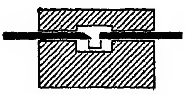

Fig. 1. Moissan's Furnace

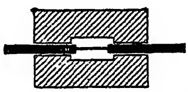

Fig. 2. Borchers' Furnace

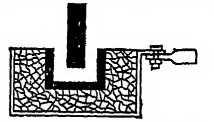

Fig. 3. Héroult Furnace

the small rod or placed under it. This type of furnace is convenient where a temperature below that of the arc is desired.

HÉROULT FURNACE (Fig. 3). A furnace that takes its name from Héroult consists in a crucible with one or more electrodes connected together above the crucible. The crucible is represented in Fig. 3 packed in carbon in an iron container. Graphite crucibles may be turned out from graphite electrodes. The charge in the crucible is usually melted by forming an arc between the crucible and the electrode above with an adjustable resistance in series with the arc. After the charge has melted the electrode may be partly immersed in the bath. In case it is desired to melt a substance in a crucible without the use of an arc, a smaller piece of carbon may be placed between the crucible and the electrode as in Borchers' furnace. If after the substance has been melted it is desired to pass the current through the bath itself, as, for example, if a salt is to be electrolyzed, the upper electrode may be raised, the thin rod removed with a pair of tongs, and the electrode then lowered into the bath. The salt will not solidify during this operation.

ARSEM FURNACE (Fig. 4). It frequently happens in the laboratory that it is desired to heat a substance to a high temperature in a vacuum or in some pure gas, such as hydrogen or nitrogen. A very convenient furnace for this purpose has been designed by Arsem (*Trans. Am. Electrochem. Soc.*, Vol. 9, p. 153, 1906), and has been extensively employed in research work. This furnace may be obtained from the General Electric Co. in two sizes. A vertical section is shown in Fig. 4. It consists of a chamber *A* and cover *B* made of a gun-metal casting turned true at the joint. A lead gasket *C*, 1/16 in. thick, forms an air-tight joint when the cover is fastened down by the cap-screws *D*. The tube *J* through which the air is removed from the furnace is soldered into the cover. The window *E* is a disk of clear white mica 0.005 in. thick clamped between lead washers *F*.

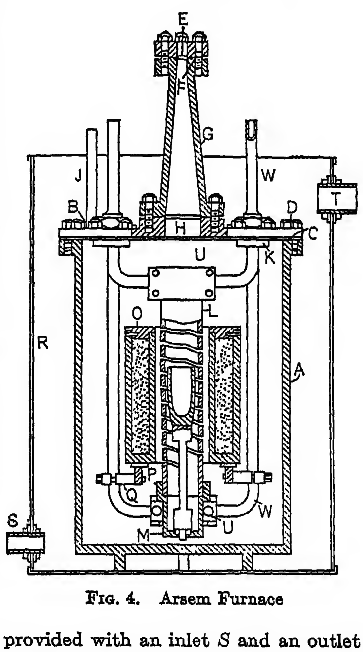

Fig. 4. Arsem Furnace

The electrodes *W* are brass tubing which are insulated from the cover. The clamps *UU* for holding the heater are copper. The heater *L* is a helix, usually of graphite, which is made by boring out a graphite electrode and cutting it along a spiral as shown. Metallic heaters may also be used. The lower end of the heater rests in the graphite cup which also holds the crucible support insulated from it by a lava ring. The screen for preventing radiation is a double-walled cylindrical box of Acheson graphite filled with graphite powder.

The water jacket *R* is a galvanized-iron tank provided with an inlet *S* and an outlet *T*. In a vacuum in the small furnace 9 to 10 kw produce a temperature of 2000 deg cent.

HIGH-PRESSURE FURNACES. The Arsem furnace may, of course, be used with the internal pressure above an atmosphere, but it is not designed for high pressures. A furnace for working up to 200 atmospheres has been designed by Hutton and Petavel. (*Phil. Trans.*, Series A, Vol. 207, p. 421, 1908, and *Electrochem. and Met. Ind.*, Vol. 6, p. 97.)

For a modified form see Pring and Fairlie (*VIII Int. Cong. App. Chem.*, Vol. 21, p. 79, 1912). For a furnace for spectroscopic work on gases at pressures up to 200 atmospheres, see King, A. S., *Astrophys. J.*, Vol. 28, p. 300, 1908.

FURNACES WITH METALLIC RESISTORS (Fig. 5). A very useful type of furnace is a tube wound in its middle portion with an electrical resistor. The ends of the tube are cooled by the air, or may be cooled by a coil of copper pipe through which water flows. The tube may be of some non-conducting substance, such as porcelain, in which case a ribbon of metal may be wound directly on the tube. Furnace tubes with grooves for winding with wire are now made by the Norton Co. of Worcester, Mass., from fused alumina. These, however, are porous and cannot be used for a vacuum furnace. Glazed German porcelain may be heated up to 1180 deg cent and a vacuum maintained. At temperatures higher than for this glazing melts and air leaks into the tube. A nickel tube may be used for higher temperatures. Muffle furnaces and pot furnaces are made on the same principle.

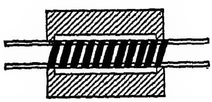

Fig. 5. Resistor Furnace

The metallic winding for carrying the current may be of platinum, nickel, nichrome, tungsten, molybdenum, or any other suitable high-resistance material (see Resistance Wires).

These furnaces are particularly useful when the temperature is to be held constant over a long period of time. For most purposes a constant current will keep the temperature sufficiently constant. For greater constancy some kind of regulator must be used (see Zabel & Hancox, *Rev. Sci. Instr.*, 1934, Vol. 5, p. 28; Henney, Electronics, 1934, Vol. 7, p. 183.

HOSKINS FURNACE. A convenient furnace for heating rather large crucibles or masses of material is made by the Hoskins Manufacturing Co. of Detroit, Mich. The heater consists of two rows of narrow, thin carbon plates which extend over two sides of the cavity which receives the substance to be heated. The contact resistance between the plates may be varied by pressure. This furnace of course requires a very large current at a low voltage.

HIGH-FREQUENCY FURNACES. These are made by the Ajax Electrothermic Corp., of Trenton, N. J. The sizes of spark-gap type are 3, 15, and 35 kv-a. Furnaces are built for use with motor generator sets from 20 to 1500 kw.

GRANULAR CARBON FURNACE (Fig. 6). A carbon or graphite crucible may be easily heated by placing it in a trough surrounded by granular gas carbon; this conducts better than coke. The current is passed through the trough, into which it is conducted by carbon rods. Clay crucibles should not be used in this way for temperatures above 1000 deg cent, as at a high temperature they are attacked by the carbon.

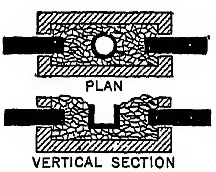

Fig. 6. Granular Carbon Furnace

GLOBAR FURNACE. The Globar Corp. of Niagara Falls, N. Y., furnished silicon carbide rods of various sizes for use as heating elements. They can be used up to 1200 deg. cent. or more if protected from the fumes of volatile materials.

16. ELECTRIC-FURNACE PRODUCTS AND INDUSTRIAL FURNACES

Some typical industrial furnaces and their products are described below.

CALCIUM CARBIDE. Willson and Horry furnaces. Calcium carbide was first made on a large scale by Willson in 1892 at Spray, N. C., by heating lime and carbon in an arc furnace. The reaction that takes place is

$$CaO + 3C = CaC_2 + CO$$

The furnace consisted of a carbon plate 3 by 2.5 ft with a carbon electrode suspended above it. The whole was surrounded by brick walls. Furnaces similar to this were at first used at Niagara Falls, with the lower electrodes mounted on a car which was removed when filled with an ingot of carbide. Later the Horry rotary furnace was used (U. S. Pat. 656,156). Carbide may be either formed in an ingot in the crucible of the furnace and removed solid, or it may be drawn off in the liquid state. When an ingot is formed it has been found better to have the current flow between two electrodes suspended over the crucible, in place of having the crucible form one electrode. (Conrad, *Electrochem. and Met Ind.*, Vol. 6, p. 397, 1908.) In practically all carbide furnaces now used the carbide is tapped. For the latest types, see Taussig, *Die Industrie des Kalziumkar-*

bides, 1930. The purity of the carbide is in the neighborhood of 80 per cent. The yield of 80 per cent carbide is about 5 kg per 24 kw hr. Söderberg self-baking continuous electrodes are used extensively in carbide furnaces. (*Trans. Electrochem. Soc.*, 1931, Vol. 60, p. 181).

SILICON PRODUCTS. Silicon Carbide. Acheson Process. Silicon carbide or carborundum was first made on a large scale by Acheson. It is produced from quartz and carbon when these substances are heated in an electric-resistance furnace, according to the reaction

$$SiO_2 + 3C = CSi + 2CO$$

The furnace has a granular carbon core around which the charge, consisting of quartz, carbon, sawdust, and sodium chloride, is packed. The latest furnaces are 9.15 meters long by 3.67 meters wide, and absorb 1600 kw. The current is 20,000 amp (*Min. Industry*, Vol. 16, p. 155, 1907; Vol. 17, p. 112, 1908). From measurements on a 750-kw furnace it was found that the carbide is formed at 1840 deg cent and decomposes when heated above 2240 deg cent. (Saunders, *Trans. Am. Electrochem. Soc.*, Vol. 21, p. 425, 1912.) The yield is about 1 kg of crystallized carbide for 8.5 kwhr. Silicon carbide is used as an abrasive and for furnace linings. (Fitzgerald, Carborundum, Vol. 13, in the Engelhardt Monographien über angewandte Elektrochemic.) For description of Norton Co.'s plant see Fitzgerald, *Met. and Chem. Eng.*, Vol. 10, pp. 519–521 (1912).

Firesand, formerly called siloxicon, is a product of the incomplete reduction of silica, and may be represented by the formula SiCO, though compounds with varying proportions of these elements are found. Firesand is used for crucible linings. It is made by heating silica and an insufficient quantity of carbon for complete reduction of the silica.

Silox is a spongy substance of about the same composition as firesand, made by the General Electric Co., by heating silica and carbon in a closed arc furnace and condensing the distillate in a large chamber. It is used for heat insulating.

Silicon is made in an arc furnace from coke and quartz. At the high temperature produced the silica is completely reduced and the melted metal is drawn off in amounts weighing from 600 to 800 lb. It varies from 90 to 97 per cent in purity (F. J. Tone, *Electrochem. and Met. Ind.*, Vol. 7, p. 192, 1900; *Min. Ind.*, Vol. 17, p. 768, 1908).

GRAPHITE. Berthelot (*Ann. de chim. et de phys.*, Series 4, Vol. 19, p. 393, 1870) defines graphite as that allotropic form of carbon which when oxidized at low temperature with powerful oxidizing agents (potassium chlorate and nitric acid) gives graphite oxide. Arsem suggests as a definition for graphite that it is that allotropic form of carbon whose density lies between 2.25 and 2.26. (*Trans. Am. Electrochem. Soc.*, Vol. 20, p. 105, 1911.)

Graphite is made by heating carbon, containing a small amount of impurity, to a high temperature in an electric resistance furnace. Anthracite coal is graphitized in bulk; electrodes and crucibles are also graphitized after molding. Acheson's theory of the formation of graphite is that the carbon first forms a carbide, which decomposes at a higher temperature, leaving the carbon in the form of graphite. This theory is not confirmed by Arsem's experiments. (See Fitzgerald, Künstlicher Graphit, Vol. 15 of the Engelhardt Monographien über angewandte Electrochemie.)

CARBON BISULFIDE is made in a specially designed furnace by heating together sulfur and carbon. Most of the disagreeable features encountered in the manufacture of this substance are thus avoided. (Taylor, *Trans. Am. Electrochem. Soc.*, Vol. 1, p. 115, 1902; Vol. 2, p. 185, 1902; Richter; *Trans. Am. Electrochem. Soc.*, 1922, Vol. 42, p. 253.)

PHOSPHORUS is a substance the production of which the use of the electric furnace has much simplified. It is made in the Readman-Parker furnace according to Wöhler's process:

$$Ca_3(PO_4)_2 + 3SiO_2 + 5C = 3CaSiO_3 + 5CO + 2P$$

(*Min. Ind.*, Vol. 6, p. 537, 1897; Vol. 7, p. 557, 1898.) Phosphoric acid is made by oxidizing the gaseous phosphorus with air, as it comes from the furnace, and absorbing the oxide in water.

ALUNDUM is the trade name of fused aluminum oxide, which is made by the Norton Co. of Worcester. Aluminum oxide is fused in an arc furnace. (*Min. Ind.*, Vol. 19, p. 28, 1910.) Fused alumina is used as an abrasive, as a refractory substance for furnace linings, and porous crucibles of this substance are used in analytical laboratories.

ALUMINUM. HALL AND HÉROULT PROCESSES. Aluminum is now produced by the electrolysis of a solution of alumina in fused cryolite ($AlF_3 \cdot 3NaF$) to which other fluorides, such as those of aluminum and of sodium, are added in some factories. The aluminum sinks to the bottom of the crucible and is drawn off. This process was discovered nearly simultaneously by C. M. Hall and Héroult. The heat developed by the current in passing through the solution is sufficient to keep the bath melted. The cathode is an iron trough lined with carbon, and the anode consists of a number of carbon rods

suspended over the crucible. The Aluminum Co. of America uses as anode for one crucible 48 carbon rods 3 in. in diameter and 15 in. long. The electromotive force applied to each crucible is 5.5 volts; the current is 10,000 amp. The yield is 1.75 lb of aluminum per horsepower-day. For further information see *Min. Ind.*, Vols. 6, 14, 15, 17, 20, Thompson & Seyl, *Trans. Electrochem. Soc.*, 1933, Vol. 64, p. 321. Aluminum is also refined by a similar process, using an anode of chromium-copper-silicon alloy. (Frary, *Trans. Am. Electrochem. Soc.*, 1925, Vol. 47, p. 275.)

SODIUM, POTASSIUM. CASTNER PROCESS. Sodium and potassium are obtained by the electrolysis of their fused hydrates, usually in the cell designed by Castner (U. S. Pat. 453,030, filed 1890; see also Becker, Die Elektrometallurgie der Alkalimettale). The production of sodium by the electrolysis of fused sodium chloride, containing additions which lower its melting point 200 to 300 deg cent, is now carried out, e.g., in the Downs cell. (U. S. Pat. 1501, 756, July 15, 1929.)

CALCIUM. Calcium is made by the electrolysis of fused calcium chloride, to which calcium fluoride is added to lower its melting point. Calcium is made at Holcomb's Rock, Va., probably in a cell devised by Seward and Von Kügelgen (U. S. Pat. 880,760, 1908; *Min. Ind.*, Vol. 16, p. 131, 1907; Vol. 17, p. 99, 1908); and abroad by the method of Rathenau (*Z. f. Elektrochem.*, Vol. 10, p. 508, 1907). In the Rathenau method the cathode is an iron rod which just touches the surface of the melted calcium chloride. The calcium solidifies when deposited on the cathode by electrolysis. As the calcium grows the rod is withdrawn so that a rod of calcium is produced. (*Trans. Am. Electrochem. Soc.*, 1920, Vol. 37, p. 465.)

BARIUM. Barium cannot be made electrolytically, because metallic barium acts on all of its fused salts. A lead alloy of barium can be made by electrolyzing barium chloride with a melted lead cathode.

ZINC. Zinc may be made by the electrolysis of fused zinc chloride. In the Swinburne-Ashcroft process sodium chloride is added to zinc chloride in such quantity that the resulting mixture contains 28 per cent zinc. The cell is a brick-lined, sheet-iron vessel. The anode is carbon, the cathode, melted zinc. Each vat takes 4.5 volts, with a cathode current density of 400 amp per sq ft. The temperature of the fused salt is 450 deg cent. (*Electrochem. and Met. Ind.*, Vol. 3, p. 65, 1905.)

MAGNESIUM. Magnesium is made by the electrolysis of fused magnesium chloride and floats to the surface. Hydrated magnesium chloride is dried in the presence of ammonium chloride ($1NH_4Cl$ to $1MgCl_2$) to prevent decomposition. The ammonium chloride is then removed by volatilization and condensed.

CERIUM. Cerium may be made by electrolyzing fused cerium chloride ($CeCl_3$). Its iron alloy sparks when filed and is used in gas lighters.

BERYLLIUM. Beryllium has been made only during recent years. It has a high modulus of elasticity and makes valuable alloys. It is also used for windows in x-ray tubes.

Beryllium can be made by the electrolysis of fused salts of beryllium, e.g., beryllium oxyfluoride and barium fluoride. (*Wiss. Veröff. aus dem Siemens-Konzern*, Vol. 8, p. 42, 1929.) The beryllium is drawn out of the bath by the method used in making calcium. The temperature is 1400 deg cent. Siemens and Halske have erected a plant in which 1 metric ton of 98.5 to 99.2 per cent beryllium is made yearly. The price is 1 mark a gram. The principal alloy made in Germany is with copper, containing 2.5 per cent beryllium. Beryllium is made by the Beryllium Corp. of America. (*Min. Ind.*, 1934, Vol. 43, p. 616.)

17. ELECTRIC FURNACES IN METALLURGY

Under special local conditions, where iron ore is plentiful, where coke is expensive, and where power is cheap, the electric reduction of iron ore is carried out commercially, as at Trolhätten and Domnarfvet, Sweden, and in Switzerland and Japan. The furnaces have a shaft resembling a blast furnace, with a crucible at the base into which the electrodes project in a slanting position. (For detailed accounts, see volumes of the *Met. and Chem. Eng.*) More recently it has been found better to have the electrodes vertical. (Taussig, *VIII Int. Congress App. Chem.*, Vol. 21, p. 105, 1912.)

Electric tin smelting has been tried on a commercial scale with apparent success (*Met. and Chem. Eng.*, Vol. 9, p. 453, 1191) as well as the smelting of copper and nickel (*Met. and Chem. Eng.*, Vol. 11, p. 22, 1913), and zinc (see volumes of the *Met. and Chem. Eng.*). The use of electric furnaces in steel refining and in the production of ferro-alloys is much more extensive for in this case power does not need to be so cheap as in the reduction of iron ore.

STEEL REFINING. Usually the steel which is refined in electric furnaces is taken from Bessemer converters or open-hearth furnaces and poured directly into the electric furnace. Steel scrap, however, is melted and refined in the electric furnace in one operation. The advantages of the electric furnaces are (Walker, *Met. and Chem. Eng.*, Vol. 10, p. 371, 1912):

1. Complete removal of oxygen.
2. Absence of oxides caused by additions, such as silicon manganese.
3. Production of electric steel ingots of 8 tons and less that are practically free from segregation.
4. Reduction of sulfur to 0.005 per cent if desired.
5. Reduction of phosphorus to 0.005 per cent as in the basic open-hearth process, but with complete removal of oxygen.

The number of electric steel refining furnaces in use was increased greatly by the war.

The recent progress in electric steel refining consists in an improvement in existing methods and in a reduction of costs. Whereas in 1911 it was considered good practice to melt and refine steel scrap in 6 hours at 750 kwhr per ton, the same operation is now carried out in 4 hours with 600 kwhr. (Héroult, *VIII Int. Cong. of App. Chem.*, Vol. 21, p. 59, 1912.)

Some of the types of furnaces used in steel refining are the following:

STASSANO STEEL FURNACE (Fig. 7). This furnace consists of a closed chamber with three electrodes connected to a three-phase system above the slag. The furnace

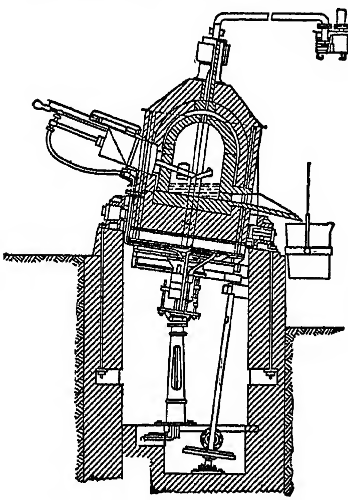

Fig. 7. Stassano Furnace

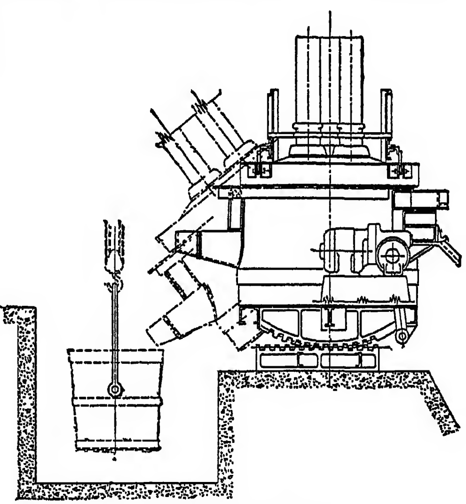

Fig. 8. Héroult Steel Furnace

rotates so as to stir all the metal. This furnace is used in Italy, Odessa, and Newcastle-on-Tyne (*Met. and Chem. Eng.*, Vol. 10, p. 66, 1912.)

HÉROULT STEEL FURNACE (Fig. 8). This furnace consists of a crucible lined with refractory material. Carbon electrodes project into it through the roof. An arc is formed where the current passes from each electrode into the slag. The power is regulated by an electrical automatic regulator which moves the electrodes up and down as required. These furnaces are made by the American Bridge Co.

GIROD STEEL FURNACE. This furnace consists of a crucible with several soft-steel rods projecting through the base. These form one electrode; the other is one or more carbon rods suspended from above. The steel electrodes of course melt several inches below the surface of the refractory lining of the crucible. The furnaces were formerly sold by C. W. Leavitt and Co., 30 Church St., New York, from whom the following data have been obtained.

These furnaces require from 65 to 70 volts at frequencies from 25 to 50 cycles. The power factor is about 80 per cent, and the duration of heat when the metal is charged in the melted state is 1 1/2 to 2 1/2 hours. The number of electrodes is from 1 to 3, according to the capacity of the furnace. For a furnace of about 12 tons' capacity, the maximum

power required is 1200 kw measured at the terminals of the electric generator; the energy consumption when the metal is charged cold is 900 kwhr per ton of steel; with a melted charge, from 150 to 250 kwhr. The electrodes are so designed that the current does not exceed 5 amp per sq in. of cross-section. The number of consecutive heats possible without repairs is:

	Linings	Cover	Electrodes
With cold charge....................	30 to 40	20 to 30	10
With melted charge..................	60 to 90	40 to 50	20

To handle the furnace 6 persons are necessary: a melter, 2 assistant melters, 2 workmen, and a boy.

Other Arc Furnaces more or less similar to the Héroult furnace are the Keller furnace (*Trans. Am. Electrochem. Soc.*, Vol. 15, p. 96, 1909), the Nathusius furnace (*Met. and Chem. Eng.*, Vol. 10, p. 227, 1912) and the Snyder furnace (*Trans. Am. Electrochem. Soc.*, Vol. 28, pp. 221–230, 1915).

LOW-FREQUENCY INDUCTION FURNACES. These furnaces are transformers in which a melted ring of steel is the secondary. The Kjellin furnace, Fig. 9, consists of a single deep ring of metal. It has only a small area of contact between the metal and slag, and the slag is not easily heated. The use of this furnace is therefore restricted in its application (Kjellin, *Trans. Am. Electrochem. Soc.*, Vol. 15, p. 175, 1909).

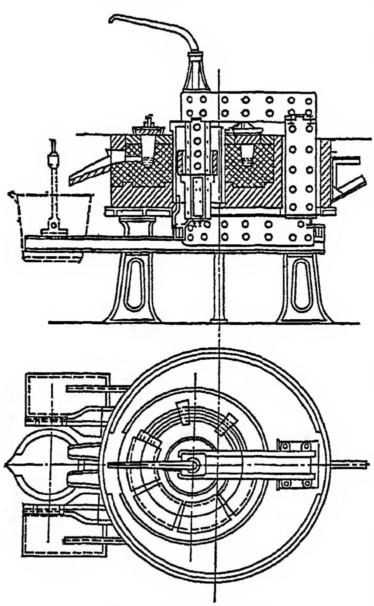

Fig. 9. Kjellin Furnace

A modified form of induction furnace is the Röchling-Rodenhauser furnace. This furnace has two annular rings combined in the form of a figure 8. The central portion carries the currents induced in both circuits, as well as a current from electrodes supplied by extra secondary coils. This current passes through the lining of the furnace, which has sufficient conductivity when hot.

These furnaces, as well as another modification, known as the Frick furnace, can be obtained from Siemens and Halske, represented in this country by Dr. G. K. Frank, New York. Another design of induction furnace is due to Hiorth (*Trans. Am. Electrochem. Soc.*, Vol. 20, p. 293, 1911).

PINCH EFFECT IN INDUCTION FURNACES. The magnitude of the current which can be sent through a trough of melted metal is limited by the so-called pinch effect (*Trans. Am. Electrochem. Soc.*, Vol. 11, p. 329, 1907). On account of the attraction of the current elements for each other, a compressing force is exerted on the metal which causes a decrease in the cross-section at some point. If the current is too great the metal may be entirely separated momentarily and the circuit broken.

FERRO-ALLOYS. Ferro-alloys were originally made from iron ore, the oxide of other metal, carbon, and flux, but on account of impurities, scrap iron and steel shavings are now used in place of iron ore (*Met. and Chem. Eng.*, Vol. 8, p. 133, 1910). Arc and resistance furnaces similar to those used for steel refining are used. For detailed information see Keeney, *Bull. Am. Inst. Mining Eng.* No. 140, pp. 1321–1373, 1918.

Ferrosilicon is the most important of the ferro-alloys. It is made from iron, quartzite, and carbon. At the Keller-Leleux works at Livet it has been found practicable to turn out 20 tons of 30 per cent ferrosilicon with 4000 hp during a day. (See preceding reference.) For an account of the uses of the other ferro-alloys, see *Electrochem. Ind.*, Vol. 1, p. 583, 1903; *Electrochem. and Met. Ind.*, Vol. 4, p. 247, 1906.

BRASS MELTING. Electric furnaces are now used to a considerable extent for brass melting. (St. John, H. M., Electric brass melting, *Elec. J.*, 16, 373, September, 1919.) Some of the types of furnaces used are the Booth rotating furnace (*Trans. Am. Electrochem. Soc.*, Vol. 33, p. 247, 1918); the Rennerfelt furnace (*Met. and Chem. Eng.*,

Vol. 12, p. 275, 1914; *Trans. Am. Electrochem. Soc.*, Vol. 29, p. 497, 1916; Vol. 31, p. 87, 1917); the Ajax-Wyatt induction furnace (*J. Franklin Inst.*, Vol. 190, p. 817, 1920); the Helberger furnace (*Met. and Chem. Eng.*, Vol. 12, p. 644, 1914, and Globar furnaces.

18. FIXATION OF ATMOSPHERIC NITROGEN

One of the most important applications of the electric furnace is the fixation of atmospheric nitrogen. The various processes employed are described below.

CARBIDE METHOD. In this method calcium carbide is heated in pure nitrogen, forming calcium cyanamide according to the reversible reaction

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C$$

The carbide is heated in iron drums by a thin carbon conductor running through the center of the drum. Heat is evolved by the reaction. The product is called "nitrolime" or "lime-nitrogen," and contains 12 to 15 per cent nitrogen. (*Met. and Chem. Eng.*, Vol. 5, p. 78, 1907.) It is used directly as a fertilizer, or may be converted into ammonia by superheated steam. The yield in nitrogen by the carbide method is about 51.6 grams per kilowatthour, including the manufacture of the carbide.

DIRECT OXIDATION METHOD. In this method the nitrogen and oxygen in air are caused to combine in a high-voltage arc accordnig to the reversible reaction

$$N_2 + O_2 \rightleftarrows 2\,NO$$

On cooling the NO is further oxidized to nitric dioxide. The nitric dioxide on treatment with water gives nitric and nitrous acids:

$$2NO_2 + H_2O = HNO_3 + HNO_2$$

Only about 1 to 3 per cent of the air treated is oxidized. The yield in the Birkeland-Eyde furnace is 12.7 grams of nitrogen fixed per kilowatt hour. (*Trans. Faraday Soc.*, Vol. 2, p. 98, 1906.) Furnaces of three different designs are now in operation for oxidizing nitrogen: that of Birkeland and Eyde, that of Schönherr, and that of H. and G. Pauling.

DIRECT SYNTHESIS OF AMMONIA. In this method, due to Haber, ammonia is formed directly from a mixture of nitrogen and hydrogen by passing over a catalyzer between 500 and 700 deg cent at 200 atmospheres, according to the reversible reaction

$$^1/_2\,N_2 + {}^3/_2\,H_2 \rightleftarrows NH_3$$

(*Zeit. f. Elektroch.*, Vol. 16, p. 244, 1910; Vol. 19, p. 53, 1913.) In the Claude process the pressure is increased to 1000 atmospheres, which gives a 25 per cent conversion, in place of 6 per cent at 200 atmospheres.

BUCHER PROCESS. This process was discovered in 1846 by Lewis Thomson (Bucher, *Am. Inst. Chem. Eng.*, Vol. 9, p. 335, 1916) and is represented by the following reaction:

$$Na_2CO_3 + 5C + N_2 = 2NaCN + 3CO$$

which requires metallic iron as catalyzer. The temperature is between 900 and 1050 deg cent. A plant was built at Saltville, Va., to make cyanide for war requirements. The furnaces were 8-in. iron tubes heated externally by hot gases. The reacting mixture is finely ground, briquetted hot with water, dried, and then treated with nitrogen. This plant was never put in operation at full capacity. See Ferguson and Manning, *J. Ind. and Eng. Chem.*, Vol. 11, p. 946, 1919; Posnjak and Merwin, *J. Washington Acad. Sci.*, Vol. 9, p. 28, 1919.

SERPEK PROCESS. In this process aluminum nitride is made by heating aluminum oxide, carbon, and nitrogen together. On heating the aluminum nitride with water, ammonia and aluminum hydrate are formed. The product obtained from the furnace is said to contain 20 to 24 per cent of nitrogen, and the power required per unit of nitrogen is said to be only one-half of that used in the calcium-carbide method. This process never got beyond the experimental stage. (*Bull. de la Soc. ind. de Mulhouse*, Vol. 79, p. 39, 1909. U. S. Pat. 996,032, 1911.)

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ELECTRIC RESISTANCE HEATING

By Frank Thornton, Jr.

20. GENERAL PRINCIPLES

There are a few general-purpose types of electric heating units but to secure most satisfactory results it has usually been necessary to design the heating elements as part of the complete appliance or apparatus. Electric heat can be released in exactly the right amount and in exactly the right place. Because of the difference in hearing characteristics, certain fundamentals of heat behavior require greater attention when applying electricity for heating than when using fuels. A brief review of these fundamentals is necessary before taking up the various applications.

The first thing that must be known is how much heat is necessary and at what rate it must be delivered. A careful study must be made of the entire process. Often, a process as conducted when using a fuel as the source of heat may not be the best process. Electricity may make it possible to improve the results greatly by changing the heating cycle. Therefore, it is important to study the fundamental requirements of each new application.

Heat is used to (*a*) raise temperature, (*b*) fuse, (*c*) evaporate, or (*d*) cause chemical change.

The weight of the material to be heated must always be known. In addition, the following facts must be known before calculation can be made of the heat required for any of the four functions mentioned above.

(a) **Raise Temperature.** The specific heat of the material must be determined at the temperature involved. It should be pointed out that this is not a constant value but may change with the temperature. For accurate computations of heat requirements at high temperature a careful check of the specific heat is important.

(b) **Fusion.** Melting point and latent heat of fusion must be known.

(c) **Evaporation.** Boiling point and latent heat of evaporation must be known for the pressure at which it is to take place.

(d) **Chemical Change.** Heating applications for the purpose of causing chemical change require special knowledge of the reactions involved. Some chemical reactions require the addition of heat; others give off heat; and still others require heat to start them and then give off heat.

After the amount of heat and the time cycle involved in the process are known it is possible to proceed with a calculation of the heating device or apparatus.

The selection of the proper length, cross-section, and surface area of resistor is governed by many different considerations which will be discussed later in connection with the various applications.

21. RESISTANCE MATERIALS

Metallic resistance materials are usually used for temperatures below 2000 deg fahr. For exceptionally high temperature furnaces a molded refractory resistance material has been successfully used.

Nickel-chromium alloys are the most generally used resistance materials for heating purposes. They have high specific resistance, reasonably low temperature coefficient, and high resistance to oxidation.

Nickel-copper alloys, such as Advance, Copel, and Monel, have some uses for lower temperatures.

Data on these alloys are given in Tables I and II. See Section 2, Art. 6 and Manufacturers' Catalogs for additional resistance data.

Table I. Temperature Limits for Resistance Materials

Material	Chemical Composition (approximate) per cent	Specific Resistance, ohms per cir mil fact. 68° F	Temperature Coefficient per deg fahr (average)	Maximum Working Temperature, deg fahr
Copper-nickel	Cu—60, Ni—40	294	0	800
Nickel-iron-chromium	Ni—60, Fe—25, Cr—15	675	0.00008	1400
Nickel-chrome	Ni—80, Cr—20	650	0.00007	2100

Table II. Wire Table

Nickel-Chromium Alloys

Size A.W.G.	Diameter, in.	Nickel Iron Chromium, Ohms per Foot	Nickel Chromium, Ohms per Foot
000	0.410	0.00402	0.00387
00	.365	0.00507	0.00488
0	.325	0.00640	0.00616
1	.289	0.00808	0.00779
2	.258	0.01013	0.00978
3	.229	0.01288	0.01240
4	.204	0.0162	0.0156
5	.182	0.0204	0.0196
6	.162	0.0257	0.0248
7	.144	0.0326	0.0314
8	.128	0.0412	0.0397
9	.114	0.0520	0.0500
10	.102	0.0650	0.0625
11	.091	0.0815	0.0786
12	.081	0.1030	0.0992
13	.072	0.1305	0.1255
14	.064	0.165	0.159
15	.057	0.208	0.200
16	.051	0.260	0.250
17	.045	0.334	0.321
18	.040	0.422	0.406
19	.036	0.521	0.502
20	.032	0.660	0.635

Size A.W.G.	Diameter, in.	Nickel Iron Chromium, Ohms per Foot	Nickel Chromium, Ohms per Foot
21	0.0285	0.830	0.802
22	.0253	1.054	1.017
23	.0226	1.323	1.274
24	.0201	1.67	1.61
25	.0179	2.11	2.03
26	.0159	2.67	2.57
27	.0142	3.35	3.23
28	.0126	4.25	4.10
29	.0113	5.28	5.10
30	.0100	6.75	6.50
31	.0089	8.53	8.21
32	.0080	10.55	10.17
33	.0071	13.4	12.9
34	.0063	17.0	16.4
35	.0056	21.7	20.7
36	.0050	27.0	26.0
37	.0045	33.3	32.1
38	.0040	42.2	40.7
39	.0035	55.1	53.1
40	.0031	70.2	67.7
	.00275	89.3	86.0
	.00250	108.	104.0
	.00225	133.	128.5
	.00200	169.	162
	.00175	220.	212.
	.00150	300.	289.

Although the temperature coefficient of nickel-chrome materials is not high, yet it should be considered in making calculations. In the production of these materials manufacturers find it impossible to duplicate the temperature coefficient exactly in succeeding batches. Table III gives average coefficients and variations from the average.

Table III. Change in Temperature Coefficient of Heater Wires

(These values are average. The excess over 100 per cent may vary plus or minus 20 per cent from these figures.)

Temperature		Resistance in per cent		Temperature		Resistance in per cent	
Deg fahr	Deg cent	Nickel-iron-chromium	Nickel-chromium	Deg fahr	Deg cent	Nickel-iron-chromium	Nickel-chromium
77	25	100.0	100.0	1200	649	108.0	103.3
500	260	104.0	....	1400	760	108.6	103.2
600	315	104.8	....	1600	871	109.7	103.6
800	427	106.3	103.7	1800	980	....	104.1
1000	539	107.4	104.0	2000	1094	....	104.8

22. ELECTRICAL INSULATION

The choice of electrical insulating material depends a great deal upon the temperature, size, and type of device being designed. Mica is very commonly used for the smaller devices such as flat-irons, toasters, waffle-irons, etc. There is a wide variety of mica both in mechanical and thermal characteristics. When it is clamped tightly in place, built-up mica may be used, the volatile bond being baked out after assembly if necessary. All mica has a definite temperature of dehydration at which the crystals give up the water of crystallization. This happens in some grades at as low as 800 deg fahr; in others it does not happen below 1200 deg fahr. The element must be so designed that the dehydration temperature will not be reached because, when that occurs, there is a rapid

deterioration of the nickel-chrome resistance material which may be in contact with the mica.

Porcelain for use in these applications not only must have sufficient electrical insulation at the working temperature, but also must resist cracking and spalling due to the changes in temperatures.

For larger apparatus, furnaces, melting pots, etc., appropriate grades of fireclay, silica, magnesia, zirconia, lava, alundum, and other materials are selected to suit the various needs.

23. INDUSTRIAL HEATING APPLIANCES

Small Appliances

Electrically heated appliances have been developed for a very large number of uses. The following list contains only a few of the more common devices.

Tailors' pressing irons.	Hatters' irons.	Hot tables.
Solder pots.	Velouring stoves.	Press blocks.
Soldering irons.	Glue pots.	Chocolate warmers.
Branding irons.	Sealing-wax heaters.	

Usually the cost of the energy to operate such devices is immaterial in view of the great advantage gained in speed, convenience, safety, and uniformity of operation.

Small solder pots of 10 to 50 lb capacity are particularly convenient because they may be located anywhere on a work bench since it is not necessary to provide for the escape of the fumes and burnt gases.

Larger solder and babbitt pots of lead, capacities up to 2000 lb, are in use. A typical example would be a 750-lb pot, rated at 13.5 kw, with automatic temperature control. Such a pot has been known to melt 4500 lb of 50–50 solder, operating at 800 deg fahr, in 9 hours with a total energy consumption of 150 kwhr.

Automatic temperature control is very desirable. A number of thermostats, such as the disk type and others, are available on the market. They are usually built into the heating appliance in such a manner as to be very responsive to changes in temperature.

24. INDUSTRIAL APPLICATION UNITS

These are small heater units, complete in themselves with supports and terminals which have been designed to meet a wide variety of applications of heat to machines and appliances to which heated surfaces or spaces are essential to their successful operation. Usually the application involves only minor changes in the construction of the machine to accommodate the electric units. Terminals are provided for readily connecting up the units.

Such units are usually provided in standard voltages of 110 and 220 volts, although the larger units for ovens are suitable for operation in series up to 550 volts. Some of the types and applications of these units are described in the following paragraphs.

CARTRIDGE HEATERS. A tubular unit having terminals at one or both ends and designed to be inserted in holes drilled in castings which require heating, such as embossing presses, hot molding presses, shoe machinery irons, hat machine irons, etc., is called a cartridge heater. Nickel-chrome resistance material is used for the heater and is wound on a central core of porcelain or lava and insulated from the outer tube of brass by cement or mica. Cartridge heaters may be used in applications up to 300 deg cent. Manufacturers' catalogs should be consulted for sizes and ratings.

STEEL-CLAD HEATERS. This is a flat type of element consisting of a mica form on which a resistance element is wound, insulated with flat plates of mica, and enclosed in a casing of sheet steel pressed flat on the element. Terminals are usually located at one end.

This unit is made in a wide variety of sizes and is suitable for hot-press heating, hot tables, heating rolls, small ovens, etc., in which the uniform distribution of heat is desired. A wide variety of sizes and ratings are available, and manufacturers' catalogs should be consulted for exact information and recommendations as to applications.

METAL-MELTING HEATERS. Special shapes of cast-in-heaters are available for immersion in lead pots, babbitt pots, type-metal pots, etc. Wattage, voltage, and shape must be designed to suit the particular application.

SPACE HEATERS. These have one terminal located at each end and run from 12 in. to 43 in. long and 220 to 1250 watts input. The 24-in. length is a standard size and is suitable for a wide variety of applications, of which the following is a partial list.

The unit is 1 $1/2$ in. wide, $3/16$ in. thick, and is supplied in two designs, one for 110 volts and the other for 220 volts, the rating being 500 watts in either case.

Small Industrial ovens.	Watchmen's houses.
Blueprint dryers.	Elevator cabs.
Hoist cabs.	Laundry dryers.
Ticket booths.	

INDUSTRIAL OVEN HEATERS. These heaters are designed for the larger sizes of industrial ovens. They are of nickel-chromium ribbon wound over porcelain bushings and assembled into a metal frame. Exposed heating ribbon is used to give free radiation into the oven space. The heating is done partly by radiation and partly by convection. Terminals are provided at the ends of connections. Units are usually made up for 110-volt and 220-volt service. Two 220-volt units may be connected in series for 440 volts. Complete sets of suitable connection straps and insulators are provided to accompany the heaters.

25. INDUSTRIAL OVENS

Electric heat has been applied to ovens for japanning, lithographing, paint drying, lacquer drying, core baking, armature baking, and many other purposes requiring temperatures up to 800 deg. fahr. The absence of fumes and gases, reduction in fire hazard, ease of control, and increased production have in many cases justified even increased costs of heat. Ovens are of the usual kiln type, the semi-continuous type, or the continuous type.

A Kiln Type Oven is usually rectangular in shape with a door at one end into which the material to be baked is loaded. When the oven has been loaded, the door is closed and current turned on. When the baking has been completed, the current is turned off, door opened, and work removed.

A Semi-continuous Oven has doors at both ends, and a new load is pushed in at one end while the finished load is removed from the other end.

In a Continuous Oven the work is carried through the oven on a continuous conveyor loading at one end and discharging at the other at a uniform speed.

Heaters are installed along the side walls or on the floor. A space of 8 to 10 in. from the wall or floor should be provided. Over the heaters a screen of expanded metal or wire mesh can be installed to prevent work dropping on the heaters and connections.

Control is usually by magnetic switches. Door switches are sometimes provided so that when the doors are opened the main switches also open. A thermostat is generally used to regulate the temperature by turning the current on or off as may be needed. This thermostat operates through a relay to control the main magnetic switches.

Either 110, 220, or 440 volts, of any frequency, and either single phase, two phase, or three phase, can be used. Direct current up to 550 volts can be used also.

HEAT REQUIRED FOR INDUSTRIAL OVEN. In calculating the heat requirements, the following losses and uses of the heat must be carefully determined.

Heat to Raise the Temperature of the Work, Including Trucks. The total weight of the material to be baked, the initial and final temperatures, and the specific heat must be known. Dividing the heat required by the time gives the heat per hour needed. The japan or paint can be ignored. The heat required to heat the trucks or hangers is calculated in the same way for each load.

Heat to Raise the Temperature of Oven Walls. The insulating material in an oven wall absorbs heat in the period of heating up even though it may be fairly good heat insulator. The usual method is to average the temperatures of the inner and outer surfaces, and subtract the initial temperature to obtain the average temperature rise. This is multiplied by the weight and specific heat to obtain the total heat required. Dividing this by the time allowed for the heating operation gives the rate at which heat must be supplied.

The interior framework and oven lining of metal are heated on the average to the same temperature as the work in the oven, and can be calculated in the same way.

In a kiln-type oven the interior cools down somewhat during the period when the doors are open for emptying and reloading. For this reason a portion of the heat must be replaced with each charge. This may be from one-quarter to one-half of that required on the first bake.

Radiation Losses from the Oven. This depends upon the wall and door construction. It is of prime importance that the amount of metal extending through the oven walls from the inner to the outer surfaces be reduced to a minimum. Average radiation loss from a metal oven with 2 in. of insulation (air-cell asbestos or magnesia) and door at one end only will be approximately 50 watts per square foot of total outside surface of oven,

with an inside temperature of 400 deg fahr. Exact figures for radiation loss cannot be given, as it is controlled to such a great extent by the design of doors, corners, vents, etc., as well as by the thickness and character of the insulation.

Ventilation. The amount of ventilation required depends upon the product to be baked. Ovens for japanning, paint drying, core baking, and all other operations that give off gases or vapors require ventilation. Where a paint or japan is used that contains a volatile solvent of combustible nature, good ventilation is necessary to reduce the danger of fire or explosion. This danger is much more remote with an electrical oven than with a gas or other fuel-heated oven because of the uniform temperature regulation obtained with an electric oven. Ventilation also removes the gases given off during baking and improves the surface obtained.

For japanning ovens from ten to twenty changes of air per hour are usually allowed. To calculate the heat required the volume of the oven is multiplied by the weight of a cubic foot of air at the oven temperature, by the temperature rise, by the specific heat of air, by the number of changes per hour.

Totals. Adding together the heats required for each of the above throughout the time allowed will give the total heat. Dividing by the number of hours gives the rate the heat will be required. If the heat is calculated in Btu, divide this rate by 3415 to get the power input in kilowatts.

It is impracticable to give average figures of inputs for various oven sizes and applications. As an example, however, it may be stated that for japanning ovens at 400 deg fahr, 2-hour baking period, the input would be around 100 watts per cubic foot of volume of the oven.

26. INDUSTRIAL RESISTANCE FURNACES

The electric furnace is much like the fuel-fired furnace in appearance and construction. More attention is usually paid to the elimination of heat losses from the electric furnace because of the higher cost of the heat units. Electric furnaces have many advantages that often more than offset the increased heat unit cost. The cost of heat may often be a very small part of the total cost of a process. It is important, therefore, to examine carefully all the factors that enter into production costs when planning the heating equipment for a process. Repair and replacement of linings, loss of material rejected on inspection after treatment, cost of handling material through the furnaces, space requirements, failures in service because of lack of uniformity—these are all elements of the total cost of a heat-treating process.

Electric furnaces may be built to meet almost any requirement as to size, shape, temperature cycle, or process. Table IV indicates roughly the classes of furnaces and some of their uses:

Table IV. Classification of Resistance Furnaces

Class	Temperature	Typical Uses
I	Low-temperature furnaces 300 to 1200° F	Temper drawing Nitriding Low-temperature annealing of steel, brass, glass, etc.
II-A	Medium-temperature furnaces 1200 to 2000° F	Hardening Vitreous enameling Carburizing Normalizing High-temperature annealing of steel
II-B	Pot-type furnaces 600 to 1650° F	Lead furnaces Cyanide furnaces Other salt furnaces
III	High-temperature furnaces 2000 to 2500° F	Forging High-speed steel hardening Ceramic firing

CONSTRUCTION. Since there is no combustion, the electric furnace requires no grates, ashpit, air or fuel inlets, or smokestack. It is usually built as a solid steel box. The outer shell is of heavy sheet steel $^3/_{16}$ or $^1/_4$ in. thick, supported and reinforced with steel angles or channels as needed. Inside this shell are several inches of heat-insulating material in the form of blocks, or wool, or powder. Next comes a wall or layer of semi-refractory slabs or bricks having good heat-insulating qualities and able to withstand the

higher temperatures of this zone. Finally, there is the inner lining of firebrick in the zone of highest temperature.

The electric heating elements are installed on this inner lining. They may be attached to the sides, roof, or bottom, as desired. In some furnaces they have even been mounted on the inside of the doors. Heaters are located as needed to give the proper distribution of heat inside the furnace. Ordinarily, location on the bottom and sides is sufficient.

In locating heaters, it is necessary to take into account the work going through the furnace and also the doors and other structural features that may conduct heat through the walls.

Heating elements for the low- and medium-temperature classes are of the best grade of nickel-chromium wire, rod, or ribbon, and are bent or formed before assembly into the furnace. They may be assembled into frames complete with insulators, or the insulators may be built into the refractory lining of the furnace and the zigzag heater mounted during the construction of the furnace.

Provision is usually made in the design for easy replacement and repair of the heating elements.

Terminals of the heaters are usually brought out through the walls and connections made outside the furnace. Terminals are usually of the same material as the resistor but of larger cross-section to reduce heating effect. They are usually welded to the resistance material.

Metal parts that may be required inside the furnace are generally of cast nickel-chromium. This is used for guards in front of side-wall heaters, cover plates over bottom heaters, rails, bumpers, conveyor parts, hanger hooks, melting crucibles, and many other purposes. Ordinary cast or wrought iron or steel is not suitable for this service because of the rapid oxidation and corrosion at these high temperatures.

For the high-temperature class of furnaces which are to operate between 2000 and 2500 deg fahr a molded resistance material is generally used in the form of rods. These units extend through the furnace walls so that terminal connections may be made on the ends. Water cooling of the terminals is often desirable. Units are available in a variety of sizes varying in length by 2-in. steps from 6 to 48 in. and in various appropriate diameters. This type of material is available under the trade names of Globar and Silit. Recent developments indicate that there may be some metallic resistance alloys capable of withstanding these higher temperatures.

Table V gives a few typical sizes of Globar units:

Table V. Typical Globar Units

Length, in.	Diameter, in.	Volts	Amperes	Watts	Length, in.	Diameter, in.	Volts	Amperes	Watts
5	5/16	30	14	420	16	1	65	85	5525
6	5/16	40	15	600	24	1 1/4	85	110	9350
8	3/8	55	20	1100	36	1 3/4	110	185	20,350
12	1/2	75	29	2175	48	1 3/4	150	200	30,000

Furnaces may be of almost any size and shape to suit the needs of the process. The following brief review will suggest the adaptability of electric furnaces.

BOX FURNACES. These are rectangular in shape with a door in the end or side. The smaller sizes are usually charged by hand, but the larger sizes are often equipped with power-operated charging apparatus. Table VI covers only some of the standard sizes. Much larger furnaces of this type are often built for special purposes. (Fig. 1.)

Table VI. Typical Box Furnaces

Type No.	Maximum Power, kw	Dimensions (inches)						Insulation, in.	Shipping Weight, lb
		Inside			Outside (overall)				
		Width	Depth	Height	Width	Depth	Height		
B–10	1.9	4	10	3	26	20	26	5	450
B–12	4.7	8	12	5	35	27	35	5	850
B–22	8.0	12	22	8	46	36	77	5	1,350
H–25	12.0	11 3/4	27 5/8	8 1/4	42 5/8	50 1/2	73 3/4	6 1/2	1,150
H–36	26.0	18	36 5/8	12 3/8	57 1/2	60	86 1/4	6 1/2	2,700
H–54	45.0	25	52 1/4	19 5/8	58 3/8	86 1/4	101 1/2	9	7,000
H–75	75.0	36	72 1/2	32 3/8	72 3/4	109	137 5/8	10 1/2	13,000

POT FURNACES. These are cylindrical or rectangular furnaces with open tops in which are suspended cast nickel-chrome crucibles which are maintained full of molten

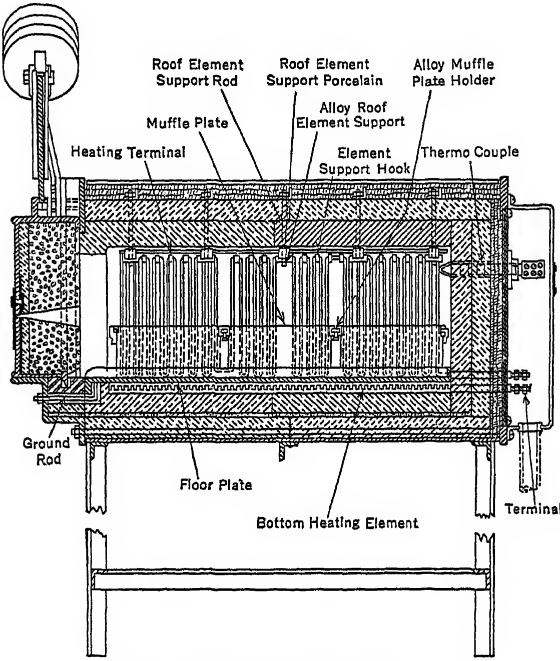

FIG. 1. Sectional View of Box Furnace

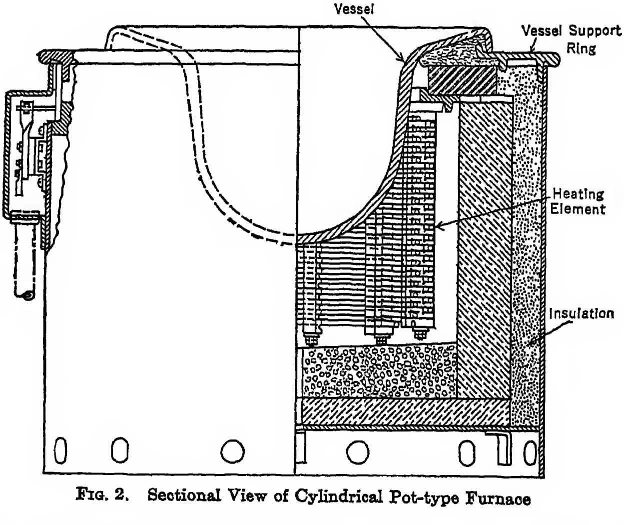

FIG. 2. Sectional View of Cylindrical Pot-type Furnace

lead or salts. Materials are immersed in these baths for heat treating. (Figs. 2 and 3.) Tables VII and VIII give some typical dimensions.

Table VII. Typical Round Pot-type Furnaces

Type	Maximum Power, kw	Capacity of Pot, lb		Dimensions (inches)				Insulation, in.	Shipping Weight, lb
				Inside		Overall			
		Lead	Cyanide	Diameter	Depth	Diameter	Height		
JC–10	21	400	54	10	17	35 1/2	41	7	1250
JC–20	21	520	72	10	20	35 1/2	41	7	1300
JC–30	30	1290	170	17	17 1/2	44	39 1/2	7	2045
JC–40	30	1840	250	17	24	44	39 1/2	7	2165

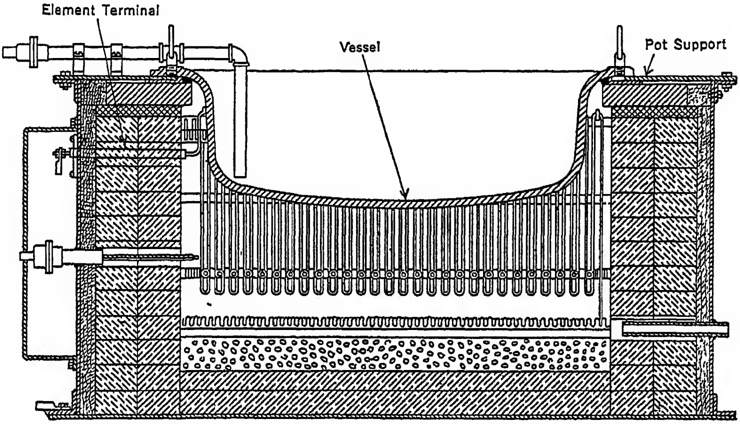

Fig. 3. Sectional View of Rectangular Pot-type Furnace

Table VIII. Typical Rectangular Pot-type Furnaces

Type	Maximum Power, kw	Dimensions (inches)						Insulation, in.	Shipping Weights	
		Inside Vessel			Overall				Furnace, lb	Cover, lb extra
		Width	Length	Depth	Width	Length	Height			
O–60	39	15	39	18	48	77	40	10 1/2	5100	700
O–70	57	12	46	12	54	86	40	13 1/2	5500	900
O–80	65	22	60	14	54	105	40	13 1/2	6900	1000

ROTARY-HEARTH FURNACES. These are cylindrical in shape with a hearth that rotates. The work is loaded on this hearth through the door and travels around through the furnace one or more revolutions and is then unloaded. Loading and unloading may be automatic if desired. Different temperature zones may be arranged to provide the proper heating cycle. Sizes vary, although hearth diameters of 5 and 7 ft have been common.

PUSHER FURNACES. These are similar to large box furnaces with automatic charging apparatus for feeding in the work and pushing it through and out the other end. The work may be pushed straight out the other end or dropped down a chute into a quenching bath. Time cycle and temperature being automatically controlled, accurate duplication of results is obtained on a mass production basis.

PIT FURNACES. These are open-top furnaces used for heat-treating large charges of small parts or large parts that can be most conveniently handled from a crane. Covers are lifted off by crane.

BELL TYPE FURNACE. The body of this type of furnace is arranged so that it can be lifted off the bottom by a crane or hoist and easily transferred to a different bottom. Several bottoms may be used so that the loads may be arranged in advance and the furnace moved from one to another, thus obtaining the full heating capacity of the furnace. The bottoms may be equipped with wheels for ease in transportation about the shop.

The furnace body or bell may be made gas tight by welding all seams and by properly sealing all terminals in case it is desired to use non-oxidizing atmosphere inside during the heating cycle. The seal, being at the bottom of the furnace, is less subject to atmosphere leakage that when at the top as in a pit furnace.

ELEVATOR-TYPE FURNACE. This is similar to the bell type in principle except that the body is fixed in a position high enough above the floor to clear the loads that may be piled on the bottom. An elevator is provided beneath this furnace which hoists the bottom up into position under the furnace.

CAR FURNACE. These are very large box-type furnaces for handling large quantities of material on cars. Rails are run inside the furnace. A specially designed car is piled full of material and pushed in through the open door. These furnaces sometimes assume very large dimensions.

CONVEYOR FURNACES. These are furnaces built around a conveyor system. They are quite generally used for vitreous enameling, which is a process involving comparatively light materials but one in which accurate conrtol of heating cycle and temperature is very important. Also the product is easily damaged in handling and by foreign particles, such as dust, falling on the surfaces. Some of these furnaces are well over 100 ft long and handle very large amounts of material per day.

PERFORMANCE. The performance of a furnace is usually stated in terms of pounds of product per kilowatthour for a certain set of conditions as to time and temperature cycle, total hours per day operated, size of charge, etc. Average economies to be expected are given in Table IX for general information only, and are not to be used as exact figures for any specific problem.

Table IX. Production per Kilowatthour

Operation	Approximate Temperature, °F	Average Pounds per Kilowatthour
Hardening carbon steel	1550	8 to 11
Tempering " "	600	20 " 25
Carburizing " "	1700	1 1/2 " 3 net
Cyaniding " "	1500	3 " 5 net
Nitriding " "	1000	2 " 4
Annealing sheet steel	1400	11 " 14
Normalizing sheet steel	1800	8 " 10
Vitreous enameling (Box furnace)	1600	6 " 8
" " (Continuous furnace)	1600	9 " 12
Forging steel	2200	2 1/2 " 5
Annealing brass	1300	18 " 22
" copper	1100	20 " 23
" aluminum	1000	12 " 16
" steel castings	1600	10 " 12
" steel forgings	1600	10 " 12

TEMPERATURE REGULATION AND CONTROL. Electric furnaces can be automatically controlled to a high degree of exactness. The type of control apparatus depends upon the requirements of the application.

Thermostats and pyrometers are available for all temperatures, and it is best to consult the manufacturer for definite recommendations. These thermostats have maximum and minimum contact points which are connected through relays to the main control panel. In some cases the entire current is turned on and off by the thermostat as required to maintain temperature. In others, only a portion of the heater circuit may be under automatic control. Sometimes it is desired that different parts of the furnace be kept at different temperatures, as in a long tunnel furnace where one portion is for preheating at low temperature and another part at high temperature to complete the process.

A temperature-limiting fuse is often a very desirable feature in a furnace to protect against overheating and possible damage to the furnace and to the work in it.

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ELECTRICAL PRECIPITATION OF SUSPENDED PARTICLES

By N. W. Sultzer

The removal and collection of finely divided solid or liquid particles carried in suspension in air or other gases may be accomplished by the application of a high-potential unidirectional electrical current to such gases. The Cottrell electrical precipitation processes embodying this principle have been applied with successful results in a large number of installations throughout the world.

28. PRINCIPLES

The essential elements of an electrical precipitator are two sets of electrodes. One set, known as the discharge electrodes, are of such form as to facilitate an electric discharge from their surface, as for instance a wire or a light chain, or a strip of metal having relatively sharp edges; the other set, known as collecting electrodes, are of such shape as to prevent as far as possible any discharge from their surface, as for instance, a flat plate or a pipe with a smooth interior surface. These electrodes are so arranged in the precipitator that the different types oppose each other, and between them a silent or glow discharge is maintained by supplying to the discharge electrode electrical energy of a unidirectional character and at a high voltage.

In practice, the collecting electrodes are grounded for reasons of convenience and safety. A simple form of precipitator, for example, consists of a grounded pipe along the central axis of which is placed a wire connected to a source of unidirectional high-voltage current.

The air or other gas carrying suspended liquid or solid particles which are to be removed is passed between the discharge and collecting electrodes. During passage the particles of suspended matter are charged and are driven away from the discharge electrodes and over to the collecting electrodes upon which they are deposited. The air or other gases pass on and out of the precipitating chamber.

It has been found that the most effective precipitating action is obtained when the potential difference between the electrode is sufficient to produce a glow known as the corona on the discharge electrodes (see Corona), and when the discharge electrodes are charged negatively rather than positively.

The voltage employed in a precipitator depends upon the size and type of discharge electrode used, the gap distance between the discharge and collecting electrodes, and the temperature and other characteristics of the gas being treated. There is a definite relation between the velocity of the gases through the precipitator and the length of the path between the electrodes. In other words, the particles must be under the influence of the electric field for a suitable period of time in order to secure satisfactory removal of these particles from the gases. The length of this period depends to a considerable extent upon the character of the particles to be precipitated. If the particles are fluffy in character, light, dry, and very finely divided, a longer time will be required than if they are relatively heavy or coarse or of a liquid or sticky nature.

29. EQUIPMENT

In electrical precipitation, the two essential groups of equipment are: the electrical equipment to generate the high-potential unidrectional current required; and the precipitator proper, in which the gases are cleaned or treated.

ELECTRICAL EQUIPMENT. To generate the high-voltage unidirectional current, electrical equipment of a more or less special type is used, developed or adapted from standard equipment. Where the source of power is a 220- or 440-volt, two- or three-phase, 25- or 60-cycle, a-c supply, a synchronous induction motor is used to drive a direct-connected mechanical rectifier, rectifying the secondary current from a special transformer energized from this same supply. Where direct current—usually at 250 volts—must be used, a motor-generator unit provides a low-voltage alternating current for the transformer and rectifier just mentioned. In this case, the rectifier is direct-connected to the alternator, which must be a four-pole machine.

Electronic tube rectifiers for precipitation work are now being designed.

SWITCHBOARD. A control panel carries all necessary switches and controls for the low-tension circuit. An a-c board has a motor starting switch with two clips for throwing the motor on the line, first through the starting resistance and then directly. In addition there are an overload circuit breaker with low-voltage trip; a single-pole, five-throw transformer switch; a double-pole, double-throw switch in the main transformer circuit for getting proper polarity on the discharge electrodes; an ammeter and voltmeter; a plug switch for connecting the voltmeter to read potential on either side of the primary resistance or rheostat in the primary circuit used to regulate the potential to the transformer and to minimize the effect of surges caused in the high-voltage circuit and reflected back into the primary circuit.

In an installation operating from a d-c supply the panel board carries, in addition to the apparatus just listed, a motor starting box with no-load release, and an alternator field rheostat. Protective fuses are also used, where required.

SYNCHRONOUS MOTOR-RECTIFIER. In this, the motor element is of the induction type with standard squirrel-cage stator for starting purposes. It is so built, however, that it has the four poles of the synchronous motor minus the windings, the magnetism being induced in the poles by a stator current. The frame is about the size of that of a 3-hp motor; and the motor may be for 220 or 440 volts, two or three phase, 25 or 60 cycles. A starting resistance is also provided.

The rectifier element is, in effect, a synchronously driven switch. Four bronze shoes, spaced 90 deg apart on a circle, are mounted on fixed insulating arms of Bakelite. The top shoe is connected to the precipitator; the bottom shoe is grounded; and each side shoe to one secondary terminal of the transformer. Rotating within these shoes, with a clearance of $^1/_8$ to $^1/_4$ in., is a rotor with four tips at 90-deg intervals, mounted on Bakelite arms or a Bakelized disk. The two adjacent tips are electrically connected, but the two pairs of tips are not connected.

TRANSFORMER. The transformer used is from 5- to 25-kv-a capacity, according to the requirements of the precipitator installation. Standard design calls for a single-phase, 25- or 60-cycle, 200-, 400-, or 500-volt primary, 50- to 85-kv secondary, with a number of primary taps (usually five) to afford a range of secondary voltages. The coils are immersed in oil and specially insulated. Choke coils and, usually, resistance rods are provided in each leg of the secondary to reduce surges. Radio interference correctors are also installed, if necessary, to prevent "static" interference with radio reception.

THE PRECIPITATOR. The type of precipitator used in any particular installation is determined largely by the nature of the operation and such factors as the kind of gas to be treated, the character of the material to be removed or recovered, and the space available for the equipment. The size of the precipitator depends primarily upon the volume of gas to be treated in unit time and upon the degree of cleaning required.

Pipe Type. This type of precipitator is used primarily in the precipitation of liquids from gas or air, and for dust and fume removal where a particularly high degree of gas cleanliness is required. It consists of a group of vertical pipes 6 to 8 in. in diameter and 9 to 12 ft long. These are nested in a shell, with connections such that the gas passes up through the pipes and emerges from the shell through a connection at the top. Within each pipe a wire is axially suspended from an insulated framework above, and weighted below. The material precipitated from the gas, in passing through the pipes, collects on the interior walls and settles into a lower header from which it is removed through drains or hoppers.

The size of a pipe precipitator is limited by mechanical and structural considerations, and for large volumes of gas, several units may be required. The material used in the construction is determined by the character of the gas treated and of the suspended matter removed, non-corroding materials being used where the nature of the treatment requires.

Plate Type. The plate type of precipitator is intended primarily for removing solids from air or gas. Here a series of parallel plates in a casing or shell form the channels through which the gas flows, and between these plates a series of wires is suspended from an insulated framework.

The material used in the construction of the plates varies with the particular conditions of each problem. In some cases they are of steel or alloys; in others, they are in the form of a concrete slab with embedded conductors at the center. Rod curtains are sometimes substituted for steel plates, offering certain structural advantages. The dust, after collecting on these plate electrodes, settles into a lower header or hopper from which it is removed and disposed of.

A TYPICAL INSTALLATION. The electrical equipment is located reasonably close to the precipitator. A single conductor carries the unidirectional current, through the bushing provided on the electrical substation, to the precipitator. Where several precipi-

tator units are provided, disconnect switches are installed on each so that any one or several may be disconnected electrically without interference with the operation of the other unit.

30. OPERATING CHARACTERISTICS

Electrical precipitation may be employed for the removal or recovery of practically any type of suspended matter from any kind of gas, with a temperature range from atmospheric to approximately 1300 deg fahr. This upper limit is determined by the ability to secure structural materials that will stand up under extreme heat.

The precipitator sets up practically no resistance to the flow of air or gas. Its introduction in a gas system, therefore, raises no added problems in draft maintenance or suitable gas flow.

The electrical power consumption is nominal. The current flow is so small as to be measured in milliamperes. Speaking in general terms, the power consumption is 5 to 8 kwhr per million cubic feet of gas cleaned.

The labor required is almost negligible. Practically no attention is needed when a liquid material is being precipitated, as in such cases the precipitator is self-cleaning. When precipitating dust, the collecting plates must be rapped and cleaned at intervals. Where labor is at a premium, mechanical and automatic contrivances are supplied to perform these functions.

RECOVERIES AND EFFICIENCIES. The phenomena of electrical precipitation are such that the degree of removal of suspended matter is a function of the time during which the suspended matter is within the electrical field. This permits the design of equipment to give practically any degree of gas cleanliness required. In certain cases, a 90 per cent removal is all that is needed. In other instances, where the demand is for practically a complete removal or recovery of suspended matter, this demand can be met, removing more than 99.99 per cent of the suspended matter.

An installation is usually made up of two or more precipitator units, which permits continuous gas treatment even when one unit is temporarily removed from service to permit cleaning—and this with but a slightly reduced efficiency of recovery or removal.

31. TYPICAL INSTALLATIONS

Electrical precipitation has been widely applied to miscellaneous gas cleaning in industrial operations where the gas after cleaning is discharged to atmosphere or utilized in subsequent operations or distributed for subsequent use. In some cases the material removed from the gas is a dry dust or fume; in others, a liquid—such as acid, tar, etc. In some cases the material collected has a value; in others, its removal simply results in abatement of a nuisance or improvement in subsequent operations. In some cases the gas is cleaned at atmospheric temperature; in others, at temperatures ranging up to 1200 deg fahr. The following are some of the more typical installations.

REMOVAL OF FLY ASH FROM PULVERIZED COAL FIRED BOILER GASES. In June, 1931, Cottrell electrical precipitation equipment was in use or under construction in thirteen different power stations. The individual installations ranged in size from one handling 50,000 cu ft per min of gas to one treating 1,500,000 cu ft per min.

DETARRING GAS. In June, 1931, electrical precipitation was in use or under construction at about seventy different gas plants, such as carburetted water gas, by-product coke oven, etc. It removes tar and oil from the gas ahead of purifiers, saturators, light oil towers, etc. The individual installations range in size from 1,000,000 to 70,000,000 cu ft per day.

COLLECTION OF METALLIC FUMES. Numerous plants in the non-ferrous metallurgical industry are using electrical precipitation for the collection of metallic compounds, such as tin, lead, zinc, etc., from gases coming from reverberatory furnaces, blast furnaces, kilns, and other smelting and refining operations. Much precious metal is also recovered.

COLLECTION OF CEMENT KILN DUST. Many of the portland cement plants are using electrical precipitation for the collection of dust from gases coming from rotary kilns producing cement clinker.

ROASTER GAS CLEANING. Where zinc or pyrites ore is roasted, electrical precipitation is used for the recovery of the dust and fume carried over in the gases, particularly where the gas is utilized in the subsequent production of sulfuric acid.

ACID MIST COLLECTION. Electrical precipitation is in use in many plants for the collection of sulfuric acid mist from exit gases coming from acid concentrators and for the purification of gas, i.e., removal of acid mist and other suspended materials, in contact sulfuric acid plants.

CLEANING IRON BLAST FURNACE GAS. Electrical precipitation is in use in several steel plants for primary cleaning of the hot gas and for secondary gas cleaning of the cooled gas directly following primary gas washers.

In addition there are numerous other applications in various industries.

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ELECTRIC WELDING

By H. M. Hobart

33. GENERAL CONSIDERATIONS AND DEFINITIONS

Among the various processes of joining metals by welding, several employ electrical energy as the source of the heat. Some one or other of these electrical methods is appropriate for almost any welding requirement.

It is proper, however, to state that in many instances there are excellent non-electrical alternative methods. Thermit welding and autogenous gas welding are notable examples. For extensive classes of work, it is generally found that electrical methods are particularly economical, rapid, and effective, probably usually much the most rapid, and, in the majority of cases, at least as effective in securing joints of high quality. Indeed, the rapidity inherent to electrical methods of welding is an important factor in obtaining high quality, since the influence of the oxygen and nitrogen of the air, at the high temperatures required,* presents one of the principal harmful factors; therefore, the more rapidly the

* In various alloys and in materials refined by heat treatment, the liability to unfavorable changes due to migration of particles of important constituents is a serious consideration.

weld is completed, the better in general will be its quality, since there is less time during which the oxygen and nitrogen are in contact with the very hot metal. High speed, of course, also contributes to low outlay for both labor and capital charges.

The great advances made in recent years in the art of welding have not only brought about its extensive use as an alternative to riveting but now permit of making joints of kinds and for conditions not heretofore possible, particularly as regards tightness, and ability to withstand extremely severe mechanical stresses.

STANDARDS AND DEFINITIONS. The American Institute of Electrical Engineers has developed *Standards* for electric welding apparatus. These Standards relate respectively to resistance welding apparatus and electric arc welding apparatus. For many years these standards contained the following definition:

"**Weld.** A solid union of metallic parts formed by either heating to a plastic or fluid state the surfaces of the parts to be joined and allowing the metals to flow together (with or without additional molten metal being supplied) without any pressure being supplied; or by uniting or consolidating by hammering or compressing, with or without previous softening by heat."

In the 1934 revision * this is replaced by the following:

"**Weld.** A localized consolidation of metals."

The writer considers this too brief to convey a useful understanding of the term. Mr. F. T. Llewellyn may be quoted (Outline of welding and allied processes, *Metal Progress*, December, 1930, and January, 1931) as follows:

"Welding may be briefly defined as a localized consolidation of metals by means of heat. While it is true that, as a laboratory experiment, the surfaces of certain metals can be united without heat, and that the several welding processes involve other factors in addition to heat, yet the importance of heat in all practical welding operations warrants its inclusion as a common factor in any useful definition."

The several processes of electric welding may be placed in three groups:

I. Resistance welding.
II. Arc welding.
III. Other welding methods having electrical features.

34. RESISTANCE WELDING

The art of welding metal parts by passing a low-voltage current (practically always an alternating current) across the surface of contact of the metal parts to be joined, was invented and also made a commercial success by Professor Elihu Thomson in the 1880's. This process, which is known as resistance welding, has been used widely for many years in a great number of manufacturing industries. In each successive year, however, new applications continue to be made, and there is as yet in sight no limit to the extent of the usefulness of resistance welding for joining metals. On the contrary, particularly during the last few years, the rate of increase in its use has been enormous.

THE PRINCIPAL ENGINEERING PROBLEMS IN RESISTANCE WELDING. For any given application of resistance welding, the engineering problems usually chiefly relate to (1) the design of machines for holding the parts; (2) bringing them into suitable juxtaposition; (3) regulating the mechanical pressure and rate of pressing together of the parts to be joined, at various stages of the process of heating the metal at the surfaces of contact; (4) arresting the process at the proper instant; (5) the regulation of the current to the most suitable amount at all stages of the process; and (6) the provision of means for cooling various parts (usually by water circulation) of the transformer, the electrodes, and the other parts carrying current. Careful consideration must also be given to the sizes, shapes, and condition of the two surfaces of the parts to be welded which are brought together and of the thermal conductivity and the proportions of the parts in the neighborhood of these surfaces.

KINDS OF JOINTS. Before further describing the resistance welding process, it is desirable to explain that the kinds of joints made by that process may be grouped in three main categories: (1) butt welded, (2) seam (or line) welded, (3) spot welded. Thus it may be said that resistance welding comprises three main varieties, and these may be defined as follows:

Butt Welding. When, in the resistance process, the parts to be welded to one another are "butted" together, the current passing through the entire section of metal at the weld, the term butt welding is often employed.

Seam Welding. When, in resistance welding, the point of introducing and carrying away the current moves along a line so as to make a continuous weld along that line, the

* See A.I.E.E. Standards Nos. 38 (arc) and 39 (resistance), January, 1934, revision.

term seam welding or line welding is employed. Seam or line welding is sometimes accomplished by making a series of spot welds adjacent to one another, in fact, most seam welding is actually a series of spot welds. (Sometimes in using this means of making a seam weld between thin sheets, the successive spot welds overlap one another.)

Spot Welding. When, in resistance welding, the current is passed transversely across two plates which it is desired to join at many places (or spots) the term spot welding is employed. In the early days spot welding was limited to relatively thin plates of metal. It is now also applied to the welding together of materials of considerable thickness.

Pressure Welding and Flash Welding

Resistance welding may be subdivided also according to two different procedures with respect to the mechanical pressure. These two varieties may be respectively designated as (1) pressure welding and (2) flash welding.

PRESSURE WELDING. In pressure welding the procedure consists in first bringing together the surfaces to be welded, then sending current across the contact between them until the material reaches the proper temperature and finally, effecting the weld by forcing the surfaces of the plastic metal into one another a short distance. The current may be cut off either before or after the last stage of the process. Pressure welding (if employed for butt welds) usually produces a somewhat enlarged cross-section at the weld, due to the upsetting of the metal. The enlargement, if objectionable in the product, is sometimes removed while hot and sometimes ground down. Great mechanical pressure is required in butt or spot welds of parts of large cross-section. As a rough indication, it may be stated that the requirements for electrical power and mechanical compression, with medium-rolled steel, are (per square inch of area to be united) respectively from 30 kv-a to 200 kv-a for 10 sec (these quantities varying with the shape, size, and thickness and grade of the material) and not less than 5000 lb nor more than 8000 lb. If the process is carried out more slowly, the electrical input required per square inch is less. But the best results usually correspond to short time and great power.

FLASH WELDING. In flash welding (as applied to butt joints), instead of immediately pressing the parts together, the current supply circuit breaker is first closed The surfaces are then brought approximately together but with little or no pressure between them. Current now flows across this poor surface of contact and arcing ensues. By continuing these conditions and this procedure for a sufficient time, the surfaces to be welded become gradually better fitted to one another. When the right temperature condition has been reached sufficiently uniformly over these surfaces, the two parts are abruptly pressed into one another. Sometimes the current supply circuit is interrupted prior to this concluding phase of the process, but in other cases, a very heavy current flows (since the resistance has greatly decreased as the result of pressing the surfaces together), and, sufficient heat for the weld having now been provided, the current is cut off. In the flash process, dirt, impurities, and oxides are ejected from the joint in molten form, or consumed, whereas with the pressure process, such impurities are unable to escape from the joint and may impair its quality.

For butt welds, particularly for high grades of steel or for very large or complicated cross-sections, flash welding often gives the best results. Flash welding may, however, also be applied to small parts. Indeed, the choice between (1) pressure welding and (2) flash welding is governed by many highly technical considerations and is based on the results of accumulated experience.

In the January, 1934, revision of A.I.E.E. Standard No. 39, entitled Standards for Resistance Welding Apparatus, there is given the following definition for flash welding:

"**Flash Welding.** A resistance butt welding process wherein the welding heat is developed by the passage of current in the form of an arc across a short gap between the surfaces to be welded, these surfaces being kept slightly separated until they have flashed off to parallelism and have reached the desired temperature. The electrical circuit is then opened and the upsetting movement takes place. The operation of the machine may be manual, semi-automatic or fully automatic. The name 'flash' arises from the fact that during the heating period, oxidizing metal is thrown off in a shower of sparks."

With the flash method, a less depth of material has to be brought to the ultimate temperature. But whereas, in the pressure method, the material is brought up only to the plastic condition, in the flash method it is heated up almost to the molten condition and thus permits the impurities to be squeezed out with the resultant improvement of the joint. The flash method is more of a surface process and occasions less change in the quality of the neighboring material. Consequently, for butt joints in steel of high quality, the flash method is sometimes the most satisfactory. The flash method is unsuitable, however, for some alloys such as stainless steel. For these it is all-important to prevent

access of air to the heated spot, and also to heat as small a mass as possible and in so short a time as to prevent any migration of carbon from the heated material. It would be very difficult for even the most experienced metallurgical and welding specialists to state any generalization as to the respective merits and fields of application for these two resistance-welding alternatives (pressure welding and flash welding). The broad term resistance welding is used below to cover both varieties.

Fields for Resistance Welding

The flash process is confined to butt welds and is usually incompatible with the conditions required in seam welding and spot welding. (However, since in some designs it is hard to decide whether the joints should be designated butt or spot, and since pressure welding shades off into flash welding, it is extremely difficult to set up rigorous distinctions to which there shall be no exceptions.) The kind of metal affects the choice. Usually the pressure process must be employed for non-ferrous metals and for most alloys and metals whose qualities are dependent on cold working or heat treatment. For such materials the pressure process with thyratron timing control (described below) is proving very effective and is coming into extensive use.

With an automatic welding machine, many hundreds (and, in some instances, even thousands) of small welds are made per hour and with relatively small * current and mechanical pressure. In one known application, the machine makes 9000 spot welds per hour.

In striking contrast to these small welds may be cited welds between enormous parts (many tens of square inches of cross-section) which may require many tens of thousands of amperes and many tens of tons of pressure.

Butt welds of iron parts of 15 sq in. cross-section can be made by the flash welding process at the rate of a few minutes per weld with an energy consumption of the general order of some 20 kwhr and with the labor of only one man and a helper. It has been pointed out that, for such a weld, there was formerly required by forge welding the labor of three or four men for a matter of half a day, and the making of such joints was an art acquired only after long training. Moreover, while, by the forge process, only low-carbon material could be joined, the flash process welds high-grade steels and various other materials which were not susceptible of welding by the forge process. The ability to weld these high-grade steels permits of their employment in machines and structures and permits of effecting great reductions in their weight in the many cases where strength is the desideratum as distinguished from mass.

METALS TO WHICH RESISTANCE WELDING IS APPLICABLE. Most metals in reasonable states of purity can be joined to one another by the resistance process. Resistance welds can also be made between many alloys. Practically all grades of steel are readily welded. Different metals can also, in many instances, be welded to one another. The current for resistance welding usually is supplied from the low-voltage secondary of an a-c transformer, but direct current can be used to advantage under certain circumstances, and recently has been employed for welding the longitudinal seams of steel tubing, the direct current being provided from an acyclic generator supplying 2.5 volts and up to 20,000 amp. (See p. 60 of the April 24, 1930, issue of the *Iron Trade Review*.)

SPOT WELDING OF THICK PLATES. Spot welding of thick plates is an alternative to the long-established process of riveting, but is as yet not extensively employed where the plate thickness is greater than $^1/_8$ or $^3/_{16}$ in. During the World War a number of experimental heavy-capacity spot welders were built, and by their use it was demonstrated that two steel plates, each 1 in. in thickness, could be successfully spot welded together. Such large resistance welders are inherently quite expensive, and the concurrent and rapid development of arc welding (considered in the next article), combined with its broader field of application, has perhaps unduly retarded the commercial development of heavy spot welding. Only a few of these heavy welders with deep jaws have been built, but since they admirably illustrate the principles involved in resistance welding (indeed better than the complicated machines used in mass-production applications of resistance welding) Figs. 1 and 2 from the preceding edition of this handbook are again included.

STATIONARY DUPLEX SPOT WELDER (Fig. 1). The construction of commercial spot welders for use in ship fabrication may in general appearance be similar to so-called bull-riveters. A large spot welder which was built for such a purpose had a 6-ft gap and was for operation from a 60-cycle circuit. This outfit was a stationary machine, in the frame of which two transformers were incorporated. The steel plates and shapes

* To avoid misconception, it is desirable to point out that even in joining such thin materials as two strips of 40-mil stainless steel, the "relatively small current" is of the order of 20,000 amp when making a spot weld. But it flows for only some 0.03 sec.

were brought to the welder. Bulkheads, frames, floors, and other parts were constructed with it, and were then transported by cranes to their places in the ship. This 6-ft-gap machine had the capacity to weld together two $^3/_4$-in.-thick steel plates. It provided a pneumatic pressure of 60,000 lb and a current of 50,000 amp. It was called a duplex welder, and it welded simultaneously two spots, each of some 1.5-in. diameter, in about 30 sec.

The arrangement is shown diagrammatically in Fig. 1. The outfit comprised two transformers, one located on each side of the plates to be welded. The current crossed the plates in one direction between two opposed electrodes and then back again between two other opposed electrodes. Thus the two secondaries and the two joints constituted four elements connected in series. *AA* represent the two primaries. *BB* represent the two secondaries (which, in the actual construction, had only one turn each). *CC* and *DD* represent water-cooled copper electrodes. *EE* represent the two steel plates to be joined.

The above-described duplex feature was introduced in this spot welder to decrease the inductance of the long loop of conductor which otherwise would have been necessary

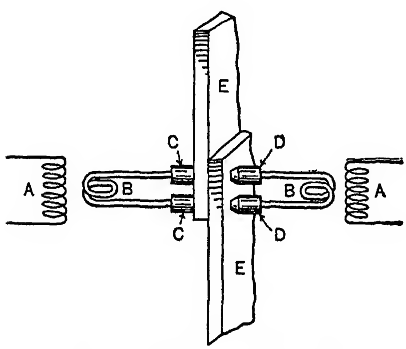

Fig. 1. Duplex Spot Welder

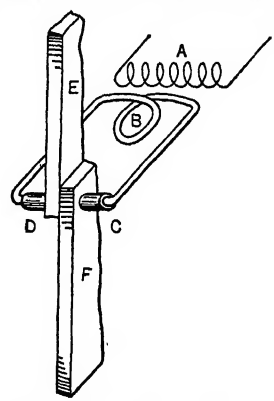

Fig. 2. Simplex Spot Welder

and which would have involved proportioning the transformer secondary for a higher voltage to overcome the inductive drop. The duplex construction thus improves the power factor.

SERIES WELDING. This method which has proved best in certain cases may be explained by referring to Fig. 1 and conceiving that everything at the left of the plates *EE* is replaced by a thick copper plate pressed against the back of the upper plate *E*. In this application, the electrodes *DD* are actually located considerably farther apart from one another than is indicated in Fig. 1. The current passing from the electrodes *DD* into the steel plates *EE* passes through these plates to the thick copper plate at the back, there completing the secondary circuit. This method usually is employed for welding thinner material than is indicated by the plates *EE* in Fig. 1.

PORTABLE SPOT WELDER. Fig. 2 is a diagram of the circuits of an ordinary (simplex) spot welder which welds only one spot at a time. In this figure: *A* represents the primary of the transformer. *B* represents its secondary. *C* and *D* represent water-cooled copper electrodes. *E* and *F* represent the two plates to be welded.

A typical 60-cycle portable welder of this kind, having a 27-in. gap, weighed only 2800 lb, including the transformer (which was embodied in the structure), and welded together steel plates of a thickness of half an inch with a current of about 30,000 amp. The pressure required for such work was some 25,000 lb.

In experimental work, currents exceeding 100,000 amp have been employed in improvised spot welding machines, and pairs of 1-in.-thick steel plates have been welded therewith.

SPOT WELDING VS. RIVETING. Spot welding has decided advantages over riveting for a great variety of important work now usually done by riveting. By employing spot welding, great savings are effected through eliminating the laying out and punching of the rivet holes, and the subsequent reaming when the holes do not match up.*

* In some naval vessels it is required that the rivet holes be drilled, thus involving still greater cost as compared with welding.

Furthermore there is avoided the reduction in cross-section and strength occasioned by the necessity for the rivet holes and thus the removal of material otherwise available for strength. With spot welding there is a great saving in material through the reduction in overlap made practicable as compared with provision for the two or three rows of rivets which are often required. Tests show that greater strength can be obtained with a single row of spot welds than with the best triple riveting.

Experience with and confidence in arc welding have so increased in recent years that proposals are now being made to *butt* weld the hull plates of ships, thus completely eliminating overlap, further reducing the weight of the hull, and providing a surface with decreased frictional resistance.

PROJECTION WELDING. The term "projection welding" is used to denote a slight modification of ordinary spot welding and is defined by Llewellyn as "a method of spot welding wherein small ridges, or other projections, are pre-formed (by rolling or otherwise), on one or both of the pieces to be joined; the projections serving to localize the current." Two examples of projection welds are shown in Fig. 3.

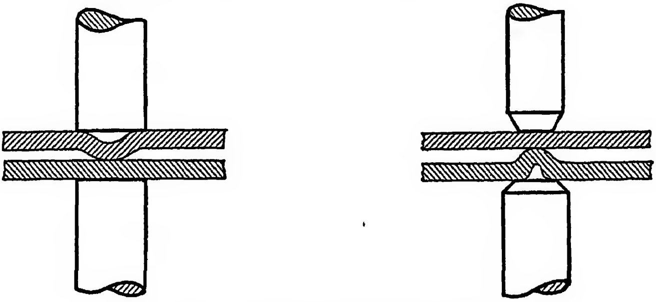

FIG. 3. Two Forms of Projection Weld

APPARATUS FOR RESISTANCE WELDING. Since resistance welding is a single-phase load of poor power factor and is usually characterized by extremely great, rapidly recurring peaks of short duration, there is particular need for careful provision that these severe characteristics shall not harmfully affect the interests of other people obtaining power from the same circuit. A single-phase load which cannot be distributed fairly equally between the phases of a polyphase system is always relatively objectionable to the power company, and it is far more objectionable when it is extremely variable in amount and when the peaks constitute a large percentage of the total load on the circuit, and particularly when the power factor is low. Unfortunately, the power factor is usually considerably lower the larger the rating of the welding machine and is as low as 0.6 in very large machines. However, with the present-day large-scale distribution systems provided in many industrial areas, the usual resistance welding load, even when large, is a smaller percentage of the total load on the distribution system than it would have been even fifteen years ago. Furthermore, it often should be quite practicable to distribute several single-phase welding transformers fairly uniformly on a polyphase system. To obtain the full advantage there would be required some interlocking plan to insure equality in the number of welding machines on each phase which would be carrying the welding current at any instant. In view of these facts, it is sometimes desirable, with particularly large resistance welding loads, to interpose, between the distribution system and the welding transformer, a motor-generator set consisting of a polyphase motor and a single-phase generator. This precludes any unbalancing between the phases of the supply system and eliminates from the distribution system all bad consequences of the poor power factor of the welding load. It does not, however, level out the peaks in the load. This can in some cases be done to a considerable extent by providing the motor-generator set with appropriate flywheel capacity, if the motor is of the induction type and designed for suitable slip. On the other hand, if the advantages of smoothing out the load are foregone, usually it will be preferable to employ a synchronous motor and operate it with overexcitation. This will be of advantage to the supply company through improving the power factor of the system and decreasing the voltage drop, and will tend to compensate for the increased cost of service occasioned by the poor load factor.

The single-phase generator of the motor-generator set is an expensive machine, much larger than a polyphase generator of the same rating, has lower efficiency, and is more prone to noise and vibration. So if the resistance welding load can be divided into two

these two kinds of electric welding should be employed. Resistance welding is applicable to joining almost all kinds of metals; electric arc welding is at present most widely applied to the welding of mild steel. However, rapid progress is being made in extending its use to the welding of high-grade steels and various other metals and ferrous and non-ferrous alloys. Indeed, the claim is made by reliable welding authorities that arc welding is already being applied with complete success to "a great variety of alloy steels, cast iron, copper, aluminum, nickel, Monel metal, bronze and a vast variety of other types of material." It is the opinion of the author of this article, however, that much remains to be accomplished in the case of many materials of these categories with respect to the uniform reliability, durability and high mechanical quality of the weld, and with respect to the speed of welding, the cost of the processes, and the general availability of artisans capable of producing such results. As already stated in this article, it is the welding of mild steel which, at present, constitutes the overwhelming preponderance of all arc welding. In the arc welding of mild steel not only are welds of uniformly unimpeachable quality readily made but also they are made cheaply and with rapidity and certainty by workmen possessed of ordinary training and ability. For most materials other than mild steel, the writer believes that, although arc welding is making rapid progress, resistance welding methods are at present farther ahead.

Before entering upon the consideration of electric arc welding, it should be pointed out that it is one of the two main subdivisions of an art which is often termed **autogenous** welding, but which will here be designated **fusion** welding.

FUSION WELDING. The term fusion welding is employed to comprise **gas welding** and **electric arc welding.** Gas welding does not come within the scope of this article, and it must suffice to state that the gas fusion process in its simplest form usually consists in simultaneously fusing (ordinarily with an oxyacetylene flame) (1) the material at and near the surfaces which it is desired to join, and (2) some material (which may or may not be similar in composition according to the object to be accomplished) in the form of a rod (termed the filler rod), the tip of which is subjected to the heat of the flame. The oxyacetylene (or other) flame is directed with one hand and the welding rod is manipulated with the other.

Kinds of Electric Arc Welding

Electric arc welding may be subdivided into several classes. One classification comprises: (*a*) carbon arc welding, and (*b*) metal arc welding.

CARBON ARC WELDING. In carbon arc welding, an arc (usually several tenths of an inch in length) is established between a carbon or graphite electrode (usually a carbon electrode) and the two pieces of metal which it is desired to join. This carbon electrode is manipulated with one hand, and material from a welding (or "filler") rod is fed into the weld by manipulating the filler rod with the other hand. The manual activities in carbon arc welding are seen to be similar to those in gas welding. In neither process is it necessary for the material of the welding rod (i.e., the filler rod) to traverse the arc.

These considerations inspired the development of a carbon arc welding method exploited under the name of the **electronic tornado system.** Its sponsors stress the less opportunity for impairment of the quality of the weld by oxygen and nitrogen inclusions when metal is not transferred *through the arc* from an electrode to the weld. In the electronic tornado system, an electromagnetic field is provided to direct and stabilize the arc and to focus the arc stream directly beneath the carbon. The system is employed in automatic welding machines, and an "autogenizer" is fed into the arc for the purpose of surrounding the hot metal with a neutral gas, thus shielding it from the oxygen and nitrogen of the surrounding air. When added metal is required, it may be fed to the weld automatically or it may be laid along the line of weld. The system is carefully described by E. W. P. Smith at p. 18 of the September, 1933, issue of the *J. Am. Welding Soc.* in an article entitled Use of shielded carbon arc in Class I welding. The electronic tornado system is employed in the automatic welding of boilers, tanks, pressure vessels, large water pipes, rear axle housings, some types of automobile frames, etc. Its use is not limited to any particular types of joints.

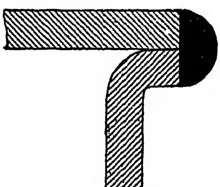

FIG. 4. Type of Arc Weld Not Requiring Additional Electrode Material

KINDS OF WORK NOT REQUIRING WELDING ROD. In carbon arc welding, the edges of the parts to be joined sometimes may be so designed as to obviate the need for any additional material; in other words, no welding rod is necessary. In Fig. 4 is shown a case where it is not necessary to use a welding rod, since it suffices simply to melt together the adjacent flanges with an arc.

METAL ARC WELDING. Metal arc welding is fundamentally different from carbon arc welding. Thus in metal arc welding of mild steel, instead of having a carbon electrode for one terminal of the circuit, the arc is established between a steel welding rod (or welding electrode) and the steel parts to be joined. The shortest practicable distance is maintained between the end of the welding rod and the work. This distance is bridged by an electric arc. It is important that the arc be maintained as short as practicable so that a minimum of oxygen and nitrogen from the surrounding air may gain access to the hot metal. Where the arc is protected (as in welding in a neutral gas or with a suitable flux-covered electrode), the length of the arc is less important and, with some heavily fluxed types of electrode, relatively long arcs have been found advantageous.

Welding Machinery

The voltage at the arc with the metal arc process in the open air is usually only from 12 to 35 volts, the lower values being used with bare electrodes and the higher with heavily coated electrodes; consequently, if a circuit of standard distribution voltage, such as 125 or 250, were employed as the power supply for the arc, with a suitable resistor in series with the arc, a great deal of power would be wasted in the resistor. For most arc welding today, however, motor-generator sets are employed to deliver direct current at considerably lower than standard distribution voltages. Two different systems of power supply from motor-generator sets are commonly employed, one known as the **multiple-operator system,** and the other as the **single-operator system.**

MULTIPLE-OPERATOR SYSTEM. In this system a motor-generator set is employed, which has a generator of a capacity sufficient for supplying several welding arcs. The generator is usually designed to operate at 50 to 75 volts and is flat compounded so as to deliver approximately constant voltage regardless of load. Then each welder employs a current-regulating resistor, and frequently an arc-stabilizing reactor, in his indiviual welding circuit. It is obvious that with this arrangement the power losses in the resistor will be very greatly reduced below what they would be if the open-circuit voltage were 125 or 250.

SINGLE-OPERATOR SYSTEM. In this system, power for each welding circuit is supplied by an individual generator which may be motor, belt, or engine driven. The generator is designed with special characteristics such that the open-circuit voltage may be from 50 to 100 volts; when the arc is struck and current drawn from the generator, the output voltage drops to the arc voltage. No resistor is ordinarily used in series with the arc, and the welding current is adjusted by various means, such as changing the shunt field rheostat, changing taps on the series field, or shifting brushes, depending on the design of the individual generator. A certain amount of reactance in the circuit is desirable in order to stabilize the arc, and this may be embodied in the design of the generator itself, or else a separate reactor may be employed.

Each of several manufacturers has placed on the market types of welding apparatus comprising single-operator d-c generators with characteristics for which various advantages are claimed and realized. Even very brief descriptions of the principal varieties of these machines would occupy too much space for this article.

MULTIPLE VS. SINGLE OPERATOR SYSTEM. Which system of power supply should be employed for any given welding operation depends upon individual conditions. If a number of welders are working in a comparatively small area continuously, the multiple-operator system may prove to be the most efficient from the standpoint of power cost. If the welding operations are scattered, or if portability is a factor, then the single-operator set should be chosen. Each installation must be studied and a decision arrived at based on the merits of the case. This subject is discussed at considerable length, introducing further important considerations, in an article in the September, 1934, issue of the *Welding Engineer*, entitled Multiple-operator welding sets or single-operator sets, which? by J. L. Jones of the Bureau of Construction and Repair of the Navy Department.

KIND OF CURRENT EMPLOYED. Direct current is most generally employed for arc welding. The generators of the motor-generator sets to which reference was made in the preceding paragraphs are d-c generators. It has been the usual conclusion in America that a d-c arc is more readily established, maintained, and manipulated, and that it permits of obtaining better welds, than an a-c arc. With direct current, a good bare-wire electrode is usually reasonably satisfactory for ordinary welding of mild steel, whereas with alternating current it has been the usual experience that a flux-covered electrode is preferable and often necessary for securing a steady arc and ready manipulation. There is, however, a disturbing influence peculiar to direct current, namely, the effect of the magnetic field to deflect the arc. This becomes quite serious when

welding with very great currents such as are often employed with automatic arc welding. This erratic behavior with direct current may be mitigated by suitable compensating arrangements, and is by no means serious with the moderate amount of current usually employed with manual welding. There has already been described the use, in the electronic tornado system, of a magnetic field imposed on the arc to increase the stability by "focusing" the arc stream.

With alternating current, especially when the current is small, there is a tendency for the arc to be extinguished every time it passes through zero when reversing. But with the greater amount of heat in a heavy current arc, this tendency is much less in evidence, particularly if a high periodicity is employed and if the current is considerably out of phase with the voltage so that, at the instant when the current is passing through zero, there is a considerable voltage to help maintain or re-establish the arc when the current is reversed. The material surrounding a covered electrode may also be of such composition and construction as to constitute a means of preventing cooling and extinction of the arc while the current is reversing. As already indicated, d-c welding is at present by far the most widely used in America; however, the development of low-cost, heavily covered electrodes suitable for use with the a-c arc has offered considerable impetus to the adoption of alternating current. It seems especially suitable for automatic welding on a large scale with very heavy currents. But in pointing out its especial suitability for automatic welding with very heavy currents, there is no intention of suggesting any limitation to the use of alternating current for manual welding. Such use has been made of it, especially in England, since the earliest days of the arc welding industry.

The choice of kind of current is not confined to direct current versus alternating current but involves the relative merits of single phase and polyphase * and of low periodicity and high periodicity. It might be supposed that the fact that the use of 60 cycles is so general would eliminate any question of periodicity. But there are all sorts of ways of transforming the periodicity, and, furthermore, the inevitable accentuation of the present enormously rapid expansion of the welding industry and the commercial importance of making the very best welds with all sorts of metals and by the most rapid and economical processes sooner or later will insure the thorough consideration and examination of all alternatives, irrespective of existing traditions, prejudices, and standards.

Several electrical manufacturers are now offering a-c welding equipments consisting inherently of high reactance transformers with suitable means of varying the reactance, and thereby the welding current. These units are usually designed to give an open-circuit or "striking" voltage of 70 to 100 volts, and with sufficient reactance so that when the welding current is flowing, the voltage drops to that required for the arc. With the heavily covered electrodes usually employed this will range from 25 to 40 volts.

Kinds of Electrodes

It will be impossible, in this brief article, to do more than mention the general situation as regards electrodes. They may be placed in two chief classes, **bare** and **covered.**

Up to a few years ago, most welding people would have agreed that bare electrodes were usually regarded in America as the most suitable for ordinary kinds of welding, particularly for welding low-carbon-steel rolled sections and plates. Bare electrodes of special compositions are also manufactured for various difficult kinds of welding, including non-ferrous metals and alloys and high-grade steels. On the whole, it has been more convenient, in automatic welding at high speeds, to employ bare welding wire automatically fed to the arc from reels. Mention has already been made of the method of automatically feeding an "autogenizer" to the arc to shield it. However, several interesting means have also been developed for the more satisfactory use of covered wire for automatic welding, when, as often is the case, it gives more adequate results.

For most work, the bare electrode is of low-carbon steel manufactured with great care and often especially treated in various ways. For work where it is more suitable, the bare electrode is of high-carbon steel, manganese steel, phosphor bronze, or sometimes of other alloys. All sorts of ingredients have been tried, sometimes with very important results on the quality of the weld.

In covered electrodes even greater diversity is found in the brands on the market. The variations are not confined to the covering but extend to the metal electrode within the covering. Some electrode purveyors standardize the enclosed electrode and confine the variations to the covering; others standardize the covering and accomplish the respective purposes by variations in the composition of the enclosed electrode; still others vary both. Metal constituents (deoxidizing or for alloying purposes) are often embodied in

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* Bethenod's polyphase current arc-welding system has been employed considerably in France. It is described in *Bull. Tech. de la Suisse Romande,* Jan. 14, 1928, p. 7.

the essentially non-metallic covering. In other cases, deoxidizing and alloying ingredients are embodied in the metal core of the electrode.

Sometimes the covering is so constituted that, when melting at the working end of the electrode, it gives off a copious supply of protecting or deoxidizing or denitrogenating gases which displace or neutralize the oxygen and nitrogen in the surrounding air as well as the gases which the heat of the arc expels from the metal being welded.

Until comparatively recently the first cost of heavily covered electrodes was so high that they were but little used. Within the past few years, however, developments in this field have been very rapid, and today such electrodes are available at reasonable cost, and the quality of the weld obtained is such that the covered electrode is rapidly displacing the bare electrode where welds of really high quality are required. This is especially true in the welding of pressure vessels and in cases where welds must have good ducility.

Gaseous Protection

There is a group of welding processes that provide for making the weld in the midst of a gaseous medium which protects the weld from the oxygen and nitrogen in the surrounding air and which may also reduce the oxygen given off from the parts being welded or from the electrode metal, or both. Again it would be impossible to describe all the methods that have been employed. As typical may be mentioned the process in which a continuous stream of hydrogen or other suitable gas is supplied around the arc. In the case of hydrogen, the arc drop is some 60 volts as against some 20 volts in air. Consequently, for the same current some three times as much heat is developed at the arc. This gives much better efficiency if the welding is done from a constant voltage supply, since it is no longer necessary, in regulating the current, to waste in the control rheostat so large a percentage of the energy supplied from the circuit. However, there are some difficulties in the way of applying this process successfully, and the welds obtainable from it have not shown sufficient superiority to justify its commercial acceptance.

Solid Protection (other than an electrode covering)

F. T. Llewellyn, in an article entitled Outline of welding and allied processes which appeared in the December, 1930, and January, 1931, issues of *Metal Progress*, describes an example of solid protection of the weld (during welding), as follows:

" In another recent process a bare metal electrode plows with oscillating motion through a thick mass of powdered fluxing material which serves both as a shield for the arc and as a blanket to permit the metal already deposited to cool slowly."

Reduction of Stresses

ANNEALING. The process above described by Mr. Llewellyn combines the advantages of affording protection while welding, and annealing the weld after it is made, and should sometimes permit of avoiding the expense of again heating the product in an annealing oven to release locked-up stresses and otherwise improve its quality. In general, when practicable, it would be of advantage to cover up promptly the welded product with some material of poor thermal conductivity before it has cooled, thus annealing it by retarding the rate of cooling. When the welded product is small, made in large quantities and quickly, it should sometimes be practicable to have each piece, while still hot from just having been welded, automatically ejected to some suitable place and then automatically covered with hot sand. Usually, with large objects, the weld has cooled at one part before other parts are welded and, if the locked-up stresses must be relieved, it is usually done in an annealing oven as a subsequent process. In the use of some electrodes with a heavy flux covering, this flux forms a thick crust on the weld when it " freezes " and serves to retard the cooling and thus anneal the deposited metal. Sometimes it is justifiable to deposit an outer bead of metal to effect a partial annealing of the lower layers, the outer bead later being machined off.

PEENING. Another way of releasing locked-up stresses, and in other respects improving the quality of the metal right after the weld has been completed and while it is very hot, is by peening (for example, with a pneumatic hammer). This is well known to be a valuable and often very economical means of reducing residual stresses. But although it has been urged repeatedly that a scientific technique should be developed by careful research tests at various temperatures and with various kinds and amounts of peening (such as the severity and periodicity of the blows), it is not known to the author that this has been done yet in an adequate manner, studying the quality and the relative freedom from internally stressed conditions after different peening techniques have been employed. At p. 33 of the March, 1934, issue of *Metal Progress*, in describing the welding of the gates for the Boulder Dam penstocks, Mr. C. H. Jennings says: " All layers of deposited weld

metal were peened with a blunt-nosed peening tool in a light chipping hammer, to reduce residual stresses and to clean the weld. The peening operation was carried out immediately after the deposition of each 6 to 8 inches of deposited metal before it and the adjoining metal became cold."

LOCKED-UP STRESSES and the **IMPORTANCE OF DUCTILITY.** By employing suitable welding technique it is often possible to avoid the creation of "locked-up stresses." Furthermore, even when they are not avoided, their amount sometimes can be controlled within permissible limits so that the product shall have the necessary strength. There is still a great deal of research work which ought to be done to increase the available knowledge in this field. Locked-up stresses will be less if the materials to be welded and the electrode material and the welding technique are all such as to provide ductility in the weld, for even when the weld will not be subjected in service to any bending stresses, its ductility will permit of internal adjustments of minute dimensions which distribute the total tensile (or shear or torsion) stress more uniformly over the entire cross-section provided to withstand the stress. This would appear to be the most scientific method of securing the best result. But there are those who contend that, unless the weld will be subjected to bending stresses, there is no virtue in striving for ductility, but rather there should be provided a maximum of rigidity, and they sometimes insure this rigidity by enlarging the cross-section at the weld sufficiently to be adequate even when locked-up stresses prevent the internal readjustments necessary to secure uniform distribution of the stress over the entire cross-section.

Recent and important additions to our knowledge of the stress concentrations and their relation to static and dynamic loading are well described by Thurm & Bantz in *Stahl u. Eisen*, Sept. 26, 1935, p. 1025.

Miscellaneous Processes

The **ARCOGEN PROCESS.** This process was developed by the Griesheimer Autogen Gesellschaft and has been described in an article entitled Das kombinierte autogen-elektrische Schweissverfahren Arcogen, by Dr. H. Munter, in Nos. 22 and 23 of *Autogene Metallbearbeitung* for 1930. The arcogen process consists basically of the use of the oxyacetylene torch held by the operator in his right hand and a flux-covered metal electrode maintaining an electric arc at its tip, the operator manipulating this electrode with his left hand. The electrode is generally of a larger diameter than would be employed for a given application with the oxyacetylene process or with the arc process. About half of the heat is provided by the oxyacetylene flame and the other half by the electric arc. The electric arc is supplied by alternating current of commercial frequency from a transformer which is smaller than would be needed for ordinary arc welding (because only half the heat is supplied electrically) and which has a higher power factor than the ordinary arc welding transformer. A somewhat higher voltage is employed for the arc, namely, some 33 volts as against 20 volts. Under these conditions, the power factor of the transformer is of the order of about 0.40 and the efficiency is about 0.75. In the arcogen process the electrode is not touched to the work in order to start the arc. It is simply necessary to bring the end of the electrode up to the cone of the oxyacetylene flame, whereupon the arc establishes itself, the no-load voltage being 80, falling to about 33 volts when the current flows. The author of the paper to which reference has been made states that, with the arcogen process, the flame (1) preheats the work, (2) brings it to the point of melting, (3) does its share in melting the welding wire, and (4) provides for the ionization of the gaseous path by burning the ionizing flux covering of the electrode. Then the arc (1) melts the electrode and (2) maintains the molten pool in fluid condition and increases the penetration. It is claimed that shrinkage stresses, as compared with those with gas welding, are less because of the less width of the seam and the higher speed of welding.

The **ATOMIC HYDROGEN PROCESS.** In this process the welding is not accomplished *directly* by means of the heat of the electric arc. The arc is established between the tips of two tungsten wires. The current is alternating and is usually supplied from a single-phase transformer connected to any ordinary distribution system. The arc is located in the path of a stream of hydrogen gas. The heat of the arc breaks up the hydrogen molecules into free atoms. When the hydrogen in the atomic form has traveled a short distance beyond the arc it has cooled sufficiently to recombine into the molecular form, and the energy thus released constitutes the heat actually employed for welding. Thus the weld is made in a predominatingly hydrogen atmosphere and is much more free from oxides and nitrides than when made in air. Secondly, the tungsten electrodes also last longer than they would with an arc in air. And this system of welding provides a third good feature which is, in the author's opinion, even more important than the first

two features: namely, it permits the operator to manipulate his " torch " in whatever manner is preferable for each particular job. If it is desirable to thoroughly " puddle " the material being welded, the " torch " can be used to do so. For some other kinds of jobs it will be better so to manipulate the torch as to bring the two surfaces to be joined just up to the " sweating " temperature and then deposit metal from the filler rod. Moreover, there can be used filler rods of whatever material is most suitable for the particular job without reference to the behavior of such material when it forms one end of an electric arc. It will be seen readily that these various features render the atomic hydrogen system of welding not only very flexible but also suitable for many kinds of jobs for which ordinary arc welding could not be used, or if used, would be less satisfactory. The process has been applied with particularly excellent results to the welding of thin sheets and for welding various ferrous and non-ferrous alloys.

ATOMIC HYDROGEN WELDING OF ALLOYS. The process may be employed for steels with any carbon content up to 1.25 per cent. In an article entitled Progress in atomic hydrogen welding, by Samuel Martin, Jr., at p. 537 of the *Iron Age* for March 3, 1932, many applications of the atomic hydrogen process are described and illustrated. In several of these applications no filler rod is required. It is stated in the article that chrome steels, including those having a chromium content up to 40 per cent, have been welded, and that if the chromium content is less than 20 per cent, the resultant welds are not particularly brittle. The article furthermore states that heat treatment after welding relieves any tendency to brittleness, giving a strong, ductile weld. Thus ascoloy (a chrome steel which has a chromium content between 12 and 16 per cent) can be welded by this system, and welds made on super-ascoloy (which has 17 to 20 per cent of chromium and 7 to 10 per cent of nickel) are strong and ductile without heat treatment. Ka–2, enduro, and Allegheny metal are further examples of ferrous alloys which may be welded by the atomic hydrogen process.

Among the non-ferrous alloys mentioned by Martin as being satisfactorily welded by the atomic hydrogen process are calorite (15 to 20 per cent chromium and 80 to 85 per cent nickel), which can be welded whether in the worked form or in the form of castings; brass (of certain compositions); and Monel metal (75 per cent nickel and 25 per cent copper). Martin mentions another successful application of the atomic hydrogen process, namely the application to drills, dies, and other equipment requiring high wear-resisting surfaces of various hard surfacing materials such as stellite, stoodite, cristite, borod, blackor, and tungsten carbide. In giving these illustrations, it is not intended to imply that it is with the atomic hydrogen method alone that these things could have been accomplished, but rather to indicate the particular adequacy of the process to several admittedly difficult welding tasks.

ATOMIC HYDROGEN PROCESS APPLIED TO AUTOMATIC WELDING. The atomic hydrogen system has also been applied to automatic welding and sometimes to the joining of thicker plates. In atomic hydrogen welding, the voltage required to start the arc is much higher than for ordinary metallic arc welding. After the arc is established, the voltage decreases to about 80 volts. It should again be emphasized that the atomic hydrogen process, the arcogen process, and the carbon arc process share in common the feature of permitting of preheating, to any desired extent, suitable portions of the materials to be welded together. The absence of this feature is a limitation (though for much work, such as welding mild steel, possibly not an important limitation) of ordinary metallic arc welding.

WELDING NICKEL AND MONEL METAL. Nickel is difficult to weld because of its property, when molten, of absorbing great quantities of gases, especially oxygen. At p. 15 of the January, 1932, issue of *Zeitschrift für Metallkunde*, will be found an article by Canzler entitled Die Nickelschweissung, which describes many methods of welding nickel and also Monel metal. It is stated in the article that, for such work, the atomic hydrogen method has been employed with success in obtaining good non-porous welds owing to the considerable prevention of oxidation by the surrounding hydrogen atmosphere which also serves as a reducing medium for such oxygen as penetrates from the surrounding atmosphere. The filling rod, stated in the article as being employed, is of Monel metal.

ARC WELDING WITH SINGLE-PHASE ALTERNATING CURRENT. It has already been pointed out that direct current has been most generally used for arc welding and that the use of a-c metallic arc welding is rapidly increasing. One difficulty with a-c welding has related to the tendency of the arc to be extinguished when the current passes through zero in reversing. At the zero instant, no heat is being supplied and the temperature tends to decrease. If the voltage is in phase with the current, the instability is increased. But if the voltage is greatly out of phase, although the stability of the arc will be greater, the low power factor may render the load objectionable to the power supply

company, unless some corrective means is adopted. Furthermore, the transformer costs more, is heavier, and is less efficient. Therefore it is important to provide additional means for stabilizing the arc. Flux coverings help a great deal.

STABILIZING THE A-C ARC BY SUPERIMPOSED HIGH FREQUENCY. In describing the arcogen process it has already been stated that the arc is established and maintained by means of the oxyacetylene flame and hence does not need to be "struck" in the ordinary way. Another way of starting and maintaining the arc, which has been employed in Europe for several years, consists in superimposing, on the welding circuit, high-frequency oscillations which maintain the ionization of the arc path and thus stabilize the arc. This means is also employed in a system now in use in America. It is described in a paper entitled *An Improved Alternating Current Arc Welder*, by A. M. Candy, presented in January, 1932, at a convention of the American Institute of Electrical Engineers.* The oscillator circuit (seen in Fig. 5 to consist of a condenser A, an air core

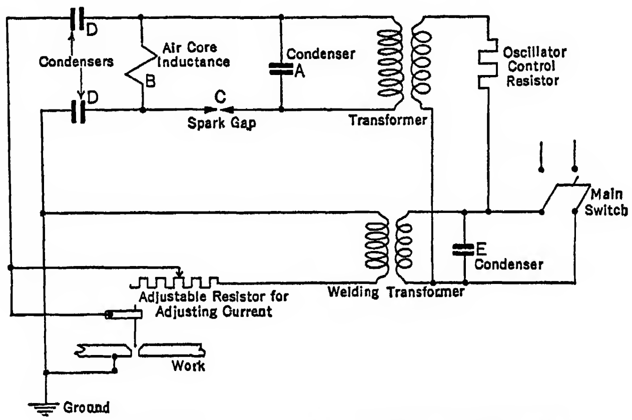

FIG. 5. Circuit Connections for an a-c Arc Welder, 100 Amperes

inductance B and a spark gap C) occasions sparks to cross the gap, reducing its resistance and enabling the low-frequency supply to establish and maintain a welding arc. The frequency in the oscillator circuit is stated to be so high as to "preclude the possibility of any annoying or injurious physiological effects upon the person using the apparatus." By providing two blocking condensers, D, placed in the circuit between the oscillator and the welding electrodes, the welding circuit is prevented from supplying any energy to the oscillator inductance, B. These blocking condensers constitute barriers to the low-frequency welding current, but they offer no impediment to the high-frequency discharge. In order that the oscillator shall not feed into the power circuit (which would be likely to cause interference with neighboring radio sets), a condenser, E, is connected across the input terminals. It is stated that a 100-amp set has an efficiency of 30 to 35 per cent (varying with the load) and a power factor above 90 per cent. Regulation of the welding current is effected by a series resistor. The open-circuit secondary voltage is only 73 volts, thus minimizing danger from shock. Successful welding is permitted by the system down to very small currents (such as are necessary for joining thin material), because the oscillator relieves the low-frequency welding arc from the duty of being self-maintaining. It is stated that, with this process, the weld has less penetration than one made with the same number of d-c amperes, and that this is a distinct advantage in welding thin plates. It is further stated that if, on heavier work, more penetration is wanted, a larger current and larger electrode can be used and the speed of welding correspondingly increases.

Welding of Various Metals and Alloys

Distributed through the preceding paragraphs are allusions to the application of various electric welding processes to the joining of specifically mentioned materials such as mild steel, high-carbon steel, various ferrous alloys, and various non-ferrous metals and

* See also Alternating Current Arc Welding Transformer and Circuit Characteristics, A. M. Candy, *J. Am. Welding Soc.*, Sept., 1934, p. 23; and at p. 405 of Feb. 27, 1932, issue of the *Elec. World*, an article by J. B. Gibbs entitled Oscillator Stabilizes a-c Welding Arc.

alloys. It has been indicated that, the greater the affinity of the metals for gases, particularly for oxygen and nitrogen, the more the need for special features to obtain good welds. Also a high percentage of carbon in steel was emphasized as requiring special features in the welding procedure employed. Brittle materials are particularly subject to acquiring internal strains, the avoidance or subsequent relief of which presents special problems. Tendencies toward porosity, which are of different degree in different metals and with different processes, must be carefully considered. In recent years the welding of stainless steels and of aluminum and its alloys (notable among which is duralumin) has been the subject of much study.

Within the limits of this article it would be futile to attempt to consider in detail these important problems, and we can do no more than suggest to the reader that, during the last few years, there have been published in the columns of the *J. Am. Welding Soc.* a good many papers and articles and informative discussions of these subjects.

WELDING AS A FOUNDRY TOOL. The Allgemeine Elektricitäts Gessellschaft's book entitled Elektrisches Schweissen, at the close of a chapter dealing with the arc welding of cast iron, states: "The foundry engineer may, by the proper application of 'hot' welding, construct iron castings of shapes and sizes which would be difficult if made in a single casting, by joining together several simple component castings." That paragraph is the only reference of which the author has knowledge, suggesting the applicability in the construction of iron castings, of the procedure of joining together two or several simple castings. Until the application of arc welding to cast iron has advanced considerably beyond present knowledge and practice, such a plan would be considerably limited because of the difficulties and slowness and consequent cost attending cast iron welding in general and because of the mediocrity of the resultant product unless slow and expensive procedures are employed.

JOINING MILD STEEL CASTINGS BY WELDING. For use in *steel* foundries, however, great economies and a better piece of work will often result if two or several small and simple mild-steel castings are joined by electric welding (arc or resistance) to form the larger and often much more complicated casting which otherwise would need to be made in a single casting. Often the cost should be much decreased, and the weight as well, and furthermore the quality should be superior to that which could be secured in a single large casting. Ever since it became widely recognized that the arc process is an especially satisfactory and economical method of welding mild steel in particular, it has been rapidly adopted in all sorts of manufacturing undertakings. But, with negligible exceptions, the designs have been worked out for using rolled material. This has tended to decrease the demand for steel castings to a serious extent, from the standpoint of the foundry's interest. For some years there occurred isolated instances of the joining of two or several steel castings to one another (or to rolled steel) by welding, and yet the great possibilities and advantages in this direction were almost completely ignored, though it is hard to see why they were not very obvious. More recently these possibilities and advantages have been pointed out, and the steel foundry industry has only itself to blame for not employing welding as a very effective tool in the steel foundry and as a means of making better, cheaper, and lighter steel castings.

The following articles dealing with this important subject should be consulted for a more complete description of the proposal:

Welding combined with foundry practice, by W. H. Namack and H. M. Hobart. *Gen. Elec. Rev.*, Dec., 1930, pp. 674–676.

The use of steel castings in welded structures, by J. G. Ritter. *J. Am. Welding Soc.*, March, 1932, pp. 27–30.

Welding's relation to steel castings, by T. S. Quinn. *Ind. and Welding*, August, 1932, p. 3.

36. MANUAL VS. AUTOMATIC WELDING

Much the greater portion of all electric resistance welding is done on automatic machines. For arc welding, manual operation was at first very usual. In many applications, notably shipbuilding, steel building, and bridge construction, much manual arc welding is at present necessary, but in the construction of trusses and girders in the shop the welds will doubtless more and more often be made automatically, the manual welding being reserved for the erection-in-place welds. For a very great many applications arc welding is done with automatic machinery, and new applications are constantly being developed where the welding is automatic.

Although the initial installation of automatic machinery is relatively expensive, the saving in time and the superior character of the welds generally justify the method in mass production. There are many alternative types of machinery. Essentially, the elec-

trode material is fed automatically to the arc, and the arc is fed along the seam to be welded. The mechanism for feeding the electrode is embodied in a part designated the "welding head." Sometimes two or more welding heads follow one another in welding a seam. The current employed may be much greater than for manual welding, and the speed of welding is also far greater. An automatic machine holds steadily an arc which is shorter than is usually attainable manually, and thus decreases the access, to the hot metal, of oxygen from the surrounding air. The machine does not need periods of rest from the effort and strain, which are very considerable for the manual arc welding operator. The machine has no eyes to smart or muscles to ache, and it is never nervous or worried. It is not subject to occasional lapses into carelessness or indifference. Often one man can run continuously more than one automatic machine. Altogether it is reasonable to expect that inventive ingenuity will gradually largely eliminate manual arc welding.

37. APPLICATIONS OF ELECTRIC WELDING

Space limitations will not permit of much more than listing some among the many very important fields in which electric welding is being applied.

USING WELDED ROLLED STEEL SECTIONS AND PLATES INSTEAD OF GRAY IRON CASTINGS. This application is placed first as being one of the largest as well as one of the earliest to be taken up.

STEEL BUILDINGS. Among the kinds of buildings which have been constructed by joining rolled sections and plates by the electric arc method of welding (instead of joining them by rivets) may be mentioned (1) hospitals and hotel buildings and buildings in their vicinity (where it is particularly desirable to avoid the noise of riveting); (2) factory buildings, including machine shops, laboratories and warehouses; (3) office buildings (one recently erected being 19 stories high). Spraragen states that several experimental steel frame dwelling houses have been erected at a cost which compares favorably with that of wooden structures. He states that general adoption would provide an outlet for more than a million tons of steel annually. Welding is already permitted by the building codes of a great many municipalities in America, and special permits are readily obtainable in many other cities. In one of the Engineering Foundation's research narratives entitled Welding Steel Structures, Spraragen mentions a very heavy welded truss made for the Forest Lawn Memorial Park Mausoleum. It weighs 60 tons and has a span of 96 ft and a height of 18 ft. In substituting welding for riveting in building construction, there is usually a material saving in weight. This saving, though rather small in welded beams and columns, may, it is stated by Spraragen, amount to as much as 36 per cent in ordinary roof trusses, and similar savings are obtained in plate girder construction.

The Report of the Structural Steel Welding Committee of the American Bureau of Welding, published in September, 1931, by the American Welding Society, describes and analyzes the results of an elaborate series of tests made for the important purpose of providing reliable data for the design of steel structures employing electric arc and gas welding. The report (comprising more than 200 pages) covers investigations which have involved the fabrication and testing of several thousand specimens welded by 61 welders at 39 shops and tested at 24 laboratories, including college and commercial institutions and the U. S. Bureau of Standards. Spraragen states that: "Tests on 2495 specimens indicated that joints commercially welded by qualified welders may be expected to possess strength within 12 per cent of general average results."

STEEL SHIPS. During the War, strenuous efforts were made in this country and in Great Britain to induce those in authority to design and build ships with the hull plates joined by welding instead of by riveting. During and immediately after the War, a completely arc-welded, cross-channel barge was built by the British Admiralty and a coasting vessel by Cammell Laird, but the inertia of the shipbuilding interests, both merchant and naval, has in all countries, until very recently, prevented much progress in joining the hull plates by welding, although welding has been used very extensively in the interiors of most ships. However, Germany has made wide use of welding (including the hull plates) on several important warships, and most countries with shipping interests are now alive to the great advantages of welding methods. Welding is now widely employed in the hull construction of barges, tankers, ferryboats, naval targets, dredgers, motor ships, lifeboats, and many other varieties of small and medium-sized craft. Recently, in the navies of Germany, Japan, America, and France, riveting is being gradually superseded by welding for the hull construction of important vessels of all classes. The very general use of welding in place of riveting for hull construction is inevitable and cannot long be delayed even by indifference and inertia.

Among the recent articles on the subject of welded ships are:

Description of all-welded auxiliary vessels being built at the U. S. Navy Yard, Mare Island, California, by Lieut. H. A. Schade. *J. Am. Welding Soc.*, May, 1931, p. 23.

The future of welded ship construction, by J. Kjekstad. *J. Am. Welding Soc.*, September, 1931, p. 13. Discussion, December, 1931, p. 12.

M. S. Shean, ferryboat *Suwa Maru*, and steam tug *Hakutaka Maru* are described in articles in *Elec. Welding* (London), for October, 1931.

Electric welding in ship construction. *Elec. Welding* (London), February, 1932, p. 19, and April, 1932, p. 13.

Welding on *U.S.S. New Orleans*, by Captain J. O. Gawne. *J. Am. Welding Soc.*, April, 1932, p. 9. Discussion, July, 1932, p. 5.

All-Welded Steel Barges, by R. B. Reid, *J. Am. Welding Soc.*, March, 1934, p. 25.

New System of Welded Ship Construction, *J. Am. Welding Soc.*, October, 1935, p. 11.

Motor Ship *Joseph Medill*. *Elec. Welding*, London, August, 1935, p. 202.

OTHER APPLICATIONS. Electric welding (resistance or arc) is being increasingly used in the construction of (1) bridges, piers, roof trusses; (2) pipe lines (for oil, gas, and steam, and for very high pressures); (3) pressure vessels, fired and unfired; (4) large storage tanks for gas and oil; (5) electric generators, motors, transformers, and switches; (6) automobiles; (7) machine tools; (8) engines; (9) locomotives; (10) other railway rolling stock; (11) rail joints; (12) all kinds of tanks and containers; (13) gears; (14) refrigerators; (15) aircraft; (16) concrete reinforcement; (17) hard facing; (18) steel mill machinery; (19) mining machinery; (20) tunnel linings; (21) towers for transmission lines, windmills, elevated water tanks; (22) in fact, in nearly all instances where metals require to be joined. This last statement (22) is possibly premature. It is already a fair statement for structures of low-carbon steel, but for many other metals and for most alloys, though developments are rapid, much remains to be done in developing techniques which will insure thoroughly satisfactory results not only as regards quality but also as regards speed and cost. A particularly important example is aircraft which, to some extent, awaits development of methods for welding light alloys.

38. BIBLIOGRAPHY

There are listed below some recent books on electric welding. Unfortunately, arc welding is stressed out of all proportion to its relative importance (in the author's estimation) as compared with resistance welding.

1935. Verein Deutscher Ingenieure, Dauerfestigkertsversuche mit Schweissverbindungen. V.D.I. Verlag, Berlin.

Selected Welded Constructions (Ausgewahlte Schweisskonstruktionen). The Committee on Welding in the Verein deutscher Ingenieure (V.D.I.) has published under the above title a series of five volumes showing by illustration how modern welding methods can be applied to various branches of engineering.
Vols. III, IV and V are in English and German.
Vol. III. Pipe and Tank Construction (" Rohrleitungs- und Behälterbau "). Edited by Dr.-Ing. H. Holler and Reg.-Baumeister a. D. W. Fink.
Vol. IV. Car and Aircraft Construction ("Fahrzeugbau"). Edited by Dipl-Ing. E. Kalisch.
Vol. V. Shipbuilding (" Schiffbau "). Edited by Obermarinebaurat Lottmann.

1935. Harris, H., Metallic Arc Welding. Longmans, Green, New York.

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Rossell, H. E., Riveting and Arc Welding in Ship Construction. 210 pp. Simmons-Boardman Pub. Co., New York.

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Fish, G. D., Arc-welded Steel Structures. McGraw-Hill Book Co., New York.

Stockton, R. C., Principles of Electric Welding. Pitman, New York.

Verein Deutscher Ingenieure, Ausgewählte Schweisskonstruktionen. Vols. 1–4. 1930–1933. Berlin. Other volumes to follow.

1932. Hubert, E. H., ed., Manual of Electric Arc Welding. McGraw-Hill Book Co., New York.

MacKenzie, L. B., and Card, H. S., Welding Encyclopedia. 8th ed. Welding Engineer Pub. Co., Chicago.

Manuel de Soudure Électrique à l'Arc. Paris, Librairie de l'Enseignement Technique.

1931. Heating and Piping Contractors National Association, Standard Manual on Pipe Welding. The Association, New York. Includes all methods of welding.

1930. Wilson, W. M. Tests of Welds. University of Illinois, Urbana.

1929. Hulse, E. P., ed., Arc Welding. McGraw-Hill Book Co., New York. Lincoln prize essays, Vol. 1.

Johnson, J. B., Airplane Welding. Goodheart-Willcox Co., Chicago.

Allgemeine Elektricitäts Gesellschaft, Berlin, Electrisches Schweissen. (Deals with resistance welding and arc welding.)

1923. Owens, J. W., Fundamentals of Welding, Gas, Arc and Thermit; a textbook for governmental engineering departments, colleges, technical schools, etc. Penton Pub. Co., Cleveland, O. A good general text.

The art of welding is undergoing such extraordinarily rapid development that, in many respects, books on the subject are obsolete before they are published. The best sources of up-to-date information are, consequently, technical periodicals. At present, the "house" journals of large corporations are on the whole (in the author's opinion) the most useful and up-to-date sources. Amongst these and professional journals the following will be found valuable as sources of current welding information:

J. Am. Welding Soc.; Die Elektroschweissung (Journal of the Deutsche Gesellschaft für Elektroschweissung); *The Welding Engineer* (Chicago); *Ind. and Welding* (The Industrial Publishing Co., Cleveland); *Elec. Welding* (published by the Quasi-Arc Co. of London); *The Welder* (published by Murex Welding Processes, Ltd., of London); *V.D.I. Zeitschrift; Stahl und Eisen; Autogene Metallbearbeitung; Zeitschrift für Metallkunde; Krupp'sche Mitteilungen; A.E.G. Mitteilungen; Siemens Zeitschrift; Gen. Elec. Rev.; Elec. J.; Engineer; Engineering; Journal of the Ateliers de Constructions Électriques de Charleroi; The Brown Boveri Review; Bulletin Oerlikon; J. Iron and Steel Institute* (London); *Metropolitan Vickers Gazette; Association Suisse des Électriciens Bulletin.*

Welding Societies. American Welding Society; Deutsche Gesellschaft für Elektroschweissung (Braunschweig); Institution of Welding Engineers (London); The Iron and Steel Institute (London) is very active in fostering the art of welding.

Welding (whether by electric or other means) involves so many departments of science and engineering that articles of value relating to welding are published in great numbers by practically all the scientific and engineering societies. In America, the relatively successful effort is made to utilize the American Welding Society to keep track of and guide developments in this enormous field. The columns of its *Journal* should consequently be consulted; they will be found to contain not only articles and reports of importance, but also references to material appearing in other publications in America and elsewhere.

Some publications which are issued by the American Welding Society or sponsored in part or wholly by that Society, and which may be obtained through its headquarters (29 West 39th Street, New York City, Miss M. M. Kelly, Secretary), are as follows:

Arc Welding Instruction Manual.
Resistance Welding Instruction Manual.
Code for Fusion Welding and Gas Cutting in Building Construction.
Structural Steel Report.
Final Report of Committee on Welded Rail Joints.
Specifications for Fusion Welding of Pressure Vessels.
Filler Metal Specifications.
Tentative Code for Fusion Welding Pressure Piping.
Part D—Rules for the Fusion Welding of Hulls and Hull Parts of Marine Code for Fusion Welding and Gas Cutting.
Rules for Fusion Welding of Gravity Tanks, Tank Towers and Risers.
Report on Welding and Cutting, Nomenclature, Definitions and Symbols.

Research always has played an important part in the activities of the American Welding Society. Up to recently, this work was done by the American Bureau of Welding, a board functioning under the joint auspices of the A. W. S. and the National Research Council. In this way, probably more research work was done in the welding field than by any corresponding similar technical organization. The published reports have formed the basis for continued advancement of welding in all branches. These reports have been accepted throughout the world as authoritative sources of information free from commercial bias. In 1935, the A. W. S. took over the A. B. of W. and now operates it under the title of Bureau of Welding Research. The B. of W. R. continues to maintain close cooperation with other leading technical and scientific bodies. At the present time (1936), the Bureau of Welding Research has two important committees—the Fundamental Research Committee and the Structural Steel Committee. The B. of W. R. is arranging to embark upon further research activities and collaboration for this purpose is now being developed between A. W. S. and Engineering Foundation. The Fundamental Research Committee has some sixty investigations under way in the various universities in this country. Each year its reports form the outstanding papers presented at the Fall Meeting of the Society. The headquarters of the F. R. C. are at 29 West 39th Street, New York City.

SECTION 19

ELECTRICITY ON THE FARM

BY

LEE C. PRICKETT

ART.		PAGE	ART.		PAGE
1.	Power Supply	02	7.	Farm Lighting *	18
2.	Dairy Farming	02	8.	Irrigation and Water Supply	19
3.	Grain and Feed	06	9.	Drying and Dehydration	21
4.	Live Stock	11	10.	Hotbeads	24
5.	Poultry—Game—Eggs	13	11.	Miscellaneous	25
6.	Fruit	18	12.	Bibliography	28

(Items, in each article, are arranged in alphabetical Order.)

* Including Ultra-Violet Light.

ELECTRICITY ON THE FARM

By Lee C. Prickett

1. POWER SUPPLY

SYSTEM. The farming regions of the United States are being rapidly covered with distribution lines reaching to the farms. Both overhead and underground distribution are used and, at the present time (1936) the designs are in a state of transition with no standardization in sight. However, underground supply as yet is little used.

The usual distribution is three phase with grounded neutral 2300/4000 volts, 60 cycles, but by far the greater number of farms are supplied by single-phase current.

RURAL LOAD CHARACTERISTICS. It is practical to carry a farm connected load of from eight to ten times the rated size of the transformer, especially with proper fusing and good general protection of motors and devices. In a typical test community, the average of the individual load factors for five farms with an average demand and monthly energy consumption of 13 kw and 207.8 kwhr, respectively, was 12.1 per cent, 14.1 per cent in summer and 10 per cent in winter. The time of maximum demand for these farms is also typical: in the evening when the load was largely lighting and household appliances; between 11 A.M. and 1 P.M. after a sizable range load was added to the line; and between 6 A.M. and 7 A.M. when the load had been still further increased by the addition of milking machines and water heaters.

For a group of farm lines the demand factor can be safely taken as 15 per cent or less of the total connected load, and well-developed farm lines should have load factors of 30 per cent or more. If special care is used to see that suitable equipment is installed, it is reasonable to expect a monthly energy consumption of 15 or even 20 kwhr for each kilowatt of connected load.

2. DAIRY FARMING

BOTTLE WASHERS. The electric power drive connection for bottle washers may be of either the direct or belt type. The consensus is that a speed of 900 rpm is more satisfactory than direct drive (1800 rpm). Screen door springs have been successfully used as (non-slipping) belts, and garden hose for direct couplings. A supply of hot water is necessary for satisfactory bottle washing.

See also Sterilizers and Water Heaters.

A $^1/_6$-hp motor is large enough for a single-brush bottle washer. For the double-brush type a $^1/_4$-hp motor is generally satisfactory. The energy consumption per 1000 bottles washed varies from 0.25 kwhr to around 0.6 kwhr respectively for single- and three-brush units.

Automatic bottle and can washers are available for use in creameries or large dairies.

BUTTERMILK WARMER. An insulated cabinet automatically keeps the air temperature around three cans of freshly skimmed warm milk at 70 deg fahr with an energy consumption of about 1 kwhr per day, a 1500-watt heating element being used. The resulting non-wheying buttermilk nets the producers as much as sweet milk.

CHURNING. A small rotating dasher type of churn, with a capacity of 12 gal of cream and operated by a $^1/_4$-hp motor, uses less than 1 kwhr per 100 lb of butter churned, the average churning and working time being 52.4 min and 17 min, respectively. A typical 6-gal barrel churn operated by a $^1/_4$-hp portable motor uses $^1/_6$ kwhr per churning, and in another case the energy consumption is about 1.85 kwhr per 100 lb of butter.

COTTAGE-CHEESE MACHINE. A typical cottage-cheese machine has a capacity of 50 gal of skim milk and is heated by a 50-gal water bath, the inner vat being made of tinned copper and the outer being insulated with 1 $^1/_2$ in. of good heat-insulating material. The water bath is kept at 70 deg for 12 hr by a thermostatically controlled 1000-watt element; the curd is then cut and the temperature immediately raised and kept at 140 deg fahr by a thermostatically controlled 2000-watt element for about $^1/_2$ hr or until the right solidity is obtained. Each vat has a thermometer attached, and the outer one a water gage.

CREAM SEPARATORS. The uniform speed of electrically driven separators results in cream of rather uniform quality and prevents heavy cream losses due to the speed dropping below the dead line. This is especially important during the winter since few separators will skim efficiently when the milk temperature is below 90 deg fahr, particularly if operated at speeds below that for which they were designed. No machine should leave more than 0.03 per cent of butterfat in the skim milk, but a survey in a typical area shows 45 per cent of the machines falling below this standard, resulting in an average annual loss of $18.74. A dirty separator enormously increases the bacterial count of milk and cream and by restricting the cream holes forces considerable cream into the skim milk. Similar losses occur when the cream opening and spout are not properly lined up.

The average farm separator requires $^1/_8$ hp (up to 1000 lb per hr) to $^1/_4$ hp (up to 1400 lb per hr), depending on machine design. The energy consumption per 1000 lb of milk separated varies widely, data showing 0.16 to 1.10 kwhr, depending particularly upon the quantity separated per operation. The average per month for each 10 cows from which all the milk is separated is not far from 2 to 2.5 kwhr.

Drip cups should furnish three or four drops of oil per minute, and the oil case should be drained about every six months (depending on amount of use), washed with kerosene, and refilled.

MILKING MACHINES. A milking machine pays its way in a herd of 15 cows where the equipment is cared for by the brush method, and in a herd of only 10 cows when the equipment is cared for by backwashing or suction washing, the brush method taking about as much time as milking four or five additional cows, and suction washing as milking one additional cow.

The average milking machine results in a labor saving of around 50 per cent in herds of 20 to 25 cows, and a reduction of 25 per cent in the total cost of milk production, the task of milking representing 41 per cent of the total labor. In fact, in herds where as many as 50 cows are milked a machine will take the place of one full-time man, an average of 20 cows (13 to 28 a typical range for various herds) being milked per man-hour. Combine and rotary combine milkers save about 70 and 95 per cent respectively of the total milking time or labor.

Power and Energy Requirements. For portable, double-unit milking machines the $^1/_6$-hp motor is practically standard, and for pipe line machines the average power requirements per cow milked range from $^1/_8$ to $^1/_2$ hp for various equipments. The energy requirements per month per 10 cows milked, for portable and pipe line machines, should not exceed 30 and 15 kwhr respectively, the consumption for the drive rod type being only a little higher than for portables. Some well-designed more recent equipment of both general types has much lower requirements.

Types of Milking Machines. Milking machines operate by either applying suction intermittently to the teats (as in less-expensive types) or a combination of suction and pressure (imitating the calf) applied to the teats.

There are six types of milking machines:

1. Pipe line. The vacuum pumps are either reciprocating or rotary, and the pulsations are either of the automatic or controlled type.
2. Drive rod. The pumps are portable and stanchion rows must be straight.
3. Portable. Less expensive and less dependable than pipe line type.
4. Relay. Usually three or four milking stalls and milk goes direct to milk room in sanitary piping.
5. Combine. Modified relay in which the milk first goes into a glass receptacle suspended from a scale where it can also be mixed by an air jet and a sample taken before it goes to the milk room to be cooled or pasteurized, or both. The milk could be bottled or canned without being touched by human hands. Manipulating the cow's udder until very little milk is seen to enter the receptacle eliminates hand stripping. However, other tests show no losses in average yearly production where no stripping is ever done.
6. Rotary combine or Rotolactor. A specially designed continuous milking plant, a rotating platform with a capacity of 50 cows, platform operated by $^3/_4$-hp motor, every rotation 12 $^1/_2$ min. Two men operate the milkers which are automatically sterilized after each milking, but 8 or 10 additional men are required to wash, dry, and care for the cows.

Milker Considerations. Vacuum pumps should have reserve capacity to take care of leaks that invariably develop from time to time.

Improper care of a milking machine results in low-quality milk, increases danger of infecting cows with mammitis or other udder disturbances, and causes a rapid deterioration of rubber parts. (See Fig. 1.)

In a typical case it took 20 min per day to wash two double units (double units seldom used), but suction or backwashing gives at least as good results. Immediately after each milking the machine is thoroughly rinsed, first with cold water and then with 1 $^1/_2$ gal per

unit, of hot water at 165 deg fahr (dousing so as to give an air brush, temperature must be no lower). The teat cups and long milk tubes are then placed on a rack and filled with disinfecting solution, preferably a 0.3 to 0.5 per cent solution of lye. In some cases they are placed in dry cold storage at 50 deg fahr instead. The teat cups, etc., are taken apart only frequently enough to inspect and renew leaking teat cup liners, etc. If the equipment is allowed to stand, this system of washing is not satisfactory until after the machine is brush washed. While not at all necessary, in some cases the milker heads are brushed when the rinsing is done, and some dairymen at milking time again rinse but this time first with hot (165 deg fahr) and then with cold water.

MILK PASTEURIZATION. An all-electric pasteurizer with a demand of 50 kw and a capacity of about 600 gal of milk at 40 deg fahr per hr (12 gal per kwhr, the pasteurized milk preheating the incoming milk) occupies only one-fourth the space required by comparable steam pasteurizers. The electricity flows through the milk and the rate of milk flow determines the pasteurizing temperature, which is 160 deg fahr. Pathogenic bacteria are killed without the usual alteration of flavor and cream line. In most types of

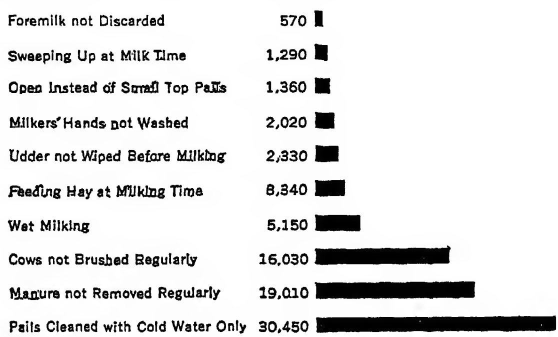

Fig. 1. Relative Contamination of Milk by Various Careless Operations, or Omissions, Showing Necessity of Clean Milking Machines

pasteurizers electricity is used only to operate the agitating equipment (batch type units). Recently (1935) electric pasteurizers with some major improvements have become available.

Milk is being pasteurized experimentally with supersonic waves, sound vibrations of around 100,000 cycles per second.

REFRIGERATION. Dairy. If fresh, good-quality milk is kept at 50 deg fahr or less, it can be stored for at least 24 hr without any marked increase in bacterial count (see Fig. 1); the critical temperature for cream is 40 deg fahr. Quality milk which is being shipped should leave the farm at a temperature of not above 40 deg fahr. On many farms, coffee-grade cream is left to accumulate for a week or even longer, and in such cases the cream should be cooled to at least 36 deg fahr in 3 hr or less by either wet storage in cans of not over 5-gal capacity, or by a tubular cooler or its equivalent, and then stored at 33 deg fahr. Fresh cream should never be mixed with old cream until completely cooled.

Methods of Cooling Milk. Milk and cream are cooled and stored by either the wet or dry methods, water being 21 times as efficient for cooling as air at the same temperature. Precooling is not necessary where the wet storage has ample cooling water at 35 to 40 deg fahr (water to milk ratios for night and morning cooling of 3 to 1 and 4 1/2 to 5 to 1 respectively), or where the dry storage has forced air circulation at around 33 deg fahr and small cans (5 gal) are used. Butter is best stored at 0 to 5 deg fahr. At this temperature and 80% humidity, butter will keep for 6–8 months. In the wet-type storage the cooling water should extend up the neck of the can at least as far as the milk.

Tubular coolers should be of the two-way type, cooling water in the upper half materially reducing energy requirements. For cooling less than 200, and from 200 to 400 gal of milk per day, 1/2- and 3/4-in. brine centrifugals respectively are satisfactory, both operated by 1/4-hp motors.

Types of Milk Boxes or Tanks and Cooling Units. Dry storage meets the needs of retail dairymen particularly and can also be used for the storage of vegetables, food, etc. For each 10-gal can stored a floor space 15 by 15 in. should be allowed, and for each quart bottle case a space 16 by 20 by 12 in. should be provided. Dry storages are cooled either by the brine system or by direct expansion, the former requiring 1 1/2 to 2 gal of

brine per gallon of milk cooled, and the latter, because of a lack of storage, having a lower capacity per horsepower.

Wet storage is favored by wholesale dairymen, the milk being cooled in the cans in which it is shipped. With forced circulation a higher water temperature can be used. (See above.)

Power Requirements and Energy Consumption. In tanks the average annual energy consumption for cooling 100 lb of milk below 50 deg fahr is 1 kwhr, and in a typical case for cooling and storing 10 gal of wholesale milk, 5 kwhr. For dry storage 60 kwhr per month per 100 qt of retail milk cooled and stored daily is typical. According to more specific averages, milk cooled and stored in 10-gal cans, by the brine system, requires about 0.12 kwhr per gal; milk cooled and stored in bottles by the brine system, 0.17 kwhr per gal; milk cooled and not stored by the brine system, 0.09 kwhr per gal; and milk when either cooled and stored, or cooled and not stored, by the direct expansion system, about 0.05 kwhr per gallon.

A $^1/_2$-hp water-cooled or $^3/_4$-hp air-cooled unit will take care of 50 to 100 gal of milk cooling and storing, or 75 to 125 gal of cooling alone, both with brine system, or 50 to 75 gal of milk cooling and storing, or storing alone, by the direct expansion method. Comparable figures for 1 hp water-cooled and 1 $^1/_2$-hp air-cooled units are 150–200 gal, 200–250 gal, and 100–150 gal respectively, and for 2-hp water-cooled and 3-hp air-cooled units, 275–350 gal, 325–400 gal, and 200–250 gal respectively. (See also Table I.)

Table I. Refrigeration Required in Retail Dairies in Southeast *

To cool 1 gal of milk from 95 to 40° F	3.5 lb of refrigeration †
To cool bottles or cans for 1 gal of milk from 85° to 40° F	1.5 " " " †
Two pounds of ice to crack over milk en route to market	3.2 " " " †
To provide cold storage and allow for losses, drinking water, etc.	2.0 " " " †
Total refrigeration needed per gallon of milk	10.2 " " " †

* Alabama Power Company averages.
† One pound of refrigeration is equivalent to the heat absorbed by the melting of 1 lb of ice.

Cost of Dairy Refrigeration. Estimating electric power at 2 cents per kwhr, and depreciation, interest, and upkeep at 10, 7, and 3 per cent respectively, a milk cooling plant will cost from $^1/_2$ to 1 cent per gallon cooled, depending on size, method of handling milk, etc. It is less expensive to cool milk in tanks with mechanical refrigeration than with ice, when electricity and ice costs are 3 cents per kwhr and $^1/_2$ cent per lb respectively. Total costs for ice refrigeration run from 45 to 156 per cent higher than for electric refrigeration, according to data from New Hampshire.

General-purpose. General-purpose boxes most frequently have a storage capacity of around 50 cu ft and will take care of ordinary food, freezing of ice cubes, and water cooling, also storing of a 5-gal can of cream, or a hog carcass, or other large piece of meat. Frequently a unit operated by a $^1/_4$-hp motor is sufficient, and nothing larger than $^1/_2$ hp should ever be required.

STERILIZERS AND WATER HEATERS. Utensils can be sterilized by exposure to: (1) dry hot air at not less than 230 deg fahr for 30 min; (2) humidified hot air at 205 deg fahr for 30 min; (3) steam or vapor at 170 deg fahr for at least 15 min; (4) steam or vapor at 200 deg fahr for not less than 5 min; (5) live steam for at least 1 min; (6) a chlorine solution of proper strength for 2 or more min, or (7) immersion in water at 170 deg fahr for not less than 2 min. However, utensils must be thoroughly washed (hot water very essential) if effective sterilization is to be assured, results with chemical solutions and live steam probably being the least and most affected, respectively, by any lack of cleanliness.

Sterilizers should be equipped with sanitary heat insulation, especially small units in cold climates or drafty locations. They should use a minimum of water with no dead compartment spaces (water level best maintained automatically so as to avoid danger of burning out heating elements), and the heat should be automatically turned off at the proper time by time switch or thermostat. Air or water circulation is essential to rapid heating, in the former case baffles and shields being required unless forced circulation (more desirable, especially for larger units) is available. It is suggested that in order to avoid the heat the fan blades be mounted on an extension shaft inside and the motor mounted outside. Temperatures much above 300 deg fahr will injure solder on pails, cans, etc. Sterilizers should be large enough to take the largest pieces of equipment such as a small 18 by 36 in. surface cooler. Probably the most popular size holds four 10-gal cans and other accessory equipment.

The heating elements for 4-can steam and dry or humidified hot air sterilizers, plus other

equipment, should be about 5 and 2 kw respectively, a 3-kw unit being ample for 7-can units of the latter type. In some areas a combination water heater and steam sterilizer with a small heating unit (low energy rate due to 100 per cent load factor) is proving satisfactory even though about one-third more energy is required than for other types.

The wash water can be heated with less energy and the water kept noticeably more sanitary through the use of a separate heater. However, the first cost of equipment is higher than where a combination type is used.

The energy consumption for sterilizing the equipment for a herd of up to 40 cows will average about 7.5 kwhr per day—for hot air types perhaps 6 kwhr. For herds of around 20 and 10 cows, the daily consumption will usually average a little under 7-kwhr and (very roughly) 3.5 to 4 kwhr (often only one batch) respectively. In somewhat typical cases for consecutive sterilizations 2.5 kwhr were required for 80 lb of metal and 3.65 kwhr for 125 lb of glass (4 cases).

About the same amount of electricity is required to heat wash water as to sterilize. From 7 to 10 gal of water at 140 deg fahr is ordinarily needed per day per 10 cows, this requiring around 25 kwhr per 100 gal heated, or 50 to 75 kwhr per 10 cows per month.

3. GRAIN AND FEED

CORN HUSKING AND SHREDDING. Shredding destroys corn borers, and the resulting stover is better than timothy hay or straw for milk cows, and better than straw for horses. The shredded stover likewise is better than straw and far superior to long unshredded stover for bedding, also much easier to feed.

The average man-hours required by hand methods and with a corn husker and shredder are 19 and 13, respectively. However, only 8 hr of man-labor are required to husk from the stalk where the stover is left in the field and the corn yields about 40 bu per acre.

Purdue data show that a 2-roll husker can be operated by a 5-hp motor, also that a 4-roll machine can be handled by a 7 $^1/_2$-hp motor by keeping the husker below full capacity. The energy consumption per 100 bu husked varies from 20 to 40 kwhr, the latter, as it happens, relating to a 2-roll machine.

CORN SHELLING. A $^1/_4$-hp motor will operate the average hand sheller with ease, almost doubling its capacity and saving the time of one man. Often the belt can be placed on the flywheel, the ordinary $^1/_4$-hp 1800-rpm motor and pulley giving about the right speed.

In a typical case a 2-hole sheller operated by a $^1/_2$-hp motor at 246 rpm had a capacity of 1536 lb per hr, using 0.02 kwhr per bu of shelled corn or 0.35 kwhr per cwt of shelled corn. The energy requirements with corn having a moisture content of 22 per cent were 12 per cent lower than with corn having a moisture content of 30.5 per cent. Shelling data for a hand sheller show 0.08 kwhr per bu of shelled flint corn.

DRYING. See Art. 9.

ELEVATING GRAIN AND SHAVINGS. Elevators are great labor savers; for example, they enabled one man to bin the grain from a thresher and reduce the cost of storage (making higher buildings practicable). The energy consumption per 1000 bu. elevated for vertical cup, tubular portable, portable drag, and chainless bucket (endless rubber belt equipped with buckets) elevators runs from a little over 1 kwhr (oats) for the vertical cup to a little more than 8 kwhr (ear corn) for the portable drag. Typical data on the energy consumption per 1000 bu per ft of lift for portable drag and vertical cup elevators show averages of 0.41 (0.28 to 0.59) and 0.13 kwhr respectively.

Ensilage cutters and fan elevators on hammer mills, as well as regular pneumatic equipment, are also used for elevating, the energy consumption running from about 10 (oats) to 33 (barley) kwhr per 1000 bu elevated. Ordinary 5-hp hammer mills will elevate 250 to 500 bu per hr to heights of 15 to 38 ft, ordinary 12-in. fans running at 2500 rpm elevating to heights of 38 ft but a maximum of 1200 to 1600 rpm, for elevations of 15 or 20 ft, is recommended if hulling or cracking of the grain is to be avoided. Horizontal runs of more than 8 or 10 ft are made by running the pipe high enough so that it has a downward slope of at least 30 deg.

Dry and wet shavings used for bedding were elevated 35 ft into a mow with a 45-in. fan 8 $^3/_4$ in. wide operated at 960 rpm by a 10-hp motor, at the respective rates of 8 and 2 tons per hour, with demands of about 10 and 5 hp respectively, and ton energy requirements of something over 1 and 2 kwhr respectively.

ENSILAGE CUTTING. See Silo Filling.

FEED AND ROUGHAGE GRINDING AND CHOPPING. **Grain.** The hard coats of ordinary grains and seeds must be broken or softened if they are to be fed efficiently, grinding being three or four times as effective as soaking. (See also under Soybean Cook-

ing.) Grinding medium fine and in some cases coarse is more profitable than finer grinding. (See Table II.) When corn is 50 cents per bushel, wheat is worth around 53 1/2 cents for sheep and poultry, and 56 cents for hogs and beef cattle.

Table II. Efficiency of Feed Utilization for Dairy Cows *

Whole corn...........	30–35% undigested
Whole oats............	20–25% "
Crushed corn..........	5–10% "
Crushed oats..........	1–2 % "
Medium corn..........	1–2 % "
Medium oats..........	0.001–1 % "
Pulverized grain.......	No findings

* Data from Purdue University; in general applies to other livestock as well.

Grinding Costs. The total cost of grinding with properly selected equipment is usually from 3 to 5 cents per 100 lb, and it is not generally profitable to own a mill unless at least 300 bu of oats or 150 bu of shelled corn (or their equivalents) are ground per year, the installation being assumed to operate without an attendant.

Grinding Considerations. 1. Grinding oats for hogs pays only when fed in amounts not to exceed one-fifth of the ration. Do not grind too fine (medium fineness preferred from the standpoint of economy), although finely ground oats are more palatable for hogs. Grinding grain for steers (cattle) and sheep frequently does not pay. Grinding grain for milk cows is usually good practice, medium fine grinding bringing the highest net returns. It is commonly thought that mash for chickens should be ground fine, but recent data favor coarser grinding because of greater palatability.

2. Hammer mills appear to be somewhat more efficient for fine and medium grinding, and burr mills for coarse work; also roller mills are ideal for cracking or coarse grinding. It is the peripheral speed of the hammers and not the rpm which is effective in hammer mill grinding.

3. Small hammer and burr mills may be successfully operated with motors of from 1 or less to 5 or more horsepower. Hammer mills are higher in first cost but are more easily adapted to semi or full automatic operation than are burr mills (less danger of injury to equipment).

4. Operation without an attendant is the main basis for the greater economy of the small-capacity mill, and therefore a feed supply and means for meal removal are essential to economical operation. The equipment overhead for a small mill is also materially lower, this being much the largest factor in determining grinding costs for fair-sized equipment unless an unusual amount of grinding is done. Burr mills operated without attention should be safeguarded against intrusion of foreign material and against running empty.

A small mill operated by a 1/2 hp motor will easily do the grinding for a herd of 40 cows or 2000 chickens with a wide margin of safety (even during the heavy grinding season), having a capacity of 30 to 200 lb per hr, depending on mill and degree of fineness. (In a few cases 1/4 hp motors have been successfully used in connection with small burr mills.) Such small mills are not adapted to grinding ear corn or any other type of roughage or semi-roughage. On a basis of 1/10 bu of ground feed per day per cow a ton of dairy feed will last a herd of 10 to 15 cows about a month.

Where grains are ground in unequal proportions, reducers or nozzles must be provided for the grain chutes, one nozzle for each rate of feed. Governor regulation can also be provided but is usually rather expensive. If the feed is fed on the same floor on which it is ground, an overhead bin should be used for storing it. A bucket type or any other suitable elevator can be used to carry the ground feed from the mill to the overhead bin. A fan elevator can be purchased as an integral part of most hammer mills, and with it feed can be lifted greater distances than with other types of elevators: 35 to 40 ft vertically but ordinarily not over 8 or 10 ft horizontally unless the pipe runs high enough so that it slopes down to the feed bin at an angle of not less than 30 deg. The use of such a fan increases the fineness of grinding, the mill capacity, and the power and energy requirements. The fan may also be used to elevate grain by removing the gooseneck (goosenecks with square or rectangular cross-sections at the hopper bottom of the mill and with smooth curves are materially more efficient than the common stovepipe type), and adding a special hopper (grain going into the fan directly), or by removing the screen and running the grain through the mill. A fan elevator should have a concentric housing with a clearance of about 1/8 in. and ample air inlets, screen opening alone being insufficient.

It is advisable to use a dust collector with most ground feed elevators; in fact, some method of dust control is almost essential with hammer mills. A variety of dust collectors are on the market, and the Agricultural Engineering Department at the University of Wisconsin has plans available for homemade dust collectors. Some farmers have found it satisfactory merely to close with sacking an otherwise airtight bin which is open at the top, or, less preferably, where the opening is a window as far as possible out of line with the delivery pipe opening.

5. Burr mills in general operate most efficiently at slower speeds, 200–500 rpm. Hammer mills are most efficient at their rated high speeds such as 2000, 3600, and 4500 rpm depending somewhat on type, etc.

6. The energy consumption for feed grinding, which does not vary materially with the type of mill, is generally 10 to 15 kwhr per ton (20 kwhr for chicken mash) including ground or cracked scratch and mash. Fine grinding consumes energy entirely out of proportion to the increase in fineness. As the moisture content of grain increases, the power required to grind increases.

In practice the energy consumption per 100 lb for medium grinding of dry shelled corn, barley, and oats is around 0.5, 1.0, and 1.5 kwhr respectively. It takes about two and one-half times as much energy to grind ear corn as to shell and then grind the same amount of shelled corn, but where bulk is needed, for example in the South where roughage is expensive, and where, as for sheep, the available concentrates would otherwise be too heavy, such grinding is fully justified.

7. By a new system of high-pressure grinding that is as yet only done locally, the grain is formed into dustless flakes resembling breakfast food. The product is very popular among rabbit feeders.

8. See also under Feed Mixing.

Roughage. Forage grinding or chopping increases gains but is profitable only when roughage is high in price or labor and power are cheap. Data from Idaho are typical, and with long, chopped, and ground hay show daily rates of gain, with steers, of 1.56, 1.82, and 1.87 lb respectively and hay losses of 10.3 per cent, 4.6 per cent, and none respectively, similar hay losses for sheep being 30.1, 13.8, and 1.8 per cent respectively. According to Kansas data, nothing is gained unless the roughage contains grain (cane fodder, Kafir fodder, etc.), most of the benefit coming from the cracking of the grain. Hay losses from tramping at the feeding racks have been reduced in California as much as 10 to 20 per cent, and refuse from low-grade hays or other roughage, for example, corn stover, coarse soybean hay, or poor alfalfa, by 15 per cent or more. It is likely that decreasing the refuse of poor quality roughages is poor economy, especially with cows. Chopped or ground hay, though dustier, is easier to feed and occupies only one-third to one-half the space required for comparable quantities of long hay. Taking account of time required to mow away, it takes about the same amount of man and horse labor to store cut as long hay. Artificially dried hay can now be compressed while still warm, into dustless and palatable briquettes. Refuse, such as cornstalks, is commonly used for bedding, and the resulting manure is much more easily handled than when the roughage is whole.

Cost of Grinding. The total cost of grinding roughage including labor, power, and equipment overhead, was found, in Kansas tests, to range from $0.99 to $1.39 per ton, depending considerably upon the number of tons ground per year and to a lesser extent on fineness of grinding. In Texas, grinding costs, exclusive of labor, of such forages as snapped corn, hegari, etc., averaged $1.75 per ton with 40-hp outfits, and $1.18 per ton with 5- to 7 $^1/_2$-hp equipment.

The average energy consumption required for grinding is around 10 kwhr per ton, and for chaffing or chopping probably 3 $^1/_2$ kwhr or less per ton, although in some cases it amounts to as much as 5 kwhr per ton. It takes about 5 kwhr to shred a ton of corn stover, very little more energy being required to husk corn at the same time. Where hay is rather finely ground, as for baby chicks, as much as 30 kwhr may be required per ton.

Size of Machine Required. It is possible rather than practical to grind or chaff roughage with a 5-hp motor, but 7 $^1/_2$-hp motors with certain types of carefully operated mills are in some cases proving fairly satisfactory. It is next to impossible to get sufficient feed into the restricted openings of some small mills, for example, 10-in. burrs being preferred to 8-in. burrs. However, a large proportion of the present installations are operated by 25-hp motors.

Type of Machine, Operation, and Fineness of Grinding. Hammer mills are especially adapted to fine grinding and plate mills to coarse grinding, the former, however, being available in a larger range of capacities and sizes than the latter. The combination mill, containing cutter head which can also be used without the mill proper, is suitable for a wider range of grinding than the other types.

An ensilage cutter can be and often is used for chaffing hay. In a typical case a 5-hp motor operating a cutter used 3 to 4 kwhr per ton of hay cut into $^1/_4$-in. lengths, hay being chopped at the rate of 1 $^1/_2$ tons per hour. *Most farmers find hay of this length fine enough for all purposes.*

Hammer mills should be operated with a peripheral speed of 14,000 to 15,000 ft per min (due to excessive bearing friction small diameter mills are sometimes best operated at lower velocities), rpm being of only incidental importance, and a burr mill should run at

600 to 850 rpm. The economical speed for ensilage cutters at most does not exceed 500 to 600 rpm, according to Kansas findings.

Keeping Qualities of Ground Roughage. Dry roughage when ground will keep longer, but is dusty and disagreeable to grind and is probably less palatable to stock than feed containing 25 per cent or more moisture. Ordinary ground corn or Kafir forage cannot be stored for more than 36 hr without heating.

FEED MIXING. Feed mixers are about as frequently found a part of a feed-grinding installation as separate equipment. The grinders together with their elevators and overhead bins are used for this purpose; the batch of material is moved back and forth as required, three moves usually being sufficient. Sometimes the concentrates and grain are mixed as ground by placing alternate layers of the required materials in the bin or container from which the mill is fed.

When used for mixing, the rotor pulley of hammer mills should preferably be disconnected or at least the gooseneck should be removed and a special hopper emptying directly into the fan should be used. Some hammer mills have a tendency to clog with ground feed.

One size of a vertical auger mixer, which carries the feed upward through the center of an upright container, the elevated feed then rolling toward the outside from the top, thoroughly mixes 600 lb of feed in 8 min (5 min actual mixing and 3 min of agitating to facilitate bagging) with an energy consumption of 0.5 kwhr per batch, with a 5-hp motor. Deducting grinding power, some data for a 5-hp hammer mill show 0.67 hp and 560 lb of ground feed elevated per hour at 3550 rpm and 1.85 hp and 2530 lb of ground feed elevated per hour at 3950 rpm. At these higher speeds, feed can be elevated vertically 35 or 40 ft, but not over 8 or 10 ft horizontally unless the pipe is high enough so that it slopes down to the storage bin at an angle of not less than 30 deg.

GRAIN AND SEED CLEANING AND GRADING. An electric motor on the fanning mill saves the time of a man or boy as operator. In addition, it makes the use of a sacker practicable, the sacker alone reducing the total cleaning time by about one-half. The unvarying speed makes for greater effectiveness; such equipment removes weed seed, trash, and undersized grain, and eliminates danger of mixing varieties in elevator cleaning. It is particularly valuable in the cleaning of vegetable and other small seeds. One-half horsepower or less, $^1/_4$ hp ordinarily, will operate most fanning mills, and the cleaning of 100 bu of grain or 20 to 30 bu of small seeds requires around 1.7 kwhr, larger and more efficient units operated by 1-hp motors apparently cleaning 1000 bu with a consumption of only 4 kwhr. With similar screens capacity is not materially influenced by weight, it being largely a matter of volume.

GREEN FEED AND STRAW CUTTING, AND ROOT SHREDDING. Apparently typical kale cutting tests show 5.6 times as much man-labor and 2.8 times longer cutting time (same amount of cutting) for hand operation as for the use of a motor on the same machine, representing a saving of $3.65 per ton of kale cut: 30 cents per hour for labor and 3 cents per kwhr for power. Energy consumption usually runs from 0.07 to 0.13 kwhr per 100 lb of kale cut, or from 0.17 to 0.51 kwhr per 100 birds per month with kale serving as the green stuff. It is recommended that from 8 to 10 lb of young kale—roots sometimes used—or 5 lb of more mature kale be fed daily per 100 layers. Unless the kale is cut, $^1/_2$-in. lengths, the midrib is left and up to 50 per cent is refuse; if cut, only up to 20 per cent becomes refuse.

For root shredding the average energy consumption is about 0.07 kwhr per 100 lb of roots—around 0.09 kwhr per month per 100 birds. In two instances with larger equipment the consumption was $^1/_3$ and 6 kwhr per ton respectively, with a 1-hp unit.

For chopping green alfalfa into $^1/_2$-in. lengths (practically one-half is lost as refuse unless cut) the average energy consumption is around 0.1 kwhr per 100 lb—about 0.15 kwhr per 100 birds per month.

Cutting straw into 2-in. lengths for baby chick litter, etc., takes around 0.11 kwhr per 100 lb cut.

Cutters with throats up to 10 in. may be operated with a $^1/_2$- or $^3/_4$-hp motor, some small machines requiring only $^1/_4$ hp. The top speed recommended is around 350 rpm, a very satisfactory speed for the larger hand machines being 200 to 250 rpm.

HAY BALING. Most balers can be satisfactorily operated by 5- or 7 $^1/_2$-hp motors, but the larger machines may require up to 25 hp. Presses should turn out from 3 to 5 tons per hp per 10-hr day and should not require, even under unfavorable circumstances, over 0.2 to 0.3 kwhr per 120-lb bale or, say, 2 to 5 kwhr per ton, in one typical case an average of 1.62 kwhr per ton being required for 75-lb bales.

HAY HOISTING. A motor on a suitable hay hoist shortens the total unloading time and saves the time of a driver for the team, a properly designed hoist being operated from a load. A motor is usually slower in elevating than horses, but this time is more than

made up in returning the fork or slings, the latter task with horses requiring about three-fourths of the total time.

Hay hoisting being an intermittent job, the motor can safely be made to carry almost 200 per cent normal load and, instead of 7 1/2 or even 10 hp, 5 hp is ample, 3 hp being used occasionally, and even 2 hp (rope speed 120 ft per min in a typical case) giving satisfaction. Slings require no more power than a fork.

The energy consumption per ton hoisted varies with the hoist, height of track, etc., ranging from 0.1 to 0.65 kwhr or an average of 0.35 kwhr—a total of about 10 kwhr per season on the average farm. Some owners of hoists cut down their overhead by using the equipment for other tasks such as hoisting sacks of feed, fertilizer, etc., lifting the dirt from a well being dug (in a typical case using an average of 1 kwhr per cu yd to hoist an average of 18 ft), and in at least one case such equipment was satisfactorily used with a grabhook to load manure.

MILLS, GRIST. It takes about 1 kwhr to grind a bushel of cornmeal.

OAT SPROUTING. A 25-watt heater will take care of a sprouter large enough to supply the average farm flock if located in a protected place, such as a cellar. A well-insulated sprouter furnishing a bushel of oats daily (enough for 1000 hens, 1 to 2 cu in. each) should not have a maximum demand of more than 125 watts or an energy consumption above 75 kwhr per month.

SEED CLEANING AND GRADING. See Grain and Seed Cleaning, etc.

SEED CORN TESTING. Operating data for a well-insulated homemade two-compartment 3 by 8 by 3 ft 1600-ear or 8000-kernal electrically heated germinator, in a protected outbuilding, are typical. The temperature was kept at 80 deg fahr, and the humidity was maintained between 95 and 98 per cent by placing a pan of water over the 400 watts of heat. The trays were staggered so that there was positive circulation of the warm air over the top of each of the four trays in each compartment. The corn kernels were placed on muslin which in turn was placed on wet-down sawdust. The energy consumption for the 7-day germinating period was around 34 kwhr, close to 1/2 bu (48 ears) being tested with 1 kwhr. Energy consumption for large germinators amounts to about 3 kwhr per bu.

SILO FILLING. There are two types of ensilage cutters: (1) cylinder, the corn being cut by a cutting head which, though an integral part of the machine, is separate from the elevating fan; (2) flywheel, the knives, varying in number from two to three or even four or more, attached to the elevating fan.

Silos have been filled with 3-hp motors and in many cases are being filled with 5-hp and to all practical heights, but in practice, especially where much help is hired, 7 1/2-hp is more satisfactory. The success of filling silos with small-horsepower motors depends more upon the use of minimum operating speeds than upon anything else. (See Table III.) Likewise there should not be over 1/8-in. clearance between the fan tips and

Table III. Minimum Speeds for Various Sized Ensilage Cutter Fans to Elevate Corn into Silos of Different Heights*

Diameter of Cutter Fan, in. (wing tip to wing tip)	Height of Silo, ft								
	25	30	35	40	45	50	55	60	75
	Revolutions per Minute of Cutter Fan								
30	500	530	575	610	650	690	720	750	835
32	465	495	540	575	610	645	675	705	780
34	440	465	510	540	570	610	635	660	735
36	415	440	480	510	540	575	600	625	695
38	390	415	450	480	510	545	570	595	660
40	370	395	430	460	485	515	540	565	625
42	355	380	410	435	465	490	515	535	595
44	340	360	390	415	440	470	490	510	570
46	325	345	375	400	425	450	470	490	545
48	310	330	360	380	405	430	450	470	520

* These speeds are based upon carefully conducted tests with a reasonable allowance to insure uninterrupted elevation and represent operating efficiencies of about 30 per cent. Good cutters will successfully operate at these speeds according to the formula; $\text{rpm} = \frac{250\,H}{\sqrt{HD}}$, where H is height of silo and D, diameter of cutter fan in feet. Experience in Oklahoma indicates that for cane or Johnson grass, or a combination of both, speeds will have to be increased at least 10 per cent. On the other hand, in Minnesota they are successfully operating cutters at lower speeds, with corn, suggesting that these data may be 5 to 15 per cent higher than necessary with their type of corn and conditions.

housing and a reasonably close fit at the sides; the pipe should be rather straight (vertical), and the cutter well anchored; the knives should be sharpened every half day (where cutter is kept reasonably busy), and the machine should be evenly fed, generally lapping the bundles about one-third and good judgment being used where a bundle is unreasonably large. The silage is usually cut into $^1/_2$-in. lengths. The cutter may be and often is fed by the wagon drivers. The normal rate of filling under these conditions is around 8 tons per hour.

THRESHING. The unvarying speed of electric motors is ideal for threshing, but in most communities portable transformers are necessary. In a typical set-up a 21 by 36 in. thresher was operated by three motors: 7 $^1/_2$-hp (could be detached from a portable truck during the threshing season) on the cylinder; 3-hp variable speed driving the racks, grain pan, shoe, tailings elevator, grain auger, and grain weigher; and 2-hp on the stacker. The operating costs were about the same as for a tractor.

Energy Consumption. Typical energy consumption data from Minnesota vary from 0.078 kwhr per bu for succotash (oats and wheat) or 0.242 kwhr per cwt, to 0.265 kwhr per bu or 0.441 kwhr per cwt for wheat. Dry and wet barley required 0.167 and 0.427 kwhr per 100 lb threshed, respectively, and poor crops full of weeds about 0.5 kwhr per 100 lb. Succotash, barley, oats, wheat, flax and wheat, timothy, and clover have energy requirements per hundred weight in the order named, ranging from 0.242 to 11.2 kwhr. The average for grain is 0.335 kwhr per 100 lb. From 5 to 8.7 bu of barley were threshed per kwhr, and an average of 7 bu of barley, oats, wheat and oats, and flax per kwhr.

Electrical equipment reduced average stack losses from 1 to 0.15 per cent; reduced dockage from 3 to 1.2 per cent; decreased the operating force from 3 to 2 men; reduced the total crew to 6 men and 5 wagons, a short haul but 900 bu of barley being threshed per day; and reduced the set-up time by 1 hr. However, the equipment had to be moved by horses or tractor from job to job.

The average demand of 13.4 kw for the thresher mentioned above was reduced about 11 per cent by roller bearing. In an older model the running light was about two-thirds the operating demand, 86 per cent on the blower motor, 96 per cent on the grain shoe and shaker, and 50 per cent on the 10-hp motor on the cylinder. However, the losses were about the same for the newer equipment.

Starting is made easier by high belt speeds, around 3400 ft per min being satisfactory in a typical case.

With a 22 by 38-in. thresher only 4 or 5 hp are required to do the actual threshing, operating speeds being too high, belts being too tight and crossed, new machines requiring almost double normal power until " run in," and in many machines the wind stacker pipe, being too large.

A small thresher in Alabama operated by a 7 $^1/_2$ hp motor had an energy consumption of 26 kw hr per 100 bu threshed, mostly oats.

4. LIVE STOCK

ANIMAL GROOMER. According to Cornell data, groomers shorten the grooming time per cow to 2 min per animal as compared to 5 or 10 min by hand methods. Not all types (knapsack, truck, etc.) are equally effective, owing to suction variations. Loose hair, dirt, and scurf constitute one of the greatest sources of milk contamination.

BARN VENTILATION. A well-ventilated barn should meet the following standards:

1. The temperature of the stable should be from 45 to 50 deg fahr with an allowable variation of 10 deg fahr for extremes in weather conditions. According to most authorities, 35 deg fahr is considered the minimum. Satisfactory temperature maintenance requires keeping the barn properly filled with animals and the use of more than a single layer of siding, or equivalent.

2. The relative humidity should be maintained at about 75 per cent and never exceed 85 per cent. (Owing to important savings in cost of insulation, etc., many do not object to some condensation during the most extreme weather if other conditions are satisfactory.)

3. The air flow through the stable may vary from 40 to 60 cu ft per cow per minute (a minimum of around one to two complete dilutions or changes per hour with fans running restricted).

4. There should be complete mixing of incoming air with stable air to insure a uniform quality of air throughout the stable. It is ordinarily assumed that this means several properly spaced and located inlets.

5. The convection currents in the stable should be constantly in operation to insure complete mixing of fresh air and stable air and to assist in the diffusing of the fresh air.

6. Intakes should be so constructed that the velocity of the incoming air is reduced, as

it enters the stable, to such a degree that no objectionable drafts can possibly result; an objectionable draft is a stream of air which is of lower temperature than the surrounding air and which blows directly onto an animal or attendant.

The forced-draft system of barn ventilation has several advantages as compared to the natural-draft system:

(*a*) Lower overall first cost ($350 as compared to $900 in a typical case in Minnesota). The operating cost of the fans is often lower than the interest and depreciation on the extra investment in a gravity system.

(*b*) Although exhaust fans should always be installed on the side of the barn away from the prevailing winds, they are largely independent of wind and wind direction.

(*c*) Operation is automatic, the fan regulating within a range of 2 or 3 deg fahr. The air outlet is restricted as the stable temperature drops; and should it drop too low, the fan stops until the temperature again rises.

Table IV. Data for Forced-draft Barn Ventilation *

State	Kwhr per Year	Kwhr per Season per 100 Head of Stock
Iowa............	2.6 †	1000
Minnesota.......	300.0	1000
Minnesota.......	36.0	144
New York.......	810.0 ‡	1250 ‡

* Data are so variable that none are really typical. Material from Iowa is more recent and probably indicative of monthly requirements during the ventilating season, preferably a period of 8 months' continuous and 4 months' intermittent operation, according to Cornell University investigation.

† Per month per cow.

‡ Many think these figures too high but they may actually be too low.

CLIPPERS, HORSE AND COW. Clippers are available in stand, suspended, and hand portable types, one being operated by the vacuum from a milking machine. One electric design operates on the vibrator principle and is very compact.

DROPPING-BOARD CLEANING. In New Jersey a flexible dropping board, similar to the canvas on binders or the apron on some manure spreaders, is operated by a motor, a stationary knife bearing against the under side at the end scraping the droppings into the box used to carry them away. The cleaning time is reduced from an hour or so to a few minutes.

In another type of installation which works fairly well, even when the boards are very dirty or the droppings frozen fast, consists of a motor-driven scraper made by attaching angle irons to common chains moving at the rate of 40 ft per min, motor and pump jack for speed reduction being mounted on a shelf above. The droppings fall into a screw conveyor which carries them outside through a hole in the back of the house, making it possible to eliminate all hand work up to unloading. This particular dropping board is only 30 ft long but of course could be made indefinitely longer if adequate power were available.

ELECTRIC HEAT FOR YOUNG PIGS AND LAMBS. In cold climates a reflecting heater sitting on a shelf so that its warmth is directed among the young animals during their first few days, if the weather is unseasonable, materially reduces cold-weather losses from chilling. In a typical instance a Minnesota hog breeder increased his income for the year by $600 ($3 per pig).

FOX-FARMING EQUIPMENT. On the fox farm, electricity lights the yards, protects against robbery, preserves and grinds the fresh meat, and in some cases cooks the rations, etc.

OX-GRUB REMOVAL WITH A VACUUM CLEANER. A special high-suction cleaner designed by the U. S. Department of Agriculture is very efficient in removing grubs from the backs of cattle.

SHEEP SHEARING. Tests with hand shears, a 10-tooth Steward hand power machine, and a 13-tooth Steward power clipper operated by a $^1/_2$-hp motor, show an average of 8 min required to shear a sheep by hand, two-thirds as long as with the hand-operated machine, and only a little more than half as long with the motor-driven machine. About 2.4 kwhr were required to shear 100 sheep with the motor-operated machine.

Water Heating for Stock Tank. Warm water as compared to cold resulted in an added average gain per steer of 8 lb during January and February, in Idaho, shelter during the same period adding another 7 lb of gain per steer. Under similar conditions the gains for lambs were not increased. The average daily midwinter energy consumption in Caldwell, Idaho, in 1930–31, for a 52-gal cylindrical tank insulated with 3 in. of granulated cork was 5.2 kwhr, about 1.8 kwhr per 100 gal (water kept at 55 deg fahr by thermostat with a 10-deg fahr range) or less than 0.25 kwhr per cow per day. The 22 cows drank an average of about 13 gal of water per day. The energy consumption for an uninsulated tank, also at Caldwell, where the water was merely kept from freezing (manually controlled—on in daytime only) was 0.15 kwhr per steer per day.

5. POULTRY—GAME—EGGS

BROODING. Brooder technique is rapidly advancing, the designs shown being indicative of general methods of heating but not necessarily representing equipment still being sold—at least in large volume. In Fig. 2–*A* is a blackheat brooder (representing

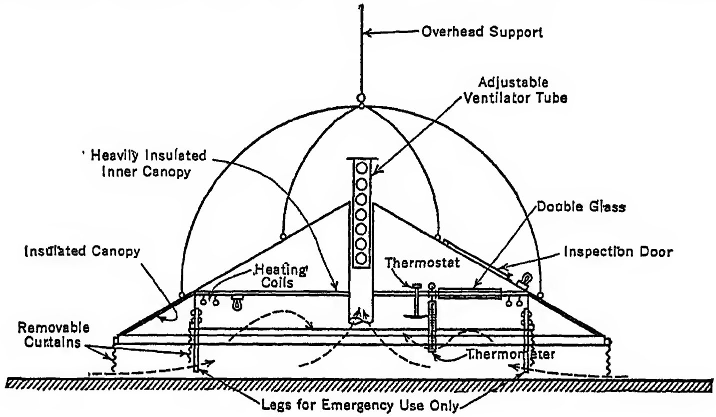

Fig. 2-A

the hover type most commonly used) the outstanding features of which are: a curtained inner compartment two-thirds of the diameter of the curtained outer compartment; heavy insulation of canopy and ceiling; and a 3-in. adjustable ventilating tube. The heat control is automatic. There are a pilot light, an attraction light on a separate circuit, an inspection door in the canopy through which the thermometer, etc., can be inspected, a glass window through which the chickens can be observed under the hover, and a suspending rope, pulleys, and counterweight for adjusting the height of the brooder and for convenience in cleaning. No brooder should be without this last feature.

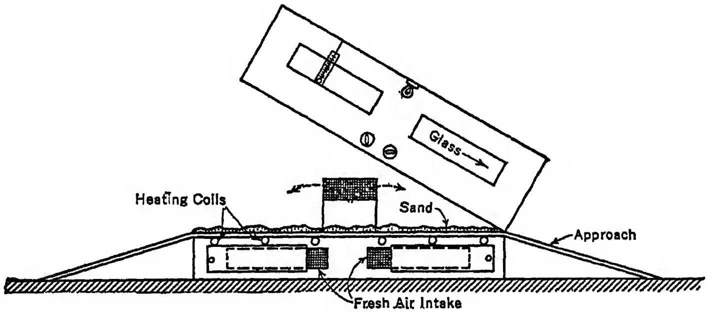

Fig. 2-B

B is an underheat brooder. Locating the heating elements beneath the false floor assures dryness, also good ventilation while some of the heating elements are on. In other fundamental respects the brooder resembles *A*.

C is an underheat brooder similar to *B* but with the heated air entering from the back. A layer of sand (1 $^1/_2$ to 2 in. thick) on the brooder floor, above the heating elements, stores a substantial amount of heat which is valuable in case of power interruption. The heating elements are controlled by three manually operated switches (Pacific Northwest), but the changeable weather of most sections of the country makes automatic control essential. Heating cable, presumably lead covered, buried in that part of the conrete brooder house floor adjacent to and directly beneath the hover of the ordinary blackheat type has been successful in keeping the litter dry.

D is a locally popular type of blackheat brooder (for example, with type *A* making up

practically all the brooder sales in certain areas of Oregon) which is insulated with 2 in. of sawdust; it is sold with slatted platform of lumber with wire netting between slats, and has an inside diameter of 28 $^{1}/_{2}$ in. and any length required to take care of the chicks to be brooded, 5.2 sq in. per chick being allowed by the manufacturer's plans.

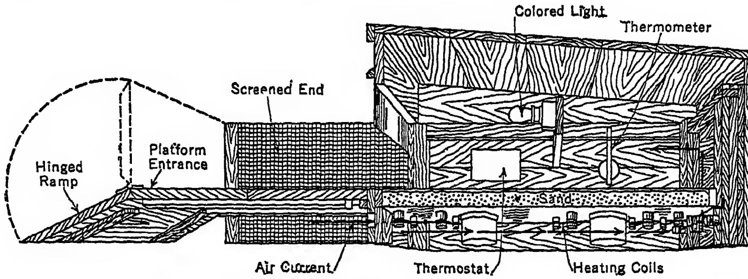

FIG. 2-C

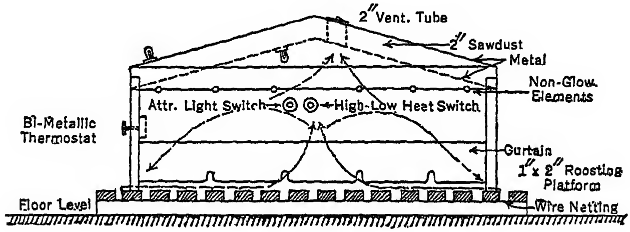

FIG. 2-D

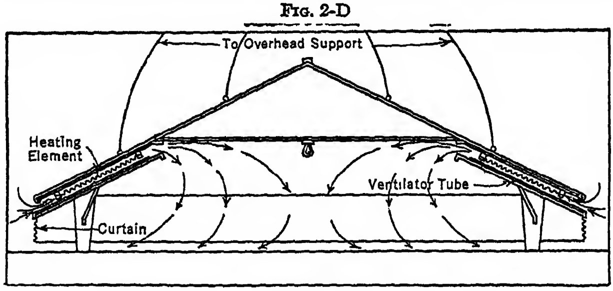

FIG. 2-E

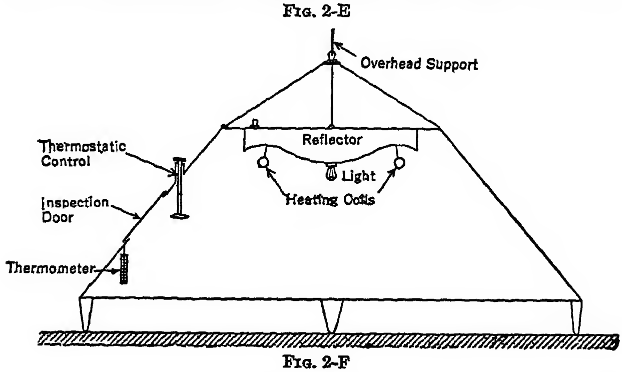

FIG. 2-F

E is a blackheat brooder with supplementing convection tubes, two tubes being used only in large units. The heating elements (glowing radiant) in the ventilating tubes are

on continually. The regular heating elements are located on porcelain knobs attached to the hover ceiling.

F is a glowing radiant-type brooder; the four elements operate at red heat. No curtains are used and there are no ventilation problems, but the energy consumption is high. Insullation on the canopy would help to lower this consumption. This general type is widely used in mild climates.

Wattage and Energy Requirements. The proper heating capacity for glowing radiant brooders varies from around 2 watts per chick in mild climates to 2.5 to 3 watts per chick where freezing temperatures are experienced, and for blackheat brooders from 1.3 to 2 watts per chick. The brooder should be able to maintain a temperature of 100 deg fahr (90 deg fahr for underheat types) with the thermometer bulb 2 1/2 to 3 in. above the brooder floor during the coldest brooding weather and before the chickens are put under the hover.

Energy consumption for the radiant type varies widely but averages between 1.2 and 1.5 kwhr per chick-season (California Bulletin No. 441); for the blackheat type the range is from less than 0.2 to 2 or more kwhr per chick-season but should not average above 0.5 kwhr per chick-season with good equipment and good operating conditions when the brooder-house temperature does not drop below 0 deg fahr. Some power companies in the Middle West and East assume an average of 1 kwhr per chick-season. The range for the underheat brooder is between 0.5 and 2 kwhr per chick-season, and for other types the averages are usually between those for the radiant and blackheat types.

The energy consumption figures for turkey brooding from Texas and Oregon show an average of a little over 1 kwhr per poult-season, poults requiring much more space (at least 12 sq in. per head recommended) than chickens but the season generally is later.

The area per chick under the hover is one of the important factors in determining the unit energy consumption. For instance, it will take only about half as much energy where 3.5 sq in. are allowed per chick as compared to 7 or 7.5 sq in., the latter being considered as the minimum for the production of husky chicks unless crowding is avoided by removing some of the chicks as they become larger. Ducks, goslings, and poults require about 14 sq in. each.

Much more space per chick is required with battery brooders. One southern (Texas) poultryman operating a heated room type of battery brooder recommends the following:

Age	Sq in. per Chick
Start	14
2 weeks	20
4 weeks	35
6—8 weeks	50

(Less space is required per chick in unheated rooms.)

One cubic foot of fresh air per minute per pound of chicks.

The use of good insulation on the brooder will materially lower the consumption when the area per chick is taken into consideration. If fibrous materials are used they must be protected or the chickens will eat them. (See Fig. 2*E*.)

The temperature in the brooder house will also materially affect the energy consumption. No data are available, but it appears that the consumption of a well-insulated brooder would be doubled by a drop from 70 deg to 0 deg fahr in the brooder house, assuming 1/2 in. of fiber insulation or its equivalent for the hover. Some tests at Manhattan, Kans., with an uninsulated brooder show the consumption doubling with a 20-deg fahr drop in temperature.

It is the opinion of at least one Petaluma, Calif., poultryman that automatic temperature control would reduce his energy consumption, as compared to manual control, by 15 to 20 per cent. The actual length of time that heat is required usually varies from 4 to 8 weeks, depending on weather conditions. At this point it is interesting to note that E. C. Easter assumes for Alabama conditions an average wattage per brooder of 500, and an average energy consumption of 60 kwhr per 100 chicks per year.

In a late season test at Schenectady, N. Y., it was found that approximately 50 per cent of the energy for brooding was consumed during the first week, and around 80 per cent during the first two weeks of operation, that is, with thermostatic control and temperature reductions as the chicks became older. However, with low outside temperatures, a somewhat larger percentage of the energy would be used later in the season.

Expense of Brooding. According to data from Oregon, electric brooding is cheaper than oil or coal brooding with kerosene at 18 cents per gallon, briquettes at $17 per ton, and electricity at 3 cents per kwhr.

CHICKEN SCALDING. An insulated scalder in rather general use on the Pacific Coast is 39 in. long, 24 in. wide, and 19 in. deep, and is divided into two compartments one

of which is twice as large as the other. Each compartment has a false floor to catch loose feathers, and is heated by a 750-watt thermostatically controlled immersion-type element, the temperatures being 190 and 140 deg fahr, respectively, in the small and large compartments. The operating costs are reported as 31 per cent of those with coal. A single-compartment insulated scalder used in the New England States is heated by a 2-kw immersion element, thermostatically controlled within $^1/_5$ deg fahr of the setting; it is used almost entirely for semi-scalding. Depending on age and type of bird, optimum semi-scalding temperatures range from 125 to 130 deg fahr and optimum immersion from $^1/_4$ minute to 1 minute.

COLD STORAGE. Egg storages should maintain a humidity of 82 to 85 percent and are usually kept at between 28 and 30 deg fahr, variations of more than 1 or 2 deg fahr resulting in watery eggs. A typical 800-case Texas plant, insulated all around with 8 in. of cork and having cottonseed hulls packed between the studs and to a depth of 6 in. over the ceiling, cost $2000. Eggs are frequently stored loose in cases until shipped, as they cool quicker, and a case holds 55 dozen loose eggs as compared to 30 dozen when packed. The average energy consumption for the 5 months' storage period of a 500-case Texas plant and an 800-case Texas plant (both plants rated in loose eggs) was 1.7 and 2.2 kwhr per case, the larger plant also serving as a domestic refrigerator.

GAME-BIRD-FARM EQUIPMENT. Electricity is used for incubation and brooding, materially reducing labor costs and disease losses. The uninsulated 100- to 150-watt 20-pheasant chick brooders on a farm at Saratoga, Calif., use during the average brooding period of 28 days (season extends from April 25 to September 25) an average of about 0.8 kwhr per chick raised.

INCUBATION. Well-designed electric incubators generally produce more and sturdier chicks, furnish no fire hazard, and have much lower operating labor requirements than types with other forms of heat.

Essentials of Incubation. While the actual proper incubating temperature (varying somewhat with type of poultry) is undoubtedly always the same, the thermometer reading varies from around 99 deg fahr (forced ventilation cabinet type) to 103 deg fahr (natural

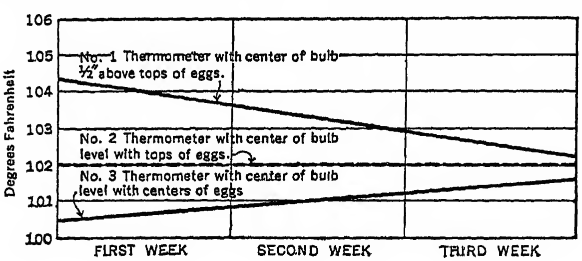

FIG. 3

ventilation flat-top type) with the thermometer location specified by the manufacturer. The center of the thermometer bulb should be just at the top of the eggs. (See Fig. 3.)

An **incubator cellar** had best be held at a temperature between 55 and 65 deg fahr, as providing proper ventilation at 85 deg fahr or above is very difficult and temperatures below 55 deg fahr greatly increase the energy requirements and with some machines lower the incubator temperature.

Emergency Heating is seldom provided (gas engine operated generators giving satisfaction), in cabinet types (forced ventilation) resulting in spotty heating. An alarm system should be provided, a combination which furnishes warning of no-voltage and high and low temperature, being best. The thermostats should keep total temperature variations below 0.5 deg fahr, some large machines having several separately controlled units in parallel. Mercury-type switches are most frequently recommended. California data indicate increasing likelihood of losses, (in case of power interruption), as the stage of incubation advances, proper temperature being particularly necessary while the chicks are hatching and are still wet; likewise the damage is usually less for 12-hr and 18-hr than for 6-hr interruptions, losses resulting from 24-hr interruptions usually exceeding those for 6-hr intervals.

After temperature, proper **humidity** is probably the most important factor in successful hatching; most machines with bad hatching records make a good showing when humidity is kept up. Higher humidity than required (can be reduced by increasing ventilation) does not lower the hatching percentage but may damage the glue and varnish of the

machine. Humidity can be increased by sprinkling the incubator cellar floor; by sprinkling the eggs; by the use of wet cloths, wet sponges, pans of water or wet sand (latter is more effective owing to greater surface) below the trays or, best of all, by regular humidifying systems now available at moderate cost. Machines should never be opened after the nineteenth day, a few beads of moisture on the glass observation door indicating proper moisture conditions. Under proper operating conditions the air cell of hens' eggs should be equivalent to a little less than one-third the size of the egg ($^{23}/_{32}$ in. deep) on the eighth day; one-third the size of the egg ($^{25}/_{32}$ to $^{13}/_{16}$ in. deep) on the fourteenth day; and two-fifths the size of the egg ($^{15}/_{16}$ to $^{31}/_{32}$ in. deep) on the nineteenth day. A more accurate way of checking up on the humidity is to weigh a tray of eggs from time to time. No egg should lose more than 0.33 oz by the nineteenth day. (See Table V)

Table V. The Normal Loss in Weight of 100 Ordinary Hen Eggs During the First Nineteen Days of Incubation

Day	Ounces	Day	Ounces	Day	Ounces	Day	Ounces
1	1.65	6	10.00	11	18.60	16	27.44
2	3.31	7	11.72	12	20.33	17	29.21
3	4.96	8	13.44	13	22.10	18	30.99
4	6.62	9	15.16	14	23.88	19	32.77
5	8.28	10	16.88	15	25.66		

Incubating chicks *breathe* through the egg pores, particularly at the air cells, and air containing more than 150 parts of carbon dioxide per 10,000 results in high mortality, actual fresh air requirements ranging from 1 to 10 cu ft per hr per 100 eggs. Little ventilation is required during the first two days, but after that, in a properly designed machine, the ventilators are gradually opened up to and including hatching so as to maintain proper humidity as the incubation period advances.

Flat-top machines are more adaptable than the cabinet type, and for farmers and small hatcherymen are less costly to operate since only as many units as are required need be used; also the full capacity of the machines set (instead of one-third as in the cabinet type) can be hatched at one time, saving growers considerable trouble in brooding. These small machines should have individual capacities of 1 $^1/_2$ to 2 cases of eggs. They can be stacked in compact tiers without legs or with very short ones. Many hatcherymen, for sanitary reasons, do no hatching in their cabinet-type machines, placing the eggs in "hatchers," or incubators made for that purpose, on the nineteenth day.

Practical units are available for converting oil-heated incubators to electric heat.

Energy Consumption, Wattage, and Cost of Incubation. The average energy consumption per 1000 eggs set of all-electric incubators ranges from 200 kwhr for small flat-top incubators to 25 kwhr for large cabinet or mammoth types. A well-insulated 200-egg machine has a consumption of 78 kwhr per 1000 eggs set as compared to the average of 134 kwhr for the same size of machine. A well-insulated 36-egg machine has a demand of about 130 watts. The average (say 600 egg) for all types is $^1/_2$ watt per egg (more for small and less for large machines). A drop of 20 deg fahr in the incubator cellar temperature apparently will almost double the energy consumption of poorly insulated machines. (See above under Incubator Cellar.) To obtain the wattage and energy requirements for turkey eggs, on account of their greater size and the extra week of incubation, the comparable data for hen's eggs must be multiplied by 1.67 (wattage) and 2.22 respectively. Kerosene-heated electrically ventilated incubators show averages per 1000 eggs set of 17.4 gal of oil, 17.0 kwhr for 2000-egg machines, and 4.3 gal of oil, 9.4 kwhr for 12,000-egg machines. California averages per hatch per 1000-egg capacity for 150- and 540-egg electric and 150- and 540-egg kerosene incubators are 250 and 200 kwhr, respectively, and 26.0 and 12.0 gallons of oil respectively.

Lighting. See Art. 7.

Water Heating. Nothing is gained by feeding chickens unless water is available, and it is apt to take several weeks to get them back into production after the slump which frequently follows any serious variation in regular routine such as a lack of water on cold days. Providing warm water instead of allowing ice to form results in an increase of about 20 per cent in water consumption, but three-fourths of this gain is due to keeping the ice melted and the other one-fourth to keeping the water temperature from falling below 40 deg fahr, higher temperatures being of little value.

Poultry water heaters are of the immersion or clamp-on or stove types. The former is more efficient than the non-immersion type, but in most cases will burn out if left turned on while not immersed in water; a 30-watt, flexible, rubber pad type now available is not so affected. Stove-type heaters generally have three available heats; some of this type

have a tendency to short to the metal after a few months' service. Homemade heaters include cylinders of sand, containing a light bulb, for immersion; box heating, etc.

For 14-ft uninsulated buckets, and possibly for even 4-gal vessels, 30-watt, 50-watt, and 75-watt immersion will prevent freezing in mild, ordinary, and severe climates respectively; for stove-type heaters, up to 100 or 125 watts would be required. In the latitude of Richmond, Va., 25 watts in a box surrounding the base of the drinking vessel are generally adequate for 3-gal vessels. Reducing the quantity of water in the vessel raises the operating temperature. Insulation consisting in the first instance of 1-in. dry boards, and in the second of 2 in. of dry shavings, reduces energy requirements by about 25 and 40 per cent respectively, 50 watts with an insulated vessel apparently being about equal to 75 watts with an uninsulated vessel.

The convenience outlets should be on a circuit separate from the lighting. Sometimes two vessels can be heated from each outlet. During cold weather the units should be left on continuously.

6. FRUIT

APPLE-BUTTER MAKING. Apple-butter stirrers are readily operated by electric motors. A Missouri farmer reports the saving of one full-time seasonal hand (12 hours a day), the $^1/_2$-hp motor consuming 6.5 kwhr per 20-gal batch of apple butter (0.3 kwhr per gal). It takes three girls 4 hr to pare and cut into eighths the 12 bus of apples usually required per batch.

DRYING. See Art. 9.

FRUIT WASHERS. Merely wiping off the visible spray residue is not sufficient, and excessive washing destroys keeping qualities. Washing in a $^1/_3$ to 2 per cent solution of hydrochloric acid reduces the arsenic (calculated as A_2O_3) to the required 0.01 grain per pound of fruit, an effective water temperature of 90 to 105 deg fahr being required for waxy or oily varieties. A typical electrically heated commercial washer has a capacity of 250 boxes per hour and a practically continuous demand of 29 kw for heating the 300 gal of washing solution. For tanks, copper, copper alloy, and lead corrode, but Monel metal and waxed wood seem to be satisfactory. Some consider rotating submergers necessary for washing and spray nozzles close to the apples as essential to proper rinsing.

GOOSEBERRY GRADING AND CLEANING. A typical gooseberry grader consists of an inclined table 6 ft long by 2 ft wide with a screened opening through which an air blast from a 4-in. fan operated by a $^1/_2$-hp motor is directed, the strength of the air blast being regulated by a sliding door on the table. The considerable amount of leaves and trash gathered with the berries in the process of stripping from the bushes, together with the wormy fruit, is blown away and the small berries are blown to one side on to a shelf directly above the inclined table. The average energy consumption is about 1 kwhr per 6900 lb of gooseberries cleaned.

GRADING. See Art. 11.

WAXING LEMONS. A typical lemon waxer has 15 kw of heating in the polishing chamber, 2 kw of heating in immersion elements serving the wax melting and atomizing unit, and 1 kw of heat for the air used by the atomizing fan.

7. FARM LIGHTING (INCLUDING ULTRA-VIOLET LIGHT)

ADVANTAGES. Chores usually can be done with electric lights in about two-thirds the time required with a lantern.

Where the ceiling and walls are dark in color the use of reflectors will increase the useful light from a given wattage lamp by as much as 100 per cent under extreme conditions. In the usual low-ceilinged barn the lights should be mounted as high as possible so as to lessen the chances of lamp breakage by livestock, etc. Workbenches and other working areas should have their own light outlets. Toggle-type wall switches are preferred, and the lighting in a large building should be divided into suitable sections with separate switch control. Only vacuum lamps should be used in granaries and other dusty locations.

Sheep feeders have increased the capacity of their feeding barns 20 per cent by keeping the lights over the feeding racks on during the night. A light spacing of 20 ft seems satisfactory.

In feed-storage barns a small floodlight with a 150-watt lamp can be mounted as high as practicable in the mows at the ends of the barn. The lighting is sometimes furnished by reflecting units (equipped with 100-watt lamps) having detachable heads which make lamp renewals from the floor possible. They are mounted over the driveway.

The lights in dairy barns usually should be located just over or back of the cows' rumps, preferably with at least one light to each four cows.

Both the silo and silo chute can be reasonably well lighted by one 100-watt lamp and reflector properly mounted at the top of the ladder.

The **average energy consumption** for farm lighting outside the house is around 10 kwhr per month, very little being used during the summer. On some dairy farms the consumption is several times higher.

Yard and Protective. Yard lighting should be controlled as required, for example, from the house, barn, and garage. The equipment should be weatherproof. Yard lights also furnish considerable protection against thievery, and in some cases 200- to 250-watt flood lights are also used.

Special. Under favorable conditions lighted electrocutors over ponds furnish ducks with considerable insect food. (See also under Insect Traps.)

Poultry House. Lights are used to lengthen the feeding day for poultry, especially during the winter. They hasten the development of pullets, control the molt of hens, increase winter egg production (may even slightly increase the total yearly production), and are also used to push hens to the limit just before they are sold for meat. All-night lighting eliminates stampeding in duck houses.

The system used varies: (1) morning lighting; (2) combination morning and evening lighting (thought by many to be most suitable); (3) evening lighting; (4) midnight lunch (feeding all the scratch grains hens will eat when the lights are turned on at, say, 9 P.M. for 20 or 30 min—longer for first few days); (5) all-night lighting (regarded by some as not suitable for hens which are to be kept another season). Some method of dimming is generally used where the lights are turned off at night, automatic or manually controlled switches and resistance (small-wattage lamps sometimes placed in series with regular lamps), or small-wattage lamps on a separate circuit being most frequently used.

A light with reflector is located every 10 ft along the length of the house (one 40-watt lamp for each 200 sq ft) half way between the edge of the dropping board and the front of the henhouse. In some cases 25-watt lamps are used; in others, where each unit serves 300 or even 400 sq ft, 100-watt lamps are used. For all-night lighting usually a 15-watt lamp is placed over each feed hopper; 60 watt CX Mazda lamps have been used with great success.

A 12-hr day is used for breeders, a 13-hr day for completely molted high-producing birds, and a 13- or 14-hr day for late August or early September molters and mature pullets of good vitality but not of the highest production quality, neither of which are to be used for breeders the following spring.

On the basis of 3 $^1/_2$ or 4 sq ft per bird the wattage per 100 birds will range from 30 or less to about 120, and the energy consumption per 100 birds per month for the lighting season from 2 to 12 kwhr (reaches the latter figure only with all-night lighting), depending on lighting plan and the particular month.

ULTRA-VIOLET LIGHT. Poultry. Ultra-violet light is having increased use by poultrymen as, under most producing conditions, it increases the vigor of birds, both young and old, lowers mortality and increases the number and hatchability of eggs.

Cows. Udder treatment hastened recovery from mastitis—15 min daily treatment with either a carbon arc or a mercury-vapor lamp. University of Wisconsin appears to have increased the vitamin D content of milk by direct irradiation of cows, but other stations have failed to substantiate this.

Hogs. Extensive treatments of both hogs and pigs seem to have no effect whatsoever.

Horses. Race-horse owners find ultra-violet lamps of considerable assistance in maintaining the health and general condition of the animals during the winter months. Such treatments also increase the rate of growth for colts. On one farm where race horses, polo ponies, etc., are kept, ultra-violet light and portable infra-red ray hand-sets have proved very helpful in speeding up the healing of cuts, bruises, and other minor injuries.

Lambs. Ultra-violet light is of considerable assistance in increasing the number and rate of growth of " hothouse " lambs.

Sheep. Treating ewes' feed with a mercury-vapor lamp at a distance of 30 in. for 30 min changed their calcium balance from negative to positive.

8. IRRIGATION AND WATER SUPPLY

IRRIGATION AND IRRIGATION DRAINAGE. Outside of the arid regions with their large developmental projects irrigation is an individual farm problem. Even in humid and semi-humid areas having water available at critical periods sometimes results in saving enough of the crop that otherwise would be lost, to entirely pay for an irrigation system in a single season. However, irrigation practice in such areas is generally limited to the raising of high-priced crops reasonably close to good markets.

Cost and Value of Irrigation. The first cost of irrigation plants pumping from adjacent streams or shallow wells does not run below \$10 or \$20 per acre, and the initial cost for the least expensive deep well pumping installations is around \$100 per acre—under less favorable conditions it may be several times higher. However, the water rights on some of the contemplated western irrigation projects would amount to \$158. For spray systems a first cost of at least \$300 per acre is expected.

For surface irrigation, estimating 20 per cent for interest, depreciation, and taxes on an average investment of \$75 per acre—\$15; \$10 per acre for fuel; \$5 per acre for repairs and incidentals, and \$5 per acre for labor (assuming a labor cost of \$4 per day, 2 acres irrigated per day per man, and 2 1/2 irrigations per season), we have a total irrigation cost of \$35 per acre. One man can irrigate from 1 1/2 to 3 acres per day by the furrow method and somewhat less with portable pipe. A conservative estimate for the total cost of spray irrigation is \$50 per acre. The average total irrigating cost for a typical sub-irrigation installation in Florida (35 acres) is \$47.60 per acre. A good average cost for surface irrigation is \$1 per acre-inch.

Not infrequently one irrigation at a critical period during a week or 10 days' drought will result in saving enough of the crop to pay for the irrigation plant in one year. A Virginia orchardist estimates that his \$4880 irrigation system, serving 50 acres of apples, paid for itself during its first year of operation (the dry year of 1929). In dry years (humid areas) small fruits without supplemental irrigation are frequently a total failure owing to a lack of moisture.

Where the irrigation system is used for emergencies (droughts) rather than regularly, efficiency of equipment is of less importance than first cost.

Energy Consumption. For surface irrigation from 2 to 4 kwhr per acre-foot (12-acre inches) are actually required for each foot of lift, theoretically only about 1 kwhr being required. Because of the more efficient use of water, the energy consumption per acre-season for spray irrigation is usually considerably lower than for surface irrigation in spite of materially higher requirements per acre-inch. The total energy consumption per installed horsepower per year (surface irrigation) runs from 200 kwhr, or even less in a few cases, to 800 or more kwhr, the latter apparently on farms being operated by the more aggressive farmers.

The seasonal distribution of irrigation energy requirements in Kansas are rather typical: 75 per cent during June, July, and August (20 to 25 per cent in July and 30 to 35 per cent in August); 15 per cent in September, and 10 per cent during the rest of the year.

WATER SUPPLY. A water system is a great labor saver, and, through the accessory plumbing, etc., a major factor in farm sanitation, water also being supplied for fire protection, lawn sprinkling, garden irrigation, milk-house use, etc. The water supply should be adequate in quantity (see Tables VI and VII) and of good quality. A large proportion of farm wells are contaminated, and where there is any chance for surface or subsurface leakage—the latter sometimes coming from a considerable distance—the water should be analyzed.

Table VI. Amount of Water Used Daily Per Individual

	Gallons
When water is carried by hand	6– 8
One pump at kitchen sink	8–10
One faucet at kitchen sink	10–15
Running hot and cold water in kitchen	15–20
Complete plumbing, and water under pressure	20–40

Table VII. Average Unit Water Requirements per Person and per Head of Livestock*

	Gallons per Day
For each member of the family for all uses including hot and cold water services in kitchen, laundry, and bath (45 to 50 gal in some cases, according to data from Illinois)	35
Schools, per person (3 to 15 gal)	7
For each horse, mule, or head of cattle (4 to 20 gal, varying especially with the season)	15
For each cow (summer as high as 40 gal)	25
For each hog	2
For each sheep	1 1/2
For each 100 chickens	2 1/2

* The gallons per person in this table may be somewhat higher than recommendations which are sometimes given, but they represent the use which is normally made of water when sufficient time has elapsed for the farmer and his family to fully appreciate the conveniences offered by the automatic water system. Water requirements preferably should be figured for maximum consumptions so as to take care of the peak demands, generally occurring in the summer.

A complete water system can be installed gradually without materially increasing its total cost, a pump jack and motor with valveless piping to the stock tank and kitchen sink being the first step. Three-way switches are located at the house and well or stock tank, and the three-way frost-proof valve is turned so that water is pumped directly to the house except when the stock tank is being filled. An overhead tank with a check valve below the faucet so that the water is available only at the kitchen sink is generally a second step.

The plumbing system should include a shower in the basement. Plans for the septic tank can usually be obtained from the State farm bureau or college of agriculture. Milk waste may require a capacity increase in the septic tank of 100 per cent or more. The disposal tile are usually 12 to 18 in. deep, 30 to 50 ft being provided for each person in the home.

Soft water can usually be supplied at lower total cost by water softeners than by cisterns and pumps, the annual total costs according to typical data being $25 and $30 respectively. Zeolite softeners readily remove common lime hardness, due to calcium and magnesium, but ozonators seem to be the only type effective with iron salts.

Pumping Units. Shallow well pumps can be used where the maximum depth to water does not exceed 22 ft. Deep well cylinders should be 15 to 20 ft below the water surface, when the water is at its lowest point for the year. Small centrifugals are not adapted to pumping against heads of more than 25 or 30 lb, and the suction lift should not exceed 15 or at most 20 ft. Air systems are comparatively inefficient but always furnish fresh water. Pump jacks should be of the quick return type. Outdoor faucets in areas subject to freezing weather should be of the anti-freeze type. A float switch set at the low-water mark on the water tank permits the windmill to pump considerable water, the motor pumping only when the water supply becomes excessively low. Ordinary water systems are easily equipped with fresh water taps.

It is seldom advisable to install a system with a capacity of less than 400 gal per hr unless the water supply is inadequate, in such cases more storage being essential. For fire protection a 2-hp motor and a tank supplying 2000 or more gallons of water probably constitute the smallest installation that will furnish reasonably adequate fire protection, though it should be remembered that some fires can be controlled only by sand, chemicals, or other suitable materials. Frost-proof outlets should be provided on every side of the barn and at other strategic points, and two $^5/_{16}$-in. nozzles, a 50-ft length and 100-ft length of 1 $^1/_2$-in. rubber hose should be available in some definite location which is free from fire risk. The wiring and the pumping equipment itself should not be subject to fire hazard.

The overall efficiency of water systems varies from 5 to 35 per cent, 25 per cent being considered very good. The efficiency of pumps, piston types particularly, depends quite largely upon the state of repair.

The energy consumption of farm water systems ranges from 0.5 to 60 kwhr per month (50 kwhr for a typical 60-cow wholesale dairy), 25 kwhr being a fair average. In the South the average is 15 to 20 kwhr per month. The average consumption for home use is 4.5 to 5 kwhr per month but may climb to 8 or 10 kwhr per month with more liberal use of water. Low lift centrifugals have very low energy requirements, for example, 0.55 kwhr per 1000 gal for a $^3/_4$-in. centrifugal lifting water 17 ft into an open stock tank.

Water pumps and systems should be on separate electric circuits and should be provided with switches and reliable inverse time relays, fuses being unsatisfactory.

9. DRYING AND DEHYDRATION

Drying

See also under Dehydration, p. 19-23.

Corn. Corn may be dried, or else the temperature may be kept low enough to prevent spoilage until it is fed. A Minnesota farmer, for example, prevented spoilage by storing his corn to a depth of 5 ft in a crib with 16-in. screened air vents at 2-ft intervals along laterals 3 or 4 ft apart, the long distributing pipe being connected with the 18-in. 144-watt 1500-cu-ft-per-min fan used to ventilate the cow stable. Outside air was used. Only 3 kwhr were consumed, the fan being operated 1 $^1/_2$ hr daily for 2 weeks. The fan was also turned on for a few days in the spring. Generally a fan used for drying is operated at intervals as necessity demands.

For real drying, heated air (above 125 deg fahr lowers germination and does not materially increase drying efficiency) is advisable—absolutely essential during moderately cold damp winters. In a typical case the moisture content was reduced in 183 hr from 38 to 22.6 per cent by forcing hot air through a longitudinal slotted core running through the center of the crib, at a cost of 9.8 cents per bushel (0.93 gal of oil at 7 cents per gal and 0.66 kwhr at 5 cents per kwhr), or $5.57 per lb of water evaporated, an improved direct heat portable dryer reducing this cost to $3.25.

Grain and Small Seed. The combine has greatly increased the demand for drying. Sweet clover has been successfully dried by permitting it to flow from a storage bin over baffles while hot air from an enclosed coke-heated brooder stove was forced up through it, an elevator carrying the seed from the bottom of the shaft to the bin. Similarly a batch dryer developed in Sweden has a sloping bottom so arranged that grain is drawn from the bottom, blown through a pipe with heated air, and scattered over the top of the grain bin.

Unless grain is stored on the farm, drying is generally done most economically at the elevator. In some cases wheat can be kept from spoiling by moving to another bin as required. In Kansas it has been found possible to reduce the temperature of the grain to about two-thirds the difference in temperature between the grain and the outside air—six movements reducing the moisture content by 2 per cent with an energy consumption per 1000 bu moved, of only 6 kwhr with chain and tubular elevators, or twice as much for blower-type elevators. Wheat, to keep safely, must not contain more than 12.5 per cent of moisture. Passing the grain through a fanning mill on its way to another bin greatly increases the drying and cooling effect. For drying, 130 deg fahr is just as effective as higher temperatures, and 5 or 6 per cent of moisture can be removed per hour with an air flow of 65 cu ft per sq ft per min, with 1/2 bu of wheat per sq ft of area.

Hay. About one-half (by weight) of good alfalfa hay is leaves, and a properly managed acre is almost equal to an acre of corn. Hay from immature cereals, as well as alfalfa and other legumes, has a protein content above 20 per cent (see Table VIII); in fact, feeders using such hay have been able to reduce the protein content of their concentrates from 18 to 12 per cent without reducing the milk flow. In two typical cases the substitution of artificially dried for sun-dried alfalfa hay resulted in increases of 2/3 and 1 quart respectively in the milk flow per cow per day. Artificially dried corn with no succulent feed produces as much milk per pound of dry matter as corn silage.

Table VIII. Average Composition of Alfalfa at Different Stages of Maturity *

Feeding Stuff	Water	Ash	Crude Protein	Carbohydrates		Fat
				Fiber	N-free Extract	
Alfalfa—all analyses	8.6	8.6	14.9	28.3	37.3	2.3
Alfalfa—before bloom	6.2	10.0	22.0	20.5	37.1	4.2
Alfalfa—in bloom	7.5	10.0	15.0	30.2	35.5	1.8
Alfalfa—in seed	10.4	7.0	12.2	27.6	40.3	2.5
Alfalfa leaves	6.6	13.6	22.5	12.7	41.2	3.4

* Data from University of Wisconsin.

The total cost of drying hay (overhead, fuel, labor, etc.) runs from $3.50 to $10 per ton. In actual practice one-half of the water is often removed in the field. The fuel requirements run as low as 26 gal of oil, 3000 cu ft of gas, or 200 lb of coke or coal per ton of dry hay. The total horsepower requirements vary with design, ranging from 5.5 to 135 per machine, the average energy consumption probably being around 40 kwhr per ton of dry hay. It is considered necessary to dry at least 500 tons of hay annually if the cost per ton is to be reasonably low, unless the dryer is also used for drying other products such as grain, small seeds, apple pomace, beet tops, orange peel and pulp, pea vines, salt, seaweed, sugar cane or corn stalk refuse, peat moss, or manure. In several cases manure is being dried. See Fig. 4.

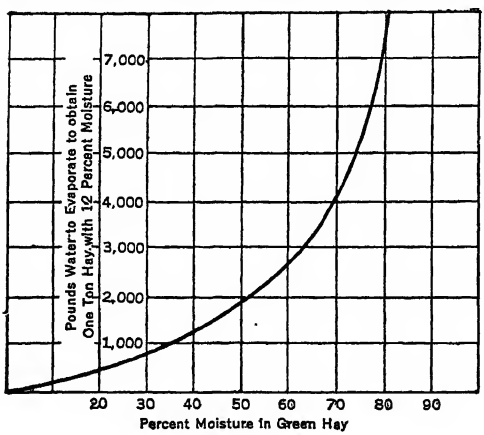

FIG. 4

Manure. Both chicken and cow manure are being successfully dried, but the drying costs per ton are higher for the latter. Peat moss is used for bedding; 100 lb absorbs 1500 lb of water as compared to 550 lb of water for straw.

In a typical case a machine ordinarily used as a hay dryer has a 6 by 40 ft drying drum, with a slope of 1/4 in. per ft towards the outlet which revolves at 11 rpm, has a capacity of 0.6 ton of dried cow manure per hour, a demand of 66 hp (94 hp when used for drying hay), a light oil consumption of 100 gal per dry ton (about 8000 lb of water evaporated per ton of dried product), and requires a crew of three. The hot air, at 1800 to 2000 deg fahr, enters at the wet end, as dry manure is very inflammable. The dried product is generally run through a hammer mill; it is odorless, and 7 bu weigh only 100 lb.

Rice. The combining and drying of rice results in a higher market price and materially lower field losses, the total cost of drying in a typical case being around 6 cents per bushel. However, unless the drying is done in steps, permitting the rice to season before it is further dried, the rice may even be lowered in milling quality and grade by checking and cracking of the kernels.

STRAW FLOWER DRYING. A 10 by 10 by 6 ft high cabinet automatically heated by 12 kw on the pipeless furnace principle (heated air diluted with fresh air and cold air circulation also provided) has a capacity of 8000 to 10,000 flowers every 8 hr (three batches each day), and an average energy consumption of a little less than 1 kwhr per hr of operation.

Dehydration

See also under Drying, p. 19-21.

Fruit. Dehydrated fruit (dried with temperature, humidity, and air velocity control) retains its natural flavor and quality to a very much greater degree than ordinary evaporated fruit; it also refreshes to almost the original weight, upon proper soaking, as compared to a loss of about 20 per cent with ordinary evaporated fruit. Apples, apricots, blackberries, blueberries, crabapples, loganberries, peaches, pears, plums, prunes, and raspberries are successfully dried, but gooseberries are not; cherries bleed badly, and drying of strawberries is not recommended. In many cases the drying plant should be operated in close cooperation with or as a part of an undertaking meeting the needs of the entire industry, for example, a fresh fruit warehouse, cider and fruit juice mill, and jam, jelly, and marmalade plant.

Most fruits, except berries, require preparation for drying. For example: the bloom on prunes and plums is cut by dipping in a lye solution; apricots are halved and pitted; apples are peeled, cored, trimmed, and sliced; peaches are split, pitted, and may or may not be peeled; pears are halved and cored and may or may not be peeled. (See also Calif. Bul. 485; Dom. of Canada Dept. of Agr. Bul. 90, Table 2.)

Grape dehydration in California is limited largely to Thompson's golden bleached and some seedless, no wine or juice grapes being dried except to prevent rain damage or to salvage slightly damaged fresh fruit. Prune dryers which are flexible enough to meet the greater heat and air demands (air velocity of 700 to 1000 lin ft per min) can be used. The finishing temperature should not exceed 165 deg fahr, at the same time relative humidity being around 25 per cent. At an average total cost of $9.51 and a 3 1/2 to 1 shrinkage ratio, drying costs amount to $35.66 per ton of dry raisins, the operating costs of $5.06 per fresh ton equaling $18.98 per ton of raisins, on the average. (See Calif. Bul. 500.)

Table IX. Drying Ratios for Fruits

Apples	8 : 1
Apricots	4–7 : 1
Figs	1 1/2 : 1
Grapes (Thompson's golden bleached)	3 3/4 : 1
Peaches	4–7 : 1
Pears	4–7 : 1
Prunes	2–2 1/2 : 1

Prune dehydrator data from Oregon on modern air-blast and natural-draft air dryers show average fixed charges of $3.65 and $3.43 and total operating costs of $7.76 and $7.64 per fresh ton, respectively (about 1 cent per lb of dried fruit); likewise for forced-draft and sun dryers a ground area of 1 acre for tray space takes care of 400 acres and 20 acres of prunes respectively. Fans increase the capacity of natural draft dryers by about 50 per cent. The fruit enters the cooler end where the temperature is kept between 120 and 140 deg fahr, and leaves at the hot end where the temperature should not exceed 165 deg fahr, the humidity being kept at around 25 per cent; the dried prunes are stored for at least 2 weeks in bins (for moisture equalization) before delivery to the packing house. The drying time ranges from 20 to 36 hr—from 28 hr up for large prunes of 40 or less per pound, and around 24 hr for 50- to 60-per-lb sizes. The energy consumption per ton of dried prunes for fan operation ranges from 80 kwhr in California to 95 kwhr in Oregon. The average horsepower requirements per ton of capacity (in fresh fruit) are 1 1/2 in California and 1.87 in Oregon; in California prune dryers have a capacity of 4 to 40 tons of fresh fruit daily—an average of 16.5 tons daily, or 350 tons during the 3 weeks' season; in Oregon the season capacities range from 30 to 800 tons and average around 300 tons

for the 3 weeks' season. California and Oregon prunes differ in size, the average weights per square foot of freshly trayed fruit being 3 1/2 and 4 1/2 lb respectively. Homemade dryers of simple design seem on the average to operate as efficiently as the more elaborate and expensive manufactured types.

Hops. Hop kilns are generally 30 ft square, one room, and the drying floor is 14 to 20 ft above the ground, the hops being placed 14 to 18 in. deep on this lattice floor. In a particular case a 15-hp motor drove the ventilating fan, a total of 50,000 lb of hops being dried in the 20-day season. It is estimated that an electrically ventilated hop dryer uses 125 to 150 kwhr per 1000 lb of hops (dry weight) dried, five 1000-lb bales being an average production per acre. Forced draft reduces the drying time per batch to 9 hr as compared to 16 hr for natural draft, enabling the grower to reduce his dryer space by 45 per cent.

Vegetables. Properly dried beans, beets, carrots, peas, pumpkin (first class for pies), potatoes, squash, spinach, and turnips regain practically all their fresh appearance and flavor when refreshed. Beans, corn, and peas are better if blanched before drying; potatoes require sulfuring. (See also under Dehydration, fruit.)

Walnuts. Large numbers of walnut dehydrators are electrically ventilated and a few are electrically heated. This method of heating is made possible by: (1) dividing up the high demand, the electricity when not needed for drying also being used for other purposes such as irrigation; (2) the small amount of moisture removed (12 to 25 per cent before and 8 per cent after curing); (3) equipment of twice the ordinary efficiency (95 per cent of air recirculated in a typical case), 44 per cent as compared to 22 per cent (however, the unit costs about $5000 as compared to $2500 for the average); (4) handling of larger tonnages per dryer, 150 as compared to 50 tons on the average. Several growers have made quite a saving in overhead by using their prune equipment for drying walnuts even though the walnuts have a maximum drying temperature of about 112 deg fahr as compared to around 165 deg fahr for prunes. The available data for electrically heated dryers shows total overall operating costs ranging from about $7 to $11 per ton of dry walnuts for the compartment type, typical energy figures per dry ton being 32 kwhr for power and 160 kwhr for heating. The connected load for heating bin and compartment-type dryers averages around 20 and 10 kw per ton of capacity in dry walnuts, respectively, and the fan load in both cases varies from 1 to 3 hp per ton of capacity (dried walnuts). It takes from 12 to 24 hr per dehydration, the season lasting about 3 weeks.

Prune, walnut, and hop dryers, if developed together, apparently offer the power company an attractive load; for example, in Oregon prune drying usually comes during the first 3 weeks of September, hop drying begins around the middle of September, and walnut drying around the first of October.

10. HOTBEDS

HOTBEDS. Seeds germinate, plants grow, and cuttings root in 20 to 30 per cent less time in electric hotbeds than with the other methods in common use. Furthermore, electric heat lends itself to automatic or manual control, and plants can also be grown in the fall when maximum heat is required at the end of the growing season. Considerable frost protection is also easily provided for. The beds do not need rebuilding each year, only the top soil being replaced every few years as required.

A cubic yard of ordinary 2-horse load of manure (enough for a 6 by 6 ft bed) furnishes about the same amount of heat as 90 kwhr of electricity. The average total first cost of equipment is about $12 per sash, and the average energy consumption (also varying widely) around 2 kwhr per day per sash, data from Ithaca, N. Y., for example, showing a range of 1.1 to 4.8 kwhr per day per sash. Each installed kilowatt is estimated to use an average of 1000 kwhr per season or in most areas from 10 to 15 or more kwhr per actual day of operation. Very little energy is required for frost protection. In cold climates good insulation will reduce operating costs about 30 per cent. Cinders outside, and some say also below the bedsoil, and two sets of single sash, one on top of the other, are recommended. In some sections banking with soil and the use of double burlap mats at night have proved satisfactory.

A thermostat saves energy and gives closer temperature regulation.

For ordinary frost protection 50 to 60 watts per sash are adequate; for ordinary service 120 to 150 watts per sash are satisfactory, and for very cold climates 200 to 250 watts per sash are used.

Cable must be used in approximately the lengths for which it is designed, less cable giving more wattage (danger of burning out) and vice versa.

Lead-covered cable is decidedly the most popular source of electric heat for hotbeds,

but space heaters, loop heaters, and open coil heaters (all but the cable located in air spaces below the bedsoil) are also used.

Table X. Maximum, Minimum, and Optimum Germinating Temperatures and the Corresponding Germination Period for Some of the Commoner Vegetables *

	Temperature, Lowest–highest, in deg fahr	Corresponding Days for Germination, Lowest–highest, days	Most Suitable Temperature, Lowest–highest, in deg fahr	Corresponding Days for Germination, Lowest–highest, days
	1	2	3	4
Radishes	39–77	23–2	50–59	7–3
Lettuce	41–79	20–3	59–68	6–3
Cauliflower	41–77	23–4	50–68	11–4
Peas	39–77	25–4	50–68	10–4
Onions	43–79	30–7	54–72	14–8
Parsley	45–77	35–9	57–72	15–9
Celery	45–82	40–9	57–75	15–9
Beans	52–81	26–3	59–72	10–4
Tomatoes	52–82	25–5	63–75	10–6
Cucumbers	57–82	16–3	72–82	5–3
Cantaloupes	61–86	20–5	72–82	6–5

* Data from Maurice W. Nixon. Columns 1 and 2 refer to maximum and minimum temperatures and corresponding average germinating periods; columns 3 and 4 refer to optimum temperature ranges and corresponding average germination periods.

Temperature is the most important consideration in the successful operation of hotbeds, cutting benches, etc. (See Table X.) Plants raised in such equipment require regular watering, and during the daytime (warm or hot days) the sash in hotbeds must be raised at the lower end, as required, so as to prevent damping off.

11. MISCELLANEOUS

ALARMS: Burglar, Fire, and No-voltage. A burglar alarm preferably should work on the closed-circuit principle, and the relay, after even the shortest possible current interruption, should require resetting by hand. In this way the alarm bell will continue to ring until it is reset.

The alarm should be so arranged that no matter how the wires are tampered with, especially by " bonding " and " cutting," the system will still give the alarm. If the resistance (high-voltage type) is placed out of reach in the protected building, the bonding of the wires would short the holding coil of the relay and blow out the fuse in the relay circuit, assuming proper relationship between resistance of relay coil and wires, and a suitable size of fuse or circuit breaker. This, of course, would give the alarm. Placing the transformer or relay in one of the protected buildings will prevent effective bonding if the system is properly installed (Fig. 5).

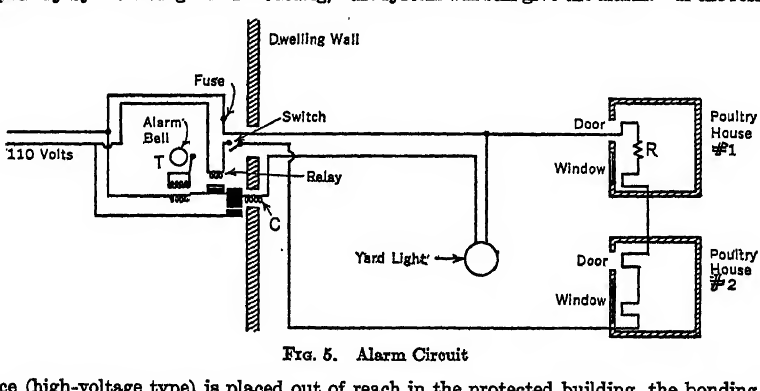

FIG. 5. Alarm Circuit

In Fig. 5 the equipment is in the closed position (switch shown open) and relay set. T is a bell-ringing transformer and bell; C a flexible wire connection, the weight keeping the contacts closed when the switch is open; R a resistance—sometimes one or more lamps in series; and at each door and window are switches the opening of which gives the alarm and at the same time instantly floods the yard with light. Resistance is not ordinarily used with low-voltage modern relays. With batteries in the secondary circuit and the use of temperature relays the alarm will also serve to warn of no-voltage, fire, and low and high temperatures.

The wiring costs as a rule are materially lower for low-voltage than for 115-volt systems. A separate system is required for each building if the building tampered with is to be indicated. An annunciator may be cheaper than separate systems for large installations.

BONE AND SHELL GRINDING. Bone grinding requires considerable power as compared to feed grinding. In a typical case a 5-hp motor ground between 100 and 150 lb of bone per hr (20 to 30 lb per hr per hp) with an energy consumption of about 1.1 kwhr per 100 lb.

Oregon data show an energy consumption of 2 kwhr per ton for grinding oyster shell for baby chicks, coarse grinding requiring still less energy. A hammer mill would be expected to produce a higher percentage of finely powdered material.

BULB COOKING. Insect pests affecting bulbs are kept in check by cooking the bulbs in a water bath at 109.5 to 111 deg fahr for 2 1/2 hr; electricity readily adapts itself to such close temperature control. The machine tested (Oregon) has a demand of 2 kw for heating, a 1/4 hp motor on the agitator, and a capacity of 150 lb of bulbs (1000 average bulbs). It used about 2 kwhr per 100 lb of bulbs treated. With the insulated tank used, the heat may be turned off for as much as an hour at a time during the cooking process. Where steam is used for the initial heating of the water and electricity for maintaining the temperature (cooker insulated with 1 in. cork board), the consumption was 1.2 kwhr per 100 pounds of bulbs cooked.

CIDER MAKING. A typical small hand-press cider mill operated by a 1/2-hp motor crushed 144 1/2 boxes of apples from which 306 1/2 gal of cider were obtained, with an energy consumption of 2.5 kwhr (0.8 kwhr per 100 gal of cider).

CONCRETE MIXING. Most small concrete mixers can be satisfactorily operated by a 1/4-hp motor, in some cases 1/2-hp being needed. From 0.4 to 0.5 kwhr will ordinarily mix a cubic yard of concrete, more energy being required with finer aggregate.

COTTON GIN. Cotton gins require three-phase service, and the season extends from September 1 to January 1, the peak load coming in October. About 0.3 hp is required per saw, and available data show an average energy consumption of 16.67 kwhr per 500-lb bale for ginning and pressing.

ELECTRIC TRUCKS. Electric trucks are more economical for daily hauls of a possible maximum of 50 miles than ordinary gasoline trucks. In a typical case a dairyman's total costs ran between 3 and 4 cents per mile (87.5 kwhr per 100 miles of travel—3 cents per kwhr).

FERTILIZER GRINDING AND MIXING. Lumpy fertilizer is easily crushed or ground by burr mills set for coarse grinding. Hammer mills could also be used. Afterward the mill should be cleaned by running rock-free soil or grain through it. Fertilizer is sometimes mixed in concrete mixers, but the same facilities used for feed mixing (see Art. 3) can also be used, the equipment subsequently requiring careful cleaning.

GRADING. See also Grain and Seed Cleaning and Grading.

Apples. A grader with a capacity of 25 boxes per hour can be operated by an experienced crew of four or five packers, two graders, and one nailer; and assuming a first cost of $800, a grower would need an average annual crop of 4000 to 6000 bu (enough for one month's operation) to justify its purchase. The energy consumption ranges from 0.5 to 1.5 kwhr (an average of 1.0) per 100 bu or boxes graded. Most graders are operated by 1/2-hp motors, although 1/3 hp is ordinarily sufficient.

Bulbs. Graders save much time on market sizes, even though the mother bulbs with the small ones attached do go in with the large ones, and are entirely satisfactory for the bulbs to be planted for the second and third year—it usually requires 3 years to grow bulbs to market size. In a typical case a machine operated by a 1/4-hp motor and equipped with 1-in., 1 1/2-in., 2-in., and 2 1/2-in. screens, graded 177 lb of bulbs in 6 1/2 min.

Potatoes. A 1/2-hp motor on a potato grader will handle from 600 to 700 or more bu of potatoes per kwhr used, and save the labor of at least one man.

HONEY EXTRACTING. A 1-hp motor will operate most honey extractors.

HORSE-RADISH SHREDDING AND MUSTARD SEED GRINDING. A combination farm plant of this type is operated by a 15-hp motor.

INSECT TRAPS. Flies while still sluggish from the early morning chill can be sucked up with a vacuum cleaner, and an ordinary fan is often used to keep them away

from open cans of milk, cream, etc. The gnat population near summer resorts has been materially reduced by luring them with lights, to the vicinity of an electric fan exhausting into a canvas bag. Electric screens and screen doors are quite effective, and a typical fly trap in Texas captured 6 gal of flies in 4 days, from that time on the number of flies about the dairy being noticeably less. Such traps (unlighted) have a demand of about 8 watts, furnish about 4000 volts, and use about 5 or 6 kwhr per month, continuous use. Some tests in apple orchards with light traps of different kinds show the electrocutor type mounted in the top of every other tree in alternate rows to be most effective. In fact, fully as good results were obtained as by spraying, and where apples were protected by both traps and spraying, twice as many good apples were obtained as by either alone. However the data have not as yet been generally accepted as completely conclusive. In Japan, lights were found to reduce materially the insect losses in the rice and tea crops and to lessen the damage to mulberry trees. (See also under Special Lighting, p. 19.).

LADYBIRD BUG HATCHING. A typical ladybird bug or Limoniera ranch (Southern California) has 25 rooms, each 8 by 16 ft with 8-ft ceiling and usually insulated with $1/2$ in. of celotex. Each room has a capacity of 125,000 beetles per crop, and an energy consumption of about 23.2 kwhr per day of operation, or 0.068 kwhr per beetle. The rooms are automatically kept at 70 deg fahr during the three weeks occupied in raising potato sprouts; at 80 deg fahr for a like period when the sprouts are infected with mealy bugs, and at 75 deg fahr during the next seven weeks when a few ladybird bugs are liberated and propagated. As the bugs mature they fly to the light and gather on a piece of muslin over the window, where they are collected and placed 10 in a capsule. They are sold for about $1/3$ cent per bug to owners of Valencia and navel orange and lemon orchards one capsule being used to a tree. The bugs are always available except during the rainy season.

MOTORS. See Portable Motors.

MUSHROOM HOUSES. Packed mushrooms and prepared spawn are held in electrically cooled storages. Electric pumps furnish water and in some cases electricity lights the houses. Some growers have installed refrigeration so as to keep their summer temperatures around 55 deg fahr and as a result are able to sell mushrooms the year around.

MUSTARD SEED GRINDING. See under Horseradish Shredding and Mustard Seed Grinding.

PAINT SPRAYING. Seven times as much area can be covered in a given length of time by an experienced operator with a good gun as by a good painter with a hand brush. Furthermore, paint losses are no higher, and with a guard, trim can be added, little retouching by hand being necessary. About 800 sq ft of straight wall surface can be covered in an hour (experienced operator) with an average energy consumption of 1 to 1 $1/2$ kwhr. An Illinois farmer, operating a machine for the first time, covered 2000 sq ft of barn surface in 8 hr, using 0.42 kwhr per 100 sq ft.

Whitewash has been successfully removed by using a $1/32$-in. nozzle and an air pressure of 160 lb. The air is also used to inflate automobile tires. Smaller and less expensive equipment is also available.

PEA VINERS. Twenty-horsepower motors are commonly used. Wisconsin figures for 1926 show a consumption of 4 kwhr per 100 lb of peas shelled, and plant consumptions ranging from 2751 to 3145 kwhr. The season lasts from the middle of June to the last week in July, the plants usually being operated from daylight until dark.

PORTABLE MOTORS. Two portables, a $1/4$- or $1/2$-hp motor and a 5- or 7 $1/2$-hp motor, will operate all the usual farm machines not provided with attached motors. In some cases the small type is provided by various companies with one or more other features such as gear reduction, a tripod, a multistep, V, or flat pulley, a belt tightening base fastened with a wing nut and flexible direct drive. Belts on the large portables are tightened and kept tight in various ways: by blocking, telescoping tongues on the truck, bases moved by screw cranks, fence stretchers, block and tackle, and by a hinged base arrangement which automatically adjusts the portion of the motor's weight resting in the belt to suit the torque required—this type of base also being used on small motors (not expensive). Flat belts should be pliable and preferably endless; at least the lacing must be light. Higher belt speeds are possible with V-belts. Owing to first cost, standard 1800-rpm motors are most commonly used. Motors having high starting torque without starting clutches give trouble less frequently than other types.

SHAVING, ELEVATING. See Elevating Grain and Shavings, Art 3.

SHELL GRINDING. See Bone and Shell Grinding.

SHOP EQUIPMENT. A 1-hp motor is large enough for most farm shops but driving several machines from inefficient line shafting may make a larger motor necessary. A large motor in a well-equipped shop on a large farm will use 4 or 5 kwhr per month, but on a moderately sized farm in South Dakota a 3-hp motor in a shop equipped with a steel

drill, wood lathe, grindstone, small circular saw, large meat grinder, two or three emery wheels, and a buffer, all operated through a line shaft, used only 52 kwhr in a year.

SOYBEAN COOKING. When soybeans are cooked (not nearly so satisfactory a feed when uncooked) they are a very good substitute for tankage and considerably cheaper. A well-insulated electric cooker holding 2 bu of raw beans and 20 gal of water is heated by 2 kw in heating elements, and about 15 kwhr are required to cook a batch, the total cooking time being 10 hr. Cooked soybeans will save cash outlay for tankage and shorten feeding period for hogs by 2 or 3 weeks.

STATIONARY SPRAY PLANTS. With stationary spray plants the spraying can be done in one-half to one-third the time that would be required with portable equipment (a saving of 38.8 per cent in a typical case); spraying can be done regardless of the ground condition (has been done from boats); in irrigated sections the irrigation ditches are not injured; fruit is not knocked off or trees injured; the soil is not packed; the plant does not jar out of adjustment, and it lasts longer; spraying can be done on extremely rough ground; and last, but certainly not least, each man can spray at his own gait.

Cost of Equipment. The average cost per acre of stationary spraying equipment ranges from $50 to $90, frequently being no greater than the cost of portable equipment for the same area. Approximately 6.35 kwhr are required per spray per acre, or close to 50 kwhr per acre per year for an average of 7.5 sprays (Washington data).

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INDEX

Note: The double folios refer to both section and page numbers. For example, 13-09 indicates page 09 of Section 13.

A

A B C cable, 14–236
A C S R, see Aluminum steel
A C S R cable, 14–164
A. S. & W. cable, 14–163
A. (steel) W. G., 1–62
A. W. G., 1–61, 1–63; 14–144
Ab (prefix), 1–37
Abampere, 1–38, 1–53
Abbreviation, 1–64
Abcoulomb, 1–37, 1–52
Abfarad, 1–37, 1–56
Abhenry, 1–56
Abohm, 1–38, 1–55
  centimeter, 1–55
Absolute electrical system, 1–37
Absolute joule, 1–51
Absorbent paper filters, 2–20
Absorption, dielectric, 4–10
  of heat, 2–52
  of light, 15–04
  of moisture, 15–61
Abvolt, 1–38, 1–54
A-c (see Alternating current)
Acceleration, 1–47
  corrected constant, 17–08
  gravity, 1–70
  retardation time curve, 17–15
  trains, 17–07
Accessory, wiring, 14–239
Accidents, effect of light on, 15–37, 15–39
Accuracy of instrument, 5–27 (and see article on particular device)
Acetylene gas, 2–03
Acheson process, 18–17
Acre, 1–42
Acroides, 2–28
Acyclic generators, 8–21
Adapter, simplex, 5–76
Adequacy of illumination, 15–33
Adhesion, 17–39
Adhesion coefficient of locomotives, 17–09, 17–24, 17–39
Adjustable inductors, 4–23
Adjustable speed motor, 16–10, 16–19
Admittance, 1–65, 3–10
  per mile, 14–26
Adsorption of moisture, 15–61
Advance, 2–09, 18–22
Advantages of a-c and d-c, 14–03
Aerial, 14–163
Aerial line calculations, 14–63 to 14–77
  ice loading, 14–60
Aerial tramways, 16–30
Aging of magnetic materials, 2–38
Agriculture, electricity in, Section 19
A. I. E. E. standard spark gap, 5–64
Air, 2–03
  brakes, 17–39
  bubbler system, 13–26
  cell, 7–02
  circuit breakers, 12–11
Air, compressors, motors for, 16–08
  conditioning, 15–60 to 15–67
    of trains, 17–41
  dielectric strength, 2–36, 14–33
  gap, 4–22
    a-c machines, 9–10, 9–19
    d-c machines, 8–09
    pull, 4–36
  heater, 14–244
  infiltration, 15–66
  inside conditions, 15–67
  leakage, buildings, 15–65
  required for manholes, 14–131
  required for ventilation, 15–66
  spaces, heat conductance of, 15–65
  spark gap, 5–63
  thermal properties of, 2–50, 2–54
  washer, 15–62
Air-blast transformer, 10–04, 10–15
  installation of, 10–20
Air-cooled transformer, 10–03
Air-core inductor, 4–16
Aircraft magnetos, 17–79
  ignition, 17–72 to 17–84
  spark-plugs, 17–83, 17–84
Air-gap measurement of induction, 5–73
Airway beacons, 5–94
Aisles, light for, 15–34
Alabaster, reflection from, 15–04
Alabaster glass, light transmission, 15–05
Alarms, burglar, 14–256, 19–25
  farm, 19–25
  fire, 14–257, 19–25
Alcohol thermometer, 5–97
Alfalfa, 19–22
Algebraic formulas, 1–02
Alkali production, 18–11
Alkaline-type storage batteries, 7–24, 17–105
Allegheny metal, welding, 18–47
Allen-Moore cell, 18–11
Alley service lines, 14–137
Alley wire construction, 14–89
All-nite lamps, 15–12
Alloy; welding, 18–47
Alloy wires, 2–09, 14–161
Alternating current, 1–64, 3–02
  and d–c combined system, 14–06
  arc welder, 18–48
  balancing coil, 14–11
  bridge, 5–19
  bus bar, 12–57
  commutator motor, 9–60
  compared with direct, 14–03
  distribution system, 14–05
  electromagnet, 4–39
  galvanometer, 5–05
  induction motor (see Motor, polyphase induction)
  line calculations, 14–16
    approximate, 14–19
    exact, 14–24
  machines, 1–66, Section 9

Alternating current, measurement, 5–26
potentiometer, 5–24
projection, 12–43
railway circuit, 17–55
series motor, 9–61
single-phase substation, 13–45
switchboard, 12–56
synchronous motor (see **Motor, synchronous**)
Alternating voltage, laboratory measurements, 5–26
Alternators, 9–02
Altitude, 12–27
correction factor, 14–36
Aluminum, 2–02, 2–07, 2–50, 6–06
cables, 14–158
coefficient of expansion, 14–58, 14–165
in car construction, 17–29
mechanical characteristics, 14–58
modulus, 14–58
plates and sheets, 1–61
plating on, 18–06
production, 18–17
reflection from, 15–04
ultimate strength, 14–58
weight, 14–58
wire, 14–149
yield point, 14–58
Aluminum-steel cables, 14–163
inductive reactance, 14–51 to 14–54
reactance, 14–52
resistance, 14–164
stress-strain curve, 14–59
tables of, 14–164
Aluminum-steel conductors, 14–58
Alundum, production, 18–17
American Steel and Wire Co. gage, 1–62
American Welding Society, 18–52
American Wire Co. gage, 1–61, 1–63, 14–144
American Zinc gage, 1–59
Amianthus, 2–18
Ammeter, 1–68, 5–27 to 5-31
Ammonia, 2–03, 2–50
Haber process for, 18–21
process, Claude, 18–21
synthesis, 18–21
Amortisseur, 9–16, 9–27
Ampere, 1–38, 1–53, 1–64
hour, 1–52, 1–64
turn, 1–57
Ampere-hour meters, 5–45
Amphibole, 2–18
Amplitude, 3–04
constant, 14–25
factor, 3–03
Anaconda I-beam cable, 14–163
Anaconda wire, 14–161
Anchor, guy, 14–92
Anchorage, 17–60
Angle of curvature, 17–12
Angstrom unit, 1–41, 1–64
Angular frequency, 1–65
Angular lag due to load, 9–32
Angular phase difference, 3–04
Angular velocity, 1–47, 3–02
Animal clippers, 19–12
Animal groomers, 19–11
Anion, 6–03
Annealed copper, 2–05
standard, 14–143
Annealing furnace, 18–26
Annealing welding, 18–45
Annular rings, 14–82
Anode, 7–02
mud, 18–07
Anti-functions, 1–08, 1–10
Antimony, 2–02, 2–03, 2–50, 6–06
Apartments, 14–247
Aperiodic motion, 5–03
Apothecaries' fluid measure, 1–44
weight, 1–49
Apple-butter making, 19–18
Appliances, heating, 18–24
power for, 14–244
Applications (see article on particular device)
of motors, Section 16, particularly 16–19
Approximate a-c line calculations, 14–20
Approximation, 1–15
Arborvitae, 14–87
Arc, carbon, 15–17
electric, 3–24, 12–02
furnace, 18–12
Keller, 18–20
light, 15–16
generator, 8–21
splitters, 12–15
temperature, 18–12
welding, 18–41
Arcing distance, 14–105
Arcing ground, 14–259
Arcing ground suppressors, 12–48
Arcing horn, 12–06
Arcogen process of welding, 18–46
Are, 1–42
Areas, 1–16, 1–42
Argon, 2–02
Arithmetical progression, 1–02
Armature, 8–03
copper loss, 9–14
core, 8–04, 8–10, 9–10
interference, 8–13
leakage reactance, 9–11, 9–20
loss, 8–18, 9–20
reaction, 3–43,
a-c machine, 9–12, 9–31
d-c machine, 8–13, 8–33
resistance, 8–11, 9–11
teeth, 8–10, 9–10
windings, 8–05, 9–05, 9–18
Armco iron, 2–43
Armor, 14–185
Armored cable, 14–236
for automobiles, 17–99
Arresters, lighting, 12–45, 14–141
Arsem furnace, 18–15
Arsenic, 2–02, 6–06
Artificial daylight, 15–26
Artificial ground, 14–243
Asbestos, 2–03, 2–18, 2–22, 2–50
covered wire, 14–240
insulation, 14–174
paper, 2–15, 2–20
Ascoloy, welding, 18–47
Asphalt, 2–15, 2–28
car resistance on, 17–104
Asphaltic varnishes, 2–33
Asphaltum, 2–03
Assembly, light for, 15–34
room, light for, 15–34
ventilation of, 16–33
of towers, 14–101
Assurance factor, defined, 14–172
Astern turbine, 17–112
Asymmetric headlamp, 15–32
Asymmetric light sources, 15–05
Asymmetric refractors, 15–28
Atmosphere, 1–50, 1–64
Atmospheric pressure, 1–70
Atom, 6–02

Atomic hydrogen welding process, 18–46
Atomic weights, 2–02
Atonic interrupter, 4–43
Attenuation constant, 3–48
  transmission line, 14–26
Auditorium lighting, 15–34, 15–38
Auto starters, 12–32
Autogenizer, 18–42, 18–44
Autogenous welding, 18–42
Automatic air brakes, 17–40
Automatic controls, air-conditioning, 15–62
Automatic equipment, 12–73
Automatic generator voltage regulators, 12–36
Automatic operation, 12–73
  regulators, 12–41
Automatic recorder, 5–95, 5–99
Automatic substation, 13–38, 13–44
Automatic welding, 18–37
  vs. manual, 18–49
Automobiles
  batteries, 7–18, 17–88, 17–105
  electric, 17–101 to 17–107
  electric equipment for internal combustion, 17–72 to 17–101
  fuses, 17–92
  generator, 17–84
  instruments, 17–92
  lamps, 5–94, 15–14, 15–32, 17–90
  lighting, 17–84
  starting, 17–84, 17–93
  transmission, 17–104
  wiring, 17–97
Auto-transformers, 10–02, 10–21
Auxiliaries, power station, 13–07, 13–24, 13–55
Average, 1–64
  values, 1–65
Avogadro's number, 1–70
Avoirdupois weight, 1–48, 1–64
A. W. G., 1–61, 1–63, 14–144
Axial ventilation, 9–17
Axle generator system, 17–41
Ayrton-Mather shunt, 5–04

B

B. and S. gage, 1–63, 14–144
Babat system, welding, 18–41
Babbit permeameter, 5–76
Backing power, 17–117
Back-spitting, 17–80
Bake oven, 14–244
Bakelite, 2–15, 2–26
Baker test ring, 5–57
Bakery, light for, 15–34
Balance coil, 14–11
Balanced polyphase voltages, 3–29
Balanced relay, 12–44
Balanced three-phase circuits, 3–32, 14–116
Balancers, 11–03
  phase, 11–17
Balancing of load, 14–10
Balbach-Thum electrolytic cell, 18–08
Ballistic galvanometer, 5–05, 5–43
Ballistic measurement of induction, 5–73
Ballistic sensitivity, 5–03
Ballroom, ventilation of, 16–33
Bandwheel drive, 16–28
Bar, 1–50
Bare electrode, welding, 18–44
Bare wire, 14–143 to 14–171
Barium, 2–02, 6–06
  production, 18–18
Barium-lead production, 18–18
Barn, car, 17–69
Barn, ventilation, 19–11
Barrel, 14–116
Barye, 1–50
Basalt, 2–03
Base, lamp, 15–10
  natural logs, 1–18
Base load hydroplant, 13–49
Basic electrical standards, 5–02
Basket-weave, 14–153
Bathroom, lighting, 15–39
  water requirements, 15–47
Battery, 1–67, Section 7
  automobile, 7–18, 17–88
    electric, 17–105
  containers, 7–11
  cutout, 17–84
  dry, 7–02
  primary, 7–02
  storage, 7–08 to 7–34
    alkaline, 7–24
    lead-acid, 7–08
  train lighting, 17–41
Battery brooder, 19–15
Bearings, frozen, resistance of, 17–07
Beater, motor for, 16–27
Bechstein photometer, 5–94
Bedroom, lighting, 15–39
Beeswax, 2–28
Bell type furnaces, 18–29
Bell wiring, 14–256
Belt conveyors, motors for, 16–29
Belt leakage, 9–11
Belts, power transmitted by, 16–16
Bending machine, motor for, 16–21
Bending radius of cable, 14–216
Bendix starter, 17–96
Benford isocandle curves, 15–05 to 15–07
Berry grading and cleaning, 19–18
Beryllium, 2–02, 18–18
Bessel's equation, 1–14, 1–16
Bethenod's welding system, 18–44
Bimetallic thermometer, 5–98
Biplane lamp filament, 15–13
Bipolar Schrage motor, 9–64
Bipost lamp bases, 15–10
Birkland-Eyde furnace, 18–21
Birmingham gage, 1–59
Birmingham wire gage, 1–62, 1–63
Bismuth, 2–02, 2–03, 2–50, 6–06
Black body, 2–52, 15–08
Blackening of lamp bulb, 15–08
Blackheat brooder, 19–13
Blast-furnace gas, cleaning, 18–34
Bleach, 18–10
Bleeding, 14–238
Blinding, by glare, 15–23, 15–41
Blotter filter press, 2–32
Blower, 13–42
  motor for, 16–32
Blue glass, 15–25
Boardmeasure, 1–43
Boiler, 13–07, 15–55
  apparatus, 13–11
Bolt cutter, motor for, 16–21
Bolt header, motor for, 16–21
Bolt holes, 14–104
Boltzmann constant, 1–70
Bonds, for drainage, 14–272
  rail, 17–64
Boost charging, 7–31
Boosted circuit, 14–10
Booth rotating furnace, 18–20
Borcher's furnace, 18–15
Boring machine, motor for, 16–21

Boron, 2–02
Bottle washers, 19–03
Boucherot circuit, 14–07
Boucherot winding, 9–53
Bougie decimal, 1–59
Bow, 17–38
Box furnaces, 18–27
Brackets, 12–59
Braids, 14–153, 14–174
 classes, 14–185
 cotton, 14–174
 weight, 14–226
Brakes and braking systems, 17–39
 on generator, 13–23
Braking of trains, 17–07
Braking period, 17–11
Branch circuit, 14–241, 14–252
Branch currents, 3–12
Brass, 2–03, 2–09, 2–50
 melting, 18–20
 plating, 18–05
 sheets, 1–61
 welding, 18–47
Brazed bonds, 17–65, 17–68
Breakdown torque (see article on particular device)
Breakdown voltage, 2–12
Breaker, circuit, 12–11 to 12–29
 motor for, 16–28
Breaker ignition, 17–72
Breaking load (of poles), 14–87
 of wires, tensile, 14–162
Breather, 10–20
Brick, 2–03
 heat conductance, 15–64
 surface, effect of, 17–104
Bridge, 5–12 to 5–19
 for cable faults, 14–224
 Schering, 5–85
Bridge transition of motors, 17–30
Briggsian logarithms, 1–18
Brightness, discrimination, 15–22
 measurement of, 5–94, 15–03
Brinell hardness, 2–06
British imperial gallon, 1–44
British imperial yard, 1–36
British Standard Wire Gage, 14–146
British thermal unit, 1–51, 1–58
Brodhun sight box, 5–89
Bromine, 2–02, 6–06
Bronze, 2–03, 2–09, 2–50
 wires, 14–161
Brooding, poultry, 19–13
Brown and Sharpe wire gage, 1–63, 14–144
Brush, carbon, 2–10, 8–12
 contact resistance, 8–12, 8–24
Brush arc machines, 8–21
Brush discharge, 3–25
Brush treatment (of poles), 14–85
BTA motors, 9–63
Btu, 1–51, 1–58
 per kw-hr, 13–03
Bucher nitrogen fixation process, 18–21
Buck arms, 14–102
Bucket conveyor, motor for, 16–29
Building, welding of, 18–50
 wiring of, 14–227 to 14–257
Building insulation, 15–54
Building materials, heat conductivities of, 15–64
Built-in lighting, 15–29
Bulb cooking, 19–26
Bulb temperature (lamps), 15–16
Bulbs and bases, 15–09, 15–10
Bulldozers, motors for, 16–08
Bunched wires, 14–153
Bunsen photometer, 5–89
Burden of instrument transformer, 5–55
Burgess light standard, 5–87
Burglar alarms, 14–256, 19–25
Burnout curve, lamps, 15–17
Burrows method of magnetic testing, 5–74
Burrows permeameter, 5–74
Bus, trolley, 17–27
Bus-bars, 12–56, 12–60, 13–47, 14–10
Bushel, 1–43, 1–44
Bushings, terminal, 10–12, 12–19
 for interior wiring, 14–231
  potential devices, 12–50
Busways, 14–235
Butt contacts, 12–17
Butt treatment (of poles), 14–85
Butt welding, 18–35
Butterfly curve, 4–21
Buttermilk warmer, 19–02
B.W.G., 1–62
B X cable, 14–236
By-pass, 14–04

C

Cable, 14–153 to 14–226
 ACSR, 14–164
 a-c versus d-c resistance, 14–215
 aerial, 14–163
 aluminum, 14–158
 aluminum steel, 14–163
 Anaconda I-beam cable, 14–163
 armored, 14–236
 A.S. and W. type, 14–163
 automobile, 17–98
 capacitance of, 14–38
 car equipment, 17–35
 central station, 12–59
 classes of, 14–170
 color code, automobile, 17–98
 concentric, 14–45
 concentric lay, 14–153, 14–172
 conductivity, 14–143
 copper, 14–156 to 14–163
 covering for, 14–223
 current-carrying capacity, which see
 definition, 14–171
 entrance, 14–140
 flexibility, 14–216
 flexible steel, 17–101
 formulas for, 14–153
 Heddenheimer, 14–163
 H.H. type, 14–163
 high voltage, 14–163
 Hochstadter pressure, 14–217
 hollow copper, 14–163
 impregnated paper cable, 14–193
 inductance, 14–51
 insulated, 14–171 to 14–226 (also see **Insulated wire**)
 ionization free, 14–217
 joints, 14–220
 long span, 14–163
 messenger, 14–165, 14–219
 network, 14–135
 oil filled, 14–217
 oilostatic, 14–217
 paper, 14–194
 P.D.C.P. hollow core, 14–162
 permissible pitch and lay, 14–171
 power factor, 14–186
 pressure cable, 14–217
 properties of, formulas, 14–155
 reactance, 14–52

Cable, reservoirs, 14–223
resistance-temperature variation, 14–160
riser, 14–238
rope lay, 14–153, 14–172
rubber insulated, 14–202
sheaths, 14–175
grounding, 14–261
steel, 14–165, 17–100
submarine, 14–138
supports, 14–175
terminal, 1–67
triplex, 14–172
underground, lightning protection of, 14–141
varnish cambric, 14–202
Cable's length, 1–41
Cadmium, 2–02, 2–51, 6–06
cell, 5–06
line, wave length of, 1–70
Cafeteria, cooking energy for, 15–46
light for, 15–34
Cal, 6–04
Calcium, 2–02, 6–06
manufacture, 18–18
Calcium carbide production, 18–16
Calculus formulas, 1–12
Calender motors, 16–30
Calomel electrode, 6–10
Calorite, welding, 18–47
Cambric tape, 2–22, 14–174
Campbell-Larsen potentiometer, 5–25
Canals, 13–18
Candle, International, 1–37, 1–59, 5–87, 15–02
Candlepower, 5–90, 15–02
measurement and curves, 5–93, 15–05
Candles per sq in., 15–03
Cane crusher, motor for, 16–27
Caoutchouc, 2–03
Capacitance, 1–56, 4–09
line calculations, 14–22
meters, 5–43
of wires and cables, 14–37 to 14–46
susceptance tables, 14–22
tables, 14–37
to neutral, 14–37
Capacitance method of transformer test, 5–59
Capacitor, 1–67, 4–08
Capacitor motor, 9–59
Capacity, electrostatic, 1–65
power (see article on particular device)
Car, electric, 17–27
railway, 17–23, 17–27
lighting for, 17–41
tractive resistance, 17–06
Car barns, 17–69
illumination, 17–70
Car bodies, 17–28
Car dumpers, 16–30
Car loading conveyors, 16–29
Car resistance, electric auto, 17–104
Car type furnace, 18–30
Carat, 1–48
Carbide method, nitrogen fixation, 18–21
Carbolineum, 14–85
Carbon, 2–02, 2–03, 2–30, 6–06, 15–17
arc, 15–17
welding, 18–42
breakers, 12–11
generator, 6–13
materials, 2–10
resistor, 4–08
steel, 2–48
Carbon bisulfide production, 18–17
Carbon dioxide, 2–36, 2–50
breakers, 12–16
Carbon monoxide, 2–50
Carbon-arc welding, 18–42
Carbon-filament lamp, 15–08
Carbon-pile regulators, 12–38
Carburizing, 18–26
Carcel, 1–59
Carding machine, motor for, 16–27
Carpenter's short rule, heat, 15–52
Carrier current relay, 12–24
Carrier current systems, 14–56
Carrying capacity, see **Current-carrying capacity**
Carter coefficient, 8–09
Cartridge fuses, 12–08, 14–239
Cartridge heaters, 18–24
Cascade control, 9–56
Cascade converter, 11–04
Cascade method of photometry, 5–91
Cast grid resistor, 12–10
Cast iron, 2–43
welding, 18–49
Cast steel, 2–43
Cast welding of rails, 17–66
Cast-in-place raceways, 14–236
Cast-lead plate, 7–09
Castner cell, 18–11
Castner process, 18–18
Castor oil, 2–36
Catenary, 1–05, 14–165, 17–51
advantages, 17–46
impedances, 17–59
Catenary suspension, 17–60
cables for, 14–165
Cat-face, 14–83
Cathode, 7–02
Cathode-ray oscillograph, 5–67, 5–69
Cation, 6–03
Caustic alkali production, 18–11
Cedar, for pole, 14–79, 14–87
Ceiling, heat transmission, 15–63
Cell, electric, Section 7
containers, 7–11
lead-acid, 7–08
primary, 7–02
standard, 5–06
Cellophane, 2–15, 2–21
Celluloid, 2–15
Cellulose acetate, 2–15, 2–21
Cement, 2–03, 2–50
dust precipitation, 18–33
Center of gravity, locomotive, 17–24
Centering machine, motor for, 16–21
Centiare, 1–42
Centigrade scale, 1–37, 5–96
Centigram, 1–48, 1–64
Centimeter, 1–41, 1–64
cubic, 1–43, 1–64
dyne, 1–51
gram, 1–51
square, 1–42, 1–65
Central web plate, 7–09
Centralized excitation, 13–52
Centrifugals, motor drive, 16–30
Centrifuge, 2–32
Ceramic furnace, 18–26
Ceresin, 2–15
Ceresine wax, 2–28
Cerium, 2–02, 6–06, 15–18,
manufacture, 18–18
Cesium, 2–02, 2–51
Cgs electric systems, 1–37
Chafing dish, 14–244
Chain, 1–41
Chain winding, 9–05
Change in load, effect of, 14–30

Characteristic (of logarithm), 1–18
Characteristic impedance, 3–48
Charcoal, 2–03
Charge on electron, 1–70
Charge per unit area, 1–53
Charging current, 13–22, 13–47
 calculation of, 14–38
Charging of a storage battery, 7–15, 7–30
Checks, 14–83
Chemical equation, 6–05
Chemical equivalent, 6–03
Chemical industries, motor drive, 16–31
Chemical plants, light for, 15–34
 motors for, 16–14
Chestnut, 14–79, 14–84
Cheval-vapeur, 1–52
Chicken scalding, 19–15
China wood oil, 2–36
Chlorine, 2–02, 2–50, 6–06
 manufacture, 18–11
Choke coils, 12–48
Chromatic aberration, 15–29
Chromel, 2–09
Chromium, 2–02, 2–51, 6–06
 plating, 18–05,
 reflection from, 15–04
 steel, 2–48
 welding, 18–47
Chubb's process, welding, 18–41
Chucking machine, motor for, 16–21
Church lighting, 15–38
Churning, 19–02
Cider making, 19–26
Cigar lighter, 14–244
Circle, 1–05, 1–16
Circle diagram, 9–50
Circuit breaker, 1–66, 12–11 to 12–29
 air, 12–11
 bushings, 12–19
 carbon, 12–11
 compressed gas, 12–15
 de-ion, 12–14, 12–17
 high-speed, 12–13
 magnetic blowout, 12–12
 oil, 12–16
 operating mechanisms, 12–20
 ratings, 12–26
 relays, 12–23
 selection of, 12–25
 tanks, 12–18
Circuits, electric, Section 3
 for light and power, 14–03
Circular inch, 1–42
Circular mil, 1–42, 1–61, 14–172
City car bodies, 17–28
City overhead lines, 14–137
Clamp, suspension, 14–113
Clamp terminals, 12–59
Classes, for poles, 14–79
Classroom, light for, 15–34
 ventilation of, 16–33
Claude ammonia process, 18–21
Clay, 2–03
Cleaning, electrolytic, 18–03
Clearance of overhead conductors, 14–73, 14–95
Cleats, 14–231
Clippers, animal, 19–12
Clock, 14–257
Closed ventilation, 13–22
Cloth, insulating, 2–22
 varnished, 2–16
Cloth factory, light for, 15–34
Club, cooking energy, 15–46
Coal, 2–04
Coal breaker, light for, 15–34
Coal handling motors, 16–29
Coal mines, motors for, 16–28
Coal tower, motors for, 16–29
Coal-tar solution, 14–85
Coasting period, 17–11
Cobalt, 2–02, 2–04, 2–50, 6–06
 steel, 2–48
Coefficient of expansion, 2–56, 14–58
 virtual, 14–58
 wire, 14–161
Coercive force, 2–38, 3–20
Coffee percolator, 14–244
Coil and condenser ignition, 17–76
Coil pitch, 9–04
Coil size, wire, 14–168
Coke, 2–04
Cold light, 15–24
Cold storage, poultry, 19–16
Cold-cathode light, 15–19
Cold-cathode oscillograph, 5–69
Cold-junction compensator, 5–99
Collector current, 17–38
Color and selective radiation, 15–25
Color code, automobile wiring, 17–98
Color equalizing filters, 5–92
Color matching, 15–26
Color temperatures for daylight and artificial light, 15–25
Colored lamps, 15–27
Columbium, 2–02
Combinations, 1–03
Combined d-c and a-c systems, 14–06
Comet, 2–09
Commercial voltages, 14–07
Common curves, equations of, 1–05
Common logarithms, 1–18
Commutating pole converter, 11–04
Commutating pole machine, 8–02
Commutating poles, 8–14, 8–16, 8–38
Commutation, 8–16
Commutator, 8–04, 8–12
 automobile, 17–72
Commutator ampere-hour meters, 5–46
Commutator interrupter, 4–43
Commutator motors, 9–60
Commutator type watthour meter, 5–47
Comparator voltmeter, 5–60
Comparison of wire gages, 1–62
Compensated series motor, 9–62
Compensated wattmeter, 5–32
Compensating winding, 9–61
Compensation method of photometry, 5–91
Compensator, 10–02, 10–21
Complex quantities, 1–06, 3–09
Components of vectors, 3–06
Composite damping constant, 14–25
Composite distribution, 14–137
Composite furnace wall, 18–14
Composition resistors, 4–08
Compound, joint filling, 14–222
Compound lenses, 15–29
Compound motor, 8–31, 16–05
Compound-wound machine, 8–02
Compressed gas breakers, 12–15
Compressed terminal bonds, 17–65, 17–68
Compression joints, 14–115
Compression sleeve, 14–115
Compressors, motors for, 16–08, 16–32
Concatenation control, 9–56
Concealed knob and tube wiring, 14–231
Concentration, 6–04
 cells, 6–12

Concentric cable, 14–45
inductance of, 14–46
Concentric wiring, 14–237
Concentric-lay cable, 14–153, 14–172
Concrete, 2–04, 2–50
conductivity of, 14–272
ducts, 14–120
electrolysis, 14–269
foundations, 14–95
heat conductance, 15–64
mixing, on farms, 19–26
Concussion, 17–06
Condenser, 4–09
rotary, 9–34
thermal, 13–08
Condenser bushing, 10–12, 12–19
Condenser motors, 16–14
Conductance, 1–65
of heat, 15–64
Conduction, electrolytic, 6–18
of heat, 2–52, 15–64
Conductivity, 1–56, 1–65
aluminum, 2–08
carbon, 18–14
copper, 2–06
electrolyte, 6–06
of heat, 15–64
standard, 14–143
wire, 14–143
Conductivity bridge, 5–18
Conductor, 2–05
concentric, 14–45
definition, 14–171
H. H. type, 14–163
I-beam, 14–163
insulation, 14–114
materials, 2–05, 14–57
size of, 14–13
solid, 14–143
splicing, 14–115
stranded, 14–153, 14–159
tables of, 2–09, 14–144 to 14–171
transmission line, 14–57
vibration, 14–63
Conduit, 14–119 to 14–132
automobile, 17–97
bank, 14–119
entrance, 14–124
flexible, 14–232
installation, 14–129
interior, 14–232
machine, 14–127
maintenance, 14–129
metallic, dimensions, 14–233
non-metallic, 17–101
offsets, 14–126
plow, 17–38
rigid, 14–232
rodding, 14–126
service, 14–119
thermal constants, 14–193
wiring, 13–54, 14–126, 14–233
Cone, 1–17
Cone winder, motor for, 16–27
Connections, transformer, 10–07
Connectors for cable joints, 14–221
Conpernik, 2–48
Constantan, 2–09, 2–50
thermocouple, 5–99
Constant-current charging, 7–30
Constant-current distribution, 14–04
Constant-current lamps, 15–13
Constant-current regulators, 12–41
Constant-current transformers, 10–02, 10–06
Constant-current transmission, 14–06
Constant-potential charging, 7–30
Constant-potential system, 14–03
Constant-potential transformer, 10–02
Constant-speed motor, 16–19
Constants, physical, 1–70
Construction (see article on particular device)
light for, 15–34
Contact, 5–12, 12–16
Contact potential, 6–08
Contact rails and wires, 17–43 to 17–63
Contact resistance, 17–68
Contactor, 12–30, 17–33
Contamination of milk, 19–04
Continuous duty, 16–19
Continuous oven, 18–25
Continuous rating, 9–38
Continuous spectra, 15–23
Continuous-current apparatus, 8–02
Contraction of cable, 14–217
Control, 12–02
a-c railway motor, 17–37
apparatus, 12–29
d-c railway motor, 17–36
industrial locomotive, 17–36
locomotive, 17–25, 17–36
machine-tool drive, 16–21
motors, 16–20
operation of circuit breaker, 12–22
panels, 12–34
railway motor, 17–29
rheostats, 12–34
ship, 17–114
slip-ring motor, 16–12
switchboards, 12–52
switches, 12–06
systems for railway motors, 17–29
three-phase railway motors, 17–37
voltage, 14–09, 17–85
wiring, 13–55
Control-bus battery, 7–22
Controller, 12–29, 17–33, 17–105
drum, 12–33, 17–30
for electric automobiles, 17–105
period, 17–11
Convection electric heater, 15–56
Convection of heat, 2–52
Convector, 15–54
Conversion factors, 1–41
Converter, 11–02
phase, 11–17
substation, 13–40
Conveyor type furnace, 18–30
Conveyors, motors for, 16–29
Cooking, bulb, 19–26
electric, 15–44 to 15–46
Cooling, air-conditioning, 15–61
conduit lines, 14–131
internal combustion engine, 13–32
milk, 19–04
resistance standard, 5–10
transformer, 10–14
Cooper-Hewitt mercury-vapor lamp, 15–19
Coordinated insulation, 14–110
Coordinated transpositions, 14–116
Copal, 2–28
Copel, 2–09, 18–22
Copper, 2–02, 2–05, 2–07, 2–50, 6–06
alloy wires, 14–153, 14–161
cables, 14–156 to 14–163 (see also **Cables**)
coefficient of linear expansion, 14–58
hard-drawn, 2–05, 14–168
hydrometallurgy, 18–09

Copper, loss (see article on particular device)
mechanical characteristics, 14–58
modulus of elasticity, 14–58
plating, 18–04
refinery electrolytic, 18–07
relative weights of systems, 14–07
sheets, 1–61
standard, 14–143
tube, electrolytic, 18–06
ultimate strength, 14–58
weight, 14–58, 14–226
wire, 14–167 to 14–171
tables, 14–144 to 14–148
yield point, 14–58
Copper-clad steel, 14–152
Copper-clad wire tables, 14–152
Copper-graphite, 2–10
Copper-nickel heating units, 18–22
Copper-oxide rectifier meter, 5–30
Cord, 14–172
Cord (of wood), 1–43
Core, armature, 8–10
carbon, 15–17
Core baker, 18–26
Core loss, 2–38, 3–21 (also see article on particular device)
measurement, 5–77
Core-type transformer, 10–03
Cork, 2–04, 2–50
Corn, seed, testing of, 19–10
Corn husking and shredding, 19–06
Corn shelling, 19–06
Corona, 2–08, 3–25, 12–59
leakage conductance due to, 14–19
limit of high-voltage lines, 14–35
on transmission lines, 14–32, 14–163
Corona voltmeter, 5–67, 14–33
Corrosion of lead sheath, 14–131, 14–267
Corrugating rolls, motor for, 16–21
Cos, 1–07, 1–20
Cosecant, 1–07, 1–64
Cosh, 1–10, 1–25
Cosine, 1–07, 1–20, 1–64
Cosine law, 15–03
Cost, relative, of cooking, 15–46
of heating, 15–55
of light, 15–16
of power station, 13–12, 13–28, 13–33
Cot (cotangent), 1–07
Coth, 1–10, 1–25
Cotton, 2–20, 2–50
braid, 14–174
Cotton insulation, 14–174
Cottrell precipitation process, 18–31
Couillie rail bond, 17–66
Coulomb, 1–37, 1–52
Counter emf, 8–32
Counterpoise ground, 14–261
Coupler, train line, 17–33
Covered electrode, 18–44
Covering, of cables, 14–223
on overhead wires, 14–136
Cow, light for, 19–19
Crackage, 15–51
Cranes, motors for, 16–08, 16–11, 16–29
Cream separators, 19–03
Creosote, 14–85
Crest factor, 3–03
Crest gates, 13–17
Crime, 15–39
Critical damping, 5–03
Critical voltage, 14–33
Crook, 14–83
Cross incline, 17–43
Cross-arms, 14–88, 14–102
wood for, 14–33
Cross-magnetization, 8–13
Cross magnetizing force, 9–12
Cross-rail motors, 16–22
Crossing of waterway, 14–138
Crucible steel cable, 14–165
Crude petroleum, 14–85
Cruising economy, 17–117
Cruising turbines for ships, 17–112
Crushers, motors for, 16–08
Csc, 1–07
Csch, 1–10, 1–25
Cubic equation, 1–03
Cubic measure, 1–43
Cubical expansion, coefficients of, 2–58
Cumulative compound motor, 16–05
Cupron, 2–09
Cup-shakes, 14–83
Curling iron, 14–244
Current, 1–53, 1–65
alternating, 3–02
collectors, 17–38
density, 1–53, 12–59
sensitivity of galvanometer, 5–03
transformers (see **Instrument transformers**)
Current-carrying capacity, 41–166, 14–192, 14–215
a-c versus d-c, 14–194
aerial cable, 14–200, 14–210
bare copper wire, 14–166
cables buried in earth, 14–214
impregnated-paper insulated cable, 14–193
insulated wires, 14–192
interior conductors, 14–214
paper cables, 14–193
rubber-insulated cable, 14–213
sheath losses, 14–215
shielded cables, 14–197, 14–203
underground cable, 14–194, 14–203, 14–213
variable load, 14–215
varnished-cambric insulated cable, 14–202
Current-limiting reactors, 4–24, 12–48
Current-limiting relay, 17–33
Current-limiting resistors, 12–11
Current-reducing devices, 16–06
Current-time curve, 17–12
Curves, and grades, 17–07
equation of, 1–05
railway, 17–60
Customer's loop, 14–133
Cut-out, 14–04
battery, 17–84
box, 14–241
Cutter, motor for, 16–27
Cyanamide process, 18–21
Cyanide furnace, 18–26
Cycle, 3–02
Cycle of ignition system, 17–81
Cycled batteries, 7–24
Cycloid, 1–16
Cylinder, 1–17
Cylinder control system, 17–29

D

Dairy farming, 19–02
bottle washers, 19–02
buttermilk warmer, 19–02
churning, 19–02
cottage-cheese machine, 19–02
cream separators, 19–03
light for, 15–34
milk pasteurization, 19–04
milking machine, 19–03

Dairy farming, refrigeration, 19–04
sterilization, 19–05
water heating, 19–05
Dam, hydroelectric station, 13–17
Dammar, 2–28
Damping, mechanical, 5–03
Damping constant, transmission line, 14–25
Damping copper, 11–09
Daniell cell, 6–11
Day, 1–37, 1–45
Daylight, artificial, 15–26
Daylight lamps, 15–13
Daylight light, 15–25
D-c (see **Direct current**)
Dead oil of coal tar, 14–85
Dead-load method, 8–24
Dead-end insulator, 14–110
Dead-man, 14–92
Decade bridge, 5–13
Decade resistance box, 5–11
Decay of timber, 14–85
Decigram, 1–48
Decimal wire gage, 1–59
Decimeter, 1–41
Decomposition, electrolytic, 6–13
Decrement, logarithmic, 5–03
Deflection and side swing, 14–66
Deflection type potentiometer, 5–24
Deflection-temperature curve, 14–68
Deformation factor, 3–03
Degree (angle), 1–45
Dehumidifier, 15–61
Dehydration of food, 19–21
De-ion breakers, 12–14
De-ion fuses, 12–06
De-ion grids, 12–17
Dekagram, 1–48
Dekameter, 1–41
Delco-Remy system, 17–86
Delivery, city, 17–102
Delta connection, 3–31
Demagnetization, 8–13
Demand factors, 14–247
form, 19–02
Demand meter, 1–68, 5–50
Demand system of charging, 5–50
Density, 2–03, 2–06
Depolarizer, 6–08, 7–02
Depreciation factor, of lamp, 15–34
Derivatives, 1–12
Design (see article on particular device)
Desks, 12–51
Detarring, gas, 18–33
Determinants, 1–04
Detinning, electrolytic, 18–09
Detonation, 17–72
Diamagnetism, 2–38
Dielectric, 4–09
admittance per mile, 14–26
constant, 1–65, 5–84
of insulating materials, 5–84
flux, 1–52, 1–65
density, 1–53, 1–65
losses, 2–15, 5–84
of insulating materials, 5–84
measurement, 5–81 to 5–86
merits of, 4–11
strength, 2–12, 2–15, 5–81
of insulating materials, 5–81
Diesel engines, 13–29
Diesel-electric drive, for ships, 17–120
Dietzel process, 18–08
Difference of potential, electric, 1–65
Differential compound motor, 16–05
Differential equation, 1–13
Differential motor, 8–31
Diffusion, 15–23
Dilute solution, 6–17
Dilution, 6–04
Dimensions (see article on particular device)
Dining room, light for, 15–38
ventilation, 16–33
Dipping and plating plants, motors for, 16–14
Direct axis synchronous reactance, 3–45
Direct control switchgear, 12–52, 12–60
Direct current, 1–64
bus bar, 12–56
compared with alternating, 14–03
distribution system, 14–05
in buildings, 14–227
galvanometer, 5–02
generator, 8–02 (see also **Generator, d-c**)
lines, design of, 14–12
measurement, 5–23, 5–63
motor, 8–30 (see also **Motor d-c**)
operation of a-c commutator motor, 17–37
potentiometer, 5–19
protection, 12–42
solenoid, 4–33
switchboard, 12–55
trolley circuit, 17–52
Direct lighting, 15–28
Direct radiator, 15–55
Direct suspension, 17–60
Direct-drive Diesel engine, 17–120
Direction of rotation (trigonometric), 1–06
Directional relay, 12–24
Dirt road, surface, 17–104
Disc, heater, 14–244
Disconnecting switches, 12–05, 12–60
Discrimination, color, 15–26
Dishwasher, 14–244
Disk, insulator, 14–108
Dispatching of trains, 17–69
Dispersion, 1–15
Disruptive critical voltage, 14–34
Dissociation, electrolytic, 6–17
Distance between substations, 13–40
Distance-time curve, railway, 17–11
Distant control switchgear, 12–60
Distributed capacitance, 4–18
Distributed core type transformer, 10–03
Distributing substation, 13–37
Distribution curve, light, 15–05
Distribution loop, 14–133
Distribution systems, Section 14
a-c, 14–05, 14–16
automobile, 17–72
building, 14–227
circuits, 14–132
conduit, 14–119
d-c, 14–05, 14–12
design of, 14–12
efficiency, 14–08
loop, 14–133
losses, 14–08
network, 14–133
overhead, 14–136
primary, 14–132
radial, 14–132
railway, 17–55
rural, 14–55
secondary, 14–134
sectionalizing, 14–141
substations, 13–37
underground, 14–142
Distributor box, 14–119
Diversity factor, 14–138

Door opener, 14–257
Dope, 17–73
Dote, 14–83
Double arms, 14–89, 14–102
Double bridge, Kelvin, 5–16
Double bus, 12–57, 13–48
Double squirrel-cage motor, 16–07
Double-acting engine, 13–30
Double-secondary transformer, 5–56
Double-truck bogie locomotive, 17–24
Double-truck car, 17–28
Double-wire automobile wiring, 17–97
Douglas fir, 14–82, 14–87
Drachm, 1–44, 1–48, 1–49
Drafting room lighting, 15–34, 15–38
Drag conveyor, motor for, 16–29
Drainage bond, 14–272
Drainage, farm, 19–19
  of manholes, 14–125
Draw-bar pull, 17–06
Drawbridge batteries, 7–23
Draw-off schedule, 15–48
Dredges, 17–122
  hopper, Diesel electric control, 17–122
  motor drive, 16–30
Drift of instruments, zero, 5–26
Drilling, 2–22
Drilling machine, motor for, 16–21
Drip-proof motor, 16–14
Drive, electric, 16–02
  automobile, 17–104
  group, 16–02
  individual, 16–02
Drop hammer, motor for, 16–21
Drop wire, 14–140
Drum controllers, 12–33, 17–30
Drum starters, 12–33
Dry arcing, 14–106
Dry air filter, 15–62
Dry battery tests, 7–04
Dry cells, 7–02
Dry flashover, 14–106
Dry measure, 1–44
Dry rot, 14–82
Dry-process porcelain, 2–20
Drying, of fruit, 19–21
  of grain, 19–21
  of paint, 18–26
  of vegetables, 19–21
  tumbler, motor for, 16–30
Drysdale potentiometer, 5–25
Duchemin formula, wind pressure, 14–61
Duck, 2–22
Duct (see Conduit)
Ductility of welds, 18–46
Dumpers, car, 16–30
Duplex cable, 14–172
Duplex reentrant cable, 8–07
Duplex spot welder, 18–37
Duplicate service, 14–133
Duprene, 2–26
Duration of head, 13–14
Dust precipitation, 18–31
Dwelling, 14–247
Dye houses, motors for, 16–14
Dynamotors, 11–02
  as balancer, 14–11
Dyne, 1–49
  centimeter, 1–50
Dysprosium, 2–02

E

ε (base of natural logs), 1–18
Earth current, 14–268, 17–56
Earth resistance, 14–268
  unit, 14–266
Earthing, see Grounding
Easement, 14–78
Ebonite, 2–04, 2–50, 2–55
Eddy current, 2–39, 3–21
Edger, motor for, 16–32
Edgewound resistor, 12–10
Edison Lalande cell, 7–02
Edison storage battery, 7–24, 17–105
Edison three-wire system, 14–04
Edison wire gage, 1–62
Effective value, 1–65, 3–03
Efficiency, 1–36, 1–64 (also see article on particular device)
Egg storage, 19–16
Einthoven string galvanometer, 5–71
Elastic limit of timber, 14–87
  of wire, 14–161, 14–167
Electric arc, 3–24
  temperature of, 18–12
Electric automobile, 17–101
Electric brake, 17–40
Electric car, 17–27
Electric circuit, 3–02
Electric conductivity, 1–56
Electric corona, 2–08, 3–25
Electric drive, 16–02
  vs. gear drive, 17–115
Electric furnace, 18–12 to 18–30
Electric gearing, 17–111
Electric heating, 15–54, 18–22
Electric hot-water radiator, 15–57
Electric ignition, 17–72 to 17–84
Electric lines, 3–02
Electric locomotive, 17–24
  acceleration and braking, 17–07
Electric manufacturing, light for, 15–31
Electric potential, 1–54, 1–65
Electric power station, Section 13
Electric propulsion of ships, 17–108
  advantages, 17–115
Electric railway, 17–02
Electric resistance, 1–55
Electric resistivity, 1–55
Electric shock, 14–258
Electric steam radiator, 15–57
Electric traction, 17–02
Electric tire, 17–106
Electric truck, 17–102
Electric units, 1–37
Electric welding, 18–34 to 18–52
Electrical conductance, 1–55
Electrical equivalent of heat, 1–70
Electrical indicating instruments, 5–27
Electrical integrating meters, 5–45
Electrical length, 3–48
Electrical machines, 3–41 (also see Generators, Motors, Converters)
Electrical network, 3–16
Electrical precipitation, 18–31
Electrical properties of insulating materials, 2–15
Electrical recording instruments, 5–95
Electrical remote-control boards, 12–52
Electrical tension, 1–65
Electrical units, 1–37, 1–70
Electrically operated switchboards, 12–56
Electric-furnace products, 18–16
Electrified steam roads, 17–04
Electrochemical equivalent, 6–03, 6–06
  of silver, 1–70
Electrochemical processes, Section 18
Electrochemistry, Section 6
Electrocuters for insects, 19–19, 19–27

Electrode, 6–03, 6–10, 7–02
boiler, 15–57
dielectric tests, 5–82
furnace, 18–14
Haber, 14–270
welding, 18–44
Electrodynamic instruments, 5–26, 5–28
Electrodynamic wattmeters, 5–31
Electroforming, 18–06
Electro-graphite brushes, 2–10
Electrolysis, chemical, 18–10
of underground structures, 14–267
of water, 18–11
Electrolyte, 6–03, 7–02, 7–10
bridge for measurement of, 5–18
Electrolytic ampere-hour meters, 5–45
Electrolytic arresters, 12–46
Electrolytic bleach, 18–10
Electrolytic cells, 6–07
Electrolytic condensers, 4–14
Electrolytic conduction, 6–18
Electrolytic decomposition, 6–13
Electrolytic detinning, 18–09
Electrolytic dissociation, 6–17
Electrolytic extraction of metals, 18–09
Electrolytic oxidation and reduction, 18–10
Electrolytic refining, 18–06
Electrolytic zone, 14–269
Electromagnet, 4–29
Electromagnetic system of units, 1–37
Electromagnetic track brakes, 17–40
Electromechanical period, 9–33
Electrometer, 5–61
quadrant, as wattmeter, 5–62
for voltage measurements, 5–62
string, 5–63
Electromotive force, 1–54, 1–64
Electron, mass of, 1–70
Electronic charges, 1–70
Electronic regulators, 12–38
Electronic tornado welding, 18–42
Electronic voltmeter, 5–30
Electroplating, 18–02
Electro-pneumatic brakes, 17–40
Electrostatic capacity, 1–65
Electrostatic flux, 1–65
density, 1–65
Electrostatic generators, 8–28
Electrostatic glow meters, 12–51
Electrostatic ground detectors, 12–51
Electrostatic system of units, 137
Electrostatic voltmeter, 5–30, 5–61
Electrostatic wattmeter, 5–63
Electrothermic voltmeter, 5–30
Electrotyping, 18–06
Element, 6–02, 6–10
Elevated car, 17–21, 17–28
Elevator, light for, 15–34
motor for, 16–08, 16–09, 16–24
Elevator-type furnace, 18–30
Ellipse, 1–05, 1–16
Ellipsoid, 1–17
Ellipsoidal reflectors, 15–30
Elmore process, 18–06
Emergency battery, 7–21
Emergent stem correction, 5–97
Emissivity, monochromatic, 5–103
Emissivity corrections of pyrometers, 5–103
Empire cloth, 2–15, 2–21
Empty cell process, 14–86
Enamel, furnace, 18–26
insulation, 14–174
reflection from, 15–04
wire, 2–33
Encased knot, 14–83
Enclosed air-jacketed motors, 16–14
Enclosed bus structure, 12–62
Enclosed fuses, 12–06
Enclosures, 13–53
End coils, 10–12
End connection leakage, 9–12
End incline, 17–43
Endosmose, 6–25
Endothermic reactions, 6–04
Enduro, welding, 18–47
Energy, 1–51, 1–65
consumption (see article on particular device)
converters, 11–02
dissipation, 4–02
distance, 14–33
magnetic and electric, 14–26
measurement of, 5–26
transfer constant, 14–25
Engineering terms, abbreviations, 1–64
Engines, internal combustion, 13–29
English candle, 1–59
English legal standard gage, 1–62
Engraving, light for, 15–34
Ensilage cutting, 19–06
fans, 19–07
Entrance cable, 14–140, 14–238
Epply potentiometer, 5–23
Epstein apparatus, 5–7
Equalizer connection, 8–26, 11–09
Equation, 1–64
algebraic, 1–02
calculus, 1–12
common curves, 1–05
complex, 1–07
differential, 1–13
exponential, 1–09
hyperbolic, 1–09
transmission line, 14–24
trigonometric, 1–07
wire span, 14–66
Equilateral hyperbola, 1–05
Equipment, ground, 14–242
switchboard instrument, 12–49
switching, 12–02
Equipment chart, 13–25
Equivalent circuit of a transformer, 10–06
Equivalent conductance, 6–04
Equivalent conductivity, 6–18
Equivalent grade, average, 17–12
Equivalent impedance, 3–12
Equivalent transducers, 3–14
Equivalent weight, 6–03
Erbium, 2–02
Erg, 1–51
Erlang potentiometer, 5–25
Erosion, water velocity to produce, 13–18
Error, of instrument, 5–27
of observation, 1–14
Ethyl gasoline, ignition of, 17–73
Europium, 2–02
Evershed megger, 5–42
Excitation, 8–08, 13–52
of hydroelectric generator, 13–26
Excitation loss, 8–18, 9–14
Exciter bus, 12–56
Exciting current, 10–18
Exhaust fan, motor for, 16–32
Exhibitive lighting, 15–38
Exide Ironclad plate, 7–10, 17–106
Exothermic reactions, 6–04
Expanded terminal bonds, 17–65
Expansion, coefficient of, aluminum, 2–07
of wire, 14–161

Expansion joints, 13–53
Expansion of cable, 14–217
Expansion shim, 17–50
Expansion voltmeter, 5–30
Explosion engine, 13–29
Explosion-tested control system, 17–34
Exponential formulas, 1–09
Exponential functions, 1–25
Exposure, 14–116
Express paper, 2–21, 2–22
Expulsion fuses, 12–06
Extended source of light, 15–03
Extraction of metals, electrolytic, 18–09
Eye, human, 15–21

F

F and F pyrometer, 5–103
Fabrics, insulating, 2–22
Face-plate starters, 12–31
Factice, 2–25
Factor of assurance, 14–172
Factor, conversion, 1–41
Factor, diversity, 14–138
Factorial, 1–02
Factory, lighting, 15–35
  ventilation, 15–66
Fahrenheit scale, 1–37, 5–96
Fahy permeameter, 5–75, 5–77
Fan, motor for, 16–32
Farad, 1–56
Faraday, 1–52, 6–03
  constant, 1–70
Faraday's laws, 6–05
Farm, electricity on, Section 19
  lighting, 19–18
  power supply, 19–02
Fathom, 1–41
Fault, insulator, 14–107
Fault location, 14–223
Fechner's law, 15–22
Feed, drying, 19–22
  grinding, 19–08
  mixing, 19–10
Feeder, 14–132
  herringbone, 14–132
  motor load, 14–242
  neutral, 14–09
  protection, 12–42
  tree, 14–132
  voltage regulator, 12–42
Feedwater heater, 13–04
Feet, 1–41
  cubic, 1–43
  square, 1–42
Ferraris theorem, 3–43
Ferric induction, 2–39
Ferro-alloy, 18–20
Ferromagnetism, 2–38
Ferrosilicon production, 18–20
Ferrous alloy, welding, 18–47
Ferrous core inductor, 4–18
Féry mirror-type pyrometer, 5–102
Feussner potentiometer, 5–23
Fiber, 2–15, 2–22
Fiber ducts, 14–120
Fiber electrometer, 5–63
Fiber stress, 14–87
Fibrous organic insulators, 2–20
Field break-up switch, 9–33
Field coils, 8–04, 9–20
Field discharge attachment breaker, 12–12
Field discharge resistors, 12–10
Field excitation, 9–29
Field forcing, 12–39
Field pole, 8–09, 9–10
Field rheostats, 12–34
Field slot factor, 9–20
Field speed control, 8–38
Field switch, 12–04
Field winding, 9–07
Field yoke, 9–10
Figure 8 wire, 14–143, 14–170
Filament diameters, lamp, 15–10
Filament materials, lamp, 15–08
Filament temperature, lamp, 15–15
Filler, 14–174
  rod, 18–42
  wire, 14–115
Filter, air-conditioning, 15–62
Finger contacts, 12–17
Finishing rate, 7–15
Fir (for poles), 14–82
Fire alarms, 14–257, 19–25
Fire hazard, 14–259
Fire protection, of farms, 19–21
  of hydroelectric generator, 13–22
Firefly, light from, 15–24
Fire-proof barrier, 12–61
Firesand, 18–17
Fish paper, 2–20, 2–22
Fisher-Hinnen method, 3–07
Fishtail burner, 15–53
Fittings, 12–59
Five-wire system, 14–06
Fixation of nitrogen, 18–21
Fixture wire, 14–240
Flame light standard, 5–87
Flame tint lamp, 15–27
Flange friction, 17–06
Flanges, 13–09
Flash point of oils, 2–59
Flash welding, 18–36
Flashboards, 13–26
Flashlight tests, 7–04
Flashover, insulator, 14–106 to 14–111
Flatiron, 14–244
Flexibility of cable, 14–216
Flexible tower, 14–94
Flicker photometer, 5–91
Flint, 2–04
Floated working batteries, 7–22
Floating battery, 7–16
Floodlight, lamps, 5–94, 15–14
Floodlighting, 15–31
Floor, heat transmission, 15–63
Floor warming, 15–60
Flour and feed pumps, motors for, 16–14
Flour mill, motor for, 16–14
Flow duration curves, 13–14
Fluid-dielectric condensers, 4–14
Flumes, 13–18
Fluorine, 2–02, 6–06
Flux, luminous, 15–02
  per pole, 8–07
Flux density, electrostatic, 1–53
  magnetic, 1–65
Fluxmeters, 5–43
Fly ash removal, 18–33
Flywheel effect of hydroelectric generator, 13–22
Flywheel on motor, 16–08
Focusing of arc, 18–44
Foil, metallic, electrolytic, 18–06
Foot, 1–41, 1–64
  cubic, 1–43, 1–64
  pound, 1–51, 1–64

Foot, square, 1–42, 1–65
Foot-candle, 15–02
  meter, 5–91
Foot-lambert, 15–03
Footlights, 15–38
Forage grinding, 19–09
Force, unit of, 1–36, 1–49
Forced oil-cooled transformer, 10–04
Forebay, 13–17
Forge shop, light for, 15–34
Forging furnace, 18–26
Form factor, 3–03
Forming of plates, 7–09
Formulas, algebraic, 1–02
  approximation, 1–15
  cables, 14–155
  calculus, 1–12
  conversion factors, 1–41
  exponential, 1–09
  hyperbolic, 1–09
  mensuration, 1–16
  trigonometric, 1–07
Foucault current, 3–21
Foundation, tower, 14–95
Foundation brake, 17–39
Foundries, light for, 15–34
  motors for, 16–14
Foundry welding, 18–49
Fourier series, 3–06
Four cycle engine, 13–29
Four-ring converter, 11–05
Four-wire three-phase system, 14–06
Four-wire two-phase system, 14–05
Fox farming, electric equipment, 19–12
Frazil ice, 13–27
Free arcing, 14–106
Freezing point, 2–50
Freight car, 17–07
Frequency, 1–65, 3–02, 9–02, 14–06
  commercial, 14–06
  control, 9–56
  converters, 11–13
  meter, 1–68, 5–37
  setter, 9–02
Fresnel lenses, 15–29
Frick furnace, 18–20
Friction cloth, 2–22
Friction loss (see article on particular device)
Frictional machine, 8–28
Frictional resistance, 17–39
Frog distance on turnout, 17–48
Frozen bearings, resistance of, 17–07
Fruit drying, 19–23
Fruit treatment, electric, 19–18, 19–26
Frustum, volume of, 1–17
Fuel, 13–07
Full cell process, 14–86
Full pitch, 9–04
Fullerboard, 2–21, 2–22
Fundamental, 3–07
Fungi, 14–85
Furfural-formaldehyde, 2–27
Furlong, 1–41
Furnace, electric, 18–12 to 18–30
Fuse, 1–67, 12–06, 14–239
  automobile, 17–92
  cartridge type, 12–08, 14–240
  motor, 14–252
  plug, 14–239
Fused electrolytes, 6–07
Fusion, heat of, 2–50
Fusion welding, 18–42
Fynn-Weichsel motor, 9–64, 16–08, 16–15

G

Gadolinium, 2–02
Gages, 1–59
  wire, 1–61
Gaining, 14–88
Gall potentiometer, 5–25
Gallium, 2–02
Gallon, 1–43, 1–44
Galvanized sheet gage, 1–60
Galvanometer, 5–02 to 5–06, 5–43
  best resistance of, 5–15
Galvanometer shunts, 5–04
Galvanoplasty, 18–06
Game-bird farm equipment, 19–16
Gang-operated switch, 12–06
Gangsaw, motor for, 16–32
Gap arresters, 12–46
Gap, spark, 5–63
Garage, 14–247
  light for, 15–34
Gas, 2–36, 2–54
  cleaning, electrical, 18–33
Gas lamps, 15–08
  hot-cathode, 15–19
Gas plants, motors for, 16–14
Gas protected welding, 18–45
Gas turbine, 13–29
Gaseous conductor lamps, 15–18 to 15–21
Gasoline engines, 13–29, 17–119
Gate capacity, 13–19
Gear cutter, motor for, 16–21
Gear drive, 16–16, 17–104
Gear for railway motor, 17–08
Geared turbine marine drive, 17–111
Gearing, 17–111
Gelatin, colored, 15–26
General Electric frequency meter, 5–38
General Electric multiple unit control, 17–32
General Electric synchroscope, 5–40
General lighting, 15–28
General purpose motor, 16–04
Generating room apparatus, 13–11
Generating voltmeter, 5–63
Generator, 1–62
  a-c (see below)
  automobile, 17–84
  d-c (see below)
  end of line, 14–12
  hydroelectric, 13–21
  marine, 17–114
  mounting for automobile, 17–88
  protection, 12–42
  series arc light, 8–21
  train lighting, 17–41
  turbine, 9–16, 17–114
  ventilation, 9–17, 13–22
  waterwheel, 13–21
Generators, a-c, Section 9
    classification, 9–02
    examples of design, 9–25
    frequency, 9–03
    hunting, 9–27
    induction, 9–02
    inductor type, 9–02
    operation, 9–25
    performance, 9–25
    salient-pole, design of, 9–04
      predetermination of performance, 9–08
    single-phase, 9–15
    specifications, 9–27
    starting, 9–26
    synchronous, 9–02
    tests of, 9–21

Generators, a-c, turbo, 9–16
ventilation, 9–17
d-c, Section 8
armature reaction, 8–13
armature resistance, 8–11
armature winding, 8–05
brush, 8–12
commutating pole, 8–02, 8–14
commutator, 8–12, 8–16
compound wound, 8–02
data on, 8–28
definitions, 8–02
electrostatic, 8–28
examples of design, 8–28
excitation, 8–08
heating, 8–19
home-polar, 8–21
installation, 8–26
interpole, 8–02
losses, 8–18
magnetic circuit, 8–08
parallel operation, 8–26
parts of, 8–03
rating, 8–03
regulation, 8–17
series, 8–02
field, 8–15
series arc-light, 8–21
shunt, 8–02
field, 8–10
special machines, 8–21
specifications, 8–25
speed, 8–27
stability, 8–14
testing, 8–23
three-wire, 8–22
weight, 8–27
Geometrical progression, 1–02
Geometrical representation of complex numbers, 1–06
Georgia pine, 14–79
German candle, 1–59
German silver, 2–04, 2–09, 2–50
Germanium, 2–02
Germination of vegetable, 19–25
Germinator, electrical, 19–07
Geyger potentiometer, 5–25
Gibbs-Helmholtz equation, 6–11
Gilbert, 1–57
Gill, 1–43
Gilsonite, 2–29
Gin pole, 14–91
Girod steel furnace, 18–19
Glare, 15–22, 15–41
reflected, 15–23
Glass, 2–04, 2–15, 2–19, 2–50
colored, 15–25
condenser, 4–14
heat transmission, 15–63
insulation, 14–105
light transmission, 15–05
Glass factories, motors for, 16–14
Glazing of porcelain, 14–107
Globar furnace, 18–16
Globar resistor, 4–08
Globar units, 18–27
Globes, 15–27
asymmetric refractors, 15–28
prismatic, 15–28
refractors, 15–28
white glass, 15–28
Glowing radiant brooder, 19–15
Gluepot, 14–244
Glyptal, 2–27
Gold, 2–02, 2–04, 2–50, 6–06
plating, 18–05
refining, 18–08
Gothic church lighting, 15–38
Governor, 13–21
Grade, average equivalent, 17–12
maximum train can ascend, 17–09
of construction (transmission line), 14–56
Graded wall, 18–14
Grades and curves, 17–07
Grading of fruit, 19–26
Grading rings, 14–110
Grain, 1–48, 1–49
Grain and feed treatment, 19–06 to 19–11
Grain drying, 19–22
Grain elevators, motors for, 16–14
Gram, 1–48, 1–49
atom, 6–03
calorie, 1–51, 1–58
centimeter, 1–50
ion, 6–03
molecule, 6–03
Granite, 2–04
Granite block surface, 17–104
Granular carbon furnace, 18–16
Grape drying, 19–23
Graphical symbols, 1–69, 14–246
Graphite, 2–10, 18–17
Grassot fluxmeter, 5–43
Gravel, 2–04
Gravity, acceleration of, 1–70
Green feed cutting, 19–10
Grid, de-ion, 12–17
Grid resistor, 4–03
Grill, 14–244
Grinding, bone and shell, 19–26
Grinding machine motors, 16–22
Grist mills, 19–11
Groomers, animal, 19–11
Grooved trolley wire, 14–143
Ground, 14–266
connections, types, 14–262
detector, 1–68, 12–51
equipment, 14–242
interior, 14–242
location of, 14–224
mass, 14–261
pipes, 14–265
poles and rods, 14–262
protection, from relays, 12–24
resistance of, 14–263
suppressors, arcing, 12–48
system, 14–242, 14–260
wires, 12–45, 14–56, 14–141
Grounded wiring, automobile, 17–97
Grounded neutral, 10–08
Grounding, 9–25, 13–55, 14–258 to 14–267
automobile, 17–98
of the neutral, 3–56
practice, 14–260
station, 13–55
Group bus, 13–49
Group drive vs. individual, 16–02
Grub remover, 19–12
Guard rings, 5–84
Guard wire, Price, 5–24
Guild flicker photometer, 5–92
Gums, 2–28
Gun bronze, 2–09
Gust velocity of wind, 14–61
Gutta percha, 2–04, 2–26, 2–51
weight of, 14–226
Guy, 14–92
Guy anchors, 14–92

Guy insulator, 14–90
Guy stub, 14–93
Guy wire, 14–92
Gymnasium, ventilation, 16–33
Gypsum, 2–04

H

H H cable, 14–163
Haber electrode, 14–270
Haber process for ammonia, 18–21
Hafnium, 2–02
Hair curler, 15–44
Hall and Heroult process, 18–17
Halowax, 2–28, 4–12
Hammer, drop, motor for, 16–21
Hammer interrupter, 4–43
Hand, 1–41
Hand brakes, 17–39
Hangar floodlights, 5–94
Harbor dredge, motor for, 16–30
Harcourt light standard, 5–87
Hard fiber, 2–22
Hard-drawn copper, 2–05, 14–168
  cables, 14–162
Harding's rule, heat, 15–52
Hardware, for insulators, 14–112
  for wood poles, 14–89
Hargreaves-Bird cell, 18–11
Harmonics, 3–06
  analysis, 3–07
Harp, 17–38
Harrington joint, 14–120
Harris electrodynamic voltmeter, 5–26
Hatching of ladybirds, 19–27
Hay handling, 19–09
Head (of water), 13–14
Head end lighting system, 17–42
Head on sight boxes, 5–88
Headsaw, motor for, 16–32
Headlamps, 17–90
Headlight photometry, 5–94
Headlighting, asymmetric, 15–32
    automobile, 15–32
    polarized light, 15–33
    prefocused bulb, 15–33
    two beam, 15–32
Headwater elevation, 13–14
Heart-shakes, 14–82
Heartwood, 14–82
Heat, 1–51
  balance, 13–03, 13–31
  conductivity, 15–64, 15–65
  electric, for animals, 19–12
  equivalent of, 1–70
  from lamps, 15–53
  from motors, 15–53
  insulation, 2–54
  losses from pipe, 15–49
  of fusion, 2–50
  of reaction, 6–11
  of vaporization, 2–51
  pump, 15–59
  resistance to, cable, 14–194
  transfer of, 2–52
  transmission coefficients, 15–63
Heat run, 8–24, 9–24
Heaters, electric, 15–56, 18–22
  portable, 15–44
  storage, thermal, 15–57
  units, 18–24
Heating (see article on particular device)
  appliances, industrial, 18–24
  household, 15–44
Heating, motors, 16–31
  of buildings, 15–50 to 15–60
  pad, 14–244
Heaviside-Lorentz system of units, 1–37
Hectare, 1–42
Hectogram, 1–48
Hectometer, 1–41
Heddenheimer cable, 14–163
Hedgehog, 14–127
Hefner candle, 1–59
Hefner light standard, 5–87, 15–02
Hefner units, 15–02
Height of lighting units, 15–37, 15–41
Helium, 2–02, 15–26
Hemisphere, 1–45
Hemp, 14–226
Henry, 1–56, 1–64
Hering method of current measurement, 14–271
Heroult furnace, 18–15, 18–19
Herringbone distribution, 14–132
Heterochromatic photometry, 5–91
Heyland circle diagram, 9–50
High and low bus system, 14–10
High-efficiency tire, 17–106
High-frequency furnace, 18–16
High-head hydroelectric stations, 13–15
High-induction materials, 2–49
High-intensity mercury lamp, 15–19
High-pressure furnace, 18–15
High-resistance fault, 14–31
High-speed breakers, 12–13
High-speed press, motor for, 16–23
High-speed relays, 12–24, 12–45
High-temperature furnace, 18–26
High-tension ignition, 17–76
High-tension magneto, 17–77
High-tension winding, 10–02
High-torque motor, 16–07
High-voltage cable, 14–163
High-voltage wiring, 13–54
Hiorth induction furnace, 18–20
Hipernik, 2–43
Hochstadter pressure cable, 14–217
Hoepfner extraction process, 18–09
Hog, motor for, 16–32
Hoist, motors for, 16–08, 16–11, 16–25
Hoisting rig, 14–91
Holding magnet, 12–14
Hole digging machine, 14–89
Hollow copper cable, 14–163
Hollow tube, inductance of, 14–45
Holmium, 2–02
Holophane light meter, 5–11
Homo-polar generators, 8–21
Honey-extractors, 19–26
Hoopes conductivity bridge, 5–18
Hop kiln, 19–24
Hopkinson load test, 8–25
Hopper dredges, Diesel-electric control, 17–122
Horizontal hyperbola, 1–05
Horizontal illumination, 15–03
Horizontal parabola, 1–05
Horizontal separation of conductors, 14–55
Horn arresters, 12–46
Horn fiber, 2–22
Horse, light for, 19–19
Horsepower, 1–52, 1–64
  hour, 1–51, 1–64
Horseshoe electromagnet, 4–38
Hoskin's furnace, 18–16
Hospital, 14–247
  cooking energy, 15–46
  ventilation, 16–33
    requirements, 15–66

Hot plate, 14–244
Hotbeds, electric, 19–24
Hot-cathode gas lamps, 15–19
Hot-cathode mercury rectifier, 11–21
Hot-cathode neon lamp, 15–20
Hot-cathode oscillograph, 5–69
Hot-water heating, 15–47, 15–54
Hot-wire instruments, 5–30
Hotel, cooking energy, 15–46
Hotel lighting, 15–38
Hour, 1–45, 1–64
House wiring, 14–243
Household electric appliances, 15–44
Household stove, 14–244
Housings, 12–64
H. T. shuttle type magnetos, 17–77
Human eye, 15–21
Humidifier, 15–61
Hundredweight (avoirdupois), 1–48
Hunting (of electrical machine), 9–27, 9–33
Hybinette process, 18–08
Hydroelectric developments, see Power stations, hydroelectric
Hydroelectric stations, 13–14
Hydrogen, 2–02, 2–04, 2–36, 2–50, 2–54, 6–06
Hydrogen electrode, 6–10
Hydrographs, 13–14
Hydrometallurgy, 18–09
Hyperbola, 1–05
Hyperbolic formulas, 1–09
  relation to trigonometric, 1–11
Hyperbolic functions, 1–25
Hyperbolic logarithms, 1–18
Hysteresis, 2–39
  dielectric, 4–10
  loop, 3–20
  loss, 3–20, 12–60
  motor, 9–33
  normal, 5–72
Hytemco, 2–09

I

I-beam type conductor, 14–163
Ice, 2–04, 2–50
  load, 14–59, 14–60
  point, 1–70
  prevention, 13–26
Ideal, 2–09
I. E. S. study lamp, 15–39
Ignition, automobile, 17–72 to 17–84
  cable for, 17–99
Illuminated signs, 15–38
Illumination, 15–02 to 15–44 (see also Light)
  car barns, 17–70
  intensity, effect of, 15–22
Illumination meter, photronic, 15–07
Illuminometers, 5–91
Images, method of, 14–46
Imaginary quantities, 1–06
Immersion heater, 14–244, 15–47
Impedance, 1–65, 3–10 (also see article on particular device)
Impedance diagram, 17–59
Impedance relay, 12–24
Imperial standard wire gage, 1–59
Imperial units, 1–36, 1–44
Imperial wire gage, 1–62
Impregnated paper, 14–174, 14–181, 14–193
Impregnation, 2–12
Impulse characteristics, 2–17
Impulse coupling, 17–79
Impulse flashover (insulator), 14–110
Impulse tests of circuit breakers, 12–28
Impulse turbine, speed, 17–114
Incandescent lamps, 15–07, 15–15, 17–91
  colored, 15–27
  for automobiles, 17–91
Inch, 1–41, 1–64
  cubic, 1–43, 1–64
  square, 1–42, 1–65
Inching, 16–22
Incident voltage, 3–47
Incident wave, 14–25
Incline, 17–43
Inclined construction, 17–60
Incremental permeability, 2–46, 4–20
Incubation, poultry, 19–16
Index, room, for light, 15–35
Indicated wind velocity, 14–61
Indicating instruments, 5–27 to 5–45
Indicial admittance, 3–49
Indirect control system, 17–29
Indirect lighting, 15–28
Indium, 2–02
Individual appliance branch circuits, 14–242
Individual drive group, 16–02
Individual excitation, 13–52
Indoor cubicles, 12–70
Indoor temperature, 15–66
Inductance, 1–56, 1–65
  formulas and tables, 14–44 to 14–54
  mutual, 1–65, 3–08; and see Vol. I
  self, 1–65, 4–16
  transmission line, 14–47
  variable, 3–25
Induction coils, 4–41
Induction frequency converter, 11–16
Induction furnaces, 18–12, 18–20
Induction generators, 9–02, 9–37
Induction meter, 5–48
Induction motor, 1–66, 9–36, 16–05
Induction type magnetos, 17–79
Induction voltage regulator, 1–66, 12–39
Induction voltmeter, 5–29
Induction watthour meter, 5–48
Induction-synchronous converters, 11–13
Inductive coordination, 17–57 (also see Vol. 5, Section 8)
Inductive interference, 14–116
Inductive reactance of transmission line, 14–47
Inductors, 4–16 to 4–24
Inductor-type generator, 9–02
Inductor-type magneto, 17–79
Industrial applications of motors, Section 16
Industrial d-c motors, 8–39
Industrial disc heater, 14–244
Industrial drive, tests, 16–16
Industrial floodlighting, 15–32
Industrial furnace, 18–16
Industrial heating appliance, 18–24
Industrial lighting, 15–37
Industrial locomotive control, 17–36
Inertaire transformers, 2–36
Inertia, thermal, 5–30
Infinite bus, 3–50
Influence machine, 8–28
Infra-red ray lamps, 15–15
In-phase component, 3–06
Insect damage, 14–83
Insect electrocuters, 19–19, 19–27
Insect traps, 19–26
Inside air temperature, 15–66
Inspection, light for, 15–34
  of overhead lines, 14–117
Inspection sheds of railways, 17–69
  pits for, 17–70
Instability, 3–49, 14–28

Installation (see article on particular device)
Instantaneous relays, 12–23
Instantaneous values, 1–65, 3–02
Instrument, 1–68, Section 5 (see also **Meter**)
Instrument switches, 12–06
Instrument transformers, 5–53, 12–50, 12–60
Insulated wires and cables, 14–171 to 14–226, 17–98
armor, 14–185, 17–98
automobile, 17–97
braids, 14–185, 17–98
oil test, 17–99
code grade insulation, 14–188
color code, automobile, 17–99
current-carrying capacity, which see
high-tension ignition, 17–99
ignition, 17–99
impregnated paper, 14–181, 14–193
insulation resistance, 5–83, 14–172, 14–189
lead sheath, 14–185
oil-filled, 14–181
pressure, 14–182
rubber, 14–176, 14–254, 17–98
rubberized tape, 14–186
S.A.E. code, 17–98
series lighting, 14–179
sheath, 14–185
shielding, 14–183
tests, 14–186
thickness of insulation, 14–175
varnished cambric, 14–178, 17–98
weights of, 14–225
Insulation, boxes, 17–62
building, 15–54, 15–64
cable, 14–171 to 14–226 (also see **Insulated wire**)
cloth, 2–22
coordination, 14–110
disks, 14–108
electrical, for resistance heater, 18–23
fabric, 2–22
heat, 2–54, 13–10, 15–64
magnet wire, 4–33
materials, 2–12, 14–173
maximum temperature, 14–175
testing, 5–81
thickness, 14–175
oil, 2–29
overhead lines, 14–105 to 14–114
paper, 2–20
resistance, 5–83, 14–172, 14–189
strain, 14–92, 14–111
stress, 14–259
supporting structure, 14–56
tests, 9–24
thickness, oil filled cable, 14–184
paper, 14–181
rubber, 14–176
series lighting cable, 14–179
signal wire, 14–179
theoretical, 14–176
varnished cambric, 14–178
varnish, 2–32
weight of, 14–226
wire (see **Insulated wire**)
wood, 14–19, 14–111
Intake, 13–17
Intake gates, 13–18
Integrals, 1–12
Integrating, 5–45
Integrating meters, see Ampere-hour meters, Watthour meters, Demand meter
Integrating sphere photometer, 5–93
Integration by parts, 1–12
Intensity of floodlighting, 15–31
Intensity of light, 15–05
Interconnecting substations, 13–37
Interconnection of systems, voltage regulation, 12–39
Interference, 14–116
Interior conductors, 14–214
Interior illumination, 15–37
Intermittent duty motor, 16–19
Intermittent water heating, 15–49
Internal combustion for ships, 17–119
Internal combustion power station, 13–29
Internal resistance of a cell, 5–08, 7–14
International candle, 1–37, 1–59, 5–87, 15–02
International copper standard, 14–143
International joule, 1–51
International temperature scale, 5–96
International units, 1–38
Interpoles, 8–16, 8–38
machine, 8–02
Interrupters, 4–43
Interrupting ratings of circuit breakers, 12–28
Interruption, of alternating current, 12–03
of direct current, 12–02
Interurban car, 17–28
Intrinsic induction, 2–39
Inverse hyperbolic sine, 1–10
Inverse sine, 1–08
Inverse square method of photometry, 5–88
Inverse time limit breaker, 12–12
Inverse time limit relays, 12–23
Inverted converter, 11–04
Inverter, 11–21
Iodine, 2–02, 6–06
Ion, 6–03
velocity of, 6–21
Ionic conductivities, 6–21
Ionization stress, occluded gas, 14–176
oil, 14–176
Ionization-free cables, 14–217
Iridium, 2–02, 2–04, 2–50
Iron, 2–02, 2–04, 2–09, 2–41, 2–50, 6–06
flat, 14–244
gage, 1–60
pressing, 15–44
refining, 18–08
tube, electrolytic, 18–06
Iron casting, welding, 18–49
Iron loss, 2–38
Iron pulley, 16–15
Iron-clad solenoid, 4–31
Iron-core inductor, 4–18
Iron-cored transformers, 3–27
Ironer, motor for, 16–30
Ironing machine, 14–244, 15–44
Irrigation, 19–19, 19–20
Isocandle curves, 15–05 to 15–07
Isovolt electrodes, 17–75
Isthmus method of magnetic testing, 5–75
Isthmus permeameter, 5–75
I T Calorie, 1–51
Ives-Kingsbury photometer, 5–92

J

J permeameter, 5–76
Japanese paper, 2–21, 2–22
Japanning, 18–26
Jeweler, light for, 15–34
Job press, motor for, 16–23
Joint, 14–220
steam pipe, 13–09
welding, 18–35
Joint compounds, 14–222

Joint use of poles, 14–77
Joule, 1–51, 1–58
Joule's law, 4–02
Journal friction, 17–06
Judgment of illumination, 15–43
Julius suspension, 5–06
Jump spark ignition, 17–76
Jump saw, motor for, 16–32
Juniper, 14–81
Jute, 2–15, 14–226

K

Keller arc furnace, 18–20
Kellner cell, 18–11
Kelvin double bridge, 5–16
Kelvin, scale, 5–97
Kelvin warming engine, 15–59
Kethanode battery, 7–13
Keyseater, motor for, 16–21
Kiln type oven, 18–25
Kilogram, 1–36, 1–48
  calorie, 1–51, 1–58
  meter, 1–50
Kilolines, 1–57
Kilometer, 1–41, 1–64
  square, 1–42
Kilowatt, 1–52, 1–64
  hour, 1–51, 1–58
Kirchhoff's laws, 3–12
Kitchen, lighting, 15–39
  ventilation, 16–33
Kjellin furnace, 18–20
Knife switch, 12–03, 14–239
Knob and tube wiring, 14–231
Knot (marine), 1–41
Knot (wood), 14–82
Koroseal, 2–26
Kraft papers, 2–21, 2–22, 2–27
Krypton, 2–02
Kva, 3–04

L

Laboratory plate, 14–244
Laboratory type furnace, 18–14
Ladybird hatching, 19–27
Lagged-demand meter, 5–50
Lamb, light for, 19–19
Lambert, 15–03
Lamp, 1–67, 15–07 to 15–21
  automobile, 15–14, 17–90
  daylight, 15–13
  hot-cathode, gas, 15–19
  I.E.S. study, 15–39
  infra-red ray, 15–15
  lenses, 17–90
  street lighting, 15–42
  train lighting, 17–42
  ultra-violet, 15–15
Lamp equipment, 17–90
Lamp-black, 2–10
Land measure, 1–42
Lanthanum, 2–02
Lap winding, 8–05, 9–06
  resistance of, 8–23
Large lathes, motors for, 16–20
Larsen potentiometer, 5–25
Latex, 2–24
Latex deposition, 14–174
Lathe, motor for, 16–21, 16–32
Laundries, motors for, 16–14
Laundry machinery, motor drive, 16–30
Lava, 2–15, 2–18
Lay and direction of lay, 14–173
Layout and structure, station, 12–60
  small substation, 13–41
Lead, 2–02, 2–04, 2–50, 6–06
  furnace, 18–26
  refining, 18–08
  weight, 14–226
Lead sheaths, 14–175, 14–185
  corrosion of, 14–131
Lead-acid cell characteristics, 7–13
Lead-acid storage batteries, 7–08
Leakage, surface, 2–14
Leakage coefficient, 8–08
Leakage current measurement, 14–270
Leakage distance, 14–105
Leakage factor, 9–04, 9–09
Leakage flux, 8–08, 10–13
Leakage reactance (see article on particular device)
League, 1–41
Leather, 2–04
Leclanché cell, 7–02
Leeds and Northrup frequency recorder, 5–39
Leeds and Northrup ohmmeter, 5–41, 5–42
Leeds and Northrup potentiometer, 5–20
Leeds and Northrup pyrometer, 5–102
Legal units of electrical measure, 1–37
Length, conversion table, 1–41
  electrical, 3–48
  standard of, 1–36
Lens, 15–29, 17–90
  of the eye, 15–21
  vs. reflector, 15–30
Letter symbols, 1–65
Leyden jar, 4–14
Life (see article on particular device)
Life-shaft drivers, motors for, 16–20
Lifting magnets, 4–35
Lift-up switchgear, 12–66
Light, 15–02 to 15–44
  absorption, 15–04
  adequacy of, 15–33
  arc, 15–16
  automobiles, 17–84
  calculation, 15–03
  code, 15–34, 15–40
  cost of, 15–16
  daylight, 15–25
  desirable intensity, 15–31, 15–34, 15–40
  distribution circuits, 14–03
    voltage regulation, 14–09
  farm, 19–18
  flux, 15–02, 15–05
  horizontal, 15–03
  intensity, 15–02
    desirable, 15–31, 15–34, 15–40
    effect of, 15–22
  lamps, 15–07
    I.E.S. study, 15–39
  mean intensity, 15–05
  measurement, 5–93
  modifying devices, 15–27
  poultry house, 19–19
  projection, 15–29
  railway car, 17–23
  recommended intensities, 15–34, 15–40
  reflection, 15–04
  refraction, 15–04
  sources, 15–07
  standards, 5–87
  street, 15–42
  trains, 17–41
  transmission, 15–04
  units, 1–37, 1–59, 15–02
  velocity of, 1–70
  vertical, 15–03
Lighting codes, 15–34

Lighting systems, 15–28 to 15–44
Lightmeters, 15–07
Lightning arrester, 1–67, 12–45
  pipe ground for, 14–266
Lightning protection, 14–141
Lime, 2–04
Lime-nitrogen, 18–21
Limestone, 2–04
  reflection from, 15–04
Limit switches, 16–13
Line, electric, 3–02, Section 14
  magnetic, 1–57
Line compounding, 11–08
Line couplers, 17–33
Line current, 3–30
Line reactance, effect of, 14–09
Line spectra, 15–23, 15–26
Line starters, 12–32
Line switching, effect of, 14–29
Linear circuit, 3–09
Linear expansion, coefficients of, 2–58
Linear velocity, 1–46
Linen, 2–20, 2–21
Lining, furnace, 18–14
Link, 1–41
Link fuses, 12–06
Linseed oil, 2–36
Liquid fuses, 12–08
Liquid in glass thermometers, 5–97
Liquid insulating materials, 2–32
Liquid measure, 1–44
Liquid rheostats, 4–08
Liter, 1–43, 1–44
Lithium, 2–02, 6–06
Live rolls, motor for, 16–32
Live stock, use of electricity, 19–11
Load, effect of change in, 14–30
Load dispatcher systems, 12–74
Load end of line, 14–12
Load factor, farm, 19–02
Load loss, 8–18, 9–14
Load runs, 9–24
Loadings, aerial line, 14–60 to 14–73
Loam, 2–04
Loblolly pine, 14–81
Local lighting, 15–28
Location of galvanometer, 5–15
Locked-up stress, weld, 18–46
Locker room, light for, 15–34
  ventilation, 16–33
Lock-out breaker, 12–12
Locomotive, 17–07 to 17–27
  adhesion coefficient of, 17–09, 17–24
  electric, 17–24
  headlight, 5–94
  resistance, 17–06
Lodge pole pine, 14–82, 14–87
Loft building, 14–247
Log *D/d* table, 14–190
Log-anchor, 14–92
Logarithmic decrement, 5–03
Logarithms, 1–18
Logging motors, 16–31, 16–32
London gage, 1–62, 1–63
Longleaf pine, 14–81, 14–87
Long-shunt connections, 8–03
Long-span cables, 14–163
Loom, motor for, 16–27
Loop, Murray, 14–224
Loop, primary feed, 14–135
Loop service, 14–133
Loop system, 12–44
Loose knot, 14–83
Loss (see article on particular device)
Loss factor of a dielectric, 4–12
Low rate charging, 7–31
Low-frequency induction furnaces, 18–20
Low-head hydroelectric stations, 13–15
Low resistance fault, 14–30
Lowry empty-cell process, 14–86
Low-temperature furnace, 18–26
Low-tension type magnetos, 17–79
Low-tension winding, 10–02
Low-torque induction motor, 16–07
Low-voltage protection, 12–31, 12–34
Lucero, 2–09
Lumen, 1–64, 15–02
  hour, 15–02
  output, 5–93, 15–10
Luminaire maintenance, 15–35
Luminosity, sources of, 15–17
Luminous efficiency, 15–24
Luminous flux, 15–02
Luminous intensity, 1–37, 5–93, 15–02
Luminous radiation, selective, 15–08
Luminous radiator, 14–244
Lummer-Brodhun sight box, 5–89
Lunch room, light for, 15–34
Lundin fault locator, 14–224
Lutecium, 2–02
Lux, 15–02

M

Macadam surface, 17–104
Macbeth, photometer, 5–90, 5–94
Machine shop, light for, 15–34
Machine tools, motors for, 16–19, 16–21
Machines, alternating-current, Section 9
Madsenell process, 18–04
Magnesium, 2–02, 6–06
  manufacture, 18–18
Magnet, 4–26
Magnetic blowout effect, 12–17
Magnetic circuit, 8–08, 9–07, 9–19
Magnetic contactor system, 17–29
Magnetic flux, 1–57, 1–65
  density, 1–57, 1–65
Magnetic hysteresis, 2–39
Magnetic intensity, 1–65
Magnetic materials, 2–38
  tests on, 5–72 to 5–81
Magnetic potential, 1–57
Magnetic quality, 4–28
Magnetic starters, 12–32
Magnetic vane type instruments, 5–29
Magnetic-shielding instrument, 5–44
Magnetite arc lamp, 15–42
Magnetization curve, 3–19, 9–07
Magnetizing current, 10–12
Magneto, 17–77 to 17–80
Magneto blowout, 12–12
Magnetometric measurement of induction, 5–73
Magnetomotive force, 1–57, 1–64
  of winding, 3–41
Magnitude of complex number, 1–06
Main, 14–132
Make-and-break ignition, 17–75
Manchester plate, 7–09
Mandrels, 14–127
Manganese, 2–02, 2–51, 6–06
Manganin, 2–09, 2–50
Mangin reflectors, 15–30
Manholes, 14–121
Mantissa (of logarithm), 1–18
Manual remote-control boards, 12–52
Manual vs. automatic welding, 18–49
Maps, 1–65

Marble, 2–04, 2–15, 2–18, 2–50
reflection from, 15–04
Marchese process, 18–09
Marine motors, 17–113
Market, light for, 15–34
Martens polarization photometer, 5–91
Masonry, 2–04
dam, 13–17
Masonry structure, 12–61
Mass, of electron, 1–70
units of, 1–36, 1–48
Mass diagram, 13–14
Masses of materials, 2–03
Master controller, 17–33
Matching, color, 15–26
Materials, properties of, Section 2
Mathematical tables, 1–18 to 1–36
Mathematics, Section 1
Matrix, sequence, 3–35
Matte finish, reflection from, 15–04
Matthiessen's standard, 14–143
Maxima, 1–12
Maximum value, 1–65, 3–02
Maxwell, 1–57
Mazda lamps, 15–11, 15–12
burnout curve, 15–17
filament temperature, 15–15
miniature, 15–14
McDonald cell, 18–11
Mean calorie, 1–51
Measurement, Section 5
alternating current, 5–26
apparatus, 5–02
brightness, 5–94, 15–03
capacitance, 5–19, 5–43
conductivity, 5–18
core loss, 5–77
current, alternating, 5–26
direct, 5–23
d-c power, 5–26
demand, 5–50
dielectric properties, 5–81 to 5–86
energy, 5–26
frequency, 5–37
high voltage, 5–63
illumination, 15–17
inductance, 5–19
induction, 5–72
insulation resistance, 5–83
laboratory, 5–26
light, 5–87
magnetic materials, 5–72
methods used by National Laboratories, 5–26
of non-electrical quantities by electric or magnetic means, 5–96
photometry, 5–87
power, 5–26, 5–36
power factor, 5–39, 5–84
quantity of electricity, 5–45
reflection factors for light, 5–94
resistance, 5–12, to 5–18, 5–41
resistivity of electrolyte, 5–18
temperature, 5–96
three-phase power, 5–36
two-wattmeter method, 5–36
transmission factor for light, 5–94
voltage, alternating, 5–26
direct, 5–19
Meat-packing plants, motors for, 16–14
Mechanical control switchboards, 12–56
Mechanical design (see article on particular device)
Mechanical equivalent of heat, 1–70
Mechanical loading of conductors, 14–59
Mechanical properties, 2–06 (see also article on particular material)
Mechanical units, 1–36
Medium-head hydroelectric station, 13–15
Medium-temperature furnace, 18–26
Megagram, 1–48
Megameter, 1–41
Megger ohmmeter, 5–42
Megmho, 1–55
Megohm, 1–55
constants of insulation, 14–189, 14–192
miles, 14–189
sensitivity, 5–03
Melting point, 2–50
certified, 5–101
Membranes, semi-permeable, 6–16
Mensuration, 1–16
Mercury, 2–02, 2–04, 2–50, 6–06
Mercury cycle, 13–12
Mercury interrupter, 4–43
Mercury lamp, 15–19, 15–26
Mercury motor ampere-hour meters, 5–46
Mercury motor watthour meter, 5–48
Mercury thermometer, 5–97
Mercury vapor, 2–54
lamp, 15–19
Mercury-arc rectifier, 11–18
Mesh connections, 3–29
Mesh currents, 3–12
Messenger cable, 14–165, 14–219
Metal, electrolytic extraction of, 18–09
Metal arc welding, 18–43
Metal fumes, precipitation, 18–33
Metal gages, 1–59
Metal graphite, 2–10
Metal molding, 14–236
Metal refining, electric, 18–18
Metal working plants, motors for, 16–14
Metal-clad switchgear, 12–64
Metallic conduit, dimensions, 14–223
Metallic foil, 18–06
Metallic resistor materials, 2–09
Metallic tubing, 14–232
Metallurgical furnaces, 18–18
Metal-melting heater, 18–24
Meter, 1–36, 1–41
cubic, 1–43, 1–64
electrical, Section 5
automobile, 17–92
demand, 5–50
indicating, 5–27 to 5–45
integrating, 5–45 to 5–53
recording, 5–95
kilogram, 1–51
square, 1–42, 1–65
Metric horsepower, 1–52
Metric system, 1–36
Metric wire gage, 1–62, 1–63
Mho, 1–55, 1–56
Mica, 2–04, 2–15, 2–19, 2–50
at high temperatures, 18–23
Mica condensers, 4–13
Micabond, 2–15
Micarta tubing, 2–14
Microfarad, 1–56, 1–64
Microfaradmeters, 5–43
Microgram, 1–48
Microhenry, 1–56
Microhm, 1–55
Micromho, 1–55
Micron, 1–41, 1–64
Microvolt, 1–54
per meter, 1–54
Midohm, 2–09

Migration of ions, 6–19
Mil, 14–172
circular, 1–42
Mile, 1–41, 1–64
square, 1–42
Milk pasteurization, 19–04
Milking machines, 19–03
Mill, grist, 19–10
motor, 16–32
Milligram, 1–48, 1–64
Millihenry, 1–56, 1–64
Millimeter, 1–41, 1–64
gage, 14–147
square, 1–42
Milling machine, motor for, 16–21
Millivolt, 1–54, 1–64
per meter, 1–54
Mineral wax, 2–28
Miner's inch, 1–46
Mines, motors for, 16–28
Miniature bus, 12–55
Miniature control switchboards, 12–55
Miniature lamps, 15–14
Miniature switchboard instruments, 12–49
Minim, 1–44
Minima, 1–12
Mining locomotive control, 17–36
explosion tested, 17–36
Mining motors, 16–28
Minute (angle and time), 1–45, 1–64
Mipor separator, 7–13
Mirror telescope pyrometer, 5–102
Mks system of units, 1–38
Modulus, of complex number, 1–06
of elasticity, 2–07, 14–58
virtual, 14–58
Moebius process, 18–08
Moissan's furnace, 18–14
Moisture content of poles, 14–84
of soil, 14–263
Mol, 6–03
Molality, 6–04
Molarity, 6–04
Molecule, 6–02
Molybdenum, 2–02, 2–04, 2–50
Moment of resistance, 14–87
Monel, 2–09, 18–22
reflection from, 15–04
welding, 18–47
Monochromatic emissivity, 5–103, 15–26
Monolithic ducts, 14–120
Monoplane lamp filament, 15–13
Montan wax, 2–28
Month, 1–45
Mortar, 2–04
Most probable value, 1–15
Motive-power batteries, 7–19
Motor, a-c, commutator, 9–61, 17–37
polyphase induction (see below)
single-phase induction, 9–58, 16–05
synchronous (see below)
adjustable speed, 16–11
ampere-hour meter, 5–45
applications, Section 16
branch circuit, 14–242
capacity for trains, 17–20
condenser, 16–14
control, 12–29, 16–13, 17–36
converter, 11–04, 11–12
d-c (see below)
drive, tests, 16–16
electric automobiles, 17–103
enclosed, 16–14
flywheel applications, 16–07
Motor, a-c, for industrial uses, Section 16
general purpose, 16–04
induction, 9–36, 9–58, 16–05
industrial applications of, Section 16
machine tool, 16–21
marine, 17–113
polyphase induction (see below)
portable, for farms, 19–27
pulleys and pinions, 16–15
railway, 8–35, 17–06 to 17–37
control of, 17–29
size limits, 17–20
rating, 16–19
reversing, 16–22
selection of, 16–03
service factor, 16–04
ship propulsion, 17–113
single-phase induction, 9–58
slip-ring induction, 16–09
control, 16–12
specific applications of, 16–19
speed classification, 16–19
split-phase, 16–14
squirrel cage, 16–05
starting current, 16–06, 16–10
starting devices, 12–31
starting torque, 16–06, 16–10
suspension, railway, 17–28
synchronous, 9–28 (and see below)
variable speed, 16–11
d-c, 8–30 to 8–42
armature reaction, 8–33
commutating poles, 8–38
compound, 8–31
use of, 16–20
control of (high-voltage), 17–36
differential, 8–31
enclosed, 8–31
interpole, 8–38
open, 8–31
performance, 8–41
railway, 8–35, 8–40
selection of, 16–04
series, 8–31, 8–34, 16–04
use of, 16–20
shunt, 8–30, 8–33, 16–04
use of, 16–20
specifications, 8–39
speed, 8–41, 16–04
speed control, 8–37, 16–04
stability factor, 8–33
starting of, 8–36
weight, 8–41
polyphase induction, 9–36, 16–05
applications, 16–20
calculation of performance, 9–49
circle diagram, 9–50
control, 17–37
definitions, 9–36
design, 9–39
double squirrel cage, 16–07
excitation, 9–41
high torque, 16–07
installation, 9–56
losses, 9–45
low torque, 16–07
magnetizing current, 9–42
marine, 17–113
operation, 9–56
performance, 9–49
power factor, 9–45
principles, 9–36
rating, 9–38
slip, 16–10

Motor, polyphase induction, speed control, 9–55
squirrel-cage rotor, 9–36
starting, 9–53, 16–10
testing, 9–47
use of, 16–20
synchronous, 9–29
advantage, 9–29
applications, 9–29
armature reaction, 9–31
hunting, 9–33
marine, 17–114
phase characteristics, 9–31
principles, 9–29
rectifier, 18–32
rotary condenser, 9–34
starting torque, 9–33
synchronizing power, 9–32
tests, 9–34
Motor car, 17–07
Motor controller, 17–33
Motor generators, 11–02
balancer, 14–11
substation, 13–40
Motor truck, 17–103
Motor-coach battery, 17–90
Motor-curve period, 17–11
Motor-operated rheostats, 12–36
Mounting, automobile headlamp, 17–91
Mounting heights for lighting units, 15–37, 15–41
Moving picture theater, light for, 15–34
Moving-coil galvanometer, 5–03
Moving-coil oscillograph, 5–67
Moving-coil voltmeter, 5–29
Moving-iron frequency meter, 5–38
Moving-iron voltmeter, 5–29
Moving-magnet galvanometer, 5–02
Muck, 2–04
Mud, 2–04
anode, 18–07
in condensers, 17–117
Mufflers, 12–19
Multiple distribution, primary feed, 14–135
system, 14–03
Multiple radial ventilation, 9–18
Multiple system of electrodes, 18–07
Multiple-conductor cable, 14–172
Multiple-drum winding, 8–05
Multiple-operator welding, 18–43
Multiple-unit control for railways, 17–32
Multiple-unit trains, 17–07
Multiplex wave windings, 8–07
Multiplier, 5–29
Multispeed motor, 16–19, 16–20
Multivoltage system, 8–37
Municipal regulations, wiring, 14–230
Murray loop, 14–224
Mushroom houses, 19–27
Mutual inductance, 1–65, 3–25
Myriagram, 1–48
Myriameter, 1–41

N

N-terminal network, 3–16
Nagoaka's constant, 4–17
Nanoampere, 5–04
Napierian logarithms, 1–18
Nathusius arc furnace, 18–20
National Board of Fire Underwriters, 14–229
National Electrical Code, 14–229
National Fire Protection Assn., 14–229
Natural damping constant, 14–25
Natural insulators, 2–18
Natural logarithms, 1–18
Natural period, 9–33
Nautical mile, 1–41
NBS gage, 1–62
Needle point gaps, 2–36, 5–64
Negative sequence components, 3–33
Nelson cell, 18–11
Neodymium, 2–02
Neon, 2–02
Neon glow lamp, 15–19
Neon ground detectors, 12–51
Neon lamps, 15–19, 15–20, 15–26
Nernst's thermodynamic theorem, 6–12
Network, 14–132 to 14–142
Network protectors, 13–38, 14–135
Network theory, 3–16
Neutral, 3–30, 14–261
capacitance to, 14–37
effect of current in, 14–09
grounding of, 3–56, 10–08
size of, 14–15
Neutral curve, 3–19
New British Standard gage, 1–62
Nichrome, 2–09
Nickel, 2–02, 2–04, 2–50, 6–06
plating, 18–03
refining, 18–08
welding, 18–47
Nickel aluminum steel, 2–48
Nickel chromium, 2–09
heating units, 18–22
Nickel-copper alloys, 18–22
Nickel silver, 2–09
Nilvar, 2–09
Nitriding, 18–26
Nitrogen, 2–02, 2–04, 2–36, 2–50
fixation, 18–21
Nitro-lime, 18–21
Nitrous oxide, 2–04, 2–50
Nobility of elements, 6–10
Noise, in telephone circuit, 14–116
No-load magnetization curve, 9–19
Nominal length for poles, 14–79
Non-ferrous alloy, welding, 18–47
Non-linear circuits, 3–19, 3–26
Non-metallic conduit, 17–101
Non-metallic resistors, 4–08
Non-metallic sheathed cable, 14–236
Non-metallic surface extensions, 14–237
Non-metallic wiring for wet places, 14–236
Non-retentive magnetic materials, 2–41
Non-reversing drum controllers, 12–33
Non-sinusoidal currents, 3–09
Normal hysteresis, 5–72
Normal induction, 2–39, 5–72
Normal permeability, 4–20
Normal solution, 6–04
Normalizing, 18–26
Northern white cedar, 14–79, 14–84
Norway iron, 2–43
Norway pine, 14–81
No-voltage alarm, 19–25
Nujol, for contacts, 5–12
Null type potentiometers, 5–19, 5–24
Number of conductors or turns, 1–65
Nut dehydration, 19–24

O

Oat sprouting, 19–06
Observations, errors in, 1–14
weighted, 1–15
Occluded gas, ionization stress, 14–176
Oersted, 1–58

Office fan, 14–244
Office lighting, 14–247, 15–34, 15–38
Office machines, 14–244
Off-peak water heating, 15–50
Offset of duct line, 14–126
Offset-end incline, 17–43
Ohm, 1–38, 1–65, 5–09
Ohmalloy, 2–09
Ohmax, 2–09
Ohmmeters, 5–41
Ohms per pound (copper wire), 14–148
Oil, 2–04, 2–15, 2–29, 10–19
  dielectric strength of, 2–13
  flash point of, 2–59
  ionization stress, 14–176
  transformer, 2–30
Oil circuit breakers, 12–16, 12–25
Oil fields, motors for, 16–28
Oil varnishes, 2–32
Oil well drilling and pumping, 16–28
Oil-cooled transformer, 10–15
Oil-filled bushing, 10–12, 12–19
Oil-filled cables, 14–181, 14–217
Oil-immersed fuses, 12–08
Oilostatic cables, 14–217
Oil-treated paper, 2–16
Old English wire gage, 1–62, 1–63
Olive oil, 2–36
On-peak water heating, 15–50
Opal glass, 15–25
  light transmission, 15–05
Opaque reflectors, 15–27
Open-arc single hearth furnace, 18–13
Open bus structure, 12–61
Open circuit, protection against, for relays, 12–23
Open construction, 12–60
Open delta connection, 10–09
Open fuses, 12–06
Open solenoid, 4–34
Open tank treatment, 14–85
Open wiring, 13–53, 14–231
Open-bottom raceway, 14–236
Open-circuit saturation curve, 9–11
Open-circuit voltage rise, 14–19
Operating mechanisms of circuit breakers, 12–20
Operation (see article on particular device)
Optical pyrometry, 5–101, 5–102
Oregon cedar, 14–87
Organic acidity, 2–30
Oscillating quantity, 3–02
Oscillating type magnetos, 17–79
Oscillographs, 5–67
  cathode ray, 5–67, 5–69
  circuits used with cathode-ray type, 5–71
  life of cathode-ray type, 5–70
  moving coil, 5–67
  recording speed, 5–70
  sensitivity, 5–68, 5–69
  string galvanometer, as oscillograph, 5–71
Osmium, 2–02, 2–04, 2–50
Osmosis, electric, 6–25
Osmotic pressure, 6–03, 6–15
Ostwald calorie, 1–51
Ounce, 1–43 to 1–50
Outdoor expulsion fuses, 12–06
Outdoor service, motors for, 16–14
Outdoor stations, 12–71
Outdoor switches, 12–05
Outdoor switchgear, 12–67
Outdoor switchhouses, 12–72
Outlet limitation, 14–243
Output coefficient, 8–04
Ovalflex, 14–237
Oven heaters, 18–25
Ovens, industrial, 18–25
Overcurrent, relays against, 12–23
Overcurrent protection devices, 14–239
Overdamping, 5–03
Overhead distribution system, 14–54 to 14–117, 14–136 to 14–142
  catenary construction, 17–60
  cities, 14–137
  direct suspension construction, 17–60
  ground wire, 14–261
  inductance of, 14–45
  loading map, 14–60
  poles for, 14–78
  railway, 17–51 to 17–61
  sag, 14–63
Overhung waterwheel generator, 13–22
Overload, relays against, 12–23
Overload capacity (see article on particular device)
Overload protection, 12–34, 17–98
Overvoltage, 6–14
  relays against, 12–23
Owen-Dyneto regulator for automobiles, 17–88
Ox-grub removal, electric, 19–12
Oxidation, electrolytic, 18–10
  method of nitrogen fixation, 18–21
Oxide film arresters, 12–47
Oxygen, 2–02, 2–04, 2–50, 6–06
Oxygen-free copper, 2–05
Ozokerite, 2–28
Ozone production, 18–11
Ozone resistance, 14–186

P

P.D.C.P. cable, 14–163
Pace, 1–41
Package conveyor, motor for, 16–29
Paint, light reflection from, 15–04
Paint dryer, 18–26
Paint shops, light for, 15–34
  motors for, 16–14
Palladium, 2–02, 2–04, 2–50
Palm, 1–41
Pan-conveyor, motor for, 16–29
Panel, 12–51
Panel board, 14,241
Panel boxes, 14–239
Panel heating, 15–60
Panels, 12–51
Pantograph, 17–38
Paper, 2–04, 2–15, 2–20, 14–186
  and pulp plants, motors for, 16–14
  impregnated, 14–174, 14–193
    weight of, 14–226
  insulating, 2–22
  oil-treated, 2–16
Paper condensers, 4–12
Paper industry, motors for, 16–27
Paper insulation, 14–182, 14–192
Paper pulleys, 16–15
Parabola, 1–05, 1–16
Parabolic reflectors, 15–29
Paraboloid, 1–17
Paraffin, 2–04, 2–15, 2–28
Parallel impedance, 3–11
Parallel operation, of generators, 8–26
  of transformers, 10–21
Parallel wires, capacitance of, 14–37
  inductance of, 14–45
Paralleling of generators, 9–26
Parallelogram, 1–16
Paramagnetism, 2–38

Passageway, light for, 15–34
Passenger car, 17–07
Passive transducer, 3–14
Passivity, 6–15
Pasted plate cell, 7–10
Pasteurization, 19–04
Pauling furnace, 18–21
P.D.C.P. hollow core cables, 14–163
Pea viners, electric, 19–27
Peak factor, 3–03
Peak load hydro plant, 13–50
Peck, 1–44, 1–64
Pedersen potentiometer, 5–25
Peening, weld, 18–45
Peltier effect, 6–09
Pennyweight, 1–48
Penstock, 13–18, 13–27
Pentane, 1–59
Pentane thermometer, 5–97
Per cent losses, 14–12, 14–18
Perch, 1–41, 1–43
Percolator, 14–244, 15–44
Percussive welding, 18–40
Period (trigonometric), 1-07, 3–02
  hyperbolic, 1–10
Periodic quantity, 3–02
Permalloy, 2–43
Permanent magnet, 2–47, 4–26
  moving-coil instrument, 5–28
Permeability, 1–65, 2–39, 4–20
Permeameter, 5–74
  Babbit, 5–76
  Burrows, 5–74
  Fahy, 5–75
    low mu, 5–77
  isthmus, 5–75
  J, 5–76
  simplex adapter, 5–76
Permeance, 1–65
Perminvar, 2–48
Permittance, 1–65
Permittor, 4–15
Permittivity, 1–65
Permutations, 1–03
Petersen earth coils, 3–56
Petroleum, 2–50
Petroleum coke, 2–10
Petticoat spread, 14–106
Petticoat tip, 17–82
***p*H**, 6–24
Phanotron, 11–21
Phase, segration, 12–62, 14–133
Phase angle, 3–04
  constant, 14–25
  control, 11–18
  wattmeter correction factor, 5–34
Phase balancer, 11–17
Phase characteristic of synchronous motors, 9–31
Phase converter, 11–17
Phase current, 3–30
Phase defect angle, 4–09
Phase displacement, 1–65
Phase meter, 5–39
Phase modifiers, 14–19
Phase shift constant, 3–48
Phase shifter, 11–18
Phenolic sheet materials, 2–27
Phonoelectric wire, 14–161
Phonograph records, 18–06
Phosphorus, 2–02, 18–17
Phot, 15–02
Photoelectric photometer, 5–95
Photoflash lamps, 15–15
Photoflood lamps, 15–14
Photographic lighting, 15–38
Photometer, 5–87 to 5–95
Photometric units, 1–37
Photronic illumination meter, 15–07
Physical characteristics of materials, Section 2
Physical constants, 1–70
π section, 3–14
Picker, motor for, 16–27
Pickling solutions, 18–03
Picture lighting, 15–38
Picture projection, 15–31
Pike Pole, 14–91
Pile, 14–96
Pilot exciter, 12–39
Pilot wire relay, 12–24
Pin, 14–112
  height, 14–106, 14–112
  pole, 14–104
Pin knot, 14–83
Pinch effect, 18–20
Pine (wood), 14–81, 14–87
Pine tar, 2–29
Pin-expanded bond, 17–65, 17–68
Pin-holes, 14–104
Pinion starter, 17–96
Pinions and pulleys, 16–15
Pint, 1–43, 1–44
Pin-type insulator, 14–107
Pipe cutter, motor for, 16–21
Pipe ground, 14–266
Pipe insulation, 13–10
Pipe line, 13–18
  pumping, 16–28
Piping, steam, 13–08
Piping loss, 15–55
Pit furnaces, 18–29
Pitch, 2–04
Pitch constant, 9–09
Pitch pocket, 14–83
Pitch streak, 14–83
Pitching of ship, 17–109
Pith, 14–82
Pith knot, 14–83
Plan of right of way, 14–74
Planck constant, 1–70
Plane angle measure, 1–45
Planers, motors for, 16–20
Planing mills, motors for, 16–14
Plant costs, 13–12, 13–28, 13–33
Planté cell, 7–08
Plaster, heat conduction of, 15–64
  reflection from, 15–04
Plaster of Paris, 2–04
Plastic insulating materials, 2–26
Plater, motor for, 16–27
Plate-type rheostat, 4–05
Plating, electro, 18–02
  motor for, 16–14
Platinum, 2–02, 2–04, 2–50, 6–06
  thermocouple, 5–99
Plenum chamber, 15–57
Plow, 17–38
Plow steel cable, 14–165
Plug fuse, 14–239
Plug resistance box, 5–11
Plunger pumps, motors for, 16–08
Plunger type instrument, 5–29
P.M.G. wire, 14–161
Pneumatic contactor system, 17–29
Pneumatic conveyor, motor for, 16–29
Point source of light, 15–03
Polaricode, 12–74
Polarity markings, 5–50
Polarization, 6–07

Pole, 1–41
dimensions, 14–79
grounding, 14–261
hole, 14–91
line, 14–54 to 14–117, 14–136 to 14–142
pin, 14–104
seasoning, 14–83
spacing, 17–51
steel, 14–93
transformer, 14–139
wood, 14–78 to 14–93, 14–138
Pole (magnetic), arc, 9–04
pieces, 8–03
pitch, 9–04
shoe, 8–03
constant, 9–08
Polyphase electromagnets, 4–39
Polyphase induction meter, 5–49
Polyphase induction motor, 9–36, 16–105
Polyphase regulator, 12–40
Polyphase synchronous motor, 9–29
Polyphase systems, 3–29
Polyphase wattmeter, 5–31
Poncelet, 1–52
Pond pine, 14–81
Poor regulation, effect of, 14–08
Porcelain, 2–04, 2–13, 2–15, 2–19
properties of, 14–105
reflection from, 15–04
Porous block arresters, 12–47
Portable heater, 15–44
Portable photometer, 5–90
Portable spot welder, 18–38
Portable substations, 12–69
Positive pressure blowers, motors for, 16–13
Positive sequence components, 3–33
Post office type bridge, 5–13
Pot furnaces, 18–26, 18–28
Potassium, 2–02, 6–06
manufacture, 18–18
Potassium chloride, 18–10
Potassium permanganate manufacture, 18–10
Potential control method, 9–53
Potential difference, 3–02
Potential gradient, 1–54, 1–68
Potential regulators, 10–02
Potential speed control, 8–37
Potential survey, 14–269
Potential transformers, 5–53, 12–50, 12–60
Potentiometer, 5–19 to 5–25
Poultry, electricity for, 19–13 to 19–19
Pound, 1–36, 1–48, 1–49
feet, 1–50, 1–64
Poundals, 1–49
Powder-filled fuses, 12–06
Power, 1–52, 1–55, 3–03
costs, 13–12, 13–28, 13–33
distribution, 14–03
for appliances, 14–244
loss (see article on particular device)
loss, per cent, 14–12
measurement of, 5–26
output (see article on particular device)
polyphase system, 3–31
to drive train, 17–10
transmission system, 14–03
Power brakes, 17–39
Power factor, 3–03 (also see article on particular device)
condenser, 4–10
correction, 4–15
effect of, on regulation, 14–09
insulation, 2–16, 5–84
meter, 1–68, 5–39
Power plant, 13–32
Power station, Section 13
circuits, 13–46
hydro-electric, 13–14
internal combustion, 13–29
light for, 15–34
steam, 13–02
wiring, 13–53
Power system stability, 3–49, 14–28
Power-angle characteristic, 3–52
Power-time curve, 17–12
Practical electrical system, 1–37
Praseodymium, 2–02
Precipitation, electrical, 18–31 to 18–34
Precision measurements by National Laboratories, 5–26
Profocused lamp bases, 15–10, 15–33
Preservative treatment, of cross-arms, 14–104
of wood poles, 14–84
Press, high speed, motor for, 16–23
Pressboard, 2–15, 2–21
Pressing iron, 15–44
Pressure, steam, 17–111
wind on wires, 14–60
Pressure cable, 14–182
Hochstadter, 14–217
Pressure gage thermometer, 5–97
Pressure treatment, 14–86
Pressure welding, 18–36
Price guard wire, 5–24
Primary batteries, 7–02
Primary distribution, system, 14–132
types of feed, 14–135
voltages, 14–08
Primary light standard, 5–87
Primary network, 14–133
Primary of induction motor, 9–36
Primary photometry, 5–87
standards (see Standards)
temperature, 5–96
Primary substation, 13–37
Primary winding, 10–02
Principal diagonal (determinant), 1–04
Principle of superposition (see Vol. 1, Electricity and Magnetism)
Printing, light for, 15–34
Printing presses, electric drive, 16–23
Printometer, 5–51
Prism, 1–16
Prismatic globes, 15–28
Prismatic reflectors, 15–27
Prison ventilation, 15–66
Probable error, 1–15
Probable value, 1–15
Profile of right of way, 14–74
Progression, 1–02
Projection apparatus, photometry of, 5–94
Projection lamps, 15–13
Projection, light, 15–29
Projection picture, 15–31
Projection welding, 18–39
Propagation constant, 3–47
Propellor characteristics, 17–108
Properties of materials, Section 2
Propulsion of ship, electric, 17–108
Protection, 12–02, 12–42
electrolysis, 14–271
fire, 13–23
interior, 14–239, 14–256
network, 13–38
overload, 12–34, 17–98
splicing chambers, 14–220
steel from electrolysis, 14–272
stray field, 5–44

Protection, underground cables, lightning, 14–141
Protection tubes, pyrometer, 5–101
Protective coverings, 14–174
Protective device, 14–256
Protective panels, 12–34
Protectors, network, 12–31, 14–135, 14–141
Protractor method, light, 15–06
Proximity effect, 3–22
Prune drying, 19–23
Pull, draw-bar, 17–06
of simple solenoid, 4–35
Pulleys and pinions, 16–15
Pull-in torque, 9–34
Pull-out point, 17–114
Pull-out torque, 9–28
Pulverizers, motors for, 16–14
Pumice stone, 2–04
Pump, motor for, 16–27, 16–28
Pump log, 14–120
Pump-back method, 8–24
Pumped storage, hydroelectric plant, 13–17
Pumps, for farms, 19–21
Punch press, motors for, 16–08, 16–21
Puncture, 2–12
Pupil of eye, 15–21
Purification of oil, 2–31
Purkinje effect, 15–21
Pusher furnace, 18–29
Pyramid, 1–17
Pyranol, 4–15
Pyrex, 14–105
Pyro radiation, pyrometer, 5–102
Pyrometer, 5–98 to 5–103

Q

Quadrant (trigonometric), 1–07, 1–45
Quadrant, as wattmeter, 5–63
Quadrant electrometer, 5–62
Quadratic equation, 1–03
Quadrature axis synchronous reactance, 3–45
Quadrature component, 3–06
Quantity, electric, 1–65
of electricity, 1–52, 1–65
Quart, 1–43, 1–44
Quarter (avoirdupois), 1–48
Quarter-phase system, 3–31
Quartz, 2–04, 2–19
Quick response excitation, 12–39
Quill drive, 17–25
Quintal, 1–48
Quotient type ohmmeter, 5–42

R

Raceway, underfloor, 14–235
Radial distribution, 12–43, 14–132
Radial main, 14–132
Radial primary feed, 14–134
Radial ventilation, 9–17
Radian, 1–45
Radiant electric heater, 15–56
Radiation, heat, 2–52
light, 15–23
selective, 15–25
luminous, 15–08
Radiation losses from oven, 18–25
Radiation pyrometry, 5–101
Radiator, 15–54
Radical, 6–02
Radio battery tests, 7–04
Radio interference, 14–108
Radiohm, 2–09
Radium, 2–02
Radius, bending, of cable, 14–216
Radon, 2–02
Rag base paper insulators, 2–20
Rail, 17–62 to 17–69
bond, 17–64
current, 17–59
section, 17–62
Railroad, 17–02
batteries, 7–19
bonds, 17–64
crossing, 14–56
distribution, three-wire, single-phase, 13–46
electric, 17–02 to 17–71
equipment, line clearance, 17–51
inspection pits, 17–70
inspection sheds, 17–69
mail car, lighting, 17–43
motor, 8–35, 8–40, 17–21
control of, 17–29
power station load, 17–24
signaling battery, 7–23
substations, 13–39
load, 17–24
track bonds, 17–64
Rake, of pole, 17–60
Ranges, electric, 15–45
Rankine scale, 5–97
Rapid response excitation, 3–57
Rapid transit control, 17–34
Rathenau process, 18–18
Rating (see article on particular device)
of motors, 16–19
of resistors, periodic, 16–13
Ratio, physical, 1–70
of electric units, 1–70
Ratio factor, of transformers, 5–54
Rationalized mks system of units, 1–38
Reactance, 1–65, 3–10
leakage, 10–05
of interior wiring, 14–254
tables, 14–49 to 14–54
transmission line, 14–49
Reactance grounded system, 14–259
Reaction, heat of, 6–04, 6–11
Reactive factor meters, 5–39
Reactive power, 3–03
meter, 5–40
Reactor, 1–67, 4–16
current-limiting, 12–48
Réaumur scale, 5–97
Reciprocal theorem, 3–13
Reciprocating pumps, motors for, 16–13
Recorder, 5–95
Recording frequency meter, 5–39
Recording instruments, 5–95
Recording pyrometers, 5–101
Recording speed, oscillograph, 5–70
Rectangular hyperbola, 1–05
Rectified currents, instruments for, 5–41
Rectifier, 11–18
substation, 13–44
Rectifier voltmeter, 5–31
Red cedar, 14–79, 14–87
Red light, emissivity of, 5–103
Reduction, electrolytic, 18–10
Redwood, 14–87
Reel (wire), 14–168
Refillable fuses, 12–08
Refining, electrolytic, 18–06
of metal, electric, 18–18
Reflected glare, 15–23
Reflected voltage, 3–47
Reflected wave, 14–25

Reflection, and transmission, 15–25
of heat, 2–52
of light, 5–94, 15–04
specular, 15–41
Reflection factor, measurement of, 5–94
of materials, 15–04
Reflection heater, 14–244
Reflectors, 15–27 to 15–31
vs. lenses, 15–30
Refraction of light, 15–05
Refractors, 15–28, 18–14
Refrigeration, 19–04
cycle reversed, 15–59
Refrigerator, 14–244
Regenerative braking system, 17–40
Regulating-pole converter, 11–04
Regulation, 8–17, 9–13, 14–08 (also see article on particular device)
evil effects of poor, 14–08
Regulators, 12–36 to 12–42
feeders, 14–10
Relative weights of copper, distribution systems, 14–07
Relay, 1–68, 4–35, 12–23
current-limiting, 17–33
Reluctance, 1–65
Reluctivity, 1–65, 2–39
Remanence, 2–39, 4–26
Remote control, railway, 17–29
Remote control switchboard, 12–56
Remote metering, selective, 12–75
Rennerfelt furnace, 18–20
Repair shops for railways, 17–69
Repulsion-induction motor, 9–62
Repulsion-starting, induction running motor, 9–62
Repulsion-type instruments, 5–29
Resaw, motor for, 16–32
Reservoir, cable, 14–223
hydroelectric station, 13–17
Residence lighting, 15–39
Residence wiring, 14–243
Residual current, 14–117
Residual induction, 2–39
Residual magnetism, 3–20
Resin bonded brushes, 2–10
Resin products, 2–26
Resistance, 1–65
a-c vs d-c, 14–215
measurement, 5–13, 5–42
rails, a-c vs d-c, 17–59
skin effect, 3–21, 14–165
standards, 5–09
steel rail, 17–64
variation with temperature, 14–160
wires, 14–143
Resistance boxes, 5–11
Resistance furnaces, 18–12, 18–26
Resistance grounded system, 14–259
Resistance heating, 18–22
Resistance leads, 9–61
Resistance materials for furnaces, 18–22
Resistance method of transformer test, 5–59
Resistance thermometers, 5–97
Resistance welding, 18–35, 18–37
Resistivity, 1–65, 2–15
of copper, 2–06
surface, 5–84
volume, 5–84
Resistor, 1–67, 4–02, 12–09
cast grid, 12–10
current limiting, 12–11
edgewound, 12–10
field discharge, 12–10
Resistor, materials, 2–09
service classification, 16–13
temperature of, 4–02
Resistor type furnace, 18–16
Resonance frequency meters, 5–38
Resorcinal-formaldehyde, 2–27
Restaurant, cooking energy, 15–46
light for, 15–34
Retentive magnetic materials, 2–47
Retentivity, 2–39
Retina of eye, 15–21
Reuping empty-cell process, 14–86
Reverse arms, 14–102
Reverse current, relays against, 12–23
Reverse current trip breaker, 12–12
Reverse phase connection, relays against, 12–23
Reverse power, relays against, 12–23
Reversed refrigeration, 15–59
Reversible electrodes, 6–08
Reversing mill motor, 16–33
Reversing motors, 16–22
Reversing turbines, 17–112, 17–118
Reversor, 17–33
Revolution (angle), 1–45, 1–64
Revolving armature generators, 9–02
Revolving field generators, 9–02
Rewinder, motor for, 16–27
Rhenium, 2–02
Rheostat, 1–67, 4–02, 12–60
control, 12–34
speed regulating, 12–35
Rheostatic brake, 17–40
Rheostatic control method, 9–54, 17–30
Rheostatic regulators, 12–38
Rheostatic speed control, 8–37
Rhodium, 2–02, 2–04, 2–50
R.I. motor, 9–63
Ribbon type resistor, 4–03
Rice drying, 19–23
Right circular cone, 1–17
Right circular cylinder, 1–17
Right pyramid, 1–17
Right-of-way, 14–78
profile and plan, 14–74
Rigid bus, 13–54
Rigid tower, 14–94
Ring bus, 13–48
Ring method of magnetic testing, 5–73
Ring system, 12–44
Rings (in trees), 14–82
Risers, 14–237, 14–238
cable, 14–238
Rising characteristic, 3–19
Riveting vs. welding, 18–38
R.L.M. reflectors, 15–27, 15–35
Rms current per railway motor, 17–19
Rms values, 3–02
Road surface, effect of, on car power, 17–104
Rochling-Rodenhauser furnace, 18–20
Rock crusher, motor for, 16–14
Rock fill dam, 13–17
Rock footings, 14–96
Rock saw, motor for, 16–32
Rock wool, heat conductivity, 15–64
Rodding, 14–126, 14–129
Roebling gage, 1–62
Roller-Smith ohmmeter, 5–41
Rolling mill motors, 16–33
Rolls, motor for, 16–32
Rood, 1–42
Roof, heat transmission, 15–63
Roofing (of poles), 14–84
Room index, light, 15–35
Root mean square, 1–64, 3–03

Rope cement paper, 2-22
Rope paper, 2-21
Rope-lay cable, 14-153, 14-172
Rosin, 2-15
Rosin oil, 2-36
Rotary condensers, 9-34
Rotary converters, 11-03
Rotary drill, 16-28
Rotary gas engine, 13-29
Rotary phase modifiers, 9-34
Rotary voltmeter, 5-63
Rotary-hearth furnaces, 18-29
Rotating carbon, 15-18
Rotating sector disk, 5-89
Rotolactor, 19-03
Rotor, 9-36
  acceleration, 3-53
  slots, 9-19
Rotten knot, 14-83
Roughage grinding, 19-06, 19-08
Round conductor, 14-172
Route of transmission line, 14-54
Roving frame, motor for, 16-26
Rubber, 2-15, 2-24, 2-51, 14-186
  insulation 14-173, 14-213
  weight of, 14-226
Rubber condensers, 4-14
Rubber mill and calender motors, 16-30
Rubber-insulated wire, 14-240, 14-254
Rubberized tape, 14-186
Rubidium, 2-02
Ruling span, 14-71
Runaway speed of hydroelectric generator, 13-21
Run-of-river hydroplant, 13-50
Rural distribution, 14-55
Rural electrification, 19-02
Rural load characteristics, 19-02
Ruthenium, 2-02

S

Safety car, 17-37
Sag calculations, aerial line, 14-63
Salient-pole generator, 9-04
Salem process, 18-09
Salt, 2-04
  furnace, 18-26
Samarium, 2-02
Sand, 2-04
Sandstone, 2-04
  reflection from, 15-04
Sapwood, 14-82
Sarlun pipe flange, 13-09
Saturated cadmium cell, 5-06
Saturation curve, 8-10, 8-24
Saturation induction, 2-39
Saw, motor for, 16-21, 16-32
Sawdust, heat conductivity, 15-64
Sawmill, motor drive, 16-31, 16-32
Scalding, poultry, 19-15
Scandium, 2-02
Schedule speed, 17-12
Schering bridge, 5-85
Schonherr furnace, 18-21
School lighting, 15-34, 15-39
School ventilation, 15-66
School wiring, 14-247
Schrage motor, 9-63
Schweitzer-Conrad fuse, 12-08
Scott connection, 10-09
SCR motors, 9-63
Screens, 15-27
Screw conveyor, motor for, 16-29
Screw machine, motor for, 16-21
Scruple, 1-49
Seam welding, 18-35
Searchlights, 5-94, 15-31
Seasoning, of cross-arms, 14-104
  of poles, 14-83
Seating capacity of railway cars, 17-28
Sec (secant), 1-07
Sech, 1-10, 1-25
Second, 1-37, 1-44
Secondary distribution system, 14-134
Secondary light standards, 5-88
Secondary networks, 14-134
Secondary of an induction motor, 9-36
Secondary winding, 10-02
Section (of land), 1-42
Sectionalizing devices, network, 14-142
Sectionalizing distribution circuits, 14-141
Sectionalizing of a station, 13-47
Sector cable, 14-172
Sector disk, 5-89
Seed cleaning, 19-10
Seed corn testing, 19-10
Segregated phase, 12-62, 14-133
Selection of motor, 16-04, 16-15
Selective radiation, 15-25
  luminous, 15-08
Selective remote metering, 12-75
Selector system, 12-57
Selenium, 2-02, 2-04, 2-51
Self-cooled transformer, 10-04
Self-inductance, 1-65, 3-25
Semi-continuous oven, 18-25
Semi-Diesel engine, 13-29
Semi-enclosed bus structure, 12-61
Semi-enclosed construction, 12-61
Semi-indirect lighting, 15-28
Semi-off peak water heating, 15-50
Sensibility curve, American standard, 15-21
Sensitivity of galvanometer, 5-03
Sensitivity of oscillograph, 5-68, 5-69
Separately excited machine, 8-02
Separation, of metals, 6-14
  of wires, 14-55, 14-231
Separators, 7-12, 14-173
Sequence matrix, 3-35
Series, 1-16
Series arc-light generators, 8-21
Series distribution, 14-04
Series electrodes, 18-07
Series impedance, 3-11
Series lighting, 14-179
Series machine, 8-02
Series motor, 8-31, 8-34, 16-04
Series system of electrodes, 18-07
Series transformer, 10-02, 10-06
Series welding, 18-38
Series-drum windings, 8-06
Series-parallel control, 17-30
Serpek process for nitrogen fixation, 18-21
Service classification of resistors, 16-13
Service duct, 14-119
Service entrance, 14-243
  cable, 14-140, 14-238
  head, 14-140
  pipe, 14-119
  wires, 14-140
Service factor of a motor, 16-04
Setting of pole, 14-89
Severity of service, 7-07
Sewing of splice bar, 17-66
Sewing machine, 14-244
Shade, 15-23
Shading coil, 4-41

Shadows, 15–23
Shaft couplings, 16–20
Shaft drive, 17–104
Shakes, 14–83
Shaking conveyor, motor for, 16–29
Shaper, motor for, 16–21
Sharpe-Millar photometer, 5–94
Shavings, heat conductivity of, 15–64
Shears, motors for, 16–08, 16–21
Sheath, lead, 14–175
corrosion of, 14–131
grounding of, 14–261
loss in, 14–215
Sheave, stringing, 14–114
Sheds, railway inspection, 17–69
Sheep, light for, 19–19
shearing, 19–12
Sheet calender, motor for, 16–27
Sheet metal gage, 1–59
Shell type transformer, 10–03
Shellac, 2–15, 2–29, 2–35
Shelmerdine process, 18–11
Shielded-conductor cable, 14–172, 14–197, 14–203
standards, 14–183
Shielding, 5–85, 14–183
"Shin," 1–10, 1–25
Shingle, heat conductivity, 15–64
Shingle mill, motor for, 16–32
Ship batteries, 7–23
Ship construction, welding in, 18–37
Ship control equipment, 17–114
Ship propulsion, electric, 17–108 to 17–123
Ship, welding, 18–50
Shipping measure, 1–44
Shipping ton, 1–44
Shock, electric, 14–258
Shoe, trolley, 17–38
Shop equipment for farms, 19–27
Short circuits, 9–27
calculations, 12–28
effect of, 14–30
limiting current in, 14–259
Short shunt connections, 8–03
Shortleaf pine, 14–81, 14–87
Shovels, motor driven, 16–25
Show window lighting, 14–247, 15–34, 15–38
Shower, water requirement, 15–47
Shredding, vegetables, 19–26
Shunt, 1–67, 5–04
galvanometer, 5–04
Shunt field, 8–10
Shunt machine, 8–02
Shunt motor, 8–30, 8–33
Shunt trip breaker, 12–12
Shuttle-type magneto, 17–77
Side clearance of overhead lines, 14–76
Side incline, 17–44
Side swing of conductor, 14–95
of insulator, 14–76
Siemens and Halske extraction process, 18–09
Siemens-Martin steel cable, 14–165
Sight box, 5–88
Sightmeters, 15–07
Sign lighting, 15–38
Signal lamps, 17–91
Signal wire, 14–179
Signal wiring, 13–55
Signals, railway, see Pender-McIlwain Communication Engineering Handbook
Silhouette vision, 15–41
Silicon, 2–02, 2–51, 18–17
Silicon carbide process, 18–17
Silicon steel, 2–43, 2–54
Silit unit, 18–27
Silk, 2–20, 2–22
Silk insulation, 14–174
Silo filling, 19–07
Silox, 18–17
Siloxicon, 18–17
Silsbee current-transformer test set, 5–88
Silsbee electrodynamic ammeter, 5–26
Silver, 2–02, 2–04, 2–50, 6–06
electrochemical equivalent of, 1–70
plating, 18–05
reflection from, 15–04
refining, 18–08
Silver-graphite, 2–10
Simple harmonic current, 3–04
Simple impedance method (a-c line calculations), 14–20
Simple lenses, 15–29
Simplex adapter, 5–76
Simplex spot welder, 18–38
Simultaneous equations, 1–03
Sin, 1–07, 1–20
Sine, 1–07, 1–20
Single bus system, 12–57, 14–09
Single circuit transformer, 10–21
Single conductor cable for three-phase lines, 14–216
Single round wire, capacitance of, 14–37
inductance of, 14–44
Single-acting engine, 13–30
Single-end condenser method (a-c line calculations), 14–22
Single-hearth open-arc furnace, 18–13
Single-operator welding, 18–42
Single-phase converter, 11–04
Single-phase distribution, 14–133
in buildings, 14–227
Single-phase generator, 9–15
Single-phase induction motors, 9–58
Single-phase regulator, 12–40
Single-phase substation, 13–45
Single-phase synchronous motors, 9–29
Single-spark ignition, 17–76
Single-spark magnetos, 17–77
Single-truck car, 17–27
Single-wire automobile wiring, 17–97
Sinh, 1–10, 1–25
Sinusoid, 1–05
Sinusoidal current, 3–04
Six-phase system, 14–06
Six-ring converter, 11–05
Six-wire, three-phase system, 14–06
Sizes (see article on particular device)
Skin effect, 3–21, 14–165,
Slasher, motor for, 16–32
Slate, 2–04, 2–15, 2–18
Sleet, 14–60
Sleeve, splicing, 14–115
Sleeve inductor type magnetos, 17–79
Sleeve joint, 14–120
Slide-wire bridge, 5–12
Sliding shoe, 17–38
Slip, 9–37, 9–46, 16–70
Slip ring, 9–02
Slipper compound, 14–174
Slip-ring induction motor, 16–09
Slip-ring motor control, 16–12
Slot factor, 9–07, 9–19
Slot pitch, 9–04
Slow-burning wire, 14–240
Sludge, 2–30
Slush pump, motor for, 16–28
Smoke precipitation, 18–31
Snake, 14–127

Snap switches, 14–239
Snow, 2–04
effect of surface, 17–104
Soapstone, 2–04, 2–19
Soapstone ducts, 14–120
Socket joint, 14–120
Sodium, 2–02, 6–06
manufacture of, 18–18
Sodium chloride, 18–10
Sodium permanganate manufacture, 18–10
Sodium vapor lamps, 15–20, 15–26
Soft annealed copper, 2–05
Soil resistance, 14–263
Solar day, 1–37
Solar heat, 15–67
Solarium lamps, 15–18
Solder pots, electric, 18–24
Soldered bonds, 17–65, 17–68
Soldering iron, 14–244
Solenoid winding, 4–29
Solid angle, 1–45
Solid filament lamps, 15–07
Solid insulating materials, 2–18
Solid masonry dam, 13–17
Solid protection of welding, 18–45
Solid wire, 14–143 to 14–153
breaking load, 14–162
Solid-vitreous insulators, 2–19
Solidly grounded system, 14–259
Solution, 6–03
of non-electrolytes, 6–15
Solvay cell, 18–11
Solvent, 2–36, 6–03
Solventless varnish, 2–29
Sorging table, motor for, 16–32
Sound knot, 14–83
Southern white cedar, 14–79, 14–84
Southern yellow pine, 14–81, 14–87
Soybean cooking, 19–28
Space factor, 4–31
Space heaters, 15–56, 18–24
Spacing of wires, 14–55
Span, 1–45
length of, 14–55, 17–60
unequal length, 14–71
Spark plugs, 17–82
aircraft, 17–83, 17–84
Spark timing, 17–81
Spark-gap, 5–63
A.I.E.E. standard, 5–64
needle gap, 5–64
spark-over voltage, 5–65
sphere gap, 5–64
Spark-over, 14–33
voltages, 5–65, 17–73
Specific conductance, 6–04
Specific gravity, 2–03
of electrolyte, 7–10
Specific heat, 2–49
Specific inductive capacity, 2–15
measurement of, 5–84
Spectra, 15–23
Spectrophotometry, 5–92
Specular reflection, 15–41
Speed (see article on particular device)
of operation of circuit breakers, 12–28
of rotation, 1–65
Speed control, 8–37, 12–33
d-c motor, 16–04
Speed regulating rheostats, 12–35
Speed regulator, 12–33
Speed-time curves, trains, 17–11, 17–14
Speed-torque curves, 16–06
Sperry process of reduction, 18–10
Sphere, 1–17
(angle), 1–45
Sphere gaps, 2–36, 5–64
Sphere photometer, 5–93
Spherical candlepower, 5–93
Spherical lenses, 15–29
Spherical reflectors, 15–30
Spherical right angle, 1–45
Spherometer, 5–64
Spike knot, 14–83
Spillway, 13–17
Spinning frame, motor or, 16–26
Spirit varnishes, 2–32
Splicing, 14–115, 14–220
chamber, 14–121
Split-conductor cable, 14–172
Split-phase motors, 16–14
Split-phase vector diagram, 9–59
Split-pole converter, 11–04
Spool (wire), 14–168
Spool insulator, 14–108
Spot welding, 18–36, 18–37
Spray plants, stationary, 19–28
Spring wood, 14–82
Springs, weakening of, in plating, 18–04
Sprocket-operated field rheostat, 12–35
Spruce, 14–87
Square, 1–42, 1–64
Squared-tangent law, 5–89
Squirrel-cage motor, 16–05
Squirrel-cage rotor, 9–36
Stability, of hydroelectric generator, 13–22
of power systems, 3–49, 14–28
Stability factor, 8–14
of a motor, 8–33
Stacker, motor for, 16–32
Stage lighting, 15–38
Stainless steel, reflection from, 15–04
Stairway, light for, 15–34
Standard (see Standards)
Standard cell, 5–06
Standard frequencies, 14–229
Standard gages, 1–59 to 1–63
Standard loading of conductors, 14–59
Standard section rail, 17–63
Standard spark gap, A.I.E.E., 5–64
Standard spark plugs, 17–83
Standard units, 1–36
Standard voltages, 14–07
in buildings, 14–220
Standard wire gage, 1–62
Standard wood poles, 14–80
Standards, 1–36
basic electrical, 1–37, 5–02
copper, 14–143
electrical, 5–02
force, 1–36
length, 1–36
light, 5–87
working, 5–88
luminous intensity, 1–37
mass, 1–36
mks system of units, 1–38
photometry, 5–87
resistance, 5–09
temperature, 5–96
time, 5–09
wood poles, 14–79
Stand-by battery, 7–20
Star connections, 3–29
Star-shakes, 14–82
Starters, motor, "across-the-line," 16–07
Starting (see article on particular device)
Starting box, 8–37

Starting current recommendations, 16–06
Starting motor for automobiles, 17–93
Starting of automobiles, 17–84
Starting sheet, 18–07
Starting switch, 12–04, 12–31
Starting system for automobiles, 17–84
Starting torque (see article on particular device)
Startix, 17–96
Stassano steel furnace, 18–19
Stat, 1–37
Statampere, 1–38, 1–53
Statcoulomb, 1–37, 1–52
Statfarad, 1–37, 1–56
Stathenry, 1–56
Static condenser welding, 18–40
Static transformer, 10–02
Station auxiliaries, 12–70
Station equipment, 12–49
Station grounding, 13–55, 14–261
Station layout, 12–69
Station stops, railway, 17–12
Station structures and layout, 12–60
Statohm, 1–38, 1–55
Stator, 9–36
Statvolt, 1–38, 1–54
Steadying resistance, 15–17
Steady-state stability, 3–50, 14–29
Steam, 2–50, 13–02
  energy in, 17–112
  heating, 15–54
  locomotive, acceleration, 17–08
  pipe sizes, 13–10
  pressure, 13–02, 17–111
  temperature, 17–118
  turbine drive, 17–111
  valve, 13–10
Steam electric power station, 13–02, 13–51
Stearin pitch, 2–29
Steel, 2–04, 2–48, 2–50, 2–54
  building, welding, 18–50
  cables, 14–92, 14–165
  coefficient of linear expansion, 14–58
  electrical, 2–42
  furnace, Stassano, 18–19
  gage, 1–60, 1–63
  hardening, 18–26
  light for, 15–34
  mechanical characteristics, 14–58
  mill motor, 16–14, 16–33
  modulus of elasticity, 14–58
  pins, 14–112
  poles, 14–93
  rails, 17–62
  refining, 18–19
  ship building, 18–50
  tower, 14–93
  ultimate strength, 14–58
  weight, 14–58
  welding, 18–47, 18–49
  wire, properties, 14–161
    tables, 14–151
  yield point, 14–58
Steel-clad heater, 18–24
Steel-core cable, reactance, 14–51
Steel-pipe conduit, 13–98
Stefan-Boltzmann constant, 1–70
Stefan-Boltzmann law, 2–52
Stem correction, 5–97
Step-by-step method of speed-time curve, 17–12
Step-down transformer, 10–02
Step-induction regulator, 12–42
Step-up transformer, 10–02
Steradian, 1–66, 15–02
Stere, 1–43
Sterilization, 18–12, 19–05
Sterilizers, dairy, 19–05
Stl. W. G., 1–62
Stock room, light for, 15–34
Stokers, motors for, 16–11
Stone, 1–48
  heat conductivity of, 15–64
Storage, 13–14, 13–15
  of cross arms, 14–104
  of water, 13–14, 15–47
Storage batteries (see Battery)
  alkaline-type, 7–24
  lead-acid, 7–08
Storage heaters, thermal, 15–57
Storage tank, water, 15–47, 15–58
Storage warehouse, 14–247
Store, 14–247
  light for, 15–34
Storeroom, light for, 15–34
Stove, 14–244, 15–45
Straight air brakes, 17–40
Straight storage lighting system, 17–42
Strain clamps, 14–114
Strain insulators, 14–92, 14–111
Strain towers, 14–57
Stranded conductor, 14–153, 14–159, 14–171
  ASTM, 14–171
  capacitance to neutral, 14–40
  IPCEA, 14–155
  susceptance to neutral, 14–42
Stranded steel cable, 14–92
Stranding, 14–173
Straw cutting, 19–10
Straw flower, drying, 19–23
Stray currents, 14–267
Stray field, 5–44
Stray power, 8–24
  loss, 8–18, 9–22
Street classification, 15–39
Street lighting, 15–39
  cables for, 14–179
Street service lines, 14–137
Strength, wire, 14–161
  in crossarms, 14–102
  in towers, 14–96
Stress (working), 14–58
  in series of spans, 14–72
  in welds, 18–45, 18–46
Stress calculation, 14–176
Stress deflection charts, 14–67
Stress temperature charts, 14–67
Stress-strain diagram, 14–59
Stretch of conductor, 14–67
Striking solution, 18–05
Striking voltage, 18–45
String electrometer, 5–63
String galvanometer as oscillograph, 5–71
Stringing sheave, 14–114
Stringing tension, over head wire, 14–63
Strings of insulators, 14–108
Strontium, 2–02, 6–06
Structural-steel foundation, 14–95
Stubbing, 14–93
Stub's iron wire gage, 1–62, 1–63
Stub's steel wire gage, 1–62, 1–63
Stucco, heat conductivity, 15–64
Stud terminals, 17–68
Study lamp, I. E. S., 15–39
Submarine cable, 14–138
Substation, 13–36 to 13–46
  synchronous converter, 12–69
Suburban car, body, 17–28
Subway car, 17–21, 17–29
Sugar mills, motor drive, 16–27

Sulfur, 2–02, 2–04, 2–15, 2–51, 6–06
Summer wood, 14–82
Sump pit, 14–125
Sunlight heat, 15–67
Sunlight lamp, 14–244, 15–20
Super-ascoloy, welding, 18–47
Super-calender, motor for, 16–27
Superheat, 17–118
Supersonic waves, pasteurization by, 19–04
Super-synchronous motor, 9–34
Supervisory control equipment, 12–74
Supports, cable, 14–125
Surface car, 17–21
Surface leakage, 2–14
Surface metal raceways, 14–236
Surface resistivity, 5–84
Surface road, effect of, 17–104
Surface wooden raceways, 14–237
Surge admittance, transmission line, 14–25
Surge impedance, 3–48, 14–26
Surge tanks, 13–27
Surgical tape, 2–22
Susceptance, 1–65, 3–10
  to neutral, 14–42
Suspension, Julius, 5–06
  of motors, 17–28
Suspension hardware, 14–113
Suspension type insulators, 14–108
Swaging, motor for, 16–21
Sweep, 14–83
SWG, 1–62
Swinburne-Ashcroft process, 18–18
Swing motor, 16–25
Swing saw, motor for, 16–32
Switch, 1–66, 12–03, 14–239
Switch oil, 2–29
Switchboards, 12–02
  a-c, 12–56
  d-c, 12–55
  instrument equipment, 12–49
  light for, 15–34
  wiring, 13–55
Switchgear, 12–02, 12–60
  lift-up, 12–66
  metal-clad, 12–64
  vertical-lift, 12–66
Switching, 12–02
Switching equipment, 12–02
Switching locomotive, 17–25
Switching substation, 13–37
Symbols, 1–64
  for electrical equipment, 1–66
  for wiring, 14–246
  graphical, 1–66
Symmetrical components, 3–32, 12–44
Synchroscope, 1–68, 5–40
Synchronization at the load, 3–56
Synchronizing, 9–27
Synchronizing bus, 13–49
Synchronizing power, 9–32
Synchronous condensers, 14–19
  line calculation, 14–23
  substation, 13–38
Synchronous converter, 1–66, 11–03
  protection, 12–43
  starting of, 13–43
  substation, 12–69, 13–39
Synchronous generators, 9–02
Synchronous impedance, 3–45, 9–21, 9–31
Synchronous machine, 1–66
Synchronous motors, 9–28, 16–14
  starting of, 9–35
Synchronous position, 9–30
Synchronous reactance, 9–13
Synchronous relay visual type supervisory control, 12–75
Synchronous speed, 9–37
Synchronous-synchronous converters, 11–14
Synchroscope, 5–40
Synthesis of ammonia, 18–21
Synthetic resin products, 2–26
System grounding, 14–242, 14–260
System of units, 1–36

T

T connection, 10–09
T section, 3–14
Tail lamps, 17–91
Tailrace, 13–18
Tailwater elevation, 13–14
Talbot's law, 5–89
Tan (tangent), 1–07, 1–20
Tangent chord, 17–60
Tanh, 1–10, 1–25
Tanks, 12–18, 15–47
  electroplating, 18–02
Tantalum, 2–02, 2–04, 2–50
Tape, rubberized, 14–186
Taper of poles, 14–79
Tapper, motor for, 16–21
Tar, 2–04, 14–85
Tarrazzo, heat conductivity, 15–64
Teeth, armature, 8–10
Telemetering, 5–96
Telephone batteries, 7–23
Telephone circuits for power lines, 14–56
  noise in, 14–116
Telescope pyrometer, 5–102
Tellurium, 2–02
Temper drawing, 18–26
Temperature, 1–37
  centigrade scale, 5–96
  coefficient, of resistivity, 2–07
    of thermal conductivity, 2–54
  conversion formulas, 1–58
  effect of, on breakdown, 2–13
  effects (see article on particular device)
  Fahrenheit scale, 5–96
  indoor, 15–66
  International scale, 5–96
  Kelvin scale, 5–97
  limits (see article on particular device)
  measurement, 5–96
  of conduit, 14–131
  of lamps, 15–15
  of precipitation, 18–33
  of steam, 13–02, 17–111
  permissible for cable insulation, 14–192
  primary standard scale, 5–96
  range, 14–59
  Rankine scale, 5–97
  Réaumur scale, 5–97
  scales, 5–96
  steam, 17–118
  unit of, 1–37
Templates for locating towers, 14–75
Tensile breaking load of wires, 14–162
Tensile strength, 2–06
  of wire, 14–161, 14–167
Tension calculations, aerial line, 14–63
Terbium, 2–02
Terminal bushings, 10–12
Terminal room, 13–55
Terra-cotta, reflection for, 15–04
Tesla coil, 4–44
Testing of d-c motors, 8–23
Test-ring, Baker, 5–57

Tests (see article on particular device)
Tetraethyl lead, ignition of, 17–73
Textile machinery, motors for, 16–26
Thallium, 2–02, 6–06
Theater lighting, 15–38
  ventilation, 15–66, 16–33
Theory, of circuit breaker oil, 12–03
  of operation (see article on particular device)
Thermal capacity, 2–49
Thermal classification of insulators, 2–12
Thermal conductivity, 2–54
Thermal constants of ducts, 14–193
Thermal emf, 5–30
  parasitic, 5–05
Thermal properties of materials, 2–49
Thermal resistivity, paper, 14–193
  rubber, 14–213
  varnished cambric, 14–202
Thermal storage demand meter, 5–52
Thermal storage heaters, 15–57
Thermionic voltmeter, 5–30
Thermit welding process, 17–66
Thermochemical equation, 6–05
Thermocouples, 5–99
  cold junction compensators, 5–100
  for special uses, 5–101
  measurement of emf, 5–100
  voltmeter, 5–30
Thermodynamic temperature scale, 5–96
Thermoelectric pyrometers, 5–98
Thermometers, 5–97
Thermostat, 15–47
Thermostatic metal, 5–98
Thermostatic regulator for automobiles, 17–88
Thévinin's theorem, 3–13
Thickness of insulation, 14–175
Thiokol, 2–26, 14–175
Third-brush regulation, 17–85
Third-rail, 17–38, 17–62
  system, 17–43, 17–46
Thomson bridge, 5–16
Thomson repulsion motor, 9–62
Thomson's rule, 6–12
Thomson-Houston arc dynamo, 8–21
Thorium, 2–02
Three parallel wires, capacitance, 14–38
  inductance, 14–46
Three-conductor cable, capacitance of, 14–39
Three-phase converter, 11–04
Three-phase measurement, 5–36
Three-phase system, 3–31, 14–06
  in buildings, 14–227
Three-phase transformer, 10–10
Three-phase wattmeter, 5–36
Three-ring converter, 11–05
Three-wire distribution, 14–04
  calculations, 14–15
  d-c line, 14–15
  railway, 17–58
  three-phase system, 14–06
  two-phase system, 14–05
Three-wire generator, 8–22
Threshing, 19–07
Thrust motor, 16–21
Thulium, 2–02
Thury system of transmission, 14–05
Thwing type pyrometer, 5–102
Thyratron, 11–21
Thyratron controlled welding, 18–41
Thyrite arresters, 12–47
Tie, motion of, 17–50
Tie wire, 14–137
Tile, 2–04
  heat conductivity, 15–64
Tile ducts, 14–120
Timber, decay of, 14–85
Timber dam, 13–17
Timber sizer, motor for, 16–32
  varieties, for poles, 14–79
Time, effect of, on breakdown, 2–13
  unit of, 1–37, 1–45
Time constant, 4–20
Time curves, acceleration and retardation of, 17–15
Time log curves, 2–17
Time standards, 5–09
Time starters, 12–32
Timer, automobile, 17–72
Timing of spark, 17–81
Timing of welding, 18–41
Tin, 2–02, 2–04, 2–50, 6–06
  detinning scrap, 18–09
  refining, 18–08
Tinplate gage, 1–59
Tipple, drive for, 16–28
Tire, for electric auto, 17–106
Titanium, 2–02, 6–06
Toaster, 14–244, 15–44
Toilet, lighting, 15–34
  ventilation, 16–33
Toluene thermometer, 5–97
Ton, 1–44, 1–48
Tone test, 14–224
Tonnage, 1–44
Tool traverse motors, 16–22
Tophet, 2–09
Torque, 1–50
  starting, 16–06
Torque-angle characteristics, 3–51
Torque-speed curves, 16–06
Tower, 14–93 to 14–101
  line, 14–54 to 14–77
Tracing current for cable fault location, 14–224
Track, trolley, 17–62
Track bond, 17–64
Track brake, 17–39
Trackless trolley, 17–27
Traction, electric, 17–02
Traction measurement of induction, 5–73
Tractive effort, 17–06 to 17–10
Tractive resistance, 17–06, 17–104
Traffic accidents, 15–39
Train, motor capacity, 17–20
Train acceleration, 17–07
Train and load diagrams, 17–23
Train control, 17–29
Train dispatching by telephone, 17–69
Train lighting, 17–41
  batteries for, 7–19
Train line couplers, 17–33
Train resistance, 17–06
Train starting resistance, 17–06
Tramways, aerial, 16–30
Transducer, 3–14
Transfer admittance, 3–13
Transfer bus, 13–49
Transfer chain, motor for, 16–32
Transfer impedance, 3–13
Transfer instruments, 5–26
Transfer of heat, 2–52
Transfer switch, 12–05
Transference numbers, 6–20
Transformer, 1–66, Section 10
  as boosters, 14–10
  instrument, 5–53
  oil, 2–29
  pole, 14–139
  protection, 12–43

Transformer substation, 13–37
Transient stability, 3–50, 14–29
Transients in transmission lines, 3–48
Transition of motors, 17–30
Transition resistance, 6–07
Translucent materials, 15–05
Translucent reflectors, 15–27
Transmission, electric, 3–45 (and see Transmission lines)
  heat, 15–63
  light, 15–04
    factor for, 5–94, 15–04
Transmission lines, 3–45 and Section 14
  automobile, 17–104
  capacitance, 14–37
  conductors for, 14–55
  conduit, 14–119
  corona, 14–34
  design, 14–12 to 14–31
  distribution, 14–132
  efficiency, 14–12
  inductance, 14–44
  railway, 17–57
  reactance, 14–48
  stability, 14–28
  vector diagram, 14–27
  voltage limits, 14–35
    usual, 14–08
Transport ratio, 6–20
Transportation, Section 17
Transpositions of transmission lines, 14–19, 14–116, 14–139
Transverse forces on overhead wires, 14–63
Trapezoid, 1–16
Trap, insect, 19–26
Trap-rock, 2–04
Trash racks, 13–18, 13–27
Traverse motor, 16–22
Tree, 14–139
Tree distribution, 14–132
Tree wire, 14–136, 14–139
Tree-type feeder, 14–132
Trenching machine, 14–129
Triangle, 1–09, 1–16
Trickle charging, 7–16, 7–31
Trigonometric formulas, 1–07
  relation to hyperbolic, 1–11
Trigonometric functions, 1–20
Trigonometry, 1–09
Trimmer, motor drive, 16–27, 16–32
Triplex cable, 14–172
Trolley, underground, 17–61
  voltages, a-c, 17–57
  wheel, 17–38
Trolley buses, 17–27
Trolley wire, 14–143 (also see Contact wire, Overhead systems, Railways)
  protection, 12–14
Troy weight, 1–48
Truck, electric, 17–102
  on farms, 19–26
Truck panels, 12–64
Truck-type switchboard, 12–65
True ratio, transformer, 5–54
Trunk lines, 14–137
Tubes, for interior wiring, 14–231
Tudor plate, 7–09
Tumbling barrel, motor for, 16–21
Tuned circuit frequency meters, 5–38
Tung oil, 2–36
Tungar rectifier, 11–21
Tungsten, 2–02, 2–04, 2–50, 6–06
  plating, 18–06
Tungsten filaments, 15–08
Tungsten steel, 2–48
Tunnels, 13–18
Turbine, gas, 13–29
  generator, 17–114
  heat cycle, 13–05
  steam, drive, 17–111
Turbo-electric drive of ships, 17–113
Turbo-generators, 9–16, 13–08
Turf, 2–04
Turning of ship, 17–110
Turnout, frog distance on, 17–48
  rail, 17–48
Twin cable, 14–172
Twin ignition, 17–80
Twin-wire, 14–172
Twisted pair, 14–172
Twisting frame, motor for, 16–26
Twisting sleeve, 14–115
Two parallel wires, capacitance, 14–37
  inductance, 14–45
Two-conductor cable, capacitance, 14–38
Two-cycle engine, 1–30
Two-phase system, 3–31, 14–05
  in buildings, 14–227
Two-ring converter, 11–04
Two-spark magnetos, 17–77
Two-stage current transformer, 5–55
Two-wattmeter method, 5–36
Two-wire d-c line, 14–12
Two-wire distribution, 14–04
  railway, 17–57
Two-wire line, inductance, 14–45
Type A.C.S.R. cable, 14–164
Type A.S. and W. cable, 14–163
Type H H cable, 14–163
Type K potentiometer, 5–20

U

Ulbricht-sphere photometer, 5–93
Ultra-violet light, for poultry, 19–19
  on farms, 19–19
Ultra-violet ray lamps, 15–15 to 15–20
Umbrella type of hydroelectric generator, 13–22
Underdamping, 5–03
Underfloor raceways, 14–235
Underground cable, 14–194, 14–203, 14–213
Underground conduits, 14–119 to 14–132 (see also Conduit)
Underground conduit plow, 17–38
Underground distribution circuits, 14–142
Underground structures, electrolysis of, 14–267
Underground trolley systems, 17–61
Underheat brooder, 19–13
Underload, relays against, 12–23
Under-plaster extensions, 14–237
Undervoltage, relays against, 12–23
Undervoltage trip breaker, 12–12
Underwriters Laboratories, 14–229
Unit air conditioner, 15–61
Unit heater, electric, 15–57
Unit switch system, 17–34
Unit ventilators, 15–61
U. S. loading map, 14–60
U. S. standard gage, 1–59
U. S. steel wire gage, 1–62
U. S. units, 1–37
Units, illumination, 15–02
  systems of, 1–36, 5–02
Unit-type press, motor for, 16–23
Unsaturated cadmium cell, 5–07
Unstacker, motor for, 16–32
Uranium, 2–02

Urban distribution, 14–54
Urea resins, 2–27
Usual magnetic densities, 9–19
Utilities regulations, wiring, 14–230
Utilization coefficient of lamp, 15–35
Uviarc lamps, 15–20

V

V-curves, 9–31
Vacuum breakers, 12–16
Vacuum cleaner, 14–244
Vacuum lamp, 15–08
Valence, 6–02
Valves, steam, 13–10
Van de Graf generator, 8–30
Vanadium, 2–02, 2–51
Vanstone pipe flange, 13–09
Vapor heating system, 15–54
Vapor velocity, 13–10
Vaporization, heat of, 2–51
Vapor-pressure thermometer, 5–97
Var, 5–40
Variable inductance, 3–25
Variable speed motor, 16–11
Variometers, 4–23
Varnished cambric, 14–174, 14–186
  cable insulation, 14–202
  insulated wire, 14–240
Varnished cloth, 2–16, 2–21, 14–174
  weight of, 14–226
Varnishes, insulating, 2–32
Varying speed motor, 16–19
Vat, electroplating, 18–02
Vector admittance, 3–10
Vector diagram of transformer, 10–05
Vector diagram of transmission line, 14–27
Vector impedance, 3–10
Vector power, 3–11
Vector representation, 1–65
  of sinusoids, 3–05
Vegetable dehydration, 19–24
Vegetable drying, 19–21, 19–24
Vegetable germination, 19–25
Vegetable shredding, 19–26
Veiling glare, 15–23
Velocity, angular, 1–47
  linear, 1–46
  of ions, 6–21
  of light, 1–70
  of propagation, transmission line, 14–25
  of wind, 14–61
Ventilation, 9–17, 15–65
  barn, 19–11
  coal mine, 16–28
  conduit lines, 14–231
  generator, 9–17, 13–22
  marine, 17–115
  motors, 16–31
  ovens, 18–26
  requirements, 15–66
Versine, 1–08, 1–64
Vertical forces on overhead wires, 14–63
Vertical hyperbola, 1–05
Vertical illumination, 15–03
Vertical lift switchgear, 12–66
Vertical parabola, 1–05
Vertical riser cables, 14–238
Vertical sag, 14–66
Vertical separation of conductors, 14–55
Vibrating reed frequency meter, 5–37
Vibrating regulator, 12–36, 17–87
Vibration, conductor, 14–63
Vibration galvanometer, 5–05
Vibrator, 14–244
Vibrator ignition systems, 17–76
Vinyl resins, 2–27
Virgin curve, 3–19
Virtual coefficient of expansion, 14–58
Virtual modulus of elasticty, 14–58
Viscosity, dielectric, 4–10
Viscous air filter, 15–62
Visibility curve of eye, 15–21
Visicode, 12–75
Vision, 15–21
Visual acuity, 15–22
Visual standards, photometry, 5–88
Volt, 1–38, 1–54
Volt amperes, 3–03
Volta effect, 6–08
Voltage, 1–65 (also see article on particular device)
  control, 14–09, 17–85
  distribution in insulator strings, 14–110
  loss, per cent, 14–12
  measurement, 5–19
  rises at junctions, 14–26
  sensitivity of galvanometer, 5–03
  sparking, 5–65, 17–73
  standard, 14–07
  to neutral, 3–30
Voltage-time curve, 17–12
Voltaic cells, 6–07
Voltmeter, 1–68, 5–30, 16–17
  corona type, 5–67
  rotary, 5–63
  used as ohmmeter, 5–42
Volt-ohmmeters, 5–42
Volume, 1–16
  of geometric figure, 1–16
  of wood poles, 14–83
Volume resistivity, 2–14, 5–84
Vulcanized fiber, 2–22
Vulcanized oil sheath, 14–175

W

Waffle iron, 15–44
Wagner repulsion starting, induction running motor, 9–62
Waidner-Burgess light standard, 5–87
Walker balanced gear drive, 17–105
Walker multiple system of refining, 18–07
Wall, furnace, 18–13
  heat transmission, 15–63
Wall switch, 14–243
Walnut drying, 19–24
Wane, 14–83
Ward-Leonard motor control, 16–05, 16–27
Warehouse, light for, 15–34
Warming engine, Kelvin, 15–59
Warped, 14–83
Washburn and Moen gage, 1–62
Washing machine, power, 14–244, 16–30
  water requirement, 15–47
Washing solution, 18–02
Watchman's clock, 14–257
Water, 2–04, 2–51
  conduit, 13–18
  density, 2–03
  electrolysis, 18–11
  heating, 14–244, 15–46 to 15–50, 19–05
  requirements, 15–47
  resistance, 14–186
  supply for farms, 19–19, 19–20
Water breakers, 12–16
Water rheostat, 4–08
Water-cooled transformer, 10–04, 10–15

Water-gas tar, 14–85
Water-pipe ground, 14–237
Waterproofing, of manhole, 14–125
Waterways, 13–17
crossing of, 14–138
Waterwheel, 13–19
generator, 13–21
Watt, 1–52, 1–64
hour, 1–51, 1–64
second, 1–51
Watthour meter, 1–68, 5–46 to 5–50
Wattless power, 3–06
Wattmeter, 1–68, 5–31 to 5–37
connections, 16–17
Wave, on transmission line, 14–25
Wave analysis, 3–07
Wave winding, 8–06
resistance of, 8–23
Wavelength, 3–48, 14–25
constant, 14–25
Wax, 2–04, 2–28, 2–50
Waxing of fruit, 19–18
Weather strips, effect of, 15–65
Weatherproof wire, 14–136, 14–240
Weatherproofing, 14–175
Web type plate, 7–09
Weber, 1–57
Weber photometer, 5–91
Wedge contacts, 12–17
Week, 1–45
Wehnelt interrupter, 4–44
Weight, of conductor, 1–48, 14–13, 14–17
of insulated cables, 14–225
of materials, 2–02 to 2–04
of wires, 14–144
of wood poles, 14–83
Weighted observations, 1–15
Weld peening, 18–45
Welded bond, 17–67
Welded joints, 13–09
Welding, 18–34 to 18–51
applications, 18–50
arc, 18–41
butt, 18–35
carbon arc, 18–42
control of timing, 18–41
electrodes, 18–44
flash, 18–36
manual vs. automatic, 18–49
metal arc, 18–43
percussive, 18–41
pressure, 18–36
projection, 18–39
protection, 18–45
reduction of stresses, 18–45
resistance, 18–35
seam, 18–35
societies, 18–52
shot, 18–36, 18–37
timing control, 18–41
track rails, 17–66
Well drilling, motor for, 16–27
Welsbach burner, 15–53
Wenner potentiometer, 5–22
Western red cedar, 14–81 to 14–87
Western yellow pine, 14–81, 14–84
Westinghouse frequency meter, 5–38
Westinghouse synchroscope, 5–40
Westinghouse unit-switch control, 17–34
Weston frequency meter, 5–38
Weston illuminometer, 5–91
Weston normal cell, 1–38, 5–06
Weston ohmmeter, 5–42
Weston synchroscope, 5–40
Wet arcing, 14–106
Wet cells, 7–02
Wet-process porcelain, 2–20
Wheatstone bridge, 5–13
Wheel trolley, 17–38
White cedar, 14–79, 14–87
White glass globes, 15–28
White lead, manufacture of, 18–10
White light, 15–25
lamps, 15–20
White northern cedar, 14–87
White potentiometer, 5–22
Whitewash removal, 19–27
Whiting cell, 18–11
Whyte classification of locomotives, 17–24
Willard's rule, heat, 15–52
Wimshurst machine, 8–29
Winchester bushel, 1–44
Wind, 14–83
Wind cracks, 14–82
Wind load, 14–60 to 14–63, 14–87
Windage, 8–18, 9–13
loss (see article on particular device)
Winding connection, 1–66
Winding distribution constant, 9–08
Window, infiltration, 15–65
ventilation, 15–61
Wind-shake, 14–83
Wire, 14–143 to 14–226
accessories, 14–239
aluminum, 14–149, 14–158, 14–163
base, 14–143 to 14–171
breaking load, 14–161
capacitance to neutral, 14–39
cleats, 14–231
control, 13–55
copper, 14–144 to 14–148, 14–156, 14–163, 14–167
copper-clad, 14–152
definition, 14–171
elasticity, 14–161
expansion coefficients, 14–161
for motor branch circuits, 14–252
gages, 1–61
comparison of, 1–63
ground, 14–56
ice loading, 14–60
insulated, 14–171 to 14–226 (also see **Insulated wires and cables**)
interior, 14–240
manufacture of, 2–05
power stations, 13–53
resistance of, 14–143
at various temperatures, 14–160
size calculation, 14–251
solid, 14–143
steel, 14–151
stranded, 14–153
strength, 14–161
susceptance to neutral, 14–42
tables, 14–143 to 14–226
nickel-chromium-alloy, 18–23
tensile breaking load, 14–161
trolley, 14–143
Wireways, 14–235
Wiring (see article on particular device)
automobile, 17–97
diagrams, 1–69
for increasing illumination, 15–33
of buildings, 14–226 to 14–257
of conduits, 14–126
residence, 14–243
systems, 14–227
Wolff potentiometer, 5–22

Wood, 2–04, 2–15, 2–22, 2–50
conductivity of, 14–270
heat conductivity, 15–64
insulators, 14–92
Wood arc-light machine, 8–21
Wood block surface, 17–104
Wood pins, 14–112
Wood poles, 14–78
preservative treatment, 14–84
seasoning, 14–83
standards for, 14–79
weight of, 14–83
Wood pulleys, 16–15
Wood's metal, 2–50
Wooden cross-arms, 14–88
Wood-stave pipe, 13–18
Woodworking industry, light for, 15–34
motors for, 16–26
Wool, 2–50
Work, 1–57, 1–65
Working light standards, 5–88
Working stress, 14–58
Worm drive, 17–105
Wrought-iron gage, 1–60
Wye connections, 3–31

X

Xenon, 2–02

Y

Y connection, 3–31
Yacht batteries, 7–23
Yard, 1–36, 1–41
cubic, 1–43, 1–64
square, 1–42, 1–65
Y-box, 5–36
Yellow longleaf pine, 14–79
Yellow pine, 14–79
cross-arms, 14–104
Yoke, 8–03, 17–62
Ytterbium, 2–02
Yttrium, 2–02

Z

Zero drift of instruments, 5–26
Zero sequence components, 3–33
Zero sequence reactance, 3–45
Zinc, 2–02, 2–04, 2–50, 6–06
plating, 18–04
production, 18–09, 18–18
Zinc chloride, 14–85
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